

ABSTRACT

OF THE THESIS ENTITLED

STUDY OF SOME PHENOMENA OF ARC DISCHARGES

by

A. E. ROBSON

ABSTRACT

The first part of this thesis presents a new approach to the understanding of the collision processes which occur in the vicinity of the cathode of the "cold cathode" arc of which Hg and Cu are taken as examples. The temperature at the cathode spots of such arcs is too low for electrons to be emitted thermionically.

None of the theories hitherto suggested, such as the field emission and thermal ionisation theories, can account satisfactorily for the well-established features of the arc: the cathode fall in potential which is often lower than the ionisation potential of the cathode vapour; the concentration of current at the cathode into a small spot where the current density is very large; the short re-ignition time; the force on the cathode; the evaporation of the cathode substance; the light emitted from the spot; the temperature of the cathode under the spot and the heat losses from the arc, and also the transition to the arc from the glow discharge.

Because of the abundance of experimental information which exists, particularly about the mercury arc, this study is essentially a theoretical one.

The starting point in the study is to consider the density distribution in the space above the cathode of the metal vapour originating from the spot. Whereas it was previously thought that atoms evaporating from the spot would cross the evacuated vessel without collisions before condensing on the walls, it is suggested here for the first time that the

evaporation is controlled by back-scattering of atoms and hence is a diffusion problem. In setting up the equation of diffusion for the steady state, allowance is made for the variation of the diffusion coefficient with the density of the vapour. The solution shows that a large density, corresponding perhaps to 10 atmospheres or more can exist close to the evaporating surface although the density elsewhere in the vessel is very low. Both the magnitude of the force on the cathode and of the net evaporation from the spot are consistent with this picture.

The excitation and ionisation processes in the cathode region are consequently regarded as taking place in a dense vapour. On account of the low cathode fall, collisions between electrons and neutral atoms can only lead to excitation: ionisation can therefore occur only by step-wise processes. Electrons from the cathode gain sufficient energy in the cathode fall to excite neutral atoms within a few mean free paths. Elastic losses are shown to be negligible. A large density of excited atoms is produced close to the cathode and these diffuse back to the cathode surface. Since the transit times of the excited atoms are shorter than their free life, they reach the cathode without losing their energy by radiation and there release electrons. This leads to a self-sustaining arc discharge.

From a simultaneous consideration of energy balance and number balance, the yield for the emission of electrons by excited atoms is estimated. It compares favourably with existing experimental and theoretical evidence.

neutralisation of the positive space charge near the cathode and the

Since the cathode fall of a mercury arc is larger than the resonance potential, electrons which have lost energy in an exciting collision can gain further energy in the remainder of the cathode fall. It is shown that because of strong interaction between the electrons an energy distribution is established, so that each electron from the cathode produces, on the average, more than one excited atom in the vapour.

Since the density of excited atoms is high, collisions between them are shown to produce electrons and molecular positive ions. The reverse process, that is recombination of ions and electrons to form excited atoms approximately balances the former process. A small difference is sufficient to account for the current of positive ions to the cathode. This current is associated with a positive space charge which predominates close to the cathode, and the ensuing electric field causes the cathode fall in potential.

From the known values of current density and cathode fall and the calculated density distribution of neutral vapour it has been possible to derive how the concentration of electrons, positive ions and excited atoms, and also the axial electric field vary as a function of the distance from the cathode.

The transition of the glow discharge into the arc is discussed. It is shown that the final size of the cathode spot can be found from stability considerations associated with the flow of excited atoms to the cathode.

The short reignition time is explained in terms of the rapid neutralisation of the positive space charge near the cathode and the

subsequent inability of the space-charge region to form when the external potential is re-applied. The light from the cathode region is attributed to the radiation from excited atoms which have diffused up into regions of lower vapour density. The observed spectrum of the spot is re-interpreted on the basis of the new picture.

The above treatment can be applied to arcs other than the mercury arc with certain minor modifications. The copper arc is taken as an example.

The second part of the thesis presents a new theory of the motion of a cathode spot in a transverse magnetic field. It is known that under certain conditions the spot will move in a direction contrary to Ampere's rule. This reverse motion usually occurs when the ambient gas or vapour pressure is low. If the pressure is increased, the direction of motion changes to the correct, or Ampere direction. The pressure at which the change of direction occurs is the greater the stronger the magnetic field and the lower the current. The effect is found only in cold-cathode arcs.

Experiments are described in which the behaviour of the mercury arc spot in a transverse magnetic field was observed. Measurements were made of the spot velocity and the arc voltage as a function of the applied magnetic field, the arc current and the pressure of the mercury vapour or of the added argon. The shape of the rotating mercury arc was observed with a stroboscope.

From these observations it is proposed that the apparent

violation of Ampere's rule is due to a local reversal of the magnetic field in the cathode region. The magnetic field acting at the cathode spot is thought to consist of two components. The first is the externally applied field, and the second is an opposing field due to the current loop in the deflected positive column. It is shown that the magnitude of the latter may exceed the applied field and thus the spot will be driven in the reverse direction. The behaviour of the arc at low pressure and with argon admitted is explained qualitatively.

It is shown here that the reverse driving effect is not confined to low-pressure arcs, but occurs in open air between closely spaced copper electrodes. The separation of the electrodes is shown to be an important parameter of the effect, which supports the view that it is due to a distortion of the arc column rather than to some process occurring in the cathode fall region.

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Foreword

This work has been supervised by Dr. R. E. Marsh, whom I wish to thank most sincerely for his encouragement and advice.

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CH. 1.

My thanks are due to Professor Lord Cherwell and Mr. T.C. Keeley for the facilities afforded me at the Clarendon Laboratory, to the Director and Sub-Committee L/G of the Electrical Research Association for a financial grant, and to the Warden and Fellows of St. Antony's College for an Exhibition.

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(3.3) The Current Density: Simple quantitative considerations

(3.4) Cathode Evaporation and Cathode Temperature

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CHAPTER 2

INTRODUCTION

(1.1) Thermionic and Field-Emission Arcs

The arc discharge is characterized by the fact that it is a few centimeters or more, together with a potential difference between the electrodes, which is of the order of the ionization potential of the gas in which the arc exists. In this thesis we are concerned with continuous, self-sustained arcs between metal electrodes in a gas at pressure. The arc is characterized with transient arcs lasting a few microseconds, and a continuous current arc or with some other discharge, but we are concerned with the arc.

PART I

THE CATHODE OF THE COLD-CATHODE ARC

Self-sustained arcs can be obtained only when the temperature of the cathode is raised sufficiently high to cause thermionic emission of electrons. The temperature of the cathode is raised electrically by the arc itself, or by an external source of heat, or by a combination of the two. The arc belongs to the first group and is known as a thermionic arc. If however the cathode is heated by an external source, the temperature necessary for thermionic emission is not reached by the cathode arc. Table 1, compares the latent heat of vaporization, as presented in all parts, the work function is of the order of the boiling point of various cathode materials. Thus the work required to vaporize the cathode is much larger than the work required to raise the cathode to the temperature necessary for thermionic emission. The arc will be a thermionic arc and vice versa. There are some exceptions.

CHAPTER I

INTRODUCTION

(1.1) Thermionic and Cold-cathode arcs

The arc discharge is characterised by its high current, of a few amperes or more, together with a relatively low potential difference between the electrodes, which is of the order of the ionisation potential of the gas in which the arc runs.^(1,2) In this thesis we are concerned with continuous, self-sustained, direct-current arcs between metal electrodes in a gas or vapour. We are not concerned with transient arcs lasting a few microseconds, with alternating-current arcs or with arcs whose cathodes are heated by some external means.

Na Self-sustained arcs can be divided into two groups according to the thermal properties of the cathode material. If as the temperature of the cathode is raised electrons are emitted thermionically before appreciable evaporation of cathode material takes place, the arc belongs to the first group and is known as a thermionic arc. If however the cathode vaporises at temperatures far below the temperature necessary for thermionic emission the arc is known as a cold-cathode arc. Table 1. compares the latent heat of evaporation, expressed in eV per atom, the work function in eV and the boiling point of various cathode materials. Where the work required to remove an atom is much larger than the work required to remove an electron the arc will be a thermionic arc and vice versa. Where these quantities

Material	Boiling Pt °K	Latent Heat of Vaporisation Cal/gm.eV/atom		Work Function eV/electron
Al	2073	2800	3.4	3.0
Cs	943	120	0.7	1.8
C	4100	12000	6.2	4.4
Cu	2570	1110	3.1	4.1
Fe	3270	1520	3.7	4.6
Hg	630	72	0.6	4.5
Mo	3830	1700	7.2	4.4
Pt	4000	600	5.2	5.3
K	1060	490	0.8	2.24
Na	1220	1000	1.0	2.2
Ta	3900	450*	3.4	4.12
W	5000	1150	9.2	4.54

Latent heats from F. Henning, Warmetechnische Richtwerte. (Berlin 1938).

Boiling points and work functions from D.A. Wright Semi-conductors. (Methuen 1950).

*calculated from boiling point by Trouton's Rule.

TABLE I. BOILING POINT, LATENT HEAT OF VAPORISATION
AND WORK FUNCTION OF SOME ELEMENTS.

are comparable we should expect the arc to show some of the characteristics of both types.

In the thermionic arc the vapour density of cathode material near the cathode surface is very low compared with the gas density, but in the cold arc there is a very high density of evaporated material near to the surface. This is evidenced by the appearance of lines of the cathode material in the spectrum of the arc, whose intensity is greatest close to the cathode. For this reason the cold arc is sometimes referred to as the vapour arc. There are however certain circumstances under which an arc having the characteristic features of a cold arc can form on a cathode which would normally support a thermionic arc. (5, 69)

(1.2) Transition from the Glow Discharge.

Both types of arc can be produced as a result of transition from an abnormal glow discharge by increasing the current ^{decreasing} or the gas pressure. (68) If, for example, the current is increased until an arc is formed one observes a reduction in discharge potential by at least an order of magnitude, accompanied by an increase in cathode current density of several orders, a decrease in cathode fall space, and an increase in the temperature of the emitting area of the cathode. The transition to a cold arc appears to be always abrupt and discontinuous; the thermionic arc however, under certain conditions of cathode geometry can be obtained by a continuous transition. (3, 66, 79)

If the current is kept constant and the transition from glow to arc is effected by a small ~~de~~crease in pressure, one observes that

the contraction takes place only in the cathode regions of the discharge. The positive column and anode regions are unaffected. An experiment in which the contraction at constant current was observed will be described later.

After contraction the current is emitted from a very small spot instead of the whole cathode surface. This applies to either of the two types of arc, the difference being that the cathode current densities in thermionic arcs are several orders of magnitude lower than in cold arcs. Also in thermionic arcs the cathode spot is practically at a standstill, whereas with the majority of cold arcs rapid random motion of the spot over the surface of the metal is observed. In Table 2 representative data of temperatures and current densities are given for some thermionic and cold arc cathodes and anodes, observed for currents of the order 1 to 10 amps.

The transfer of current at the cathode of a thermionic arc can be explained in terms of thermionic emission of electrons from a small area of the cathode heated to a high temperature by the arc itself. The mechanism of such an arc is quite well understood.^(2,3) In the case of the cold arc however it is hard to believe that temperatures necessary for thermionic emission could be maintained on the low-boiling-point material, and so a number of other theories of current transfer have been advanced. These theories will be discussed in Chapter 2.

TABLE 2 CATHODE AND ANODE TEMPERATURES AND
CURRENT DENSITIES

Cathode	Anode	Gas	T_c °K	T_a °K	j_c A/cm ²	j_a A/cm ²
C	C	Air	3500	4200	470	65
C	C	N ₂	3500	4000	500	70
Cu	Cu	Air	2200	2400	$\sim 10^5$	$\sim 10^3$
W	W	Air	3000	4200		
Al	Al	Air	3400	3400		
Zn	Zn	Air	3000	3000		
Hg	C	Vac	~ 600		10^5-10^6	
Ni		N ₂	2370			
Fe		N ₂	2430		7000	

From Ionised Gases by A. von Engel (Oxford 1955).

Also references 2, 3.

(1.3) Features of the cold-cathode arc

Cathode The subject of this work is the cathode of the cold arc in general and in particular the arc with a liquid mercury cathode in a vessel which contains only mercury vapour at low pressure (of the order 10^{-2} mm Hg). The mercury arc was chosen because the low boiling point of mercury would seem to eliminate any possibility of thermionic emission from the cathode. Also the mercury arc appears to have one of the lowest cathode falls and the highest cathode current density. As the only substance in the arc vessel (apart from the anode) is mercury it was thought that this would simplify the theoretical study of the arc. However, in order not to confine the discussion to one particular case, the arc with a copper cathode running in air will also be considered. Some features of the cold arc will now be outlined.

The electric parameters of the mercury and copper arcs, that is to say the cathode and anode falls of potential and the current densities at the electrodes have been the subject of many investigations during the last half-century. Much attention has been paid to measuring the current density in the cathode 'spot', and as techniques have improved, estimates of the current density have become larger. The principal measurements are listed in Table 3. In all the experiments the size of the spot has been measured either optically or by marks left on cathodes and the lower values previously obtained were probably due to the inability of the earlier techniques to resolve the rapid random motion of the cathode spot. The very low values obtained by Slepian

TABLE 3

CURRENT DENSITY MEASUREMENTS

Cathode	Gas	Duration	Technique	j_c A/cm ²	Ref.
Hg	Hg	long	Cine film	$4 \cdot 10^3$	4
Hg	Hg	$\sim 10^{-2}$ sec	optical scanning	$5 \cdot 10^4 - 2 \cdot 10^5$	5
Hg	Hg	1-100 μ sec	Kerr cell photography	$2 \cdot 10^6$	6,7
Hg	H ₂	0.25 μ sec	photography	$3 \cdot 10^5$	11
Hg	H ₂	5 μ sec	photography	$3 \cdot 10^4$	11
Cu	Air	1 μ sec	marks on cathode	$8 \cdot 10^5$	8
Cu	Air	50 μ sec	marks on cathode	$9 \cdot 10^4$	8
Cu	Air	.06 sec	marks on cathode	$< 10^3$	10
Cu	Air	long	marks on cathode	1.2	5
Al	A	1 μ sec	marks on cathode	$2 \cdot 10^5$	8
Al	A	50 μ sec	marks on cathode	$4 \cdot 10^4$	8
Al	A	.06 sec	marks on cathode	26	9
Zn	A	.06 sec	marks on cathode	30	9

and others from cathode markings are almost certainly due to the motion of the spot during the arcing period. Most investigators tacitly assume that the current-carrying area of the cathode is identical with the luminous area or with the area which leaves marks on the cathode, and this assumption may not be entirely justified. The latest and highest estimates of 10^6 A/cm^2 for mercury have been obtained by high-speed photography of the arc and the apparent boundary of the arc spot might well depend on the exposure and processing of the film. Furthermore the photographs were of transient arcs lasting not more than 100 microseconds and although Froome claims⁽²⁴⁾ that only 10 microseconds after initiation an arc can be considered as 'normal' (i.e. having the characteristics of a more prolonged arc), this claim has been disputed by Bertele^(25,26) who considers that the high current densities of 10^6 A/cm^2 occur only in the initial stages of arc formation, and that the current density decreases with time to a value of the order 10^4 A/cm^2 for a prolonged arc.

Having examined the photographs taken by Froome it would appear that although the current density is undoubtedly high it is probably not as high as is claimed, especially in the case of a prolonged, continuous arc. We therefore will take a figure of about 10^5 A/cm^2 as representing most likely value of current density at the mercury and copper arc cathodes. This is the value obtained by Cobine and Gallagher.⁽⁵⁾

There seems to be agreement that the current density is independent of current, but there is no evidence to show that it

remains constant over a wide range of pressure. As the current density in the glow discharge shows a strong variation with pressure, a similar dependence might be expected in the arc.

The cathode fall in potential is difficult to measure since it is impossible to put probes into the cathode region on account of its small size. However by taking probe measurements at some distance from the cathode and extrapolating or measuring the total discharge potential while varying the electrode separation and then extrapolating to zero separation it is possible at least to set an upper limit to the cathode fall. Table 4. gives some cathode falls measured in this way, and for comparison the work functions of the surface, the first excitation potential and the ionisation potential of the cathode vapour are given.

Measurements of the cathode fall in potential of a mercury arc vary from 8v to 11v. (30-32) In this thesis the lower value of about 8v. will be used in calculations on the cathode fall, and conditions under which a slightly larger cathode fall may exist will be discussed.

The cathode spot is many times brighter than any other part of the arc both a line spectrum and a continuous spectrum originate from it. (5,28,29) It appears that the light is emitted from a layer of dense vapour immediately above the cathode rather than from the cathode itself and an intrinsic brightness of 10^6 candles/cm² has been estimated from photographic measurements. (11)

The spot is constantly in random motion over the cathode surface at speeds of the order 10^3 cm/sec. (27) This motion can however be suppressed by 'anchoring' the spot at the junction of the mercury

TABLE 4.

Cathode fall V_c , Work function of cathode ϕ ,
First excitation potential V^* , First ionisation
potential V_i and Second ionisation potential V_i'
of cathode vapour.

Cathode	V_c	ϕ	V^*	V_i	V_i'
Cu	8-9	4.1	1.4	7.7	20.3
Fe	8-12	4.6	2.4	7.9	16
Hg	7-10	4.5	4.9	10.4	19
Na	4-5	2.2	2.1	5.1	47.5
K		2.24	1.6	4.34	31.7
Ag		4.7	3.65	7.56	21.9
Ni	11.7	5.0	3.18	7.63	18.2

All values in volts.

V_c from Ionised Gases by A.von Engel (Oxford 1955).

ϕ from Semi-Conductors by D.A. Wright (Methuen 1950).

V^* , V_i , V_i' from Landolt-Bornstein Tables, 5th edition
3rd. Supplement, 2nd part, 1935. p.871.

and another metal, such as molybdenum, (13,14,19,32,51-55) The phenomenon of anchoring does not appear to have a great effect on the electrical characteristics of the arc, except to cause a small decrease in arc voltage and a reduction of the fluctuations of this voltage.

If the current in a free spot is reduced below about 5 A the spot will extinguish spontaneously after a certain 'statistical lifetime'. (26,27,114) An anchored spot can however be maintained at currents as low as 50 mA. (6) If the arc current is raised above about 10 A the current will be carried by several spots each carrying about 6-8 A. (5,26)

Cathode material is evaporated from the cathode at a rate which is approximately proportional to the current. Table 5 shows various measurements of the evaporation from mercury and copper cathodes. The larger values for mercury are found when the spot is free and in this condition the loss of cathode material is principally due to droplets ejected from the 'spot'. If the spot is anchored the ejection of droplets ceases and the total mass transfer falls by an order of magnitude. The loss of material from an anchored spot is considered to be due to true evaporation.

The cathode surface beneath the spot experiences a downward force which results in a depression of the mercury surface, or in the case of a solid copper cathode, a measurable force on the cathode. When the gas or vapour pressure in the arc vessel is raised it is observed in the case of mercury that the 'spot' digs in to a lesser extent, and in the case of copper that there is a smaller force on the

TABLE 5EVAPORATION FROM ARC CATHODES

Cathode	Condition of spot	Current A	Evaporation gm/coulomb	Reference
Hg	free		2.2×10^{-3}	Guntherschulze 4
	free		$< 1.3 \times 10^{-3}$	v. Issendorf 12
	anchored	35	1.7×10^{-5}	Kobel 13
	anchored	0.5-5	2.5×10^{-4}	Tonks 14
Cu	air pressure $< 50\mu$	70-150	7×10^{-6}	Easton et al 15
		11-32	1.5×10^{-5}	Tanberg 16
		15	8.7×10^{-5}	Robertson 17

TABLE 6FORCE ON ARC CATHODES

Cathode	Gas	Pressure MM Hg	Current amp	Force dynes/A	Reference
Cu	air	760	10	1	Duffield 18
Cu	air	$< 10^{-2}$	10-30	17	Tanberg 16
Cu	N ₂	< 1	2-15	15	Robertson 17
Cu	N ₂	> 10	2-15	3	Robertson 17
Hg	Hg	low	35	40	Kobel 13
Hg	Hg	low	0-10	35-66	Tonks 19

See also refs. 158, 159.

cathode. Table 6 gives some measurements of the force on copper and mercury cathodes.

If the current is interrupted for as little as 10^{-8} sec the arc will go out and not re-ignite. (21,22) The arc will never start spontaneously but must either result from a transition from a spark or glow discharge or be initiated by some external means such as bringing the electrodes together and separating them. Although a mercury arc will start on a perfectly clean surface, it is found that the presence of a thin layer of oxide appears to be necessary before an arc can be run on a copper surface. (5,23,56)

Finally, under certain conditions the arc spot in a magnetic field will move in a direction contrary to Ampere's rule.

(1.4) The problems of the cold-cathode arc.

The principal problem of the cold arc is to account for the transfer of current at the cathode surface and through the region immediately above the cathode. A number of theories have been proposed but for reasons which will be discussed in the next chapter, none has proved entirely acceptable. Any new proposal must not only account for the high current density, but must be consistent with the low value of cathode fall as well as explaining such features as the extremely short re-ignition time, the spectrum of the arc spot, the splitting of spots and their motion in a magnetic field.

It is generally realised that since the cathode fall is only of the order of the ionisation potential of the gas or vapour in which the

are is run, ionisation of a neutral atom by a single collision with an electron is a most unlikely process. The most likely single-collision process will be excitation, and ionisation will occur by cumulative processes. (64) What is not always realised is that in the vicinity of the cathode these processes take place in the vapour which has been evaporated from the cathode rather than in the gas which fills the discharge tube. Furthermore this is probably at a considerably greater density than the gas, and so virtually excludes any gas from the cathode region. Also as the excitation and ionisation potentials of metal vapours are generally lower than the corresponding potentials of gases, electrical processes will occur preferentially in the vapour.

Calculations of other workers based on the assumption that gas conditions extend down to the cathode surface must necessarily be invalid, as we shall endeavour to show in the following chapters. In the case of the thermionic arc, where there is little or no cathode evaporation, there is probably only gas above the cathode, and this is at a lower density than elsewhere in the arc vessel owing to the high temperature of the arc.

The unusual behaviour of an arc spot in a transverse magnetic field is treated in some detail in the second part of this thesis. It was originally hoped that investigation of this behaviour would throw some light on the mechanism of current transfer at the cathode, but the explanation we propose does not involve any specific electron emission process. It does however give certain information as to the probable dimensions of the cathode region.

(1.5) The size of the cathode region.

If we consider a typical arc spot carrying 5 A at a current density of 10^5 A/cm² the radius of the spot, assumed circular, will be about $4 \cdot 10^{-3}$ cm. The extent of the cathode region in the axial direction will now be discussed. We define the cathode region of the arc as that part of the discharge in which the processes responsible for maintaining the current transfer at the cathode take place. The potential difference across this region is the cathode fall in potential, and is due to the space-charge of the positive ion current flowing to the cathode.

The proportion of the current at the cathode which is carried by positive ions is an important quantity when considering the cathode fall and also the energy balance at the cathode. In the positive column, where the concentrations of ions and electrons are approximately equal, the ratio of the ion current to the electron current is the ratio of the ion drift velocity to the electron drift velocity in a given electric field. For a mercury positive column this ratio is about 1:400 (see ^{ref. 137} Appendix 1). Since the cathode is negative with respect to the positive column, plasma electrons will be reflected and positive ion space charge will predominate near the cathode. We can therefore estimate that the positive-ion current at the cathode will certainly be greater than 1% of the total current.

In order to estimate an upper limit, we assume that all the energy that an electron gains in the cathode fall is used, through cumulative processes, to produce ions. Then if the cathode fall is

8 v and the ionisation potential is 10 v, each electron will produce on the average 0.8 ions. Under these circumstances the ion current would be $0.8/1.8 = 45\%$ of the total current. This is the extreme upper limit. Such complete conversion of cathode fall energy into ionisation is most unlikely, and such a large ion current would represent a heat input to the cathode which, as we shall show, would be inconsistent with the energy balance. At this stage therefore we shall assume that the ion current is between 5% and 20% of the total current, and calculate a more exact figure later.

If we assume that positive ions fall freely through the cathode fall, the relation between the potential V , the positive ion current density j^+ and the distance x is given by the Child-Langmuir space-charge equation (75,76)

$$j^+ = \frac{1}{9\pi} \left(\frac{2e}{M} \right)^{\frac{1}{2}} \frac{V^{\frac{3}{2}}}{x^2} \quad (1.1)$$

which we may write

$$V^{\frac{3}{2}} = \frac{81\pi^2 M}{2e} \cdot j^{+2} x^4 \quad (1.2)$$

where e is the electronic charge and M the ionic mass.

If on the other hand we assume that the vapour density in the cathode fall region is large enough to cause the ions to make collisions with gas atoms during their transit, (and we shall show later that this is almost certainly the case) a different relation will hold which is

derived in Appendix 2, viz

Although the measured cathode fall is about 8v, we have included in the tables calculations with $V = 5v$. We shall show later

$$V^3 = \frac{77p}{k^2} \cdot j^{+2} x^5 \quad (1.3)$$

where p is the vapour pressure in mm Hg reduced to 0°C , and k is the constant in the equation for the drift velocity of ions

$$v_d^+ = k \sqrt{\frac{x}{p}} \quad (1.4)$$

The value of k for mercury is given in Appendix 1.

In Table 7 are given values of x calculated from equation (1.1) for $V = 5v$ and $8v$ and $j^+ = 50\%$, 20% and 5% of the total current density (10^5 A/cm^2).

In Table 8 are given values of x calculated from equation (1.3) for the same values of V and j^+ as in Table 6, but with $p = 3$ to 3000 mm Hg . Also in this table are given the values of the concentration of atoms N , and the mean free path λ corresponding to each value of p .

Since equation (1.3) is only applicable if the ions collide with gas atoms as they fall through the potential V , the values in Table 8 are only valid if $x \gg \lambda$. This is the case only when $p > 1000 \text{ mm Hg}$. For lower vapour densities equation (1.1) will hold. It can be seen from the tables that the value of x is very little different when derived from (1.1) or (1.3) at high p .

Although the measured cathode fall is about $8v$, we have included in the tables calculations with $V = 5v$. We shall show later

Values of the distance x over
which a potential difference V exists due to the space
charge of a positive ion current density j^+

Table 7 High Vacuum Case (equation 1.2)

Values of x

j^+	for $V = 5v.$	for $V = 8v.$
$5 \cdot 10^4 A/cm^2$	$9.2 \times 10^{-7} cm$	$1.3 \times 10^{-6} cm$
$2 \cdot 10^4 A/cm^2$	$1.5 \times 10^{-6} cm$	$2.1 \times 10^{-6} cm$
$5 \cdot 10^3 A/cm^2$	$2.9 \times 10^{-6} cm$	$4.1 \times 10^{-6} cm$

Table 8 Vapour Case (equation 1.3)

Values of x in cm for $V = 5v.$

j^+	$p=3mm\ Hg$	$30mm\ Hg$	$300\ mmHg$	$3000mmHg$
$5 \cdot 10^4 A/cm^2$	4.7×10^{-6}	3.0×10^{-6}	1.9×10^{-6}	1.2×10^{-6}
$2 \cdot 10^4 A/cm^2$	6.8×10^{-6}	4.3×10^{-6}	2.7×10^{-6}	1.7×10^{-6}
$5 \cdot 10^3 A/cm^2$	1.2×10^{-5}	7.5×10^{-6}	4.8×10^{-6}	3.0×10^{-6}
$\lambda =$	$6 \cdot 10^{-4} cm$	$6 \cdot 10^{-5} cm$	$6 \cdot 10^{-6} cm$	$6 \cdot 10^{-7} cm$
$N =$	$10^{17}/cm^3$	$10^{18}/cm^3$	$10^{19}/cm^3$	$10^{20}/cm^3$

Values of x in cm for $V = 8v.$

j^+	$p=3mm\ Hg$	$30mm\ Hg$	$300\ mmHg$	$3000mmHg$
$5 \cdot 10^4 A/cm^2$	6.2×10^{-6}	4.0×10^{-6}	2.5×10^{-6}	1.6×10^{-6}
$2 \cdot 10^4 A/cm^2$	8.9×10^{-6}	5.8×10^{-6}	3.6×10^{-6}	2.3×10^{-6}
$5 \cdot 10^3 A/cm^2$	1.5×10^{-5}	1.0×10^{-5}	6.2×10^{-6}	4.0×10^{-6}

that the distance over which the first 5v of the cathode fall occurs is an important parameter in the theory of the arc cathode. From Tables 7 and 8 we see that taking $j^+ = 5 \cdot 10^3 - 2 \cdot 10^4 \text{ A/cm}^2$, that is 5-20% of the total current density, the first 5v will fall off within $1.7 \times 10^{-6} - 3.0 \times 10^{-6} \text{ cm}$ from the cathode surface.

The first excitation potential of mercury vapour is 4.9 volts. Hence electrons from the cathode which have fallen through 5v will be able to excite vapour atoms and, as we shall describe later, ionisation will occur by cumulative processes. Equations (1.1) and (1.3) are not applicable to a region in which ions are being produced and so are strictly valid only for the first 5v of the cathode fall. The total cathode fall of 8v will fall off over a distance which is certainly greater than 10^{-5} cm , but probably no greater than 10^{-4} cm .

G.G. Smith has published photographs of a mercury arc moving in a magnetic field in which there would appear to be a dark space extending to a distance 10^{-3} cm from the cathode. Smith infers that this is a cathode dark space similar to the cathode dark space of a glow discharge, and that the cathode fall in potential takes place over this distance. However there is no evidence to support this inference except the analogy to the glow discharge, and it will be shown later that a dark space may in fact occur some distance from the cathode fall region. Also the conditions of the experiment (arc being driven by a magnetic field) may have given rise to an apparent dark space, as will be discussed in Part II. This measurement should not therefore be regarded as being at variance with the calculations given in Tables 7 and 8.

CHAPTER 2.

PREVIOUS THEORIES OF THE CURRENT TRANSFER AT THE CATHODE

(2.1) Thermionic Emission Theories.

The early theory of J. Stark⁽⁷⁴⁾ supposed that electrons were emitted thermionically from the cathodes of all arcs. It can be seen from Table 2 that the temperature of the cathode of a carbon arc is about $3,500^{\circ}\text{K}$ and that this is sufficient to release a current density of about 500 A/cm^2 . In order to account for the higher current densities observed in mercury and copper arcs, temperatures greater than $4,000^{\circ}\text{K}$ would be necessary, and it is hard to imagine how such temperatures could be maintained at the surface of a low boiling point metal, particularly in the case of mercury whose normal boiling point is about 350°C .

Recently C.G. Smith⁽²²⁾ put forward an ingenious suggestion that the electrons in the cathode might be raised to enormous temperatures while the cathode lattice remained cool and hence 'thermionic' emission might occur from a comparatively cold metal. His theory of how such a difference in electron and lattice temperature could be maintained suffered from serious fundamental errors which were later acknowledged.⁽³³⁾

Some recent experiments⁽³⁴⁾ have indicated that a type of thermionic emission takes place from the surface of a thin wire through which a large current pulse of very short duration is passed. Emission

currents corresponding to temperatures above the boiling point of the wire have been measured, yet the wire remains cool. This suggests that the temperature of the electrons has been raised and that they have been emitted before they had time to share their heat energy with the lattice. This is however essentially a transient phenomenon and is unlikely to apply to a steady arc.

Loeb⁽³⁵⁾ has suggested that on account of the large influx of energy to the very small 'spot' region conditions may exist there which cannot be described in macroscopic terms of 'temperature', 'work function' etc. He appears to favour the idea that a microvolume might be heated to such an extent that a type of thermionic emission could take place.

A similar picture has been put forward by Rothstein⁽³⁷⁻³⁹⁾ who has suggested that if the density of vapour immediately above the cathode were sufficiently high ($10^{22}/\text{cm}^3$) metallic conduction would take place from cathode to vapour, and then "thermionic emission" could occur from the hot vapour.

Such arguments in favour of thermionic emission can be dismissed by reference to Table 1. Here the energies required to release an electron and to evaporate a neutral atom from a metal are compared. If the work function is much greater than the latent heat of evaporation of an atom, the lattice will surely disintegrate rather than release electrons. A large heat input might then produce electrons by thermal ionisation of the evaporated metal, but thermionic emission from the

metal itself seems to be impossible.

There are at present two main theories of the cold arc cathode, each of which has its supporters. These are the field-emission theory and the thermal ionisation theory respectively.

(2.2) Field Emission Theory.

It was first proposed by Langmuir^(36,40) that electrons might be drawn from the cathode by the high electric field produced at the cathode surface by the space charge of the positive ion current to the cathode.⁽⁶⁷⁾ This suggestion was made before the theory of field emission had been fully developed and at the time may have seemed feasible, but with the advent of the Fowler-Nordheim theory^(72,73) serious quantitative difficulties were encountered.

The Fowler-Nordheim equation for field emission is

$$j_- = \frac{6.2 \times 10^{-6}}{\mu + \phi} \left(\frac{\mu}{\phi} \right)^{\frac{1}{2}} \cdot X^2 \exp \left\{ - \frac{6.8 \times 10^7 \phi^{\frac{3}{2}}}{X} \right\} \quad (2.1)$$

where j_- = electron current density in A/cm².

μ = Fermi energy limit of electrons in the metal.

ϕ = work function in eV.

X = electric field in volts/cm.

For mercury $\phi = 4.5$ eV and $\mu \sim 10$ eV so the equation becomes:

$$j_- = 6.4 \times 10^{-5} X^2 \exp \left\{ - \frac{6.5 \times 10^8}{X} \right\} \quad (2.2)$$

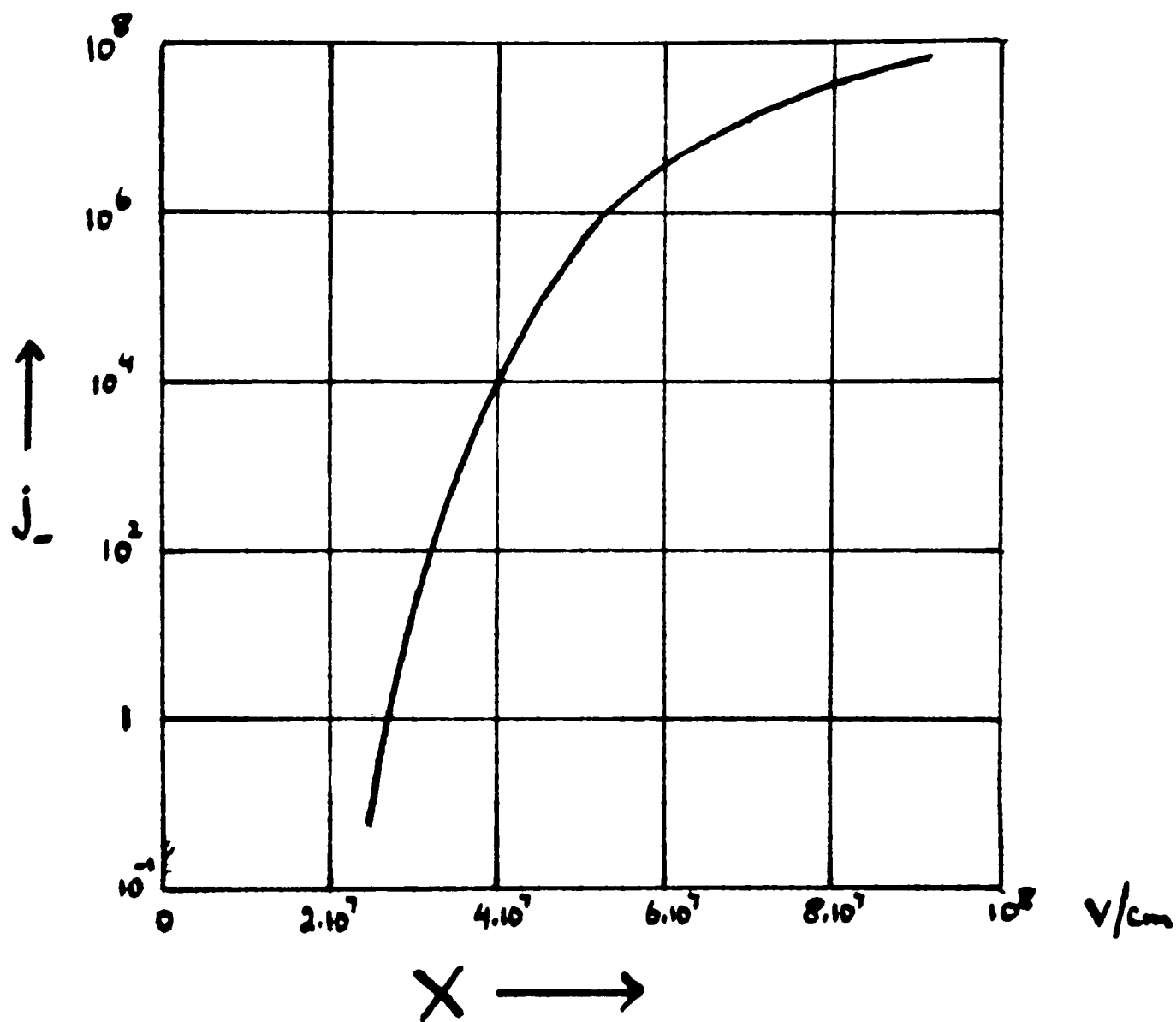


FIG. 1 CURRENT DENSITY OF ELECTRONS j_- DRAWN BY ELECTRIC FIELD X FROM MERCURY.

In Fig.1 j^- is plotted as a function of x in the range $j^- = 1 \text{ A/cm}^2$ to $j^- = 10^8 \text{ A/cm}^2$. It can be seen that the electric field need vary only by a factor 3 to produce a change of eight orders of magnitude in j^- , but that in order to extract an appreciable current density an electric field of the order 10^7 V/cm is required. To extract 10^5 A/cm^2 , the field must be about $4.5 \times 10^7 \text{ V/cm}$.

By differentiating equation (1.1) with respect to distance we obtain the field at the cathode due to the ion space charge is

$$X_c = \frac{4V}{3x} \quad (2.3)$$

Similarly, differentiating (1.3) gives

$$X_c = \frac{5V}{3x} \quad (2.4)$$

Both the high-vacuum and the vapour treatments of the cathode fall therefore lead to approximately the same relation.

From Table 8 we see that for $V = 5\text{v}$ and $j^+ = 5 \cdot 10^3 - 2 \cdot 10^4 \text{ A/cm}^2$ $x = 1.7 \times 10^{-6} - 3 \cdot 10^{-6} \text{ cm}$. With these values in (2.4) we have

$$X_c = 2.8 \times 10^6 - 4.9 \times 10^6 \text{ V/cm.}$$

It can be seen from Fig.1 that such a field would extract a quite negligible current density of electrons. In order to produce a current density of 10^5 A/cm^2 an electric field of $4.5 \times 10^7 \text{ V/cm}$ at the cathode would be required. We see from equation (2.3) or (2.4) that x would have to be about 10^{-7} cm . Since this is probably smaller than the

mean free path in the vapour we can apply equation (1.2) to find the positive ion current density. Putting $V = 5v$ and $x = 10^{-7}$ we obtain $j^+ = 4 \cdot 10^6 \text{ A/cm}^2$. If the total current density is 10^5 A/cm^2 , such a positive ion current density is impossible.

The simple field emission theory therefore fails ^{quant} qualitatively to account for the high current densities of the cold arc cathode. In order to make the theory agree with observations two suggestions have been put forward. The first ⁽⁷⁸⁾ is that the work function might be lowered by the conditions existing at the surface and so, from (2.1), a higher j_- would be obtained from a given field. The second is that surface irregularities either in the liquid surface itself or due to minute insulating particles on it ⁽⁷⁷⁾ would cause local enhancement of the field and hence increased emission of electrons.

In order that an electric field of $3 \cdot 10^6 \text{ V/cm}$ should extract 10^5 A/cm^2 of electrons, we calculate from equation (2.1) that ϕ must be less than one volt. It is surely incorrect however to consider that the work function would be lowered if, on account of the high vapour density above the surface, the liquid-vapour interface became indefinite. The work necessary to extract an electron would in this case lie between the work function of the metal and the ionisation potential of the metal vapour, and so the 'effective work function' would only be increased.

In order that surface irregularities should produce appreciably enhanced fields they must be very small and also well separated from each other. This means that although the current densities from

the points may be greatly increased the contribution of the points to the total current will be small and so the average current density will only show a small increase. (35)

Experiments on field-emission have shown that the emission varies greatly with surface conditions, but that for clean, smooth surfaces the Fowler-Nordheim equation is applicable. (70,71,57) Since the cathode-surface beneath the 'spot' is likely to be kept very clean by the continuous evaporation, there is no reason to suppose that the equation cannot be applied. If this is so, field emission cannot be accepted as the mechanism of the cold arc cathode.

(2.4) Thermal Ionisation Theory.

In the thermal ionisation theory, first proposed by Slepian⁽⁴¹⁾ and currently favoured by many German workers,⁽⁴²⁾ the difficulty of accounting for a large electron emission from the cathode is avoided by assuming that the entire arc current is carried to the cathode by positive ions.

Slepian considered that ion-pairs would be produced in the gas above the cathode by thermal ionisation and that the electric field would then move the electrons towards the anode and the positive ions to the cathode (Fig. 2). He assumed that the degree of ionisation in the gas could be calculated by assigning a temperature T to the gas and applying the Lindemann-Saha equation⁽⁸⁵⁾

$$\frac{s^2}{1-s^2} = 2.4 \times 10^{-4} \frac{T^{5/2}}{p} \exp \left\{ - \frac{eV_i}{kT} \right\} \quad (2.5)$$

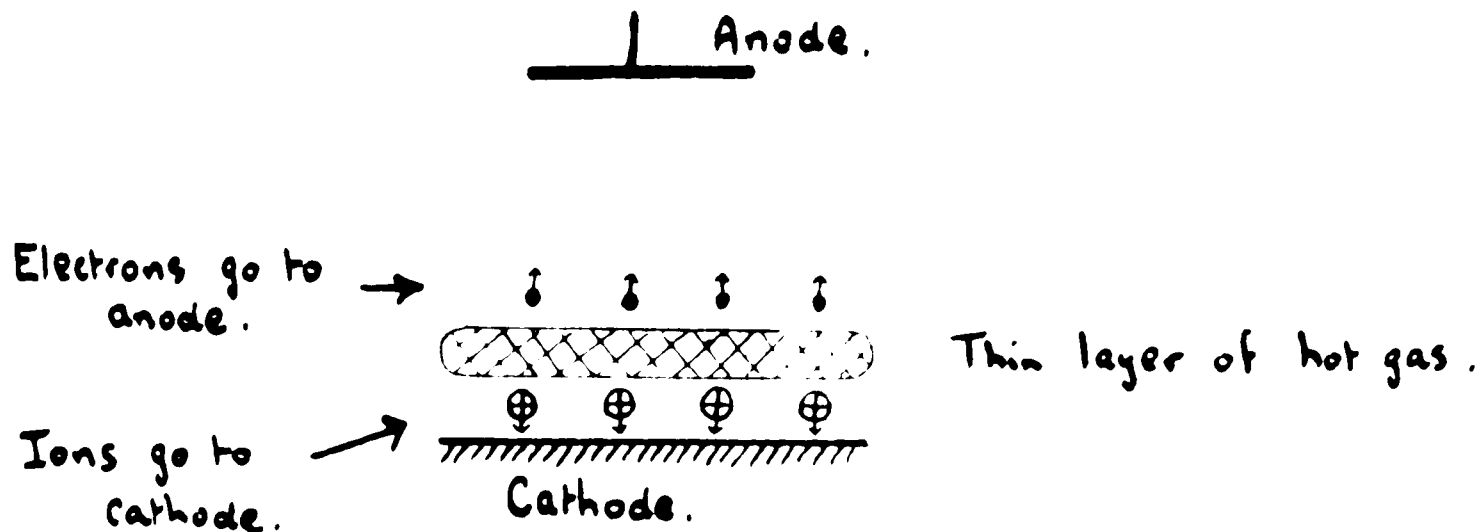


FIG. 2. THERMAL IONISATION THEORY.

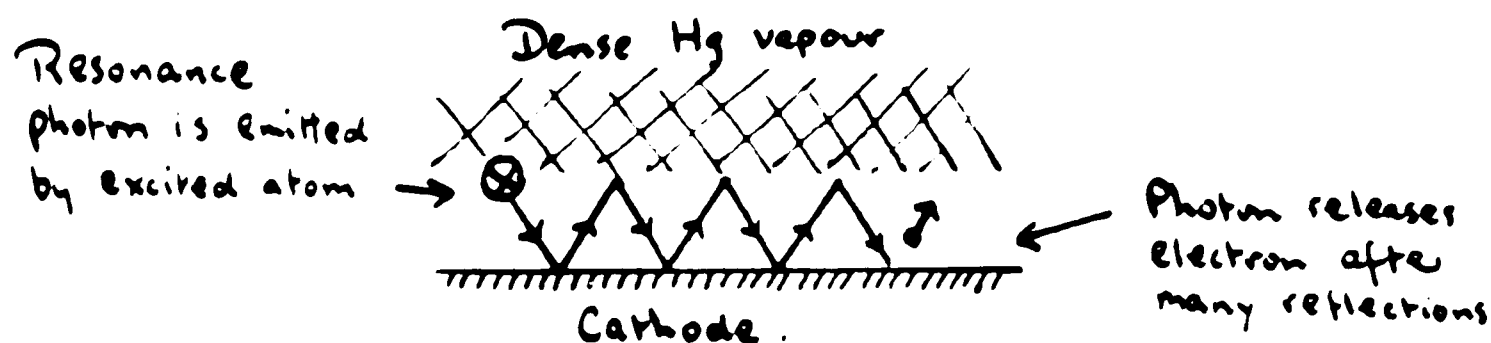


FIG. 3. SPECULAR REFLECTION THEORY.

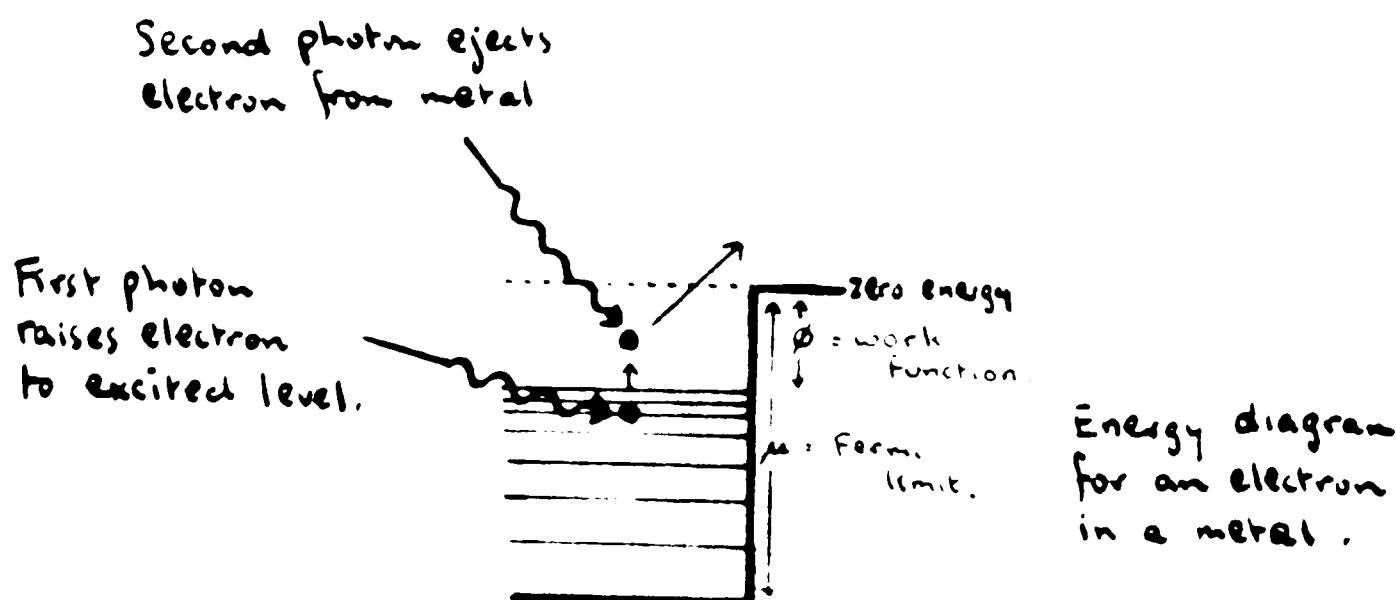


FIG. 4. CUMULATIVE PHOTOELECTRIC THEORY.

approach. The cathode was considered to act as a negative probe, drawing off the positive ions from the ionised gas. The current density was given by the order 10^{-6} cm (see Table 8) from the relatively cool cathode surface.

$$j^+ = \frac{1}{4} N^+ \bar{c}. \quad (2.6)$$

Also, although Saha's equation may be used to calculate the where $\bar{c}$ is the mean random velocity of the ions and N^+ the positive ion density given by

$$N^+ = s N. \quad (2.7)$$

In Slepian's original theory the current density was taken as only 100 A/cm^2 and gas temperatures of about 5000°K were estimated. If the theory is considered in the light of more recent measurements of current density it can be shown that the simple concept of ionisation in the gas cannot account for the observed current density by orders of magnitude, especially if the gas pressure is low.

If $p = 1$ atmosphere and $T = 5000^\circ\text{K}$ we have $N = 2.10^{18}/\text{cm}^3$, $\bar{c} = 8.10^4 \text{ cm/sec}$ and $s = 2.10^{-4}$ (these figures are calculated for Hg vapour, $V_1 = 10.4\text{eV}$). From equations (2.6) and (2.7) we find

$$j_+ \sim 1 \text{ A/cm}^2.$$

For a low pressure arc with $p = 1 \text{ mm Hg}$ and $T = 5000^\circ\text{K}$ we have $N = 3.10^{15}$, $\bar{c} = 8.10^4$ and $s = 4.10^{-3}$, hence

$$j_+ = 4.10^{-2} \text{ A/cm}^2.$$

It would appear then that to account for current densities

approaching the observed values would require temperatures in excess of $10,000^{\circ}\text{K}$ and furthermore such temperatures would have to exist at a distance of the order 10^{-6} cm (see Table 8) from the relatively cool cathode surface.

Also, although Saha's equation may be used to calculate the degree of ionisation in a thermal positive column, where there are approximately conditions of thermal equilibrium, in the region close to the cathode the enormous losses to the cathode render thermal equilibrium impossible. The conditions are rather those of a steady state and so Saha's equation is not strictly valid there. Further, as there are no cathode electrons to be accelerated in the cathode fall, it is hard to see how energy is supplied to keep the gas or vapour in a state of thermal ionisation in the face of the large losses which would occur from the hot region to the cathode surface.

Finally, to produce one ampere of positive ion current to the cathode, V_1 watts must be supplied to the gas, where V_1 is its ionisation potential. Thus if we assume no other losses at all, the discharge potential could not be less than V_1 volts. Other losses, such as heat conduction to the cathode, radiation etc. would increase the discharge potential still further, and as it is well known that arcs will run at potentials below ionisation potential the thermal ionisation theory is rendered untenable.

therefore considered that a photon which had failed to release a photo-electron upon its first arrival at the cathode might be repeatedly

(2.5) Photoelectric Theories.

In the course of the present work the author considered the possibility that electrons might be released from the cathode by the impact of photons from the vapour above the cathode, as was originally suggested by Stolt. (100)

The intense luminosity of the 'spot' suggests that there is a layer of dense vapour immediately above the active cathode area, and that a considerable proportion of the vapour atoms are in excited states. On account of the low cathode fall, ionisation by single collision between electrons and atoms must be ruled out, and the primary collision process will be excitation. Ionisation in the vapour will occur either by further excitation of an excited atom or by a collision of the second kind between two excited atoms. A large number of photons will be produced by excited atoms returning to the ground state, and the majority of these will probably be resonance photons, whose quantum energy is, for Hg, 4.9 volts. Since this is only 0.4 volts above the work function of mercury the probability that a quantum will release a photoelectron from the metal at a single incidence is very small, probably 10^{-3} to 10^{-4} . (97)

It has been shown by Wood that mercury vapour at a pressure greater than about 100 mm Hg has the property of reflecting mercury resonance radiation like a mirror (specular reflection). (80-84)

We therefore considered that a photon which had failed to release a photoelectron upon its first arrival at the cathode might be repeatedly

reflected between the cathode surface and the dense vapour above it until it eventually did release an electron, (Fig.3). In this way the effective photo-electric yield would approach unity and this would be a highly efficient emission process. Unfortunately for this theory the reflection coefficient for $\lambda 2537$ light on a mercury surface is only 60%^(101,102) so the chance that a photon will be absorbed at each incidence is 0.4. Its energy would be lost to the cathode as heat.

Since about 10^3 reflections would be required to produce a photoelectron, far too much energy would be lost by absorption to make the efficiency of emission as high as we need. Furthermore, the phenomenon of specular reflection applies only to resonance radiation, and mercury is the only metal whose resonance photons have energy greater than the work function. The photoelectric theory would not therefore apply to cold arcs other than mercury.

Another possibility which we considered was that if the flux density of photons on the cathode were sufficiently high, cumulative effects might occur in the metal, analogous to step-wise processes in a gas. A metal electron would receive the energy of one photon and before it could share its energy with other electrons another photon would release it from the metal. If this could happen, Einstein's law of proportionality between the photoelectric current and the photon flux might be expected to break down at very large flux densities, and photoelectric yields up to a possible maximum of 0.5 be obtained. Also electrons might be released by photons with energy less than the work function, if the flux density were high enough to cause such cumulative

processes to occur (Fig.4). However after a discussion with H. Frohlich we decided that the time interval in which a metal electron would share its energy with other metal electrons was short compared with the time interval in which two photons might act on the electron, for such photon flux densities which we might expect to fall on the cathode of an arc. We should therefore not expect cumulative effects in the metal.

It would appear that no theory has yet been proposed which can account for both the high current density and the low cathode fall. In the next chapter all possible cathode processes will be considered with these two requirements in mind.

of the available energy to the metal by means of particles, photons etc. and a certain proportion of this energy is used to extract electrons. We can therefore define an extraction efficiency for any process.

$$\eta_e = \frac{\text{Energy used in extraction of electrons}}{\text{Energy supplied to the metal}}$$

$$\eta_e = \frac{W_{ex}}{W_{in}} \quad (3.1)$$

Consider the processes by which electrons can be released from a metal. Energy is supplied to the metal by particles, photons etc. and a certain proportion of this energy is used to extract electrons. We can therefore define an extraction efficiency for any process.

$$\eta_e = \frac{\text{Energy used in extraction of electrons}}{\text{Energy supplied to the metal}}$$

CHAPTER 3

A NEW SUGGESTION FOR THE COLD CATHODE ARC

(3.1) The cathode fall and possible emission processes,

The outstanding difficulty to be overcome by a satisfactory arc theory is the low value of the cathode fall. If we consider a typical mercury arc carrying 1A with a potential of 10 v between cathode and anode, 10 watts are dissipated in the discharge. We assume for reasons already discussed that the current at the cathode is predominantly electronic, so as the work function of Hg is 4.5 volts, 4.5 out of the available 10 watts must be used in extracting electrons from the cathode. We can therefore define a total emission efficiency η_c of the arc:

$$\begin{aligned}\eta_c &= \frac{\text{energy used in extracting electrons}}{\text{energy available in arc}} \\ &= \frac{\text{work function}}{\text{discharge potential}}\end{aligned}\tag{3.1}$$

Consider now any process by which electrons can be released from a metal. Energy is supplied to the metal by particles, photons etc and a certain proportion of this energy is used to extract electrons. We can therefore define an emission efficiency for any process.

$$\eta = \frac{\text{energy used in extracting electrons}}{\text{energy supplied to the metal}}\tag{3.2}$$

It should be noted that if a proportion of the incident particles are reflected with their full energy, this energy is returned to the discharge and may be used again to produce electron emission. We must deduct this reflected energy from the denominator of equation (3.2).

If E is the energy ($K.E + P.E$) of the incident particle, y the yield of electrons per particle, ϕ the work function and R the fraction reflected with full energy, we have

$$\eta = \frac{\phi y}{E(1-R)} \quad (3.3)$$

It is clear that for whatever process we consider may be operative at the arc cathode, the energy supplied to the cathode cannot be greater than the energy available in the arc. Hence the emission efficiency of the cathode process must be greater than the total emission efficiency of the arc.

From (3.1) we find that for a mercury arc with $V_c = 8-10$ v
 $\eta_c = 45\% - 55\%$.

Let us now consider what values of η can be obtained for various emission processes.

1. Field emission: since no energy is supplied by the field except when an electron is extracted, η should be 100%. This is strictly the case only when the field is maintained by some external, static means, e.g. a potential difference between electrodes. When the field is created by positive-ion space charge we should take into account the energy required to maintain the positive-ion current, and η then

becomes a function of the current density.

The energy required to produce and maintain a positive ion current density j_+ across the cathode fall V_0 is $j_+(V_0 + V_i)$ where V_i is the ionisation potential. Hence from (3.2)

$$\eta = \frac{\phi j_-}{j_+(V_0 + V_i)} \quad (3.4)$$

We have shown in section (2.2) that if $j^- = 10^5 \text{ A/cm}^2$, then $j^+ = 4.10^6 \text{ A/cm}^2$. Hence with $V_0 = 8 \text{ v}$ and $V_i = 10 \text{ v}$ we have $\eta = 0.6\%$.

This is another way of showing that field emission cannot account for the mechanism of the cold cathode arc.

2. Thermionic emission: heat energy is supplied to the metal to maintain it at a temperature T . If heat losses by evaporation and conduction are minimised (e.g. by using a spherical cathode of a refractory metal on the end of a thin rod) the principal heat losses will be by the 'evaporation' of electrons and by radiation. At about 3000°K the losses by these two processes are approximately equal, the radiation loss predominating at lower temperatures and the electron cooling predominating at higher temperatures. Hence at 3000°K , $\eta \approx 50\%$.

Thus it appears that thermionic emission would be energetically consistent with the low cathode fall if cathode temperatures greater than 3000°K could be attained. We have already discussed why this is not possible in the case of a low boiling point cathode.

3. Photoelectric emission: yields of the order 0.1 electrons/photon are considered high⁽⁹⁷⁾ and are only obtained with high energy photons ($E = 20$ eV). A fraction of these (say 50%) may be reflected and used again, so putting $y = 0.1$, $R = 0.5$, $E = 20$ in equation (3.3) we have $\eta = 4\%$.

4. Secondary emission by positive ions: yields as great as 1 electron/ion can be obtained, but only with ions whose kinetic energy is of the order 1000 v, whereas for slow ions the yields fall to 0.1 or less.⁽⁹⁷⁾ Hence $\eta \approx 0.5\%$. For mercury in particular the yield for slow ions is very low.⁽⁹⁸⁾ Electron emission by photons or positive ions cannot therefore be the operative process at the cold arc cathode.

5. Secondary emission by excited neutral atoms: there is some controversy as to the yield obtained by this process, but it seems certain that it is greater than 0.1 and may in fact be nearly 1.0. For a mercury atom in its resonance state $E = 4.9$ eV. Hence in equation (3.3) putting $y = 0.5$ and $R = 0$ we obtain $\eta = 45\%$. If $y = 0.75$, $\eta = 68\%$.

However a proportion of the excited atoms which do not release electrons may be reflected with their excitation energy. Thus if $R > 0$ a value of $y < 0.5$ might still lead to $\eta > 50\%$. In the limit, if all the incident excited atoms either released electrons or were reflected we should have $R = 1 - y$, and hence from (3.3) $\eta = 92\%$, independent of the value of y .

It would appear from the above analysis of emission processes that, if we discount any entirely new and hitherto undiscovered mechanism of electron emission from a metal, out of the known processes only electron emission by excited atoms can be energetically consistent with the low cathode fall. The possibility that this process is in fact responsible for the electron emission at the cold arc cathode⁽⁹⁹⁾ will now be discussed.

(3.2) Electron Emission by Excited neutral atoms.

It has been known for some time that excited atoms in metastable states can release electrons from a metal, provided that their excitation energy exceeds the work function.

Early experiments showed that the effectiveness of this process was orders of magnitude greater than photoelectric emission by quanta of comparable energy.^(43,44,45) A number of investigations have since been made,^(46,57,59-62) but on account of experimental difficulties there has been no attempt to measure the yield by a direct method. There is general agreement however that the yield is large.

Unlike measurements on the secondary emission of electrons by positive ions, where the flux of impacting particles can be determined by measuring a current, the absence of charge on a metastable atom renders it difficult to estimate with any accuracy how many metastables fall on a given metal surface. Consequently most of the experiments have been confined to measuring the energy distribution of the ejected electrons, and estimates of the yield have been of a tentative nature.

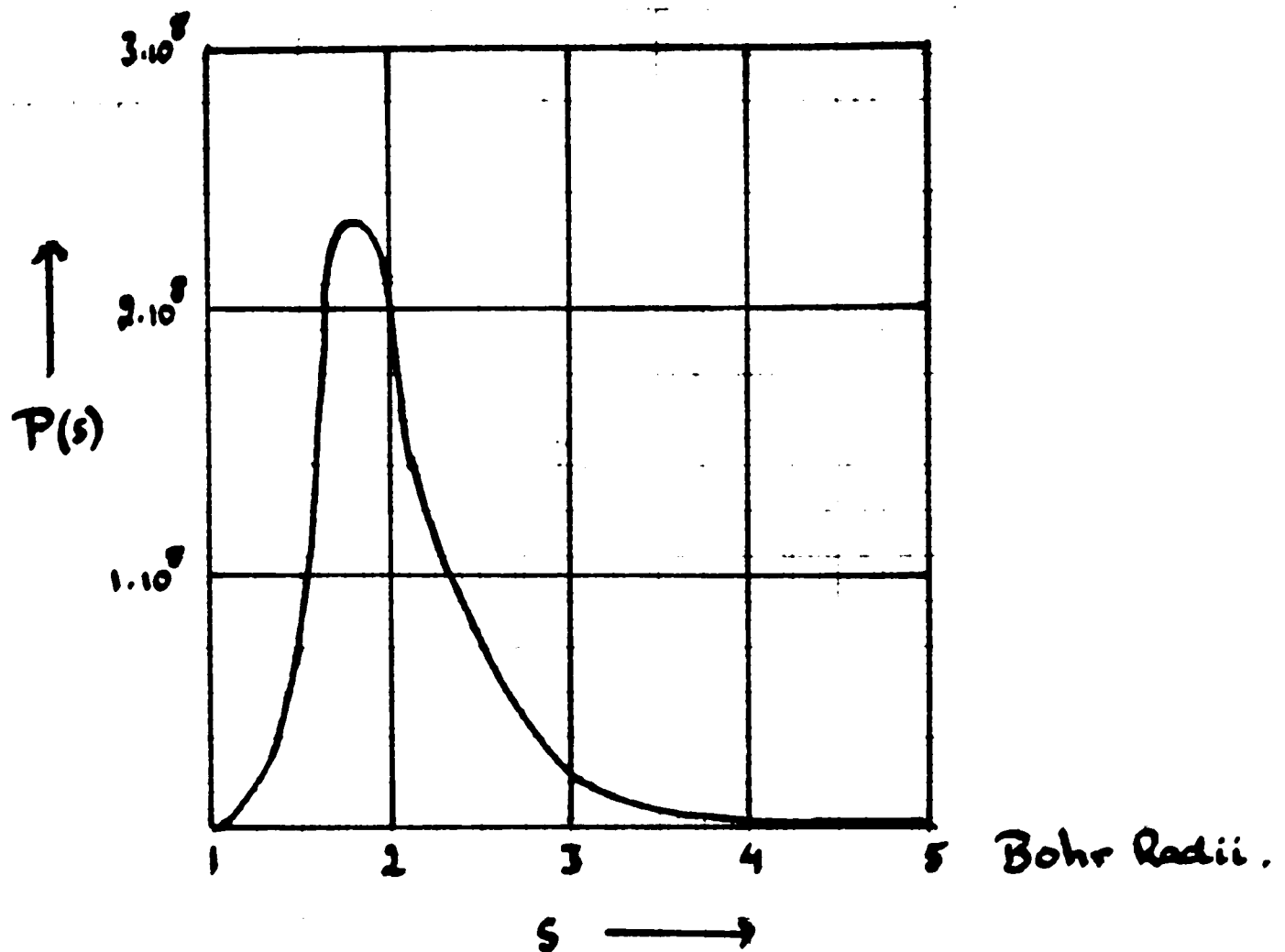
Both Oliphant,⁽⁴⁶⁾ using He metastables on a Ni surface, and Sonkin⁽⁵⁷⁾ using Hg metastables on a W surface have estimated yields of the order unity on clean, outgassed surfaces. Dorrestein⁽⁶¹⁾ has estimated a yield of about 0.4 for He metastables. None of these experiments however were of sufficient accuracy to establish an exact value of the yield.

Several theories of the emission of electrons by positive ions and metastable atoms have been put forward, based on a wave-mechanical treatment of the interaction of the incident particle with the electrons in the metal. Following the earlier work of Oliphant^(46,47) and Massey,^(48,49) Cobas and Lamb⁽⁵⁰⁾ developed a theory in which the rate of decay of a metastable atom was calculated as a function of its distance from the metal surface. A velocity was then attributed to the atom and the probability $P(s)$ of its decaying at a distance s from the metal was calculated. For He metastables approaching a Mo surface a function of the form shown in Fig.5 was obtained, having a maximum at a distance equal to a few Bohr radii. The integral

$$\int_0^{\infty} P(s) ds.$$

gave the probability that one atom would release an electron: for the particular case considered the yield was approximately unity.

Hagstrum⁽⁶³⁾ developed a theory of the ejection of electrons by positive ions. He assumed that an ion, on approaching the metal, first neutralised itself by capturing an electron into an excited level and then acted on the metal in the same way as an excited atom (Fig.6). The probability that it would eject another electron from the metal was then



He metastable atom approaching Mo surface at 10^5 cm/sec.
 Probability of decay within ds at distance $s = P(s) ds$.

FIG. 5. $P(s)$ AS A FUNCTION OF s .

1. Ion neutralised by capturing electron into an excited state.



2. Ion becomes an excited atom.



3. Excited atom returns to ground state and ejects electron from metal.

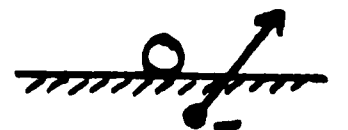


FIG. 6. SECONDARY ELECTRON EMISSION BY IONS.

the probability that an excited atom would eject an electron, and so he assumed that the yields for slow ions and uncharged metastable atoms were about the same. His theory was more complicated than that of Cobas and Lamb in that it involved a greater number of mathematical operations. However instead of a rigorous calculation of some of the matrix elements involved in the interaction of the ion with the metal he deduced their value from experimental results. He then calculated the energy distribution of the ejected electrons and integrated the distribution curve to obtain the yield. He estimated a yield of about 0.15 - 0.20 for He^+ on W and concluded that the yield for He metastables would be about the same. In a private communication he expressed the opinion that for Hg metastables on Hg, the yield would not exceed 0.1.

We do not however agree with Hagstrum that the ejection of an electron by an excited atom which has been formed on the metal surface by neutralisation of an ion is an identical process with the ejection of an electron by an excited atom which has diffused to the surface, since in the former case the electrons in the metal will have been disturbed by the charge on the ion before neutralisation. It would seem that although Hagstrum's theory is successful in accounting for the energy distribution and yield of electrons ejected by ions, so far as a calculation of the yield by metastables is concerned the treatment of Cobas and Lamb is preferable on account of its relative simplicity and because it does not rely on experimental data for its evaluation of matrix elements.

Gurevich and Yavorski⁽⁵⁸⁾ have used a method almost identical to that of Cobas and Lamb to calculate the yield of Hg metastables (energy

4.66 eV) on a Ta surface (work function 4 eV). They estimate that the yield is certainly greater than 0.5 and probably 0.75. No calculations have been made for Hg atoms on a Hg surface, but we should expect a yield not greatly different from the one obtained for Ta.

It might be argued that an electron in the metal which receives the energy of the metastable atom has an equal probability of either going into the metal or of coming out, and so the yield could never be expected to exceed 0.5. However, if the metastable atom arrives with only thermal velocity, the interaction takes place in the surface layer of the metal and there is a possibility that an electron which starts going into the metal will be reflected out, which will explain how the yield could be greater than 0.5.

The discussion has so far been confined to metastable atoms, since in the majority of discharges and experiments the long lifetime of these states enables them to travel considerable distances to deliver their energy to metal surfaces, whereas normally excited atoms radiate within $10^{-7} - 10^{-8}$ sec. However there is no reason to doubt that ordinary excited atoms are equally effective in releasing electrons from a metal if they arrive at the surface without having radiated their energy. In the experiments of Greene⁽⁵⁹⁾ there appears to be some evidence of the effect of normal excited atoms.

The interaction between a metal and a normally excited atom which is capable of spontaneous radiation to its ground state may be slightly different to the interaction involving a metastable atom. Since in the latter case a direct transition to the ground state is

Max $E_k = \mu - \phi$

$\therefore$ Max $E = v^2 - \phi$

- ① Metal electron goes into ground state of atom
- ② Electron in metastable level is ejected from the atom.

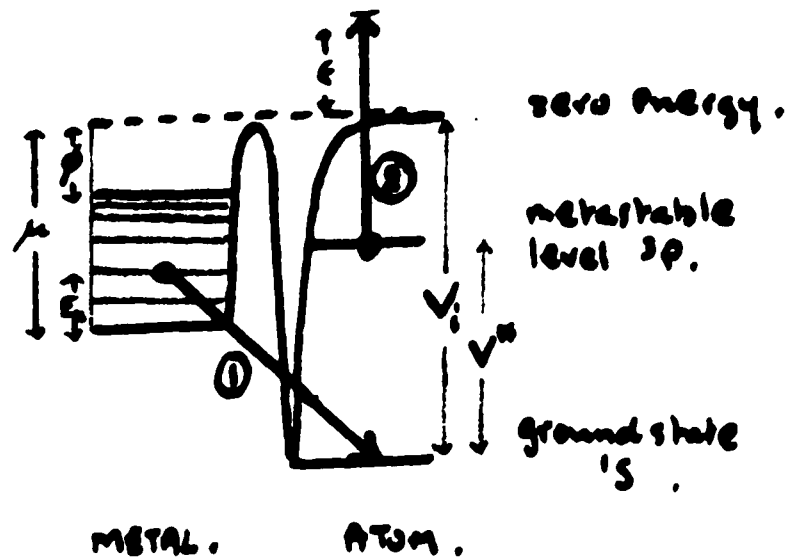


FIG. 7. EJECTION OF AN ELECTRON FROM A METAL BY A METASTABLE ATOM.

- ① Excited atom returns to ground state.
- ② Metal electron is ejected.

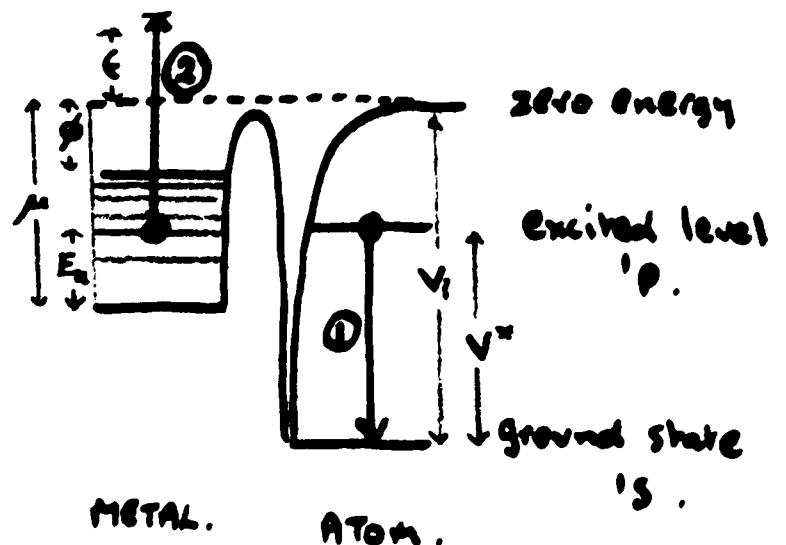


FIG. 8. EJECTION OF AN ELECTRON FROM A METAL BY A NORMALLY EXCITED ATOM.

μ : Fermi limit . ϕ : work function . V_i : ionisation potential .
 V^* : excitation potential . E_k : energy of metal electron above Fermi limit . ϵ : energy of ejected electron .

$$\epsilon = E_k + V^* - \mu$$

$$\text{Max } E_k = \mu - \phi \quad \therefore \text{Max } \epsilon = V^* - \phi$$

forbidden by the selection rule $\Delta S = 0$, the metastable must first capture a metal electron into its ground state and then eject the excited valence electron. In the normally excited atom the valence electron could return to the ground state and the metal electron would be ejected. These two processes are illustrated on an energy-level diagram in Figs. 7 and 8.

Whether or not there is a difference in yield between these processes could only be discovered by laborious calculation, and we consider that any difference is probably insignificant. It should be noted that the mercury resonance line $\lambda 2537$ is the result of a "forbidden" transition ($^3P_1 \rightarrow ^1S_0$) and that the interaction of mercury resonance atoms with a metal may be identical with the interaction of metastables.

Since excited atoms can only move towards a surface by virtue of their random velocity, which for normal temperatures is of the order $10^4 - 10^5$ cm/sec, the only atoms which are likely to reach a metal without having radiated their excitation energy must have been excited within $10^{-3} - 10^{-4}$ cm from the surface. This would be considered a very short distance for the majority of experiments, but as we have seen, the extension of the cathode region of a cold arc is considerably smaller and so we must consider the effect of both metastable and normally excited atoms arriving at the arc cathode.

It can be seen from the dimensions given in Fig. 5 that a layer of gas atoms on a metal surface might prevent an incident excited atom from approaching as far as the maximum of the probability-of-decay curve.

This means that gas layers or other surface contamination would drastically reduce the yield, as was shown experimentally by Sonkin.⁽⁵⁷⁾ In the case of a cold arc where continuous evaporation of cathode material takes place, it is likely that the surface beneath the 'spot' is kept very clean, and so the yield is likely to have its highest value.

All the theoretical treatments of the interaction of excited atoms with a metal have considered a single excited atom approaching a surface at which there is no electric field. In the case of the cold arc, large numbers of excited atoms would arrive on a small area of the cathode and any cumulative effects which might occur in the metal would tend to increase the effective yield. Also there is a strong electric field at the cathode surface which, although not sufficient to cause appreciable field emission, might assist the excited atoms in ejecting electrons. The electric field has the effect of reducing the work function by an amount $\Delta\phi$ given by

$$\Delta\phi = (eX_c)^{1/2} \quad (3.5)$$

This equation is derived from the theory of the Schottky effect.^(123,124)

In Section (2.2) we calculated $X_c = 3.10^6$ v/cm - 5.10^6 v/cm. Hence from (3.5)

$$\Delta\phi = 0.65 - 0.85 \text{ volts.}$$

The effect of the electric field is therefore to reduce the work function of mercury (normally = 4.5 v) to about that of Ta (4.0 v). Hence the yield calculated by Gurevich and Yavorski for Hg on Ta probably applies in our case. In the following discussion therefore we shall consider

that the yield lies between 0.5 and 0.75.

(3.3) The current density: simple quantitative considerations.

We have shown that electron emission by excited neutral atoms is a cathode process which would be energetically consistent with the low cathode fall. We must now see whether it can account for the observed high current density.

A current density of 10^5 A/cm² is equivalent to 6×10^{23} electrons/cm² and sec. If this current density is released by excited atoms with a yield of 0.75, about $8 \cdot 10^{23}$ excited atoms/cm² and sec must fall on the cathode. Since the atoms are neutral and therefore unaffected by the electric field, the only way they can reach the cathode is by virtue of their random velocity, which for a mercury atom at a temperature of 1000°K has a value $\bar{v} = 4 \times 10^4$ cm/sec. Applying the familiar equation:

$$S^* = \frac{1}{4} N^* \bar{v} \quad (3.6)$$

where S^* is the number of excited atoms striking the cathode per cm² and sec and N^* is the number density of excited atoms immediately above the cathode surface, we find that for $S^* = 8 \cdot 10^{23}$ /cm² and sec, $N^* = 8 \cdot 10^{19}$ /cm³.

At first sight this appears to be an impossibly large density since the vapour pressure in a low pressure arc vessel is usually of the order 10^{-2} mm Hg corresponding to a number density of the order 10^{14} /cm³. However the intense brightness of the cathode 'spot' suggests that the density of atoms in this region has been augmented by evaporation from

the cathode and a rough estimate of a lower limit can be made as follows.

The brightness of the spot has been measured by Haynes⁽¹¹⁾ and has been estimated as 10^6 candles/cm². This is equivalent to 2×10^4 watts/cm² of visible light,⁽¹¹²⁻¹¹³⁾ and taking the average photon energy as 2.5 eV, represents very roughly 5×10^{22} visible photons/cm² and sec. The number of resonance photons is likely to be at least of the same order of magnitude. These photons originate from a region which, we believe extends no more than about 10^{-4} cm from the cathode. Assuming a lifetime of 10^{-7} sec for the excited states we calculate that a number density of at least 10^{19} excited atoms/cm³ exists in the 'spot'. This is an average value, and the density immediately above the cathode may be still higher.

The light emission from the spot will be discussed in greater detail later, but this simple calculation, based on the brightness measurement, can be taken as evidence that a high density of excited atoms exists above the cathode. A density of 10^{20} /cm³, which would be sufficient to release the observed current density of electrons, appears to be quite feasible. How this density can be maintained will now be discussed.

(3.4) Cathode evaporation and cathode temperature.

Attempts have been made to measure the temperature of the cathode surface by pyrometric methods.⁽⁸⁶⁻⁸⁸⁾ At the time of these measurements it was not realised that the active cathode area is covered by a thin but dense layer of vapour which is emitting not only a strong

line spectrum but a faint continuous spectrum as well, and hence measurements taken through this layer must be in error. The values of cathode temperature given in Table 2 were measured in this way and probably represent underestimates of the true cathode temperature of cold-cathode arcs.

A number of workers have measured the rate of loss of cathode material from the cathode of mercury and copper arcs. A summary of these measurements is given in Table 5. Observations on a mercury arc are complicated by the random motion of the 'spot' over the cathode and by the spray of droplets which is ejected from the 'spot'. These droplets may account for over 90% of the total mass transfer from the cathode, but if the random motion is suppressed by 'anchoring' the spot the droplets are eliminated and under these circumstances the mass transfer is assumed to be due to evaporation only. An anchored mercury arc 'spot' evaporates about 3×10^{-4} gm/coul, while a vacuum copper arc evaporates about 2×10^{-5} gm/coul. These rates of evaporation correspond to 10^{18} and 2×10^{17} atoms/coul respectively.

Some investigators have attempted to estimate the temperature of the cathode under the 'spot' from the measured rate of evaporation. The rate of evaporation per unit area W was calculated from the total evaporation and the measured area of the spot and, assuming evaporation into vacuum, the value of T was determined from the equation. (6)

$$W = \frac{P}{4} \left(\frac{3M}{\kappa T} \right)^{\frac{1}{2}} \quad (3.7)$$

where M is the atomic mass and p the vapour pressure corresponding to temperature T . Using Kobel's values of evaporation (2×10^{-5} gm/coul) and current density (1900 A/cm^2) Compton calculated $T \sim 200^\circ\text{C}$ for a mercury arc. However if we use Tonks' value of the evaporation (3×10^{-4} gm/coul) and modern values of the current density (10^5 A/cm^2) we get $T \sim 350^\circ\text{C}$.

Another method of estimating cathode temperature is from the force exerted by the arc on the cathode, assuming that this is due to the momentum of evaporating atoms. The force has been measured for mercury and copper arcs and Table 6 gives some of the results. The measurements on mercury must be accepted with caution, since the spot was anchored and therefore it is hard to say how much of the momentum was transferred to the anchor and how much to the mercury. Tanberg however measured simultaneously a force of 20 dynes/ampere on a copper cathode and a loss of mass of 2×10^{-5} gm/coul. From this he calculated that the average velocity of the evaporating atoms was 1.6×10^6 cm/sec, corresponding to a cathode temperature of about $5 \cdot 10^5 \text{ K}$. Similar measurements by other workers^(90,91) were interpreted in the same way.

This value of the temperature was criticised as absurdly high, since the energy of each atom would be about 60 eV. Attempts were made to interpret Tanberg's observations in terms of reflected ions,^(89,92,94,95) reflected plasma electrons⁽⁹³⁾ etc. However, the fundamental error in both the above methods of estimating cathode temperature lies in the assumption that atoms evaporate into free space. An atom which leaves the cathode is not only likely to collide with an atom of the

residual gas in the arc vessel, (although the chance of this at low pressure is very small) but also with other atoms which have been evaporated from the cathode. The greater the rate of evaporation, the greater will be the density of atoms above the cathode and hence the greater the likelihood of an evaporated atom suffering collisions. As a result of such collisions many atoms will be scattered back to the cathode. The net evaporation will be the difference between the gross evaporation and the back-scattering, and if the difference is small compared with the gross evaporation the layer of vapour immediately above the cathode can be considered in equilibrium with the liquid. The transfer of atoms from the cathode to the condensing surface of the arc vessel is due to density gradients in the vapour and must be treated as a diffusion problem.

Atoms which pass from the liquid to the vapour, and also atoms returning to the liquid after being back-scattered in the vapour will communicate momentum to the cathode and so exert a pressure on it. If the vapour immediately above the cathode is in equilibrium with the liquid (and we shall show later that this is approximately the case) this pressure will be the vapour pressure corresponding to the cathode temperature.

(3.5) The Force on the Cathode.

The pressure on the cathode can be estimated from measurements of the cathode reaction force and the current density. If F is the force in dynes/amp and j the current density in amps/cm², the pressure

on the cathode is given by

$$p = Fj \text{ dynes/cm}^2 \quad (3.8)$$

For copper we see from Table 6 that $F \sim 20$ dynes/amp and so taking $j = 10^5$ A/cm² we find $p \approx 2 \cdot 10^6$ dynes/cm² = 2 atmospheres. For mercury $F \sim 50$ dynes/amp giving $p \approx 5$ atmospheres.

This pressure on the cathode is the resultant of the pressures due to various processes which occur at the cathode.

1. The vapour pressure p_v . This is due to the evaporation and back-scattering of neutral atoms, as discussed in the previous section.

2. The electron pressure p_e . This is due to the transfer of momentum to the cathode by the electrons as they leave the surface. (18,20)
Each electron of mass m emitted with velocity v^- will communicate momentum mv^- to the cathode. If the electron current density is j^- we derive

$$p_e = \frac{j^- m v^-}{e} \quad (3.9)$$

3. The ion pressure p_i . If the ion current density is j^+ and each ion of mass M arrives at the cathode with velocity v^+ the pressure on the cathode due to the arriving ions is, by analogy with equation (3.9)

$$p_i = \frac{j^+ M v^+}{e} \quad (3.10)$$

4. The electric field. The electric field at the cathode exerts a force on the cathode in the opposite direction to the pressure due to the processes described above. If the field is X_0 we may write this 'upward pressure'

$$p_x = -\frac{X_c^2}{8\pi} \quad (3.11)$$

The measured resultant pressure p is given by

$$p = p_v + p_e + p_i + p_x \quad (3.12)$$

Putting $j^- = 10^5 \text{ A/cm}^2$, $\frac{e}{m} = 1.7 \times 10 \text{ coul/gm}$, $v^- = 2.10^7 \text{ cm/sec}$

(corresponding to an initial K.E. of about 0.1 eV) we have from equation (3.9)

$$p_e \approx 10^4 \text{ dynes/cm}^2 = 0.01 \text{ atmospheres}$$

Putting $j^+ = 5.10^3 - 2.10^4 \text{ A/cm}^2$, $\frac{e}{M} = 500 \text{ coul/gm}$ (see Appendix 1) and assuming that, on account of their making many collisions in the cathode fall space, the positive ions arrive at the cathode with thermal velocities corresponding to about 1000°K , so $v^+ = 4.10^4 \text{ cm/sec}$, we have from (3.10)

$$p_i = 0.4 \text{ to } 1.6 \text{ atmospheres}$$

We have estimated in Section 2.2 that the electric field at the cathode is about 3.10^6 V/cm to 5.10^6 V/cm . From equation (3.11) we have

$$p_x = -4 \text{ to } -11 \text{ atmospheres}$$

In equation (3.12) the only unknown quantity is p_v , so by substitution we find

$$p_v = 5.5 \text{ to } 11 \text{ atmospheres for copper.}$$

$$p_v = 8.5 \text{ to } 14.5 \text{ atmospheres for mercury.}$$

It can be seen that this estimate of p_v is very sensitive to the value chosen for X_c . Since it is not possible to estimate X_c very

accurately, and the measured value of the force is probably not particularly accurate, especially in the case of mercury, we can do no more than estimate the vapour pressure at the cathode as of the order 10 atmospheres.

In Fig. 9 is plotted the saturated vapour pressure of mercury as a function of temperature. In Appendix 3 we derive the density of vapour corresponding to the vapour pressure, and this is plotted in Fig. 10 as a function of temperature. We see that a pressure of 10 atmospheres corresponds to a number density of about 10^{20} atoms/cm³ and a temperature of 520°C. If the vapour above the cathode is in equilibrium with the liquid, this represents the temperature of the active cathode area.

The force on the cathode therefore provides additional evidence that a high density of vapour exists immediately above the active cathode area. It is now apparent how the cathode may be bombarded by excited atoms. Atoms which evaporate from the cathode will, for the most part, be in their ground state. The majority will be back-scattered to the cathode, only a comparatively small number being lost by diffusion. These back-scattered atoms will have spent some time in the space above the cathode, and there they will have experienced collisions with electrons which have gained sufficient energy in the cathode fall to be able to excite them. Since the excitations will have occurred very close to the cathode the majority of the excited atoms will reach the cathode before having time to radiate their energy, and will be able to release electrons, provided that their potential energy exceeds the work function

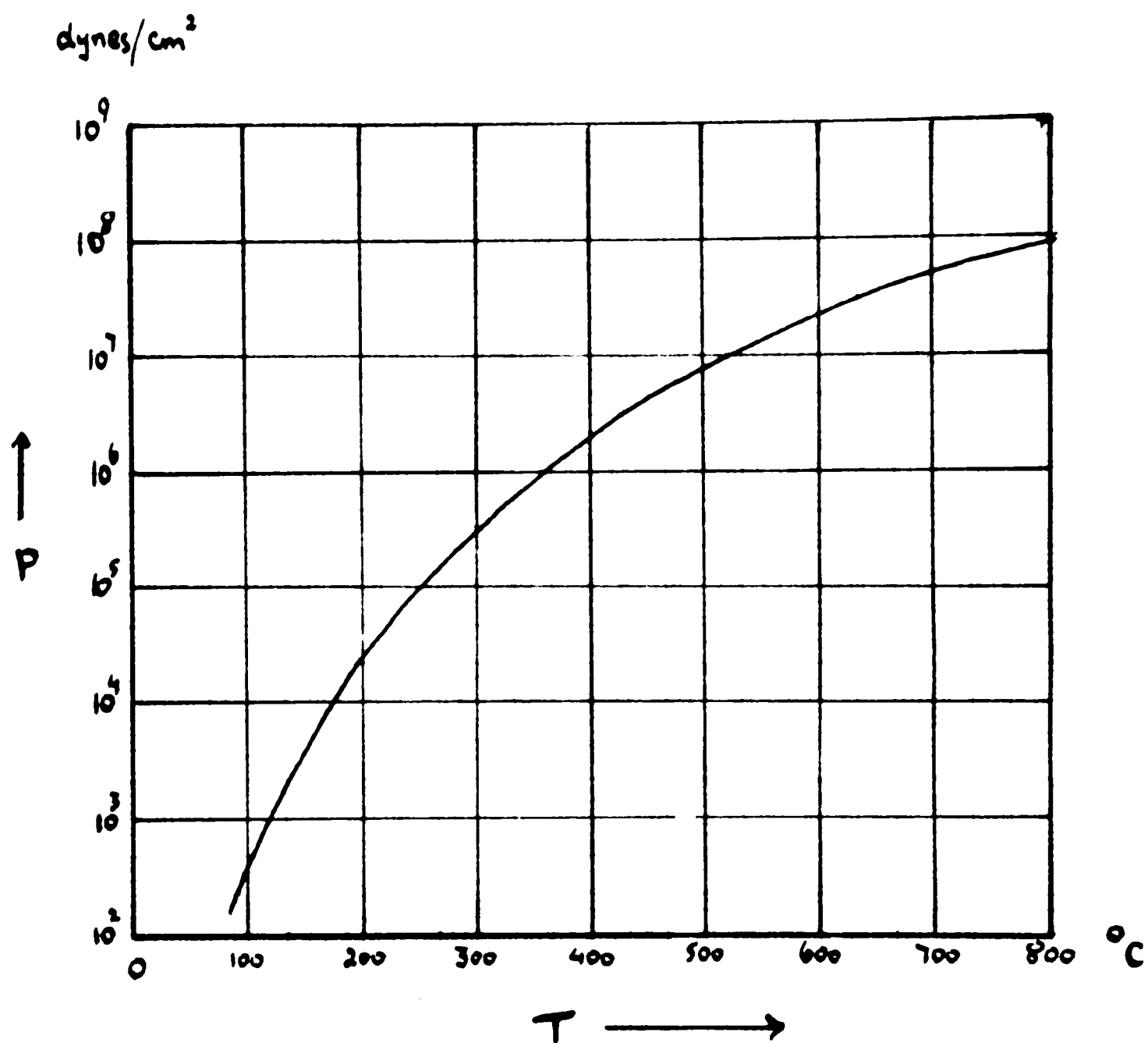


Fig. 9. VAPOUR PRESSURE p OF MERCURY VAPOUR IN EQUILIBRIUM WITH LIQUID AT A TEMPERATURE T .

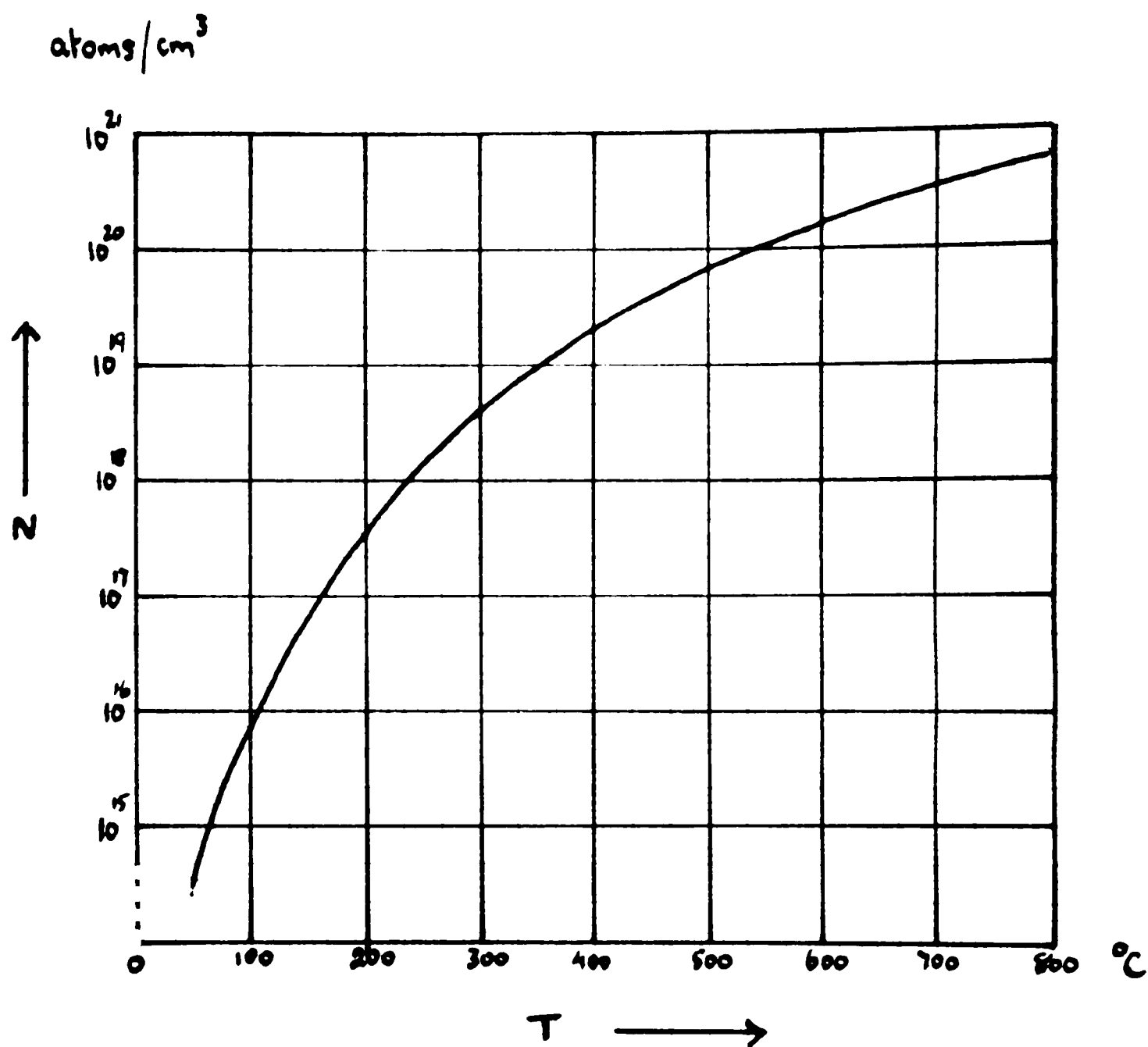


Fig. 10. VAPOUR DENSITY N OF MERCURY VAPOUR
IN EQUILIBRIUM WITH LIQUID MERCURY
AT A TEMPERATURE T .

of the cathode.

Table 4 gives a list of the work function, lowest excitation potentials and ionisation potentials of a number of metals. Mercury is seen to be the only metal having all its excited states of higher energy than the work function. For all other metals, atoms in the lower excited states could not release electrons from the cathode. We shall show later that this does not represent a serious objection to our new suggestion for the cold arc cathode. On account of its simplicity the mercury arc will be treated first, and we shall show that in, for example, the copper arc, the majority of excited atoms are probably in states of higher energy than the work function.

At any point in the vapour space the electron current will be a flow of vapour determined by the density gradient and the diffusion coefficient at that point

$$j = -D \nabla n \quad (4.1)$$

where j is the flow vector, D the diffusion coefficient and n the concentration.

Now on the simple kinetic theory

$$D = \frac{\lambda \bar{v}}{3} \quad (4.2)$$

where λ is the mean free path and $\bar{v}$ the mean random velocity of a vapour molecule. $\bar{v}$ is a function of the temperature only and λ is inversely proportional to the concentration n . Hence for constant temperature

CHAPTER 4

THE VAPOUR DISTRIBUTION

(4.1) The diffusion problem.

We consider that the back-scattering of evaporating atoms causes a large density of vapour to exist immediately above the evaporating area of the cathode. This would seem to be borne out by observations of the luminosity and the cathode force. The distribution of vapour will now be derived by considering the transfer of cathode material in the space above the evaporating area as a problem in diffusion.

At any point in the region above the cathode there will be a flow of vapour determined by the density gradient and the diffusion coefficient at that point

$$\underline{f} = -D \text{ grad } N \quad (4.1)$$

where $\underline{f}$ is the flow vector, N the concentration and D the diffusion coefficient.

Now on the simple kinetic theory

$$D = \frac{\lambda N \bar{c}}{3} \quad (4.2)$$

where λ is the mean free path and $\bar{c}$ the mean random velocity of a vapour molecule. $\bar{c}$ is a function of the temperature only and λ is inversely proportional to the concentration N . Hence for constant temperature

From (4.3), (4.5) $D = \frac{A}{N}$ where A is a constant (4.3)

Constant temperature is a simplification as the temperature will vary throughout the space, but the assumption is made in order to obtain an easily soluble differential equation.

The problem is therefore one of a small circular area evaporating into a semi-infinite region in which the diffusion coefficient at any point is inversely proportional to the concentration at that point. A concentration N_0 is maintained at the evaporating surface: the concentration of neutral atoms of vapour is to be found as a function of position in space.

(4.2) Take cylindrical polar co-ordinates such that the surface of the mercury is the plane $z = 0$ and the z -axis passes through the centre of the circular evaporating area $0 \leq r \leq a$ in the plane $z = 0$. (Fig.11)

The general flow equation in the steady state for the region where z is positive is

$$\text{div } \underline{f} = 0 \quad (4.4)$$

Expressed in cylindrical polar coordinates equation (4.4) becomes

The boundary

$$\frac{\partial f_r}{\partial r} + \frac{1}{r} f_r + \frac{\partial f_z}{\partial z} = 0 \quad (4.5)$$

where f_r and f_z are the flows per unit area in the r and z directions respectively, and are given by

familiar problem in heat conduction

$$f_r = -D \frac{\partial N}{\partial r} ; f_z = -D \frac{\partial N}{\partial z} \quad (4.6)$$

From (4.3), (4.5) and (4.6) we obtain

$$\frac{\partial}{\partial r} \left(\frac{1}{N} \frac{\partial N}{\partial r} \right) + \frac{1}{rN} \left(\frac{\partial N}{\partial r} \right) + \frac{\partial}{\partial z} \left(\frac{1}{N} \frac{\partial N}{\partial z} \right) = 0 \quad (4.7)$$

The boundary conditions of this equation are:

1. $N = N_s$ at infinity, where N_s is the vapour density corresponding to the residual pressure in the arc vessel. In the case of the mercury arc the residual pressure is determined by the temperature of the condensing surface.

2. $N = N_0$ for $0 < r < a$ in the plane $z = 0$.

(4.2) Solution of the diffusion equation.

If we make the substitution

$$Y = \ln \left(\frac{N}{N_s} \right) \quad (4.8)$$

equation (4.7) becomes

$$\frac{\partial^2 Y}{\partial r^2} + \frac{1}{r} \frac{\partial Y}{\partial r} + \frac{\partial^2 Y}{\partial z^2} = 0 \quad (4.9)$$

The boundary conditions become

(1) $Y = 0$ (corresponding to $N = N_s$) at infinity

(2) $Y = Y_0 = \ln \frac{N_0}{N_s}$ (corresponding to $N = N_0$) for $r < a$, $z = 0$.

Equation (4.9) and its boundary conditions are analogous to a familiar problem in heat conduction: to find the temperature distribution in an infinite solid due to a circular disc maintained at

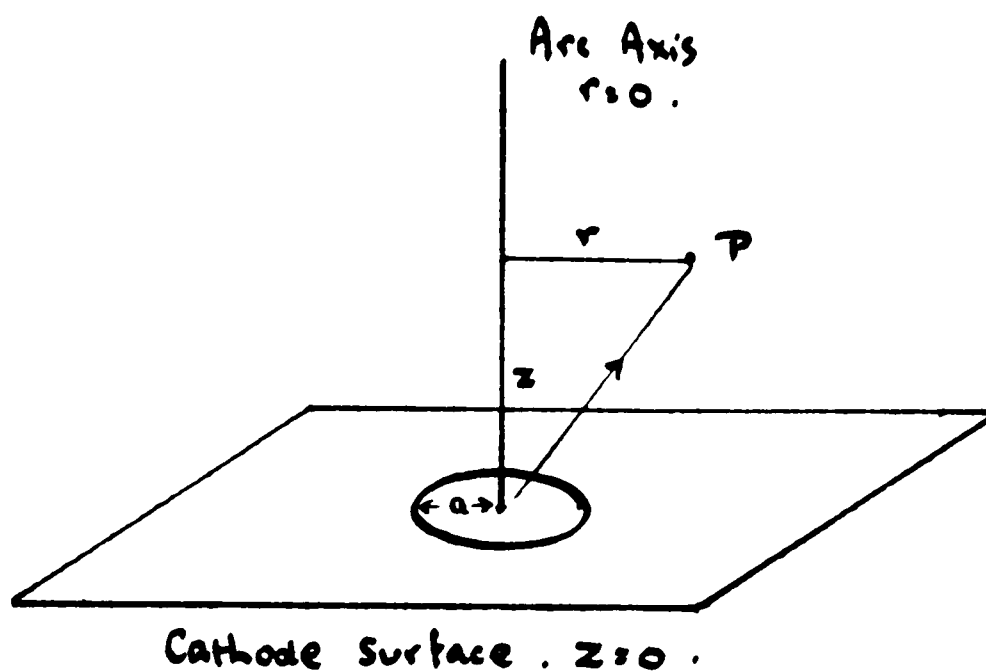


Fig. 11. CYLINDRICAL CO-ORDINATES.

$N_s = 10^{18}$		$N_s = 10^{17}$	
N	K	N	K
10^{20}	0.00	10^{20}	0.00
10^{19}	0.11	10^{19}	0.33
10^{18}	0.53	10^{18}	3.00
10^{17}	1.88	10^{17}	∞
10^{16}	9.47	-	-
10^{15}	∞	-	-

TABLE 9. TABLE OF N AND K.

temperature Y_0 , the temperature at infinity being zero (see Appendix 4).

The solution is known⁽¹⁰³⁾ and is

solds.

$$\frac{Y}{Y_0} = \frac{2}{\pi} \int_0^\infty e^{-\lambda z} J_0(\lambda r) \frac{\sin \lambda a}{\lambda} d\lambda. \quad (4.10)$$

$N_0 = 10^{15}$ (corresponding

for $N_0 = 10^{15}$ The integral has been evaluated⁽¹⁰⁴⁾ and the solution is then

and H_g) and from these values

$$\frac{Y}{Y_0} = \frac{2}{\pi} \tan^{-1} \frac{a}{Z} \quad (4.11)$$

has already been discussed.

where

$$2Z^2 = r^2 + z^2 - a^2 + \left((r^2 + z^2 - a^2)^2 + 4z^2 a^2 \right)^{1/2} \quad (4.12)$$

Substituting back from (4.8) we have the solution

$$\frac{Z}{Z_0} = \left(\frac{Z}{Z_0} \right)^{2/\pi} \tan^{-1} \frac{a}{Z} \quad (4.13)$$

In Fig. 1, the distribution of density in the case is plotted for

equation (4.10) as a function of Z/Z_0 for $N_0 = 10^{15}$ and for the two values

If we put

of N_0 , $N_0 = 10^{15}$ and $N_0 = 10^{17}$.

$$K = \cot^2 \frac{\pi Y}{2Y_0} \quad (4.14)$$

$a = 0$ falls off rapidly as z increases while for $a \neq 0$ the density is

we have from (4.11), (4.12) and (4.14)

essentially the residual density N_0 . The effect of increasing N_0 is to

extend the region of high density further from the cathode. For ex-

$$\frac{r^2}{a^2(1+K)} + \frac{z^2}{a^2 K} = 1 \quad (4.15)$$

ample, when $N_0 = 10^{15}$ the density falls from $N = 10^{15}$ at $z = 0$ to $N = 10^{18}$ at $z = 0.75a$, but when $N_0 = 10^{17}$ we find $N = 10^{18}$ at $z = 2a$.

This equation represents a family of ellipses in the co-

ordinates (r, z) for different values of K . From (4.14) and (4.8) we
distribution of vapour if a density N_0 is maintained over a circular area

see that each value of K corresponds to a value of N , hence the surfaces of constant concentration in the space above the cathode will be ellipsoids.

Values of K and N are given in Table 9 for $N_0 = 10^{20}$ and $N_s = 10^{15}$ (corresponding to a residual pressure of 3×10^{-2} mm Hg) and for $N_0 = 10^{20}$ and $N_s = 10^{17}$ (corresponding to a residual pressure of 3 mm Hg) and from these values the ellipsoids of constant concentration have been plotted in Figs. 12 and 13. The reason for taking $N_0 = 10^{20}$ has already been discussed.

Consider now the distribution of vapour on the axis of z only.

Putting $r = 0$ in equation (4.12) we find $Z = z$, so from (4.13)

$$\frac{z}{a} = \left(\frac{N_0}{N_s} \right)^{\frac{2}{\pi}} \tan^{-1} \frac{z}{a} \quad (4.16)$$

In Fig. 14 the concentration N on the axis is plotted from equation (4.16) as a function of z/a for $N_0 = 10^{20}$ and for the two values of N_s , $N_s = 10^{15}$ and $N_s = 10^{17}$.

It can be seen from Fig. 14 that the initial high density at $z = 0$ falls off rapidly as z increases until for $z > 7a$ the density is essentially the residual density N_s . The effect of increasing N_s is to extend the regions of high density further from the cathode. For example, when $N_s = 10^{15}$ the density falls from $N = 10^{20}$ at $z = 0$ to $N = 10^{18}$ at $z = 0.75a$, but when $N_s = 10^{17}$ we find $N = 10^{18}$ at $z = 2a$.

In the foregoing treatment we have obtained the density distribution of vapour if a density N_0 is maintained over a circular area

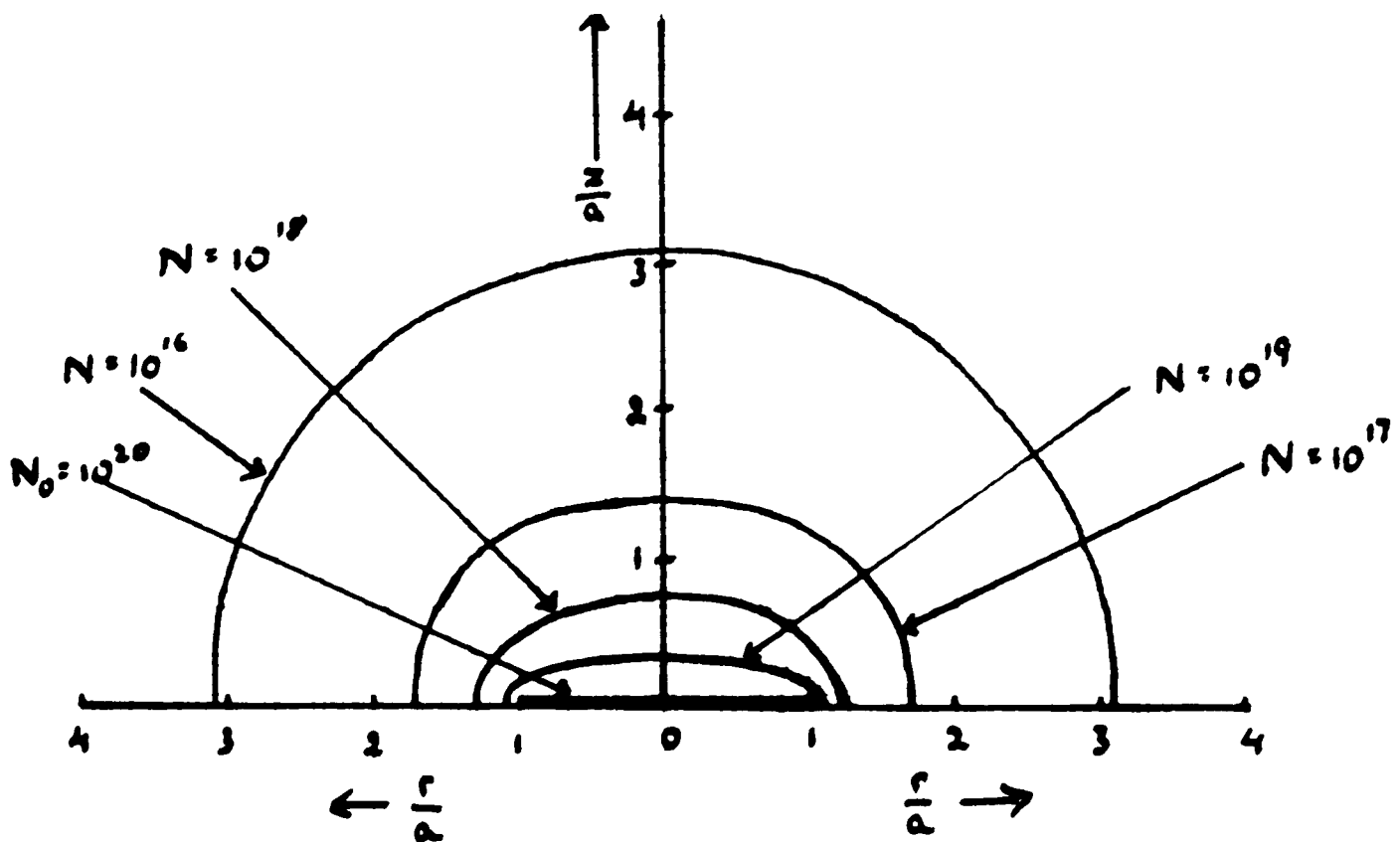


Fig 12. SURFACES OF CONSTANT CONCENTRATION
FOR $N_0 = 10^{20}$, $N_s = 10^{15}$.

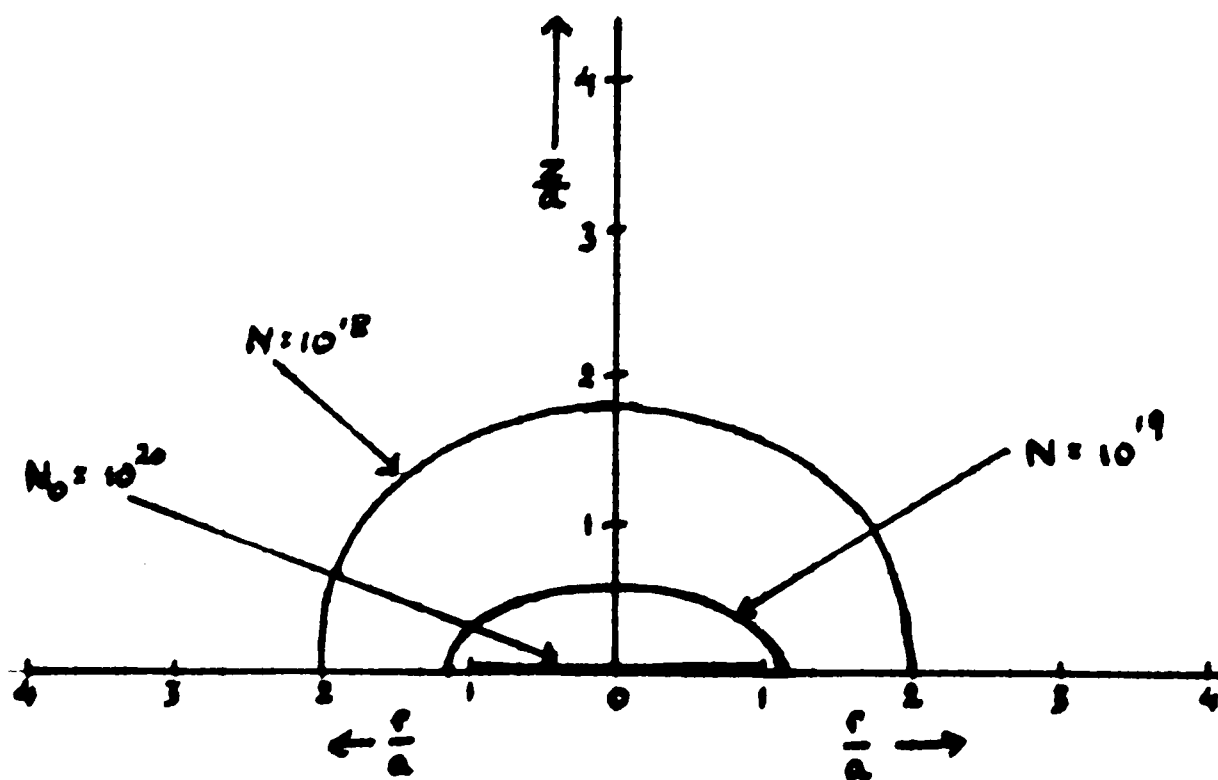


Fig 13. SURFACES OF CONSTANT CONCENTRATION
FOR $N_0 = 10^{20}$, $N_s = 10^7$.

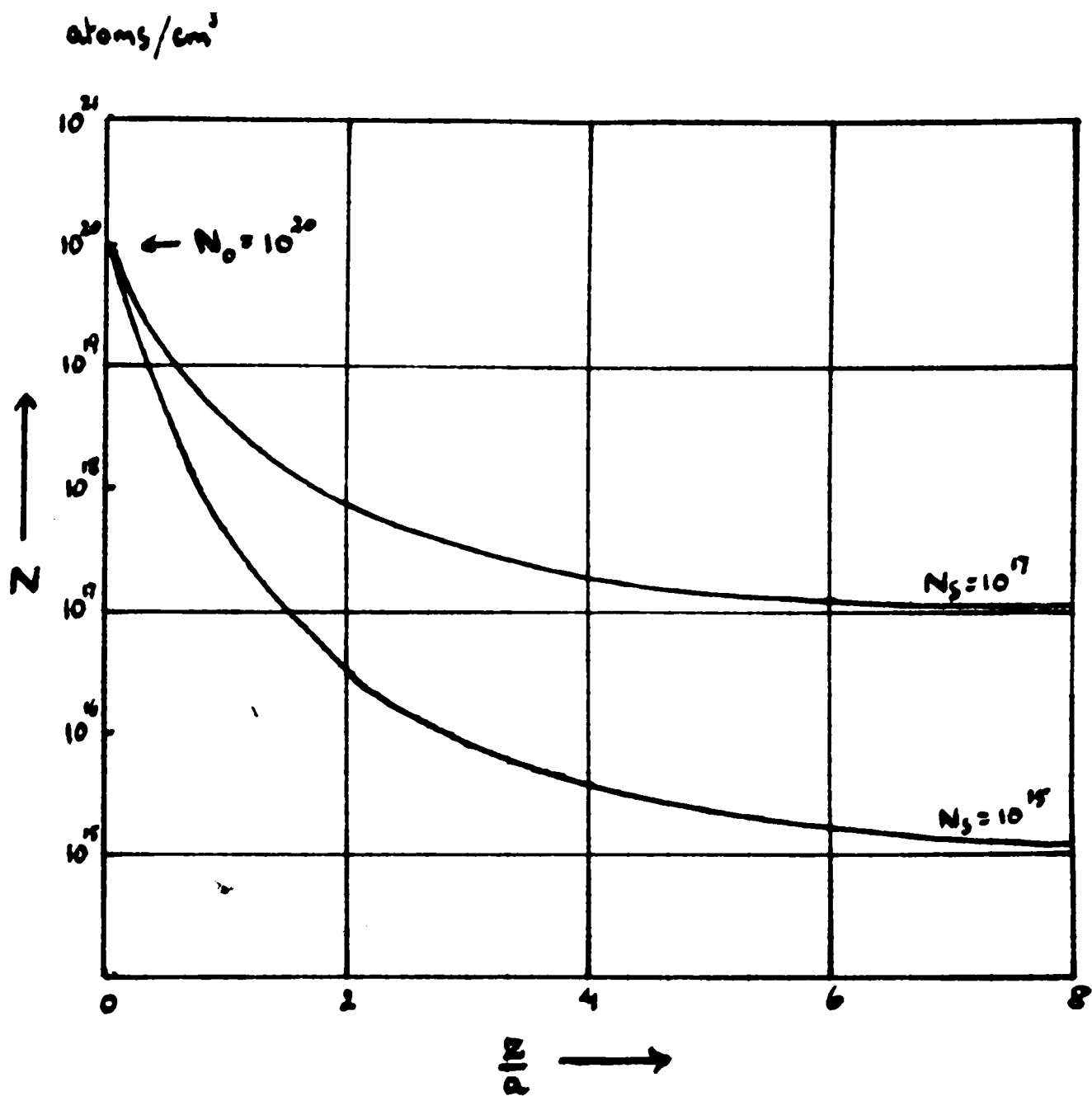


FIG. 14. CONCENTRATION OF NEUTRAL ATOMS N AS A FUNCTION OF $\frac{r}{a}$ ON THE AXIS $r \geq 0$.

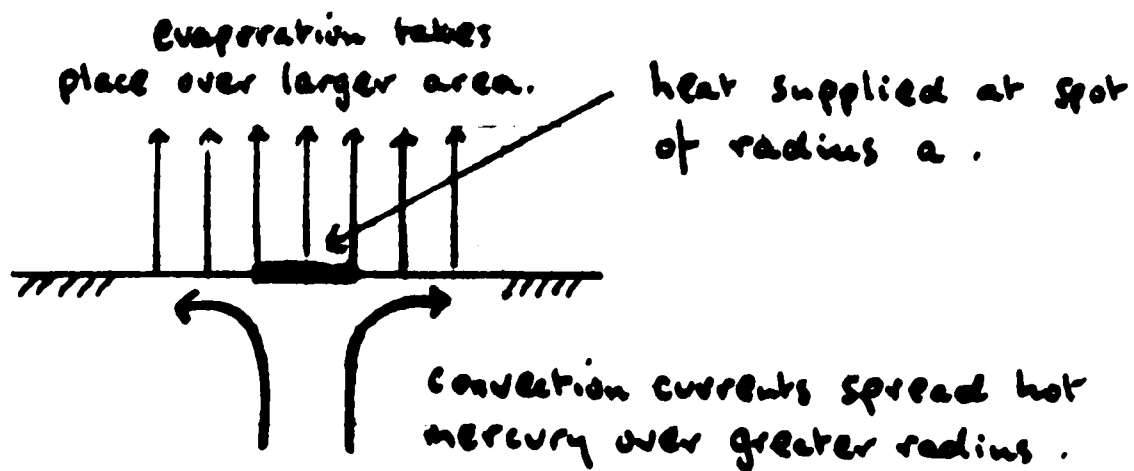


FIG. 15. CONVECTION CURRENTS IN MERCURY CATHODE.

of radius a on the surface of a liquid and a density N_s exists at infinity. In order to maintain the density N_0 at the cathode surface a net transfer of molecules must take place from the liquid to the vapour to replace those lost by diffusion to infinity. The net transfer will now be derived.

(4.3) The net evaporation.

The normal flow per unit area from the plane $z = 0$ is, from

(4.3) and (4.6)

$$f_z = - \frac{A}{N} \cdot \left(\frac{\partial N}{\partial z} \right)_{z=0} \quad (4.17)$$

From (4.8) this becomes

$$f_z = - A \left(\frac{\partial Y}{\partial z} \right)_{z=0} \quad (4.18)$$

Differentiating (4.10) with respect to z and putting $z = 0$ we have

$$\left(\frac{\partial Y}{\partial z} \right)_{z=0} = - \frac{2Y_0}{\pi} \int_0^\infty J_0(\lambda r) \sin \lambda a \cdot d\lambda \quad (4.19)$$

On evaluating the integral⁽¹⁰⁵⁾ we have

$$\begin{aligned} \left(\frac{\partial Y}{\partial z} \right)_{z=0} &= - \frac{2Y_0}{\pi (a^2 - r^2)^{1/2}} \quad \text{for } r < a \\ &= 0 \quad \text{for } r > a. \end{aligned} \quad (4.20)$$

The total normal flow is therefore given by

$$F = - \int_0^a 2\pi A \left(\frac{\partial \gamma}{\partial z} \right)_{z=0} r dr. \quad (4.21)$$

that is

$$F = 4AY_0 \int_0^a \frac{r dr}{(a^2 - r^2)^{1/2}} \quad (4.22)$$

On evaluating the integral we have

$$F = 4AY_0 a \quad (4.23)$$

From (4.2) and (4.3)

$$A = \frac{\lambda N \bar{c}}{3} \quad (4.24)$$

therefore we can finally write the equation for the net evaporation as

$$F = \frac{4(\lambda N) \bar{c}}{3} a \ln \left(\frac{N_0}{N_s} \right) \quad (4.25)$$

For mercury vapour $(\lambda N) = 6.10^{13} \text{ cm}^{-2}$ (see Appendix 1) and if the temperature is of the order 1000°K , $\bar{c} = 4.10^4 \text{ cm/sec}$. If $j = 10^5 \text{ A/cm}^2$ the radius $a = 4.10^{-3} \text{ cm}$ for a 5A arc spot.

(4.25). Taking $N_0 = 10^{20}$ we have, from (4.25)

$$F = 3.0 \times 10^{16} \text{ atoms/coulomb for } N_s = 10^{15}$$

$$F = 1.8 \times 10^{16} \text{ atoms/coulomb for } N_s = 10^{17}$$

Comparing the values of F calculated from equation (4.25) with

the measured rate of evaporation of 10^{18} atoms/coulomb for mercury (see p. 37) we see that the theory predicts an evaporation rate about 30-50 times smaller than is observed. The reason for this discrepancy is almost certainly due to the initial assumption that evaporation takes place only from a stationary circular area of radius a , where a is the measured radius of the cathode spot. Even if the spot is anchored there is always some random motion along the junction of the mercury and the anchor, and as the hot spot moves over the mercury it leaves behind it a warm area which will continue to evaporate after the spot has passed. The value of a which must be used in equation (4.25) is therefore an 'effective radius' of the evaporating area and may be many times larger than the current carrying area of the cathode.

The effect of convection currents in the liquid mercury must also be considered.⁽⁵⁴⁾ In conjunction with the flow of mercury representing the net evaporation, the convection currents will cause a circulation of the mercury in the vicinity of the spot as illustrated in Fig. 15. Although the discharge heats the mercury only over an area of radius a , representing the radius of the cathode spot, the figure shows how hot mercury may extend over a considerably larger radius as a result of convection. This increased radius must be used in equation (4.25).

In deriving the value of $\bar{c}$, a temperature of 1000°K was assumed. This is probably the order of the temperature of the evaporating mercury surface, but in the vapour above the cathode the temperature is certainly higher due to the elastic collisions made by the atoms

with electrons and ions which have gained energy in the cathode fall. However it would seem unlikely that the temperature would exceed 1000°K by as much as an order of magnitude, since the heat loss to the close cathode surface would then be very large. Since $\bar{c}$ is proportional to the square root of the absolute temperature it seems therefore that it might vary by a factor two or three over the region under consideration.

If we now take a value of a which is ten times the radius of the arc spot and a value of $\bar{c}$ corresponding to $T = 4000^{\circ}\text{K}$ and substitute these values in (4.25) we obtain

$$F = 6 \cdot 10^{17} \text{ atoms/coulomb for } N_s = 10^{15}$$

This is very much closer to the measured rate of evaporation.

For the arc with a copper cathode the cathode spot does not exhibit such rapid random motion as in the case of mercury, and since the cathode is solid except probably for the area immediately under the spot we should not expect convection currents to be important. For a copper cathode therefore we should not expect the evaporating area to be much greater than the current carrying area. No figures exist for the value of (λN) for Cu vapour, but as the copper atom is somewhat smaller than the mercury atom we should expect (λN) to be rather larger for Cu than for Hg, so we shall take $(\lambda N) = 10^{14} \text{ cm}^{-2}$. If $\bar{c} = 10^5 \text{ cm/sec}$ (corresponding to a temperature of 3000°K in Cu vapour) and if the effective radius of the evaporating area is taken as twice the radius of the arc spot we derive from (4.25) that for the copper arc

$$F = 2.4 \times 10^{17} \text{ atoms/coulomb for } N_s = 10^{15}$$

This is in good agreement with the measured evaporation of $2 \cdot 10^{17} \text{ atoms/}$

coulomb.

From equation (4.25) we should expect that as N_s is increased the net evaporation would decrease. As N_s appears in the logarithmic term the effect is probably small, but it has however been observed for a copper arc.⁽¹⁷⁾ Since F is proportional to the radius a , and a is proportional to the square root of the current we should expect the evaporation rate in atoms/coulomb to decrease with increasing current. It appears however from observations on mercury that the evaporation rate increases slightly with increasing current⁽¹⁴⁾ but this is probably due to the general heating of the mercury by the higher currents.

(4.4) Limitations of the simple theory.

In the foregoing simple diffusion treatment the assumption was made that the diffusion coefficient could be written

$$D = \frac{A}{N} \quad (4.3)$$

where A is a constant given by

$$A = \frac{\lambda N \bar{c}}{3} \quad (4.24)$$

In order that A should be constant in the space above the cathode, uniform temperature was assumed and in the calculation a value $T = 1000^\circ K$ was taken. We have explained that T may be larger in the vicinity of the cathode and smaller at the condensing surface, but $\bar{c}$ would vary only by a factor two or three in the region under consideration. Since in a first treatment we are primarily interested in an order of magnitude

calculation the assumption of constant temperature is a justifiable simplification, but the value 1000°K is probably an under-estimate of the temperature in the vapour.

Another consideration is the validity of equation (4.3) at large values of N , assuming constant temperature. For mercury vapour, the constant N is equal to $6.10^{13} \text{ cm}^{-2}$, so if $N = 10^{20}/\text{cm}^3$, $\lambda = 6.10^{-7} \text{ cm}$. The mean free path is thus not large compared with the size of the atom and at these high densities the use of the expressions (4.3) and (4.24), which are derived from simple kinetic theory, becomes questionable.

There is as yet no comprehensive kinetic theory of dense gases⁽¹¹⁸⁾ and no measurements appear to have been made on diffusion coefficients at large gas densities. However, we may come to some conclusions about the behaviour of the diffusion coefficient by examining the coefficient of viscosity, which has been measured at large gas pressures.⁽¹¹⁹⁾ The coefficient of viscosity η is, in the simple kinetic theory given by

$$\eta = \frac{m(\lambda N) \bar{c}}{3} \quad (4.26)$$

where m is the mass of the atom. From (4.24) we have

$$A = \frac{\eta}{m} \quad (4.27)$$

On the simple theory we should expect both A and η to be independent of density. However it is found that for nitrogen at 100 atmospheres η is about 50% greater than at one atmosphere. A should therefore show

a corresponding increase, but only by a small factor 1.5. The simple theory would therefore seem to be applicable even up to densities of the order $10^{21}/\text{cm}^3$.

The fundamental limitation of the foregoing treatment is however the boundary condition of equation (4.7):

$$N = N_0 \text{ for } 0 < r < a \text{ in the plane } z = 0.$$

From (4.11) and (4.12), putting $z = 0$, we have the distribution of vapour in the cathode plane

$$\left. \begin{aligned} N &= N_0 \text{ for } r < a \\ \frac{N}{N_0} &= \left(\frac{z}{z_0} \right)^{\frac{2}{\pi} \sin^{-1} \frac{a}{r}} \text{ for } r > a. \end{aligned} \right\} \quad (4.28)$$

In Figure 15, N is plotted as a function of $\frac{r}{a}$ for $N_0 = 10^{20}$ and $N_s = 10^{15}$.

In order that a uniform density N_0 should exist over the area of radius a and be approximately in equilibrium with the liquid surface below it, the temperature of the cathode over this area must be uniform. The value of the temperature for a given N_0 can be found by reference to Fig. 10. The temperature distribution on the cathode surface will therefore be a flat-topped curve similar in shape to the density distribution curve given in Fig. 16. A sketch of the temperature distribution is given in Fig. 17.

We see from equation (4.20) that the rate of evaporation per unit area which is required to give the density distribution of Fig. 14 is proportional to $(a^2 - r^2)^{-\frac{1}{2}}$ for $r < a$. Similarly it is shown in Appendix

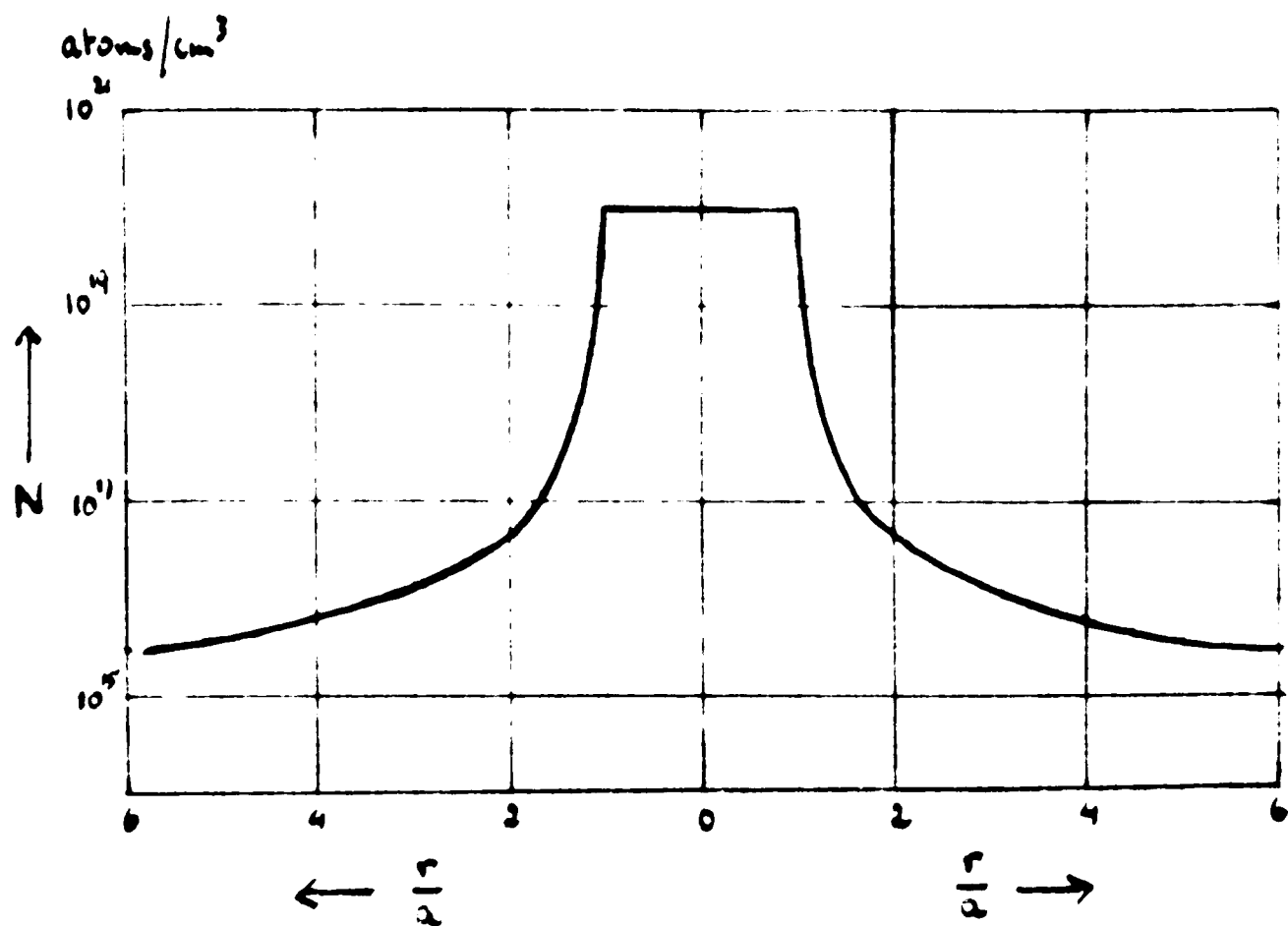


FIG. 16. CONCENTRATION OF ATOMS N AS A FUNCTION OF $\frac{r}{a}$ ON THE SURFACE $z=0$

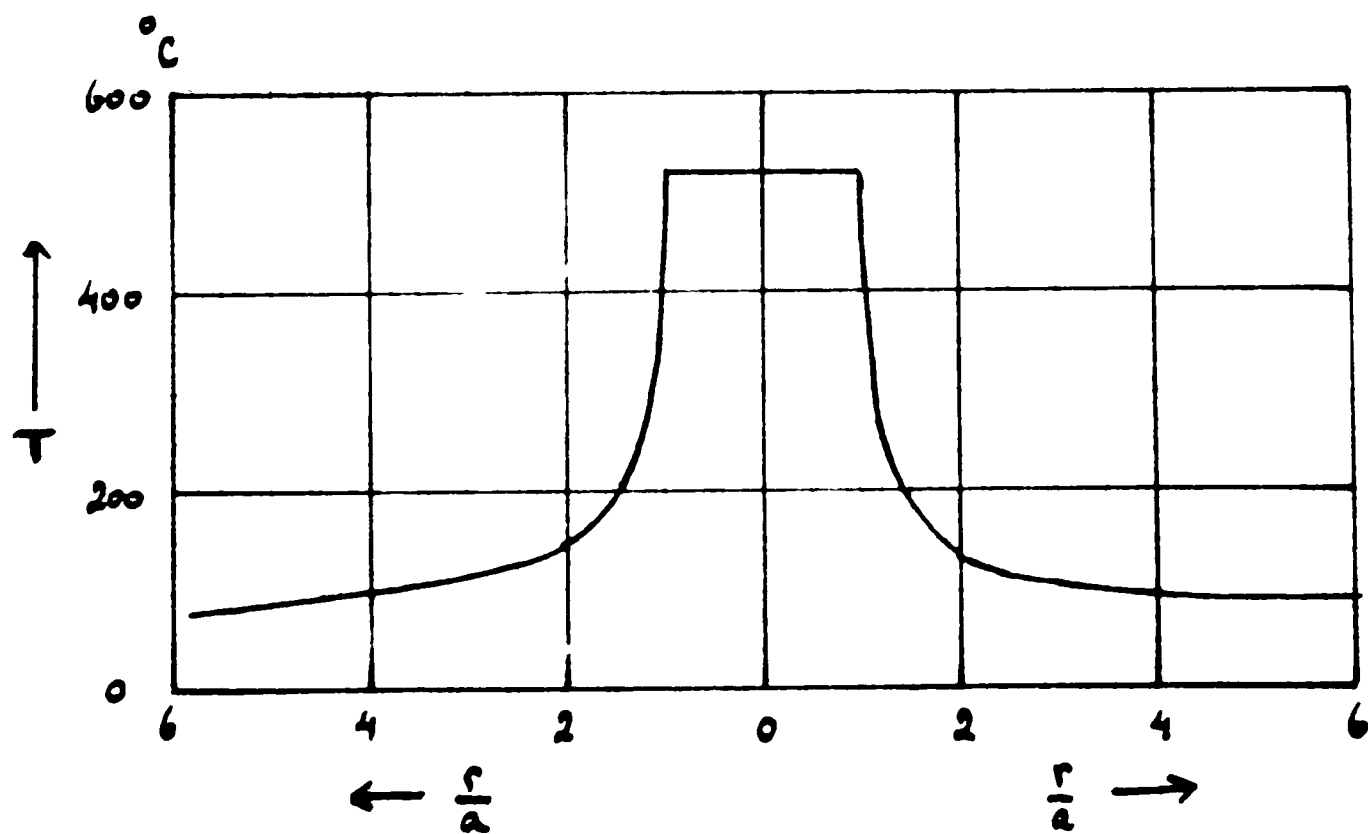


FIG 17. TEMPERATURE OF SURFACE $z=0$ AS A FUNCTION OF r/a

4 that to maintain the area of radius a at a uniform temperature in the face of losses by conduction through the cathode, heat must be supplied at a rate per unit area which also varies as $(a^2 - r^2)^{-\frac{1}{2}}$. Therefore if the density distribution $N = N_0$ for $0 < r < a$ is to be maintained, the cathode must be supplied with heat at a rate Q/cm^2 where Q is given by

$$\left. \begin{aligned} Q &\propto (a^2 - r^2)^{-\frac{1}{2}} \text{ for } r < a \\ Q &= 0 \text{ for } r > a \end{aligned} \right\} \quad (4.29)$$

In Fig.18 Q is shown as a function of r . Clearly more energy per unit area has to be supplied at $r = a$ than at $r = 0$ and it is unlikely that the discharge would supply energy to the cathode in this manner. It would rather supply more energy per unit area at the centre than at the circumference, thus leading to a rounded temperature and density distribution curve (Fig.19). This will be discussed in greater detail in the chapter on the contraction of the arc spot.

However by simplifying the problem so that it can be treated by reasonably simple mathematics we have demonstrated that strong evaporation from a small area on a cathode gives rise to a large density of atoms at the cathode surface. Transfer of material from the cathode to the condensing surface, assumed to be at infinity, takes place by diffusion, under the action of density gradients in the vapour. Calculation shows that a density of $10^{20}/\text{cm}^3$ (corresponding to a cathode temperature of 520°C) could be maintained over an area of radius $4 \cdot 10^{-3}$ cm at the cathode surface (corresponding to the area of a 5A arc spot) at the

Fig 19. MORE PROBABLE DISTRIBUTION OF N AS A FUNCTION OF r , LEADING TO A ROUNDED TEMPERATURE AND DENSITY DISTRIBUTION

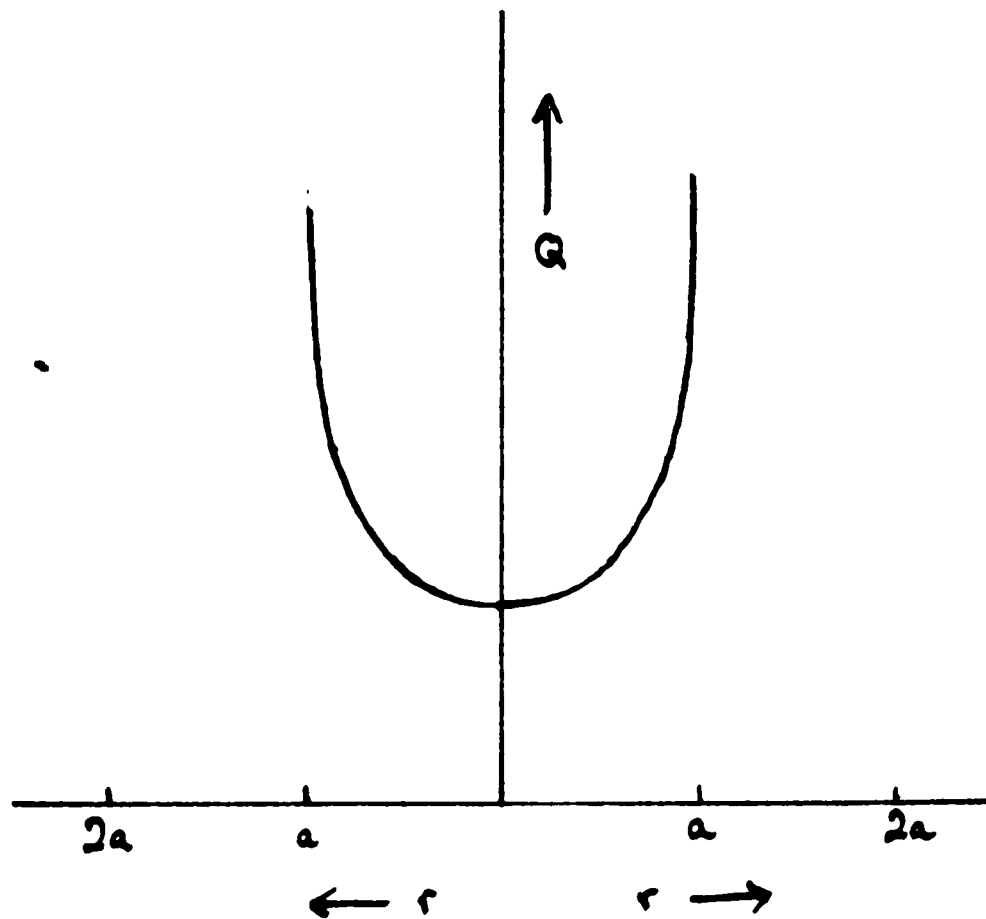


Fig. 18. HEAT INPUT Q / unit area AS A FUNCTION OF r WHICH IS NECESSARY TO MAINTAIN THE TEMPERATURE DISTRIBUTION OF FIG 17.

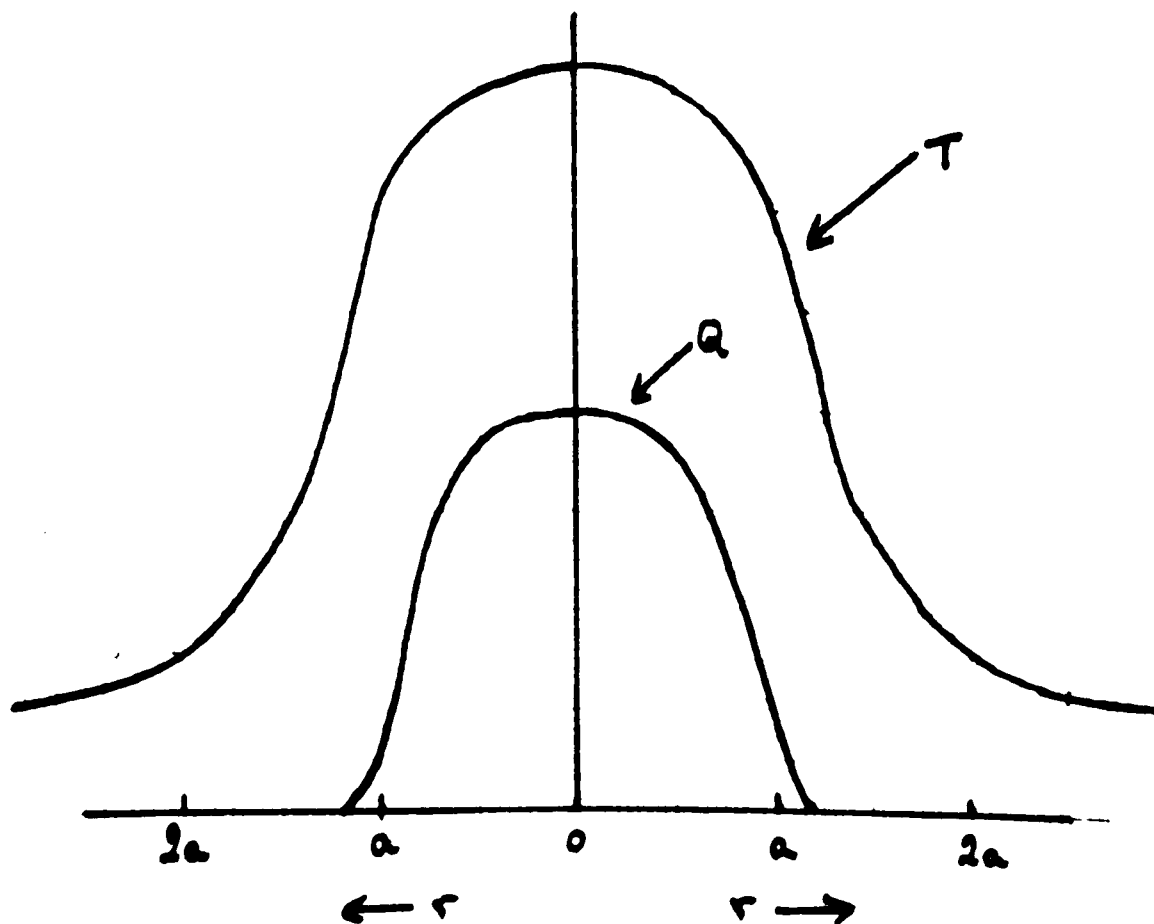


Fig 19. MORE PROBABLE DISTRIBUTION OF Q AS A FUNCTION OF r , LEADING TO A ROUNDED DISTRIBUTION CURVE OF T AS A FUNCTION OF r .

expense of a net loss of about $3 \cdot 10^{17}$ atoms/sec. Since the measured evaporation from a 5A arc spot is about $5 \cdot 10^{18}$ atoms/sec we conclude that the effective evaporating area has a radius of about $4 \cdot 10^{-2}$ cm.

It is instructive to compare the evaporation loss from this area if we would assume evaporation into high vacuum, that is if the atoms move to infinity without colliding with each other and every atom which leaves the cathode surface reaches the condensing surface. Since mercury at a temperature of 520°C is in equilibrium with a density of $10^{20}/\text{cm}^3$ of vapour, the evaporation would be given by

$$\frac{1}{4} N \bar{c} = 10^{24}/\text{cm}^2$$

The total evaporation from a circular area of radius $4 \cdot 10^{-2}$ cm into vacuum would therefore be $5 \cdot 10^{21}$ atoms/sec. that is 1000 times greater than the evaporation derived by the diffusion treatment. The assumption that the vapour immediately above the cathode is in equilibrium with the liquid mercury is therefore justified.

In the foregoing discussion we have neglected the presence of positive ions which, as they move towards the cathode represent a transfer of atoms in the opposite direction to the evaporation. If the positive-ion current is 5-20% of the total current the flow is $3 \cdot 10^{17}$ - 1.2×10^{18} ions/coulomb. If these condense on the cathode they represent a back flow of the same order of magnitude as the measured evaporation. In order to account for the measured evaporation, the true evaporation as calculated in this chapter must be larger by a factor 2. If however the ions do not condense but are reflected as neutral atoms they

will not represent a net transfer of material. Probably a proportion are reflected and the rest are condensed. If the ions arrive at the cathode with approximately thermal velocities, the number density of ions at the cathode is about $10^{14}/\text{cm}^3$, which is negligible compared with the neutral density, and so for the purposes of this simple diffusion theory we have neglected the influence of the ions.

In this chapter we have explained how a large density of vapour can exist in the immediate vicinity of the cathode spot although the density elsewhere in the arc vessel is very low. In the next chapter we shall consider the excitation and ionisation processes which occur in the dense vapour.

CHAPTER 5

EXCITATION AND IONISATION PROCESSES

5.1. The current density in the mercury arc.

A typical mercury arc discharge tube is sketched in Fig. 20. The cathode is a pool of mercury at the bottom of a glass tube about 8 cm in diameter. The anode is an iron disc and its distance from the cathode is large compared with the diameter of the tube. The tube is evacuated and contains only mercury vapour at a pressure determined by the temperature of the condensing surface of the tube. We shall take this to be about 60°C , corresponding to a pressure of about 2.5×10^{-2} mm Hg and a number density of 10^{15} atoms/cm³.

Consider an arc discharge in this tube carrying a current of 5A. The current density at the cathode spot will be 10^5 A/cm², while at large distances from the cathode the positive column will fill the tube and the current density will have fallen to about 0.1 A/cm². The cathode spot is assumed to be at the centre of the mercury pool and in Fig. 21 the current density j is plotted as a function of the distance z from the cathode on the axis of the arc. This curve is derived from purely geometrical considerations, as follows: when z is small compared with the spot diameter j will be approximately constant and equal to 10^5 A/cm² (line 1). When z is large compared with the tube diameter, j will again be constant and equal to 0.1 A/cm² (line 2). In between these extreme values we should expect j to vary as the inverse square of z .

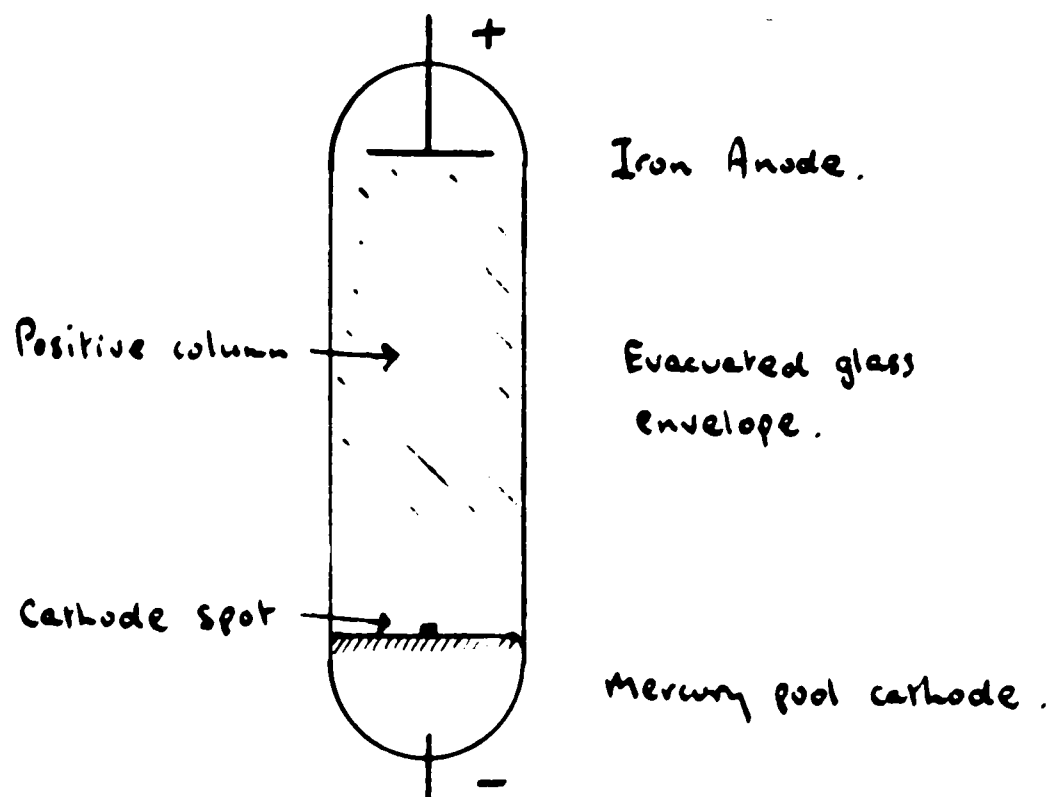


FIG 20. MERCURY ARC DISCHARGE TUBE.

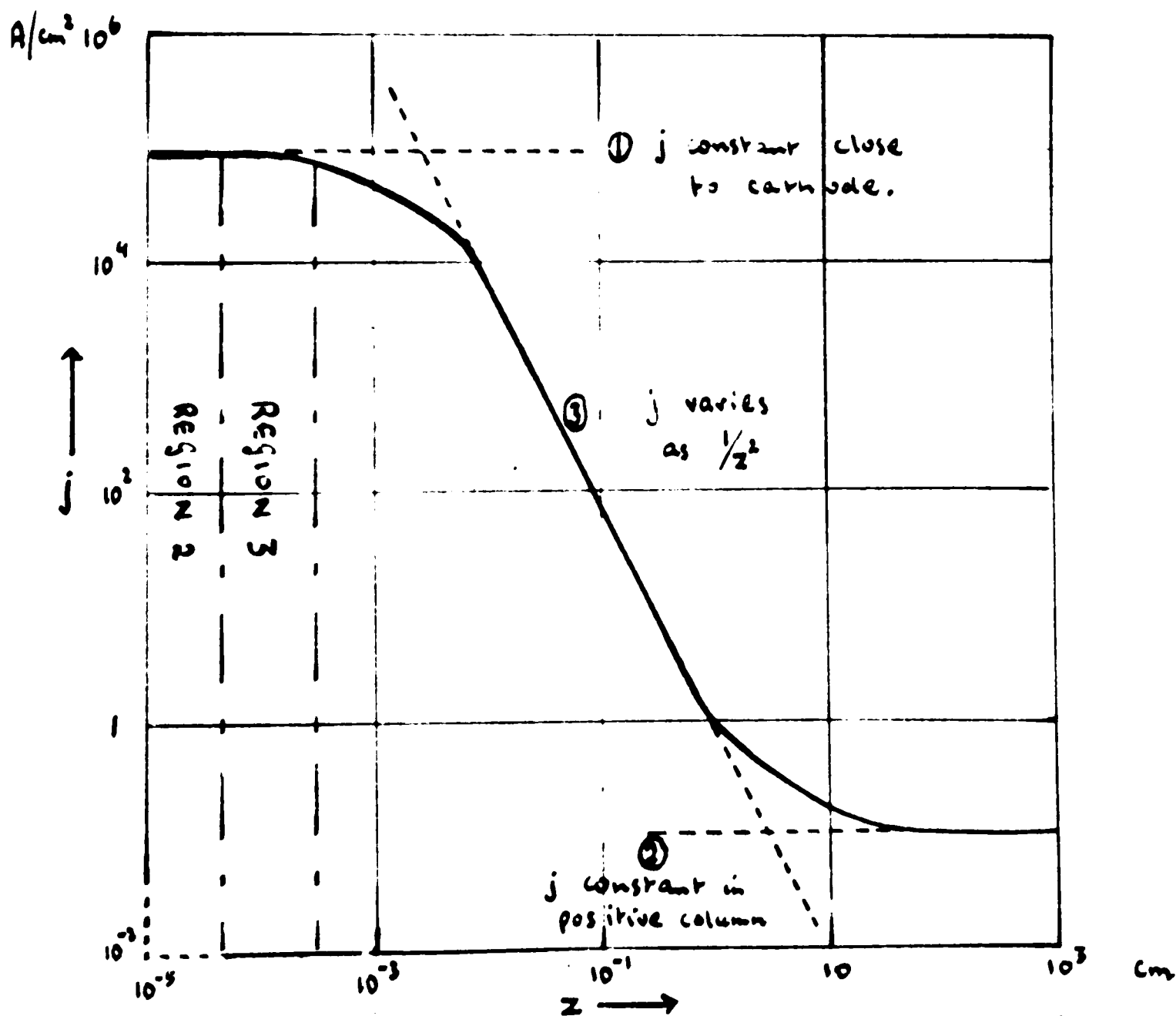


FIG. 21. CURRENT DENSITY j AS FUNCTION OF z .

Here we have

$$j = \frac{i}{2\pi z^2} \quad (5.1)$$

where the current i is taken to be 5A. The solid curve joining these three lines represents the probable variation of j with z .

In discussing processes which occur up to distances of about $5 \cdot 10^{-4}$ cm from the cathode we see from Fig. 21 that we can take $j = 10^5$ A/cm and neglect its variation with z . Furthermore we see from the results of the previous chapter (Fig. 14) that over this region we may consider the density of neutral atoms to be constant and equal to $10^{20}/\text{cm}^3$. Also the radius of a 5A arc spot is about $4 \cdot 10^{-3}$ cm, so for distances up to $5 \cdot 10^{-4}$ cm from the cathode we shall, for the purposes of this chapter consider the problem as being in one co-ordinate only (the axial, or z co-ordinate) since the extension in the z direction is small compared with the spot diameter.

5.2. Excitation in the vapour.

For the purpose of the following discussion we shall divide the discharge into a number of regions by drawing planes parallel to the cathode surface. These regions will be numbered in order from the cathode, as shown in Fig. 22.

Region 1 extends from the cathode to a plane where the potential is 5v positive with respect to the cathode. We have seen from Table 8 that this region will extend to about $2 \cdot 10^{-6}$ to $3 \cdot 10^{-6}$ cm from the cathode surface. In this region electrons from the cathode will

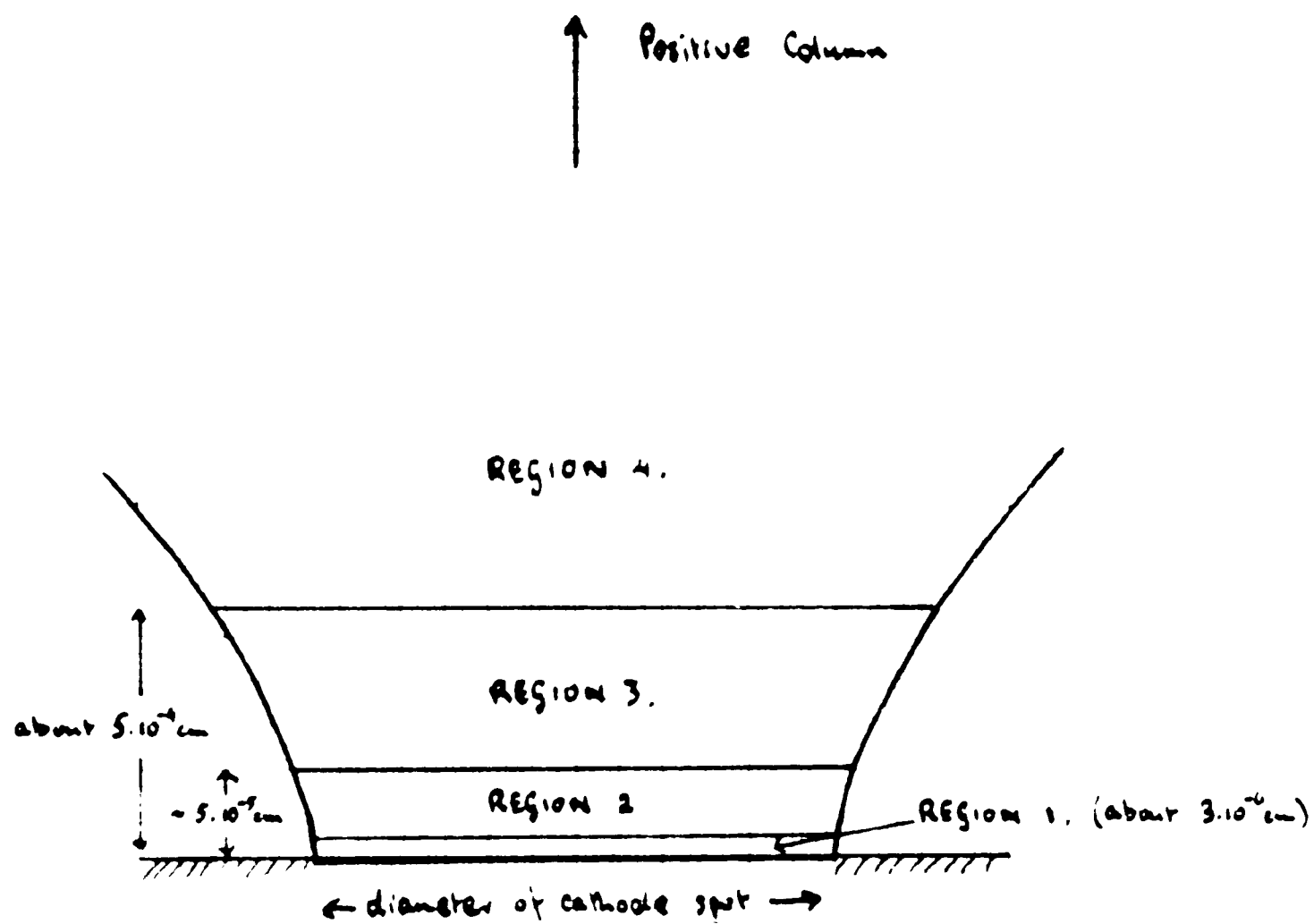


Fig 22. REGIONS OF THE ARC. [not to scale].

gain energy in the electric field of the positive ion space charge and will suffer only elastic collisions with neutral atoms. As the mass of a mercury atom is 3.6×10^5 that of the electron (see Appendix 1) at each collision an electron will lose on the average, a fraction $\frac{2m}{M} = 6.10^{-6}$ of its energy. Since the electron mean free path in mercury vapour of density $10^{20}/\text{cm}^3$ is about 10^{-6} cm (see Appendix 1) the electrons will make very few collisions as they traverse Region 1. The energy they lose by elastic impacts is therefore negligible.

Electrons which are released from the cathode (work function 4.5v) by excited atoms of energy 4.9eV will have an energy distribution with a maximum of 0.4eV. The average initial kinetic energy of an electron will therefore be about 0.2eV. The approximate initial energy distribution is sketched in Fig. 23. The electrons will then acquire approximately 5eV each as they cross Region 1. They will be scattered by elastic impacts with vapour atoms, but as the electric field is high they will be scattered preferentially in the forward direction. (135) They will therefore enter Region 2 practically as a beam, with a narrow energy distribution which is sketched in Fig. 24.

Electrons which enter Region 2 will have gained sufficient energy in Region 1 to be able to excite mercury atoms into the 3P_1 resonance state (4.9eV) or the metastable 3P_0 state (4.6eV). The probability of their exciting the metastable state is very small, and so for simplicity of the discussion we shall consider that mercury has one excited state only, the resonance state of 4.9eV. The other resonance

STATE OF MERCURY
ENERGY.

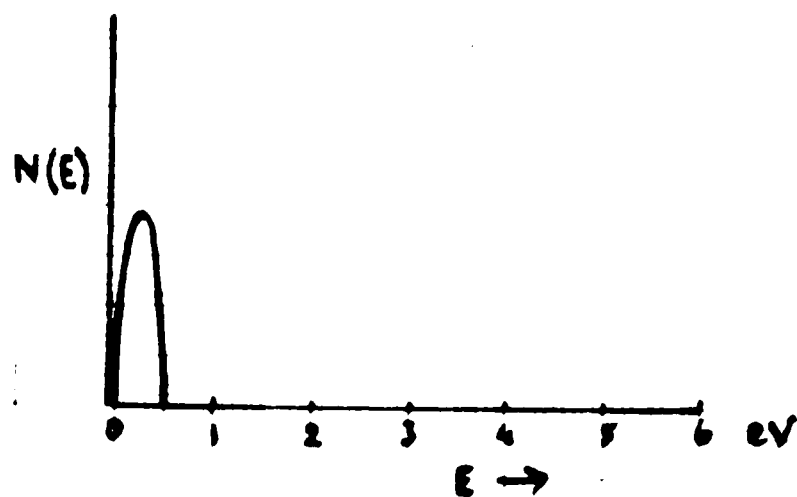


FIG. 23. ENERGY DISTRIBUTION OF ELECTRONS LEAVING CATHODE.

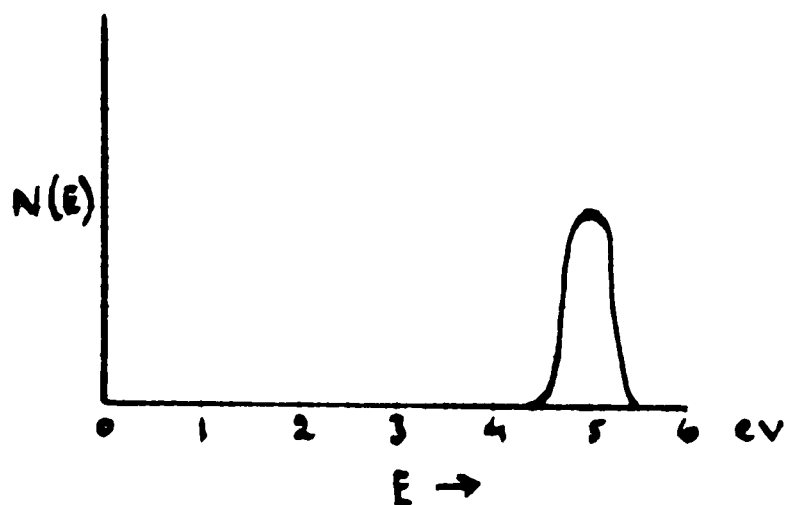


FIG 24. ENERGY DISTRIBUTION OF ELECTRONS ENTERING REGION 2.

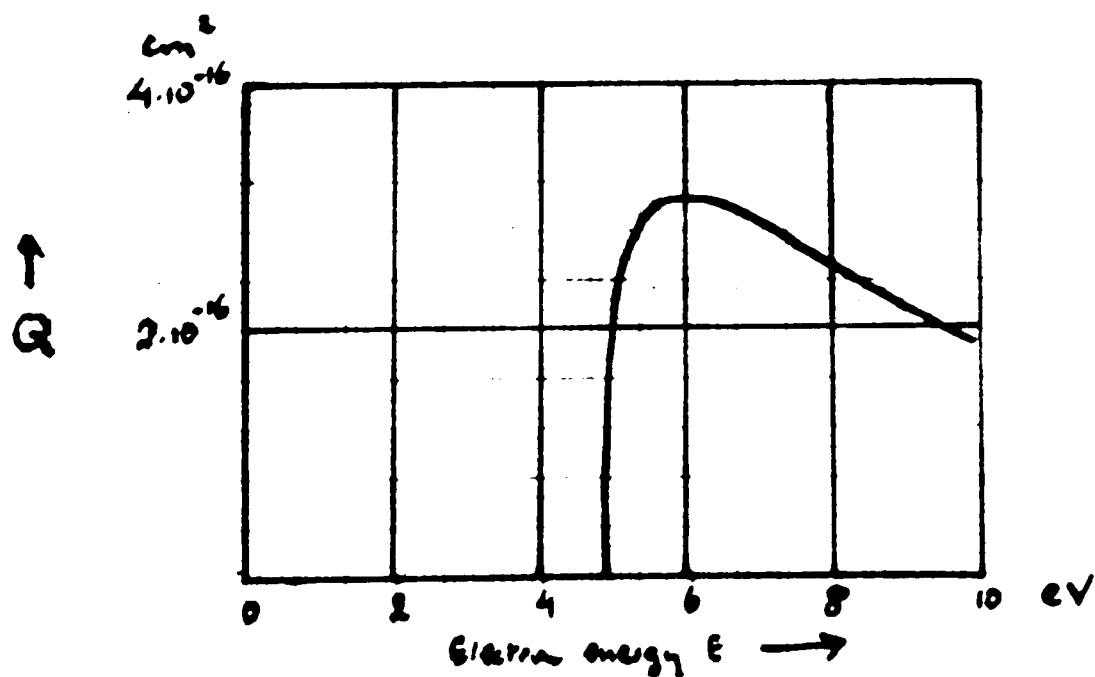


FIG 25. EXCITATION CROSS-SECTION OF 1P_1 (RESONANCE) STATE OF MERCURY AS A FUNCTION OF ELECTRON ENERGY.

state (1P_1 , energy 6.7eV) cannot be excited by electrons of only 5v energy and so will be neglected in the first discussion.

The cross-section Q for excitation of the 3P_1 state of mercury varies with the electron energy, rising from zero at 4.9eV to a maximum of $3 \cdot 10^{-16} \text{ cm}^2$ at 5.6eV and then decreasing. ⁽¹⁵¹⁻¹⁵²⁾ This is sketched in Fig.

25. For simplicity we shall approximate this curve to a simple step function of height $Q = 3 \cdot 10^{-16} \text{ cm}^2$.

Since the electrons from the cathode enter Region 2 as a beam, we can calculate that the fraction f of cathode electrons which have excited within a distance z' after entering Region 2 is given by

$$f = 1 - e^{-NQz'} \quad (5.2)$$

where N is the number density of neutral atoms. With $N = 10^{20}/\text{cm}^3$ $Q = 3 \cdot 10^{-16} \text{ cm}^2$ we find that $f = 80\%$ when $z' = 5 \cdot 10^{-5} \text{ cm}$.

Since the electrons are probably not truly a beam but have a certain amount of random motion, this simple treatment leads for a given z' to a value of f which is too small. We may therefore conclude that practically every cathode electron makes one exciting collision within $5 \cdot 10^{-5} \text{ cm}$ after entering Region 2. We now define Region 2 as the region in which all the electrons from the cathode make their first exciting collision, and take its extent as $5 \cdot 10^{-5} \text{ cm}$ (Fig. 22).

Having made inelastic collisions in Region 2, the electrons will move on into Region 3 with reduced energy. We define Region 3 as extending from Region 2 to a plane where the potential is 8 volts positive with respect to the cathode: that is to say, the measured

cathode fall occurs over Regions 1-3. We shall show later that Region 3 for a 5A arc probably extends to a distance of about $5 \cdot 10^{-4}$ cm from the cathode, and as we have already stated, we may consider the current density as constant up to this distance. In Region 4 the current density decreases with increasing z and we define this region as joining Region 3 to the positive column (Figs. 21 and 22).

5.3. Ionisation in the Vapour.

Electrons entering Region 2 have energies of only about 5eV and so are not capable of ionising a vapour atom (ionisation potential 10.4eV) at a single collision. Ionisation may occur however by two possible processes.

1. An electron with energy $> 5.5\text{eV}$ may make an inelastic collision with an atom in the 3P_1 state (4.9eV), forming an atomic ion Hg^+ (ionisation energy 10.4eV). (Fig. 26).

2. Two atoms in the 3P_1 state, or one in the 3P_1 state and another in the 3P_0 state, may collide and a molecular positive ion Hg_2^+ (ionisation energy = 9.65eV) may be formed (Fig. 27).

The first process requires slightly more energy than the excitation of an atom from the ground state to the resonance state. On the other hand the cross-section is likely to be much larger, on account of the increased 'size' of the excited atom. No measurements are available on this process in mercury vapour, but it appears that in hydrogen the cross-section for ionisation of an excited atom rises steeply from

Fig 28. Cross-sections for Excitation, Ground State, and Ionisation as a Function of Electron Energy

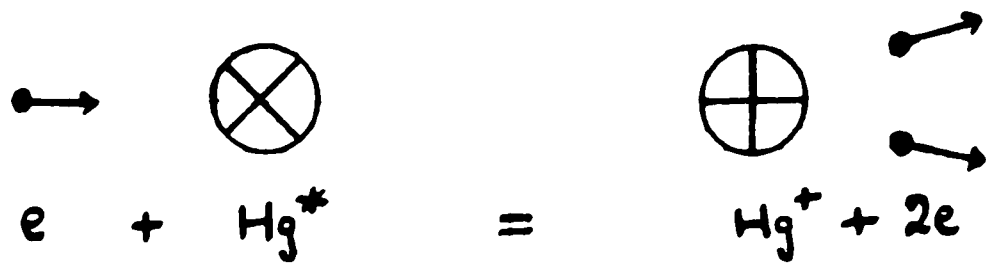


FIG 26. IONISATION OF AN EXCITED ATOM.

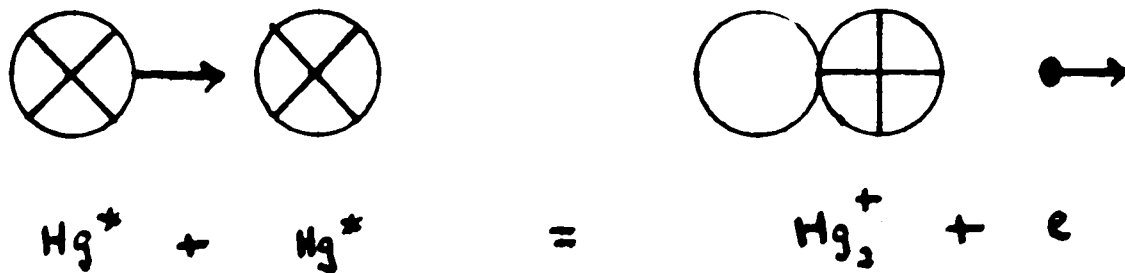


FIG 27. IONISATION BY COLLISION OF EXCITED ATOMS.

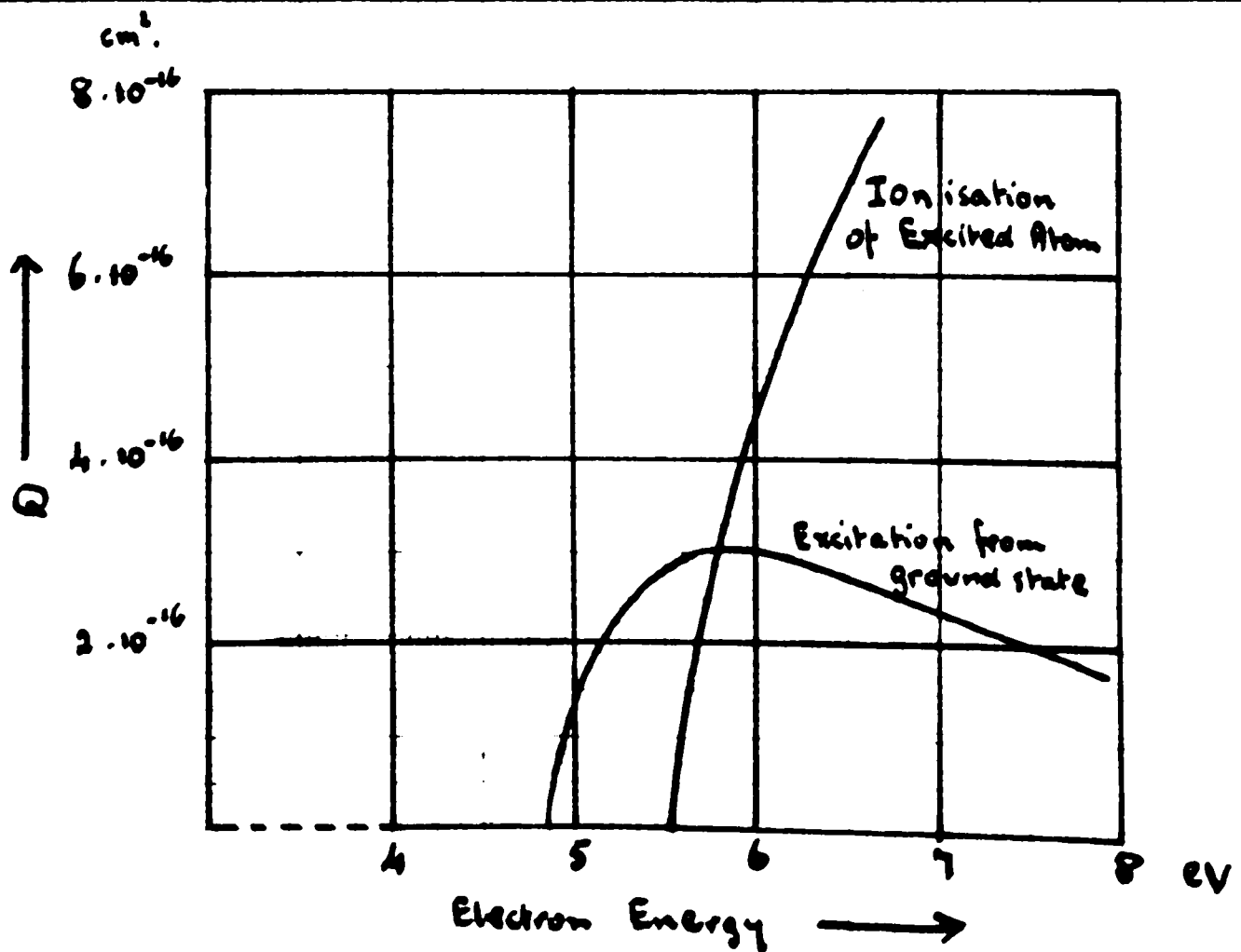
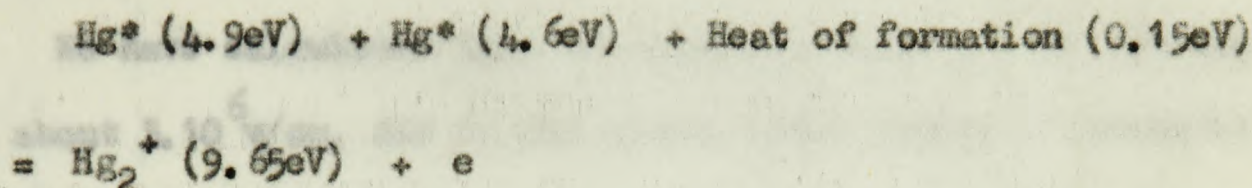


FIG 28. CROSS-SECTIONS FOR EXCITATION OF 3p, STATE FROM GROUND STATE, AND IONISATION OF 3p, STATE AS A FUNCTION OF ELECTRON ENERGY.

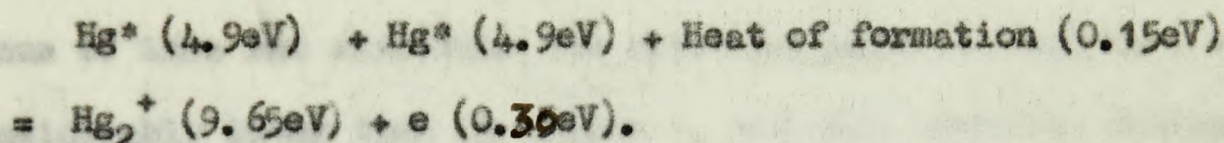
zero at the critical energy to a maximum which occurs for an electron energy on few eV above the critical energy. This maximum cross-section in hydrogen is about ten times the maximum cross section for excitation from the ground state.⁽¹³⁰⁾ We may therefore make an estimate of the probable cross-section for ionisation of an excited mercury atom: this⁽¹⁵³⁻¹⁵⁵⁾ has been done in Fig.28, where the cross-section for excitation of the 3P_1 state from the ground state is also shown. It can be seen from the figure that in the energy range 5-6eV the cross-section for excitation from the ground state is larger than the cross-section for ionisation of an excited state, but above 6eV the cross-section for ionisation is the greater.

The second process, ionisation by collisions between excited atoms, has been investigated by a number of workers.⁽¹³¹⁻¹³³⁾ It is found that if resonance light ($\lambda 2537$) from a mercury lamp is shone into mercury vapour, ions and electrons are produced in the vapour. Since the light can only produce excited atoms of quantum energy 4.9eV, some stepwise process must operate to produce ions in the vapour. The ionisation potential of the atom is 10.4V, so two excited atoms would not have quite sufficient energy to ionise one of them. However the mercury molecular ion Hg_2^+ is thought to have an ionisation potential of only about 9.65eV, and it has been shown experimentally that the process which probably occurs at low pressure is a collision between a 3P_1 resonance atom (4.9eV) and a 3P_0 metastable atom (4.6eV). The combined energy of these two excited atoms, together with the heat of formation of a mercury molecular ion (0.15eV, see ref.134) will be just sufficient to ionise the

molecule. The reaction is



At high densities, collisions between $^3\text{P}_1$ (resonance) atoms are likely, and this will result in ionisation of the molecule with a small excess of energy which is carried away by the electron. This reaction is shown in Fig. 27:



Since there is an almost exact energy resonance for this process there is a high probability that it will occur when two excited atoms collide. There have been no measurements of the probability but we shall take it to be of the order 0.1 - 1.

It is now apparent that whenever conditions exist where mercury atoms are excited into their lowest excited states, ionisation is likely to occur by cumulative processes. Such cumulative processes are most likely to occur at high current densities, since then the excited atoms have no time to radiate their energy before they are involved in an ionising collision. It should be noted however, that in the majority of other metal vapours, such as copper, the lowest excited states have energies less than half the ionisation potential, and so only the higher excited states will be able to produce ionisation by cumulative processes.

5.4. The axial electric field.

We have calculated that the electric field at the cathode surface is about $3 \cdot 10^6$ v/cm, due to the strong space charge of positive ions in Region 1.

From equations (1.3) and (2.4) we find that the field in Region 1 decreases with increasing z . In Region 2 there will be excitation, and hence production of ions by cumulative processes as described in the previous section. Since this will tend to equalise the concentrations of ions and electrons the electric field in Regions 2 and 3 will be considerably lower than in Region 1, and will probably decrease slowly with increasing z .

The electric field in Region 2 must fulfill the condition that the voltage drop across the region is only 5 volts. There we have

$$\int_0^x X dz = 5 \text{ volts} \quad (5.3)$$

where x is the extension of Region 1, i.e. about $3 \cdot 10^{-6}$ cm.

Also the field in Regions 1 - 3 must fulfill the condition that the total voltage drop across them is the cathode fall of 8 volts. Therefore we have

$$\int_0^d X dz = 8 \text{ volts} \quad (5.4)$$

where d is the extension of Region 3 from the cathode i.e. about $5 \cdot 10^{-4}$ cm.

The variation of the axial electric field X with the distance from the cathode z is represented roughly by Fig. 29, in which the boundaries of

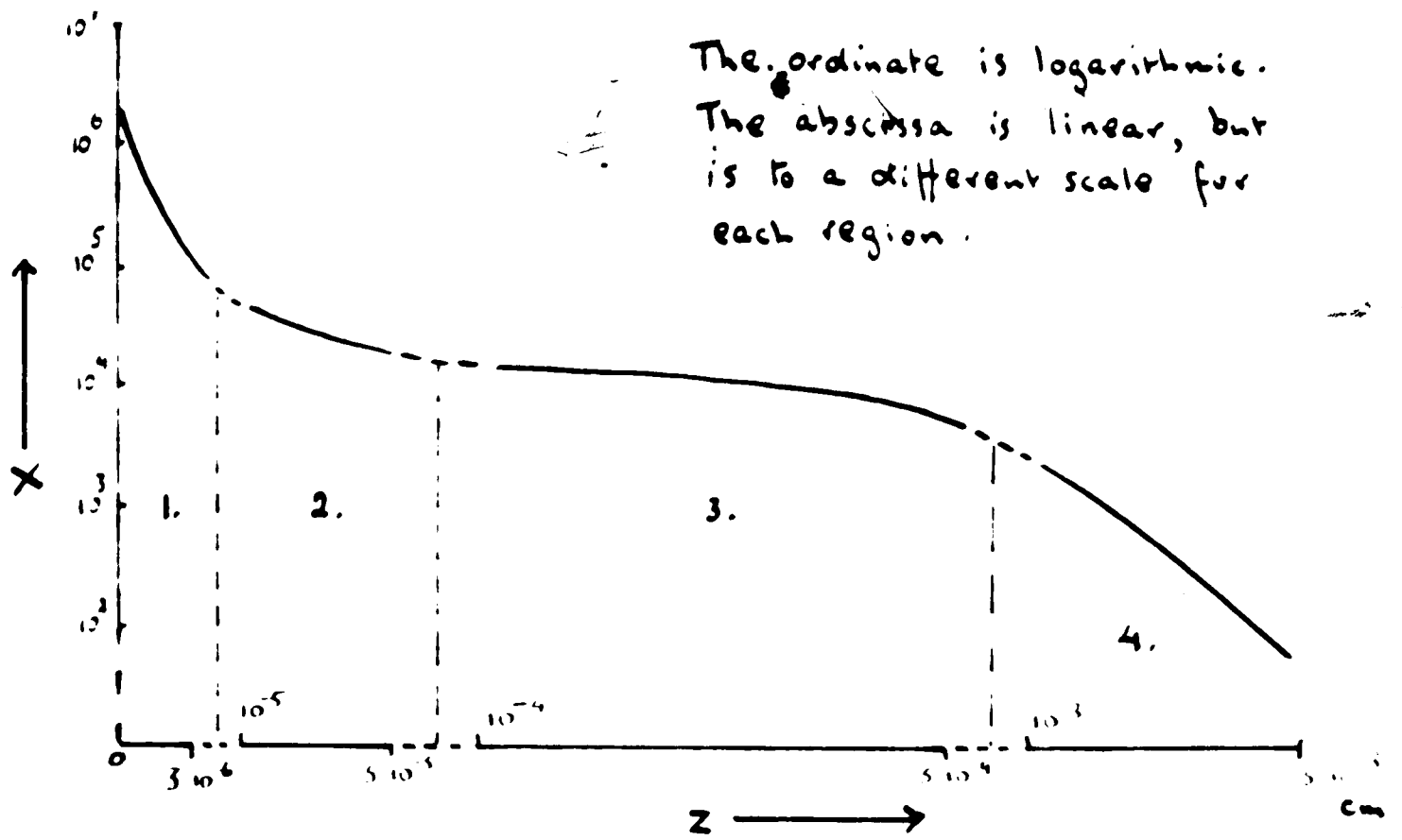


FIG 29. AXIAL ELECTRIC FIELD X AS A FUNCTION OF z .

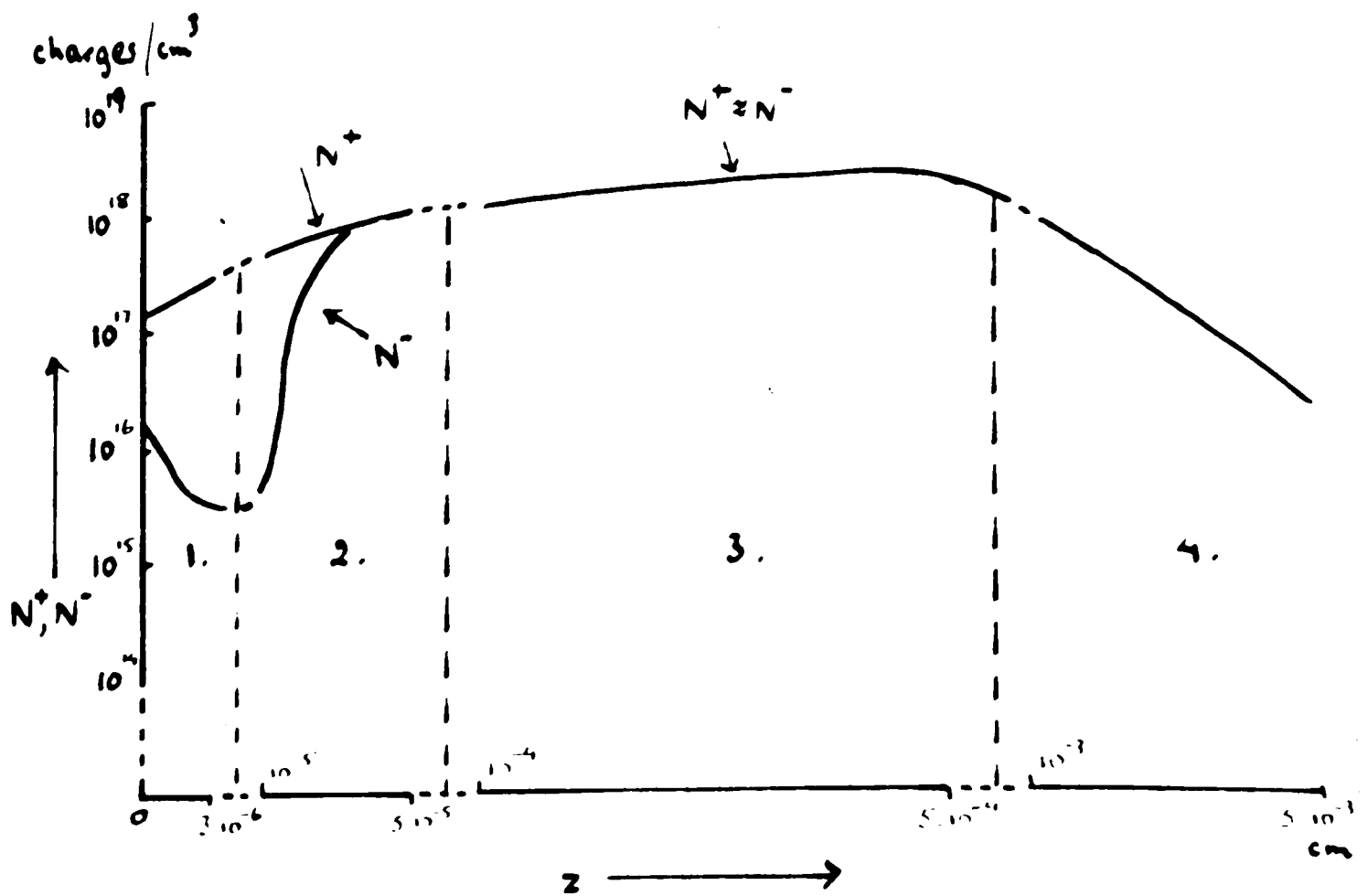


FIG 30. DENSITY OF CHARGED PARTICLES AS A FUNCTION OF z .

the various regions are marked. The integrals in equations (5.3) and (5.4) represent areas under the curve. Thus we can deduce that the average electric field in Region 1 is about 10^6 v/cm while the average electric field in Regions 2 and 3 is about 10^4 v/cm.

In Region 4 the discharge is expanding radially and j decreases with increasing z , probably according to an inverse square law (equation 5.1). For the purpose of an approximate calculation, let us assume that the field X in Region 4 also varies as the inverse square of z . Then the potential drop across Region 4 is given by

$$V_4 = \int_d^g \frac{X_3 d^2}{z^2} dz \quad (5.5)$$

where X_3 is the field at the boundary of Regions 3 and 4, d is the distance of this boundary from the cathode and g is the extension of Region 4. If we now put $d = 5 \cdot 10^{-4}$ cm, $g \sim 5$ cm (see Fig. 21) and take $X_3 = 5 \cdot 10^3$ to 10^4 V/cm we have

$$V_4 = 2.5 \text{ to } 5 \text{ volts}$$

It would therefore appear that, the cathode fall of 8v having fallen off in Regions 1-3, a further potential difference amounting to a few volts would probably fall off over Region 4. This is in agreement with the observation that the total discharge potential of a mercury arc of the type shown in Fig. 20 is usually about 10 - 15v. In the positive column the electric field is very low (probably 0.1 - 1.0 V/cm) and so unless the column is very long it only makes a small contribution to the total discharge potential.

5.5. The distribution of ions and electrons.

An estimate of the density of charged particles will now be made. The distribution of the electron density N^- and the ion density N^+ is plotted as a function of z in Fig. 30 which will be explained in the following paragraphs. The current density of electrons j^- is given by

$$j^- = N^- e v_d^- \quad (5.6)$$

where v_d^- is the drift velocity of the electrons in the direction of the electric field.

In Region 1 the electrons move as a beam and so their velocity at any point can be derived from the energy they have acquired by the relation (see Appendix 1).

$$v = 6.10^7 V^{1/2} \text{ cm/sec} \quad (5.7)$$

where V is the potential through which they have fallen.

At the cathode the electrons will be emitted with an average energy of about 0.2eV corresponding to a velocity of about 3.10^7 cm/sec. In equation (5.6) putting $j^- = 10^5 \text{ A/cm}^2$ (since, as we have discussed in the previous chapters the current in the arc is carried mainly by electrons) and $v_d^- = 3.10^7 \text{ cm/sec}$, we derive $N^- = 2.10^{16} / \text{cm}^3$ at the cathode.

As the electrons enter Region 2 they will have velocities of about $1.4 \times 10^8 \text{ cm/sec}$, corresponding to an energy of 5eV. With this velocity in equation (5.6) we derive that $N^- = 4.10^{15} / \text{cm}^3$ at the boundary of Regions 1 and 2.

In Region 2 the electrons will lose nearly all their energy in inelastic collisions, and so will be slowed down. Since j^- does not vary with z in this region, we see from equation (5.6) that N^- must therefore rise sharply. Since the electric field in Regions 2 and 3 is much lower than in Region 1 the electrons will now move as a swarm rather than as a beam. They will have a mean random velocity $\bar{v}$ corresponding to their average energy, and a drift velocity $\bar{v}_d$ in the field direction which will be much lower than $\bar{v}$.

Since the probability that an electron will excite the 4.9 volt state of mercury is zero for electrons of exactly 4.9eV energy, but rises steeply with increasing electron energy, (see Fig. 25) it is likely that the majority of electrons which excite atoms in Region 2 will have a small excess of energy above 4.9eV. This excess energy will be retained by the electrons after making inelastic collisions: we shall take it to be about 0.5eV, which will give the electrons a mean random velocity of about $4 \cdot 10^7$ cm/sec as they pass on into Region 3.

The drift velocity of an electron swarm in the field direction is given by

$$\bar{v}_d = \frac{e}{m} \cdot \frac{\lambda_e}{\bar{v}} \cdot X \quad (5.8)$$

where λ_e is the mean free path for electron-atom collisions. If the density of neutral atoms is $10^{20}/\text{cm}^3$ we see from Appendix 1 that for electrons of 0.5eV energy $\lambda_e \approx 5 \cdot 10^{-7}$ cm. With this value in equation (5.4) and $\bar{v} = 4 \cdot 10^7$ cm/sec we derive

$$\bar{v}_d = 20X \quad \text{where } X \text{ is in V/cm} \quad (5.9)$$

As we have already discussed, X in Region 3 is probably about 10^4 V/cm. Hence $v_d^- \approx 2 \cdot 10^5$ cm/sec and from (5.3) $N^- \approx 3 \cdot 10^{18}$ /cm³.

In region 3, X probably decreases as z increases (Fig. 29) so N^- will increase with increasing z . In Region 4 however the discharge is expanding and j^- and X decrease as z increases. (see Figs. 21 and 29). Since however the density of neutral atoms in this region decreases rapidly with increasing z (see Fig. 14) λ_e will increase rapidly with increasing z . Hence from equations (5.6) and (5.8) we should expect N^- to decrease also until it reaches the low uniform value of the electron density in the positive column. N^- will therefore go through a maximum, and we shall show later that it occurs at the boundary of Regions 3 and 4, where the potential is 8v positive with respect to the cathode.

This approximate account of how N^- varies with z is illustrated in Fig. 30. In order that the electric field shall not be inordinately large N^+ must be nearly equal to N^- in Regions 3 and 4. In Region 1 however, there is a positive space charge, so N^+ exceeds N^- . Since they make collisions with vapour atoms in Region 1 the ions will arrive at the cathode with appreciably less energy than the 5 volts through which they have fallen. If we take their energy as 1eV, corresponding to a velocity of 10^5 cm/sec we calculate that if the ion current density in Region 1 is about 10^4 A/cm², then $N^+ = 6 \cdot 10^{17}$ /cm³ at the cathode. Since X decreases as z increases, N^+ will increase across Region 1. The probable variation of N^+ with z is included in Fig. 30. In Region 1 we have taken the ion current to be about 10-20% of the total current. In Region 3 since $N^+ \approx N^-$, the ion current will be only about 0.2% of the

total (see p.10) and so in this region we may regard the current as carried by electrons only.

The simple expression for the electron drift velocity given by equation (5.8) is strictly applicable only to conditions of low current density. For the large densities of charged particles which occur near the cathode of an arc the simple treatment must be modified. The mean free path for the electrons can no longer be taken as being simply for electron-atom collisions but the mean free path for electron-ion collisions must be considered. This is given by Gvosdover's equation. (120)

$$\frac{1}{\lambda_e} = \frac{\pi N^+ e^4}{2 k^2 T_e^2} \ln \frac{1.5 k T_e}{e^2 (N^+)^{1/3}} \quad (5.10)$$

If we take $kT_e = 0.5\text{eV}$, $N^+ \approx 3.10^{18}/\text{cm}^3$ we obtain from this equation that in Region 3, $\lambda_e \approx 6.10^{-7}\text{cm}$. It appears then that the mean free paths for electron-atom collisions and electron-ion collisions are in this case approximately equal. This is because there is a high density of neutral atoms as well as a high density of charged particles. The use of the simple classical treatment given above is therefore justifiable to obtain at least the orders of magnitude of the quantities involved. In the above treatment we have assumed that the electrons are in thermal equilibrium with the ions and the neutral atoms.

5.6. The electron energy distribution in Region 3.

When a swarm of electrons moves in an electric field through a gas, the electrons reach a steady state energy distribution, gaining energy in the electric field and handing it over to the atoms by collisions. In discharges where the density of charged particles is low, the electrons do not interact with each other and so no electron in the energy distribution has an energy greater than eV' , where V' is the potential difference through which the swarm has moved. If this were the case in Region 3, no electron which has excited an atom in Region 2 could gain further energy greater than $3eV$, and so no electron from the cathode could produce more than one excited atom as it fell through the $3V$ of the cathode fall. However, on account of the high density of electrons in Region 3 we should expect that the electrostatic fields of the electrons would interact with each other and that the electrons would exchange energy by means of 'collisions'. In this way, although the swarm has moved through only 3 volts, electrons with energies greater than 3 volts will be found in the energy distribution.

A theory of electron interaction in a region where the concentration of positive and negative charges is approximately equal has been developed by Davydov.⁽¹²¹⁾

In the steady state electrons gain energy from the electric field and deliver it to the atoms by elastic and inelastic collisions. The time interval t_e in which an electron can share its energy with another electron, i.e. a 'collision time' for electron-electron collisions,

is compared with the time t in which an electron can hand over the same amount of energy to the atoms. The ratio t/t_e is called γ .

If $\gamma \ll 1$, $t_e \gg t$ and so the energy exchange between electrons has little effect on the energy distribution of the electrons, which is then obtained simply by consideration of electron-atom collisions. If $\gamma \gg 1$ however, the electrons exchange energy with each other before they can hand it over to the atoms. In this case a Maxwellian energy distribution will be set up and in the steady state the electron temperature T_e is determined by the condition that all energy gained by the electrons from the field must be handed over to the atoms in collisions.

Davydov derives the equation

$$\gamma = \frac{\pi e^2 N^-}{X^2 \lambda_e} \ln \left(\frac{\epsilon r}{e^2} \right) \quad (5.11)$$

where N^- is the electron concentration

X the electric field

λ_e the electron mean free path

ϵ the mean electron energy

r the Debye Radius, given by

$$\frac{1}{r} = \left(\frac{8\pi N^- e^2}{k T_e} \right)^{\frac{1}{2}} \quad (5.12)$$

If we substitute in equations (5.11) and (5.12) the values $X = 10^4$ v/cm; $N^- = 3 \cdot 10^{18}$ /cm³; $\lambda_e = 5 \cdot 10^{-7}$ cm; $\epsilon = 0.5$ eV = $k T_e$ we find

$$\gamma > 10^4$$

There is little doubt therefore that under the conditions which exist in Region 3 there will be strong electron-electron interaction and the electrons will have a Maxwellian energy distribution. Thus although the electron swarm moves through a potential difference of only $3v$, electrons will be found in the distribution which have energies greater than $5v$ and are therefore capable of exciting an atom.

If N^- is the electron density the number of collisions between electrons and atoms per cm^3 and sec is given by

$$n = \frac{N^- \bar{v}}{\lambda_e} \quad (5.13)$$

where $\bar{v}$ is the mean random velocity of the electrons and λ_e is the mean free path for electron-atom collisions.

If f^* is the fraction of electrons with energy $> V^*$ and P^* is the probability that an electron with energy greater than V^* will excite an atom upon collision, we have the number of exciting collisions per cm^3 and sec

$$n^* = n f^* P^* \quad (5.14)$$

If the average energy of the electron swarm in Region 3 is about 0.5eV , the fraction f^* with energy greater than 5eV will, if the distribution is Maxwellian, be $2 \cdot 10^{-4}$ (see ref.122).

If now in equations (5.13) and (5.14) we put $\lambda_e = 5 \cdot 10^{-7} \text{cm}$, $\bar{v} = 4 \cdot 10^7 \text{cm/sec}$ $f^* = 2 \cdot 10^{-4}$ $P^* = 2.5 \times 10^{-2}$ (see Appendix 1) and $N^- = 3 \cdot 10^{18} / \text{cm}^3$ we have

$$n^* \approx 10^{27} / \text{cm}^3 \text{ and sec.}$$

If we assume that in the steady state the electrons lose all the energy they gain in the electric field by making inelastic collisions with atoms, we may equate the energy per cm^3 and sec taken from the field with the energy lost in inelastic collisions per cm^3 and sec. We have therefore

$$j^- X = n^* V^* \quad (5.15)$$

Putting $n^* = 10^{27}$ $V^* = 5\text{eV}$ $j^- = 10^5 \text{ A/cm}^2$ we find $X \approx 10^4 \text{ V/cm}$.

Our original estimate that the average electric field in Region 3 is about 10^4 V/cm and that the mean electron energy is about 0.5eV is therefore consistent with the assumption that the energy gained by the electrons in the electric field is entirely lost by inelastic collisions with atoms. The effect of elastic collisions must also be considered.

We have calculated that the number of inelastic collisions is $n^* = 10^{27}/\text{cm}^3$ and sec. The total number of collisions n is derived from equation (5.13) and is $2 \cdot 10^{32}/\text{cm}^3$ and sec. We have derived in Section 5.2 that an electron loses on the average a fraction $6 \cdot 10^{-6}$ of its energy in an elastic collision with a mercury atom. If the mean energy of the electrons is 0.5eV we see that the energy lost by elastic collisions is about $6 \cdot 10^{26} \text{ eV/cm}^3$ and sec. The energy lost by inelastic collisions is about $5 \cdot 10^{27} \text{ eV/cm}^3$ and sec, so to a first approximation we may consider that the electrons lose all their energy in inelastic collisions.

The rate of production of excited atoms by electron impact n^* is plotted as a function of z in Fig. 31. In Region 1, $n^* = 0$. In Region 2 each electron makes one inelastic collision in $5 \cdot 10^{-5} \text{ cm}$. As

inelastic collisions/
cm³ and sec

Ordinates are logarithmic
Abcissae are linear, but
drawn to a different scale
for Regions 1 and 2.

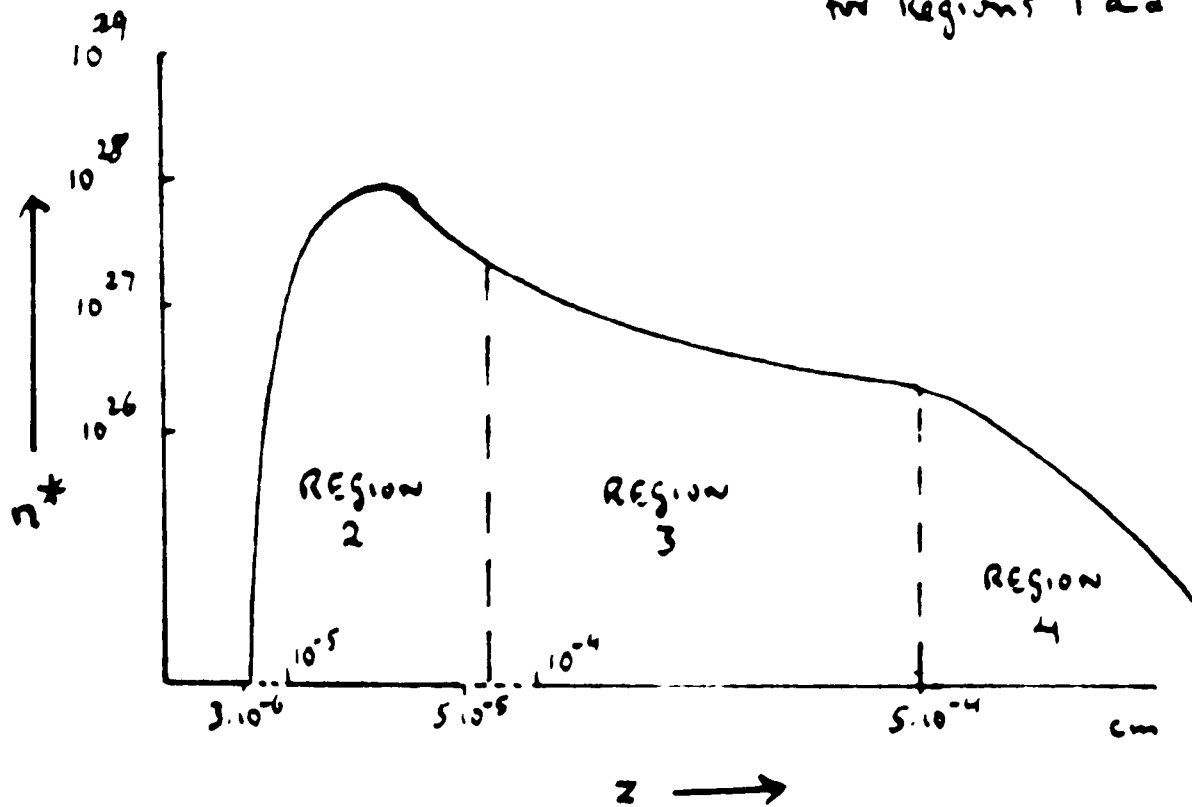


Fig 31. n^* AS A FUNCTION of z .

the current density is 10^5 A/cm² the average n^* is 10^{28} /cm³ and sec. In Region 3, the average n^* is 10^{27} /cm³ and sec and decreases with increasing z , since from (5.15) n^* is proportional to the field X .

Each electron passing through Region 3 will therefore expend the 3 volts it gains, if there are no other losses, in producing on the average 0.6 excited atoms. If we took losses by elastic collisions into account this figure would be slightly smaller. This is of course in addition to the one excited atom the electron has produced in Region 2 with the energy it has gained in Region 1. Each electron will therefore, on the average, produce 1.6 excited atoms as it falls through the 8 volts of the cathode fall.

5.7. The transfer of excited atoms.

Excited atoms will be lost by:

1. Radiation

2. Diffusion

3. Collisions of the second kind in which an ion-pair is formed (Fig. 27).

4. Ionisation by electron impact (Fig. 26).

The loss of excited atoms will be balanced by two production processes.

5. Excitation of an atom by electron impact. This has been discussed in the previous section.

6. Production of excited atoms by recombination of an ion and an electron. This is the reverse of process (3) and will

be discussed later.

The transfer of excited atoms by radiation and diffusion will be considered first. On account of the high density of atoms, the radiation from an excited atom returning to its ground state will be absorbed by a neighbouring unexcited atom. Since the cross-section for the absorption of a resonance photon is greater than the collision cross-section of an atom with another atom,⁽¹³³⁾ the excitation energy will simply be transferred by radiation to an adjacent atom. The lifetime of the 3P_1 state of mercury is about 10^{-7} sec. The collision frequency of atoms at a density of $10^{20}/\text{cm}^3$ and 1000°K is about $10^{11}/\text{sec}$, so an excited atom makes about 10^4 collisions during its lifetime. It may of course hand over its excitation energy at one of these collisions, but if the collisions are perfectly random this event is indistinguishable from a collision in which the energy is not handed over. If the excited atom makes 10^4 collisions and then hands its energy by radiation to an adjacent atom, which then makes 10^4 collisions and so on, the excitation energy will be transferred mainly by diffusion of excited atoms and the effect of radiation will be negligible. This is known as the imprisonment of resonance radiation, and has been investigated by a number of workers.^(140,141)

At the edges of the cathode region, where the densities are lower, radiation may escape and energy will be lost. However, since the extension of the cathode region is small compared with its diameter, we shall neglect radial losses and consider only the diffusion of excited

atoms in the axial direction.

The excited atoms will diffuse under the influence of density gradients and the flow F^* per unit area will be given by

$$F^* = -D^* \frac{dN^*}{dz} \quad (5.16)$$

where D^* is the diffusion coefficient for excited atoms, which we shall take as equal to the coefficient of self-diffusion of the vapour. Any difference due to the increased size of the excited atoms, the effect of radiation etc. will be neglected.

Differentiating equation (5.16) with respect to z we obtain

$$\frac{dF^*}{dz} = -D^* \frac{d^2N^*}{dz^2} \quad (5.17)$$

But in the steady state, $\frac{dF^*}{dz}$ is equal to the net rate of production of excited atoms per cm^3 and sec, which we shall call U^* . Therefore

$$U^* = -D^* \frac{d^2N^*}{dz^2} \quad (5.18)$$

From this equation it should be possible to derive the distribution of N^* as a function of z , if U^* is known as a function of z . In the previous section the production of excited atoms by electron impact was discussed: we must consider also the production of excited atoms by the recombination of ions and electrons.

5.8. Production and Recombination of Ions.

On account of the electron interaction in Regions 2 and 3 there will be a few electrons which have sufficient energy to ionise an unexcited atom at a single impact. If the average energy is about 0.5eV and the ionisation requires 10.4eV, the fraction of electrons capable of ionising directly will however be very small. A more likely process by which ions will be formed is by ionisation of an excited atom by an electron (this requires only $10.4 - 4.9 = 5.5$ volts) or by a collision of the second kind between two excited atoms in which a molecular ion is formed.

The number of collisions an excited atom makes per second with other atoms is $\frac{\bar{c}}{\lambda}$ and of these a fraction $\frac{N^*}{N}$ will be with excited atoms. The total number of collisions between excited atoms per cm^3 and sec is therefore

$$\left(\frac{\bar{c}}{\lambda N}\right) N^{*2} \quad (5.19)$$

We will assume that the probability of an ion-pair being formed at such a collision is about 0.1 (see Section 5.3) so the rate of production of molecular ions by this process is

$$\left(\frac{\partial N^+}{\partial t}\right)_1 = \left(\frac{\bar{c}}{10\lambda N}\right) N^{*2} \quad (5.20)$$

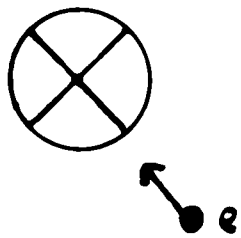
We have discussed in Section 5.3 the relative probability that an electron of given energy will ionise an excited atom or excite an atom from the ground state. The cross-sections are sketched in Fig.28. We see that electrons between 5 - 6eV energy will rather excite atoms from the ground

state, and electrons of energy greater than 6eV will rather ionise excited atoms. For a Maxwellian distribution of average energy 0.5eV a fraction $2 \cdot 10^{-4}$ of the electrons will have energy greater than 5eV and a fraction $2 \cdot 10^{-5}$ will have energy greater than 6eV (see Ref.122). The rate of production of ions by ionisation of excited atoms under the conditions which exist in Region 3 is thus about one tenth of the rate of production of excited atoms by electrons, times the fraction of excited atoms present. We may write therefore

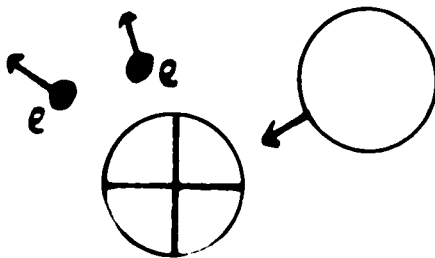
$$\left(\frac{\partial N^+}{\partial t} \right)_2 = \frac{1}{10} \cdot \frac{N^+}{N} \quad (5.21)$$

from which it appears that the majority of inelastic collisions which the electrons make result in excitation of unexcited atoms, rather than ionisation of excited atoms. The process represented by equation (5.21) will lead to the production of atomic ions Hg^+ . However if the ionisation takes place near a neutral unexcited atom, the ion may associate with it and form Hg_2^+ , the excess energy being carried away by the electrons or by another adjacent atom (Fig.32A). Also it is possible for a mercury atom in the resonance state to associate with an unexcited atom to form an excited molecule⁽¹⁴²⁻¹⁴³⁾ which may then be ionised by electron impact (Fig.32B). At high densities these processes seem likely, so for simplicity we shall assume that all the ions which are produced are molecular ions, Hg_2^+ .

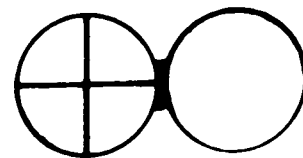
A proportion of the ions which are produced in Regions 2 and 3 will be drawn to the cathode by the electric field. Ions will also be



1. Electron ionises excited atom.

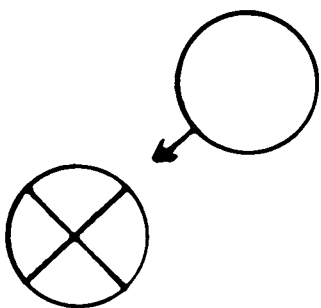


2. Atomic ion associates with neutral atom.

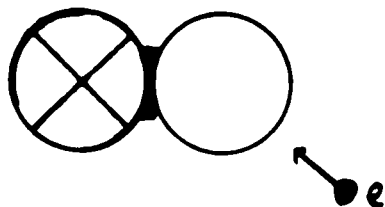


3. Molecular ion is formed.

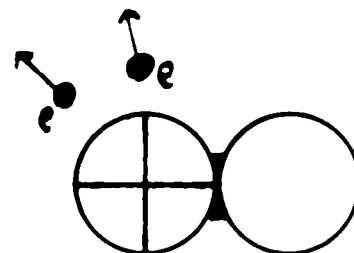
A.



1. Excited atom associates with unexcited atom.



2. Electron ionises excited molecule



3. Molecular ion is formed.

B.

FIG 32. PRODUCTION OF MOLECULAR IONS BY ELECTRONS

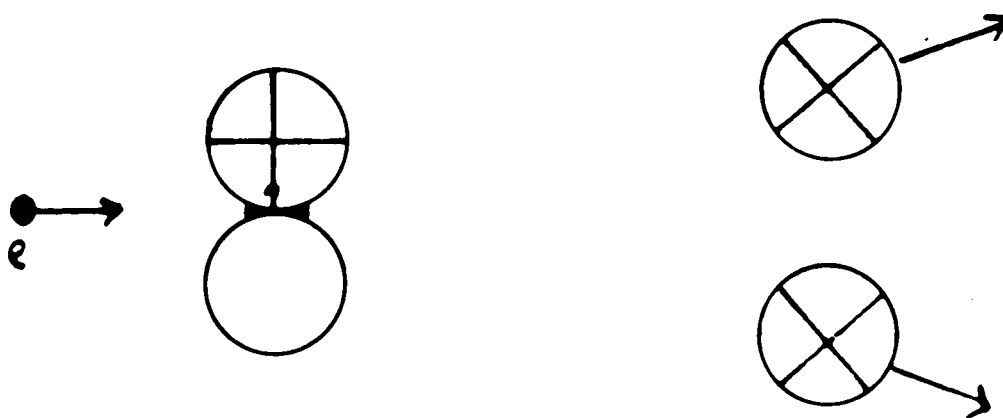


FIG 33. PRODUCTION OF EXCITED ATOMS BY DISSOCIATIVE RECOMBINATION OF A MOLECULAR ION.

lost by recombination with electrons. If a slow electron combines with a molecular ion, the molecule will dissociate, since it is unstable when neutral. The energy released in the recombination will be the ionisation energy of the molecule plus the dissociation energy, that is about $9.65 + 0.15 = 9.8$ eV. This energy will be carried away as kinetic and excitation energy of the two atoms. It is most unlikely that all the energy goes into kinetic energy of the atoms, and since the first excited state of Hg has an energy of 4.9eV, the only two ways in which the energy can be shared are either for one atom to be unexcited and the other to carry all the energy, or for each atom to be excited to the 4.9eV state. (Fig. 33). It would seem that the latter process is the more likely and so we shall take it to be the only one which occurs. The loss of ions by recombination is thus a production process for excited atoms.

The rate of loss of ions by recombination per cm^3 is given by

$$\left(\frac{\partial N^+}{\partial t}\right)_3 = -\alpha N^{+2} \quad (5.22)$$

This results in a rate of production of excited atoms given by

$$\left(\frac{\partial N^*}{\partial t}\right)_i = 2\alpha N^{+2} \quad (5.23)$$

The factor 2 occurring because each recombination produces two excited atoms.

No measurements have been made of the recombination coefficient of Hg_2^+ at high densities but since the recombination results in the dissociation of the molecule, α is likely to be high, probably of the order 10^{-7} to 10^{-8} (see ref.136).

Ions are taken from Regions 2 and 3 to form the positive current to the cathode. If we assume, for the purpose of this calculation, that this represents a uniform volume loss of ions, the ion current represents a loss given by

$$\left(\frac{\partial N^+}{\partial t}\right)_4 = -\frac{j^+}{ed} \quad (5.24)$$

where d is the extension of Regions 2 and 3 in the axial direction.

In the steady state, the rates of production and loss of ions must balance. Therefore

$$\left(\frac{\partial N^+}{\partial t}\right)_1 + \left(\frac{\partial N^+}{\partial t}\right)_2 + \left(\frac{\partial N^+}{\partial t}\right)_3 + \left(\frac{\partial N^+}{\partial t}\right)_4 = 0 \quad (5.25)$$

Consider now the orders of magnitude of the terms in equation (5.25).

From equation (5.21), since N^* must be less than N , and n^* as calculated in Section (5.6) is about $10^{27}/\text{cm}^3$ and sec we have

$$\left(\frac{\partial N^+}{\partial t}\right)_2 < 10^{26}/\text{cm}^3 \text{ and sec}$$

From equation (5.22), putting $\alpha = 10^{-7}$ to 10^{-8} and $N^+ \sim 3 \cdot 10^{18}/\text{cm}^3$ we have

$$-\left(\frac{\partial N^+}{\partial t}\right)_3 \sim 10^{29} - 10^{30}/\text{cm}^3 \text{ and sec}$$

From equation (5.24), putting $j^+/e = 10^{22}/\text{cm}^2$ and sec and $d = 5 \cdot 10^{-4} \text{ cm}$ we have

$$\left(\frac{\partial N^+}{\partial t}\right)_4 \sim 5 \cdot 10^{18}/\text{cm}^3 \text{ and sec}$$

The only unknown term in equation (5.25) is therefore $\left(\frac{\partial N^+}{\partial t}\right)_1$, and we see that in order that the equation should be satisfied this term must be of the same order of magnitude as $\left(\frac{\partial N^+}{\partial t}\right)_3$, i.e. $10^{29} - 10^{30}/\text{cm}^3$ and sec.

$\left(\frac{\partial N^+}{\partial t}\right)_1$ represents the production of ions by collisions between excited atoms, and $\left(\frac{\partial N^+}{\partial t}\right)_3$ represents the reverse process, namely the production of excited atoms by recombination of ions. Since the other terms in equation (5.25) are small compared with these two terms, there is approximately a state of equilibrium between the two processes (detailed balancing). We have therefore

$$\left(\frac{\partial N^+}{\partial t}\right)_1 \approx -\left(\frac{\partial N^+}{\partial t}\right)_3 \quad (5.26)$$

From equations (5.20), (5.22) and (5.26) we have

$$\left(\frac{\bar{c}}{10\lambda N}\right) N^{+2} = \alpha N^{+2} \quad (5.27)$$

from which

$$\frac{N^+}{N^+} = \left(\frac{10\lambda N \alpha}{\bar{c}}\right)^{\frac{1}{2}} \quad (5.28)$$

Under these conditions therefore the ratio of N^+ to N^* is a constant.

With $(\lambda N) = 6 \cdot 10^{13}$, $\bar{c} = 4 \cdot 10^4$, $\alpha = 10^{-7}$ to 10^{-8} we have from (5.28)

$$\frac{N^+}{N^+} \sim 10 \text{ to } 40. \quad (5.29)$$

We should now be able to derive from this relation the density distribution of excited atoms from the curve of the density distribution of charged particles given in Fig. 30.

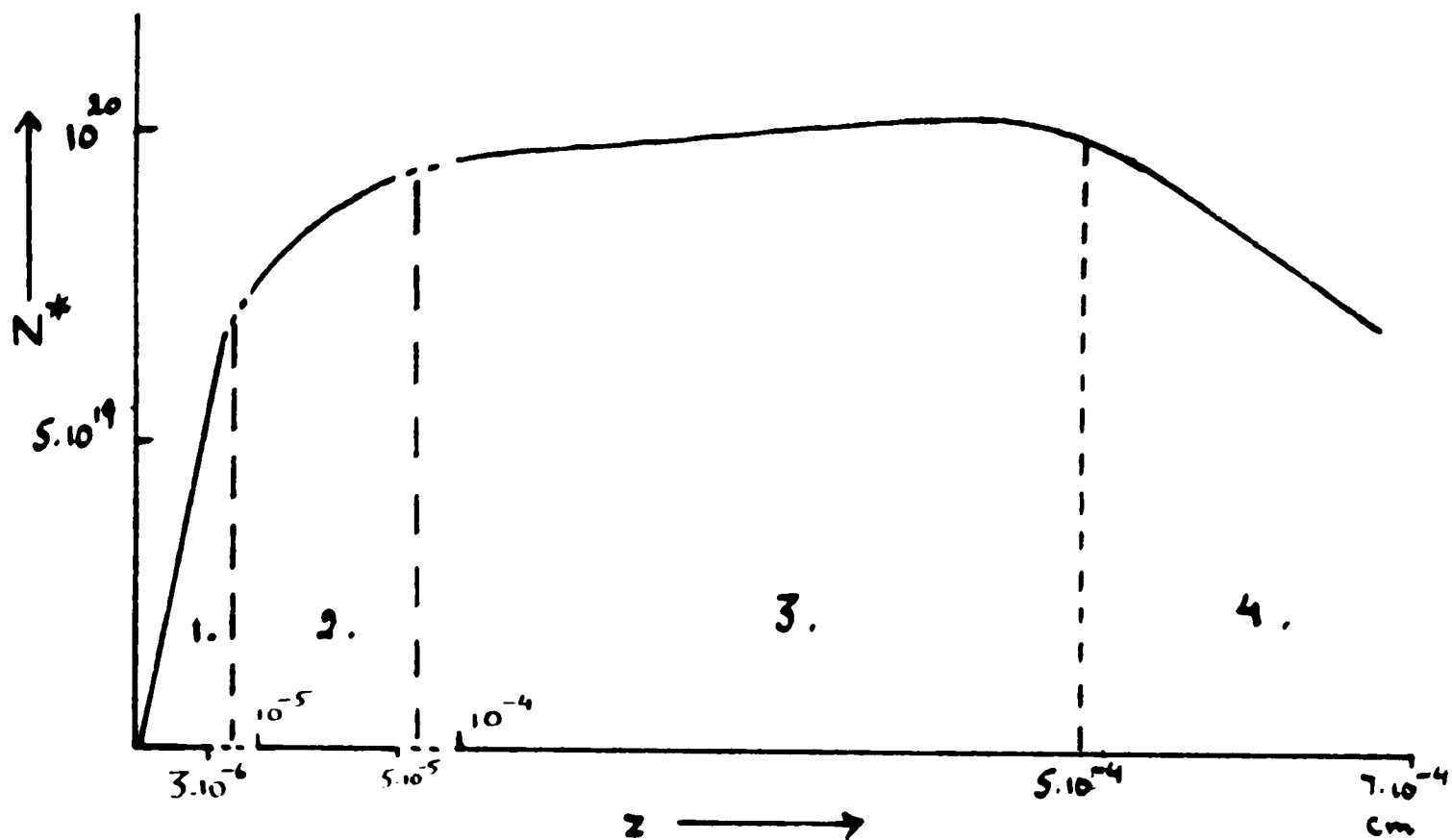
5.9. The distribution of excited atoms.

In Fig. 34 the density of excited atoms N^* is shown as a function of z . This has been derived as follows.

At the cathode, most of the excited atoms will be de-excited so $N^* = 0$ at $z = 0$. In Regions 2 and 3 we see from Fig. 30 that $N^+ \sim 3 \cdot 10^{18}/\text{cm}^3$ and so from equation (5.29) $N^* \sim 3 \cdot 10^{19}$ to $10^{20}/\text{cm}^3$. In Region 1 there is no production of excited atoms, hence from equation (5.18) $\frac{dN^*}{dz}$ is constant. N^* thus rises linearly from zero at the cathode to a large value in Region 2.

The shape of the curve of N^* in Region 3 must be the same as the shape of the curve of N^+ , since the ratio N^*/N^+ is constant. Since N^+ goes through a maximum at $z \approx 5 \cdot 10^{-4} \text{ cm}$, N^* must also go through a maximum at the same distance. The direction in which excited atoms diffuse is determined by the sign of the density gradient (equation 5.16) and on the cathode side of the maximum the flow will be towards the cathode while on the other side of the maximum the flow will be away from the cathode. (Fig. 35).

Excited atoms which diffuse away from the cathode in Region 4 will move into regions of lower vapour density and will eventually radiate their energy: this radiation provides the observed luminosity of the cathode spot. Excited atoms which diffuse towards the cathode have a high probability of releasing electrons when they arrive at the surface. It will be shown in the next section that the diffusion flow of excited atoms to the cathode is sufficient to release the observed current density



Both axes are linear, but the abscissa is plotted to a different scale for each region.

FIG 34. DENSITY OF EXCITED ATOMS N^* AS FUNCTION OF z .

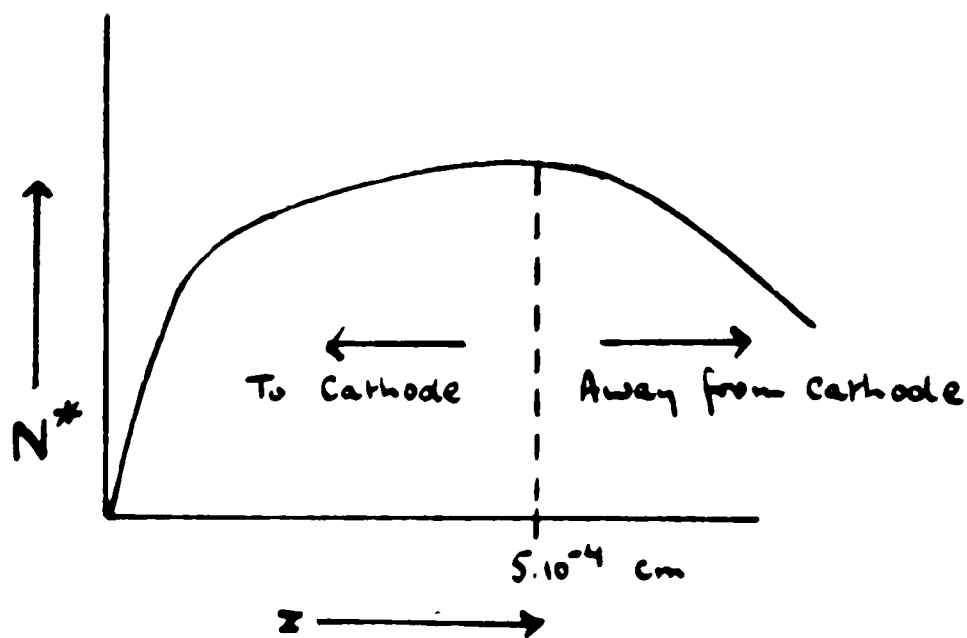


FIG 35. DIFFUSION OF EXCITED ATOMS.

of electrons. The self-sustaining mechanism of the cathode is therefore contained on the cathode side of the maximum in the curve of Fig. 34, and hence this maximum defines the limit of the cathode fall region. The assumption made earlier in this chapter that the 8v cathode fall extends over about $5 \cdot 10^{-4}$ cm is now justified. It should be remembered that the maximum in the curve of N^+ was derived from purely geometrical considerations and the maximum in the curve of N^* was then derived from the curve of N^+ .

It should theoretically be possible to run an arc consisting only of Regions 1 to 3, that is to say with the anode brought down to a distance of about $5 \cdot 10^{-4}$ cm from the cathode. However it would not be possible to do this without disturbing the distribution of excited atoms and so in all practical cases it is necessary to run an arc which includes part of Region 4. Thus although the true cathode fall in potential occurs over Regions 2 to 3, the measured cathode fall determined by the usual methods may include some of the potential across Region 4, which, as we estimated in Section (5.4) is probably a few volts. It is for this reason that we have taken the lowest measured value of 8v for the cathode fall in the foregoing calculations, since the higher values of about 10v probably included some of the potential across Region 4.

It is instructive at this stage to take the curve of N^* as a function of z and obtain its first and second derivatives with respect to z . This is done in Fig. 36. Curve A gives N^* against z , and is copied from Fig. 34. Curve B gives $\frac{dN^*}{dz}$ against z , derived from Curve

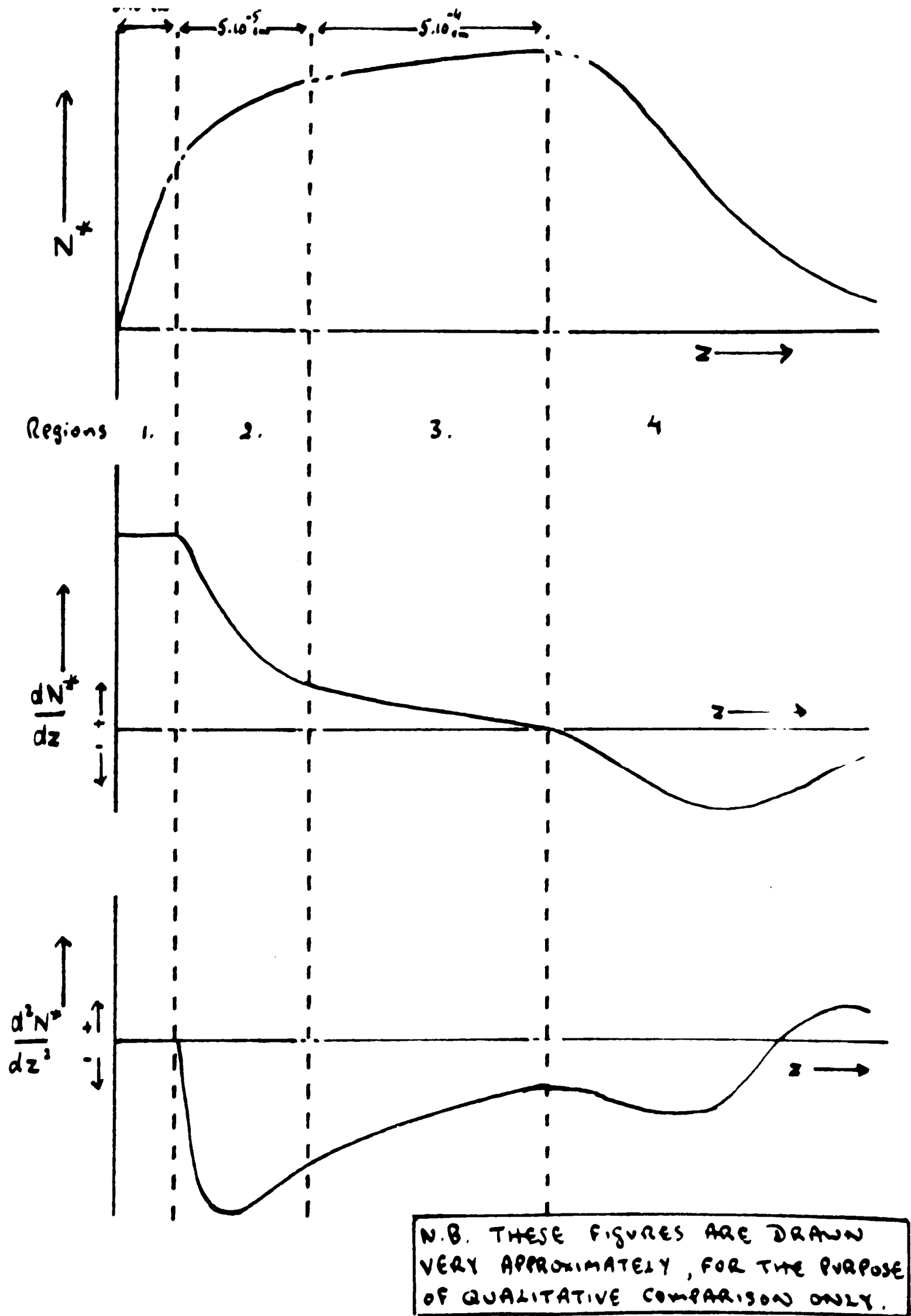


FIG 36 . DERIVATIVES OF THE N^*-z CURVE

A, and curve C gives $\frac{d^2 N^*}{dz^2}$ derived from Curve B. These curves will be discussed from a qualitative point of view only, and no numerical calculations will be made from them.

From Curve B it is seen that dN^*/dz is large, positive and constant in Region 1, decreases rapidly in Region 2, less rapidly in Region 3, goes through zero as N^* goes through its maximum, is negative in Region 4 and finally is zero in the positive column.

From Curve C it is seen that $\frac{d^2 N^*}{dz^2}$ is zero in Region 1, becomes large and negative in Region 2 and after going through a minimum increases steadily in Regions 3 and 4. It goes through zero somewhere in Region 4 and then becomes positive and goes through a maximum before finally becoming zero in the positive column.

From equation (5.18) we see that $\frac{d^2 N^*}{dz^2}$ is proportional to the net rate of production of excited atoms U^* . This equation is valid for the region close to the cathode where there is no appreciable radial diffusion of excited atoms. Consider now the production and loss of excited atoms per cm^3 and sec. We have

$$U^* = n^* - \left\{ \left(\frac{\partial N^*}{\partial r} \right)_2 + 2 \left(\frac{\partial N^*}{\partial r} \right)_1 + 2 \left(\frac{\partial N^*}{\partial r} \right)_0 \right\} \quad (5.30)$$

Of the terms on the right hand side, n^* represents the rate of production by electron impact, and the bracket represents the rate of loss by collisions which involve production of ions. The factor 2 occurs when two excited atoms are involved in the production or recombination of one ion (Figs. 27 and 33).

From equations (5.25) and (5.30) we have

$$U^* = n^* + \left(\frac{\partial N^*}{\partial r}\right)_2 + 2 \left(\frac{\partial N^*}{\partial r}\right)_4 \quad (5.31)$$

Hence from (5.21), (5.24) and (5.31) we have

$$U^* = n^* \left(1 + \frac{N^*}{10N}\right) - \frac{2j^+}{e\alpha} \quad (5.32)$$

We see from the previous section that the last term is small compared with n^* and so can be neglected, and as N^* must be less than N we may write

$$U^* \approx n^* \quad (5.33)$$

That is to say, the net rate of production of excited atoms is due almost entirely to electron impact. From equations (5.18) and (5.33) we have

$$\frac{d^2 N^*}{dz^2} \propto -n^* \quad (5.34)$$

If now we compare the curve of $\frac{d^2 N^*}{dz^2}$ given in Fig. 36 with the curve of n^* given in Fig. 31 we see that, taking the negative sign into account, that they are exactly similar over the regions in which a comparison is valid, i.e. Regions 1 to 3. The purpose of this comparison is to show that the distribution of excited atoms given in Fig. 34 is consistent with the rate of production of excited atoms as derived in Section 5.6. The positive $\frac{d^2 N^*}{dz^2}$ in part of Region 4 indicates that there is a net loss of excited atoms in the volume: this is probably due to radial diffusion,

since at these distances from the cathode it is no longer permissible to consider the problem as being in the z co-ordinate only.

5.10. The Diffusion of Excited Atoms to the Cathode.

When excited atoms diffuse across Region 1 to the cathode surface, three processes may occur there

1. The excited atom may release an electron
2. The excited atom may give up its energy to the cathode as heat.
3. The excited atom may be reflected without losing its excitation energy.

The probability of the first process is the electron yield and has been discussed in Chapter 3. We take it to be about 0.50 - 0.75. Very little is known about the relative probability of the second and third processes. Oliphant⁽⁴⁶⁾ has conducted experiments on the reflexion of He metastables from a Mo surface and estimated reflexion coefficients for fast (K.E. = 200eV) metastables which varied from 10 - 30% for normal incidence to as great as 90% for near-grazing incidence. The reflexion coefficient increased with decreasing atomic velocity. It does not seem unreasonable therefore to expect a mean reflexion coefficient of 25 - 50% for slow excited atoms of mercury on a mercury surface. This would mean that practically every excited atom which did not release an electron would be reflected with its excitation energy, and therefore that process 2 would be insignificant. We shall take this to be the case in the following discussion.

Since there is no production of excited atoms in Region 1, the

density gradient of excited atoms will be constant. If $N^* = N_2^*$ in Region 2 and $N^* = N_0^*$ at the cathode the density gradient in Region 1 is

$$\frac{dN^*}{dz} = \frac{N_2^* - N_0^*}{x} \quad (5.35)$$

where x is the extension of Region 1.

From equations (4.3) and (5.16) we have the flow to the cathode F_c^* is given by

$$F_c^* = \frac{A}{N} \cdot \frac{N_2^* - N_0^*}{x} \quad (5.36)$$

where N is the total density of neutral atoms and A is a constant.

If all the excited atoms which reached the cathode were de-excited there, we could write $N_0^* = 0$ and hence equation 5.36 becomes

$$F_c^* = \frac{A}{N} \cdot \frac{N_2^*}{x} \quad (5.37)$$

This would be the flow only if all the excited atoms were de-excited at the cathode. If a proportion R are reflected, we cannot put $N_0^* = 0$ but must take N_0^* as given by the equation

$$\frac{1}{4} N_0^* \bar{v} = R F_c^* \quad (5.38)$$

This expresses the fact that if $R F_c^*$ atoms are reflected away from the cathode, the conditions are the same as if a density N_0^* excited atoms exists there.

Combining equations (5.36) and (5.38) and solving for F_c^* we obtain

$$F_c^* = \frac{A}{N} \cdot \frac{N_2^*}{x} \left(1 + \frac{4RA}{xN\bar{v}} \right)^{-1} \quad (5.39)$$

This is similar to equation (5.37) except for the factor in brackets which represents the effect of reflexion of excited atoms.

In Chapter 4 we first took $A = 8.10^{17}$, corresponding to $\bar{c} = 4.10^4$ cm/sec but we showed that it was more likely to be larger by about a factor 2, corresponding to $\bar{c} = 8.10^4$ cm/sec. If therefore we substitute in equation (5.39) the values $A = 1.6 \times 10^{18}$, $\bar{c} = 8.10^4$ cm/sec, $x = 2.10^{-6}$ cm and $N = 10^{20}$ and take $R = 25\%$ we find

$$\frac{4RA}{\pi N \bar{c}} \sim 0.1 \quad (5.40)$$

The reflexion of excited atoms therefore has only a small effect on the diffusion. We have calculated in the previous section that N^* in Region 2 is probably about 3.10^{19} - 10^{20} /cm³, that is, of the same order as N .

If therefore we take $N_2^* = N$, representing 100% excitation we have from (5.9)

$$F_c^* \sim 7.10^{23}/\text{cm}^2 \text{ and sec.}$$

The flow of excited atoms to the cathode represents the number per cm² and sec which are de-excited there, and we have assumed that all these release electrons. Excited atoms which do not release electrons are reflected and so are not included in the flow. The flow of excited atoms calculated above would then be more than sufficient to release 10^5 A/cm² of electrons ($1 \text{ A} = 6.10^{18}$ electrons per sec.)

In the next Chapter we shall show that the positive ion current is probably about 20% of the total current, so that if the total current density is 10^5 A/cm², the electrons need carry only 8.10^4 A/cm². The flow

of excited atoms would therefore need to be only about $5 \cdot 10^{23}/\text{cm}^2$ and sec. to release this current density of electrons.

The calculation of the flow of excited atoms to the cathode which has been given above is not sufficiently accurate to give more than the order of magnitude of the flow, since it is not possible to assign accurate values to A and N_2^* in equation (5.39). Also the assumption that all excited atoms which fail to release electrons are reflected is probably an over simplification, since a proportion are likely to give up their excitation energy to the cathode as heat.

We have shown however that each electron from the cathode will produce on the average about 1.6 excited atoms as it falls through the 8v of the cathode fall (see Section 5.6) and furthermore that the production of excited atoms by electrons is balanced almost entirely by the loss due to diffusion to the cathode, other losses being small in comparison (see equations 5.33, 5.34). We should therefore expect

$$F_c^+ = \frac{1.6 j^-}{e} \quad (5.41)$$

which, if we put $j^- = 8 \cdot 10^4 \text{A}/\text{cm}^2$, leads to

$$F_c^+ = 8 \cdot 10^{23} \text{ atoms}/\text{cm}^2 \text{ and sec.}$$

This figure will be reduced slightly if other losses are considered, but as it stands it would be more than sufficient to release the observed current density of electrons from the cathode.

An effect which might assist the flow of excited atoms to the cathode is the action of the highly non-uniform electric field in Region

1 on the electric dipole moments of the excited atoms. This has been worked out in Appendix 5 and the effect is shown to be very small.

5.11. The Copper Arc.

In the foregoing discussion we have described how a self-sustaining discharge can be maintained at the cathode of an arc by means of excitation processes occurring in the dense vapour close to the cathode surface. The calculations show that the proposed mechanism is consistent with the observed features of the mercury arc.

The mercury arc is however a special case, as all the excited states of mercury have greater energy than the work function, and hence all excited atoms can release electrons from the cathode. We shall now consider a more general case, and take as an example the arc with a copper cathode.

The work function of copper is 4.1eV and the first excitation potential is 1.4eV (Table 4). Atoms in the lower excited states therefore cannot release electrons: if they strike the cathode there is a high probability that they will be de-excited, in which case they will give up their energy to the cathode as heat. If a large number of such atoms reached the cathode they would represent a large energy loss from the discharge, and the cathode fall would have to be large to balance this loss. We see however from Table 4 that the cathode fall of a copper arc is only 8-9 volts, which is about the same as the cathode fall of the mercury arc.

At the beginning of this chapter we defined Region 1 as the region in which electrons from the cathode made only elastic collisions with vapour atoms. It so happens that in the mercury arc this corresponds with the region in which no ionisation occurs, since as we have shown, cumulative ionisation occurs simultaneously with excitation in mercury vapour. Region 1 would be better defined as the region of strong positive space charge which extends from the cathode to where ionisation occurs in the vapour. In copper vapour, ionisation potential 7.7 volts, ionisation will occur by stepwise processes when atoms are excited to energies of about 4eV, and hence the potential across Region 1 of a copper arc will be about 4 volts. Since the current density, cathode fall and other features of the copper arc are very similar to the mercury arc we should expect the dimensions of the cathode region and the density of vapour there to be about the same as for mercury, and so electrons from the cathode will make only a few collisions with atoms in Region 1. The probability that they will excite an atom into one of the lower states while crossing Region 1 is therefore very small. The faster the electron, the less likely it is to excite the lower states; in Region 2 therefore the electron will probably excite atoms into states of 3-4eV energy rather than states of 1.5-2v energy.

In Region 3, where the electrons have an energy distribution with a mean energy of a fraction of an electron volt most of the excitations of atoms from the ground state will be to the lower excited states. However, the cross section for further excitation of the lower states to states of higher energy is almost certainly larger than the cross-section

for excitation of the lower states from the ground state: hence atoms in the lower excited states will readily be raised to higher states by stepwise processes. It seems likely therefore that in the steady state most of the excited atoms in the vapour will have energies of about 4eV or more: this is supported by the spectrum of the cathode region of the copper arc⁽¹⁴⁴⁾ which indicates a large population of atoms in the higher excited states. When these atoms diffuse to the cathode they will have sufficient energy to release electrons. It appears therefore that the cathode processes described in this chapter will apply to all cold-cathode arcs.

The potential difference across the cathode region is the cathode fall in potential V_c . The radius of the cathode is r_c and the extension in the axial direction is l_c . The current through the cathode is I_c . At the cathode end of the cathode region the current consists of an electron current I_e and a positive ion current I_i which satisfy the relation

$$I_e + I_i = I_c$$

at the other end, as has been pointed out in the previous chapter, the current is carried almost entirely by electrons in the cathode region and will be neglected. These features of the cathode are illustrated in Fig. 37.

Electrons which are released from the cathode surface with a velocity v , by excited atoms of energy V^* will have a maximum energy (see p. 64) up to a maximum of $(V^* - \phi)$. The assumption is made that all the electrons are emitted with a velocity

CHAPTER 6

THE ENERGY BALANCE

6.1. The energy balance in the vapour.

For the purpose of the following discussion the cathode region of the arc will be divided into two parts:

1. The vapour region: this consists of Regions 1 to 3 as defined in the previous chapter, but excludes the cathode itself.

2. The cathode.

The potential difference across the vapour region is the cathode fall in potential V_0 . The radius of the region is the spot radius a , the extension in the axial direction is the cathode fall distance d and the current through the region is i . At the cathode end of the vapour region the current consists of an electron current i^- and a positive ion current i^+ which satisfy the relation

$$i^- + i^+ = i \quad (6.1)$$

At the other end, as has been explained in the previous chapter the current is carried almost entirely by electrons so the ion current at this end will be neglected. These features of the vapour region are illustrated in Fig. 37.

Electrons which are released from the cathode, of work function ϕ , by excited atoms of energy V^* will have an energy distribution (see p. 64) up to a maximum of $(V^* - \phi)$. For simplicity however we shall assume that all the electrons are emitted with kinetic energy $(V^* - \phi)$.

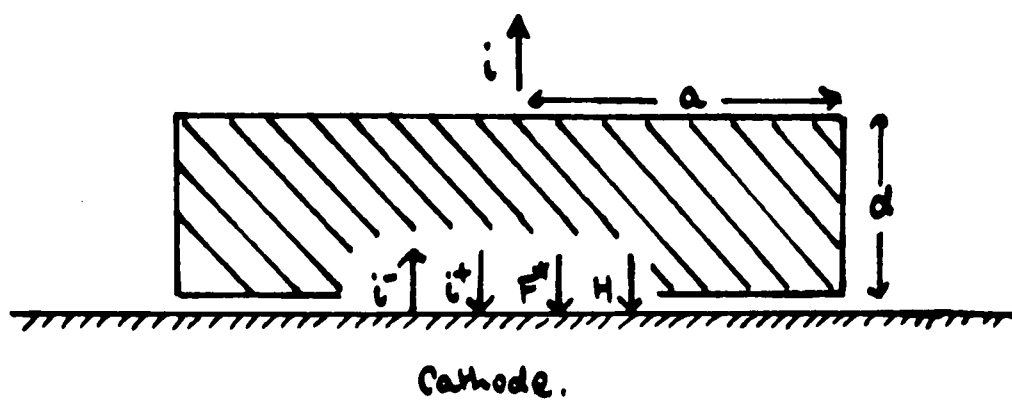


FIG 37. THE VAPOUR REGION.

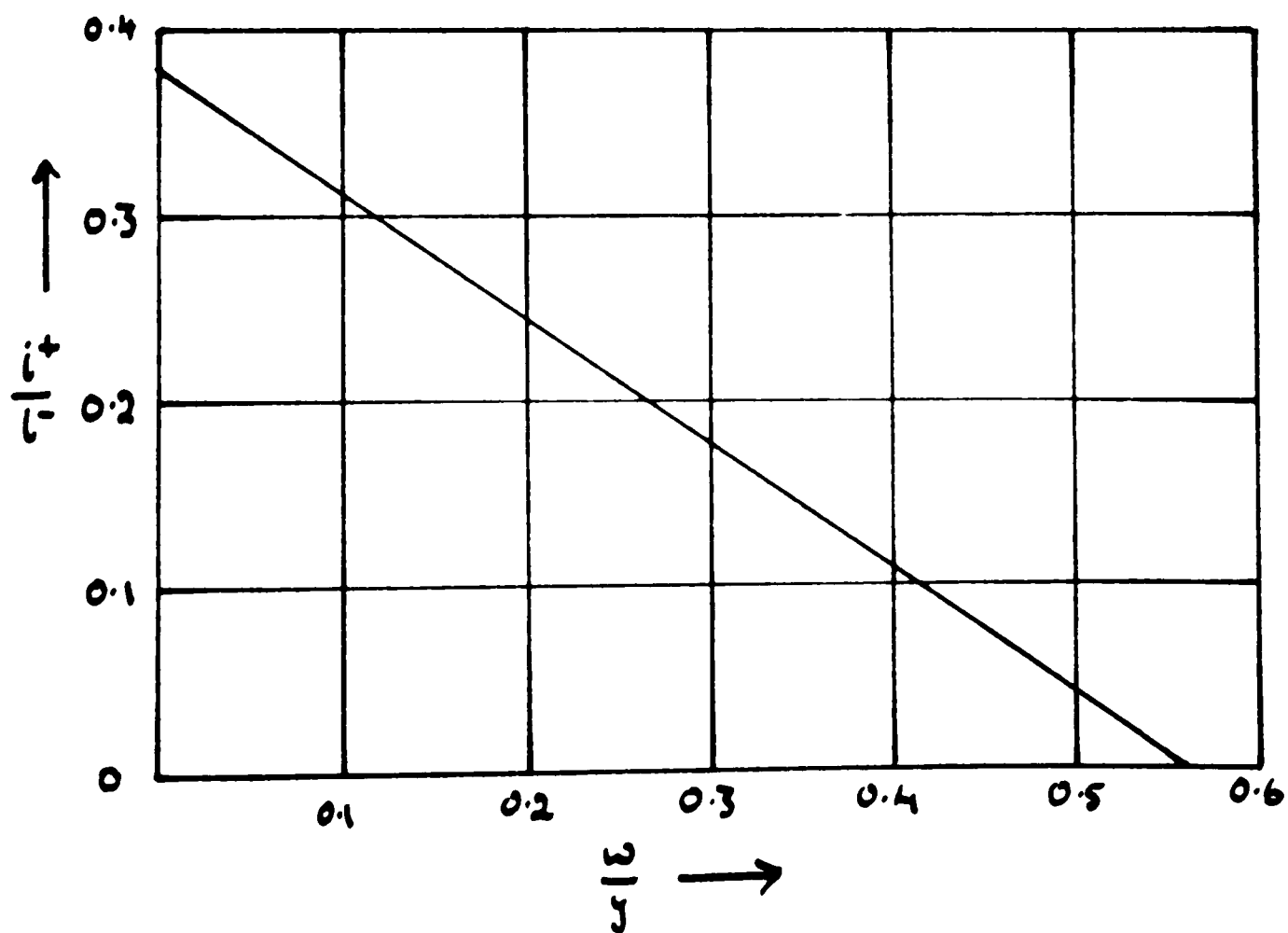


FIG 38. GRAPH OF $\frac{i^+}{i^-}$ AGAINST $\frac{z}{y}$.

A current i^- of electrons from the cathode enter the vapour region and represent an energy input of $i^-(V^* - \phi)$. In addition, the current i passing through the potential difference V_c dissipates energy iV_c in the vapour. The total energy input W to the vapour region is therefore given by

$$W = iV_c + i^-(V^* - \phi) \quad (6.2)$$

Energy is lost from the vapour region in the following ways:

A. To the cathode.

1. The ion current to the cathode. Each ion produced in the vapour takes energy eV_i where V_i is its ionisation potential. If the ion current is i^+ , the energy loss is therefore i^+V_i .

2. The flow of excited atoms to the cathode. If the total flow to the cathode by diffusion is F^* and each atom carries excitation energy eV^* , the energy loss is eF^*V^* .

3. Heat loss by conduction. Charged particles gain energy in the electric field and give up some of this energy in elastic collisions with vapour atoms. The temperature of the vapour is therefore certain to be higher than the temperature of the cathode and so a heat flow will occur from the vapour region to the cathode. This heat flow will be called H .

B. Other losses.

4. Radiation. We assume that in the high densities of vapour which exist near the cathode radiation is 'imprisoned' and the energy spends most of its time as excitation energy of vapour atoms. The energy is therefore transferred mainly by diffusion of excited atoms (see p.77).

We have shown in the previous chapter that the cathode fall region extends to where the excited atom density N^* is a maximum, and since here is zero, there will be no transfer of excitation energy from the upper end of the vapour region (see Fig. 35). However, since the extension of the vapour region in the axial direction, although small is finite, there will probably be a small radial diffusion of excited atoms, which when they reach the lower densities of vapour in the periphery of the arc spot will be able to radiate their energy. A rough estimate of this radiation loss can be made from the brightness measurement as follows. The area of the curved surface of the vapour region is $2 \pi a d$ (Fig. 37) so if the brightness of the spot is B photons/cm² and the photon energy is V^* the total radiation energy loss is $2 \pi a d B V^*$.

5. The electron current to the anode. The current from the top of the vapour region will consist of electrons with a mean energy ϵ . The electron current will therefore represent an energy loss of $i \epsilon$.

6. Heating of the evaporated cathode material. Evaporated atoms will leave the cathode with the temperature of the surface, which we have estimated to be about 500°C. They will be raised to a higher temperature as they pass through the vapour region, and so will represent an energy loss from the region. If F is the evaporation in atoms/sec and ΔT the rise in temperature, the energy loss will be $1.5 k F \Delta T$.

We may now equate energy input to energy loss to obtain the energy balance for the steady state in the vapour

$$i V_c + i^-(V^* - \phi) = (i^+ V_1 + e V^* F^* + H) + (i \epsilon + 1.5 k F \Delta T + 2 \pi a d B V^*)$$

(6.3)

The first bracket on the right hand side of this equation represents the energy lost from the vapour region to the cathode. The second bracket represents the energy lost in other ways. A rough quantitative estimate of the second bracket will now be made, taking the current $i = 1$ A.

In the previous chapter ϵ was estimated as 0.5eV. Hence $i\epsilon = 0.5$ watts. The evaporation F is 10^{18} atoms/coul (p.37). If $\Delta T \approx 3000^\circ\text{K}$ (say), we have $1.5kF\Delta T \approx 0.06$ watts. If $B = 5 \cdot 10^{22}$ photons/sec, corresponding roughly to a brightness of 10^6 candles/cm² (see p.36) and $V^* = 5\text{eV}$, $a = 2 \cdot 10^{-3}\text{cm}$ and $d \sim 2 \cdot 10^{-4}\text{cm}$ (for a 1A arc spot) we have $2\pi ad V^*B = 0.1$ watt. The total loss of energy from the vapour region other than to the cathode therefore amounts to about 0.7 watts/ampere.

We shall now define a reduced cathode fall in potential V'_0 as that part of the cathode fall which supplies energy to balance losses to the cathode only. From the preceding calculation we see that V'_0 is about 0.7 volts less than the true cathode fall V_0 . In equation 6.3 we now put V'_0 for V_0 and neglect the second bracket on the right hand side, and obtain the equation

$$iV'_0 + i^-(V^* - \phi) = (i^+V_1 + eV^*F^* + H) \quad (6.4)$$

6.2. The energy balance at the cathode.

Energy is supplied to the cathode from the vapour region in three ways:

1. Positive ions which arrive at the cathode with ionisation energy

V_1 are neutralised by extracting an electron from the metal. The rest of their energy is then given up to the cathode, representing an energy input $i^+(V_1 - \phi)$.

2. Excited atoms are de-excited at the cathode surface and give up their energy to the cathode. Some of this energy will be used to emit electrons and the rest will be given up as heat, but at this stage we are concerned only with the total energy input, which is eF^*V^* .

3. Heat energy is supplied by conduction from the hot vapour, representing an energy input H .

Energy is lost from the cathode in three principal ways:

1. Evaporation of neutral atoms. If the evaporation is F atoms/sec and the latent heat of vaporisation is L /atom, the energy loss is LF .

2. Emission of electrons with energy $(V^* - \phi)$. This represents an energy loss $j^-\phi + j^-(V^* - \phi) = j^-V^*$.

3. Conduction of heat through the cathode. This loss will be called C and will be discussed in greater detail later.

Equating energy input to energy loss we obtain for the steady state the energy balance at the cathode:

$$i^+(V_1 - \phi) + eF^*V^* + H = j^-V^* + LF + C \quad (6.5)$$

6.3. The total energy balance.

By adding equations (6.4) and (6.5) we obtain

$$iV_0' = i\phi + LF + C \quad (6.6)$$

This equation expresses that, taking the cathode region as a whole and neglecting losses other than to the cathode, the energy input iV'_0 is used to produce a current i of electrons and to dissipate $LF + C$ to the cathode as heat. This is the complete energy balance equation, and from it we derive that the heat input to the cathode is

$$LF + C = i(V'_0 - \phi) \quad (6.7)$$

We must now discuss what proportion of the heat input to the cathode is due to heat conduction from the vapour, positive ions and excited atoms.

If an excited atom arrives at the cathode there is a probability y that it will release an electron, where y is the yield for the process. The probability that it will be de-excited and give up its energy to the cathode in the form of heat will be called w , and the probability that it will be reflected without losing its energy will be called R . Since it must do one of these three things we have

$$y + w + R = 1 \quad (6.8)$$

As we have already explained, the diffusion flow of excited atoms F^* consists only of those which are de-excited at the cathode; those which are reflected do not contribute to the flow. In the previous chapter we took for simplicity $w = 0$, which meant that all which did not release electrons were reflected. We shall now consider the more general case when $w > 0$. If the flow to the cathode is F^* , the electron current which is released will be given by

$$j^- = \frac{e F^* y}{y + \omega} \quad (6.9)$$

By eliminating F^* between equations (6.5) and (6.9) we obtain

$$V^* i^- \frac{\omega}{y} + i^+ (V_1 - \phi) + H = LF + G \quad (6.10)$$

The three terms on the left hand side represent the heat input to the cathode by excited atoms, by ions and by conduction from the vapour respectively. An estimate of the conduction term H can be made by considering how heat energy is supplied to the vapour. Since the mass of the electron is very small compared with the mass of a mercury atom the electrons transfer their energy principally by inelastic collisions and so have only a small heating effect in the vapour. An ion however will lose on the average half its energy in an elastic collision with a vapour atom. The heating of the vapour will therefore be due almost entirely to the ions. As described in Chapter 5 the ion current is small compared with the electron current, except in Region 1 where positive space charge predominates. If the potential across Region 1 is taken to be the excitation potential V^* , the energy gained by the ions is $i^+ V^*$. Most of this energy will be given to the vapour atoms in collisions, thereby raising the temperature of the vapour. As we have shown, the heat carried away from the vapour by the evaporating atoms is small. In the steady state therefore the heat energy given to the vapour by the ions will be conducted to the cathode. In addition, any kinetic energy the ions have on arrival at the cathode will be given up as heat. We may therefore write

but in the absence of further information, no definite estimate of i^+

$$H = i^+ V^* \quad (6.11)$$

From (6.10), (6.11) and (6.7) we have

$$\frac{i^+}{i^-} (V^* + V_i - V'_0) = V'_0 - \phi - \frac{w}{y} V^* \quad (6.12)$$

This equation gives the ration w/y as a function of the ration i^+/i^- .

If now we put the values $V^* = 4.9$ $V_i = 9.7$ (the ionisation potential of a mercury molecular ion), $\phi = 4.5\text{eV}$ and $V'_0 = 8.0$ minus $0.7 = 7.3\text{v}$ equation (6.12) becomes

$$7.3 \frac{i^+}{i^-} = 2.8 - 4.9 \frac{w}{y} \quad (6.13)$$

In Fig.38 i^+/i^- is plotted from equation (6.13) as a function of w/y . It can be seen from this graph that the greatest value of i^+/i^- would occur if $w = 0$, in which case $i^+ = 0.38i^-$. The ion current at the cathode cannot therefore exceed $0.38/1.38 = 27.5\%$ of the total current. Furthermore we see that w/y cannot exceed 0.56 , since for this value $i^+ = 0$. In the absence of any evidence as to the probable value of the ratio w/y it is not possible to do more than set an upper limit to i^+ from the energy balance. If however we take $i^+ = 0.2i^-$ as we have done in the previous chapters, it appears that $w/y = 0.3$ which is consistent with

$$y = 0.5, \quad w = 0.15 \quad R = 0.35$$

$$\text{or } y = 0.75 \quad w = 0.23 \quad R = 0.02$$

A more likely set of values would seem to be:

$$y = 0.6 \quad w = 0.2 \quad R = 0.2$$

but in the absence of further information, no definite estimate of i^+

can be made.

A significant result from this treatment is however that the energy balance equations are independent of y : they insist however that $w < 0.5 y$. The proposed mechanism of the arc cathode would therefore be valid energetically if y was low (say ~ 0.1) provided nearly all the excited atoms which did not release electrons were reflected. The lower limit to y is however set by physical considerations. The number of excited atoms arriving at the surface of the cathode cannot be greater than the total number of atoms striking the surface. If the density of neutral atoms N_0 is $10^{20}/\text{cm}^3$ (this was estimated from the force on the cathode see p.41) and $\bar{c} = 4 \cdot 10^4 \text{ cm/sec}$, the total number of atoms striking the cathode is $10^{24}/\text{cm}^2$ and sec. The number of excited atoms arriving at the cathode cannot be greater than this, hence we have

$$j^- / e < 10^{24} y \quad (6.14)$$

so if $j^- \approx 8 \cdot 10^4 \text{ A/cm}^2$, y must be greater than 0.48.

6.4. The heat loss from the mercury cathode.

From equation (6.7), putting $V'_0 = 7.3v$, $\phi' = 4.5v$, we derive that the heat loss from the cathode by evaporation and conduction is

$$LF + C = 2.8 \text{ watts/amp} \quad (6.15)$$

To calculate the loss by evaporation we take the evaporation F as 10^{18} atoms/coulomb (see p.37) and $L = 0.6 \text{ ev/atom}$ (table 1) from which

$$LF = 0.1 \text{ watts/amp}$$

Hence from equation (6.15) the conduction loss is

$$C = 2.7 \text{ watts/amp.}$$

This is in good agreement with the generally accepted value of 2.68 W/A measured by Guntherschulze⁽⁴⁾ for the heat conducted through the cathode pool under free spot conditions occurring in mercury arc rectifiers.

Other measurements^(54,138) range from 1.5 W/A to 5 W/A according to the conditions under which the measurements were made. It is unfortunate that measurements of the cathode fall were not made simultaneously with the measurements of heat transfer, since under conditions in which the surface is strongly cooled, e.g. by stirring⁽⁵⁴⁾ or with low heat resistance anchors,⁽¹³⁸⁾ the heat loss from the cathode would be greater and to maintain the cathode temperature a greater heat input from the vapour region would be necessary. This could only be accomplished at the expense of an increase in the cathode fall in potential (see equation 6.6). The discrepancies which exist between the measurements of the cathode fall by different workers (see p. 6) may therefore be due to the different arrangements they used to cool the cathode.

The rate of supply of heat C necessary to maintain a circular area of radius a on the surface of a metal at a temperature T_o is derived in Appendix 4 and is given by

$$C = 4\Lambda a (T_o - T_s) \quad (6.16)$$

where Λ is the thermal conductivity of the metal and T_s is the temperature at infinity.

Consider a mercury arc spot carrying 5A. The temperature T_o

is about 500°C (see p.42), the spot radius $a = 4 \cdot 10^{-3} \text{ cm}$ and $\lambda = 0.019 \text{ cal/cm}^{\circ}\text{C}$ (see Appendix 1). Hence from (6.16) with $T_s = 50^{\circ}\text{C}$

$$G = 0.12 \text{ watts/amp}$$

This is smaller than the measured value of the heat loss by a factor 20. In dealing with the evaporation from the spot (Chapter 4) a similar discrepancy was encountered between the observed and calculated values of the net evaporation, and the difficulty was resolved by regarding the evaporation as taking place over an 'effective area' having a radius about ten times that of the arc spot. If now in equation (6.16) we take a to be about 20 times the radius of the spot, satisfactory agreement between theory and experiment is obtained.

It would seem therefore that both evaporation and heat conduction measurements are consistent with the assumption that the effective area of the cathode which is at a temperature of 500°C can be considered as having a radius of 10 - 20 times the radius of the spot. However the concept of such an 'effective area' must be treated with caution. Since it is probably due to the random motion of the spot over the surface, and to convection currents in the liquid, it probably does not represent a physically definable area, and for an accurate evaluation of the problem the heat loss by evaporation, conduction and convection from a moving spot would have to be calculated.

A possible process is the Thomson

effect, which may be described as follows. When a temperature difference exists between parts of a circuit carrying a current, in addition to the heat flow by thermal conduction

6.5. The heat loss from a copper cathode.

If we consider that the vapour pressure in the cathode spot of a copper arc is about the same as that in the mercury arc, that is about 10 atmospheres, the temperature of the evaporating area of the cathode would be about 3000°K . In Chapter 4 we showed that to account for the observed evaporation from a copper arc it was not necessary to assume that the evaporation took place from an area greater than the spot itself, so we need not consider an augmented area for the conduction problem.

If we take $a = 4 \cdot 10^{-3} \text{ cm}$ (for a 5A spot), $T_0 = 3000^{\circ}\text{K}$ and $\lambda = 0.9 \text{ cal/cm}^{\circ}\text{C}$ we find from equation (6.16)

$$C = 32 \text{ watts/amp.}$$

This value is absurdly large, since with the values $V_0 = 9\text{v}$ and $\phi = 4$ volts for copper, equation (6.7) gives no more than 5 watts/amp for the heat input to the cathode. It is necessary to assume the large value of T_0 in order to provide the vapour density necessary for the processes which occur above the cathode: furthermore the effective radius for conduction can hardly be less than the radius of the spot. We must therefore consider whether the simple conduction equation (6.7) is applicable to the transfer of heat in the cathode, or whether some other process affects the heat flow. A possible process is the Thomson effect, which may be described as follows.

When a temperature difference exists between parts of a circuit carrying a current, in addition to the heat flow by thermal conduction

and in addition to the Joule heating effect, there is a flow of heat associated with the current itself. If i is the current and T_1 and T_2 are the two different temperatures, the heat flow due to the Thomson effect is given by

$$Q = \sigma i (T_1 - T_2) \quad (6.17)$$

where σ is the Thomson coefficient. If σ is positive, heat is transferred in the direction of the positive current, if σ is negative the heat transfer is in the opposite direction. Since σ varies with temperature equation (6.17) should be written

$$Q = i \int_{T_1}^{T_2} \sigma dt \quad (6.18)$$

Few measurements exist of the Thomson coefficient for metals at high temperatures. (139) It would appear however that for copper is positive, at least up to 1500°K . This means that in the case of the copper arc cathode, the Thomson effect assists the heat flow from the spot. The average value of σ for copper in the range 300 to 1500°K is about $+5 \cdot 10^{-6}$ watts/amp $^\circ\text{C}$. Hence with this value in equation 6.17 and $T_1 - T_2 \approx 3000^\circ\text{C}$ we have

$$Q = 1.5 \times 10^{-2} \text{ watts/amp.}$$

Even if at higher temperatures σ for copper assumed a negative value, which is not entirely impossible, the magnitude of the Thomson effect is far too small to influence the conduction of heat from the spot. A theory of the arc cathode put forward by C.G. Smith⁽²²⁾ assumed that

large temperature differences could be maintained by the Thomson effect at the expense of a small heat flow, but the theory was based on a misunderstanding of the effect.

A possible reason for the large value of C calculated for copper from equation (6.17) may be the value chosen for Λ . The value $0.9 \text{ cal/cm}^{\circ}\text{C}$ is the generally accepted value for solid copper at normal temperatures. In the vicinity of the cathode spot, if temperatures of about 3000°K exist, the copper will be liquid and it is likely that the thermal conductivity of the liquid is somewhat lower than of the solid. Since the thermal resistance of the cathode is mainly in the vicinity of the spot, this would reduce the heat flow. In order that C should be about 5 watts/amp, Λ would have to be about $0.15 \text{ cal/cm}^{\circ}\text{C}$, but whether the conductivity decreases by as much as a factor 6 when the copper melts is open to question. It seems however that this is the most likely explanation of the discrepancy in the heat flow calculations for the copper cathode.

CHAPTER 7

THE CONTRACTION OF THE SPOT

7.1. The Glow to Arc Transition.

A comprehensive account of the experiments in which the evolution of an arc from a glow discharge has been observed and a summary of current theories, may be found in Ref. 68.

For the purpose of this chapter the glow to arc transition will be described with the aid of a voltage-current diagram (Fig. 39). Consider a normal glow discharge between metal electrodes in a gas at a few mm Hg pressure. If the pressure is kept constant and the current increased, the current density at the cathode will be constant, so that the area of the cathode covered by the glow increases. The voltage across the discharge remains constant and the glow discharge is called normal. On the voltage-current curve it is represented by AB. At B the glow has covered the cathode completely, and a further increase in the current causes an increase in the current density at the cathode and an increase in the voltage. This is the abnormal glow discharge and represented by BC. If the current is increased beyond C an arc develops, as described on p. 2 and the current is carried by a small spot on the cathode instead of the entire surface. The arc, which has a negative V-i characteristic, is represented in the figure by DE.

The transition CD is usually discontinuous if the cathode is a plane surface, and takes place within a time interval of 10^{-4} sec if the

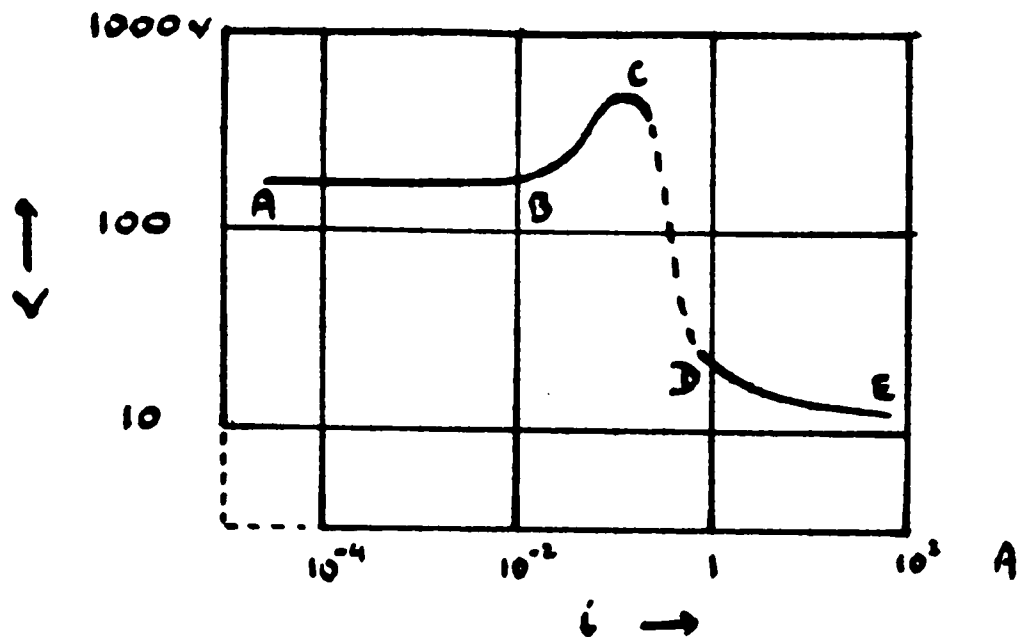


Fig 39. VOLTAGE-CURRENT DIAGRAM FOR GLOW-ARC TRANSITION.

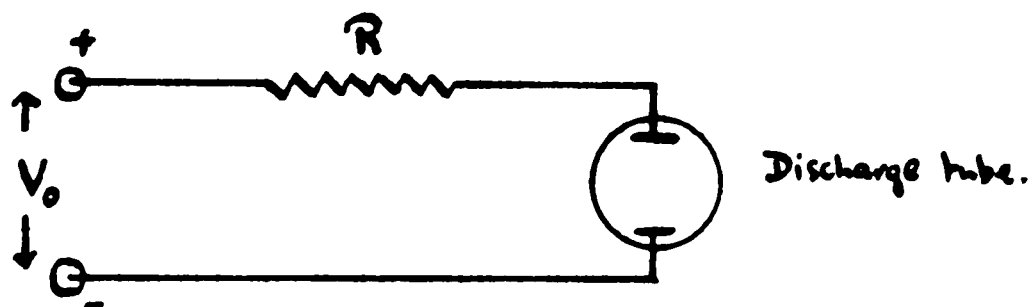


Fig 40. CIRCUIT FOR OBSERVING GLOW-ARC TRANSITIONS.

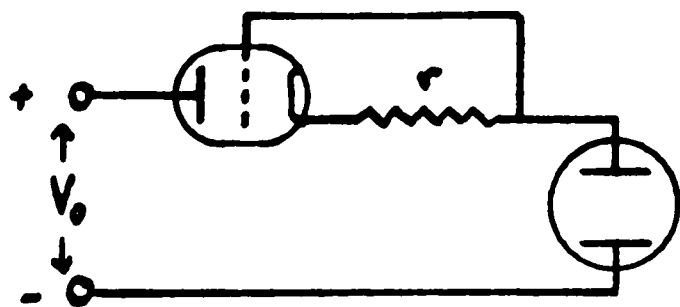


Fig 41. PRINCIPLE OF CURRENT STABILISER.

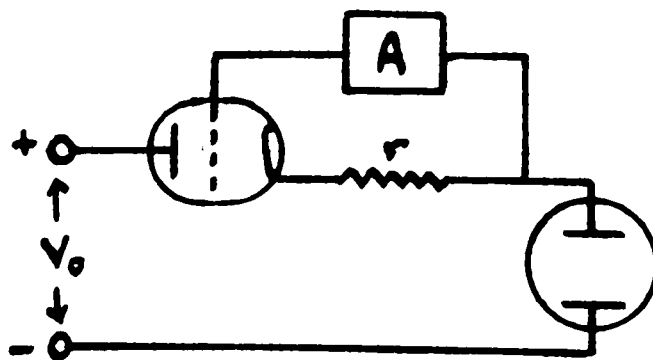
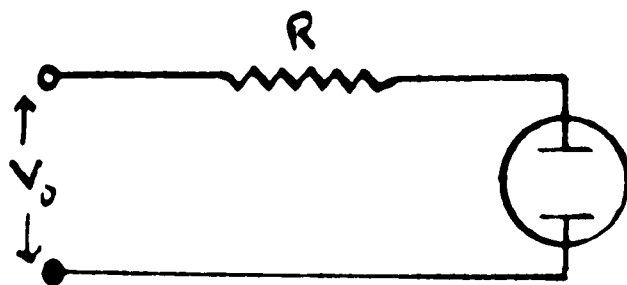


Fig 42. CURRENT STABILISER WITH FEEDBACK AMPLIFIER.



Circuit.

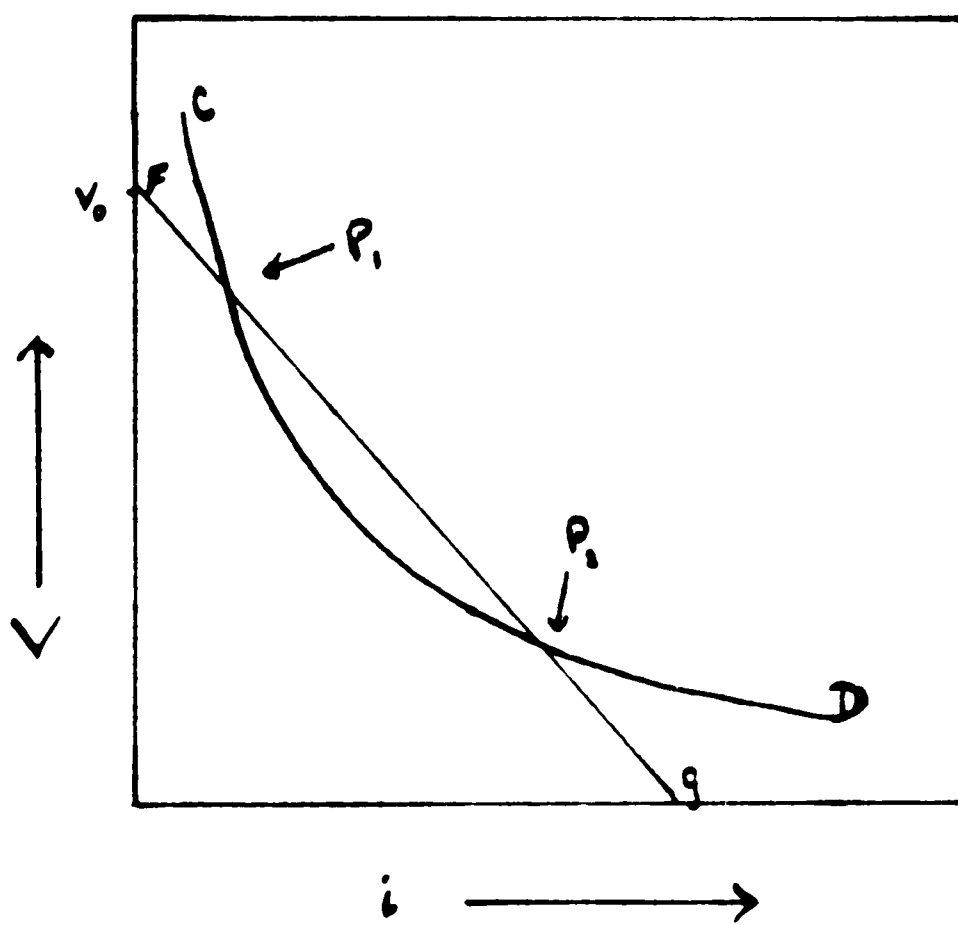


FIG 40A. EQUILIBRIUM OF CIRCUIT
(Kaufmann's Criterion)

resulting arc is thermionic, or 10^{-8} sec if it is a cold cathode arc. (129)

If however the cathode is a small tungsten sphere on the end of a thin rod it is possible to obtain a continuous glow to arc transition. (3, 66, 79)

The discharge may be maintained at points on the curve between C and D, and in this condition part of the current at the cathode is carried by the glow discharge mechanism and part by the arc mechanism, which in this case is thermionic emission. No such continuous transition has ever been observed in the case of a cold cathode arc, and the smallest increase of current beyond C immediately results in a fully developed arc at D.

A typical circuit arrangement for observing glow to arc transitions is shown in Fig. 40. The discharge tube is connected to a source of potential V_0 through a resistor R. The current through the tube is determined as follows: in Fig. 40A CD represents the V-i characteristic of the discharge. The line FG has the equation

$$V = V_0 - iR \quad (7.1)$$

and represents the voltage available at the discharge tube when a current i is being drawn. The currents for which the system is in equilibrium are given by the points P_1, P_2 where FG cuts CD. The equilibrium is stable if the slope of FG at the point of intersection is steeper than the slope of CD, as at P_1 and unstable if the slope of CD is steeper than the slope of FG as at P_2 (see refs. 125, 126). The current in the circuit can be altered by varying V_0 or R, which alters the position of P_1 on the V-i curve.

The discharge characteristic for the transition from the glow to the cold arc is very steep (Fig. 39) so in order to obtain points of stable equilibrium between C and D the line FG must be made even steeper. This can only be done with the arrangement of Fig. 40 by making R very large, and in order to obtain the necessary current, V_0 would need to be very large also.

In order to overcome the difficulty we constructed an electronic current stabiliser which had a very high impedance and thereby gave effectively a large R without the need for a large V_0 . With this device we attempted to obtain a stable discharge intermediate between a glow and cold arc.

7.2. The current stabiliser.

7.3. The principle of current stabilisation is illustrated in Fig. 41. A triode valve is connected in series with the source of potential and the discharge tube. The voltage developed across a resistor r in the cathode lead is fed back to the grid of the valve. If the current in the circuit rises by a small amount δi the voltage of the grid with respect to the cathode becomes more negative by an amount $r \delta i$. The voltage across the valve therefore increases by $\mu r \delta i$ where μ is the amplification factor. The impedance R of the circuit is therefore

$$R = r_a + (\mu + 1)r \quad (7.2)$$

where r_a is the anode impedance of the valve.

The impedance R may be increased by including a d.c. amplifier

in the feedback to the grid (Fig. 42). If the amplification of the d.c. amplifier is A, the impedance of the circuit is then given by

$$R = r_a + (A\mu + 1)r \quad (7.3)$$

To obtain the maximum impedance it is preferable to use a pentode valve, since r_a and μ are larger for a pentode than a triode. In the final circuit arrangement shown in Fig. 43, the stabiliser valve consisted of three 813 pentodes in parallel and the d.c. amplifier was a 6AC7. The stabilised current could be varied by altering the bias on the grid of the d.c. amplifier, and when set at any given current the impedance of the arrangement was estimated as about 10^7 ohms. The current could be adjusted to any value between 50 and 500 mA.

7.3. Experiments with the stabiliser.

The stabiliser was connected in series with a source of 2kV and a discharge tube. This tube is drawn in Fig. 44, and consisted of a mercury cathode and an iron anode in an evacuated pyrex envelope. The space between the electrodes contained only mercury vapour, the pressure of which was derived by measuring the temperature of the wall of the tube with a copper-constantan thermo-couple.

The experimental procedure was as follows. The tube was heated in a flame to a temperature of about 200°C , corresponding to a mercury vapour of about 20 mm Hg. The voltage was applied with the stabiliser set for low current, and a normal glow discharge formed. At this pressure the positive column was highly contracted. The current was

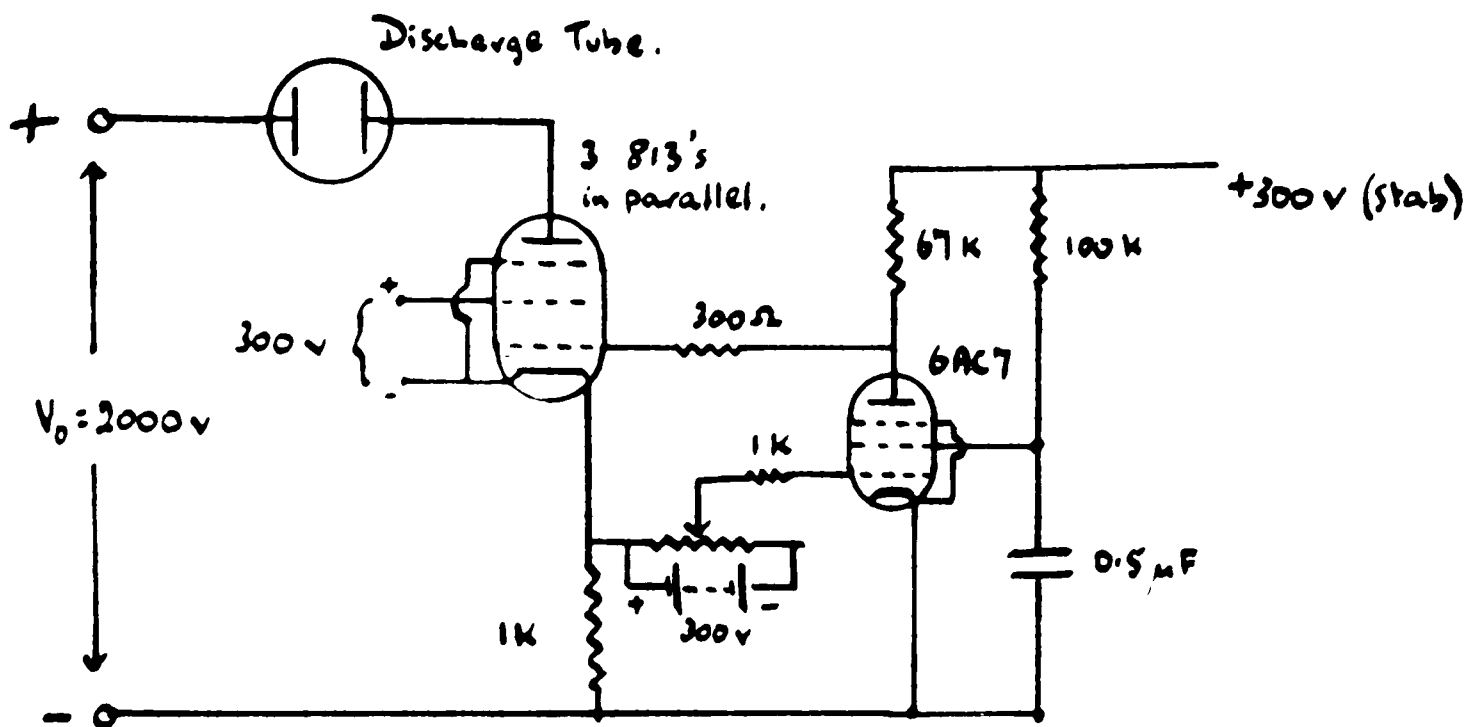


FIG 43. FINAL CIRCUIT OF STABILISER.

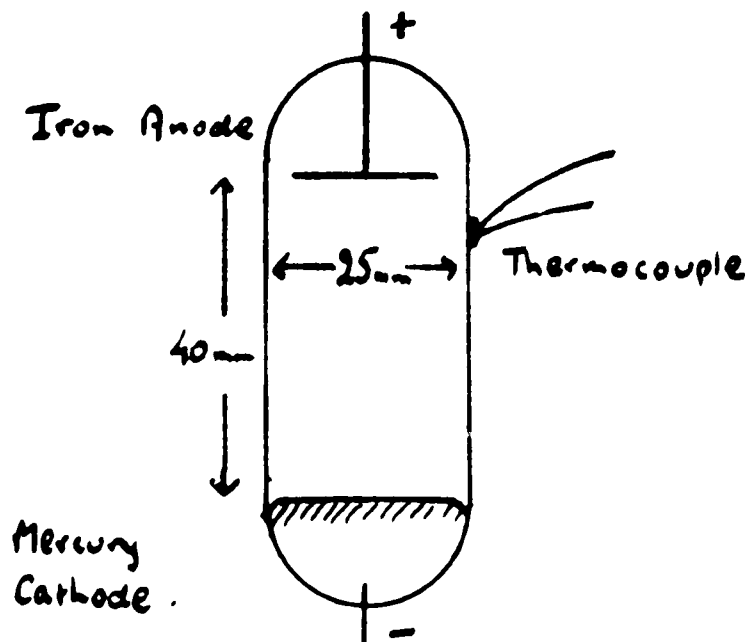


FIG. 44 . EXPERIMENTAL DISCHARGE TUBE.

increased until the glow covered the entire cathode, and on increasing the current still further an arc formed. At this stage transitions from glow to arc and back again took place spontaneously. Since the spot was free and the current was only about 100 - 200 mA the lifetime of the arc spot was short (see later) and as soon as the arc extinguished the glow discharge re-struck. It was found that the current at which glow to arc transitions occurred decreased as the vapour pressure decreased. Some results are given in Fig. 45 where the critical current is plotted against vapour pressure. In the shaded region glow/arc transitions took place spontaneously. Below this region only a glow discharge existed, and above this region there was only an arc.

It was not possible to obtain a continuous transition between a glow discharge and an arc. The failure may have been due to the inability of the stabiliser to follow the rapid change of voltage in the transition, since as this takes place in 10^{-8} sec the amplifier would need to have a frequency response up to 100 Mc/s to follow it. The stabiliser was probably not sensitive at such a high frequency. However it seems more likely that the very nature of the transition from a glow discharge to a cold arc renders a continuous and controlled transition impossible.

The experiments were however of some value. If the apparatus was adjusted until glow to arc transitions took place spontaneously it was possible to observe the difference between a glow discharge and an arc for the same current and vapour pressure. Previous experiments which have been made with circuits of the type shown in Fig. 40 always involved a change in current when the transition took place, and so a direct comparison

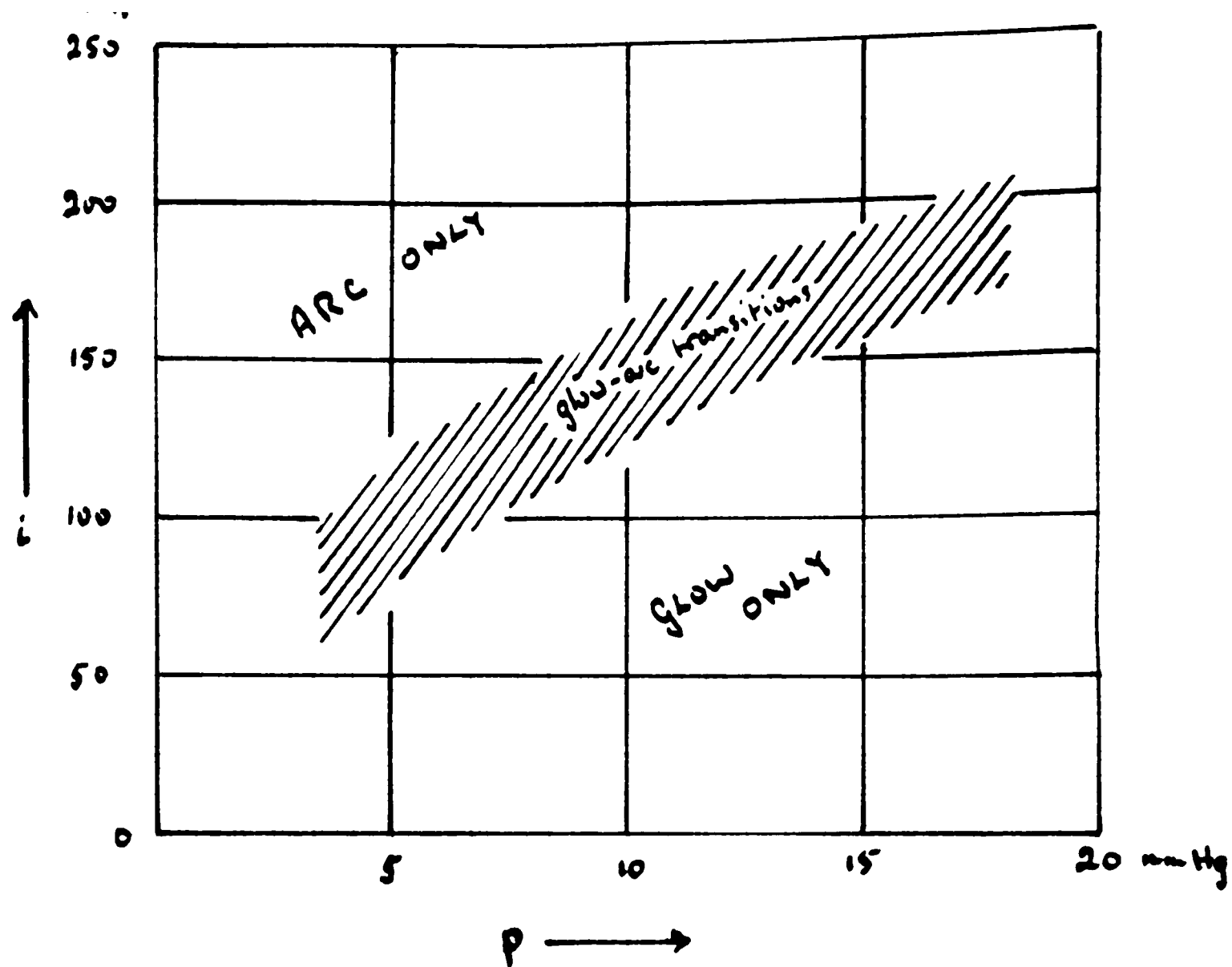


FIG 45. CRITICAL CURRENT AT WHICH GLOW DISCHARGE BECOMES AN ARC, AS A FUNCTION OF THE VAPOUR PRESSURE.

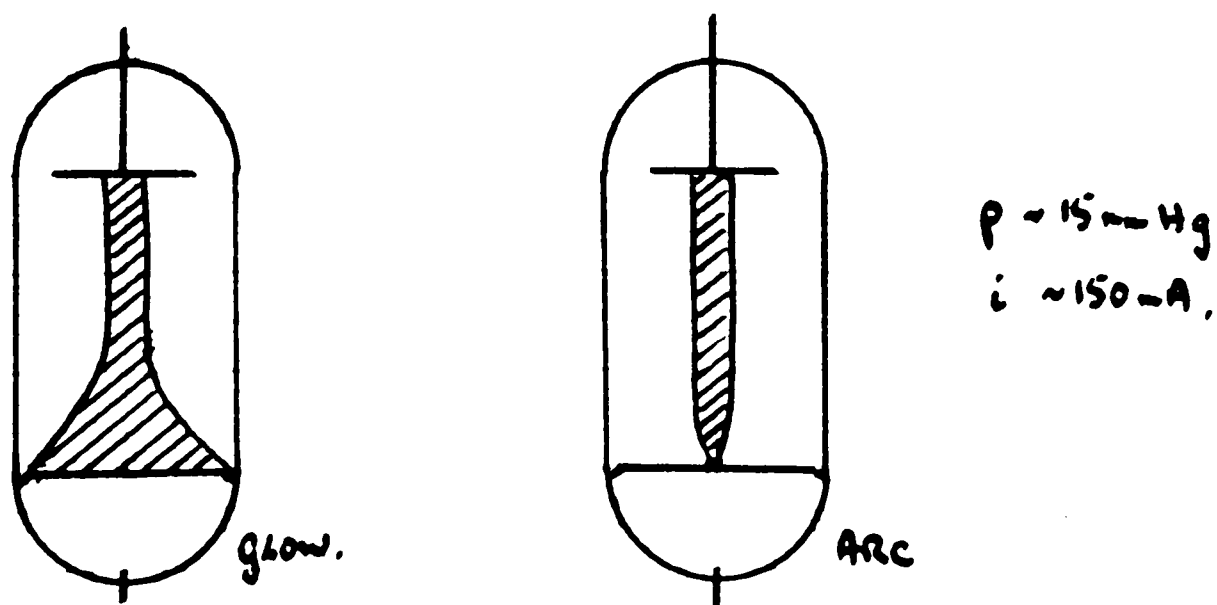


FIG 46. GLOW AND ARC DISCHARGES AT THE SAME CURRENT AND VAPOUR PRESSURE.

of the two discharges at the same current has not been made.

The appearance of the two discharges is shown in Fig. 46. The difference between them lies in the cathode region only: the constricted positive column and the anode region were identical in both cases. This establishes that the glow-arc transition is a phenomenon connected only with the cathode of the discharge.

7.4. Description of the glow-arc transition.

Let us now consider the processes involved in the glow-arc transition in the light of the new arc mechanism which has been proposed in the previous chapters.

In the glow discharge the current at the cathode is carried almost entirely by positive ions. When the ions reach the cathode they are neutralised and a small fraction release electrons: the majority give up their potential and kinetic energy to the cathode as heat. In the normal glow discharge both cathode fall and current density remain constant as the current is increased, and hence the heat input per unit area of the cathode remains constant. When the glow becomes abnormal both current density and cathode fall increase, and hence the heat input per unit area of the cathode is increased.

Consider first a discharge with a mercury cathode and running in mercury vapour, as described in the previous section. As the current in the abnormal glow is increased, two effects will occur at the cathode. Owing to the increased heat input the cathode will evaporate and the density of mercury vapour near the surface will increase. Also the

electric field at the cathode will increase. On account of the increased field, the distance over which the first 5v of the cathode fall occurs will be reduced, and electrons from the cathode will acquire sufficient energy to excite vapour atoms closer to the cathode. On account of the increased density of atoms the probability that an electron will excite an atom close to the cathode is increased. Since the cathode fall of the glow discharge is large the electron may produce several excited atoms close to the cathode, and if some of these diffuse back to the cathode surface before having time to radiate their energy, they will release electrons.

Once conditions have been established at one point on the cathode surface such that each electron from the cathode produces sufficient excited atoms in the vapour to return to the cathode and produce there more than one electron, a multiplication will occur and the spot where the process started will emit an increasing electron current.

If the current in the external circuit is kept constant, the glow discharge mechanism will be required to supply less current and hence the discharge will contract about the spot. The evaporation at the spot will increase, and on account of the higher vapour density radiation will be 'trapped' and the diffusion of excited atoms to the cathode will be more likely. Since the emission of electrons by excited atoms is a more efficient process than the glow discharge mechanism, the cathode fall will decrease, until the discharge assumes the high current density and low cathode fall of a fully developed arc.

The variation with pressure of the critical current at which

the glow goes over into the arc (Fig. 45) can now be understood. The heat input to unit area of the cathode is roughly proportional to jV_c where j is the current density and V_c the cathode fall. Since the glow discharge is abnormal, the cathode fall V_c is roughly proportional to j/p^2 , where p is the pressure. The heat input per unit area is therefore proportional to j^2/p^2 . The lower the pressure, the lower will be the current necessary to supply the cathode with sufficient heat to vaporise it, and hence the lower the current at which the glow goes over into the arc.

Consider now a glow discharge with a copper cathode running in air at a pressure of a few mm Hg. The transition to an arc is in this case slightly different to the case of the discharge with a mercury cathode in mercury vapour. The electrical processes of the normal glow discharge take place in the gas which fills the discharge tube, and no vapour from the cathode is present (this can be confirmed spectroscopically). When the glow is made abnormal, cathode sputtering occurs and atoms of the cathode material enter the discharge. Details and theories of cathode sputtering⁽¹⁴⁹⁾ will not be discussed here, but it is sufficient to regard the phenomenon as intense localised evaporation of cathode metal. Since the excitation and ionisation potentials of copper vapour are considerably lower than the corresponding potentials of air, the evaporated copper atoms will have a greater probability of being excited close to the cathode and hence of diffusing to the surface without having radiated their energy. Once a small spot on the cathode has developed where sufficient excited copper atoms are diffusing back to

cause a multiplication of the electron current, the discharge will contract about this spot. The increasing heat input at the spot will evaporate more vapour and so on until the arc is fully developed. The arc then runs in copper vapour which virtually excludes the gas from the cathode region, as is evidenced by the spectrum of the arc.⁽¹⁵⁰⁾ This account is consistent with the experiments of Flesse⁽¹⁹⁵⁾ who showed that an arc could be formed by directing a stream of metal vapour onto the cathode of a glow discharge.

The foregoing description is a qualitative account of how an arc which is sustained by excited-atom emission can develop from a glow discharge. When the arc is fully developed the current density at the cathode is found to be about 10^5 A/cm^2 , and in the previous chapters we have shown that the proposed mechanism is valid at this high value. We have not however discussed the reason why the arc, when it contracts from the glow, assumes this particular density. It must be that at about 10^5 A/cm^2 the processes tending to contract the arc are balanced by the processes resisting further contraction. The equilibrium value of the current density will now be discussed.

7.5. Radial equilibrium.

The current i , the current density j and the radius of the arc spot a are related by the equation

$$i = \pi a^2 j \quad (7.4)$$

There are two ways of approaching the problem of the equilibrium value of j . The first way is to consider the effects which act on the

discharge in the radial direction tending to constrict it, and find the radius a at which these effects are balanced by other effects acting in the radial direction but in the opposite sense tending to expand the discharge. The current density at equilibrium can then be calculated from a , i and equation (7.4).

The second approach is to consider the processes occurring in the vicinity of the cathode, where as we have shown, the problem is one of particles flowing in the axial direction only. If it can be shown that the arc will be self-sustaining only for a certain value of j , and if i is fixed by the circuit constants, then the spot must assume a radius a given by equation (7.4).

We shall consider the problem first in terms of radial effects.

A powerful contracting process at high current density is the force on the electrons due to the self-magnetic field of the arc current. This is sometimes referred to as the 'pinch effect' and is observed in both liquid and gaseous conductors. (127,128) This is illustrated in Fig.47. Consider a long cylindrical discharge in which the densities of ions and electrons are approximately equal. The direction of the current is along the axis of the cylinder and the current will be carried almost entirely by the electrons.

If the current density varies across the radius of the discharge and has the value j_r at radius r , then the current enclosed by the cylinder of radius r is given by

$$i_r = \int_0^r 2\pi j_r r dr \quad (7.5)$$

Ions and electrons moving perpendicular to H_ϕ are deflected towards the axis.

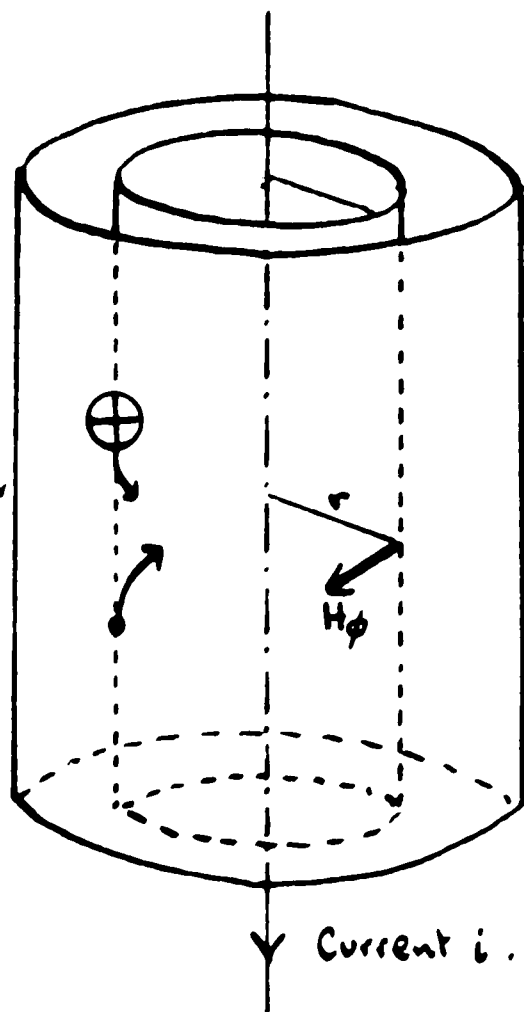
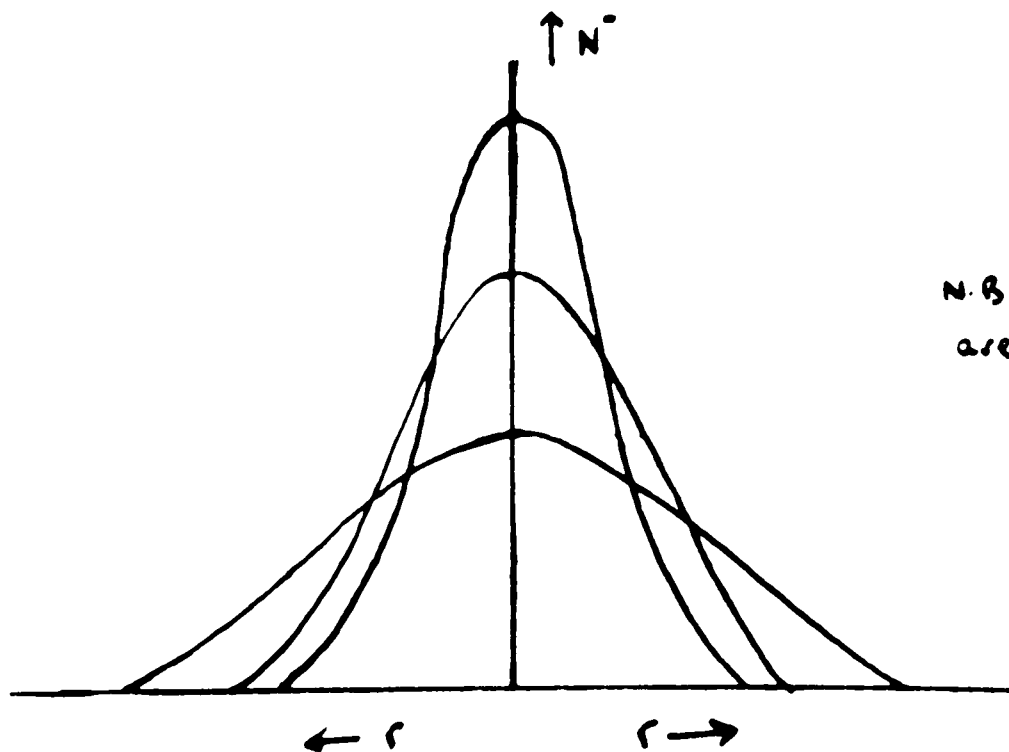


FIG 47. THE PINCH EFFECT IN A DISCHARGE.



N.B. These curves are approximate.

FIG 48 DISTRIBUTION OF CHARGED PARTICLES AS A FUNCTION OF THE RADIUS FOR DIFFERENT VALUES OF N_0^- .

At a radius r from the axis there will therefore be an azimuthal magnetic field H_ϕ given by

$$(7.5) \text{ is given } H_\phi = \frac{2i_r}{r} = \frac{4\pi}{r} \int_0^r j_r r' dr' \quad (7.6)$$

Electrons carrying the current move perpendicular to this magnetic field, and so experience a radial force which is directed towards the axis. The force f on an electron is

$$\text{pressure gradient } f = \frac{H_\phi e v}{c} \quad (7.7)$$

where v is the electron drift velocity in the axial direction. The positive ions will experience a similar force towards the axis, but as their drift velocity is small compared with the electron drift velocity the electromagnetic force on the ions will be small. However on account of the strong electrostatic attraction which exists between the ions and electrons at large charge densities, the force on the electrons will be shared by the ions also. The electromagnetic 'pinch effect' will therefore tend to move charges of both signs towards the axis, that is to contract the discharge.

Consider the annular region formed by the cylinders of radii r and $r + dr$. If N^- is the density of electrons (and ions) at radius r , the current density is given by

$$j_r = N^- e v \quad (7.8)$$

Hence the pressure difference across the walls of the annular region due to the self-magnetic field is

$$dp = \frac{N^- e v}{c} \cdot H_p dr \quad (7.9)$$

This represents a pressure gradient which, from equations (7.6) (7.8) and (7.9) is given by

$$\left(\frac{dp}{dr}\right)_{e.m.} = \frac{4\pi N^-}{r} \cdot \left(\frac{ev}{c}\right)^2 \cdot \int_0^r N^- r dr. \quad (7.10)$$

This radial pressure gradient will tend to increase the density of charged particles on the axis. In equilibrium it will be balanced by the radial pressure gradient due to the radial density distribution of charged particles, which will tend to expand the discharge by diffusion. This diffusion pressure gradient is given by

$$\left(\frac{dp}{dr}\right)_{diff} = \frac{d}{dr} (N^- kT) = kT \frac{dN^-}{dr} \quad (7.11)$$

where T is the electron temperature, which is taken to be independent of r.

In equilibrium no net flow of particles takes place in the radial direction, hence

$$\left(\frac{dp}{dr}\right)_{e.m.} + \left(\frac{dp}{dr}\right)_{diff} = 0 \quad (7.12)$$

From (7.10) (7.11) and (7.12) we have

$$kT \frac{dN^-}{dr} = - \frac{4\pi N^-}{r} \cdot \left(\frac{ev}{c}\right)^2 \cdot \int_0^r N^- r dr. \quad (7.13)$$

which leads to the differential equation

$$\frac{d}{dr} \left(\frac{r}{N^-} \cdot \frac{dN^-}{dr} \right) = - Ar N^- \quad (7.14)$$

where A is a constant given by

$$A = \frac{4\pi}{kT} \cdot \left(\frac{ev}{c}\right)^2 \quad (7.15)$$

The boundary conditions of equation (7.14) are $N^- = N_0^-$ and $\frac{dN^-}{dr} = 0$ at $r = 0$. The solution is

$$\frac{N^-}{N_0^-} = \frac{1}{\left(1 + \frac{A}{8} N_0^- r^2\right)^2} \quad (7.16)$$

The radial distribution of N^- is therefore dependent on the value of N_0^- at $r = 0$. Some curves are sketched in Fig. 48 for different values of N_0^- . The total current in the discharge is

$$i = \int_0^\infty 2\pi ev N^- r dr = \frac{8\pi ev}{A} \quad (7.17)$$

The total current is thus independent of N_0^- , ~~that is to say the area under each of the curves of Fig. 48 is the same.~~ Each of the curves represents a distribution in which the effect of the self magnetic field is balanced by the radial density gradient of charged particles. For a given current an infinite number of distributions satisfying this condition are possible, depending on the value chosen for N_0^- .

The radial balance therefore can decide only the shape of the distribution curve: if the current density is high the slope of the sides of the curve will be steep, and vice versa. The equilibrium value of the current density however is still left undetermined by this treatment.

7.6. The criterion for a self-sustained arc.

Since it appears that a radial balance does not lead to an

equilibrium value of the current density, let us now consider the processes which occur in the axial direction in the vicinity of the cathode.

We start by assuming that electrons are released from the cathode by excited atoms. These electrons produce new excited atoms in the vapour, which diffuse back to the cathode and there release more electrons. The arc will be in equilibrium when one cathode electron produces just sufficient excited atoms to release one further electron from the cathode. It is not proposed to make a detailed quantitative evaluation of the equilibrium current density. We shall simply suggest in qualitative terms how a stable equilibrium value might be achieved.

From equation (5.37) on p.89 we see that the flow of excited atoms to unit area of the cathode is given approximately by

$$F_c^* = \frac{A}{N} \cdot \frac{N_2^*}{x} \quad (7.18)$$

where x is the extension of Region 1, N_2^* is the density of excited atoms in Region 2 and N the density of neutral vapour.

If the current density j is increased, the density of charged particles in Region 2 will also increase, although not necessarily in proportion, since the electron drift velocity might also be affected (for example, by a change in the electric field). Since the excited atom density N^* is proportional to the density of ions and electrons (see p.87), N_2^* will also rise when j increases.

If we assume the positive ion current density to increase in proportion to the total current density, we see from equation (1.3) that x will decrease as j increases. We may therefore write

$$\frac{N_1}{x} \propto j^b \quad (7.19)$$

where b probably lies between 1 and 2.

As j increases therefore, the density gradient of excited atoms in Region 1 will probably increase more than linearly with j , tending to increase the flow of excited atoms to the cathode.

As j increases however the cathode temperature will rise. Since this determines the density of neutral atoms, N will rise also. From equation (7.18) we see that this will tend to reduce the flow of excited atoms to the cathode. Since the relation between vapour density and temperature is of the form

$$N \propto e^{-B/T} \quad (7.20)$$

where B is a constant, a small increase in T may result in a large increase in N . This should counteract the increase of the density gradient with j (equation 7.19) and from equation (7.18) we should therefore expect F_c^* to increase less than linearly as j increases. In Fig. 49 F^* is plotted as a function of j . The curve A represents approximately the flow F_c^* of excited atoms to unit area of the cathode as a result of the current density j passing through the vapour. This curve is concave towards the axis of j . The straight line B represents the current density j which is released from the cathode by the flow F_c^* of excited atoms. The equilibrium value of the current density is determined by the point of intersection of A and B. By considering a small displacement from equilibrium it can be seen that, as the slope of B is greater than the slope of A at

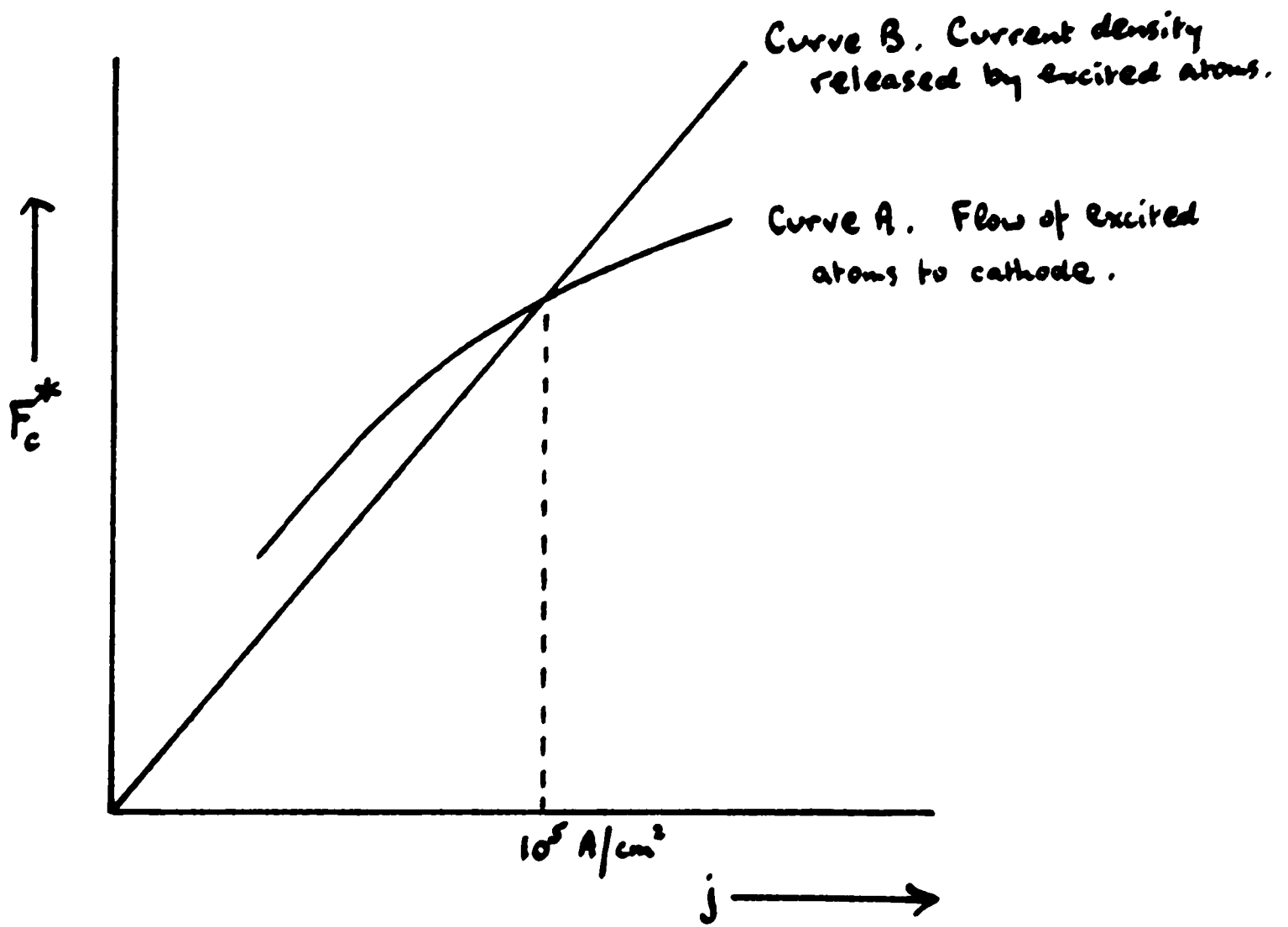


FIG 49. STABILITY CONDITION OF SELF-SUSTAINED ARC .

this point, the equilibrium is stable.

It seems likely therefore that a more rigorous mathematical treatment might enable the curve A to be derived from the constants of the vapour, and hence the point of equilibrium might be accurately calculated. At this stage however we can do no more than indicate the method by which the equilibrium can be determined.

The probable origin of the positive action can be described qualitatively as follows.

In Chapter 3 it was explained that the pressure of the vapour in the spot exerts a downward force on the cathode. In the case of mercury this results in a depression of the liquid surface. It has been shown⁽⁷⁾ that the spot does not depart from the surface if it is moving over the surface faster than the rate of depression. This is presumably because the force of the arc does not act on any one place for sufficient time to displace the mercury. If however the spot takes a rest in the liquid, it can be seen from Fig. 3 that for the arc to remain vertical in the dent is a position of unstable equilibrium. A small displacement results in a vapour blast which will drive the dent out of the spot. If it leaves the dent at over 10° to the vertical it will not depress the mercury until it has slowed down, when it will form another dent and the process will be repeated. Since on each occasion it is driven out in a completely random direction, this results in the observed erratic action over the cathode surface.

OTHER FEATURES OF THE ARC

8.1. The Random Motion of the Cathode Spot.

A feature of nearly all cold-cathode arcs is the random motion of the cathode spot over the surface of the cathode. This phenomenon is most pronounced in the mercury arc and makes detailed observation of a free spot very difficult. The probable origin of the random motion can be described qualitatively as follows.

In Chapter 3 it was explained that the pressure of the vapour in the spot exerts a downward force on the cathode: in the case of mercury this results in a depression of the liquid surface. It has been shown⁽⁷⁾ that the spot does not depress the mercury if it is moving over the surface faster than about 10^3 cm/sec. This is presumably because the force of the arc does not act at any one place for sufficient time to displace the mercury there. If however the spot forms a dent in the liquid, it can be seen from Fig. 50 that for the spot to remain central in the dent is a position of unstable equilibrium. A small displacement results in a vapour blast which will drive the spot out of the dent. If it leaves the dent at over 10^3 cm/sec it will not depress the mercury until it has slowed down, when it will form another dent and the process will be repeated. Since on each occasion it is driven out in a completely random direction, this results in the observed erratic motion over the cathode surface.

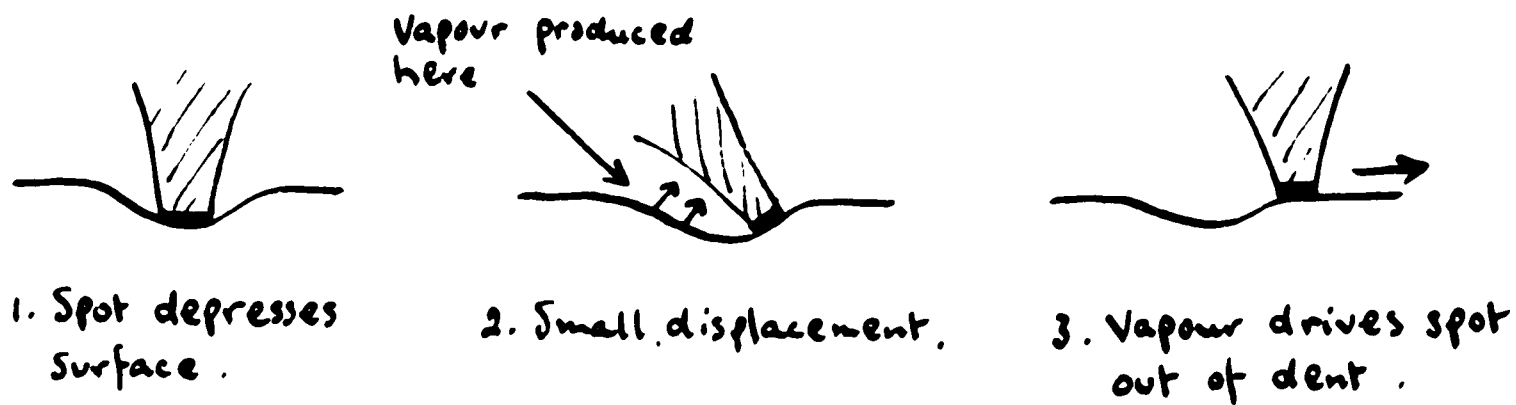


Fig. 50. ORIGIN OF THE RANDOM MOTION.

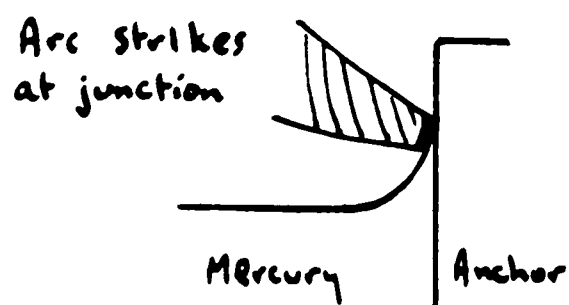


Fig 51. ANCHORED SPOT.

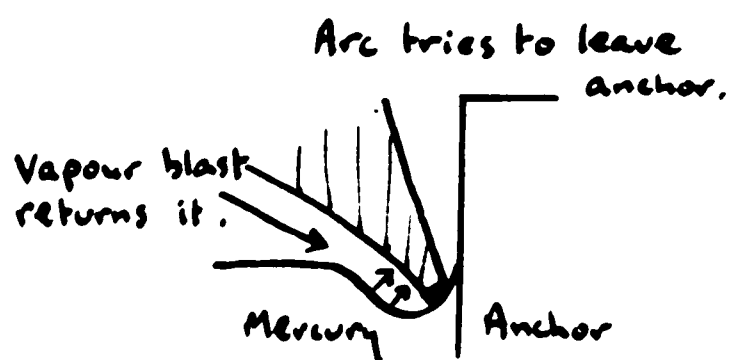


Fig 52. ORIGIN OF ANCHORING.

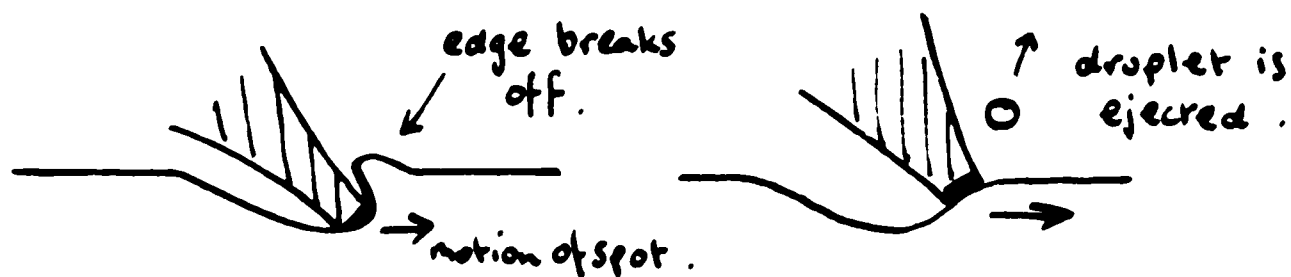


Fig 53. ORIGIN OF DROPLETS.

If it were possible to hold the spot stationary on the liquid surface it would dig in to a great depth, since the pressure at the surface is probably of the order of 10 atmospheres. On account of the random motion however, it never stays sufficiently long at one place to dig in very far. It has been found that under high current conditions such as occur in large rectifiers, the spot may dig into the liquid up to depths of about 12 mm, ⁽¹³⁸⁾ but for a low current the depression is usually no more than a millimetre or two.

When a spot 'anchors' at the junction of mercury and another metal, it strikes on a thin layer of mercury on the surface of the anchor (Fig.51). Since the latter is solid, the arc cannot depress the surface appreciable. If however, the spot tries to leave the anchor it will depress the mercury surface in such a way as to produce vapour blasts which tend to drive it back onto the anchor, as is shown in Fig.52. Once the spot has struck on the anchor it will therefore stay there: its only degree of freedom will be along the 'wetting line' of the mercury and the anchor, and it is observed that the spot is constantly in motion along this line. It is significant that a spot will only stay on those parts of an anchor which are wetted by the mercury. A thin layer of mercury on a solid base appears to be essential, which is consistent with this explanation of the phenomenon.

When the spot is free it is found that mercury droplets are ejected from the surface, and that these droplets account for most of the loss of mercury from the cathode. A possible explanation of this droplet evaporation is illustrated in Fig.53. When the spot depresses the

mercury, and is then driven out of the dent by the vapour, the edge of the dent may be broken off and driven upwards by the vapour stream. The droplets would seem therefore to be a consequence of the random motion of the spot. This is supported by the fact that when the spot is anchored, no droplets are ejected. Also on the above picture we should expect that the maximum radius of the droplets would be no larger than the radius of the spot. It is found experimentally⁽¹⁴⁾ that the radii of the droplets vary from minute values up to about 10^{-2} cm. The upper limit represents the radius of a typical arc spot carrying 8-10A, and is therefore consistent with the proposed explanation.

8.2. The reignition time.

Consider If the current in the thermionic arc is interrupted for as long as 0.1 sec the arc will strike again as soon as the circuit is closed, since after this time interval the temperature of the cathode hot spot is still sufficient to support thermionic emission. A thermionic arc can therefore be run from ordinary A.C. supply mains.

If however the current in a mercury arc is interrupted for as little as 10^{-8} sec, the arc will go out and not reignite. According to the theory described in the previous chapters, this arc depends for its maintenance on the back diffusion of excited atoms in the dense vapour of the spot. We must therefore consider how conditions in the spot will change within 10^{-8} secs of the current being switched off.

It has been shown in Chapter 6 that the loss of heat from the hot area on the cathode surface takes place mainly by conduction through

the cathode, the heat loss by evaporation being small in comparison. Consider now an area of radius a on the cathode surface and assume for the purpose of this calculation that the hemisphere in the liquid of this radius is at a temperature T . (see Fig. 54). In the steady state, heat is supplied to this hemisphere by the discharge and lost by conduction through the cathode. The rate of flow of heat is C .

If the specific heat of the cathode is s and the density ρ , the heat capacity of the hemisphere is $\frac{2}{3}\pi a^3 \rho s$. When the heat input ceases the initial rate of loss of heat will be C . The initial rate of fall of temperature is therefore

$$\frac{dT}{dt} = - \frac{3C}{2\pi a^3 \rho s} \quad (8.1)$$

Consider now an arc spot carrying 5A. The radius a will be about $4 \cdot 10^{-3}$ cm, the conduction loss for a mercury arc is about 2.7 watts/amp so $C = 13.5$ watts = 3.2 calories/sec. For mercury $\rho = 13.6$ gm/cm³ and $s = 3.28 \times 10^{-2}$ cal/gm°C. With these values in equation (8.1) we obtain

$$\frac{dT}{dt} = -5 \cdot 10^7 \text{ } ^\circ\text{C/sec.}$$

Thus in 10^{-8} sec after the heat supply has ceased, the temperature of the cathode hot spot will have fallen by less than one degree centigrade.

Furthermore, the value for a substituted in (8.1) was the actual radius of the arc spot, and it has been shown in the previous chapters that in considering the problem of heat transfer it is necessary to assume that the hot spot on the cathode has a radius about ten times larger. If now we put $a = 4 \cdot 10^{-2}$ cm in equation (8.1) we have

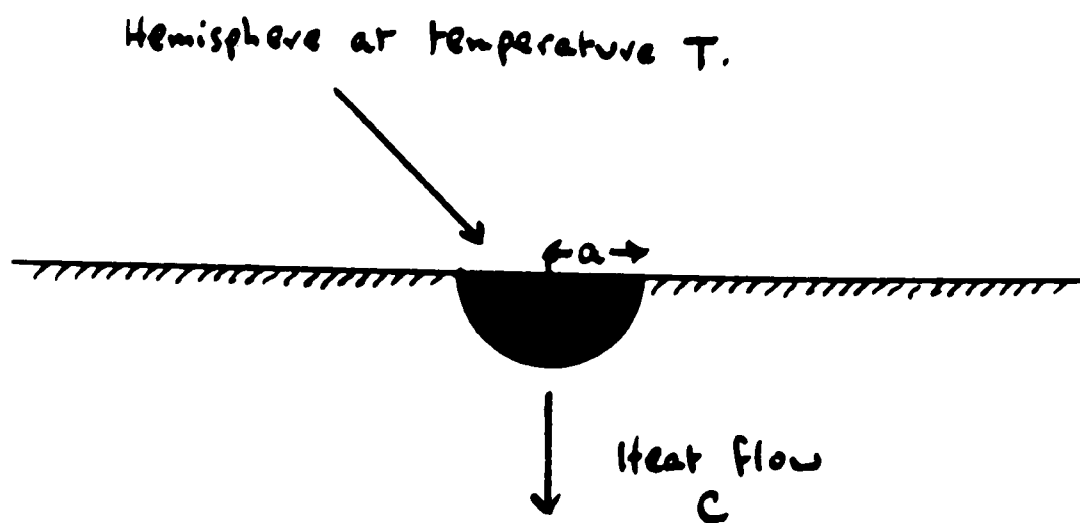


FIG 54. COOLING OF CATHODE HOT SPOT.

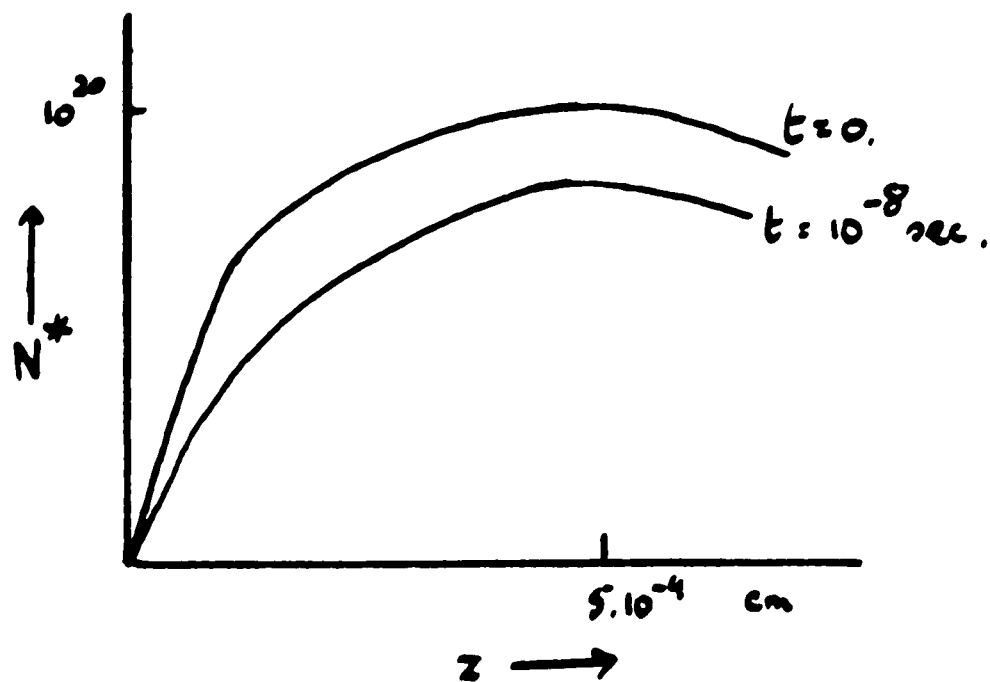


FIG 55. DECREASE IN EXCITED ATOM CONCENTRATION AFTER 10^{-8} sec.

after 10^{-8} secs that the change in the excited atom concentration will be sufficient to prevent the arc from re-igniting.

$$\frac{dT}{dt} = -5.10^{40} \text{ C/sec.}$$

It appears therefore that within 10^{-8} secs of interrupting the heat input, the temperature of the hot spot, and hence the density of vapour above the cathode will remain unchanged. The short reignition time is therefore not due to the dispersal of the neutral vapour cloud.

Consider now how the density of excited atoms will vary after the current is interrupted. The density of excited atoms N^* is given as a function of the distance z for the steady state in the upper curve of Fig. 55. When the current ceases, we assume that the production of excited atoms ceases, but the loss by diffusion to the cathode continues.

Therefore N^* in Regions 2 and 3 will decrease, and the distribution after a short time t will be of the form shown in the lower curve of Fig. 55.

If the flow of excited atoms to the cathode in the steady state is F^*/cm^2 and sec and the extension of Regions 2 and 3 from the cathode is d , the initial rate of fall in the density of excited atoms is given approximately by

$$\frac{dN^*}{dt} = -\frac{F_c^*}{d} \quad (8.2)$$

putting $F_c^* \sim 7.10^{23}/\text{cm}^2$ and sec (see p. 90) and $d = 5.10^{-4} \text{ cm}$ we have

$$\frac{dN^*}{dt} = -10^{27}/\text{cm}^3 \text{ and sec}$$

In 10^{-8} secs N^* will have fallen by about $10^{19}/\text{cm}^3$, and so if $N^* = 10^{20}/\text{cm}^3$ when the current was interrupted, this represents a decrease of about 10% in N^* . It seems unlikely therefore that when the circuit is restored

after 10^{-8} secs that the change in the excited atom concentration will alone be sufficient to prevent the arc from re-striking.

Finally let us consider the movement of electrons after the current is interrupted. We see from Fig. 30 that in the steady state there is a strong positive space charge in Region 1 which is maintained by the passage of the current. When the current in the external circuit ceases, electrons will continue to be drawn into Region 1 by the field of the space charge until sufficient electrons have entered the region to neutralise the excess positive charge there, whereupon the flow will cease.

The density of positive ions in Region 1 is about $10^{18}/\text{cm}^2$ (see Fig. 30), the depth of Region 1 is about $3 \cdot 10^{-6}$ cm and the rate of flow of electrons from the cathode into Region 1 is about $5 \cdot 10^{23}/\text{cm}^2$ and sec. The time taken for the positive space charge to be neutralised is therefore of the order $10^{-11} - 10^{-12}$ seconds.

When the external circuit is restored after 10^{-8} secs the electric field in the cathode region will be due to the electrodes only, since the space charge has been neutralised. It is unlikely under the usual experimental conditions that this field will be greater than a few hundred volts/cm. The first 5 volts will therefore fall off at a distance of about $5 \cdot 10^{-2}$ cm from the cathode instead of the $3 \cdot 10^{-6}$ cm of the established arc. Furthermore the density of neutral atoms is, as we have shown, practically unchanged during 10^{-8} sec, and so the $\frac{x}{p}$ near the cathode will be very low (probably of the order 0.1 v/cm. mm Hg) and incapable of separating the positive and negative charges. The space-

charge region will therefore not be re-established and the arc will not re-ignite.

On account of the experimental difficulty of producing extremely short pulses, the shortest interruptions of mercury arcs have been about 10^{-8} sec, and the arc has always been extinguished. It seems likely from the above treatment that pulses as short as 10^{-11} sec would have the same effect.

8.3. Mean lifetime of a cathode spot.

If the current in a free spot is reduced below about 5A the arc is liable to go out after a short lifetime. It has been shown⁽¹¹⁴⁾ that the extinction of the spot is a purely random occurrence, that is to say the probability of the arc going out within a small time interval Δt is proportional only to Δt . The probability $P(t)$ of a spot having a lifetime greater than t is then given by

$$P(t) = \exp(-t/L) \quad (8.3)$$

where L is the mean lifetime, which is found to vary with the current according to the empirical relation

$$L = 10^{-2} \exp(2.5 i) \quad (8.4)$$

for a free spot, and according to the relation

$$L = 10^{-3} \exp(20 i) \quad (8.5)$$

for an anchored spot.

It can be seen from these equations that for a free spot carrying more than 5A or an anchored spot carrying more than 0.5A the mean

lifetime is sufficiently long for the arc to be regarded as permanently established.

In the light of the new arc theory, two possible explanations of the random extinction can be put forward, both associated with the random motion of the spot. The first is concerned with the droplet evaporation.

If a droplet which is ejected by the moving arc should fall back into the spot, in passing through the highly excited vapour it represents a large, cold area on which excited atoms may condense. The passage of the droplet may depopulate the spot of so many excited atoms that the self-sustaining mechanism of the discharge will be interrupted. The larger the current, the greater the energy stored in the spot, and hence the greater the probability that the spot will survive if a droplet falls into it. An anchored spot, which does not evaporate droplets, would be expected to exist at lower currents, as is observed.

The other possible explanation is connected with the establishment of the dense vapour cloud in which the excitation processes occur. When the spot moves at random it is liable to be driven onto a cold area of the cathode which it must raise to a sufficient temperature to establish the vapour cloud above it. For a given temperature the heat loss by conduction through the cathode is proportional to the radius of the spot (equation 6.17): the heat supplied by the arc is proportional to the current and hence, if the current density is constant, to the square of the spot radius. More watts/ampere must therefore be used to overcome the heat conduction loss from a low current spot than a high current one.

It is likely therefore that if a low current spot is driven suddenly by a vapour blast onto a cold area of the cathode, that there may not be sufficient energy to establish the vapour cloud in the face of the large conduction loss and so the arc will go out. An anchored spot, which does not exhibit random motion, is therefore likely to survive at lower currents than the free spot.

8.4. The importance of Oxide layers on the cathode.

It has been found that whereas an arc can readily be established on a clean mercury surface, an arc cannot be struck on a clean, oxide-free solid cathode such as Cu, Fe, Ag etc. in an oxygen-free gas. (5,23,56,110,111,116) An arc may form momentarily when the electrodes are separated, but it goes out immediately. If there is sufficient voltage available a glow discharge will form rather than an arc, even up to currents of 10A. (56) If however the cathode has a thin layer of oxide, or if there is appreciable oxygen present in the gas in the discharge tube, there is no difficulty in forming an arc. Thermionic arcs may be struck on refractory cathodes whether there is an oxide layer or not, but arcs of the cold cathode type will form on tungsten only if oxide is present. (5) It appears therefore that an oxide layer is essential if a stable cold-cathode arc is to be formed on a solid cathode.

It has been suggested that the thin layer of oxide, being an insulator, allows a charge of positive ions to build up on it and thus to produce a high electric field at the cathode sufficient to draw the arc current by field emission. (77,106-109) It has also been suggested that

the oxide layer, by virtue of its low thermal conductivity may allow a high temperature to exist at the active cathode area without excessive heat loss by conduction. This temperature, in conjunction with the lowered work function which might be expected at the composite oxide-metal surface, might be sufficient to cause thermionic emission of the arc current. (5,23,117)

These suggestions assume that the arc actually strikes on the oxide, whereas we should expect that on account of the intense evaporation from the hot spot, the oxide layer would be rapidly driven off. This is confirmed by the tracks left by arc spots on oxidised cathodes which seem to indicate that the oxide has been removed where the spot has passed, exposing the bare metal underneath.

When a mercury arc spot moves at random over the cathode surface, it establishes a hot spot on the cathode at each point of its travel at a sufficient temperature to provide the vapour cloud necessary for the spot mechanism. It was suggested in the previous section that if there was insufficient energy available to establish the hot spot the arc would go out. Consider now the arc on a copper cathode. The energy required to establish a hot spot on a cold copper cathode is very much greater than in the case of mercury. Not only must the temperature of the spot be higher (3000°K instead of 800°K), but the specific heat and thermal conductivity of copper are an order of magnitude greater than of mercury. Once the spot has been established at one point however, it appears that the steady state heat loss through the cathode is no greater than in the mercury arc. As discussed in Section 6.5, this is probably

due to a decrease in the thermal conductivity of copper when it is in the liquid state. Until the arc has melted the cathode however, losses by conduction will be very large.

If an arc has been initiated on an oxide free copper cathode, for example by touching and then separating the electrodes, the spot will, on account of its random motion, move rapidly away from the point of its formation (even though the cathode is solid there will be random motion on account of directional fluctuations in the evaporation from the cathode). The spot will move too rapidly onto a cold area of the cathode and the heat losses will make a greater demand than the arc can supply, so the spot will extinguish.

If however there is a layer of oxide on the cathode the mobility of the arc spot will be greatly reduced. The spot must first burn off the oxide before it can move, and this gives it time to melt the copper underneath. The molten copper provides a larger heat resistance than the solid and enables the vapour cloud to be produced without excessive conduction loss. The oxide layer, by restricting the motion of the spot, thus enables a stable arc to be formed.

It appears therefore that the oxide layer is not an essential part of the emission mechanism, as has been previously suggested, but rather provides conditions under which the emission mechanism can operate. In confirmation of this suggestion, it has been shown experimentally that the presence of an oxide layer reduces the speed at which an arc spot moves in a given magnetic field by two orders of magnitude.⁽⁵⁶⁾ In the case of the mercury arc, where the cathode temperature is low and the thermal

conductivity of the cathode is also low, no oxide layers are necessary.

8.5. Cold arcs on refractory cathodes.

Examples have been found of arcs having all the characteristics of cold-cathode arcs forming on metals which would normally support a thermionic arc: the most frequent example of this is found on tungsten (5, 69, 116, 145-148). The arc exhibits the high current density, short reignition time and random motion typical of the cold arc. Furthermore the temperature of the cathode is found to be too low for thermionic emission to occur, although after a short time a hot spot may form on the cathode and the discharge will then go over into a thermionic arc, having a lower current density, longer reignition time and a steady spot. It appears that the presence of occluded gas is necessary if a cold-cathode arc is to form: if the tungsten is thoroughly outgassed, only thermionic arcs can be struck on it.

This phenomenon can be understood in terms of the new arc theory. If the cathode is too cold to provide thermionic emission, it will certainly not evaporate cathode material. If however there is occluded gas, this will be driven off where the arc strikes and will represent, for a short time, a strong local evaporation of gas atoms from the cathode. These will be back-scattered and will build up a layer of dense gas close to the cathode in which excited atoms will be produced. These will diffuse to the cathode and release electrons, so a cold cathode type of arc will form until the supply of occluded gas is exhausted. By this time the cathode temperature has probably risen sufficiently to

provide thermionic emission.

It is found that when a cold-cathode type of arc strikes on tungsten the cathode fall is somewhat lower than when the arc is thermionic. This is to be expected, since in the case of the cold arc the temperature of the cathode is much lower and losses by heat conduction through the cathode will be smaller.

8.6. The light from the spot.

It has been described in Section 5.7 how on account of the high density of neutral atoms, resonance radiation will be imprisoned and excitation energy will be transferred by diffusion of excited atoms rather than by radiation. Resonance radiation only was considered, but since there is likely to be a large number of excited atoms in states other than the resonance state, the phenomenon of successive absorption and re-emission of radiation will apply to spectral lines other than the resonance line, although probably to a lesser extent on account of the smaller cross-sections for absorption of the other lines. The light which is observed to originate from the spot is therefore emitted by excited atoms which have diffused to the periphery of the arc where the density of neutral atoms is lower, and hence the likelihood of re-absorption of a quantum is less.

The diffusion of excited atoms in Region 4 is in the direction of the anode (see Fig. 35), and in this direction the density of neutral atoms is decreasing rapidly with increasing distance. From Fig. 14, putting $a = 4 \cdot 10^{-3}$ cm for a 5A spot, we see that at a distance of $4 \cdot 10^{-3}$ cm

from the cathode of a low pressure arc ($N_g = 10^{15}/\text{cm}^3$) the density has fallen by more than two orders of magnitude from the density at the cathode. Most of the visible light, for which the cross-section for absorption is small, is likely to have escaped within this distance. In Regions 1 - 3 however, the excited atoms diffuse mainly to the cathode, since as this represents a condensing surface there are large density gradients in this direction. The diffusion in the radial direction is therefore likely to be small, so Regions 1 - 3 when viewed in a direction perpendicular to the axis of the arc, are likely to appear darker in comparison with the lower part of Region 4. This is confirmed by C.G. Smith's micro-photographs of the arc spot⁽¹⁶⁰⁾ which show an apparent dark space extending about 10^{-3} cm from the cathode and then a bright luminous zone extending about $4 \cdot 10^{-3}$ cm from the cathode. Considering the probable experimental error and the approximate nature of our calculations, these measurements are in good agreement with the estimated dimensions of the cathode region given in Chapter 5.

The spectrum of the cathode region of the mercury arc has recently been examined in some detail by St. John and Winans.⁽²⁹⁾ In order to overcome the difficulty of observation inherent with the free spot, the spot was driven along an anode by a magnetic field. The spot was made to travel in a horizontal path past the vertical slit of a spectro-scope, and in this way it was possible to see how the spectrum varied with the distance from the cathode. It was found that close to the cathode the spectrum was continuous, with intensity maxima centred on the mercury lines (this was probably Regions 2 and 3 of our definition) but at larger

distances from the cathode there was only the mercury line spectrum (this was probably Region 4).

The continuous spectrum is probably due to the broadening of spectral lines under the conditions of high electric field and high density of atoms and charged particles which exist in the spot. Lines which are not too widely spaced may become sufficiently broad to overlap. In addition we might expect some contribution to the spectrum from the bands of Hg_2 . St. John and Winans measured the broadening of a number of lines in the continuous spectrum and compared their results with Stark effect investigations in a mercury glow discharge. (156) They assumed that the broadening in the arc spectrum was due to a second order Stark effect and estimated that an axial electric field of the order $5 \cdot 10^6 \text{ V/cm}$ existed in the spot.

The validity of such a comparison with glow discharge measurements is however most questionable, since the current density in the arc was probably at least five orders of magnitude greater than in the glow, resulting in entirely different conditions. The broadening is probably a combination of pressure broadening, due to the high density of atoms, and the Stark effect in the high inter-atomic micro-fields due to the large density of charged particles, rather than to the Stark effect in a high axial electric field. The average micro-field is given by (191)

$$X_m = 20e N_e^{2/3} \quad (8.6)$$

where N_e is the number density of ions and electrons. If we take $N_e \approx 10^{18} / \text{cm}^3$, which is the probable density in Region 3, we have

$$X_m \approx 3 \cdot 10^6 \text{ V/cm.}$$

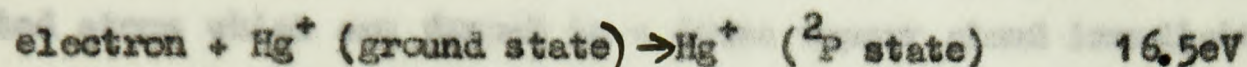
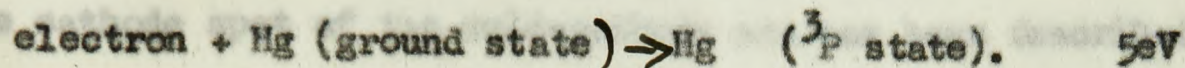
This would be sufficient to account for the observed broadening of the lines.

The bands of Hg_2 appeared to be absent from St. John and Winans spectra, and so they concluded that the pressure of mercury vapour in the spot was no greater than about 50 mm Hg. However it may have been that the intensity of the bands relative to the lines was so low that they escaped detection, especially as they are likely to originate only from a very narrow region. On the other hand if temperatures of more than a thousand degrees absolute exist in the vapour it is likely that most Hg_2 molecules, whose binding energy is only 0.15eV, will be dissociated, and hence their contribution to the spectrum will be very small. The apparent absence of Hg_2 bands is not therefore evidence against the existence of a vapour cloud of high density.

Another feature of the spectrum from the arc is the presence of HgII lines, which originate from the excited levels of the mercury ion. The first excitation potential of Hg^+ is about 6eV (see Ref.190), so if an excited ion is to be produced by the impact of an electron with an atom in the ground state, the electron would need to have an energy of at least 16eV. Some workers have therefore taken the spark lines as evidence that electrons of such energy exist in the discharge and have concluded that there must be a potential difference of 20 to 30 volts in the spot. (29)

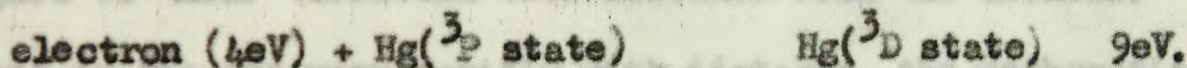
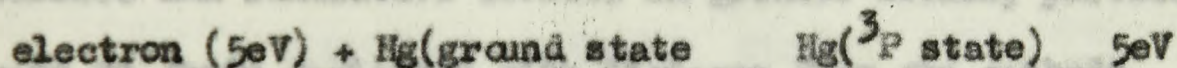
This is an entirely erroneous conception. We have described in chapter 5 how the conditions of high vapour density and high current density which exist in the spot favour cumulative effects. The excitation

of the spark lines is therefore likely to be the result of stepwise processes. Successive impacts on an atom by electrons of only 6eV energy may excite the atom in the following stages:



Thus Hg^{++} , of ionisation energy 19eV may be formed by electrons with no more than 6eV energy.

Another likely cumulative process involves collisions of the second kind between neutral mercury atoms in the higher excited states. Atoms may be excited to the ${}^3\text{D}$ state of 9eV energy as follows:



Two ${}^3\text{D}$ atoms may now collide. Their combined energy of 18v will be sufficient to produce an excited ion. On further collision between an excited Hg^+ and a neutral excited atom may then result in the production of Hg^{++} .

It is seen therefore that cumulative processes can account for the appearance of HgII lines in the spectrum of the arc. The spark lines are found to be enhanced when the arc is subjected to a strong magnetic field and HgIII lines then appear: it will be shown in Part 2 that these lines are emitted from Region 4, and their origin will be discussed.

8.7. Conclusion.

In the foregoing chapters a new picture of the conditions in the cathode spot of the cold-cathode arc has been described. The transfer of current at the cathode is attributed to electron emission by excited atoms which are formed in a dense vapour cloud immediately above the cathode, and diffuse back to the surface. It would appear that this picture is consistent with practically all the observed features of the arc spot.

At this stage the theory is necessarily rather primitive, as the processes have been simplified in order to understand the essential details involved. A more refined theory would treat the excitation processes and cumulative effects in greater detail, particularly with regard to their variation with distance from the cathode. Since conditions in the arc vary rapidly with distance, the mathematics are likely to be more complicated than have been used here.

In the light of the new theory it would be valuable to have more experimental information of the conditions in the spot. It would be necessary to resolve distances of the order 10^{-4} cm from the cathode and to examine the spectrum in greater detail as a function of distance. Also some means of measuring the density of vapour as a function of distance to the same degree of resolution might be devised. Finally it would be of great interest if more accurate determinations were made of the yield of electrons when excited atoms strike a metal surface.

PART II

CHAPTER 9

THE INVERSE DRIVING EFFECT

9.1. Introduction.

The work to be described in this part of the thesis was undertaken before the theoretical study which has been the subject of Part I. The apparent violation of Ampere's rule by a low pressure arc in a transverse magnetic field is one of the more remarkable features of the cold-cathode arc. The phenomenon was investigated in the hope that it would throw some light on the processes occurring in the cathode spot, but although the explanation was not found, some information as to the probable dimensions of the cathode spot was obtained. It does not lead to any conclusions concerning the electron emission mechanism. Further experimental work on the arc in a magnetic field was therefore abandoned in favour of the theoretical investigation of the cathode processes.

PART II THE MOTION OF THE ARC IN A TRANSVERSE MAGNETIC FIELD

9.2. The phenomenon of reverse driving.

If a transverse magnetic field acts on a direct current, cold-cathode arc, the positive column of the arc is always deflected in the direction given by Ampere's rule. Under certain conditions however, the cathode spot is found to move in the opposite direction, dragging the deflected column with it. The arc as a whole therefore moves contrary to Ampere's rule.

PART II

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9.2. The phenomenon of reverse driving.

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The reverse driving effect was first observed in a low pressure mercury arc by J. Stark⁽¹⁶¹⁾ in 1903. Since then it has been the subject of numerous investigations.⁽¹⁶²⁻¹⁸⁹⁾ The effect has been studied in low current arcs between a variety of electrodes in different gases: in all cases the motion of the cathode spot is in the reverse direction if the gas pressure is sufficiently low. A change to the Ampere direction occurs if the pressure is increased above a critical value, other conditions being constant. This critical reversing pressure is the greater the lower the current and the stronger the magnetic field: it depends on the nature of the gas, but appears to be almost independent of the cathode material.^(176,178) If the arc length becomes small, the reversing pressure has been found to increase with decreasing electrode separation.^(180,181)

The reverse driving effect is found only in cold cathode arcs. In the case of a tungsten cathode where, as has been explained in Chapter 8, both thermionic and cold-cathode types of arc spot can form, only the cold-cathode spot gives reverse driving. When the cathode reaches a temperature at which thermionic emission occurs, the arc cannot be driven in the reverse direction.^(170,177)

9.3. Previous Theories.

Stark⁽¹⁶¹⁾ considered that the neutral vapour which was evaporated from the spot acted as a conductor moving perpendicular to the magnetic field. An e.m.f. would be induced in the vapour which would act parallel to the mercury surface in the opposite direction to the

electromagnetic force, and would thus tend to drive the arc in the reverse direction (Fig. 56). However, in the modern view it is not permissible to regard the neutral vapour as a conductor.

Minorsky⁽¹⁶⁴⁾ investigated the effect with a mercury arc in which the spot was driven round an annular cathode pool by a radial magnetic field. He proposed that electrons moved right round his annular tube in the Ampere direction as a result of elastic collisions with gas molecules in the magnetic field, and built up a space charge on the reverse side of the arc. This attracted the positive ion space charge, and hence the spot, in the reverse direction (Fig. 57). This theory cannot account for the observed reverse driving of arcs along straight paths.

Tanberg⁽¹⁶⁵⁾ proposed that the vapour blast from the spot carries positive ions against the electric field. These are deflected in the reverse direction and returned to the cathode, forming a new spot. However it is hard to believe that a sufficient number of ions take this course to counteract the effect of the majority of the positive ions which move towards the cathode and are deflected in the Ampere direction.

Smith⁽¹⁶⁶⁾ developed a theory of the mercury arc cathode in which he assumed that the conduction electrons in the liquid were heated to 4000°K while the lattice remained at only 400°K . There was therefore a large electron temperature gradient perpendicular to the applied magnetic field. Smith then invoked the Righi-Leduc effect which would give a thermo-motive force on the current, parallel to the cathode surface. This would give a reverse motion to the spot if the cathode material had a negative Righi-Leduc coefficient. This theory assumes that the reverse

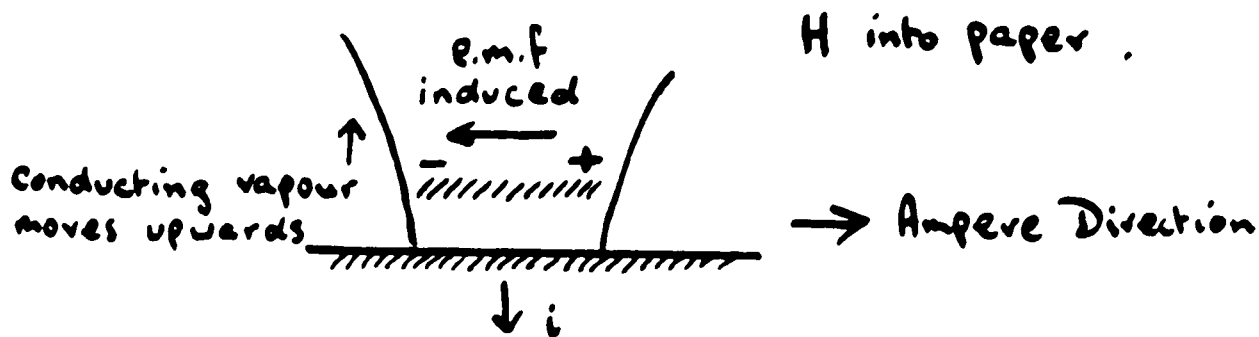


FIG. 56 STARK'S THEORY.

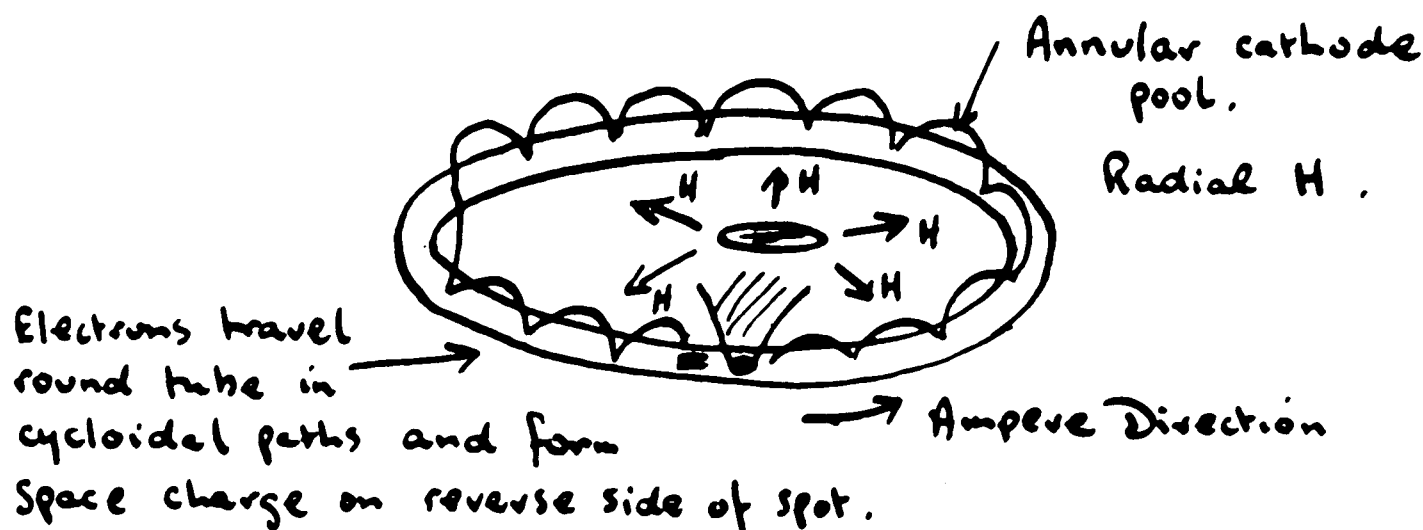
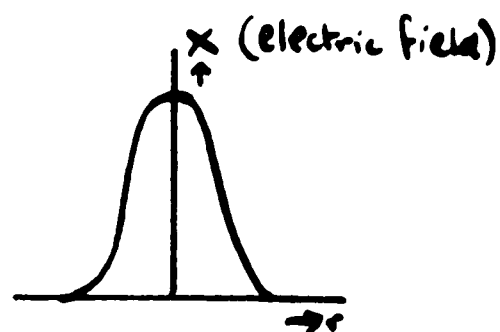
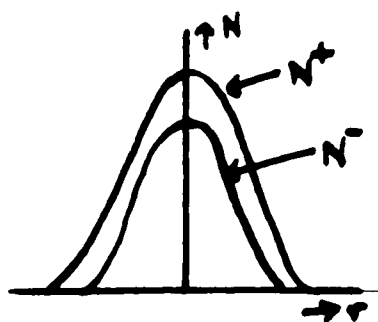
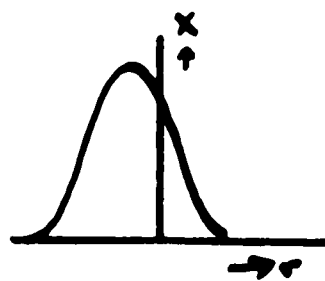
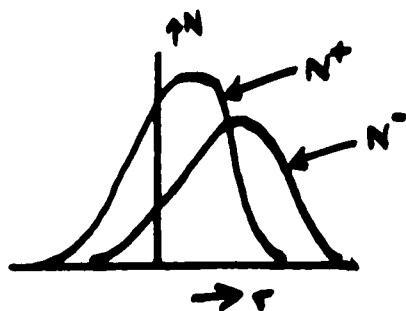


FIG. 57. MINORSKY'S THEORY.

- ① Charge distribution and field as a function of r .
No magnetic field.



- ② Magnetic field moves charges in A direction.
Peak of field moves in R direction.



H into Paper .

→ Ampere direction .

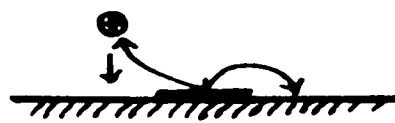
FIG 58 LONGINI'S THEORY

Electrons emitted from spot at random.



$H=0$

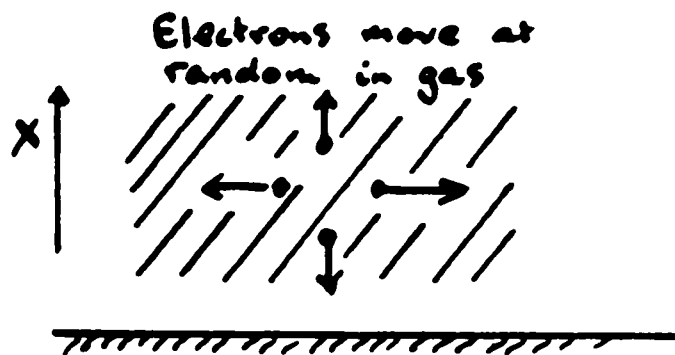
Only those emitted in R direction form ions.



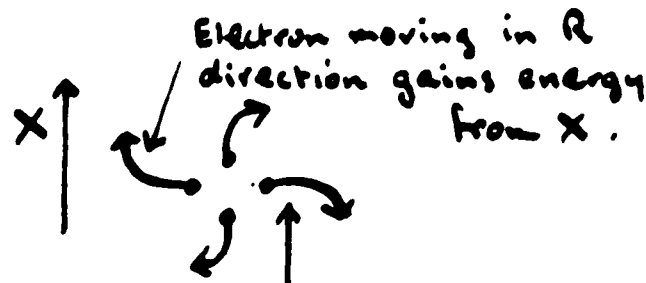
H into paper.

Ampere Direction $\rightarrow$

FIG 59. HIMLER AND COHN'S THEORY.



Electrons move at random in gas



Electron moving in R direction gains energy from X.

H into paper

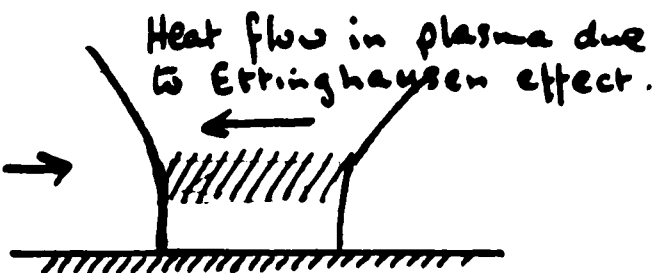
A $\rightarrow$

Electron moving in A direction loses energy.

FIG 60. YAMANURA'S THEORY.

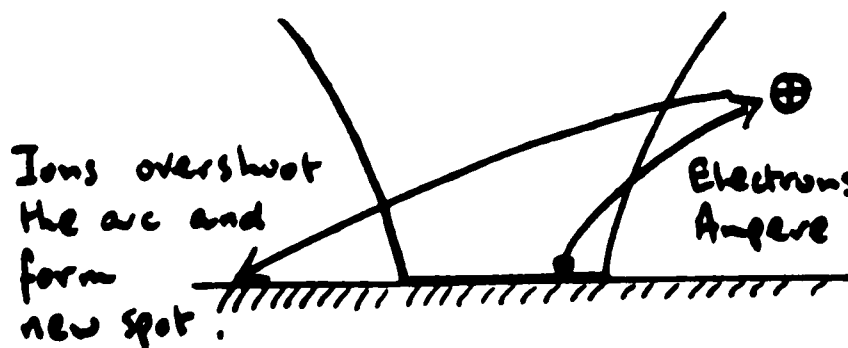
H into paper.

Ampere direction $\rightarrow$



Heat flow in plasma due to Ettinghausen effect.

FIG. 61. WARE'S THEORY.



Ions overshoot the arc and form new spot.

Electrons produce ions on Ampere side of the arc

FIG 62. ST. JOHN AND WINANS' THEORY.

driving effect occurs in the cathode itself. It does not therefore conform with the observations of Cobine and Gallagher that conditions of reversal do not depend greatly on the cathode material, and cannot account for the effect of gas pressure on reversal. The theory also suffered from errors of a fundamental nature. (168)

Longini (182) considered the relative displacements of the electron and ion space charges over the cathode spot when a magnetic field is applied. Although both space charge zones are moved in the Ampere direction, the electrons are assumed to move more than the ions, and so leave a greater positive charge on the reverse side. The peak of the resultant electric field at the cathode is thus moved in the reverse direction (Fig. 58). Assuming that the electron current from the cathode is due to field emission, this peak determines the spot position.

This theory assumes an independence of the electron and ion space charges. However at the high charge densities in the spot it is not possible to neglect their strong interaction, which will resist separation by the magnetic field. Furthermore the theory relies for support upon the observation by Smith that at a large magnetic field (about 5000 g) a reversal of the direction of motion (R to A) takes place even at low pressure. This observation has not been confirmed by any other investigator and we would suspect it was due to some other cause.

Himler and Cohn (180) assumed that the electron emission from the spot was random in direction. Electrons emitted with a velocity component in the reverse direction would be deflected towards the anode

while those emitted with a component in the Ampere direction would be returned to the cathode (Fig. 59). Only the former would produce positive ions in the vapour which in turn produce the spot. However there is no justification for the initial assumption as on account of the high electric field in the vicinity of the spot the electrons would be emitted perpendicular to the cathode surface.

Yamamura⁽¹⁸¹⁾ proposed a similar theory to that of Himler and Cohn, only applied to the region above the spot. Electrons with a high electron temperature are assumed to ionise preferentially when moving in the reverse direction, since here the magnetic field deflects them in such a direction that they gain energy from the electric field (Fig. 60). Consideration shows that such energy differences would be extremely small compared with the random energy of the electrons and so the effect would be negligible.

Rothstein⁽¹⁸⁷⁾ considers that on account of the high vapour density in the spot, metallic conduction takes place from liquid to vapour, and assumes that 'positive holes' carry some of the current. As in the anomalous Hall effect such holes would be deflected by the magnetic field in the reverse direction.

However the assumption of metallic conduction is unsupported by experimental evidence, and no quantitative theory on these lines has been proposed.

Ware⁽¹⁸⁸⁾ has suggested that the Ettinghausen effect, which produces a heat flow parallel to the cathode but in a direction opposite to the Ampere force, might act in the plasma immediately above the cathode.

The cathode beneath the spot would then be heated preferentially on the reverse side of the spot and this was assumed to produce reverse motion (Fig. 61). This might be the case if the cathode depended for its electron emission on heat received from the discharge and so would strictly apply only to thermionic arcs, which are known not to exhibit reverse driving. In addition, in order that the Ettinghausen effect should be appreciable he assumes that the vapour density in a low pressure arc is as low in the cathode spot region as elsewhere in the arc. We have shown in Part I that the opposite is true.

St. John and Winans⁽¹⁸⁴⁾ propose that cathode electrons are accelerated in the cathode fall and deflected in the Ampere direction, where they form ions on the Ampere side of the spot. These are then attracted electrostatically back into the spot, which they overshoot and fall on the cathode on the 'reverse' side of the spot (Fig. 62). Here they are assumed to create a new spot, thus moving the arc in the reverse direction.

This very crude picture of conditions in the cathode spot assumes that electrons produce ions by single collision, that the density of atoms in the spot is as low as elsewhere in the arc and that positive ions are responsible for electron emission from the cathode. As we have seen, none of these assumptions is valid.

Thus in spite of these numerous suggestions, no really satisfactory theory of the reverse driving effect has been proposed which can account even qualitatively for the observed phenomena. In particular none of the above theories deals with the observed variation of reversal

pressure with electrode separation, since their treatments apply only to the immediate vicinity of the cathode or to the cathode itself and so neglect a parameter which is, as we shall show, of considerable importance.

10.1. The Rotating Mercury Arc

In order to study the motion of a mercury arc in a transverse magnetic field we followed the example of Minorsky and Smith and constructed a mercury arc tube with the cathode in the form of an annular pool. The lower part of the tube fitted into an electromagnet of the loudspeaker type, which produced a radial magnetic field of up to 4000 gauss. Fig. 63 is a scale drawing of the tube and magnet. As can be seen, the magnetic field is confined to the cathode region of the discharge.

The tube could carry currents of up to 10A and was cooled by blowing air onto the condensing surfaces and up the centre pole of the magnet. In later experiments, the cathode was cooled by circulating water instead of air.

The tube was connected to a vacuum system which allowed the tube to be pumped continuously, in which case the pressure of residual gas could be kept below 10^{-5} mm Hg, or alternatively the tube could be isolated from the pumps and pure argon could be admitted up to a pressure of 500 mm Hg.

The tube was connected to a source of 230 volts d.c. through a variable resistor. The circuit is shown in Fig. 64. For some experiments a radio-frequency current (2.5 Mc/s) could be super-imposed on the steady d.c. current by means of the circuit shown in Fig. 65.

SOME EXPERIMENTS ON THE MERCURY ARC10.1. The Rotating Mercury Arc.

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The tube was connected to a source of 250 volts d.c. through a variable resistor. The circuit is shown in Fig. 64. For some experiments a radio-frequency current (2.5 Mc/s) could be super-imposed on the steady d.c. current by means of the circuit shown in Fig. 65.

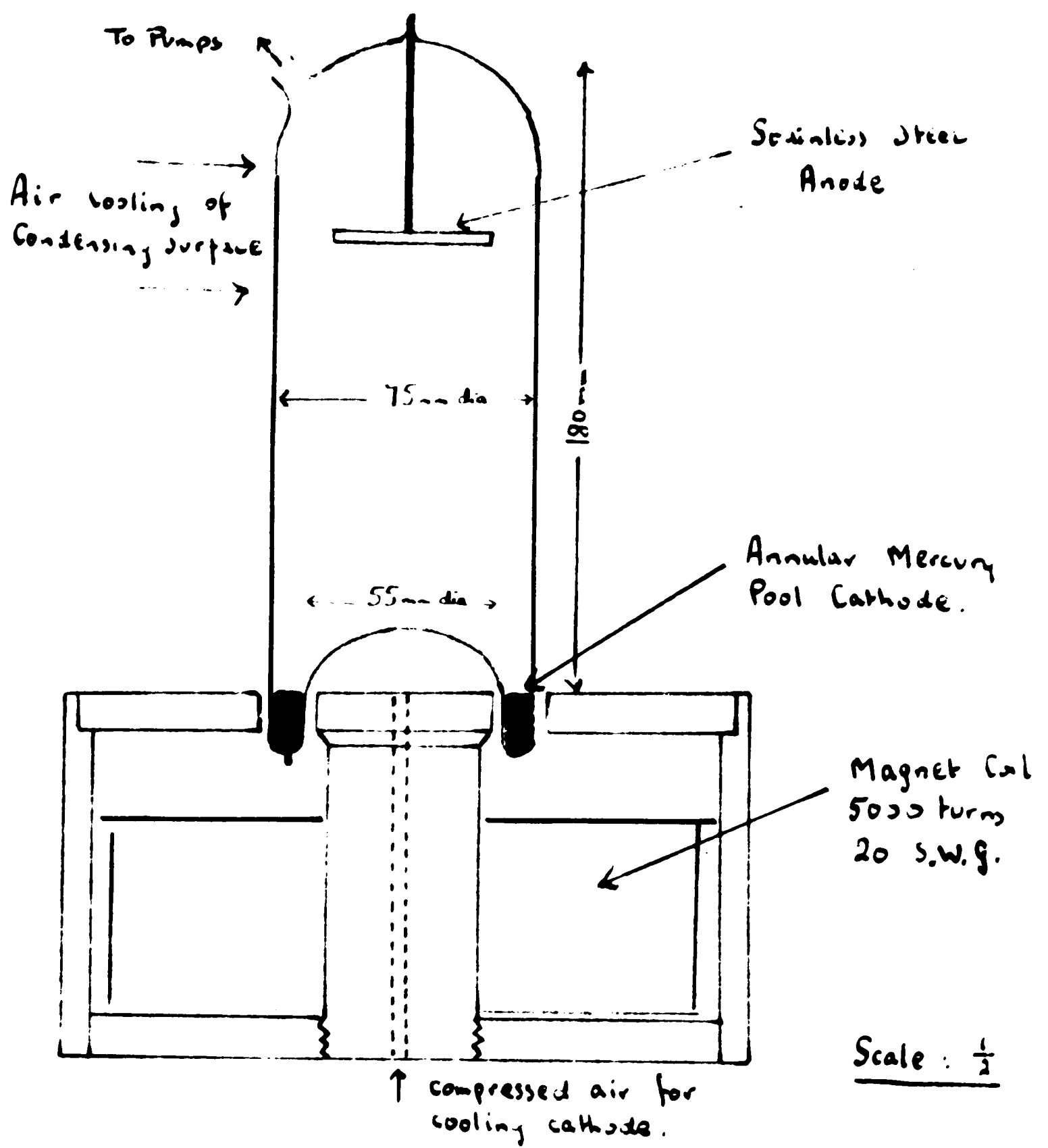


FIG 63. DISCHARGE TUBE AND MAGNET.

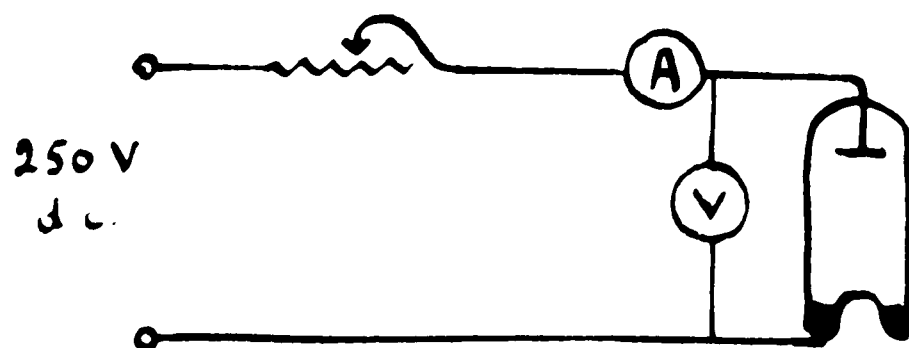


Fig. 64. THE CIRCUIT.

The arc was started by a high voltage impulse applied to the outside of the glass near the cathode surface. When the magnet was switched on, the arc spot rotated at a steady speed round the annular mercury track.

The parameters which could be varied were

1. The magnetic field
2. The d.c. arc current
3. The r.f. current
4. The pressure of argon in the tube
5. The degree of cooling applied to the tube

Observations were made of

1. The speed of the arc spot
2. The voltage across the tube
3. The appearance of the discharge

FIG 66. ARRANGEMENT FOR MEASURING SPOT VELOCITY

The speed of the spot was measured by an apparatus shown schematically in Fig. 66. A photocell picked up the light from the spot each time it came round the circular track. The photocell current was amplified and displayed on the Y-plates of a cathode-ray oscilloscope. The output of a beat-frequency oscillator was fed into the X-plates, and the frequency of the oscillator adjusted to give a stationary closed loop on the screen. The frequency of the oscillator was then equal to the frequency of rotation of the spot. The linear speed of the spot was derived by multiplying the frequency of rotation by the circumference of the cathode pool.

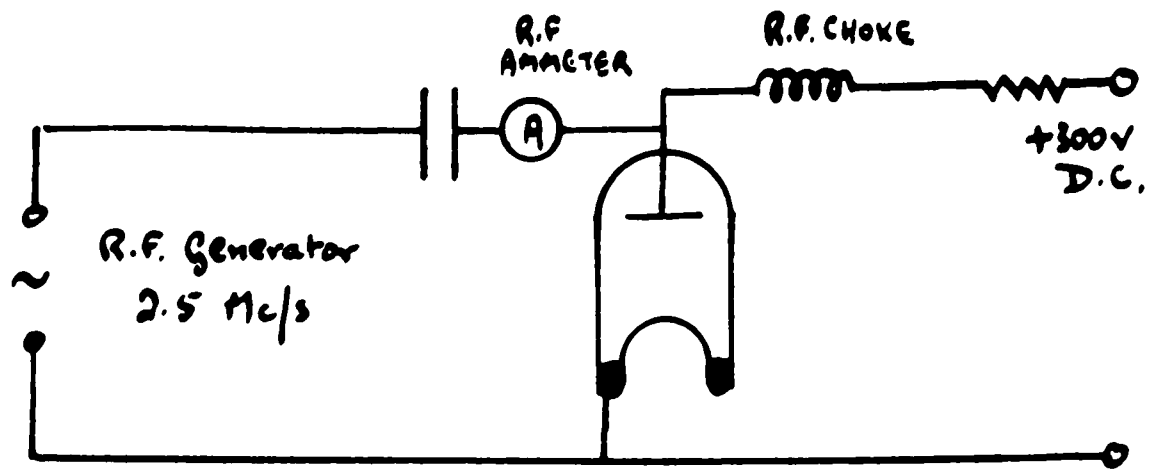


FIG 65. CIRCUIT FOR APPLYING R.F. TO ARC.

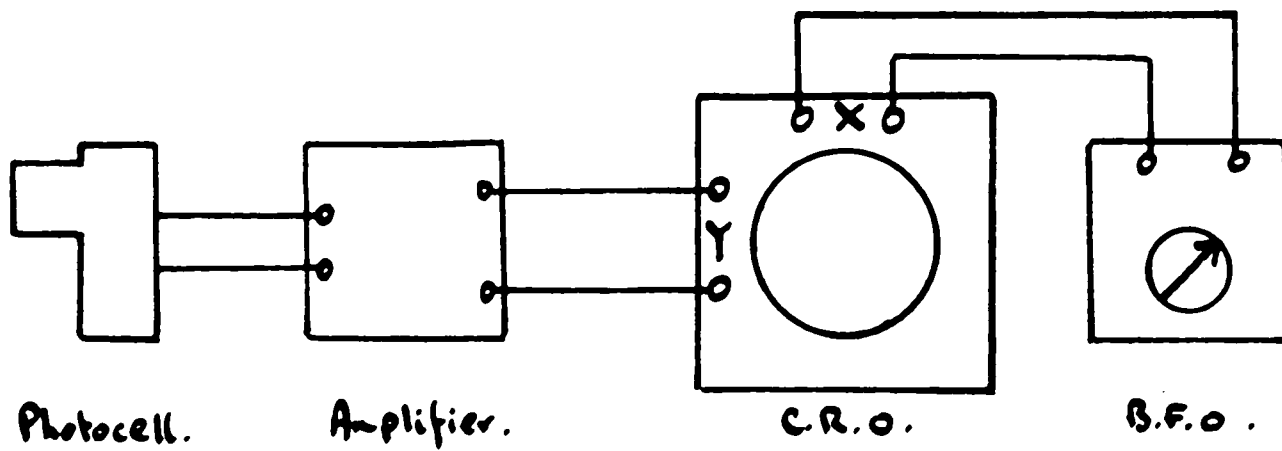


FIG 66. ARRANGEMENT FOR MEASURING SPOT VELOCITY.

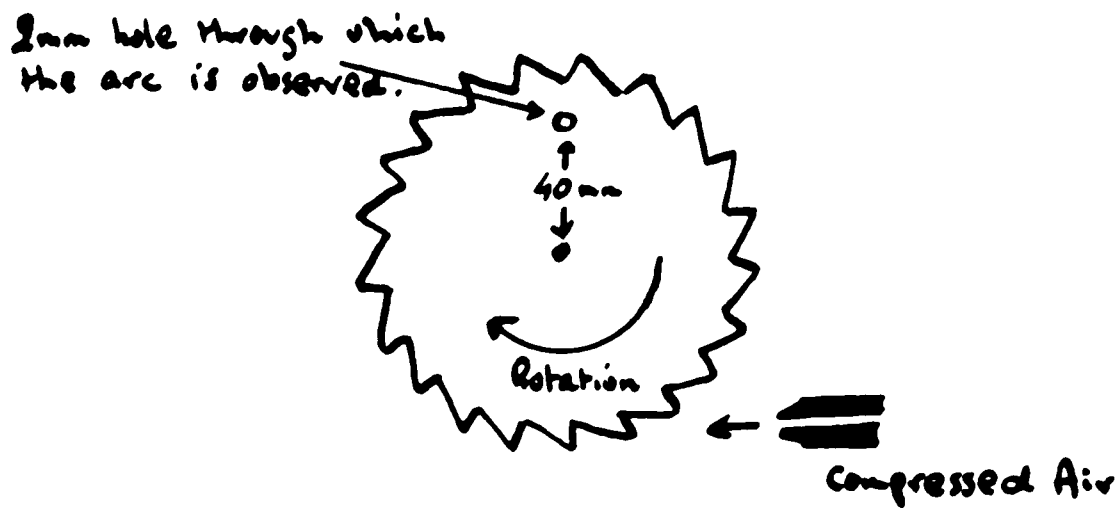


FIG 67. THE STROBOSCOPE.

80 The moving arc was observed through a rotating-wheel stroboscope shown in Fig. 67. The wheel could be driven by compressed air up to a speed of 14,000 r.p.m. and the arc was in view for about one hundredth of each revolution. By adjusting the speed of the wheel until the arc appeared stationary, the shape of the discharge could be seen very clearly.

10.2. Observations at low pressure.

In the first experiments no argon was added to the tube and the arc ran in mercury vapour only. The vapour pressure was determined by the temperature of the condensing surface of the tube, which was usually about 60°C corresponding to a vapour pressure of about $2 \cdot 10^{-2}$ mm Hg. The condensing surface was kept cool by blowing air onto it; the cathode was cooled by blowing air up the centre pole of the magnet (see Fig. 63).

20 The arc current was kept constant and readings were taken of the spot velocity and arc voltage as a function of the magnetic field. Some results are shown in Figs. 68 and 69 for several values of the current. There was no r.f. applied in this experiment.

As the pressure was low the motion of the spot was always in the reverse direction. The cooling of the tube was kept constant and after altering the magnetic field, readings of the velocity and voltage were taken quickly.

FIG 68 The curves of arc voltage against magnetic field at constant current, which have not been observed by other investigators, showed a

AT CONSTANT CURRENT

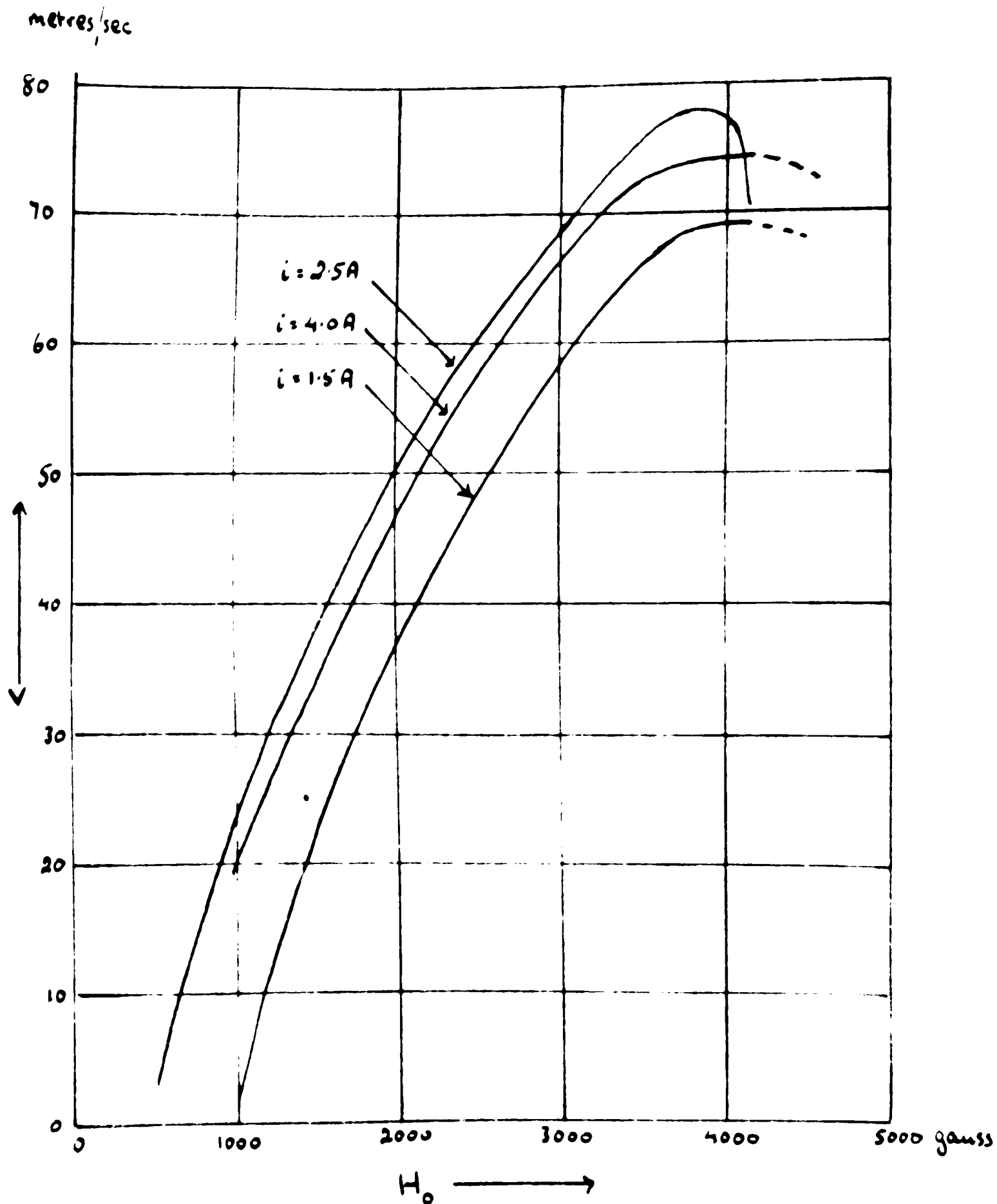


FIG 68 SPOT VELOCITY v IN REVERSE DIRECTION AS A FUNCTION OF APPLIED MAGNETIC FIELD H_0 AT CONSTANT CURRENT AND LOW PRESSURE

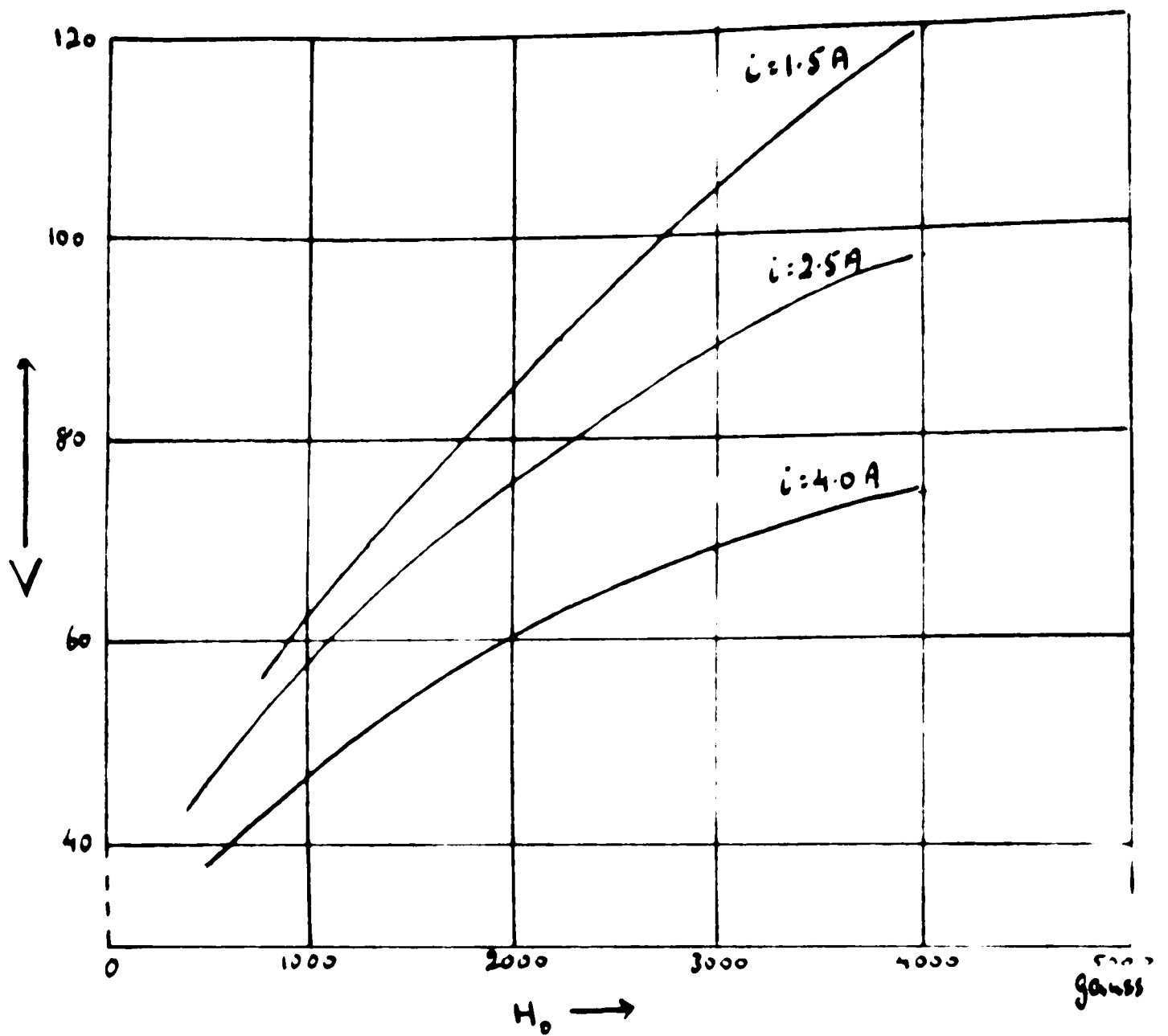


Fig 69. ARC VOLTAGE V AS A FUNCTION OF THE APPLIED MAGNETIC FIELD H_0 AT CONSTANT CURRENT AND LOW PRESSURE.

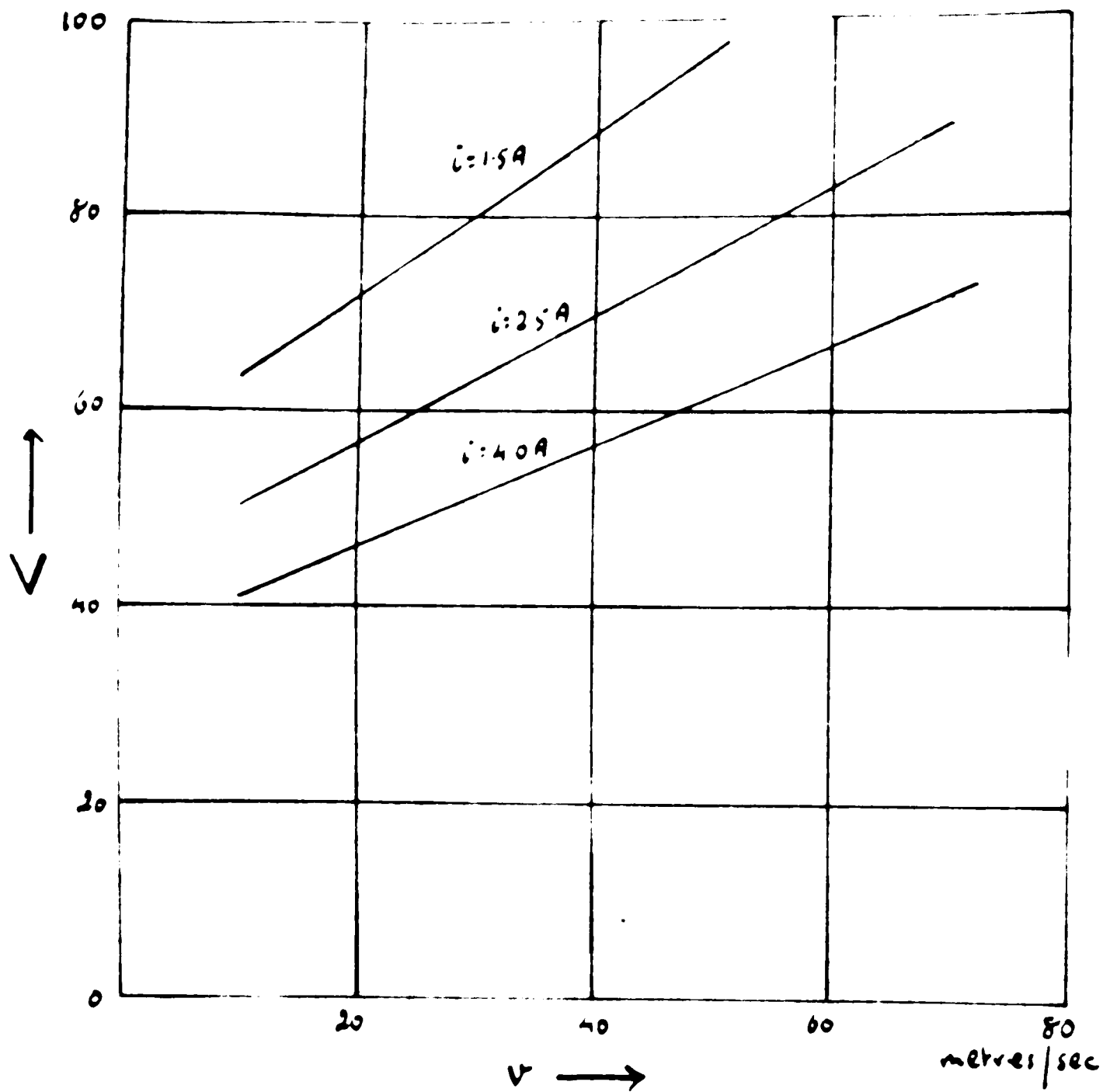


Fig 70 ARC VOLTAGE AS A FUNCTION OF SPOT VELOCITY AT CONSTANT CURRENT AND LOW PRESSURE.

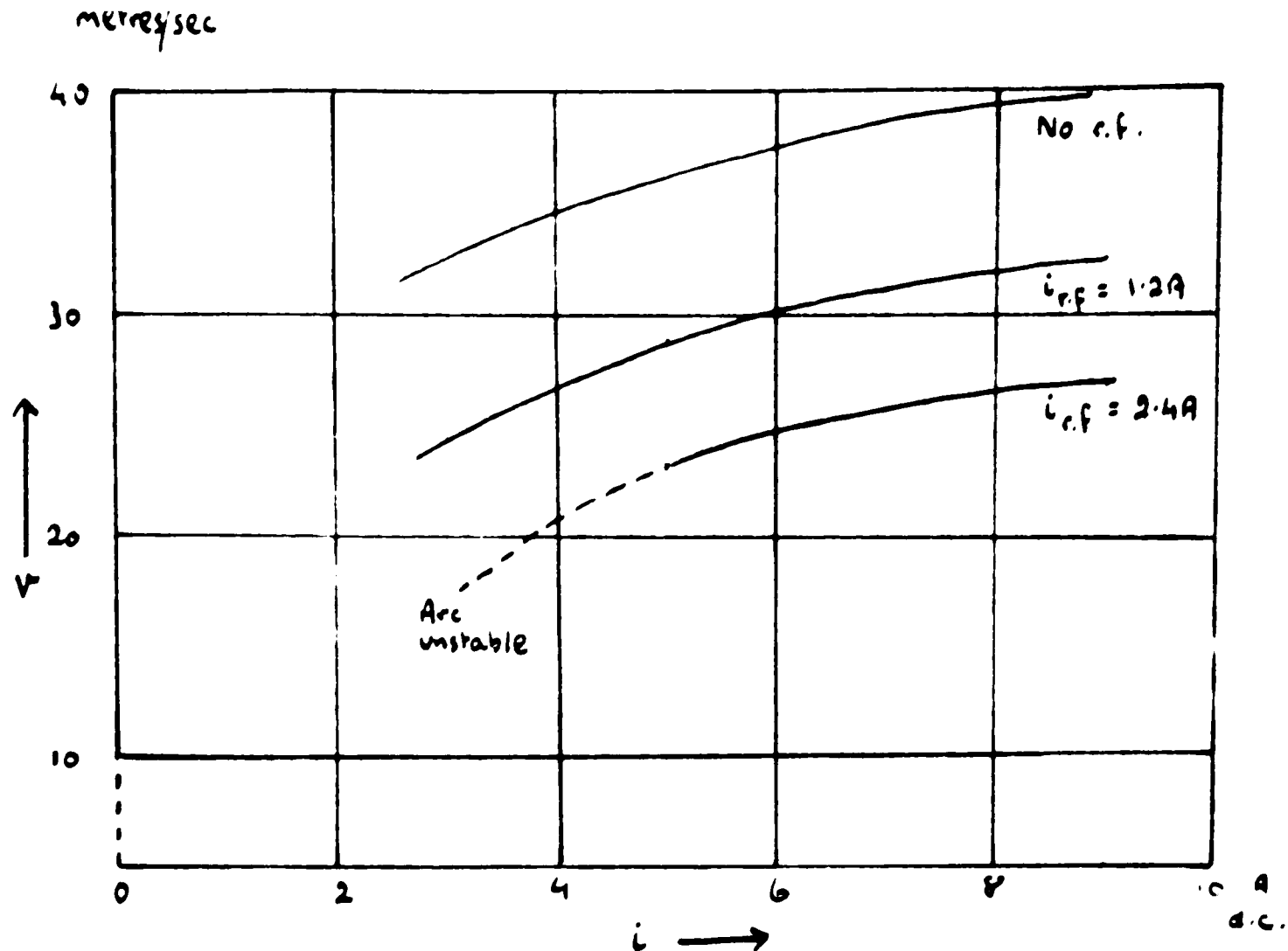


Fig 71 SPOT VELOCITY v IN REVERSE DIRECTION AS A FUNCTION OF CURRENT i , AT CONSTANT MAGNETIC FIELD 1290 gauss

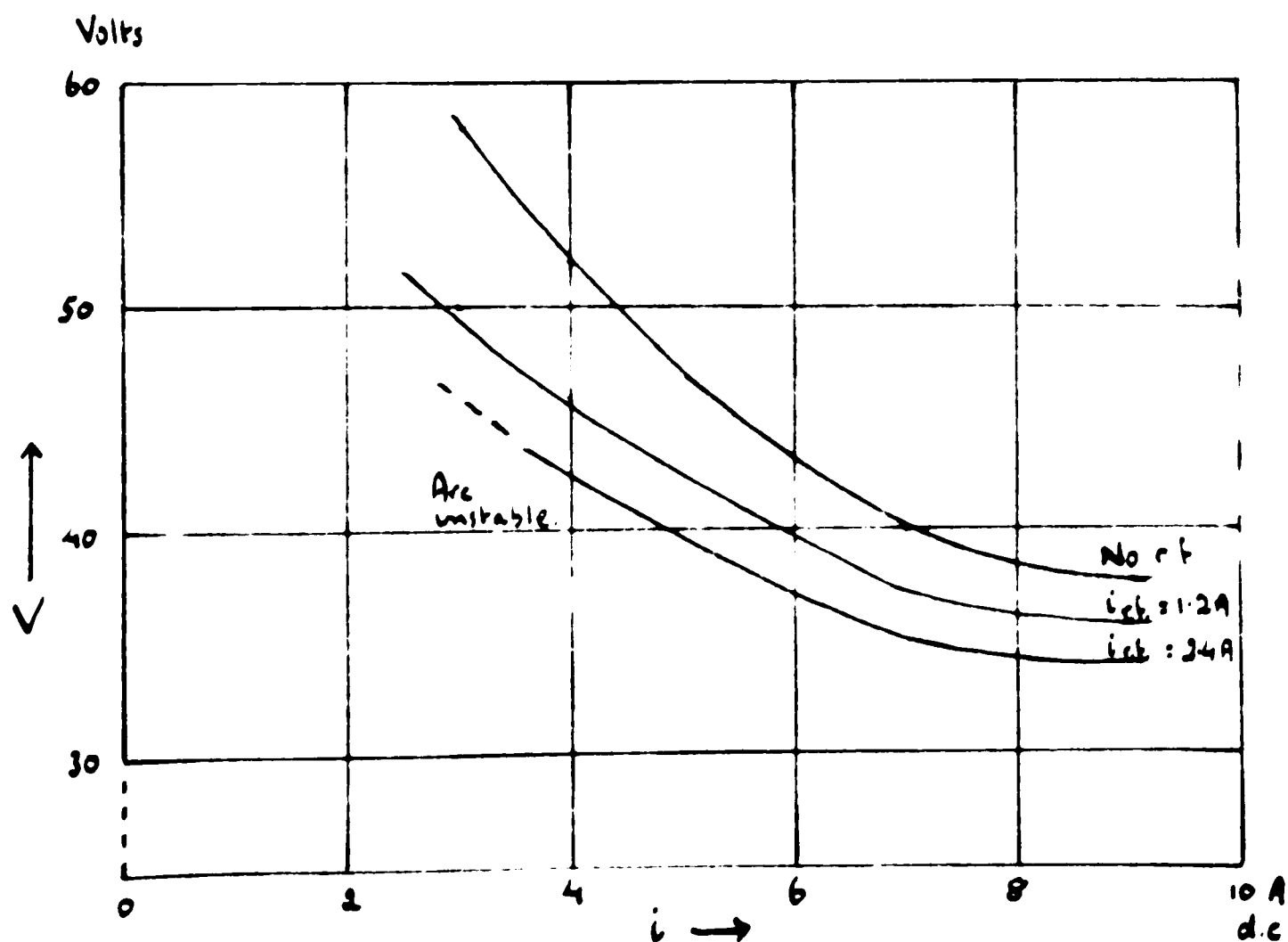


Fig 72. ARC VOLTAGE V AS A FUNCTION OF CURRENT i , AT CONSTANT MAGNETIC FIELD 1290 gauss.

marked similarity to the velocity curves. By plotting voltage against velocity for constant current, straight line graphs are obtained (Fig. 70).

The magnetic field was then kept constant and readings taken of the spot velocity and arc voltage as a function of the current, with and without superimposed r.f. The results for a magnetic field of 1290 gauss are shown in Figs. 71 and 72.

10.3. Discussion of observations at low pressure.

The curves of the spot velocity in the reverse direction against magnetic field at constant current (Fig. 68) are similar in shape to those obtained by Gallagher under conditions of carefully controlled temperature. Whereas Gallagher found that the curve for each value of the current lay above the curves of lower current, it can be seen that our 4A curve lies slightly below the curve for 2.5A. On the other hand, it can be seen from Fig. 71 that when the field was kept constant, the velocity increased steadily with increasing current. There is therefore an inconsistency between the results shown in Fig. 68 and the results shown in Fig. 71.

This inconsistency was found to be due to variations in the temperature of the cathode. Since the cooling was by air blasts only, variations in the energy dissipated in the tube caused variations in the temperature of both the condensing surface and the cathode. It was found that for a constant current and magnetic field, altering the flow of air onto the condensing surfaces had little effect on velocity or

voltage. Altering the flow onto the cathode however, altered the velocity a little and produced considerable changes (sometimes as much as 20%) in the arc voltage. Reducing the cathode cooling caused the velocity and voltage to fall, and vice-versa. In order to obtain reproducible results the air flow to the cathode had to be carefully metered, but even then the thermal time-constant of the system was so long that consistent results were hard to obtain. The reason why the 4A curve of Fig. 68 lies below the 2.5A curve is probably that the cathode was heated to a higher temperature by the greater current.

It appears therefore that the cathode temperature is an important parameter in determining the spot velocity and arc voltage for the mercury arc in a magnetic field. This is probably because the cathode temperature controls the density of mercury vapour immediately above the cathode: the spot has to be driven through this vapour, and for a given current and magnetic field it would seem likely that the greater resistance to the motion offered by the higher density would reduce the speed of the spot. This supported by the observations when an r.f. current was superimposed upon the d.c. arc current (Fig. 71). Since the r.f. current reverses its direction every half-cycle it can have no resultant effect on the electromagnetic forces on the arc. It does however increase the heat dissipation in the discharge, in particular in the cathode region. This raises the cathode temperature and increases the density of vapour through which the cathode spot has to move. This results in the observed reduction in the spot velocity when the r.f. is applied.

10.4. Fluctuations at high speeds.

It can be seen from Fig. 68 that at constant current the velocity increases with increasing magnetic field up to a value of 70 - 75 metres/sec for a field of 3500 gauss. Above 3500 gauss the velocity levels off and appears for some values of the current to decrease as the field is increased.

For magnetic fields up to 3500 gauss the spot velocity was extremely stable, but at greater fields severe fluctuations set in and it was difficult to obtain a steady picture on the oscilloscope screen.

This decrease in the velocity and the apparent instability of the spot speed which accompanies it has been observed by Gallgher, using a similar experimental arrangement. ⁽¹⁷⁶⁾ In his case however, the onset of the phenomenon occurred at only 2000 gauss.

At the same time as the fluctuations set in, a hydrodynamic phenomenon was observed in the mercury cathode pool. As the spot raced in the reverse direction, the liquid mercury circulated slowly in the Ampere direction (electro-magnetic pumping). At low fields the mercury surface was quite smooth, and the path of the spot could be seen as a bright line pressed against the outer wall of the tube. At high fields however, ripples appeared on the mercury surface perpendicular to the path of the spot. These ripples remained almost stationary and built up to heights of several mm. The track of the spot was then no longer pressed against the outer wall but showed an irregular path over the corrugated cathode surface.

A possible explanation of this phenomenon is as follows. As

the spot moves over the mercury it may occasionally make a dent in the surface, although if the spot is being driven at high speed the size of the dent is likely to be small. The spot will be rapidly driven out of the dent it has made, and the level of the surface will be restored in a short but finite time. If however the spot, having travelled once round the circular cathode, arrives at the dent again before the level has had time to be restored, it is likely to stay momentarily in the dent and depress the surface still further before passing on. In this way the depression increases at each passage of the spot until the observed large, stationary ripples are built up. At the same time, the momentary 'sticking' of the spot in the dent causes an apparent reduction in the velocity of the spot. It would appear therefore that this phenomenon is a consequence of the experimental arrangement of an arc being driven round a circular track, and the onset is likely to occur at a critical frequency of rotation rather than at a critical speed.

Gallagher did not observe the hydrodynamic effect since his tube was completely enclosed in an oil-bath. It is significant that his curves show a levelling-off of the velocity beginning at 2000 gauss and decreases of the velocity at fields of less than 3000 gauss. We observed neither effect until the field exceeded 3000 gauss. No dimensions of Gallagher's discharge tube were given, but it would seem likely that our tube was larger than his, which would make the onset of the hydro-dynamic phenomenon occur at higher spot speeds.

It is significant that in the experiments of St. John and Winans,

(184-185) who used an arrangement in which the cathode spot was driven around a circular anchor, no fluctuations or decrease of velocity were observed at high fields. In this case the spot was presumably unable to depress the mercury surface. They found however that after tending to a constant value for magnetic fields above about 7000 gauss, the velocity (in the reverse direction) appeared suddenly to double at fields above 10,000 gauss. This doubling has been observed by others using an anchored cathode spot^(171-173,186) and will be discussed later.

10.5. Observations with argon added to the mercury vapour.

In the experiments described so far, the discharge tube contained only mercury vapour at low pressure, and the motion of the spot was always in the reverse direction. If an inert gas such as argon is admitted to the tube, the velocity in the reverse direction decreases as the gas pressure is increased. At a critical pressure, which depends on the current, the magnetic field and the nature of the gas, the arc is not driven at all. This is known as the point of reversal. If the gas pressure is increased still further, the arc begins to move again, but now in the Ampere direction.

Some experiments were made in which argon was added to the discharge tube, and the spot velocity and arc voltage were measured as a function of the argon pressure for different currents and magnetic fields. To improve the reproducibility of the results and to prevent excessive variation of the cathode temperature, the air cooling of the cathode was replaced by circulating water. The air cooling of the condensing

surface was retained.

The shape of the curve of spot velocity as a function of argon pressure for constant current and magnetic field is sketched in Fig. 73, and the corresponding curve for the arc voltage in Fig. 74. Some results are plotted in Figs. 75 and 76 for magnetic fields of 2000 gauss and 4000 gauss and currents of 2A and 5A. In order to show the details of the curves, the section for pressures greater than 50 mm Hg has been drawn to a different scale.

The curves of velocity against argon pressure are similar to those obtained by St. John and Winans for an anchored spot. Unlike the results at low pressure, the voltage curves show practically no similarity to the velocity curves.

A significant feature is that the curves of velocity against pressure for the same magnetic field but different currents intersect at a certain value of the pressure. At lower pressures increasing the current at constant field will increase the spot velocity in the reverse direction, as was found in the low pressure experiments. At greater pressures, increasing the current will decrease the velocity in the reverse direction and so tend to move the arc in the Ampere direction.

Another important feature is the effect of admitting a small pressure of argon to the arc when rotating at high speed in the reverse direction. Adding 1 mm Hg of argon caused practically no change in the spot speed, but produced a marked reduction of arc voltage, in some cases by nearly 30%.

A change in current at constant field altered the spot speed,

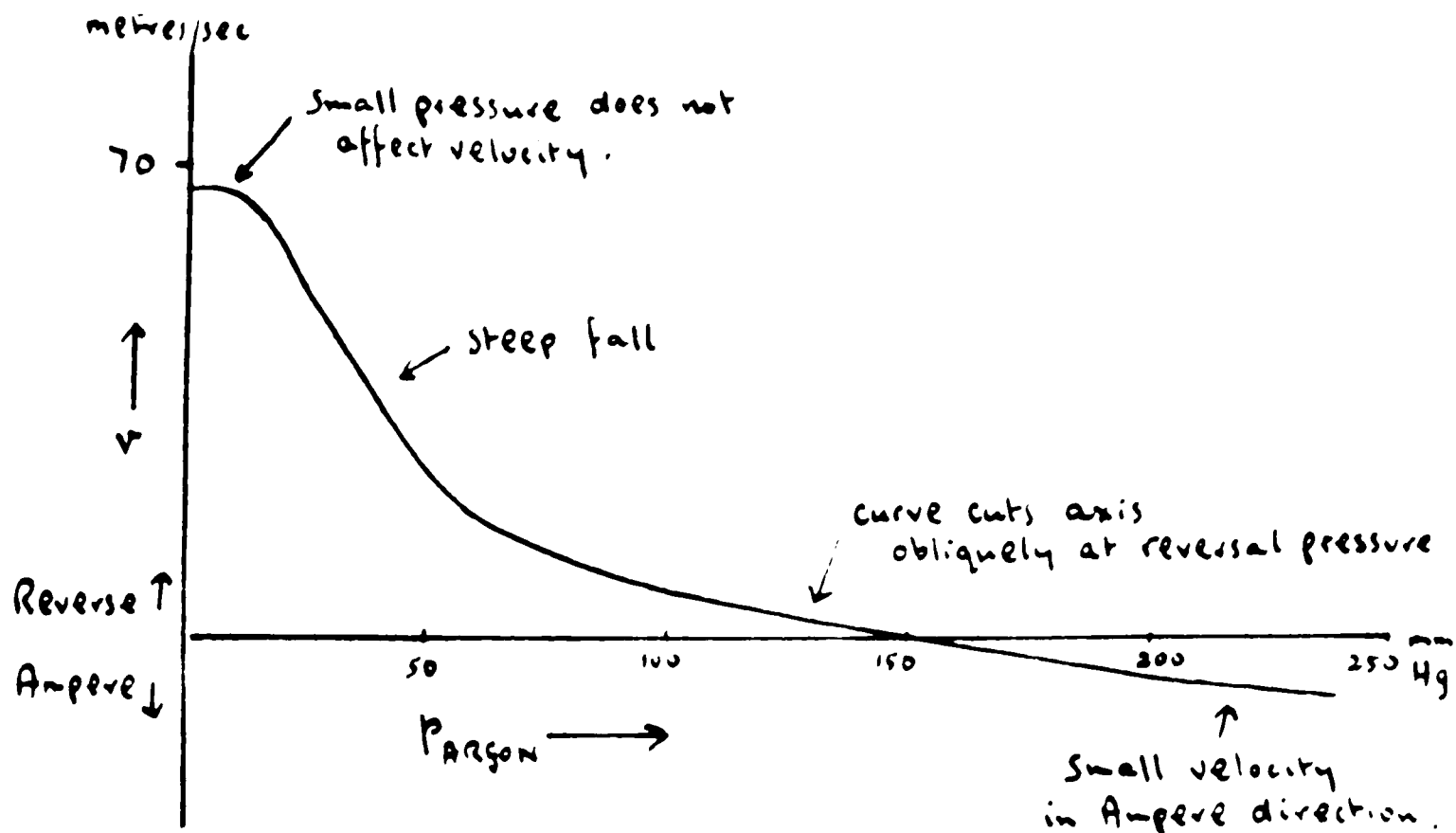


FIG 73. FEATURES OF THE CURVE OF SPOT VELOCITY AGAINST ARGON PRESSURE [$H_{0,i}$ constant]

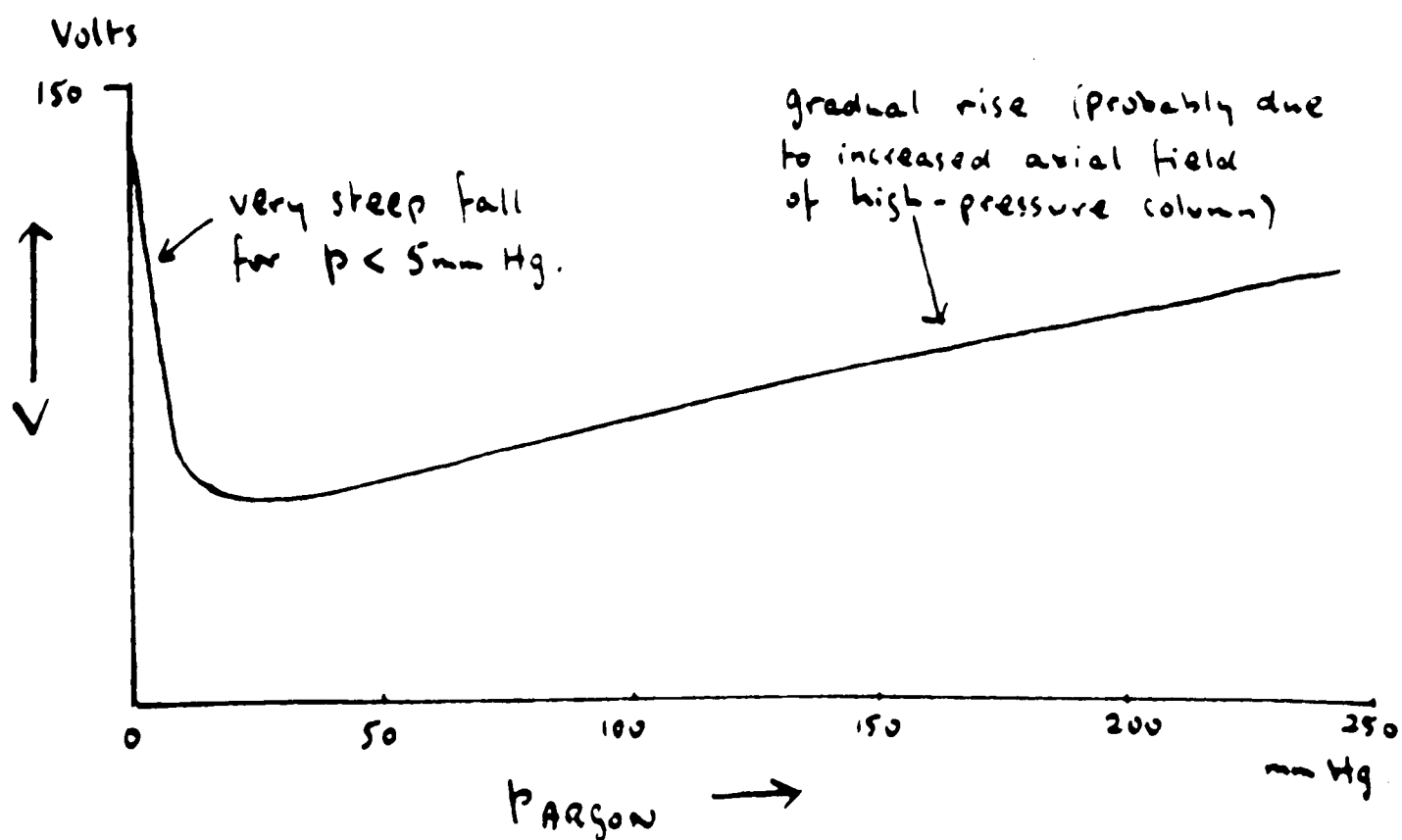


FIG 74. FEATURES OF THE CURVE OF ARC VOLTAGE AGAINST ARGON PRESSURE [$H_{0,i}$ constant]

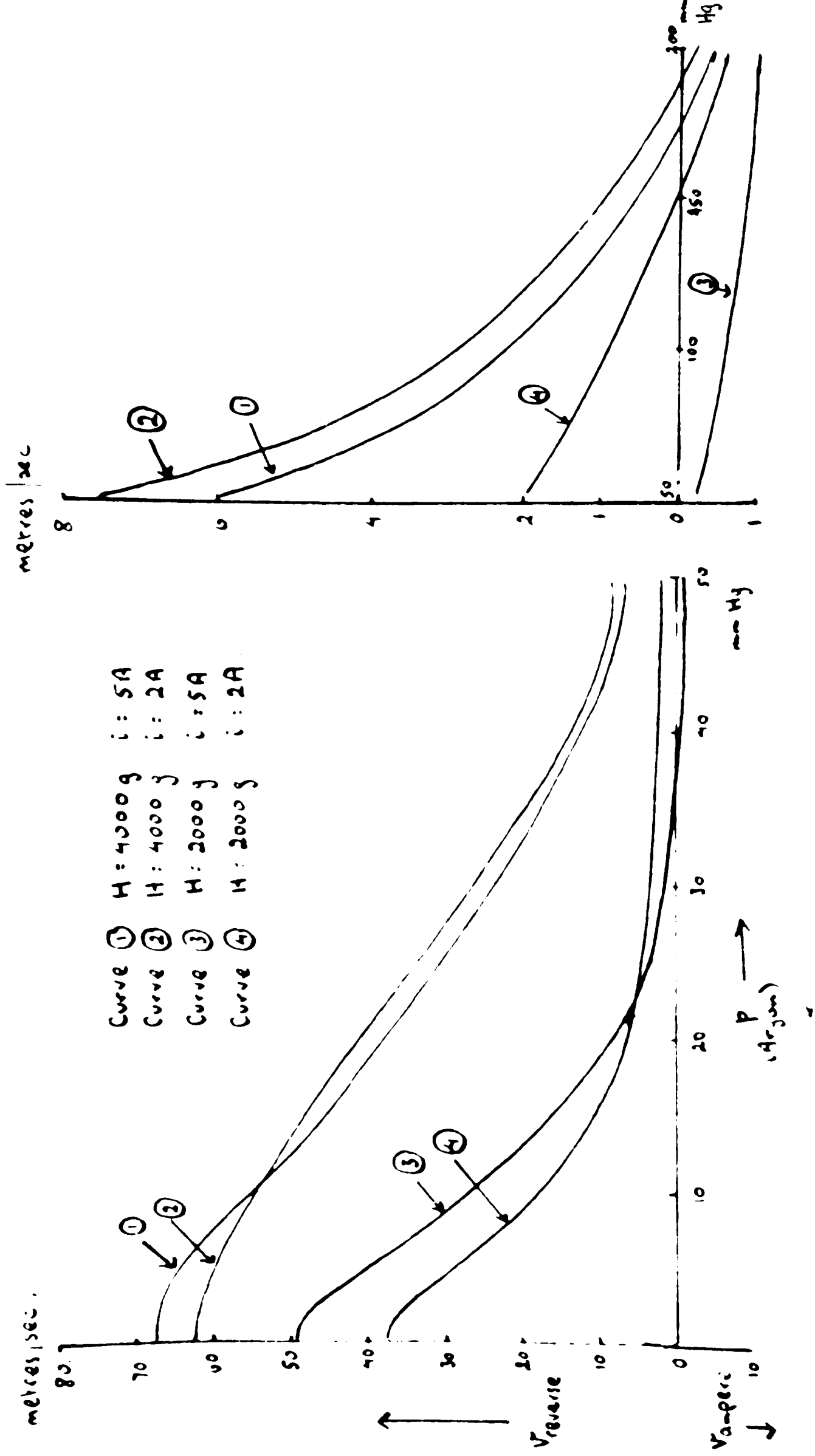
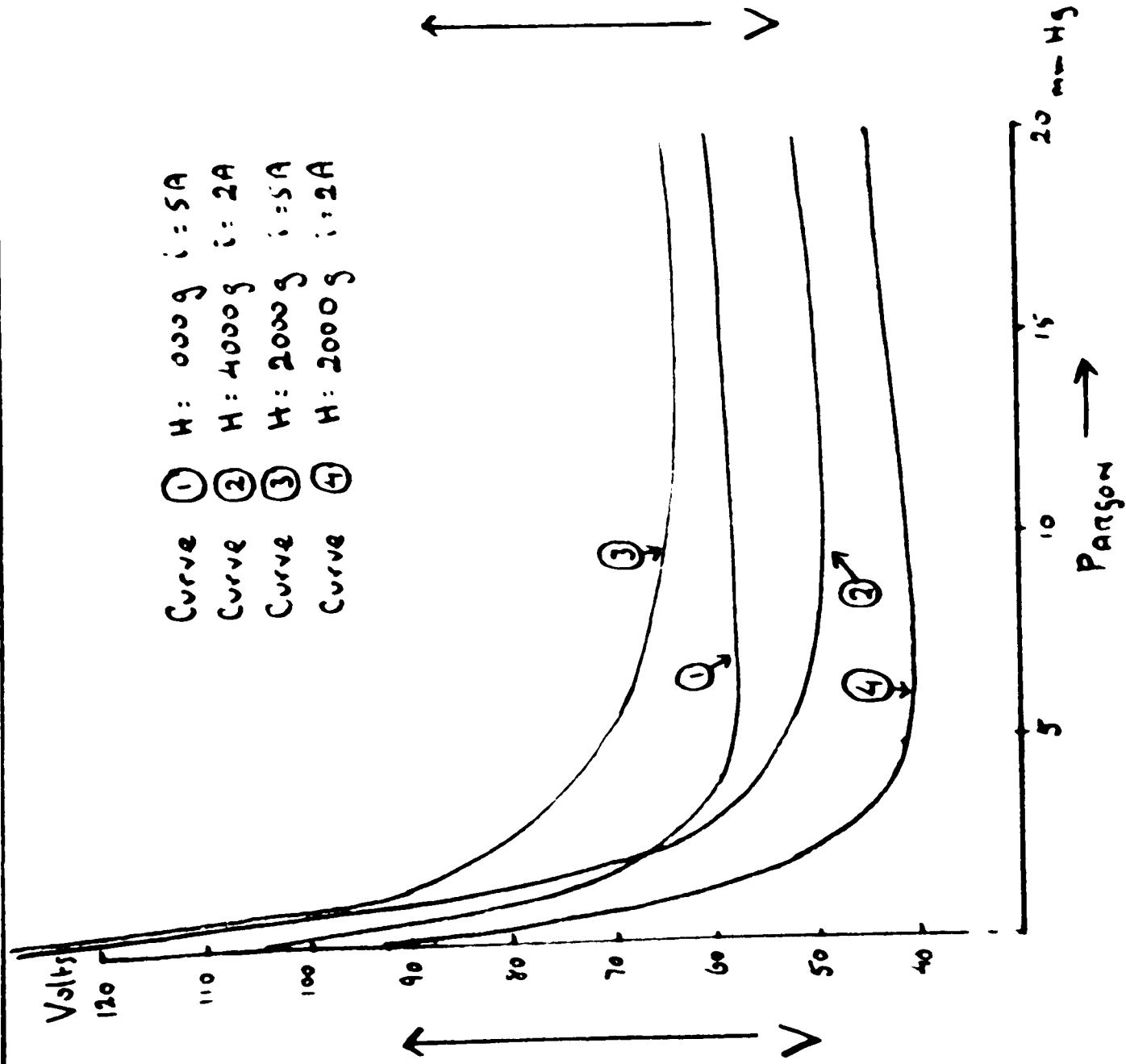


Fig 75 SPOT VELOCITY v AS A FUNCTION OF ARGON PRESSURE. [H_0 and i CONSTANT]



- Curve ① H: 000g i: 5A
 Curve ② H: 4000g i: 2A
 Curve ③ H: 2000g i: 5A
 Curve ④ H: 2000g i: 2A

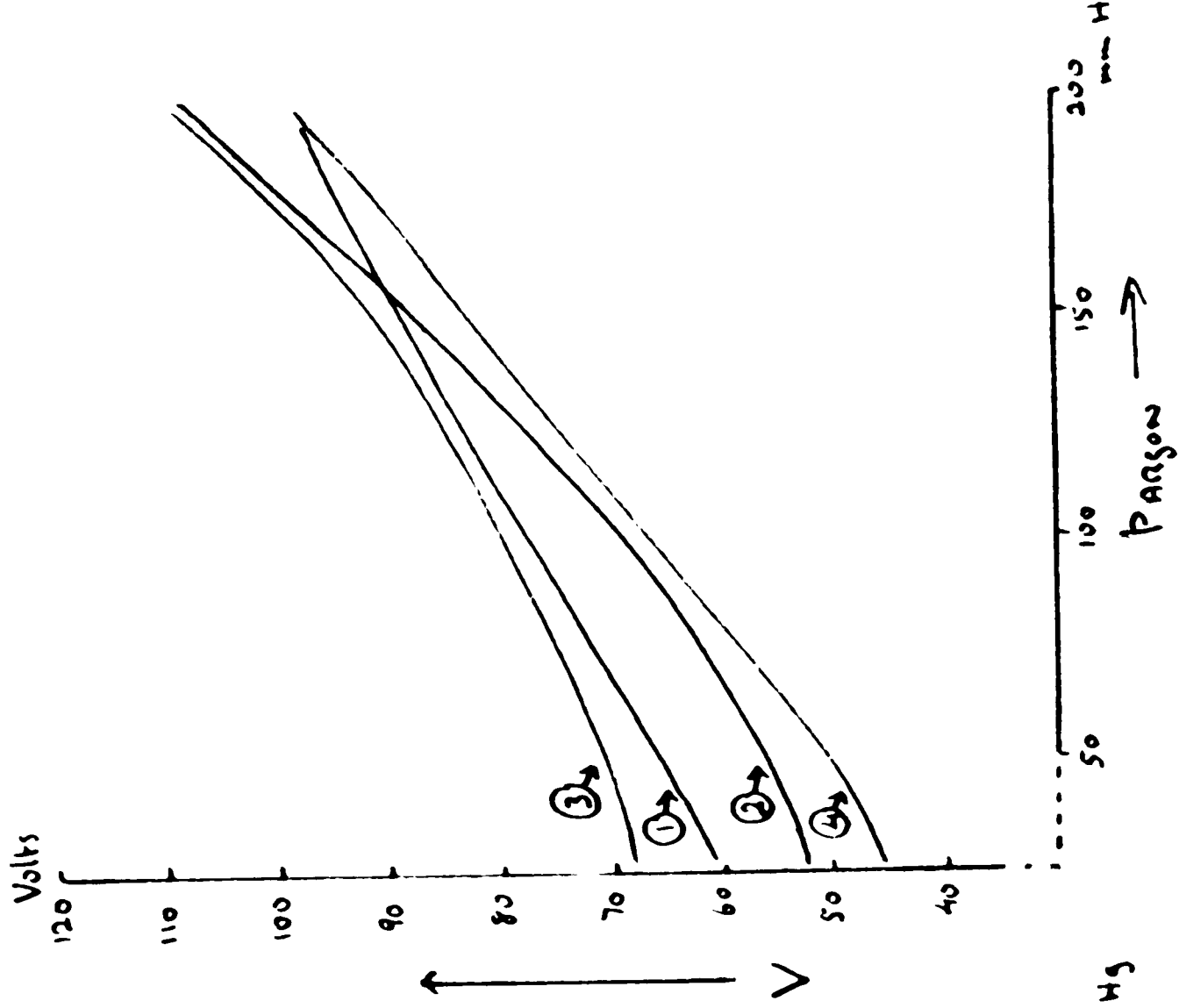


FIG 76. ARC VOLTAGE AS A FUNCTION OF ARC LENGTH [H₀ and i CONSTANT]

which settled down immediately to its new value. The arc voltage took longer (up to 30 secs) to stabilise itself. If the current had been decreased the voltage gradually rose to its new value, and vice-versa. This is attributed to the change in temperature of the cathode when the current is altered. If the current was decreased, the cathode temperature decreased slowly to its new equilibrium value; as was described in the section on the low pressure experiments, this causes an increase in the arc voltage.

10.6. The appearance of the rotating arc.

In the absence of the magnetic field and without argon the arc carrying about 5A at low pressure had the appearance sketched in Fig.77. Apart from the bright cathode spot only a uniform plasma filled the tube. When the magnetic field of about 2000 gauss was applied the arc rotated and became very bright. When observed through the stroboscope it could be seen that the moving spot trailed a bright column which was pressed down onto the cathode surface for some distance behind the spot (Fig.78). The column was deflected in the Ampere direction, while the motion of the spot was in the reverse direction.

If the current and magnetic field were kept constant and a small pressure of argon (about 10 mm Hg) was admitted to the tube the arc still moved in the reverse direction, but with reduced velocity. The column appeared to be more constricted than at low pressure and was not pressed onto the cathode for as great a distance behind the spot (Fig.79).

When the pressure was increased further to about 100 mm Hg the

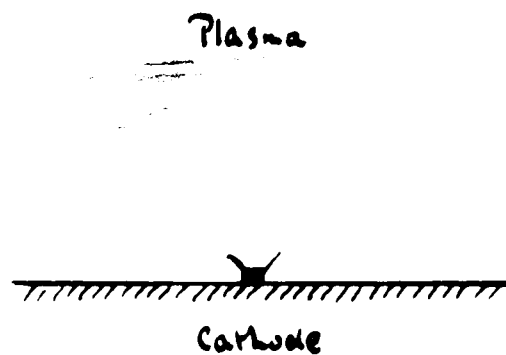


FIG. 77. APPEARANCE OF ARC AT LOW PRESSURE, $H_0 = 0$

H_0 into paper.
Ampere direction $\rightarrow$

$\leftarrow$ Motion



FIG 78. H_0 APPLIED, p LOW.

Same H_0 .

$\leftarrow$ Motion



FIG 79. 10 mm Hg ARGON ADMITTED.

Same H_0

No Motion

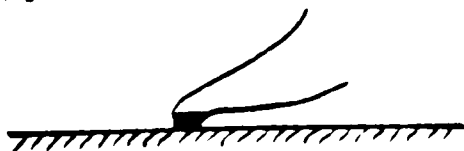


FIG 80. 100 mm Hg ARGON :
POINT OF REVERSAL.

Same H_0

Motion $\rightarrow$

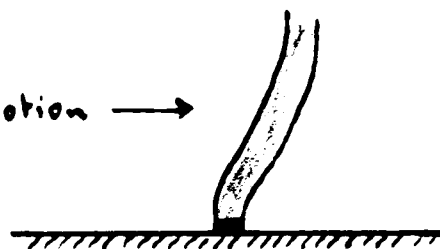


FIG 81. 150 mm Hg. ARGON :
AMPERE MOTION

arc came to the point of reversal. The arc column was more constricted than at lower pressures but was still deflected strongly in the Ampere direction close to the spot, although only a short section appeared to be pressed to the cathode (Fig. 80).

With a still larger pressure of argon (about 150 mm Hg) the arc moved in the Ampere direction. The whole arc consisted of a bright filament from cathode to anode, which was only slightly deflected by the magnetic field. (Fig. 81).

in the direction of Ampere's rule (Fig. 82).

These observations will be discussed in some detail in the next chapter. In a gaseous conductor, that is to say an ionized conducting path through a gas, the current is carried by charges of both sign which move under the influence of an applied electric field. If a transverse magnetic field H is applied, a particle with charge e moving with a drift velocity v in the direction of the electric field will experience an average force F given by

$$F = \frac{Hev}{c}$$

(91.4)

Since in a given electric field the drift velocity of the electrons is very much greater than the drift velocity of the ions (for mercury the ratio is 400:1) the electrons experience the greater force, which tends to deflect them in the direction of Ampere's rule.

There are now two ways of considering how the conducting path in the gas is deflected. First, if the density of charges is low and hence the electrostatic interaction between them is small, electrons may

CHAPTER 11.

A NEW THEORY

11.1. The bending of the Arc.

When a transverse magnetic field acts on a solid conductor carrying a current, the conduction electrons experience an electromagnetic force which they transfer to the lattice. The conductor as a whole therefore experiences a mechanical force tending to move it in the direction of Ampere's rule (Fig. 82).

In a gaseous conductor, that is to say an ionised conducting path through a gas, the current is carried by charges of both sign which move under the influence of an applied electric field. If a transverse magnetic field H is applied, a particle with charge e moving with a drift velocity v in the direction of the electric field will experience an average force F given by

$$F = \frac{Hev}{c} \quad (11.1)$$

Since in a given electric field the drift velocity of the electrons is very much greater than the drift velocity of the ions (for mercury the ratio is 400: 1) the electrons experience the greater force, which tends to deflect them in the direction of Ampere's rule.

There are now two ways of considering how the conducting path in the gas is deflected. First, if the density of charges is low and hence the electrostatic interaction between them is small, electrons may

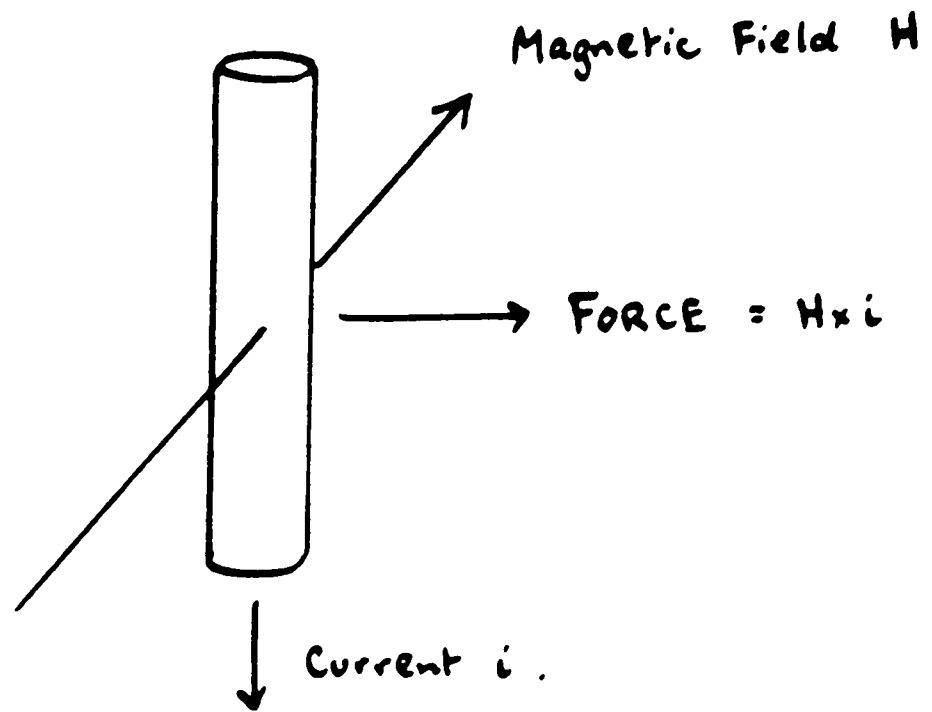
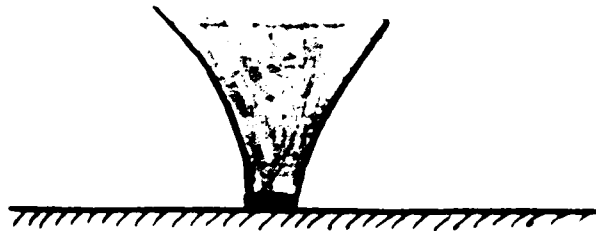
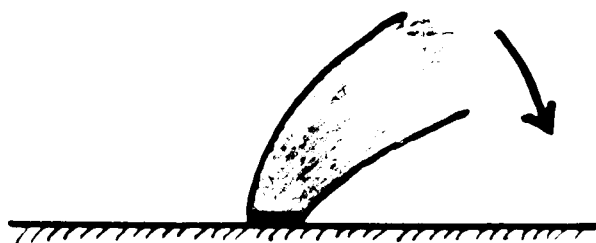


FIG 82. AMPERE'S RULE FOR A SOLID CONDUCTOR.

①. No magnetic field



② Magnetic field applied.
Column pivots about spot.



H into
paper.

③ Column pressed onto
cathode surface.



FIG 83 STAGES IN THE DEFLECTION OF THE ARC

be deflected independently of the ions. They will therefore produce new ions in the gas on the Ampere side of the discharge, and in this way the conducting path will be moved in the Ampere direction at a speed which is determined by the velocity component the electrons have in that direction, which is valid for moderate values of β . Here β is the ratio of the electron velocity to the speed of light.

The second approach is applicable to a discharge in which the density of charged particles is so high that there is strong electrostatic interaction between the ions and electrons, and hence the electrons can no longer be deflected independently of the ions. The electromagnetic force on the electrons will in this case be shared by the ions also, and so the speed at which the conducting path will move is determined by the speed at which the ions can be dragged through the gas. In the low pressure mercury arc the electron drift velocity v and the vapour pressure p vary greatly with distance from the cathode. The electron drift velocity in the positive column is likely to be somewhat higher than in the dense vapour of the cathode spot. The vapour pressure in the column is several orders of magnitude lower than in the cathode region of a mercury arc is strong, the second approach must be used when considering the motion of the cathode spot in a magnetic spot. Consequently, if a uniform transverse magnetic field is applied to the arc it can be seen from equation (11.1) that the positive column will be deflected faster than the cathode region. The column will now the electron drift velocity $\bar{v}$, is regarded as acting on the ion, therefore pivot about the spot until it is arrested by the cathode, and which is moved with a transverse velocity v_t . The force on the ion is will then be pressed down by the magnetic field into the cathode surface the same as would be exerted by an equivalent transverse electric field on the Ampere side of the spot (Fig. 93). This is in fact what is observed (see Fig. 78). This bending of the column will set up a local magnetic field due to $\bar{v} \times H = \frac{Hv}{c}$ which will act on the cathode (11.2) region of the discharge in the opposite direction to the applied field.

The force on an electron, given by equation (11.1) where v is now the electron drift velocity $\bar{v}$, is regarded as acting on the ion, therefore pivot about the spot until it is arrested by the cathode, and which is moved with a transverse velocity v_t . The force on the ion is will then be pressed down by the magnetic field into the cathode surface the same as would be exerted by an equivalent transverse electric field on the Ampere side of the spot (Fig. 93). This is in fact what is observed (see Fig. 78). This bending of the column will set up a local

$$\text{magnetic field due to } \bar{v} \times H = \frac{Hv}{c} \text{ which will act on the cathode (11.2)}$$

region of the discharge in the opposite direction to the applied field.

The transverse velocity of the ion can now be found from the equation

$$v_c = k^+ \frac{X}{p} \quad (11.3)$$

which is valid for moderate values of $\frac{X}{p}$. Here k^+ is the mobility of the ions and p the pressure of the gas in which they move (reduced to 0°C).

From (11.2) and (11.3), the velocity with which the conducting path is driven by the magnetic field H is given by

$$v_t = \frac{k^+ v^-}{c} \frac{H}{p} \quad (11.4)$$

The direction of motion will be given by Ampere's rule.

In the low pressure mercury arc the electron drift velocity v^- and the vapour pressure p vary greatly with distance from the cathode. The electron drift velocity in the positive column is likely to be somewhat higher than in the dense vapour of the cathode spot: the vapour pressure in the column is several orders of magnitude lower than in the spot. Consequently, if a uniform transverse magnetic field is applied to the arc it can be seen from equation (11.4) that the positive column will be deflected faster than the cathode region. The column will therefore pivot about the spot until it is arrested by the cathode, and will then be pressed down by the magnetic field onto the cathode surface on the Ampere side of the spot (Fig. 83). This is in fact what is observed (see Fig. 78). This bending of the column will set up a local magnetic field due to the arc current which will act on the cathode region of the discharge in the opposite direction to the applied field.

The origin of the self-magnetic field can be understood with the aid of Fig. 84. Consider a current of i e.m.u. flowing in a thin rectangular loop ABCD. There will be a magnetic field at P, the mid point of BC, due to the current in the arms AB, CD of the rectangle. The current in BC itself will not contribute to this field. If the distance $BC = 2R$ and AB and CD are long compared with BC the arms AB and CD will each contribute $\frac{i}{R}$ gauss to the field at P, so the resultant field at P will be

$$H_p = \frac{2i}{R} \quad (11.5)$$

Consider now the section PCD replaced by the cathode surface XY and let PBA now represent the deflected arc (Fig. 85). If we assume that the current in the cathode makes no contribution to the field, the self magnetic field at P will be given by

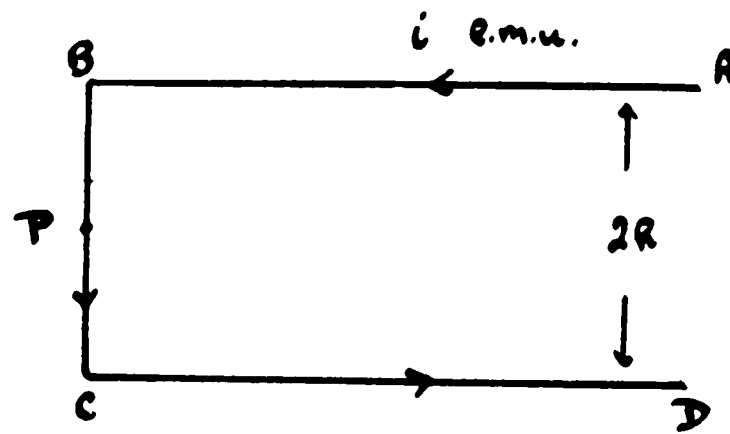
$$H_s = \frac{i}{R} \quad (11.6)$$

It can be seen from Fig. 86 that R can be regarded as an effective radius of curvature of the current where the latter is bent at the cathode spot.

The self-magnetic field H_s of the deformed arc will act on the cathode spot in opposition to the applied field H_o which has produced the deformation of the column. The resultant field H on the cathode region is therefore given by

$$H = H_o - H_s \quad (11.7)$$

Fig 86 RADIUS OF CURVATURE OF DEFLECTED ARC
that is



$$H_P = \frac{2i}{R}$$

OUT of paper.

FIG 84. MAGNETIC FIELD OF CURRENT LOOP.

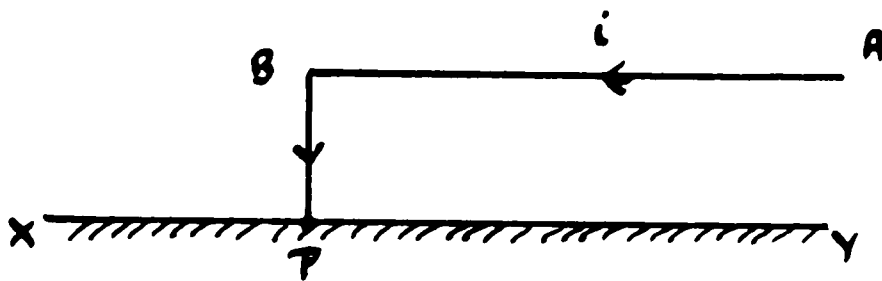


FIG 85. CURRENT LOOP AT CATHODE.

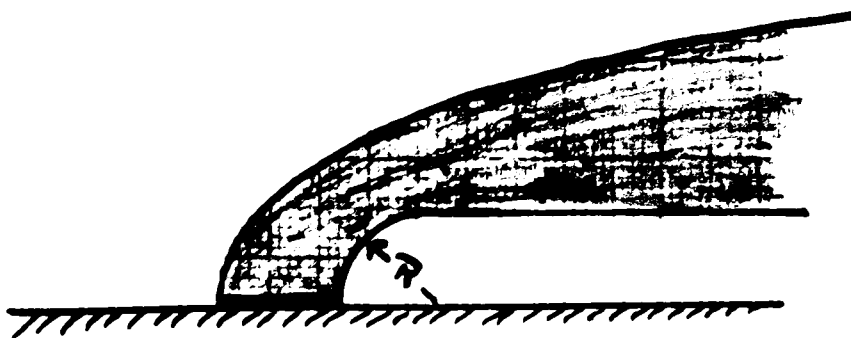


FIG 86 RADIUS OF CURVATURE OF DEFLECTED ARC

1). The electrons enter Region 2 with axial speeds of about 10^6 cm/sec and leave with a drift velocity $H = H_0 - \frac{i}{R}$ about $2 \cdot 10^5$ cm/sec (see Chapter (11.8))

The average drift velocity v will be taken as $10^6 - 10^7$ cm/sec. It seems possible that a strong H_0 might bend the arc column to such an extent that the current from the cathode would emerge from the side of Region 3 instead of from the top (see Fig. 22). The lower limit to R seems therefore to be of the order of the extension of Region 2 from the cathode, that is about $5 \cdot 10^{-5}$ cm. From equation (11.6) if $i = 5A$ we derive the maximum self-magnetic field

$$H = 10^{10} - 10^{11} \text{ gauss/cm}^2 \text{ and take } H \approx H_0 = 10^4 \text{ gauss we have from}$$

equation 11.4 $H_s \approx 10,000 \text{ gauss}$

It seems probable therefore that under conditions in which the arc is strongly deformed that the self-magnetic field can exceed the applied field in the cathode region, and that the resultant field there is therefore opposite in direction to the applied field. The arc spot will in fact obey Ampere's rule with respect to the resultant field, while appearing to violate it with respect to the applied field. This is proposed as a new explanation of the reverse driving effect, and has been published. (189)

A calculation can now be made from equation 11.4 or the speed at which the self-magnetic field is likely to drive the arc spot. The equation will be applied to Region 2 of the arc, since it is here that the majority of excited atoms are produced which maintain the electron emission from the cathode. Any movement of Region 2 will therefore result in a movement of the cathode region as a whole.

For mercury vapour $k^+ = 500$ cm/sec per v/cm.mm Hg (see Appendix

1). The electrons enter Region 2 with axial speeds of about 10^8 cm/sec and leave with a drift velocity of about $2 \cdot 10^5$ cm/sec (see Chapter 5). The average drift velocity v^- will be taken as $10^6 - 10^7$ cm/sec.

If the pressure in the arc vessel is low, the radial density gradient of neutral atoms in the cathode region is very steep (Fig.16) and so the ions and electrons will be driven by the magnetic field into vapour which is probably at least an order of magnitude less dense than on the axis. If we therefore take $p \approx 30 - 300$ mm Hg corresponding to $N = 10^{18} - 10^{19}$ atoms/cm³ and take $H \approx H_g = 10^4$ gauss we have from equation 11.4

$$v_t = 10^2 - 10^4 \text{ cm/sec}$$

Velocities up to about $8 \cdot 10^3$ cm/sec were observed in the experiments described in the previous chapter. Thus in spite of the very approximate nature of the calculation made above, it shows that the proposed explanation of the reverse driving effect is consistent with the observed high spot speeds in the reverse direction.

For $H_g < 3000$ gauss we find from experiment that v_{rev} is approximately proportional to H_g . For low magnetic fields it appears therefore that

11.2. The arc at low pressure. When the pressure in the arc vessel is low ($\sim 10^{-2}$ mm Hg) the magnetic field is observed to press the arc column right down onto the cathode surface on the Ampere side of the spot (Fig.78). On account of the increased losses from the column to the cathode surface it is to be expected that the electron temperature and axial field in the column will rise. (193) This will cause the discharge to become brighter and the voltage across the arc to increase.

Under these conditions R is assumed to be sufficiently small to make the self magnetic field H_s exceed the applied field H_0 and so the motion of the spot will be in the reverse direction. On account of the complicated geometry of the distorted cathode region no attempt will be made here to give more than a qualitative account of how the velocity of the spot varies with current and magnetic field.

The velocity of the spot in the reverse direction will be called v_{rev} . We assume that if the ambient pressure p_a in the arc vessel is low, the pressure p in the spot will be independent of the current and the applied magnetic field and we assume also that the drift velocity of the electrons, at least in Region 2, will be constant.

Hence from (11.4) and (11.8) we have

$$v_{rev} \propto \frac{1}{R} - H_0 \quad (11.9)$$

At constant current R will clearly decrease as H_0 is increased, since the greater magnetic field presses the column closer to the cathode.

For $H_0 < 3000$ gauss we find from experiment that v_{rev} is approximately proportional to H_0 (Fig. 63). For low magnetic fields it appears therefore that

$$R \propto \frac{1}{H_0} \quad (11.10)$$

Since R cannot decrease indefinitely, at larger H_0 the decrease in R will be less rapid as H_0 increases, and the velocity will level-off, as is observed. Another reason for the velocity tending to a constant value may be that at these speeds (10^4 cm/sec) the transverse velocity imparted

to the ions becomes comparable with their thermal velocity so that the mobility treatment (equation 11.4) is no longer applicable, and the velocity is likely to increase less than proportionally with H . Further-
 ing the mercury or by admitting argon, two effects each which tend to more at such speeds the arc may not be able to establish the vapour cloud increase R . The first can be seen by referring to Fig. 74. As H in-
 necessary for the cathode processes, and so the emission mechanism limits creases, the high densities near the cathode extend further from the the spot speed.

Consider now the variation of velocity with current, the applied is likely to increase R .
 magnetic field being kept constant. From (11.9) we should expect, if R is a function of H_0 only that v_{rev} will be proportional to i . Observa-
 the pressure increases the column tends to contract, until at pressures tions show however that v_{rev} increases less than proportionally to i .
 of the order 100 mm Hg the arc takes the form of a bright filament from (Fig. 71)

cathode to anode. In this condition the arc behaves like a stretched
 It has been shown (Fig. 14) that the density distribution of elastic string, tending to resist deformation from the straight line
 vapour on the arc axis is a function of $\frac{z}{a}$ where a is the radius of the between the electrodes. The applied magnetic field produces little
 arc spot. If the current density is constant deformation of the arc at high pressure (Fig. 59).

$$a \propto i^{\frac{1}{2}} \quad (11.11)$$

This resilience, or resistance to deformation of a contracted
 If the current is increased therefore, the regions of high vapour density column is probably due to the surplus negative charge which exists at
 will be moved further away from the cathode, and this is likely to cause the boundary of the column (Fig. 57). Any attempt to bend the column
 an increase in R . If we consider R is approximately proportional to a , will result in a compression of these charges on the inside of the bend
 we have from (11.9) and (11.11), if H_0 is constant and a rarefaction on the outside, which will tend to restore the column

$$v_{rev} \propto i^{\frac{1}{2}} - \text{const} \quad \text{The greater the pressure} \quad (11.12)$$

to its original straight shape.
 the arc vessel the greater the degree of contraction (i.e. the smaller
 This is a better agreement with observations.

the radius of the column) and hence the greater the resistance to defor-
 The new theory can thus account qualitatively for the observed mation. Increasing the current also increases the resistance to defor-
 velocity variation at low pressure. The above treatment answers the ori-
 nation. This can be demonstrated quite simply by blowing a jet of air ticsisms by St. John and Winans (184) of this theory when it was first pro-
 posed.

11.3. The reversal of motion.

When the pressure in the arc vessel is raised, either by heating the mercury or by admitting argon, two effects occur which tend to increase R . The first can be seen by referring to Fig. 14. As N_s increases, the high densities near the cathode extend further from the cathode surface, and we have suggested in the previous section that this is likely to increase R .

The other effect is a property of the positive column. As the pressure increases the column tends to contract, until at pressures of the order 100 mm Hg the arc takes the form of a bright filament from cathode to anode. In this condition the arc behaves like a stretched elastic string, tending to resist deformation from the straight line between the electrodes. The applied magnetic field produces little deformation of the arc at high pressure (Fig. 81).

This resilience, or resistance to deformation of a contracted column is probably due to the surplus negative charge which exists at the boundary of the column (Fig. 87). Any attempt to bend the column will result in a compression of these charges on the inside of the bend and a rarefaction on the outside, which will tend to restore the column to its original straight path (Fig. 88). The greater the pressure in the arc vessel the greater the degree of contraction (i.e. the smaller the radius of the column) and hence the greater the resistance to deformation. Increasing the current also increases the resistance to deformation. This can be demonstrated quite simply by blowing a jet of air

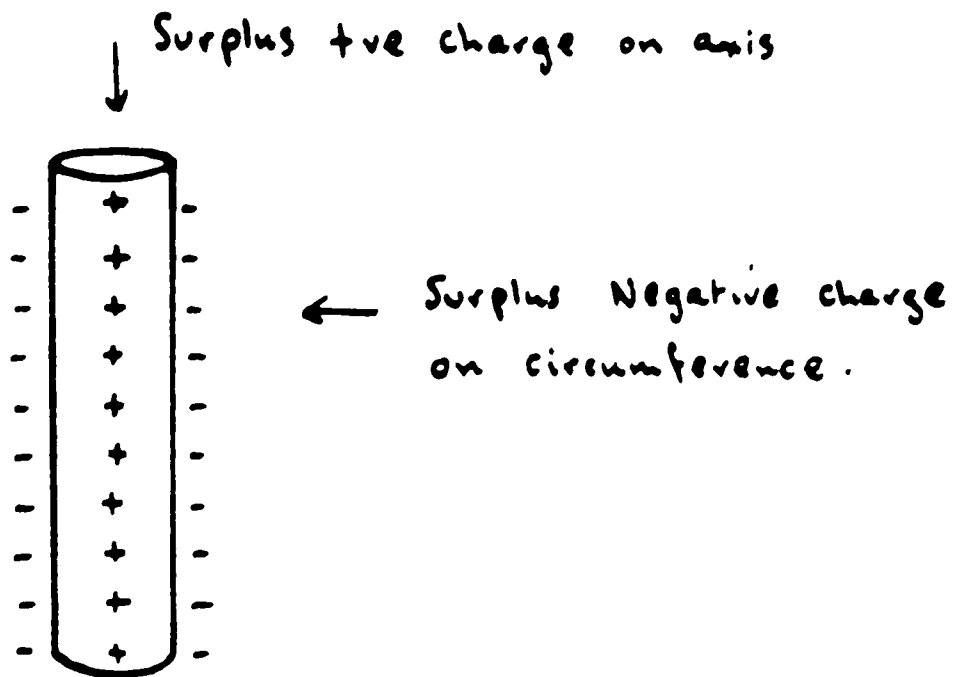


FIG 87. THE CONTRACTED POSITIVE COLUMN.

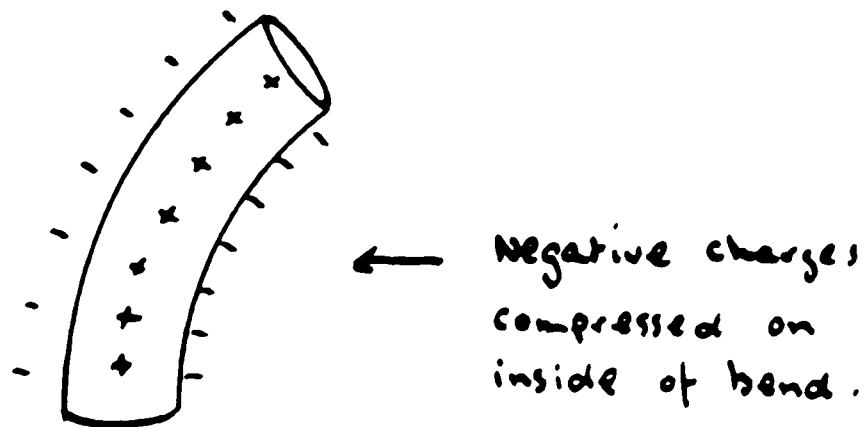


FIG 88 RESISTANCE OF COLUMN TO DEFORMATION

onto an arc maintained in open air between pointed electrodes. The higher the current, the less the deflection.

Consider now the curves of velocity against pressure of argon given in Fig. 75. Admitting a small pressure of argon (up to 1 mm Hg) had no effect on the velocity of the spot: this is to be expected, since the vapour conditions in the vicinity of the spot are determined mainly by the evaporation from the spot itself and the density there corresponds to a pressure much greater than 1 mm Hg.

As the argon pressure is increased, the velocity falls for two reasons. The first is that the radial density gradient of atoms in the spot will become less steep and so the magnetic field will drive the ions into a region which is denser than was the case at very low pressure. From equation (11.4) this will cause a decrease in velocity. In addition, R is likely to increase as the high density regions now extend further from the cathode: this will reduce the self-magnetic field and hence decrease the reverse velocity.

At higher pressures the contraction of the column will take place and this will be the major factor tending to increase R .

The onset of contraction of the column, and the corresponding increase of R can be seen in the velocity - pressure curves of Fig. 75. The curves for $H = 4000$ g and $i = 2A$ and $5A$ cross at a pressure of about 10 mm Hg. Below this pressure v_{rev} increases with current for the reasons described in the previous section. Above it v_{rev} decreases with current, since, as has been explained, the resistance to deformation of a contracted column is greater the larger the current, so R will

increase rapidly with increasing i . When the pressure is increased to about 100 mm Hg R will be so large that $\frac{1}{R} = H_0$ and the resultant magnetic field on the cathode region will be zero. The arc will therefore move in neither direction. This is the point of reversal and the pressure is known as the reversal pressure. If the pressure is increased still further R increases and H_s is now less than H_0 . The arc therefore moves in the Ampere direction. If the arc is at the point of reversal and H_0 is increased the arc will move again in the reverse direction; this indicates that R must decrease with increasing H_0 at such a rate that $\frac{1}{R}$ becomes greater than H_0 .

11.4. The rise in arc voltage in the magnetic field.
If the arc is at the point of reversal and the current is increased the arc will move in the Ampere direction; this indicates that R must increase faster than linearly with i so that H_s becomes less than H_0 .

For a given current and magnetic field, the reversal pressure depends on the nature of the gas which is admitted to the arc. Gallagher measured the reversal pressures for a number of gases and tried to correlate them with the resonance potentials of the gases. (176) In the light of the new theory, a comparison should be made between the reversal pressures and the pressures at which constriction of the column becomes pronounced. Gallagher's measurements of reversal pressures, and Fowler's measurements of the pressure at which a column contracts (194) are

compared in Table 10. It can be seen that He and A, which exhibit high reversal pressures, are also highest on the list of contraction pressures. O_2 , being electronegative, causes contraction in air at low pressure, and the corresponding reversal pressure for O_2 is low. Hydrogen appears to be an exception, having a high reversal pressure and a low contraction pressure. Since however the contraction measurements were made in a long cylindrical column where the current density was low, and the important region of the arc is close to the cathode where the current density is high, this comparison must be accepted with caution. At this stage no more can be said except that a correlation between reversal pressure and contraction pressure for a given gas seems likely.

11.4. The rise in arc voltage in the magnetic field.

We have found (Sect.10.2) that although the voltage across the mercury arc tube was of the order 20 v in the absence of a magnetic field, it would rise to as high as 120 v. when a field was applied, provided that the pressure was low. The voltage was found to be proportional to the speed of the spot (Fig.70) and thus it seemed likely that the increase in voltage was due to some loss mechanism in the moving spot itself, for example an increased heat conduction loss from the moving spot to the cathode surface.

It was shown in Chapter 6 that the heat conduction loss from a mercury arc spot to the cathode in the absence of a magnetic field is about 2.7 watts/ampere. The spot is, under these conditions, moving with a random velocity of about 10^3 cm/sec. When driven by the magnetic

TABLE 10. COMPARISON OF THE PRESSURE p_r AT WHICH AN ARC REVERSES WITH THE PRESSURE p_c AT WHICH A POSITIVE COLUMN CONTRACTS.

Gas	p_r mm Hg	p_c mm Hg
He	160	45
A	110	35
Ne	-	25
H ₂	80	3
N ₂	70	-
Air	-	8
O ₂	2	-
Hg	1	-

p_c referred to a tube of 1cm radius

p_r measured for $H_0=100g$, $i=4A$.

field the spot attains speeds of about $8 \cdot 10^3$ cm/sec. It seems unlikely therefore that the conduction loss will increase by more than a factor 8: this would make it about 20 watts/ampere, and would therefore only increase the arc voltage by about 17 volts. It appears therefore that the motion of the spot cannot produce sufficiently large conduction losses to account for the observed increase of 100 v in the arc voltage. Furthermore it was found that changing the cooling conditions at the cathode greatly affected the voltage but not the speed of the spot. The relatively small change in vapour pressure near the cathode due to a small change in cathode temperature would be unlikely to affect conditions in the spot where the density of vapour is already high: it could only have an effect on the arc column at some distance from the cathode spot. (172, 173, 184, 185) have found that the Hall

It is proposed therefore that the magnetic field presses the column onto the cathode surface and increases the losses from it. The electron temperature and axial electric field both rise to combat this. The increased electron temperature is evidenced by the increased brightness of the column and the increased axial electric field accounts for the increased voltage. It was observed that at low pressure the column was pressed down onto the cathode round the entire circumference of our cathode pool. The circumference was about 20 cm, so that an average increase in the axial electric field of about 5V/cm would account for the increase of 100 v. in the arc voltage.

Warming the cathode either by reducing the cooling or increasing the current would tend to lessen the degree with which the

column is pressed down onto the cathode by increasing the vapour pressure near the surface. The arc voltage would therefore tend to fall as was observed (Sec.10.2). Admitting a small pressure (1 mm Hg) of argon likewise had no effect on the speed, but reduced the voltage by 30% (Sect.10.3).

In the experiments of St. John and Winans, in whose arc the magnetic field was for the most part parallel to the discharge, no increase in voltage was observed as presumably only a short section of the column near to the spot was pressed to the cathode. Also, C.G. Smith⁽¹⁶⁹⁾ using a very short mercury arc observed a voltage of only 15 v. although his spot speed was 70 m/sec. There was in this case presumably hardly any column to be deflected.

Smith and others^(172,173,184,185) have found that the HgII lines in the spectrum of the arc are enhanced and HgIII lines appear when the magnetic field is applied. These lines were attributed to the spot itself, but in the light of the new proposals it seems likely that they come from the part of the column close to the spot which is pressed down onto the cathode. The increased electron temperature in this region will assist the cumulative processes by which these lines are produced (see Section 8.6).

When the separation was about 2mm the arc ran always in the same direction throughout the range of currents and fields at our disposal. However, when the separation was decreased to 0.5 mm it was found that for low current and high field the arc ran in the reverse direction. For a separation of 0.5mm the notion of a 5A arc was in

Magnet

THE REVERSE DRIVING OF A SHORT ARC

12.1. The Short Copper Arc.

The experiments of Himler and Cohn⁽¹⁸⁰⁾ indicated that the reversal pressure could be increased by decreasing the distance between the electrodes, although the pressures at which they observed reversal were well below atmospheric. We decided to attempt to drive an arc in the reverse direction in open air, and for this purpose the most favourable conditions for reverse motion were chosen, namely low current, large magnetic field and small electrode separation.

Two wedge-shaped copper electrodes were set up in air with their sharp edges parallel and transverse to the field of an electromagnet which could produce a steady magnetic field of up to 5000 gauss (Fig. 89). The electrodes were connected to a D.C. generator through a resistance, and the current could be varied between 0.5A and 5A. An arc was struck by touching the electrodes at one end with a carbon rod. The arc was driven the length of the electrodes by the magnetic field and blew out at the end of its run.

When the separation was about 2mm the arc ran always in the ampere direction throughout the range of currents and fields at our disposal. However, when the separation was decreased to 0.5 mm it was found that for low current and high field the arc ran in the reverse direction. For a separation of 0.5mm the motion of a 3A arc was in

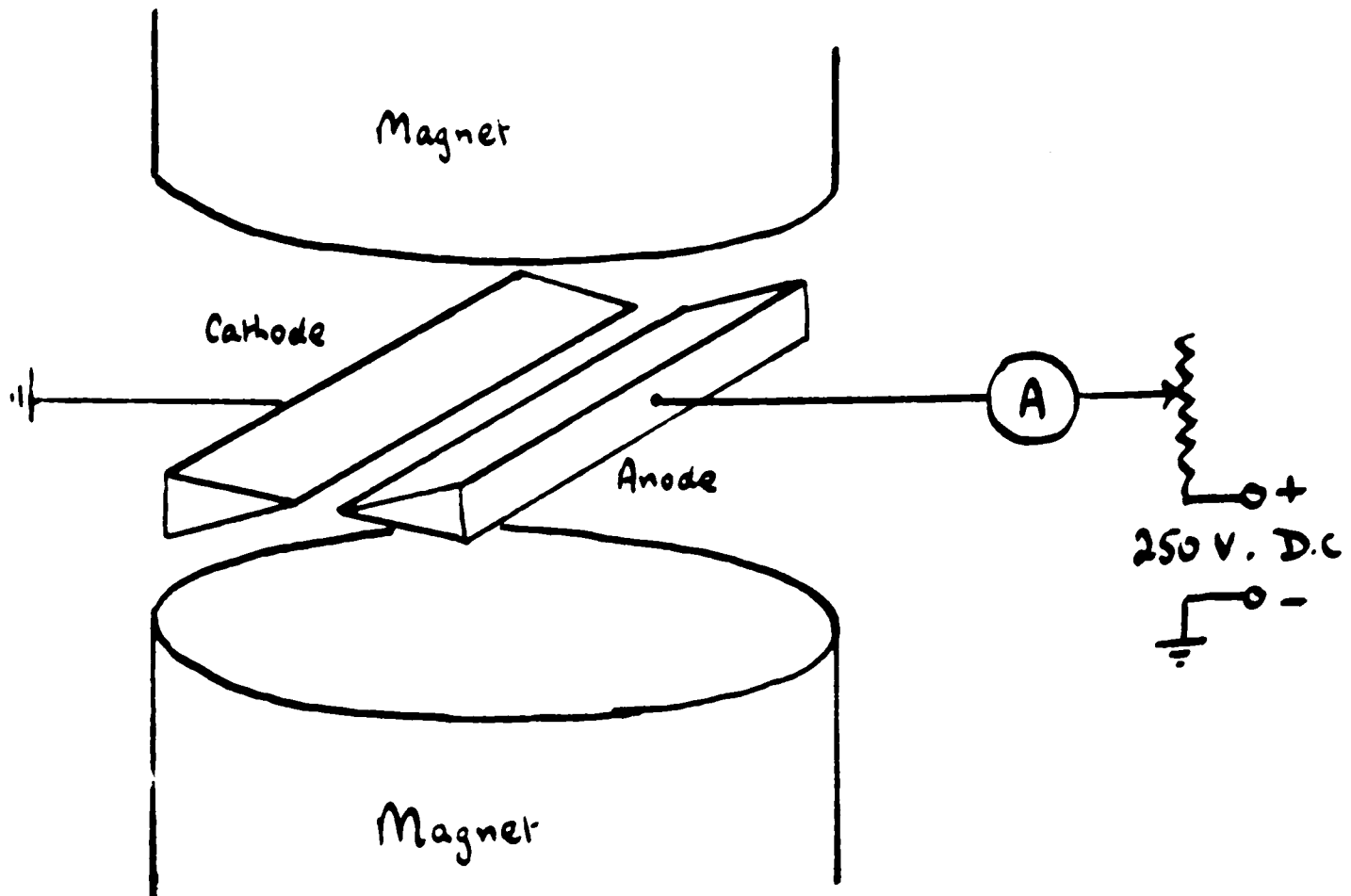


Fig. 89. THE SHORT COPPER ARC.

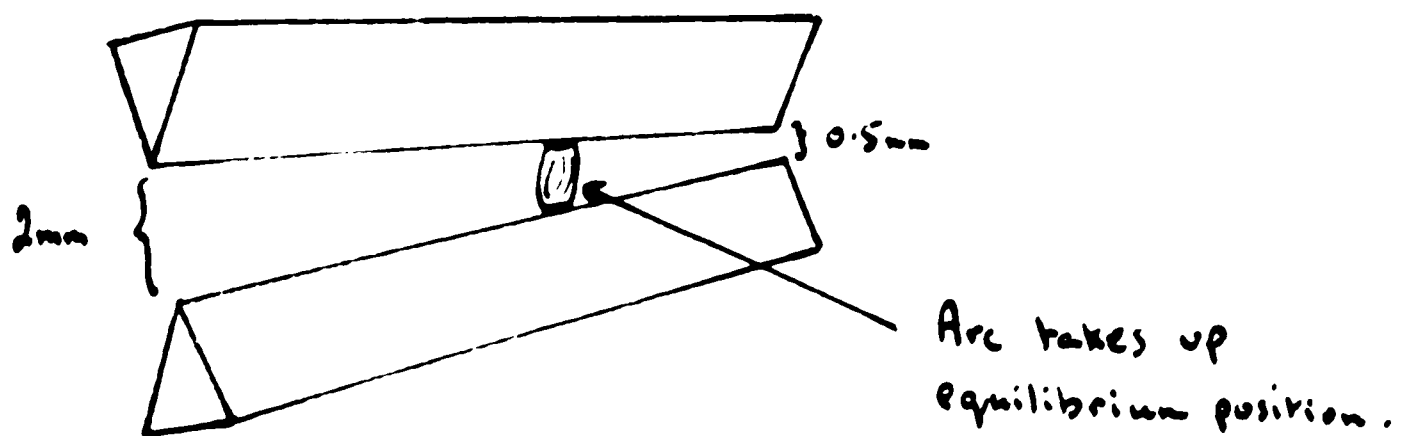


Fig 90. THE TAPERED GAP.

the ampere direction at low fields but changed to the reverse direction when the field exceeded 3,000 gauss. Also for a field of 4,000 gauss the motion was reverse at low current but changed to the ampere direction when the current exceeded 5A. The reversal of the arc in these cases was not sharply defined and immobility occurred over a definite region of current and magnetic field.

In another experiment (Fig. 90) the wedge-shaped copper electrodes were not set parallel but formed a gap which tapered from 2 mm at one end to less than 0.5 mm at the other. The taper was such that an arc moving in the Ampere direction would move towards the narrow end. When an arc of 3A was started at one end in a magnetic field of 5,000 gauss it was observed that the arc moved quickly away from the end and took up an equilibrium position at some point along the electrodes. When either current or field were varied the arc took up a new equilibrium position: the arc would move towards the narrow end of the gap if the field was decreased or if the current was increased, and vice-versa.

The dependences of reversal on current and field correspond exactly to the conditions of reversal of long arcs at much lower pressures. The significant feature is that by decreasing the electrode separation, the reversing pressure for a low-current arc in a strong magnetic field can be increased above one atmosphere, so that reverse driving, which is normally regarded as a low pressure effect, can occur in open air. The importance of the electrode separation has been overlooked in previous investigations of the effect.

12.2. Theory of the Short Arc.

H_2O
The importance of the electrode separation will now be examined in the light of the new theory.

Consider an arc in air between parallel copper electrodes a few mm apart. In the absence of a magnetic field the anode and cathode spots will be opposite each other (Fig.91). When a transverse magnetic field is applied, the column is deflected in the Ampere direction with respect to the cathode spot. The anode spot will move with the column and so will be displaced in the Ampere direction with respect to the cathode spot (Fig.92). At atmospheric pressure such an arc would normally move in the Ampere direction.

If now the electrode separation is decreased the arc will be compressed. (Fig.93) It is suggested that on account of the relative displacement of the anode and cathode regions the anode will now act as a confining wall for the column near the cathode, since now the electrode separation is smaller than the usual diameter of the column. The column will therefore tend to move away from the anode and so the radius of curvature of the column near the cathode will decrease. The proximity of the anode will assist the applied field to decrease R , and therefore may cause the self-magnetic field to predominate. The arc will then move in the reverse direction, as has been described.

The importance of the short arc experiment is that it confirms that the reverse driving effect is connected with the deformation of the arc column and is not an intrinsic property of the cathode spot. We

FIG 93. SHORT ARC IN MAGNETIC FIELD

$H_0 = 0$

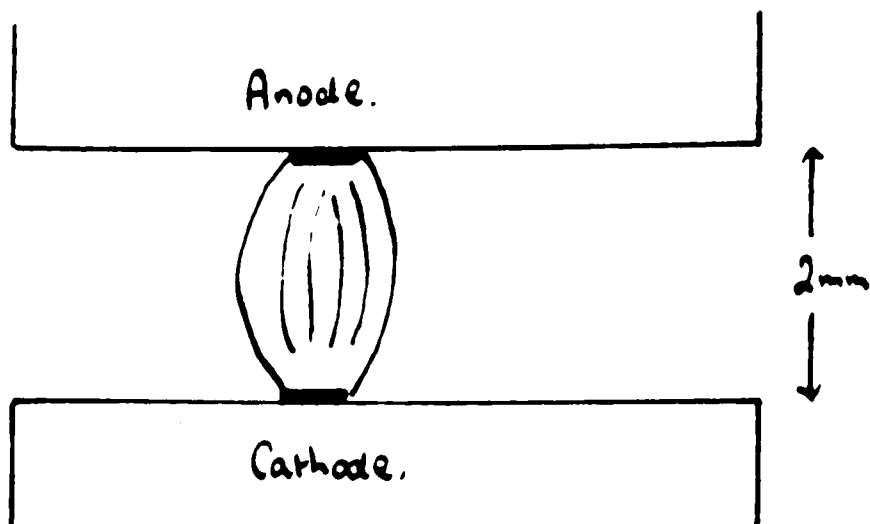


FIG 91. COPPER ARC WITHOUT MAGNETIC FIELD

H_0 into paper.
Ampere Direction $\rightarrow$

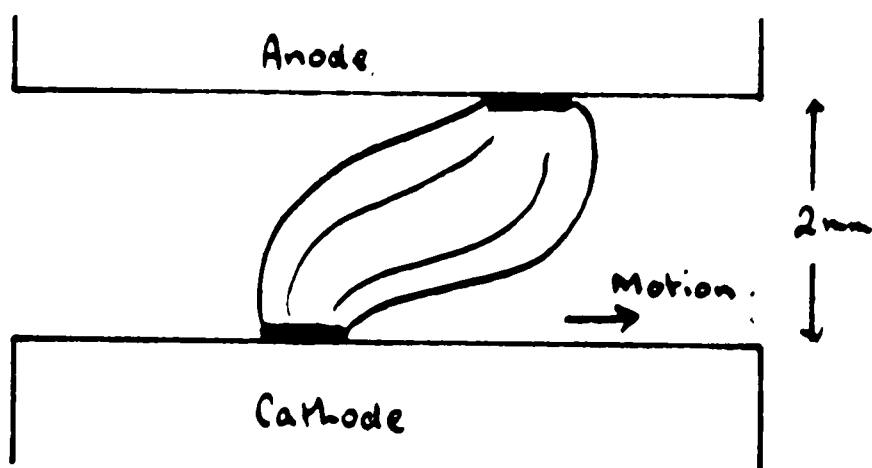


FIG 92. COPPER ARC IN MAGNETIC FIELD.

H_0 into paper.
Ampere direction $\rightarrow$

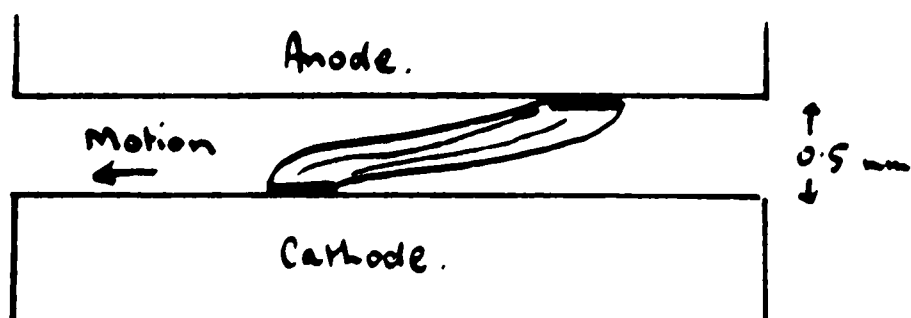


FIG 93. SHORT ARC IN MAGNETIC FIELD.

have shown that an arc can be reversed by decreasing the separation from 2mm to 0.5mm. This can have no direct effect on the cathode spot since the dimensions of the cathode region are two orders of magnitude smaller: it can however act on the cathode region through the deformation of the column. No other theory has been proposed which takes the electrode separation into account. (185)

12.3. Conclusion.

We conclude that the reverse driving effect is caused by the self-magnetic field of the deflected arc column acting on the cathode region. This opposes the applied field and may exceed it. Evidence that the column is deflected is provided by the appearance of the arc and the increase in voltage. Since the reversal pressures correspond to densities orders of magnitude less than the vapour densities in the cathode region the effect of admitting gas must be principally on the column. Finally the influence of electrode separation emphasises that the cause of reverse driving must lie in the column and not the cathode spot. conditions under which it has been observed.

The only phenomenon not explained by the above theory is the doubling of spot speeds at very large magnetic fields which has been reported. (171-173, 184-186) This has only been observed with experimental arrangements in which the arc was driven round a circular anchor by a magnetic field which was parallel to the arc current except in the immediate cathode region. We would suggest three possible explanations of this effect:-

1. Spot splitting. If the spot split into 2 spots at a certain current and if the spots set themselves at diametrically opposite points on the cylindrical anchor, the photocell method of measurement (as described in Sec. 10.1) would record a doubling of speed. The discontinuity of the speed change and the almost exact doubling suggest this, although recent experiments have shown a continuous change. (185)

2. Spot runs into the arc 'tail'. If the column were bent right round the anchor by a strong magnetic field the arc might well be running into a highly ionised region which facilitates the creation of a new arc spot.

3. Change of rotational mode of the column. As the arc current is for the most part parallel to the magnetic field, yet the source of current (the spot) is rotating, there may be modes of rotation of the column which would change discontinuously.

It would be instructive to see whether the doubling of the velocity occurred for an arc running in a straight path. This would determine whether or not the effect was due to the particular experimental conditions under which it has been observed.

This varies with the electron energy E (see Ref. 196).

$$\text{If } E = 0.5 \text{ eV } \lambda_c = 1.4 \times 10^{-3} \text{ cm } (\lambda_c E = 2.0 \times 10^{13})$$

$$E = 1.0 \text{ eV } \lambda_c = 2.5 \times 10^{-3} \text{ cm } (\lambda_c E = 9.0 \times 10^{13})$$

$$E = 2.0 \text{ eV } \lambda_c = 4.3 \times 10^{-3} \text{ cm } (\lambda_c E = 2.3 \times 10^{14})$$

$$E = 25 \text{ eV } \lambda_c = 1.7 \times 10^{-3} \text{ cm } (\lambda_c E = 6.1 \times 10^{14})$$

Drift velocity v_d^+ of mercury ions (probably Hg_2^+) in mercury vapour is

APPENDIX 1

Some data on Mercury and Copper.

Mercury vapour.

Mass of mercury atom $M = 3.33 \times 10^{-22}$ gm.

$\frac{e}{M}$ for mercury ion $Hg^+ = 1.44 \times 10^{12}$ esu/gm = 480 coul/gm.

Velocity v of a mercury ion which has fallen through a potential difference V is given by

$$v = 0.98 \times 10^5 V^{\frac{1}{2}} \quad (V \text{ in volts})$$

Velocity v of an electron which has fallen through a potential difference V is given by

$$v = 6 \times 10^7 V^{\frac{1}{2}} \quad (V \text{ in volts})$$

Mean free path λ of mercury atoms at 1 mm Hg and $0^\circ C$

$$\lambda = 1.65 \times 10^{-3} \text{ cm}$$

If N is the number density of atoms

$$\lambda N = 6 \times 10^{13}$$

Mean free path λ_e of electrons in mercury vapour at 1 mm Hg and $0^\circ C$.

This varies with the electron energy E (see Ref. 196).

$$\text{If } E = 0.5 \text{ eV } \lambda_e = 1.4 \times 10^{-3} \text{ cm } (\lambda_e N = 5.0 \times 10^{13})$$

$$E = 1.0 \text{ eV } \lambda_e = 2.5 \times 10^{-3} \text{ cm } (\lambda_e N = 9.0 \times 10^{13})$$

$$E = 4.0 \text{ eV } \lambda_e = 6.3 \times 10^{-3} \text{ cm } (\lambda_e N = 2.3 \times 10^{14})$$

$$E = 25 \text{ eV } \lambda_e = 1.7 \times 10^{-3} \text{ cm } (\lambda_e N = 6.1 \times 10^{14})$$

Drift velocity v_d^+ of mercury ions (probably Hg_2^+) in mercury vapour is

given by (196)

$$v_d^+ = k^+ \frac{X}{p}$$

where $k^+ = 500$ cm/sec per V/cm.mm.Hg for X/p less than 20.

and by Let the current originate from the plane $x = 0$ where the

potential is zero. $v_d^+ = k^+ \frac{X}{p}^{\frac{1}{2}}$ current density be j^+ .

In the plane x , let the potential be V , the electric field E , the charge density ρ and the ion drift velocity be v . If k is the mobility

Mercury metal sure we have the four equations

Density. 13.6 gm/cm^3

Specific Heat. $3.28 \times 10^{-2} \text{ cal/gm}^\circ\text{C.}$

Thermal conductivity at 50°C. $0.019 \text{ cal/sec.cm.}^\circ\text{C.}$

Copper

Density. 8.9 gm/cm^3

Specific Heat. $0.09 \text{ cal/gm}^\circ\text{C.}$

Thermal conductivity at 100°C. $0.9 \text{ cal/sec.cm}^\circ\text{C.}$

Vapour pressure of copper

$P =$ 1 At. 10 At. 1000 At.

$T =$ 2660°K 3070°K 3500°K

Integrate $N =$ and $p = 2.85 \times 10^{18}$ 2.47×10^{19} $2.17 \times 10^{20} \text{ atoms/cm}^3$.

$$X^{\frac{1}{2}} = \frac{6\pi j^+ p^{\frac{1}{2}}}{k} x$$

From (7) and (4) we have

$$\frac{dV}{dx} = - \left\{ \frac{6\pi j^+ p^{\frac{1}{2}}}{k} \right\}^{\frac{2}{3}} x^{\frac{2}{3}}$$

Integrating and putting $V = 0$ at $x = 0$ we have

APPENDIX 2

Derivation of equation 1.3

Consider a current of positive ions flowing between infinite parallel planes. Let the current originate from the plane $x = 0$ where the potential is zero. Let the current density be j^+ .

In the plane x , let the potential be V , the electric field X , the charge density ρ and the ion drift velocity be v . If k is the mobility and p the pressure we have the four equations

$$j^+ = \rho v \quad (1)$$

$$v = k \left(\frac{X}{p} \right)^{1/2} \quad (2)$$

$$\frac{dX}{dx} = 4\pi\rho \quad (3)$$

$$X = -\frac{dV}{dx} \quad (4)$$

From (1) and (3)

$$\frac{dX}{dx} = \frac{4\pi j^+}{v} \quad (5)$$

From (5) and (2)

$$\frac{dX}{dx} = \frac{4\pi j^+}{k} \quad (6)$$

Integrating and putting $X = 0$ at $x = 0$ we have

$$X^{3/2} = \frac{6\pi j^+ p^{1/2}}{k} x \quad (7)$$

From (7) and (4) we have

$$\frac{dV}{dx} = - \left\{ \frac{6\pi j^+ p^{1/2}}{k} \right\}^{2/3} x^{2/3} \quad (8)$$

Integrating and putting $V = 0$ at $x = 0$ we have

$$V^3 = \frac{77 j^2 p x^5}{k^2}$$

(9)

This is equation (1.3).

where p is in dynes/cm², k is in atoms/cm³ and

Since $1 \text{ atm Hg} = 1.36 \times 10^6 \text{ dynes/cm}^2$ and $1 \text{ atm} = 760 \text{ mm Hg}$

hence, we have from (1.1)

$$k = 2.7 \times 10^{18} \frac{p}{T}$$

where p is in atm Hg and T is in $^{\circ}\text{K}$.

The p as a function of T is found from tables of vapor pressure of mercury.

is then derived from equation (1.2).

APPENDIX 3

N as a function of T

Consider the familiar equation

$$p = NkT \quad (3.1)$$

where p is in dynes/cm², N is in atoms/cm³ and T is in °K.

Since 1 mm Hg = 1.36 x 981 dynes/cm² and $k = 1.38 \times 10^{-16}$ ergs/degree, we have from (3.1)

$$N = 9.7 \times 10^{18} \frac{p}{T} \quad (3.2)$$

where p is in mm Hg and T is in °K.

The p as a function of T is found from tables. N as a function of T is then derived from equation (3.2).

$$T - T_0 = (T_0 - T_1) \frac{2}{\pi} \tan^{-1} \frac{a}{2} \quad (4.2)$$

where

$$2a^2 = r^2 + s^2 - a^2 + \left\{ (r^2 + s^2 - a^2)^2 + 4s^2 a^2 \right\}^{\frac{1}{2}} \quad (4.3)$$

It can be shown that the isothermal surfaces in the solid are ellipsoids.

The flow of heat per unit area in the plane $z = 0$ is given by

$$Q = -\Lambda \left(\frac{\partial T}{\partial z} \right)_{z=0} \quad (4.4)$$

where Λ is the thermal conductivity of the solid.

Differentiating (4.1) with respect to z and putting $z = 0$ we have

$$\left(\frac{\partial T}{\partial z} \right)_{z=0} = - \frac{2(T_0 - T_1)}{\pi} \int_0^\infty J_0(\lambda r) \sin \lambda s \alpha \lambda \quad (4.5)$$

On evaluating the integral (109) we have

APPENDIX 4

Heat conduction in the cathode

Consider a semi-infinite solid bounded by the plane $z = 0$. Take cylindrical co-ordinates and let the circular area $0 < r < a$ in the plane $z = 0$ be at a temperature T_0 , and let the temperature at infinity be T_s .

The distribution of temperature in the solid for the steady state is known⁽¹⁰³⁾ and is given by

The total flow by conduction from the area of radius a through the solid is therefore

$$T - T_s = (T_0 - T_s) \frac{2}{\pi} \int_0^\infty e^{-\lambda z} J_0(\lambda r) \frac{\sin \lambda a}{\lambda} d\lambda \quad (4.1)$$

The integral has been evaluated⁽¹⁰⁴⁾ and the solution is then

$$T - T_s = (T_0 - T_s) \frac{2}{\pi} \tan^{-1} \frac{a}{z} \quad (4.2)$$

where

$$2z^2 = r^2 + z^2 - a^2 + \left\{ (r^2 + z^2 - a^2)^2 + 4z^2 a^2 \right\}^{\frac{1}{2}} \quad (4.3)$$

It can be shown that the isothermal surfaces in the solid are ellipsoids.

The flow of heat per unit area in the plane $z = 0$ is given by

$$Q = -\Lambda \left(\frac{\partial T}{\partial z} \right)_{z=0} \quad (4.4)$$

where Λ is the thermal conductivity of the solid.

Differentiating (4.1) with respect to z and putting $z = 0$ we have

$$\left(\frac{\partial T}{\partial z} \right)_{z=0} = - \frac{2(T_0 - T_s)}{\pi} \int_0^\infty J_0(\lambda r) \sin \lambda a d\lambda \quad (4.5)$$

On evaluating the integral⁽¹⁰⁵⁾ we have

$$\left(\frac{\partial T}{\partial z}\right)_{z=0} = \begin{cases} \frac{-2(T_0 - T_s)}{\pi} \cdot \frac{1}{(a^2 - r^2)^{1/2}} & \text{for } r < a \\ 0 & \text{for } r > a \end{cases} \quad (4.6)$$

Hence from (4.4) and (4.6)

$$Q = \begin{cases} \frac{2\Lambda(T_0 - T_s)}{\pi} \cdot \frac{1}{(a^2 - r^2)^{1/2}} & \text{for } r < a \\ 0 & \text{for } r > a \end{cases} \quad (4.7)$$

The total flow by conduction from the area of radius a through the solid is therefore

$$C = \int_0^a 2\pi Q r dr \quad (4.8)$$

From (4.7) and (4.8) we find by integration that

$$C = 4a\Lambda(T_0 - T_s) \quad (4.9)$$

and acts towards the cathode. Substituting $E = 3 \cdot 10^{-18}$ esu/cm and $\frac{dE}{dz} = 10^3$ esu/cm² we obtain $f = 3 \cdot 10^{-2}$ dynes.

The drift velocity which this force would give to an atom may be estimated by comparing the action of the atom with the action of an ion in a uniform electric field. We have

$$v_d^* = k^* E \quad (5.3)$$

where v_d^* is the drift velocity of the ion and k^* the mobility. Since the force p^* on the ion is given by

$$p^* = eE \quad (5.4)$$

APPENDIX 5

The Action of the Electric Field on the Excited Atoms.

An excited atom has a permanent electric dipole moment M which is given approximately by the equation, (192)

$$\frac{1}{\tau} = \frac{4\pi^4}{3} \cdot \frac{M^2}{h\lambda^2} \quad (5.1)$$

where τ is the lifetime of the excited state and λ the wavelength of the emitted radiation. For the 3P_1 state of mercury $\lambda = 2.5 \times 10^{-5}$ cm. $\tau \approx 10^{-7}$ sec. Hence $M \approx 3.10^{-18}$ e.s.u.cm.

In Region 1 there is an electric field which varies from 2.10^6 v/cm at the cathode to about 10^4 v/cm at a distance 5.10^{-6} cm. The average field gradient $\frac{dX}{dz} = 4.10^{11}$ v/cm² $\approx 10^9$ e.s.u./cm².

The force on the dipole M is given by

$$f = M \frac{dX}{dz} \quad (5.2)$$

and acts towards the cathode. Substituting $M = 3.10^{-18}$ esu/cm and $\frac{dX}{dz} = 10^9$ esu/cm² we obtain $f = 3.10^{-9}$ dynes.

The drift velocity which this force would give to an atom may be estimated by comparing the motion of the atom with the motion of an ion in a uniform electric field. We have

$$v_d^+ = k^+ X \quad (5.3)$$

where v_d^+ is the drift velocity of the ion and k^+ the mobility. Since the force p^+ on the ion is given by

$$p^+ = eX \quad (5.4)$$

we have, from (5.3)

$$v_d^* = \frac{k^+ f}{e} \quad (5.5)$$

For mercury k^+ at 1 mm Hg and $0^\circ\text{C} = 500 \text{ cm/sec per V/cm}$.

at $N = 10^{20}$ and $T \approx 1000^\circ\text{K}$ $k^+ \approx 0.1 \text{ cm/sec per V/cm}$

$\approx 30 \text{ cm/sec per esu/cm}$.

Thus substituting this value in (5.5), with $f = 3 \cdot 10^{-9}$ dynes, we obtain

$$v_d^* \approx 100 \text{ cm/sec}$$

The flow of excited atoms due to the action of the electric field on the dipoles is therefore not greater than $N_2^* v_d^*$, i.e. $10^{22}/\text{cm}^2$ and sec. This is therefore a 1% effect and will not make a substantial contribution to the flow of excited atoms to the cathode.

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Abbreviations

P.R.	Physical Review
J.A.P.	Journal of Applied Physics
Brit.J.A.P.	British Journal of Applied Physics
Helv.Phys.Act.	Helvetica Physica Acta
C.R.	Comptes Rendus
P.Z.	Physikalische Zeitschrift
Z.f.P.	Zeitschrift fur Physik
J.Phys.Rad.	Journal de Physique et Le Radium
Ann d.P.	Annalen de Physik.