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Putting OPEC out of business

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Putting OPEC out of business

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Abstract

I develop a differential game between an oil cartel, and an importer investing in R&D to reduce the cost of a substitute to oil. The equilibrium dynamics mirror the recent oil price collapse: prices first increase, but eventually fall as the cartel is forced to deter unconventional (shale) oil. Interpreting the substitute as clean energy, I assess future climate policy. Credible carbon taxes are below the Pigovian level, implying the importer certainly cannot capture resource rents. Without a tax instrument, the importer curtails long-run pollution using a costly R&D programme. Normatively, carbon taxes are necessary to tackle climate change efficiently.

Keywords: exhaustible resources, OPEC, oil prices, alternative fuels, limit pricing, climate change

JEL Classification: D42, O32, Q31, Q40, Q54

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1 Introduction

> "An increase in supply [by 2014] led to a spectacular plunge in the oil price [causing] energy companies to lay off workers and cut or delay investment projects. (...) Bits of green-energy world are wilting under the impact of low oil prices [but] the story may not be so much what falling oil prices mean for clean energy than what the prospect of clean energy will mean for the oil price." – *The Economist, Jan 17, 2015*[2]

Since May 2014, the world has witnessed a dramatic fall in oil prices. Should low prices persist, they will contradict the predictions of many analysts who had suggested that high prices, in the neighbourhood of $100/bbl, were "here to stay" (Hamilton, 2014; Spiro, 2014). In December 2014, Ali al-Naimi, the former oil minister of Saudi Arabia—the dominant member of the petroleum supply cartel OPEC—reportedly claimed that the rationale behind letting prices fall was to force high-cost oil producers to scale back, and indeed suggested $100/bbl oil might never return.[3] This is a new phenomenon: the technologies required for so-called tight oil production have matured in the past decade. 'Demand destruction' was not a concern in 2008, when prices soared to nearly $140/bbl. Since 2011, when prices rose to their post-Lehman peak of $120/bbl, it has been.

In this paper, I construct a model of oil cartel behaviour and strategic, gradual technological development which makes substitutes cheaper and cheaper. Unlike other intertemporally optimising models of resource use, the model is consistent with both observed oil price dynamics and with the idea that oil prices may have peaked for good. I then study the positive implications of the model on climate policy and the future of oil.

The first question I ask tackles al-Naimi's statement: can the fall in oil prices be explained in terms of OPEC trying to put unconventional oil producers out of business? It can. Before the threat of substitute entry is acute, increasing scarcity and intertemporal optimisation drive up prices; while advances in extraction tech-

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[2] *'Let there be light', Special report on energy and technology*, The Economist, Jan 17 2015.

[3] *'Full MEES Interview With Saudi Oil Minister Ali Naimi', Oilpro / Middle East Economic Survey*, Dec 21 2014 (http://oilpro.com/post/9223/mees-interview-saudi-oil-minister-ali-naimi).

nologies lower the production cost of substitutes. Eventually, the prices meet, and large quantities of unconventional oil threaten to enter the market. OPEC can stave off entry, however, by keeping its prices just below the level at which the unconventional producers break even. Continuing technological progress in unconventional extraction technologies will force OPEC to cut its margins, lowering prices in lockstep with the falling substitute production cost. The net effect is a price peak, as observed in the past decade, culminating in the dramatic fall since 2014. The cartel's pricing power is constrained, but the outcome remains inefficient: oil use is too conservative, and substitute development too aggressive.

My second question is the flip-side of this phenomenon, and reflects environmental concerns: can oil importers put OPEC out of business by developing clean substitutes to oil? Yes, they can, but this is costly if carbon pricing is not possible. Oil producers supply as long as their margins for selling oil are positive. If importers are concerned about climate change, carbon taxes can be used to cut these margins, and in equilibrium the oil business will be closed once its social value becomes nil. If the oil importing countries are unable to agree on a carbon pricing regime, they will instead resort to a crash research and development (R&D) programme. Such a programme seeks to push substitute production costs down very rapidly, in order to make oil uneconomic and to so shut a larger fraction of oil reserves underground for good. The programme will be more expensive, however, than closing the oil market with a carbon tax while using R&D for its primary purpose of lowering the cost of energy. Thus carbon taxes cannot be fully substituted for by R&D-based climate policies.

Third, can oil importers use carbon taxes to appropriate some of OPEC's oil rents? Not after OPEC starts limit pricing. Limit pricing severely curtails the oil importer's ability to credibly tax resource rents. Indeed, the equilibrium carbon tax is below the Pigovian level (i.e. the present value of future marginal costs due to the emission of a tonne of carbon), instead reflecting the social cost of carbon should the importer take the 'outside option' of immediately shutting down the oil market. The importer's welfare is constrained to the level of this outside option; the exporter only gives away sufficient resource rents to induce the oil importer to remain a customer.

To tackle these questions, I extend the Hoel (1978) model of a limit-pricing resource monopolist into a differential game.[4] A resource cartel maximises profits from selling a fixed stock of a resource over time. There exists a perfect substitute to the resource. The substitute is costly to produce, but an oil-importing coalition can lower this cost by R&D investment. I also incorporate a climate externality and oil extraction costs. The cartel, faced with a perfect substitute that can be produced with constant returns to scale, will eventually limit price; setting prices to just keep the substitute out of the market for a prolonged period of time. With elastic demand and a high initial resource endowment, this period may be preceded by a Hotelling-type stage, with resource prices rising due to scarcity. In this stage, the substitute is not used, but convex costs ensure R&D already takes place in anticipation of the technology's eventual use.

For an illustration of the logic of limit pricing, consider Figure 1, which displays the dominant automotive engine technology as a function of automotive battery prices and US fuel prices (Hensley et al., 2012). In 2011, fuel prices were just below the level at which hybrid-electric vehicles would become competitive. Improvements in battery technology would shift the US economy to the left, with battery-electric vehicles the dominant technology once battery prices fell to \$350/kWh. However, the oil producers have a substantial margin they are able to cut were this to happen, shifting down along the border to only just stay in the region in which the internal combustion engine remains dominant. Even at the depressed crude oil prices in November 2014, almost 50% of the US gasoline price consisted of margins to a Middle East exporter, so the exporter could be pushed all the way to the bottom edge of the graph before going out of business.[5] There

[4]Dockner et al. (2000) provide a good introduction to differential games. These games have been widely applied in environmental and resource economics; Long (2011, 2012) provides exhaustive surveys. Harstad and Liski (2012) provide a useful introduction to games with resources, including an example with substitute development, in a two-period framework.

[5]According to data from the U.S. Energy Information Administration, average finding and lifting costs in the Middle East are \$17/bbl, or 22% of the November 2014 WTI price of \$76/bbl. Crude oil makes up 62% of the price of a gallon of gasoline, so finding and lifting costs comprise 14% of the price of a gallon of fuel. Refining and marketing costs of 23% mean total production costs account for 37% of the price of a gallon of gasoline. Taxes make up another 15%.

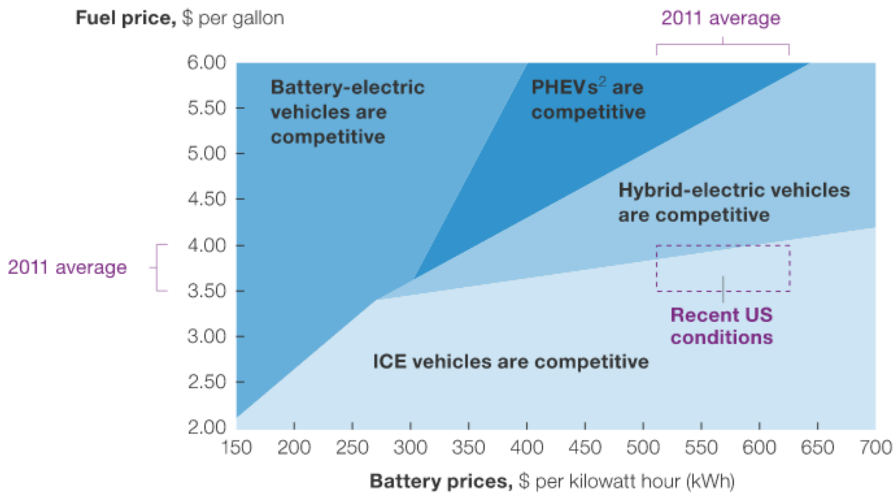

Figure 1: Electric vehicles' projected competitiveness versus internal combustion engine vehicles, based on total cost of ownership. Exhibit from "Battery technology charges ahead", July 2012, McKinsey Quarterly, www.mckinsey.com/insights/mckinsey_quarterly. Copyright (c) 2012 McKinsey & Company. All rights reserved. Reprinted by permission.

is a lot of room for OPEC to cut, were importers trying to push the cartel out with an R&D program alone. This paper describes the process of R&D shifting the economy towards the left, oil exporters responding by reducing their margins, and a carbon tax closing the difference once oil becomes socially worthless.

From a technical perspective, I demonstrate how two properties of the model—monotonic evolution of the state variables and a kinked objective function for one player—make the game particularly simple to analyse. Jointly, these assumptions effectively kill off strategic effects in one region of the state space. I can then characterise the equilibrium of the game very precisely, in an asymmetric, bivariate set-up. With inelastic demand, the equilibrium can be shown to be unique. I also extend previously developed numerical methods to such a set-up, to calculate the equilibrium in the region of the state space in which strategic effects are present.

The paper is structured as follows. I conclude the introduction with a literature review. Section 2 develops the basic model. Section 3 obtains the efficient benchmark. Section 4 defines the strategic set-up and solves for the Nash equilibrium. Section 5 adds climate change, extraction costs, and carbon taxes. Section 6 demonstrates the numerical solution to the model. Section 7 concludes.

1.1 Related literature

The limit pricing result of Hoel (1978) has not received much attention in the literature; most of the exceptions have related to strategic substitute development.[6] Typically R&D investments are assumed to be indivisible (e.g. Dasgupta et al., 1983; Gallini et al., 1983; Olsen, 1993; Gerlagh and Liski, 2011; Michielsen, 2014). In other words, the decision to begin developing or adopting the 'backstop technology', once taken, cannot be adjusted, even if the R&D process takes time.[7] The backstop is also considered to be of fixed quality. These models produce price peaks and subsequent (discontinuous) price falls, associated with the start of the innovation process or the arrival of the innovation. In all of the models, the price crashes result from the assumption that, before the innovation, the substitute does not exist at all.

However, many technologies undergo a prolonged process of development after the initial fundamental innovations have been made. This leaves open the question of how prices would evolve under a more gradual R&D process. I assume that the substitute exists from the start, that the R&D process is both gradual and observable, and that the agents can respond to the observable state of the world. All these assumptions are arguably realistic.[8] I demonstrate that an oil price peak, followed by a gradual and continuous decline in prices, can result from strategic interaction between fully rational agents, one limit pricing and the other conducting incremental R&D. This is a novel theoretical explanation for gradually

[6]Some authors consider how limit-pricing monopolists would respond to 'green' policies, such as subsidies or taxes: I discuss Van der Ploeg and Withagen (2012) and Andrade de Sá and Daubanes (2016) below. Van der Meijden et al. (2015) assess the case with multiple substitutes.

[7]As an exception, Harris and Vickers (1995) model a probabilistic R&D process in which effort takes place continuously; instead, the stochastic arrival of the innovation is discrete. Fully incremental development of backstop technologies has been considered by Tsur and Zemel (2003), who limit their investigation to the socially optimal R&D process.

[8]Former Saudi oil minister Ali Al-Naimi again (see footnote 3):

> "A lot of effort is being exerted worldwide, whether in research, or boosting efficiency, or using non-fossil fuels. All these might witness a breakthrough one day. Any strategy must be done in a way that allows it to be changed continuously. (...) But one thing is for sure. Current prices do not support all producers."

falling resource prices.[9]

The closely related paper by Gerlagh and Liski (2011) also produces a non-monotonic supply path. In their model, the R&D process is discrete, but involves a period of delay from the R&D decision until actual adoption. The exporter behaves as a Stackelberg leader at each stage, with the importer able to immediately end the relationship by triggering the adoption process. Once adoption is triggered, the exporter sells the remaining stock during the delay period. Resource scarcity reduces the quantity of the resource available for consumption in this period, making continuation ever costlier for the latter. The exporter must thus increase resource supply prior to the beginning of the R&D process, 'bribing' the importer to remain in the relationship.

The present paper advances a complementary view into strategic oil supply. First, while the basic intuition on how substitutes can limit resource owners' market power is similar to that in Gerlagh and Liski (2011), my empirical predictions are more specific. In particular, the present model clearly links the dynamics of oil prices to the observable cost of producing substitutes.[10] In contrast, it is not immediately clear what kind of data could be used to test the model of Gerlagh and Liski (2011). Second, while also discussing the behaviour of oil prices, I go further and address the negative externalities of oil, carbon (and rent) taxation, as well as the question of limiting cumulative extraction. These are issues on which their model is silent. Finally, both models remain somewhat stylised, and it is

[9]Other causes for falling prices include: falling extraction costs; unexpected supply shocks; strategic manipulation of the importer's R&D efforts (Harris and Vickers, 1995); or a finite, rolling time horizon considered by the resource owner (Spiro, 2014). The long oil price decline in 1985–1998 is better explained by the entry of a fringe of resource owners with large exhaustible deposits and high fixed costs (Newbery, 1981). Current tight oil extraction is more flexible, with the typical well in operation for a few years only. Chakravorty et al. (2012) derive cycles in resource prices from the interaction of a pollution ceiling and learning-by-doing in backstop technologies, but this cannot explain observed oil price behaviour, as a pollution ceiling has not been imposed to date.

[10]Behar and Ritz (2017) provide a calibrated empirical analysis of OPEC's price-setting in a static model, showing that the optimal response to higher shale output may involve limit pricing. Kilian (2016) advances a skeptical view, suggesting that up to December 2014 the price fall was mostly not driven by changes in supply.

possible to foresee potential extensions, with which the model mechanisms and predictions might diverge further.[11]

The importance of stock-dependent extraction costs and cumulative extraction, in tempering the 'green paradox' of Sinn (2008)—the perverse outcomes of supposedly environmental policies resulting from intertemporal supply-side responses by owners of fossil fuels—has been demonstrated by Gerlagh (2011) and Van der Ploeg and Withagen (2012). Cumulative extraction matters in the present paper also due to a supply-side effect, namely limit pricing; but this response is not related to intertemporal reallocation.

I also contribute to the literature on rent taxation (Bergstrom, 1982). Andrade de Sá and Daubanes (2016) show how the response of a limit-pricing monopolist may neutralise the effect of carbon taxes on extraction, or even lead to higher extraction. Their normative analysis takes the carbon tax and the price path of the substitute as exogenously given. The present paper takes a complementary, positive, stance and considers taxation and R&D in equilibrium. Liski and Tahvonen (2004) also consider equilibrium taxation, showing that, in the absence of a backstop, an oil-importing government uses carbon taxes to capture some of the resource rents (see also Wirl and Dockner, 1995). The limit pricing associated with a backstop leads to strong and opposite conclusions: in equilibrium, once limit pricing begins, carbon taxes cannot even fully internalise the pollution damages, much less capture resource rents.

2 The model

Time is continuous: $t \in [0, \infty)$. All variables are functions of time; I omit notation which explicitly indicates this dependence. There is perfect information.

All agents live forever and discount future utility flows at the common rate ρ. Utility is obtained from consumption of energy, denoted q, and of other goods:

[11] As an example, both models assume there is effectively a single sector which uses the resource. I discuss in the concluding section potential extensions to multiple sectors or multiple importers, which are left for future work. Gerlagh and Liski (2014) consider resource dependence under asymmetric information.

Assumption 1. Utility and demand. The representative consumer has a quasilinear utility function $v(q, M) = u(q) + M$, with $u' > 0$, $u'' < 0$. M denotes consumption of other goods, the exogenously given flow of which is normalised to zero. Denoting the energy price by p, demand is $q(p) \equiv (u')^{-1}(p)$. The function $u(\cdot)$ satisfies either $u'''(q)q + 2u''(q) < 0$, or $u''(q)q + u'(q) < 0$, for all q.

The last two restrictions imply that either revenue is strictly concave, or marginal revenue is always negative. Either assures the existence of a unique optimum to the monopolist's problem.

Energy comes from two sources. The first is a natural resource—to fix ideas, a fossil fuel, called henceforth 'oil'—the consumption of which is denoted by q_F.

Assumption 2. Exhaustibility. Oil is nonstorable and exhaustible, with the remaining stock denoted by $S \geq 0$, and the (given) initial stock by $S_0 \geq 0$. The evolution of the oil stock is given by $\dot{S} = -q_F$.

There is a perfect substitute for the resource—for example, biofuels or unconventional oil—called the *backstop technology*.[12] The production rate of the backstop is denoted q_B. Total energy consumption is thus $q = q_F + q_B$. The backstop is produced under perfect competition and using a constant returns to scale technology. The unit production cost depends on the accumulated knowledge of technologies used in backstop production:

Assumption 3. Backstop technology. The backstop unit production cost, in terms of other goods, is a function of accumulated knowledge K, given by $x = x(K)$, $x' \leq 0$, $x'' \geq 0$. The knowledge stock is normalised so that $K \geq 0$, and the initial stock K_0 is known, with $\lim_{q \to 0} u'(q) > \overline{x} \equiv x(0)$. There exists $\overline{K} > K_0$ with $\underline{x} \equiv x(\overline{K})$, $\lim_{K \uparrow \overline{K}} x'(K) = 0$, $x'(K) = 0$ for $K > \overline{K}$.[13]

I thus assume that using the backstop is always preferable to zero energy use.

The R&D stock is built up by investment:

[12] Perfect substitutability makes the intuition clearer, but is not crucial: a sufficiently high degree of substitutability would yield similar behaviour (Van der Meijden and Withagen, 2016).

[13] I could easily allow for $\underline{x}$ being reached asymptotically as $K \to \infty$, or for $\lim_{K \uparrow \overline{K}} x'(K) < 0$, without affecting the key results; however, the proposition statements would become cumbersome and additional technical assumptions would be required for some of the proofs.

Assumption 4. R&D process. R&D investment is denoted $d(t) \in [0, \overline{d}]$, $\overline{d} > 0$, and builds up the knowledge stock according to $\dot{K} = d$. There are strictly convex monetary costs to conducting research $c(d)$: $c \geq 0$, $c' \geq 0$, $c'' > 0, c(0) = 0$, $c'(0) = 0$. The upper bound $\overline{d}$ satisfies $c'(\overline{d}) > \frac{-x'(0)q(\underline{x})}{\rho}$.[14]

Thus, knowledge does not depreciate. The gradual, predictable improvements imply that investments can be thought of as development and deployment work, rather than fundamental research, which could be seen as producing discontinuous 'breakthroughs'. I will refer to investment, for brevity, as 'research' or 'R&D'.

The backstop producers set their price equal to the constant marginal cost, so that given oil price p_F, the energy price p is

$$p(p_F, K, S) = \begin{cases} \min\{p_F, x(K)\} & \text{if } S > 0; \\ x(K) & \text{otherwise,} \end{cases}$$

and oil and backstop demands are given by

$$q_F(p_F, K, S) = \begin{cases} q(p(p_F, K, S)) & \text{if } p_F \leq x(K), S(t) > 0, \\ 0 & \text{otherwise;} \end{cases} \tag{1}$$

$$q_B(p_F, K, S) = q(p(p_F, K, S)) - q_F(p_F, K, S). \tag{2}$$

In words, oil supplies all demand at prices $p_F \leq x(K)$ (as long as $S > 0$); otherwise the backstop is used at price $p = x(K)$. Thus, the residual demand faced by the exporter has an effective price ceiling $x(K)$ (Figure 2). I define the revenue function $R(q) \equiv qu'(q)$. Assumption 1 guarantees either $R'(q) > 0$, $R''(q) < 0$; or $R'(q) < 0$.

For now, I abstract from extraction costs. This is an adequate approximation to recent history when considering conventional oil supplied by OPEC countries, as the marginal extraction cost of such supplies is very low. I also abstract from pollution, as to date, willingness to pay to avoid carbon emissions has been very low. I introduce both features in Section 5.

[14]The assumption of an upper bound on d is technical and simplifies the proofs; it will never be binding. Relaxing the assumption of zero marginal cost at $d = 0$ is straightforward but yields no interesting intuition.

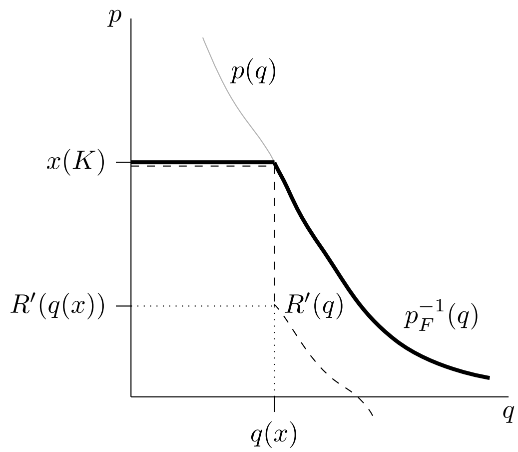

Figure 2: Energy (grey) and oil (solid) demand, and marginal revenue (dashed).

3 Social optimum

I first present the social optimum. This both provides a benchmark for efficiency, and is also used to determine the behaviour of the economy in later stages of the non-cooperative equilibria.

The social planner chooses energy consumption directly to solve

$$\max_{q_F, q_B, d} \int_0^\infty \mathrm{e}^{-\rho t} \left(u(q_F + q_B) - q_B x(K) - c(d) \right) \mathrm{d}t, \tag{3}$$

subject to the dynamics

$$\begin{aligned} \dot{K} &= d, & K(0) &= K_0, \\ \dot{S} &= -q_F, & S(0) &= S_0, \quad S \geq 0. \end{aligned} \tag{4}$$

Let λ_K and λ_S denote the costate variables (shadow values) associated with K and S. As K grows linearly with d, optimal R&D is given by $c'(d) = \lambda_K$. I first define a trajectory which describes the ultimate behaviour of K in all solutions:

Definition 1. The *terminal path* $\{K, d^{\mathrm{T}}(K), \lambda_K^{\mathrm{T}}(K)\}$ is the optimal R&D process when the exhaustible resource is not used, and solves problem (3)–(4) for $S_0 = 0$. The associated value is denoted $V^{\mathrm{T}}(K)$.[15]

[15] d^{T} and λ_K^{T} can be expressed as functions of K as $K(t)$ is weakly monotonic and as problem (3) is autonomous.

Clearly, along the terminal path, $q_B = q(x(K))$ and $d^{\mathrm{T}}(K) = (c')^{-1}(\lambda^{\mathrm{T}}(K))$.

Lemma 1. *The terminal path is unique and satisfies*

$$\lim_{K\to\overline{K}} \lambda_K^T(K) = \lim_{K\to\overline{K}} d^T(K) = 0.$$

Proof. All proofs are in the Appendix. □

The terminal path describes the efficient R&D process, given that the energy price equals $x(K)$, once resource extraction has stopped for good. It also provides a boundary condition for the problem in the resource-using stage.

R&D intensity $d^{\mathrm{T}}(K)$ may behave non-monotonically.[16] As $\lambda_K^{\mathrm{T}} > 0$ for $K < \overline{K}$, and as $\overline{K}$ is reached only asymptotically, the assumptions on $c(\cdot)$ assure research effort is always strictly positive.

The following proposition characterises the social optimum (Figure 3).

Proposition 1. *The social optimum consists of two stages:*

Stage H (Hotelling). $t \in [0, T)$, $T \geq 0$. *Only the exhaustible resource is used, with rate of extraction decreasing monotonically and marginal utility rising at the discount rate. R&D intensity is strictly positive and increases monotonically. Exhaustion of the resource stock occurs at the switching date T, satisfying $u'(q_F(T)) = \lambda_S(T) = x(K(T))$.*

Stage B (Backstop). $t \in [T, \infty)$. *In the second stage, the economy uses only the substitute and moves along the terminal path. Substitute use increases monotonically as the unit cost falls asymptotically towards the lower bound on the backstop cost. Research effort is strictly positive, with* $\lim_{t\to\infty} d(t) = 0$.

The economy initially uses only oil, with the resource rent (reflecting the marginal utility of oil) rising at the discount rate (satisfying the Hotelling Rule). The extraction level is set so that at exhaustion, this equals the cost of using the

[16] Integrating the equation of motion for the costate variable $\dot{\lambda}_K = \rho\lambda_K + q_B x'(K)$, the marginal benefit of knowledge is the present value of the associated stream of cost reductions: $\lambda_K(t) = -\int_t^\infty e^{-\rho(s-t)} q_B(s) x'(K(s))\,\mathrm{d}s$. Capital gains may be low if the backstop is consumed in small amounts (q_B small), *or* if a marginal unit of knowledge only reduces the costs a little ($x'(K)$ small). When capital gains are low, the shadow value mostly represents future benefits and will fall more slowly, or even rise over time.

substitute, and the economy switches to the backstop. Energy consumption does not jump at the switching point. To emphasise: socially optimal oil use is never associated with falling prices.

With convex R&D costs, substitute development begins immediately, even though the knowledge stock only provides value after oil is exhausted. After the switch, the marginal utility of energy decreases as R&D brings energy costs down.

4 Non-cooperative equilibrium

I now develop the Nash equilibrium for the above economy. The outcome is consistent with both recent peaking oil price dynamics and the popular narrative (and the claim of the former Saudi oil minister referred to above) on their cause.

Divide the model economy into two countries. The players are the governments of the resource exporting country, Country E ('she'), which owns the resource; and of the resource importing country, Country I ('he'), which owns the technology to utilise the resource, as well as the backstop technology. Country E chooses the resource price p_F.[17] Given this and (K, S), Country I consumers' oil demand is given by (1). I now proceed to define the game more precisely.

Definition 2. Let the *state domain* be $\mathcal{D} \subseteq [0, \overline{K}] \times [0, \overline{S}]$, $\overline{S} > 0$, satisfying $\forall (K, S) \in \mathcal{D} : K' \in [K, \overline{K}], S' \in [0, S] \Rightarrow (K', S') \in \mathcal{D}$. A *Markov strategy* ω^j, $j \in \{d, p_F\}$, is a piecewise-continuous mapping $\mathcal{D} \to \mathbb{R}^+$.[18]

I thus focus on stationary Markov-perfect Nash equilibria (MPNE), with strategies not dependent on the full history, and also not directly dependent on the calendar time t (Maskin and Tirole, 2001). The domain is defined so that all points potentially reachable from any $(K, S) \in \mathcal{D}$ are also included in $\mathcal{D}$. $\overline{S}$ just denotes some upper bound to the oil stock that we want to consider.

Definition 3. A strategy profile $\omega \equiv (\omega^d, \omega^{p_F})$ is *admissible* if, given (1)-(2), setting $d(t) = \omega^d(K(t), S(t))$ and $p_F(t) = \omega^{p_F}(K(t), S(t))$ generates a unique,

[17] The price instrument for Country E ensures the equilibrium is well-defined once I allow Country I to have a tax instrument in Section 5.

[18] To make the proofs easier, I exclude pieces such that the edges form cusps.

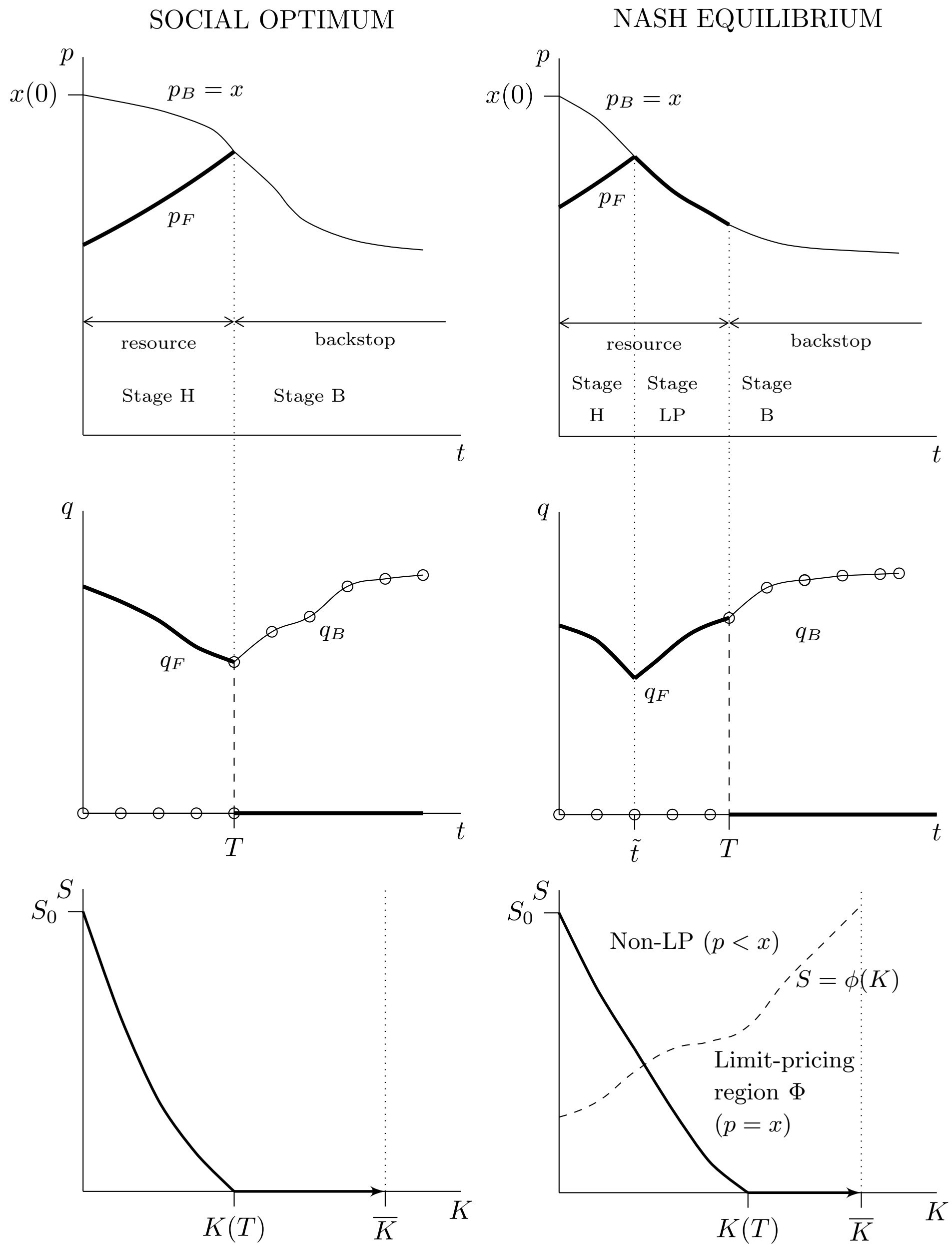

Figure 3: (*top*) Time paths of the backstop price (thin line) and the oil price (solid line) under the social optimum (*left*) and the Markov-perfect Nash equilibrium (*right*); (*middle*) Oil extraction (solid line) and backstop production (thin line with circles); (*bottom*) Trajectories in (K, S)-space (solid line). Terminal path R&D begins at the K-axis (social optimum) or once limit-pricing starts (non-cooperative case; region Φ).

absolutely continuous solution to the initial value problem (4), for any initial $(K_0, S_0) \in \mathcal{D}$.[19]

Thus, admissible strategies generate well-defined dynamic trajectories of the game.

Utility functions map trajectories to welfare of Countries I and E:

Definition 4. Given a pair of piecewise continuous control paths $d(t)$, $p_F(t)$, and the associated trajectories $q_F(t)$, $q_B(t)$, $K(t)$, $S(t)$ generated according to (1)-(2) and (4), the utilities of the importer and exporter are, respectively,

$$\begin{aligned}\tilde{\pi}^{\mathrm{I}}(d, p_F, q_F, q_B, K, S) = \int_0^\infty e^{-\rho t} &\left(u\left(q_F + q_B\right)\right. \\ &\left.-R(q_F) - x(K)q_B - c(d)\right) \mathrm{d}t,\end{aligned} \tag{5}$$

$$\tilde{\pi}^{\mathrm{E}}(d, p_F, q_F, q_B, K, S) = \int_0^\infty e^{-\rho t} R(q_F) \, \mathrm{d}t. \tag{6}$$

I can now define the payoffs as a function of strategies:

Definition 5. Given $\mathcal{D}$, for any admissible strategies ω^d, ω^{p_F} and for $i \in \{\mathrm{I}, \mathrm{E}\}$, the *payoff functions* are denoted by $\pi^{\mathrm{i}}(\omega^d, \omega^{p_F})$, and satisfy, for all $(K_0, S_0) \in \mathcal{D}$,

$$\pi^{\mathrm{i}}(\omega^d, \omega^{p_F}) = \tilde{\pi}^{\mathrm{i}}(d, p_F, q_F, q_B, K, S),$$

in which the trajectories d, p_F, q_F, q_B, K, S are generated as in Definition 3.

In equilibrium, each player chooses a strategy which is a best response to the other player's strategy:

Definition 6. Given domain $\mathcal{D}$, a *stationary Markov-perfect Nash equilibrium* (MPNE) is a strategy profile $(\omega^{d*}, \omega^{p_F*})$ such that, for any admissible strategy

[19] Restricting the strategies to be Lipschitz-continuous would guarantee admissibility of any strategy pair, but this is a strong assumption (Fudenberg and Tirole, 1991, p. 525-6; Başar and Olsder, 1999, p. 226-7). As I derive an equilibrium which features a discontinuous strategy in Section 5, such a restriction would be particularly inappropriate. I skirt the problem by imposing the requirement of admissibility directly (as in Dockner and Sorger, 1996; Dockner et al., 2000). This approach could, in principle, imply some artificial equilibria might exist in which a player's actions were limited by the admissibility requirement. My goal is not to characterise the entire equilibrium set, but to rather study the properties of one particularly intuitive equilibrium. The requirement of admissibility is innocuous in the context of this exercise.

pairs $(\tilde{\omega}^d, \omega^{p_F*})$, $(\omega^{d*}, \tilde{\omega}^{p_F})$,

$$\pi^{\mathrm{I}}(\tilde{\omega}^d, \omega^{p_F*}) \leq \pi^{\mathrm{I}}(\omega^{d*}, \omega^{p_F*}),$$
$$\pi^{\mathrm{E}}(\omega^{d*}, \tilde{\omega}^{p_F}) \leq \pi^{\mathrm{E}}(\omega^{d*}, \omega^{p_F*}).$$

I now construct a MPNE for the above economy. Such an equilibrium will induce value functions $V^{\mathrm{I}}(K,S)$ and $V^{\mathrm{E}}(K,S)$. To unburden the notation, I will omit the arguments of the value functions and strategies, and of their partial derivatives, in what follows.[20] Given that the players follow Markovian strategies, the MPNE must satisfy the following Hamilton-Jacobi-Bellman equations at all points such that the value functions are differentiable:

$$\rho V^{\mathrm{E}} = \max_{p_F \in \mathbb{R}^+} R(q_F(p_F, K)) + \omega^{d*} V^{\mathrm{E}}_K - q_F(p_F, K) V^{\mathrm{E}}_S, \tag{7}$$

$$\begin{aligned}\rho V^{\mathrm{I}} = \max_{d \in \mathbb{R}^+} \Big\{ & u(q(\omega^{p_F*}, K)) - R(q_F(\omega^{p_F*}, K)) - x(K) q_B(\omega^{p_F*}, K) \\ & - c(d) + d V^{\mathrm{I}}_K - q_F(\omega^{p_F*}, K) V^{\mathrm{I}}_S \Big\},\end{aligned} \tag{8}$$

with the strategies corresponding to optimal choices at all (K,S):

$$\begin{aligned}\omega^{p_F*}(K,S) \in \arg\max_{p_F} \ & R(q_F(p_F), K) + \omega^{d*}(K,S) V^{\mathrm{E}}_K(K,S) \\ & - q_F(p_F, K) V^{\mathrm{E}}_S(K,S), \\ \omega^{d*}(K,S) \in \arg\max_{d} \ & \Big\{ u(q(p(\omega^{p_F*}, K))) - R(q_F(\omega^{p_F*}, K)) \\ & - x(K) q_B(\omega^{p_F*}, K) - c(d) + d V^{\mathrm{I}}_K(K,S) - q_F(\omega^{p_F*}, K) V^{\mathrm{I}}_S(K,S) \Big\}.\end{aligned}$$

The next lemma emphasises the key mechanism of this paper, limit pricing.

Lemma 2. *Consider a point (K', S') such that $V^{\mathrm{E}}_S(K', S')$ exists. Then, if $V^{\mathrm{E}}_S(K', S') \in [R'(q(x(K'))), x(K')]$, the exporter's equilibrium strategy at (K', S') is $\omega^{p_F*}(K', S') = x(K')$.*

In words, there is a continuum of values of the opportunity cost V^{E}_S for which the exporter sets $p_F = x(K)$. Limit pricing is easiest to understand by reference to

[20] To keep the notation manageable, I also omit here the dependence of the demand functions on S. Of course, at zero oil stock, the exporter cannot sell anything.

Figure 2. As a monopolist, the exporter's static problem is to equate marginal revenue with marginal cost, i.e. the scarcity rent of the resource: $R'(q_F(p_F, K, S)) = V_S^{\mathrm{E}}$. For $V_S^{\mathrm{E}} < R'(q(x(K)))$, this yields an interior solution with $p_F < x(K)$. However, the marginal revenue curve is discontinuous at $q(x(K))$, the level at which the price ceiling imposed by the backstop begins to bite. Without the substitute, the exporter would price above $x(K)$ as soon as $V_S^{\mathrm{E}} > R'(q(x(K)))$; but the presence of the substitute implies doing so would collapse demand to zero. She will optimally satisfy the full demand at price $p_F = x(K)$ as long as $x(K) \geq V_S^{\mathrm{E}}$.

Figure 2 now makes clear the first contribution of this paper: the MPNE exhibits peaking price dynamics. Heuristically, the scarcity rent tends to rise over time, because of Hotelling-type concerns, while the backstop price will fall with R&D, shifting the discontinuity down with it. With a large enough initial resource stock, the initial rent is low, so that an interior solution obtains. In this interior region, a gradual increase of the scarcity rent goes together with an increase, over time, in the price. As soon as the scarcity rent meets the discontinuous section of the marginal revenue curve, the exporter will find it optimal to limit price. As the limit price $x(K)$ is falling with the increasing knowledge stock, however, the oil price is forced down, until exhaustion, which occurs exactly when the shadow value V_S^{E} meets the backstop price $x(K)$.

I will now make this heuristic argument more precise. I first define a region of the state space such that, were the importer able to commit to conducting terminal path R&D in this region, then the optimal response of the exporter would be to limit price, setting the oil price equal to the backstop production cost.[21]

Definition 7. Denote by Φ the set of $(K_0, S_0) \subseteq [0, \overline{K}] \times [0, \overline{S}]$ such that: (i) for trajectories $S(t)$, $K(t)$ induced by $d(t) = d^{\mathrm{T}}(K(t))$, $p_F(t) = x(K(t))$, (1)-(2) and (4); and (ii) for T given by $\int_0^T q_F(x(K))\,\mathrm{d}t = S_0$; (iii) it holds that $R'(q(x(K_0))) \leq e^{-\rho T} x(K(T))$.

[21] An open-loop Nash equilibrium—with strategies functions of the current date only, implying commitment—does not exist. In such an equilibrium, an exporter would limit price, matching the backstop price; but given this, the importer would always prefer to relax R&D efforts, allowing the backstop price to drift above the oil price. Details are available from the author.

Lemma 3. $\Phi = \{(K,S) : K \in [0,\overline{K}], S \in [0,\phi(K)]\}$*, with* $\phi(K)$ *defined as the locus along which inequality (*iii*) of Definition 7 holds with equality.*

The region Φ is illustrated in the bottom right panel of Figure 3 and covers the southeast corner of the state domain.[22] Given terminal path R&D, Φ is the set of (K_0, S_0) such that Country E would prefer to wait to sell the marginal barrel of oil at the time the stock is exhausted, rather than immediately, which would lower inframarginal revenues. For any K_0, however, a high initial stock S_0 implies that the exhaustion date is far in the future, so that the revenues thus obtained are heavily discounted. Thus the marginal revenue at $q(x(K))$ may exceed the discounted profit of retaining a marginal barrel until the exhaustion date and selling it at the backstop price prevailing then. For such high S_0, $(K_0, S_0) \notin \Phi$. On the other hand, if oil demand were inelastic, marginal revenue would be negative everywhere, the inequality in *(iii)* would be strict everywhere, and Φ would cover the entire state domain.

The next lemma shows that limit-pricing in Φ is an MPNE:

Lemma 4. *Suppose* $\mathcal{D} = \Phi$*. Then there exists an MPNE with* $\omega^{d*} = d^{\mathrm{T}}(K)$*,* $\omega^{p_F*}(K,S) = x(K)$*. This is the unique MPNE with differentiable* $V^{\mathrm{I}}, V^{\mathrm{E}}$*.*

Thus, in the region Φ, it is an equilibrium for the exporter to limit price, and for the importer to conduct R&D according to the terminal path. In fact, this equilibrium is the only MPNE in this region when value functions must be differentiable; any other equilibria would require coordinated discontinuities in behaviour.[23] The intuition is straightforward. Suppose that the exporter is expected to limit price in all possible future states, by setting $p_F = x(K)$. The importer then knows that, irrespective of what he does, the energy price is given by $x(K)$. As we have

[22]Typically $\phi' \geq 0$; sufficient for this is e.g. that utility is either isoelastic or quadratic, and that the relative effectiveness of R&D $|x'(K)/x(K)|$ is decreasing in K.

[23]Such other equilibria might be seen as less plausible than the obvious focal point of limit pricing everywhere. Differential games often admit a continuum of MPNE, and evaluation of whether multiple equilibria exist can be difficult. The partial uniqueness result here stems from two features of my model. First, the monotonic evolution of the state pins down a boundary condition (as in e.g. Karp, 1996). Second, the discontinuity of Country E's objective function means the same action is optimal for a range of V^{E}_S.

(for now) assumed there are no externalities, from his perspective, it is as if he were already using the backstop; so the best response is to set R&D according to the the terminal path, or $d = d^{\mathrm{T}}(K)$. This, conversely, implies that the future development of the knowledge stock is out of the hands of the exporter: the path $K(t')|_{t'>t}$ is fully determined given $K(t)$, so that the exporter cannot manipulate the R&D process. Following the intuition given after Definition 7, it is then optimal for the exporter to limit price.

Above, I have talked about the MPNE only in the set Φ. These equilibria also extend outside Φ, where oil prices are strictly below the backstop cost. It is difficult to give crisp characterisations of these extensions analytically, but there is no reason to doubt e.g. the existence of such extended equilibria. Numerical experiments show that these equilibria are well-behaved (see Section 6).

I will use the term *limit-pricing MPNE* to refer to any equilibrium which features limit pricing in the set Φ. I now state the first key result, describing the behaviour of the oil price at the boundary of Φ.

Proposition 2. *Suppose a limit-pricing MPNE exists for some $\mathcal{D}$ s.t. $\Phi \subset \mathcal{D}$. Then, for all K: (*i*) in any non-empty intersection of some neighbourhood of $(K, \phi(K))$ and $S > \phi(K)$, $\omega^{p_F*}(K,S) < x(K)$; and (*ii*) the oil price has a local maximum at $S = \phi(K)$, unless $\lim_{S\downarrow\phi(K)} V_K^E \omega_S^{d*}$ is sufficiently large.*

In words, any equilibrium featuring limit pricing inside Φ will feature oil pricing strictly below the backstop production cost just before entering Φ (part (i)). The oil price typically peaks at this point (part (ii)). The reason is simple: before limit pricing begins, the scarcity rent of the resource evolves according to

$$\frac{\mathrm{d}V_S^{\mathrm{E}}}{\mathrm{d}t} = \rho V_S^{\mathrm{E}} - V_K^{\mathrm{E}} \omega_S^{\mathrm{I}}. \tag{9}$$

The first 'Hotelling' term reflects the increasing scarcity of the resource, and it pushes the scarcity rent, and thus the oil price, up over time. After crossing $S = \phi(K)$, the exporter limit prices and the importer's R&D effort forces the exporter to lower oil prices in lockstep with the backstop cost. Before the peak, the second term in (9) reflects the strategic motive of saving more oil to discourage the importer from doing R&D. This term would have to be sufficiently large to outweigh the upward trend resulting from the increasing scarcity. Numerical experiments in Section 6 verify the strategic term is far too weak to achieve this.

Figure 3 depicts the qualitative behaviour of the limit-pricing equilibria, including the peaking prices. There are three regimes. Initially, there may be a Hotelling-style regime ('Stage H') with rising prices. This regime is degenerate if demand is not sufficiently elastic, or if the initial stock is too low. The Hotelling regime is followed by a limit-pricing regime ('Stage LP') as described in Lemma 4, with oil prices falling in lockstep with the backstop price. The importer understands he is in control of the energy price, and consequently intensifies R&D efforts. Finally, the economy switches to the backstop and R&D continues along the terminal path ('Stage B'). Comparing the equilibrium to the efficient outcome derived in the previous section, the limit-pricing equilibrium is clearly inefficient: while oil stocks remain positive, efficient energy consumption must always exceed the rate corresponding to limit pricing.

The peaking behaviour is consistent with the observed long-run behaviour of oil prices over the past 17 years, and the mechanism also corresponds closely to statements given by market participants regarding the causes of the collapse in oil prices since 2014: namely, that improved extraction technologies for tight oil have imposed a price ceiling on the OPEC cartel.

5 Extraction costs and climate change

I now introduce extraction costs, climate damages and the possibility of carbon taxation (used to limit externalities and/or capture resource rents). I then demonstrate that subsidies to R&D on clean technologies are an imperfect substitute to carbon taxes, and that equilibrium taxes cannot fully internalise the social cost of pollution, much less capture rents.

Assumption 5. Resource extraction costs. The marginal and average cost of oil extraction at a point in time is given by $C(S) \geq 0$, $C' < 0$, with $C(0) > x(0)$.

Country E incurs the extraction costs, with flow payoff $R(q_F) - q_F C(S)$ (omitting the arguments of q_F). The unit cost depends on the remaining stock of the resource, increasing as the stock diminishes (Heal, 1976).[24] I assume exhaustion is always economic, i.e. it is never profitable to extract the last unit of oil.

Climate damages affect only Country I. This is a stylised way of modelling the asymmetry between the countries in terms of size: if the exporting country's population is very small, it bears a very small burden of overall damages resulting from its resource exports, but receives all the revenues (List and Mason, 2001). I assume the harmful impacts of climate change to be gradual:[25]

Assumption 6. Climate change. Climate change impacts are an increasing and strictly convex function $Z(G)$ ($Z' \geq 0$, $Z'' > 0$) of the cumulative emissions $G(t) \equiv \tilde{S} - S(t)$, with $\tilde{S} \geq S_0$.

I denote the pre-industrial carbon stock by $\tilde{S}$. The flow payoff to the importer is thus $u(q) - R(q_F) - q_B x(K) - c(d) - Z(G)$. Importantly, backstop consumption does not produce emissions, implying it is based on e.g. renewable energy sources.[26]

I proxy climate change by cumulative carbon emissions to make the model more tractable. Climate science indicates the deviation in the Earth's mean surface temperature is roughly linear in cumulative emissions.[27] I ignore sources of emissions other than oil as these would distract from the key insights.[28]

5.1 Equilibrium without taxes

As in Section 4, I assume carbon taxes are not feasible, say due to political constraints. The exhaustion locus is $S = \overline{\phi}(K)$, given by $x(K) = C(S)$: below this locus, oil extraction cannot be profitable. The locus satisfies $\overline{\phi}' = \frac{x'(K)}{C'(S)} > 0$.

[24] Hart (2016) considers a more detailed model of increasing extraction costs.

[25] In other words, I assume there are no tipping points as in e.g. Lemoine and Traeger (2014).

[26] Explicit consideration of externalities related to the backstop (Chakravorty et al., 2008) or of its exhaustibility (Newbery, 1981; Groot et al., 2003) would complicate the dynamics.

[27] Higher atmospheric concentrations reduce the marginal warming effect of a ton of CO_2 but this is offset by changes to carbon uptake by the biosphere (Matthews et al., 2009). Lemoine and Rudik (2017) consider decay of atmospheric carbon and inertia in the climate system.

[28] On the margin, a tonne of carbon should be taxed equally whether it is sourced from coal or oil, so that the qualitative results are not affected by omitting emissions from other sources. Including e.g. exogenous coal emissions would introduce a third state variable.

I will now introduce my second key result. To avoid unnecessary complications, I will assume that demand is everywhere inelastic.[29]

Proposition 3. *With $\tau \equiv 0$, the unique MPNE features limit pricing, with*

$$\begin{aligned} S \geq \overline{\phi}(K) \quad &\Rightarrow \quad \omega^{p_F *} = x(K), \quad \omega^{d*} = (c')^{-1}\left(V_K^I\right), \\ S < \overline{\phi}(K) \quad &\Rightarrow \quad \omega^{p_F *} > x(K), \quad \omega^{d*} = d^T(K). \end{aligned}$$

For $S \geq \overline{\phi}(K)$, ω^{d*} is generically different from $d^{\mathrm{T}}(K)$, with

$$\lim_{S \downarrow \overline{\phi}(K)} \omega^{d*}(K,S) > d^{\mathrm{T}}(K).$$

Exhaustion occurs at time T. If for some (K_0, S_0), $\omega^{d*}(K_0,S_0) < d^{\mathrm{T}}(K_0)$, then $\exists t'$ s.t. on the equilibrium path starting at (K_0,S_0):

$$\begin{aligned} t \in [0,t') \quad &\Rightarrow \quad \omega^{d*}((K(t),S(t)) < d^{\mathrm{T}}(K(t)), \\ t \in (t',T) \quad &\Rightarrow \quad \omega^{d*}((K(t),S(t)) > d^{\mathrm{T}}(K(t)). \end{aligned}$$

The first claim of the proposition says that, without taxes, R&D is distorted away from the terminal path; in particular, close to exhaustion, R&D is increased to account for environmental concerns (Figure 4). To see why, consider the difference in the marginal value of knowledge (to the importer) at the moment of exhaustion:

$$\begin{aligned} \Delta V_K^{\mathrm{I}} &\equiv \lim_{t \uparrow T} V_K^{\mathrm{I}} - \lim_{t \downarrow T} V_K^{\mathrm{I}} \\ &= \lim_{t \uparrow T} \left(\frac{\mathrm{d}T}{\mathrm{d}K(t)}\right)\left(-\frac{Z'}{\rho} q_T + \left[\left(V_K^{\mathrm{T}} \omega^{d*} - c(\omega^{d*})\right) - \left(V_K^{\mathrm{T}} d^{\mathrm{T}} - c(d^{\mathrm{T}})\right)\right]\right), \end{aligned}$$

in which the damage, value functions and policies are all evaluated at $(K(T), S(T))$. The sum in the brackets is the excess flow of overall welfare the importer receives in the limit-pricing stage, relative to the post-exhaustion stage. The first term reflects the permanent damage due to the additional extraction during this period, and is negative. The term in the square brackets reflects the difference in

[29] The key intuition is unchanged with elastic demand, but would require more precise definitions of the domain for which the equilibrium is defined. The construction would be as in the previous section: the limit pricing equilibrium holds in a region which can be clearly defined and computed, outside of which the equilibrium will feature pricing strictly below the backstop cost.

the net benefit of R&D, considering the *long-run* value of knowledge. This is also negative, as d^{T} is optimal given the marginal value V_K^{T}. The difference in the marginal values is just this negative flow value of staying in the pre-exhaustion regime for a moment longer, times the marginal effect of knowledge on the date of exhaustion (the term outside the brackets). As a higher knowledge stocks brings exhaustion forward, the difference in the marginal values is positive: knowledge has strictly higher value before exhaustion than after.

In other words, in addition to lowering the cost of energy, driving backstop costs down ensures lower cumulative extraction of the polluting resource, and thus lower long-run pollution damages. This encourages the importer to conduct more R&D, to bring the fossil stage to a rapid close. These incentives are exacerbated by the fact that, looking at the long-run fundamentals, the crash R&D program is wasteful. This wastefulness makes the end of the fossil era even more appealing, amplifying the effect resulting purely from climate concerns. As soon as the backstop cost reaches extraction cost, the remaining reserves are guaranteed to be locked underground. Thus, the additional R&D incentives vanish and investment jumps down at the moment of economic exhaustion, as R&D spending is refocused to efficiently bring down the energy price.

In the short run, the importer may conduct less R&D than on the terminal path. In the limit-pricing stage, there is an extra cost to research investment: lowering the price of the backstop simply lowers the price of oil, increasing extraction and thus pollution. The cost of the resulting medium-run damages may outweigh the benefit of lower long-run pollution. If this is the case, there exists a unique date t' on which the crash programme begins: before t', R&D effort is strictly below the terminal path, and after t', strictly above it.

The above effects as well as strategic motives related to energy prices, including consideration of pollution effects, will all be present in the non-limit pricing stage. The magnitudes of the strategic effects will be affected by both climate change and extraction costs. This stage can be solved numerically as in Section 6; I do not explicitly investigate this, as such experiments do not yield additional intuition.

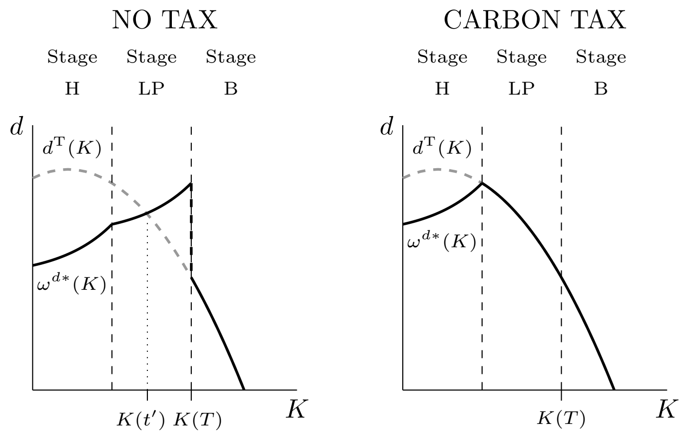

Figure 4: R&D as function of capital and the terminal path.

5.2 Equilibrium with taxes

I now consider the equilibrium in which Country I can impose a unit tax τ on resource consumption, so that the consumer price of the resource is $p_F + \tau$. Tax revenues are recycled lump-sum. Country I's strategy is extended in the obvious way, with $\omega^{\mathrm{I}} = (\omega^d, \omega^\tau)$, as are the definitions of payoffs and of MPNE. I assume Country E can subsidise sales, should she want to do so, up to the tax rate: $p_F \in (-\tau, \infty)$.[30]

The inclusion of climate damages makes extraction costs a key feature, as well as raising the importance of taxes. As CO_2 is a stock pollutant, long-term damages are driven by cumulative oil extraction, which depends on extraction costs, backstop prices and carbon taxes. In particular, extraction can only be profitable if the cost of extraction plus the carbon tax τ is less than the substitute production cost, i.e. $C(S) + \tau \leq x(K)$. Thus, the tax can be used to affect cumulative extraction (as in Van der Ploeg and Withagen, 2012).

I exclude 'self-defensive' equilibria, in which there is scope for trade, yet both countries price themselves out, happy to do so only because each expects the other to behave similarly.[31]

Country I values underground carbon and sets taxes accordingly:

[30]The assumption of potentially negative prices is technical and intended to simplify the proof of Proposition 4. Subsidisation never occurs in equilibrium.

[31]Fudenberg and Tirole (1985) discuss such 'self-defensive' equilibria. See the proof of Proposition 4 for a precise definition.

Lemma 5. *Country I's equilibrium tax rate is* $\omega^{\tau *} = V_S^{\mathrm{I}}$.

Consumers equate the consumer price $p_F + \tau$ with marginal utility. Country I government wants to equate marginal utility to the overall cost to its consumers including a shadow value term, $p_F + V_S^{\mathrm{I}}$, as the tax revenues are recycled and so net out of the government's objective function.

Country I can always shut down the oil market by setting a high enough tax. I define a region in which the social cost of carbon plus oil extraction cost exceeds the backstop price:

Definition 8. Define the region $\tilde{\Phi} \equiv \left\{(K, S) : x(K) < C(S) + \frac{Z'(S_0 - S)}{\rho}\right\}$, and denote the boundary of this region by the function $S = \tilde{\phi}(K)$.

It is trivial to show $\tilde{\phi}' > 0$, and that the locus lies above $S = \overline{\phi}(K)$: given the stock of knowledge at exhaustion $K(T)$, cumulative extraction will be less socially valuable when climate damages are taken into account.

I now state the third key result, characterising the unique equilibrium in the limit-pricing stage and showing how the requirement of time consistency limits the ability of the importer to use carbon taxes to appropriate rents. I again consider the case with inelastic demand.[32]

Proposition 4. *The unique MPNE involves strategies* $\omega^{d*} = d^{\mathrm{T}}(K)$, $\omega^{\tau *} = Z'(S_0 - S)/\rho$, *and*

$$\begin{aligned} S > \tilde{\phi}(K) \quad &\Rightarrow \quad \omega^{p_F *} = x(K) - Z'(\tilde{S} - S)/\rho, \\ S \leq \tilde{\phi}(K) \quad &\Rightarrow \quad \omega^{p_F *} > x(K). \end{aligned}$$

The associated value functions satisfy

$$V^{\mathrm{I}}(K_0, S_0) = V^{\mathrm{T}}(K_0) - \tilde{\Xi}(S_0), \tag{10a}$$

$$\begin{aligned} V^{\mathrm{E}}(K_0, S_0) = &\int_0^T e^{-\rho t} \left(x(K) - C(S)\right) q(x(K))\, \mathrm{d}t \\ &- \left(\Xi(K_0, S_0) - \tilde{\Xi}(S_0)\right), \end{aligned} \tag{10b}$$

[32]For the case with elastic demand, a similar limit-pricing equilibrium exists and is straightforward to characterise, but cumbersome to express precisely. Also, the uniqueness result has proven more difficult to show. See footnote 29.

with $\Xi(K_0, S_0) \equiv \int_0^{\infty} e^{-\rho t} Z(\tilde{S} - S(t))\,\mathrm{d}t$, $\tilde{\Xi}(S_0) \equiv Z(\tilde{S} - S_0)/\rho \leq \Xi(K_0, S_0)$, *and the paths* $K(t)$, $S(t)$ *determined by* (1)-(2) *and* (4). *Limit pricing stops with economic exhaustion at the locus* $S = \tilde{\phi}(K)$.

In words, in the limit pricing regime, Country I conducts terminal path R&D (Figure 4). It obtains terminal path welfare, less $\tilde{\Xi}$, which represents the present value of climate damages, were concentrations permanently stabilised at the current level. Such stabilisation could always be obtained by setting taxes high enough to shut down the oil market immediately; Country I's welfare is driven down to this 'outside option'. Carbon taxes are curtailed to below the social cost of carbon (SCC), or the sum of discounted marginal damages along the future emission path, denoted Ξ. Country E obtains the producer surplus for oil (the first line of (10b)), less the transfer required to keep the importer in the oil market (the second line of (10b)). Total welfare equals consumer's surplus $V^{\mathrm{T}}(K)$ plus producer's surplus, less the SCC.

Exhaustion takes place once the cost of extraction plus the associated marginal environmental damage equals the cost of producing the substitute. In other words, the resource is used up to the point at which it still provides economic value. While the backstop is adopted at the efficient level of cumulative extraction, the overall equilibrium is inefficient, even with two instruments: an efficient outcome cannot feature limit pricing. However, a carbon tax enables R&D to be chosen in a second-best way, used for its primary purpose of lowering energy costs (and given by the terminal path): there is no need for a crash R&D programme to lock carbon underground, as taxes will serve that purpose. Thus, climate policy is still second-best, but less costly than without carbon taxes.

In equilibrium, the credible tax must be below the Pigovian level. To see why, consider an alternative candidate equilibrium, in which the importer sets a Pigovian tax, or $\omega^{\tau} = \tau^{\mathrm{P}} = \int_0^{\infty} e^{-\rho t} Z'(\tilde{S} - S(t))\,\mathrm{d}t$, and conducts R&D according to the terminal path. The exporter limit prices by setting the oil price equal to the difference between the backstop production cost and the tax. Given these strategies, and integrating the importer's HJB equation, at $t = 0$,

$$V^{\mathrm{I}}(K_0, S_0) = V^{\mathrm{T}}(K) - \Xi(K_0, S_0) + \int_0^{T} e^{-\rho t} q(x(K)) \tau^{\mathrm{P}}(K(t), S(t))\,\mathrm{d}t.$$

That is, the value equals just the terminal path value for energy consumption, less the present value of damages, partly offset by future tax revenue (i.e. the beneficial terms of trade). Differentiating this with respect to S_0, one can obtain

$$V_S^{\mathrm{I}}(K_0, S_0) = \tau^{\mathrm{P}} + \int_0^T e^{-\rho t} q(x(K(t))) \frac{\partial \tau^{\mathrm{P}}(K(t), S(t))}{\partial S(t')} \,\mathrm{d}t < \tau^{\mathrm{P}},$$

where the inequality follows because $\tau^{\mathrm{P}}(K(t), S(t))$ is decreasing in $S(t)$.[33] Leaving the marginal barrel of oil in the ground avoids future marginal climate damages, which is worth exactly the Pigovian tax. However, it also reduces future tax revenues; so by Lemma 5, the current Pigovian tax is then set too high, as $\tau^{\mathrm{P}} > V_S^{\mathrm{I}} = \omega^{\tau *}$. The importer would thus want to lower taxes today, to increase current extraction, thus improving the future terms of trade (differentianting the maximand of the HJB equation gives with the marginal effect, on importer welfare flow, of tightening the tax as $-(V_S^{\mathrm{I}} - \tau^{\mathrm{P}}) \partial q_F / \partial p_F < 0$). The conjectured Pigovian taxes are too conservative, and not credible in equilibrium.

Summing up, in the absence of commitment and with a backstop, carbon taxes cannot neutralise the entire flow of pollution damages. They certainly cannot capture additional resource rents, as they can without a backstop technology (Liski and Tahvonen, 2004). Without environmental concerns (i.e. with $Z(G) \equiv 0$), equilibrium taxes are zero in the limit-pricing stage.

With elastic demand, a non-limit pricing stage exists for a large resource stock. This stage can be solved numerically as in the next section; I do not explicitly investigate this.

6 Numerical results

To confirm that the price peak is not neutralised by strategic effects, and to demonstrate how to solve the Hotelling stage of the MPNE, I now compute an extension of a limit pricing equilibrium outside the set Φ (as discussed in Proposition 2).

[33] A change in $S(t)$ also changes the date of exhaustion T; further, the a conserved barrel increases the long-term underground carbon stock $S(T)$, in general, less than proportionally. These effects cancel out, so that a marginal barrel left in the ground will unequivocally lower the social cost of carbon, and hence the tax, in the future.

I solve the HJB equations (7)–(8) numerically, using Chebyshev polynomials to approximate the value functions outside the set Φ (as in Besanko and Doraszelski, 2004), imposing continuity of value functions along the locus $S = \phi(K)$.[34]

I present one of potentially many possible equilibria to the game.[35] Calculation of numerical solutions even without uniqueness theorems is a common approach in the literature: see Salo and Tahvonen (2001) and Harris et al. (2010) for applications to resource games. Varying the starting point of the numerical algorithm has not yielded alternative equilibria.[36]

Let utility be isoelastic, $u(q) = \frac{q^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}$; backstop cost and R&D cost be quadratic, $x(K) = \underline{x} + \frac{\gamma}{2}(\overline{K} - K)^2$, $c(d) = \frac{\xi}{2}d^2$. To illustrate the qualitative results, I parameterise with $\eta = 2$, $\xi = .01$, $\gamma = 1.6(-4)$, $\overline{x} = 10$, $\underline{x} = 5$.

The value functions are displayed in Figure 5. The economy travels towards the bottom right in the state space. In the limit pricing regime, importer value by construction depends only on the knowledge stock, reaching a maximum at $\overline{K} = 250$. In the non-limit pricing regime, a higher resource stock implies higher value: the importer captures part of the surplus of higher oil sales. Higher knowledge also increases Country I value. Exporter value increases with the resource stock. A higher knowledge stock reduces exporter value, but the effect is very weak.

Optimal investment and oil extraction are shown in Figure 6. By construction, in the limit-pricing stage R&D coincides with the terminal path and resource extraction is given by the backstop price (both functions only of K). In the non-limit pricing stage, R&D decreases and resource extraction increases with the remaining resource stock; resource extraction responds only weakly to changes in the knowledge stock. The terminal path is non-monotonic, so that in the limit-pricing regime R&D investment can either increase or decrease as the knowledge

[34]The numerical algorithm is detailed in Appendix A.2.

[35]Showing equilibrium uniqueness by using a 'natural boundary condition' relies on uniqueness results for ordinary differential equations, in which the state variable is a scalar (Karp, 1996). Applicable uniqueness results to systems of partial differential equations (PDEs), which would be required to use the same approach in the present model, are not readily available.

[36]Convergence to the same equilibrium may also reflect the interaction of the numerical algorithm and the equilibrium problem. Fujii and Karp (2008) demonstrate a numerical algorithm systematically converging to one out of the continuum of equilibria they show must exist.

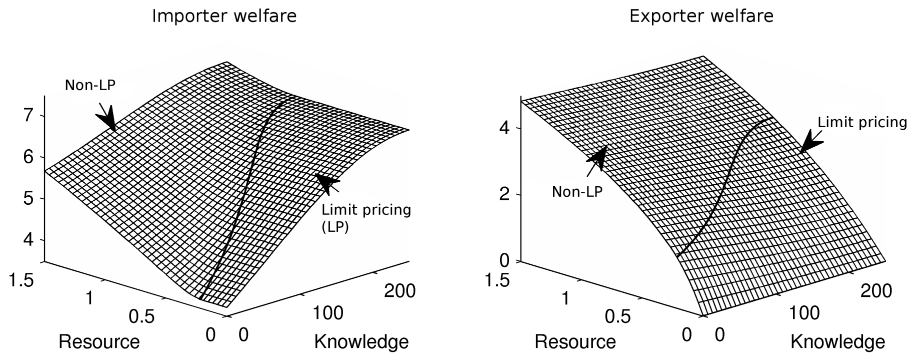

Figure 5: Equilibrium value.

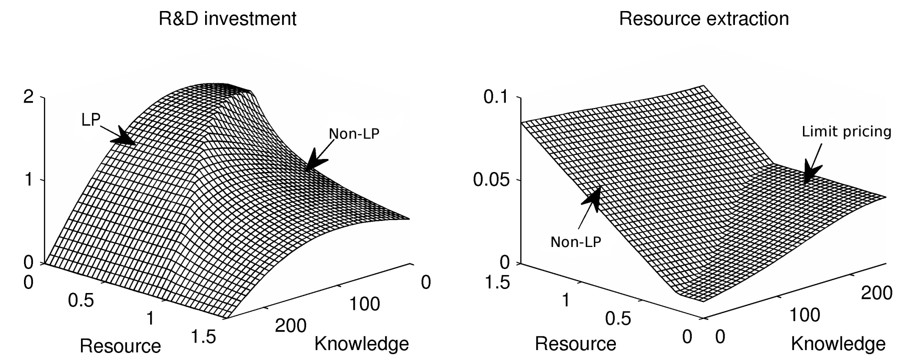

Figure 6: Investment and oil extraction. (*left*: reversed axes.)

stock increases; the non-monotonicity is echoed in the non-limit pricing stage.

It is clear that extraction first falls along the equilibrium path, as the resource stock diminishes and the knowledge stock accumulates, so that the oil price indeed increases in the Hotelling stage and peaks at the moment when limit pricing begins. Thus, the strategic effects discussed in Proposition 2 are much too weak to eliminate the peak; this results from the fact that exporter value is nearly independent of the knowledge stock, so that distorting oil extraction plans, to strategically manipulate future R&D efforts, is not particularly worthwhile. R&D investment approaches the terminal path from below.

7 Conclusions

I have analysed strategic competition between a resource exporter, selling an exhaustible resource, and a resource-importing country, able to gradually reduce the production cost of a perfect substitute to oil. The model produces a theoretically novel explanation of peaking prices, in line with recent observations, and which is—encouragingly—consistent with widely accepted narratives on the cause of the price peak.

The overall outcome is inefficient: oil is used too conservatively, and R&D efforts are too aggressive. Further, while the backstop technology curtails the cartel's market power, it also curtails the importer's ability to credibly tax resource rents. With climate change, the equilibrium tax is below the Pigovian level, so that even the pollution externality is not fully corrected for. Without climate concerns, the equilibrium tax is zero. Limit pricing thus puts oil importers at a disadvantage, even were they to cooperate on resource taxation.

Eventually, the importer switches to the backstop technology. With climate concerns, carbon taxation ensures this switch happens when oil loses its social value. Without taxes, R&D efforts are distorted before the switch: they may at first be reduced to limit short-run pollution, but will eventually accelerate to curtail long-run impacts. Thus, carbon taxes and green R&D are not substitutes, as often implied in the public discussion, but rather complements. Oil importers can put OPEC out of business. This is best done when oil loses its social value, by pricing the carbon externality of oil. Without carbon pricing, oil importers will end up paying too much for expensive technology policies.

The present model could be extended in several directions. It would be relatively straightforward to consider a heterogeneous importer bloc, with cooperative decisionmaking by many countries or sectors, each with their own backstop (for the non-strategic case, see Hoel, 1984). The exporter would limit-price against each sector in turn, giving up on technologically advanced markets one by one. The R&D efforts of technological laggards would accelerate once they became limit-pricing targets. Alternatively, the importing bloc could be split into several countries, to see how an oil cartel might be able to play several importers off against each other; a similar logic would follow, but with additional strategic incentives. Finally, decentralising backstop development is an interesting avenue for further research. This would require e.g. monopolistic competition or differentiated energy demand, to ensure private agents have an incentive to develop technologies which will be soon superseded by even cheaper alternatives.

A Online Appendix

A.1 Proofs

Proof of Lemma 1. To obtain the terminal path, I solve the planner's problem (3) with $S_0 = 0$. Commonly used sufficient conditions do not apply, so I first prove existence and then show that the necessary conditions have a unique solution.

Note that the objective function is bounded above, and the controls are bounded; and that, denoting $q_B^*(p_B) \equiv (u')^{-1}(p_B)$, the function $\tilde{u}(d) \equiv \max_d u(q_B^*(x(K))) - q_B^*(x(K))x(K) - c(d)$ is concave in d. By Theorem 15, Chapter 3 in Seierstad and Sydsæter (1987), a solution then exists.

Employing the Maximum Principle, and denoting the costate variable for K by λ_K, the necessary conditions are

$$c'(d) = \lambda_K \tag{A11a}$$

$$\dot{\lambda}_K = \rho\lambda_K + x'(K)q_B \tag{A11b}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = 0 \tag{A11c}$$

I will show that the solution will satisfy $K \to \overline{K}$, $\lambda_K \to 0$. Suppose first that $K \to K' < \overline{K}$, so that $d^{\mathrm{T}}(K') = 0$. Consider instead, at $K = K'$, setting $d = \epsilon$ for ϵ units of time. For small ϵ, the quantity of knowledge developed is $\epsilon^2 < \overline{K} - K'$. As $\epsilon \to 0$, the cost per unit of knowledge is $\epsilon c(\epsilon)/\epsilon^2 \to c'(\epsilon) \to 0$. The benefit per unit is approximately $-\int_0^\infty e^{-\rho t} x'(K) q_B^*(x(K))\,\mathrm{d}t$, which is strictly positive in the same limit. Thus it is never optimal to leave the knowledge stock below $\overline{K}$. Clearly it is also never optimal to have $d^{\mathrm{T}}(K) > 0$ for $K > \overline{K}$.

Uniqueness follows from the fact that, around the steady state $(\overline{K}, 0)$, the eigenvalues of the linearised system are $\pm x''(\overline{K}) q_B^*(x(\overline{K}))/c''(0)$; thus $(\overline{K}, 0)$ is a saddle point and the solution approaching it must be unique. □

Proof of Proposition 1. From the Maximum Principle, the necessary conditions are given by (A11), complemented with

$$u'(q_F + q_B) \le \lambda_S, \quad q_F \ge 0, \quad \text{C.S.} \tag{A12a}$$

$$u'(q_F + q_B) \le x(K), \quad q_B \ge 0, \quad \text{C.S.} \tag{A12b}$$

$$\dot{\lambda}_S = \rho \lambda_S \tag{A12c}$$

$$\lim_{t\to\infty} e^{-\rho t} \lambda_S(t) S(t) = 0 \tag{A12d}$$

There cannot be a time interval of simultaneous use of oil and the backstop. Suppose otherwise; then in such an interval $\lambda_S = x(K)$. Differentiating both with respect to time, $0 < \rho\lambda_S = \dot{\lambda}_S = x'(K)\dot{K} < 0$, a contradiction.

The final stage (Stage B) will always involve backstop use as $u'(0) > x(0) \ge x(K)$. The economy follows the terminal path, with $\lambda_K > 0$, $d > 0$ for all t (Lemma 1). The asymptotic behaviour of R&D follows from the steady state being a saddle point. There cannot be two stages of oil use separated by a stage of backstop use. To see why, observe that for $q_F > 0$ to be optimal, $\lambda_S \le x(K)$; as soon as this inequality no longer holds, it can never hold again. Thus, there is an initial Stage H of oil use, followed by Stage B. The behaviour of marginal utilities, energy use rates, and other variables follow immediately from the necessary conditions and continuity of costate variables at $t = T$. □

Proof of Lemma 2. The exporter's static problem is to maximise $R(q_F) - V_S^{\mathrm{E}} q_F$, with

$$R(q_F(p_F, K, S)) = \begin{cases} p_F q_F(p_F, K, S) & \text{if } p_F \le x(K), \\ 0 & \text{otherwise.} \end{cases}$$

As $\partial q_F/\partial p_F < 0$, an interior solution is given by $R'(q_F) = V_S^{\mathrm{E}}$; Assumption 1 guarantees the maximand is quasi-concave for $p_F < x(K)$. Possible corner solutions are $p_F = x(K)$, in which case the static maximand takes the value $q_F(x(K), K, S)\left(x(K) - V_S^{\mathrm{E}}\right)$; or $p_F \to 0$, which yields a negative maximand. Setting $p_F > x(K)$ yields zero. The claim follows immediately. □

Proof of Lemma 3. Fix K_0. By the definition of Φ and by Lemma 1, the path of $K(t)$, $t \ge 0$, is unique.

Consider a monopolist who maximises $\tilde{\pi}^{\mathrm{E}}$ given by (6), subject to the path

$K(t)$. It is a trivial extension of the proof of Hoel (1978) to show that the solution features limit pricing. Frame the problem as one of choosing the control path $p_F(t) \leq x(K(t))$ and terminal time T. Denoting the costate on the resource stock by λ_S^{E} and the Hamiltonian by $\mathcal{H} = R(q_F) - \lambda_S^{\mathrm{E}} q_F$, the necessary conditions are

$$R'(q_F) \leq \lambda_S^{\mathrm{E}}, \quad p_F \leq x(K), \quad \text{C.S.} \tag{A13a}$$

$$\dot{\lambda}_S^{\mathrm{E}} = \rho \lambda_S^{\mathrm{E}} \tag{A13b}$$

$$\lambda_S^{\mathrm{E}}(T) S(T) = 0 \tag{A13c}$$

$$\mathcal{H}(T) = 0 \tag{A13d}$$

where (A13d) relates to optimality of T, and with (A13a) implies $\lambda_S^{\mathrm{E}}(T) = x(K(T))$. Equation (A13c) then implies $S(T) = 0$ and thus, from (A13b), $\lambda_S^{\mathrm{E}} = \mathrm{e}^{\rho(t-T)} x(K(T))$. As $S_0 \to 0$, $T \to 0$ with T given by the resource constraint (ii) in Definition 7. Further, for small S_0, by Lemma 2 and the facts that $\dot{\lambda}_S^{\mathrm{E}} > 0$, $\dot{K} \leq 0$, we must have $p_F = x(K)$ for all $t \in [0, T]$. It is apparent that T increases, and $x(K(T))$ (weakly) decreases, with S_0. Thus, *a fortiori*, $e^{-\rho T} x(K(T))$ decreases in S_0. Thus $\phi(K)$ is just given by the maximal S_0 which features immediate limit pricing; by (A13a), this is characterised by $R'(q_F) = e^{-\rho T} x(K(T))$ as claimed. □

Proof of Lemma 4. If Country I follows the given strategy, the proof of Lemma 3 implies that, by definition of Φ, Country E's best response is to limit price.

If Country E limit prices, then $p = x(K)$ even when oil is used. Country I's objective is then identical to that of the social planner for $S_0 = 0$, and by Lemma 1, it is optimal to conduct R&D according to the terminal path.

Uniqueness in differentiable value functions follows from the fact that, in the interior of Φ, $V_S^{\mathrm{E}} > R'(q(x(K)))$. Suppose there is an alternative equilibrium in which Country E is not limit pricing in some region within Φ; in this region, $V_S^{\mathrm{E}} \leq R'(q(x(K)))$. There must be a kink where these regions join up.

Note that the supposition $\mathcal{D} = \Phi$ implicitly requires that $\phi' \geq 0$ (by the definition of $\mathcal{D}$). This holds for typical functional forms (see footnote 22). □

Proof of Proposition 2. The supposition of equilibrium existence refers to the existence of an equilibrium extending beyond Φ; by Lemma 4 the equilibrium exists in the limit-pricing region. I show (i) by *reductio ad absurdum*. Suppose the equilibrium features limit pricing in the intersection of $S > \phi(K)$ and some neighbourhood of $S = \phi(K)$. By arguments used in the proof of Lemma 4, Country I's best response is to set $\omega^{d*} = d^{\mathrm{T}}(K)$. By the definition of Φ and following the arguments used in the proof of Lemma 3, Country E's best response to this is to set $p_F < x(K)$ so that the strictly interior $R'(q_F) = V_S^{\mathrm{E}}$ obtains, a contradiction.

To show (ii), differentiate the exporter's HJB equation (7) with respect to S, to get (9) (as the solution is interior, we can use the envelope theorem). As $V_S^{\mathrm{E}} > 0$, clearly $\dot{V}_S^{\mathrm{E}} > 0$ unless the second term is large; and thus $\dot{p}_F > 0$. $\square$

Proof of Proposition 3. For $S < \overline{\phi}(K)$, the equilibrium is trivial. As $C(S) > x(K)$, the exporter cannot make positive profits; it is optimal to set $p_F > x(K)$, so that oil demand is zero. The importer then faces, effectively, problem (3), and by Lemma 1 follows the terminal path as claimed.

For $S \geq \overline{\phi}(K)$, the exporter can make positive profits. With inelastic demand, no interior solution satiesfies the FOC $R'(q_F) = C(S) + V_S^{\mathrm{E}}$, as $R' < 0$ and $V_S^{\mathrm{E}} \geq -C(S)$. To see the latter inequality, note that the exporter can always sell at an arbitrarily high rate, equivalent to an immediate jump from S to $S' < S$. The payoff for such a jump will be bounded in $V^{\mathrm{E}}(S') - V^{\mathrm{E}}(S) \in [(S - S')C(S), (S - S')C(S')]$; taking the limit as $S' \to S$, this implies $V_S^{\mathrm{E}} \geq -C(S)$.[37] Then, as long as $x(K) \geq V_S^{\mathrm{E}}$, the best response is to limit price irrespective of what the importer does and, supposing this is the case, ω^{p_F*} as given is a best response to ω^{d*}.[38]

[37]See also Polasky et al. (2011) and Jaakkola and Van der Ploeg (2017).

[38]With elastic demand, it is possible to show that there exists an MPNE such that above the locus $S = \overline{\phi}(K)$, the exporter limit prices, and the importer conducts R&D as according to Proposition 3. It is straightforward to calculate the upper boundary of this region.

Supposing that the exporter follows ω^{p_F*}, the importer faces the problem[39]

$$
\begin{aligned}
V^{\mathrm{I}}(K_0, S_0) = \max_{d(t),T} \int_0^T & e^{-\rho t} \left(u(q(x(K), K, S)) - q(x(K), K, S)x(K)\right. \\
& \left. -c(d) - Z(G)\right) \mathrm{d}t + e^{-\rho T} \left(V^{\mathrm{T}}(K_T) - Z(G_T)/\rho\right), \\
\text{s.t. } x(K_T) = C(S_T), &
\end{aligned}
$$

subject to (1)–(2) and (4). I denote the state variables at $t = T$ by K_T, S_T, G_T.

The Hamiltonian for this problem is

$$
\mathcal{H} = u(q(x(K), K, S)) - x(K)q(x(K), K, S) - c(d) + \lambda_K d - (\lambda_S - \lambda_G)q(x(K), K, S),
$$

where λ_K, λ_S, λ_G denote the costate variables. The one-to-one correspondence between S and G implies that only the difference $\lambda_S - \lambda_G$ matters, but keeping these two costates separate makes the necessary conditions more intuitive.

The necessary conditions are[40]

$$
c'(d) = \lambda_K, \tag{A14a}
$$

$$
\dot{\lambda}_K = \rho\lambda_K + x'(K)\left(q(x(K), K, S) + (\lambda_S - \lambda_G)\frac{\partial q(x(K), K, S)}{\partial p}\right), \tag{A14b}
$$

$$
\dot{\lambda}_S = \rho\lambda_S, \tag{A14c}
$$

$$
\dot{\lambda}_G = \rho\lambda_G + Z'(G), \tag{A14d}
$$

$$
\lambda_K(T) = V_K^{\mathrm{T}}(K(T)) - \mu x'(K), \tag{A14e}
$$

$$
\lambda_S(T) = \mu C'(S(T)), \tag{A14f}
$$

$$
\lambda_G(T) = -Z'(G)/\rho, \tag{A14g}
$$

$$
\mathcal{H}(T) = \rho\left(V^{\mathrm{T}}(K(T)) - Z(G)/\rho\right), \tag{A14h}
$$

where μ is a shadow value related to the constraint $C(S(T)) = x(K(T))$, so that at the date of exhaustion, additional underground carbon and additional knowledge stocks are also valuable because they affect the date of exhaustion. Equation

[39] I use an optimal control approach instead of dynamic programming. This requires solving an ordinary differential equation, which is easier and more intuitive than solving a PDE.

[40] See Note 2, Chapter 6.2 in Seierstad and Sydsæter (1987).

(A14h) is for the stopping time T being chosen optimally.

It is well-known that $\lambda_K(t) = \partial V^{\mathrm{I}}(K(t), S(t))/\partial K$. We are particularly interested in the sign of the difference $\lim_{t\uparrow T} d(t) - d^{\mathrm{T}}(K(T))$; this has the same sign as $\lim_{t\uparrow T} \lambda_K(t) - V_K^{\mathrm{T}}(K(T))$. From the HJB equation in the terminal path stage, I can obtain $\rho V^{\mathrm{T}}(K(T))$, and substituting this into (A14h), cancelling out equal terms as $q(x(K), K, S)$ does not jump at exhaustion,

$$\begin{aligned}\lim_{t\uparrow T}&\left(-c(d) + \lambda_K d - (\lambda_S - \lambda_G) q(x(K), K, S)\right)\\ &= -c(d^{\mathrm{T}}(K_T)) + d^{\mathrm{T}}(K_T) V_K^{\mathrm{T}}(K_T).\end{aligned} \tag{A15}$$

Writing $d_T \equiv \lim_{t\uparrow T} d(t)$ and $q_T \equiv q_F(x(K_T), K_T, S_T)$, this can be rearranged to

$$\begin{aligned}d_T \left(\lambda_K(T) - V_K^{\mathrm{T}}\right) &= c(d_T) - c(d^{\mathrm{T}}(K_T)) + V_K^{\mathrm{T}}(K_T)(d^{\mathrm{T}}(K_T) - d_T)\\ &\quad + \left(\lambda_S(T) - \lambda_G(T)\right) q_T\end{aligned} \tag{A16}$$

On the other hand, using (A14e) and (A14f), I can write

$$\lambda_K(T) - V_K^{\mathrm{T}} = -\frac{x'(K_T)}{C'(S_T)} \lambda_S(T) = -\overline{\phi}'(K_T) \lambda_S(T). \tag{A17}$$

Solving (A17) for $\lambda_S(T)$ and substituting into (A16), and then solving for the difference $\lambda_K(T) - V_K^{\mathrm{T}}(K_T)$ using (A14g), I get

$$\begin{aligned}\lambda_K(T) - V_K^{\mathrm{T}}(K_T) &= \frac{-\overline{\phi}'}{q_T + d_T \overline{\phi}'} \Bigg(-\frac{Z'(S_0 - S_T)}{\rho} q_T \\ &\qquad - \left(c(d_T) - c(d^{\mathrm{T}}(K_T))\right) + V_K^{\mathrm{T}}(K_T) \left(d_T - d^{\mathrm{T}}(K_T)\right) \Bigg) \\ &> 0.\end{aligned}$$

The interpretation of this equation is given in the text after Proposition 3. The negativity of the second line follows from an envelope argument, as $d^{\mathrm{T}}(K)$ is optimal given cost $V_K^{\mathrm{T}}(K)$, and by (A14e) implies $\mu > 0$. As the surplus of R&D is increasing in its marginal value, together with (A15) this also implies $\lambda_S(T) - \lambda_G(T) > 0$: moving the last tonne of carbon extracted back into the ground would improve welfare, as it allows a little more R&D to lock a fraction of that tonne in permanently. Then $\lambda_S - \lambda_G > 0$, $\forall t$, as (A14c) and (A14d) imply that were this quantity to become negative, it would stay negative, so that $\lambda_S(T) - \lambda_G(T) > 0$ could not be met.

I will finally establish the property that the optimal trajectory crosses the terminal path at most once; and, so, that there are two distinct phases of R&D, the first (if it exists) with R&D lower, and the latter with R&D higher, than on the terminal path. Suppose there exists a point in time when the optimal trajectory coincides with the terminal path. Then, along the optimal path, $\dot{\lambda}_K$ must be higher than $\dot{\lambda}_K^{\mathrm{T}}$ (comparing (A11b) with (A14b) and recalling $\lambda_S - \lambda_G > 0$) while clearly the R&D rate, and hence $\dot{K}$, is equal in both cases. Thus the optimal trajectory will cross to above the terminal path and stay there until exhaustion.

There can be no other equilibrium, in particular no equilibrium such that $\omega^{p_F *} > x(K)$ for $S > \overline{\phi}(K)$. The arguments used to show uniqueness in Proposition 4 also apply here; I refer the reader to the proof of that proposition. □

Proof of Lemma 5. The importer solves the HJB equation

$$\rho V^{\mathrm{I}} = \max_{\tau, d} u(q_F + q_B) - p_F q_F - x(K) q_B - c(d) - Z(G) + V_K^{\mathrm{I}} d - V_S^{\mathrm{I}} q_F$$

subject to (1)–(2), with p_F taken as given (for fixed K,S).

Optimal $d = (c')^{-1}(V_K^{\mathrm{I}})$ is straightforwardly obtained. I will show the importer's FOC is $\tau^* = V_S^{\mathrm{I}}$. To see this, denote the maximand (with optimal d) by $\Gamma(\tau)$. The tax affects the maximand only via its effect on q_F. Differentiating the maximand and using the consumers' FOC,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\tau} = \begin{cases} \left(\tau - V_S^{\mathrm{I}}\right) \frac{\partial q_F(\omega^{p_F *} + \tau, K, S)}{\partial p_F} & \text{if } \tau < x(K) - p_F, \\ 0 & \text{if } \tau > x(K) - p_F. \end{cases}$$

For $\tau < x(K) - p_F$, the maximand is quasi-concave, as the second derivative at any critical point is $1/u''(q_F) < 0$. Thus the optimal τ will either be an interior optimum with $\tau = V_S^{\mathrm{I}} < x(K) - p_F$, if one exists; equal $\tau = x(K) - p_F \equiv \tau'$; or take any value above τ'. Now $\lim_{\tau \uparrow \tau'} \Gamma - \lim_{\tau \downarrow \tau'} \Gamma = q(x(K)) \left(\tau' - V_S^{\mathrm{I}}\right)$. If $\tau' \leq V_S^{\mathrm{I}}$, this is weakly negative, and Γ jumps (weakly) up at $\tau = \tau'$, but also $\lim_{\tau \uparrow \tau'} \mathrm{d}\Gamma / \mathrm{d}\tau \geq 0$, so that the optimal solution is $\tau^* \geq \tau'$, say, $\tau^* = V_S^{\mathrm{I}}$. If

$\tau' > V_S^{\mathrm{I}}$, the expression is positive, and Γ jumps down but also $\lim_{\tau \downarrow \tau'} \mathrm{d}\Gamma / \mathrm{d}\tau < 0$, so the interior solution $\tau^* = V_S^{\mathrm{I}}$ is optimal. $\square$

Proof of Proposition 4. A unit tax τ shifts the perceived inverse demand and marginal revenue curves down by τ. An interior equilibrium price would require $C(S) + V_S^{\mathrm{E}} + \tau = R'(q_F) < 0$. The assumption that the exporter is able to set a negative price, up to $-\tau$, guarantees the ability to dispose of the resource instantaneously. Adapting the argument used in the proof of Proposition 3, $V_S^{\mathrm{E}} > -C(S) - \tau$, ruling out an interior price. Thus Country E either sets $p_F = x(K) - \tau$ (limit pricing), or sets $p_F > x(K) - \tau$ (not extracting at all).

A *self-defensive equilibrium* has, in some region, $\omega^{p_F} > x(K)$ and $\omega^{\tau} > \max\left\{x(K) - C(S), V_S^{\mathrm{I}}\right\}$. I rule out these uninteresting equilibria, in which there might be scope for trade (if $x(K) \geq C(S) + Z'(\tilde{S} - S)/\rho$), but each party is happy to price itself out only because of expecting the other party to do the same. The MPNE is unique only ignoring such equilibria.

The rest of the proof proceeds as follows. I first show that an exhaustion locus exists, and must be given by $S = \tilde{\phi}(K)$. I then show that immediately to the left of this locus, the equilibrium strategies are as claimed. Finally, I show that this region in fact extends all the way to the left of the exhaustion locus.

Assuming the domain $\mathcal{D}$ contains a point such that $S > \tilde{\phi}(K)$, clearly it is not an equilibrium to never sell the resource (which would imply $V_S^{\mathrm{E}} = 0$, so that there is clearly scope for trade above $S = \tilde{\phi}(K)$). Assumption 5 guarantees economic exhaustion, i.e. $S = 0$ can never be reached. Piecewise continuity of strategies also implies piecewise continuity of the exhaustion locus. By definition, there is no extraction to the right of the exhaustion locus, and inelastic demand guarantees limit pricing immediately to the above and left of it. Furthermore, as after exhaustion the stock S does not interact with the optimal R&D process (the terminal path), it is clear that along the exhaustion locus $V^{\mathrm{I}}(K, S) = V^{\mathrm{I}}(K, \tilde{\phi}(K)) = V^{\mathrm{T}}(K) - Z(\tilde{S} - \tilde{\phi}(K))/\rho$.

In some region above and to the left of the exhaustion locus, Country I's HJB equation, in equilibrium, is

$$\rho V^{\mathrm{I}} = u(q(x(K))) - (x(K) - \omega^{\tau *})\, q(x(K)) - c(\omega^{d*}) - Z(\tilde{S} - S)$$
$$+ V_K^{\mathrm{I}} \omega^{d*} - V_S^{\mathrm{I}} q(x(K))$$

where I have inserted the equilibrium price $\omega^{p_F *} = x(K) - \omega^{\tau *}$. As $\omega^{\tau *} = V_S^{\mathrm{I}}$ by Lemma 5, the last term cancels out with $\omega^{\tau *}$. Partially differentiating and using the envelope theorem,

$$\rho V_K^{\mathrm{I}} = -x'(K) q(x(K)) + V_{KK}^{\mathrm{I}} \omega^{d*}, \tag{A18}$$

which implies that ω^{d*} evolves according to the same equation as $d^{\mathrm{T}}(K)$ for a fixed S (compare with equation (A11b)). Thus, if the equilibrium R&D rate is continuous at the exhaustion locus, then for all S, it follows the terminal path on both sides of the locus, at least near the locus.

I will show continuity of R&D at the exhaustion locus together with the fact that this locus is, in fact, $S = \tilde{\phi}(K)$. First note that the exhaustion locus cannot lie to the left of the locus $S = \tilde{\phi}(K)$. Suppose it did; then in between the two loci, $V_S^{\mathrm{E}} = 0$, $V_S^{\mathrm{I}} = Z(\tilde{S} - S)/\rho$, and $x(K) - V_S^{\mathrm{I}} > C(S)$, i.e. the exporter could set p_F in between these values and make further profits.

Denote the exhaustion locus by $S = f(K)$. Suppose, for some K, $f(K) < \tilde{\phi}(K)$. As assumptions guarantee exhaustion will always happen somewhere, some sections of the locus must be either horizontal, $f'(K) = 0$, or have a positive slope, $f'(K) > 0$. Denote Country I's value function before exhaustion by V^{I} and after exhaustion by $\tilde{V}^{\mathrm{I}}$, and the corresponding taxes by τ and $\tilde{\tau}$. Suppose $f'(K) > 0$. If $V_K^{\mathrm{I}} \leq \tilde{V}_K^{\mathrm{I}}$; then continuity of the value function implies that, for (K, S) in a small neighbourhood of $(K^\circ, S^\circ) = (K^\circ, f(K^\circ))$, $-f'(K)\left(V_S^{\mathrm{I}} - \tilde{V}_S^{\mathrm{I}}\right) = V_K^{\mathrm{I}} - \tilde{V}_K^{\mathrm{I}} \leq 0$ (obtained by totally differentiating $V^{\mathrm{I}} = \tilde{V}^{\mathrm{I}}$ along $S = f(K)$). Then $\tilde{\tau} = \tilde{V}_S^{\mathrm{I}} \leq V_S^{\mathrm{I}} = \tau$, and $x(K^\circ) - \tau \leq x(K^\circ) - \tilde{\tau} \approx x(K^\circ) - Z'(\tilde{S} - S^\circ)/\rho < C(S^\circ)$, where the last inequality holds as I have supposed $S = f(K)$ lies below $S = \tilde{\phi}(K)$. Thus Country E cannot profitably limit price just prior to exhaustion, a contradiction.

Suppose instead that $f'(K) > 0$ and $V_K^{\mathrm{I}} > \tilde{V}_K^{\mathrm{I}}$, so that $V_S^{\mathrm{I}} < \tilde{V}_S^{\mathrm{I}} = Z'(\tilde{S} - S)/\rho$. The equilibrium trajectory starting at $(K^{\circ\circ}, S^{\circ\circ})$ near the exhaustion locus will cross the locus, say at (K°, S°). Continuity of value along the equilibrium path implies $V^{\mathrm{I}}(K^{\circ\circ}, S^{\circ\circ}) + V^{\mathrm{I}}_K\,\mathrm{d}K + V^{\mathrm{I}}_S\,\mathrm{d}S \approx \tilde{V}^{\mathrm{I}}(K^\circ, S^\circ)$, with $\mathrm{d}K \approx d\,\mathrm{d}t$ and $\mathrm{d}S \approx -q(x(K))\,\mathrm{d}t$ where $d = (c')^{-1}\left(V^{\mathrm{I}}_K\right)$ and I have used $\omega^{p_F} = x(K) - \omega^\tau$. Thus

$$
\begin{aligned}
&V^{\mathrm{I}}(K^{\circ\circ}, S^{\circ\circ}) + V^{\mathrm{I}}_K(K^{\circ\circ}, S^{\circ\circ}) d\,\mathrm{d}t - V^{\mathrm{I}}_S(K^{\circ\circ}, S^{\circ\circ}) q_F\,\mathrm{d}t \\
&\qquad \approx V^{\mathrm{T}}(K^\circ) - Z(\tilde{S} - S^\circ)/\rho \\
&\qquad \approx V^{\mathrm{T}}(K^{\circ\circ}) - \frac{Z(\tilde{S} - S^{\circ\circ})}{\rho} + V^{\mathrm{T}\prime}(K^{\circ\circ}) d\,\mathrm{d}t - \frac{Z'(\tilde{S} - S^{\circ\circ})}{\rho} q(x(K^{\circ\circ}))\,\mathrm{d}t.
\end{aligned}
$$

Consider an alternative strategy for Country I: using the tax instrument to price oil out just before the exhaustion locus. This strategy would yield an excess of value, over the posited equilibrium strategy, given by

$$
\begin{aligned}
&\Big(V^{\mathrm{T}}(K^{\circ\circ}) - Z(\tilde{S} - S^{\circ\circ})/\rho\Big) - V^{\mathrm{I}}(K^{\circ\circ}, S^{\circ\circ}) \\
&\qquad \approx \Big(V^{\mathrm{I}}_K - V^{\mathrm{T}\prime}(K^{\circ\circ})\Big)\, d\,\mathrm{d}t + (\tilde{V}^{\mathrm{I}}_S - V^{\mathrm{I}}_S) q(x(K^{\circ\circ}))\,\mathrm{d}t \\
&\qquad > 0,
\end{aligned}
$$

where the inequality follows from the suppositions. Thus Country I could do better by using a strategy which featured exhaustion already from $(K^{\circ\circ}, S^{\circ\circ})$, and so the posited strategy is not a best response. Finally, suppose that $f'(K) = 0$. Continuity of value implies $V^{\mathrm{I}}_K = \tilde{V}^{\mathrm{I}}_K$ along this locus. The assumption that there is extraction just before the locus implies $V^{\mathrm{I}}_S < \tilde{V}^{\mathrm{I}}_S$, and the above inequality holds again.

Thus the exhaustion locus $S = f(K)$ cannot lie below $S = \tilde{\phi}(K)$ either; and we can conclude it must coincide with $S = \tilde{\phi}(K)$. The above arguments also rule out a strict discontinuity in the R&D strategy at $f(K) = \tilde{\phi}(K)$, so that $\lim_{K \uparrow \tilde{\phi}(K^\circ)} \omega^{d*}(K, S) = d^{\mathrm{T}}(K^\circ)$: as $f'(K) > 0$, $V^{\mathrm{I}}_K < \tilde{V}^{\mathrm{I}}_K$ implies extraction could not be profitable just before exhaustion, and $V^{\mathrm{I}}_K > \tilde{V}^{\mathrm{I}}_K$ would imply the importer could do better by setting a higher tax, shutting down oil extraction, before the exhaustion locus is reached.

Thus the R&D process is continuous at the exhaustion locus, and (A18) then implies it must follow the terminal path just to the left of the exhaustion locus. In this limit-pricing region, $V^{\mathrm{I}}_K = \lambda^{\mathrm{T}}_K(K) = V^{\mathrm{T}}_K(K)$ as defined in Lemma 1, and

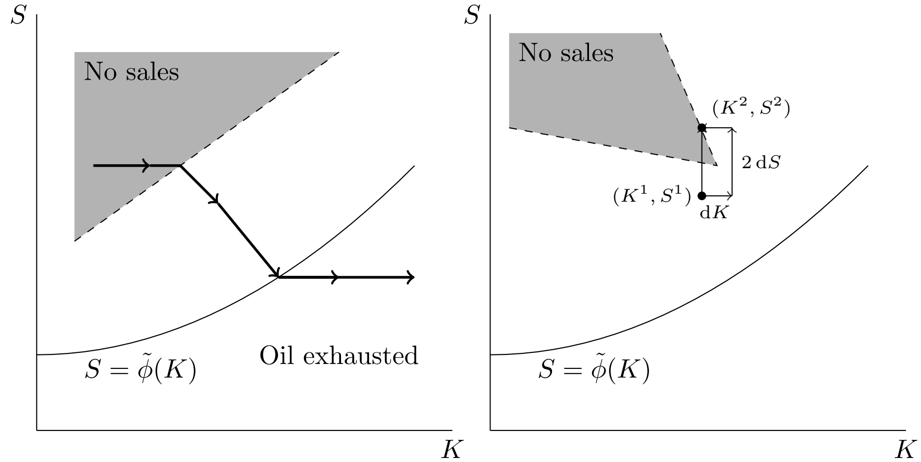

Figure 7: Conjectured no-sales region with a sample trajectory of the economy.

consequently $V^{\mathrm{I}}(K,S) = V^{\mathrm{T}}(K) + g(S)$, where $g(\cdot)$ is an unknown function. From the continuity of value, we obtain $V^{\mathrm{I}}(K, \tilde{\phi}(K)) = V^{\mathrm{T}}(K) + Z(\tilde{S} - S)/\rho$, so that $g(S) = Z(\tilde{S} - S)/\rho \equiv \tilde{\Xi}(S)$. This gives us the value function V^{I} as claimed, and differentiating we obtain $\omega^{\tau*} = V^{\mathrm{I}}_S = Z'(\tilde{S} - S)/\rho$ as claimed. The value function of Country E is easy to obtain by simply integrating the payoff function in $t \in [0, T]$ (using integration by parts on the term $\int_0^T e^{-\rho t} q_F Z'(\tilde{S} - S)/\rho \,\mathrm{d}t$).

The only remaining task is to show that the limit-pricing region covers the entire region $S > \tilde{\phi}(K)$. Suppose it doesn't; then there is some region inside the limit-pricing stage with no extraction. In such a 'no-sales region', the economy moves parallel to the K-axis, and clearly the exporter is making no flow profits. Denote Country E's value in the no-sales region by V^{E}, and in the limit-pricing region by $\tilde{V}^{\mathrm{E}}$. As higher K just brings the economy closer to exiting the no-sales region, so $V^{\mathrm{E}}_K > 0$. Near the boundary of the limit-pricing region, it must be that $V^{\mathrm{E}}_S > \tilde{V}^{\mathrm{E}}_S$ as we have supposed it is optimal for Country E to not extract in the no-sales region.

The lower right boundary of the no-sales region can either be 'smooth', described by some function $S = f(K)$, $f'(K) > 0$ for some point on the boundary; or the boundary has to be 'sharp', featuring an acute or right angle pointing southeast (Figure 7). I will now show that either will lead to a contradiction.

Suppose there is a point (K°, S°) on the boundary of the region such that, in the neighbourhood of the point, the boundary is given by $S = f(K)$, $f'(K) > 0$.

Then $V_S^{\mathrm{E}} > \tilde{V}_S^{\mathrm{E}}$, and (as above) continuity of the value function implies $0 < V_K^{\mathrm{E}} < \tilde{V}_K^{\mathrm{E}} < 0$, a contradiction (with the last inequality being immediate from the value function in the limit-pricing region). So the smooth boundary is ruled out.

Suppose instead there is an angle. If the upper arm of the angle is vertical (parallel to the S-axis), continuity of the value function implies $V_S^{\mathrm{E}} = \tilde{V}_S^{\mathrm{E}}$, a contradiction. Suppose instead that the angle is acute, with the upper arm non-vertical. If the lower arm of the angle is not horizontal, the we can choose points (K^1, S^1) and (K^2, S^2) in the neighbourhood of the vertex, below and above the angle respectively (Figure 7), so that

$$V^{\mathrm{E}}(K^2, S^2) - V^{\mathrm{E}}(K^1, S^1) \approx \tilde{V}_S^{\mathrm{E}}\,\mathrm{d}S + V_S^{\mathrm{E}}\,\mathrm{d}S.$$

However, we also have

$$\begin{aligned} V^{\mathrm{E}}(K^2, S^2) - V^{\mathrm{E}}(K^1, S^1) &\approx \tilde{V}_K^{\mathrm{E}}\,\mathrm{d}K + 2\tilde{V}_S^{\mathrm{E}}\,\mathrm{d}S - \tilde{V}_K^{\mathrm{E}}\,\mathrm{d}K \\ &= 2\tilde{V}_S^{\mathrm{E}}\,\mathrm{d}S \\ &< \tilde{V}_S^{\mathrm{E}}\,\mathrm{d}S + V_S^{\mathrm{E}}\,\mathrm{d}S, \end{aligned}$$

as the value function has to be continuous also along the lower arm of the angle (a discontinuity would imply a profitable deviation would exist, as the exporter can control the economy either into or out of the no-sales region).

Finally, suppose the lower arm of the angle is horizontal, so that the exporter cannot force the economy into the no-sales region from below. Now there could be a discontinuity ΔV^{E} along the lower arm of the angle. Then

$$\begin{aligned} 2\tilde{V}_S^{\mathrm{E}}\,\mathrm{d}S &= V^{\mathrm{E}}(K^2, S^2) - V^{\mathrm{E}}(K^1, S^1) \\ &\approx \tilde{V}_S^{\mathrm{E}}\,\mathrm{d}S + \Delta V^{\mathrm{E}} + V_S^{\mathrm{E}}\,\mathrm{d}S, \end{aligned}$$

implying $\Delta V^{\mathrm{E}} = (\tilde{V}_S^{\mathrm{E}} - V_S^{\mathrm{E}})\,\mathrm{d}S < 0$. However, a negative discontinuity would imply that near the lower arm of the angle, but inside the no-sales region, the exporter could profitably deviate by driving the economy out of the no-sales region. Hence the 'sharp' boundary to the no-sales region also produces a contradiction.

Thus the equilibrium cannot have a no-sales region before exhaustion, so that the limit-pricing region covers the entire domain above the locus $S = \tilde{\phi}(K)$. $\square$

A.2 A Chebyshev collocation method to solve for the MPNE

I will approximate the continuous-time value functions outside the region Φ.

The first-order conditions to the problems (7) and (8), with limit pricing not binding, are $d^* \equiv d^*(V^{\mathrm{I}}_K) = (c')^{-1}(V^{\mathrm{I}}_K)$ and $p^*_F \equiv p^*_F(V^{\mathrm{E}}_S) = p((R')^{-1}(V^{\mathrm{E}}_S))$. These are uniquely defined, given the assumptions made, for any V^{I}_K and V^{E}_S. Without taxes, I can solve equivalently for the optimal q^*_F. Substituting the optimal actions into the HJB equations, the optimal value functions satisfy

$$\begin{aligned}\rho V^{\mathrm{I}} &= u(q^*_F) - q^*_F p(q^*_F) - c(d^*) + V^{\mathrm{I}}_K d^* - V^{\mathrm{I}}_S q^*_F,\\ \rho V^{\mathrm{E}} &= q^*_F p(q^*_F) + V^{\mathrm{E}}_K d^* - V^{\mathrm{E}}_S q^*_F,\end{aligned} \tag{A19}$$

in which I have omitted the dependence of q^*_F and d^* on V^{E}_S and V^{I}_K, respectively. This is a system of two nonlinear, first-order partial differential equations, the unknowns being V^{I} and V^{E}. The boundary conditions will be given by the continuity of the value functions at the upper boundary of the set Φ.

I solve the system using the Chebyshev collocation method for solving partial differential equations: I find k-dimensional approximations $\overline{V}^{\mathrm{I}}$, $\overline{V}^{\mathrm{E}}$ which satisfy the above system at k points. This approach to PDEs is briefly described by Judd (1998) and has been used to tackle symmetric bivariate differential games by Doraszelski (2003) and Saini (2012).[41] I apply the method to tackle an asymmetric, bivariate diffential game, extending the method to solve a pair of coupled PDEs.

By choosing Chebyshev polynomials, I am imposing differentiability of an arbitrary degree in the set $\mathcal{D} \setminus \Phi$, ruling out coordinated switches of strategies in the interior of this set. Chebyshev nodes require the approximation domain to be rectangular, so I transform the set $\mathcal{D} \setminus \Phi$ into a rectangular set in the space (K, s), by using

$$s \equiv \frac{S - \phi(K)}{\overline{S} - \phi(K)}, \tag{A20}$$

implying $s \in [0, 1]$. I choose n basis functions in the K-dimension and m basis

[41] Similar methods have been also used in univariate stopping problems by Dangl and Wirl (2004), Caporale and Cerrato (2010), Balikcioglu et al. (2011) and Mosiño (2012).

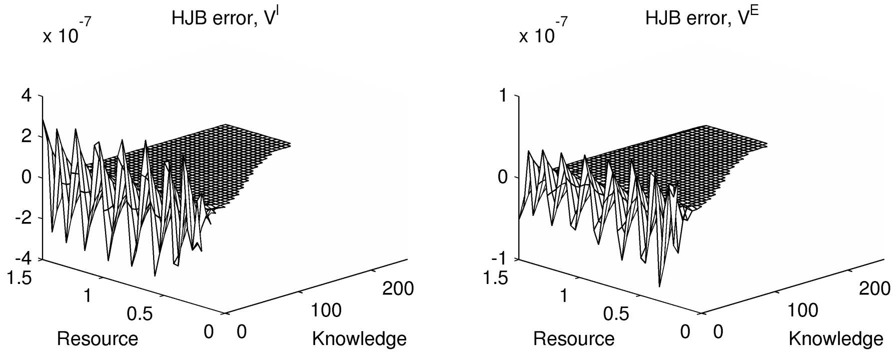

Figure 8: HJB equation errors relative to the HJB equation LHS.

functions in the s-dimension and form a tensor product of Chebyshev polynomials to approximate each value function.

I will thus approximate transformed value functions $v^{\mathrm{I}}(K,s)$, $v^{\mathrm{E}}(K,s)$, the partial derivatives of which satisfy, for $i \in \{\mathrm{I},\mathrm{E}\}$,

$$\begin{aligned} v_K^i &= V_K^i(K,S) + V_S^i(K,S)(1-s)\phi'(K), \\ v_s^i &= V_S^i(K,S)(\overline{S} - \phi(K)). \end{aligned}$$

The function approximations $\overline{v}^i$ will be of the form $\overline{v}^i(K,s) = V^{\phi,i}(K) + sA^i g(s,K)$, where $V^{\phi}(K)$ is the relevant value function at the limit-pricing boundary $S = \phi(K)$, A^i is a coefficient matrix with dimensions $(1, nm)$, and $g(\cdot)$ is an (nm)-vector of Chebyshev polynomials. The boundary condition is satisfied by construction.

I use the COMPECON package developed by Miranda and Fackler (2002). I set $n = m = 20$, to get a system with 800 equations and unknowns. I calculate the open-loop solution in $\mathcal{D}\backslash\Phi$, imposing the boundary condition of continuity of value functions and their first derivatives at $S = \tilde{\Phi}(K)$, and back out the coefficients which match the associated value functions to use as my initial guess.

The system is solved rapidly by a standard non-linear rootfinding algorithm in Matlab, probably largely due to the good initial guess. For initial guesses 'near' the open-loop equilibrium values, the system converges to effectively identical results; for very different initial guesses, the algorithm typically fails to converge. HJB equation errors are small, of order 10^{-7} relative to the HJB equation values (Figure 8).

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