1 Introduction

1.1 Offshore foundations

The development of offshore foundations has been mainly driven by the needs of the petroleum industry. Reservoirs of oil and gas are located in geological structures offshore as well as onshore. Oil has been extracted from offshore locations for about half a century. But the world's two main offshore production areas, in the Gulf of Mexico and in the North Sea, have only been substantially developed over the last thirty years. The development of some offshore fields has posed huge engineering problems because of the water depth and the sometimes severe loading conditions (mainly wave loading).

The offshore extraction process has required the construction of large structures (platforms) that are fixed to the seabed. There are different types of platforms but these can be grouped into three general categories. Steel jacket structures are usually supported by steel piles that are driven deep into the seabed. Concrete gravity base structures are founded on the surface of the seabed. Tension leg platforms are used in very deep water and the tethers are fixed to piled foundations in most cases. Large offshore platforms are only cost effective when the reservoirs are large and when they are expected to be in production for decades.

Many large fields have recently reached (or will soon reach) the end of their production lives. This has led the petroleum industry to develop marginal (smaller) oil and gas reservoirs. The development of marginal fields has required novel engineering solutions. Smaller platforms have been used, and in some cases even totally sub-surface production systems have been used. Piled or large gravity base foundations are usually not cost effective solutions for small platforms. Small shallow foundations, such as suction caisson foundations, have been shown to be viable alternatives. Small shallow foundation systems can also be used for other purposes such as supporting wind turbines.

1.1.1 Suction caissons and partially-drained behaviour

Some aspects of the behaviour of a small shallow foundation on a clay seabed can be predicted using established design procedures. This is because the response of such a foundation/soil system under severe wave loading is mainly undrained. The behaviour of a small shallow foundation on a sand seabed is neither fully drained or undrained, but is partially-drained. The partially-drained behaviour of sand is not well understood, and consequently, design procedures have not been established.

The foundations of offshore structures are subject to vertical, horizontal, and moment loads. Foundations also have to withstand tensile loads (negative vertical loads) in some cases. To withstand tensile loads, small shallow foundations have to be designed to induce negative pore fluid pressures on their bases. The factors affecting the development of pore pressures in the foundation soil (or the partially-drained behaviour) are the loading rate, the permeability of the soil, and the drainage length. The loading rate is dependent on the wave period, and the permeability is dependent on the size of the sand particles. These two factors cannot be altered but the drainage length of the foundation can be set to ensure that the required degree of partial-drainage is attained.

The drainage length of a shallow foundation can be increased by incorporating skirts around the periphery. Skirts not only increase the drainage length, they also increase the vertical, horizontal, and moment capacity of the foundation as well. Circular foundations with skirts are generally referred to as suction caisson foundations (or skirt-plate or bucket foundations). This type of foundation has been only recently used to support steel jacket platforms on sand. That is, in the Europipe and Sleipner West fields in the North Sea (Aas & Andersen, 1992; Bye *et al.*, 1995; Erbrich, 1996). The recent use of suction caisson foundations required extensive offshore and laboratory modelling because of the lack of knowledge of partially-drained behaviour. The understanding of partially-drained behaviour is poor because soil mechanics research has focused on the two extremes of soil behaviour (drained and undrained behaviour) about which simplifying assumptions can be made.

1.1.2 Modelling of foundation behaviour

In the offshore industry, bearing capacity methods are commonly used to calculate the ultimate drained and undrained capacity of foundations under vertical, inclined, and eccentric loads (Brinch Hansen, 1970 and Vesic, 1975). Recent research has successfully interpreted the drained and undrained behaviour of shallow foundations in terms of plasticity theory (Tan, 1990; Nova & Montrasio, 1991; Martin, 1994; Gottardi *et al.*, 1997). Foundation models based on plasticity theory are simple and powerful methods for analysing the behaviour of shallow foundations. These models can incorporate vertical as well as horizontal and moment loads, and also link loads and displacements.

1.1.3 Current knowledge of the behaviour of shallow footings

The behaviour of circular footings subject to vertical loads and under the limiting conditions of drained and undrained loading will be outlined. The discussions below serve as an introduction to the current knowledge associated with some aspects of the problem addressed by this thesis.

Mechanisms of failure

Experimental evidence in the literature indicates that mechanisms can be categorised as general shear, local shear, or punching shear. These mechanisms differ in the extent of the failure surface or slip lines. General shear refers to mechanisms with extensive slip lines, where the failure surface extends all the way to the surface of the foundation material. Punching shear is the opposite extreme, where slip lines are only evident beneath the footing. Local shear refers to mechanisms where the development of the failure surface is between that of general shear and punching shear.

The failure mechanisms that are expected with various combinations of material characteristics and loading conditions are listed in Table 1.1 (note that geometrical effects are not mentioned in Table 1.1; these will be discussed below). There is substantial evidence which indicates that under drained loading conditions, a general shear mechanism is probable in materials of low compressibility (dense sand and over-consolidated clay). While a punching shear mechanism is probable in materials of high compressibility (loose sand and normally-consolidated clay). This evidence is summarised by Vesic (1975). These characteristics can be explained by the volumetric strains associated with drained shearing. Dilation results in volumetric expansion and general movement in a material of low compressibility. Conversely, contraction results in volumetric compression and local movement in a material of high compressibility.

Under undrained loading conditions, a general shear mechanism is probable in a material of high compressibility. Vesic (1975) indicates that there is substantial evidence for this with regard to normally-consolidated clay. The failure mechanism of a footing on loose sand under undrained conditions should be similar considering the constant volume condition and that a relatively low undrained shear strength is characteristic of loose sand.

In the case of undrained loading of footings on materials of low compressibility, the tests of Vesic *et al.* (1965) on dense sand indicate that punching shear is probable. Vesic (1975) indicated that this was the case with over-consolidated clay as well. The undrained loading of a shallow footing on a dilatant material can result in very high effective stresses due to negative pore pressures; where the undrained strength might be dependent on the cavitation pressure of the pore fluid.

The size of a footing also affects the failure mechanism. It is well documented that the average shear strength along a slip line decreases with footing size (Vesic, 1975). This manifests itself as an increase in the relative compressibility of the foundation material with increasing footing size. Thus for a particular soil, a small footing might develop a general shear mechanism while a large footing might develop a punching shear mechanism.

Response on elastic soil

It is sometimes acceptable to assume that the foundation soil can be modelled as a linear elastic material. In such cases, standard elastic solutions can be used to estimate the response of footings. Some of these solutions are outlined in Poulos and Davis (1974) for example.

The elastic parameters which are usually used in these solutions are the Young's modulus E' and the Poisson's ratio ν'. Under axi-symmetric conditions with the radial stress held constant, the Young's modulus is deduced from the ratio of the increments of effective axial stress and elastic axial strain. Under similar conditions, the Poisson's ratio is deduced from the ratio of the increments of elastic radial strain and elastic axial strain.

The drained and undrained cases are modelled using appropriate values for Young's modulus and Poisson's ratio. Under drained loading conditions, the values of E' and ν' which are representative of the effective stress state are applicable. Under undrained loading conditions, values of E_u and $\nu_u = 0.5$ which are representative of undrained or constant volume loading are applicable. The constant volume condition implies that the Poisson's ratio is 0.5 since the undrained bulk modulus is infinite.

The relationships between the parameters of an isotropic elastic material imply that the undrained and drained values of Young's modulus are not independent (most notably, the relationship between the drained and undrained values of shear modulus). Hence the undrained value of Young's modulus can be deduced from the drained parameters as follows,

$$E_u = \frac{3E'}{2(1+\nu')} \qquad [1.1]$$

Since the settlement of a surface footing on an elastic base is a function of Young's modulus and Poisson's ratio (and of the applied load and footing geometry), the ratio between the undrained and drained values of settlement can be expressed as,

$$\frac{w_u}{w_d} = \frac{1}{2(1-\nu')} \qquad [1.2]$$

Taking a typical value of $\nu' = 0.2$ implies that $w_u = 0.625w_d$. The difference between the drained and undrained settlement is equal to the settlement which occurs during consolidation. Note that this is for a footing on an infinite base. Solutions do exist, however, for footings on finite bases. For the extreme case of one-dimensional loading, the undrained settlement is zero.

<u>Load:displacement response</u>

The load:displacement response of a footing under conditions of drained and undrained loading are discussed below. The response of a footing on an elastic soil and on an inelastic soil will be addressed separately.

It was mentioned previously that the ratio between the undrained and drained settelment of a footing on an infinite elastic base could be deduced using the value of the drained Poisson's ratio. For a given load increment, the ratio between the undrained and drained load:displacement response could be deduced from the ratio of w_u/ w_d. Using the example given previously, where $\nu' = 0.2$ and $w_u = 0.625w_d$, the undrained stiffness of the footing would be 1.6 times higher than the drained stiffness. Thus the stiffness of a footing under undrained loading conditions should be higher than under drained loading conditions.

Soils are highly non-linear and inelastic. In most practical situations, the elastic solutions cannot be used to predict the load:displacement response of footings. Some insight into the effect that loading conditions have on the load:displacement response of footings on inelastic soil can be gained, however, by considering the relationship between soil stiffness and effective stress. If one assumes that the load:displacement response (or footing stiffness) is proportional to some value of soil stiffness (be it the shear modulus or bulk modulus; shear and volumetric effects being coupled because the soil is inelastic), then the response of a footing will be dependent on the effective stress. In soils that are prone to dilate, undrained loading might result in higher effective stresses and stiffer response than for drained loading. Conversely, undrained loading might result in lower effective stresses and less stiff response than drained loading in soils which are prone to contract.

<u>Pore pressures under conditions of undrained loading</u>

The pore pressure distribution on the base of a footing under conditions of undrained loading is discussed below. The distribution on a footing on an elastic soil and on an inelastic soil will be addressed separately.

On an elastic soil, where shear and volumetric effects are not coupled, undrained conditions imply that any change in total stress results in an equivalent change in pore pressure. The change in the total stress distribution, which can be deduced using the standard solutions of elasticity outlined in Poulos and Davis (1974), is equal to the change in the pore pressure distribution. Thus a saddle shaped total stress and pore pressure distribution, where the total stress and pore pressure is highest around the perimeter of the footing, is expected for a rigid footing.

Undrained loading of a rigid footing on an inelastic soil should result in a saddle shaped total stress distribution similar to that for an elastic soil. The pore pressure distribution is unlikely to be similar to the total stress distribution, however, because of the coupling between shear and volumetric effects.

<u>Bearing capacity and the effect of embedment</u>

Simple bearing capacity equations for drained and undrained loading and the effects of footing embedment will be introduced.

The bearing capacity of a shallow foundation under conditions of drained loading is given by,

$$q_f = \tfrac{1}{2}\gamma' B N_\gamma + \gamma' D N_q \qquad [1.3]$$

where N_γ is the bearing capacity factor for soil self-weight, N_q is the bearing capacity factor for surcharge, γ' is the effective unit weight of the soil, B is the foundation width, and D is the depth of embedment. These two bearing capacity factors are dependent on the friction angle of the soil and increase with the value of the friction angle. It is apparent from Equation 1.3 that for a constant value of friction angle and effective unit weight, the bearing capacity is proportional to the depth of embedment. If Equation 1.3 were used to estimate load:displacement response by calculating the bearing capacity at different depths, for a range of friction angle values, then a series of linear load:displacement curves would be produced. Each of these curves would start from the q_f axis at a value dependent on the friction angle and equal to the term in Equation 1.3 which is representative of the soil self-weight. The gradients of the load:displacement curves would increase with the value of the friction angle. A limiting load:displacement curve equivalent to penetration of a foundation in a material of zero frictional resistance (water for example) could also be postulated.

The bearing capacity under conditions of undrained loading is given by,

$$q_f = s_u N_c + \gamma D \qquad [1.4]$$

where N_c is the bearing capacity factor for cohesion, s_u is the undrained strength of the soil, and γ is the total unit weight of the soil. The bearing capacity factor N_c depends on the shape and depth of the foundation, and increases with embedment to a constant value at a D/B ratio of about four. Equation 1.4 predicts a non-linear load:displacement response at values of embedment of relevance to a shallow foundation. If the undrained strength were zero and the unit weight were constant, Equation 1.4 would predict a linear load:displacement curve.

1.1.4 The need for more research

The traditional bearing capacity methods and the more recent plasticity foundation models can only be used to analyse the drained and undrained behaviour of shallow foundations. The use of small shallow foundations to support offshore structures on sand requires an understanding of the partially-drained behaviour. Although the partially-drained behaviour of suction caissons has been investigated, previous studies have mainly focused on the design for specific prototypes and were not generic studies of the partially-drained foundation problem. The partially-drained behaviour of a simple flat foundation should also be investigated before that of the more complex geometry of a suction caisson.

1.2 Aims of present work

This investigation was intended to provide a database of experimental evidence of partially-drained foundation behaviour. The vertical, horizontal, and moment partially-drained response of a model footing under 1-g conditions was to be explored. Partially-drained behaviour was to be investigated by varying the displacement rates of the footing.

More specifically, the aim of this project was the investigation of the effects of partial drainage on the monotonic load:deformation behaviour of a flat circular footing. The following aspects of behaviour were to be investigated:

— Virgin-penetration behaviour. The effect of the vertical penetration rate on the virgin-penetration behaviour was to be explored.

— Consolidation behaviour. A simple partially-drained consolidation model was to be developed and compared to the results of vertical loading partially-drained tests.

— Combined load behaviour. The effect of the horizontal and rotational displacement rates on the combined load yield surfaces was to be studied.

Because of the novelty of this research, it was difficult to postulate what the outcome would be. It was expected, however, that the partially-drained behaviour would be rate dependent.

1.3 Outline of thesis

In **Chapter 2**, information of relevance to this investigation is reviewed. Literature on the drained and partially-drained response of footings is presented. At the end of Chapter 2, the scaling of model footing tests is discussed and scaling relationships are presented. **Chapter 3** describes the apparatus used in the footing tests, as well as that used in the miniature cone penetrometer tests and the sample preparation process. In **Chapter 4**, the properties of the sand and of the oil-saturated sand samples are discussed. The results of element tests, and of the miniature cone penetrometer results, are analysed. **Chapter 5** outlines the test programme and describes the procedures involved in some typical tests. **Chapter 6** analyses the vertical load results and **Chapter 7** the combined load results. **Chapter 8** summarises the findings of this investigation. Suggestions for further research are also made.

2 Literature Survey

2.1 Introduction

The aspects of footing behaviour that are of relevance to the investigation of partially-drained behaviour are reviewed. The drained elastic and plastic behaviour of footings are discussed. Theoretical elastic solutions are compared with unload-reload data in the literature. The traditional bearing capacity methods and the more recent combined load solutions based on plasticity theory are outlined. Evidence of partially-drained behaviour and the factors affecting this type of behaviour are investigated. Theoretical and empirical data on the pore pressure and effective stress distributions on footings is outlined. Solutions for the consolidation behaviour of footings are detailed. The issues associated with the scaling of model behaviour are also addressed.

2.2 Elastic behaviour

The working loads that shallow offshore foundations are subjected to are usually lower than the loads required to induce significant plastic deformation. The working loads are generally below the combined load bearing capacity and therefore within the combined load failure surface. Loads resulting in plastic deformation are usually only encountered during installation of the structure and during periods of severe environmental loading. The load:deformation response is governed by essentially elastic soil behaviour for loads within the combined load failure surface.

2.2.1 Available elastic solutions

Estimates of deformation can be made for unload-reload behaviour assuming the soil to be an isotropic homogeneous elastic material. Short-term static loading results in this form of deformation. It is more or less the instantaneous response of the foundation to loading and primarily results from shear straining of the soil. The elastic solutions of Poulos and Davis (1974) for a rigid flat circular footing provide a useful means for estimating deformation and have been adopted by the American Petroleum Institute in their offshore platform design guidelines (API, 1993). These solutions are as follows.

$$V=\left[\frac{4GR}{1-\nu}\right]w;\qquad H=\left[\frac{32GR(1-\nu)}{7-8\nu}\right]u;\qquad M=\left[\frac{8GR^3}{3(1-\nu)}\right]\theta \qquad [2.1;\ 2.2;\ 2.3]$$

where the loads and displacements are defined in Figure 2.1, and G and ν are the elastic shear modulus and Poisson's ratio. Gazetas *et al.* (1985) proposed an empirically derived equation for the vertical stiffness that is almost identical to Equation 2.1. An empirically derived equation for the horizontal stiffness was also proposed by Gazetas *et al.* (1987), which predicts stiffness values that are not less than 95% of those that are predicted by Equation 2.2 (for a Poisson's ratio of 0.2). Equations 2.1 and 2.3 are applicable to smooth footings and Equation 2.2 is applicable to rough footings. Spence (1968) derived the following solution for the case of a rough footing subjected to vertical loading,

$$V=\left[\frac{4GR\ln(3-4\nu)}{1-2\nu}\right]w \qquad [2.4]$$

Bell (1991) used the finite element method to investigate the effect of the depth of footing penetration and the variation of Poisson's ratio on the elastic stiffness of circular footings. He found that the vertical, horizontal, and rotational stiffness increased with the depth of penetration. Bell also detected coupling between the horizontal and rotational displacements of rough footings. In general, a horizontal load does not only displace the footing horizontally, but also results in rotation. Similarly, a pure moment load results in some horizontal displacement. The degree of coupling was found to increase with footing penetration, to be higher for lower Poisson's ratios, and to be dependent on the position of the load reference point. Bell expressed this mathematically as,

$$\begin{Bmatrix} V \\ H \\ M/R \end{Bmatrix} = GR\begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & K_4 \\ 0 & K_4 & K_3 \end{bmatrix}\begin{Bmatrix} w \\ u \\ R\theta \end{Bmatrix} \qquad [2.5]$$

Ngo-Tran (1996) investigated the effect of the cone angle of conical footings, footing penetration, and Poissons ratio using the finite element method. He derived values for the factors in Equation 2.5 and a selection of these are shown in Table 2.1. Shallow foundations on dense sand generally satisfy the surface condition. Thus the effect of footing penetration can be ignored.

2.2.2 Elastic parameters

There is great uncertainty in the choice of appropriate elastic shear moduli for soils, especially for sands. Soils are not isotropic or elastic materials as assumed in the derivation of Equations 2.1-2.3. Soils exhibit non-linear and stress path dependent behaviour. The elastic moduli of soils are also dependent on the density, the state of effective mean stress, and the magnitude of shear strain. Wroth and Houlsby (1985) found that the elastic shear modulus was generally proportional to the square root of the effective mean stress. Atkinson (1992) noted that the elastic shear modulus reduced with increasing shear strain. Hicher (1996) found that the unload-reload behaviour of granular materials ceased to be linear and reversible at strain amplitudes of 0.001% to 0.01%. The effect of the soil density on the elastic shear modulus is less dramatic. The findings of Iwasaki *et al.* (1978) and Lo Presti (1987) indicate that the elastic shear modulus of a dense soil is approximately twice that of a loose soil. An appropriate elastic shear modulus is best estimated by carrying out "laboratory tests in which the field state and stress path (including cyclic loading) is reproduced as closely as possible", as stated by Poulos (1988).

Barltrop and Adams (1991) suggested relationships for deriving the elastic shear moduli for very small strains. These values of the elastic shear moduli are compared with values derived from other relationships in the literature in Figure 2.2. This comparison is based on a stress level and strain magnitude that is representative of the model foundation problem, where $R = 0.075$ m and $V = 1000$ N. The moduli shown in Figure 2.2 are representative of axial strain magnitudes of 0.025% and 0.05%, where the representative strain of a footing is assumed to be equal to $w/2R$. The predicted small-strain elastic shear moduli range from 10 MPa to 20 MPa.

The influence of the effective mean stress on the elastic shear moduli measured in the model footing tests of Gottardi and Houlsby (1995), and calculated from the relationship suggested by Iwasaki *et al.* (1978), are compared in Figure 2.3. Although the moduli from the footing tests are greater than the values calculated using the relationship of Iwasaki *et al.*, they can still be modelled using a power relationship.

Elastic theory can be used to predict that $H/u = 0.86V/w$ and $M/(R^2\theta) = 0.71V/w$. These relationships are derived from Equation 2.5 and the values in Table 2.1 for $\nu = 0.2$ and $w/R = 0$.

The horizontal and rotational stiffness values from the model footing tests of Gottardi and Houlsby (1995) were approximately one fifth of the vertical stiffness values, and therefore less than implied from the theory. The vertical stiffness was about 17 kN/mm, and the horizontal and the moment stiffness were both about 3 kN/mm.

2.3 Bearing capacity of shallow foundations

Bearing capacity analyses can be used to determine a suitable foundation size for a specified load and soil type. Load:deformation behaviour cannot be estimated using bearing capacity theory but the calculated failure load can be used in the development of a load:deformation model. The bearing capacity of shallow foundations on sand subjected to vertical and combined loads will be discussed.

2.3.1 Central vertical loading

There are only a few specific bearing capacity problems for which analytical solutions have been derived. Prandtl (1921) solved the problem of a strip footing on a weightless, perfectly plastic material. The axisymmetric case was solved by Shield (1955) for a Tresca material and by Cox *et al.* (1961) for a weightless Mohr-Coulomb material. Cox (1962) extended the solution of Cox *et al.* (1961) to include Mohr-Coulomb material with self-weight, but results were not generated for a wide range of friction angle (ϕ) values.

Terzaghi (1943) used the results of Prandtl to develop the well known semi-empirical bearing capacity formula for shallow strip footings. He suggested that the effects of self-weight, cohesion, and surcharge could be separated and superimposed. Bearing capacity factors N_c, N_γ, and N_q, which were dependent on the effective friction angle and the undrained strength, were used to represent the effects of cohesion, self-weight, and surcharge respectively. Shape factors were introduced so circular and square footings could be analysed using the plane-strain bearing capacity formula. Terzaghi's formula is still widely used although various workers have suggested different bearing capacity factors (Meyerhof, 1963; Brinch Hansen, 1970).

Terzaghi's formulation was extended by Brinch Hansen (1970) and Vesic (1975) to account for load inclination, vertical load eccentricity, footing shape, depth of embedment, and the inclination of the base. Most workers have adopted the empirically derived shape factors of de Beer (1970). The bearing capacity formula and the factors suggested by some researchers are given in Appendix A. Vesic also includes a reduction factor to account for soil compressibility. Vesic's formula is recommended by the API (1993) guidelines for estimating the foundation capacity of offshore structures.

Recent research by the Cambridge Soil Mechanics Group (Lau, 1988; Shi, 1988; Tan, 1990) has extended the Cox solution for axisymmetric footings on Mohr-Coulomb material with weight by calculating bearing capacity factors for a range of friction angles. Bolton and Lau (1993) summarise this work and compare their analytical results with empirical results.

Values of N_γ derived from the bearing capacity formula and values suggested by Bolton and Lau (1993) are compared with some experimental results in Figure 2.4. The experimental results taken from the literature are of tests using rough circular footings. The factors derived from the Brinch Hansen (1970) formula are compared in Figure 2.4[a] and the rough axisymmetric factors proposed by Bolton and Lau (1993) are compared in Figure 2.4[b]. The factors derived from the theoretical methods are approximately double most of the experimental factors. The experimental factors of Montrasio and Nova (1997) and of Byrne and Houlsby (1999), however, are larger than the theoretical predictions. One would expect all of the theoretical predictions to be conservative, as is generally accepted. But in this comparison it is obvious that the experimental failure loads are generally less than the theoretical values. The various methods employed in the derivation of the friction angles are listed in Figure 2.4. The difference between the results of Montrasio and Nova (1997), of Byrne and Houlsby (1999), and of the other investigators cannot be explained by the effect of footing size on the bearing capacity factor. It should be noted that the friction angles for most of the tests are limited by the restriction on the dilatancy index implied in Bolton's (1986) method. If this restriction were ignored, the friction angles of Byrne's tests would be about two degrees greater than shown in Figure 2.4—or almost equal to the relevant theoretical values.

A number of assumptions are made in the solution of the bearing capacity problem. The foundation material is assumed to be rigid perfectly plastic. Only very dense sand develops the *general shear* mechanism (shown Figure 2.5) that is expected of a rigid perfectly plastic material.

Full mobilisation of the shear strength in the plastic zone is assumed. Sand is not a homogeneous material. The friction angle is dependent on the density and the stress level (de Beer, 1965a; Bolton, 1986; Hettler & Gudehus, 1988; Simonini, 1993).

The bearing capacity formulae are very sensitive to variations in the friction angle. This problem is exacerbated as there are a number of factors that contribute to the uncertainty in the friction angle. There is also confusion on whether to use plane strain or triaxial friction angles. Meyerhof (1963) recommended plane strain angles for strip footings and triaxial angles for circular footings. Brinch Hansen (1970), however, suggested that the plane strain angle be used as the theoretical bearing capacity factors were developed for the plane strain case. The plane-strain angle is approximately 10% larger than the triaxial angle. Ingra and Baecher (1983) and Basma (1994) performed statistical analyses to investigate the effect of uncertainty in the friction angle on the bearing capacity. The variation in the bearing capacity coefficient N_γ was found to be as much as 100% for a standard deviation of 2° in the friction angle.

The scale effects of foundation size are not taken into account in the general bearing capacity formulae. de Beer (1965b) investigated the scale effect, and Vesic (1975) found that an increase in the foundation size caused an increase in the relative compressibility of soils and a substantial decrease in N_γ. These effects are greatest for small surface footings ($B < 100$ mm) and care is needed when extrapolating the results from small model tests to large footings. The scaling of the foundation problem is discussed in Section 2.8.

Some workers have investigated the problem of smooth footings but the smooth case is usually only applicable to model tests, where machined footings are used. At the prototype level, footings can generally be classed as rough or semi-rough. Experiments have also shown that foundation roughness sometimes has practically no effect on bearing capacity (Vesic, 1975). The effect, however, does increase with increasing cone angle for conical footings (Tan, 1990).

Meyerhof (1961) found the bearing capacity of shallow cones to be very similar to that of flat footings. Investigations by Tan (1990) led to similar conclusions but differences were detected for the case of sharp cones on dense sand. Noble Denton and Associates (1987) suggested methods for analysing cones in the joint industry report based on work performed by the Cambridge Soil

Mechanics Group.

The finite element method has been used to predict bearing capacity loads, and in some cases the results have been in agreement with traditional bearing capacity predictions (Griffiths, 1982; Simonini, 1993).

2.3.2 Combined loading: Introduction

The horizontal and moment components of load in the offshore environment are substantially greater than the components on onshore structures. Offshore combined loads are caused by environmental forces, such as current, wave, and wind forces, acting on the platform structure. Footings have to withstand simultaneous vertical, horizontal, and moment loads.

The bearing capacity formulae of Meyerhof (1963), Brinch Hansen (1970), and Vesic (1975) can be used to calculate the degrading effect of horizontal and moment load on the vertical capacity. And thereby derive failure surfaces for the load combinations. Meyerhof (1953) suggested that the load in situations involving V, H, and M could be modelled as an equivalent inclined and eccentric point load acting on an effective foundation area. The inclined load tests conducted by Meyerhof were on strip footings. He suggested that the results could be applied to circular footings using a shape factor. Most researchers have retained the concept of an equivalent inclined and eccentric point load. Brinch Hansen (1970) and Vesic (1975), however, suggested the modified effective foundation area shown in Figure 2.6 to account for load eccentricity. The bearing capacity formulae and the factors of Meyerhof, Brinch Hansen, and Vesic are summarised in Appendix A. The API (1993) recommendations on the calculation of the effective dimensions are also included in Appendix A.

Roscoe and Schofield (1956) may have been the first researchers to use plasticity theory to analyse foundations subjected to combined loading. Butterfield and Ticof (1979) and Butterfield (1981) used a similar approach to develop interaction diagrams for describing the combined load problem of footings on granular materials and of foundations of offshore gravity platforms. A number of researchers have recently developed footing models based on plasticity theory, and calibrated with experimental data. Tan (1990) developed a model based on the results of centrifuge tests of circular footings embedded in loose sand. Nova and Montrasio (1991) analysed the results of 1-g tests of strip footings on the surface of loose sand. A plasticity approach was outlined by Gottardi *et al.* (1997) for circular footings on dense sand. And Martin (1994) developed a plasticity model for circular footings embedded in clay.

2.3.3 Combined loading: *V*:*H* failure surfaces

Combinations of vertical and horizontal load can be modelled as a single load, with an inclination angle α to the vertical, acting at the centre of the footing as shown in Figure 2.6. Meyerhof (1953) noted that the bearing capacity reduced to zero and that sliding failure occurred when the inclination angle approached the soil-footing friction angle. The inclination factors in the bearing capacity formulae were derived from a combination of experimental evidence and theoretical analyses of the strip footing problem.

The following relationship, which describes a V:H failure surface for a circular footing on sand, can be deduced from Meyerhof's formulae (listed in Table A.1),

$$\frac{V}{V_m} = \left[1 - \frac{\tan^{-1}(H/V)}{\phi}\right]^2 \qquad [2.6]$$

where V is the vertical load, V_m is the bearing capacity under a central load, H is the horizontal load, and ϕ is the friction angle of the sand. Corresponding relationships can be derived from Brinch Hansen's bearing capacity formula,

$$\frac{V}{V_m} = \frac{1}{3}\left[5 - 2\left(1 - 0.7\frac{H}{V}\right)^5\right]\left(1 - 0.7\frac{H}{V}\right)^5 \qquad [2.7]$$

and from Vesic's formula,

$$\frac{V}{V_m} = \left[1 - \frac{H}{V}\right]^{5/2} \qquad [2.8]$$

The normalised failure surfaces obtained using Equations 2.6-2.8 are compared in Figure 2.7. The Brinch Hansen method predicts a larger normalised capacity against bearing failure than the other two methods. The Meyerhof formula predicts a lower capacity than the methods of Brinch Hansen and Vesic.

Andersen (1972) reviewed experimental data and performed simple stability analyses before concluding that the inclination factor was not a function of the friction angle. Ingra and Baecher (1983) suggested relationships for the inclination factor based on statistical analysis of available data. They noted that the inclination factor was dependent on the footing shape and was less for strip footings than for square footings. The failure surfaces derived above must be applied with caution, as the bearing capacity formulae were not developed with combined loading in mind.

There is a great deal of experimental evidence (Georgiadis & Butterfield, 1988; Ricceri & Simonini, 1989; Nova & Montrasio, 1991; Gottardi & Butterfield, 1993) from plane-strain load controlled tests on sand that supports the approximately parabolic form of the V:H failure surface predicted by Equations 2.6-2.8 . Butterfield and Gottardi (1994) found that a simple parabola,

$$\frac{h}{h_o} = 4v(1 - v) \qquad [2.9]$$

where $h = H/V_m$, $v = V/V_m$ and $h_o = 0.13$ fit the experimental data from three different series of plane-strain tests of rough footings on dense sand. The peak horizontal load occurred at $V/V_m = 0.5$. Butterfield (1981) noted that the peak normalised horizontal load h_o of a smooth footing on dense sand was dependent on the embedment, and increased to 0.15 at $w/R = 1$. Equation 2.9 can also be used to fit parabolas in V_o-normalised space, where h and v are defined as H/V_o and V/V_o respectively.

Tan (1990) investigated the V:H failure surface of flat circular and conical footings using a geotechnical centrifuge apparatus. Tan's tests were performed on medium dense and loose sand at w/R ratios of 0.2 to 1.8. The V_m parameter is not applicable to situations where a general shear failure mechanism does not occur, such as situations involving loose sand and to post bearing capacity behaviour. Thus Tan normalised his results by the maximum vertical load previously experienced by the footing. This parameter will be referred to as V_{oi} in this thesis. Tan found that a V:H failure surface, for a particular ratio of w/R, could be traced in one test by subjecting a footing to a horizontal displacement whilst maintaining a constant vertical displacement. Tan termed this type of displacement controlled test a *sideswipe*. The typical path of a sideswipe test and of a constant-V test in w:V:H space are shown in Figure 2.8[a]. The details of the vertical load:deformation behaviour is also illustrated in Figure 2.8[b]. Plastic vertical displacement occurs even in a constant vertical displacement sideswipe as shown in Figure 2.8[b]. Tan used the predictions from an elastic-plastic model to illustrate how the plastic vertical displacement during a sideswipe, and consequent expansion of the failure surface, was negligible when the ratio of elastic to plastic stiffness was high.

Tan (1990) found that the peak of the *V*:*H* failure surface occurred at a normalised vertical load of approximately 0.4. The peak normalised horizontal load was found to be a function of embedment; h_o was equal to 0.15 for surface footings and to 0.24 for footings at an embedment of $w/R = 1$. Tan's peak normalised horizontal load for an embedded footing was higher than that recorded by Butterfiled (1981). It should be noted that Tan (1990) did not measure the moment load on his footings. Thus his *V*:*H* surfaces surfaces may in fact be *V*:*H*:*M* surfaces.

Martin (1994) performed a comprehensive series of displacement controlled tests in which the combined load failure surface of a spudcan footing on clay was defined. His sideswipe tests indicated that h_o was equal to 0.127 at a V/V_{oi} value of 0.464. As with Tan (1990), Martin used V_{oi} values to normalise his results since his sideswipe tests were performed on a soft material. Martin used the following modified parabola to fit his data,

$$\frac{h}{h_o} = \left[\frac{(\beta_1+\beta_2)^{\beta_1+\beta_2}}{(\beta_1)^{\beta_1}(\beta_2)^{\beta_2}}\right](v)^{\beta_1}(1-v)^{\beta_2} \qquad [2.10]$$

where $\beta_1 = 0.764$ and $\beta_2 = 0.882$. β_1 and β_2 are constants to adjust the curvature of the failure surface at low and high vertical load. Martin (1994) utilised the Cam-clay soil model to suggest that the expansion of a failure surface during a sideswipe test on normally consolidated clay was negligible when the ratio of elastic to plastic stiffness was about 50.

Gottardi and Houlsby (1995) investigated the pre-V_m combined load behaviour of a rough circular footing on the surface of dense sand. Their tests were also reported in Cassidy (1996) and Gottardi *et al.* (1997). The results of the sideswipe tests are shown normalised by V_o in Figure 2.9[a]. Equation 2.9 with $h_o = 0.12$ is also shown. The experimental values of V_o were not equal to the V_{oi} values because of the increase in the vertical penetration that occurred during the tests. This was caused by the flexibility of the experimental rig. The symbol V_{oi} is used to denote the maximum vertical load that the footing was previously subjected to and V_o is used to denote the extrapolated virgin-penetration load that is representative of the current state. These terms are illustrated in Figure 2.8[b]. Plastic straining during a sideswipe test results in expansion or contraction of the surface, depending on the slope of the virgin penetration curve. The sideswipe tests do not track along a single surface but traverse adjacent surfaces as illustrated in Figure 2.8[a], and described in Gottardi *et al.* (1997). The constant vertical penetration sideswipe ($\delta w = 0$) is labelled "E" and the penetrating sideswipe ($\delta w > 0$) is labelled "D".

Two sideswipe tests on overconsolidated ("OC") samples, which were unloaded from V_{oi} before commencing the sideswipes, and a constant-V test are also shown in Figure 2.9[a]. Constant-V tests are not usually plotted using V_o values, but are usually plotted using the constant V_{oi} value. Plotting against V_o produces a path similar to a sideswipe as the test tracks up and across adjacent surfaces in w:V:H space. This is illustrated by the path labelled "B" in Figure 2.8. The peak of the constant-V path in Figure 2.9[a] represents a point on the V_m:H failure surface. This point is represented by the intersection of path "B" and "C" in Figure 2.8[a].

The peak normalised horizontal load in Figure 2.9[a] seems to be dependent on the value of V_{oi}. The peak value is larger for smaller values of V_{oi}. The observed relation between h_o and V_{oi}/V_m is illustrated in Figure 2.10. The results of sideswipe tests conducted by Byrne and Houlsby (1999) on Baskarp Cyclone sand with a relative density of 95% are also included in Figure 2.10. The following equation is a fit to the data in Figure 2.10,

$$h_o = -0.045\ln\left(\frac{V_{oi}}{V_m}\right) + 0.11 \qquad [2.11]$$

Normalisation by V_{oi} may not be applicable to sideswipe tests initiated at vertical loads less than V_m. The data of Gottardi and Houlsby (1995) is shown normalised by V_m in Figure 2.9[b]. The V_m values used in the normalisation were estimated from vertical bearing capacity tests on similar samples. One would expect all the load paths from tests initiated at vertical loads less than V_m to be inside the V_m failure surface. This is the case as all the load paths converge on the parabola described by Equation 2.9 with h_o = 0.12. Even the tests from low V/V_m values and the overconsolidated tests converge. The shape of the sideswipe surfaces in Figure 2.9[b] is dependent on the ratio of V_{oi}/V_m at which the sideswipes were initiated. Cassidy (1996) used the failure surface defined by Equation 2.10 with $h_0 = 0.116$, $\beta_1 = 0.9$ and $\beta_2 = 0.99$ to fit the data of Gottardi and Houlsby (1995).

The variation in size may have been due to changes in the mechanism occurring beneath the footing. The development of slip lines under a general failure mechanism beneath a vertically loaded footing should be proportional to the value of V_{oi}/V_m. Thus a sideswipe from a high V_{oi}/V_m value will result in a different failure mechanism than a sideswipe from a low V_{oi}/V_m value. After bearing capacity failure and full slip line development, the mechanism should remain relatively unchanged and V_{oi} should work as a normalising parameter.

Sideswipe tests were initially employed by Tan (1990) and Martin (1994) to define the post-bearing failure surface of footings on soft work-hardening materials. Their results are of particular relevance to the design of jackup structures that undergo significant vertical penetration. Knowledge of the failure surface allows the factor of safety to be set by pre-loading the foundation. The design of foundations on stiff materials, in which a general shear mechanism is probable, cannot employ significant pre-loading because of the risk of work-softening at the vertical failure load. These foundations are usually designed without pre-loading, and it is usually assumed that the behaviour within the V_m failure surface is mainly elastic. The pre-V_m sideswipe tests of Gottardi and Houlsby (1995) indicate that significant yielding occurs within the V_m failure surface. The term *yield surface* is used in this thesis to describe the path of sideswipe tests performed within the V_m *failure surface*. The yield surfaces tracked by sideswipe tests can be used to differentiate between areas of predominantly elastic and plastic behaviour. They can also be used to gauge the effect of pre-loading within the failure surface.

Nova and Montrasio (1991), Montrasio and Nova (1997), and Tan (1990) found that sand density did not affect the shape of the failure surface. Tan also made a similar conclusion about the effects of footing roughness but Nova and Montrasio (1997) noted that the normalised horizontal capacity of smooth footings was 73% that of rough footings. Nova and Montrasio also investigated the effects of footing shape and embedment. They concluded that the shape of the V:H failure surface was not affected by the shape of the footing but increased in size with embedment.

2.3.4 Combined loading: *V*:*M* failure surfaces

The bearing capacity of footings subjected to vertical and moment loads has traditionally been calculated using the statically equivalent eccentric load and effective area concept that is shown in Figure 2.6. The calculation of effective area, length, and breadth is outlined in Appendix A. The following *V*:*M* interaction equation can be derived using the aforementioned procedure in combination with Meyerhof's (1963) method,

$$\frac{V}{V_m} = \frac{A'B'}{AB}\left[\frac{1+0.1\tan^2(45+\phi/2)(B'/L')}{1+0.1\tan^2(45+\phi/2)}\right] \qquad [2.12]$$

where the prime indicates an effective dimension. Moment load is incorporated into Equation 2.12 via the effective dimensions that are functions of the load eccentricity. A similar equation can be deduced using the methods of Brinch Hansen (1970) and Vesic (1975),

$$\frac{V}{V_m} = \frac{A'B'}{3AB}\left(5-2\frac{B'}{L'}\right) \qquad [2.13]$$

Normalised failure surfaces obtained using Equations 2.12-2.13 are compared in Figure 2.11. The curves are all approximately parabolic and thus similar to the *V*:*H* failure surfaces shown Figure 2.7. The most notable difference being the lower magnitude of the peak normalised moment load as compared to that in *V*:*H* space. The Brinch Hansen and Vesic method predict a larger normalised capacity against bearing failure than that predicted by the Meyerhof method. At low vertical load, the normalised failure surfaces converge. Ingra and Baecher (1983) suggest a relationship for an eccentricity factor based on statistical analysis of available data. Their method predicts a peak normalised moment load of 0.084 at V/V_m = 0.446. This is greater than that predicted by the methods shown in Figure 2.11. It is widely accepted that the equivalent area method is conservative.

Most of the researchers mentioned in the previous review of *V*:*H* surfaces also addressed the *V*:*M* problem. The main difference between *V*:*M* and *V*:*H* surfaces being the smaller size of the *V*:*M* surfaces, when normalised by V_m and using the *M*/2*R* convention. Equation 2.9 can be altered to represent the *V*:*M* surface,

$$\frac{m}{m_o} = 4v(1-v) \qquad [2.14]$$

where $m = M/2RV_m$ and m_o is the peak normalised moment load. Butterfield and Ticof (1979) found that a peak normalised moment load of 0.1, occurring at $V/V_m = 0.5$, was representative of model footing tests on sand. Recent research conducted by Butterfield and Gottardi (1994) and Montrasio and Nova (1997) on strip footings suggests a lower peak value of 0.0875. Montrasio and Nova also found that the size of the normalised failure surface increased with footing embedment and suggested that $m_o = 0.125$ at $w/R = 1$. Sand density and footing roughness were found not to affect the V:M failure surface.

The V:M sideswipe tests of Gottardi and Houlsby (1995) are shown normalised by V_o in Figure 2.12[a]. Equation 2.14 with $m_o = 0.09$ is also shown in Figure 2.12[a]. The changes in the vertical displacement that occurred during the sideswipe tests are accounted for in the values of V_o. A sideswipe test on an overconsolidated sample and a constant-V probe are also shown in Figure 2.12[a]. The peak normalised moment load in Figure 2.12[a] seems to be dependent on the value of V_{oi}. This is consistent with V:H behaviour. The following expression is used to fit the m_o:V_{oi}/V_m data shown in Figure 2.10,

$$m_o = -0.039\ln\left(\frac{V_{oi}}{V_m}\right) + 0.07 \qquad [2.15]$$

The moment sideswipe results of Byrne and Houlsby (1999) are included in the derivation of Equation 2.15.

The data of Gottardi and Houlsby (1995) is normalised by V_m in Figure 2.12[b]. All the load paths from tests initiated at vertical loads less than V_m are inside the parabola described by Equation 2.14 with $m_o = 0.09$. Cassidy (1996) used the failure surface defined by Equation 2.10 (with h and h_o replaced by m and m_o respectively) with $m_o = 0.086$, $\beta_1 = 0.9$ and $\beta_2 = 0.99$ to fit the data of Gottardi and Houlsby.

2.3.5 Combined loading: *V*:*H*:*M* failure surfaces

The V:H:M problem can be modelled using the traditional bearing capacity methods and an equivalent inclined load centred on a reduced footing area as shown in Figure 2.6 and described in

Appendix A. Meyerhof 's (1963) method may be used to derive the following V:H:M failure surface,

$$\frac{V}{V_m} = \frac{A'B'}{AB}\left[\frac{1+0.1\tan^2(45+\phi/2)(B'/L')}{1+0.1\tan^2(45+\phi/2)}\right]\left[1-\frac{1}{\phi}\tan^{-1}\left(\frac{H}{V}\right)\right]^2 \qquad [2.16]$$

Similarly, the theory of Brinch Hansen (1970) gives,

$$\frac{V}{V_m} = \frac{A'B'}{3AB}\left[5-2\left(1-0.7\frac{H}{V}\right)^5\frac{B'}{L'}\right]\left(1-0.7\frac{H}{V}\right)^5 \qquad [2.17]$$

and the theory of Vesic (1975) gives,

$$\frac{V}{V_m} = \frac{A'B'}{3AB}\left(5-2\frac{B'}{L'}\right)\left(1-\frac{H}{V}\right)^{m+1} \qquad [2.18]$$

where m is defined in Table A.1. It should be noted that Equations 2.16-2.18 are only valid in the 1st and 3rd quadrants of H:M space. The H:M surfaces in the 1st quandrant are shown in Figure 2.13. The degrading effect that a horizontal load has on the moment capacity at $V/V_m = 0.5$, and vice versa, is evident.

Butterfield and Ticof (1979) used the results of numerous combined load tests of strip footings on sand to derive parabolic shaped V:H and V:M surfaces. They also postulated a cigar shaped V:H:M surface as shown in Figure 2.14. The cigar shaped V:H:M failure surface has been verified in $(+H)$:$(+M)$ space by Georgiadis and Butterfield (1988) and Nova and Montrasio (1991) for strip footings on sand, and by Dean *et al.* (1992a) for conical footings on sand. Butterfield and Gottardi (1994) and Gottardi *et. al* (1997) have since verified the applicability of a cigar shaped failure surface to both positive and negative combinations of horizontal and moment load, for strip and circular footings respectively. Both of these investigations indicated an elliptical H:M surface rotated by about 14°. Gottardi *et al.* used the following expression to fit their V:H:M data,

$$\left(\frac{h}{h_o}\right)^2 + \left(\frac{m}{m_o}\right)^2 - 2a\frac{hm}{h_o m_o} - [4v(1-v)]^2 = 0 \qquad [2.19]$$

where a defines the rotation of the failure surface in H:M space. The latter two surfaces mentioned above are compared in Figure 2.15. Butterfield and Gottardi assumed that anti-clockwise moments were positive and Gottardi *et al.* assumed the opposite. The effect of the sign convention on the rotation of the H:M ellipse is illustrated in Figure 2.16. The surface of Butterfield and Gottardi shown in Figure 2.15 is plotted assuming clockwise moment positive. These failure surfaces are very similar considering that one represents a strip footing and the other a circular footing. The rotation of the ellipse in H:M space is dependent on the sign convention used to describe the combined loads and also on the location of the point to which these loads are referenced. The above is also true for displacements, and the effect of the load/displacement reference point is illustrated in Figure 2.17. At some reference point R, the ellipse is symmetrical. At points above R the ellipse is rotated anti-clockwise, and at points below R the rotation is clockwise. Butterfield *et al.* (1996) discussed the different standardised sign conventions for footings subjected to combined loads.

2.4 Modelling of foundation behaviour

2.4.1 Introduction

The load:deformation behaviour and stability of an offshore structure is dependent on the interaction between the structure and the soil on which it is founded. Understanding the foundation:soil interaction problem and developing a means (or a model) for predicting load:deformation response is essential. A complete foundation:soil interaction model should incorporate all aspects of a foundation's behaviour. This is not usually possible because of the complexity of the loads and resulting foundation response. The important aspects of foundation behaviour and some foundation models in the literature will be discussed.

Soils do not behave in a purely elastic or plastic way, or in a linear way. For a foundation:soil interaction model (as for a soil model of triaxial tests) to be accurate, it would have to capture non-linear elastic, plastic work-hardening, and plastic work-softening load:deformation response. The boundary conditions of soil beneath a foundation and in a triaxial test are broadly similar. The elements of a soil model derived from triaxial tests should also be applicable to foundation behaviour. The basic elements of foundation behaviour which have to be modelled are described in the framework of an elastic-plastic soil model (Wood, 1990) as follows:

— Vertical load:deformation response links the absolute magnitude of vertical plastic deformation to the changing size of the failure surface. A *hardening rule* can be used to model this link.

— There is a boundary in *V:H:M* load space within which behaviour can be described as elastic or non-linear elastic. A description of this *failure surface* is required.

— The vertical, horizontal, and rotational unload and reload foundation response within the failure surface can usually be modelled assuming *elastic behaviour*.

— The relative magnitudes of the various components of plastic deformation (δw, δu, and $\delta\theta$) can be describe by a *plastic potential*. The absolute magnitudes can be deduced from the plastic vertical displacement δw; this value is given by the hardening rule.

The elastic behaviour of foundations and the shape of failure surfaces were discussed in Sections 2.2 and 2.3.

2.4.2 Vertical and combined loading

The vertical load:deformation behaviour of footings on sand is non-linear even at small displacements. Butterfield (1981) suggested that a modified exponential relationship deduced from plate bearing tests be used to model this behaviour. The three variables in his relationship were the initial slope of the load:deformation curve k, the bearing capacity load V_m, and the slope of the curve after bearing capacity failure. Butterfield did not differentiate between plastic and elastic displacement, as the influence of elastic displacement was negligible. Butterfield adopted the technique commonly used in plasticity theory of plotting loads and their associated displacement vectors on the same diagram, so as to model the displacements associated with combined loading. It is likely that Roscoe and Schofield (1956) were the first to use this procedure to analyse the foundation problem. The results of strip footing tests on dense sand (reported by Butterfield, 1981) showed that displacement vectors along a radial load path in *V:H* space were generally parallel and that the magnitude of the vectors increased as the load path approached the parabolic failure surface. An elliptic, and hence non-associated, plastic potential was found to fit the displacements associated with radial load increments close to the failure surface. The displacement vectors were also found to be approximately the same irrespective of the load path taken to reach the failure surface. Butterfield made an analogy to the plastic potentials used in plasticity theory.

Tan (1990) used the bearing capacity theory of Terzaghi to predict vertical load:deformation and develop a hardening rule for circular footing tests on loose sand. He assumed that the depth of overburden was equivalent to the penetration of the footing. This was coupled with the elastic solutions of Poulos and Davis (1974). Tan's model predicts that the footing does not penetrate the soil before reaching the bearing capacity load V_m. He was only trying to model post-V_m work-hardening behaviour of footings on loose sand and his approach worked well. A non-associated flow rule was required to model vertical and horizontal displacement. He derived a plastic potential, based on the Cam-clay constitutive soil model, that predictes a change from work-softening to work-hardening behaviour at $V/V_o = 0.135$ as shown in Figure 2.18[a]. The point of transition was called a *parallel point*. His experiments indicated that there was indeed a load state at which horizontal displacement could take place with no change in vertical displacement or vertical and horizontal load. Plastic displacement on the left side of the parallel point resulted in heave whilst on the right side settlement resulted. The term critical state is used to refer to such states in this thesis. Tan (1990) combined a failure surface, plastic potential, theoretical definitions of elastic behaviour, and a hardening rule to produce a work-hardening plasticity model. His model gave good retrospective predictions of both load and displacement controlled tests.

Nova and Montrasio (1991) used a similar hardening rule to that suggested by Butterfield (1981) in their rigid plastic work-hardening model for strip footings on loose sand. Only three tests were required to calibrate the model: one under central vertical loading, one with $M = 0$ and one with $H = 0$. Experimental results, such as an increase in vertical displacement with decreasing vertical load, indicated that a non-associated flow rule was required. Also, it was noted that an associated flow rule would over predict the foundation heave linked to sliding at low V/V_m. Nova and Montrasio adjusted their failure surface to create a non-associated plastic potential by incorporating constants to alter the horizontal and moment load. The failure surface and plastic potential incorporated a history parameter that was not only a function of vertical displacement but of horizontal and rotational displacement as well. This indicates that a hardening rule may be a function of the three components of displacement and not just of vertical displacement. The model accurately predicted combined load tests that included load reversals.

More recently, Nova and Montrasio (1997) investigated the effects of foundation shape on the behaviour of rectangular footings on dense sand and noted that the hardening rule was more representative of square footing behaviour. This was for a constant value of the non-dimensional ratio $V_m/2Rk$. It was suggested that $V_m/2Rk$ was a function of sand density only. Normalised experimental load:deformation curves were also found to be very similar.

Martin (1994) developed an incremental work-hardening plasticity model from the results of spudcan footing tests on clay. Theoretical stiffness factors were used to define elastic behaviour. A theoretical vertical load:deformation relationship was derived using the Method of Characteristics to solve the bearing capacity problem. An elliptical paraboloid shape was used to represent the failure surface. Martin noted that an associated flow rule would over predict vertical footing displacements occurring during failure in the V:H and V:M planes. He used a non-associated flow rule in which only the vertical plastic displacement was adjusted so as to be different than that predicted by normality. His experimental evidence supported the applicability of associated flow in the H:M plane.

Gottardi and Butterfield (1995) noted that the displacement increments of a strip footing on sand in the H:M plane were non-associated. Their w:u and w:θ displacement increments were also non-associated but differ in form from Tan's non-associated flow rule. For example, the displacement increments from a constant-V test remained parallel although the relative position along the failure surfaces changed. On the other hand, the displacement increments from a radial load path test did not remain parallel although the relative position along the failure surfaces did not change. The difference in behaviour mentioned above may be due to the fact that Tan's tests were post-V_m and the tests of Gottardi and Butterfield were pre-V_m.

Cassidy (1996) and Gottardi *et al.* (1997) used the following empirical expression relating vertical load to the plastic component of displacement to fit data from circular footing tests on dense sand,

$$V_0 = \frac{k_p w_p}{1 + \left(\frac{k_p w_{pm}}{V_m} - 2 \right) \left(\frac{w_p}{w_{pm}} \right) + \left(\frac{w_p}{w_{pm}} \right)^2} \qquad [2.20]$$

where k_p is the initial plastic stiffness and w_{pm} is the plastic displacement at the bearing capacity load V_m. The total displacement w is equal to the sum of the plastic and elastic components of displacement (w_p and w_e respectively). Equation 2.20 incorporates one more parameter, w_{pm}, than the modified exponential relationship of Butterfield (1981). This makes it more representative of footing response because it can be used to model work-softening behaviour that is common beyond V_m.

Some of the experimental data shown in Figure 2.19[a] suggests that the vertical bearing capacity V_m is inversely proportional to w_{pm}. Also, the k_p:V_m data shown in Figure 2.19[b] supports the evidence of Montrasio and Nova (1997) that V_m is proportional to k_p. The linear relationship between the normalised initial penetration stiffness and the relative density that was suggested by Montrasio and Nova is shown in Figure 2.20. Although the data of de Beer (1970) supports the evidence of Montrasio and Nova, the other data were inconclusive because of the small ranges of the relative density values. The considerable variation between the different tests shown in Figure 2.20 does not seem to be dependent on the footing size.

Cassidy (1996) developed an incremental work-hardening plasticity model similar to Martin's (1994) model. Equation 2.20 was used as the hardening rule and a more sophisticated form of Equation 2.19 was used to represent the failure surface. The normalised V:H failure surface and plastic potentials are shown in Figure 2.18[b]. A transition from work-softening to work-hardening behaviour occurs at V/V_m = 0.265. Cassidy used this model to perform retrospective predictions of the footing tests conducted by Gottardi and Houlsby (1995) on dense sand.

The V:H experimental evidence of Gottardi and Houlsby does not support the existence of a critical state point at V/V_m = 0.265. If a V:H critical state point did exist, then its value would be less than V/V_m = 0.05. This is indicated by the failure surfaces shown in Figure 2.9[b] and by the u:H/V_m response of the sideswipe tests shown in Figure 2.21[a]. The horizontal load in the sideswipe with V_{oi} = 1800 N is still reducing linearly at the end of the test. The load:deformation paths of the other sideswipes shown in Figure 2.21[a] are of a similar shape to that of the test with V_{oi} = 1800 N and probably would have reduced in a similar fashion as well.

On the other hand, the V:M sideswipe tests shown in Figure 2.12[b] converge at $V/V_m \cong 0.25$. This convergence was is evident in load:deformation space as shown in Figure 2.21[b]. The moment load of the sideswipe tests tend to a $M/2RV_m$ value of 0.06 at large rotational displacements. These results suggest that there is a critical state point in V:M space.

Intuitively, one would not expect a critical state point in V:H space, since there can be no horizontal resistance when sliding from zero vertical load at a constant vertical displacement. In V:M space, however, rotation from zero vertical load at a constant vertical displacement would result in moment resistance. The above findings indicate that different plastic potentials may be required for V:H and V:M behaviour.

2.5 Partially-drained behaviour

2.5.1 Introduction

The loads that an offshore foundation are subjected to can be categorised as static, transient, or cyclic. The terms static and transient refer to situations where the load changes by a single increment. A static load is applied in an instant and a transient load is applied over a finite time. During the application of a static load one assumes that the change in effective stress is equal to the change in total stress (drained condition) or is equal to the difference between the total stress and the pore pressure (undrained condition). The load is said to be transient (partially-drained condition) if the effective stress changes during the period that the load is applied as illustrated in Figure 2.22. Cyclic loading is similar to transient loading but there is a continuous change of load by alternating positive and negative increments. Foundation response is different for each type of load and becomes increasingly complex as the load type changes from static, to transient, through to cyclic.

The sand beneath a shallow foundation of an offshore platform is likely to behave in a partially-drained manner mainly because of the extreme wave loading experienced during storm conditions. Wave loading, which typically has cyclic periods in the range of 10-20 seconds, results in simultaneous cyclic vertical, horizontal, and moment foundation loads. The extent to which the foundation soil behaves in a partially-drained manner is dependent on the size of the foundation, the loading rate, and the drainage and deformation characteristics of the soil. Under partially-drained conditions, the effective stresses of the foundation soil may decrease or increase.

The partially-drained behaviour of submerged shallow foundations has been investigated by a number of researchers. Vesic *et al.* (1965) investigated the effect of loading rate on the drained and partially-drained bearing capacity of model sized footings on dense sand. Research into the transient loading of foundations has been mainly focused on the behaviour of suction caissons. In sand, suction caissons rely on the partially-drained capacity to generate uplift resistance. Goodman *et al.* (1961) investigated the feasibility of vacuum anchorage in a range of soils. Wang *et al.* (1977) studied the effect of an applied suction on the breakout capacity of caissons and Byrne and Finn (1978) investigated the effect of pullout rate on suction caissons in clay. In terms of the more conventional types of offshore foundations, McNeilan (1985) noted partially-drained effects during the installation of spudcans in silt, and Finnie (1993) investigated the effect of loading rate on the behaviour of spudcans on carbonate sands. Dean (1991) suggested approximate methods for estimating the pore pressures on spudcans. Recently, there has been considerable interest in the use of suction caissons for fixed offshore platforms founded on dense sand (Aas & Andersen, 1992; Bye *et al.*, 1995; Erbrich, 1996).

2.5.2 Evidence of partially-drained behaviour with sand

Vesic *et al.* (1965) performed dynamic loading tests on submerged sand and an apparent cohesion was noticed for rapid loading. The fastest displacement rates shown in Figure 2.23[a] result in bearing capacities twice that recorded at the drained displacement rate. The API (1993) guidelines state that an undrained bearing capacity analysis is required in cases where the rate of loading is such that no drainage occurs. And in such cases, the friction angle should be assumed to be zero and an appropriate undrained strength parameter be used. Janbu (1985) notes that undrained tests on sand indicate an apparent cohesion a and a frictional resistance. Janbu recommends using undrained a-ϕ soil data to calculate effective stress bearing capacities using limit equilibrium analyses. A dilatancy parameter derived from triaxial tests was also used to account for the effects of shear deformation on the excess pore pressures along the failure surface.

The potential for the dilatancy of sand to affect the excess pore pressures will not be fully realised where loading rates are such that the conditions are not undrained. Drainage will result in bulk deformation, and dilatancy will have a reduced effect on the excess pore pressures. The results of partially-drained element tests on sand could not be found in the literature. Intuitively, one would expect the partially-drained bearing capacity to be bounded by the drained and undrained bearing capacity. And for it to be inversely proportional to the degree of drainage (proportional to the rate of loading). This proportionality is evident in Figure 2.23[a]. The undrained bearing capacity can be less than the drained capacity in collapsible soils, such as in loose and medium dense sand. The effect of density on the partially-drained bearing capacity is illustrated in Figure 2.23[a]. A test in which the relative density is nine percentage points less than the average, has a bearing capacity a factor of about 2.5 less than the tests on the denser sand.

Kolk and Campbell (1997) discussed the findings of Finnie (1993) relating to the partially-drained bearing capacity of spudcan foundations on carbonate sands and silts. Finnie defined the displacement rate in terms of a non-dimensional relative penetration velocity,

$$v_n = \frac{v\,D}{c_v} \qquad [2.21]$$

where v was the spudcan penetration velocity, D the diameter, and c_v the coefficient of consolidation. The bearing stress under undrained conditions, which was normalised with respect to the initial vertical effective stress at unit depth, was about half that under drained conditions. The response was found to be fully drained when $v_n < 0.01$ and fully undrained when $v_n > 10$.

The partially-drained model footing tests of Vesic *et al.* (1965) shown in Figure 2.23[b] illustrate the effect of the penetration rate on the ultimate settlement. As the degree of drainage reduces, the load:deformation behaviour becomes more ductile and the ultimate settlement dramatically increases. The maximum settlement was taken to be representative of the ultimate settlement in the fast partially-drained tests as the peak value was not reached. The initial slope of the load:deformation curves of the fastest partially-drained tests was also slightly less than the slope of the drained tests.

The pull-out capacity (tension capacity) of suction caissons is dependent on the rate of displacement. The effect of the pull-out rate on the stress:strain behaviour of laboratory and offshore tests of suction caissons is shown in Figure 2.24. The values on the vertical scales are not shown for reasons of confidentiality. The pull-out velocity and total stress are shown in non-dimensional form. Only the Oxford and NGI tests are at equivalent stress and time scales. From Figure 2.24[a] it is evident that there was a general trend of increasing total stress with increasing velocity. The values from each test were measured at a constant value of strain. The dotted line is drawn between the Sleipner T and B tests because these were performed at different sites with slightly different drainage characteristics. Actual stress:strain curves from some of the Oxford and NTH/SINTEF tests are shown in Figure 2.24[b] and [c] respectively. The Oxford and NTH/SINTEFF tests were not at equivalent scales; the size of the NTH/SINTEFF caisson was a factor of about 1.5 greater than the effective size of the Oxford caisson. This difference in size may have explained the larger stiffness and load response observed in the Oxford tests. But the general trend of increasing stiffness with velocity was clearly evident in both sets of tests. Note that the fastest of the Oxford tests shown in Figure 2.24[b] might have resulted in cavitation of the pore fluid, as the pore pressure at the base of the footing was very low.

2.5.3 Factors affecting effective stress

Terzaghi's (1943) Principle of Effective Stress states that the effective stress is related to total stress as follows,

$$\sigma' = \sigma - u \qquad [2.22]$$

where σ' is the effective stress, σ is the total stress, and u is the pore fluid pressure. The application of this principle to the loading of a foundation is illustrated in Figure 2.22. The loading of sand results in particle rearrangement and a change in the volume of inter-particle voids. The change in volume may induce excess pore fluid pressures if the rate of volume change is greater than the rate of fluid flow between the voids. The induced excess pore pressures cause changes in effective stress and result in partially-drained behaviour.

The characteristics of the volume change arising from a load increment are dependent on the relationship between the shear and volumetric behaviour of the sand. It is dependent on the *dilatancy* when sand tries to increase in volume during shearing, and on the *contractancy* when sand tries to reduce in volume during shearing. Sand will tend to dilate, thereby inducing negative excess pore pressures under rapid shearing, when the ratio of shear stress to normal stress is high. And sand will tend to contact, thereby inducing positive excess pore pressures under rapid shearing, when the ratio is low. The density of the sand also affects this tendency. Dense sand is likely to dilate and loose sand is likely to contract when sheared. The rate of pore fluid flow is controlled by the permeability of the sand, by the size of the foundation, and by the rate of loading. The lower the permeability, the longer the drainage length, and the faster the loading rate, the lower is the rate of pore fluid flow. And, therefore, the greater is the tendency for excess pore pressures to develop.

Yamamuro and Lade (1998) noted that silty sands could exhibit *reverse* dilatant behaviour. Their specimens of silty sand became more dilatant with increasing confining stress under undrained conditions.

In dense sands, the coupling of dilatancy with a low rate of pore fluid flow can induce enormous effective stresses. The effective stress has an upper bound that is dependent on the cavitation pressure of the pore fluid, which in turn is dependent on the ambient water pressure at foundation level. Cavitation of the pore fluid in triaxial tests was investigated by McManus and Davis (1996), and Os and Van Leussen (1987) detailed the effects of cavitation on the response of a submerged plough.

2.5.4 Calculation of effective stress

As outlined previously, the effective stress state of a soil is dependent on the interaction between the soil skeleton and the pore fluid. The fundamental principles of mechanics, which are the conditions of equilibrium and compatibility, can be applied to analyse this situation. Equilibrium ensures that the forces (and stresses) acting on a body do not result in accelerations, and compatibility ensures that the displacements (and strains) in a body do not result in discontinuities. The relationship between stress and strain is governed by the characteristics of the material. The governing equations (in two dimensions) for equilibrium, compatibility and material behaviour for a saturated soil mass are outlined below.

Equilibrium

Consider a body in which the stress components vary with x and z, and on which there is a body force acting in the z direction. Equilibrium is satisfied if,

$$\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} - i_x \gamma_w = 0 \qquad [2.23a]$$

$$\frac{\partial \sigma'_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} - \gamma' - i_z \gamma_w = 0 \qquad [2.23b]$$

where τ is shear stress and i is hydraulic gradient. Note that $i_x = -(\partial h/\partial x)$ where h is total head which is assumed increasing in directions x and z (x direction positive rightwards and z direction positive downwards).

Compatibility

If the components of displacement in the x and z directions are denoted by u and w respectively, then the normal strains are given by,

$$\varepsilon_x = -\frac{\partial u}{\partial x} \qquad \varepsilon_z = -\frac{\partial w}{\partial z}$$

and the shear strains by,

$$\gamma_{zx} = -\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

The above strains must be compatible with each other if the body is to remain continuous. This leads to the following relationship,

$$\frac{\partial^2 \varepsilon_x}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial x^2} - \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} = 0 \qquad [2.24]$$

Compatibility also requires continuity of fluid flow through the body. The following relationship applies for a body in which there is a net outflow of an incompressible fluid having velocities v_x and v_z,

$$\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}\right) \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = -\frac{\mathrm{d}V}{\mathrm{d}t} \qquad [2.25]$$

where dV/dt is the change in volume per unit time.

Material characteristics

A constitutive relation for the soil skeleton is required. This can be defined incrementally in terms of effective stress as follows,

$$d\sigma' = f(d\varepsilon) \qquad [2.26]$$

A relationship for fluid flow is also required. The following relationships for an isotropic soil mass are applicable if Darcy's law is assumed to be valid,

$$v_x = k\,i_x = -k\frac{\partial h}{\partial x} \qquad v_z = k\,i_z = -k\frac{\partial h}{\partial z} \qquad [2.27a; 2.27b]$$

where k is permeability and $h = z + u/\gamma_w$, with u representing the pore pressure in this case.

The equation for continuity of fluid flow can now be written as,

$$\frac{k}{\gamma_w}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{1}{1+e}\frac{\partial e}{\partial t} \qquad [2.28]$$

Equation 2.28 links the pore pressure to the volumetric changes (or strain). But the pore pressure is also linked to the strain by Equation 2.26. This closed loop (strain→pore pressure→effective stress→strain) or coupled problem has only been solved for a few limited cases. Biot (1941) solved the equations for a saturated linear-elastic porous solid. Many practitioners have used numerical techniques to model this type of behaviour (Small *et al.*, 1976; Hicks & Smith, 1988). But numerical solutions require external inputs and for the loop to be broken.

2.6 Stress distribution on footings

The pore pressure distribution on the base of a footing was required for the calculation of the pore pressure load U, which was used to derive the effective load V'. Terzaghi's theory of effective stress states that the effective load is equal to the total load minus the pore pressure load. To derive U, the function describing the pore pressure distribution has to be integrated over the area of the footing. The shape of the pore pressure distribution on a footing is dependent on the rate of loading and the drainage characteristics of the soil.

Under undrained conditions, the displacements and stresses in a consolidating material are the same as those in an elastic material with $\nu = 0.5$ (Davis & Poulos, 1963; Chiarella & Booker, 1975). At $t \cong 0$, the pore pressure distribution beneath a rigid circular footing on an isotropic elastic semi-infinite mass would be equal to the contact stress distribution defined by Bossinesq (1885), and later described by Poulos and Davis (1974). The undrained pore pressure distribution is shown in Figure 2.25[a] and the elastic contact stress distribution is defined as follows,

$$q = \frac{V}{2\pi R\left(R^2 - r^2\right)^{1/2}} \qquad [2.29]$$

where V is the applied load, R is the radius, and r is the distance from the centre. Distributions calculated using the above equation should also be valid for footings on a finite layer when the thickness of the layer is greater than twice the radius of the footing (when $h/R \geq 2$).

Contact stress distributions similar to the Bossinesq solution have been measured on footings founded on clay. Faber (1933) measured a Bossinesq shaped contact stress distribution on a rigid circular footing that was resting on stiff London clay. Conclusive experimental data on the undrained and the partially-drained pore pressure distribution beneath footings on sand could not be found in the literature, but one would expect the undrained distribution to be similar to that described by Equation 2.29.

The shape as well as the magnitude of the pore pressure distribution beneath a consolidating footing is a function time. The distribution gradually changes from the initial undrained shape to a uniform distribution, and finally to a parabolic distribution as drainage progresses. Yue *et al.* (1994) found that the pore pressure distribution was approximately parabolic in shape after a non-dimensional time factor T_v value of about 0.1. They conducted numerical analyses of a poro-elastic semi-infinte mass subjected to a uniform stress. The parabolic distribution is shown in Figure 2.25[b].

In the model tests of this programme, where the pore pressure was only measured at the centre of the footing, the following expression was used to estimate the pore pressure load,

$$U = C_p \pi R^2 u_c \qquad [2.30]$$

where u_c is the pore pressure measured at the centre of the footing and C_p is a parameter that described the pore pressure distribution. C_p is defined as a function of time, and is equal to two for an undrained distribution and to 0.5 for a parabolic shaped pore pressure distribution. The following relationship between C_p and the non-dimensional time factor T_v is proposed,

$$\begin{array}{ll} C_p = 2 & T_v \cong 0 \\ C_p = 1.51e^{-50T} + 0.49 & 0 < T_v < 0.1 \\ C_p = 0.5 & T_v \geq 0.1 \end{array} \qquad [2.31]$$

The constants in Equation 2.31 for $0 < T_v < 0.1$ were chosen assuming that the exponential decay shown in Figure 2.26 was valid between $C_p = 2$ and $C_p = 0.5$. Equations 2.30 and 2.31 indicate that the rate of decay of U is greater than that of u_c. Therefore a more realistic assessment of the effective load V' could be made by assuming that the pore pressure distribution on a consolidating footing is a function of time. Assuming a uniform distribution of pore pressure for all stages of the consolidation process, as is commonly done, underestimates the rate of increase of the effective load.

In the discussions above, it was assumed that a load increment was applied in zero time. Fully undrained loading of sand is only likely in situations involving large footings and very rapid rates of loading. If the load increment is partially-drained, the initial C_p value would be less than two. An elastic base was also assumed, but penetration of a footing on sand may result in local volumetric expansion as well as contraction. Davis and Poulos (1972) found that non-elastic effects on the rate of consolidation were negligible for practical engineering purposes. Thus it was assumed that similar effects on the pore pressure distribution would be negligible.

In terms of drained behaviour, there exists a wealth of information on the problem of a footing interacting with a sand base. Selvadurai (1979) and Kerr (1989) reported the findings of numerous investigations that indicated a tendency for a Bossinesq shaped distribution beneath footings where there had been little plastic deformation and stress re-distribution. They also stated that as the sand beneath the edge of a footing deformed, the contact stress distribution tended to be parabolic in shape. Therefore the contact stress distribution beneath a footing that is undergoing plastic penetration should be similar to that described by the following equation,

$$q = \frac{2V}{\pi R^2}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad [2.32]$$

A similar parameter to C_p, which was introduced to describe the shape of the pore pressure distribution on a footing, could be used to describe the total stress distribution on the footing. And hence account for the degree of elasticity. This parameter was labelled C_q and defined as follows,

$$q = \frac{V}{C_q \pi R^2} \qquad [2.33]$$

where q is the total stress at the centre of the footing, V is the applied load, and R is the radius of the footing. One would expect C_q to take similar values as C_p for similarly shaped stress distributions. Therefore $C_q = 2$ for an elastic stress distribution (shown in Figure 2.25[a]) and $C_q = 0.5$ for a plastic stress distribution (shown in Figure 2.25[b]). For an elastic material, C_q would be constant throughout the consolidation process but C_p would decrease with dissipation of the pore pressure.

2.7 Consolidation behaviour

2.7.1 Introduction

The one-dimensional consolidation equation is defined as,

$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \qquad [2.34]$$

and can be derived by combining the appropriate forms of the continuity and stress:strain equation (see Equations 2.26 and 2.28 in Section 2.5.4). Terzaghi (1943) developed his well known theory for one-dimensional consolidation by solving Equation 2.34 and expressing the solution in terms of the variation of excess pore pressure with depth and time. The one-dimensional coefficient of consolidation c_v in Equation 2.34 is expressed as follows,

$$c_v = \frac{k}{\gamma_w m_v} = \frac{2kG}{\gamma_w} \frac{(1-\nu)}{(1-2\nu)} \qquad [2.35]$$

where k is the permeability and m_v, the coefficient of compressibility, is equal to the inverse of the constrained modulus M'. Equation 2.34 is a form of diffusion equation that is applicable to many physical problems, and has been solved for a variety of initial and boundary conditions. Solutions, be they Taylor's (1948) analytical solution or the method of parabolic isochrones, are usually expressed in non-dimensional terms. In non-dimensional terms Equation 2.34 becomes (Scott, 1980),

$$\frac{\partial^2 U_v}{\partial Z^2} = \frac{\partial U_v}{\partial T_v} \qquad [2.36]$$

where U_v is equal $(u_i—u)/(u_i—u_f)$ and is called the degree of consolidation (or degree of pore pressure dissipation; subscripts i and f refer to the initial and final states). The non-dimensional depth Z is equal to the vertical dimension divided by the length of the maximum drainage path (z/d), and the time factor T_v is defined as,

$$T_v = \frac{c_v t}{d^2} \qquad [2.37]$$

In practical problems it is the average degree of consolidation $\overline{U}_p$ over the depth of the layer that is of interest and the consolidation process is usually expressed in terms of $\overline{U}_p$ and T_v. The degree of consolidation settlement U_s is also equal to $\overline{U}_p$ in the one-dimensional case. This is because the total stress, as well as the variation between the volume and the effective stress, is assumed constant.

Terzaghi's (1943) theory of one-dimensional consolidation is applicable to oedometer tests and to foundation problems where the depth of the layer is small compared with the width of the foundation. For circular footings on deep layers, where the thickness of the layer is greater than the radius of the footing, three-dimensional effects result in a considerable increase in the rate of consolidation as pore pressures can dissipate horizontally as well as vertically.

Davis and Poulos (1972) discussed the rate of settlement under three-dimensional conditions. They mentioned that theoretical solutions of the Biot (1941) equations for the three-dimensional problem have been only formulated for a few foundation scenarios. Such as for uniformly loaded footings on semi-infinite and finite masses with relatively simple boundary conditions. Nevertheless, Davis and Poulos suggest that solutions for more complex foundation scenarios obtained from simple diffusion theory should be sufficiently accurate for practical use.

2.7.2 Three-dimensional consolidation settlements

The settlement of a footing on a saturated porous elastic medium results from distortion of the medium as a consequence of shear stresses imposed by the applied load, and from volume changes due to the dissipation of excess pore pressures (consolidation). The settlement of a footing on a consolidating soil is usually calculated assuming that the soil behaves as an elastic medium. The total or drained settlement w_d is given by,

$$w_d = w_u + w_c \qquad [2.38]$$

where w_u is the immediate or undrained settlement and w_c is the final consolidation settlement. The undrained as well as the drained settlement of a rigid circular footing on a finite layer can be calculated using the following equation suggested by Poulos and Davis (1974),

$$w = \frac{V}{2\pi G R(1+\nu)} I_p \qquad [2.39]$$

where I_p is an influence factor that is dependent on the Poisson's ratio and the ratio h/R. For $h/R \cong 3$, the ratio relevant to this enquiry, the influence factor is approximately equal to 1.16 for drained loading ($\nu = 0.2$) and equal to 0.80 for undrained loading ($\nu = 0.5$). These values indicate that the undrained settlement is less than the drained settlement by a factor of about 0.55 (note that $G' = G_u$ for an elastic material in which shear and volumetric effects are not coupled). The final consolidation settlement is taken as the difference between the drained and the undrained settlements. The settlement during the consolidation process at any time t after the application of the load increment is given by,

$$w_t = w_u + U_s\, w_c \qquad [2.40]$$

where U_s is the degree of consolidation settlement for the appropriate geometrical conditions.

In the one-dimensional or oedometer case there is no undrained settlement and the influence factor I_p is approximately equal to 1.25 for $\nu = 0$ and $h/R \cong 3$. Davis and Poulos (1968) stated that a Poisson's ratio of zero should be used to derive the influence factor but that $\nu = \nu'$ should be used in the following one-dimensional settlement equation,

$$w_{oedo} = \frac{V}{\pi M' R} I_p \qquad [2.41]$$

where M' is the constrained modulus which is given by,

$$M' = \frac{2(1-\nu)G}{1-2\nu} \qquad [2.42]$$

Equation 2.41 can be rewritten as,

$$w_{oedo} = \frac{V}{2\pi G R}\frac{(1-2\nu)}{(1-\nu)}I_p \qquad [2.43]$$

Comparing the total settlements under three and one-dimensional conditions (Equations 2.39 and 2.43) indicates that they are approximately equal for $\nu = 0.2$ and $h/R \cong 3$. But using the one-dimensional theory to predict the settlement in three-dimensional situations underestimates the total settlement when the Poisson's ratio is greater than 0.25 (Davis and Poulos, 1968).

The methods outlined in the preceding paragraphs for the calculation of footing settlements where based on the assumption that the foundation soil could be modelled as ideal elastic medium. The stiffness of soil is dependent on the effective stress level. Thus stiffness parameters should be experimental values measured at an appropriate stress level.

2.7.3 Three-dimensional rate of consolidation settlement

The degree of consolidation settlement is usually expressed as a function of the time factor T_v that is dependent on the coefficient of consolidation. The definition of the coefficient of consolidation is dependent on the geometry of the problem, and appropriate coefficients should be used in the solutions of the one-dimensional and three-dimensional consolidation equations. The one-dimensional coefficient of consolidation was outlined in Equation 2.35 and the three-dimensional form is given by,

$$c_3 = \frac{2}{3}\frac{kG}{\gamma_w}\frac{(1+\nu)}{(1-2\nu)} \qquad [2.44]$$

Comparing the coefficients for a Poisson's ratio of 0.2 indicates that $c_3 = ½\, c_1$ (note that $c_1 = c_v$). Davis and Poulos (1972) stated that the theoretical one-dimensional and three-dimensional $T_v : \overline{U}_p$ curves would be the same if the appropriate coefficients of consolidation were used (for example when comparing an oedmeter and a triaxial test).

In the three-dimensional problem solved using simple diffusion theory, the degree of settlement is not equal to the average degree of pore pressure dissipation ($U_s \neq \overline{U}_p$). Davis and

Poulos (1972) suggested that $U_s \cong \overline{U}_p$ if the time scale is appropriately adjusted by defining T_s in terms of c_1 for both the one and three dimensional cases. The $T_s : \overline{U}_p$ curves obtained by Davis and Poulos using simple diffusion theory are shown in Figure 2.27. Also shown is an estimate of the curve for $h/R \cong 3$. Although the curves in Figure 2.27 were calculated assuming a Poisson's ratio of 0.5 and a flexible footing, Davis and Poulos stated that their solutions are applicable for all values of ν' and could be applied to rigid footings with reasonable accuracy.

2.8 Scaling

The scaling of the foundation problem will be reviewed although the scaling relationships were not applied to the results of this investigation. The scaling relationships are outlined so as to guide readers who may wish to scale the results according to their own specifications.

2.8.1 Introduction

Scaling is an import element in the design of foundations because of the lack of rigorous theoretical design techniques and the high cost of prototype tests (full-scale tests). Designs rely heavily on observed behaviour, and even the established theories, such as bearing capacity theory, require reference to physical tests. Model (physical) tests are usually used to derive design parameters that have to be scaled to the prototype situation. The scaling of model tests conducted at 1-g is not straightforward, as the behaviour of soil is dependent on the effective stress level. One must consider the effective stress level at homologous points in the soil of the model and prototype. The effective stress level in the model can be increased to the prototype value by conducting the model test in a centrifuge or by imposing a downward gradient on the pore fluid. These methods are more costly and time consuming than tests conducted at 1-g. The cost benefit of using the results of 1-g tests for foundation design has led researchers to propose various ways of overcoming the problems of scaling 1-g model tests. An outline of the scaling of 1-g model tests will be given. The details of scaling the stress:strain response, the elastic stiffness, and the drainage characteristics of sand will also be discussed in the following sections.

2.8.2 Model footings on sand

A satisfactory design will need to predict the stress:strain behaviour of the sand beneath a prototype foundation before the load:deformation response of the foundation can be deduced. The rate of change of stress and strain may also need to be incorporated into the design when dealing with saturated sand. The results from a model test will only be useful if the relation between stress and strain is similar in the prototype and the model. Rate effects must also be similar where these affect the stress:strain response. Dimensional analysis is used to define the set of similarity requirements that have to be satisfied (Isaacson and Isaacson, 1975; Douglas *et al.*, 1985). The similarity can involve length, mass and time or any combination of these. Complete similarity is achieved when all dimensionless groups attain the same values in the model and the prototype. This is usually not feasible, and any departure from complete similarity has to be justified.

An example of the problems encountered when scaling 1-g tests is highlighted by the effect of footing size on the dimensionless bearing capacity factor for self-weight N_γ. It is well known that N_γ decreases with increasing footing size for footings on dense sand. de Beer (1965b) suggested that the progression of a rupture surface beneath a footing might be dependent on footing size. He also suggested that the pressure dependency of the shear strength parameters of sand might be the cause of the size effect. The dependency of the mobilised friction angle on the stress level along a rupture surface beneath a footing was confirmed by Yamaguchi *et al.* (1977). Hettler and Gudehus (1988) also found that the size effect could be explained by the pressure dependence of the shear strength parameters of sand. Thus for a model to be representative, the effective stress state has to be increased to the prototype value by testing in a centrifuge or by imposing a downward hydraulic gradient. Alternatively, the stress:strain response of a 1-g test could be made similar to that of the prototype by reducing the density of the sand in the model. The latter technique is possible because the stress:strain response of sand is a function of void ratio, as well as of confining stress.

Hettler and Gudehus (1988) have shown that departure from similarity with regard to the scaling of the size of sand particles is unimportant. A ratio of footing diameter to sand particle diameter above a threshold value of about 50 has little influence on the bearing capacity. In the scaling of model tests the effects of chamber size have to be assessed. In centrifuge tests, Ovesen (1979) found that a chamber diameter to footing diameter ratio of five was too small. Lau (1988) found that a chamber diameter to footing diameter ratio of 8.5, and also a height of sand layer to footing radius ratio of 3.5, resulted in no significant chamber size effects. A comprehensive literature review of the scaling of models tests is given in Corté (1989).

2.8.3 Scaling of stress:strain behaviour

The scaling of footing load:deformation behaviour is best illustrated by considering the stress:strain behaviour of an element of soil beneath a footing. This could, for instance, be at a depth equivalent to the radius of the footing. The comparison of the load:deformation behaviour of a footing and the stress:strain response of soil in an element test was utilised by Altaee and Fellenius (1994). The initial vertical stress before application of the foundation load can be used to represent the reference stress state. If the effective mean stress p' is assumed to be representative of the initial vertical stress beneath the footing and the void ratio is used to represent density, then the scaling of a soil element may be illustrated using the concepts of critical state soil mechanics and the state parameter (Roscoe & Poorooshasb, 1963; Been & Jeffries, 1985). The stress:strain behaviour of three model reference soil states, representing three different methods of scaling, will be compared to a prototype reference soil state. The reference soil states, which are shown in Figure 2.28, are characterised by the void ratio and the effective mean stress.

The equivalent void ratio model represents a conventional 1-g model test and the equivalent state parameter model utilises the evidence that the stress:strain behaviour of a soil is similar if the state parameter is the same. Altaee and Fellenius (1994) used the results from drained triaxial tests to illustrate the importance of the state parameter. The results of their tests, which correspond to states M1, M2 and P in Figure 2.28, are shown in Figure 2.29. The similarity between M1 and P is evident in the plots of normalised shear stress and volumetric strain against axial strain, as is the difference between M2 and P. The stress:strain response of the equivalent void ratio model does not scale because of the increased dilation effect. The drained critical state stress of this model will, however, scale by $1/n$. The stress:strain response of the centrifuge model will be the same as that of the prototype. A model state similar to the prototype state is attained in the centrifuge model by increasing the body acceleration by the geometric scale factor n.

If triaxial stress:strain behaviour is representative of the soil beneath a footing, then the scaling factors listed in Table 2.2 can be deduced. Most of the scaling factors of a downward hydraulic gradient model will be similar to those of the centrifuge model, assuming that the prototype stress state can be attained with a downward hydraulic gradient. The acceleration will, however, be the same in the downward hydraulic gradient model and the prototype.

The scale factors for force from Table 2.2 can be used to scale vertical and horizontal drained capacity. The drained moment capacity of the equivalent state parameter model will scale by $1/n^3$ because of the additional length variable in the definition of a moment. The drained moment capacity of the equivalent void ratio model will scale by a factor of the order of $1/n^4$.

The undrained strength scale factor of the equivalent state parameter model will be the same as the drained factor. The undrained strength scale factor of the equivalent void ratio model will depend on the definition of failure, but it would be between about $1/n$ and 1. If both the model and prototype are sheared to cavitation, then the undrained scale factor would be influenced by the difference in the ambient pore pressure at foundation level between the model and prototype. The scaling of partially-drained strength is even more complex. One would need to know the drained and undrained strengths of the model and prototype, and the degree of drainage in the model and prototype would also have to similar.

In terms of model tests conducted at 1-g, it would seem that the equivalent state parameter model is the most appropriate scaling technique. This is because it correctly models dilatancy. Poorooshasb (1995) drew similar conclusions in his analysis of the problem of scaling in 1-g tests. He used an elasto-plastic constitutive soil model in conjunction with the equilibrium equation to prove that similarity, under quasi static loading conditions, could be achieved with the equivalent state parameter model. This was only if the viscosity of the pore fluid was also appropriately scaled. Under dynamic loading conditions, however, he found that similarity could only be achieved if the soil skeleton was assumed to be poro-elastic.

2.8.4 Scaling of elastic stiffness

The elastic stiffness of sand is highly dependent on the magnitude of strain. A ten-fold variation in stiffness occurs in the $10^{-4}\%$ to 1% strain range. This review of stiffness will be limited to small strain "linear" elastic behaviour and the initial stress state. The API (1993) recommendations on elastic foundation stiffness (see Equations 2.1-2.3) give the following relationships:

vertical stiffness	$\propto GR$
horizontal stiffness	$\propto GR$
rotational stiffness	$\propto GR^3$

The shear modulus is known to be approximately proportional to the square root of effective mean stress. The radius will scale by the geometric scaling factor $1/n$. The scale factor for p' will depend on the scaling of initial effective stresses, which is $1/n$ for both the equivalent state parameter and void ratio models. This results in a factor of $1/n^{1.5}$ for vertical and horizontal stiffness. The factor for moment stiffness is $1/n^{3.5}$.

2.8.5 Scaling of drainage

Soil behaviour that is not fully drained or undrained will be affected by the drainage characteristics of the sand. Drainage can be scaled using the dimensionless time factor,

$$T = \frac{c_v\,t}{H^2} \qquad [2.45]$$

where c_v is the coefficient of consolidation, t is time, and H is the drainage length. The radius of a footing can be used as a representative drainage length. The coefficient of consolidation can be expressed by,

$$c_v = \frac{k\,M'}{\gamma_w} \qquad [2.46]$$

where k is the permeability and M' is a representative constrained modulus (one-dimensional compression modulus). The work of Poorooshasb (1995) indicated that stress:strain similarity under quasi static loading conditions is achieved if the viscosity of the pore fluid in the model is increased by the geometric scale factor n. This would result in a scale factor of $1/n$ for permeability. If the constrained modulus represents plastic stress:strain behaviour, an appropriate scale factor for this variable would be $1/n$. Thus the coefficient of consolidation would scale by

$1/n^2$ and time would scale by a factor of one for the same dimensionless time factor in Equation 2.45. If on the other hand the process is principally elastic, the modulus would scale by $1/n^{1/2}$ and the time would scale by $1/n^{1/2}$.

3 Apparatus

3.1 Introduction

The diameter of the model footing was 150 mm. The behaviour of the footing was investigated using a displacement controlled rig. The rig and all of the data logging processes were controlled from a PC. The miniature cone penetrometer tests were also performed using the same rig and data logging equipment. The sample preparation procedures were novel and required special equipment.

3.2 Model footing tests: Rig

3.2.1 Design of the rig

The test rig was designed by Martin (1994) for model spudcan footing tests on clay. The control of the rig was fully automated and allowed independent control over vertical (w), horizontal (u), and rotational (θ) displacements in a single plane. Displacement control facilitates exploration of the complete V:H:M failure surface using sideswipe tests, which involve either horizontal or moment displacement at a constant vertical displacement. Simultaneous measurement of the resulting load components (V, H and M) was achieved using a Cambridge type load cell.

Independent control over the three components of displacement was accomplished by using separate bearing arrangements, and by superposition of the different motion systems. This is illustrated in Figure 3.1. Linear slides guide the vertical and horizontal systems, and a rotary slide guides the rotational system. The vertical plate displaces along the vertical slide that is attached to the back plate. The back plate is fixed to the specimen container. The horizontal plate displaces along the horizontal slide that is attached to the vertical plate. The bearings guiding the rotational slide are attached to the horizontal plate. The shaft supporting the model footing was attached directly to the rotary slide. Each plate, and the rotary slide, were driven by a motors that were fixed to the plate immediately behind. LVDTs positioned between the adjacent systems measured displacements.

The displacement and load design specifications adopted by Martin were calculated for a 100 mm diameter footing. The choice of this diameter was influenced by the use of 450 mm diameter specimen containers. The vertical, horizontal, and rotational displacement ranges adopted were 300 mm, 50 mm, and 30 ° respectively. The corresponding maximum displacement rates were 1 mm/s, 0.25 mm/s, and 0.25 deg/s respectively. The displacement rates were chosen to achieve an undrained footing response with kaolin clay specimens. Martin used Brinch Hansen's (1970) bearing capacity method to estimate the required load capacity of the footing, and hence of the rig. The rig and the load cell were designed for working loads of $V = 1000$ N, $H = 300$ N, and $M = 12.5$ Nm. A safety factor of 1.5 was used in the design of the load cell.

The rig, as designed by Martin for tests on clay, was not wholly suited for footing tests on dense sand specimens. For a similar sized footing, a dense sand specimen results in stiffer behaviour and larger capacities. The investigation of partially-drained behaviour also required an apparatus that could produce a large range of displacement rates. Slow displacement rates are required for drained loading and fast displacement rates for undrained loading. The rig was adapted to cope with the increased stiffness and displacement rates required for partially-drained footing tests on dense sand. This entailed replacement of some structural elements, a faster and more powerful vertical motion system, an altogether different and more sensitive displacement measuring system, and a new load cell of increased capacity.

The load capacity of the rig was dictated by the capacity of the vertical and horizontal motion systems and by components in the rotational motion system. The motion systems imposed limits of 970 N on the vertical load and 350 N on the horizontal load, for non-lubricated slides. The limits could be increased by a factor of four with lubrication of the slides. The author used an alternative and less effective method of lubrication than that recommended by the manufacturers. Load limits a factor of about two greater than the values for dry slides mentioned above were deemed to be acceptable. A moment capacity of 200 Nm was dictated by the strength of the plastic rack in the rack and pinion system used to rotate the rotary slide.

The vertical load capacity of the rig restricts the maximum footing size that can be used if one is interested in investigating bearing capacity failure. The investigation of partially-drained behaviour, however, was of overriding importance. Partially-drained considerations dictated a minimum footing size, in addition to rates of displacement and the permeability of the sand specimen. The degree of drainage under partially-drained conditions is dependent on the drainage length, the rate of loading, and the drainage properties of the sand base. To achieve undrained conditions (or a low degree of drainage) at a specific loading rate and permeability, the drainage length has to be sufficiently long. Hence the specification of a minimum footing size.

Rapid loading trials with footings of various sizes on an oil-saturated sand specimen indicated that a 100 mm diameter footing was too small. It was not possible to develop undrained footing response with a 100 mm diameter footing. The degree of drainage was estimated from a pore pressure measurement at the centre of the footing. With a 150 mm footing, the increase in pore pressure was similar to the increase in total stress under rapid loading. A footing diameter of 150 mm was chosen. The estimated bearing capacity load of a 150 mm footing on dense sand was approximately 6000 N. This was three times higher than the capacity of the rig. The footing diameter was ultimately restricted by the 450 mm diameter specimen container and the influence of possible chamber size effects. Minimum and maximum vertical displacement rates of 0.001 mm/s and 5 mm/s (for drained and undrained response respectively) were deemed necessary for the choice of footing size and sand permeability. The permeability of the sand was reduced by using a silicone oil pore fluid with a higher viscosity than that of water.

Martin's (1994) displacement measurements were affected by the flexibility of the rig since his measurements were not recorded at the footing, but between the relevant sliding plates. By measuring the stiffness of the rig under various load combinations, he was able to account for this effect. The relevant flexibility coefficients were as follows,

$$w = -2.75 \times 10^{-4}\ V \qquad \text{under vertical load only} \qquad [3.1a]$$

$$u = -2.25 \times 10^{-3}\ H \qquad \text{under horizontal load only} \qquad [3.1b]$$

$$\theta = -4.63 \times 10^{-3}\ M \qquad \text{under moment load only} \qquad [3.1c]$$

where the units were mm, degrees, N and Nm. The author developed a displacement measurement system that was attached directly to the footing. This removed the influence of the rig flexibility on the displacement measurements. It was still deemed necessary to increase the rig stiffness because sideswipe tests ideally require a constant vertical displacement. A flexible rig extends on the relaxation of load that is encountered during such a test. It was not possible to reduce the vertical flexibility of the rig without having to perform major alterations. This flexibility was generated in the numerous bearings and sliding parts, and could not be altered by simply replacing the structural elements. The horizontal and rotational flexibility were reduced by approximately 8% by replacing the shaft, onto which the footing was attached, by a stiffer element. A larger shaft was also needed to fit the new load cell. The new shaft and associated connections are shown in Figure 3.2. A removable U-shaped element was machined to fit the base of the shaft and to connect to the top of the load cell. The shaft was positioned so that the centre of the base of the footing was at the centre of rotation of the rotary slide. It was possible to maintain a constant vertical displacement during the slowest sideswipe tests by using a load feedback mechanism. This facility was not adopted because the feedback mechanism would not work in the fast tests.

The vertical stiffness of the rig between the vertical stepper motor and the footing, shown in Figure 3.3, was calculated by comparing the displacement at the stepper motor and at the footing. Note that this stiffness was different from that specified by Martin (1994) as he measured the stiffness between the vertical plate and the footing. There was considerable flexibility in the vertical lead screw and associated bearings situated between the vertical stepper motor and the vertical plate (see Figure 3.3). The slack in the vertical displacement system was also measured. This occurred in the lead screw system. The load increments and displacements are shown in Figure 3.4[a]. The average value of the rig stiffness was 1944 N/mm. The stiffness measurements that were affected by slack are not included in the estimation of the rig stiffness. The slack was approximately 0.2 mm and from Figure 3.4[b] it is clear that the slack occurred at a vertical load of about 450 N.

The range of vertical displacement rates required for partially-drained footing response could not be achieved with the existing vertical motor. In addition, a single motor could not cover the full velocity range of 0.001 mm/s to 5 mm/s. Two different motors were used in tests requiring both drained and undrained response. The vertical drive system was adapted so that the motor could be changed during a test with minimum disturbance to the footing. The position of the vertical motor was raised above the confined space within the rig so as to accommodate a new larger motor required for undrained loading. An extended couple was introduced between the motor and the lead-screw that drove the vertical plate. This arrangement is shown in Figure 3.3.

3.2.2 Displacement control system

Three stepper motors and an intelligent stepper motor controller were used to control the displacement of the vertical and horizontal plates, and of the rotary slide, and hence the displacement of the footing. The intelligent stepper motor controller received position, velocity and acceleration commands from a PC via an RS232 interface. The vertical and horizontal plates were connected to the appropriate stepper motors by a lead screw and nut arrangement whilst a rack and pinion arrangement was used to drive the rotary slide. The general layout of the stepper motors and drive systems is shown in Figure 3.3.

The stepper motors were supplied by McLennan Servo Supplies Ltd and were all of type 23HS-108, which produce 400 steps of 0.9 ° per revolution. These motors operate best at 400-1000 r.p.m., which result in linear and rotational displacement rates higher than the footing displacement rates required by Martin (1994). Martin incorporated gearboxes to reduce the r.p.m. from all three stepper motors. Gearbox reduction ratios of 50:1, 250:1 and 500:1 were used on the vertical, horizontal and rotational stepper motors. These gearbox reduction ratios, combined with the reductions due to the lead screw/nut and rack/pinion systems, produce the conversion factors in Table 3.1. These relate motor steps and footing displacements. It was mentioned in the previous section that a larger vertical stepper motor was needed to generate the fast displacement rates required for undrained footing response. A 34HS-209 stepper motor (without a gearbox) was used and the conversion factor relating motor steps to footing displacement for this motor is also listed in Table 3.1.

An important characteristic of stepper motors are their working ranges in terms of speed and acceleration. The limits of the working ranges of the three small stepper motors, under minimal footing load, are shown in Figure 3.5. These working ranges reduce with increased footing load. The range of velocities used in this test programme is listed in Table 3.1. Although an undrained displacement rate of 5 mm/s could be attained using the smaller vertical motor with a lower gearbox reduction ratio, the larger and more powerful motor was required to cope with the anticipated torque. It was also possible to develop faster displacement rates than 5 mm/s with the larger motor, but this would have required a faster data logger to achieve an acceptable data logging resolution.

The SM9343 triple-axis intelligent stepper motor controller was supplied by McLennan Servo Supplies Ltd and included three PM301 stepper motor controllers and three PM164C translators. Each axis corresponds to a motor. Axis one was the rotational motor, axis two was the horizontal motor, and axis three was the vertical motor. The power supply module was replaced by a more powerful 50 V d.c. module to cope with the increased torque required for this test programme. External current switches for each axis were also placed on the intelligent stepper motor controller to accommodate changing of the vertical motor. The three PM301 controllers were daisy chained on the single RS232 serial interface to the host PC. Simple ASCII commands, specifying motor position, velocity, and acceleration were used in the two-way communication between the PM301 controllers and the host PC. These commands could also be stored in the on-board RAM of the controllers and executed as a sequence of moves. Separate PM301 controllers for each motor allowed independent control of the three different displacement components. The PM301 controllers supplied the required direction signal and step pulses to the TM164C translators. The translators converted these into drive currents to drive the stepper motors.

The ASCII commands for the PM301 stepper motor controllers were generated using a QuickBASIC program on an IBM compatible 286 PC. A typical Move Relative (MR) velocity path and the associated PM301 commands are shown in Figure 3.6. The MR command rotates the stepper motor by a fixed number of steps. This move starts at a programmed Base Speed (SB) and ramps up linearly at a rate defined by the Set Acceleration (SA) command, until the desired Slew Speed (SV) is reached. The motor will continue at this SV speed until it decelerates at the SA rate back down to the SB speed. The Creep Steps (CR) command defines the number of final steps at the SB rate. The PM301 controller calculates when to decelerate the motor so as to achieve the programmed steps, and hence footing displacement. Another commonly used feature was the

Constant Velocity (CV) command, which was used to rotate the stepper motors at a specified SV speed for an unspecified number of steps. Many more stepper motor commands were available in addition to those mentioned above. It was possible to simulate sinusoidal cyclic displacements and to have load control via a feedback routine in the QuickBASIC control program. Use of the feedback routine was restricted by the speed of the host PC. Limit switches were attached to the rig to stop the stepper motors from displacing the moving parts of the rig beyond their limits.

3.2.3 Model footing and connection plate

The factors affecting the decision to use a 150 mm diameter footing were discussed in Section 3.2.1. The first factor was the need for a sufficiently large footing to ensure undrained response. The vertical displacement rate of the rig and the permeability of the sand specimen had already been altered as much as possible to reduce drainage rates. The second and conflicting factor was the need for a sufficiently small footing, imposed by the relative size of the specimen container and the desire for the bearing capacity load of the footing to be within the load limits of the test rig. The shape of the footing was circular because most shallow offshore foundations are circular in plan. A circular flat footing shape, as opposed to one with skirts around the periphery, was chosen because it was felt that the partially-drained behaviour of the simplest geometry had to be investigated before that of a more complicated sort.

A 150 mm diameter footing was chosen based on the results of a series of rapid loading preliminary tests. Different size footings and pore pressure measurement arrangements were investigated. The pore pressure was measured at the centre of the footing. This was achieved by attaching the transducer to the centre of the footing or by measuring the pressure at the centre via a tube to an externally located transducer. The nine preliminary tests were conducted on the oil-saturated sand used in the main series of tests. Different combinations of loading apparati, footing sizes, transducer locations, and effective stress arrangements were tested. The preliminary tests and their results are summarised in Table 3.2 and the apparatus is shown in Figure 3.7.

Tests TP01, TP02 and TP03 comprised a 100 mm diameter footing with an external transducer. This arrangement is shown in Figure 3.7[a]. The footing in these tests was loaded using the rig described in the preceding sections. The larger stepper motor was not installed when the preliminary tests were performed, thus the maximum vertical displacement rate of the rig was about 1 mm/s. The measured pore pressure response was low in the above mentioned tests, even when the length of tubing to the transducer was reduced and the effective stress of the sand was increased by downward hydraulic gradient. The ratio of the increments of pore pressure load to vertical load averaged only 5% and the footing response could not be classified as undrained. This ratio, and hence the magnitude of the pore pressure, was used to indicate how undrained the tests were. The pore pressure load was calculated assuming a uniform pore pressure distribution on the footing.

A faster method of load application was needed to investigate the effect of loading rate, as opposed to footing size, on the pore pressure response. A hanger and weights system was devised that resulted in displacement rates varying between 5 mm/s and 10 mm/s. The results of test TP04, using the hanger and weights arrangement, again resulted in a low pore pressure response ratio. The effect of directly attaching the pore pressure transducer to the centre of the footing was apparent in tests TP06 and TP07 (see Table 3.2). The pore pressure in test TP05 was anomalous. A direct connection reduces the effects of the capillary action in the tubing. A pore pressure response ratio of about 25% was attained with the 100 mm diameter footing, when loaded with the hanger and weights loading system.

A pore pressure response ratio of 64% was attained in tests TP08 and TP09 by increasing the footing diameter to 150 mm. The apparatus of these tests is shown in Figure 3.7[b]. The displacement rate in these two tests averaged about 8 mm/s. This combination of footing size and displacement rate probably resulted in an undrained response. Therefore a footing diameter of 150 mm was chosen for the main series of tests.

The design of the 150 mm diameter footing used in the main series of tests incorporated a wall around the periphery. This was to prevent inundation of the instrumentation in and above the footing. A connecting plate, termed the base plate, was needed between the footing and the load cell. The base plate acted as a spacer for the pore pressure transducer, which was screwed into the centre of the footing, and also facilitated removal of the footing without disconnection of the load cell. Toyoura sand with a mean grain size of 0.16 mm was glued to the base of the footing to create a rough footing. The footing displacement measuring devise was attached to the wall of the footing. A fine stainless steel woven wire-cloth (supplied by Potter & Soar, nominal aperture = 0.05 mm) was attached at the base of the footing at the pore pressure transducer port. It was felt that a less permeable VYON F filter (supplied by Porvair plc, thickness = 0.75 mm, average pore size = 0.09 mm) or stone filter might have reduced the transducer response. The footing and base plate are shown in Figure 3.8.

3.3 Model footing tests: Instrumentation

The instrumentation consisted of ten transducers, a Mowlem Microsystems "Autonomous Data-acquisition Unit" (ADU) supplied by Measurements Group UK, and an additional "RDP Dataspan 2000" signal-conditioning unit supplied by RDP Electronics. The ten transducers required 12 channels and consisted of six LVDT transducers for measuring displacement, three pore pressure transducers (PPT), and a load cell with three channels. The ADU was connected to the host PC via a RS232 interface.

3.3.1 Data-acquisition and signal-conditioning

The ADU was used to log 12 channels and to provide excitation for six channels. The six LVDTs required a.c. excitation, which was provided using the RDP Dataspan unit. The ADU incorporated a MM720 analogue to digital 12 bit converter (which had an accuracy of 1 in 4096 bits), MM724 and MM740 analogue input boards, a MM718 memory module, and a MM712 microprocessor module with an RS232 interface. The analogue input boards accepted a full scale input range of ±10 mV to ±10 V, and the MM724 module was also capable of supplying an excitation of up to ±10 V. The six load cell and PPT channels were connected to the MM724 module and the six LVDT channels were connected to the MM740 module. The MM720 A/D converter module had gain settings, with a maximum gain of 1024. The MM718 module had an onboard storage capacity of 256 kBytes. The ADU was commanded and interrogated using simple ASCII commands sent from the host PC to the MM712 module.

Two forms of data logging were utilised, fast logging (up to 100 scans per second) of relatively short tests by the MM718 module and slow logging by the host PC (up to 1 scan per second). Each test, which could be up to 12 hours long, included a series of sub-tests in which fast logging was used. Data from the sub-tests was transferred (and cleared) from the ADU to the host PC immediately after each sub-test. Each test produced an average of 2 MBytes of data.

3.3.2 Load cell

The load cell was designed to measure vertical, horizontal, and moment loads acting on the footing and was similar to the contact stress transducers developed at Cambridge University. Bransby (1973) described the development and design of these transducers. The cell consists of two blocks attached by an arrangement of thin strain-gauged webs as shown in Figure 3.9. The lower block, or active face, is connected to the surface on which loads are measured. The upper block is attached to the loading apparatus. In this study, the active face of the load cell was connected to the base plate and the upper block was connected to the shaft. This is illustrated in Figure 3.2 and Figure 3.8.

Two pairs of vertical webs at each end of the cell (labelled LCV1 and LCV2 in Figure 3.9) measure the vertical and moment load. The moment load is deduced from the difference between the forces in the two pairs of vertical webs. Four horizontal webs (labelled LCH in Figure 3.9) measure the horizontal load. Two horizontal webs were located on each side of cell close to the active face. A compressive vertical load results in compressive strains in both of the vertical web arrangements (LCV1 and LCV2), whilst a pure moment results in compressive strains in one of the vertical web arrangements and tensile strains in the other. A horizontal load results in strains in the horizontal webs. There is some degree of cross coupling depending on the position of the Load Reference Point (LRP).

Three full Wheatstone bridge strain-gauged circuits measure strain in the webs. All of the webs were strain-gauged on both sides, with each circuit comprising eight gauges. There were four "dummy" gauges in each of the vertical circuits that were attached to the solid part of the cell. The three load cell channels were connected to the MM724 module on the ADU that supplied an excitation of 10 V d.c.. The signal from each vertical channel had a load sensitivity of approximately 0.01 mV/N for the specified excitation (with respect to the vertical load acting on each web arrangement, and not the total vertical load). The sensitivity of the horizontal web arrangement was approximately 0.036 mV/N. A gain of 256 was used in both of the vertical channels, which meant a voltage input range of ±20 mV. In the horizontal channel, a gain of 128 and a voltage input range of ±40 mV was used.

The cell was machined from a single block of dural (E = 70000 MPa and 0.1% proof stress = 220 MPa). The webs were sized to achieve 1000 $\mu\varepsilon$ under the web design loads. A vertical web design load of 450 N was chosen to give a vertical footing design load of 1800 N. This vertical web design load results in the following relationship between allowable moment footing load and vertical footing load,

$$M = 46.08 - 0.0256\,V \qquad [3.2]$$

where the units are Nm and N. A permanent strain of 0.1% would be expected at a vertical web load a factor of 3 greater than the web design load. The factor of safety calculated using Rankine's theory (Stephens, 1970) was 2.7 assuming a fixed web. The failure load derived from Rankine's theory is a function of the direct compression and Euler failure loads, and is valid for struts of intermediate length. A horizontal web design load of 125 N was chosen to give a horizontal footing design load of 500 N. The factors of safety of the horizontal webs were the same as for the vertical webs. Cambridge In-situ Ltd supplied the strain-gauged and wired load cell.

The load cell was calibrated by applying a load (V, H or M) at the load reference point whist keeping the other two loads constant. The calibration apparatus is shown in Figure 3.10. The load cell was calibrated with the active face directed upwards. A calibration block was connected to the active face of the cell and the loads were applied to this block. Moment load was applied by varying the eccentricity of a constant vertical load on a hanger. Horizontal load was applied by a wire positioned at a height above the active face of the load cell equivalent to the LRP. The distance from the active face to the LRP was 33 mm and represented the thickness of the base plate plus the footing (shown in Figure 3.8). Grooves were made in the calibration block along its axis for positioning the hanger to simulate moment loading. Negative and positive calibrations under horizontal and moment loads were performed separately. The negative and positive calibration gradients were averaged together but these were almost identical. Calibration of the cell with each load resulted in three coefficients, one for each set of webs, relating load and bits. This was because vertical load also generated strain in the horizontal webs and vice versa. There was some cross coupling.

The gradients of the calibration curves, which relate the output from the load cell to the applied loads, form the entries in the matrix below,

$$\begin{Bmatrix} LC_{v1}\,(\text{bits}) \\ LC_{v2}\,(\text{bits}) \\ LC_{h}\,(\text{bits}) \end{Bmatrix} = \begin{bmatrix} 5.952\times10^{-1} & 8.329\times10^{-1} & -23.188 \\ 5.854\times10^{-1} & -8.127\times10^{-1} & 22.920 \\ -2.760\times10^{-3} & 1.866 & -4.283\times10^{-1} \end{bmatrix} \begin{Bmatrix} V\,(\text{N}) \\ H\,(\text{N}) \\ M\,(\text{Nm}) \end{Bmatrix} \qquad [3.3]$$

where LC_{v1}, LC_{v2} and LC_{h} are the outputs from the three load cell channels and V, H, and M are the applied loads. To derive loads from the outputs of the load cell (in bits), the above matrix is inverted. Inversion results in the following matrix,

$$\begin{Bmatrix} V\,(\text{N}) \\ H\,(\text{N}) \\ M\,(\text{Nm}) \end{Bmatrix} = \begin{bmatrix} 8.422\times10^{-1} & 8.519\times10^{-1} & 4.890\times10^{-3} \\ -3.720\times10^{-3} & 6.332\times10^{-3} & 5.404\times10^{-1} \\ -2.164\times10^{-2} & 2.210\times10^{-2} & 1.929\times10^{-2} \end{bmatrix} \begin{Bmatrix} LC_{v1} \\ LC_{v2} \\ LC_{h} \end{Bmatrix} \qquad [3.4]$$

The accuracy of the load cell calibration matrix was checked by applying various known combinations of the three load components, and comparing these with the values output by the load cell. The results of these checks are listed in Table 3.3 and indicate that the measured loads were accurate to ±5 N, ±3 N and ±0.3 Nm for V, H and M respectively. The standard error in the calibration coefficients of the vertical webs during vertical loading, and of the horizontal webs during horizontal loading, was not more than 0.12%.

Measurements of the three load components were affected by what seemed to be random noise, with ranges of ±4 N, ±3 N and ±0.2 Nm for V, H and M respectively. The load cell readings were monitored after each test to check to see if they returned to their initial values. Any differences were generally less than the accuracy ranges quoted in the preceding paragraph. The load cell was calibrated twice before being used in this project.

3.3.3 Displacement measurement

There were two forms of displacement measurement: coarse measurements by three long LVDTs positioned to measure the relative movement between the three adjacent moving elements of the rig; and fine measurements by an arrangement of three short LVDTs connected to the footing and referenced to the specimen container. All of the LVDTs were provided with an excitation of 5 V r.m.s. at 5 kHz from the RDP Dataspan 2000 unit. The a.c. output signal from these transducers was transformed to a d.c. signal by the RDP Dataspan 2000 unit before being transferred to the MM740 module on the ADU.

The long vertical LVDT (DLV) was fixed to the back plate and its armature was attached to the vertical plate. The long horizontal LVDT (DLH) was fixed to the vertical plate and its armature was placed against the horizontal plate. The long rotational LVDT (DLM) was pinned at one end to a beam attached to the horizontal plate and at the other end it was pinned onto the shaft connecting the footing to the rig. Linear displacement measured by this LVDT was transformed to rotational displacement using a trigonometric calculation. The specifications of the long LVDTs are listed in Table 3.4 and their positions on the rig are shown in Figure 3.11[a]. The long vertical LVDT was calibrated using a ruler and the other two long ones were calibrated using a micrometer. The random noise levels were ±0.10 mm, ±0.05 mm and ±0.01 ° for vertical, horizontal and rotational displacements respectively. There was no detectable drift in the readings from DLV and DLM but the channel on the RDP Dataspan 2000 unit onto which DLH was connected was subject to drift. This resulted in a typical variation of 0.1 mm in the reading from DLH over the period of a test.

Additional fine displacement measurements of the footing were required for the following reasons:

— Lack of sensitivity in the existing coarse LVDT measurements. The stiffness of dense sand meant that the displacements that were required to produce the necessary variations in load were of the order of a few millimetres to fractions of a millimetre.

— There was considerable flexibility in the rig between the LRP and the points at which the coarse displacement measurements were recorded. The difference between the vertical displacement of the LRP and of the vertical plate was of the order of 0.5 mm under large loads.

Preliminary tests on dense sand indicated that vertical penetrations were unlikely to exceed 10 mm at the maximum load (under a pure vertical displacement path). The fine displacement measurement system consisted of three short LVDTs (±5 mm range) which were pinned to a beam connected to the footing and pinned to a L-shaped frame. The frame was fixed to the specimen container. The fine displacement measurement system is shown in Figure 3.11[b]. The specifications of the small LVDTs are listed in Table 3.4. The three linear displacement measurements from the two vertically aligned small LVDTs (DSV1 and DSV2) and the approximately horizontally aligned one (DSH) were transformed using a trigonometric calculation to give the vertical (w), horizontal (u) and rotational displacement (θ) of the LRP. The procedure for calculating the displacements from the small LVDTs is outlined in Appendix B.

The non-linearity of these transducers was rated at 0.1%, which translates to 0.01 mm, and the random noise levels were ±0.0075 mm, ±0.0100 mm and ±0.0050 ° for w, u and θ respectively. The three short LVDTs were calibrated using a micrometer. The displacements measured by the LVDT frame were compared with the displacements expected for specified revolutions of the stepper motors, and with dial gauge measurements of the footing displacements. The small LVDT system was accurate and the absolute errors in the measurements of w, u, and θ were similar to the noise levels.

3.3.4 Pore pressure measurement

In addition to the pore pressure transducer in the footing (pp_1), two transducers were placed in the walls of the specimen container (pp_2 and pp_3). The position and dimensions of pp_1 are shown in Figure 3.8, and those of the other two transducers are shown in Figure 3.12. Transducer pp_1 was screwed directly into the centre of the footing, whilst special adapters for connecting pp_2 and pp_3 to the specimen containers were machined. pp_1 was designed for this project and manufactured by Gaeltec Ltd and the other two transducers were standard PDCR910 units supplied by Druck Ltd.. The specifications of the pore pressure transducers are listed in Table 3.5.

Before installing pp_1 in the centre of the footing, an arrangement whereby the transducer was connected to the port by copper tubing was investigated (see Figure 3.7). This arrangement proved unsatisfactory because of the poor transmission of pressure by the tubing. The VYON filter used in the preliminary tests was also replaced by a more permeable wire-cloth. The response time of the transducers in the specimen container was not a vital factor and the VYON filter was used in the adapters for pp_2 and pp_3.

The transducers were calibrated using positive and negative air pressure. A DPI600 (Druck Ltd.) calibration unit was used to measure the calibration air pressure. The transducers were all supplied with an excitation of 10 V d.c. by the MM724 module on the ADU. The non-linearity of the transducers was ±0.1% and the random noise level was ±0.05 kPa. The signal of the Gaeltec transducer (pp_1) was subject to spurious jumps in a few of the tests. The spurious output was easily corrected since the magnitude of the jump was large and consistent.

3.4 Model footing tests: Control and monitoring

Tests were controlled and monitored using a QuickBASIC program (filename CONT_1.BAS) on the host IBM compatible 286 PC. Houlsby (1995) wrote the program for a previous project but some alterations were performed to the program for this investigation. RS232 serial links were used to communicate with the data logging unit (ADU) and with the displacement control unit (SM9343). A schematic diagram of this arrangement is shown in Figure 3.13.

The control program performed two main functions: real-time control and data logging of testing; and autonomous testing using parameters stored in previously created input files (filename extension: .INP). The autonomous method of testing was used to perform the sub-tests that required fast logging. The term sub-test is used to refer to special events that were performed in the main-tests. The transducer calibration coefficients, the LVDT frame constants, and the sample details were stored in a definition file (filename extension: .DEF) that was loaded by the control program at the start of each main-test. The program logged raw data from the twelve channels of the ADU into files with extension .OUR and also performed real-time processing of the raw data. The processed data was stored in files with extension .OUT. The rate of logging to .OUR and .OUT files (termed background logging) could be varied but was restricted by the maximum background logging rate of the ADU, which was one scan per second.

The scan rate of the ADU could be increased by operating it in an autonomous testing mode. In this mode, the ADU could be configured to perform tests with a maximum logging rate of 100 scans per second. This mode was used to log the sub-tests that required fast logging rates. The sub-tests were stored onboard the ADU and downloaded to the PC immediately after each sub-test. The raw sub-test data transferred from the ADU was stored in files with extension .RAW. The control program was also used to create processed data files with extension .DAT. In addition to the storage of raw and processed background logging data, the data was displayed on the monitor. The data displayed on the monitor was updated at the background logging rate.

Soft limits for the transducers could be set using the control program. The soft limits on the three components of load were used to control the loading of the footing. Hard limits were also permanently set in the program. These limits were set in bits and responded to the unprocessed data. The hard limits ensured that the transducers were not accidentally pushed over their working ranges by unchecked loading. The control program contained a menu system for setting the parameters required by the numerous control functions. There were functions to control constant velocity moves, fixed displacement moves, combined fixed displacement moves (moves involving all three components of displacement), load holding routines, ADU sub-test configuration, data retrieval from the ADU, reading of .INP files, and the zeroing of the fine displacement measuring system, to name but a few. All the necessary real-time control and data logging adjustments could be performed from the control program on the host PC.

3.5 Miniature cone penetrometer tests

Cone tests were performed with a small penetrometer to gauge the relative density and uniformity of the sand samples. The penetrometer was developed by McBean and West (1987) to test pressurised soil samples in the laboratory. The diameter of the cone was 8 mm (cross sectional area of 50 mm^2) and the cone angle was 60 °. The cone was designed to yield at 2500 N and was strain gauged to produce an output signal of 4.8×10^{-3} mV/N with an excitation of 2.5 V.

The transducer on the cone tip was connected to the ADU. The penetrometer loads encountered in this investigation were generally less than 100 N but a resolution of 1 N was produced by setting the relevant channel on the ADU to its maximum gain. Although the resolution of the A/D conversion was adequate, the noise in the signal from the strain gauged cone was ±20 N. Thus the signal from the cone tip was not used. The measurement from the load cell on the rig was used to represent the penetrometer load instead. Consequently, the penetrometer load comprised the resistance due to the wall friction as well as the tip resistance

The connection of the penetrometer to the rig is shown in Figure 3.14. The penetrometer was clamped to a bracket that was attached to the base plate below the load cell on the rig. The penetrometer tests were controlled and logged using the same apparatus as for the footing tests.

3.6 Preparation of sand samples

Three sand specimens were prepared. One of the specimens consisted of dry sand and the other two consisted of oil-saturated sand. The two similar oil-saturated specimens were prepared to enable concurrent testing and sample preparation. The oil-saturated sand specimens required a novel preparation procedure. The high cost of the silicone oil, and the imported Baskarp Cyclone sand, necessitated recycling of the specimens. The recycling process had to ensure that all disturbances caused by a footing test could be removed, and that the specimen could be reconstituted to the required uniform density. A fluidisation process involving liquefaction and mechanical agitation of the specimens was adopted. A process that entailed applying alternate components of vibration and downward hydraulic gradient was used to densify the specimens. The dry specimen was only used for a single test and was prepared by vibrating the container and the sand itself.

3.6.1 Sand specimen containers

The specimen containers were produced by Gue (1984) for preparing clay specimens and were also used by Martin (1994) for model footing tests on clay. The dural containers consisted of a flanged cylindrical section and a removable base. All three containers had an internal diameter of 450 mm. The effective internal height of the two containers with oil-saturated specimens was 421 mm and that of the container with the dry specimen was 446 mm. The dimensions of the containers are shown in Figure 3.15[a].

A number of alterations were required for this project. The containers for the oil-saturated specimens were made watertight. The incorporation of o-ring seals satisfied this objective. Two pore pressure transducer ports were incorporated into the walls of the oil-saturated specimen containers. Filters were attached to the base of the containers. The bases were machined to facilitate drainage, and to attach the associated plumbing. The filtering system consisted of a VYON F filter (supplied by Porvair plc; thickness = 0.75 mm and average pore size = 0.09 mm) placed on top a plastic geo-grid. These were screwed to the base by a circular bracket around the periphery of the container. The strong connection of the filter to the base was needed because of the intense vibrations applied to the containers during the densification process. The top flange of the containers was adapted to take the LVDT frame. The layout of the specimen containers and the associated plumbing, required for the liquefaction and densification processes, are shown in Figure 3.15[b].

3.6.2 Densification of sand samples

The dry sand specimen was poured in two layers, which were both vibrated. For the oil-saturated specimens, the Baskarp Cyclone sand had to be saturated with the viscous silicone oil. This process was complicated because of the low permeability (of the order of 10^{-7} m/s) of the resulting mixture. Some methods for saturating low permeability fluid/soil mixtures were suggested in the literature. Tan (1990) used a process of percolation under vacuum for a mixture with a similar permeability. The whole system was also heated to reduce the viscosity of the pore fluid and took approximately four days to complete. Lau (1988) found that 24 hours of vibration was sufficient to de-air a water/silt mixture and assumed that the specimen was fully saturated when bubbles stopped surfacing.

Pilot saturation tests were performed in a perspex tube to assess the feasibility of saturating Baskarp Cyclone sand with silicone oil. A saturation front was evident but air ingress into the apparatus under high vacuums was a problem. Vacuum was not used in the saturation process of the main specimens because of the risk of air ingress, and because the vacuum pump could not be activated for extended periods. Instead, the oil was percolated up through the sand and the specimen was vibrated until bubbles stopped surfacing.

A densification process employing alternating downward hydraulic gradient and vibration was investigated by the NGI (1993) and deemed to be effective. Homogeneous samples with relative densities of 80% were obtained. The effectiveness of this method in densifying the soil used in this project was assessed. The result of a pilot densification test performed in a perspex tube is shown in Figure 3.16. The surcharge was induced by applying a negative fluid pressure to the base of the sample. A relative density of 97% was achieved in less than two hours. The densification of the main samples, however, took longer. This method of densification would only be efficient if the initial relative density of the specimen is greater than 0%. Vibration is only effective if soil has strength (effective stress greater than zero). Full sedimentation of the sand particles out of the suspension was deemed a necessary precursor to the application of the downward hydraulic gradient and vibration.

The sedimentation of Baskarp Cyclone sand in oil was investigated. The settlement of the oil/sand interface is shown in Figure 3.17. The maximum void ratio (e = 1.075, R_D = 0%) is attained after about two hours. The velocity of the solidification front is about 7×10^{-6} m/s. It is difficult to estimate what the velocity of the solidification front should have been during sedimentation in the large specimen containers (used for the footing tests). The density of the fluidised sand in the large containers was higher and the distance travelled by the solidification front was greater.

Compaction of sand is known to be dependent on the shear strain. The degree of compaction increases with the number of cycles (to a limiting density) for a particular shear strain amplitude (Youd, 1972). The frequency of the shear strain cycles is thought not to be important. In saturated sand, vibration reduces the effective stresses and does not itself cause the particles of sand to settle. Settlement is dependent on the vertical force on the particles. Seepage forces generated by a downward hydraulic gradient increase the vertical force on the particles and induce settlement. An existing pneumatic powered vibrator was adapted to apply horizontal vibrations to the walls of the specimen containers. This apparatus is shown in Figure 3.15[b]. The amplitude of the vibrations could not be controlled but varying the pneumatic pressure altered the intensity as well as the frequency. The downward hydraulic gradient was induced by applying an air pressure of about 15 kPa to the oil surface above the sand. An asynchronous timing switch and two pneumatic solenoid valves were used to automatically control the air supply that drove the vibration and downward hydraulic gradient systems. The air supply to the specimen container was regulated and relief valves were installed in the lid to prevent the container being over pressurised. The automated densification system was only employed from test T106 onwards. In the previous oil-saturated tests, switching between the alternate densification components was done manually. The different methods of densification used to prepare the specimens are listed in Table 3.6.

The progress of the densification could only be monitored by removing the lid on the container. Density measurements (and cone penetrometer tests in some instances) were taken to chart the densification process. It was not possible to obtain the same relative density measurements in all of the samples prepared for each test. Approximately three days of sedimentation and five days of densification were required to obtain a relative density of 90%. The uniformity of the samples prepared for each test was investigated by measuring the hydraulic gradient along the vertical axis using the two pore pressure transducers in the wall of the container. A linear hydraulic gradient was taken to indicate that the sample prepared for a test was uniform (assuming that the permeability was dependent on the density). The uniformity was also gauged from the miniature cone penetrometer tests.

The footing response and cone penetrometer results of the initial tests suggested that the density distributions in the oil-saturated samples were not uniform. In an attempt to increase the density in the top layer of the samples, a surcharge was applied to the sand surface during the densification process of samples T107 and T309 (listed in Table 3.6). The surcharge consisted of a heavy dural disc with a filter on the base and around the edge. In test T107, a VYON F filter was attached to the base of the disc and a brush filter was placed around the edge. The filter around the edge of the disc was needed as the sand was in a suspended state and was susceptible to flow. On removal of the surcharge, it was evident that sand adhered to the VYON F filter although the surcharge was removed slowly. This indicated the possibility of uplift forces being applied to the sample, and consequent disturbance. It was deemed necessary to use a more permeable filter. In the densification of the sample for test T309, a fine stainless-steel woven wire cloth was used. This was the same material as used on the pore pressure port in the model footing. This filter proved more beneficial but it was still not possible to raise the surcharge at a sufficiently slow rate, and also to keep the disc level during the retraction process. Thus the surcharge system was not used in the preparation of the later samples. Use of the surcharge system would have entailed the production of a retraction apparatus, incorporating a rigid frame and a stepper motor drive system for example.

3.6.3 Fluidisation of sand samples

The term fluidisation is used to describe a state in which the effective stress of the sand was reduced to zero after total rearrangement of the sand particles by stirring. The fluidisation process was needed to remove heterogeneities, such as localised soil densification under the footing, caused by previous tests. NGI (1993) concluded that full liquefaction of water-saturated fine-grained sand was possible if the hydraulic gradient was applied with caution. In this project, it was not possible to consistently induce full liquefaction of the samples, and agitation was needed to reduce the samples to a fluidised state. In some of the initial attempts at liquefaction, localised piping occurred. Once piping occurred, it was impossible to achieve full liquefaction. The fact that the samples could not be fully liquefied may have been due to the low permeability. The estimated vertical total stresses and the measured pore pressures (at the pore pressure ports) during one of the liquefaction processes are shown in Figure 3.18. The effective stresses reduced to zero between t = 2.5 hr and t = 3 hr. The data in Figure 3.18 implies that the effective stresses are negative after about t = 3 hr. Since effective stresses cannot be negative, the estimates of total stress in Figure 3.18 are probably not representative. The resistance due to wall friction, acting against the upward movement of the sand specimen, may explain this discrepancy. The coefficient of permeability during the liquefaction process was estimated at 7×10^{-7} m/s (assuming a constant head). This value was similar to the results of the permeability tests ($k = 1.8 \times 10^{-7}$ m/s).

An automatic stirring apparatus was built to agitate the samples as manual agitation proved too demanding. The stirrer is shown in Figure 3.19 and consisted of a rotating paddle that was slowly lowered into the sand. Each rotation of the paddle scraped a thin layer of sand from the sample. This sand remained fluidised by the movement of fins that were placed above the paddle. The whole sample was gradually fluidised as the paddle was lowered to the bottom of the container. An electric motor with a variable speed control was used to power the paddle. This motor incorporated a trip switch to prevent overloading of the motor and was fixed to an elevated platform that was guided by two vertical bearings. It was necessary to devise a cooling system for the electric motor as it was operating at full capacity for long periods. The paddle was connected to the motor by an extended shaft. The lowering of the platform and the paddle was controlled by a lead screw and stepper motor arrangement. A 23HS-108 stepper motor was connected to the stepper motor control box (SM9343), which in turn was controlled by the host PC. Trip switches were incorporated to ensure that the paddle ceased penetration about 30 mm above the base of the tank to protect the filter. The paddle was lowered at approximately 0.01 mm/s and rotated at about 20 r.p.m.. This resulted in an average time for fluidisation of five hours. The fluidisation apparatus in Figure 3.19 proved highly effective but a more powerful electric motor for turning the paddle would have improved performance. It was necessary to manually agitate the bottom 30 mm of sand but this was only done from test T309 onwards. The samples that were fully fluidised before being densified are indicated in Table 3.6.

4 Sand Samples

4.1 Introduction

The Baskarp Cyclone sand that was used as the foundation soil was supplied by A.B. Baskarpsand of Habo, Sweden. Three different sand specimens were prepared: one dry and two oil-saturated. The two saturated sand specimens were saturated with viscous silicone oil. Only one test was performed on the dry specimen. The two oil-saturated specimens were recycled *in-situ* to facilitate multiple testing. Baskarp Cyclone sand was used previously at Oxford and at the Norwegian Geotechnical Institute (Houlsby, 1995; and NGI, 1994b).

4.2 Index properties

The Baskarp Cyclone sand was a silty fine sand produced using a cyclone separation process. The grains were sub-angular to angular and were 84.3% Quartz (NGI, 1994b). The relationship between the void ratio and the coefficient of angularity, suggested by Holubec and D'Appolonia (1973), indicated that the coefficient of angularity for Baskarp Cyclone sand was probably 1.65.

4.2.1 Particle size characteristics

The particle size of soil is usually characterised by a particle size distribution curve. In situations involving submerged sand, the distribution curve can be used as an indicator of drainage related behaviour such as permeability and consolidation. A particle size analysis was performed on Baskarp Cyclone sand at Oxford University using a "CILAS Granulometer" situated in the Geography Department. The sand was mixed with water using an ultrasonic mixer and a calgon dispersing agent, before being placed in the Granulometer. The particle size distribution curves obtained by the aforementioned procedure and by the NGI (1994b) are shown in Figure 4.1. The median grain size d_{50} from the Granulometer curve was 0.058 mm. The d_{60} and d_{10} sizes were 0.069 mm and 0.018 mm, and the uniformity coefficient (defined as the ratio of d_{60} to d_{10}) was 3.8. The value of the uniformity coefficient was on the border between the well-graded and uniform classifications (Jackson and Dhir, 1988).

4.2.2 Specific gravity of solids

The unit weight of the solid fraction was determined to be 26.40 kN/m^3 by the NGI (1994b). This corresponds to a specific gravity of G_s = 2.69. The author measured the volume of solids required for the calculation of G_s using a simple water displacement method with vibration to remove the air bubbles. G_s was found to be 2.71 but the value determined by the NGI was used in this investigation.

4.2.3 Relative density

It is generally accepted that the relative density is a useful parameter for characterising density, and it is defined as follows,

$$\begin{aligned} R_D &= 100\left(\frac{e_{max}-e}{e_{max}-e_{min}}\right) \\ &= 100\frac{\gamma_{max}}{\gamma}\left(\frac{\gamma-\gamma_{min}}{\gamma_{max}-\gamma_{min}}\right) \end{aligned} \qquad [4.1]$$

where e is the void ratio, and γ represents the dry unit weight. The maximum and minimum dry unit weights were determined by the NGI (1994b) using the following methods. A minimum dry unit weight of γ_{min} = 12.72 kN/m^3 was determined by pouring sand into a mould whilst maintaining a minimal free-fall height for the sand particles. Dry and wet tamping, in combination with vibration, was used to determine a maximum dry density of γ_{max} = 16.85 kN/m^3. These values correspond to e_{max} = 1.07 and e_{min} = 0.57. The author calculated a maximum void ratio of 1.01 using the "tilt" method suggested by Kolbuszewski (1948), and a minimum void ratio of 0.59 using a combination of downward hydraulic gradient and vibration. The NGI values were used in this investigation.

4.3 Drainage properties

The use of viscous pore fluids in model testing was initially driven by the need to scale consolidation behaviour in the centrifuge (Tan, 1990). But it becomes increasingly difficult to saturate, densify, and fluidise fine sand samples as the viscosity of the pore fluid increases. Tan used circulatory systems oil with a kinematic viscosity of 437 mm^2/s as the pore fluid in his fine sand samples. He investigated the cyclic behaviour of spudcan footings in the centrifuge, using the circulatory systems oil instead of silicone oil because of the reduced cost. Tan utilised the temperature dependency of the circulatory systems oil in his sample preparation procedure. He heated the oil to reduce its viscosity during sample preparation.

Allard *et al.* (1994) used a viscous pore fluid developed at Delft Geotechnics in their centrifuge tests of a model skirted gravity platform. This fluid was stated to have similar physical and chemical properties to that of water, but its actual constituents have not been made public. The constitutive behaviour of sand saturated with the Delft fluid and of sand saturated with water is similar. Use of the fluid resulted in consistent samples and overcame some of the problems of saturating sand samples with a viscous pore fluid. The viscous pore fluid was only percolated into the sample (under an applied hydraulic gradient) after full saturation and densification had been achieved. Laboratory tests indicated linearity between the permeability of water-saturated and Delft fluid-saturated samples. The permeability was found to be inversely proportional to the kinematic viscosity. The Delft fluid can be prepared to a range of viscosity values and a value of 300 mm^2/s was used in the model tests of Allard *et al.*.

Watson and Randolph (1997) found that by using silicone oil as the pore fluid in their tests on a model caisson in the centrifuge, they could induce undrained behaviour. Their tests were conducted on calcareous silt and utilised the findings of Finnie (1993) on the relationship between loading rate and sample permeability.

The main disadvantages with using oils were the high cost (especially of silicone oils) and the difficult sample preparation procedures. Silicone oils do, however, possess many beneficial properties: such as being inert; non-hazardous; and having well defined values of viscosity. A water-soluble pore fluid would overcome the sample preparation problems but such fluids were not readily available. The Delft fluid was not widely available and was classified as a hazardous chemical, for the purposes of handling and disposal. Glycerine could be a water-soluble alternative, but information on the use of this fluid in model testing could not been found.

4.3.1 Pore fluid characteristics

Silicone oil was used as the pore fluid in the two saturated sand samples. The choice of the viscosity of the oil was influenced by the need to achieve a low permeability sand sample, and the difficulties associated with saturating such samples. The silicone oil was a Dow Corning® 200 Fluid with a kinematic viscosity of 100 mm^2/s. This type of water-clear silicone oil was noted for its thermal and chemical stability, low vapour pressure, low surface tension, shear-breakdown resistance, and essentially non-toxic nature. The 100 mm^2/s oil had a specific gravity of 0.96, and its viscosity reduced slightly with increasing temperature. The viscosity of this oil was approximately 100 times that of water, which has a viscosity of 0.897 mm^2/s at 25° C. The bulk modulus of the silicone oil was approximately 800 MPa at a relevant stress level, as compared with a value of about 2300 MPa for water. Dow Corning could not supply information on the cavitation pressure of the silicone oil but Houlsby (1995) concluded that cavitation was probable at low pressures.

4.3.2 Coefficient of permeability

Three constant head permeability tests were performed on oil-saturated Baskarp Cyclone sand. The permeability apparatus is shown in Figure 4.2. The applied head varied by only 0.15% and the results from the permeability tests are shown in Table 4.1. The tests indicated an average coefficient of permeability of 1.8×10^{-7} m/s. The relative density of the samples was approximately $R_D = 84\%$.

The NGI (1994b) performed permeability tests with water-saturated Baskarp Cyclone sand at relative densities of 63% and 89%. The flow rate in a modified triaxial apparatus was measured during loading and unloading, at consolidation stresses from 20 kPa to 110 kPa. The consolidation stress and the state of stress (virgin loading or unloading) did not affect the permeability. If one assumed a linear R_D:k relationship then the NGI data suggests a permeability of 8.0×10^{-6} m/s for water-saturated Baskarp Cyclone sand at a relative density of 84%.

The permeability of the oil-saturated sand measured by the author was approximately 44 times less than the NGI estimate for water-saturated sand. One would have expected the two values to differ by a factor of approximately 100, as the coefficient of permeability defined by the Kozeny-Carman Equation is inversely proportional to the fluid viscosity (Scott, 1980).

4.3.3 Coefficient of consolidation

The one-dimensional coefficient of consolidation c_v of water-saturated Baskarp Cyclone sand was derived from element tests conducted by the NGI (1994b). The values of c_v are listed Table 4.2. In the one-dimensional tests, the coefficient of consolidation was calculated using Equation 2.35. In the three-dimensional tests, the coefficient of consolidation was calculated using Equation 2.44 and then converted to the one-dimensional value. The average one-dimensional coefficient of consolidation for loading was 3.6 x 10^{-2} m^2/s and for unloading it was 2.7 x 10^{-1} m^2/s. The coefficient of permeability of water-saturated Baskarp Cyclone sand was a factor of 44 greater than that of oil-saturated sand (Section 4.3.2). One would expect the coefficient of consolidation of the oil and water-saturated sand to differ by a similar factor, assuming that the modulus was constant. Thus for oil-saturated sand the one-dimensional coefficient of consolidation should have been approximately 8 x 10^{-4} m^2/s for loading and 6 x 10^{-3} m^2/s for unloading (for elastic shear moduli of about 10 MPa and 70 MPa respectively).

The coefficient of consolidation is a function of the effective stiffness and the permeability of a soil. For a given relative density the permeability does not change significantly but the change in stiffness can be great. For example, consider the difference between the virgin penetration stiffness and the unloading stiffness. Thus the coefficient of consolidation is sensitive to the effective stress level and the state of stress. Wood (1990) states that the coefficient of consolidation is not a satisfactory parameter for describing the movement of pore fluid in general transient problems and recommends the use of the permeability in conjunction with an effective stress model.

4.4 Stiffness properties

The elastic shear moduli G of Baskarp Cyclone sand were derived from element tests reported in NGI (1994b). Moduli for unloading are compared with predictions from the literature in Figure 4.3. The triaxial and oedometer data shown in Figure 4.3 is also listed in Table 4.2. The moduli derived from the NGI tests were all greater than the comparable small-strain theoretical values. At a relative density of 85%, the experimental values were a factor of approximately two to three greater than the relevant theoretical values. At a maximum effective mean stress of 87 kPa and a strain level of 0.04%, an elastic shear modulus of 90 MPa was estimated from triaxial tests BC10 to BC12. The oedometer tests (BC13 and BC14) indicated an elastic shear modulus of about

35 MPa at a maximum effective mean stress of 30 kPa. The elastic shear moduli of the oedometer tests were calculated from the constrained moduli M', using Equation 2.42 and assuming a Poisson's ratio of 0.2. From the data for the triaxial tests in Table 4.2, it is evident that the elastic shear modulus for unloading was a factor of about seven greater than the modulus for loading.

The Poisson's ratio from tests BC10 to BC12 was checked using the following relationship,

$$\nu = \frac{\sigma_r + \frac{\sigma_a}{2}\left(1 - \frac{\varepsilon_v}{\varepsilon_a}\right)}{\sigma_r\left(2 - \frac{\varepsilon_v}{\varepsilon_a}\right) + \sigma_a} \qquad [4.2]$$

where σ_a and σ_r are the axial and radial effective stresses, and ε_a and ε_v are the axial and volumetric strains. The average value of the Poisson's ratio during loading was found to be less in the denser samples and was approximately equal to 0.25 at a relative density of 90%. The Poisson's ratio from the oedo-triaxial test NTH93-2 was calculated using Equation 2.42 and the measured values of the elastic shear modulus and the constrained modulus. Test NTH93-2 indicated a Poisson's ratio of 0.2 during loading for a relative density of 85%.

4.5 Strength properties

Shear strength parameters were derived from triaxial tests. These parameters are compared with predictions of Bolton's (1986) method, and a low stress adjustment to Bolton's method is proposed.

4.5.1 Shear strength of sand

The results from static triaxial tests conducted by the NGI (1994b) on Baskarp Cyclone sand were used to derive strength parameters. The results from six drained and seven undrained compression tests were analysed. A drained and an undrained test with silicone oil as the pore fluid (the same fluid as used in the footing tests, Section 4.3.1) were included in the analysis. Two of the drained tests and four of the undrained tests were performed on samples that had previously been subjected to undrained cyclic tests. These tests are sometimes referred to as tests on "disturbed samples". The results of the drained and undrained triaxial tests are listed in Table 4.3 and Table 4.4 respectively. The drained and undrained tests are also shown in Figure 4.4 and Figure 4.5 respectively. The results from the tests on the disturbed samples are only included in Figure 4.4[d] and Figure 4.5[d].

The drained tests indicated a peak friction angle, which reduced with increasing effective mean stress, of 43 ° at an effective mean stress of 100 kPa. The peak friction is shown in Figure 4.6. The relative density of the drained samples was approximately 80%. The peak friction angle from the oil-saturated test was about 2 ° less than that from the comparable water-saturated test (tests BC23 and BC15 respectively). The difference in the dilation angle, which is defined as $\beta = \tan^{-1} d\varepsilon_q/d\varepsilon_p$, between tests BC15 and BC23 is evident in Figure 4.4[f]. This reduction in the peak friction angle is consistent with the findings of Eypton (1982) and Bielby (1989), which were reported in Tan (1990). The peak and critical states were highlighted in all the plots in Figure 4.4. The peak state shown in Figure 4.4[c] consistently occurred at a volumetric strain of about -2%. The stress ratio η shown in Figure 4.4[e] tended to a value of approximately 1.3 at the critical state. The stress ratio is equal to q/p'. The critical state friction angles are also shown in Figure 4.6. A critical state friction angle of 33 ° was representative of an effective mean stress of about 100 kPa if one ignores the high value from test BC16. The presence of silicone oil does not seem to affect the critical state friction angle. One would expect the critical state friction angle to reduce with increasing effective mean stress, especially at low levels of stress (Chu, 1995). It was probable that the critical state had been attained at the end of the drained tests. The dilation angle β shown in Figure 4.4[f] is constant and approximately equal to 90 °. The critical state points shown in Figure 4.4[d] indicate a probable critical state line in $\ln(p'):e$. The parameters of the best-fit line are $\lambda = -0.016$ and $\Gamma = 0.75$. These values are similar to those of the steady state lines of some of the sands referred to by Altaee and Fellenius (1994). It should be noted that the sands referred to consisted of larger grain sizes than Baskarp Cyclone sand, and that the steady state is usually derived from undrained tests and the critical state from drained tests. Been *et al.* (1991) found that these lines were coincident for some sands.

The results from the undrained tests are shown in Figure 4.5. The difference in the shear stress and pore pressure responses between the two undrained water-saturated tests, shown in Figure 4.5[a] and [c], is due to the difference in relative density. In test BC1 the relative density was 50% and in test BC2 it was 80%, as listed in Table 4.4. The dilatancy that is evident in the drained tests manifests itself as negative pore pressures in the undrained tests. The effect of the silicone oil pore fluid is evident in the undrained tests as it is in the drained tests. The oil-saturated undrained test BC24 exhibits a less intense shear stress and pore pressure response than the comparable water-saturated test BC2. The stress ratio response shown in Figure 4.5[e] of the oil-saturated test is also lower. This reinforces the observation from the drained tests that the silicone oil had a degrading effect on stiffness and strength. It is probable that the undrained tests had not reached a steady state even though the values of the stress ratio at the end of the tests are similar to the critical state values observed in the drained tests. The stress paths (shown in Figure 4.5[b) are moving up and along the critical state line as the pore pressures (shown in Figure 4.5[c]) are still decreasing at the end of the tests. A probable steady state line is plotted in $\ln(p')$:e space in Figure 4.5[d]. The parameters of the best-fit line are λ = -0.072 and Γ = 0.87. These values are similar to those of the steady state lines of some of the sands referred to by Altaee and Fellenius (1994). They are different, however, from the critical state line deduced from the drained tests.

The critical state line of the drained tests and the steady state line of the undrained tests are compared in Figure 4.7. The two state lines do not seem to be coincident. But if one ignores test BC1, the undrained test with $e \cong 0.82$, it is possible that the two state lines are coincident. A reduction in the slope of the critical state line at $p' \cong 1000$ kPa was observed by Been *et al.* (1991). Therefore it is possible that the critical state line of Baskarp Cyclone sand might have a similar shape.

It was probable that the pore pressures in the undrained tests would have continued to decrease until cavitation of the pore fluid occurred. McManus and Davis (1997) found that dilatant sand samples exhibited signs of pore fluid cavitation when the pore fluid pressure reduced to an absolute value of -100 kPa. On the onset of cavitation, their samples behaved as if undergoing drained shearing. The plot of ε_q:u in Figure 4.5[c] does not indicate if cavitation is probable in the tests on Baskarp Cyclone sand. An indication of the likelihood of cavitation is gained by normalising the excess pore pressure by the pore pressure increment u_{cav} required for an absolute pressure of -100 kPa. The value of u_{cav} is assumed to be equal to the initial excess pore pressure, plus the backpressure, and plus 100 kPa. The samples should cavitate at a value of $u/u_{cav} = -1$. The normalised pore pressures are shown in Figure 4.8. From this figure it is evident that the absolute pore pressures in the undrained tests do not reach a value of -100 kPa. Therefore cavitation was not probable. Tests BC3 and BC8, which were performed on disturbed samples, exhibited reduced rates of pore pressure change after shear strains of about 16%. The reduction in the rate of pore pressure change in these tests was probably due to the effects of disturbance, as opposed to those of cavitation.

4.5.2 Drained triaxial peak states and Bolton's method

The triaxial friction angles of Baskarp Cyclone sand are compared with predictions of Bolton's (1986) empirically derived R_D:p':ϕ'_{max} relationship. Bolton defined a relative dilatancy index as

$$I_R = R_D\left(Q - \ln p'\right) - R \qquad [4.3]$$

where R_D is the relative density and p' is the effective mean stress at failure, measured in kPa. The constants Q and R are fitting parameters. Bolton found that values of Q = 10 and R = 1 fit the observed behaviour well (of element tests on quartz and feldspar sands, with fine to coarse sizes). The relationship between the peak and critical state friction angles, and the relative dilatancy index, was defined as

$$\phi'_{max} - \phi'_{crit} = 3I_R \qquad [4.4]$$

for triaxial strain and for $0 < I_R < 4$. The upper bound to I_R of four, which was equivalent to a lower bound to p' of approximately 150 kPa at R_D = 100%, was necessary because the dilatancy of sand below effective mean stresses of 150 kPa was found to be relatively insensitive to changes in stress. It should be noted, however, that some investigators observed no reduction in sensitivity above I_R values of four. This was the case for Berlin and Mol sand (de Beer, 1965a).

An abrupt cessation of further increases in dilatancy at points defined by I_R = 4 probably does not occur in practice. But this limitation provided a conservative check on an aspect of behaviour that was not well defined, and that was of little relevance to prototype-size geotechnical problems. In model-size footing tests on dense sand, the relative dilatancy index of the foundation sand is commonly greater than the limiting value of four. The "at rest" horizontal effective stresses associated with the miniature cone penetrometer tests performed in this investigation where also so low as to produce I_R values greater than four. It was therefore necessary to formulate a theoretical framework for predicting the $R_D:p':\phi'_{max}$ behaviour of Baskarp Cyclone sand at low stress states. This theoretical framework is outlined in Section 4.5.3 below.

A comparison of the experimental and the theoretical dilation angles is shown in Figure 4.9. The experimental angles are derived from the drained triaxial tests on Baskarp Cyclone sand and the theoretical angles are assumed equal to $3I_R$. Two different methods are used to estimate the experimental angles: one method using the peak and critical state friction angles listed in Table 4.3; and the other using the volumetric and major principal strain increments. The relationship between the dilation angle and the strain increments

$$\phi'_{max} - \phi'_{crit} = 10\left(-\frac{d\varepsilon_v}{d\varepsilon_1}\right)_{max} \qquad [4.5]$$

is derived from the following relationship given by Bolton (1986)

$$\left(-\frac{d\varepsilon_v}{d\varepsilon_1}\right)_{max} = 0.3I_R \qquad [4.6]$$

and Equation 4.4. The quantities $d\varepsilon_v$ and $d\varepsilon_1$ represent the volumetric and major principal strain increments respectively. The experimental results shown in Figure 4.9 are lower than the predictions but the difference is not greater than about 2 °. The experimental dilation angles for the water-saturated tests that are calculated by the two different methods are also similar. The experimental results for the oil-saturated test differed considerably. One would expect the dilation angle in the oil-saturated test to be approximately 2 ° less than in the equivalent water-saturated test, as is the case when the dilation angle is calculated from the friction angles.

The peak friction angles from the drained triaxial tests are compared with the predictions of Bolton's (1986) method in Figure 4.10. A critical state friction angle of 31.5 ° is used in Bolton's method although the angle measured in the triaxial tests is about 33 °. This is because the lower angle produced a better fit between the triaxial data and Bolton's prediction. It is also probable that the critical state friction angles used in the derivation of Bolton's method were measured at larger values of effective mean stress than those measured in the triaxial tests on Baskarp Cyclone sand. From Figure 4.10, it is evident that the difference between the experimental and theoretical values of the peak friction angle is not greater than 2 °. They were of a similar accuracy as observed by

Bolton.

4.5.3 Low stress adjustment to Bolton's method

It was necessary to formulate a low stress adjustment to Bolton's (1986) R_D:p':ϕ'_{max} relationship to predict the friction angle of the sand samples from the miniature cone penetrometer tests. The cone penetrometer was penetrated to a maximum depth of about 200 mm. The maximum horizontal effective stress of relevance to the cone penetrometer tests was 3.4 kPa for R_D = 100%. Substituting these values into Equations 4.3 and 4.4 results in a relative dilatancy index of 7.8 and a peak friction angle of 55 °. This value of I_R was greater than the suggested limit of four and the estimated friction angle is unreasonably high. If the limit of I_R = 4 is adhered to, the estimated friction angles of all of the sand samples would be 43.5 ° (as deduced from Figure 4.10). Thus it was necessary to devise a low stress R_D:p':ϕ'_{max} relationship.

The chosen low stress adjustment to Bolton's (1986) method is shown in Figure 4.11. The friction angles from the triaxial tests and an additional friction angle estimated from the bearing capacity tests of Byrne and Houlsby (1999) (Section 2.3.1 and Figure 2.4) are also included in Figure 4.11. The characteristics of the adjustment are influenced by the following factors:

— In Bolton (1987), the peak friction angle was shown to be less sensitive to changes in the effective mean stress when I_R > 4. An abrupt cessation of further increases in ϕ'_{max} at a specified value of I_R, however, seemed unreasonable. A reduction in the slope of the ln(p'):ϕ'_{max} curve at low stresses was probably more realistic and this characteristic was adopted.

— In Bolton (1987), the effect of the relative density on the peak friction angle (at a constant effective mean stress) was found to be generally the same at low and high values of stress.

The two additional parameters that are required to define the adjusted relationship are: the value of I_R at R_D = 100% at which the slope of the ln(p'):ϕ'_{max} curve changed, denoted I_{RC}; and a parameter a that affected the slope of the adjusted section. The slope of the ln(p'):ϕ'_{max} curves in Bolton's method is sensitive to the factor that I_R was multiplied by in Equation 4.4. This factor is labelled m_H and is equal to three in Equation 4.4. The equivalent parameter in the adjusted section is labelled m_L, and a is defined as the ratio of m_L to m_H. The subscripts "L" and "H" are used to differentiate between the low and high stress sections, or the adjusted and normal sections shown in Figure 4.11. It is necessary to alter the value of the parameter Q in the adjusted section. The parameter Q_L is defined as

$$Q_L = I_{RC}\left(\frac{1}{a} - 1\right) + Q_H \qquad [4.7]$$

where Q_H is equal to ten. The effective mean stress at the point at which the slope changed is defined as

$$p' = \exp\left[\frac{R_D(aQ_L - Q_H) + R(1-a)}{R_D(a-1)}\right] \qquad [4.8]$$

where R is equal to one.

Sufficient data were not available to rigorously substantiate the chosen characteristics of the low stress adjustment. Only three data points from triaxial tests, and a point that was estimated from the 16 bearing capacity tests performed by Byrne and Houlsby (1999), were available. Thus the chosen values of the parameters I_{RC} and a (I_{RC} = 5.0 and a = 0.2) are somewhat arbitrary. The value of I_{RC} is probably representative if one assumes that the estimated values of ϕ'_{max} and p' for the bearing capacity tests are accurate.

4.6 Characterisation of samples

The relative density parameter is commonly used in the characterisation of sand behaviour. In the case of footings on sand, the bearing capacity is usually correlated to the relative density, via the peak friction angle. The elastic shear modulus, and hence the elastic response of footings, is also estimated from the relative density.

In this investigation, the relative density of the sand base was initially estimated from the volume and the mass of the samples (it was a bulk relative density measurement). Three different sand specimens were prepared. One of the specimens consisted of dry sand and the other two consisted of oil-saturated sand. The specimen used in each of the tests was identifiable by the second character in the test names; for example T**1**01, T**2**04 and T**3**09. The properties of the three sand specimens are listed in Table 4.5. The oil-saturated specimens were recycled *in-situ* to create a number of different samples.

The relative density measurements could not be used to characterise the sand samples because the correlation between the R_D measurements and the footing response was not good. The characterisation stiffness K_{ch} (which was the virgin-penetration stiffness of the footing measured between 400 N and 500 N) was found to be representative of the load:deformation behaviour of the footing. This parameter was used to characterise the sand samples and is plotted against the relative density in Figure 4.12. A similar load:deformation response was observed in tests on different samples in which the relative density measurements ranged from 88% to 102% (compare tests 6, 12, 19 and 20 in Figure 4.12). At these high values of R_D one would expect the response of the footing to be very sensitive to changes in the value of R_D. It was probable that the bulk relative density measurements were not representative of the state of the sand in the top layer of the samples. The samples may not have been uniform. Byrne and Houlsby (1999) found that even when there was a high degree of confidence in their relative density measurements, where the standard deviation of 16 samples (with $R_D \cong 97.3\%$) was only 1.7%, the initial stiffness and bearing capacity varied by ±33% and ±22% respectively.

In the sections below, it is attempted to qualify the degree of uniformity of the samples, and to use the cone penetrometer results to characterise the top layer of the samples.

4.6.1 Uniformity of samples and possible errors in R_D

The uniformity of the density distribution in the samples (along the vertical axis) was investigated using the pore pressures measured in the downward hydraulic gradient tests. The effect on the relative density measurements of a very dense layer of sand is checked. This layer was at the bottom of some of the samples and was not fluidised. The possible sand losses through the filter are also assessed.

A downward hydraulic gradient was induced in the samples by applying an air pressure to the oil surface and allowing the oil to drain from the base of the container. The pore pressure was measured at two points along the wall of the container and it was assumed that the base of the sample was at atmospheric pressure. The permeability of the VYON F filter at the base of the samples was calculated from the manufacturer's specifications. It was also deduced from permeability tests. This was to ensure that the head loss through the filter was relatively small. The permeability of the filter was estimated to be six times greater than that of the sand samples (Mangal, 1994).

The results from the downward hydraulic gradient tests are shown in Figure 4.13. It was known that there was a 30 mm thick layer of sand at the bottom of some of the samples that were not fully fluidised. In the preparation of the samples for tests T102, T103, and T105 to T108, the samples were not fully fluidised (Section 3.6.3 and Table 3.6). The effect of the dense layer is evident in the downward hydraulic gradient tests shown in Figure 4.13, as the pore measurements in samples T309 to T311 are closer to the theoretical predictions. The downward hydraulic gradient tests seem to indicate that the samples for test T309 and later tests were uniform. Although downward hydraulic gradient tests were not performed on the samples for test T112 onwards, it is assumed that the degree of uniformity in these samples was similar to that in samples T309 to T311. This is because the preparation procedures did not differ. Even though it is probable that the degree of uniformity was similar in the samples of tests T309 onwards, there were large variations in the bulk density measurements of samples on which the load:deformation footing behaviour was similar. Compare tests T311 and T114 in Figure 4.12.

The effect of the layer of sand that was not fluidised on the bulk relative density measurements was estimated. If it is assumed that the relative density of the layer was 100%, then the relative density of the remainder of the sample would have been about 3 % less than the bulk relative density measurement. This error cannot account for the large difference in the bulk relative density measurements of samples T105 and T311 for instance, which exhibited similar load:deformation footing behaviour as shown in Figure 4.12.

Traces of sand were evident in the drainage system beneath the sample containers and it was possible that sand might have leaked into the drainage channels in the base of the sample containers. This could not be verified, however, as the containers were not dismantled. If there was leakage, the dry weight of the samples would have been reduced by only 1%, lowering the relative density measurements by about three points. If the filter was damaged, the probability of this damage occurring would have been highest during the latter stages of the test programme. The large difference between the relative density measurements of samples T317 and T319 (the penultimate and last tests performed on specimen three) could not be explained by the possible effects of leakage. Since only trace quantities of sand were evident in the external drainage system, it was not probable that the internal drainage channels were filled with sand.

The variability in the values of R_D could not be explained by the possible sources mentioned in the preceding paragraphs. It was most probable that the samples were not uniform. The downward hydraulic gradient method was too crude an instrument. More pore pressures transducers, and a well defined relationship between the density and the permeability, would have increased the usefulness of this method.

4.6.2 Cone penetrometer results

The miniature cone penetrometer tests were performed at the centre and at the sides of the samples, and the majority of these were performed after the footing tests were conducted. Since none of the penetrometer tests were before-centre tests (performed at the centre of the samples and before the footing tests), it was necessary to estimate the before-centre cone tip resistance values. The estimated cone tip resistance values are compared with the behaviour of the footing and used to estimate the relative density of the top layer of the samples.

The relative density, the characterisation stiffness, and the cone tip resistance measurements are listed in Table 4.6. The characterisation stiffness K_{ch} is the vertical virgin-penetration stiffness of the footing measured between V = 400 N and V = 500 N. The values of the cone tip resistance q_c are the average of the various penetrometer tests that were performed on each sample (Section 5.7 and Table 5.2).

The tip resistance between 50 mm and 100 mm of penetration is taken to be representative, as it is assumed that the characteristics of this layer of the sample had a significant effect on the response of the footing. The values of q_c estimated from the penetrometer tests performed at the centre of the samples, and before the footing tests, are also taken to be representative of the undisturbed characteristics of the samples (the before-centre penetrometer tests are taken to be representative). The footing tests might have resulted in disturbance of the samples, and the cone tip resistance from tests performed at the sides of the samples was probably influenced by the boundary conditions of the sample containers.

Estimation of the before-centre cone tip resistance

The measured q_c values shown in Figure 4.14[a] from the tests on samples 2 to 12 indicate a possible linear relationship between the results from the tests at the side and at the centre of the samples. The cone tip resistance at the side is approximately 43% greater than the resistance at the centre of the samples. This relationship is used to estimate the before-centre values for samples 12 to 20, as listed in Table 4.6. A similar relationship shown in Figure 4.14[c] can be deduced when all of the after-centre and after-side results are plotted against the characterisation stiffness.

The results in Figure 4.14[c] indicate that the variation between the cone tip resistance and the characterisation stiffness could be approximated by a linear relationship. This is expected as it is assumed that $\ln(q_c)$ is proportional to ϕ when σ'_h is constant (Houlsby and Hitchman, 1988), and that $\ln(K_{ch})$ is roughly proportional to ϕ as well. Note that a linear relationship is assumed between K_{ch} and the bearing capacity, and hence between K_{ch} and N_γ (Figure 6.9). The standard error in q_c, when predicted from K_{ch}, is less than 200 kPa in the three relationships shown in Figure 4.14[c].

Both before and after penetrometer tests were only performed on two samples (samples 12 and 13) and the q_c values from the tests on one of these samples were spurious. The before-side and after-side results from the test on sample 12 are compared in Figure 4.14[b]. Due to the low confidence in the results shown in Figure 4.14[b], the relationship deduced by comparing all of the before-side and after-side results in Figure 4.14[c] is taken to be representative. Ideally, one would prefer to have a relationship based on a direct comparison of before-side and after-side data. The degree of variability in the K_{ch}:q_c relationships are, however, acceptable. The before/after ratio shown in Figure 4.14[c] is used to estimate the before-centre q_c values for samples 2 to 11, as listed in Table 4.6. The before/after ratio indicates that the disturbance caused by the footing tests resulted in a 40% reduction in the cone tip resistance. Note that the penetrometer results from the tests on the dry sample (the tests on sample T204) are also assumed to be affected by the footing test. It is probable, however, that the disturbance was mostly due to partially-drained loading effects as opposed to the effects of drained shearing. The majority of the disturbance probably occurred at the end of the footing tests when the footing was retracted, inducing uplift forces in the samples.

It is possible to compare two different estimates of some of the before-centre cone tip resistance values. That is, values estimated from measured before-side values and from measured after-centre values. This comparison is shown in Figure 4.15 and indicates that the estimates from the before-side values are generally larger than those from the after-centre values. This discrepancy might have been due to localisation of the disturbance effects and/or the effect of disturbance on the centre/side relationship. The estimated before-centre values of q_c are plotted against K_{ch} in Figure 4.16. This comparison indicates that the data could be fit by a linear relationship. The variability in the q_c values, resulting from the variability in centre/side and before/after relationships shown in Figure 4.14[a] and [c], is acceptable.

Relative density of the top layer of the samples

The bulk relative density measurements were not consistent with the response of the footing. The results from the cone penetrometer tests, however, indicate that the relationship between the cone tip resistance and the characterisation stiffness could be approximated by a linear expression.

Using the cone tip resistance values, it is possible to estimate the relative density of the top layer of the samples (R_D at a depth of about 75 mm). Two bulk measurements of R_D that were assumed to be accurate are used in the development of a R_D:q_c relationship. The two R_D:q_c points are derived from the dry sand sample. This sample consisted of two layers with different values of density.

The methodology of Houlsby and Hitchman (1988) is used to derive a relationship between the relative density and cone tip resistance. Houlsby and Hitchman found that the cone tip resistance (from calibration chamber tests of a cone penetrometer in sand) could be expressed as a function of the effective horizontal stress and of a cone tip resistance factor,

$$q_c = N_h \, \sigma'_h \qquad [4.9]$$

The cone tip resistance factor N_h is related to the peak friction angle as follows,

$$\ln N_h = 0.16\left(\phi'_{max} - 9\right) \qquad [4.10]$$

The above expression was found to fit the results from calibration chamber tests performed by different investigators on a variety of sands. It is necessary to alter the constants in Equation 4.10 to fit the results from the cone penetrometer tests of this investigation. This difference is probably due to the lower stress states encountered in this investigation, and to the smaller diameter of the cone penetrometer that was used. The peak friction angle of the Baskarp Cyclone sand is estimated using the low stress adjustment to Bolton's (1986) method that is postulated in Section 4.5.3.

The results from the cone penetrometer test at the centre of the dry sample (CP04C) are used to derive two points in ϕ'_{max}:ln(N_h) space as shown in Figure 4.17[b]. The q_c values shown in Figure 4.17[a] are equal to the measured after-centre values multiplied by 1.66. The effective horizontal stress is also plotted in Figure 4.17[a], and the boundary between the two layers is shown. The horizontal effective stress of the sand samples is assumed to be equal to the vertical effective stress, as the earth pressure coefficient K_0 is assumed equal to one. The vibration that was applied during the sample preparation procedure should have countered any overconsolidating effects. The average values of ln(N_h) and ϕ'_{max} from two sections of the cone penetrometer profile (from a section in the top layer and from a section in the denser bottom layer) are shown in Figure 4.17[b]. These two values are used to derive the following expression

$$\ln N_h = 0.6\left(\varphi'_{max} - 33.8\right) \qquad [4.11]$$

which is taken to be representative of the penetrometer tests on dry as well as oil-saturated sand. The two ϕ'_{max}:ln(N_h) data points from the test on the dry sample and Equation 4.11 are shown in Figure 4.18[a]. The similar expression given in Houlsby and Hitchman (1988) (Equation 4.10) and the results from the penetrometer tests on the oil-saturated samples are also plotted in Figure 4.18[a]. The peak friction angles of the oil-saturated samples are estimated from the bulk relative density measurements, which are deemed to be unrepresentative, hence the scatter. The oil-saturated peak friction angles estimated using the low stress adjustment to Bolton's (1986) method are reduced by 1.5 ° to account for the degrading effect of the silicone oil. The triaxial tests discussed in Section 4.5.1 indicated that the silicone oil resulted in a 2 ° reduction in the peak friction angle. This was based on a comparison of only two triaxial tests, hence a reduction of 1.5 ° was reasonable.

Although there is considerable scatter in the results from the tests on the oil-saturated samples shown in Figure 4.18[a], it is probable that Equation 4.11 represented the relationship between the peak friction angle of the sand and the cone tip resistance (at a depth of about 75 mm).

The bulk relative density measurements are either greater than or equal to the actual values of the relative density at 75 mm, assuming that the density did not decrease with depth. Thus the peak friction angles deduced from the bulk relative density measurements were either greater than or equal to the actual values of the peak friction angle at 75 mm. The data in Figure 4.18[a] satisfies this criterion.

The penetrometer results, at a depth of 75 mm, and the proposed relationships are also plotted in R_D:q_c space in Figure 4.18[b]. The dry and the oil-saturated tests, as well as the results from different depths, could be described by one expression in ϕ'_{max}:ln(N_h) space (Equation 4.11). In R_D:q_c space, however, the result of changes in the effective unit weight, of changes in the depth of penetration, and of the degrading effect of the silicone oil, is evident. Hence the different curves for the dry and oil-saturated tests. The proposed relationship between the cone tip resistance and the relative density for the tests on the dry sample is,

$$q_c = 0.174 e^{0.11 R_D} \qquad [4.12]$$

and for the tests on the oil-saturated samples is,

$$q_c = 0.044 e^{0.11 R_D} \qquad [4.13]$$

The above equation is an approximation of the actual relationship, which is derived from Equation 4.11 and Bolton's method. The relative density values estimated from the cone penetrometer tests are listed in Table 4.6.