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PAPER

Which theories have a measurement problem?

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Abstract

It is shown that any theory that has certain properties has a measurement problem, in the sense that it makes predictions that are incompatible with measurement outcomes being *absolute* (that is, unique and non-relational). These properties are *Bell Nonlocality*, *Information Preservation*, and *Local Dynamics*. The result is extended by deriving Local Dynamics from *No Superluminal Influences*, *Separable Dynamics*, and *Consistent Embeddings*. These results are achieved using a framework of *perspectival theories* that generalizes the notion of a Heisenberg cut beyond quantum theory. As well as explaining why the existing Wigner's-friend-inspired no-go theorems hold for quantum theory, the results also shed light on whether a future theory of physics might overcome the measurement problem. In particular, they suggest the possibility of a theory in which absoluteness is maintained, but without rejecting relativity theory (as in Bohm theory) or embracing objective collapses (as in GRW theory).

Contents

Suppose Alice performs a measurement, and observes the outcome that a light flashes red. Is it an absolute fact that this is what she saw? Or is there some other world, context, or perspective, in which she saw it flash some other color?

Remarkably, recent no-go results suggest that sometimes there is, at least given some well-motivated assumptions [1–6][4]. One assumption is that measurements can, at least in principle, be treated as unitary interactions. In [2, 5, 6], the other is that one can nevertheless apply the Born rule to obtain the statistics for a pair of simultaneous measurements in any inertial frame of reference. Thus the two pillars of modern physics—quantum theory and relativity—conspire to impose some kind of relativity of observed events.

Yet, this does not prove that observed events in *nature* fail to be absolute. The claim that measurements using macroscopic apparatus are unitary interactions lies well beyond the current scope of empirical confirmation. Therefore, a legitimate response to the existing no-go results is to maintain that observed events are absolute by rejecting the version of quantum theory that suggests otherwise. One can then hope that a future, post-quantum theory of physics will avoid a similar no-go theorem, and thus overcome the infamous measurement problem of quantum theory.

Here, we investigate whether this is wishful thinking. We identify very general physical principles that inevitably lead a theory to make predictions that are inconsistent with the absoluteness of observed events. Once these principles are identified, one can judge the prospects for a future physical theory to avoid a measurement problem by dropping one of them.

To this end, we first recap the particularly simple quantum no-go theorem from [6], and discuss which features of quantum theory appear to make the argument go through (section 1). We then construct a framework of more general theories that may contain an analogue of quantum theory's

[4] A closely related no-go theorem in [7] concerns the consistency between various agents' beliefs about the outcomes they observe.

'Heisenberg cut', deploying the notion of a memory update from [8] (section 2). We identify three properties that may be satisfied by a theory in the framework, whose conjunction is inconsistent with the absoluteness of observed events (section 3). These three properties are:

1. Bell Nonlocality;
2. Information Preservation;
3. Local Dynamics.

Local Dynamics is stronger than the prohibition of superluminal influences required by relativity. However, it may be derived from three further principles (section 4), namely

1. No Superluminal Influences;
2. Separable Dynamics;
3. Consistent Embeddings.

Substituting these three principles for Local Dynamics, we arrive at a deeper no-go theorem, which derives a contradiction with absolute observed events from five fundamental principles. Our results are summarized graphically in figure 3.

In deriving these results, we shed light on the precise features of quantum theory that are responsible for the existing no-go theorems, illustrating, for example, that unitarity or reversibility per se is not essential[5], whereas quantum theory's peculiar combination of dynamical locality and Bell nonlocal correlations is crucial. The proof of our main result is constructive, meaning that readers can see for themselves exactly how the incompatibility with the absoluteness of observed events comes about as a result of our assumptions.

If one believes that observed events are absolute, then our first result shows that one cannot believe in a theory that fits in our framework, violates Bell inequalities, preserves information, and has Local Dynamics. Since the framework is rather minimal, and Bell nonlocality has already been observed, we conclude that believers in absoluteness face a dilemma between accepting the irretrievable loss of information (i.e. fundamental stochasticity) and embracing some sort of dynamical nonlocality (section 5).

We note, however, that accepting dynamical nonlocality does not necessarily mean rejecting relativity theory, since one can reject Separable Dynamics instead of No Superluminal Influences. This suggests a strategy for overcoming the measurement problem (or at least the aspect of the measurement problem that is revealed extended Wigner's friend scenarios). That is, develop a theory in which the idea of nonseparability, familiar from quantum states, is extended to the dynamics, in a way that allows one to retain the absoluteness of observed events. One might then be able to avoid a measurement problem while conserving much of what is fundamental to the modern physical picture, including relativity theory and the preservation of information (section 5).

However, with this approach one does still have to give up the idea that unitary dynamics are fundamental. For this reason, many will prefer to simply reject the absoluteness of observed events. This raises both an ontological question, of how 'relative events' might be understood and modeled mathematically, and an epistemological one, of how agents might communicate, without error, beliefs about such events[6]. At the end, we reflect briefly on the clues for these problems provided by our results (section 6).

1. A simple quantum no-go theorem

To begin with, we will provide a gentle introduction to the original Wigner's friend scenario [11], and an extended version thereof. As we do so, we will aim to set the stage for what is to come, by highlighting some aspects of some familiar quantum-theoretical arguments that we will eventually seek to generalize to arbitrary physical theories.

1.1. The original thought experiment

Imagine yourself standing outside a laboratory with thick, concrete walls. Inside the lab is your friend; her only company is a qubit prepared in the state $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$. She measures the qubit in the $\{|0\rangle,|1\rangle\}$ basis.

[5] This corroborates a result from [8], in which a similar no-go theorem is derived in the context of PR boxes [9].

[6] The epistemological question was taken up in [10] in the context of quantum theory; the results here might inspire a generalization of that approach to other perspectival theories.

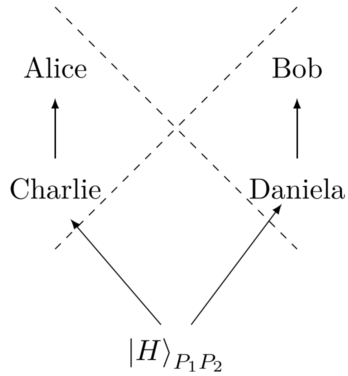


Figure 1. The scenario that proves the quantum no-go theorem, reproduced from [6]. Dashed lines denote the possible paths of light rays. Reproduced from [6]. CC BY 4.0.

She will presumably experience herself observing one of two possible outcomes. In which case, we can model the outcome—the *observed event*—as a bit taking on the value $F=0$ or $F=1$.

But your friend is nothing special: at the end of the day, she is just a quantum system like everything else. So presumably, you should *also* be able to treat her as having a quantum state. And, if all time evolution is unitarity, then presumably you should describe her interaction with the particle roughly as follows:

$$\frac{|0\rangle_P+|1\rangle_P}{\sqrt{2}}|\mathrm{ready}\rangle_F\mapsto\frac{|0\rangle_P|0\rangle_F+|1\rangle_P|1\rangle_F}{\sqrt{2}}. \tag{1}$$

Intuitively, the state on the right side says that if the particle is in its $|i\rangle_P$ state, then your friend is in a state of having observed $F=i$. (In an abuse of notation, we are here re-using the same letter F to denote the friend as a quantum system, and the outcome of her measurement.) She therefore ends up entangled with the particle. And this statement is by no means purely theoretical: given a sufficiently powerful experimental architecture, *you* could actually confirm or falsify the entanglement (given enough trials) by performing a measurement of a basis for the lab that includes $\frac{|0\rangle_P|0\rangle_F+|1\rangle_P|1\rangle_F}{\sqrt{2}}$.

But this entanglement would appear to be in contradiction with the earlier claim that your friend observes some particular outcome, *either* $F=0$ *or* $F=1$. For in that case, intuition suggests that your friend and the particle should have ended up in some product state, *either* $|0\rangle|0\rangle$ *or* $|1\rangle|1\rangle$.

If the quantum state is a complete description of an agent-independent reality, then we have a choice. Either we accept that there is not a single, particular outcome that the friend experiences. Or, we deny that the fundamental dynamical law of quantum theory—that time evolution is unitary—holds inside the laboratory.

However, this dilemma can potentially be avoided if the quantum state is understood differently. If the state is epistemic, then a deeper theory may reveal that the information about the friend and system summarized by an entangled quantum state is entirely compatible with just one or the other outcome having happened. And the dilemma can certainly be avoided if the quantum state is supplemented with additional variables, as shown by the Bohm-theoretic account of the experiment. There, all time evolution is unitary, but only one outcome is observed, depending on which branch of the wavefunction is picked out by the positions of the Bohmian particles. It follows that the argument above cannot have amounted to any no-go theorem ruling out the combination of unitarity and unique outcomes; at best it illustrates a tension between these ideas.

However, it turns out that one *can* derive a no-go theorem if one (a) extends the scenario above to incorporate two spacelike separated agents, each with their own friend inside of an isolated lab, and (b) adds a locality assumption [1–6]. This is shown in the next subsection.

1.2. A relatively simple no-go theorem

Now, let us sketch the result from [6]. We consider the scenario depicted schematically in figure 1. At the start, a 'Hardy state' [12, 13]

$$|H\rangle_{P_1P_2}=\frac{1}{\sqrt{3}}\left(|00\rangle_{P_1P_2}+|01\rangle_{P_1P_2}+|10\rangle_{P_1P_2}\right) \tag{2}$$

of two particles is prepared. One particle is sent to Charlie, the other is sent to another, spacelike separated agent, called Daniela. Once they receive the particle, their labs are isolated from the rest of the Universe.

These agents then each perform a measurement in the $\{|0\rangle, |1\rangle\}$ basis. As is evident from (2), the Born probability for Charlie and Daniela both seeing the '1' outcome on the same run of the experiment is 0.

$$p(C = \text{`1'}, D = \text{`1'}) = 0. \tag{3}$$

Now, let us assume that a 'superobserver' outside the labs can treat these measurements as unitary interactions. A reasonable choice of unitary in this case would be the quantum CNOT gate[7], where the target system is a qubit representing the agent's memory (see section 2.3 for a discussion of the arbitrariness of this choice)[8]. Once the memory of the agent is prepared in the 'ready' state $|0\rangle$, these CNOTs give rise to an isometry U that coherently copies the logical value of the particle onto the agent's memory. For example, for Charlie, we have

$$U|i\rangle_{P_1} = |i\rangle_C |i\rangle_{P_1}. \tag{4}$$

We consider two superobservers, Alice and Bob, who are each capable of performing any measurement they like on a composite system consisting of one agent's memory and the corresponding particle after the initial measurement has taken place. Given the unitary/isometric description of the initial measurements just proposed, the four-partite quantum state they will measure is given by $(U \otimes U)|H\rangle$. Alice measures an orthonormal basis including $U\frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$ on $\mathcal{H}_C \otimes \mathcal{H}_{P_1}$, and Bob performs a similar measurement on $\mathcal{H}_D \otimes \mathcal{H}_{P_2}$. Since $U^\dagger U = I$ for an isometry, it is easy to see that the Born probabilities for this measurement are equivalent to those obtained by measuring the basis $\frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$ directly on $|H\rangle$. It follows that the probability of both of the agents seeing the outcome for $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$ is

$$p(A = \text{`}-\text{'}, B = \text{`}-\text{'}) = 1/12. \tag{5}$$

So far, we have calculated joint probabilities for two pairs of measurements. In each case, we assumed that the quantum state evolved unitarily right up to the moment of the pair of measurements in question. Let us assume that we can do this for any pair of measurements conducted at the same time. As figure 1 indicates, we are also assuming that Charlie's measurement is spacelike separated from Bob's measurement, and likewise that Daniela's measurement is spacelike separated from Alice's measurement.

Now here is the key point: if relativity theory provides the correct way of thinking about simultaneity, then *it is just as legitimate to think of Alice and Daniela, or of Bob and Charlie, as measuring at the same time*.

In that case, to calculate joint probabilities for Alice's and Daniela's outcome, we should assume Charlie's measurement can be treated unitarily, so Alice and Daniela measure the state $(U \otimes I)|H\rangle$. It follows that the probability for the agents seeing the outcomes for $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$ and $|0\rangle$ respectively is

$$p(A = \text{`}-\text{'}, D = \text{`0'}) = 0 \tag{6}$$

and, by similar reasoning, the probability for Bob and Charlie to see these outcomes is

$$p(B = \text{`}-\text{'}, C = \text{`0'}) = 0. \tag{7}$$

But if we think of the observed outcomes A, B, C, and D as *absolute*—that is, we assume that on any run of the experiment, there is some unique global assignment $(A = a, B = b, C = c, D = d)$ for which the quantum probabilities are predictions—then we get a contradiction. Figure 2 illustrates the argument, which is summarized as follows. Suppose that on some run, Alice saw the '–' outcome. By (6), Daniela saw '1' on the same outcome. This implies via (3) that Charlie saw '0', which implies via (7) that Bob

[7] This gate either does nothing to or flips the logical value of a target qubit based on the logical value of a control qubit. Formally, we have $\texttt{CNOT}|i\rangle|j\rangle = |i\rangle|j + i\rangle$, where '+' denotes addition modulo 2.

[8] A more sophisticated analysis of the measurement process would involve explicitly accounting for the role of many different systems involved in the measurement, including environmental systems that induce decoherence. While this additional rigor does not change anything important in our analysis, it does make things a lot more complicated, hence our decision to ignore it.

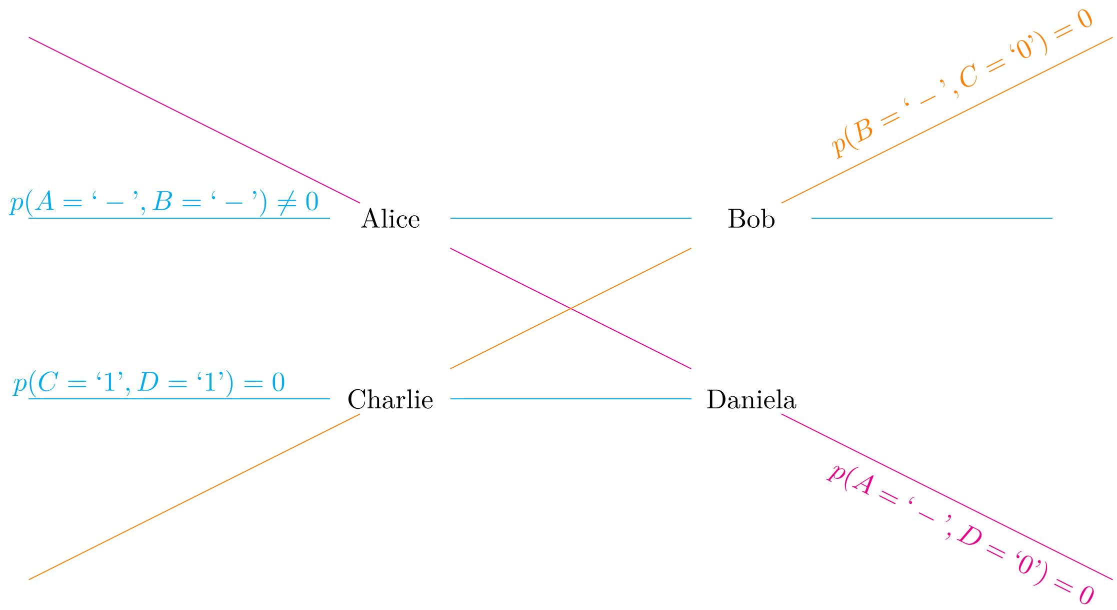


Figure 2. A visual summary of the argument, adapted from [6]. The different colors represent different reference frames. Combining the quantum's predictions for measurements that are simultaneous in different frames implies that there cannot be a single global assignment of the observed outcomes on each run of the experiment. Reproduced from [6]. CC BY 4.0.

saw '+'. So, every time Alice sees '−', Bob sees '+'. Yet (5) implies that Alice and Bob sometimes both see '−'![9,10].

1.3. What makes the argument go through?

This argument relies on the following key ideas.

1. *Universal measurability.* Alice was capable of performing any quantum measurement she liked on $\mathcal{H}_C \otimes \mathcal{H}_{P_1}$, despite the fact that $\mathcal{H}_C \otimes \mathcal{H}_{P_1}$ was a macroscopic system containing a human observer.
2. *A duality of perspectives on measurements.* Charlie's measurement was thought of from the 'inside' as projecting onto $\{|0\rangle, |1\rangle\}$, and from the 'outside' as a unitary interaction.
3. *Nonlocal correlations.* The way that the four quantum predictions are chained together to obtain a contradiction in the argument above is precisely analogous to Hardy's proof of nonlocality [13]. The argument before that can be viewed as a series of manoeuvres designed to bring that nonlocality into contradiction with the absoluteness of observed events.
4. *Information preservation.* Since information is preserved by isometries, Alice was able to effectively perform a measurement on P_1 that was incompatible with Charlie's measurement on the same particle.
5. *Dynamical locality.* We assumed that any pair of spacelike separated measurements could be thought of as simultaneous, allowing us to apply the Born rule to calculate joint probabilities.

A slight modification of the result in [8] demonstrates that essentially the same properties in a model inspired by PR boxes [9] leads to a breakdown in the absoluteness of observed events.

The central question of this paper is: do similar properties lead to similar no-go theorems in *any* reasonable physical theory? Before we can give an answer, we need a suitable framework of general physical theories.

[9] A similar no-go theorem [2] has been criticized on the grounds that the four predictions it makes that together imply a contradiction with absoluteness can never be simultaneously observed by a single agent [14]. As argued in [15], this misses the point. The result in [2], like the ones from [5, 6], aims to demonstrate that the absoluteness of observed events is *logically inconsistent* with a particular combination of quantum theory and relativity. The fact that the relevant predictions of that theory cannot be confirmed by a single agent poses no obstacle to this.

[10] In this exposition, we have prioritized conciseness above rigor and clarity. Readers who want a more detailed understanding of the no-go result are referred to [6], or to [2, 5] for similar results.

2. Perspectival theories

The no-go theorem above relied on a strange feature of quantum theory: that the same measurement can be thought of as either giving rise to classical outcomes, or as a unitary interaction (which does not formally involve any classical variables). The goal of this section is to formalize the idea that measurements can be viewed from two different perspectives, and to generalize it beyond quantum theory.

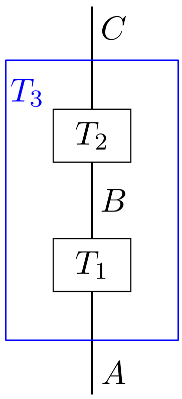
But if the reader is not used to, or is uninterested in, category theory, there is no need to worry! Categorical probabilistic theories are a simple idea, which we will introduce here without any heavy mathematical machinery (though a more formal introduction can be found in appendix A).

We will then supplement categorical probabilistic theories with a little bit of extra structure, allowing us uniquely associate every model of a measurement incorporating classical outcomes with another model of the same measurement that might not explicitly incorporate any classical outcomes. This gives us the framework of *perspectival theories*. In some perspectival theories, such as certain classical theories or 'collapse' theories, the connection between the two perspectives is very natural; they might be essentially identical. But in others, such as a unitary quantum one, the connection might feel very unnatural indeed.

2.1. Categorical probabilistic theories

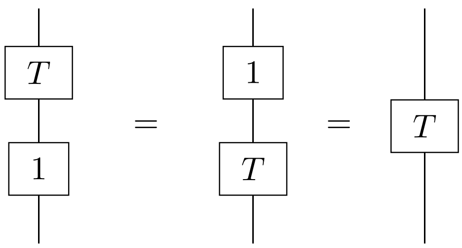
In a nutshell, a categorical probabilistic theory is a theory that can be described by circuits, that contains a notion of normalization and the reduced states of subsystems, and that includes classical systems, so that it can directly model measurement settings and outcomes. For readers who are familiar with circuit theories, this sentence alone should permit a reasonable understanding of the next subsection; such readers should feel free to skip ahead. For the uninitiated, we introduce the main ideas here, leaving a more formal treatment for appendix A.

Circuit theories. First, we explain what we mean by a 'circuit theory' (formally, a *symmetric monoidal category*; see e.g. [16] for a pedagogic introduction). Such a theory includes a set of systems and a set of transformations between each pair of systems. If the theory contains a transformation $T_1 : A \to B$ whose output is the system B, and another one $T_2 : B \to C$ that accepts B as its input, then we require that the theory also contains a third transformation $T_3 := T_2 \circ T_1$, corresponding to 'doing T_1 first and then T_2'. Diagrammatically, this means that if we draw T_1 and T_2 as boxes plugged into each other, we can think of the resulting shape as another transformation:



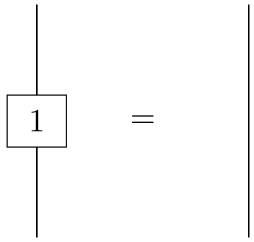
(8)

We further require that this *sequential composition* has the identity transformation $1 : A \to A$ for each system A as a unit, meaning that



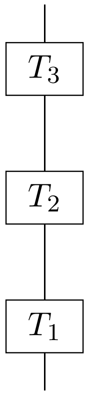
(9)

for all T. The identity transformation can therefore be thought of as simply as a wire.



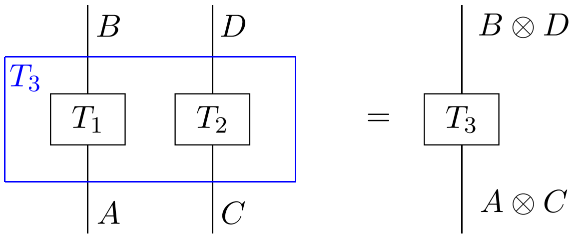
(10)

We also require that sequential composition is associative, meaning that the picture below is well defined; there is no need to put brackets around T_1 and T_2 or T_2 and T_3.



(11)

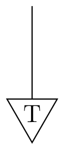
We do not only want to be able to perform transformations one after the other; we also want to be able to perform any pair of transformations independently. Thus we require that we can take a *tensor product* any pair of transformations $T_1 : A \to B$ and $T_2 : C \to D$ to form another transformation in our theory: $T_3 = T_1 \otimes T_2 : A \otimes C \to B \otimes D$. To make sense of this, we also need to require that we can take a tensor product of any pair of systems to form another system in our theory. All this means that a diagram of a pair of transformations placed side-by-side always defines a new transformation:



(12)

We also stipulate that the tensor products on systems and on transformations are associative and unital. The unit 1_I of the tensor product on transformations is the identity transformation on a special trivial unit system I. Associativity implies that three boxes lined up in a row have a well-defined meaning.

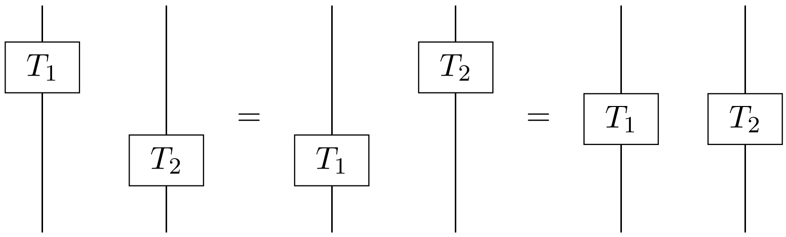
A transformation $T : I \to A$ from the trivial system to some system A is called a 'state' on A. Diagrammatically, the wire corresponding to the trivial system can be safely omitted, meaning that a state has the following representation.



(13)

The last key rule is known as the interchange law. This is the requirement that the sequential composition and tensor product distribute over one another, in the sense that $(T_1 \otimes T_2) \circ (R_1 \otimes R_2) =$

$(T_1 \circ R_1) \otimes (T_2 \circ R_2)$. Diagrammatically, this means we needn't worry about the order of boxes on different wires:



(14)

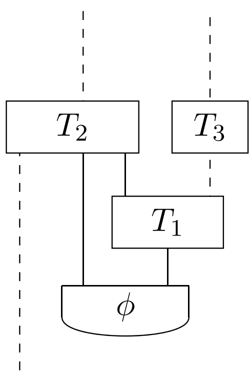
Although these diagrams are yet to be given any spatiotemporal meaning, note that (14) is reminiscent of the idea that inertial frames are equivalent. We will return to this point when we introduce our Local Dynamics assumption in section 3.

If all these requirements are satisfied, plus some others about the existence of a 'swap' transformation for each pair of systems (which are not required for the proofs in this paper), then we have a symmetric monoidal category. It is known that such theories always admit a circuit representation [16, 17], so we can always think of them as 'circuit theories'.

Categorical probabilistic theories. A categorical probabilistic theory is a circuit theory that also satisfies the following three requirements.

1. It contains a full subtheory of classical systems that also forms its own circuit theory (i.e. a symmetric monoidal category).
2. Sums of transformations are well-defined, and include convex combinations of probability distributions in the case of classical systems. The $\circ$ and $\otimes$ operations are bilinear.
3. The theory comes with a special 'trace' transformation $\mathrm{Tr}_A : A \to I$ for any system A, given by marginalization in the case of classical systems.

Let us explain each of these in turn, starting with (1). The idea there is that there is some subset of the systems in the larger theory such that all the transformations between them define their own circuit theory. This circuit theory is classical (formally speaking, it is equivalent to $\mathbb{R}^+$-Mat, the symmetric monoidal category of positive real-valued matrices), and can therefore explicitly incorporate conditional probability distributions for measurement outcomes given settings within our theory. For example, consider the diagram below, in which dashed wires denote classical systems.



(15)

Out of all the boxes in the picture, only T_3 represents a transformation from the classical circuit subtheory, since only it acts exclusively on classical systems. Nevertheless, the combination of all the boxes into a circuit gives rise to an overall transformation that does live in the classical circuit subtheory. This overall transformation might admit an interpretation as a conditional probability distribution for a pair of measurement outcomes given a single choice of setting.

Moving onto (2), by sums being well-defined we mean we can take a sum of any pair of transformations, $T_1, T_2 : A \to B$, that act on the same systems to form another transformation $T_3 := T_1 + T_2 : A \to$

B that also lives in our theory[11]. Given this summation structure, $\circ$ and $\otimes$ are required to be bilinear operations[12].

Finally, the trace in (3) must satisfy $\mathrm{Tr}_A \otimes \mathrm{Tr}_B = \mathrm{Tr}_{A\otimes B}$ and the trace on the trivial system Tr_I must equal the identity transformation on that system, $\mathrm{Tr}_I = 1_I$. The trace operation in quantum theory provides an example. This operation allows us to define a *normalized state* $T: I \to A$ as one satisfying $\mathrm{Tr}_A \circ T = \mathrm{Tr}_I$, and a *normalization-preserving* transformation $T: A \to B$ as one satisfying $\mathrm{Tr}_B \circ T = \mathrm{Tr}_A$.

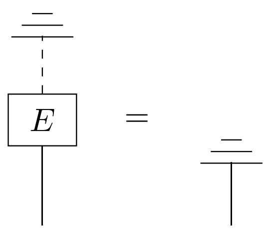
2.2. Perspectival theories

From the 'inside' perspective on Charlie's measurement in section 1, outcomes are written onto classical systems with various probabilities. Formally, this process can be represented with a linear map E from quantum states to probabilities,

$$\rho \quad \overset{E}{\mapsto} \quad p(C), \tag{17}$$

where $p(C)$ is the probability distribution given by the Born rule, $p(C=i) = \mathrm{Tr}(\rho|i\rangle\langle i|)$. E is an example of what we will call a *probability extractor*.

In an arbitrary categorical probabilistic theory, a probability extractor is any normalization-preserving transformation with a classical output, i.e. one satisfying:



(18)

where $\top$ represents the trace. Note that the input to a probability extractor is permitted to be classical; the solid wire in (18) denotes only a system that *might* be nonclassical. Such a transformation always outputs a probability distribution when fed a normalized state.

We now make an assumption about the relationship between a categorical probabilistic theory and any possible world it accurately describes. In the quantum no-go theorem above, the realizability of Alice's and Bob's measurements can be justified by a *universal measurability* assumption. In the context of quantum theory, universal measurability is the assumption that if some physical system is represented by the Hilbert space $\mathcal{H}_S$, then any measurement (POVM) that is mathematically defined on $\mathcal{H}_S$ could, in principle, be carried out by a human observer. Crucially, this is assumed to be the case regardless of how large, or complicated the system is, and regardless of whether it includes conscious observers.

In any categorical probabilistic theory, we will also assume universal measurability: if a physical system is represented by S in our theory, and $E: S \to X$ is an extractor, then E represents a measurement that could, in principle, be carried out by a human observer, regardless of how large, complicated, or conscious S is[13]. (From here on, we refer to this property of a measurement as *physical realizability.*)

[11] Note that T_3 may not be a physical transformation. The physical transformations are picked out by the requirement that the trace is preserved, or, more generally, decreased. For example, the quantum perspectival theory discussed in section 2.3 consists of completely positive maps, not all of which are physical. But they are elements of quantum instruments when they are trace-decreasing, and they are quantum channels when they are trace-preserving.

[12] It follows from bilinearity that the possibility of taking probabilistic combinations of classical states leads to the possibility of taking probabilistic combinations of arbitrary states. For example, suppose we have a transformation T from a classical system (i.e. a variable) to a nonclassical system that prepares the state ρ_0 when fed the classical state [0] (i.e the variable taking the '0' value), and prepares ρ_1 when fed the state [1]. Then the bilinearity of $\circ$ means that inputting a probabilistic combination of [0] and [1] leads to a probabilistic combination of the nonclassical outputs:

$$\begin{aligned} T\circ(p[0]+(1-p)[1]) &= T(p[0]) + T((1-p)[1]) \\ &= pT([0]) + (1-p)T([1]) \\ &= p\rho_0 + (1-p)\rho_1. \end{aligned} \tag{16}$$

[13] One could perhaps argue that universal measurability is already part of what we should mean when we claim that a categorical probabilistic theory accurately describes the world: if a theory says that a physical system is represented by S, and there exists an extractor $E: S \to X$ in the theory that cannot, even in principle, be realized on that physical system, then the theory does not accurately reflect what operations on that system are physically possible. Not wishing to be dogmatic on this point, however, we will persist in to referring universal measurability as a separate assumption.

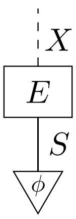
Now, in quantum theory, we are used to associating probability extractors with isometries, which can be thought of as representing an 'outside' perspective on the measurement. Indeed, this is precisely what we do whenever we apply the von Neumann measurement scheme to model a projector-valued measurement unitarily (assuming that the probe is initially prepared in some pure state). In Charlie's case, we associated his extractor with the isometry defined in (4). Informally, a *perspectival theory* is just a categorical probabilistic theory which comes with a similar sort of mapping between inside and outside perspectives on a measurement. In some theories, these two perspectives will look more or less the same (one would expect this in a perspectival formulation of a typical classical theory, for example). But in others, they might appear very different.

More formally, perspectival theories are categorical probabilistic theories that uniquely associate each probability extractor of type $E: S \to X$ with a transformation of type $U: S \to M \otimes S$, called a *memory update* (to borrow an idea from [8]). We think of S as the system being measured, and M as the 'memory' of the agent doing the measurement. Both S and M can be nonclassical.

Even more formally:

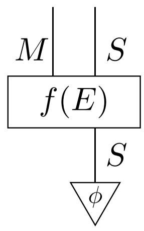
Definition 1 (Perspectival theories.). A perspectival theory is a categorical probabilistic theory equipped with a set of memory updates $\mathcal{U}$; and a bijective function $f: \mathcal{P} \to \mathcal{U}$, where $\mathcal{P}$ is the set of probability extractors.

Let us explain how a perspectival theory accommodates a duality of perspectives on a measurement. From the 'inside' perspective, one can obtain probabilities for measurement outcomes by applying a probability extractor to a state:



(19)

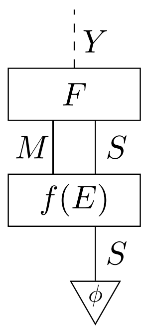
This will be the appropriate model for the agent who actually does that measurement, who presumably takes herself to observe a unique actual outcome, among many possible outcomes that can be assigned probabilities. But in a perspectival theory, this model is uniquely associated with another model, featuring a memory update:



(20)

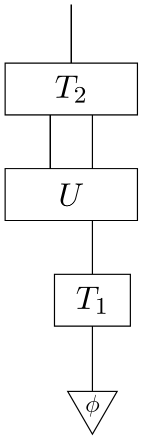
(Recall that solid wires denote systems that may or may not be classical; there is no requirement that the inputs or outputs of a memory update are nonclassical.)

Equation (20) will be the appropriate model for a superobserver (that is, someone like Alice or Bob from section 1) about to carry out a supermeasurement on $M \otimes S$. Explicitly, the probabilities for the data seen by a superobserver who carries out a measurement associated with an extractor F are given by a circuit of the following form.



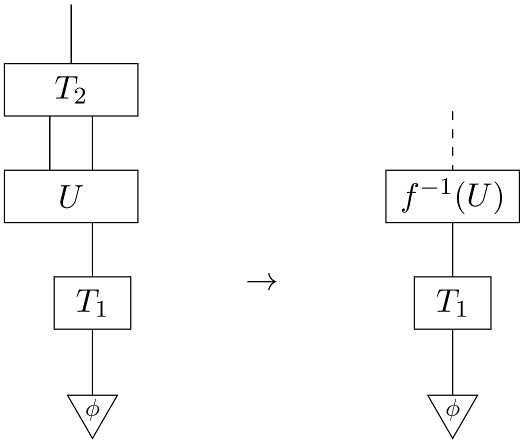
(21)

Above, we began with the inside perspective on a measurement, and applied f to obtain the outside perspective. But we can also go the other way around. Suppose that we have some circuit involving a memory update U:



(22)

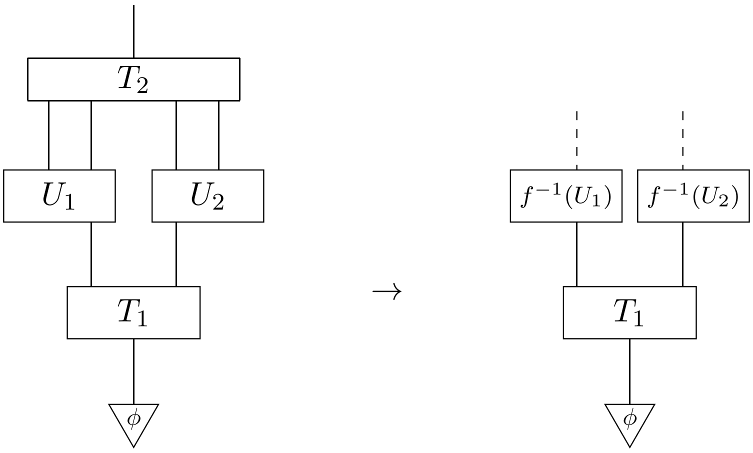
Applying f^{-1} at the appropriate point in the circuit gives us a model from the inside perspective:



(23)

Formally, the diagram on the right is a probability distribution—the distribution over possible outcomes of the measurement associated with U. (These probabilities are associated with the time at which U was implemented, so they are unaffected by T_2.)

More generally, if a set of memory updates are applied in parallel, joint probabilities for those measurements are obtained by applying f^{-1} to each one[14]. For example:



(24)

[14] One might therefore wish to require that the tensor product of a set of memory updates is itself a memory update, which is associated with an extractor that is operationally equivalent to the tensor product of the extrators associated with each member of the set. We will not bother with this requirement since it is not necessary for our proofs.

The model on the right can be used to predict the correlations between the observations of agents associated with U_1 and U_2.

We emphasize that in some cases there will be little difference between the inside and outside perspectives. To take a classical example, the probability extractor $E : S \to X$ might simply be the identity transformation, setting the outcome variable X equal to the value of the system variable S. $f(E) : S \to M \otimes S$ might then be the copy map, which preserves the probability distribution over S while perfectly correlating it to M. If we ignore the S output of $f(E)$, we are left with a map of the same form as E. In fact, we could even suppose that $M = X$, in which case one is left with E itself. In this case, the terminology of 'inside' and 'outside' perspectives is a little grandiose. $f(E)$ only provides a different perspective on the measurement to E in the banal sense that it includes what happens to S after the measurement.

In other cases, the difference between the inside and outside perspectives will be more dramatic. Indeed, it might be that S and M are both nonclassical systems, and that the form of $f(E)$ gives no clear sign that a classical outcome is produced at all. Such a situation may be bizarre, but it is familiar from quantum theory.

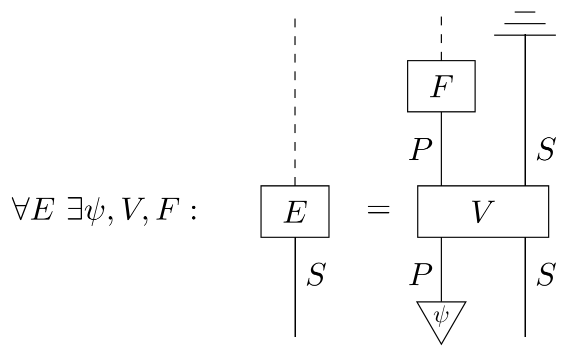
Since any perspectival theory is a categorical probabilistic theory, our universal measurability assumption applies-but for perspectival theories, universal measurability also carries over to memory updates. To see how, suppose that a physical system is represented in a perspectival theory by S, and $U : S \to M \otimes S$ is a memory update. There must exist a probability extractor $f^{-1}(U)$ which, by universal measurability, is physically realizable. U is then the outside perspective on that same physically realizable measurement. Thus if S represents a physical system, U represents a physically realizable measurement on that system.

2.3. A quantum perspectival theory

Recalling section 1, a natural memory update to associate with the quantum measurement of the basis $\{|i\rangle\}_i$ is $U = \sum_i |ii\rangle\langle i|$. One can extend this idea to form a fully-fledged quantum perspectival theory, in which measurements can alternatively be viewed as POVMs or isometric interactions (which in turn arise from unitary interactions). This theory can be seen as the rigorous formalization of the 'quantum theory' assumed in various no-go theorems inspired by Wigner's friend [2–7].

Let us briefly describe how this works. As shown in [18], one can formulate quantum theory as a categorical probabilistic theory. In this formulation, transformations between quantum systems are provided by completely positive maps, and transformations from quantum systems to classical ones include the mappings to probabilities afforded by POVMs.

To construct a quantum perspectival theory, one then only needs to associate the probability extractors with memory updates. One natural way of doing this relies on the Naimark dilation theorem for quantum measurements. This theorem says that any measurement on a quantum system can be implemented by letting that system unitarily interact with another quantum probe, initialized in some pure state, and then (projectively) measuring the probe. Or, diagrammatically:



(25)

holds for extractors E and F, a unitary V, and a pure state ψ. One could take as a measurement update for E the isometry obtained by inserting ψ into V.

This isometry is not unique, since the unitary dilation V is not unique. One therefore needs to make an arbitrary choice. Note however that if one wishes to associate two different memory updates with the 'same' extractor E, one can always duplicate E in the theory, and associate the different copies with different updates.

Those who do not worship at the church of the larger Hilbert space may wish to formulate other sorts of quantum perspectival theories. For example, objective collapse theorists will want to use non-unitary channels that induce wavefunction collapse as memory updates, and identify the probabilities for outcomes in the inside perspective with probabilities for collapse in the outside perspective. This is another illustration of the fact that the two perspectives in a perspectival theory are not always in tension with one another.

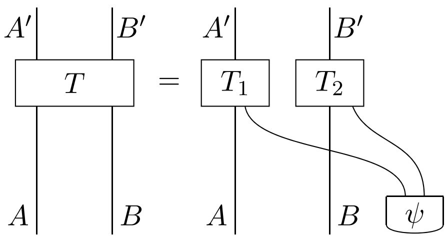
3. A generalized no-go theorem for absolute observed events

In this section, we prove that certain perspectival theories are inconsistent with the absoluteness of observed events. We call them the 'BIL' theories, because they violate Bell inequalities, they preserve information, and they are dynamically local.

3.1. BIL theories

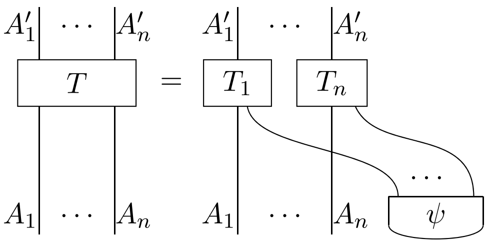
Let us explain in more detail what we mean by a BIL theory, one letter at a time.

Local Dynamics. The 'L' in BIL is for a dynamical sort of locality. The basic idea is straightforward. Suppose one region contains the systems A and A', and another, spacelike separated, region contains B and B'. Consider a transformation $T: A \otimes B \to A' \otimes B'$ taking place across both regions. In any dynamically local theory, the output at A' should not depend on the input at B, nor should the output at B' depend on the input at A. Assuming the diagrammatic representation of T is 'faithful' to these independences, we should then be able to write T in the following way.



(26)

In this diagram, there is no directed path of wires from A to B' or from B to A', making the lack of dependencies clear. Generalizing to n pairs of mutually spacelike separated systems, we have the following.



(L)

We consider that a theory has Local Dynamics just in case transformations on spacelike separated systems always decompose like this.

To make this definition more precise, we must formalize the idea of spacelike separation in the context of perspectival theories. Following [19], we consider an embedding function $\mathcal{E}$ that maps the input and output subsystems of a transformation to points on a Lorentzian manifold $\mathcal{M}$.

Given a transformation in our theory, we assume that an embedding function $\mathcal{E}$ on its subsystems is either 'valid' or 'invalid'. Local Dynamics can then be cast as a restriction on the valid embeddings of transformations. More precisely, suppose we embedded a transformation $T: A_1 \otimes \ldots \otimes A_n \to A_1' \otimes \ldots \otimes A_n'$ in such a way that every pair $(\mathcal{E}(A_i), \mathcal{E}(A_i'))$ of spacetime points is spacelike separated from every other pair. Then Local Dynamics requires that this sort of embedding is valid only if T decomposes as in (L).

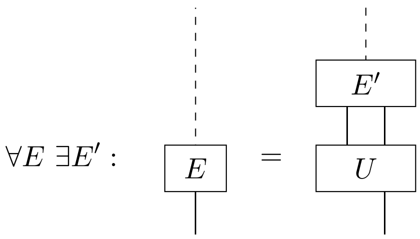
Definition 2 (Local Dynamics.). A perspectival theory has **Local Dynamics** if all transformations of the form $T : A_1 \otimes \ldots \otimes A_n \to A_1' \otimes \ldots \otimes A_n'$ can be validly embedded into a Lorentzian manifold $\mathcal{M}$ in such a way that each pair $(\mathcal{E}(A_i), \mathcal{E}(A_i'))$ of spacetime points is spacelike separated from every other pair only if T decomposes as in (L).

When Local Dynamics holds, a transformation that takes place across spacelike-separated regions must decompose into local boxes. Then the interchange law (14) can be applied, effectively telling us that various different inertial frames are equivalent.

On the other hand, suppose that Local Dynamics does *not* hold. Then the interchange law, while still true, cannot in general be meaningfully applied to a transformation that takes place across a spacelike separated region. Therefore, if Local Dynamics does not hold, the interchange law does not commit us to the equivalence of inertial frames. This subtle point will be important when we come to assess the implications of our no-go theorems: frame-independence does not follow from the interchange law itself, but from the *applicability* of the interchange law, which is guaranteed by Local Dynamics.

So much for the 'L' in BIL theories. 'I' is for...

Information Preservation. Information Preservation is the idea is that no information is irretrievably lost in the process of measurement, at least when we take the outside perspective on that measurement. The following is the formal condition for an individual memory update U to be *information-preserving*.



(IP)

The interpretation of this condition is that any extractor E that could have been applied to the input system S of U had U *not* been applied can still be effectively implemented *after* U by means of some other extractor E' on $M \otimes S$. The condition is inspired by a similar one from [8].

Our definition of an information preserving memory update naturally extends to a definition of an information-preserving *theory*.

Definition 3 (Information Preservation.). A perspectival theory is **Information Preserving** just in case all of its memory updates are information-preserving; that is, all memory updates satisfy (IP).

Taken together with universal measurability, Information Preservation generalizes the idea of the *universal applicability of quantum theory*. Universal measurability tells us that any mathematically defined measurement can be physically performed—independently of the size, complexity, or consciousness of the system to be measured—generalizing the idea that superobservers can perform arbitrary measurements on their quantum systems. Information Preservation tells us that all of these physically realizable measurements have an information-preserving representation from the outside, generalizing the idea that all measurements have a unitary representation.

Finally, 'B' is for…

Bell Nonlocality. This is the requirement that our perspectival theory violates Bell inequalities in a nontrivial way. More precisely, it demands that the theory can model a situation in which n pairwise spacelike separated[15] agents choose a measurement setting and record an outcome, in which the resulting data converge on a conditional probability distribution $p(A_1 \ldots A_n | X_1 \ldots X_n)$ that does not admit a local hidden variable model. Not admitting a local hidden variable model just means that it cannot be written in the form

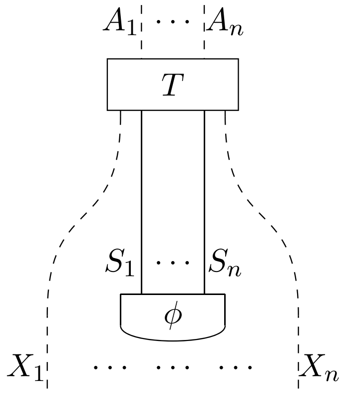
$$p(A_1 \ldots A_n | X_1 \ldots X_n) = \sum_\lambda p(A_1 | X_1 \lambda) \ldots P(A_n | X_n \lambda)\, p(\lambda) \tag{B1}$$

for some λ, $p(\lambda)$, and $p(A_i | X_i \lambda)$.

[15] Note the importance of this requirement: even a classical and relativistic theory can violate Bell inequalities locally.

To formalize this, we consider a circuit model provided by a perspectival theory in which an n-partite normalized state ϕ is fed into a *classically controlled* probability extractor, T:

$$p(A_1...A_n|X_1...X_n) \quad = \tag{B2}$$



By calling T a classically controlled extractor, we simply mean that plugging in probability distributions on each of the n classical inputs leads to a probability extractor on $S_1 \otimes \ldots \otimes S_n$. We require that T admits a valid embedding $\mathcal{E}$ where every triplet $(\mathcal{E}(S_i), \mathcal{E}(X_i), \mathcal{E}(A_i))$ of spacetime points is spacelike separated from all the others. This leads to the following definition, in which 'normalized circuit model' means a circuit built exclusively out of normalized states and normalization-preserving transformations[16].

Definition 4 (Bell Nonlocality.). A perspectival theory is **Bell Nonlocal** just in case it leads to a normalized circuit model of the form (B2), where

1. each triplet of systems (X_i, S_i, A_i) of T can be validly embedded into mutually spacelike separated regions; and
2. the resulting conditional probability distribution does not admit a local hidden variable model of the form (B1).

To grasp the results of this paper, it will help to understand that Bell nonlocality is a kind of *contextuality*. Consider the Bell scenario where there are $n = 2$ parties. The outcome variables are denoted A_1 and A_2, and take values in $\{1,2\}$. The settings are denoted X_1 and X_2, and also take values in $\{1,2\}$. It turns out that in this case the existence of a hidden variable model is equivalent to global distribution $q(A_1^1 A_1^2 A_2^1 A_2^2)$ that 'contains $p(A_1 A_2|X_1 X_2)$ in its marginals' [20, 21]. What this means is that, for each pair of settings $(X_1 = x_1, X_2 = x_2)$, the marginal distribution $q(A_1^{x_1} A_2^{x_2})$ is equal to $p(A_1 A_2|X_1 = x_1, X_2 = x_2)$. Explicitly,

$$\begin{aligned}
p\left(Ar_1 = a_1, A_2 = a_2|X_1 = 1, X_2 = 1\right) &= \sum_{a_1' a_2'} q\left(A_1^1 = a_1, A_1^2 = a_1', A_2^1 = a_2, A_2^2 = a_2'\right) \\
p\left(Ar_1 = a_1, A_2 = a_2|X_1 = 1, X_2 = 2\right) &= \sum_{a_1' a_2'} q\left(A_1^1 = a_1, A_1^2 = a_1', A_2^1 = a_2', A_2^2 = a_2\right) \\
p\left(Ar_1 = a_1, A_2 = a_2|X_1 = 2, X_2 = 1\right) &= \sum_{a_1' a_2'} q\left(A_1^1 = a_1', A_1^2 = a_1, A_2^1 = a_2, A_2^2 = a_2'\right) \\
p\left(Ar_1 = a_1, A_2 = a_2|X_1 = 2, X_2 = 2\right) &= \sum_{a_1' a_2'} q\left(A_1^1 = a_1', A_1^2 = a_1, A_2^1 = a_2', A_2^2 = a_2\right).
\end{aligned} \tag{27}$$

This result generalizes to arbitrary Bell scenarios. To see how, it is worth streamlining our notation. Let $\mathbf{a}$ and $\mathbf{x}$ respectively denote a full list of n outcomes and settings:

$$\begin{aligned}
\mathbf{a} &:= (A_1 = a_1, \ldots, A_n = a_n) \\
\mathbf{x} &:= (X_1 = x_1, \ldots, X_n = x_n).
\end{aligned} \tag{28}$$

[16] We note that the predictions of a Bell Nonlocal perspectival theory do not strictly imply that Bell's local causality condition is not respected by nature, since, for *that*, one also needs to assume that measurement settings can be freely chosen. However, rejecting free choice does not allow one to evade the no-go theorems from *this* paper, since these are proved using scenarios that do not involve any choices of measurements.

The conditional probability of a list of outcomes given a list of settings can then be written $p(\mathbf{a}|\mathbf{x})$. Given a global distribution

$$q\left(A_1^1,\ldots,A_1^{x_1^{\max}},\ldots,A_n^1,\ldots,A_n^{x_n^{\max}}\right), \tag{29}$$

we introduce a shorthand for the marginal probability

$$q\left(\mathbf{a}^{\mathbf{x}}\right) := q\left(A_1^{x_1}=a_1^{x_1},A_2^{x_2}=a_2^{x_2},\ldots,A_n^{x_n}=a_n^{x_n}\right). \tag{30}$$

The conditional distribution p admits a local hidden variable model if and only if there exists a global distribution q such that

$$p\left(\mathbf{a}|\mathbf{x}\right)=q\left(\mathbf{a}^{\mathbf{x}}\right) \tag{C1}$$

for all $\mathbf{a}$ and $\mathbf{x}$.

Thus Bell nonlocality can be understood as the impossibility of combining into a consistent whole all the data corresponding to different choices of measurements. Now, in a Bell experiment, only one choice can be made per run, meaning that this does not lead directly to a contradiction with the absoluteness of observed events. But we will soon see that, in a BIL theory, all of the different choices can effectively be in a single run.

3.2. A no-go theorem

We aim to show that BIL theories are incompatible with the following assumption about reality, quoted from [3].

Definition 5 (absoluteness of observed events (AOE) [3]). An observed event is a real single event, and not relative to anything or anyone.

Let us spell out the implications of AOE for a perspectival theory. By universal measurability, any extractor or update represents a measurement that can, in principle, be carried out by a human observer, and this measurement leads to an observed outcome. By definition, AOE tells us that this outcome exists objectively, not relative to anything or anyone. In particular, it does not exist only relative to the inside perspective on the measurement. It could well be that the outcome does not explicitly feature in our description of the physics when we represent the measurement from the outside perspective, but if so, its absence should be interpreted epistemically, as a limitation of our description, rather than metaphysically, as indicating that the event only exists relative to certain perspectives.

Now, consider a circuit model featuring N memory updates. Universal measurability implies that each of these memory updates can be physically realized. It follows that the circuit as a whole can be taken to represent a situation in which each of N human observers carries out a measurement represented by one of the memory updates. In this context, AOE further implies that an outcome c_i of one measurement exists independently of the perspectives taken on all others. It implies that all of the outcomes can be described collectively, as a unique global assignment $\{C_i=c_i\}_{i=1}^N$ of a collection of variables C_i representing the possible outcomes of each measurement.

For our first theorem (but not for theorem 2 below), we also assume that the relative frequencies of the global assignments converge to, over many realizations, a probability distribution $q(\{C_i\}_i)$, and that an accurate perspectival theory would correctly predict some of the marginals of this distribution. In particular, if an accurate perspectival theory predicts a distribution for the measurements corresponding to some subset $S\subseteq\{C_i\}_{i=1}^N$, then this distribution must coincide with the corresponding marginal $q(S)$. All of this leads to the following theorem.

Theorem 1 (BIL theories are incompatible with AOE.). *Any perspectival theory that is Bell Nonlocal, Information Preserving, and has Local Dynamics makes some predictions that are incompatible with AOE.*

Before proving the theorem, we make three brief comments on its implications. Firstly, the theorem shows us that unitarity or reversibility per se is not essential for a measurement problem—what really matters about the unitary dynamics of quantum theory is that they preserve information, in the sense of equation (IP).

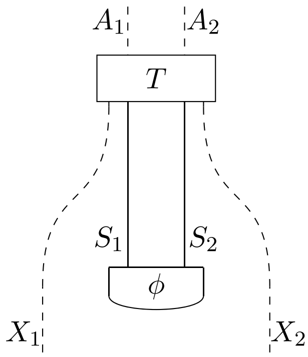
Secondly, it is interesting that the theorem relies on both a locality assumption and a *non*locality assumption. It is sometimes suggested that quantum theory involves only a weak, middle-of-the-road form of nonlocality (on one intuitive gloss, 'passion', but not 'action', at a distance [22]). And this weak nonlocality is often thought of as a kind of gift, as it appears to allow us to reconcile the quantum-theoretical Bell inequality violations with the theory of relativity. And yet theorem 1 suggests that this intermediate status of quantum nonlocality is part of what is responsible for the difficulty with reconciling it with the absoluteness of observed events. We will return to this theme in section 5.

Finally, it is interesting to note that for the nonlocality assumption, it suffices to assume that the theory violates Bell inequalities. In the perspectival framework, nonlocality in Bell's sense is always enough to ensure a breakdown of AOE when it is assumed that dynamics are local and information is preserved. (One does not have to assume, for example, that the less restrictive Local Friendliness inequalities are also violated [3].)

3.3. A proof: constructing a measurement problem

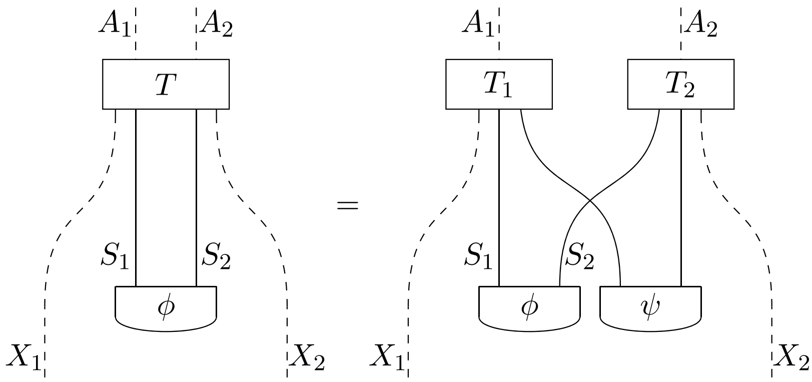
This subsection will prove theorem 1 by constructing a circuit model in an arbitrary BIL theory that is inconsistent with AOE. We will do this explicitly for a theory that yields a Bell-nonlocal probability distribution $p(A_1A_2|X_1X_2)$ for two outcome variables and binary choices, before explaining how the argument generalizes. Our proof will be diagrammatic, but perfectly formal—everything we will do can be directly translated into standard linear algebra by interpreting putting boxes one after the other and next to each other as performing the $\circ$ and $\otimes$ operations respectively.

Constructing the model. Suppose some BIL theory provides a Bell nonlocal conditional distribution (i.e. one that does not satisfy (B1)) in the following way:

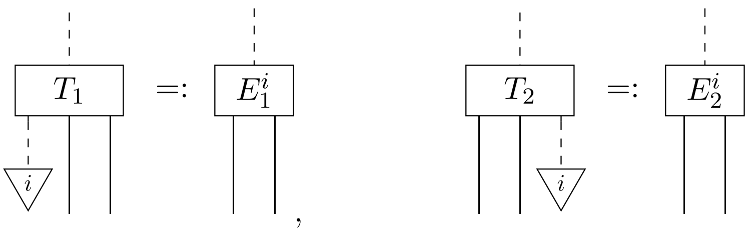
$p(A_1A_2|X_1X_2) =$  (31)

where (S_1, X_1, A_1) and (S_2, X_2, A_2) admit a valid embedding into spacelike separated regions, and X_1 and X_2 take values in $\{1,2\}$.

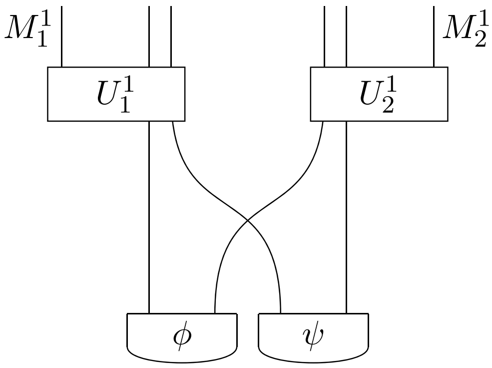
This circuit model provides raw materials which, with the help of Information Preservation and Local Dynamics, can be used to construct a scenario much like the one in figure 1, in which the nonlocality of $p(A_1A_2|X_1X_2)$ is converted into the global inconsistency of four agents' results. The first step is to use Local Dynamics to rewrite the circuit.

 (32)

Next, we streamline notation.

 (33)

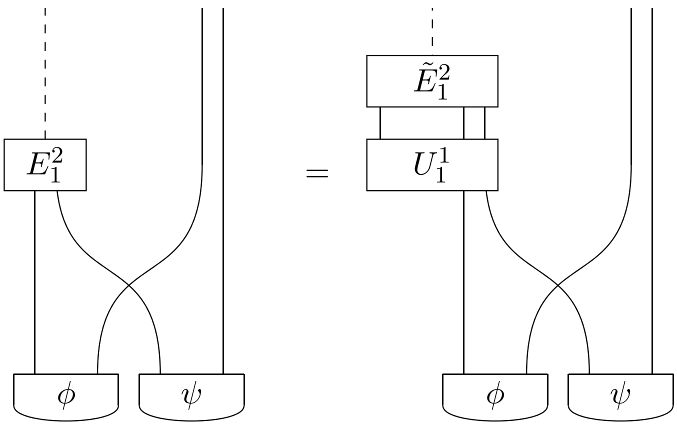
where $i \in \{1,2\}$ labels the two possible measurement settings for each party. Note that each E^i_j is a probability extractor, and therefore comes with an associated memory update. Defining $U^1_j := f(E^1_j)$ as the memory update for E^1_j, we consider the following circuit.



(34)

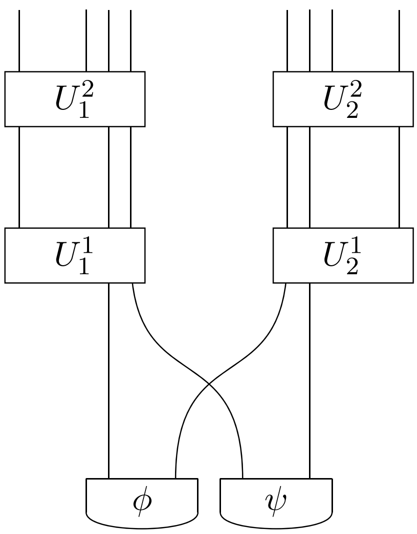
This circuit describes a pair of measurements corresponding to $X_1 = 1$ and $X_2 = 1$ from an outsider's perspective. By universal measurability, these measurements can in principle be carried out by a human observer, so let us assume that they are. We now want to find a way of effectively performing a pair of measurements corresponding to $X_1 = 2$ and $X_2 = 2$ on the original systems even after these first two measurements have taken place.

To do this, we need to find appropriate supermeasurements on the outputs of each memory update U^i_j. Information Preservation guarantees that such supermeasurements exist. For example, the following circuits are equivalent, for some probability extractor $\tilde{E}^2_1$.



(35)

We define $U^2_1 := f(\tilde{E}^2_1)$. Setting $U^2_2 := f(\tilde{E}^2_2)$ for a similarly defined $\tilde{E}^2_2$, we construct the following circuit.



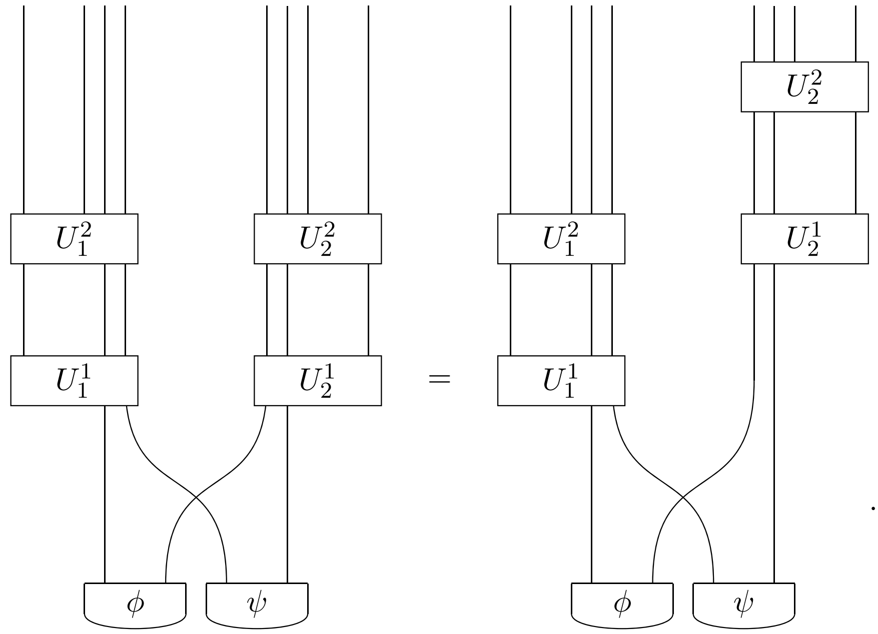
(36)

By universal measurability, the measurements represented by U_1^2 and U_2^2 can be in principle be carried out by human observers, even though they are measurements of systems that include the human observers associated with U_1^1 and U_2^1. So let us assume that this circuit of four memory updates represents a situation in which four human observers perform measurements, two of them superobservers.

Deriving a contradiction with AOE. There are four memory updates in our circuit, hence four measurements in the scenario it represents. By AOE, there exists a probability distribution over the four outcomes. Denoting by A_i^j the outcome of the measurement associated with U_i^j, we write this distribution $q(A_1^1 A_1^2 A_2^1 A_2^2)$. Our task is now to show that this distribution contains $p(A_1 A_2 | X_1 X_2)$ in its marginals, i.e. that (27) holds.

First of all, we can apply (24) to find our theory's predictions for the marginal distribution $q(A_1^1, A_2^1)$. This involves applying f^{-1} to U_1^1 and U_2^1. It is easily inferred that the top line of (27) holds.

Secondly, we rearrange (36) using the interchange law (14) to obtain



. (37)

Then we apply f^{-1} to U_1^2 and U_2^1, and deduce the second line of (27). In a similar way, we derive that the third and fourth lines also hold. It follows that $p(A_1 A_2 | X_1 X_2)$ admits a local hidden variable model, which is a contradiction.

Generalizing the argument. This argument can be straightforwardly generalized for any BIL theory that has some Bell nonlocal distribution $p(A_1 \ldots A_n | X_1 \ldots X_n)$. Again, one constructs a circuit model, analogous to (36), for a measurement scenario, with one measurement for each value of each measurement setting X_i. This circuit model will again describe the scenario from an outsider's perspective by using a memory update for each measurement.

To this end, for n variables, one proceeds in a very similar way to construct the first 'layer' of measurements, analogously to (34), and then the second layer, analogously to (36). (If for the ith party, there is only one setting, then the memory update in the second layer is the identity.) To construct the third layer, note that the property of information-preservation is closed under sequential composition. In other words, if U and V are both information-preserving in the sense of (IP), and $V \circ U$ is well-defined, then $V \circ U$ is also information-preserving in the sense of (IP). This means that one can construct the third layer from the second in much the same way that one constructs the second layer from the first, and so on.

Having constructed the circuit model, one can then use the interchange law (14) to think of any set of measurements for a global choice of settings $(X_1 = x_1, \ldots, X_n = x_n)$ as performed in parallel, before applying f^{-1} to predict joint probabilities for subsets of the purportedly absolutely events. It is then easily verified that these predictions are equivalent to $p(A_1 \ldots A_n | X_1 = x_1, \ldots X_n = x_n)$, implying that $p(A_1 \ldots A_n | X_1 \ldots X_n)$ admits a local hidden variable model, violating our initial assumption. □

Before proceeding, let us summarize the argument and the role of each assumption. Bell Nonlocality implies the existence of a controlled extractor T, implementable at spacelike separation, that violates Bell inequalities. By Local Dynamics, T decomposes into local extractors E^i_j, one for each party and measurement setting. Although extractors for the same party j and different settings i, i' cannot be implemented simultaneously, Information Preservation guarantees they can all be *effectively* implemented, in the sense of (IP), within a single run of the experiment. Collectively, these three assumptions allow us to construct a circuit of memory updates (such as (36)) whose probability distributions, obtained from (24), cannot arise from marginalizing any single global distribution $q(\{A^i_j\})$ over all outcomes. In other words, these probability distributions are incompatible with the distributed variables being absolute. Finally, universal measurability implies that all memory updates correspond to measurements that could, in principle, be performed by a human observer, establishing a no-go theorem for the absoluteness of *observed* events.

3.4. Closing a loophole

The argument above assumed the applicability of probability theory. In particular, it assumed that the relative frequencies for events converged to a probability distribution. One might therefore speculate that a BIL theory might be reconciled with AOE via a nonclassical generalization of probability theory.

But it is doubtful that such a strategy could succeed in reconciling quantum theory with AOE. This is because quantum theory is not only Bell nonlocal at the level of probabilities, but also *possibilistically* Bell nonlocal. The punchline of the argument at the end of section 1 was essentially that there exists no global distribution q such that

$$\begin{aligned} q(A=-,B=-) &\neq 0 \\ q(A=-,D=0) &= 0 \\ q(B=1,C=1) &= 0 \\ q(C=0,D=-) &= 0. \end{aligned} \tag{38}$$

Hence AOE was shown not only to be incompatible with the specific numerical probabilities predicted by quantum theory, but with its predictions for which pairs of outcomes are *possible*.

A similar argument can be made for any *PIL* theory. A PIL theory is Information Preserving, has Local Dynamics, and also has the following property.

Definition 6 (Possibilistic Bell Nonlocality.). A perspectival theory is **Possibilistically Bell Nonlocal** just in case it leads to a normalized circuit model of the form (B2), where

1. each triplet of systems (X_i, S_i, A_i) of T can be validly embedded into mutually spacelike separated regions; and
2. the resulting conditional possibility distribution does not satisfy

$$p(\mathbf{a}|\mathbf{x}) = 0 \qquad \Longleftrightarrow \qquad q(\mathbf{a}^{\mathbf{x}}) = 0 \tag{39}$$

for any q.

Theorem 2 (PIL theories are incompatible with AOE, without probabilities.). *Any perspectival theory that is Possibilistically Bell Nonlocal, Information Preserving, and has Local Dynamics makes some predictions that are incompatible with AOE—even when the applicability of probability theory is not assumed.*

The proof of theorem 2 is almost identical to the proof theorem 1, so we will not give it explicitly.

Theorem 2 suggests that a future theory of physics will not successfully recover AOE via a modification of classical probability theory—at least not unless the classical approach to *possibilities* is modified alongside it.

3.5. A trilemma?

Bell Nonlocality is experimentally well-supported. Therefore, believers in AOE can only adopt perspectival theories that either fail to be Information Preserving or else lack Local Dynamics.

This might suggest that one faces a trilemma between (1) rejecting AOE; (2) allowing information to be destroyed; and (3) rejecting relativity theory. But this would be too quick a judgment, since Local Dynamics is not simply the prohibition of superluminal influences suggested by relativity. The next section shows that this prohibition can indeed be used to derive Local Dynamics, but only in combination with the assumption of a sort of dynamical separability (together with a rather minimal consistency constraint).

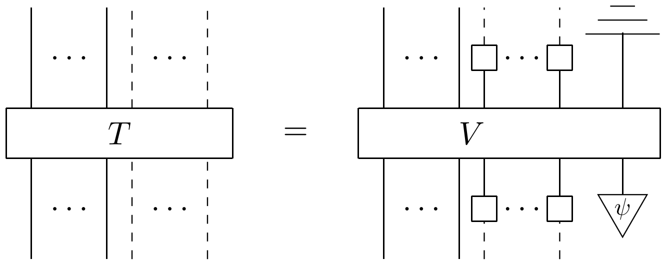
And, as we will discuss in section 5, this suggests a relatively conservative way of retaining AOE.

4. From deeper principles

In this section, we will derive a contradiction with AOE from deeper physical principles. To this end, we will show that all *NSC perspectival theories*—those with what we will call No Superluminal Influences, Separable Dynamics, and Consistent Embeddings—have Local Dynamics (theorem 3). The theorem suggests an interesting approach to maintaining AOE in which one rejects Local Dynamics and yet avoids superluminal influences by rejecting Separable Dynamics.

4.1. NSC theories have Local Dynamics

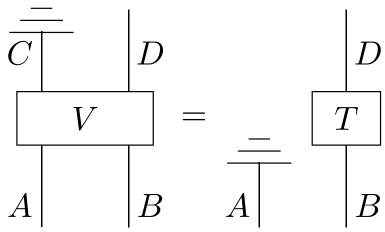
To define the three NSC properties, we must first assume that there is a subset of the transformations in our perspectival theory that can be considered *fundamental*. We stipulate that any trace-preserving transformation T in the theory should be obtained from a fundamental one V and some state ψ in the following way



(40)

For example, suppose that we have some trace-preserving transformation T in a perspectival formulation of quantum theory. Some of the inputs and outputs of that transformation might be classical. But we can always recover the transformation from some quantum channel (whose inputs and outputs are all quantum systems) by (1) placing probability extractors on some of the outputs of the channel and (2) placing 'encoding maps', that store classical information in a preferred basis of a quantum system, on some of the inputs of the channel. And this channel will always admit a Stinespring dilation. It follows that T can be written in the form of (40), where V is a unitary. (Note however that in the general case there is no assumption that ψ is a 'pure' state.)

We must also assume that for each input subsystem of a fundamental transformation, there is a fact of the matter about whether it exerts a causal influence on any given output subsystem. The precise definition of causal influence is irrelevant to the derivation, though a natural candidate definition is that A causally influences D through a fundamental transformation V if and only if the following holds, where T is some trace-preserving transformation.



$\nexists T$ (41)

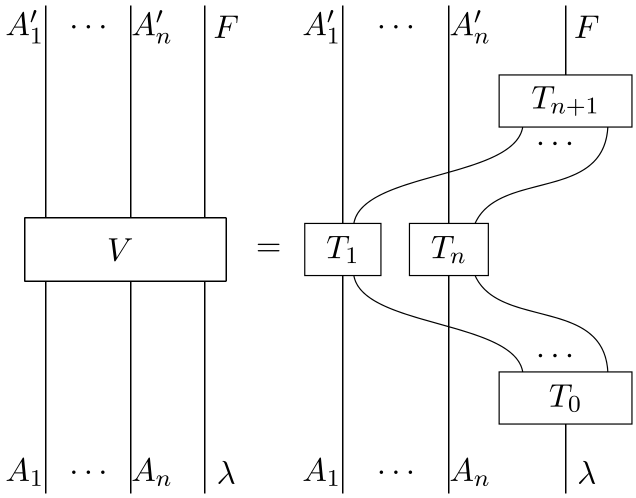
Then No Superluminal Influences may be defined as follows.

Definition 7 (No Superluminal Influences). A perspectival theory respects **No Superluminal Influences** just in case there is no valid embedding of any fundamental transformations such that an input subsystem exerts a causal influence on a spacelike separated output subsystem.

Our next principle concerns a fundamental transformation of the type

$$V: \quad A_1 \otimes \ldots \otimes A_n \otimes \lambda \quad \rightarrow \quad A_1' \otimes \ldots \otimes A_n' \otimes F \tag{42}$$

with a particular sort of causal structure. Namely, a causal structure such that each input A_i does not influence A_j' for any $j \neq i$. In that case, we want to assume that V *separates* into a circuit of transformations, such that there are only directed paths of wires between systems that might influence each other:



(43)

Note that we do not need to assume that the T_i are fundamental transformations. Our second NSC principle is summarized as follows.

Definition 8 (Separable Dynamics). A theory has **Separable Dynamics** just in case any fundamental transformation $V : A_1 \otimes \ldots \otimes A_n \otimes \lambda \to A_1' \otimes \ldots \otimes A_n' \otimes F$ with the property that A_i does not influence A_j' for all $j \neq i$ decomposes as in (43).

The final principle we need to derive Local Dynamics is just a consistency constraint on the valid embeddings. The idea is that, for any valid embedding of a non-fundamental transformation, there is a consistent valid embedding of a fundamental one from which it can arise. For example, suppose the input of a quantum channel is a system A at the location x, and its output is a system B at the location y. If this channel actually arises from a more fundamental, unitary, channel, then obviously the input subsystem A and the output subsystem B of that unitary are located at x and y respectively (though its other subsystems might be elsewhere). So the fact that we can embed the original channel a certain way means we must also be able to embed some unitary in a corresponding way.

Consistent Embeddings is a generalization of this idea. If an input (output) of T is embedded at a given spacetime point then the corresponding input (output) of V should be embeddable at the same point. (As (40) indicates, a classical input (output) of T is related to an input (output) of V via a local transformation, but we assume this local transformation could take place arbitrarily quickly.) More formally, let us denote the inputs of T by B_i and the outputs by B_i', and the corresponding inputs and outputs of V by A_i and A_i' respectively. (If B_i is classical then it might be that $B_i \neq A_i$.) We then have the following.

Definition 9 (Consistent Embeddings.). A perspectival theory has **Consistent Embeddings** just in case for any trace-preserving transformation T that can be validly embedded with the function $\mathcal{E}$, there is a fundamental transformation V from which T arises via (40) that can be validly embedded using a function $\mathcal{F}$ satisfying

$$\begin{aligned} \mathcal{E}(B_i) &= \mathcal{F}(A_i) \\ \mathcal{E}(B_i') &= \mathcal{F}(A_i'). \end{aligned} \tag{44}$$

We then obtain the following theorem.

Theorem 3 (NSC theories have Local Dynamics.). *Any perspectival theory with No Superluminal Influences, Separable Dynamics, and Consistent Embeddings also has Local Dynamics.*

Proof. If the theory has Consistent Embeddings, then any transformation $T : B_1 \otimes \ldots \otimes B_n \to B_1' \otimes \ldots \otimes B_n'$ can be embedded such that each pair (B_i, B_i') is spacelike separated only if it can be obtained via (40) from a fundamental transformation $V : A_1 \otimes \ldots \otimes A_n \otimes \lambda \to A_1' \otimes \ldots \otimes A_n' \otimes F$, which can itself be embedded in such a way that each pair (A_i, A_i') is spacelike separated. In that case, given No Superluminal Influences, the antecedent condition of the Separable Dynamics condition is satisfied by V. Thus, given Separable Dynamics, V decomposes as in (43). Inserting a state on λ and tracing out F in (43) to obtain T via (40) leads to a circuit diagram that can easily be simplified to one of the form (L). □

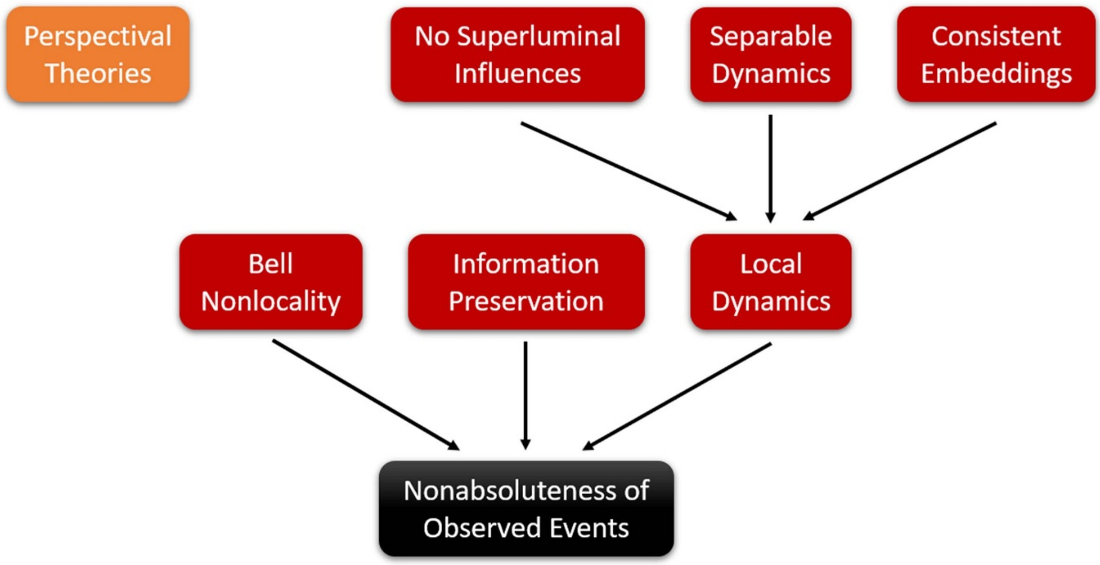


Figure 3. A graphical summary of theorems 1, 3, and 4.

Quantum theory as a BINSC theory. Before stating another result, let us show that quantum theory can be formulated as a BINSC theory (and hence, by theorem 3, as a BIL theory). As discussed in section 2.2, one can devise a quantum perspectival theory in which all memory updates are isometries. This theory is clearly Information Preserving, and will be Bell Nonlocal for any sensible specification of the valid embeddings. Let us designate the unitary channels as the fundamental transformations, and adopt the definition of causal influences in (41). We can then impose by fiat that the valid embeddings are restricted so that both No Superluminal Influences and Consistent Embeddings are respected. Appendix B shows that the resulting perspectival theory has Separable Dynamics, implying, by theorem 3, that it also has Local Dynamics.

4.2. Recasting the no-go result

Combining theorems 1 and 3 leads to the following no-go result.

Theorem 4 (BINSC theories are incompatible with AOE.). *Any perspectival theory that*

- *is Bell Nonlocal;*
- *is Information Preserving;*
- *satisfies No Superluminal Influences;*
- *has Separable Dynamics; and*
- *has Consistent Embeddings*

makes some predictions that are incompatible with AOE.

Thus in order to maintain AOE one would have to either embrace a perspectival theory that lacked one of these five properties, or else adopt a theory that could not be formulated in the perspectival framework. We note that a similar theorem can be derived to the effect that 'PINSC' theories are incompatible with AOE, by swapping Bell Nonlocality for Possibilistic Bell Nonlocality, and that this theorem would not rely on the validity of probability theory.

Our no-go results admit a graphical representation, in the style of [23, 24]. In figure 3, each principle with parents is implied by their conjunction. The disconnected orange node denotes the background assumption that one's theory can be formulated as a perspectival theory, which is required to make sense of the other principles. Rejecting a principle in the graph that has parents requires one to reject at least one of its ancestors and/or the disconnected node.

4.3. Notions of dynamical nonlocality

Since we will argue that distinguishing between different notions of dynamical (non)locality may hold the key to solving the measurement problem posed by extended Wigner's friend scenarios, it is worth elaborating on these notions a bit.

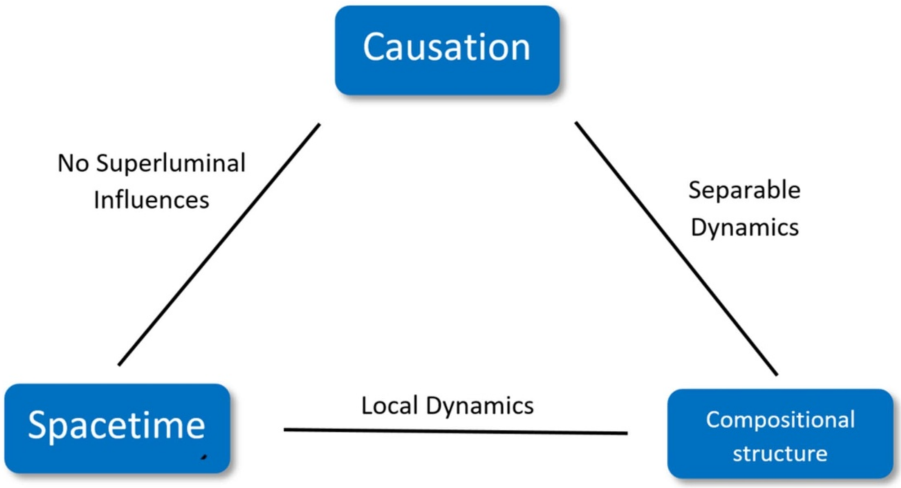


Figure 4. Different notions of dynamical locality.

Local Dynamics is the assumption that transformations must decompose a certain way when they take place across spacelike separated regions: it is a connection between *spacetime* and *compositional structure* (i.e. the set of all facts about how transformations can be combined to create other transformations in the theory). Given Consistent Embeddings, one can split up Local Dynamics into two further dynamical locality assumptions. No Superluminal Influences says that causal influences are local: it links *spacetime* with *causation*. On the other hand, Separable Dynamics says that transformations with certain causal structures must decompose a certain way: it links *causation* with *compositional structure*. Therefore, one can think of the connection between spacetime and compositional structure provided by Local Dynamics as mediated by causation, as in figure 4.

Relativity theory strongly suggests that spacetime constrains causation. On the other hand, it is not obvious that it suggests causation should constrain compositional structure. This observation is key to the argument we will give in section 5 that rejecting Separable Dynamics is a possible way of avoiding a measurement problem.

5. Roads back to absoluteness

In this penultimate section, we assess the prospects for a future theory of physics to avoid a measurement problem, or at least the aspect of the measurement problem that is revealed by the absoluteness no-go theorems.

One way a future theory might evade our no-go theorems is by failing to fit into the perspectival framework. It seems to us that this is likely to come at great cost. Recall that in many perspectival theories, the inside and outside perspectives are essentially the same, meaning that one cannot reject the framework simply by rejecting the idea that there are different perspectives on measurements. Instead, rejecting the perspectival framework will likely involve rejecting the framework of categorical probabilistic theories.

But a categorical probabilistic theory is essentially any circuit theory containing a classical subtheory that handles probabilistic empirical predictions. Theorem 2 suggests that one will not be able to recover absoluteness by denying that empirical predictions should be dealt with using classical probability theory. Therefore, the strategy of denying that the theory should be a circuit theory is likely to involve rejecting *compositionality*, the idea underlying a circuit theory that if a set of transformations can be performed, so too can combinations of them.

One might object that the interchange law (14) of categorical probabilistic theories encodes a frame-independence assumption, which one might reject without rejecting the idea of compositionality. But this is not right: as noted in section 3, the interchange law does *not* by itself suggest frame-independence. Rather, it only suggests frame-independence when it is combined with Local Dynamics. The natural option for those who reject the equivalence of inertial frames is not to reject the interchange law, but rather to reject Local Dynamics. For if one does reject the interchange law, then one faces the challenge of explaining (1) what the expression $T = T_1 \otimes T_2$ means if not that T is the combination of two independent transformations, and (2) what 'independent' here means if not that the order in which T_1 and T_2 are applied does not matter.

What can one do without rejecting the perspectival framework? Since the violation of Bell inequalities is an experimental fact, recovering AOE involves rejecting either Information Preservation or Local Dynamics. It therefore involves rejecting one of

- Consistent Embeddings
- Information Preservation
- No Superluminal Influences
- Separable Dynamics.

We find it difficult to imagine a justification for rejecting Consistent Embeddings. Rejecting Information Preservation or Local Dynamics has long been advocated respectively by objective collapse theorists and pilot-wave theorists. An interesting alternative, however, is to reject Separable Dynamics. Note that although relativity theory would appear to rule out superluminal influences, it does not obviously rule out nonseparable dynamics.

An interesting comparison can be made between the strategy of rejecting Separable Dynamics and a common response to Bell's theorem. Bell's theorem appears to reveal a tension between quantum correlations and relativity theory. But in response, it is often argued that the correlations are achieved by a middle-of-the-road nonlocality, poetically called *passion* rather than action at a distance [22], and often cashed out in terms of the nonseparability of states rather than the nonlocality of causal influences. Passion at a distance is supposed to be nonlocality enough to violate Bell inequalities, but locality enough to save relativity theory.

Analogously, we are suggesting here that the nonseparability of dynamics might be nonlocality enough to save AOE, but locality enough to save relativity theory. Perhaps the lesson of Bell is that the states of distant particles are inextricably linked, and the lesson of Wigner and the new no-go results is that their dynamics are too.

Of course, this suggestion is no silver bullet. Embracing dynamical separability amounts to rejecting unitary dynamics, which *are* separable (see appendix B). For many, rejecting unitarity will feel equally or even more difficult than rejecting relativity theory. One might therefore be inclined to simply accept the failure of AOE, with all of its problematic consequences.

6. Roads to relativity

It is important to recognize that simply stating that observed events are not absolute is *not* a complete response to the absoluteness no-go theorems. One then has to answer a question: if observed events are not absolute, what *are* they? If they are 'relative', then relative to *what*? After answering this question, one then needs to demonstrate that the new notion of events does not create even greater problems than it was intended to solve.

Existing answers to the question of what events are relative to diverge considerably. Observed events are taken by some to be relative to emergent quasi-classical 'worlds'; by others to be relative to consistent sets of histories; or by others still to systems. But neither Everett, nor consistent histories, nor Rovelli's relationalism has been able to command consensus, and it is even controversial whether any one of these approaches constitutes a precise physical theory that can recover the predictions of Copenhagen interpretation (see, for instance, [25–27]).

The basic problem is that if *events* are relative to X, then *predictions* about events are also relative to X. Thus an ontology of relational events creates the epistemological problem of working out, in any given situation, which X I should choose when making my predictions. And it is not always clear that this can be done in a principled way that ultimately leads to the same predictions as the Copenhagen interpretation.

We note that there is a closely related problem here, of a more purely epistemological variety. Namely: in a world where observed events are not absolute, how can inter-subjective agreement be achieved? This question was already tackled from a quantum perspective in [10], which resolves paradoxes arising from multiple agents combining their beliefs about non-absolute events. An open question is whether similar resolutions could be provided for arbitrary BINSC theories. To tackle this question, one could attempt to extend the quantum circuits framework for the subjective perspectives of agents from [10] to arbitrary BINSC theories[17].

[17] That framework might also provide hints about which sorts of relational ontology can permit inter-subjective agreement.

Our results also provide a potential clue for approaching a conception of relative events. In a perspectival formulation of quantum theory, it is *assumed* that a circuit of memory updates corresponds to a number of different 'inside' perspectives, featuring various sets of probability extractors. This raises the question of whether it is possible to *derive* these inside perspectives in a principled way. This is the approach taken by the interpretation of quantum theory proposed in [28], in which 'inside' perspectives are derived from the causal structure of a unitary circuit. Observed events are held to be relative to sets of systems because they emerge from the causal relations between them.

Data availability statement

No new data were created or analysed in this study.

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Appendix A. Categorical probabilistic theories done more formally

Here, we provide a relatively rigorous definition of categorical probabilistic theories that does not assume any prior knowledge of category theory. We do so in four steps. First, we define a (strict) *symmetric monoidal category* (SMC), which is the categorical term for what we have called a circuit theory. Then, we introduce the concept of traces. We then offer a particular example of an SMC, known as 'matrices over the positive reals', or $\mathbb{R}^+$-Mat. This will put us in a good position to define categorical probabilistic theories, which are SMCs that include (a category that is equivalent to) $\mathbb{R}^+$-Mat as a special *sub*-SMC.

A.1. Strict symmetric monoidal categories

We will define a *strict symmetric monoidal category* in the stages: first by defining a category, then a strict monoidal category, and then finally a strict symmetric monoidal category. We will then discuss how these relate to symmetric categories more generally.

Category. In a nutshell, a category is a collection of transformations that can be performed in sequence. More formally, a *category* $\mathcal{C}$ consists of

1. A set of *systems* $\mathrm{Sys}(\mathcal{C})$.
2. For each $A, B \in \mathrm{Sys}(\mathcal{C})$, a set of *transformations* $\mathcal{C}(A, B)$.
3. In $\mathcal{C}(A, A)$ for each $A \in \mathrm{Sys}(\mathcal{C})$, an *identity transformation* 1_A.
4. For each $A, B, C \in \mathrm{Sys}(\mathcal{C})$, a *sequential composition function* $\mathcal{C}(B, C) \times \mathcal{C}(A, B) \to \mathcal{C}(A, C)$ that is associative and unital,

$$(V \circ U) \circ T = V \circ (U \circ T)$$
$$1_B \circ T = T \circ 1_A, \qquad (45)$$

where $T : A \to B$.

Strict monoidal category. In a strict monoidal category, one also has a notion of performing a pair of transformations 'at the same time'. More formally, a *strict monoidal category* $\mathcal{C}$ is a category with *tensor product* functions on systems and transformations

$$\otimes : \mathrm{Sys}(\mathcal{C}) \times \mathrm{Sys}(\mathcal{C}) \to \mathrm{Sys}(\mathcal{C})$$
$$\otimes : \mathcal{C}(A,B) \times \mathcal{C}(C,D) \to \mathcal{C}(A \otimes C, B \otimes D) \tag{46}$$

that are associative, unital, and 'play well' with $\circ$:

$$\begin{aligned}
&(A \otimes B) \otimes C = A \otimes (B \otimes C)\\
&A \otimes I = A = I \otimes A\\
&(T \otimes U) \otimes V = T \otimes (U \otimes V)\\
&T \otimes 1_I = T = 1_I \otimes T\\
&(T_1 \otimes T_2) \circ (R_1 \otimes R_2) = (T_1 \circ R_1) \otimes (T_2 \circ R_2).
\end{aligned} \tag{47}$$

(In a non-strict monoidal category, these equalities are only required to hold up to isomorphism.)

Strict symmetric monoidal categories. In a strict symmetric monoidal category, when two transformations are performed 'at the same time', it does not much matter 'which way round' they are performed. More formally, a *strict symmetric monoidal category* is a strict monoidal category in which, for each $A, B \in \mathrm{Sys}(\mathcal{C})$, there exists a *swap* transformation $\mathtt{SWAP}_{A,B}$ satisfying:

$$\begin{aligned}
\mathtt{SWAP}_{A,B}(T \otimes U) &= (U \otimes T)\,\mathtt{SWAP}_{A,B}\\
\mathtt{SWAP}_{B,A} \circ \mathtt{SWAP}_{A,B} &= 1_A \otimes 1_B\\
(1_B \otimes \mathtt{SWAP}_{A,C})(\mathtt{SWAP}_{A,B} \otimes 1_C) &= \mathtt{SWAP}_{A,B\otimes C}.
\end{aligned} \tag{48}$$

Many if not most interesting SMCs are *not* strict ones. Non-strict SMCs are defined similarly to strict ones, except that many of the equalities we have imposed above are required to hold only up to isomorphism. Defining the relevant isomorphisms is a complicated business that we have chosen to avoid here. Fortunately, however, it turns out that any SMC is *categorically equivalent* to a strict SMC, meaning that one can often think about a non-strict SMC as if it were strict without running into trouble.

Before moving on, we permit ourselves a brief tangential remark on what is conceptually distinctive about category theory. Often, in mathematics, we think of a transformation as a transformation between sets, whose elements are defined independently of that transformation. And often, in physics, we think that the fundamental object is a physical state, and the job of a transformation is just to tell that state how to change. Category theory is motivated by a different perspective. Although any category $\mathcal{C}$ does include a set $\mathrm{Sys}(\mathcal{C})$ of systems, a system is not defined as a set of states. As far as the categorical axioms are concerned, a system is merely a label that helps us to keep track of different *types* of transformations in the category. In other words, they allow us to define different sets $\mathcal{C}(A,B)$ of transformations. And a state of A is merely a special type of transformation; namely, one found in $\mathrm{Sys}(I,A)$. Category theory thus provides a perspective on mathematics and physics in which the role of transformations is elevated, and the roles of systems and states are diminished.

A.2. The trace

If we want to ensure that states are normalized and transformations are normalization-preserving, or if we want to ignore a part of a system and focus on a smaller subsystem, then it is useful to introduce a *trace* transformation, Tr: $A \to I$, for each system $A \in \mathrm{Sys}(\mathcal{C})$. We require that tracing out two subsystems individually is equivalent to tracing out the composite system

$$\mathrm{Tr}_A \otimes \mathrm{Tr}_B = \mathrm{Tr}_{A\otimes B} \tag{49}$$

and that tracing out the trivial system is equivalent to doing nothing at all

$$\mathrm{Tr}_I = 1_I. \tag{50}$$

Let us call a transformation $T : A \to B$ *causal* (i.e. normalization-preserving) just in case the trace pulls through

$$\mathrm{Tr}_B \circ T = \mathrm{Tr}_A. \tag{51}$$

It is not hard to see that the causal transformations in an SMC form a *sub-SMC*. This means that they form an SMC in their own right, using the identity and swap transformations from the larger SMC. One example of an SMC and its sub-SMC of causal transformations is the matrices over positive numbers and stochastic matrices, to which we now turn.

A.3. $\mathbb{R}^+$-Mat

For our purposes, a particularly important SMC is $\mathbb{R}^+$-Mat. In this SMC, a system A is a natural number, i.e. $\mathrm{Sys}(\mathcal{C}) = \mathbb{N}$, and $\mathcal{C}(A,B)$ is the set of all $\mathbb{R}^+$-valued matrices with A columns and B rows. The $\circ$ and $\otimes$ operations are given by matrix multiplication and the Kroenecker product respectively. The trivial system is the number '1', transformations on which are one dimensional $\mathbb{R}^+$-valued matrices with one entry, giving us a notion of positive numbers. The identity transformations are identity matrices, and $\mathtt{SWAP}_{A,B}$ is defined in an obvious way.

The traces in this theory are row matrices with a '1' for each entry. Multiplying a matrix with the trace from the left results in a new row matrix where each entry is the sum of all of the elements in the corresponding column of the original matrix. This ensures that the causal transformations are stochastic matrices and that the causal states are probability distributions. Hence $\mathbb{R}^+$-Mat, and especially its sub-SMC of causal transformations, provide useful tools for modeling operational procedures, in which measurement settings may be chosen and outcomes obtained with various probabilities.

A.4. Categorical probabilistic theories

We can now finally define categorical probabilistic theories. These are SMCs with three specific features. Firstly, they contain $\mathbb{R}^+$-Mat, or an equivalent category, as a full sub-SMC[18]. We will call the smaller theory the 'classical sub-SMC'.

Secondly, a categorical probabilistic theory comes with traces for all its systems, and the traces in the classical sub-SMC are just (equivalent to) the usual traces from $\mathbb{R}^+$-Mat.

Finally, a categorical probabilistic theory must come with a notion of summing a pair of transformations $T, U \in \mathcal{C}(A,B)$ to form another transformation $T + U \in \mathcal{C}(A,B)$, satisfying three requirements. Firstly, there is a unit of summation $0 \in \mathcal{C}(A,B)$ for each A and B. Secondly, the sums on the classical sub-SMC are given by the usual notion of adding matrices. And thirdly, the $\circ$ and $\otimes$ operations are bilinear with respect to the sums, in the sense that the following equations are respected.

$$\begin{aligned}
&(T+U)\circ V = T\circ V + U\circ V\\
&T\circ(U+V) = T\circ U + T\circ V\\
&T\circ 0 = 0\\
&0\circ T = 0\\
\\
&(T+U)\otimes V = T\otimes V + U\otimes V\\
&T\otimes(U+V) = T\otimes U + T\otimes V\\
&T\otimes 0 = 0\\
&0\otimes T = 0.
\end{aligned}\tag{52}$$

Finally, let us state the definition more compactly.

Definition 10. A categorical probabilistic theory is an SMC with a full classical sub-SMC, with traces and sums. When one restricts to the classical sub-SMC, the traces are row matrices with unit entries, and summation is matrix-addition.

Appendix B. Quantum theory has Separable Dynamics

Here, we show that unitary transformations of the form

$$V: \quad \mathcal{H}_{A_1}\otimes\ldots\otimes\mathcal{H}_{A_n}\otimes\mathcal{H}_\lambda \quad\rightarrow\quad \mathcal{H}_{A_1'}\otimes\ldots\otimes\mathcal{H}_{A_n'}\otimes\mathcal{H}_F, \tag{53}$$

with the property that each A_i does not influence A_j' for any $j \neq i$, are separable, in the sense of satisfying (43). The notion of influence used here is given by (41), which is equivalent to the possibility of signalling through the unitary, as well as a number of other natural conceptions of causal influence (see theorem 3.1 of [29]). An immediate corollary is that any perspectival theory that has unitaries as fundamental transformations, and the same definition of influence, has Separable Dynamics. This includes the

[18] A sub-SMC is an SMC comprised of some of the systems from a larger SMC, and some of the transformations between them. The sub-SMC inherits the relevant compositional structure from the larger SMC, and must share its trivial system, identity transformations, and swap transformations. A sub-SMC is *full* just in case it contains *all* of the transformations from the larger SMC that go between any pair of systems that they share.

quantum perspectival theory with isometric memory updates introduced in section 2.3 and developed in section 4.1.

We begin with the case of $n=2$. In this case, the result follows theorem 3 of [30]. In more detail, suppose the Hilbert spaces of λ and F each admit a certain decomposition into a direct sum of m tensor product spaces:

$$\begin{aligned}\mathcal{H}_\lambda &\cong \bigoplus_{i=1}^m \mathcal{H}^i_{X_1} \otimes \mathcal{H}^i_{X_2}\\ \mathcal{H}_F &\cong \bigoplus_{i=1}^m \mathcal{H}^i_{X_1'} \otimes \mathcal{H}^i_{X_2'}.\end{aligned} \tag{54}$$

Given a unitary transformation

$$W: \quad \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2} \otimes \left(\bigoplus_{i=1}^m \mathcal{H}^i_{X_1} \otimes \mathcal{H}^i_{X_2}\right) \quad \to \quad \mathcal{H}_{A_1'} \otimes \mathcal{H}_{A_2'} \otimes \left(\bigoplus_{i=1}^m \mathcal{H}^i_{X_1'} \otimes \mathcal{H}^i_{X_2'}\right), \tag{55}$$

we say that V and W are isomorphic,

$$V \cong W, \tag{56}$$

if and only if there exists a pair of unitary transformations

$$\begin{aligned}S:& \quad \mathcal{H}_\lambda \quad \to \quad \bigoplus_{i=1}^m \mathcal{H}^i_{X_1} \otimes \mathcal{H}^i_{X_2}\\ R:& \quad \bigoplus_{i=1}^m \mathcal{H}^i_{X_1'} \otimes \mathcal{H}^i_{X_2'} \quad \to \quad \mathcal{H}_F\end{aligned} \tag{57}$$

such that

$$V = \left(I_{A_1'} \otimes I_{A_2'} \otimes R\right) W \left(I_{A_1} \otimes I_{A_2} \otimes S\right). \tag{58}$$

Theorem 3 of [30] shows that, for any V of the form (53), if A_i does not influence A_j' for all $j \neq i$ then there exist decompositions of the form (54) and unitary transformations of the form

$$\begin{aligned}V_1^{(i)}: &\quad \mathcal{H}_{A_1} \otimes \mathcal{H}^i_{X_1} \quad \to \quad \mathcal{H}_{A_1'} \otimes \mathcal{H}^i_{X_1'}\\ V_2^{(i)}: &\quad \mathcal{H}_{A_2} \otimes \mathcal{H}^i_{X_2} \quad \to \quad \mathcal{H}_{A_2'} \otimes \mathcal{H}^i_{X_2'}\end{aligned} \tag{59}$$

for $i \in \{1, \ldots, m\}$ such that

$$V \cong \bigoplus_{i=1}^m V_1^{(i)} \otimes V_2^{(i)}. \tag{60}$$

Let X_1 be a system with the Hilbert space $\mathcal{H}_{X_1} := \bigoplus_{i=1}^m \mathcal{H}^i_{X_1}$, and analogously X_2, X_1', and X_2'. (60) implies that V has the form in (43), where T_0 is the isometry obtained by extending the codomain of $\mathcal{S} := S(\cdot)S^\dagger$ to the full space

$$\mathcal{H}_{X_1} \otimes \mathcal{H}_{X_2} \cong \bigoplus_{i,j=1}^m \mathcal{H}^i_{X_1} \otimes \mathcal{H}^j_{X_2}, \tag{61}$$

and T_3 is the any channel obtained by extending the domain of the unitary channel $\mathcal{R} := R(\cdot)R^\dagger$ to the full

$$\mathcal{H}_{X_1'} \otimes \mathcal{H}_{X_2'} \cong \bigoplus_{i,j=1}^m \mathcal{H}^i_{X_1'} \otimes \mathcal{H}^j_{X_2'} \tag{62}$$

by direct-summing it with an arbitrary channel acting on the rest of the space.

We now prove the result for $n=3$. We note that if A_1 does not influence A_2 or A_3, then it does not influence the composite system $A_{23}' := A_2' \otimes A_3'$ [31]. Similarly, if A_2 and A_3 each do not influence A_1', then $A_{23} := A_2 \otimes A_3$ does not influence A_1. Therefore, we can apply the result for the $n = 2$ case to write:

$$V \cong \bigoplus_{i=1}^{m} V_1^{(i)} \otimes V_{23}^{(i)}. \tag{63}$$

Since A_2 does not influence A_3 through V, A_2 does not influence A_3 through $V_{23}^{(i)}$ for each i. Similarly, A_3 does not influence A_2 through $V_{23}^{(i)}$ for each i. For each i, we can therefore apply the result from the $n = 2$ case again to find that

$$V_{23}^{(i)} \cong \bigoplus_{j_i=1}^{m_i} V_2^{(ij_i)} \otimes V_3^{(ij_i)}. \tag{64}$$

Then substituting equation (63) into equation (64) and applying the isomorphism

$$\mathcal{H}_A \otimes (\mathcal{H}_B \oplus \mathcal{H}_C) \cong (\mathcal{H}_A \otimes \mathcal{H}_B) \oplus (\mathcal{H}_A \otimes \mathcal{H}_C) \tag{65}$$

gives a decomposition of the form

$$\begin{aligned} V &\cong \bigoplus_{i=1}^{m} \bigoplus_{ij_i=1}^{m_i} V_1^{(i)} \otimes V_2^{(ij_i)} \otimes V_3^{(ij_i)} \\ &\cong \bigoplus_{k=1}^{m'} V_{A_1X_1}^{(k)} \otimes V_{A_2X_2}^{(k)} \otimes V_{A_3X_3}^{(k)} \end{aligned} \tag{66}$$

where in the second line we have defined a new summation index $k := ij_i$ and $m' := \sum_{i=1}^{m} m_i$. This implies that the result holds for the $n = 3$ case. The argument generalizes in an obvious way to arbitrary n.

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