ISSN 1471-0498

DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

AMBIGUITY AVERSION AND INCOMPLETENESS OF FINANCIAL MARKETS

Sujoy Mukerji and Jean-Marc Tallon

Number 46

December 2000

Manor Road Building, Oxford OX1 3UQ

Ambiguity Aversion and Incompleteness of Financial Markets*

Sujoy Mukerji
Department of Economics
University of Southampton
Highfield, Southampton SO17 1BJ
United Kingdom
e-mail: sm5@soton.ac.uk

Jean-Marc Tallon
CNRS– EUREQua
106-112 bld de l'Hôpital
75647 Paris Cedex 13
France
e-mail: jmtallon@univ-paris1.fr

February 15, 1999

*We thank A. Bisin, S. Bose, P. Ghirardato, I. Gilboa, R. Guesnerie, B. Lipman, J. Malcomson, M. Marinacci, M. Piccione, H. S. Shin and J. C. Vergnaud for helpful discussions. The paper has also benefitted from the responses of seminar members at the Univ. of British Columbia, Univ. of Essex, Johns Hopkins Univ, NYU, UPenn, Univ. of Paris I, ENPC-Paris and the ESRC Economic Theory Conference at Kenilworth.

Extended Abstract

Subjective uncertainty is characterized by ambiguity if the decision maker has an imprecise knowledge of the probabilities of payoff relevant events. In such an instance, the decision maker's beliefs are better represented by a set of probability functions than by a unique probability function. An ambiguity averse decision maker adjusts his choice on the side of caution in response to his imprecise knowledge of the odds. The non-additive expected utility model allows a formal characterization of such behavior. This paper uses ambiguity aversion to understand aspects of behavior of financial markets.

More particularly, the paper focuses on the question, "What prevents the typical bond-equity finance economy from offering sufficient opportunities for Pareto optimal risk sharing? In other words, why should the theorems of general equilibrium with incomplete markets (GEI), rather than general equilibrium with complete markets (GE), be a more compelling description of the typical bond-equity economy?" To analyze the question, we consider a stylized bond-equity economy, which though incomplete per se, has a rich enough set of assets available for trade such that given standard assumptions about behavior under uncertainty, the equilibrium allocation would arbitrarily approximate a complete market (GE) allocation. We show, however, that given 'sufficient' ambiguity aversion, a certain subset of the available assets will not be actually traded in equilibrium, even though available. Hence it is proved that, given 'sufficient' ambiguity aversion, provided the non-traded securities are non-redundant, equilibrium allocation of the bond-equity economy is a GEI equilibrium. Thus we show how ambiguity aversion may endogenously limit the scope of risk sharing obtainable through the bonds/equities actually traded in a typical economy, and therefore, explain why the actual behavior of such an economy is better described by the GEI model, rather than the GE model.

The formal analysis in this paper basically involves a reconsideration of the general equilibrium of a simple 'finance economy' while allowing for ambiguity aversion. The underlying objective is to identify the class of assets whose trade is vulnerable to ambiguity aversion: assets that will be traded if agents are subjective expected utility maximizers but not if the agents' common beliefs about payoffs of the assets is sufficiently ambiguous and the agents are ambiguity averse. We find that what determines an asset's vulnerability to ambiguity aversion is whether its payoffs have an *idiosyncratic component*, i.e., if at least some component of the payoff is independent of the realized endowment vector and of the payoff of any other asset as well. It turns out that if, (1) the range of variation of the payoff's idiosyncratic component is 'large' relative to the range of the variation of the component correlated with the endowment vector and, (2) the ambiguity of the agents' common belief about the idiosyncratic component is sufficiently high, then the asset will not be traded in any general equilibrium of the finance economy. Moreover, we also find that the effect of idiosyncracy cannot simply be 'washed away' by the standard techniques of diversification relying on the laws of large numbers, as it would be if the agents' beliefs were not ambiguous. Indeed, it is shown that given unambiguous beliefs the idiosyncracy does not hinder risk sharing in the sense that a complete market allocation may be obtained using the given set of assets. However, if a subset of (non-redundant) primitive assets satisfies the two conditions mentioned above then, with ambiguity aversion, the number of contingencies spanned by the assets actually traded in any equilibrium (as opposed to the assets that are in principle available for trade) shrinks endogenously and it becomes impossible to obtain complete market risk sharing with primitive assets.

1 Introduction

Typically, economic agents are endowed with income streams that are not evenly spread over time or across uncertain states of nature. A financial contract is a claim to an income stream– hence the logic of the financial markets: by exchanging such claims agents change the shapes of their income streams, obtaining a more even consumption across time and the uncertain contingencies. A financial market is said to be *complete* if contingent payoffs from the different marketed financial contracts are varied enough to span all the contingencies. However, casual empiricism suggests that in just about every financial market in the real world the span is less than the full set of contingencies, i.e., the markets are *incomplete.* The primary implication of incompleteness of financial markets is that agents may transfer income only across a limited set of contingencies and are thus left exposed to risk in a suboptimal manner. And indeed formal empirical investigations overwhelmingly confirm that the data on individual consumption are more consistent with incomplete than complete markets[1].

Incompleteness of financial markets is a compelling economic phenomenon not just because it is a pervasive empirical reality, but because of its very significant economic consequences. This, in fact, is the inspiration for what is perhaps the most comprehensive model of the market economy: general equilibrium with incomplete markets (GEI). Analysis of GEI has uncovered a host of novel insights about competitive markets. If asset markets are incomplete, a competitive equilibrium allocation is typically constrained inefficient, in the sense that there will exist a reallocation of the available assets such that if the agents were to trade goods after the assets have been reallocated they would achieve an allocation which would Pareto dominate the original equilibrium (Geanakoplos and Polemarchakis (1986)). Incompleteness also typically leads to real indeterminacy of competitive equilibrium (Cass (1985), Geanakoplos and Mas-Colell (1989)). In turn, this leads to the non-neutrality of money (Magill and Quinzii (1992)). Other serious macroeconomic consequences include the preservation of memory in macroeconomic aggregates and the existence of a Philips trade-off (Dutta and Polemarchakis (1990)). Many pervasive features of financial markets that are difficult to account for in the complete (competitive) markets paradigm are insightfully explained in the GEI framework. Innovation of new securities by decomposing and repackaging primary securities (i.e., stock equities and bonds) to obtain various 'derivatives' is the major activity of financial investment houses. This activity would not even arise if the market for primary securities were a complete market to begin with. Consider, next, the case of bankruptcy. In GEI, bankruptcy can be understood as an equilibrium, as opposed to a disequilibrium phenomenon, and thus widely observed lenient bankruptcy and default penalties may be rationalized in this framework, whereas they remain wholly inexplicable in a competitive world with complete markets (Dubey, Geanakoplos, and Shubik (1994), Zame (1993)).

The key role of incompleteness of asset markets in explaining diverse and important economic phenomena underscores the importance of understanding what causes the incompleteness in the first place. In the archetypal GEI model, though, the unavailability of classes of assets and market incompleteness is a primitive and not derived from other considerations. As Geanakoplos (1990) pointed out, "Perhaps the most unexplored part of the GEI model is a theory explaining which markets are open and which are closed. This may be viewed ... as a challenge for a research program ... ". The challenge then, is to obtain a model that yields conditions on primitives explaining why certain assets would not be traded even if they were available and could easily be developed. This paper does precisely that.

[1] Among others, see Zeldes (1989), Cochrane (1991), Carroll (1992), Deaton (1992), Deaton and Paxson (1994) and Hayashi, Altonji, and Kotlikof (1996). The evidence, however, is not unanimous, see e.g., Mace (1991) and Townsend (1994).

Given all this it is hardly surprising that formal identification of circumstances that render financial markets incomplete has been a focal point of recent research (see Duffie and Rahi (1995) for a survey). The work completed so far has essentially been inspired by the twin strands of transactions costs and asymmetric information and, by and large, focus on the question of creation of derivative securities and why private incentives may prevent the creation of the full set of derivatives required to complete a market. This theme has been studied by Allen and Gale (1994), Duffie and Jackson (1989), Bhattacharya, Reny, and Spiegel (1995), Ohashi (1995), Rahi (1995), and Bisin (1998), among others. Bisin and Gottardi (1997) show that the presence of asymmetric information generates endogenous restrictions on the set of tradable contracts: the set of agents' insurance opportunities are restricted endogenously by the agents' incentive compatibility constraints and the associated conditions for the viability of markets. The findings in this paper complements the existing literature explaining the incompleteness financial markets, in that it provides an alternative explanation based on the hypothesis that decision behavior under subjective uncertainty is affected by *ambiguity aversion*. Also, unlike the literature cited above (with the notable exception of Bisin and Gottardi (1997)), which concerns itself with the question of endogenizing the incompleteness of a market of *derivative* securities, the present paper provides an explanation of incompleteness of a financial market consisting of *primary* securities.

Possible contingencies are, quite literally, endless. Matters arising from transactions costs, asymmetric information and verifiability of complicated contractual clauses are compelling enough to explain why "customized" contracts promising appropriate deliveries conditional on the realization of *any and every* contingency one can conceive of, corresponding to every permutation of every slight detail, are not available. This paper will not provide any new insight about why such "personalized" contingent delivery contracts are typically not available. Instead this paper focuses on the question, what prevents the typical bond-equity finance economy from offering sufficient opportunities for Pareto optimal risk sharing? In other words, the focus is on the pure risk-sharing opportunities provided by the portfolio of bonds and equities available in the economy: to what extent may households be able to share their risk by appropriately going short or long on these assets. Characteristically, contingent payoffs from the typical equity will only be imperfectly correlated with households' income risks. However, that would not explain why bonds and equities actually traded in the market are not adequate to span all 'relevant' contingencies and thereby provide for optimal risk sharing. Indeed, all it takes is that given any income risk there be a set of corporate bonds/equities whose payoffs are also affected by that risk. Seemingly, as long as every household income risk is even partly correlated with the payoffs of a large enough number of such bonds/equities, the equilibrium risk-sharing would closely approximate a prototypical complete markets economy, even though the bond-equity economy is not complete *per se* (in the sense that *every conceivable contingency* is not 'spanned'). Hence, why should the theorems of GEI, rather than GE, be a more compelling description of the typical bond-equity economy? To analyze the question, we consider a stylized bond-equity economy, which though incomplete per se, has a rich enough set of equities available for trade such that given standard assumptions about behavior under uncertainty, the equilibrium allocation would arbitrarily approximate a complete market allocation. We show however, given 'sufficient' ambiguity aversion, a certain subset of the available equities will not be traded in equilibrium. Provided the non-traded securities are non-redundant, equilibrium allocation under ambiguity aversion is a GEI equilibrium. Thus we show how ambiguity aversion may *endogenously* limit the scope of risk sharing obtainable through the bonds/equities *actually traded* in a typical economy, and therefore, explain why the *actual behavior* of such an economy is better described by the GEI model, rather than the GE model.

Suppose an agent's subjective knowledge about the likelihood of contingent events is consistent with more than one probability distribution. And further that, what the agent knows does not inform him of a precise (second order) probability distribution over the set of 'possible' probabilities. We say then that the agent's beliefs about contingent events are characterized by *ambiguity*. If ambiguous[2], the agent's beliefs are captured not by a unique probability distribution in the standard Bayesian fashion but instead by a set of probabilities. Thus not only is the particular outcome of an act uncertain but *also* the expected payoff of the action, since the payoff may be measured with respect to more than one probability. An agent's ambiguity of belief about an event is said to be greater, the greater the difference between the maximum and minimum probability estimate of the event, consistent with the agent's knowledge. An *ambiguity averse* decision maker evaluates an act by the minimum expected value that may be associated with it: the decision rule is to compute all possible expected values for each action and then choose the act which has the best minimum expected outcome. The idea being, ceteris paribus, the more an act is affected adversely by ambiguity the less its appeal to the ambiguity averse decision maker.

The formal analysis in this paper basically involves a reconsideration of the general equilibrium of a simple 'finance economy' while allowing for ambiguity aversion. The underlying objective is to identify the class of assets whose trade is vulnerable to ambiguity aversion: assets that will be traded if agents are subjective expected utility (SEU) maximizers but not if the agents' common beliefs about payoffs of the assets is sufficiently ambiguous, and the agents are ambiguity averse. The finance economy is a two period exchange economy wherein agents consume a single good in the final period. There are a (finite) number of possible contingent states in the final period and endowments may vary across contingencies. There is no consumption in the initial period; the agents' activity in this period consists of exchanging financial assets which deliver varying amounts of the consumption good in the final period depending on the contingency realized. We find that in such an economy what determines an asset's vulnerability to ambiguity aversion is whether its payoffs have an *idiosyncratic* component. We say an asset's payoff has an idiosyncratic component if at least some component of the payoff is independent of the realized endowment vector *and* of the payoff of any other asset as well. It turns out that if, (*1*) the range of variation of the payoff's idiosyncratic component is 'large' relative to the range of the variation of the component correlated with the endowment vector and, (*2*) the ambiguity of the agents' common belief about the idiosyncratic component is sufficiently high, then the asset will not be traded in *any* general equilibrium of the finance economy. Moreover, and what is perhaps as curious, we also find that the effect of idiosyncracy cannot simply be 'washed away' by the standard techniques of diversification relying on the laws of large numbers, as it would be if the agents' beliefs were not ambiguous. Indeed, given unambiguous beliefs the idiosyncracy does not hinder risk sharing and a complete market allocation may be obtained using the given set of assets. However, if a subset of (non-redundant) primitive assets satisfies the two conditions mentioned above then, given ambiguity aversion, the number of contingencies spanned by the assets *actually traded in any equilibrium* (as opposed to the assets that are in principle available for trade) shrinks endogenously and it becomes impossible to obtain complete market risk sharing with primitive assets. Plausibility of the explanation naturally rests on the plausibility of the assumptions. Next, we discuss in turn the three key assumptions our explanation draws on.

[2] To preempt misunderstandings it is emphasized that the term "ambiguity" as used in this paper, refers purely to the fuzzy perception of the likelihood subjectively associated with an event (e.g., when asked about his subjective estimate of the probability of an event, the agent replies, "It is between 50 and 60%."). It **does not** refer to a lack of clarity in the description of contingent events and actions. Also note, some authors and researchers refer to ambiguity as "Knightian Uncertainty" or even simply as "uncertainty". As it is used in this paper, the word "uncertainty" is simply the defining characteristic of *any* environment where the consequence of at least one action is not known for certain.

The first assumption is about the presence of idiosyncratic effects on asset payoffs. Essentially, it is assumed that there is a set of assets whose payoffs are believed to be affected not only by some of the same risks/shocks/factors that affect individual households' endowment income and common to many assets but *also* by risks specific to each asset. These assets are modeled in this way in order to incorporate, what is arguably an important and pervasive feature of many equities. While most firms' profits will naturally be affected by aggregate or sectorial demand shifts and supply shocks, other factors, more idiosyncratic to the firm, will typically also affect profits. For instance, suppose a firm introduces a new product line, an innovation, into the market. In such a case, typically, it is not just the shocks commonly affecting firms in the same trade that will affect the sales of the new product but also more specific elements, e.g. whether (or not) the innovation has a 'special' appeal for the consumers. Or maybe a firm experiences a slip up (or finds a serendipitous bout of efficiency) in its organizational hierarchy/production process that makes difference to the production time, costs or even product quality[3]. As it appears in the formal model, a household's endowment income is *distinct* from the household's income obtained from the ownership of assets. We may think of the endowment income as income generated by the individual's labor endowment (wages or more broadly, returns to human capital) and bequests and gifts. To summarize, therefore, the households have two sources of (random) income in the final period. The first part is exogenous and corresponds to income from their endowment in goods (say, labor income). The second part, which is endogenous, is the income derived from their portfolio holding. The assumption that the second part has some idiosyncracy attached to it is to reflect the fact that, typically, financial assets returns are not perfectly correlated with household endowments. It is worth clarifying that we do not model firms, and hence, equities, *per se*, since we have the assets in zero net supply: the assets are pure (contingent) claims and promises. In particular, as modelled in this paper, an asset does not have an explicit 'physical' entity, it is not a claim to a firm which actually 'creates' output. Essentially, we do this to analyze the nature of *pure risk sharing opportunities* offered by primary assets like equities[4]. The point is to abstract away from the fact that by owning a (part of) a firm an agent changes the shape of his (contingent) income stream by simply laying claim to the contingent physical output of the firm. Instead, the paper studies how an agent may use an equity to *share his risk* by selling the equity short to another agent; how agents may alter their risk positions through trade and exchange rather than by relying on 'physical fixes', i.e., actual creation of goods.

Next, consider the assumption that households' have ambiguity averse beliefs about asset payoffs. While the results in the paper hold if people are 'generally' ambiguity averse (i.e., across all events, all that is actually necessary is that agents' beliefs are ambiguous with respect to the idiosyncratic events affecting payoffs. Indeed, in an asset pricing context, Barsky and DeLong (1992) present convincing empirical evidence to argue that there is substantial uncertainty about the structure of the aggregate dividend process in the U.S. over the last century, even on the part of current analysts who have the benefit of hindsight[5].

[3] Allen and Gale (1994), chapter 10, contain further examples and an insightful discussion of such firm specific idiosyncratic uncertainty and its role in financial innovation.

[4] Since the securities considered in this paper are "inside" assets, in the sense of having zero aggregate demand/supply for each of them in equilibrium, the asset trading that we analyze includes all trade in corporate bonds and all trades in any kind of assets (including equities) which involve one side of the market going short (i.e., the trade involves a forward contract on the security in question). In this context it is worth noting that it is reported almost 70% of corporate borrowing in the U.S. is through bonds. Default rates on bonds are also significant. *Financial Times,* 13 October 1998, in its report headlined "US corporate bond market hit," notes, "the rate of default on US high-yield bonds was running at 10% in the early 1990s...today the default rate is hovering around 3% but creeping higher".

[5] Significantly enough, Allen and Gale (*op. cit.*, pp 332-333) in their pioneering analysis of financial innovations suggest that it would be of interest to model the idiosyncratic uncertainty in assets in a non-standard way using

The final significant assumption is the unavailability of customized contingent delivery contracts that allow explicit conditioning of deliveries on idiosyncratic shocks/events. (Though we formally assume that such contracts are not available corresponding to *any* given idiosyncratic event, as will be evident from the analysis, what is actually crucial is that such contracts be not available for at least *some* idiosyncratic events.) Given what an idiosyncratic event represents in our model, standard arguments invoking transactions costs, verifiability issues and asymmetric information, we believe, are a reasonable defence for this assumption. Moreover as has already been mentioned, and will be formally demonstrated in a subsequent section, given what assets are available, the unavailability of such contracts does not imply that the market equilibrium corresponds to that of an incomplete market in any 'real' sense, so long as agents' beliefs and behavior conform with the standard norm of (Savage) rationality: If agents are SEU maximizers, then the equilibrium behavior and exchange is as if the market were actually complete. To the extent exchange and consumption behavior departs from the complete market benchmark to correspond to an actual GEI equilibrium it does so on account of the introduction of ambiguity aversion to the model which endogenously causes some securities not to be traded. Hence, market incompleteness, in so far as it is actually reflected in (potentially) observable behavior, arises completely endogenously in the model and is in no way assumed at the outset.

The rest of the paper is organized as follows. In order to make the paper as self contained as possible the next section provides an introduction to the formal model of ambiguity aversion applied in this paper. Section 3 explains through an example the simple intuition behind the main result in the paper and its relation to the existing literature on the application of ambiguity aversion to financial markets. Section 4 contains the formal model of the finance economy and the main result. Section 5 explores the robustness of the results presented in the preceding section to a relaxation of some of the maintained assumptions. Section 6 concludes the paper with a discussion of the economic and empirical significance of the results. All formal proofs are in the appendix.

2 An introduction to the model of decision making under ambiguity aversion

It is often the case that a decision maker's (DM) perception of the uncertain environment is *ambiguous* in the sense that his knowledge is consistent with more than one probability function. The theory of ambiguity aversion is inspired by two simple hypotheses about decision making in such situations. First, that *behavior* is influenced by ambiguity: i.e., DM's behavior actually reflects the fact that his guess about a likelihood may be given by a probability interval. By presumption agents do not necessarily behave as if they have reduced all their ambiguity to a belief consistent with a unique probability using a 'second order' probability over the different probability distributions consistent with their knowledge. Second, that agents are ambiguity *averse*. That is, *ceteris paribus*, the more ambiguous their knowledge of the uncertainty the more conservative is their choice. Schmeidler (1989) pioneered an axiomatic derivation of a model of DMs with preferences incorporating ambiguity aversion (see also Gilboa and Schmeidler (1989)). The proposed research will apply Schmeidler's model, termed the Choquet expected utility (CEU) model, in the formal arguments.

The DM's domain of uncertainty is the finite state space $\Omega = \{\omega_i\}_{i=1}^{N}$. The DM chooses between acts whose payoffs are state contingent: e.g., a financial asset z, $z : \Omega \to \mathbb{R}$. In the CEU model an ambiguity averse DM's subjective belief is represented by a *convex non-additive*

sets of probabilities, since such an approach would do more to incorporate the essence of the idea of idiosyncratic risk.

probability function, ν. Like a standard probability function it assigns to each event a number between 0 and 1, and it is also true that, (i)$\nu(\emptyset) = 0$ and (ii)$\nu(\Omega) = 1$. Where a convex non-additive probability function differs from a standard probability is in the third property, (iii)$\nu(X \cup Y) \geq \nu(X) + \nu(Y) - \nu(X \cap Y)$, for all $X, Y \subset \Omega$. By this third property[6] the measure of the union of two disjoint events may be greater than the sum of the measure of each individual event. A convex non-additive probability function may be interpreted as a parsimonious representation of the full range of probabilities compatible with the DM's knowledge. $\nu(X)$ is interpreted as the minimum possible probability of X. This is readily seen from the fact that a given convex non-additive probability ν corresponds to a unique convex set of probability functions identified by the *core*[7] of ν, denoted by $\mathcal{C}(\nu)$ (notation: $\Delta(\Omega)$ is the set of all additive probability measures on Ω):

$$\mathcal{C}(\nu) = \{\mu \in \Delta(\Omega) \mid \mu(X) \geq \nu(X), \text{ for all } X \subset \Omega.\}$$

Hence, $\nu(X) = \min_{\mu \in \mathcal{C}(\nu)} \mu(X)$. The convex non-additive probability representation allows us to express the notion of ambiguity precisely. We say ν is *ambiguous* if there are two events X, Y such that axiom (iii) holds with a strict inequality; ν is *unambiguous* if axiom (iii) holds as an equality everywhere. (A DM with unambiguous belief is an SEU maximizer.) The *ambiguity*[8] of the belief about an event X is measured by the expression $\mathcal{A}(X;\nu) \equiv 1 - \nu(X) - \nu(X^c)$. The relation between ν and $\mathcal{C}(\nu)$ shows that the $\mathcal{A}$ is indeed a measure of the 'vagueness' of the belief, since, $\mathcal{A}(X;\nu) = \max_{\mu \in \mathcal{C}(\nu)} \mu(X) - \min_{\mu \in \mathcal{C}(\nu)} \mu(X)$.

Like in SEU, a *utility function* $u : \mathbb{R}_+ \to \mathbb{R}$, $u'(\cdot) \geq 0$, describes DM's attitude to risk and wealth. The DM evaluates *Choquet expected utility* of each act with respect to the non-additive probability, and chooses the act with the highest evaluation. Given a convex non-additive probability ν, the Choquet expected utility[9] of an act is simply the minimum of all possible 'standard' expected utility values obtained by measuring the contingent utilities possible from the act with respect to each of the additive probabilities in the core of ν :

$$\mathbb{C}\mathbb{E}_\nu u(z) = \min_{\mu \in \mathcal{C}(\nu)} \left\{ \sum_{\omega_i \in \Omega} u(z(\omega_i))\, \mu(\omega_i) \right\}.$$

The Choquet expected utility of an act is just its standard expected utility calculated with respect to a 'minimizing probability' corresponding to this act. Hence, in the Choquet method, the DM's appraisal is not only informed by his knowledge of the odds but is also automatically

[6] In general, a non-additive probability (or *capacity*) ν obeys the axioms (i), (ii), and the condition that $X \supseteq Y \Rightarrow \nu(X) \geq \nu(Y)$. The axiom (iii) applies to the special case of a *convex* non-additive probability. In Schmeidler's model, it is when the non-additive probability is convex that the CEU decision rule corresponds to ambiguity aversion.

[7] This follows from the celebrated theorem in Shapley (1965) which asserts the existence of a core allocation corresponding to any convex characteristic value function defined on possible coalitions in a cooperative game.

[8] Fishburn (1993) provides an axiomatic justification of this definition of ambiguity and Mukerji (1997) demonstrates its equivalence to a more primitive and epistemic notion of ambiguity (expressed in term's of the DM's knowledge of the state space).

[9] The Choquet expectation operator may be directly defined with respect to a non-additive probability. Label ω_i such that $z(\omega_1) \leq \ldots \leq z(\omega_N)$. Then,

$$\mathbb{C}\mathbb{E}_\nu u(z) = u(z(\omega_1)) + \sum_{i=2}^{N} [u(z(\omega_i)) - u(z(\omega_{i-1}))]\nu(\{\omega_i, \ldots, \omega_N\})$$

$$= \sum_{i=1}^{N-1} u(z(\omega_i))\, [\nu(\{\omega_i, \ldots, \omega_N\}) - \nu(\{\omega_{i+1}, \ldots, \omega_N\})] + z(\omega_N)\nu(\{\omega_N\})$$

adjusted downwards to the extent it may be affected by the imprecision of his knowledge[10]. The classic experiment which identifies the qualitative features of ambiguity aversion, due to Ellsberg (1961), runs as follows:

> There are two urns each containing one hundred balls. Each ball is either red or black. The subjects are told of the fact that there are fifty balls of each color in urn I. But no information is provided about the proportion of red and black balls in urn II. One ball is chosen at random from each urn. There are four events, denoted IR, IB, IIR, IIB, where IR denotes the event that the ball chosen from urn I is red, etc. On each of the events a bet is offered: \$100 if the events occurs and \$0 if it does not.

The modal response is for a subject to prefer every bet from urn I (IR or IB) to every bet from urn II (IIR or IIB). That is, the typical revealed preference is $IB \succ IIB$ *and* $IR \succ IIR$. (The preferences are strict.) The DM's beliefs about the likelihood of the events, as revealed in the preferences, cannot be described by a unique probability distribution. The story goes: People dislike the ambiguity that comes with choice under uncertainty; they dislike the possibility that they may have the odds wrong and so make a wrong choice (*ex ante*). Hence they go with the gamble where they know the odds — betting from urn I. It is straightforward to check that the choice is consistent with convex non-additive probabilities: For instance, let $\nu(IR) = \nu(IB)$ and $\nu(IR) + \nu(IB) = \nu(IR \cup IB) = 1$; also let $\nu(IIR) = \nu(IIB)$, but allow $\nu(IIR) + \nu(IIB) < \nu(IIR \cup IIB) = 1$. It follows that the expected payoff from betting on $IR \equiv \mathbb{CE}_\nu(IR) = \mathbb{CE}_\nu(IB) = 50$; and, $\mathbb{CE}_\nu(IIR) = \mathbb{CE}_\nu(IIB) = \nu(IIR) \times 100 = \nu(IIB) \times 100 < 50$.

The fact that the same additive probability (in the core of the relevant non-additive probability) will not in general 'minimize' the expectation for two different acts, explains why the Choquet expectations operator, unlike the standard operator, is not additive:

Fact 1 *For any two acts z, w : $\mathbb{CE}(z) + \mathbb{CE}(w) \leq \mathbb{CE}(z + w)$.*

The above property holds as an equality when the two acts z and w are *comonotonic*, i.e., if $(z(\omega_i) - z(\omega_j))(w(\omega_i) - w(\omega_j)) \geq 0$. Two comonotonic acts cannot be used to mutually hedge each other. As a consequence, combining them does not reduce the uncertainty attached to them and the Choquet expected value of that combination is simply the sum of their Choquet expected value.

The following example (inspired by Dow and Werlang (1992)) explains a consequence of the failure of additivity that will be particularly crucial to the arguments to be advanced in the following sections.

Example 1 Suppose a risk neutral investor is considering a transaction involving a unit of a financial asset z with contingent payoffs. Specifically, the investor is comparing the expected payoff from buying one unit of the asset to that from short selling one unit of the asset. The following table indicates the (non-additive) probability describing the common information about

[10] Ghirardato (1994) and Mukerji (1997) point out how the DM's awareness that the precise implication of some contingencies is inevitably left unforeseen, may lead to beliefs that have non-additive representation. The papers explain the Choquet decision rule as a 'procedurally rational' agent's means of 'handicapping' the evaluation of an act to the extent the estimate of its 'expected performance' is adversely affected by his imprecise knowledge of the odds.

the uncertainty and the contingent payoffs:

Possible states	ω_L	ω_H
Non-additive probability ν	$\nu(\omega_L) = 0.3$	$\nu(\omega_H) = 0.4$
State contingent payoff to buying	1	3
State contingent payoff to selling	-1	-3

The expected payoff of buying an unit of z, let us call it the act z_b, $CE_\nu(z_b)$ is obtained by taking expectations w.r.t. the relevant minimizing probability in the core of ν. Notice, the payoff from the act z_b is lower at ω_L than at ω_H. Hence, the relevant minimizing probability when evaluating $CE_\nu(z_b)$ is that probability in $C(\nu)$ that puts most weight on ω_L. Therefore,

$$\mathbb{CE}_\nu(z_b) = \min_{\mu \in \mathcal{C}(\nu)} \left\{ \sum z_b(\omega_i)\,\mu(\omega_i) \right\} = 0.6 \times 1 + 0.4 \times 3 = 1.8$$

On the other hand, the payoff from going short on an unit of z (the act z_s) is higher at ω_L than at ω_H. In other words, buying and selling are non-comonotonic acts. Hence, the relevant minimizing probability when evaluating $CE_\nu(z_b)$ is that probability in $C(\nu)$ that puts most weight on ω_H. Thus,

$$\mathbb{CE}_\nu(z_s) = \min_{\mu \in \mathcal{C}(\nu)} \left\{ \sum z_s(\omega_i)\,\mu(\omega_i) \right\} = 0.3 \times (-1) + 0.7 \times (-3) = -2.4$$

An 'economic' interpretation would run as follows. Given the ambiguity in the investor's subjective assessment of the uncertainty, more than one probability is consistent with his knowledge. Being ambiguity averse, he 'shades' the valuation to the extent it may be affected by the ambiguity. The switch in the relevant minimizing probability implicit in the evaluation when comparing a buying position to a selling, is simply a reflection of the 'shading effect'. ■

Next, we consider the formal modeling of the idea of stochastic independence of random variables when the beliefs are ambiguous. Let y be a function from a given space τ to the real line $\mathbb{R}$, and $\sigma(y)$ be the smallest $\sigma-$algebra that makes y a random variable. τ_n denotes the n-fold Cartesian product of τ, and $\sigma(y_1, ..., y_n)$ the product $\sigma-$algebra on τ_n generated by the $\sigma-$algebras $\{\sigma(y_i)\}_{i=1}^n$. Let ν_i be a convex non-additive probability defined on $\sigma(y_i)$. There is more than one way to define an independent product non-additive on $\sigma(y_1, ..., y_n)$. The following definition of independence was proposed by Gilboa and Schmeidler (1989), and also figures in Marinacci (1996) (definition 7.2), Dow and Werlang (1994) and Walley and Fine (1982).

Definition 1 *Let ν_i be a convex non-additive probability defined on $\sigma(y_i)$. The independent product, denoted $\bigotimes_{i=1}^n \nu_i$ is defined as follows*

$$\bigotimes_{i=1}^{n} \nu_i(A) = \min\left\{ (\mu_1 \times ... \times \mu_n)(A) : \mu_i \in \mathcal{C}(\nu_i) \ \text{ for } 1 \le i \le n \right\}$$

for every $A \in \sigma(y_1, ..., y_n)$, and where $\mu_1 \times ... \times \mu_n$ is the standard additive product measure. We denote by $\otimes\nu_i$ any non-additive probability on $\sigma(y_1, ..., y_n, ...)$ such that for any finite class $\{y_{t_1}, ..., y_{t_n}\}$ it holds $\bigotimes_{i \ge 1} \nu_i(A) = \bigotimes_{i=1}^n \nu_{t_i}(A)$ for every $A \in \sigma(y_1, ..., y_n)$.

In other words, the independent product is the lower envelope of the set of additive product measures generated by choosing a probability from each of the cores of the marginals. The computation of the Choquet expectation operator using product non-additive beliefs is particularly simple for a large class of functions, called *slice comonotonic,* defined below. The class was first identified and so named in Ghirardato (1997). As the name suggests, a function is slice comonotonic if all its "slices", along a given axis, are a comonotonic family. Let $X_1, ..., X_n$ be n (finite) sets and let $\Omega = X_1 \times ... \times X_n$. Correspondingly, let ν_i be convex non-additive probabilities defined on algebras of subsets of X_i, $i = 1, ..., n$.

Definition 2 *Let $f : \Omega \to \mathbb{R}$. We say that f has comonotonic x_i-sections if for every $(x_1, ..., x_{i-1}, x_{i+1}, ..., x_n)$, $(x'_1, ..., x'_{i-1}, x'_{i+1}, ..., x'_n) \in X_1 \times ... \times X_{i-1} \times X_{i+1} \times ... \times X_n$, $f(x_1, ..., x_{i-1}, \cdot, x_{i+1}, ..., x_n) : X_i \to \mathbb{R}$, and $f(x'_1, ..., x'_{i-1}, \cdot, x'_{i+1}, ..., x'_n) : X_i \to \mathbb{R}$ are comonotonic functions. f is called slice-comonotonic if it has comonotonic x_i-sections for every $i \in \{1, ..., n\}$.*

The following fact, which greatly facilitates the computation of the Choquet operator, follows[11] from Proposition 7 and Theorem 1 (taken together) in Ghirardato (1997).

Fact 2 *Suppose that $f : \Omega \to \mathbb{R}$ is slice comonotonic. Then*

$$\int_{\Omega} f(x_1, ..., x_n)\, d(\otimes \nu_i) = \int_{X_1} ... \int_{X_n} f(x_1, ..., x_n)\, d\nu_n ... d\nu_1$$

The following fact applies directly to the computation of agents' expected utility in the model presented in sections 4 and 5. For later reference, it helps to think of z as the monetary payoffs from a portfolio of 2 assets, e as an agent's contingent endowment vector and of u as a vN-M utility index describing her attitude to risk.

Fact 3 *Let $z : X_1 \times X_2 \to \mathbb{R}$, $e : X_3 \to \mathbb{R}$, $\Omega = X_1 \times X_2 \times X_3$, and $u : \mathbb{R} \to \mathbb{R}$. Define $f \equiv u \circ (z + e) : \Omega \to \mathbb{R}$. Further, suppose that u is strictly monotone. Then f is slice comonotonic.*

3 The main idea and the related literature

In this section we give the simple intuition of the idea behind our main result and place it in the context of the existing literature on the application of ambiguity aversion to financial markets.

Dow and Werlang (1992) identified an important implication of Schmeidler's model about optimal financial decision making. The paper showed, in a static model with one risky and one riskless asset, that given ambiguous beliefs and ambiguity aversion there exists a *set* of asset prices that support the optimal choice of a riskless portfolio. The intuition behind this finding may be grasped by referring back to example 1 in the last section. It is evident from our computations in that example that if the price of the asset z were to lie in the open interval $(1.8, 2.4)$, then the investor would strictly prefer a zero position to either going short or buying. Unlike in the case of unambiguous beliefs there is no single price at which to switch from buying to selling. Taking a zero position on the risky asset has the unique advantage that its evaluation is not affected by ambiguity. Thus price has to rise (fall) sufficiently to allow the investor feel secure in going short (long) by meeting the test of his conservative estimate–'shading' of valuations due to ambiguity aversion is what results in the 'inertia' zone.

It is however important to note that Dow and Werlang's demonstration was simply a statement about optimal portfolio choice corresponding to *exogenously* given prices. Their result is not a description of an equilibrium since the model is not closed to obtain asset prices endogenously. An asset is said to be not traded in a market equilibrium if no participant has a

[11] Proposition 7 and Theorem 1 (and indeed, the definition of slice-comonotonicity) as they appear in Ghirardato (1997) are stated for the the case where the product space is a product of *two* spaces, rather than n. However, the author does note explicitly (pp268) that the results extend to a product of n spaces.

non-zero holding of the asset in the equilibrium. Hence, given the ‘inertia’ zone found by Dow and Werlang it would appear that we were but a step away from obtaining a no-trade result: all we have to do is close the model. However, this conjecture is false. This fact is illustrated in the Edgeworth box diagram in Figure 1.

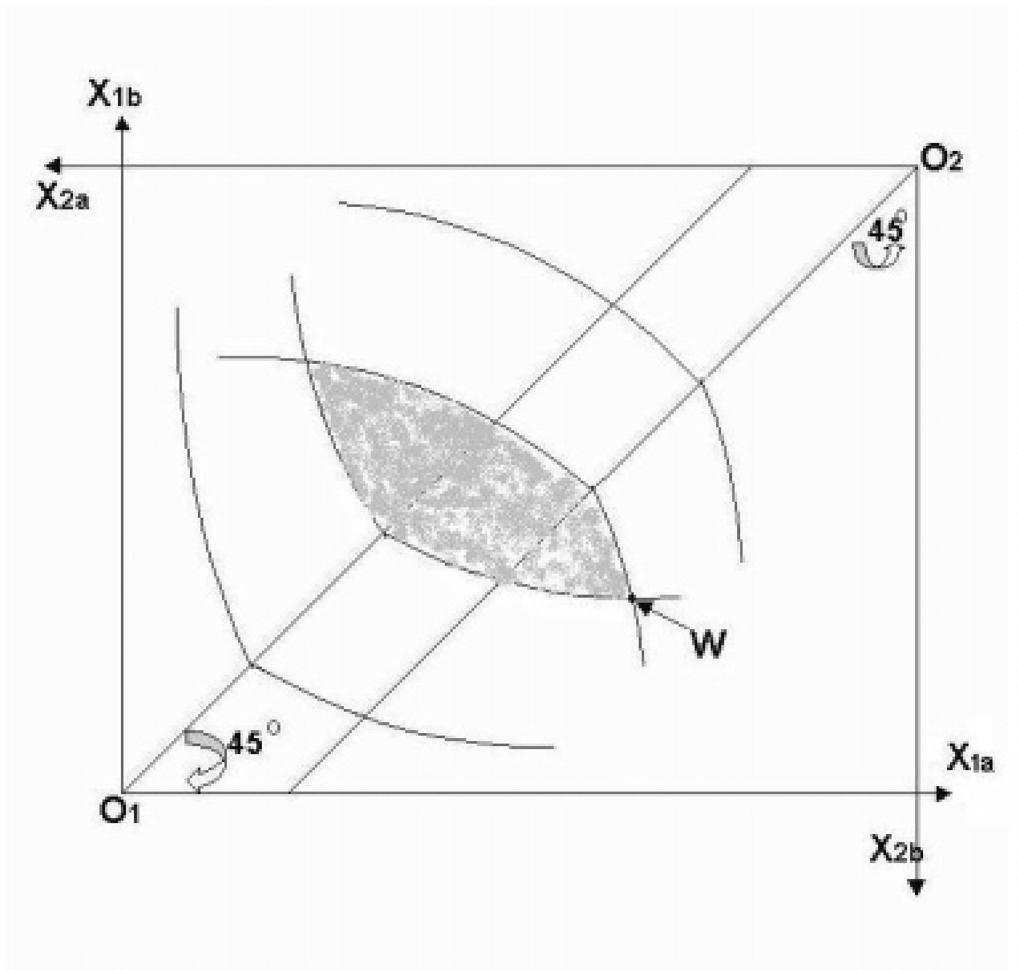

Figure 1:

The diagram depicts the possibilities of risk-sharing between two risk averse and ambiguity averse agents, $h = 1$, 2, with uncertain endowment income in the two possible states ω_a and ω_b. The horizontal axis measures the allocation x_{ha} in state ω_a, while the vertical axis measures the allocation x_{hb} in state ω_b. Each set of indifference curves represents the Choquet expected utility functional of an agent: any two allocations on a given curve obtain identical *ex ante* utility for the agent. The utility functional for agent h, is given by

$$\mathbb{CE}_{\nu} u_h (x_{ha}, x_{hb}) = \min_{\mu \in \mathcal{C}(\nu)} \{ u_h (x_{ha}) \mu (\omega_a) + u_h (x_{hb}) \mu (\omega_b) \}$$

where $\nu : 2^{\{\omega_a, \omega_b\}} \rightarrow [0, 1]$, represents the common beliefs of the two agents. It is assumed that $\nu (\{\omega_a\}) + \nu (\{\omega_b\}) < 1$ and $u''_h < 0$, $h = 1, 2$. **W** shows the endowment vector for the economy. Notice, because of ambiguity aversion, the indifference curves are kinked at the point of intersection with the 45° ray through the origin[12]. Roughly put, an ambiguity averse individual wants to equate the level of utility in different states, whereas a risk averse individual simply wants to equate the marginal utilities of income across states. Hence, introduction of ambiguity aversion implies individuals will want to trade more (in insurance) if anything, since ambiguity aversion simply compounds the desire of risk averse individuals to share risk. Accordingly, the shaded area in the diagram represents the area of mutually advantageous trade. No-trade is an equilibrium outcome in this economy if and only if endowment is Pareto optimal to begin with–something that is not only non-generic, but is true irrespective of ambiguity aversion. Introduction of ambiguity aversion in an economy, seemingly, would not impede the trade in risk sharing contracts and would not be a reason for incomplete risk sharing[13]. As this paper will demonstrate, the situation in the 2-state world may be misleading in general. The clue that this might well be the case is found in a subsequent contribution to this literature, Epstein and Wang (1994).

Consider a 2-state, 2 agent world as discussed above but suppose that the endowment vector is such that the Edgeworth box is a square and that the agents' initial position is riskless. In this case the endowment point is on the diagonal and the indifference curves through this point are mutually 'tangent' at a point of kink. Hence, the asset price supporting the equilibrium consisting of the initial endowment is indeterminate (Tallon (1998))[14]. Epstein and Wang extend Dow and Werlang's analysis to an infinite-horizon, multiple-asset framework and find that the non-uniqueness of supporting prices is not restricted to riskless positions. More specifically, they generalize the Lucas (1978) asset pricing model by allowing for ambiguous beliefs and ambiguity aversion and find that the non-uniqueness of supporting prices in general extends to the case where "there exist state variables affecting dividends that do not influence consumption". The main finding of our paper, that it is idiosyncratic riskiness of asset payoffs which is the route by which ambiguity aversion causes no-trade, incomplete risk-sharing and incomplete asset markets, owes its inspiration to this observation by Epstein and Wang. However, Epstein and Wang do not formally trace the implications of their finding to incomplete risk-sharing and/or incomplete asset markets, or even informally suggest the possibility of such a link. Indeed, like the Lucas model, theirs is a representative agent economy; such an economy by its very definition involves only trivial risk sharing. Also, as is actually demonstrated by Epstein and Wang, a formal extension of their analysis to the case of heterogeneous agents requires an explicit assumption of complete asset markets. Next we expound a simple example to explain how idiosyncratic effects on an asset's payoffs may inhibit trade in the asset and, consequently, risk sharing when agents are ambiguity averse.

Example 2 Consider a 2-period finance economy with two risk averse agents and one good, similar to the one sketched in the introductory section. Agents consume in period 1, while they merely exchange financial claims in period 0. Agents' period 1 endowment is random; suppose

[12] To see this, recall the slope of the indifference curve is given by

$$\frac{u_h'(x_{hb})\,\mu(\omega_b)}{u_h'(x_{ha})\,\mu(\omega_a)}$$

If $x_{hb} > x_{ha}$, $\mu_j(\omega_b) = \nu(\{\omega_b\})$ and $\mu(\omega_a) = 1 - \nu(\{\omega_b\})$; if $x_{ha} > x_{hb}$, $\mu(\omega_b) = 1 - \nu(\{\omega_a\})$ and $\mu(\omega_a) = \nu(\{\omega_a\})$.

[13] This is actually noted by Dow and Werlang (op. cit., pp198): "In terms of empirical implications of the Schmeidler-Gilboa model, broadly similar types of behavior could be caused by transactions costs or asymmetric information, or by the preferences in Bewley's (1986) model. The main difference is that in each of those three cases there is a tendency not to trade, whereas in Schmeidler-Gilboa there is a tendency not to hold a position. In other words, the agent's frame of reference here is the safe allocation, rather than the *status quo*."

[14] If the endowment point is off the diagonal, i.e., individual endowment is uncertain while there is no aggregate uncertainty, then the (complete market) equilibrium allocation becomes indeterminate as well (Tallon (1997)).

there are just two (distinct) possible endowment vectors for the economy, defining, what we call the two economic states of the world, α and β. There are two assets available, one of which is the safe asset b. The safe asset delivers one unit of the good in each of the two economic states. The other asset, z has uncertain deliveries. The uncertain asset's payoff has two components: one that is dependent on the realization of the economic state, y^s, $s = \alpha, \beta$, and another that is independent of it (the idiosyncratic component)[15], $y(t)$, $t \in \{0,1\}$. The payoff of the asset in α is given by $y^\alpha + y(t)$; the payoff in β is given by $y^\beta + y(t)$. Fix, $y^\alpha = 1$, $y^\beta = 2$, $y(0) = 0$, $y(1) = 2$. Hence, in the economic state[16] the uncertain asset has two possible payoffs; at α it may be 1 or 3, at β it may be 2 or 4. The agents' endowments are denoted e_h^s, $h = 1,2$. The agents' (common) beliefs are as follows. The beliefs about the economic states are given by $\pi(\alpha)$ and $\pi(\beta)$ and beliefs about the idiosyncratic components are given by $\nu(t)$. Assume $\pi(\alpha) + \pi(\beta) = 1$ and $\nu^0 + \nu^1 \leq 1$. Beliefs on the full state space $\{(s,t)\}$ are described using the notion of independence explained in the last section.

First consider the case where beliefs about idiosyncracies are unambiguous, i.e., $\nu^0 + \nu^1 = 1$. The model reduces to a standard incomplete market equilibrium with two assets and four states, in which, for 'generic' endowments, there is trade, i.e., some partial insurance among agents[17]. Next suppose, to simplify matters drastically, that $\nu^0 = \nu^1 = 0$. Consider an agent h contemplating buying the uncertain asset at a price q^z, given the safe asset is priced q^b. h may buy z_h units of the uncertain asset and take a position b_h in the safe asset such that $b_h + q^z z_h = 0$. His utility functional is then given by:

$$
\begin{aligned}
& \mathbb{CE}_{\pi \otimes \nu} u_h \left(e_h^s + z_h \left(y^s + y(t)\right) + b_h\right) \\
= \; & u_h \left(e_h^\alpha + z_h \left(y^\alpha + y(0)\right) + b_h\right) \pi(\alpha) \left(1 - \nu^1\right) \\
& + u_h \left(e_h^\alpha + z_h \left(y^\alpha + y(1)\right) + b_h\right) \pi(\alpha) \nu^1 \\
& + u_h \left(e_h^\beta + z_h \left(y^\beta + y(0)\right) + b_h\right) \pi(\beta) \left(1 - \nu^1\right) \\
& + u_h \left(e_h^\beta + z_h \left(y^\beta + y(1)\right) + b_h\right) \pi(\beta) \nu^1
\end{aligned}
$$

Once we substitute in $\nu^1 = 0$, it is clear from the above functional that the payoff matrix (from the two assets) the agent (as a buyer of z) will consider is:

$$
\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}
$$

If $q^z \geq 2$, any balanced portfolio (i.e. such that $b_h + q^z z_h = 0$) with $z_h > 0$ yields negative payoffs in the final period and is therefore not worth buying. Thus, an agent will wish to buy the uncertain asset only if $q^z < 2$. Next consider an agent h' who contemplates going short on

[15] To fix ideas, it might help to think of z as a claim to a share of the future profits of a designer perfume manufacturing firm, about to market a new perfume. While the sales revenue of the new perfume is affected by aggregate income shocks, it nevertheless has an idiosyncratic component ("that special appeal!") which is independent of any income shock.

[16] Notice, there are actually four states in this model, as given by the set $\{(s,t) \mid s = \alpha, \beta; t = 0, 1\}$.

[17] This has to be qualified since there exists some non-generic constraints among endowments in different states, namely $e_h^{s,t} = e_h^{s,t'} \equiv e_h^s$.

asset z. His utility functional is therefore:

$$\begin{aligned}
& \mathbb{CE}_{\pi\otimes\nu} u_{h'}\left(e^s_{h'} + z_{h'}\left(y^s + y\left(t\right)\right) + b_{h'}\right) \\
= \; & u_{h'}\left(e^{\alpha}_{h'} + z_{h'}\left(y^{\alpha} + y\left(0\right)\right) + b_{h'}\right)\pi\left(\alpha\right)\nu^0 \\
& + u_{h'}\left(e^{\alpha}_{h'} + z_{h'}\left(y^{\alpha} + y\left(1\right)\right) + b_{h'}\right)\pi\left(\alpha\right)\left(1-\nu^0\right) \\
& + u_{h'}\left(e^{\beta}_{h'} + z_{h'}\left(y^{\beta} + y\left(0\right)\right) + b_{h'}\right)\pi\left(\beta\right)\nu^0 \\
& + u_{h'}\left(e^{\beta}_{h'} + z_{h'}\left(y^{\beta} + y\left(1\right)\right) + b_{h'}\right)\pi\left(\beta\right)\left(1-\nu^0\right)
\end{aligned}$$

Notice now the functional is dependent on ν^0 since the agent is going short, i.e., $z_{h'} < 0$. Substituting $\nu^0 = 0$, we find the payoff matrix the agent h' will consider:

$$\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$$

For $q^z \leq 3$ any balanced portfolio with $z_{h'} < 0$ yields negative payoffs in the second period. Thus, an agent will wish to sell the risky asset only if $q^z > 3$. If there is going to be trade at equilibrium, both sellers and buyers of the risky asset must exist. However, buyers of asset 1 will not want to pay more than 2, while sellers will not sell it for less than 3. Hence, there does not exist a price such that agents will have a non-zero holding of the uncertain asset. ■

It is instructive to understand why ambiguity aversion may impede trade in the presence of idiosyncracies (as in example 3) but not otherwise (as, for instance, in the situation depicted in the Edgeworth box). No-trade arises in the above example because ambiguity aversion makes the buyer 'undervalue' the idiosyncratic component of the asset by concentrating probability weight on the lower payoffs of the idiosyncratic component, and makes the seller 'overvalue' the asset by concentrating probability weight on the higher payoffs of the idiosyncratic component. In the Edgeworth box, one may think of the risk sharing as being obtained through the exchange of two Arrow securities, one for each contingency. In contrast to example 3, in the Edgeworth box where there is no idiosyncracy of asset payoffs, at an equilibrium the buyer and the seller of an asset would evaluate their respective prospects using the *same* (additive probability); there is no switch in the 'minimizing' probability. Notice from the figure that Pareto optimal allocations lie within the 'tramlines', the 45^0 rays through each origin. In other words, Pareto optimal allocations of the two agents are comonotonic. Hence, at a Pareto optimal allocation, the ranking of the states is identical for both agents and is given by the ordering of aggregate endowment. Now with complete markets, as it is in this case, equilibrium allocations are Pareto optimal and therefore comonotonic as well. Thus, agents use the same 'minimizing probability' at equilibrium, and agree on asset valuation. Risk sharing proceeds, just as in an economy with SEU economy (see Chateauneuf, Dana, and Tallon (1997)).

Bewley (1986) applied a model of behavior under Knightian uncertainty that is distinct from the one applied in this paper. Bewley essentially drops Savage's assumption that preferences are complete and adds an axiom of the 'status quo'. Bewley shows that his model may imply the incompleteness of markets for contingent deliveries. In his findings idiosyncracies are not seen to play any crucial role, unlike in ours. In our model the status quo is endogenously derived and arises only because of the interaction between ambiguity aversion and the idiosyncratic shocks, whereas in Bewley's model the preference for status quo is exogenously imposed as a part of the definition of ambiguity aversion.

4 The model and the main result

The specific goal of the model is to find what general conditions on asset structure ensures that a given set of primitive assets (vis., bonds, equity share contracts) available in an economy is such that *both* of the following statements are satisfied:

1. Provided their beliefs are unambiguous, symmetrically informed agents will be able to obtain a Pareto optimal risk-sharing allocation by trading the available set of assets, just as they would *if* the economy were endowed with a complete set of simple contingent contracts (Arrow securities). In other words, given unambiguous beliefs, this economy works in accordance with the propositions of general equilibrium with complete markets.

2. Ceteris paribus, if the agents' beliefs were ambiguous and agents were ambiguity averse, the *same* set of assets may allow *at best* a Pareto sub-optimal and incomplete risk sharing. That is, in this case the allocation will be demonstrably identical to one obtained in an equilibrium of an economy endowed with only an incomplete set of Arrow securities; working of this economy is described by the propositions of the GEI model.

4.1 The model

The backdrop of our formal results is a model of a stylized two period finance economy which we call an *n-financial asset economy with idiosyncracy.* Agents/households ($h = 1, \ldots, H$) trade assets in period 0, before uncertainty is resolved, and consume the one (and only) good in the economy in period 1. The assets available for trade are claims, possibly contingent, on deliveries of the consumption good in period 1.

There are two sources of uncertainty. First, there is some 'economic uncertainty': agents do not know their endowments tomorrow. An *economic state of the world*, s, $s = \{1, 2, ..., S\}$, is completely identified by the endowment vector for that state $(e_1, ..., e_h, ..., e_H)$, where each component of the vector, $e_h \in \mathbb{R}_+$, gives a particular household's endowment the consumption good in period 1. Secondly, there is idiosyncratic financial uncertainty. An *idiosyncratic state of the world* completely characterizes the realization of the idiosyncratic components of payoffs of the available financial assets (defined below); it is identified by the vector $t = (t_1, t_2, ..., t_n)$, where $t_i \in \{0, 1\}$, $i = \{1, 2, ..., n\}$, and n is the total number of financial assets. τ_n denotes the set of all t's, i.e., $\tau_n \equiv \{0, 1\}^n$. Hence, to obtain a complete description of a state of the world, complete in the sense of exhausting all uncertainty relevant to the model, the economic states s must each be further partitioned into cells denoted (s, t). A typical state of the world is denoted by the letter ω, $\omega \in \Omega \equiv \left\{(1, t)_{t \in \tau_n}, (2, t)_{t \in \tau_n}, ..., (S, t)_{t \in \tau_n}\right\}$.

The assets available for trade at date 0 are as follows:

1. Financial assets, $z^i, i = 1, ..., n$, with payoffs that have idiosyncratic components. The occurrence of a particular "economic state" may not completely fix the outcome of these assets. An asset z^i yields a payoff of $y^s + y(t_i) > 0$ units of the good; $s = \{1, 2, ..., S\}$ $t_i \in \tau \equiv \{0, 1\}$. $y(t_i)$ is the idiosyncratic component, in the sense that it is independent of the realized economic state and independent of the realization of the payoff from any other financial asset, i.e., assets z^j, where $j \neq i$. It is assumed that $y(1) > y(0)$ and that $y^1 \neq y^2$. Price of an asset z^i is denoted by $q_n^{z^i}$.

2. A safe asset, b, which delivers one unit of the good irrespective of the realized state of the world. Price of this security is normalized to 1.

3. An Arrow security, b^s, which is a promise to deliver one unit of the good in state s and nothing otherwise, is available for each of the economic states $3, 4, ..., S$. Price of this security is denoted by q_n^s.

The point behind modeling the asset structure as above is to ensure that there are at least two economic states, such that, to transfer resources across these states the agents would have to rely on financial assets whose payoffs are affected by idiosyncratic shocks. In other words, there exists a non-empty set of *non-redundant* financial assets whose payoffs have an idiosyncratic component.

Prior to the resolution of uncertainty, in period 0, agents are endowed with a common belief about the likelihood of the state ω being realized in period 1. It is assumed that the realization of the payoff of a financial asset z^i is commonly believed to be independent of the realization of any other financial asset $z^{i'}, i \neq i'$, and the realized economic state. The (marginal) beliefs about particular idiosyncratic component t_i are described by ν_i, $\nu_i(0) + \nu_i(1) \leq 1$. To model the assumption that the realization of t_i and t_j are believed to be independent, the beliefs on τ_n are described by independent product $\nu \equiv \bigotimes_{i=1}^n \nu_i$. For simplicity, we shall assume however, that $\nu_i(0) = \nu_j(0) = \nu^0$, and that $\nu_i(1) = \nu_j(1) = \nu^1$, $i, j \in \{1, ..., n\}$. The belief on an economic state s is given by $\pi(s)$. To make it transparent that it is the ambiguity of beliefs about the idiosyncratic realizations (and not the ambiguity of beliefs about the economic states) which is responsible for the possibility of no-trade in financial assets, and also to make the computation less tedious, we assume $\pi(1) + \pi(2) + ... + \pi(S) = 1$. As will be explained in the next section, our findings are robust to the possibility that the beliefs about the economic states are ambiguous. Since it is believed by the agents that the realization of an idiosyncratic state is independent of the realization of the economic state, the belief on Ω is given by $\pi \bigotimes \nu$.

Let $e_{h,n}^\omega$ and $x_{h,n}^\omega$ be the hth household's endowment and consumption, respectively, in state ω, given the total number of financial assets in the economy is n. Note, the definition of an economic state implies $e_{h,n}^{(s,t)} = e_{h,n}^{(s,t')}$. Holding of the asset b^s by h is denoted $b_{h,n}^s$. Similarly, holding of the asset b by h is denoted $b_{h,n}$ and holding of the asset z^i by h is denoted $z_{h,n}^i$. Agent h has a von-Neumann Morgenstern utility index $u_h : \mathbb{R}_+ \to \mathbb{R}$, which is assumed to be strictly increasing, smooth and strictly concave. Furthermore, $u_h'(0) = \infty$ and $e_{h,n}^\omega > 0$ for all h and all ω.

Let $\mathcal{P}_{hn}$ denote the maximization program of agent h,which is as follows:

$$\max_{b_{h,n}, b_{h,n}^3, ..., b_{h,n}^S, z_{h,n}^1, ..., z_{h,n}^n} \mathbb{CE}_{\pi \otimes \nu} u_h\left(x_{h,n}^{s,t}\right)$$

$$s.t. \quad \begin{cases} b_{h,n} + \sum_{s=3}^S q_n^s b_{h,n}^s + \sum_{i=1}^n q_n^{z^i} z_{h,n}^i = 0 \\ x_{h,n}^{s,t} - e_{h,n}^s = b_{h,n} + \sum_{i=1}^n \left(y^s + y(t_i)\right) z_{h,n}^i, \ \forall s \in \{1, 2\}, \, t \in \tau_n \\ x_{h,n}^{s,t} - e_{h,n}^s = b_{h,n} + b_{h,n}^s + \sum_{i=1}^n \left(y^s + y(t_i)\right) z_{h,n}^i, \ \forall \ s \in \{3, ..., S\}, \, t \in \tau_n \end{cases}$$

An equilibrium consists of a set of asset prices, $\mathbf{q}_n \equiv \left\{1, q_n^3, ..., q_n^S, q_n^{z^1}, ..., q_n^{z^n}\right\}$, a set of asset holdings,

$$(\mathbf{b}_n, \mathbf{z}_n) \equiv \left\{\left(b_{h,n}, b_{h,n}^3, ..., b_{h,n}^S, z_{h,n}^1, ..., z_{h,n}^n\right)_{h=1}^H\right\},$$

and a consumption vector,

$$\mathbf{x}_n \equiv \left(x_{h,n}^\omega\right)_{h=1,...,H; \omega \in \Omega},$$

such that, given $\mathbf{q}_n$ all agents solve their maximization program $\mathcal{P}_{hn}$, and the asset markets clear, i.e.,

$$\sum_h b_{h,n} = \sum_h b^s_{h,n} = \sum_h z^i_{h,n} = 0, \ \forall \ s \in \{3, ..., S\} \text{ and } i \in \{1, ..., n\},$$

and the consumption vector is feasible at each state, i.e.,

$$\sum_h x^{\omega}_{h,n} = \sum_h e^{\omega}_{h,n}.$$

Notice, a tuple $(\mathbf{q}_n, (\mathbf{b}_n, \mathbf{z}_n))$uniquely pins down the equilibrium, hence we may denote an equilibrium of an n-financial asset economy using such a tuple.

4.2 The main result

The focus of the analysis of the model of n-financial asset economy with idiosyncracy will be to compare two cases: one, where beliefs about idiosyncratic outcome is unambiguous $\left(\nu^0 + \nu^1 = 1\right)$ and the other where beliefs about the idiosyncracy is ambiguous $\left(\nu^0 + \nu^1 < 1\right)$. In order to make the comparison stark, the analysis will relate the two cases to two benchmarks. One benchmark is a complete market economy which we call *an economy without idiosyncracy*. An economy without idiosyncracy is an economy that is identical to the n-financial asset economy with idiosyncracy described in the last section in every respect except that there is only a single financial asset z which pays off $y^s + \mathbb{E}_{\nu} y(t) \equiv \bar{y}^s$ units in the economic states $s = \{1, ..., S\}$. Correspondingly, q^z denotes the price of z and z_h denotes the amount held by household h. (Note, when denoting endogenous variables in the economy without idiosyncracy we may omit the subscript n.) The second benchmark is an incomplete market economy which is identical to the n-financial asset economy with idiosyncracy described in the last section in every respect except that the only assets available are the Arrow securities for states $3, ..., S$, and the safe asset.

To facilitate the analysis we first obtain a simple characterization of the equilibrium portfolio holding of the n-financial asset economy with idiosyncracy. Let $\widetilde{\mathcal{P}}_{hn}$ denote the following maximization problem faced by an agent h in the economy with idiosyncracy, where $\tilde{z}_n$ denotes an unit of a portfolio composed of $\frac{1}{n}$ unit of the asset z^i, $i = 1, ..., n$, $\tilde{z}_{h,n}$ is the amount held of this portfolio by h and $\tilde{q}_n$ is the price of an unit of this portfolio:

$$\max \mathbb{CE}_{\pi \otimes \nu} u_h \left(x^{s,t}_{h,n} \right)$$

$$s.t. \begin{cases} b_{h,n} + \sum_{s=3}^{S} q^s_n b^s_{h,n} + \tilde{q}_n \tilde{z}_{h,n} = 0 \\ x^{s,t}_{h,n} - e^s_{h,n} = b_{h,n} + \tilde{z}_{h,n} \left[\frac{\sum_{i=1}^{n} (y^s + y(t_i))}{n} \right], \ \forall s \in \{1, 2\}, t \in \tau_n \\ x^{s,t}_{h,n} - e^s_{h,n} = b_{h,n} + b^s_{h,n} + \tilde{z}_{h,n} \left[\frac{\sum_{i=1}^{n} (y^s + y(t_i))}{n} \right], \ \forall \ s \in \{3, ..., S\}, \ t \in \tau_n \end{cases}$$

Let $\widetilde{\mathcal{V}}_{h,n}\left(\widetilde{\mathbf{q}}_n\right)$ be the value of $\widetilde{\mathcal{P}}_{h,n}$ at the asset price vector $\widetilde{\mathbf{q}}_n$.

Lemma 1 *Let $(\mathbf{q}_n, (\mathbf{b}_n, \mathbf{z}_n, \mathbf{x}_n))$ be an equilibrium of the n-financial assets economy with idiosyncracy. Suppose $\nu^0 + \nu^1 \leq 1$. Then, $z^i_{h,n} = z^{i'}_{h,n}$, $\forall i, i' \in \{1, ..., n\}$, $\forall h \in \{1, ..., H\}$.*

Thus, at an equilibrium, agents will hold all the financial assets in the same proportion. This is essentially a consequence of the fact that agents are risk averse and that the n financial assets are simply 'independent replicas'. The primary implication of the lemma is that it is without loss of generality that we may assume that it is only the (portfolio) asset $\tilde{z}_n$, instead of the individual assets z^i, that is available for trade in the economy. Hence, an equilibrium of an n-financial assets economy with idiosyncracy, $(\mathbf{q}_n, (\mathbf{b}_n, \mathbf{z}_n))$, may be equivalently denoted by the tuple $(\widetilde{\mathbf{q}}_n, (\mathbf{b}_n, \widetilde{\mathbf{z}}_n))$, where,

$$\widetilde{\mathbf{q}}_n \equiv \left\{1, q_n^3, ..., q_n^S, \tilde{q}_n\right\} \text{ and } (\mathbf{b}_n, \widetilde{\mathbf{z}}_n) \equiv \left\{\left(b_{h,n}, b_{h,n}^3, ..., b_{h,n}^S, \tilde{z}_{h,n}\right)_{h=1}^H\right\}.$$

The above characterization of the equilibrium in turn facilitates a simple definition of what it means to satisfy the conditions of equilibrium when n is arbitrarily large. We say $(\widetilde{\mathbf{q}}_\infty, \mathbf{b}_\infty, \widetilde{\mathbf{z}}_\infty, \mathbf{x}_\infty)$ satisfies the conditions of equilibrium of the n-financial assets economy with idiosyncracy where n is arbitrarily large, i.e., $n \to \infty$ if

1. Given $\widetilde{\mathbf{q}}_\infty$, $(\mathbf{b}_\infty, \widetilde{\mathbf{z}}_\infty, \mathbf{x}_\infty)$ is a solution to the problem $\widetilde{\mathcal{P}}_{h,\infty}$ defined as follows:

$$\max \mathbb{CE}_{\pi \otimes \nu} u_h\left(x_{h,n}^{s,t}\right)$$
$$s.t. \begin{cases} b_{h,\infty} + \sum_{s=3}^S q_\infty^s b_{h,\infty}^s + \tilde{q}_\infty \tilde{z}_{h,\infty} = 0 \\ x_{h,\infty}^{s,t} - e_{h,\infty}^s = b_{h,\infty} + \tilde{z}_{h,\infty}\left[\lim_{n\to\infty} \frac{\sum_{i=1}^n (y^s + y(t_i))}{n}\right], \\ s = 1, 2, \text{ with probability } 1, \\ x_{h,\infty}^{s,t} - e_{h,\infty}^s = b_{h,\infty} + b_{h,\infty}^s + \tilde{z}_{h,\infty}\left[\lim_{n\to\infty} \frac{\sum_{i=1}^n (y^s + y(t_i))}{n}\right], \\ s = 3, ..., S, \text{ with probability } 1. \end{cases}$$

2. $\sum_h b_{h,\infty} = \sum_h b_{h,\infty}^s = \sum_h \widetilde{z}_{h,\infty} = 0$, $\forall\ s \in \{3, ..., S\}$ and the consumption vector is feasible at each state, i.e., $\sum_h x_{h,n}^\omega = \sum_h e_{h,n}^\omega$ with probability 1.

Werner (1997) considers a finance economy of which the above is just a special case. There are standard arguments that ensure the existence of equilibria of such economies (op.cit., pp.100).

Next consider our first benchmark, the economy without idiosyncracy. Let $\mathcal{P}_h$ denote the following maximization problem faced by an agent h in the economy without idiosyncracy:

$$\max \mathcal{U}_h(x_h) \equiv \mathbb{E}_\pi u_h(x_h^s)$$
$$s.t. \begin{cases} b_h + \sum_{s=3}^S q^s b_h^s + q^z z_h = 0 \\ x_h^s - e_h^s = b_h + z_h \bar{y}^s,\ s \in \{1, 2\} \\ x_h^s - e_h^s = b_h + b_h^s + z_h \bar{y}^s,\ s \in \{3, ..., S\} \end{cases}$$

Call $\mathcal{V}(\mathbf{q})$ the value of $\mathcal{P}_h$ at the asset price vector $\mathbf{q}$. Theorem 1, below, relates equilibria of an n-financial assets economy with idiosyncracy, where n is arbitrarily large, to that of the economy without idiosyncracy.

Theorem 1 *Suppose $\nu^0 + \nu^1 = 1$. Then $(\widetilde{\mathbf{q}}_\infty, \mathbf{b}_\infty, \widetilde{\mathbf{z}}_\infty)$ satisfies the conditions of equilibrium of the n-financial assets economy with idiosyncracy where n is arbitrarily large, if and only if, $(\widetilde{\mathbf{q}}_\infty, (\mathbf{b}_\infty, \widetilde{\mathbf{z}}_\infty))$ describes an equilibrium of an economy without idiosyncracy, wherein the price of a unit of z is equal to $\tilde{q}_\infty$, and a household's holding of the asset z, z_h, is equal to $\tilde{z}_{h,\infty}$.*

Theorem 1 shows that equilibrium allocations of the n-financial assets economy with idiosyncracy are essentially identical to that of the economy without idiosyncracy, the latter we recall is actually an economy in which financial markets are complete, provided the number of available financial assets is large enough and agents' beliefs are unambiguous. The result follows from an application of the usual diversification principle, stating that, in the limit idiosyncracies are 'washed away', in conjunction with the assumption that payoffs of financial assets are affected by the realization of economic states, in particular states $s = 1, 2$. However, if the model of the n-financial assets economy with idiosyncracy were to be reconsidered with the sole amendment that beliefs about idiosyncracies were ambiguous, i.e., $\nu^0 + \nu^1 < 1$, then the result no longer holds. In such an economy the equilibrium allocation may be bounded away from Pareto optimal risk-sharing however large the n, and indeed actually coincides with the allocation of an incomplete market economy. This in fact is the subject of our main theorem, below.

Main Theorem *Consider the n-financial assets economy with idiosyncracy. Suppose $\nu^0+\nu^1 < 1$. Let $y^{\underline{s}} \equiv \min_s \{y^s\}$ and $y^{\bar{s}} \equiv \max_s \{y^s\}$ and suppose that $y^{\underline{s}} + y(1) > y^{\bar{s}} + y(0)$. Then there exists an $\bar{\mathcal{A}}, 0 < \bar{\mathcal{A}} < 1$, such that if $1 - \nu^0 - \nu^1 > \bar{\mathcal{A}}$, $\tilde{z}_{h,n} = 0$ for all $h \in \{1, ..., H\}$ and $x_n^{s,t} = x_n^{s,t'}$, $s = 1, 2, ..., S$, $t \neq t'$ at every equilibrium $(\mathbf{q}_n, (\mathbf{b}_n, \mathbf{z}_n, \mathbf{x}_n))$, for all $n \in \mathbb{N}$.*

Stated differently, this says that if the range of variation of the idiosyncratic component of the financial asset is greater than the range of variation due to the economic shocks, if the beliefs over the idiosyncratic states are ambiguous enough, and if agents are ambiguity averse then the equilibrium of an n-financial assets economy with idiosyncracy is an equilibrium of the economy with one safe asset and $S - 2$ Arrow securities, i.e., an economy with incomplete markets, since the financial assets are not traded in equilibrium. Note, the statement holds *whatever the value of n.*

The formal proof of the main theorem appears in the appendix; here we present the basic intuition for the result. The two more significant ways in which the theorem generalizes the demonstration in Example 3 are: one, it shows that no-trade obtains even when beliefs have a degree of ambiguity strictly less than 1; two, it allows for any arbitrary number of financial assets, in particular, for $n \to \infty$. We consider the intuition for each of these generalizations in turn. First, consider a 2-(economic)-state, 2 agent, 1-financial asset economy with idiosyncracy, in which the financial asset's payoffs are as in Example 3. The economy also has a safe asset which delivers one unit irrespective of the contingency. Consider an agent thinking of buying the financial asset. The maximum average payoff he expects in an economic state is the amount, $2 + 0 \times (1 - \nu^1) + 2 \times \nu^1 \equiv V(B)$, that he expects in state β. This implies, whatever his utility function, whatever his endowment vector, he will not want to buy the asset for more than this amount. To see why this is so suppose the financial asset costs $p > V(B)$. Note, if the agent invested the amount p in the safe asset he will be returned p for *sure* both at α and β. Hence, irrespective of his risk preferences the agent would strictly prefer the safe asset. Now, instead, consider an agent who contemplates going short with the asset. This agent expects to have to repay, at least, $1 + 0 \times \nu^0 + 2 \times (1 - \nu^0) \equiv V(S)$, and therefore, will not want to sell the asset if the price is less than this. Clearly, if ν^0 and ν^1 were small enough, $V(B) < V(S)$. Therefore, if ν^0 and ν^1 were small enough, agents will not trade in the financial asset.

Next, suppose there are n independent replicas of the financial asset with payoffs as above, where $n \to \infty$. To obtain an intuition for this case it is necessary to bear in mind the way the law of large numbers work for non-additive beliefs. Specifically, let us consider an i.i.d. sequence $\{X_n\}_{n \geq 1}$ of $\{0, 1\}$ −valued random variables. Suppose, $\nu(\{X_n = 0\}) = \nu(\{X_n = 1\}) = \frac{1}{4}$ for all $n \geq 1$. As is usual with laws of large numbers, the question is about the limiting distribution of the sample average, $\frac{1}{n}\sum_{i=1}^{n} X_i$. The result[18] is

$$\nu\left(\frac{1}{4} \leq \liminf_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i \leq \limsup_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i \leq \frac{3}{4}\right) = 1.$$

This shows that the DM has a probability 1 belief that the limiting value of the sample average lies in the interval $\left(\frac{1}{4}, \frac{3}{4}\right)$. However, unlike in the case of additive probabilities, the DM is not able to further pin down its value. Thus, even with non-additive probabilities the law of large numbers works in the usual way, in the sense that here too the tails of the distribution are 'cancelled out' and the distribution 'converges on the mean'. But of course here, given that the DM's knowledge is consistent with more than one prior, there is more than one mean to converge on; hence, the convergence is to the *set* of means corresponding to the set of priors consistent with the DM's knowledge. The result would not appear counter intuitive, if we care to recall our intuition for ambiguous beliefs. The interpretation of the law of large numbers in the case of ambiguous beliefs is that the DM realizes that the sample average converges (in probability) to the 'true' mean. However, given the DM's vague knowledge of the 'true' mean (he at best knows the interval in which the true mean lies), he does not know at which precise point on the support the convergence occurs—at best he may narrow it down to an interval. The less ambiguous his belief, the 'tighter' the interval. In other words, *any* (additive) probability distribution whose support is a subset of the interval $\left(\frac{1}{4}, \frac{3}{4}\right)$ is consistent with the DM's subjective belief of the limiting distribution. Hence, a DM whose (ex post) utility is increasing in X (e.g., when the DM is a buyer of an asset with payoff X) will behave as if the convergence of the sample average occurred at the lower boundary of the interval (his conservative estimate), while a DM whose utility is increasing in $-X$ (e.g., when the DM is a seller of an asset with payoff X) will behave as if the convergence of the sample average occurred at the upper boundary of the interval (his conservative estimate).

Let us return to our simplified financial economy with n fold replicas of the financial asset (payoffs as in Example 3) where $n \to \infty$. We consider trade between 'two' assets, one the safe asset and the other the 'portfolio' asset, consisting in equal proportion each of the independent replica assets. The result explained above implies that any agent contemplating going long on the portfolio asset will *behave as if* he believes that each component asset of the portfolio has i.i.d. payoffs $y^s + 0$ and $y^s + 2$ with the (additive) probability $1 - \nu^1$ and ν^1, respectively, and applies the laws of large numbers for additive probabilities to fix belief that an unit of the portfolio will payoff $y^s + \left[0 \times \left(1 - \nu^1\right) + 2 \times \nu^1\right]$ with probability 1 in economic state s. On the other hand, any agent contemplating going short will *behave as if* he believes that a unit of the portfolio will payoff $y^s + \left[0 \times \nu^0 + 2 \times \left(1 - \nu^0\right)\right]$ with probability 1 in economic state s. Hence, exactly the same argument as before applies: for ν^0 and ν^1 sufficiently small, $V(B) < V(S)$ and there will not be any trade in the portfolio. The important insight here is that while agents are fully aware that a 'well diversified' portfolio 'averages out' the idiosyncracies, they only have an imprecise knowledge of *what it averages out to*, due to the fact that their priors are imprecise. That is why, under ambiguity aversion, even though diversification ensures that idiosyncracies cancel out, the effect of the presence of idiosyncracies on the decision making 'outlives' the idiosyncracies and the effect of idiosyncracies are not simply washed away, as it happens with unambiguous beliefs[19].

[18]This is merely an informal statement of the result, various versions of which may be found in Dow and Werlang (1994), Marinacci (1996) and Walley and Fine (1982). A full account of the theorem is provided in Appendix B.

[19]One might be tempted to conjecture that results of the paper may be replicated by simply assuming heterogeneous beliefs among agents. The conjecture is, however, incorrect. To see this, note, what is at work in the

How small do ν^0 and ν^1 need to be to ensure $V(B) < V(S)$? In the example of the financial asset employed in the preceding discussion, a level of ambiguity (of belief about the idiosyncratic state, $1-\nu^0-\nu^1$) greater than $\frac{1}{2}$ is enough to ensure no-trade, *irrespective of the utility functions of the agents and the endowment vector.* The bound follows from the general closed form expression for $\bar{\mathcal{A}}, \bar{\mathcal{A}} = \frac{y^{\overline{s}}-y^{\underline{s}}}{y(1)-y(0)}$, derived in the formal proof. But appropriate care must be exercised in interpreting $\bar{\mathcal{A}}$; the expression derived for it may be misleading on two counts. First, it might appear that obtaining no-trade hinges on $y(1)-y(0) > y^{\overline{s}}-y^{\underline{s}}$. Second, it might seem that neither the degree of risk aversion of the trading agents nor their endowments have any role in the logic connecting ambiguity aversion to no-trade. Recall, however, that $\bar{\mathcal{A}}$ is the answer to the question, "how much of ambiguity would be enough to cause no trade, *irrespective of the utility functions of the agents and the endowment vector*?" Given that $\bar{\mathcal{A}}$ is an upper bound (i.e., the supremum across all the possible combinations of parameters of utilities or endowments), it is apposite that an expression for it should be independent of any parameter of utility or endowment. Typically, once we specify the utilities and the endowments, the ambiguity required for no trade would be less than $\bar{\mathcal{A}}$. More importantly, the requisite ambiguity, given specific utilities and endowments, *would* depend on the utilities and endowments. Taking a position on a financial asset exposes an agent to ambiguity (i.e., effect of the 'error in his estimate' of the 'actual' probabilities). Whereas, not taking a position would leave the agent exposed to the risk inherent in his endowment. Thus, given idiosyncratic effects, there is a tension between the effect of ambiguity aversion on one hand, and the riskiness of their endowment and the agents' desire to smooth this risk, on the other. The following example, is intended to provide an illustration of this.

Example 3 Consider a 2-period finance economy with two assets, b and z, traded in period 0 and two possible economic states, $s=\alpha$ or β, in period 1, inhabited by two agents B and S whose utility functions are given by

$$u_h\left(x_h^{\omega}\right) = -\left(\tfrac{1}{\lambda}\right)\exp\left(-\lambda x_h^{\omega}\right), \quad \lambda > 0;\ h \in \{B, S\}.$$

Further, suppose that $e_B^{\alpha} > e_S^{\alpha}$ and $e_S^{\beta} > e_B^{\beta}$. The asset b delivers one unit in each of the two economic states. The other asset, z has uncertain deliveries with an idiosyncratic component: $y(s,t) = y^s + y(t)$; assume $y^{\alpha}=0$, $y^{\beta}=2$, $y(0)=0$, $y(1)=1$ (notice the lack of an 'overlap', i.e., it is not true that $y^{\alpha}+y(1) > y^{\beta}+y(0)$). Let $\pi(\alpha)=\pi(\beta)=0.5$ and $\nu^0+\nu^1<1$.

Note, the given endowment vector, asset payoff matrix and the fact that the two agents have the same utility function, together imply that there is no equilibrium in this economy where B goes short and S goes long on z. Denote $\underline{\mu}(t) = \left(1-\nu^1\right)^{1-t} \times \left(\nu^1\right)^t$ and $\overline{\mu}(t) = \left(\nu^0\right)^{1-t} \times \left(1-\nu^0\right)^t$, $t=0,1$. It follows from the first order conditions (evaluated in the neighborhood of z_h=0), that if the inequality (1), below, holds then no-trade is an equilibrium in this economy :

$$\frac{\mathbb{E}_{\pi\times\underline{\mu}}\left[\left(y^s+y(t)\right)u_B'\left(e_B^s\right)\right]}{\mathbb{E}_{\pi\times\underline{\mu}}\left[u_B'\left(e_B^s\right)\right]} < \frac{\mathbb{E}_{\pi\times\bar{\mu}}\left[\left(y^s+y(t)\right)u_S'\left(e_S^s\right)\right]}{\mathbb{E}_{\pi\times\bar{\mu}}\left[u_S'\left(e_S^s\right)\right]} \tag{1}$$

This is so, since the l.h.s. of (1) is the <u>*maximum*</u> price of the uncertain asset that will induce B to move to a positive asset holding, while r.h.s. shows the <u>*minimum*</u> price of the uncertain asset required to induce S to go short. Substituting in the assumed parameter values and functional

paper is not that different agents have different beliefs but that agents behave as if they evaluate different actions (e.g. going short, going long) with different (probabilistic) beliefs.

form, we may rewrite (1) as the inequality (2)

$$\begin{aligned}
&\Rightarrow \quad \frac{0.5\exp\left(-\lambda e_B^\alpha\right)\left[\nu^1\right]+0.5\exp\left(-\lambda e_B^\beta\right)\left[2+\nu^1\right]}{0.5\exp\left(-\lambda e_B^\alpha\right)+0.5\exp\left(-\lambda e_B^\beta\right)} \\
&< \quad \frac{0.5\exp\left(-\lambda e_S^\alpha\right)\left[1-\nu^0\right]+0.5\exp\left(-\lambda e_S^\beta\right)\left[2+\left(1-\nu^0\right)\right]}{0.5\exp\left(-\lambda e_S^\alpha\right)+0.5\exp\left(-\lambda e_S^\beta\right)} \qquad (2) \\
&\Rightarrow \quad \frac{\nu^1\left[\exp\left(-\lambda e_B^\alpha\right)+\exp\left(-\lambda e_B^\beta\right)\right]+\exp\left(-\lambda e_B^\beta\right)}{\left(1-\nu^0\right)\left[0.5\exp\left(-\lambda e_S^\alpha\right)+0.5\exp\left(-\lambda e_S^\beta\right)\right]+\exp\left(-\lambda e_S^\beta\right)} \\
&< \quad \frac{0.5\exp\left(-\lambda e_B^\alpha\right)+0.5\exp\left(-\lambda e_B^\beta\right)}{0.5\exp\left(-\lambda e_S^\alpha\right)+0.5\exp\left(-\lambda e_S^\beta\right)}. \qquad (3)
\end{aligned}$$

Denote $0.5\exp\left(-\lambda e_B^\alpha\right)+0.5\exp\left(-\lambda e_B^\beta\right) \equiv Eu'(B)$ and $0.5\exp\left(-\lambda e_S^\alpha\right)+0.5\exp\left(-\lambda e_S^\beta\right) \equiv Eu'(S)$. Then the inequality in (3) may be written as follows:

$$\frac{\nu^1 Eu'(B)+\exp\left(-\lambda e_B^\beta\right)}{\left(1-\nu^0\right)Eu'(S)+\exp\left(-\lambda e_S^\beta\right)} < \frac{Eu'(B)}{Eu'(S)} \qquad (4)$$

By inspecting (4) it may be seen that there exists a $\bar{\lambda}, \bar{\lambda} > 0$, such that for all $\lambda \in \left(0, \bar{\lambda}\right)$ the inequality in (4) will hold. To see this consider the function φ, where

$$\varphi\left(\lambda; e_B^\beta, e_S^\beta\right) \equiv \frac{\exp\left(-\lambda e_B^\beta\right)}{\exp\left(-\lambda e_S^\beta\right)} = \exp\left[\lambda\left(e_S^\beta - e_B^\beta\right)\right], \ \textit{and } e_S^\beta - e_B^\beta > 1.$$

Clearly, $\varphi\left(\lambda; e_B^\beta, e_S^\beta\right)$ is monotonic in λ, and moreover, $\varphi \downarrow 1$ as $\lambda \downarrow 0$. Notice, we may rewrite (4) as follows

$$\frac{\nu^1 Eu'(B)+\varphi\left(\lambda; e_B^\beta, e_S^\beta\right)\exp\left(-\lambda e_S^\beta\right)}{\left(1-\nu^0\right)Eu'(S)+\exp\left(-\lambda e_S^\beta\right)} < \frac{Eu'(B)}{Eu'(S)} \qquad (5)$$

Hence, given that $1-\nu^0 > \nu^1$, (5) will hold provided λ is sufficiently close to 0.

This shows that we may obtain no trade even when the "over-lap" condition does not hold. Further, ceteris paribus, larger the value of the risk aversion parameter λ, the greater the ambiguity of belief $\left(1-\nu^0-\nu^1\right)$ required for the inequality (4) to hold (of course, for high values of λ the inequality may fail to hold altogether). Finally, note, greater the difference $e_S^\beta - e_B^\beta$, ceteris paribus, the greater the $\left(1-\nu^0-\nu^1\right)$ required for the inequality to hold. In other words, greater the gain from sharing the risk the greater the level of ambiguity required to "kill off" the trade.

It is relatively straightforward to extend the exercise to allow for n independent replicas of the financial asset, z^i,with $n \to \infty$. In that case, we consider trade of 'two' assets, one the safe asset and the other the 'portfolio' asset, consisting of $\frac{1}{n}$ unit of each z^i. Given that we are evaluating the first order conditions at (or, in the 'neighborhood' of) z^i, the analysis would obtain with precisely the same conditions and inequalities as above. ■

5 Robustness of the main result

The following remarks are intended to clarify that many of the assumptions of the model presented in the last section were principally put in place to ensure transparency of the analysis, and could be substantially relaxed, without losing the substance of the analytical results. As we shall see, relaxing the assumptions does not necessarily pose a conceptual or a technical challenge. By and large, it would merely multiply the notational complexity and the tedium of the analysis without adding substance.

Remark 1 The first remark concerns the matter of introducing a more general belief structure. As we have explained, what is crucial in obtaining our main result is the two ways in which the beliefs about idiosyncracies are independent and the fact that beliefs about the idiosyncracies are sufficiently ambiguous. In particular, it does not matter whether the beliefs about the economic states are ambiguous. The following exercise essentially reanalyzes the finance economy model by relaxing the assumption on additive beliefs about economic states to validate this claim.

Consider the financial economy with idiosyncracy analyzed in the previous section with the following amendments:

- There are two economic states, $s = 1, 2$, (and each economic state is further divided into two idiosyncratic states $t = 0, 1$). $\omega \in \Omega \equiv \{(s,t) \mid s = 1, 2 \text{and } t = 0, 1\}$.
- There is one safe asset, b, whose price is normalized to 1 and <u>one</u> uncertain asset, z, whose payoffs are affected by idiosyncratic events. The uncertain asset's payoffs are as before: $y^s + y(t)$. Assume w.l.o.g. $y^1 < y^2$ and that $y^1 + y(1) > y^2 + y(0)$.
- Beliefs: For idiosyncratic uncertainty, ν^t, with $\nu^0 + \nu^1 < 1$. For economic uncertainty, μ^s, with $\mu^1 + \mu^2 < 1$. Beliefs on ω, π, are defined as the independent product of ν and μ: $\pi \equiv \nu \bigotimes \mu$.

Call the financial economy with idiosyncracy amended as explained in the bulleted items above, $\hat{\mathcal{E}}$.

Proposition 1 *Consider the economy $\hat{E}$. Then there exists an $\bar{A}, 0 < \bar{A} < 1$, such that if $1 - \nu^0 - \nu^1 > \bar{A}$, then $z_h = 0$ for all $h \in \{1, ..., H\}$ and $x^{s,t} = x^{s,t'}$, $s = 1, 2$, $t \neq t'$, at every equilibrium.*

Remark 2 Since the diversification argument -based on the law of large numbers- "does not work" in the presence of idiosyncratic uncertainty and high enough ambiguity aversion, one might wonder if the assumption that the assets introduced are all 'replicas' is not too demanding. To make the point somewhat differently, in the case of standard beliefs considering the case with replicas is sufficient since that is enough to drive out the effect of idiosyncracy. But given that diversification with replica assets doesn't work with ambiguous beliefs, the question remains as to whether diversification can be achieved through assets which are not replicas (in terms of payoffs). It turns out that it does not make any difference (to the result) if we were to relax the assumption about 'strict' replicas. To obtain an intuition of why that is so, consider the financial economy with idiosyncracy analyzed in the previous section with the following amendments:

- There are two economic states, $s = 1, 2$, (and each economic state is further divided into two idiosyncratic states $t = 0, 1$). $\omega \in \Omega \equiv \{(s,t) \mid s = 1, 2 \text{and } t = 0, 1\}$.

- There is one safe asset, b, whose price is normalized to 1 and n uncertain assets, z^i, $n \to \infty$,whose payoffs are affected by idiosyncratic events. However, instead of replicating the same asset, we introduce new ambiguous assets, z^i has payoffs given by $y_i^s + y_i(t)$, $s = 1, 2$, $t = 0, 1$, with the maintained assumption: $y_i(t_i = 0) < y_i(t_i = 1)$. Note, unlike the case of the model in the previous section, we now allow that $y_i^s \neq y_{i'}^s$ and $y_i(t_i = 0) \neq y_{i'}(t_{i'} = 0)$, $y_i(t_i = 1) \neq y_{i'}(t_{i'} = 1)$. However, suppose the following generalization of the 'overlap condition' holds:

$$\left[y_i^1 + y_i(0), y_i^1 + y_i(1)\right] \cap \left[y_i^2 + y_i(0), y_i^2 + y_i(1)\right] \neq \emptyset$$

- Beliefs: Beliefs on the idiosyncratic part of asset z^i can be represented by ν_i^0 and ν_i^1. Further, assume that all these assets are independent (in the sense that beliefs over the joint realization of the idiosyncratic part is simply the independent product of the capacities). Beliefs on ω, are defined as the independent product of π and ν, i.e.,$\pi \bigotimes_{i=1}^n \nu_i$.

Call the financial economy with idiosyncracy amended as explained in the bulleted items above, $\check{\mathcal{E}}$.

Proposition 2 *Consider the economy $\check{E}$. Then there exists an $\bar{A}, 0 < \bar{A} < 1$, such that if $1 - \nu^0 - \nu^1 > \bar{A}$, then $z_h = 0$ for all $h \in \{1, ..., H\}$ and $x^{s,t} = x^{s,t'}$, $s = 1, 2$, $t \neq t'$, at every equilibrium.*

Remark 3 Finally, the Arrow securities, whose essential function is to enable transfer contingent resources among the states $s, s \geq 3$, may be replaced by a set of arbitrary assets (with payoffs not affected by idiosyncratic effects) without affecting any of our results as long as the set satisfied the following two conditions:

1. the payoff matrix corresponding to the set of assets is at most of rank $S - 2$

2. the payoff matrix corresponding to the entire set of assets available in the economy, including the safe asset and the financial assets with their idiosyncratic payoffs 'averaged out' is of full rank (S) with respect to the economic states.

The conditions are sufficient to ensure that the economy with the idiosyncracies 'washed out' is a complete market economy and the economy with a set of financial assets missing (due to ambiguity aversion) is an incomplete market economy.

6 Concluding discussion

In this section we sketch out some of the broader, economically and empirically significant, implications of the results obtained in the paper.

In the first instance, the results provide a new insight on the issue of whether a "complete market level of risk sharing" can be achieved with a financial market consisting purely of primary securities, i.e., bonds and equities. The question does not have a trivial answer because, arguably, a fundamental and distinctive characteristic of equities is that their payoff have idiosyncratic components. This attribute of equities has the implication that a financial market consisting purely of bonds and equities will be, by definition, an incomplete market *per se*. However, teachings of modern finance allows us to establish, assuming agents have Savage/Bayes-rational beliefs, assuming the absence of significant asymmetric information and transactions costs, that it is possible to achieve optimal risk sharing as long as there are no exogenous restrictions limiting the availability of "enough" equity share contracts. But as the main result shows, if a proportion of the exogenously available assets have 'large enough' idiosyncratic components in their payoffs, ambiguity aversion may cause such assets not to be traded. The paper, subsequently, also alerts us to the fact that no amount of 'exogenous availability' of equities is enough to guarantee a "complete market level of risk sharing", if agents are ambiguity averse. Assuming ambiguity aversion, a combination of beliefs and risk attitudes may always arise to *endogenously* prevent trade in *any* of the exogenously given equities and thereby cause "genuine" incompleteness. In other words, ambiguity aversion may endogenously limit the scope of risk sharing obtainable through the exogenously available assets by determining what assets are *actually* traded. Hence, so long as we accept that presence of idiosyncratic effects are a definitive characteristic of bond/equity payoffs, ambiguity aversion provides a reason as to why we might think that the GEI model and its propositions are well founded in an economy where the financial market consists purely of primary securities.

Another related finding of import in the paper, we believe, is with regard to the role of idiosyncratic risk in a finance economy. A, perhaps naive, but nevertheless widely drawn inference obtained from modern financial analysis is that such idiosyncracies ultimately 'do not matter'–standard diversification techniques ensure that these will (almost) surely be 'washed' away. As this paper demonstrates, this is again something that is not robust to ambiguity aversion[20]. Hence, something more than standard diversification techniques are required–perhaps hedging possibilities obtained through derivative financial instruments–to deal with idiosyncracies in presence of ambiguity aversion. The presence of idiosyncratic effects on payoffs and the inability to write contingent delivery contracts on idiosyncratic events is of no major consequence in a world of 'standard' beliefs; it is merely an irrelevant 'friction'. But this 'small friction' of not being able to trade across contingencies agents do not care to trade anyway (in an SEU world), has a novel 'domino effect' in the world of ambiguity averse agents and leads to the endogenous closure of markets which are necessary for transferring resources across those contingencies at which agents' endowments are actually affected. That the seed of 'small frictions' can lead to a 'big loss' in the realization of gains from trade is a theme that has been visited earlier in economics: recall, for instance, the menu costs literature in 'New-Keynesian' macroeconomics. However, a more interesting parallel is the work applying ambiguity aversion to explain incomplete (bilateral) contracts (Mukerji (1998)). That paper showed how restrictions on contractibility that are insignificant when agents have additive subjective beliefs may result in contractual incompleteness of actual significance when agents are ambiguity averse.

The findings do have empirically significant implications. A basic tenet of modern day financial regulation and managing of financial exchanges is to put in place regulations and procedures that minimize the effect of asymmetries of information (via insider trading, for instance) and transactions costs. To the extent such procedures are successful, one might be tempted to conclude that such 'controlled' environments are not significantly vulnerable to the scourge of market incompleteness. What the results in this paper suggest is that even in such 'controlled' environments, risk sharing achieved through primitive assets may be far from complete and hence financial innovation, by introducing new hedging possibilities using derivative securities, will continue to have an important role. An implication of our results is that risk sharing transactions may not be freely achieved even in circumstances involving

[20] At the first instance our result might appear to be in conflict with the finding reported in Kelsey and Milne (1995) that the 'standard' conclusions of the arbitrage pricing theory (APT) model holds even when agents have non-additive expected utility preferences. What Kelsey and Milne do, in terms of the framework of the present paper, is that they model beliefs on the economic states (which are like 'factors' in a standard APT set up) as non-additive while fixing the beliefs on the idiosycratic states as additive. As is explained in this paper, what drives our result is the non-additivity of beliefs over the idiosycratic states, and hence, the result reported in Kelsey and Milne (1995) is quite consistent with ours.

trades of equities between relatively anonymous agents where one would not normally expect strong asymmetries of information. This is supported by the results reported from an empirical investigation of the conundrum why "so few hold stocks (among the general population)?" by Haliassos and Bertaut (1995), who conclude that it is departures from the expected utility paradigm of the kind incorporated in this paper, that holds the key to that question (rather than risk aversion, heterogeneity of beliefs, habit persistence, etc.).

The results in our paper identify asset characteristics which make its trade vulnerable to ambiguity aversion: essentially, the point of vulnerability is the strength of the idiosyncratic component in its payoffs. This connection between the extent of idiosyncratic component and no-trade is the key empirical content of the theory advanced in this paper and deserves to be considered more fully. Consider an increase in the ambient uncertainty in an economy with a set of assets and ambiguity averse agents. More precisely, ceteris paribus, suppose that the increase in uncertainty manifests as an increase in the ambiguity of belief over contingent events, including an increase in the ambiguity of belief over idiosyncratic events, though not necessarily exclusively so. The analysis and results in this paper suggest that if the increase in uncertainty were sufficiently great then trade in a certain subset of the assets will thin out (in particular, trade in those corporate bonds and forward contracts on equities for which the ratio of the range of variation of the idiosyncratic component to the range of variation due to the economic shocks is greater). It is instructive to note the distinction between the empirical content of a theory of no-trade based on the 'lemons' problem (e.g., Morris (1997)) and the theory based on ambiguity aversion .The primitive of the former theory is asymmetric uncertainty between the transacting parties, and more importantly, the asymmetric uncertainty may cause no-trade *irrespective of the nature of the idiosyncratic component* of the payoff of the asset concerned. Indeed, no trade may result even if there were no idiosyncratic component. Thus that theory, *per se*, does not link the presence and extent of idiosyncratic component to no trade. To obtain such a link, one has to assume, *a priori*, that there is sufficient asymmetric information only in the presence of idiosyncratic information. On the other hand, the theory based on ambiguity aversion does not require that one assume that ambiguity is present only with idiosyncracies, or that agents have ambiguous beliefs especially with respect to payoffs of assets with idiosyncratic components. One may well begin with the primitive that ambiguity is present in a 'general' way, across all contingencies. However, as has been shown, ambiguity aversion will only generate no trade for those assets whose payoffs are substantially affected by idiosyncracies, and not otherwise. Hence, since ambiguity aversion *selectively* attacks only those assets whose payoffs have idiosyncratic components, the link between idiosyncracy and no trade is *endogenously* generated in the theory based on ambiguity aversion. This positive understanding is of significance. History of financial markets is replete with episodes of increase in uncertainty leading to a thinning out of trade (or even seizing up completely) *peculiarly* in assets such as high yield corporate bonds ('junk' bonds) and bonds issued in "emerging markets"(vis., Latin America, Eastern Europe and East Asia)[21]. It appears only natural to interpret these episodes as one of drastic increase in the *common* uncertainty faced by investors, rather than as an increase in the asymmetry of information. Also, it seems eminently demonstrable that the high risk bonds which appear to be so sensitive to attacks of uncertainty are precisely those bonds which have high idiosyncratic components in their payoffs. Thus the theory of ambiguity aversion provides an endogenously generated 'natural' explanation of why only this certain class of assets, and not all assets, will be affected by the increase in uncertainty. The explanation is also useful

[21]Consider, for instance, the widely reported recent paralysis afflicting the junk bond markets in the U.S. and in Europe in the aftermath of the Russian and East Asian crises. For related press reporting see, "US corporate bond market hit," *Financial Times,* 13 October, 1998 and "Virgin arm abandons junk bond issue" *Financial Times,* 29 October, 1998.

in providing a novel understanding of the role of certain institutions of financial contracting in facilitating the transaction of corporate bonds. A recent paper, Anderson (1999), investigates the institutions of financial contracting under extreme uncertainty. The volatile and uncertain Brazilian economic environment presents many obstacles for firms seeking debt financing, yet a nascent corporate bond market survives. The paper analyzes the common features of a sample of successful Brazilian indenture agreements to understand what holds the key to the success of these debentures. That key is found to be a set of contingent-maturity mechanisms built into the design of successful contracts. Specifically, the investors enjoy rights to (periodically) recontract/renegotiate the original financial contract. Similarly, the debtors may renegotiate or call the debentures. Long-term bonds are periodically marked to market according to prevailing conditions and firm-specific risk. Consequently, the interest-rate risk of a long-term bond that recontracts is lower than for a fixed-rate issue. Most interestingly, the analysis observes (pp. 61), that 72% of the bonds with recontracting provisions in the indenture include other provisions that allow it to effectively pay floating interest, adjusted in accordance with the changes in the market rate. This leaves us to conclude, like Anderson, that recontracting may viewed primarily as a mechanism to adjust interest rates for firm-specific risks, *not changes in the general (i.e. market) real rates of interest.* The right of debtors to call serve a related function. As was observed by Jensen and Meckling (1976), larger the amount of risky debt in the capital structure of the firm greater the incentive of shareholders to enhance the value of their equity by choosing investments of higher risk but lower value. Hence, perversely, ceteris paribus, firms with higher outstanding debt would tend to have higher firm specific risks, since these firms would be opting for the kind of risky investments not favored by other firms. A call provision addresses this so called asset-substitution problem identified by Jensen and Meckling. Barnea, Haugen, and Senbet (1980) show that the value of a call provision to shareholders increases with total firm value. Consequently, increased equity value due to shifting to risky but inefficient investments is offset by a decline in the value of the call provision to the share holders. Thus, the call provision reduces the incentives to initiate investment that would lead to firm specific risks. In this context, it is worth mentioning that empirical evidence based on US debt suggests that firms with high growth potential and higher default risks (as compared to other firms) are more likely to include call provisions in their indentures (Thatcher (1985); Kish and Livingston (1992)). The theory presented in this paper gives us an understanding of the role of such contractual institutions in facilitating the working of bond markets. The lesson is particularly important in the light of the recent spate of episodes of seizure of bond markets brought upon by instability of the emerging financial markets.

Financial economists have observed that even though barriers to international investment have fallen dramatically, investors continue to allocate only a very small fraction of their portfolio to foreign investments. Indeed, the allocation is much smaller than would be expected in the absence of barriers to international investment. This evidence constitutes what is generally referred to as the home-bias puzzle[22]. It is suggested that ambiguity aversion may well hold an important key to unlocking the puzzle. There are two factors which suggest that ambiguity aversion may be the reason which leads investors to favor domestic over foreign assets in their portfolios. First, an investor may well have less information about payoff prospects of foreign assets as compared to domestic assets. Shiller, Kon-Ya, and Tsutsui (1991) provide some survey evidence consistent with this view. Less information, may well mean more ambiguous information. Secondly, foreign assets may typically have more relatively pronounced idiosyncratic components in relation to the endowments of the domestic investor. Baxter and Jermann (1997) report that returns to human capital (which may be identified with the agents'

[22]See Frankel (1995) and Lewis (1995) for references, as well as French and Poterba (1991), Cooper and Kaplanis (1994), and Tesar and Werner (1995).

stochastic endowment in the model presented in this paper) and physical capital are much more correlated within countries as compared to the correlation across countries. Indeed, it is the connection with idiosyncracy that suggests that ambiguity aversion may be a more wide ranging explanation of the phenomenon than a theory of asymmetric information. Kang and Stulz (1997) studies stock ownership in Japanese firms by non-Japanese investors from 1975 to 1991. The paper documents that the two outstanding factors which explain foreign investors' holdings are, first, the sectoral association of the firm and second, the largeness of the firm. Asymmetric information may explain the second factor as well as ambiguity aversion since larger firms may be better known outside the country. But asymmetric information does not provide an obvious explanation of the finding that foreign investors hold disproportionately more shares of firms in manufacturing sector while almost none at all in sectors such as utilities, transport and construction. Casual empiricism suggests that the Japanese fortunes of the Japanese manufacturing sector would be well in tune with the vagaries of the world market, given that most units in the manufacturing sector would be directly or indirectly reliant on the world market. The same cannot be said of the other sectors mentioned above; the final products of those sectors are much more geared towards domestic consumption. This would imply that returns in the Japanese manufacturing sector would be far more correlated with the returns to human capital in, say, the U.S., than returns in any of the other mentioned sectors. If the casual empiricism is vindicated, then we may assert that ambiguity aversion provides more complete explanation of the facts of the home-bias puzzle than a theory based on asymmetric information. This is an open question for future empirical research.

An empirical test of the theory advanced in this paper may be constructed along the following lines. We take a set of corporate bonds and equities and rank them in order of the idiosyncratic effects associated with the respective time series of their payoffs. (It would appear such an ordering is potentially determinable.) If it is then found that this ranking is significantly correlated with the ranking of the securities with respect to their individual trading volume (in case of equities one should consider trades involving forward contracts) then we would obtain a (partial) validation of the theory. Note, a theory of asymmetric information does not yield such a testable hypothesis since it merely connects the potentially *unobservable* primitive of beliefs about an asset to the possibility of no-trade in the asset. On the other hand, ambiguity aversion connects the potentially *observable* primitive of idiosyncratic effects associated with an asset to the occurrence of no-trade in the asset.

Finally, while there is a vast literature on ambiguity aversion, and indeed on the many other departures from SEU, that convincingly establishes their importance in laboratory settings (see Camerer and Weber (1992)), this work has had little impact on the way that economics is done. In large part this is because there have been so few demonstrations showing that the presumption of such departures can generate novel and significant insight into the working of important economic institutions. This paper, we believe, makes a contribution towards filling this gap in the literature.

7 Appendix

Appendix A

Proof of Lemma 1:

Suppose w.l.o.g. $q^{z^i} \geq q^{z^{i'}}$ for some $i, i' \in \{1, ..., n\}$. In a first step we show that this implies $z^i_{h,n} \leq z^{i'}_{h,n}, \forall h \in \{1, ..., H\}$. Indeed, assume $z^i_{h,n} > z^{i'}_{h,n}$ for some $h \in \{1, ..., H\}$. Then construct the new portfolio holding $\bar{z}_{h,n}$ as follows:

$$\begin{aligned}\bar{z}^{i}_{h,n} &= z^{i}_{h,n} - \varepsilon \\ \bar{z}^{i'}_{h,n} &= z^{i'}_{h,n} + \frac{q^{z^i}}{q^{z^{i'}}}\varepsilon \\ \bar{z}^{j}_{h,n} &= z^{j}_{h,n}\ \forall j \neq i, i'.\end{aligned}$$

where ε is small enough so that $\bar{z}^{i}_{h,n} > \bar{z}^{i'}_{h,n}$. Note, $\bar{z}_{h,n}$ is budget feasible. Let

$$\begin{aligned}\bar{x}^{s,t}_{h,n} &\equiv e^s_h + b_{h,n} + \sum_{i=1}^{n} z^{i}_{h,n}(y^s + y(t_i) \text{ for } s = 1, 2,\ \forall t \in \tau_n \\ \bar{x}^{s,t}_{h,n} &\equiv e^s_h + b_{h,n} + b^s_{h,n} + \sum_{i=1}^{n} z^{i}_{h,n}(y^s + y(t_i) \text{ for } s = 3, ..., S,\ \forall t \in \tau_n\end{aligned}$$

Because $x^{s,t}_{h,n}$ and $\bar{x}^{s,t}_{h,n}$ are comonotonic, and u_h is strictly increasing, it follows from definition 1 that there exists an *additive* product measure μ, where $\mu \equiv \times_{i=1}^{n}\mu_i$, and $\mu_i : 2^{\{0,1\}} \to [0,1]$ are *additive* measures, such that,

$$\begin{aligned}\mathbb{CE}_{\pi\otimes\nu}\left(x^{s,t}_{h,n}\right) &= \mathbb{E}_{\pi\times\mu}\left(x^{s,t}_{h,n}\right), \mathbb{CE}_{\pi\otimes\nu}\left(\bar{x}^{s,t}_{h,n}\right) = \mathbb{E}_{\pi\times\mu}\left(\bar{x}^{s,t}_{h,n}\right), \text{ and,} \\ \mathbb{CE}_{\pi\otimes\nu}\left(u_h\left(x^{s,t}_{h,n}\right)\right) &= \mathbb{E}_{\pi\times\mu}\left(u_h\left(x^{s,t}_{h,n}\right)\right), \\ \mathbb{CE}_{\pi\otimes\nu}\left(u_h\left(\bar{x}^{s,t}_{h,n}\right)\right) &= \mathbb{E}_{\pi\times\mu}\left(u_h\left(\bar{x}^{s,t}_{h,n}\right)\right),\ \forall s = 1, 2, ..., S, \forall t \in \tau_n.\end{aligned}$$

Furthermore,

$$\mathbb{E}_{\mu}\left(x^{s,t}_{h,n} \mid s\right) = \mathbb{E}_{\mu}\left(\bar{x}^{s,t}_{h,n} \mid s\right) + \mathbb{E}_{\mu_i}\varepsilon y(t_i) - \mathbb{E}_{\mu_{i'}}\varepsilon y(t_{i'}),\ \forall s = 1, 2, ..., S.$$

Next, notice $\mathbb{E}_{\mu_i}\varepsilon y(t_i) - \mathbb{E}_{\mu_{i'}}\varepsilon y(t_{i'}) \leq 0$. Indeed, either $z^{i}_{h,n}$ and $z^{i'}_{h,n}$ have the same sign, in which case $\mu_i = \mu_{i'}$ and $\mathbb{E}_{\mu_i}\varepsilon y(t_i) - \mathbb{E}_{\mu_{i'}}\varepsilon y(t_{i'}) = 0$. Or $z^{i}_{h,n} > 0 > z^{i'}_{h,n}$ and then

$$\begin{aligned}\mathbb{E}_{\mu_i}\varepsilon y(t_i) - \mathbb{E}_{\mu_{i'}}\varepsilon y(t_{i'})) &= \varepsilon\left[(1-\nu^1)y(0) + \nu^1 y(1) - \nu^0 y(0) - (1-\nu^0)y(1)\right] \\ &= \varepsilon[1 - \nu^0 - \nu^1][y(0) - y(1)] \leq 0\end{aligned}$$

Hence, $\bar{x}^s$ stochastically dominates x^s. Given $u'' < 0$, therefore, $\mathbb{E}_{\pi\times\mu}u_h(\bar{x}^{s,t}_{h,n}) > \mathbb{E}_{\pi\times\mu}u_h(x^{s,t}_{h,n})$. As a consequence, $\mathbb{CE}_{\pi\otimes\nu}u_h(\bar{x}^{s,t}_{h,n}) > \mathbb{CE}_{\pi\otimes\nu}u_h(x^{s,t}_{h,n})$. But this is a contradiction to the hypothesis that $(\mathbf{q}_n, (\mathbf{b}_n, \mathbf{z}_n, \mathbf{x}_n))$ is an equilibrium. $\therefore\ z^{i}_{h,n} \leq z^{i'}_{h,n}, \forall h \in \{1, ..., H\}$.

Since, $(\mathbf{q}_n, (\mathbf{b}_n, \mathbf{z}_n, \mathbf{x}_n))$ is an equilibrium, $\sum_{h=1}^{H} z^{i}_{h,n} = \sum_{h=1}^{H} z^{i'}_{h,n} = 0$. Therefore, using the fact that $z^{i}_{h,n} \leq z^{i'}_{h,n}$ for all h, we get that $z^{i}_{h,n} = z^{i'}_{h,n}$ for all h. ∎

Proof of Theorem 1:

The maximization problem $\widetilde{\mathcal{P}}_{h\infty}$ given asset prices $\widetilde{\mathbf{q}}_{\infty}$ may be written as follows:

$$\begin{gathered}\max \mathbb{E}_{\pi(1)\otimes\pi(2)\otimes\nu}u_h\left(e^s_h + b_{h,\infty} + \tilde{z}_{h,\infty}\left[\lim_{n\to\infty}\sum_{i=1}^{n}\frac{y^s + y(t_i)}{n}\right]\right) \\ +\mathbb{E}_{\otimes_{s\geq 3}\pi(s)\otimes\nu}u_h\left(e^s_h + b^s_{h,\infty} + b_{h,n} + \tilde{z}_{h,\infty}\left[\lim_{n\to\infty}\sum_{i=1}^{n}\frac{y^s + y(t_i)}{n}\right]\right) \\ s.t.\ b_{h,\infty} + \sum_{s=3}^{S} q^s_{\infty}b^s_{h,\infty} + \tilde{q}_{\infty}\tilde{z}_{h,\infty} = 0\end{gathered}$$

And the maximization problem $\mathcal{P}_h$ given asset prices $\mathbf{q} = \widetilde{\mathbf{q}}_{\infty}$:

$$\max \sum_{s\in\{1,2\}} \pi(s)\, u_h\left(e_h^s + b_{h,n} + z_h \bar{y}^s\right) + \sum_{S\geq s\geq 3} \pi(s)\, u_h\left(e_h^s + b_{h,n} + b_{h,n}^s + z_h \bar{y}^s\right)$$
$$s.t.\ b_h + \textstyle\sum_{s=3}^S q_\infty^s b_h^s + \tilde{q}_\infty z_h = 0$$

If $n \to \infty$, by the law of large numbers, with probability 1 an unit of the portfolio $\tilde{z}_n$ yields a payoff of $y^s + \mathbb{E}_{t\in\{0,1\}} y(t) \equiv \bar{y}^s$ units. That is, $\lim_{n\to\infty} \sum_{i=1}^n \left(\frac{(y^s+y(t_i))}{n}\right) \xrightarrow{a.s} \bar{y}^s$. Recall, the financial asset z yields $\bar{y}^s$ units of the good in the economic states $s = 1, 2, ..., S$.

Hence, $(\mathbf{b}_\infty, \widetilde{\mathbf{z}}_\infty)$ solves the maximization problem $\widetilde{\mathcal{P}}_{h\infty}$ at prices $(\widetilde{\mathbf{q}}_\infty)$, if and only if $(\mathbf{b}_\infty, \widetilde{\mathbf{z}}_\infty)$ also solves the maximization problem $\mathcal{P}_h$ at prices $(\widetilde{\mathbf{q}}_\infty)$.

Finally note, if $(\widetilde{\mathbf{q}}_\infty, (\mathbf{b}_\infty, \widetilde{\mathbf{z}}_\infty, \mathbf{x}_\infty))$ describes an equilibrium of the n-financial assets economy with idiosyncracy it must be that $(\mathbf{b}_\infty, \widetilde{\mathbf{z}}_\infty)$ satisfies the conditions of (asset) market clearing at the price vector $\widetilde{\mathbf{q}}_\infty$. Hence, $(\widetilde{\mathbf{q}}_\infty)$ will also clear asset markets in the economy without idiosyncracy. Conversely, if $(\widetilde{\mathbf{q}}_\infty, (\mathbf{b}_\infty, \widetilde{\mathbf{z}}_\infty, \mathbf{x}_\infty))$ describes an equilibrium of the economy without idiosyncracy then $(\widetilde{\mathbf{q}}_\infty)$ will also clear asset markets in the n-financial assets economy with idiosyncracy. ■

Proof of the Main Theorem:

Consider $\widetilde{\mathcal{P}}_{hn}$, the maximization problem faced by an agent h in the n-financial asset economy with idiosyncracy. Suppose that, at equilibrium there exists h' such that $\tilde{z}_{h',n} \neq 0$, say $\tilde{z}_{h',n} > 0$. Since, at equilibrium $\sum_h \tilde{z}_{h,n} = 0$, there must be h'' such that $\tilde{z}_{h'',n} < 0$.

Next, since $\tilde{z}_{h',n} > 0$, and $y(0) < y(1)$, by Facts 2 and 3 (the fact that given that $\tilde{z}_{h',n} > 0$, $u_{h'}\left(x_{h',n}^{s,t}\right)$ is slice-comonotonic), we may calculate $\mathbb{CE}_{\pi\otimes\nu} u_{h'}\left(x_{h',n}^{s,t}\right)$ just as a standard expectation operator with respect to the standard additive measure $\pi \times \underline{\mu}(t)$, where

$$\underline{\mu}(t) = \left(1-\nu^1\right)^{n_0} \times \left(\nu^1\right)^{n-n_0},$$

where n_0 is the number of financial assets whose idiosyncratic payoff is $y(0)$ at the idiosyncratic state (s,t). This is because agent h' will have a contingent consumption, $x_{h',n}^{s,(t_i,t_{-i})}$, that is necessarily smaller at a state $(s,(0,t_{-i}))$ than at the state $(s,(1,t_{-i}))$, $s = 1, ..., S$.

The first order conditions of the problem $\widetilde{\mathcal{P}}_{h'n}$ (for agent h') then give:

$$\tilde{q}_n = \frac{\mathbb{E}_{\pi\times\underline{\mu}}\left[\sum_{i=1}^n \left\{\left(\frac{y^s+y(t_i)}{n}\right) u'_{h'}\left(x_{h',n}^{s,t}\right)\right\}\right]}{\mathbb{E}_{\pi\times\underline{\mu}}\left[u'_{h'}\left(x_{h',n}^{s,t_i}\right)\right]}$$

Notice, for all $s \in \{1, ..., S\}$

$$x_{h',n}^{s,t} \text{ and } \sum_{i=1}^n \left(\frac{y^s + y(t_i)}{n}\right)$$

are positively dependent given s (see Magill and Quinzii (1996)) since $\tilde{z}_{h',n} > 0$. Hence, because $u''(\cdot) < 0$,

$$Covariance\left(\sum_{i=1}^n \left(\frac{y^s + y(t_i)}{n}\right), u'_{h'}\left(x_{h',n}^{s,t}\right)\right) < 0, \text{ given } s.$$

Now,

$$\mathbb{E}_{\underline{\mu}}\left[\sum_{i=1}^{n}\left\{\left(\frac{y^{s}+y\left(t_{i}\right)}{n}\right)u'_{h'}\left(x_{h',n}^{s,t}\right)\right\}\right] = Covariance\left(\sum_{i=1}^{n}\left(\frac{y^{s}+y\left(t_{i}\right)}{n}\right),u'_{h'}\left(x_{h',n}^{s,t}\right)\right) +\mathbb{E}_{\underline{\mu}}\left[\sum_{i=1}^{n}\left(\frac{y^{s}+y\left(t_{i}\right)}{n}\right)\right]\mathbb{E}_{\underline{\mu}}u'_{h'}\left(x_{h',n}^{s,t}\right).$$

Thus,

$$\mathbb{E}_{\underline{\mu}}\left[\sum_{i=1}^{n}\left\{\left(\frac{y^{s}+y\left(t_{i}\right)}{n}\right)u'_{h'}\left(x_{h',n}^{s,t}\right)\right\}\right] < \mathbb{E}_{\underline{\mu}}\left[\sum_{i=1}^{n}\left(\frac{y^{s}+y\left(t_{i}\right)}{n}\right)\right]\mathbb{E}_{\underline{\mu}}u'_{h'}\left(x_{h',n}^{s,t}\right).$$

Hence,

$$\begin{aligned}\tilde{q}_{n} &< \frac{\sum_{s=1}^{S}\pi\left(s\right)\mathbb{E}_{\underline{\mu}}\left[\sum_{i=1}^{n}\left(\frac{y^{s}+y\left(t_{i}\right)}{n}\right)\right]\mathbb{E}_{\underline{\mu}}u'_{h'}\left(x_{h',n}^{s,t}\right)}{\sum_{s=1}^{S}\pi\left(s\right)\mathbb{E}_{\underline{\mu}}\left[u'_{h'}\left(x_{h',n}^{s,t}\right)\right]} \\ &\Rightarrow \tilde{q}_{n} < \max_{s}\left\{\mathbb{E}_{\underline{\mu}}\left[\sum_{i=1}^{n}\left(\frac{y^{s}+y\left(t_{i}\right)}{n}\right)\right]\right\} \\ &\Rightarrow \tilde{q}_{n} < y^{\overline{s}}+\left(1-\nu^{1}\right)y\left(0\right)+\nu^{1}y\left(1\right) \qquad (6)\end{aligned}$$

Consider next, h'' such that $\tilde{z}_{h'',n} < 0$. Since $\tilde{z}_{h'',n} < 0$, and $y\left(0\right) < y\left(1\right)$, by Fact 2 and 3 (the fact that given that $\tilde{z}_{h'',n} < 0$, $u_{h''}\left(x_{h',n}^{s,t}\right)$ is slice-comonotonic), we may calculate $\mathbb{CE}_{\pi\otimes\nu}u_{h''}\left(x_{h',n}^{s,t}\right)$ just as a standard expectation operator with respect to the standard additive measure $\pi\times\overline{\mu}\left(t\right)$, where,

$$\overline{\mu}\left(t\right)=\left(\nu^{0}\right)^{n_{0}}\times\left(1-\nu^{0}\right)^{n-n_{0}}$$

where n_0 is the number of financial assets whose idiosyncratic payoff is $y\left(0\right)$ at the idiosyncratic state (s,t). This is because agent h'' will have a contingent consumption, $x_{h'',n}^{(s,(t_i,t_{-i}))}$, that is necessarily greater at a state $(s,(0,t_{-i}))$ than at the state $(s,(1,t_{-i}))$. Proceeding in the same way as for the agent h', we consider the first order conditions for agent h'' :

$$\tilde{q}_{n}=\frac{\mathbb{E}_{\pi\times\overline{\mu}}\left[\sum_{i=1}^{n}\left\{\left(\frac{y^{s}+y\left(t_{i}\right)}{n}\right)u'_{h''}\left(x_{h'',n}^{s,t}\right)\right\}\right]}{\mathbb{E}_{\pi\times\overline{\mu}}\left[u'_{h''}\left(x_{h'',n}^{s,t}\right)\right]}$$

Notice that, for h'', given that $\tilde{z}_{h'',n} < 0$, it must be that $x_{h'',n}^{s,t}$ and $\sum_{i=1}^{n}\left(\frac{(y^{s}+y(t_{i}))}{n}\right)$ are negatively dependent given s and thus,

$$Covariance\left(\sum_{i=1}^{n}\left(\frac{\left(y^{s}+y\left(t_{i}\right)\right)}{n}\right),u'_{h''}\left(x_{h'',n}^{s,t_{i}}\right)\right)>0,\text{ given }s.$$

Hence, by a reasoning similar to that followed for the agent h',

$$\tilde{q}_{n} > y^{\underline{s}}+\left(1-\nu^{0}\right)y\left(1\right)+\nu^{0}y\left(0\right). \qquad (7)$$

Therefore, a necessary condition for having an equilibrium with $\tilde{z}_{h,n} \neq 0$ for at least some $h, h \in \{1, ..., H\}$, is that

$$\begin{aligned} y^{\overline{s}} + \left(1-\nu^1\right) y(0) + \nu^1 y(1) &> y^{\underline{s}} + \nu^0 y(0) + \left(1-\nu^0\right) y(1) \\ \Leftrightarrow y^{\overline{s}} - y^{\underline{s}} &> \left(1-\nu^0-\nu^1\right)\left(y(1)-y(0)\right) \end{aligned}$$

Set $\bar{\mathcal{A}} = \frac{y^{\overline{s}}-y^{\underline{s}}}{y(1)-y(0)}$, $\bar{\mathcal{A}}$ will belong to the *open* interval $(0,1)$ by our assumptions on the asset payoffs (in the statement of the theorem). Hence, if $1-\nu^0-\nu^1 > \bar{\mathcal{A}}$, then $\tilde{z}_{h,n} = 0$ for all $h, h \in \{1, ..., H\}$, at any equilibrium.

Finally, note if $n \to \infty$, by the law of large numbers for non-additive probabilities (see Appendix B, below) we may calculate $\mathbb{CE}_{\pi\otimes\nu} u_{h'}\left(x^{s,t}_{h',n}\right)$ just as a standard expectation operator with respect to the standard additive measure $\pi \times \underline{\mu}(t)$, where $\underline{\mu}(t)$ is such that

$$\underline{\mu}\left(\left\{t : \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{(y^s + y(t_i))}{n}\right) = y^s + \left(1-\nu^1\right) y(0) + \nu^1 y(1)\right\}\right) = 1.$$

Hence,

$$Covariance\left(\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{y^s + y(t_i)}{n}\right), u'_{h'}\left(x^{s,t}_{h',n}\right)\right) \leq 0, \text{ given } s.$$

Thus the proof would proceed as in the case of finite n except that the inequality (6) would read

$$\lim_{n\to\infty} \tilde{q}_n \leq y^{\overline{s}} + \left(1-\nu^1\right) y(0) + \nu^1 y(1)$$

and the inequality (7) would read

$$\lim_{n\to\infty} \tilde{q}_n \geq y^{\underline{s}} + \left(1-\nu^0\right) y(1) + \nu^0 y(0).$$

■

Appendix B

Let y be a function from a given space Ω to the real line $\mathbb{R}$, and $\sigma(y)$ the smallest $\sigma-$algebra that makes y a random variable. Ω^n denotes the n-fold Cartesian product of Ω, and $\sigma(y_1, ..., y_n)$ the product $\sigma-$algebra on Ω^n generated by the $\sigma-$algebras $\{\sigma(y_i)\}_{i=1}^n$. Set $S_n = \frac{1}{n}\sum_{i=1}^n y_i$.

Law of Large Numbers for Non-additive Probabilities (Marinacci (1996) Theorem 7.7, Walley and Fine (1982) Theorem 4.1 and Remark (i), Dow and Werlang (1994) Theorem 3.) Let Ω be (countably) finite. Let each ν_i be a convex capacity on $\sigma(y_i)$, and let $\{y_i\}_{i\geq 1}$ be a sequence of random variables independent and identically distributed relative to $\bigotimes \nu_i$. Suppose both $\mathbb{CE}_{\nu_1}(y_1)$ and $\mathbb{CE}_{\nu_1}(-y_1)$ exist. Then

1. $\bigotimes \nu_i \left(\left\{ \begin{array}{c} \omega \in \Omega^\infty : \mathbb{CE}_{\nu_1}(y_1) \leq \liminf_n S_n(\omega) \leq \limsup_n S_n(\omega) \\ \leq -\mathbb{CE}_{\nu_1}(-y_1) \end{array} \right\}\right) = 1.$
2. $\bigotimes \nu_i \left(\left\{\omega \in \Omega^\infty : \liminf_n S_n(\omega) < \mathbb{CE}_{\nu_1}(-y_1)\right\}\right) = 0$
3. $\bigotimes \nu_i \left(\left\{\omega \in \Omega^\infty : -\mathbb{CE}_{\nu_1}(-y_1) < \limsup_n S_n(\omega)\right\}\right) = 0$
4. $\bigotimes \nu_i \left(\left\{ \begin{array}{c} \omega \in \Omega^\infty : \mathbb{CE}_{\nu_1}(y_1) < \liminf_n S_n(\omega) \leq \limsup_n S_n(\omega) \\ < -\mathbb{CE}_{\nu_1}(-X_1) \end{array} \right\}\right) = 0.$

5. $\bigotimes \nu_i \left(\{\omega \in \Omega^\infty : \mathbb{CE}_{\nu_1}(y_1) \neq \liminf_n S_n(\omega)\}\right) = 0.$
6. $\bigotimes \nu_i \left(\{\omega \in \Omega^\infty : -\mathbb{CE}_{\nu_1}(-y_1) \neq \limsup_n S_n(\omega)\}\right) = 0.$

Appendix C

Proof of Proposition 1:

We first compute explicitly the capacity π, where π is defined as follows:

$$\pi(A) = \min\{(p \times q)(A) \mid p \in \mathcal{C}(\nu),\ q \in \mathcal{C}(\mu)\}$$

This gives (the minimum is taken s.t. the constraints $(p^1, 1-p^1) \in \mathcal{C}(\mu)$ and $(q^0, 1-q^0) \in \mathcal{C}(\nu)$):

$$\begin{aligned}
\pi(1,0) &= \min p^1 q^0 = \mu^1\nu^0 \\
\pi(1,1) &= \mu^1\nu^1 \\
\pi(2,0) &= \mu^2\nu^0 \\
\pi(2,1) &= \mu^2\nu^1 \\
\pi((1,0)\cup(1,1)) &= \min p^1q^0 + p^1q^1 = \min p^1 = \mu^1 \\
\pi((1,0)\cup(2,0)) &= \nu^0 \\
\pi((1,0)\cup(2,1)) &= \min p^1q^0 + (1-p^1)(1-q^0) \\
\pi((1,1)\cup(2,0)) &= \min p^1(1-q^0) + (1-p^1)q^0 \\
\pi((1,1)\cup(2,1)) &= \nu^1 \\
\pi((2,0)\cup(2,1)) &= \mu^2 \\
\pi((1,0)\cup(1,1)\cup(2,0)) &= \min p^1q^0 + p^1(1-q^0) + (1-p^1)q^0 = \mu^1 + (1-\mu^1)\nu^0 \\
\pi((1,0)\cup(1,1)\cup(2,1)) &= \nu^1 + (1-\nu^1)\mu^1 \\
\pi((1,0)\cup(2,0)\cup(2,1)) &= (1-\mu^2)\nu^0 + \mu^2 \\
\pi((1,1)\cup(2,0)\cup(2,1)) &= 1-(1-\mu^2)(1-\nu^1)
\end{aligned}$$

We now replicate the reasoning of the proof of the main theorem . The only complication is that the "orders" for consumption that matter to compute Choquet expected value are more numerous.

To simplify notation, drop subscript h and denote $u(s,t) = u(e^s + b^{1,2} + z(y^s + y(t)))$ for $s = 1, 2$ and $t = 0, 1$.

If $z > 0$, the four following orders are the only ones compatible with the structure of the model:

$$u(1,0) < u(1,1) < u(2,0) < u(2,1) \tag{8}$$
$$u(1,0) < u(2,0) < u(1,1) < u(2,1) \tag{9}$$
$$u(2,0) < u(1,0) < u(2,1) < u(1,1) \tag{10}$$
$$u(2,0) < u(2,1) < u(1,0) < u(1,1) \tag{11}$$

[Observe that, with $z > 0$, it is not possible to have $u(s,0) > u(s,1)$. Furthermore, it is not possible to have at the same time $u(s,0) < u(s',0)$ and $u(s,1) > u(s',1)$.]

Consider these cases in turn:

Case 1 and 2

The Choquet expected value then takes the following form:

$$(1-\mu^2)\Big[(1-\nu^1)u(1,0) + \nu^1 u(1,1)\Big] + \mu^2\Big[(1-\nu^1)u(2,0) + \nu^1 u(2,1)\Big]$$

Using the same reasoning as in the proof of the main theorem, if $z > 0$ is part of an equilibrium, then the FOC yield:

$$q^z = \frac{(1-\mu^2)\mathbb{E}_{(1-\nu^1,\nu^1)}\Big((y^1+y(t))u'(1,t)\Big) + \mu^2\mathbb{E}_{(1-\nu^1,\nu^1)}\Big((y^2+y(t))u'(2,t)\Big)}{(1-\mu^2)\mathbb{E}_{(1-\nu^1,\nu^1)}\Big(u'(1,t)\Big) + \mu^2\mathbb{E}_{(1-\nu^1,\nu^1)}\Big(u'(2,t)\Big)}$$

where $\mathbb{E}_{(1-\nu^1,\nu^1)}\Big((y^s+y(t))u'(s,t)\Big) \equiv (1-\nu^1)\Big((y^s+y(0))u'(s,0)\Big) + \nu^1\Big((y^s+y(1))u'(s,1)\Big)$

But since $y(t)$ and $c(s,t)$ are positively dependent (recall $z_h > 0$) and $u'' < 0$, we can conclude that:

$$\mathbb{E}_{(1-\nu^1,\nu^1)}\Big((y^s+y(t))u'(s,t)\Big) < \mathbb{E}_{(1-\nu^1,\nu^1)}\Big((y^s+y(t)\Big)\mathbb{E}_{(1-\nu^1,\nu^1)}\Big(u'(s,t)\Big) \quad s = 1,2.$$

Hence,

$$q^z < (1-\nu^1)\Big(y^2+y(0)\Big) + \nu^1\Big(y^2+y(1)\Big)$$

Case 3 and 4

Everything goes through. The expression for the Choquet expected value is the same as for Cases 1 and 2 except that one has to replace μ^2 by $(1-\mu^1)$. The same conclusion holds, i.e.,

$$q^z < (1-\nu^1)\Big(y^2+y(0)\Big) + \nu^1\Big(y^2+y(1)\Big)$$

If, at an equilibrium, there are agents buying the risky asset, there must be some agents going short on that asset ($z < 0$). Now, going through the same steps as above (four possible orders are here again possible), it is straightforward to show that a necessary condition for an agent to go short is that:

$$q^z > \nu^0(y^1+y(0)) + (1-\nu^0)(y^1+y(1))$$

Therefore, if, at an equilibrium, there exists h such that $z_h \neq 0$, it must be the case that

$$y^2 + (1-\nu^1)y(0) + \nu^1 y(1) > y^1 + \nu^0 y(0) + (1-\nu^0)y(1)$$

As a consequence, if

$$(1-\nu^0-\nu^1) > \bar{A} \equiv \frac{y^2-y^1}{y(1)-y(0)}$$

such an equilibrium cannot exist. Hence, under that condition, the only possible equilibrium is one with no financial trade. ■

Proof of Proposition 2 (Sketch): An argument similar to that of main theorem establishes that if, at an equilibrium, $z_h^i > 0$ then:

$$q^{z^i} < \max_s \mathbb{E}_{(1-\nu_i^1,\nu_i^1)}(y_i^s + y(t))$$

and, if at an equilibrium $z_h^i < 0$ then:

$$q^{z^i} > \min_s \mathbb{E}_{(\nu_i^0,1-\nu_i^0)}(y_i^s + y(t))$$

Hence, if $1-\nu_i^0-\nu_i^1$ is large enough and

$$\big[y_i^1 + y_i(0), y_i^1 + y_i(1)\big] \cap \big[y_i^2 + y_i(0), y_i^2 + y_i(1)\big] \neq \emptyset$$

then $z_h^i = 0$ for all h at equilibrium. Note: the proof is indeed identical to that of the main theorem, except that in the f.o.c. there is no summation over i appearing explicitly, and the y have now i indices. ■

References

Allen, F., and D. Gale (1994): *Financial innovation and risk sharing.* MIT Press, Cambridge, Mass.

Anderson, C. W. (1999): "Financial Contracting under Extreme Uncertainty: An Analysis of Brazilian Corporate Debentures," *Journal of Financial Economics*, 51, 45–84.

Barnea, A., R. Haugen, and L. Senbet (1980): "A Rationale for Debt Maturity Structure and Call Provisions in the Agency Theoretic Framework," *Journal of Finance*, 35, 1223–1234.

Barsky, R., and J. DeLong (1992): "Why does the stock market fluctuate?," Working Paper 3995, NBER.

Baxter, M., and U. J. Jermann (1997): "The International Diversification Puzzle is Worse Than You Think," *American Economic Review*, 87(1), 170–180.

Bewley, T. (1986): "Knightian decision theory: Part I," Discussion Paper 807, Cowles Foundation.

Bhattacharya, U., P. Reny, and M. Spiegel (1995): "Destructive interference in an imperfectly competitive multisecurity market," *Journal of Economic Theory*, 65, 136–170.

Bisin, A. (1998): "General equilibrium with endogenously incomplete financial markets," *Journal of Economic Theory*, 82(1), 19–45.

Bisin, A., and P. Gottardi (1997): "General competitive analysis with asymmetric information," mimeo.

Camerer, C., and M. Weber (1992): "Recent developments in modeling preferences: uncertainty and ambiguity," *Journal of Risk and Uncertainty*, 5, 325–370.

Carroll, C. (1992): "The buffer stock theory of savings; some macroeconomic evidence," *Brookings Papers on Economic Activity*, 2, 61–135.

Cass, D. (1985): "On the "Number" of equilibrium allocations with incomplete financial markets," Working paper 85-16, CARESS University of Pennsylvania.

Chateauneuf, A., R. Dana, and J. Tallon (1997): "Optimal risk-sharing rules and equilibria with Choquet expected utility," Cahiers EcoMaths 97-54, Universite Paris I.

Cochrane, J. (1991): "Volatility tests and efficient markets," *Journal of Monetary Economics*, 27, 463–485.

Cooper, I. A., and E. Kaplanis (1994): "What Explains the Home Bias in Portfolio Investment.," *Review of Financial Studies*, 7, 45–60.

Deaton, A. (1992): *Understanding consumption.* Clarendon Press, Oxford.

Deaton, A., and C. Paxson (1994): "Intertemporal choice and inequality," *Journal of Political Economy*, 102(3), 437–467.

Dow, J., and S. Werlang (1992): "Uncertainty aversion, risk aversion, and the optimal choice of portfolio," *Econometrica*, 60(1), 197–204.

——— (1994): "Learning under Knightian uncertainty: the law of large numbers for non-additive probabilities," mimeo, London School of Business.

Dubey, P., J. Geanakoplos, and M. Shubik (1994): "Default and efficiency in a general equilibrium model with incomplete markets," mimeo.

Duffie, D., and M. Jackson (1989): "Optimal innovation of future contracts," *Review of financial studies*, 52, 647–663.

Duffie, D., and R. Rahi (1995): "Financial market innovation and security design: an introduction," *Journal of Economic Theory*, 65(1), 1–42.

Dutta, J., and H. Polemarchakis (1990): "Asset markets and Equilibrium processes," *Review of Economic Studies*, 57, 229–254.

Ellsberg, D. (1961): "Risk, ambiguity, and the Savage axioms," *Quarterly Journal of Economics*, 75, 643–669.

Epstein, L., and T. Wang (1994): "Intertemporal asset pricing under Knightian uncertainty," *Econometrica*, 62(3), 283–322.

Fishburn, P. (1993): "The axioms and algebra of ambiguity," *Theory and Decision*, 34, 119–137.

Frankel, J. A. (1995): *The Internationalization of Equity Markets*. University of Chicago Press, Chicago, IL.

French, K. R., and J. M. Poterba (1991): "Investor Diversification and International Equity Markets," *American Economic Review*, Papers and Proceedings, 222–226.

Geanakoplos, J. (1990): "An introduction to general equilibrium with incomplete financial markets," *Journal of Mathematical Economics*, 19, 1–38.

Geanakoplos, J., and A. Mas-Colell (1989): "Real indeterminacy with financial assets," *Journal of Economic Theory*, 47, 22–38.

Geanakoplos, J., and H. Polemarchakis (1986): "Existence, regularity, and constrained suboptimality of competitive allocations when markets are incomplete," in *Uncertainty, information and communication, Essays in honor of Kenneth Arrow*, ed. by W. Heller, R. Ross, and D. Starett, vol. 3. Cambridge University Press, Cambridge.

Ghirardato, P. (1994): "Coping with ignorance: unforeseen contingencies and non-additive uncertainty," Discussion paper, University of California at Berkeley.

——— (1997): "On independence for non-additive measures, with a Fubini theorem," *Journal of Economic Theory*, 73, 261–291.

Gilboa, I., and D. Schmeidler (1989): "Maxmin expected utility with a non-unique prior," *Journal of Mathematical Economics*, 18, 141–153.

Haliassos, M., and C. Bertaut (1995): "Why do so few hold stocks ?," *Economic Journal*, 105, 1110–1129.

Hayashi, F., J. Altonji, and L. Kotlikof (1996): "Risk sharing between and within families," *Econometrica*, 64(2), 261–294.

Jensen, M., and W. Meckling (1976): "The Theory of the Firm: Managerial Behavior, Agency Costs, and Capital Structure," *Journal of Financial Economics*, 3, 305–360.

Kang, J.-K., and R. M. Stulz (1997): "Why is There a Home Bias? An Analysis of Foreign Portfolio Equity Ownership in Japan," *Journal of Financial Economics*, 46, 3–28.

Kelsey, D., and F. Milne (1995): "The Arbitrage Pricing Theorem with Non-Expected Utility Preferences," *Journal of Economic Theory*, 65(2), 557–574.

Kish, R., and M. Livingston (1992): "Determinants of the Call Option on Corporate Bonds," *Journal of Banking and Finance*, 16, 687–703.

Lewis, K. (1995): "Puzzles in international finance," in *Handbook of international econcomics*, ed. by G. Grossman, and K. Rogoff. North Holland.

Lucas, R. (1978): "Asset prices in an exchange economy," *Econometrica*, 46, 1429–1445.

Mace, B. (1991): "Full insurance in the presence of aggregate uncertainty," *Journal of Political Economy*, 99(5), 928–956.

Magill, M., and M. Quinzii (1992): "Real effects of money in general equilibrium," *Journal of Mathematical Economics*, 21, 301–342.

——— (1996): *Theory of incomplete markets*, vol. 1. MIT Press.

Marinacci, M. (1996): "Limit laws for non-additive probabilities, and their frequentist interpretation," mimeo.

Morris, S. (1997): "Risk, Uncertainty and Hidden Information," *Theory and Decision*, 42(3), 235–269.

Mukerji, S. (1997): "Understanding the nonadditive probability decision model," *Economic Theory*, 9(1), 23–46.

——— (1998): "Ambiguity Aversion and Incompleteness of Contractual Form," *American Economic Review*, 88(5), Forthcoming.

Ohashi, K. (1995): "Endogenous determination of the degree of market-incompleteness in futures innovation," *Journal of Economic Theory*, 65(1), 198–217.

Rahi, R. (1995): "Optimal incomplete markets with asymmetric information," *Journal of Economic Theory*, 65(1), 171–197.

Schmeidler, D. (1989): "Subjective probability and expected utility without additivity," *Econometrica*, 57(3), 571–587.

Shapley, L. (1965): "Notes on n-person games VII: Cores of convex games," Discussion Paper RM-4571-PR, The Rand Corp.

Shiller, R. J., F. Kon-Ya, and Y. Tsutsui (1991): "Speculative Behavior in the Stock Markets: Evidence Form the U.S. and Japan.," *National Bureau of Economic Reseach Working Paper*, 3613.

Tallon, J.-M. (1997): "Risque microeconomique, aversion a l'incertitude et indetermination de l'equilibre," *Annales d'Economie et de Statistique*, 48, 211–226.

——— (1998): "Do sunspots matter when agents are Choquet-expected-utility maximizers?," *Journal of Economic Dynamics and Control*, 22, 357–368.

Tesar, L. L., and I. M. Werner (1995): "Home Bias and High Turnover," *Journal of International Money and Finance*, 14(4), 467–492.

Thatcher, J. (1985): "The Choice of Call Provision Terms: Evidence of the Existence of Agency Costs of Debt," *Journal of Finance*, 40, 549–561.

Townsend, R. (1994): "Risk and insurance in village India," *Econometrica*, 62(3), 539–592.

Walley, P., and L. Fine (1982): "Towards a frequentist theory of upper and lower probability," *Annals of Statistics*, 10, 741–761.

Werner, J. (1997): "Diversification and equilibrium in securities markets," *Journal of Economic Theory*, 75, 89–103.

Zame, W. (1993): "Efficiency and the role of default when security markets are incomplete," *American Economic Review*, 83(5), 1142–1164.

Zeldes, S. (1989): "Consumption and liquidity: an empirical investigation," *Journal of Political Economy*, 97, 305–346.