Heisenberg limited quantum metrology under the effect of dephasing

Yuichiro Matsuzaki,[1,2,*] Simon Benjamin,[3] Shojun Nakayama,[4] Shiro Saito,[1] and William J. Munro[1,2,4]

[1]*NTT Basic Research Laboratories, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi, Kanagawa, 243-0198, Japan.*
[2]*NTT Theoretical Quantum Physics Center, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi, Kanagawa 243-0198, Japan.*
[3]*Department of Materials, University of Oxford, OX1 3PH, United Kingdom*
[4]*National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan.*

Quantum sensors have the potential to outperform their classical counterparts. For classical sensing, the uncertainty of the estimation of the target fields decreases $1/\sqrt{T}$ with measurement time T. On the other hand, by using quantum resources, one can decrease the uncertainty with time to $1/T$ - the Heisenberg limit. However as quantum states are susceptible to dephasing, it has been not clear whether one can achieve the Heisenberg limit for a measurement time scaling longer than the coherence time of those states. Here, we propose a scheme that estimates the amplitude of globally applied fields with Heisenberg limited sensitivity in principle for an arbitrary time scale under the effect of dephasing. We use one way quantum computing based teleportation between qubits to prevent the correlation between the quantum state and its local environment from increasing, and have shown that such a teleportation protocol can suppress the local dephasing while the information from the target fields keeps growing. Our methods has the potential to realize a quantum sensor with a sensitivity far beyond that of any classical sensors.

It is well known that two-level systems are attractive candidates to realize ultrasensitive sensors as the frequency of the qubit can be shifted by a coupling to target fields. Such a frequency shift induces a relative phase between the qubits basis states which can be simply measured in a Ramsey type experiment. This method has been used to measure magnetic fields, electric fields, temperature, and even gravity waves [1–5]. With the typical classical sensor measurement device (including SQUID's [6], Hall sensors [7], and force sensors [8]), the uncertainty in the estimation of the target fields decreases only slowly as $1/\sqrt{T}$ with measurement time T. This scaling is known as the shot noise limit [9]. Quantum and especially qubit-based sensors can in principle decrease the uncertainty with time to $1/T$ - the Heisenberg limit [10–12]. This can be achieved using the coherence properties of the qubit and several experimental demonstrations using quantum feedback have shown sub-shot noise sensitivity measuring the amplitude of target fields [10–12]. However as quantum states are fragile to decoherence, it has generally been considered that such sub-shot noise scaling can only be realized if the measurement time T is much smaller than the coherence time of the qubit [10, 13]. Recently several approaches using quantum error correction [14] and dynamical decoupling [15–17] have been proposed to circumvent this limitation. Using quantum error correction, one can measure an amplitude of the target field with the Heisenberg limited sensitivity under the effect of specific decoherence such as bit flip errors [18–24], while dynamical decoupling makes it possible to estimate the frequency of time-oscillating fields with sensitivity beyond the shot-noise limit on time scale longer than the coherence time [25, 26]. However, there is no currently known metrological scheme to achieve Heisenberg limited sensitivity when measuring the amplitude of the target fields with dephasing.

In this letter we propose a scheme to realize a Heisenberg-limited sensing of the amplitude of target fields under the effect of dephasing. We use a similar concept to the quantum Zeno effect (QZE) [27–29]. For shorter time scales than the correlation time of the environment τ_c, the interaction with the environment induces a quadratic decay rate that is much slower than an exponential decay [30]. Hence frequent measurements to reset the correlation with the environment can keep the states in the initial quadratic decay region, which suppresses the decoherence [27–29]. However, if we naively apply the QZE in quantum metrology, the frequent measurements freezes all dynamics so that the quantum states cannot acquire any information from the target fields. Instead, we use quantum teleportation (QT) based on the concepts from one-way quantum computation [31–35] to reset the correlation between the system and environment [36]. If we transfer the quantum states to a new site, we can prevent the correlation between the system and environment in the previous site from increasing, and the quantum state are then only affected by a slow quadratic decay due to the local environment in the new site. This noise suppression has been proposed and demonstrated by superconducting qubits [36]. The crucial idea in this paper is to use this one-qubit teleportation-based noise suppression for quantum metrology. Interestingly, although the QT protocol eliminates the deteriorate effect due to the dephasing from the local environment, we can accumulate the phase information from the global target fields during this protocol. We have shown that, as long as nearly perfect QT is available, we can achieve the Heisenberg limit sensing with dephasing. Moreover, we have found that, even when QT is moderately noisy, the sensitivity of our protocol can be much better than that of the standard Ramsey measurement.

Our system and environment can be described by a Hamiltonian of the form $H = H_{\rm S} + H_{\rm I} + H_{\rm E}$ [37] where $H_{\rm S} = \sum_{j=1}^{L} \frac{\omega}{2}\sigma_z^{(j)} \otimes \mathbb{1}_{\rm E}^{(j)}$ ($H_{\rm E} = \sum_{j=1}^{L} \mathbb{1}_{\rm S}^{(j)} \otimes C_j$) denotes the system (environmental) Hamiltonian while $H_{\rm I} = \sum_{j=1}^{L} \lambda\sigma_z^{(j)} \otimes B_j$ denotes an interaction between the system and environment. Here $\sigma_z^{(j)}$ is the usual pauli Z operator of the j-th qubit with frequency ω, while B_j and C_j denote the environmental operator at that j-th site. $\hat{\mathbb{1}}_{\mathrm{S}}^{(j)}$ ($\hat{\mathbb{1}}_{\mathrm{E}}^{(j)}$) denotes an identity operator for the system (environment). Transforming to an interaction picture, we have $H_{\mathrm{I}}(t) = \lambda \sum_{j=1}^{L} \sigma_z^{(j)} \otimes \tilde{B}_j(t)$ where $\tilde{B}_j(t) = e^{iH_{\mathrm{E}}t} B_j e^{-iH_{\mathrm{E}}t}$. The initial state is given as $\rho(0) = \bigotimes_{j=1}^{L} (\rho_{\mathrm{S}}^{(j)}(0) \otimes \rho_{\mathrm{E}}^{(j)})$ where we further assume $\rho_{\mathrm{E}}^{(j)}$ is in thermal equilibrium such that $[\rho_{\mathrm{E}}^{(j)}, H_{\mathrm{E}}] = 0$ and our noise is non-biased such that $\mathrm{Tr}[\rho_{\mathrm{E}}^{(j)} B_j] = 0$ for all j. Since the initial state is separable, we consider only the first site by tracing out the other sites. Solving Schrodinger equation, we obtain

$$\rho_{\mathrm{I}}^{(1)}(\tau) \simeq \rho^{(1)}(0) - i\lambda \int_0^{\tau} dt' \lambda [\sigma_z^{(1)} \otimes \tilde{B}_1(t'), \rho^{(1)}(0)]$$
$$-\lambda^2 \int_0^{\tau} \int_0^{t'} dt' dt'' [\sigma_z^{(1)} \otimes \tilde{B}_1(t'), [\sigma_z^{(1)} \otimes \tilde{B}_1(t''), \rho^{(1)}(0)]]$$

using a second order perturbation expansion in λ where τ denotes a time [37]. Tracing out the environment, we obtain

$$\rho_{\mathrm{I}}^{(1)}(\tau) \simeq \rho_{\mathrm{S}}^{(1)}(0) - \lambda^2 \int_0^{\tau} \int_0^{t'} dt' dt'' C_{t'-t''}^{(1)} \left[\hat{\sigma}_z^{(1)}, \left[\hat{\sigma}_z^{(1)}, \rho_{\mathrm{S}}^{(1)}(0)\right]\right]$$

where we define the correlation function of the environment as $C_{t'-t''}^{(1)} \equiv \mathrm{Tr}[(\tilde{B}_1(t')\tilde{B}_1(t'') + \tilde{B}_1(t'')\tilde{B}_1(t'))\rho_{\mathrm{E}}^{(1)}]$. If we are interested in a time scale much shorter than the correlation time of the environment, we can approximate the correlation function as $C_{t'-t''}^{(1)} \simeq C_0^{(1)}$. For most of the solid state systems, the correlation time is much longer than the coherence time of the qubit [38–41], and so this condition is readily satisfied for many systems. In this case $\rho_{\mathrm{S}}^{(1)}(\tau) \simeq (1-\epsilon_\tau) U_{1,\tau} \rho_{\mathrm{S}}^{(1)}(0) U_{1,\tau}^{\dagger} + \epsilon_\tau \hat{\sigma}_z^{(1)} U_{1,\tau} \rho_{\mathrm{S}}^{(1)}(0) U_{1,\tau}^{\dagger} \sigma_z^{(1)}$ where $\epsilon_\tau = \lambda^2 C_0 \tau^2/2$ denotes an error rate for $\lambda^2 C_0(\tau)^2/2 \ll 1$ and $U_{j,\tau} = e^{-i\omega\tau\hat{\sigma}_z/2}$ denotes a unitary operator at a site j. Since the error rate has a quadratic form against the time t, the decoherence effect is negligible for short time scales $t \ll 1/\lambda\sqrt{C_0}$, which has been discussed in the field of the QZE [27–29]. On the other hand, if we consider longer time scales $t \gg 1/\lambda\sqrt{C_0}$ with the same environment, error accumulation will destroy the quantum coherence of the qubit.

Let us now describe the noise suppression technique using QT. It begins with allowing free evolution of the qubit for a time $\tau = t/n$ (where t is the total time and n is the number of time QT to be performed). After this, QT sends $\rho_{\mathrm{S}}^{(1)}$ to a different site 2. The quantum states then starts interacting with a new local environment described by a density matrix $\rho_{\mathrm{E}}^{(2)}$. The error rate will be suppressed due to the quadratic decay [36]. Performing QT n times (each time to a fresh qubit) yields $\rho_{\mathrm{S}}^{(n)}(t) \simeq (1 - \frac{\lambda^2 C_0 t^2}{2n}) U_{n,t} \rho_{\mathrm{S}}(0) U_{n,t}^{\dagger} + \frac{\lambda^2 C_0 t^2}{2n} \hat{\sigma}_z U_{n,t}^{\dagger} \rho_{\mathrm{S}}(0) U_{n,t}^{\dagger} \sigma_z$ at site n. For large n this approaches the pure state $\rho_{\mathrm{S}}(t) \simeq U_{n,t} \rho_{\mathrm{S}}(0) U_{n,t}^{\dagger}$ meaning the QT approach can suppress the local dephasing.

Our general approach described above has used a perturbative analysis typically valid only for a short time scale. However we need to examine the dynamics of our system for arbitrary time scales, and so we will consider a more specific noise model given by $\rho_{\mathrm{S}}^{(1)}(\tau) = \frac{1+e^{-\gamma^2\tau^2}}{2} U_{1,\tau} \rho_{\mathrm{S}}^{(1)}(0) U_{1,\tau}^{\dagger} + \frac{1-e^{-\gamma^2\tau^2}}{2} \hat{\sigma}_z^{(1)} U_{1,\tau} \rho_{\mathrm{S}}^{(j)}(0) U_{1,\tau}^{\dagger} \sigma_z^{(1)}$ during the evolution for a time τ (with γ giving the dephasing rate). This model is consistent with the general short time scale noise model described above by choosing $\gamma = \lambda^2 C_0$. Typical dephasing models [38–40, 42] show this behavior if the correlation time of the environment is much longer than the dephasing time.

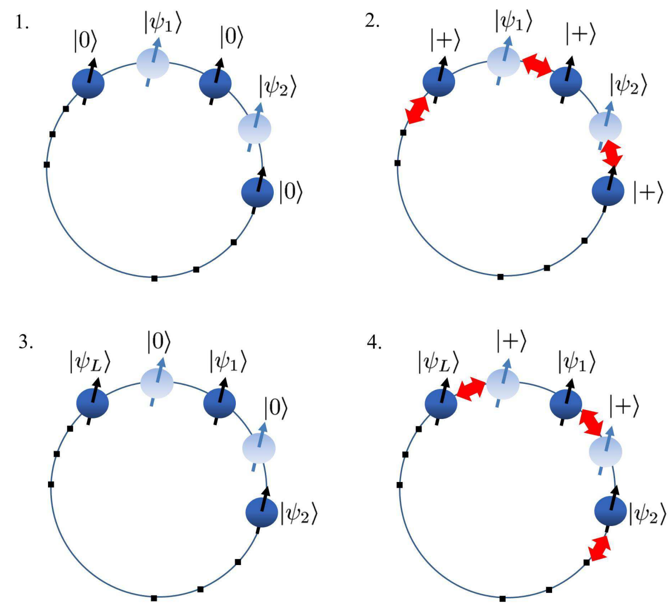

FIG. 1: Schematic illustration of $2L$ qubits in a ring structure to measure globally applied fields at the Heisenberg limit. Half of the qubits contain an information of the target fields as a probe while the remaining half are used as an ancilla's for one qubit teleportation. By performing a controlled phase gate between the probe qubit and ancillary qubit, single qubit measurements on the probe qubit with feedforward operations depending on the measurement results, one can teleport a quantum state $|\psi\rangle$ from the original site to the right neighboring site [31, 43]. After the teleportation, the measured qubit becomes the new ancilla which we initialize into $|0\rangle$. By implementing the teleportation frequently, dephasing from the local environment can be suppressed while quantum states keep accumulating a phase information from the target global fields.

Now let us turn our focus to using the QT scheme to enable quantum metrology with Heisenberg-limited sensitivity. Consider the situation in which the qubit frequency ω is shifted depending on the amplitude of the target fields, and so a measurement of the qubit's frequency shift allows us to infer the amplitude of the target field. Such a qubit frequency shift is estimated from a relative phase between quantum states. The key idea is to use the QT in a ring arrangement where a qubit has a tunable interaction with it's nearest neighbor qubits, as illustrated in Fig. 1. Half of the qubits are used to probe the target fields while the remaining qubits are used as ancilla for QT (probe qubits are located between two ancillary qubits). The QT is enacted by performing a control-phase gate between a probe qubit and an ancilla qubit, followed by a $\hat{\sigma}_x$ measurement on the probe qubit (and single qubit corrections depending on the measurement result). This QT approach has been widely used in one-way quantum computation [31, 43].

Our scheme to measure the amplitude of target fields is as follows: First, we prepare a probe state of $\bigotimes_{j=1}^{L}|+\rangle_{2j-1}$ located at the site $2j-1$ $(j=1,2,\cdots,L)$. Second we then let the state evolve for a time $\tau = t/n$ and then teleport (using QT) the state of the probe qubit to the next site using the ancillary qubit (we assume that our gate operations are much faster than τ). Third we then in step 3 repeat the second step $(n-1)$ times while in the fourth step we let this state evolves for a time $\tau = \frac{t}{n}$, and readout the state by measuring $\hat{M}_y = \sum_{j=1}^{L}\hat{\sigma}_y^{(j)}$. Finally, we repeat these steps N times during the measurement time T where $N \simeq T/t$ is the repetition number. When n is even, the density matrix before the readout is described as $\rho_{\rm S}(t) \simeq \bigotimes_{j=1}^{L}\rho_{\rm S}^{(2j-1)}(t)$ where $\rho_{\rm S}^{(2j-1)}(t) = \frac{1}{2}(\mathbb{1}_{2j-1} + |1\rangle_{2j-1}\langle 0|e^{-i\omega t-\gamma^2t^2/n} + |0\rangle_{2j-1}\langle 1|e^{i\omega t-\gamma^2t^2/n})$. Here, $\gamma/\sqrt{n}$ can be interpreted as an QT improved coherence time. If n is odd, we obtain the same density matrix for the probe qubits at a site $2j$ $(j=1,2,\cdots,L)$. In the situation that the target field to be sensed is weak ($L\omega T \ll 1$) we can estimate the uncertainty in our estimator as

$$\delta\omega_{n,t} = \frac{\sqrt{\langle\delta\hat{M}_y\delta\hat{M}_y\rangle}}{|\frac{d\langle\hat{M}_y\rangle}{d\omega}|}\frac{1}{\sqrt{N}} \simeq \frac{e^{\gamma^2t^2/n}}{\sqrt{LTt}} \tag{1}$$

where $\delta\hat{M}_y = \hat{M}_y - \langle\hat{M}_y\rangle$. Setting $t=T$ we obtain $\delta\omega_{n,T} \simeq 1/T\sqrt{L}$ for large n. The $1/T$ scaling shows we can achieve the Heisenberg limit.

Let us now analyze how imperfect QT affects the performance of our sensing scheme. In our previous ideal QT analysis, the uncertainty of the estimation monotonically decreases as the number of the QT increases. However in realistic situations, errors caused during the QT operations will limit our achievable sensitivity and there will be an optimal number of QT's that we can perform. We will consider a simple error model, a depolarization channel that makes the state completely mixed with a probability p after the QT operation. In such a case, after the imperfect QT is performed on say the j th site, the state ρ_j evolves to $(1-p)\rho_{j+1} + p\mathbb{1}_{j+1}/2$. The uncertainty of the QT based metrology estimation for our weak target field is then $\delta\omega_{n,t} \simeq e^{\gamma^2t^2/n}/\sqrt{TtL}(1-p)^{n-1}$. For $T \leq 1/4\sqrt{p}\gamma$, we obtain $\delta\omega_{T,1/4p} \simeq e^{1/4+\gamma^2T^2/n}/T\sqrt{L} \leq e^{1/2}/T\sqrt{L}$ when $t=T$ and $n=1/4p$. This allows us to achieve Heisenberg limited sensitivity.

In quantum metrology one typically considers the scaling law in the limit of long T, which we will now discuss. We can minimize this uncertainty by setting $t_{\rm opt} = \sqrt{n}/2\gamma$ as long as $T \gg t_{\rm opt}$ is satisfied. In such a case we obtain $\delta\omega_{n,t_{\rm opt}} = \frac{\sqrt{2}e^{1/4}}{(1-p)^{n-1}}\sqrt{\frac{\gamma}{\sqrt{n}TL}}$ which for $n=1$ gives the standard Ramsey uncertainty $\delta\omega_{\rm R} = \frac{e^{1/4}\sqrt{\gamma}}{\sqrt{TL}}$ [9] where we replace L with $2L$ (because the standard Ramsey scheme can utilize every qubit to probe the target fields without ancillary qubits). For $n \gg 1$. we can treat n as a continuous variable, and we can analytically minimize the uncertainty as $\delta\omega_{\rm opt} \simeq 2\sqrt{\frac{e\sqrt{p}\gamma}{LT}}$ for $1/16\gamma^2T^2 \ll p \ll 1$ where we choose $n_{\rm opt} = -1/4\log(1-p) \simeq 1/4p$. In this case, we have a constant factor improvement over the standard Ramsey scheme for a longer T. Actually, as long as $p < 0.0251$, our scheme is better than that standard Ramsey scheme ($\delta\omega_{\rm R}/\delta\omega_{\rm opt} > 1$). Also, if we have $p = 10^{-4}$, we obtain $\delta\omega_{\rm R}/\delta\omega_{\rm opt} \simeq 3.89$. So our sensor has an advantage with finite errors by the QT.

There are of course other sources of decoherence that can not be suppressed by the QT protocol. For instance, If our quantum systems are affected by high-frequency noise with a short correlation time, the decay is not quadratic in nature but more exponential like. Energy relaxation in a high temperature environment is known to induce such noise [44]. Consider the situation in which an initial state $|+\rangle_j$ evolves under the effect of both low-frequency dephasing and high-frequency noise for a time τ. In such a case $\rho_{\rm S}^{(j)}(\tau) = \frac{1}{2}(\mathbb{1}_j + |1\rangle_j\langle 0|e^{-i\omega\tau-\gamma^2\tau^2-\Gamma\tau} + |0\rangle_j\langle 1|e^{i\omega\tau-\gamma^2\tau^2-\Gamma\tau})$ where Γ denotes the decay rate associated with the high-frequency noise ($\Gamma = 0$ gives the same noise model we used previously). It is then straightforward to calculate the uncertainty of the estimation under the effect of this noise with imperfect QT as $\delta\omega_{n,t} = \frac{e^{\Gamma t+\gamma^2t^2/n}}{(1-p)^{n-1}}\frac{1}{\sqrt{TtL}}$. Choosing $t_{\rm opt} = \frac{-\sqrt{n}m+\sqrt{nm^2+4}}{4(\gamma/\sqrt{n})}$ (where $m = \Gamma/\gamma$), we minimize this with respect to time as

$$\delta\omega_{n,t_{\rm opt}} = \frac{2\sqrt{\gamma}e^{\frac{1}{4}+\frac{-nm^2+m\sqrt{n(nm^2+4)}}{8}}}{(1-p)^{n-1}\sqrt{(-nm+\sqrt{n(nm^2+4)})LT}}.$$

A numerical minimization of the uncertainty with n as $\delta\omega_{\rm opt} = \min_n \delta\omega_{n,t_{\rm opt}}$ can be done. In Fig. 2, we plot $\delta\omega_{\rm R}/\delta\omega_{\rm opt}$ versus p and Γ/γ where $\delta\omega_{\rm R}$ is the uncertainty for the standard Ramsey scheme. Our plots shows that our scheme has a better performance than the standard Ramsey scheme for $\Gamma/\gamma < 0.130$ and $p < 0.0123$.

A natural question we have not addressed so far is whether entanglement improves our teleportation-based sensing. We could for instance create a GHZ state $|\psi\rangle_M = \frac{1}{\sqrt{2}}\left[\bigotimes_{j=1}^{M}|0\rangle_j + \bigotimes_{j=1}^{M}|1\rangle_j\right]$ composed of M qubits. For a given L qubits, we create GHZ states with this size, and the number of the GHZ states is L/M. By letting the GHZ states evolves with low-frequency dephasing for a time τ where we

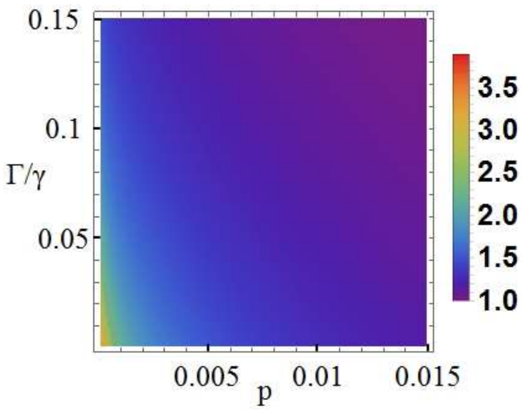

FIG. 2: Performance of our teleportation based scheme against the standard Ramsey scheme. We plot $\delta\omega_{\rm R}/\delta\omega_{\rm opt}$ against $\frac{\Gamma}{\gamma}$ and p. Our scheme has a better performance than the standard Ramsey scheme for $\Gamma/\gamma < 0.130$ and $p < 0.0123$.

ignore the high-frequency decoherence for simplicity, we have

$$\rho_k(\tau) = \frac{1}{2}\bigotimes_{j=1+kM}^{M+kM}|0\rangle_j\langle 0| + \frac{1}{2}\bigotimes_{j=1+kM}^{M+kM}|1\rangle_j\langle 0|e^{-iM\omega\tau - M\gamma^2 t^2} + \frac{1}{2}\bigotimes_{j=1+kM}^{M+kM}|0\rangle_j\langle 1|e^{iM\omega\tau - M\gamma^2\tau^2} + \frac{1}{2}\bigotimes_{j=1+kM}^{M+kM}|1\rangle_j\langle 1| \quad (2)$$

for $k = 0, 1, \cdots, (\frac{L}{M} - 1)$ where γ denotes the dephasing rate for a single qubit. To readout the GHZ states, we measure a projection operator defined by $\hat{\mathcal{P}}^{(k)}_{\pm} = |\psi^{(\pm)}_{\perp}\rangle_k\langle\psi^{(\pm)}_{\perp}|$ where $|\psi^{(\pm)}_{\perp}\rangle_k = \frac{1}{\sqrt{2}}\bigotimes_{j=1+kM}^{M+kM}|0\rangle_j \pm i\frac{1}{\sqrt{2}}\bigotimes_{j=1+kM}^{M+kM}|1\rangle_j$. We consider an imperfect QT as follows. If we teleportate a state of ρ_1 from $j = 1, 2, \cdots, M$ sites to $j' = 1 + M, 2 + M, \cdots, 2M$ sites, we obtain a state of $\rho'_2 = (1-p)^M\rho_2 + (1-(1-p)^M)\rho_2^{(\rm error)}$ where p denotes the error rate on a single qubit, ρ_2 denotes the ideal state (that we could obtain by a perfect QT), and $\rho_2^{(\rm error)} = \frac{1}{2}(\bigotimes_{j=1+M}^{2M}|0\rangle_j\langle 0| + \bigotimes_{j=1+M}^{2M}|1\rangle_j\langle 1|)$ denotes a decohered state. Here, we assume that any error on a single qubit makes the probe state into $\rho_2^{(\rm error)}$ (The $M = 1$ corresponds to our previous case where we used separable states). It is straightforward in this GHZ entangled situation to estimate our sensitivity as

$$\delta\omega^{(\rm GHZ)}_{n,t,M} = \frac{\sqrt{\langle\delta\hat{\mathcal{P}}_{\pm}\delta\hat{\mathcal{P}}_{\pm}\rangle}}{|d\langle\hat{\mathcal{P}}_{\pm}\rangle/d\omega|}\frac{1}{\sqrt{N}} = \frac{e^{M\gamma^2 t^2/n}}{(1-p)^{M(n-1)}\sqrt{MLTt}}$$

where $\delta\hat{\mathcal{P}} = \hat{\mathcal{P}} - \langle\hat{\mathcal{P}}\rangle$ and $N \simeq TL/tM$. By setting $n = 1$, we reproduce the results discussed in [45, 46] as an entanglement based sensor with low-frequency dephasing. We can minimize this uncertainty with $t_{\rm opt} = \frac{\sqrt{n/M}}{2\gamma}$ to obtain $\delta\omega^{(\rm GHZ)}_{n,t_{\rm opt}} = \frac{\sqrt{2}e^{1/4}\sqrt{\gamma/\sqrt{Mn}}}{(1-p)^{M(n-1)}\sqrt{LT}}$. For the ideal perfect QT ($p = 0$), we achieve the Heisenberg limit $\delta\omega^{(\rm GHZ)}_{n,t_{\rm opt},M} = \Theta(L^{-1}T^{-1})$ by choosing $M = \Theta(L)$ and $n = \Theta(MT^2)$. However, for $p > 0$ and $n > 1$, with $M_{\rm opt} = -1/4\log(1-p) \simeq 1/4p$ and $n_{\rm opt} = 2$, we can minimize the uncertainty as $\delta\omega^{(\rm GHZ)}_{\rm opt} = 2^{\frac{3}{4}}\sqrt{\frac{e\sqrt{p}\gamma}{LT}}$. This is a constant factor improvement over our scheme using separable states. This is consistent with the fact arbitrary small noise can sometimes make the entanglement sensor almost equivalent with separable sensors [9].

In conclusion, we have proposed in this letter a scheme to achieve Heisenberg limited quantum sensing of the amplitudes of globally applied fields. We have found that frequent implementations of quantum teleportation provide a suitable circumstance for sensing where the dephasing is suppressed while the information from the target fields is continuously accumulated. If perfect quantum teleportation is available, we can achieve the Heisenberg limit. Moreover, even when quantum teleportation is moderately noisy, our protocol still shows quantum enhancement over the standard Ramsey scheme.

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* Electronic address: matsuzaki.yuichiro@lab.ntt.co.jp

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