Supplemental Document

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# Tomographic refractive index profiling of direct laser written waveguides: supplement

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# Tomographic refractive index profiling of direct laser written waveguides: supplemental document

We provide a detailed schematic of our imaging setup and describe the tomography algorithm we developed in order to reconstruct the 2D refractive index (RI) profiles associated to shift-invariant laser written waveguides. We first describe the forward model from which we obtain the intensity images to compare to the experimental data. Then, we derive the gradient expression associated with our error metric by following a backpropagation scheme. Finally, we present the proximal gradient descent we use for the reconstruction.

## 1. TOMOGRAPHY SETUP

Figure S1 shows a detailed sketch of our tomography setup. Light from a blue LED is focused onto a pinhole of 150 µm diameter and collimated by a lens of 200 mm focal length. The light is reflected off a motorized mirror at a user definable angle. The beam is then focused into the condenser back aperture (CBA), where it forms a slightly demagnified image of the pinhole. The lateral position of the pinhole image in the CBA defines the angle of illumination in the focal plane (FP).

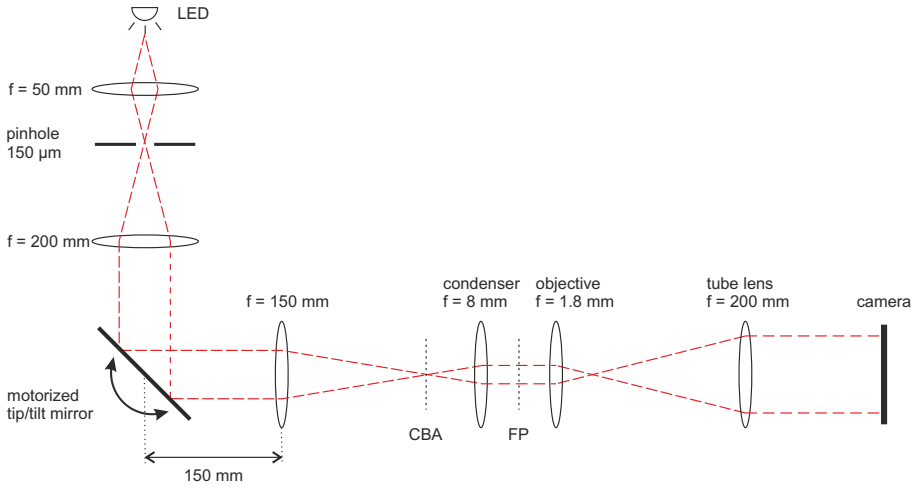


**Fig. S1. Layout of our optical tomography setup.** CBA = condenser back aperture; FP = focal plane.

## 2. FORWARD MODEL

The recorded experimental data consists of a set of $N$ intensity images denoted $I_{\text{exp}}^{(k)}$, corresponding to intensity diffraction patterns of tilted plane waves with angles $\{\theta_k\}_{1\leq k\leq N}$ sent across the sample. In the particular case of the shift-invariant waveguides we are dealing with, the recorded intensities $I_{\text{exp}}^{(k)}$ are assumed to be 1–dimensional. As already stated in the main text, these 1–dimensional views are obtained from the 2D camera images by averaging over a hundred pixels along the invariant direction of the considered waveguide.

In order to reconstruct the unknown RI profile, we first need to define a forward model which takes a tilted plane wave with angle $\theta_k$ as an input, and outputs a 1D intensity image $I_{\text{sim}}^{(k)}$ to be compared to $I_{\text{exp}}^{(k)}$. This forward model takes the RI profile and the aberration correction as parameters, on which a joint optimization will be performed later.

### A. Propagation model

We consider the propagation of tilted plane waves across a medium $\mathrm{RI}(x,z) = n_0 + \Delta\mathrm{RI}(x,z)$, where $n_0$ is the refractive index of the bulk glass at the wavelength $\lambda$, and $\Delta\mathrm{RI}(x,z)$ is the RI change induced by the laser writing process. The direction $x$ corresponds to the camera imaging plane whereas the orthogonal direction $z$ defines the optical axis of the imaging system. The propagation angle $\theta = 0$ corresponds to straight illumination.

We model the propagation through such medium thanks to the well known split-step Fourier Beam Propagation Method (BPM) [1], to which we make two important changes. First, in order to avoid Fourier issues due to wrong wrapping over the field boundaries, we integrate the information about the wave angle $\theta_k$ into the propagator instead of the wave field $E_k$. Second, in order to be more accurate when propagating to large angles, we re-scale the phase contributions of the thin medium slices according to the cosine of the angle of the incoming wave, as justified in [2].

More formally, we define the propagation of an incoming wave $E_k$ with angle $\theta_k$ in an homogeneous medium $n_0$ of thickness $z$ by the operator:

$$\mathcal{P}_z(E_k) = \mathcal{F}^{-1}\left(\mathcal{F}\left(E_k\right)\exp\left(i2\pi z\sqrt{\frac{n_0^2}{\lambda^2} - \left(f_x - \frac{n_0\sin\theta_k}{\lambda}\right)^2}\right)\right). \tag{S1}$$

Moreover, at a given transverse position $z = cst$, the phase contribution of a thin slice of differential refractive index $\Delta\mathrm{RI}$ with thickness $\Delta z$ is modeled by the operator:

$$\mathcal{N}_{\Delta\mathrm{RI}}(E_k) = E_k\exp\left(i\frac{2\pi}{\lambda}\Delta z\frac{\Delta\mathrm{RI}}{\cos\theta_k}\right). \tag{S2}$$

This allows to define the propagation through a thin slice $\Delta\mathrm{RI}$ with thickness $\Delta z$ as the composition of the previous operators:

$$\mathcal{P}_{\Delta\mathrm{RI}} = \mathcal{P}_{\frac{\Delta z}{2}}\circ\mathcal{N}_{\Delta\mathrm{RI}}\circ\mathcal{P}_{\frac{\Delta z}{2}}. \tag{S3}$$

With this approach, we have to set $E_k(x)_{|z=0} = 1$, whatever the incoming propagation angle. Then, a sample of total thickness $L_z$ needs to be split into $N_l$ layers satisfying $L_z = N_l\Delta z$, so that the final field distribution $E_k(x)_{|z=L_z}$ can be obtained by successive applications of the propagator $\mathcal{P}_{\Delta\mathrm{RI}}$ (S3). Without taking aberration correction into account, the result of the forward model is:

$$I_{\mathrm{sim}}^{(k)} = \left|\mathcal{P}_{\Delta\mathrm{RI}}^{(N_l)}\left(E_{k|z=0}\right)\right|^2 = \left|E_{k|z=L_z}\right|^2. \tag{S4}$$

### B. Aberration correction

A simple way of taking the system aberrations into account is to consider that they can be modelled by a single phase mask in the pupil plane of the microscope objective. With the concern of not increasing the ill-posedness of the problem, we restrain the aberration mask to a linear combination of a few even polynomials ($N_a \leq 8$) in the Fourier domain:

$$\Phi_k(f_x) = \begin{cases} \sum_{p=1}^{N_a}\alpha_p\left(f_x - \frac{n_0\sin\theta_k}{\lambda}\right)^{2p} & \text{if } |f_x| \leq \frac{\mathrm{NA}}{\lambda} \\ 0 & \text{otherwise,} \end{cases} \tag{S5}$$

where $f_x$ refers to the 1D Fourier domain variable, NA is the numerical aperture of the objective lens and $\{\alpha_p\}_{1\leq p\leq N_a}$ are unknown coefficients to be optimized along with $\Delta\mathrm{RI}$. We note that in order to be consistent with the propagation method defined in S1 we also have to translate the aberration mask according to the incoming wave angle.

The previous forward model can simply be extended by applying the following operator to the final field distribution $E_{k|z=L_z}$ before computing its intensity:

$$\mathcal{A}_\Phi(E_k) = \mathcal{F}^{-1}\left(\mathcal{F}(E_k)e^{i\Phi_k}\right). \tag{S6}$$

The result of the forward model becomes:

$$I_{\mathrm{sim}}^{(k)} = \left|\mathcal{A}_\Phi\left(E_{k|z=L_z}\right)\right|^2. \tag{S7}$$

## 3. ERROR DISTANCE AND GRADIENT BACKPROPAGATION

We pose the RI reconstruction problem as the joint minimization of the $l_2$ distances between the experimental and simulated intensities:

$$\begin{aligned} e &= \frac{1}{N}\sum_{k=1}^{N} \| I_{\text{sim}}^{(k)} - I_{\text{exp}}^{(k)} \|_2^2 \\ &= \frac{1}{N}\sum_{k=1}^{N} \int \left( I_{\text{sim}}^{(k)}(x) - I_{\text{exp}}^{(k)}(x) \right)^2 \mathrm{d}x. \end{aligned} \tag{S8}$$

We can derive the expressions of the gradients of the error $e$ with respect to $\Delta\text{RI}(x,z)$ and $\{\alpha_p\}_{1\leq p\leq N_a}$ by following the algorithmic differentiation rules presented in [3]. In the following, we also employ the overbar notation to denote these gradients.

To do so, we first introduce the error-field $E_k^{\text{err}}$ associated to the field $E_k$ by:

$$\begin{aligned} E_k^{\text{err}} &= \overline{\mathcal{A}_\Phi\left(E_{k|z=L_z}\right)} \\ &= \frac{4}{N}\left(I_{\text{sim}}^{(k)} - I_{\text{exp}}^{(k)}\right)\mathcal{A}_\Phi\left(E_{k|z=L_z}\right). \end{aligned} \tag{S9}$$

From $E_k^{\text{err}}$ defined in S9, we can immediately derive the gradients $\{\overline{\alpha_p}\}_{1\leq p\leq N_a}$:

$$\overline{\alpha_p} = \int \sum_{k=1}^{N} \left(f_x - \frac{n_0 \sin\theta_k}{\lambda}\right)^{2p} \Im\left\{\left(\mathcal{F}\left(E_{k|z=L_z}\right)^\star e^{-i\Phi_k}\mathcal{F}\left(E_k^{\text{err}}\right)\right)(f_x)\right\} \mathrm{d}f_x. \tag{S10}$$

Finally, we can successively compute the backpropagated gradients $\overline{\Delta\text{RI}}(x,l_z)$ with $l_z = \frac{\Delta z}{2} + (N_l - 1 - l)\Delta z$ for $l \in \{0,\dots,N_l-1\}$, by applying the hermitian adjoints of the operators defined in S3:

$$\overline{\Delta\text{RI}}(x,l_z) = \sum_{k=1}^{N} \frac{2\pi}{\lambda}\frac{\Delta z}{\cos\theta_k} \Im\left\{E_{k|z=l_z}^\star \left(\mathcal{P}^{\dagger(l)}_{\Delta\text{RI}} \circ \mathcal{P}^\dagger_{\frac{\Delta z}{2}} \circ \mathcal{A}^\dagger_\Phi\right)\left(E_k^{\text{err}}\right)\right\}. \tag{S11}$$

## 4. ITERATIVE ALGORITHM

The reconstruction algorithm can be formulated as a minimization of the error distance defined in S8, according to the spatial $\Delta$RI distribution and the aberration coefficients $\{\alpha_p\}$. Such minimization is typically achieved through a gradient descent. In order to deal with the missing cone problem, due to a limited range of illumination angles, and to recover from noisy experimental data, we add a regularization term to the error distance. The regularized minimization problem reads as:

$$\min_{\Delta\text{RI},\{\alpha_p\}} \frac{1}{N}\sum_{k=1}^{N} \| I_{\text{sim}}^{(k)} - I_{\text{exp}}^{(k)} \|_2^2 + \kappa \left\|\Delta\text{RI}\right\|_{\text{TV}_l}, \tag{S12}$$

where $\kappa$ is a positive constant and $\|\|_{\text{TV}_l}$ designs the isotropic TV–norm [4], which is typical in tomographic reconstruction problems.

We perform the minimization S12 with a proximal gradient descent [4, 5] meant to take into account this regularization term in combination with the gradients already computed in S10 and S11. Eventually, we implement a convergence acceleration thanks to the classical Nesterov's momentum algorithm [6, 7].

## REFERENCES


1. M. Feit and J. Fleck, "Light propagation in graded-index optical fibers," Appl. Opt. **17**, 3990–3998 (1978).
2. Y. Bao and T. K. Gaylord, "Clarification and unification of the obliquity factor in diffraction and scattering theories: discussion," JOSA A **34**, 1738–1745 (2017).
3. A. S. Jurling and J. R. Fienup, "Applications of algorithmic differentiation to phase retrieval algorithms," JOSA A **31**, 1348–1359 (2014).
4. A. Beck and M. Teboulle, "Fast gradient-based algorithms for constrained total variation image denoising and deblurring problems," IEEE Transactions on Image Process. **18**, 2419–2434 (2009).

5. N. Parikh and S. Boyd, "Proximal algorithms," Foundations Trends Optim. **1**, 127–239 (2014).
6. Y. E. Nesterov, "A method for solving the convex programming problem with convergence rate $o(1/k^2)$," in *Dokl. akad. nauk Sssr,* vol. 269 (1983), pp. 543–547.
7. A. Beck and M. Teboulle, "A fast iterative shrinkage-thresholding algorithm for linear inverse problems," SIAM J. on Imaging Sci. **2**, 183–202 (2009).