Scale-separated AdS vacua and holography

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Statement of originality

This thesis is based on the following publications by the author, the majority of which were written in collaboration,

1. *Comments on Classical AdS Flux Vacua with Scale Separation*, F. Apers, M. Montero, T. Van Riet, T. Wrase, *JHEP* 05 (2022) [1]
2. *Integer Conformal Dimensions for Type IIA Flux Vacua*, F. Apers, J. Conlon, S. Ning, F. Revello, *Phys. Rev. D* 105 (2022) [2]
3. *Aspects of AdS Flux Vacua with Integer Conformal Dimensions*, F. Apers, *JHEP* 05 (2023) [3]
4. *Backtracking AdS Flux Vacua*, F. Apers, M. Montero, I. Valenzuela (2024) [4]

The results of [1] are presented in Sections 2.2.3, 3.1.3, and 3.2. Section 3.1.4 is based on [2], while Section 3.3.2 and Chapter 4 are from [3]. Chapter 6 is based on [4].

All chapters represent original contributions by the author, with the following exceptions, which are included for completeness:

- Section 3.1.3 was carried out by Timm Wrase;
- the paragraph *Spectrum* in Section 3.1.4 was written by Filippo Revello;
- the introduction (Section 5.1, Section 5.2), and conclusion (Section 5.5) of Chapter 6 were written by Miguel Montero and Irene Valenzuela.

The following additional publications by the author are not included in this thesis:

1. *Kination, Meet Kasner: On the Asymptotic Cosmology of String Compactifications*, F. Apers, J. Conlon, M. Mosny, F. Revello, *JHEP* 08 (2023) [5]
2. *String Theory and the First Half of the Universe*, F. Apers, E. Copeland, J. Conlon, M. Mosny, F. Revello, *JCAP* 08 (2024) [6]
3. *A Note on 4D Kination and Higher-Dimensional Uplifts*, F. Apers, J. Conlon, M. Mosny, *Eur. Phys. J. C* 85 (2025) 3, 337 [7]

Abstract

This thesis explores potential holographic duals of AdS vacua in string theory with a parametric separation between the AdS curvature scale and the scale of the internal dimensions. Our primary focus is on the four-dimensional vacua constructed by DeWolfe, Giryavets, Kachru, and Taylor (DGKT, 2005). We analyze the spectrum of low-lying scalar operators and find a universal structure: all dimensions are integer-valued, independent of flux choices or intersection numbers.

For similar three-dimensional scale-separated AdS vacua, we find that integer conformal dimensions are not always present there. We explain how integer dimensions come together with exotic shift symmetries for the scalar fields that depend polynomially on spacetime coordinates. Additionally, we observe how vacua with integer dimensions seem to correspond to a simplified holographic brane set-up, where large N scalings can be derived in a simple way from the near-horizon geometry, similarly to the well-understood Freund-Rubin vacua.

Finally, we present a systematic method for deriving the holographic brane setup from the flux-induced scalar potential and apply this framework to several examples, including the DGKT model.

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1
Introduction

Contents

1.1 Introduction

Quantum field theory provides an important framework for particle physics by describing particles as excitations of some field. Particle interactions are considered point-like. When gravity is included, however, non-renormalisable divergences appear in the theory. At very small distances (at the order of the Planck length), quantum field theory does not give an accurate description of nature. We need a more fundamental theory.

String theory is a promising candidate for such a theory. It replaces point-like particles by one-dimensional extended objects called strings. This way, point-like interactions are avoided.

The dynamics of a string is described by an action principle defined on the two-dimensional worldsheet that the string sweeps out as it propagates through spacetime. Depending on the boundary conditions imposed on their endpoints, strings can be classified as *open* or *closed*. Open strings have a massless spin-1 mode in their spectrum, which, in an appropriate limit, gives rise to Yang-Mills gauge theory [8]. The quantized spectrum of closed strings, on the other hand, includes a massless spin-2 mode whose low-energy effective description is precisely general relativity. It is remarkable that gauge theory and gravity are described within a single framework.

String theory has demonstrated its power in several concrete ways. One success lies in black hole physics, where it provides a microscopic explanation for the Bekenstein-Hawking entropy by counting the quantum states of certain D-brane configurations. [9]. Another breakthrough is the AdS/CFT correspondence, which establishes a duality between quantum gravity in asymptotically anti-de Sitter (AdS) spacetimes and conformal field theories (CFTs) on their boundary [10]. This duality offers a non-perturbative formulation of quantum gravity and has also become a useful tool for analyzing strongly coupled quantum field theories.

However, it remains challenging to describe *our universe* in string theory.

In its best-understood form, string theory operates in 10 dimensions. To recover our 4-dimensional universe, one typically considers compactifications in which 6 spatial dimensions form a compact internal manifold too small to be observed. In the simplest case, spacetime takes the product structure

$$\mathcal{M}_{1,9} = \mathcal{M}_{1,3} \times X_6, \tag{1.1}$$

where $\mathcal{M}_{1,3}$ is our non-compact 4d universe and X_6 is the compact internal space.

Upon dimensional reduction of the 10d supergravity action, the resulting 4d effective action typically contains multiple massless scalar fields, called *moduli*. One such modulus is the dilaton, while others parameterize the size and shape of the internal manifold.

A distinctive feature of moduli is their universal coupling to other fields. For example, Standard Model parameters, such as gauge couplings $g_i = g_i(\phi)$ and Yukawa interactions $Y_{ab} = Y_{ab}(\phi)$, are functions of the moduli ϕ. Also physical scales, including the electroweak scale and the cosmological constant, are moduli-dependent.

The existence of massless moduli is problematic for several reasons. Their unfixed values would correspond to undetermined constants of nature. Moreover, because moduli typically couple to Standard Model fields, the exchange of a light modulus would mediate an unobserved long-range fifth force [11]. In the early universe, light moduli could dominate the energy density and potentially disrupt processes like Big Bang Nucleosynthesis, unless their masses satisfy $m_\phi \gtrsim 10$ TeV [12–15].

It is therefore essential to generate a potential for the moduli, thereby providing them with a mass and fixing their vacuum expectation values at the potential minimum, see Figure 1.1. To ensure the validity of the supergravity approximation, the moduli must be stabilised in a regime where the string coupling is weak and the internal space is large compared to the string scale. We say that a setup has *parametric control* if these quantities can be tuned to be arbitrarily small or large.

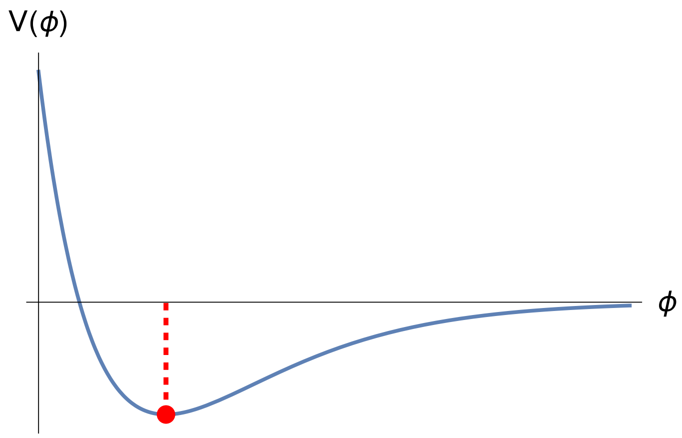

Figure 1.1: *Moduli Stabilisation with potential $V(\phi)$*

Observations indicate that our universe is isotropic and undergoing accelerated expansion [16, 17]. This motivates the search for 4d de Sitter (dS) solutions:

$$\mathcal{M}_{1,3} = dS_4. \tag{1.2}$$

In addition, a realistic model must reproduce the particle content of the Standard Model. Constructing a string compactification that simultaneously obtains a 4d dS spacetime, stabilises all moduli in a regime of control, and incorporates the Standard Model spectrum is extremely challenging and there are no known constructions that achieve this.

We will focus on a smaller set of constraints: controlled 4d Anti-de Sitter (AdS) spacetimes where all moduli are stabilised. Despite being a major simplification, this still turns out to be challenging to obtain. Nevertheless, there exist promising constructions that might achieve this which we will discuss below. A crucial requirement, though seemingly obvious, is that the AdS_4 vacua exhibit scale separation: the AdS curvature radius must be much larger than the size of the internal space,

$$L(\mathrm{AdS}_4) \gg L(X_6). \tag{1.3}$$

This hierarchy is essential for the validity of a 4-dimensional effective field theory. We will occasionally consider AdS_d vacua in other dimensions d to investigate whether constraints and structures are specific to the dimensionality of spacetime.

One reason for interest in AdS vacua is that in typical constructions [18, 19], a de Sitter solution is obtained by first finding an AdS vacuum with a large separation of scales and then uplifting it.

A second reason is the AdS/CFT correspondence. A valid string theory background in AdS must be dual to a well-defined conformal field theory. Since the consistency conditions for CFTs are exceptionally clear, this duality offers an unambiguous test of which AdS vacua are consistent. By contrast, proposals for de Sitter holography (such as [9]) remain far less developed and are still largely conjectural.

The main examples of consistent AdS backgrounds are of the Freund-Rubin type. Here, string theory or M-theory is compactified on a sphere that is supported by flux, resulting in a d-dimensional AdS vacuum,

$$\mathrm{AdS}_d \times S^n, \quad d + n = 10 \text{ or } 11. \tag{1.4}$$

However, the typical length scale of the internal sphere is always of the same order as the AdS length scale, so these solutions are far from being scale-separated. These examples also form the primary examples of the AdS/CFT correspondence. The canonical example is $\mathrm{AdS}_5 \times S^5$ obtained by compactifying type IIB string theory on a sphere threaded by a 5-form flux. Its dual theory is the 4-dimensional super Yang Mills theory with $\mathcal{N} = 4$ supersymmetry. [10]. All well-understood realisations of this correspondence in string theory share the structure of an AdS factor combined with an internal manifold of comparable size. This observation raises the question of whether the term AdS/CFT correspondence may be misleading since the gravity side always involves more than an AdS space on its own.

A few constructions have been proposed for scale-separated AdS vacua with full moduli stabilisation in a controlled regime, such as the DGKT vacua [20], the LVS vacua [21], and the KKLT vacua [18], along with some candidate scale-separated AdS in 3 dimensions [22–24]. However, these are not yet understood to the same extent as the non scale-separated examples mentioned above. Moreover, no explicit holographic dual has been found for any of these scale-separated vacua. If such CFTs exist, they are expected to exhibit unusual features, such as a large gap in the spectrum of single-trace operators, that have not been seen in any known example so far.

The DGKT vacua contain a flux that is unconstrained. By increasing this flux (proportional to a parameter N), parametric control can be achieved in a way similar to the Freund-Rubin vacua. In the same limit, scale separation is obtained. The LVS and KKLT constructions rely on a combination of effects to obtain control and scalar separation. Their putative CFT duals, if they exist, are expected to be significantly more complex, without a clear large N structure.

This thesis aims to contribute to the initial exploration of holographic duals for scale-separated AdS vacua. The broader goal is to move towards a holographic understanding of vacua that more closely resemble our universe than the well-studied AdS/CFT cases, and to investigate whether genuinely scale-separated AdS/CFT dualities exist, where the gravity side involves pure AdS without a large internal space. We focus primarily on the DGKT vacua, as they are structurally closer to the known examples. The structure of the thesis is as follows:

- Chapter 1 reviews foundational aspects of string theory, flux compactification and holography.
- Chapter 2 surveys the literature on scale-separated AdS vacua, focusing on the DGKT construction by DeWolfe, Giryavets, Kachru, and Taylor (2005), and related constructions.
- Chapter 3 investigates the holographic spectra of these vacua, based on results from [1, 2].
- In Chapter 4, we discuss the holographic central charge of such vacua and relate it to the D-branes that source these vacua, which is a result from [3].
- In Chapter 5, we describe an algorithm to compute the holographic brane picture from the flux potential, as was presented in [4].
- We conclude in Chapter 6.

1.2 String theory

1.2.1 The web of string theories

The simplest string theory is the *bosonic string* [25], which requires the spacetime dimension to be $d = 26$ to cancel the conformal anomaly. Its spectrum contains a tachyonic ground state, indicating an instability, as well as the following massless fields: the spacetime metric $g_{\mu\nu}$, an antisymmetric two-form field $B_{\mu\nu}$ (the *Kalb–Ramond field*), and the dilaton ϕ.

The *superstring* [26] also takes into account fermionic modes on the worldsheet in a supersymmetric way, reducing the critical dimension to 10. Unlike the bosonic string, several perturbative superstring theories exist, each handling worldsheet fermions differently. For instance, in the *heterotic string* only the right-moving fermions are retained, yielding a theory with 16 supercharges in 10 dimensions; the gauge group can be either $SO(32)$ or $E_8 \times E_8$. The *type II* theories preserve both left- and right-moving fermions, resulting in a maximally supersymmetric theory with 32 supercharges in 10 dimensions.

The type II theories are divided into two classes, IIA and IIB, distinguished by the chirality of their worldsheet fermions. In type IIA, the left- and right-moving fermions have opposite chirality, while in type IIB, both sets of fermions share the same chirality. In addition to the massless fields of the bosonic string, the type II theories include Ramond–Ramond (RR) p-form potentials C_p, whose degrees are odd in type IIA and even in type IIB:

$$\text{IIA}: \quad g_{\mu\nu},\ B_2,\ \phi,\ C_1,\ C_3, \tag{1.5}$$

$$\text{IIB}: \quad g_{\mu\nu},\ B_2,\ \phi,\ C_0,\ C_2,\ C_4. \tag{1.6}$$

Lastly, *type I* string theory can be obtained from IIB string theory via an orientifold projection (defined in Eq. (1.45)), resulting in a theory with 16 supercharges. The different superstring theories are related by dualities (see Figure 1.2). For instance, *T-duality* connects the type II theories as well as the two heterotic theories. T-duality interchanges momentum and winding modes, which implies in the simplest form that a circle of radius R is equivalent to a circle of radius l_s^2/R, where l_s is the fundamental string length. Similarly, *S-duality* relates theories with string coupling g_s to those with coupling $1/g_s$, mapping the heterotic SO(32) theory to type I string theory, and type IIB to itself.

Type IIA string theory does not have a known S-dual among the perturbative string theories. Instead, it is conjectured that its strong coupling limit (as well as that of the heterotic $E_8 \times E_8$ theory) corresponds to an 11-dimensional, non-perturbative theory known as *M-theory* [27, 30]. The low-energy effective theory of M-theory is 11-dimensional $\mathcal{N} = 1$ supergravity, but beyond this, the full formulation of M-theory remains to be understood.

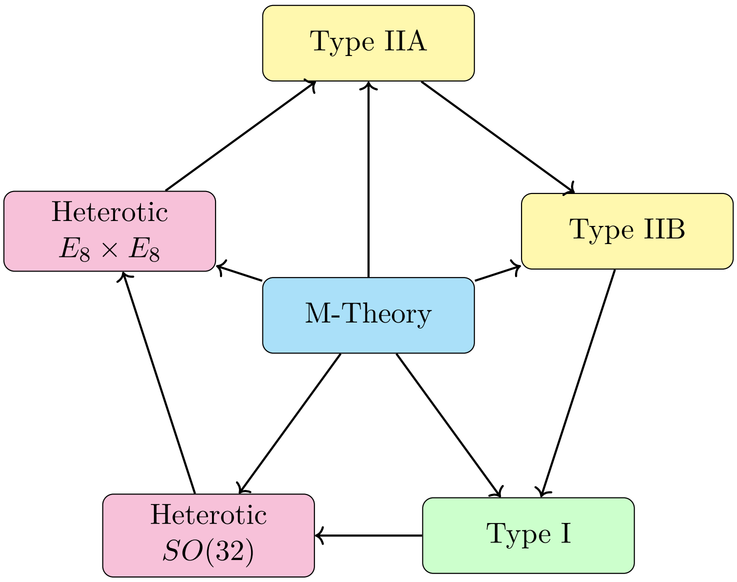

Figure 1.2: *Duality web of string theories [27–29].*

1.2.2 10d supergravity

In practice, we do not work with the full 10d string theory. At energies below the string scale $M_s = l_s^{-1}$, we can focus on the massless sector, for which the corresponding 10d supergravity theories provide an effective description. These actions are valid in the weak-coupling regime and when higher-derivative corrections can be neglected. Below, we briefly summarize the two type II supergravity theories.

The 10d type IIA supergravity action

The 10d Type IIA supergravity is a non-chiral $(1,1)$ theory [31–33]. The field strengths for the p-form fields are given by

$$\begin{aligned} H_3 &= dB_2, \\ F_2 &= dC_1, \\ F_4 &= dC_3 - C_1 \wedge H_3, \end{aligned} \tag{1.7}$$

in terms of which the bosonic part of the 10d Type IIA supergravity action in string frame is

$$\begin{aligned} S_{IIA} = &\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[e^{-2\phi} \left(R + 4\,(\partial\phi)^2 - \frac{1}{2}\,|H_3|^2 \right) - \frac{1}{2}\,|F_2|^2 + |F_4|^2 \right] \\ &- \frac{1}{4\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4, \end{aligned} \tag{1.8}$$

where

$$\frac{1}{2\kappa_{10}^2} = \frac{2\pi}{(2\pi l_s)^8}. \tag{1.9}$$

The last term is the Chern-Simons action, a topological contribution. This action can be derived via dimensional reduction from the 11d supergravity action.

The bosonic content of the 11d supergravity consists of the metric g and a three-form C_3, with field-strength $F_4 = dC_3$. The action reads

$$S_{11} = \frac{1}{2\kappa_{11}^2}\left[\int d^{11}x\,\sqrt{-g}\,\left(R - \frac{1}{2}|F_4|^2\right) - \frac{1}{12}\int C_3 \wedge F_4 \wedge F_4\right]. \tag{1.10}$$

The 10d type IIB supergravity action

The type IIB supergravity is a chiral $(2,0)$ theory with the following field strengths [34, 35]:

$$\begin{aligned} F_1 &= dC_0, \\ F_3 &= dC_2 - C_0 dB_2, \\ F_5 &= dC_4 - \frac{1}{2}C_2 \wedge dB_2 + \frac{1}{2}B_2 \wedge dC_2. \end{aligned} \tag{1.11}$$

In string frame, the bosonic part of the Type IIB supergravity action is given by

$$\begin{aligned} S_{IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\,&\left[e^{-2\phi}\left(R + 4\,(\partial\phi)^2 - \frac{1}{2}\,|H_3|^2\right) - \frac{1}{2}\,|F_1|^2 - \frac{1}{2}|F_3|^2 - \frac{1}{4}|F_5|^2\right] \\ - \frac{1}{4\kappa_{10}^2}\int C_4 \wedge H_3 \wedge F_3& \end{aligned} \tag{1.12}$$

We also need to impose the self-duality condition

$$F_5 = *F_5. \tag{1.13}$$

1.2.3 D-branes

In addition to fundamental strings, string theory contains other extended objects known as Dp-branes [36]. A Dp-brane has a $(p+1)$-dimensional worldvolume and is defined by the property that open strings can end on it. This implies that gauge theories live on D-branes, with the gauge fields arising as the massless modes of open strings attached to the brane. The tension of a D-brane scales as $1/g_s$, which blows up in the weak-coupling limit, indicating that D-branes are non-perturbative objects.

Dp-branes are electrically charged under the $(p+1)$-form Ramond–Ramond potentials C_{p+1} and magnetically charged under their 10d Hodge duals, the $(7-p)$-form potentials. In Type IIA string theory, D-branes have even-dimensional worldvolumes: D0-, D2-, D4-, D6-, and D8-branes. In Type IIB, they are odd-dimensional: D1-, D3-, D5-, D7-, and D9-branes.

The stable D-branes in type II string theory are BPS objects, meaning that their tension T_p equals their Ramond–Ramond charge μ_p, and they preserve half of the spacetime supersymmetry.

D-brane stacking

Because of their BPS nature, the attractive gravitational force and the repulsive R-R force between D-branes cancel exactly. This cancellation allows one to *stack* multiple Dp-branes. In a stack of N coincident Dp-branes, open strings can stretch between different branes. Consequently, the spectrum is enhanced: rather than a $U(1)$ gauge field, the massless modes now form an adjoint representation of the enhanced gauge group $U(N)$. When the D-branes are slightly separated, the off-diagonal string modes acquire masses proportional to the separation, and the full $U(N)$ symmetry is spontaneously broken to $U(1)^N$.

D-brane action

The effective dynamics of the massless modes on the worldvolume of a Dp-brane is captured by the D-brane action, which consists of two parts: the Dirac-Born-Infeld (DBI) action and the Chern-Simons (CS) term. The DBI-action for a single Dp-brane is given by

$$S_{DBI} = -\mu_p \int_{\Sigma_{p+1}} d^{p+1}x \, e^{-\phi} \sqrt{-\det\left(g_{ab} + \mathcal{F}_{ab}\right)}, \tag{1.14}$$

where

$$\mu_p = 2\pi l_s^{-(p+1)} \tag{1.15}$$

denotes the charge (and tension) of the Dp-brane, Σ_{p+1} is the brane worldvolume, and g_{ab} is the pullback of the spacetime metric onto the brane. The gauge-invariant field strength $\mathcal{F}$ is defined as

$$\mathcal{F}_{ab} = B_{ab} + 2\pi\alpha' F_{ab}, \tag{1.16}$$

with B_{ab} being the NS-NS two-form and F_{ab} the field strength of the worldvolume gauge field. This action generalizes both the Nambu-Goto action for a brane and the Yang-Mills action for the gauge fields on the brane.

The Chern-Simons term, responsible for the topological coupling to the R-R gauge potentials, reads

$$S_{CS} = \mu_p \int_{\Sigma_{p+1}} \left(\sum_n C_n \wedge e^{\mathcal{F}} \right). \tag{1.17}$$

Here, the sum runs over the appropriate R-R potentials.

Supergravity description of D-branes

The supergravity description of N Dp-branes in type II string theories is characterized by a metric that respects the symmetries of the brane. A general ansatz for the metric is [37]

$$ds^2 = e^{2A(r)} \, dx_{\parallel}^2 + e^{2B(r)} \, dx_{\perp}^2, \qquad e^{\phi} = e^{\phi(r)}, \tag{1.18}$$

where r denotes the radial distance from the brane, $x_{\parallel}$ are the coordinates parallel to the brane, and $x_{\perp}$ the transverse coordinates. The metric is required to approach the Minkowski metric at large r ($r \to \infty$), with a constant dilaton $\phi \to 0$ in this limit.

It turns out that solutions can be written in terms of a single harmonic function $H(r)$,

$$ds^2_{\mathrm{D}p} = H(r)^{-1/2}\, dx^2_{\parallel} + H(r)^{1/2}\, dx^2_{\perp}, \tag{1.19}$$

which is valid in the string frame for $p \leq 6$. The dilaton and the R-R $(p+1)$-form field strength are given by

$$e^{\phi} = H(r)^{\frac{3-p}{4}}, \qquad F_{m0\cdots p} = \partial_m H(r)^{-1}. \tag{1.20}$$

The harmonic function takes the form

$$H(r) = 1 + \frac{\alpha}{r^{7-p}}, \qquad \alpha = (4\pi)^{\frac{5-p}{2}}\Gamma\Big(\frac{7-p}{2}\Big)(\alpha')^{\frac{7-p}{2}} g_s N. \tag{1.21}$$

For future reference, we also quote the result for an NS5-brane. An NS5-brane is a brane with a 6-dimensional worldvolume that couples magnetically to B_2. Its metric is

$$ds^2_{\mathrm{NS5}} = dx^2_{\parallel} + H(r)\, dx^2_{\perp}, \tag{1.22}$$

with the dilaton and three-form field strength

$$e^{\phi} = H(r)^{1/2}, \qquad H_{mnp} = \epsilon_{mnpq}\, \partial_q H(r), \tag{1.23}$$

and the harmonic function

$$H(r) = 1 + \frac{\alpha}{r^2}, \qquad \alpha = N\alpha'. \tag{1.24}$$

M-branes

Similarly, within 11d supergravity, extended objects known as *M-branes* can be described. These include the *M2-brane*, with a $(2+1)$-dimensional worldvolume, and the *M5-brane*, with a $(5+1)$-dimensional worldvolume.

The M2-brane couples electrically to the 3-form potential C_3. Its metric and field strength are given by [37]

$$ds^2_{\mathrm{M2}} = H(r)^{-2/3}\, dx^2_{\parallel} + H(r)^{1/3}\, dx^2_{\perp}, \qquad F_{r012} = -\partial_r H(r)^{-1}, \tag{1.25}$$

with

$$H(r) = 1 + \frac{\alpha}{r^6}, \qquad \alpha = 2^5\pi^2 l_{11}^6 N, \tag{1.26}$$

where l_{11} is the 11d Planck length.

The M5-brane couples magnetically to C_3. Its solution is

$$ds^2_{\mathrm{M5}} = H(r)^{-1/3}\, dx^2_{\parallel} + H(r)^{2/3}\, dx^2_{\perp}, \qquad F_{mnpqr} = \epsilon_{mnpqrs}\partial_s H, \tag{1.27}$$

with,

$$H(r) = 1 + \frac{\alpha}{r^3}, \qquad \alpha = \pi l_{11}^3 N. \tag{1.28}$$

1.3 Flux compactification and moduli stabilisation

1.3.1 Calabi-Yau manifolds and their moduli

To obtain a 4d theory from compactification of a 10d supergravity, we decompose the metric as

$$ds_{10}^2 = ds_{4,\text{universe}}^2 + ds_{6,\text{compact}}^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu + g_{mn}(y)\, dy^m dy^n, \qquad (1.29)$$

where x^μ ($\mu = 0, 1, 2, 3$) denote coordinates on the non-compact 4d spacetime, and y^m ($m = 1, \ldots, 6$) are coordinates on the compact internal manifold. For simplicity, we do not consider any warping here.

In the absence of additional fields, this 10d metric should solve the vacuum Einstein equations $R_{MN} = 0$ ($M = 0, \ldots, 10$), which decouple into separate conditions for the 4d and 6d metrics,

$$R_{\mu\nu}(x) = 0 \quad \text{and} \quad R_{mn}(y) = 0.$$

For the non-compact dimensions, a Minkowski metric is the maximally symmetric Ricci-flat solution.

In 6 dimensions, Calabi-Yau manifolds are compact Ricci-flat manifolds. These are defined as compact Kähler manifolds with vanishing first Chern class, and by Yau's theorem, they admit a Ricci-flat metric that is unique in each Kähler class. The holonomy group of these manifolds is contained in $SU(3)$.

The *holonomy group* of a Riemannian manifold is the group of transformations experienced by a tangent vector upon parallel transport around closed loops. For an n-dimensional manifold, this group is a subgroup of $O(n)$. A manifold is said to have *special holonomy* if its holonomy group is a strict subgroup of $O(n)$; for example, $SU(3)$ is a strict subgroup of $O(6)$ [38].

It is generally expected that the only simply-connected compact Ricci-flat manifolds are those with special holonomy, though there is no proof for this. In 6 dimensions, this expectation singles out Calabi-Yau manifolds as the natural candidates for compactification.

A particular feature of special holonomy manifolds is that they admit parallel spinors. This ensures that some amount of supersymmetry is preserved after compactification. The smaller the holonomy group, the more parallel spinors exist and hence the more supersymmetries are preserved. 6-dimensional $SU(3)$ manifolds have exactly one parallel spinor and keep $\mathcal{N} = 2$ supersymmetry in 4d when compactifying 10d type II string theory. With extra ingredients such as fluxes and orientifolds, supersymmetry can then be broken to $\mathcal{N} = 1$ (or $\mathcal{N} = 0$) in 4d.

From a phenomenological standpoint, minimal $\mathcal{N}=1$ supersymmetry in 4d is especially appealing. It retains some benefits, like controlling divergences, while allowing for supersymmetry breaking at relatively low scales. $\mathcal{N}=2$ supersymmetry results in non-chiral matter content, which is incompatible with the chiral structure of the Standard Model.

Keeping $\mathcal{N}=1$ supersymmetry also makes calculations more tractable, especially when dealing with quantum dynamics. There are different non-renormalization theorems, one of which ensures that the $\mathcal{N}=1$ superpotential does not receive corrections at any order in perturbation theory due to its holomorphicity.

Additionally, non-supersymmetric compactifications have been argued to suffer from instabilities [39, 40], providing yet another reason to preserve some degree of supersymmetry.

Next, let us examine the moduli a Calabi-Yau manifold has. We search for transformations of the Calabi-Yau metric g that preserve Ricci-flatness. A simple example is a uniform rescaling

$$g' = c\, g, \tag{1.30}$$

where c is a positive constant. This modulus is the *breathing mode* (also referred to as the *volume modulus*) and is often the most important modulus together with the dilaton. To be more systematic, consider an infinitesimal deformation of the metric,

$$g' = g + \delta g. \tag{1.31}$$

To preserve Ricci-flatness, any variation δg must satisfy the linearized Einstein equations, which can be written as

$$\nabla^m \nabla_m \delta g_{pq} + 2\, R_{p\ q}^{\ r\ s}\, \delta g_{rs} = 0, \tag{1.32}$$

known as the *Lichnerowicz equation.* Its solutions naturally split into two decoupled classes.

The first class consists of the *mixed deformations* $\delta g_{i\bar{j}}$. These deformations can be organized into a variation of the Kähler form,

$$\delta J = \delta g_{i\bar{j}}\, dz^i \wedge d\bar{z}^{\bar{j}}, \tag{1.33}$$

which is harmonic when the Lichnerowicz equation is satisfied. Hence, the mixed deformations are in one-to-one correspondence with elements of the Dolbeault cohomology group $H^{1,1}(X)$ and describe the Kähler moduli, including the overall volume rescaling.

The second class contains the *pure deformations* of the form δg_{ij} (along with their complex conjugates). These deformations can be expressed as

$$\Omega^{\bar{\nu}}_{\kappa\lambda}\, \delta g_{\mu\nu}\, dx^{\kappa} \wedge dx^{\lambda} \wedge dx^{\bar{\mu}}, \tag{1.34}$$

which corresponds to harmonic $(2,1)$-forms in $H^{2,1}(X)$. These solutions are the complex structure deformations.

$H^{1,1}(X)$ and $H^{2,1}(X)$ are indeed the only two non-empty and non-trivial Dolbeault cohomology groups for a Calabi-Yau manifold. In summary, a Calabi–Yau manifold has $h^{1,1}$ Kähler moduli and $h^{2,1}$ complex structure moduli.

Lower-dimensional AdS vacua cannot be obtained as a vacuum solution of the Einstein equation without extra sources, such as fluxes. Fluxes are non-zero background configurations of p-form field strengths over the internal dimensions. The simplest way to solve the Einstein equations is then to consider a positively curved Einstein manifold, such as a sphere, as the internal manifold, and a flux threading this sphere. We discuss these *Freund-Rubin vacua* in the box below. This way, too much supersymmetry is typically preserved though. With some more subtle balancing it is possible to find AdS vacua with an internal Calabi-Yau manifold and hence minimal supersymmetry ([18, 20, 21]).

1.3.2 Stabilisation of moduli

It is essential to stabilise the Kähler moduli, complex structure deformations and the dilaton. In this section, we outline how fluxes, orientifold planes and non-perturbative effects contribute to the moduli potential.

Fluxes

Because the energy density of fluxes depends on the moduli, fluxes contribute non-trivially to the effective scalar potential.

To illustrate the moduli dependence of the lower-dimensional potential induced by fluxes, let us focus on a simplified scenario in type II string theory with two universal moduli: the dilaton ϕ and the breathing mode or overall internal volume $\mathcal{V}$. The relation between the 10d and 4d metrics is given by

$$ds_{10}^2 = e^{2\phi}\mathcal{V}^{-1} ds_4^2 + \mathcal{V}^{1/3} ds_6^2, \tag{1.35}$$

which implies

$$\sqrt{-g_{10}} = \sqrt{-g_4}\,\sqrt{g_6} = e^{4\phi}\mathcal{V}^{-1}. \tag{1.36}$$

To construct *Freund-Rubin vacua*, we consider a single contribution from an n-flux F_n threading the n-dimensional internal manifold. The action with this flux is given by

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-g_D} \left(R_D - \frac{1}{2\, n!} F_n^2 \right), \tag{1.37}$$

where $D = d + n$. The ansatz for the flux is

$$F_{m_1 \cdots m_n} = f\, \epsilon_{m_1 \cdots m_n}, \tag{1.38}$$

with f a constant and $\epsilon_{m_1 \cdots m_n}$ the Levi–Civita tensor on the internal manifold. With the product metric ansatz

$$ds_D^2 = ds_d^2 + ds_n^2 = g_{\mu\nu}(x) dx^\mu dx^\nu + g_{ab}(y) dx^a dx^b, \tag{1.39}$$

the equations of motion decouple into

$$R_{\mu\nu} = -\frac{n-1}{2(d+n-2)} f^2\, g_{\mu\nu}, \qquad R_{ab} = \frac{d-1}{2(d+n-2)} f^2\, g_{ab}. \tag{1.40}$$

Thus, the noncompact space must have negative curvature (i.e. it is an AdS space) while the internal space is a positively curved Einstein manifold (with, for example, the round sphere S_n as the simplest case). In fact, the radii of the AdS space and the internal sphere are given by

$$L_{\mathrm{AdS}}^2 = \frac{2(d+n-2)(d-1)}{n-1} f^{-2}, \qquad L_{S_n}^2 = \frac{2(d+n-2)(n-1)}{d-1} f^{-2}, \tag{1.41}$$

which shows that the two length scales are comparable: hence the compactification does not lead to a lower-dimensional effective theory. Nevertheless, these vacua are very important in the context of the AdS/CFT correspondence.

Moreover, the norms of the p-form fluxes depend on the internal volume. In particular,

$$H_3^2 \sim \mathcal{V}^{-1}, \quad F_p^2 \sim \mathcal{V}^{-p/3}, \tag{1.42}$$

so that the effective potentials generated by these fluxes scale as (see Eqs. (1.8), (1.12))

$$V_{H_3} \sim e^{2\phi}\, \mathcal{V}^{-2}, \quad V_{F_p} \sim e^{4\phi}\, \mathcal{V}^{-(p+3)/3}. \tag{1.43}$$

These different moduli dependences provide the possibility of balancing the contributions to create a minimum in the total potential.

Furthermore, if the internal manifold possesses non-zero curvature, its contribution to the potential typically scales as

$$V_{\mathrm{curvature}} \sim \sqrt{-g_{10}} e^{-2\phi} R_6 \sim e^{2\phi}\, \mathcal{V}^{-4/3}, \tag{1.44}$$

and it is sometimes useful to view this curvature contribution as a generalized flux.

Orientifold planes

In a Calabi–Yau compactification of type II string theory, the 4d effective theory will have $\mathcal{N} = 2$ supersymmetry. To reduce the supersymmetry further to $\mathcal{N} = 1$, one common approach is to introduce an orientifold projection. This projection combines a worldsheet parity transformation Ω with a spacetime parity transformation $\mathcal{R}$, and includes the extra factor $(-1)^{F_L}$ (where F_L denotes the left-moving spacetime fermion number) to ensure the proper transformation of fermionic fields. In symbolic form, the orientifold projection is given by

$$\Omega\,\mathcal{R}\,(-1)^{F_L}. \tag{1.45}$$

This action breaks exactly half of the supersymmetries. The fixed locus of this orientifold action defines an *orientifold plane*, which carries negative charge and can contribute to the stabilisation of moduli.

The contribution to the 10d action of an orientifold Op-plane is of the same form as for Dp-branes (1.14), (1.17),

$$S_{\mathrm{O}p} = -\mu_{\mathrm{O}p}\int_{\Sigma p+1} d^{p+1}x e^{-\phi}\sqrt{-g} + \mu_{\mathrm{O}p}\int C_{p+1} \tag{1.46}$$

but where the charge $\mu_{\mathrm{O}p}$ is related to the Dp-brane charge μ_p as follows,

$$\mu_{\mathrm{O}p} = -2^{p-5}\mu_p. \tag{1.47}$$

In contrast with D-branes, O-planes do not have worldvolume fields. From the 4d perspective, the presence of Op-planes introduces a negative term in the scalar potential with the following dependence on the universal moduli,

$$V_{\mathrm{O}p} \sim -e^{3\phi}\mathcal{V}^{\frac{p-15}{6}}. \tag{1.48}$$

Non-perturbative effects

Other contributions to the low-energy potential can come from non-perturbative effects. Typical sources include instanton effects from Euclidean D-branes and gaugino condensation on branes within the internal manifold. Such effects contribute to the 4d superpotential in the following way,

$$W_{\mathrm{np}} = Ae^{iaT}, \tag{1.49}$$

where A and a are real constants and the complex modulus $T \equiv x + iy$ contains an axionic component x and a modulus y. For a typical Kähler potential that depends logarithmically on T, the resulting contribution to the scalar potential shows a double-exponential dependence on the canonically normalized moduli. This behavior contrasts with the single-exponential dependence in the contributions from fluxes, curvature, and orientifolds.

1.3.3 Axions in flux compactifications

Aside from moduli, there is another type of particle that naturally arises in string compactifications: the axion. Axions are pseudo-scalar fields with a compact field range, i.e. there is a shift symmetry

$$a \equiv a + 2\pi f_a, \tag{1.50}$$

where f_a is called the axion decay constant. Because of their compactness, axions are naturally light particles; any contribution to the potential should respect the shift symmetry. Shift symmetric pseudo-scalars typically arise in string theory by compactififying the p-form fields C_p (see e.g. [41]). Except for local gauge invariance,

$$C_p \to C_p + d\lambda_{p-1}, \tag{1.51}$$

for some $(p-1)$-form λ, these fields are also invariant w.r.t. large gauge transformations. These are topologically non-trivial transformations that arise when winding around extra dimensions. They take the form

$$C_p \to C_p + 2\pi \sum_{i=1}^{b_p(M)} n^i \omega_i^p, \tag{1.52}$$

where n^i are integers, b_p is the p-th Betti number of the internal manifold M and ω_i^p are the basis p-forms of the p-th cohomology, dual to the p-cycles Σ_i^p. Integrating such a p-form over the p-cycles, then leads to $b_p(M)$ axions,

$$a_i = \int_{\Sigma_i^p} C^p, \tag{1.53}$$

as these will inherit the shift symmetry

$$a_i \equiv a_i + 2\pi \tag{1.54}$$

from the large gauge transformations. After normalizing the kinetic terms of the axion field, the shift symmetry takes the more general form (1.50).

1.4 AdS/CFT correspondence

1.4.1 Holographic correspondence

As mentioned in Chapter 1, the framework of string theory naturally captures both gravitational theories (arising from closed strings and leading to Einstein's equations) and gauge theories, which emerge from open strings attached to D-branes. This connection reflects a deeper duality built into string theory: a non-perturbative equivalence between certain gravitational and large N gauge theories.

This duality is a strong-weak coupling correspondence [10, 42]. When the radius of curvature R of the gravitational background is large compared to the string length l_s,

$$\frac{R}{l_s} \gg 1, \tag{1.55}$$

the supergravity approximation becomes valid. In this regime, the dual gauge theory becomes strongly coupled, meaning that perturbative techniques no longer apply. For a large N Yang-Mills theory, this condition translates into

$$g_{\mathrm{YM}}^2 N \gg 1, \tag{1.56}$$

where the product $g_{\mathrm{YM}}^2 N$ reflects the effective 't Hooft coupling, with N accounting for the number of colour degrees of freedom running in loop diagrams.

Conversely, when the gauge theory is weakly coupled ($g_{\mathrm{YM}}^2 N \ll 1$), the dual gravitational theory is strongly curved in string units ($R/l_s \ll 1$), and a classical supergravity description is no longer reliable.

More concretely, there is a correspondence between string theory backgrounds of the form $\mathrm{AdS}_d \times M_n$ and $(d-1)$-dimensional conformal field theories (CFTs). This duality is referred to as *holographic*, as the CFT resides on the boundary of the AdS spacetime, and in particular, the entropy in an AdS spacetime is proportional to the area of its boundary [43].

1.4.2 Motivating the canonical AdS/CFT example through a stack of D3-branes

The first concrete example of the AdS/CFT correspondence [10] relates type IIB string theory on $\mathrm{AdS}_5 \times S^5$ to 4d $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory. This duality can be motivated by considering a stack of N D3-branes in flat 10d spacetime from two different viewpoints.

From the first perspective, the D3-branes are seen as end-points of open strings. In the low-energy limit $E \ll 1/\ell_s$, the system decouples into two independent sectors. The first is described by the gauge theory on the branes, which turns out to be $\mathcal{N} = 4$ SYM in 4 dimensions. The second consists of bulk closed string modes, corresponding to type IIB supergravity in flat spacetime.

The second perspective treats the D3-branes as classical sources in supergravity. Their backreaction gives rise to the metric (1.19), (1.21)

$$ds^2_{D_3} = H(r)^{-1/2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + H(r)^{1/2}(dr^2 + r^2 d\Omega_5^2), \tag{1.57}$$

$$H(r) = 1 + \frac{\alpha}{r^4}, \quad \alpha \sim N, \tag{1.58}$$

with a constant dilaton. The energy E of an object measured at infinity is redshifted relative to the local energy E_p measured at radial coordinate r:

$$E = H(r)^{-1/4} E_p \sim r E_p. \tag{1.59}$$

This implies that low-energy excitations can come either from bulk massless modes, or from excitations localized near $r = 0$, whose redshifted energy becomes small at infinity.

Taking the near-horizon limit ($r \ll \alpha^{1/4}$), the geometry becomes

$$ds^2_{\mathrm{NH}} = \left(\frac{r}{R}\right)^2 (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \left(\frac{R}{r}\right)^2 dr^2 + R^2 d\Omega_5^2, \tag{1.60}$$

where $R = \alpha^{1/4} \sim N^{1/4}$. This is the geometry of $\mathrm{AdS}_5 \times S^5$, where both the AdS and the sphere radius are given by R.

The observation is that both descriptions of the D3-branes contain two decoupled sectors: one describing type IIB supergravity in flat space, and the other either $\mathcal{N} = 4$ SYM or IIB supergravity on $\mathrm{AdS}_5 \times S^5$. Since the same low-energy theory arises from both pictures, Maldacena conjectured that type IIB string theory on $\mathrm{AdS}_5 \times S^5$ is dual to $\mathcal{N} = 4$ super-Yang–Mills theory in 4 dimensions [10].

1.4.3 Conformal field theories

CFTs form a special class of quantum field theories that remain invariant under conformal transformations. Under a conformal transformation, the coordinates and the metric transform as

$$x \to x', \quad g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = e^{f(x)}\, g_{\mu\nu}(x), \tag{1.61}$$

where $f(x)$ is a function that characterizes the local rescaling. The full symmetry group includes Poincaré transformations (translations and Lorentz transformations), as well as dilatations and special conformal transformations.

$$\begin{aligned}
&\text{Translations:} \quad x_\mu \to x'_\mu = x_\mu + a_\mu, \\
&\text{Lorentz transformations:} \quad x^\mu \to x'_\mu = \Lambda^\mu{}_\nu\, x^\nu, \\
&\text{Dilatations:} \quad x_\mu \to x'_\mu = \lambda\, x_\mu, \\
&\text{Special conformal transformations:} \quad x_\mu \to x'_\mu = \frac{x_\mu - b_\mu\, x^2}{1 - 2b \cdot x + b^2\, x^2}.
\end{aligned} \tag{1.62}$$

We denote the generators of these transformations as P_μ, $M_{\mu\nu}$, D, and K_μ. The algebra generated by these transformations is isomorphic to $\mathrm{SO}(d, 2)$.

Primary and descendant operators Representations of the conformal group can be labeled by the scaling dimension Δ, defined by the eigenvalue of the dilatation operator D. The operators P_μ and K_μ act as raising and lowering operators, respectively, for the scaling dimension. Conformal primary operators are defined as those that are annihilated by the special conformal generators K_μ. Under a scale transformation $x \to \lambda x$, a primary operator $\mathcal{O}_i$ transforms as

$$\mathcal{O}_i(\lambda x) = \lambda^{-\Delta_i}\, \mathcal{O}_i(x). \tag{1.63}$$

Descendant operators are obtained by applying derivatives to the primary operators.

Correlation functions Physical observables in a CFT are expressed in terms of correlation functions of these operators. Conformal invariance strongly constrains the form of these correlators. For example, the two-point function of primary operators is given by

$$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) \rangle = \frac{\delta_{\Delta_1,\Delta_2}}{|x_1 - x_2|^{\Delta_1+\Delta_2}}, \tag{1.64}$$

Similarly, the three-point function takes the form

$$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2)\, \mathcal{O}_3(x_3) \rangle = \frac{f_{123}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}\, |x_{23}|^{\Delta_2+\Delta_3-\Delta_1}\, |x_{31}|^{\Delta_3+\Delta_1-\Delta_2}}, \tag{1.65}$$

with f_{123} constant coefficients and $|x_{ij}| = |x_i - x_j|$.

Operator product expansion (OPE) The operator product expansion (OPE) is a powerful tool in CFTs. It asserts that the product of two local operators can be expanded as an infinite sum over local operators,

$$\mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) = \sum_k \mathcal{F}_{12k}(x_1 - x_2)\, \mathcal{O}_k(x_2), \tag{1.66}$$

where the functions $\mathcal{F}_{12k}(x_1 - x_2)$ encode the dependence on the separation and include contributions from both primary and descendant operators. Conformal symmetry determines the descendant coefficients uniquely in terms of the primary ones, so that the OPE is often recast as

$$\mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) = \sum_k f_{12k}\, \frac{\mathcal{O}_k(x_2)}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_k}} + \text{descendants}, \tag{1.67}$$

where f_{12k} are the three-point coefficients appearing in (1.65).

CFT data and crossing symmetry The complete set of *CFT data* consists of the quantum numbers of the operators (their scaling dimensions Δ_i, charges, and spins) and the three-point function coefficients f_{ijk}. This data uniquely fixes all higher-point correlation functions through repeated applications of the OPE.

For instance, consider a four-point function,

$$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2)\, \mathcal{O}_3(x_3)\, \mathcal{O}_4(x_4)\rangle. \tag{1.68}$$

One may first apply the OPE in the *s-channel* (combining $\mathcal{O}_1 \times \mathcal{O}_2$ and $\mathcal{O}_3 \times \mathcal{O}_4$) or in the *t-channel* (combining $\mathcal{O}_1 \times \mathcal{O}_3$ and $\mathcal{O}_2 \times \mathcal{O}_4$). Crossing symmetry is the requirement that these different expansions lead to the same result. This condition serves as a important consistency constraint on the CFT data (Δ_i, f_{ijk}).

Unitarity further constrains the theory. For example, for operators with spin $l > 0$, unitarity requires

$$\Delta \geq d - 2 + l. \tag{1.69}$$

In the *conformal bootstrap* program (see [44] for a review), these symmetry and unitarity constraints are employed to narrow down the space of consistent CFTs. Numerical studies have yielded impressive results, such as the determination of operator dimensions for 3d CFTs, with the Ising model lying near the boundary of the allowed region [45].

1.4.4 The AdS/CFT dictionary

In this section, we give a brief overview of how observables in a conformal field theory relate to fields in a gravity theory and vice versa [46, 47].

The partition functions on both sides of the duality match,

$$Z_{\text{AdS}} = Z_{\text{CFT}}. \tag{1.70}$$

On the gravity side, this is a path integral over bulk fields (often approximated semiclassically by $\exp(-S_{\text{AdS}}/\hbar)$, where S_{AdS} is the Einstein action), while on the CFT side it is the generating functional for correlation functions. To make this precise, we need to fix boundary conditions for the bulk fields.

CFT observables are correlation functions of gauge-invariant operators, and each of them corresponds to a bulk field in AdS. To see how this works, consider a scalar field ϕ in AdS_{d+1}, written in Poincaré coordinates:

$$ds^2 = \frac{R^2}{x_0^2}\left(-dt^2 + \sum_{i=1}^{d-1} dx_i^2 + dx_0^2\right), \tag{1.71}$$

with the boundary located at $x_0 \to 0$. The scalar satisfies the Klein-Gordon equation,

$$\Box\phi - m^2\phi = 0, \tag{1.72}$$

which near the boundary has solutions of the form

$$\phi(x_i, x_0) \sim x_0^{\Delta_\pm}\phi_0(x_i), \quad \Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2R^2}, \tag{1.73}$$

with $\phi_0(x_i)$ identified as the boundary value. Requiring scale invariance under

$$x_0 \to \lambda x_0, \quad x_i \to \lambda x_i \tag{1.74}$$

(the dilatations) imposes that

$$\phi_0(\lambda x_i) = \lambda^{-\Delta}\phi_0(x_i), \tag{1.75}$$

so ϕ_0 acts as a source for a primary operator $\mathcal{O}$ in the CFT with scaling dimension Δ. In standard quantization $\Delta = \Delta_+$ and the mode decaying with exponent Δ_- is treated as the source. For masses in the range

$$-\frac{d^2}{4} < m^2R^2 < -\frac{d^2}{4} + 1, \tag{1.76}$$

alternative quantization is possible, where the roles of the two modes are swapped.

This leads to a more refined version of the partition function identity:

$$Z\left[\phi(x_i, x_0 = 0) = \phi_0(x_i)\right]_{\text{AdS}} = \left\langle \exp\left(\int d^dx\, \phi_0(x)\mathcal{O}(x)\right)\right\rangle_{\text{CFT}}. \tag{1.77}$$

This same logic applies to other fields. For instance, a bulk gauge field A_μ is dual to a conserved current J^μ in the CFT:

$$Z\left[A^\mu(x_i, x_0 = 0) = A_0^\mu(x_i)\right]_{\text{AdS}} = \left\langle \exp\left(\int d^dx\, A_0^\mu(x)J_\mu(x)\right)\right\rangle_{\text{CFT}}. \tag{1.78}$$

Gauge invariance in the bulk translates to current conservation on the boundary: $D_\mu J^\mu = 0$.

Similarly, the boundary value of the bulk metric $g_{\mu\nu}$ acts as the source for the CFT stress-energy tensor $T_{\mu\nu}$.

Correlation functions in the CFT are computed by functional differentiation of the AdS partition function. For example, from (1.77), correlation functions of scalar operators $\mathcal{O}$ are given by

$$\langle\mathcal{O}(x_1)\ldots\mathcal{O}(x_n)\rangle = \left.\frac{\delta^n Z_{\text{AdS}}[\phi_0(x)]}{\delta\phi_0(x_1)\ldots\delta\phi_0(x_n)}\right|_{\phi_0=0}. \tag{1.79}$$

2

Scale separation in AdS vacua

Contents

In this chapter, we introduce scale-separated AdS vacua. We first comment on general features of holographic duals to AdS vacua with scale separation, and very briefly review the literature on scale separation in AdS/CFT. We then present explicit examples of string compactifications that realize this property. The focus is on cases where there is still an unbounded flux (which generates the separation of scales and the parametric control), accompanied by additional, yet still *classical*, ingredients such as multiple bounded fluxes and orientifold planes. The primary example is the DGKT [20] AdS_4 construction. We also discuss related 4d models connected via T-duality [48], as well as a closely analogous 3d construction [22]. We do not consider constructions that rely on inherently quantum, non-perturbative effects, such as the KKLT [18] or LVS [21] vacua.

An obvious requirement for flux compactification vacua to resemble our universe is that the Hubble length scale be much larger than the Kaluza-Klein scale of the internal dimensions:

$$\frac{L_{KK}}{L_{\text{Hubble}}} \ll 1. \tag{2.1}$$

We refer to this as the *scale separation* condition. Equivalently, one can express it in terms of the cosmological constant Λ as

$$\frac{m_{KK}^2}{\Lambda} \gg 1, \tag{2.2}$$

where both m_{KK} and Λ are expressed w.r.t. the Planck scale.

This condition is crucial for obtaining a well-defined lower-dimensional effective theory: the KK modes must be sufficiently heavy to be integrated out, leaving only a finite number of light fields in the action.

Although our ultimate goal is to obtain de Sitter vacua with scale separation, achieving any dS vacuum in string theory is already challenging. Therefore, it is worthwhile to first investigate the possibility of scale separation in AdS vacua, which often serve as an intermediate step in constructing dS vacua.

Nevertheless, obtaining scale-separated AdS vacua remains a significant challenge, and there is currently no consensus regarding the consistency of the known examples, despite several promising constructions [18, 20, 21]. These constructions either involve exotic ingredients (such as intersecting O-planes, for which a complete 10d description is lacking) or rely on steps that are not fully explicit and may require extreme fine-tuning. This contrasts with the Freund-Rubin vacua, where the AdS and KK scales are naturally of the same order.

2.1 Holographic duals of scale-separated AdS vacua

An interesting new perspective on the existence of scale-separated AdS vacua is offered by their holographic duals. The CFT duals of such vacua are expected to have an unconventional operator spectrum, unlike that from any known CFT.

For a scale-separated CFT, the spectrum of single-trace primary operators generally follows this pattern (with single-trace operators corresponding to single-particle states in AdS):

- A vacuum operator,
- The energy-momentum tensor and a few light scalar operators (representing moduli and light axions), possibly accompanied by some light fermions,
- A large gap that separates these light operators from a tower of heavy operators: such as those dual to Kaluza-Klein (KK) modes, wrapped D-brane states, and potentially black hole states.

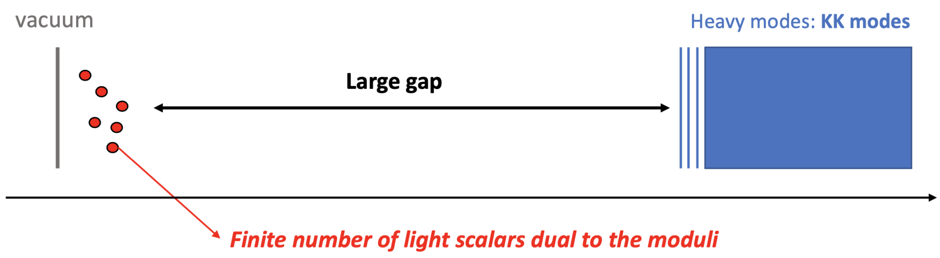

Figure 2.1: *A schematic scale-separated spectrum: a finite set of light scalars (moduli) is separated by a large gap from heavy KK modes. The conformal dimension increases from left to right.*

The presence of such a large gap in the single-trace spectrum is a very peculiar feature of scale-separated CFTs. No known CFT exhibits this kind of gap [49], which underlines the novelty of scale-separated CFTs, if they were to be found.

In a sense, these spectra can be viewed as a less extreme version of the pure gravity spectrum. In pure gravity, aside from the energy-momentum tensor, there are essentially no light operators until one reaches the black hole states which are heavier than the KK modes. This sparsity potentially makes duals to pure gravity easier to rule out [50] and could come together with the presence of global symmetries [51].

It would be highly interesting to either rule out the existence of scale-separated CFTs using bootstrap-like techniques or to prove the existence of an entirely new class of CFTs.

Early literature on scale separation in AdS/CFT includes [52–54]. In [52], an argument is made for capturing the KKLT degrees of freedom holographically via string junctions. Reference [53] presents a first holographic analysis of DGKT vacua, discussing coarse features of the dual spectrum and examining the moduli space of D4-brane domain walls. [54] attempts to construct a new scale-separated AdS/CFT pair by introducing 7-branes into a Freund-Rubin setup.

More recently, the holographic KKLT degrees of freedom have been revisited [55, 56], with suggestions that the central charge is too large to permit well-controlled vacua. The DGKT D4-brane moduli space was also reanalyzed in [57], which points to a potential tension with the Weak Gravity Conjecture for membranes [58, 59].

From a bootstrap perspective, progress was made in [60, 61]. In particular, [60] derives a formula that links the number of large AdS dimensions to CFT data via AdS loop amplitudes. Reference [61] places nontrivial bounds on scale separation in 4d $\mathcal{N} = 2$ superconformal field theories (SCFTs). It is expected that scale-separation can be ruled out in SCFTs with a non-trivial R-symmetry [60, 61].

2.2 Examples of Scale-Separated AdS Vacua

2.2.1 DGKT vacua

The DGKT vacua by DeWolfe, Giryavets, Kachrua and Taylor [20] are a class of $\mathcal{N} = 1$ supersymmetric 4d AdS vacua that are constructed by compactifying *massive IIA string theory* on a Calabi-Yau manifold in the presence of fluxes.

Massive type IIA string theory *Massive IIA string theory* [62] extends ordinary IIA string theory by introducing the Romans mass parameter, m. Through a Higgs-like mechanism, this parameter gives the NS-NS field B_2 a mass proportional to m. The Romans mass can be interpreted as a F_0-form or equivalently a F_{10}-form field strength, representing a flux uniformly distributed throughout 10d spacetime. Massive IIA string theory can be connected to ordinary IIA string theory via a domain wall consisting of a D8-brane.

The Romans mass effectively acts as a 10d cosmological constant, facilitating scale separation in compactifications of the form $\text{AdS}_d \times M$. In the absence of such a cosmological constant, the curvature of M would need to balance the negative curvature of AdS to satisfy the 10d Einstein equations.

However, the inclusion of the Romans mass also introduces challenges. Massive IIA string theory is believed to lack a strongly coupled regime [63], precluding its uplift to M-theory. We will discuss related challenges in more detail later.

Internal manifold The DGKT construction works for any Calabi-Yau manifold as an internal manifold. To give a concrete example, we will mostly focus on the

$$\frac{T^6}{\mathbb{Z}_3 \times \mathbb{Z}_3} \tag{2.3}$$

orientifold. This manifold is constructed starting from a six-torus $T^6 = T^2 \times T^2 \times T^2$, defined by the following identifications in complex coordinates z^i,

$$z_i \sim z_i + 1 \sim z_i + \alpha, \quad \alpha = e^{\frac{\pi i}{3}}. \tag{2.4}$$

Consequently, this T^6 is quotiented by two $\mathbb{Z}_3$ actions,

$$(z_1, z_2, z_3) \to \alpha^2 (z_1, z_2, z_3), \tag{2.5}$$

$$(z_1, z_2, z_3) \to \left(\alpha^2 z_1 + \frac{1+\alpha}{3}, \alpha^4 z_2 + \frac{1+\alpha}{3}, z_3 + \frac{1+\alpha}{3}\right). \tag{2.6}$$

In addition, we should also mod out by

$$\mathcal{O} = \Omega_p (-1)^{F_L} \sigma, \tag{2.7}$$

where Ω_p is the worldsheet parity, F_L is the number of left-moving fermions, and σ is the projection,

$$\sigma : z_i \to -\overline{z}_i. \tag{2.8}$$

This will also ensure that supersymmetry is broken to $\mathcal{N} = 1$ in four dimensions. The fixed locus of σ (a three-cycle in the internal manifold) corresponds to an O6-plane that wraps this cycle and fills the non-compact dimensions.

The resulting set of moduli is very simple. There are three Kähler moduli, corresponding to the sizes of the T^2's, v_i, which come together with B_2-axions. There is one dilaton, that is paired up with the C_3-axion, and there are no complex structure moduli (this is a rigid manifold). Additionally, there are 9 singular orbifold points, whose resolution comes together with 9 additional moduli, leading to a total of $h^{1,1} = 12$.

Fluxes So far, we have considered two ingredients: the Romans mass F_0 and the presence of O6-planes. The Bianchi identity for F_2 in the presence of these ingredients takes the form

$$dF_2 = H_3 \wedge F_0 + \delta_{O6}, \tag{2.9}$$

where δ_{O6} represents the localized contribution from O6-planes, which act as magnetic sources for F_2. As a consequence, the flux H_3 must be nonzero, since integrating both sides of the Bianchi identity over the compact space gives

$$\int H_3 \wedge F_0 = -\int \delta_{O6} \neq 0. \tag{2.10}$$

This imposes a constraint on the allowed values of H_3 and F_0: since the number of O6-planes is finite, these fluxes cannot grow arbitrarily large. Consequently, their values are bounded.

To achieve full moduli stabilisation, an additional ingredient is required: the F_4 flux. Unlike H_3 and F_0, the F_4 flux is not subject to any Bianchi identity constraints, meaning it remains unbounded.

Effective scalar potential from toroidal compactification In the presence of Romans mass m, the p-form fields (1.7) are modified to

$$\begin{aligned}\tilde{H}_3 &= H_3^0 + dB_2\,,\\ \tilde{F}_2 &= dC_1 + m\,B_2\,,\\ \tilde{F}_4 &= F_4^0 + dC_3 - C_1\wedge H_3 - \frac{m}{2}\,B_2\wedge B_2\,.\end{aligned} \tag{2.11}$$

where H_3^0 and F_4^0 denote the background value of the fluxes which are turned on. The 10d action (1.7) then becomes

$$S_{10d} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\,\left[e^{-2\phi}\left(R + 4\,(\partial\phi)^2 - \frac{1}{2}\,|\tilde{H}_3|^2\right) - \frac{1}{2}m^2 - \frac{1}{2}\,|\tilde{F}_2|^2 + |\tilde{F}_4|^2\right] + \text{Chern-Simons terms} + S_{O_6}\,, \tag{2.12}$$

where we have included a contribution from the O6-planes given by (1.46).

After dimensional reduction on a toroidal $T^6/\mathbb{Z}_3^2$ orbifold, the low-energy scalar potential for the moduli is

$$V = \frac{h^2}{4}\frac{e^{2\phi}}{\mathcal{V}^2} + \frac{1}{2}\left(\sum_{i=1}^{3} f_i^2 v_i^2\right)\frac{e^{4\phi}}{\mathcal{V}^3} + \frac{m^2}{2}\frac{e^{4\phi}}{\mathcal{V}} - \sqrt{2}|mh|\frac{e^{3\phi}}{\mathcal{V}^{3/2}}, \tag{2.13}$$

where f_i ($i = 1,2,3$ for the three sub-tori) and h are the quantized magnitudes of the F_4 and H_3 fluxes and

$$\mathcal{V} = \kappa v_1 v_2 v_3 \tag{2.14}$$

is the overall internal volume in string units, κ being a constant. In (2.13) we have omitted the contributions involving axionic fields, since all axion vevs vanish. We have also left out the terms from blow-up modes, as these can be stabilised by fluxes localized on the blow-up cycles, and in the limit of large F_4-flux their interaction with the other moduli becomes negligible [20].

In Figure 2.2, we show an illustrative plot of this potential as a function of the volume modulus and dilaton. We see that there indeed is a stable mimimum with negative vacuum energy. This minimum will turn out to be a supersymmetric minimum (see next subsection). The minimum is located at,

$$v_i = \frac{1}{|f_i|}\sqrt{\frac{5}{3}\left|\frac{f_1 f_2 f_3}{\kappa m}\right|}, \quad e^{\phi} = \frac{3}{4}|h|\left(\frac{5}{12}\frac{\kappa}{|m f_1 f_2 f_3|}\right)^{1/4}, \tag{2.15}$$

with corresponding vacuum energy

$$V_0 = -\sqrt{\frac{4}{15}}\left(\frac{27}{160}\right)^2 \kappa^3 \frac{h^4|m|^{5/2}}{|f_1 f_2 f_3|^{3/2}}. \tag{2.16}$$

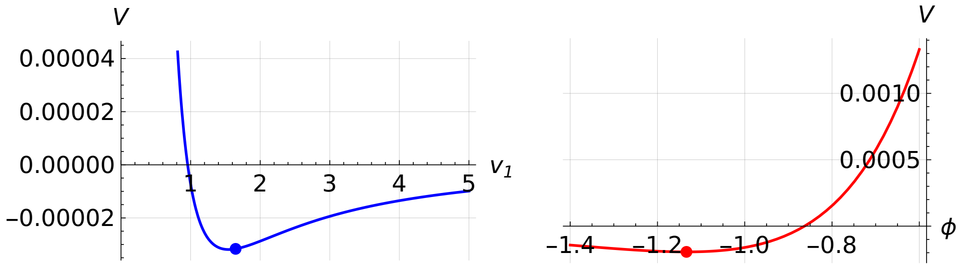

Figure 2.2: *Potential $V(v_1)$ and $V(\phi)$ with minimum points marked.*

Effective potential for a general Calabi–Yau internal manifold The procedure to obtain AdS minima with all moduli stabilised, as discussed in the previous subsection, can be extended to arbitrary Calabi–Yau internal manifolds. It is convenient to formulate this in a manner that makes the underlying $\mathcal{N}=1$ supersymmetry manifest, using a Kähler potential and a superpotential. The internal manifold contains $h^-_{1,1}$ orientifold-odd two-cycles, with associated complex Kähler moduli T_i, as well as the axio-dilaton S:

$$T_i = b_i + iv_i, \qquad S = \xi + i\frac{e^{-D}}{\sqrt{2}}, \tag{2.17}$$

where b_i are the B_2-axions and ξ is a C_3-axion. The internal volume is expressed as

$$\mathcal{V} = \kappa_{abc}\, v_a v_b v_c, \tag{2.18}$$

with κ_{abc} the triple intersection numbers.

The Kähler potential then takes the form

$$K = -\log \mathcal{V} - 4\log e^{-D}, \tag{2.19}$$

where D denotes the 4d dilaton. The superpotential is given by

$$W = f_a T^a - \frac{m}{6}\,\kappa_{abc}\, T_a T_b T_c - hS. \tag{2.20}$$

This formulation provides a general expression for the scalar potential, explicitly including the axion dependence.

In addition, the compactification has $h^{2,1}$ complex structure moduli,

$$U_\alpha = a_\alpha + iu_\alpha, \tag{2.21}$$

where a_α are the remaining C_3-axions. These moduli do not appear explicitly in the superpotential.

The F-term equations

$$D_{T^a}W = D_S W = D_{U^\alpha}W = 0 \tag{2.22}$$

admit solutions in general. In particular, for the simple case of intersection numbers $\kappa_{123} = \kappa$ with all others vanishing, the solution reproduces the result (2.15).

Parametric Control and Scale Separation As the F_4-flux is unconstrained, we can take the limit $f_i \to \infty$. From (2.15), we see that this results in a parametrically large internal volume and a parametrically small string coupling, allowing us to enter a regime of parametric control where higher-derivative and string-loop corrections are suppressed. The vacuum energy is also parametrically small in this limit.

The string scale is given by

$$M_s = M_p \frac{e^{\phi}}{\sqrt{\mathcal{V}}} = M_p \frac{3\sqrt{\frac{3}{10}} h\kappa m}{4\sqrt{f_1 f_2 f_3 \kappa m}}, \tag{2.23}$$

where M_p is the 4d Planck scale. From this, we deduce the Kaluza-Klein length scale in Planck units,

$$l_{KK} = \kappa^{1/6} \max_i \left(\sqrt{v_i}\right) l_s = \frac{4\left(\frac{5}{3}\right)^{3/4}\sqrt{2}}{3hm^{3/4}\kappa^{7/12}} \frac{(f_1 f_2 f_3)^{3/4}}{\min_i (f_i)^{1/2}} l_p, \tag{2.24}$$

which we take to be the radius of the largest cycle.

From equation (2.16), we also find that the AdS length scale is

$$l_{\text{AdS}} = \sqrt{\frac{-3}{V_0}} l_p = \frac{80}{9}\sqrt{\frac{2}{3}} 15^{1/4} \frac{|f_1 f_2 f_3|^{3/4}}{\kappa^{3/2} h^2 |m|^{5/4}} l_p, \tag{2.25}$$

which leads to the ratio

$$\frac{l_{KK}}{l_{\text{AdS}}} = \frac{1}{4}\sqrt{\frac{3}{5}} h\kappa^{11/12} \sqrt{\frac{m}{f_1}}. \tag{2.26}$$

This ratio is indeed small in the limit $f_i \to \infty$, indicating scale separation.

For future reference, we summarize the leading scalings in this large-flux limit for the isotropic case $f_i = N$,

$$\mathcal{V} \sim N^{3/2}, \quad e^{\phi} \sim N^{-3/4}, \quad \frac{l_{\text{AdS}}}{l_p} \sim N^{9/4}, \quad \frac{l_{KK}}{l_{\text{AdS}}} \sim N^{-1/2}. \tag{2.27}$$

Non-supersymmetric DGKT-like vacua From (2.13), we observe that the moduli potential is independent of the signs of the fluxes f_i. This implies that new vacua with the same energy can be obtained by flipping these signs. However, the supersymmetry conditions are sign-sensitive, requiring

$$\text{sgn}(m f_1 f_2 f_3) < 0, \quad \text{sgn}(m f_i) < 0 \tag{2.28}$$

for supersymmetry to be preserved.

The non-supersymmetric vacua obtained in this manner are perturbatively stable and also remain non-perturbatively stable within the thin-wall approximation [64]. However, recent work [65] suggests a potential instability mediated by D8-branes, which becomes visible only beyond the approximation where the O6-planes are considere fully smeared.

Potential issues The main concern regarding the DGKT vacua is that the solutions are obtained within 4d effective field theory, without a verification that the full 10d equations are satisfied [66, 67].

This issue is complicated by the presence of Romans mass. In general, fluxes induce deformations in a Calabi-Yau manifold, causing it to deviate from strict Calabi-Yau conditions. However, fluxes typically dilute over space, allowing the internal manifold to remain approximately Calabi-Yau in the large-volume limit. The Romans mass is an exception: it does not dilute.

It was shown in [68] that the 4d effective solutions satisfy the 10d equations if the O6-sources are smeared. This approximation assumes a uniform distribution of the O6-plane energy density across the internal manifold rather than localisation at the orientifold locus. Progress has been made in demonstrating that the equations remain valid at higher orders in the smearing expansion [69–71]. At these higher orders, the internal manifold is no longer Calabi-Yau but instead a more general $SU(3) \times SU(3)$ structure is required.

While the backreaction of a single O6-plane in massive IIA may be unproblematic [72], the DGKT solution involves intersecting O6-planes, significantly complicating the analysis. However, once the orbifold singularities are blown-up they do not need to be intersecting [73].

In summary, although substantial progress has been made in addressing these consistency issues, the validity of DGKT vacua in fully localized 10d massive IIA backgrounds remains an open question.

2.2.2 Performing a double-T-duality on the DGKT vacua

The presence of Romans mass obscures some of the analysis of the DGKT vacua. Fluxes turn into different kind of fluxes after a T-duality, so we can try to find T-duals of DGKT that do not involve Romans mass. After a single T-duality, we end up in IIB string theory, but it turns out that there will be cycles in the internal manifold that are too small for the supergravity analysis to be valid [48]. With a double-T-duality on the other hand, we end up in IIA string theory again, but without the Romans mass turned on. New scale-separated AdS_4 solutions were obtained this way [48]. In some limits of the fluxes, also strongly coupled solutions can be obtained which can then be uplifted to M-theory, potentially providing an interesting new perspective. As a result, the backreaction of O-planes is better understood in this context.

Flux potential and scaling relations To illustrate this transformation in the simplest setting, consider a toroidal internal manifold. The DGKT superpotential initially takes the form

$$W \;=\; c_{F_{4,1}}\, T_1 + c_{F_{4,2}}\, T_2 + c_{F_{4,3}}\, T_3 + c_{F_0}\, T_1 T_2 T_3 + c_{H_3}\, S, \tag{2.29}$$

where the coefficients c are linearly dependent on the fluxes. Up to additive constants, the Kähler potential is

$$K \;=\; -\log\Big(s^4\, u_1 u_2 u_3\Big)\,. \tag{2.30}$$

Performing a double T-duality on the first two-torus, T_1^2, acts as $t_1 \mapsto 1/t_1$ on the real part of T_1. In terms of the chiral multiplet T_1, this leads to the effective replacement $T_1 \mapsto 1/T_1$. The superpotential and Kähler potential then become

$$W \;=\; \frac{c_{F_{4,1}}}{T_1} + c_{F_{4,2}}\, T_2 + c_{F_{4,3}}\, T_3 + c_{F_0}\, \frac{T_2 T_3}{T_1} + c_{H_3}\, S, \qquad K \;=\; -\log\bigg(s^4\, \frac{u_2 u_3}{u_1}\bigg)\,. \tag{2.31}$$

This expression for K looks different from the original one. However, the scalar potential is invariant under Kähler transformations of the form

$$K \;\to\; K + f(T_1) + \overline{f(T_1)}, \tag{2.32}$$

$$W \;\to\; e^{-f(T_1)} W, \tag{2.33}$$

with f a holomorphic function. Choosing $f(T_1) = \log T_1$ precisely restores the original functional form of the Kähler potential, at the cost of redefining the superpotential. Explicitly, this yields

$$W \;=\; c_{F_{4,1}} + c_{F_{4,2}}\, T_1 T_2 + c_{F_{4,3}}\, T_1 T_3 + c_{F_0}\, T_2 T_3 + c_{H_3}\, S T_1, \qquad K \;=\; -\log\Big(s^4\, u_1 u_2 u_3\Big)\,. \tag{2.34}$$

One leg of the original F_4-flux now behaves like an F_6-flux, while the other legs transform into F_2-fluxes. The Romans mass itself behaves like an F_2-flux, and the original H_3-flux now plays the role of geometric flux, introducing curvature into the internal manifold. This transformation is summarized in Figure 2.3. For simplicity, let's set $t_2 = t_3$ and $F_{4,2} = F_{4,3}$, and relabel fluxes for clarity

$$W = c_{F_6} + c_{F_{2,1}} T_1 T_2 + c_{F_{2,2}} T_2^2 + c_{\text{curv}} S T_1, \quad K = -\ln s^4 u_1 u_2^2. \tag{2.35}$$

The internal manifold, now an Iwasawa manifold with $SU(3)$-structure, loses the simpler Calabi-Yau structure. The distribution of F_2-fluxes is uneven: $F_{2,2}$ is constrained by the tadpole condition, while $F_{2,1}$ (and its equivalent leg) can be made arbitrarily large. This

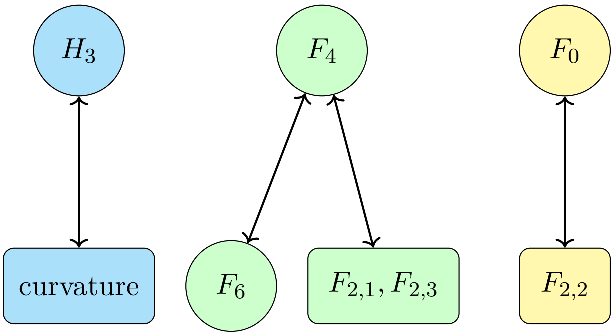

Figure 2.3: *Double-T-transformation of fluxes*

asymmetry inevitably leads to an anisotropic geometry, with one sub-torus becoming parametrically larger than the others.

To understand scaling behaviors, we label fluxes explicitly

$$c_{F_6} \sim N \to \infty, \quad c_{F_{2,1}} \sim k_1 \to \infty, \quad c_{F_{2,2}} \sim k_2 = \text{fixed}, \tag{2.36}$$

allowing N and k_1 to grow arbitrarily while keeping k_2 fixed due to tadpole constraints. At the potential minimum, moduli scale as

$$u_1 \sim \frac{N^{1/2}\, k_2^{1/2}}{k_1}, \quad u_2 \sim \frac{N^{1/2}}{k_2^{1/2}}, \quad s \sim N^{1/2}\, k_1\, k_2^{-1/2}. \tag{2.37}$$

This leads to the following scaling for overall volume and string coupling

$$\mathcal{V} \sim u_1 u_2^2 \sim \frac{N^{3/2}}{k_1\, k_2^{1/2}}, \quad g_s \sim s^{-1}\mathcal{V}^{1/2} \sim N^{1/4}\, k_1^{-3/2}\, k_2^{1/4}. \tag{2.38}$$

The relation between the string length and the 4d Planck length scale is

$$l_s = l_{p,4}\sqrt{\mathcal{V}}e^{-\phi} = l_{p,4}N^{1/2}k_1 k_2^{-1/2}. \tag{2.39}$$

If we associate the KK scale with the largest cycle (u_2), we get

$$l_{KK} = u_2^{1/2} l_s = N^{3/4} k_1 k_2^{-3/4} l_{p,4}, \tag{2.40}$$

while the AdS scale behaves as

$$l_{\mathrm{AdS}} = N^{3/4} k_1^{3/2} k_2^{-5/4} l_{p,4}. \tag{2.41}$$

The unbounded F_2-flux (k_1) acts as the driver for scale separation, with the scale separation ratio given by

$$\frac{l_{AdS}}{l_{KK}} \sim k_1^{1/2} k_2^{-1/2}. \tag{2.42}$$

Purely geometric scale separation? From an M-theory perspective, all fluxes but F_6 will be geometrized. Interestingly, this solution is then of the simple Freund-Rubin type. The F_2-flux responsible for the separation of scales is turned into geometry. Hence, the question of whether flux vacua can truly be scale-separated boils down to a purely geometric question: does there exist a 7-dimensional manifold with the appropriate weak G_2-structure? Progress towards these questions can be found in [74].

2.2.3 Scale-separated AdS_3 vacua

In [48], scale-separated AdS_3 vacua were obtained from massive IIA string theory in a way that mimics the DGKT [20] construction: the involved fluxes are unbounded F_4-fluxes, together with H_3-flux and Romans mass F_0 that are bounded by a tadpole, and there are O6-planes as well. Instead of compactifying on a Calabi-Yau three-fold, we instead reduce on a manifold of G2 holonomy. By taking the limit of large F_4-flux, parametric weak coupling, large volume and scale separation are obtained.

In this section, we review this construction [22] and extend the discussion of the spectrum of light fields to the case of vectors.

The internal G2 Holonomy Manifold G2 holonomy manifold are 7-dimensional Ricci-flat manifolds typically used in M-theory compactifications to 4d that preserve some amount of supersymmetry [39].

A precise definition is as follows. A G_2 *holonomy manifold* is a 7-dimensional Riemannian manifold (X, g) whose holonomy group is the exceptional Lie group G_2. These manifolds admit a *torsion-free* G_2*-structure*, i.e. a smooth 3-form Φ such that

$$d\Phi = 0, \qquad d{*}\Phi = 0,$$

and at each point $p \in X$, the triple (T_pX, g, Φ) is isomorphic to $(\mathbb{R}^7, g_0, \Phi_0)$, where Φ_0 is the standard G_2 3-form on $\mathbb{R}^7$, given by

$$\Phi_0 = dy_{123} + dy_{145} + dy_{167} + dy_{246} - dy_{257} - dy_{347} - dy_{356},$$

$$*\Phi_0 = dy_{4567} + dy_{2367} + dy_{2345} + dy_{1357} - dy_{1346} - dy_{1256} - dy_{1247}.$$

For concreteness, we will consider an explicit G2 manifold obtained by orientifolding and orbifolding a 7-torus T^7 (see [75] for the first constructions of such manifolds). The coordinates on T^7 are y^m $(m = 1, \ldots, 7)$, with the following identifications,

$$y^m \simeq y^m + 1. \tag{2.43}$$

We consider the quotient of T^7 by the orbifold group $\Gamma \simeq \mathbb{Z}_2^3$, generated by the involutions

$$\Theta_\alpha : (y^1, \dots, y^7) \mapsto (-y^1, -y^2, -y^3, -y^4, y^5, y^6, y^7), \tag{2.44}$$

$$\Theta_\beta : (y^1, \dots, y^7) \mapsto (-y^1, c - y^2, y^3, y^4, -y^5, -y^6, y^7), \tag{2.45}$$

$$\Theta_\gamma : (y^1, \dots, y^7) \mapsto (c - y^1, y^2, c - y^3, y^4, -y^5, y^6, -y^7). \tag{2.46}$$

For simplicity, we set $c = 0$ in what follows. Since these involutions do not act freely on one another, the resulting quotient is a singular orbifold without a known geometric way to resolve the orbifold singularities. By contrast, for $c = 1/2$ one obtains a smooth G2 Joyce orbifold [38], whose AdS_3 vacua were studied in [76].

All of these manifolds share a universal untwisted sector with Betti numbers

$$b_0 = 1, \quad b_1 = 0, \quad b_2 = 0, \quad b_3 = 7. \tag{2.47}$$

Hence, the untwisted spectrum contains eight real scalars: the dilaton together with seven volume moduli. The contribution from the twisted sectors depends on the resolution of the orbifold singularities.

We express the internal metric in Einstein frame as

$$ds^2 = \sum_m (r^m)^2 (dy^m)^2, \tag{2.48}$$

where the radii r^m of the different cycles appear explicitly. The G2 calibration is then written as

$$\Phi = s^i \Phi_i, \quad \Phi_i = \left(dy^{127}, -dy^{347}, -dy^{567}, dy^{136}, -dy^{235}, dy^{145}, dy^{246}\right), \tag{2.49}$$

with combinations of radii $s^1 = r^1 r^2 r^7$, $s^2 = r^3 r^4 r^7$, and so forth. The co-associative calibration is then

$$*\Phi = \sum_{i=1}^{7} \frac{V_E}{s^i} \Psi_i, \quad \Psi_i = \left(dy^{3456}, -dy^{1256}, -dy^{1234}, dy^{2457}, -dy^{1467}, dy^{2367}, dy^{1357}\right), \tag{2.50}$$

with the Einstein-frame volume given by $V_E = r^1 r^2 \dots r^7$.

The orbifold T^7/Γ is further quotiented by the following involution

$$\sigma : (y^1, \dots, y^7) \to (-y^1, -y^2, -y^3, -y^4, -y^5, -y^6, -y^7), \tag{2.51}$$

which has 2^7 fixed points, leading to 2^7 O2-planes at the points $y^i = 0, 1/2$. Combining this involution with the orbifold group induces the O6-involutions

$$\begin{aligned} \Theta_\alpha \sigma &: y^i \to (y^1, y^2, y^3, y^4, -y^5, -y^6, -y^7), \\ \Theta_\beta \sigma &: y^i \to (y^1, y^2, -y^3, -y^4, y^5, y^6, -y^7), \\ \Theta_\gamma \sigma &: y^i \to (y^1, -y^2, y^3, -y^4, y^5, -y^6, y^7), \end{aligned} \tag{2.52}$$

and products thereof. This results in 7 different directions for O6-planes

$$\begin{pmatrix} \mathrm{O6}_{\alpha}: & \times & \times & \times & \times & - & - & - \\ \mathrm{O6}_{\beta}: & \times & \times & - & - & \times & \times & - \\ \mathrm{O6}_{\gamma}: & \times & - & \times & - & \times & - & \times \\ \mathrm{O6}_{\alpha\beta}: & - & - & \times & \times & \times & \times & - \\ \mathrm{O6}_{\beta\gamma}: & - & \times & \times & - & - & \times & \times \\ \mathrm{O6}_{\gamma\alpha}: & - & \times & - & \times & \times & - & \times \\ \mathrm{O6}_{\alpha\beta\gamma}: & \times & - & - & \times & - & \times & \times \end{pmatrix}. \tag{2.53}$$

which are intersecting. With the introduction of these O-planes, the resulting 3d theory will be minimally supersymmetric.

Fluxes The Bianchi identities in the presence of O2– and O6–planes,

$$dF_2 = H_3 \wedge F_0 + \delta_{O6}\,, \tag{2.54}$$

$$dF_4 = H_3 \wedge F_2\,, \tag{2.55}$$

$$dF_6 = H_3 \wedge F_4 + \delta_{O2}\,, \tag{2.56}$$

show that one may take

$$H_3 \neq 0, \quad F_0 \neq 0, \quad F_2 = 0.$$

The O2–tadpole in the third equation can be canceled by adding D2–branes, leaving the residual condition

$$H_3 \wedge F_4 = 0\,.$$

Expanding the fluxes in a harmonic basis.

$$H_3 = h\sum_i \Phi_i\,, \qquad F_4 = \sum_i f^i\,\Psi_i\,, \qquad F_0 = m\,, \tag{2.57}$$

the tadpole constraints imply

$$h\,m = \mu_{O6}\,, \qquad \sum_i f^i = 0\,, \tag{2.58}$$

where μ_{O6} is the total O6–plane charge. The integers h, m and f^i must satisfy the usual quantization conditions (see [22]). A simple choice of fluxes satisfying this condition is

$$f^i = (-f,-f,-f,-f,-f,-f,+6f). \tag{2.59}$$

Stabilisation of Scalars The scalar fields to be stabilised include the seven size moduli s_i for $i = 1, \dots, 7$, along with the dilaton ϕ. The total internal volume in Einstein frame is given by

$$V_E = \prod_{i=1}^{7} r_i = \left(\prod_{i=1}^{7} s_i\right)^{1/3} = \frac{1}{7}\int \Phi \wedge *\Phi \equiv e^{7\beta v}, \quad \text{with} \quad \beta = \frac{1}{4\sqrt{7}}. \tag{2.60}$$

To simplify notation, we redefine the moduli by introducing two orthonormal combinations of the dilaton ϕ and the volume modulus v, defined as:

$$\frac{x}{\sqrt{7}} = -\frac{3\phi}{8} + \frac{\beta v}{2}, \qquad 2y = -21\beta v - \frac{\phi}{4}. \tag{2.61}$$

The six remaining shape moduli are rescaled as

$$\tilde{s}^a = V_E^{3/7} s^a, \quad a = 1, \dots, 6. \tag{2.62}$$

We also define $s^7 = \prod_{a=1}^{6} \frac{1}{\tilde{s}^a}$ such that the $\tilde{s}$ represent rescaled moduli of a unit-volume internal space. In 3d $\mathcal{N} = 1$ supergravity (with two real supercharges), the scalar potential is derived from a real superpotential $P(\phi)$ via

$$V(\phi) = G^{IJ} P_I P_J - 4P^2, \tag{2.63}$$

where $P_I \equiv \partial_I P$ and G_{IJ} is the scalar field space metric, given by

$$G_{IJ} = \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & G_{ij} \end{pmatrix}, \qquad G_{ij} = \frac{1}{4(\tilde{s}^i)^2}\delta_{ij}. \tag{2.64}$$

The general form of the superpotential reads

$$P = \frac{m^2}{2} e^{\frac{y}{2} - \frac{\sqrt{7}}{2}x} + \frac{1}{8} e^{y + \frac{x}{\sqrt{7}}} \int \star\Phi \wedge H_3\, e^{-4\beta v} + \frac{1}{8} e^{y - \frac{x}{\sqrt{7}}} \int \Phi \wedge F_4\, e^{-3\beta v}. \tag{2.65}$$

For our specific flux background, this reduces to:

$$P = \frac{m}{8} e^{\frac{y}{2} - \frac{\sqrt{7}x}{2}} + \frac{h}{8} e^{y + \frac{x}{\sqrt{7}}} \sum_{i=1}^{7} \frac{1}{\tilde{s}^i} + \frac{1}{8} e^{y - \frac{x}{\sqrt{7}}} \sum_{i=1}^{7} f^i \tilde{s}^i. \tag{2.66}$$

The simplest supersymmetric AdS vacua studied in [22] assume all rescaled moduli $\tilde{s}^a$ (with $a = 1, \dots, 6$) take the same value σ, i.e.

$$\langle \tilde{s}^a \rangle = \sigma. \tag{2.67}$$

We then define the dimensionless combinations

$$a = \frac{h}{f} V_E^{1/7} e^{-3\phi/4}, \quad b = \frac{m}{f} e^{\phi} V_E^{4/7}, \tag{2.68}$$

which evaluate numerically to

$$a = 0.515696\ldots, \quad b = 3.43111\ldots, \quad \sigma = 1.32691\ldots \tag{2.69}$$

The vacuum energy is then given by

$$\langle V \rangle = -\frac{1}{64a^6b^4}\left(6\sigma^2 + \frac{36}{\sigma^{12}}\right)\frac{m^4h^6}{f^8}. \tag{2.70}$$

The stabilisation of the blow-up moduli has not yet been studied, but it is expected, by analogy with the DGKT vacua, that their stabilization will have little effect on the stabilisation of the remaining moduli.

Scale separation We find the following f-scalings from (2.68),

$$g_s = e^{\phi} \sim f^{-3/4}\,, \quad V_E \sim f^{49/16}\,, \tag{2.71}$$

and thus our vacuum corresponds to weak string coupling and to large volume for large f. The volume in 10D string frame scales as

$$\mathcal{V} \sim f^{7/4} \tag{2.72}$$

which also grows large. The Kaluza-Klein length scale compared to the 3d Planck scale l_p then reads

$$L_{KK} = \mathcal{V}^{1/7}l_s = \mathcal{V}^{1/7}(l_p \cdot \mathcal{V}/e^{2\phi}) \sim f^{7/2}l_p, \tag{2.73}$$

while the AdS radius scales as

$$L_{AdS} \sim l_p\sqrt{\langle V \rangle}^{-1} \sim f^4 l_p, \tag{2.74}$$

resulting in an arbitrarily large scale separation

$$\frac{L_{KK}}{L_{AdS}} \sim f^{-1/2}\,, \tag{2.75}$$

at arbitrary *small coupling and large volume.*

Axion content Reference [22] did not investigate the axion content of the resulting vacua. We fill this gap by demonstrating that all axions are removed by the background fluxes. Axions can originate from several sources: D-brane position moduli, dualization of D-brane worldvolume vectors, dualization of spacetime vectors arising from the reduction of C_3 over even 2-cycles, and from the reduction of B_2 over odd 2-cycles. However, since RR tadpole cancellation can be achieved purely via fluxes, no D-branes are needed, and the only possible supergravity-originating axions would arise from the reduction of C_3.

In our setup, the only non-trivial 2-cycles are expected to reside in the *twisted sector*, if present. We now argue that any spacetime vectors arising from the reduction of C_3 over these 2-cycles become massive through Chern-Simons couplings, and therefore no massless axions are generated.

The relevant 10D Chern-Simons term that induces a mass term upon dimensional reduction is

$$S_{10} = -\frac{1}{2}\int_{10} C_3 \wedge H_3^{\rm BG} \wedge dC_3\,. \tag{2.76}$$

Inserting the expansions

$$H_3^{\rm BG} = \sum_{i=1}^{b_3} h^i \Phi_i\,, \quad C_3 = \sum_{a=1}^{b_2} A_\mu^a\, dx^\mu \wedge \omega_a\,, \tag{2.77}$$

we obtain

$$\mathcal{L}_3 = -\frac{1}{2} m_{ab} A^a \wedge dA^b\,, \quad m_{ab} = \lambda_{abj} h^j\,, \tag{2.78}$$

where $a, b = 1, \ldots, b_2$ label the even 2-cycles and $j = 1, \ldots, b_3$. The intersection numbers are given by

$$\lambda_{abj} = \int_7 \omega_a \wedge \omega_b \wedge \Phi_j\,. \tag{2.79}$$

If we use that the flux vacuum has all h_i equal; $h_i = h$, and that we can choose calibrated representatives for the basis of 2-cycles (satisfying $\star\omega = \omega \wedge \Phi$), we find

$$m_{ab} \sim h \int \star\omega_a \wedge \omega_b\,, \tag{2.80}$$

which is generically of full rank. Hence, all vectors obtain a mass and no axions remain in the spectrum.

3
Spectroscopy

Contents

In this chapter, we will analyse the spectrum of the DGKT vacua and some other scale-separated AdS vacua. We will outline the overall scaling of the spectrum, with a particular emphasis on the light sector (highlighted in red in Figure 2.1). Remarkably, for DGKT, the scaling dimensions of the light operators take integer values. We note that this feature cannot be explained by supersymmetry alone and explain that it implies specific polynomial shift symmetries of the moduli in AdS.

3.1 DGKT spectroscopy

3.1.1 Overall spectrum

In this section we provide an overview of the spectrum of the DGKT vacua, with a focus on how the operator dimensions scale with the unbounded F_4-flux (which is proportional to N). This was first discussed in [53]. As expected, the moduli fields have conformal dimensions of order one,

$$\Delta_{\text{moduli}} = N^0. \tag{3.1}$$

The KK-modes have conformal dimensions that scale as

$$\Delta_{KK} \sim \sqrt{N}, \tag{3.2}$$

see Equation (2.27) The DGKT background does not support particle-like states from Dp-branes wrapped on p-cycles. Indeed, in the presence of a nonzero Romans mass m, the Freed–Witten effect [77] requires that a D0-brane must have m fundamental strings ending on it, while a D6-brane must have m NS5-branes attached. On the other hand, D2-branes wrapped on 2-cycles and D4-branes wrapped on 4-cycles are projected out by the orientifold.

3.1.2 Light spectrum: toroidal internal manifold

The light spectrum of operators dual to the moduli and their axions was first computed for a toroidal internal manifold $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$ in [78]. There are four moduli: the dilaton and the sizes of the three sub-tori. This is a rigid manifold, meaning that there are no complex structure moduli. The result for the moduli is

$$\Delta_{\text{saxion}} = (10, 6, 6, 6). \tag{3.3}$$

The axions on the other hand have conformal dimensions equal to

$$\Delta_{\text{axion}} = (11, 5, 5, 5). \tag{3.4}$$

The conformal dimensions do not depend on any specific flux choices.

Also for the non-supersymmetric cousins of the DGKT vacua, simple expressions for the conformal dimensions are found. In particular, by flipping the sign of either one or all three of the F_4-fluxes, the saxion dimensions remain unchanged while the axionic dimensions become

$$\Delta_{\text{axion}} = (8, 8, 8, 1 \text{ or } 2). \tag{3.5}$$

On the other hand, when flipping two fluxes, the spectrum is identical to the supersymmetric one.

3.1.3 Light spectrum: other examples of internal manifolds

It is surprising that [78] found integer conformal dimensions, given the quadratic relation (1.73) between masses in AdS and conformal dimensions. On the other hand, they consider a highly symmetric setting with a rather simple relation between the 2-cycle volumina v_i and the overall volume $\mathcal{V} = v_1 v_2 v_3$. There is an obvious exchange symmetry between the size-moduli here. One might wonder whether this example is special. Furthermore, this example does not include complex structure moduli with $h^{2,1} = 0$. Hence this motivates us to study a broader class of massive type IIA flux compactifications and calculate the operator dimensions for different internal manifolds, also those with complex structure moduli.

We focus on the abelian toroidal orbifolds listed in Table 3.1 (see [79]). Following [79], we restrict ourselves to the bulk moduli, leading to 11 different classes of models (some of which are subclasses of others, with one class corresponding to $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$).

#	$h^{2,1}$	$h^{1,1}_+$	$h^{1,1}_-$	$\kappa,\ \hat\kappa$
I	0	0	3	$\kappa_{123} = 1$
II	0	1	2	$\kappa_{122} = 1,\ \hat\kappa_{111} = -1$
III	0	1	4	$\kappa_{123} = 1,\ \kappa_{144} = -1,\ \hat\kappa_{111} = -1,$
IV	0	2	3	$\kappa_{122} = 1, \hat\kappa_{1\alpha\alpha} = -1,\ \alpha \in \{1,2,3\}$
V	0	3	6	$\kappa_{123} = 1, \kappa_{456} = -2,\ \hat\kappa_{432} = \hat\kappa_{513} = \hat\kappa_{612} = 1,$ $\hat\kappa_{111} = \hat\kappa_{222} = \hat\kappa_{333} = -1,\ \kappa_{144} = \kappa_{255} = \kappa_{366} = -2,$
VI	1	0	3	$\kappa_{123} = 1$
VII	1	1	2	$\kappa_{122} = 1, \hat\kappa_{111} = -1,$
VIII	1	1	4	$\kappa_{123} = 1,\ \kappa_{144} = -1,\ \hat\kappa_{111} = -1,$
IX	1	3	2	$\kappa_{122} = 1, \hat\kappa_{1\alpha\alpha} = -1,\ \alpha \in \{1,2,3\}$
X	3	0	3	$\kappa_{123} = 1$
XI	3	1	2	$\kappa_{122} = 1, \hat\kappa_{111} = -1,$

Table 3.1: *Abelian orbifolds with non-vanishing triple intersection numbers*

Across all these models, we observe a universal complex direction that combines the complex dilaton and the overall volume. This direction has dual dimension equal to 10, while its axionic partner has dimension 11. We will refer to it as the 'dilaton direction', keeping in mind that it also involves the Kähler moduli via the overall volume. The Kähler moduli all have dimensions equal to 6 (with axion dimensions 5), in agreement with (3.3) and maintaining the degeneracy.

For the complex structure moduli, which appear together with massless axions, we also obtain integer conformal dimensions. Their mass squared is $-2/R^2_{AdS}$, which naively

Modulus	Operator dimension Δ
1. $h^{1,1}_{-}$ saxionic Kähler moduli from J	6
1. $h^{1,1}_{-}$ axionic Kähler moduli from B_2	5
2. The dilaton direction	10
2. The C_3-axion appearing in W	11
3. $h^{2,1}$ saxionic complex structure moduli from $Re(\Omega)$	2
3. $h^{2,1}$ massless C_3-axions	3

Table 3.2: *Summary of integer operator dimension of a putative CFT_3 dual for generic supersymmetric DGKT type AdS_4 vacua.*

allows for operator dimensions $\Delta = 1$ or $\Delta = 2$. However, supersymmetry selects $\Delta = 2$ for these operators [80]. The complete spectrum is summarized in Table 3.2.

While the degeneracy among the complex structure moduli is expected due to the presence of massless axions, the degeneracy among the Kähler moduli is more surprising. Even in cases where the internal volumes lack an obvious exchange symmetry, this degeneracy persists, suggesting a deeper underlying reason.

The supersymmetric DGKT AdS vacua have closely related non-supersymmetric AdS vacua that are related to the supersymmetric vacua by sign flips of F_4 flux quanta [20]. The original $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$ model had three F_4 flux quanta and corresponding non-supersymmetric vacua that were obtained by flipping the sign of any one, two or all three of these quanta. Whenever one would flip the sign of one or three F_4- flux quanta, the masses of the axionic directions would change so that the conformal scaling dimensions of their dual operators are 8 for the B_2 axions and 1 or 2 for the C_3-axion. We find that, while more complicated models do not have non-supersymmetric vacua for a sign flip of any choice of F_4-flux quanta, they have non-supersymmetric vacua that can be obtained by flipping the sign of all F_4-flux quanta. In that case we find in all the abelian toroidal orbifold models the following dual operator dimensions shown in table 3.3.

Modulus	Operator dimension Δ
1. $h^{1,1}_{-}$ saxionic Kähler moduli from J	6
1. $h^{1,1}_{-}$ axionic Kähler moduli from B_2	8
2. The dilaton direction	10
2. The C_3-axion appearing in W	1 or 2
3. $h^{2,1}$ saxionic complex structure moduli from $Re(\Omega)$	1 or 2
3. $h^{2,1}$ massless C_3-axions	3

Table 3.3: *Summary of integer operator dimension of a putative CFT_3 dual for non-supersymmetric DGKT type AdS_4 vacua obtained by flipping the signs of F_4-flux quanta. Note that all non-flat axionic directions have different masses now.*

3.1.4 Light Spectrum: General Calabi–Yau Manifold

The results of the previous section suggest that the conformal dimensions in the DGKT vacua are independent not only of the flux choices but also of the specific triple intersection numbers. In this section we prove that the results summarized in Table 3.2 remain valid for any choice of internal Calabi–Yau manifold. As T-dualities preserve the mass spectrum, this results extends to the scale-separated vacua in massless IIA [48].

We begin by writing the second derivative of a generic supersymmetric 4d scalar potential at the supersymmetric locus as

$$\begin{aligned}\partial_m\partial_n V &= -3e^K\Big(K_{mn}|W|^2 + K_m\partial_n|W|^2 + \partial_m\partial_n|W|^2\Big)\\ &\quad + 2e^K K^{i\bar{j}}(\partial_n D_i W)(\partial_m D_{\bar{j}}\overline{W})\\ &= -3e^K\Big[(K_{mn} - \frac{1}{2}K_m K_n)|W|^2 + W_{mn}\overline{W} + W\overline{W}_{mn}\Big]\\ &\quad + 2e^K K^{i\bar{j}}(\partial_n D_i W)(\partial_m D_{\bar{j}}\overline{W}),\end{aligned} \tag{3.6}$$

where the subscripts m and n denote derivatives with respect to the real moduli.

Complex structure moduli mass matrix

The superpotential does not depend explicitly on any of the complex structure moduli U_a, i.e. $\partial_{U_a}W = 0$. Hence, the supersymmetry condition $D_{U_a}W = 0$ then implies that $K_{u_a} = 0$ at the minimum. Thus, the expression for the Hessian (3.6) simplifies to

$$\begin{aligned}\partial_{u_a u_b}V &= -3e^K K_{u_a u_a}|W|^2 + 2e^K K^{i\bar{j}}K_{iu_a}K_{\bar{j}u_a}|W|^2\\ &= -e^K K_{u_a u_a}|W|^2,\end{aligned} \tag{3.7}$$

with $\partial_{u_a u_b}V$ vanishing for $a \neq b$. Similarly, cross-terms with the dilaton or Kähler moduli vanish. As a result, the mass matrix for the complex structure moduli is diagonal, with all eigenvalues equal to

$$m_u^2 = -2/3V_{\rm min} = -2/R_{\rm AdS}^2. \tag{3.8}$$

Dilaton and Kähler moduli mass matrix

We note that, at the potential minimum,

$$\begin{aligned}&3m_0^2\kappa_{abc}v^b v^c + 10m_0 e_a + 5\kappa_{abc}m^b m^c = 0,\\ &s = -\frac{2\,m_0}{15\,p}\mathcal{V}, \quad b_a = \frac{m_a}{m_0}, \quad \xi = -\frac{e_0}{2p},\end{aligned} \tag{3.9}$$

where m_a denote F_2-fluxes that can optionally be turned on. This implies that at the minimum [20]

$$W = \frac{2i}{15}m_0\mathcal{V}. \tag{3.10}$$

Eqs (3.9) and (3.10) imply the second derivatives of the superpotential take the simpler form

$$\begin{aligned}\partial_{v_a}\partial_{v_b}W &= -\partial_{t_a}\partial_{t_b}W \\ &= -\kappa_{cab}m_c + m_0\kappa_{abc}t_c = im_0\kappa_{abc}v_c \\ &= i\frac{m_0\mathcal{V}}{6}(K_aK_b - K_{ab}) = \frac{5}{4}(K_aK_b - K_{ab})W.\end{aligned} \tag{3.11}$$

when Eqs (3.9) are satisfied. To evaluate the derivatives in the first line of (3.6), note that

$$\begin{aligned}&\partial_s^2|W|^2 = \frac{1}{2}K_s^2|W|^2, \\ &\partial_s\partial_b|W|^2 = \frac{1}{2}K_sK_b|W|^2, \\ &\partial_b\partial_a|W|^2 = 2(\partial_{v_a}\partial_{v_b}W)\overline{W} + 2(\partial_{v_a}W)(\partial_{v_b}\overline{W}) = \left(3K_aK_b - \frac{5}{2}K_{ab}\right)|W|^2.\end{aligned} \tag{3.12}$$

For the second line in (3.6), we have [1]

$$\begin{aligned}&K^{i\bar{j}}(\partial_n D_iW)(\partial_m D_{\bar{j}}\overline{W}) \\ &= K^{i\bar{j}}(W_{in} + K_{in}W - \frac{K_iK_n}{2}W)(\overline{W}_{\bar{j}m} + K_{\bar{j}m}\overline{W} - \frac{K_{\bar{j}}K_m}{2}\overline{W}) \\ &= K^{i\bar{j}}W_{in}\overline{W}_{\bar{j}m} + 2K^{i\bar{j}}W_{in}K_{\bar{j}m}\overline{W} - K^{i\bar{j}}W_{in}K_{\bar{j}}K_m\overline{W} \\ &+ \left[K^{i\bar{j}}K_{in}K_{\bar{j}m} - K^{i\bar{j}}K_{in}K_{\bar{j}}K_m + \frac{1}{4}(K^{i\bar{j}}K_iK_{\bar{j}})K_nK_m\right]|W|^2 \\ &= K^{i\bar{j}}W_{in}\overline{W}_{\bar{j}m} + 4W_{nm}\overline{W} - K^{i\bar{j}}W_{in}K_{\bar{j}}K_m\overline{W} + \left[K_{nm} + \frac{3}{4}K_nK_m\right]|W|^2.\end{aligned}$$

If $n, m = a, b$ are both size moduli,

$$K^{i\bar{j}}W_{in}\overline{W}_{\bar{j}m} = \frac{25}{16}(K_aK_b + K_{ab})|W|^2, \tag{3.13}$$

and if $n = a$ is a size modulus

$$K^{i\bar{j}}W_{in}K_{\bar{j}}K_m\overline{W} = 5K_aK_b|W|^2 \tag{3.14}$$

by substituting (3.11), and otherwise these terms vanish. Combining these results leads to

$$\partial_a\partial_bV = e^K(9K_{ab} + 8K_aK_b)|W|^2, \tag{3.15}$$

$$\partial_a\partial_sV = e^K(-2K_aK_s)|W|^2, \tag{3.16}$$

$$\partial_s^2V = e^K(-K_{ss} + 3K_s^2)|W|^2. \tag{3.17}$$

Hence, the mass matrix for the moduli in AdS units is given by

$$R_{\text{AdS}}^2\partial_m\partial_nV = \begin{pmatrix} 9K_{ab} + 8K_aK_b & -2K_aK_s \\ -2K_aK_s & -K_{ss} + 3K_s^2 \end{pmatrix}, \tag{3.18}$$

[1] Note that because $K = K(t_a + \bar{t}_a, S + \overline{S})$, the derivatives w.r.t. the complex moduli are $\partial_{t_a}K = \partial_{v_a}K/2i$, $\partial_S K = \partial K_s/2i$. The superpotential is holomorphic and so $\partial W_{t_a} = -i\partial_{v_a}W$, $\partial_S W = -i\partial_s W$.

where $R^2_{\text{AdS}} = -3/V_{min}$. We can proceed similarly with expression (3.6) for the axions b_a, ξ, where now all derivatives of the Kähler potential will vanish, and for the superpotential derivatives we have

$$\partial_{b_a} W = -i\partial_{v_a} W, \quad \partial_\xi W = -i\partial_s W. \tag{3.19}$$

This results in

$$\partial_{b_a}\partial_{b_b} V = e^K(5K_{ab} + 12K_aK_b)|W|^2, \tag{3.20}$$

$$\partial_{b_a}\partial_\xi V = -3e^K K_a K_s |W|^2, \tag{3.21}$$

$$\partial^2_\xi V = 2e^K K_s^2 |W|^2, \tag{3.22}$$

and so the mass matrix is

$$R^2_{\text{AdS}}\partial_m\partial_n V = \begin{pmatrix} 5K_{ab} + 12K_aK_b & -3K_aK_s \\ -3K_aK_s & 2K_s^2 \end{pmatrix}. \tag{3.23}$$

These expressions agree with the results obtained in [65] (see in particular Appendix B). However, this computation provides a more compact way to arrive at these results, using the ordinary $\mathcal{N} = 1$ structure of the potential and without having to rely on the formalism developed in [65, 81, 82]. Moreover, the derivation provided above makes it clear that the only properties feeding into the final result are the no-scale relations for the Kähler potential and the specific form of the superpotential leading to (3.11).

Spectrum

Let us define the two block matrices

$$K_{mn} = \begin{pmatrix} K_{ab} & 0 \\ 0 & K_{ss} \end{pmatrix} \quad L_{mn} = \begin{pmatrix} 4K_aK_b & -K_aK_s \\ -K_aK_s & \frac{K_s^2}{4} \end{pmatrix}, \tag{3.24}$$

where K is simply the Kähler metric for the moduli (in real components) evaluated at the minimum of the potential. Then, from the results of the previous section the Hessians of the moduli and axion potential can be expressed as

$$H^M_{mn} \equiv R^2_{AdS} a^2 \partial_m\partial_n V = 9K_{mn} + 2L_{mn} \tag{3.25}$$

and

$$H^A_{mn} \equiv R^2_{AdS}\partial_m\partial_n V = 5K_{mn} + 3L_{mn}. \tag{3.26}$$

respectively. The physical masses can be obtained as the eigenvalues of $M = 2K^{-1}H$, which is related to the mass matrix

$$\tilde{M} = 2\sqrt{K^{-1}}^T H\sqrt{K^{-1}} \tag{3.27}$$

by a similarity transformation.

In principle, the stabilized values of the v_i's and the dilaton can be extracted by the system of $h_-^{1,1}+1$ equations in the first line of (3.9). Knowing the value of the axions at the minimum, it would then be possible to write the mass matrix as a function of the flux data only. Although the equations cannot be solved explicitly for arbitrary triple intersection numbers κ_{abc} and fluxes e_a, m_a, the "implicit" form of the matrix derived above is not only sufficient for our purposes, but it is more helpful to elucidate the surprising simplifications in the final expressions.

To compute the eigenvalues of $K^{-1}L$, one can begin by noticing that any vector of the form

$$x = (x_i, x_0) \quad \text{with} \quad x_0\, K_s = 4K_a x^a \tag{3.28}$$

satisfies $K^{-1}\, L\, x = 0$. This essentially descends from the fact that M can be expressed as a tensor product $L_{mn} = A_m A_n$, with $A_m = (2K_a, -K_s/2)$. One therefore has a basis of $h_-^{1,1}$ eigenvectors with an eigenvalue of $\lambda_x^i = 0$. Furthermore, the no-scale relations imply that the vector

$$y = (v_i, K_s^{-1}), \tag{3.29}$$

is also an eigenvector of L, with eigenvalue $\lambda_y = 13$.

Then, there is a basis in which the mass matrices in AdS units read [2]

$$M^M_{mn} = 18\delta_{mn} + 52\, \delta_{1,m} \tag{3.30}$$

$$M^A_{mn} = 10\delta_{mn} + 78\, \delta_{1,m}, \tag{3.31}$$

also in agreement with the results of [65]. Through the relation $\Delta(\Delta - d) = m^2 R^2_{AdS}$, they correspond to conformal dimensions of

$$\Delta_1 = 10, \qquad \Delta_{2\ldots h_-^{1,1}+1} = 6 \tag{3.32}$$

for the saxions, and

$$\Delta_1 = 11, \qquad \Delta_{2\ldots h_-^{1,1}+1} = 5. \tag{3.33}$$

for the corresponding axions. Similar conclusions can be drawn for all the complex structure moduli as well. From Eq. (3.8), there are in principle two distinct possibilities for the saxions as both solutions to $\Delta(\Delta - d) = m^2 R^2_{AdS}$ satisfy the unitarity bound. However, $\mathcal{N} = 1$ superconformal symmetry excludes the option $\Delta_{u_\alpha} = 1$,[3] as the axion and saxion are in the same $3d$ $\mathcal{N} = 1$ supermultiplet [80]. Combined with the fact the complex structure axions are all massless, one arrives at the conclusion that

$$\Delta_{u_\alpha} = 2, \quad \Delta_{a_\alpha} = 3. \tag{3.34}$$

for all complex structure moduli U_α, where $\alpha = 1 \ldots h^{2,1}$.

[2]Since $M_{mn} = 2K^{mp}H_{pn} = 2K^{mp}\,(9K_{pn} + 2L_{pn}) = 18\delta_{mn} + 4K^{mp}L_{np}$.

[3]It would be interesting to understand how supersymmetry implements this from an AdS perspective, where the choices correspond to different boundary conditions for the scalar field.

3.1.5 Other type IIA AdS$_4$ vacua

Type IIA AdS$_4$ vacua with metric fluxes

Given that the DGKT setup with Ricci flat Calabi-Yau manifolds is a special subclass within the compactifications on more general $SU(3) \times SU(3)$ manifolds, it is naturally to ask whether the spectrum of integer conformal dimensions generalises.

One interesting model is discussed in [83] (see also [84]). This is a model that contains metric fluxes, i.e. have non-trivial curvature, but is not T-dual to a model that is Ricci flat. The squared masses in this model are

$$m^2 R_{AdS}^2 = (18, 22/9, -2, 10, -8/9, 0)\,. \tag{3.35}$$

While these are not integers as was the case before, it seems equally surprising that they are rational numbers and so are the dual operator dimensions

$$\Delta = (6, 11/3, 1 \text{ or } 2, 5, 8/3, 3)\,. \tag{3.36}$$

So, it seems there might be a larger class of models that has potentially rational dual operator dimensions and the Calabi-Yau compactifications are a subclass with integer dimensions.

3.2 Spectroscopy for Scale-separated Type IIA AdS$_3$ vacua

3.2.1 Model-independent part: flux dependence of the overall spectrum

We can derive the flux dependence of the overall spectrum from the generic flux-scalings of the uiversal moduli. The contributions to the scalar potential from the different fluxes goes in the following way

$$V_{F_4} \sim N^2 \cdot e^{6\phi} \cdot \mathcal{V}^{-22/7}, \quad V_{H_3} \sim h^2 \cdot e^{4\phi} \cdot \mathcal{V}^{-20/7}, \quad V_{F_0} \sim m^2 \cdot e^{6\phi} \cdot \mathcal{V}^{-2}, \tag{3.37}$$

which is derived in a similar way as in Section (1.3.2). Requiring that each contribution has the same flux-scaling leads to

$$\mathcal{V} \sim \left(\frac{N}{m}\right)^{7/4}, \quad e^{\phi} \sim \frac{h}{N^{3/4} m^{1/4}}. \tag{3.38}$$

The potential energy at the minimum then scales like

$$V_{min} \sim \frac{h^6 m^4}{N^8}, \tag{3.39}$$

which implies that the central charge goes like

$$c \sim \frac{N^4}{h^3 m^2}. \tag{3.40}$$

The masses of the Kaluza-Klein modes are

$$m_{KK} = \mathcal{V}^{-1/7} l_s^{-1} = \mathcal{V}^{-1/7} (l_p \cdot \mathcal{V}/e^{2\phi})^{-1} = \frac{h^2 m^{3/2}}{N^{7/2}} l_p^{-1}, \tag{3.41}$$

where l_s is the string scale, and thus

$$\Delta_{KK} \sim \frac{N^{1/2}}{hm^{1/2}}. \tag{3.42}$$

Other states in the CFT can come from wrapping D-branes on p-cycles in AdS. The only options that are not projected out by the orientifold are D2-branes wrapped on 2-cycles, if the twisted sector contains such cycles. Their mass is given by

$$m_{D_2} \sim e^{-\phi} l_s^{-1} \mathcal{V}^{2/7} \sim \frac{hm}{N^2} l_p^{-1}, \tag{3.43}$$

and so the dimension of the dual operator scales with the F4-flux like

$$\Delta_{D_2} \sim \frac{N^2}{h^2 m}. \tag{3.44}$$

A summary of the spectrum is shown in Table 3.4.

Central charge	$c \sim N^4$
Operator dimensions	
light	
graviton, gravitino, moduli	$\Delta \sim \mathcal{O}(1)$
medium	
KK modes	$\Delta_{KK} \sim N^{1/2}$
wrapped D2 branes	$\Delta_{D2} \sim N^2$
heavy	
BTZ black hole	$\Delta_{BH} \gtrsim N^4$

Table 3.4: *The spectrum.*

3.2.2 Model-dependent part: operators dual to the moduli

To compute the conformal dimensions of the operators dual to the 8 scalars originating from 7 metric fluctuations and the dilaton, we need to be careful with normalizations and conventions. We will follow [22] and then the relevant part of the Lagrangian is given by:

$$e^{-1}\mathcal{L} = \frac{1}{2}\mathcal{R} - \frac{1}{4}\delta_{ab}\partial\phi^a\partial\phi^b + \frac{1}{L^2} - \frac{1}{4}\sum_a m_a^2 \phi_a^2\,, \tag{3.45}$$

where a, b runs from $1, \ldots, 8$. We only displayed the potential to quadratic order and in a basis that diagonalizes the mass matrix and kinetic terms. In these conventions the operator dimensions in the CFT are:

$$\Delta_a = 1 + \sqrt{1 + m_a^2 L^2}. \tag{3.46}$$

The kinetic terms in the Lagrangian are almost of the canonical form (3.45), but not quite:

$$e^{-1}\mathcal{L}_{kin} = -\frac{1}{4}(\partial x)^2 - \frac{1}{4}(\partial y)^2 - \frac{1}{4}\frac{(\partial \tilde{s}^i)^2}{(\tilde{s}^i)^2}\,. \tag{3.47}$$

To go to canonically normalized moduli (3.45), we define $S^i \equiv \log \tilde{s}^i$. The cosmological constant $-1/L^2$ equals the value $-4P^2$ for the extremum of the real (or fake) P for the (fake) SUSY vacuum. The masses in the Lagrangian above are obtained from diagonalizing the Hessian of $8(\delta^{ab}P_aP_b - P^2)$, where again the indices are related to the canonically normalized scalars defined above. We then find [85]:

$$m_a^2L^2 = (52.625, 7.326, 3.384, 3.384, 3.384, 3.384, 3.384, 3.128), \tag{3.48}$$

such that:

$$\Delta_a = (8.323, 3.885, 3.094, 3.094, 3.094, 3.094, 3.094, 3.032). \tag{3.49}$$

These are the light single trace operators in the CFT and our main observation is that they are not integer, unlike the operators in the CFT dual to the DGKT vacua [78].

The operator dimensions are independent of the overall scalings N, m, h of the fluxes but do depend on the precise distribution of the fluxes across the internal manifold. For example, the dimensions (3.49) were computed for the following choice of H_3 and F_4-flux

$$h^i = h(1,1,1,1,1,1,1), \quad f^i = N(-1,-1,-1,-1,-1,-1,6). \tag{3.50}$$

Different flux distributions lead to different light spectra. For instance, with

$$h^i = h(1,1,1,0,0,0,0), \quad f^i = N(0,0,0,1,1,1,1), \tag{3.51}$$

as considered in [76], we find

$$\Delta_a = (8,4,4,4,4,2,2,2), \tag{3.52}$$

which are all integers. However, not all moduli are stabilised in this case, and when additional ingredients are introduced to stabilise the scalars, the dimensions generally become non-integer [76].

Without going into detail, we also mention the results for recently constructed scale-separated AdS$_3$ vacua in IIB. There are supersymmetric models [23] with

$$\Delta = (8,8,8,4,4,4,2), \tag{3.53}$$

and two non-supersymmetric models [24, 86] with 64 scalars with

$$\Delta = (10,8,6,4,2), \quad \Delta = (8,4,2), \tag{3.54}$$

with different multiplicities.

3.3 Integer conformal dimensions

We have seen that the DGKT vacua generically lead to integer conformal dimensions in their putative holographic dual. Given the relation between field masses in AdS units and these dimensions, this is somewhat surprising. Not only must the AdS masses be integers, but they must also take on very specific integer values. The fact that these integers arise independently of the specific flux choices and internal manifold suggests the presence of an underlying structure yet to be understood. In the remainder of this section, we highlight some observations related to this structure, emphasizing that its origin remains mysterious.

The calculation itself does not reveal much about the reason behind these integers. It is only at the very end when everything is taken into account (no-scale relations, precise form of the second derivatives of the superpotential, ...) that these integers appear.

3.3.1 Minimal Supersymmetry

The appearance of integer (or half-integer) conformal dimensions in a CFT often signals the presence of high supersymmetry. For instance, in $d = 3$ SCFTs with $\mathcal{N} \geq 3$, there exist short multiplets in which the conformal dimension of the primary operator is determined by the R-charge, which is quantized as a positive integer in such theories [80]. For instance, for $\mathcal{N} = 3$, short multiplets have primaries satisfying

$$\Delta = \frac{1}{2}R, \quad \text{or} \quad \Delta = \frac{1}{2}R + 1, \tag{3.55}$$

naturally leading to integer (or half-integer) dimensions in such settings.

However, in the DGKT case, we only have minimal $\mathcal{N} = 1$ supersymmetry. With such limited supersymmetry, there is no immediate reason to expect integer dimensions. Nevertheless, we can at least explain why the conformal dimensions of saxions and axions differ by exactly one unit. Indeed, the axion and saxion must reside in a long multiplet (denoted L' in [80]), which contains two scalar operators (along with one vector), with their dimensions differing by precisely 1. Thus, if we can establish why the axions have integer dimensions, the saxions will necessarily follow suit - or vice versa.

One speculative possibility is that the DGKT CFTs may secretly exhibit an enhanced supersymmetric structure beyond the manifest $\mathcal{N} = 1$. However, at present, there is no concrete evidence for such a higher-supersymmetric origin.

3.3.2 Shift symmetries

The most straightforward way of obtaining integer conformal dimensions is by having a massless field on AdS_{d+1}, for which

$$\Delta = d. \tag{3.56}$$

These scalars enjoy a constant shift symmetry

$$\phi \to \phi + c, \quad c \in \mathbb{R}. \tag{3.57}$$

Could there be different shift symmetries for other integer values?

For a free massless field in Minkowski space, there is not only the constant shift symmetry, but also an infinite sequence of shift symmetries that are polynomial in the spacetime coordinates X^μ,

$$\phi \to \phi + c + c_\mu X^\mu + c_{\mu\nu} X^\mu X^\nu + \dots, \tag{3.58}$$

where $c_{\mu_1 \dots \mu_k}$ are rank-k symmetric traceless constant tensors [87]. In AdS space, this is not true anymore, but still, each polynomial shift symmetry of *level* k can be kept separately if the free field ϕ has a particular mass depending on k [88, 89]. More precisely, in AdS_{d+1} a free field ϕ [4] has a symmetry

$$\phi \to \phi + c_{\mu_1 \dots \mu_k} X^{\mu_1} \dots X^{\mu_k}|_{AdS}, \tag{3.59}$$

where X^μ are coordinates on an embedding $(d+2)$-dimensional flat spacetime, if the mass of the field equals

$$m_\phi^2 = \frac{k(k+d)}{R_{AdS}^2}, \tag{3.60}$$

with R_{AdS} the radius of AdS. In a dual CFT, such masses correspond to integer dimensions

$$\Delta_+ = k + d, \quad \text{or} \quad \Delta_- = -k. \tag{3.61}$$

We conclude that there are indeed polynomial shift symmetries for the moduli and axions in DGKT related to the integer dimensions:

$$\begin{aligned} \Delta_\phi &= \Delta_+ = k + 3, \quad k = 3, 7, \\ \Delta_a &= \Delta_+ = k + 3, \quad k = 2, 8. \end{aligned} \tag{3.62}$$

For the massless axions, there are the ordinary constant ($k = 0$) shift symmetries, but for the complex structure moduli with $\Delta = 2$ such a symmetry is absent. A microscopic explanation of these shift symmetries, possibly related to the domain walls discussed later in this thesis or the discrete symmetries of [90] in the large N limit, would be very

[4] The interactions in DGKT are $1/N$ suppressed, and so the fields are free in the large flux $N \to \infty$ limit [78].

interesting.
The non-susy DGKT vacua obtained from the original ones by a change in the sign of the F_4-flux still have a full integer spectrum [91], and so the shift symmetries are there as well for all the moduli. There is another class of scale-separated non-susy vacua, described in [91], with both irrational and an integer $\Delta = 6$ in the spectrum, so there the shift symmetries are only present for a subset of the moduli.

To check what these symmetries look like on the boundary of AdS, we consider Poincaré coordinates

$$ds^2_{AdS} = \frac{R^2_{AdS}}{z^2}\left(-dx_0^2 + \sum_{i=1}^{d-1} dx_i^2 + dz^2\right), \tag{3.63}$$

the boundary being at $z = \epsilon$, $\epsilon \to 0$. Then the polynomial shifts on the boundary reduce to

$$c_{\mu_1 \dots \mu_k} X^{\mu_1} \dots X^{\mu_k}|_{AdS} \xrightarrow{z \to 0} \left(\frac{R_{AdS}}{z}\right)^k c_{i_1 \dots i_k} X^{i_1} \dots X^{i_k}|_{boundary}, \tag{3.64}$$

with X_i coordinates of the flat embedding restricted to the boundary. Considering the near-boundary behaviour of the scalar field

$$\phi \sim \phi_0 z^{\Delta_-} + \phi_1 z^{\Delta_+} = \phi_0 z^{-k} + \phi_1 z^{k+d}, \tag{3.65}$$

we see that the shift acts only on ϕ_0 as

$$\phi_0 \to \phi_0 + R^k_{AdS} c_{i_1 \dots i_k} X^{i_1} \dots X^{i_k}|_{boundary}. \tag{3.66}$$

In standard quantization ϕ_0 is fixed, and this means that the shift symmetry is broken on the boundary [92]. This should be investigated more precisely when more is known about an explicit DGKT CFT dual.

3.3.3 A universal conformal dimension for the breathing mode

We note that the DGKT and other AdS$_4$ vacua contain an operator in the spectrum with

$$\Delta = 6, \tag{3.67}$$

while the AdS$_3$ vacua with integer conformal dimensions do have

$$\Delta = 4. \tag{3.68}$$

For the IIB AdS$_5 \times S^5$ vacua, the breathing mode has

$$\Delta = 8. \tag{3.69}$$

This points towards a common operator for parametric AdS$_d$ vacua with integer conformal dimensions equal to,

$$\Delta = 2(d-1). \tag{3.70}$$

which might correspond to the breathing mode. In DGKT, none of the dimension-6 operators clearly align with the breathing mode, but in its double-T dual version, one of these operators corresponds to the 7-dimensional internal volume from the M-theory perspective. Below, we show that this breathing mode operator is present in all Freund-Rubin vacua.

Breathing mode in Freund–Rubin vacua

To study the dynamics of the internal volume (breathing mode), we rewrite the metric in the form

$$ds_D^2 = \mathcal{V}^{-\frac{2}{d-2}}\, ds_d^2 + \mathcal{V}^{\frac{2}{n}}\, d\tilde{s}_n^2, \tag{3.71}$$

where the factor $\mathcal{V}$ (which depends on the internal moduli) has been explicitly factored out so that $d\tilde{s}_n^2$ is a metric of unit volume. The prefactor on ds_d^2 ensures that the d-dimensional action is written in Einstein frame.

Neglecting derivatives of $\mathcal{V}$, the Ricci scalar splits as

$$R_D = \mathcal{V}^{\frac{2}{d-2}} R_d + \mathcal{V}^{-\frac{2}{n}} \tilde{R}_n, \tag{3.72}$$

and the flux term scales according to

$$F_n^2 = f^2\, \mathcal{V}^{-2n}\, n!\,. \tag{3.73}$$

Thus, the effective scalar potential (in d dimensions) becomes

$$V(\mathcal{V}) = -\tilde{R}_n\, \mathcal{V}^{-\frac{2(n+d-2)}{n(d-2)}} + \frac{1}{2}\, f^2\, \mathcal{V}^{-\frac{2(d-1)}{d-2}}, \tag{3.74}$$

where the first term originates from the internal curvature and the second term from the flux.

Extremizing the potential leads to a stabilised volume

$$\mathcal{V} = \left(\frac{n(d-1)f^2}{2(n+d-2)\tilde{R}_n} \right)^{\frac{n}{2(n-1)}}. \tag{3.75}$$

Lastly, we want to compute the mass of the volume modulus. For this, we need to know the kinetic terms as well. By successive application of a Weyl rescaling

$$g_{\mu\nu} = e^{2A}\tilde{g}_{\mu\nu}, \quad R_x = e^{-2A}\left(\tilde{R}_x - 2(x-1)\tilde{\nabla}^2 A - (x-1)(x-2)(\tilde{\nabla}A)^2\right), \tag{3.76}$$

first in $x = D$ dimensions with $A = \frac{1}{n}\ln\mathcal{V}$ and then in $x = d$ dimensions with $A = \left(\frac{1}{n} - \frac{1}{d-2}\right)\ln\mathcal{V}$, one finds that the kinetic term is given by

$$\frac{-(d+n-2)}{n(d-2)}\frac{1}{\mathcal{V}}\partial_\mu\mathcal{V}\partial^\mu\mathcal{V}, \tag{3.77}$$

such that the canonically normalised modulus is

$$\varphi = \sqrt{\frac{d+n-2}{n(d-2)}} \ln \mathcal{V}. \tag{3.78}$$

Computing the second derivative if the potential (3.74) w.r.t. this modulus, we find that

$$m^2 V^{-1} = -4\frac{d-1}{d-2} \tag{3.79}$$

at the potential minimum. We conclude that

$$\Delta = 2(d-1). \tag{3.80}$$

4

Large N scaling of central charge

Contents

In the previous chapter, we analyzed the spectrum of scale-separated AdS flux vacua. Here, we focus on another fundamental property: the central charge.

In two-dimensional CFTs, the central charge can be defined via the Weyl anomaly. While classically the energy-momentum tensor is traceless, quantum effects induce an anomaly given by:

$$T^{\mu}_{\mu} = -c\frac{\mathcal{R}}{2}, \tag{4.1}$$

where $\mathcal{R}$ denotes the Ricci scalar of the spacetime. In the context of the AdS_3/CFT_2 correspondence [93], the central charge relates directly to the AdS_3 length scale as:

$$c = \frac{3R_{AdS}}{2G_N}, \tag{4.2}$$

with G_N the three-dimensional gravitational constant. Thus, the central charge encodes the AdS scale and hence the number of degrees of freedom in the theory.

However, this specific definition of central charge does not straightforwardly generalize to higher dimensions. In even-dimensional CFTs, multiple anomaly coefficients appear, while odd-dimensional CFTs do not exhibit local conformal anomalies. Nevertheless, one can introduce a generalized notion of a central charge via the holographic free energy. For simplicity, we will continue to refer to this free energy as the central charge.

The holographic free energy F is defined as the logarithm of the sphere partition function:

$$F = -\log Z_{S^{d-1}}, \tag{4.3}$$

which is well-defined for CFTs in arbitrary dimensions. This free energy continues to characterize the number of degrees of freedom and scales with the AdS length as:

$$c \sim R_{AdS}^{d-2}, \tag{4.4}$$

when expressed in Planck units.

For example, in the case of two-dimensional CFTs, the relationship between the sphere partition function and the central charge is explicitly given by:

$$\partial_{\log r} \log Z_{S^2} = \frac{c}{3}, \tag{4.5}$$

with r denoting the radius of the sphere. The holographic free energy $-\log Z_{S^{d-1}}$ and the central c have the same overall dependence on the large N parameter. For a more detailed discussion, see [94].

With the relation (4.4), we can easily find the large N scaling of the DGKT central charge. It is given by

$$c \sim N^{9/2}. \tag{4.6}$$

At first sight, this central charge is extremely large when $N \gg 1$, and indeed, no known CFT exhibits such a high dependence on N.

In contrast, consider the standard example of $\mathrm{AdS}_5 \times S^5$, where the central charge scales as

$$c \sim N^2.$$

A straightforward explanation for this scaling is provided by examining the near-horizon limit of N D3-branes in flat spacetime. In this case, the degrees of freedom are associated with open strings stretching between the branes. These open-string modes transform in the adjoint representation of the $U(N)$ gauge group, whose dimension scales as N^2. Hence, a perturbative counting naturally leads to $c \sim N^2$.

For the ABJM theories [95], which are dual to Freund–Rubin vacua of the form $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$, the central charge behaves as

$$c \sim N^{3/2} k^{1/2},$$

which arises from the degrees of freedom of a stack of M2-branes for which no perturbative methods are available. Interestingly, in the 't Hooft limit (with the 't Hooft coupling fixed as $\lambda = N/k$), the gauge theory expectation of $c \sim N^2$ is recovered.

The highest known scaling in CFTs is for the six-dimensional (2,0) superconformal field theory, which describes the worldvolume dynamics of a stack of N M5-branes. The central charge scales like

$$c \sim N^3 \tag{4.7}$$

here. This N^3 scaling can be heuristically understood by considering the degrees of freedom associated with membranes stretching between three distinct sets of M5-branes [96].

In this chapter, we work toward an understanding of the $N^{9/2}$ scaling observed in DGKT. Interestingly, the scale-separated AdS_3 vacua [22] also exhibit a remarkably large scaling,

$$c \sim N^4. \tag{4.8}$$

To address these scalings, we need to understand what D-branes are involved in a holographic setting for DGKT vacua. In Section 4.1, we outline a general method to map flux vacua to a system of domain walls formed by (possibly wrapped) D-branes. We then apply this framework specifically to the DGKT vacua.

Afterwards, we discuss how to derive large N scalings from the supergravity description of D-branes and apply it to different flux vacua. Our analysis leads to two main conclusions. First, there seems to be a correspondence between the supergravity scalings of D-branes and the flux scalings, provided that the vacua exhibit integer conformal dimensions. Second, a configuration consisting of three sets of intersecting D4-brane domain walls appears to capture the essential DGKT degrees of freedom at the supergravity level. A microscopic counting of these degrees of freedom remains necessary to fully confirm this picture.

4.1 Flux - Domain Wall Correspondence

Dp-branes are sources of F_{8-p}-form flux: by placing a Dp-brane in flat spacetime, you are turning on flux around the brane. In the context of flux compactifications, it is useful to consider domain walls consisting of (possible wrapped) D-branes as an alternative description of the flux vacuum. By probing the near-horizon region of the domain walls, you find an AdS vacuum where fluxes are turned on. More concretely, a system of N Dp-branes placed in 10d flat spacetime (consisting of d flat non-compact dimensions and $10-d$ flat compact toroidal dimensions, $(p+2-d)$ of which are wrapped by the brane) is equivalent to a d-dimensional AdS flux vacuum. These wrapped D-branes are domain walls in AdS and essentially cap off the spacetime, interpolating it with an asymptotically flat region.

A similar correspondence exists for NSNS 3-form flux H_3, which can be sourced by NS5-branes. Additionally, we can consider geometric fluxes (associated with non-trivial internal curvature) as sourced by Kaluza-Klein monopoles [97].

When multiple fluxes are involved, we can consider the flux vacuum as a system of multiple orthogonally intersecting branes in the simplest settings. There are harmonic superposition rules to find the supergravity description for such systems [98]. One can not arbitrarily compose branes, there are stringent rules of which branes can be combined. Table 4.1 summarizes these rules in terms of the number of relative transverse directions (i.e., directions that are transverse to some but not all of the branes).

intersecting branes	**# of relative transverse directions**
Dp/Dq	$0 \mod 4$
NS5/NS5	$0 \mod 4$
Dp/NS5	$7-p$ or $11-p$
Dp/KK	$5-p$ or $9-p$
KK/KK	$0 \mod 4$
NS5/KK	4 or 8

Table 4.1: *Rules for supersymmetric orthogonal brane intersections, from [99]. Relative transverse directions are directions that are transverse to some, but not all of the involved branes.*

Applying this procedure to the DGKT vacua, we find a configuration of orthogonally intersecting D4-branes (wrapped on 2-cycles), NS5-branes (wrapped on 3-cycles) and D8-branes (wrapped on the entire internal manifold), shown in Table 4.2. The DGKT AdS vacua can be found in the near-horizon region of this brane set-up [100]. The D4-branes have been highlighted because they are dual to the unbounded flux and will be the primary focus of this chapter.

4.2 Large N scalings from D-brane domain walls

In this section, we concentrate on the D-brane domain walls that are dual to unbounded fluxes in AdS flux vacua. We derive the large N behavior of the near-horizon geometry of these branes and use it to determine the scaling relations for the central charge as well as the universal moduli, namely the internal volume and the dilaton. We then validate these scaling relations against various flux vacuum examples and apply the method to both the scale-separated AdS_4 (DGKT) and AdS_3 cases.

	t	x^1	x^2	x	y_1	y_2	y_3	y_4	y_5	y_6
D4	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$				
D4	$\otimes$	$\otimes$	$\otimes$				$\otimes$	$\otimes$		
D4	$\otimes$	$\otimes$	$\otimes$						$\otimes$	$\otimes$
NS5	$\otimes$	$\otimes$	$\otimes$		$\otimes$		$\otimes$		$\otimes$	
NS5	$\otimes$	$\otimes$	$\otimes$		$\otimes$			$\otimes$		$\otimes$
NS5	$\otimes$	$\otimes$	$\otimes$			$\otimes$		$\otimes$	$\otimes$	
NS5	$\otimes$	$\otimes$	$\otimes$			$\otimes$	$\otimes$			$\otimes$
D8	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$

Table 4.2: *Brane configuration for DGKT. The coordinates $\{t, x^1, x^2, x\}$ span a 4-dimensional Minkowski space, with x representing the direction transverse to the domain walls. The y_i coordinates parametrize a 6-dimensional torus.*

4.2.1 General Method

If there are N units of unbounded flux, F_{8-p}, we can change this for N Dp-branes, wrapped on $(p+2-d)$-cycles, in a 10d flat spacetime consisting of d non-compact directions, the other directions forming a toroidal geometry. These branes can be considered as domain walls in the d-dimensional spacetime, with coordinates $(t, x_1, x_2, \dots x_{d-2}, x)$, and where x will be the direction transverse to the branes. We denote the wrapped internal coordinates by $(y_1, y_2, \dots y_{p+2-d})$, and the unwrapped ones by $(z_1, z_2, \dots z_{8-p})$, as shown in Table 4.3.

	t	x^1	$\dots$	x^{d-2}	x	y_1	$\dots$	y_{p+2-d}	z_1	$\dots$	z_{8-p}
N Dp	$\otimes$	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$	$\otimes$			

Table 4.3: *N Dp-brane domain walls in d dimensions*

This system is described by [42]

$$ds_{10}^2 = f_p^{-\frac{1}{2}} \left[-dt^2 + dx_1^2 + \dots + dx_{d-2}^2 \right] + f_p^{\frac{1}{2}} dx^2 + f_p^{-\frac{1}{2}} dy_i dy^i + f_p^{\frac{1}{2}} dz_j dz^j, \tag{4.9}$$

and the string coupling is

$$g_s = f_p^{\frac{3-p}{4}}, \tag{4.10}$$

with

$$f_p \sim 1 + \frac{N}{r^{7-p}}, \quad r^2 = \sum_i y_i^2 + \sum_j z_j^2, \tag{4.11}$$

a harmonic function in the transverse coordinates that scales like $f_p \sim N$ in the near-horizon region.

Letting $N \to \infty$, the near-horizon geometry will be of the following schematic form,

$$ds_{NH}^2 = \alpha' \left[N^{\frac{1}{2}} ds_{X_d}^2 + N^{-\frac{1}{2}} dy_i dy^i + N^{\frac{1}{2}} dz_j dz^j \right], \tag{4.12}$$

where X_d is the d-dimensional spacetime and

$$\boxed{g_s \sim N^{\frac{3-p}{4}}}. \tag{4.13}$$

We see that there is a separation of scales between the wrapped internal dimensions and the scale of X_d, and we can read off the internal volume in string units:

$$\boxed{\mathcal{V} \sim (N^{-\frac{1}{2}})^{\frac{p+2-d}{2}} \cdot (N^{\frac{1}{2}})^{\frac{8-p}{2}} \cdot l_s^{10-d} = N^{\frac{d+6-2p}{4}} \cdot l_s^{10-d}}. \tag{4.14}$$

From this, we deduce the d-dimensional Planck length

$$l_{p,d} = (g_s^2 \mathcal{V}^{-1})^{1/(d-2)} l_s = N^{-\frac{d}{4\,(d-2)}} l_s. \tag{4.15}$$

Hence, the metric in Planck units is

$$ds^2_{NH} = N^{\frac{1}{d-2}} \left[N ds^2_{X_d} + dy_i dy^i + N dz_j dz^j\right] l^2_{p,d}. \tag{4.16}$$

Then, the AdS radius is

$$R_{AdS} \sim N^{\frac{d-1}{2\,(d-2)}} l_{p,d}, \tag{4.17}$$

and so the central charge will be

$$\boxed{c \sim (R_{AdS}/l_{p,d})^{d-2} \sim N^{\frac{d-1}{2}}}. \tag{4.18}$$

Repeating the same calculation for domain walls from NS5-branes, we find

$$g_s \sim N^{\frac{1}{2}}, \quad \mathcal{V} \sim N^{\frac{3}{2}}, \quad c \sim N^{\frac{d-1}{2}}, \tag{4.19}$$

or for F1-strings (only for $d = 3$),

$$g_s \sim N^{-\frac{1}{2}}, \quad \mathcal{V} \sim N^{\frac{d-3}{2}}, \quad c \sim N^{\frac{d-1}{2}}. \tag{4.20}$$

In an M-theory setting, we could also consider M2-branes, with

$$\mathcal{V} \sim N^{\frac{7}{6}}, \quad c \sim N^{\frac{3}{2}}, \tag{4.21}$$

or M5-branes with

$$\mathcal{V} \sim N^{\frac{4}{3}}, \quad c \sim N^{3}, \tag{4.22}$$

where the volume is now expressed in 1dd Planck units. In the examples that we discuss later, the setup often involves multiple stacks of D-branes (as illustrated in Table 4.3), each dual to unbounded flux. In such cases, we can assign a function f_{p_i} to each stack of D$_{p_i}$-branes, which depends only on the overall transverse directions. For each of the coordinate directions $\zeta = t, x, y$ or z, the metric components are modified according to

$$\prod_i f_{p_i}^{1/2} \cdot \prod_j f_{q_j}^{-1/2} \cdot (d\zeta)^2, \tag{4.23}$$

where the p_i-branes are orthogonal, and the q_j-branes are parallel to the ζ-direction. Using the harmonic superposition rules [98], we obtain the supergravity description of the brane configuration. The string coupling will be given by

$$g_s = \prod_i f_{p_i}^{\frac{3-p_i}{4}}, \tag{4.24}$$

with the product taken over all branes in the system. From the resulting near-horizon geometries as specified in Eq. (4.23), we can then extract the scaling behavior of both the central charge and the internal volumes.

4.2.2 Freund-Rubin Vacua

As a consistency check, we verify whether the procedure outlined in the previous section correctly reproduces the expected scalings for the $\mathrm{AdS}_5 \times S^5$, $\mathrm{AdS}_4 \times S^7$, and $\mathrm{AdS}_7 \times S^4$ vacua. From the scalar potential (3.74) and the volume relation (3.75), we deduce that the volume and central charge scale as

$$\mathcal{V} \sim N^{\frac{n}{n-1}}, \quad c \sim N^{\frac{n+d-2}{n-1}}. \tag{4.25}$$

Setting $d = n = 5$, we recover the expected $p = 3$ scalings (4.14), (4.18) for D3-branes. Similarly, for $d = 4, n = 7$, the result aligns with (4.21) for M2-branes, while the $d = 7, n = 4$ case matches the M5-brane scalings in (4.22). We remind that the volume modulus is dual to an operator of integer conformal dimension

$$\Delta = 2(d-1) \tag{4.26}$$

in these vacua.

Similarly, we find back the ABJM scalings in the type IIA description from a stack of N D2-branes,

$$g_s \sim N^{\frac{1}{4}}, \quad \mathcal{V} \sim N^{\frac{3}{2}}, \quad c \sim N^{\frac{3}{2}}. \tag{4.27}$$

Having checked these well-known examples, we now turn to the examples of scale-separated AdS vacua, for which no dual is known.

4.2.3 Scale separated AdS_4 vacua in massive IIA string theory (DGKT)

The DGKT [20] potential is given by

$$V = \frac{1}{s^3}\left[\frac{A_{F_4}}{us} + \frac{A_{F_0}u^3}{s} + \frac{A_{H_3}s}{u^3} - A_{O6}\right], \tag{4.28}$$

where $u^3 = \mathcal{V}$ and $s = e^{-D} = e^{-\phi}\sqrt{\mathcal{V}}$, and the A's are coefficients depending on the values of the fluxes or the orientifold charge, with $A_{O6}^2 = 16A_{F_0}A_{H_3}$. The Kähler potential equals

$$K = -3\log u - 4\log s. \tag{4.29}$$

The conformal dimensions of the dual fields are

$$\Delta_1 = 6, \quad \Delta_2 = 10. \tag{4.30}$$

The mass eigenstates are the following fields

$$\Phi_1 = logs^{-4}u, \quad \Phi_2 = \log su^3. \tag{4.31}$$

Imposing the scaling $A_{F_4} \sim N^2$, we find that, at the minimum of the potential

$$V \sim N^{-\frac{9}{2}}, \quad s \sim N^{\frac{3}{2}}, \quad u \sim N^{\frac{1}{2}}, \tag{4.32}$$

which means that

$$c \sim N^{9/2}, \quad g_s \sim N^{-3/4}, \quad \mathcal{V} \sim N^{3/2}. \tag{4.33}$$

We now check whether we can reproduce this result from the near-horizon geometry of domain walls. We only focus on the D4-brane domain walls dual to the unbounded F_4-fluxes (Table 4.4), while the full brane configuration is shown in Table 4.2. As

	t	x^1	x^2	x	y_1	y_2	y_3	y_4	y_5	y_6
N_1 **D4**	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$				
N_2 **D4**	$\otimes$	$\otimes$	$\otimes$				$\otimes$	$\otimes$		
N_3 **D4**	$\otimes$	$\otimes$	$\otimes$						$\otimes$	$\otimes$

Table 4.4: *D4-brane domain walls in DGKT*

$N_i \sim N \to \infty$, using the harmonic superposition rules, we find

$$\begin{aligned} ds^2_{NH} = \alpha' [&N_1^{\frac{1}{2}} N_2^{\frac{1}{2}} N_3^{\frac{1}{2}} ds^2_{X_4} + N_1^{-\frac{1}{2}} N_2^{\frac{1}{2}} N_3^{\frac{1}{2}} (dy_1^2 + dy_2^2) \\ &+ N_1^{\frac{1}{2}} N_2^{-\frac{1}{2}} N_3^{\frac{1}{2}} (dy_3^2 + dy_4^2) + N_1^{\frac{1}{2}} N_2^{\frac{1}{2}} N_3^{-\frac{1}{2}} (dy_5^2 + dy_6^2)], \end{aligned} \tag{4.34}$$

with

$$g_s \sim N_1^{-\frac{1}{4}} N_2^{-\frac{1}{4}} N_3^{-\frac{1}{4}} \sim N^{-\frac{3}{4}}. \tag{4.35}$$

The internal volume can be read off,

$$\mathcal{V} \sim N_1^{\frac{1}{2}} N_2^{\frac{1}{2}} N_3^{\frac{1}{2}} \sim N^{\frac{3}{2}}, \tag{4.36}$$

along with the degree of scale separation,

$$\frac{L^2_{KK}}{L^2_H} \sim \max_i (N_i^{-1}) \sim N^{-1}. \tag{4.37}$$

The 4d Planck scale is given by

$$l_{p,4} = \frac{g_s}{\sqrt{\mathcal{V}}} l_s \sim N_1^{-\frac{1}{2}} N_2^{-\frac{1}{2}} N_3^{-\frac{1}{2}} l_s, \tag{4.38}$$

and so it follows that the central charge is given by

$$c \sim \frac{R_{X_4}^2}{l_{p,4}^2} \sim N_1^{\frac{3}{2}} N_2^{\frac{3}{2}} N_3^{\frac{3}{2}} \sim N^{\frac{9}{2}}. \tag{4.39}$$

We obtain the same scalings as the ones (4.33) from the scalar potential. This match is non-trivial and suggests that the DGKT CFT duals can indeed be found on the D-branes dual to the fluxes. In the next sections we will see that something similar holds for other vacua with integer conformal dimensions.

Scale separated AdS$_4$ vacua in massless IIA string theory

We can proceed similarly for scale-separated AdS$_4$ vacua without Romans mass. These are obtained by performing two T-dualities on massive DGKT and then re-scaling fluxes to get a solution under control [48, 99]. The internal manifold will now have curvature becoming a more general SU(3) structure manifold. The flux distribution is anisotropic, with only unbounded flux on 2 out of 3 2-tori in the toroidal case. That is why we now write explicitly $\mathcal{V} = u_1 u_2 u_3$. There is unbounded F_6-flux, both bounded F_2-flux ($F_{2,1}$) and unbounded F_2-flux ($F_{2,2}, F_{2,3}$), a curvature R contribution and again there are O6-planes. The scalar potential is given by

$$V = \frac{1}{s^3}\left[A_{F_6}\frac{1}{su_1u_2u_3} + A_{F_{2,1}}\frac{u_2u_3}{su_1} + A_{F_{2,2}}\frac{u_1u_3}{su_2} + A_{F_{2,3}}\frac{u_1u_2}{su_3} + A_R\frac{su_1}{u_2u_3} - A_{O6}\right], \tag{4.40}$$

with Kähler potential

$$K = -\log u_1u_2u_3 - 4\log s. \tag{4.41}$$

Conformal dimensions do not change under T-duality:

$$\Delta_{1,2,3} = 6, \quad \Delta_4 = 10. \tag{4.42}$$

Letting $F_6 \sim N$, $F_{2,2} \sim M_1$ and $F_{2,3} \sim M_2$, it follows that

$$c \sim N^{\frac{3}{2}} M_1^{\frac{3}{2}} M_2^{\frac{3}{2}}, \quad g_s \sim N^{\frac{1}{4}} M_1^{-\frac{3}{4}} M_2^{-\frac{3}{4}}, \quad \mathcal{V} \sim N^{\frac{3}{2}} M_1^{-\frac{1}{2}} M_2^{-\frac{1}{2}}. \tag{4.43}$$

For N large enough compared to M_1 and M_2, the coupling will be strong and this solution can be uplifted to M-theory. The first dimension-6 operator can then be interpreted as being dual to the (7d) internal volume.

Now we change the F_6-flux for D2-branes, and the unbounded F_2-fluxes for D6-branes wrapped on 4-cycles, as in Table 4.5.

The following near-horizon geometry can be read off:

$$\begin{aligned} ds_{NH}^2 = \alpha' [&N^{\frac{1}{2}} M_1^{\frac{1}{2}} M_2^{\frac{1}{2}} ds_{X_4}^2 + N^{\frac{1}{2}} M_1^{-\frac{1}{2}} M_2^{-\frac{1}{2}} (dy_1^2 + dy_2^2) \\ &+ N^{\frac{1}{2}} M_1^{-\frac{1}{2}} M_2^{\frac{1}{2}} (dy_3^2 + dy_4^2) + N^{\frac{1}{2}} M_1^{\frac{1}{2}} M_2^{-\frac{1}{2}} (dy_5^2 + dy_6^2)], \end{aligned} \tag{4.44}$$

with

$$g_s \sim N^{\frac{1}{4}} M_1^{-\frac{3}{4}} M_2^{-\frac{3}{4}}, \quad \frac{L_{KK}^2}{L_H^2} \sim \max_i(M_i^{-1}), \tag{4.45}$$

fully in accordance with the scalings from the scalar potential (4.43).

	x^0	x^1	x^2	x	y_1	y_2	y_3	y_4	y_5	y_6
N **D2**	$\otimes$	$\otimes$	$\otimes$							
M_1 **D6**	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$	$\otimes$	$\otimes$		
M_2 **D6**	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$			$\otimes$	$\otimes$

Table 4.5: *D2- and D6-brane domain walls*

4.2.4 AdS$_3$ vacua

Scale-separated AdS$_3$ in massive IIA string theory

The vacua of [22] are scale-separated AdS$_3$ vacua, from compactification of massive IIA string theory on a G2 holonomy manifold. There are F_4-, H_3- and F_0- fluxes, together with O6-planes, and the scalar potential is:

$$V = \frac{A_{F_4}}{u^{\frac{1}{2}} s^3} + \frac{A_{F_0} u^{\frac{7}{2}}}{s^3} + \frac{A_{H_3}}{u^3 s^2} - \frac{A_{O6} u^{\frac{1}{4}}}{s^{\frac{5}{2}}}. \tag{4.46}$$

with $A_{O6}^2 = 12 \cdot A_{f_0} \cdot A_{h_0}$, and where $u^{7/2} = \mathcal{V}$ and $s = e^{-2\phi}\mathcal{V}$ [1]. The conformal dimensions are irrational

$$\Delta = 1 + \sqrt{\frac{191 \pm 8\sqrt{277}}{7}}. \tag{4.47}$$

With $F_4 \sim N$, we find from (4.46) that

$$c \sim N^4, \quad g_s \sim N^{-3/4}, \quad \mathcal{V} \sim N^{7/4}. \tag{4.48}$$

There are such F_4-fluxes on seven different 4-cycles, and interchanging them for D4-branes wrapped on the dual 3-cycles, we find the system of Table 4.6.

	t	x^1	x	y_1	y_2	y_3	y_4	y_5	y_6	y_7
N_1 **D4**	$\otimes$	$\otimes$		$\otimes$	$\otimes$					$\otimes$
N_2 **D4**	$\otimes$	$\otimes$				$\otimes$	$\otimes$			$\otimes$
N_3 **D4**	$\otimes$	$\otimes$						$\otimes$	$\otimes$	$\otimes$
N_4 **D4**	$\otimes$	$\otimes$		$\otimes$		$\otimes$			$\otimes$	
N_5 **D4**	$\otimes$	$\otimes$			$\otimes$	$\otimes$		$\otimes$		
N_6 **D4**	$\otimes$	$\otimes$		$\otimes$			$\otimes$	$\otimes$		
N_7 **D4**	$\otimes$	$\otimes$			$\otimes$		$\otimes$		$\otimes$	

Table 4.6: *D4-brane domain walls for scale-separated AdS$_3$*

There should be further domain walls from NS5-branes and D8-branes, dual to the bounded H_3 - and F_0-fluxes. If there is indeed an AdS$_3 \times T_7$ geometry in the near-horizon limit of this set of domain walls, the schematic flux dependence should be like

$$\begin{aligned} ds^2_{NH} &= \alpha' \big[(N_1 \cdot \dots \cdot N_7)^{\frac{1}{2}} ds^2_{X_3} + (N_1 N_4 N_6)^{-\frac{1}{2}} (N_2 N_3 N_5 N_7)^{\frac{1}{2}} dy_1^2 + \dots \big] \\ &= \alpha' \big[N^{\frac{7}{2}} ds^2_{X_3} + N^{\frac{1}{2}} (dy_1^2 + dy_2^2 + dy_3^2 + dy_4^2 + dy_5^2 + dy_6^2 + dy_7^2) \big], \end{aligned} \tag{4.49}$$

[1] To make contact with the notation in [22]: $u = e^{\frac{\phi}{2} + 2\beta v}$, $s = e^{-\frac{\phi}{4} + 7\beta v}$.

with $N_i = N, i = 1, \ldots 7$. The string coupling (4.13) would be

$$g_s = (N_1 \cdot \ldots \cdot N_7)^{-\frac{1}{4}} = N^{-\frac{7}{4}}, \tag{4.50}$$

and the internal volume is

$$\mathcal{V} = (N_1 \cdot \ldots \cdot N_7)^{\frac{1}{4}} = N^{\frac{7}{4}}. \tag{4.51}$$

Finally, the central charge would be

$$c = N_1 \cdot \ldots \cdot N_7 = N^7. \tag{4.52}$$

Remarkably, the scalings of the central charge and the string coupling, derived from the brane system do not match with (4.48). Given that each of the seven internal lengths should scale in the same way with N [22], and taking into account that each pair of stacks of D4-branes should have 0 or 4 relative transverse directions [99], the set-up of Table 4.6 is the only possible set-up consisting of orthogonally-intersecting D4-branes. Given the irrational dimensions (4.47) for these vacua, it seems that this simple holographic set-up with orthogonal D-branes, dual to fluxes, only works for vacua with integer conformal dimensions, which go together with the polynomial shift symmetries discussed in Section 2. This suggests that the large N structure of these 3d vacua differs from that of other parametric vacua. For all other cases with a large N flux quantum, this procedure for deriving the scaling relations applies. The fact that it fails here may indicate an inconsistency in these vacua, which future work should clarify.

Parametric AdS$_3$ vacua in IIB string theory

To stress that this failure to obtain the scalings from the near-horizon geometry of a simple system of orthogonally-intersecting branes is not generic to asymptotic vacua in 3d, we mention another example: AdS$_3$ vacua from compactification of IIB string theory on G2-structure manifolds (so with curvature R) [101] with F_7-, F_3-fluxes and O5-planes and possibly D5-branes. These do not allow scale separation, but the moduli can take parametric values. The scalar potential equals

$$V = \frac{A_{F_7}}{u^{\frac{7}{2}} s^3} + \frac{A_{F_3} u^{\frac{1}{2}}}{s^3} + \frac{A_R}{u s^2} - \frac{A_{O5/D5}}{u^{\frac{1}{4}} s^{\frac{5}{2}}}, \tag{4.53}$$

with $A_{O5/D5}^2 = 16 A_{F3} \cdot A_R/3$. The conformal dimensions are now integer and rational

$$\Delta_1 = 4, \quad \Delta_2 = \frac{20}{7}. \tag{4.54}$$

Choosing $F_7 \sim N$, the scalings

$$c \sim N, \quad g_s \sim N^{1/2}, \quad \mathcal{V} \sim N^{7/4}. \tag{4.55}$$

from the scalar potential (4.53). These coincide with the scalings (4.13), (4.14), (4.18) for N D1-domain walls in 3d, see Table 4.7.

	t	x^1	x	y_1	y_2	y_3	y_4	y_5	y_6	y_7
N **D1**	$\otimes$	$\otimes$								

Table 4.7: *D1-brane domain walls for parametric* AdS_3

4.3 Conclusions

In this chapter, we have studied brane configurations that provide a dual description of unbounded fluxes in AdS flux vacua. From the supergravity perspective of the near-horizon region, we derived large N scalings for the central charge and universal moduli. These scalings consistently matched those of AdS vacua in all cases where a dual operator with integer dimension exists,

$$\Delta = 2(d-1). \tag{4.56}$$

A deeper understanding of the connection between the mass spectrum and the brane configuration remains an open question.

The DGKT AdS_4 vacua have a remarkably high central charge,

$$c \sim N^{9/2}, \tag{4.57}$$

but this scaling seems consistent with the large N near-horizon limit of three intersecting stacks of N D4-branes wrapped on 2-cycles. A microscopic count is needed to confirm this scaling precisely.

The scale-separated AdS_3 vacua are unlike their 4-dimensional DGKT counterparts, despite being built from similar ingredients, with irrational dimensions and scalings that cannot be obtained from orthogonal D4-domain walls. Moreover, a massless version ($F_0 \neq 0$) does not exist here [102], preventing an M-theory uplift [63]. Future research will clarify the origin of the failed scalings for the AdS_3 vacua and whether they imply any inconsistency.

5

Backtracking AdS flux vacua

Contents

In this chapter, we introduce an algorithm (dubbed "flux backtracking") to reverse-engineer the brane picture from an AdS flux vacuum. Given an AdS flux vacuum as input, the algorithm outputs a singularity in 10 or 11 dimensions. This singularity has the property that when probed with the appropriate stack of branes (and after taking the near-horizon limit), one recovers the initial AdS vacuum. After testing the procedure on a number of known AdS/CFT pairs, we apply it to AdS flux vacua without known holographic dual, notably the scale-separated DGKT solution. In this case, flux backtracking produces a certain strongly coupled singularity in massive IIA; we conjecture that the worldvolume CFT of D4-branes probing this singularity should be the holographic CFT dual to DGKT (if it exists). Applying the procedure to the DGKT-related scale-separated AdS_4 solutions without Romans mass, we find instead a conical and weakly coupled singularity. We also comment on the results and limitations of applying the procedure to KKLT.

5.1 Introduction

The AdS/CFT correspondence [10, 46, 47] is an exact match between a conformal field theory (CFT) and an AdS quantum gravity vacuum. The original statement is supported by a vast amount of evidence (e.g. [10, 42, 95, 103–105], which are some of the examples that we will discuss here) in the form of concrete AdS/CFT pairs, where a particular AdS quantum gravity (in practice, an AdS vacuum of string theory) is matched to a known CFT. There is a standard way to produce AdS/CFT pairs from string theory: First, one identifies a suitable stack of branes, perhaps probing a singularity in string theory. If the stack has the right properties, its low-energy worldvolume effective field theory will be a CFT. On the other hand, when this happens, the near-horizon geometry of the brane system provides the dual AdS geometry, thus completing the pair.

Given a brane stack, obtaining the near-horizon AdS is a straightforward (if sometimes cumbersome [106]) procedure. The *reverse* problem of finding the brane picture given an AdS solution is much harder, and no systematic technique exists, to our knowledge. It is also a much more interesting direction, since there are several known AdS geometries (e.g. [18, 20, 22, 48]) with no known holographic dual CFT, including the proposed DGKT vacuum [20], which is of independent interest since it exhibits scale separation between the AdS scale and the size of the internal extra dimensions.

In this chapter, we outline a simple procedure, well-known in many concrete examples in holography, to reverse-engineer a brane picture given an AdS flux compactification. The idea is simple: A typical AdS solution comes out of balancing of curvature and flux terms in an effective action. Fluxes that can be rescaled arbitrarily are precisely those sourced by the probe branes in the dual brane picture; if one sets all of these fluxes to zero, which corresponds to removing all brane charges, the AdS solution disappears; however, one can find instead a running, horizonless solution. Indeed, this is a dynamical cobordism in the sense of [107–109], which in examples coincides with the original ten-dimensional geometry probed by the stack of branes. Therefore, to find the brane picture, all one needs to do is find the running solution, and probe it with the stack of branes. Therefore, in some sense, in this chapter we are just outlining a systematic procedure to determine particularly interesting examples of dynamical cobordism in flux compactifications. For ease of reference, we dub this procedure “flux backtracking”, since it allows one to trace back the steps that led to an AdS flux vacuum.

By following the steps above, we are able to recover many known AdS/CFT pairs. For instance, we recover a non-compact Calabi-Yau from compactification on a Sasaki-Einstein manifold, an M-theory orbifold for ABJM or a type I' background from the theories in [104]. As we will see below, this procedure cannot be applied straightforwardly to all cases, since one needs the explicit form for the full, off-shell effective action for arbitrary values of the fluxes, which is not always available. But such an off-shell effective action is available for DGKT [20] (or the massive IIA vacua of [105]), where the brane picture is not known. Applying the procedure to this case results in an unfamiliar singularity in massive IIA. We do not know how to study its low-energy physics, or how to find the worldvolume QFT of D-branes probing it, but this object is a natural candidate for the exotic object providing the brane picture for DGKT. Similar results are found for the other scale-separated proposals [22, 48], although the singularity associated to the vacuum in [48] turns out to be weakly coupled, so it could in principle be analysed using worldsheet techniques.

This chapter is structured as follows: In Section 5.2 we explain the details and limitations of the flux backtracking procedure. In Section 5.3 we test the procedure, successfully recovering the known brane picture for several AdS/CFT pairs. Section 5.4 describes the result of flux backtracking in the DGKT solution and other examples where the holographic dual is not known. Finally, Section 5.5 contains some concluding remarks.

5.2 Flux backtracking: general strategy

Our starting point will be a d-dimensional AdS$_d$ vacuum obtained from a flux compactification in string theory. This means that we actually have a higher-dimensional geometry AdS$_d \times X_n$, where X_n is an internal compactification manifold, such that $d + n = 10$ (for perturbative string theory constructions) or $d + n = 11$ in the case of M-theory. In many cases of interest, the AdS$_d$ vacuum can be obtained as a solution of a lower d-dimensional effective action arising from dimensional reduction on X_n,

$$S_d = \int d^d x \sqrt{-g} \left(\frac{R}{16\pi\, G_d} + \mathcal{L}_d(\Phi_i, \vec{n}) \right), \tag{5.1}$$

where $\{\Phi_i\}$ denotes the complete set of KK modes (spins $0, 1, 2$) of the fields obtained by dimensional reduction of the higher-dimensional theory on X^n, including the moduli, and $\vec{n}$ is a (properly quantized) vector of integer fluxes threading the compactification manifold X_n, which affect the couplings of the lower dimensional action [110, 111]. Each choice of $\vec{n}$ is a different low-energy effective field theory, and a different compactification. In examples coming from near-horizon geometries of brane stacks, $\vec{n}$ roughly counts the number of branes (of different kinds) in the stack.

$\mathcal{L}_d(g, \Phi_i, \vec{n})$ is then a very general Lagrangian including kinetic and mass terms for the Φ_i, couplings to the metric g, and possibly an infinite tower of higher-derivative couplings arising from dimensional reduction of the higher-dimensional theory. Attempting to find solutions to the equations of motion arising from $\mathcal{L}_d(g, \Phi_i)$ is in general a hopeless task, since they involve infinite towers of fields coupled to one another. There are, however, two circumstances in which the problem simplifies and only a finite number of fields becomes relevant:

- When there is a *consistent truncation*, it is possible to truncate to zero all but finitely many of the Φ_i and still solve the equations of motion. This usually happens due to supersymmetry of the low-energy effective action [112], and it is the case in e.g. the $\mathrm{AdS}_5 \times S^5$ compactification of IIB string theory [10], where only the lowest modes of the KK tower are switched on, in spite of the fact that they couple strongly to every other KK mode (since the solution is not scale-separated).
- When the AdS_d solution is *scale-separated*, there is a large gap between the (finitely many) lightest Φ_i and the rest (see [113] for a review). In this case, one can integrate out all heavy fields to obtain the low energy effective theory including only a finite number of light fields. However, this is typically impossible or extremely hard in realistic string theory compactifications. Instead, if the scale separation is parametrically large, the heavy fields are expected to have a very small effect on the light ones at low energies, so that one can in principle ignore them and consider the d-dimensional effective field theory truncated to the light fields.[1] In particular, the DGKT solution of [20] is famously scale-separated and the low-energy effective field theory of the light fields is well-known.

In both these cases, one can further truncate to zero all spin-1, and spin-2 fields, and look for solutions involving the scalars ϕ^i only. At the two-derivative level, the action for the scalars simply looks like

$$\mathcal{L}_{\text{eff}}^{\text{Scalars}} = \frac{1}{2} G_{ij}(\phi^i, \vec{n}) d\phi^i \wedge *d\phi^j - V(\phi^i, \vec{n}), \tag{5.2}$$

where $V(\phi^i)$ is a scalar potential. If there is a critical point of $V(\phi^i, \vec{n})$ with $V(\phi^i) < 0$, then (5.1) admits an AdS_d solution, with cosmological constant given by

$$\Lambda = 8\pi\, G_d\, V_{\text{min}}(\vec{n}). \tag{5.3}$$

[1] When we obtain a vacuum from simply truncating the theory to the light fields and ignore the effect of the heavy fields, we say that the vacuum does not have a full top-down description in string theory, since it is yet to be proven that such effective theory is indeed the low energy limit of the full theory after properly integrating out all the heavy fields (including exotic stuff like degrees of freedom at singularities, higher order supersymmetry breaking effects, etc). At the moment, all proposed scale-separated AdS vacua in string theory have been obtained via this truncation, and this is why it is under debate whether they can be lifted to vacua of the full theory.

Since $V_{\min}(\vec{n})$ depends on $\vec{n}$, we get a family of AdS vacua; the usual dependence is such that $\Lambda \to 0$ as the fluxes get very large.

Our goal is to find out what is the precise geometry that has to be probed by branes, which source the fluxes $\vec{n}$, in order to produce the given AdS$_d$ family of flux vacua as their near-horizon geometry. In other words, to backtrack the flux compactification in this way, we need to get rid of the flux; so we simply set $\vec{n} = 0$ in the above. In this case, the scalar potential $V(\phi^i, \vec{n})$ does *not* have a critical point and hence there is no AdS solution. Instead, in this case one can obtain a running solution, where the metric takes the form

$$ds_d^2 = dr^2 + e^{2A(r)} ds_{d-1,\text{flat}}^2 \tag{5.4}$$

where $ds_{d-1,\text{flat}}$ is a flat metric, and where the profiles of all scalar fields Φ_i depend only on r. When $A(r)$ is a linear function of r, this metric (5.4) describes (the Poincaré patch) of AdS$_d$ [114]. But when no AdS solution exist, one has more general profiles, similar to the dynamical cobordisms of [107–109]. The resulting running solution with $\vec{n} = 0$ is the geometry we are looking for. A general lesson from those works is that one should expect A to become singular at a particular value of r, where the geometry ends in a singularity. Putting back the branes that source the fluxes $\vec{n}$ on the geometry on top of this singularity recovers the original AdS solution we started with, as illustrated in Figure 5.1.

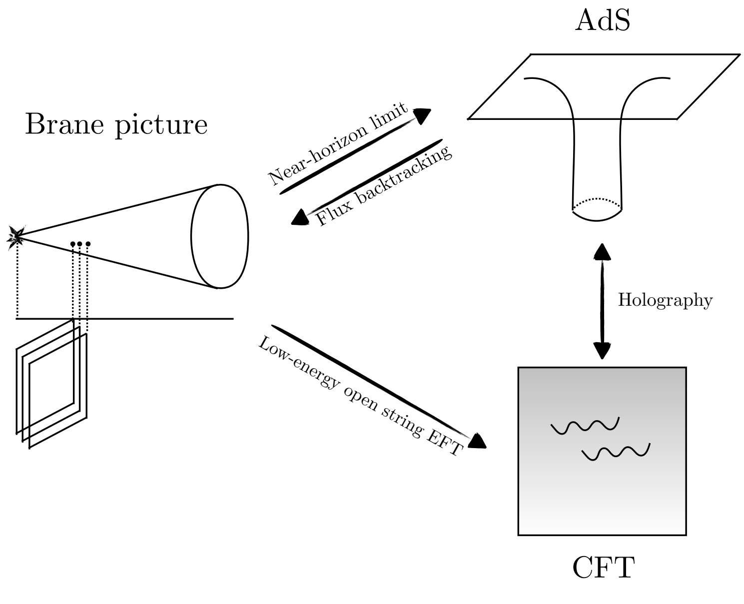

Figure 5.1: *Flux backtracking is the inverse operation of taking a near-horizon limit of a brane stack probing a singularity (which produces an AdS geometry under favorable circumstances). It may be of interest in cases where the brane picture and/or the holographic dual are unknown, such as in DGKT or the vacua in [48].*

In the next Section we will illustrate the flux backtracking procedure in some very well-known examples of the AdS/CFT correspondence, to check that it produces meaningful answers. We close this Section with some general issues that will appear when applying the procedure:

- The process of flux backtracking is not unique except in very simple cases. In general, it can be done in several ways, corresponding to the different ways there may be to obtain a brane picture for a given AdS vacuum. For instance, the $\mathrm{AdS}_3 \times S^3 \times T^4$ vacuum dual to the D1-D5 SCFT [9] is supported by two fluxes $\vec{n} = (n_1, n_2)$, sourced by D1- and D5-branes respectively. One can get the vacuum by probing the singular geometry sourced by n_1 D1-branes with D5-branes, or by probing with D1 branes the *different* singularity sourced by n_2 D5-branes. Consequently, there are at least two different ways to implement the flux backtracking procedure.

- The output of flux backtracking is a singular configuration which is, topologically, a boundary for the starting compactification on X_n – it is an example of the defects predicted in [108, 109, 115]. In principle, there are infinitely many ways to do this; for instance, one could consider a cobordism to a different compactification manifold Y_n, or change the way that tadpoles are satisfied in X_n by adding additional branes to the compactification, etc. All of these operations change the effective Lagrangian $\mathcal{L}_{\text{eff}}(g, \Phi_i, \vec{n})$. In this chapter, we restrict to flux backtracking within a given effective theory, meaning that the fields in $\mathcal{L}_{\text{eff}}^{\text{Scalars}}$ are all the same for any value of $\vec{n}$. As a result, we do not allow for more exotic cobordisms, and the only fluxes in $\vec{n}$ that we can change freely are those that do not appear in tadpoles of the compactification. It would be interesting to lift these restrictions in the future.

- When the scalar potential $V(\phi^i, \vec{n})$ is obtained via a consistent truncation, one must be sure that this truncation remains consistent for all $\vec{n}$, including $\vec{n} = 0$; otherwise, the $\vec{n} = 0$ running solution might require one to leave the truncation, and an extended problem should be considered.

Related to the second point above, when testing the flux backreaction procedure in concrete examples, we will find several instances where the lower d-dimensional scalar potential is not known – instead, only the AdS solution is known, and it was obtained directly from the higher-dimensional supergravity. Absent the scalar potential, we are forced to do some informed guesswork about which scalars are relevant and run once the fluxes are switched off. We get the right result because in these examples we know the brane picture anyway; but for any interesting example where the brane picture is not known, we must be sure to include all relevant scalars. This will be a potential issue in some of the examples in [106], but importantly, it is *not* an issue for the DGKT solution, precisely because it is supposed to be scale-separated [20]. As described above, in this case there is an honest lower-dimensional EFT with a finite number of scalar fields, and as long as these are all taken into account, the description is complete.

5.3 Vacua with holographic dual

We are now ready to test the flux backtracking procedure against known holographic examples. We will start with the simplest case where the relevant effective field theory contains a single scalar. As in Section 5.2, we consider a d-dimensional action with a scalar potential $V(\phi)$, expressed as

$$S_d = \int d^d x \sqrt{-G_d} \left(\frac{1}{2}\mathcal{R}_d - \frac{1}{2} G_{ij} (\partial \phi^i)(\partial \phi^j) - V(\phi) \right), \tag{5.5}$$

where G_{ij} is a metric on field space. After removing the flux $\vec{n}$, we will be looking for solutions of the form

$$ds_d^2 = dr^2 + e^{2A(r)} ds_{d-1}^2, \quad \phi_i = \phi_i(r), \tag{5.6}$$

for the metric and scalar field, where just like in Section 5.2 ds_{d-1}^2 is a flat metric. This ansatz is precisely what is considered in the study of supersymmetric domain wall solutions [116], and we will benefit from this fact. Substituting this ansatz into the action yields

$$S_d = \int d^d x \, e^{(d-1)A} \left(\frac{1}{2}(d-1)(d-2)A'^2 - \frac{1}{2} G_{ij} (\phi^i)'(\phi^j)' - V(\phi) \right). \tag{5.7}$$

From this, we derive the second-order equations of motion[2]:

$$(\phi^i)'' + (d-1)(\phi^i)' A' = \partial_{\phi^i} V(\phi), \tag{5.8}$$

$$(d-2)\left[2A'' + (d-1)A'^2 \right] = -G_{ij}(\phi^i)'(\phi^j)' - 2V(\phi), \tag{5.9}$$

$$(d-2)(d-1)A'^2 = G_{ij}(\phi^i)'(\phi^j)' - 2V(\phi). \tag{5.10}$$

To simplify the analysis, following the techniques from supersymmetric domain walls [116] we aim to recast these into first-order flow equations. A good choice is

$$\frac{d\phi^i}{dr} = \alpha G^{ij} \frac{dP}{d\phi^j}, \quad \frac{dA}{dr} = \beta P, \tag{5.11}$$

where α and β are constants given by

$$\alpha = \pm 2(d-2), \quad \beta = \mp 2 \tag{5.12}$$

and P is a potential function. The change of coordinates $r \to -r$ flips the sign choice in (5.12), so without loss of generality we can take the positive sign for α. For these first-order equations to be consistent with the second-order equations of motion (5.8), we specify that P relates to the scalar potential $V(\phi)$ by

$$V = \frac{1}{2}\alpha^2 \left[\sum_i \left(\frac{dP}{d\phi_i} \right)^2 - \frac{(d-1)}{(d-2)} P^2 \right]. \tag{5.13}$$

[2] Assuming the field space metric has vanishing Christoffel symbols, which is always true in the one-dimensional case under consideration here.

Under these conditions, solutions to the first-order flow equations automatically satisfy the second-order equations of motion in (5.8) [116]. Such a potential function P is generally present in supersymmetric frameworks, but it can also arise in contexts involving fake supersymmetry [117, 118]. We are now ready to implement flux backtracking. We will be assuming that the d-dimensional effective field theory arises from a higher $d+n$ dimensional theory, such that the metric can be uplifted to something of the form

$$ds^2_{d+n} = f(\phi_i)\left(dr^2 + e^{2A(r)}ds^2_{d-1}\right) + g(\phi_i)ds^2_n, \tag{5.14}$$

for some moduli-dependent functions f and g. Here, ds^2_n is the metric of the internal compactification X_n, which remains the same whether we turn fluxes on or off in all examples considered in this chapter (but this will not be the case in more general examples).

Finally, we wish to provide a word of caution about low codimension cases. For the picture of a stack of branes probing a singularity to give a field theory description of the system, it is important that bulk low energy modes decouple from the degrees of freedom described by the brane stack+singularity system. This happens automatically when the codimension of the singularity and/or brane probes is higher than two. Otherwise, the question of bulk mode decoupling is subtler and needs to be analysed on a case by case basis. If bulk modes do not decouple, spin two or higher states may remain in the low energy theory, thereby spoiling a pure field theoretical description (which can only involve fundamental fields of spin ≤ 1). Hence, even if the singularity+branes do describe the system in some sense, we will not be able to use them to learn anything about the system that we did not understand from the original gravity solution. Fortunately, all examples considered in this chapter will be of higher codimension when viewed from the 10 or 11 dimensional perspective.

5.3.1 Freund-Rubin vacua

We will first consider Freund-Rubin vacua, which are arguably the simplest class of holographic AdS vacua in string theory [10, 119, 120]. In this case, the $(d+n)$-dimensional action includes contributions from both the curvature of the compact space and a flux threading the n internal dimensions. The action is given by

$$S_{d+n} = \int d^{d+n}x\sqrt{-G_{d+n}}\left(\frac{1}{2}\mathcal{R}_{d+n} - \frac{1}{2n!}F_{M_1\ldots M_n}F^{M_1\ldots M_n}\right), \tag{5.15}$$

following the notation in [121]. In a holographic context, the flux threading the internal dimensions is replaced by D-brane domain walls, with an AdS vacuum emerging as their near-horizon limit. To determine the geometry experienced by these branes, we remove the flux term, leaving a pure Einstein-Hilbert action,

$$S_{d+n} = \int d^{d+n}x\sqrt{-G_{d+n}}\frac{1}{2}\mathcal{R}_{d+n}. \tag{5.16}$$

Upon dimensional reduction, we obtain

$$S_d = \int d^dx\sqrt{-G_d}\left(\frac{1}{2}\mathcal{R}_d - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right), \tag{5.17}$$

where ϕ is the canonically normalized volume modulus,

$$\phi = \sqrt{\frac{d+n-2}{n(d-2)}}\ln\mathcal{V}, \tag{5.18}$$

defined in terms of the internal volume $\mathcal{V}$ in string units. The potential for ϕ, sourced by the curvature term, reads

$$V(\phi) = -Ce^{-2\frac{\sqrt{n+d-2}}{\sqrt{n(d-2)}}\phi}, \tag{5.19}$$

where C is a positive constant. This curvature potential can be rewritten in the form of (5.13) by defining

$$P = pe^{-\frac{\sqrt{n+d-2}}{\sqrt{n(d-2)}}\phi}. \tag{5.20}$$

where $p^2 = \frac{nC}{2(n-1)(d-2)}$. Using the ansatz for the metric in (5.6), the flow equations are solved by

$$\phi(r) = \sqrt{\frac{n(d-2)}{n+d-2}}\ln r, \quad A(r) = \frac{n}{n+d-2}\ln r, \tag{5.21}$$

up to constants. Uplifting the resulting metric to $d+n$ dimensions yields

$$\begin{aligned} ds^2_{d+n} &= \mathcal{V}^{-\frac{2}{d-2}}ds^2_d + \mathcal{V}^{\frac{2}{n}}ds^2_n & (5.22)\\ &= r^{-\frac{2n}{n+d-2}}\left(dr^2 + r^{\frac{2n}{n+d-2}}ds^2_{d-1}\right) + r^{\frac{2(d-2)}{d+n-2}}ds^2_n & (5.23)\\ &= dy^2 + ds^2_{d-1} + y^2 ds^2_n, & (5.24)\end{aligned}$$

where $y \sim r^{\frac{d-2}{d+n-2}}$ defines a new radial coordinate. This metric describes a conical singularity, implying that all these Freund-Rubin vacua can be described as the near-horizon geometry of the D-branes, that source the flux, probing a conical singularity. This is a well-known fact of Freund-Rubin AdS vacua [122, 123]. Therefore, the flux-backtracking procedure successfully recovers the geometry that must be probed by the branes in this case.

Example: AdS$_5 \times S^5$ and Sasaki-Einstein compactifications

The simplest example are the AdS$_5 \times S^5$ vacua in IIB string theory, where a five-form flux F_5 threads an internal sphere S^5 [10]. To find the brane picture, the N units of five-form flux should be replaced with N D3-branes. Following the results that we obtained above using the flux backtracking method, these D3-branes should probe the following geometry,

$$ds_{10}^2 = dy^2 + ds_4^2 + y^2 ds_{S^5}^2, \tag{5.25}$$

which is just the 10-dimensional flat space metric. Indeed, it is well known that the near-horizon geometry of a stack of D3 branes in flat space corresponds to AdS$_5 \times S^5$, so the flux backtracking procedure has yielded the correct answer. Moreover, the central charge of the CFT scales as N^2, which is precisely the result that one would get from counting perturbative degrees of freedom for a stack of N D3-branes. Since the string coupling does not run in this solution, it can be taken as very small in the solution, so that the worldsheet techniques to count degrees of freedom remain valid (we will see that this is not always the case in more complicated examples).

This solution can be generalized e.g. for the Klebanov-Witten vacua [103] of the form AdS$_5 \times X^5$, where X^5 is an Einstein space. Using the backtracking flux procedure, the metric then becomes

$$ds_{10}^2 = dy^2 + ds_4^2 + y^2 ds_{X^5}^2, \tag{5.26}$$

with X^5 being the base of an actual conical singularity. This geometry is no longer flat space, but indeed corresponds to a conical Calabi-Yau whose base is the Sasaki-Einstein space X^5, as expected. A stack of N D3-branes probing such conical singularity yields the AdS$_5 \times X^5$ vacuum as their near-horizon geometry. We can also apply the same logic to any Sasaki-Einstein compactification, yielding the appropriate conical singularity.

Example: AdS$_4 \times S^7$ and AdS$_7 \times S^4$

Other well-known examples of Freund-Rubin vacua are the AdS$_4 \times S^7$ and AdS$_7 \times S^4$ solutions of M-theory [10]. In these cases, the higher dimensional spheres are supported by N units of the seven-form flux F_7 and four-form flux F_4 respectively. We can then replace these fluxes by the corresponding stack of N M2-brane and M5-brane domain walls, respectively, and solve the above flow equations. The result is that these branes should probe an eleven-dimensional flat spacetime with metrics

$$ds_{11}^2 = dy^2 + ds_3^2 + y^2 ds_{S^7}^2, \quad \text{and} \quad ds_{11}^2 = dy^2 + ds_6^2 + y^2 ds_{S^4}^2. \tag{5.27}$$

recovering the brane picture of these AdS vacua. Here, the central charge grows as $N^{3/2}$ and N^3 respectively, corresponding to a stack of N M2 branes and a stack of N M5 branes, respectively.

5.3.2 ABJM theories in IIA string theory

Let us move on now to a slightly more complicated family of vacua: the ABJM vacua [95]. The ABJM field theories arise as the worldvolume description of N M2-branes placed at a $\mathbb{C}^4/\mathbb{Z}_k$ orbifold singularity, rather than flat space. In the strong coupling limit, where $N \gg k^5$, these are dual to M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$. In the 't Hooft limit, though, where $N, k \to \infty$ with $\lambda = N/k$ fixed but small, they are better described by Type IIA on $\mathrm{AdS}_4 \times \mathbb{C}P^3$. In this section, we check whether our method recovers the aforementioned orbifold singularity starting from the bulk supergravity description.

We start from type IIA string theory with N units of F_6-flux, k units of F_2-flux and internal curvature. We remove the contributions from the F_6-flux, dual to the M2-branes, from the effective potential and consider the resulting flow generated by F_2-flux and internal curvature. In Appendix A.1 we perform the general dimensional reduction of Type IIA and the derivation of the superpotential P for the flux-induced scalar potential. Particularizing it to the case of ABJM, we obtain that the four-dimensional scalar potential is given by

$$V = A_R \frac{1}{us^2} + A_{F_2} \frac{u}{s^4} + \text{local contributions} \tag{5.28}$$

where the first contribution is from the curvature of the internal space and the second one corresponds to the F_2 flux. The functions A_R and A_{F_2} are independent of the non-compact moduli so they will not play any role in our discussion. The universal moduli u, s are related to the overall volume $\mathcal{V}$ and the 10-dimensional string dilaton ϕ as follows,

$$\mathcal{V} = u^3 \qquad e^{-2\phi} = s^2 u^{-3}. \tag{5.29}$$

Recall that this is not a proper effective field theory since there is no scale separation; however, thanks to supersymmetry, we can consider a truncation of the theory to the sub-sector of zero modes, which will suffice for our purposes.

Using (A.6), the superpotential P is given by

$$P = c_R u^{-\frac{1}{2}} s^{-1} + c_{F_2} u^{\frac{1}{2}} s^{-2}, \tag{5.30}$$

where we again do not specify the value of the constants as we only need the moduli dependence.

We find that the flow equations (5.11) are solved by

$$s(r) \sim r^{2/3}, \quad u(r) \sim r^{2/3}, \quad A(r) = \frac{7}{9} \ln r. \tag{5.31}$$

The ten-dimensional uplift of the domain wall metric is then given by

$$ds_{10}^2 = [s(r)]^{-2} \left(dr^2 + e^{2A(r)} ds_3^2 \right) + [u(r)] ds_{\mathbb{C}P^3}^2 \quad = dy^2 + y^{2/3} ds_3^2 + y^2 ds_{\mathbb{C}P^3}^2, \tag{5.32}$$

where we have defined $y = r^{1/3}$. This metric represents a deformed conical singularity with a running string coupling

$$g_s(y) = e^{\phi(y)} = u^{3/2}(y)s^{-1}(y) = y. \tag{5.33}$$

When uplifting to eleven dimensions, with z parametrizing the M-theory circle, we have

$$\begin{aligned} ds^2_{11} &= e^{-2\phi(y)/3}ds^2_{10} + e^{4\phi(y)/3}dz^2 \\ &= d\tilde{y}^2 + ds^3_3 + \tilde{y}^2 ds^2_{S^7/\mathbb{Z}_k}, \end{aligned} \tag{5.34}$$

redefining $\tilde{y} = y^{2/3}$. Therefore, we indeed recover the conical orbifold singularity probed by the stack of M2-branes in the brane picture. In conclusion, the solution goes to strong coupling for large $\tilde{y}$, matching the M-theory on $S^7/\mathbb{Z}_k$ description. In the opposite limit $\tilde{y} \to 0$, the string coupling decreases as we approach the brane stack, so that the system can be analyzed via a D2-brane picture [124]. The central charge of the IR CFT grows as $N^{3/2}$ times a k-dependent function [95], generalizing the result of the previous example of $AdS_4 \times S^7$. As it is well-known, the tree level N^2 naive expectation from a stack of N D2-branes is reduced due to strong coupling effects, but still upper bounded by the perturbative result of the UV worldvolume theory of the D2's since the singularity is weakly coupled.

5.3.3 AdS vacua in massive IIA string theory

Let us next investigate AdS vacua arising from massive Type IIA. We are interested in these vacua as they also include Romans mass, which plays a key role in the DGKT vacuum that we aim to analyse in Section 5.4. We will see that the flux backtracking procedure provides the correct result in these vacua with Romans mass.

We first consider the singularity probed by D2-branes in the Guarino-Jafferis-Varela set-up [105]. This is a squashed AdS$_4 \times S^6$ background in massive type IIA, with dual super-Chern-Simons-matter theories that have a $SU(N)$ gauge group and level m equal to the Romans mass. The CFT lives on the D2-branes which have Romans-mass induced Chern-Simons coupling, which deforms the D2-brane near-horizon geometry.

The lower dimensional potential of the bulk theory will receive contributions from the F_6 flux, the Romans mass and the curvature (see [125]). In order to find the geometry probed by the D2-branes that source the F_6 flux, we need to find the running solution to the potential upon setting to zero the F_6 flux. Using the result for the effective potential in (A.5) of Appendix A.1, we obtain that the potential driving this running solution is given by

$$V = A_R \frac{1}{us^2} + A_{F_0}\frac{u^3}{s^4} + \text{local contributions} \;\to P = c_R u^{-\frac{1}{2}}s^{-1} + c_{F_0}u^{\frac{3}{2}}s^{-2}, \tag{5.35}$$

where again the moduli u, s are related to the overall volume and the dilaton as in (5.29). The flow generated by the Romans mass and the curvature is solved by

$$s(r) \sim r^{4/5}, \quad u(r) \sim r^{2/5}, \quad A(r) = \frac{19}{25} \ln r, \tag{5.36}$$

and this corresponds to the following ten-dimensional metric,

$$ds^2_{10} = [s(r)]^{-2} \left(dr^2 + e^{2A(r)} ds^2_3\right) + [u(r)] ds^2_{\tilde{S}^6} \quad = dy^2 + y^{-2/5} ds^2_3 + y^2 ds^2_{\tilde{S}^6}, \tag{5.37}$$

with $y = r^{1/5}$ and running coupling $g_s(y) \sim y^{-1}$. This describes a deformed conical singularity, as perhaps could have been expected from the fact that massive IIA does not admit 10-dimensional flat space as a solution. The central charge is known to grow as $N^{\frac{5}{3}}$. Since the singularity is strongly coupled, unlike in previous examples, we do not expect to be able to use worldsheet techniques to derive this result.

The result (5.37) is, to our knowledge, the first attempt in the literature at constructing the massive IIA configuration dual to the theories in [105]. However, we emphasize that the result should be taken with a grain of salt. As explained in Section 5.2, the simple flux backtracing procedure that we perform relies crucially on us including all light scalars that are relevant to the problem. Since we only started from a four-dimensional effective field theory view, from which then we switched off some fluxes, it is conceivable that doing so turns on scalars not included in our original consistent truncation. The way we are doing things, we are blind to this possibility (which will not impact scale-separated setups such as the DGKT scenario that we discuss in Section 5.4.1), and so (5.37) should only be taken as an educated guess for further exploration. We have checked that (5.37) satisfies directly the massive IIA equations of motion in ten dimensions provided that $c^2_{F_0}/c^2_R = (m/(6\sqrt{30}))^2$, and is compliant with the bound on the size of strongly coupled regions in massive IIA proposed in [63] (although it exactly saturates it since the inverse of curvature radius and the string dilaton decrease at the same rate as $y \to 0$), so at least there's that.

5.3.4 More general examples

We end this Section with a few comments about examples with known holographic duals that are more involved, either because they have many fields and/or non-trivial field profiles in the internal dimensions.

Consider the AdS$_6$ vacua of Jafferis-Pufu [104] arising in Type I' string theory. The CFT comes from a system of D4-branes intersecting D8-branes and O8-planes. The bulk gravitational theory contains an AdS$_6 \times S^3$ factor arising upon compactification of Type I' including F_4-flux. Recall that Type I' is equivalent to Type IIA on an interval, with two

$O8^-$-planes at the endpoints of the interval and D8-branes to cancel the tadpole. The D8-branes induce a non-trivial value of the Romans mass in the interval. Consequently, the low energy theory contains two geometric moduli (in addition to the dilaton) which parametrize the volume of the S^3 and the size of the Type I' interval. The Romans mass, the curvature of S^3 and the F_4-flux supporting the internal space will all contribute to the effective bulk potential. However, this is not all. This example exhibits a feature that did not appear in previous examples: the dilaton has a non-trivial profile in the internal dimensions, so it is not a homogeneous solution. In particular, the dilaton non-trivial profile occurs along the interval in between the O8's. This induces an additional contribution to the six-dimensional potential coming from the spatial gradient of the dilaton. In order to find the geometry probed by the D4-branes, we would need to solve the flow equations taking all these ingredients into account (except for the F_4-flux sourced by the D4's), which becomes quite involved as the running of the scalars will occur along both external and internal directions.

The basic issue we are facing is again that we do not have an off-shell scalar potential including all relevant fields in this case. However, there is a way to simplify the computation by performing the flux backtracking procedure in two separate steps (which secretly use the fact that we know what the final answer should be). Notice that we actually have two independent flow directions: the non-compact radial direction of the six-dimensional space and the nine-th coordinate associated to the internal Type I' interval. Hence, we can first consider the Type I' flow in nine dimensions driven by the Romans mass, and then further compactify to consider the six-dimensional flow driven by the curvature of the S^3 as in a Freund-Rubin setup.

Let's start with the first step and look for a running ten-dimensional solution of the form

$$ds_{10}^2 = dr^2 + e^{2A(r)} ds_9^2, \tag{5.38}$$

where ds_9^2 is a Minkowski metric. The flow is only driven by the Romans mass since it is the only ingredient affecting the warp factor $A(r)$ and the dilaton. Using (5.11), this leads to $P(\phi) = e^{5\phi/4}$, with $e^\phi = r^{-4/5}$ and $A(r) = \frac{1}{25}\ln r$. After going to string frame and redefining the radial coordinate $r \mapsto r^{4/5}$, the result is the very well-known Polchinski-Witten solution of Type I' given by

$$ds_{10}^2 = dr^2 + r^{-2/5} ds_9^2, \quad e^\phi \sim r^{-1}. \tag{5.39}$$

One can also check that this is the near-horizon geometry in a background induced by D8-branes.

Next, we further compactify this nine-dimensional Minkowski space on S^3. The combination of the F_4-flux and the curvature yields the AdS$_6$ solution. However, to find the geometry probed by the D4-branes, we get rid of the F_4-flux and simply find the running solution driven by the curvature term. The computation is analogous to that of Freund-Rubin compactifications in Section 5.3.1, so we can borrow the results to obtain

$$ds_9^2 = \mathcal{V}^{-1/2}(d\tilde{r}^2 + e^{2A(\tilde{r})}ds_5^2) + \mathcal{V}^{2/3}ds_{S^3}^2 = dy^2 + ds_5^2 + y^2 ds_{S_3}^2, \tag{5.40}$$

with $y = \tilde{r}^{4/7}$, $A(\tilde{r}) = \frac{3}{7}\ln r$ and e^ϕ being a constant function of $\tilde{r}$. The result is that the nine-dimensional metric is still flat space, so when combining everything together we get that the geometry probed by the D4's is

$$ds_{10}^2 = dr^2 + r^{-2/5}ds_9^2, \quad e^\phi \sim r^{-1}, \tag{5.41}$$

with ds_9^2 corresponding to Minkowksi space as in (5.40).

Another example that we did not discuss about is the AdS$_7$ vacua of [106]. These vacua are notoriously complicated, involving "football" geometries with an anisotropic internal manifold on the bulk side, and intersecting D6-NS5-brane stacks on the other. This is yet another example of the cases (mentioned at the end of Section 5.2) where we do not have an off-shell flux superpotential. As a result, there is no guarantee that sticking to just one scalar (e.g. the volume of the compactification manifold, as we have done in previous cases) is a consistent truncation, and in fact, we have checked that if one does so, the flux backtracking procedure gives wrong results. Since we know that the internal manifold is anisotropic, it makes sense that we should include at least another modulus (and possibly more). While in principle this resolves the contradiction, it would be nice to explore multi-field cases in detail.

5.4 Vacua with unknown holographic dual

Now that we have some confidence in the validity of flux backtracking, we apply it to the cases of real interest: AdS vacua without a known holographic dual.

5.4.1 DGKT scale separated AdS$_4$ vacua in massive IIA

The DGKT [20] vacua are compactifications of massive IIA string theory on a Calabi-Yau manifold in the presence of unbounded F_4-fluxes, bounded H_3-flux (and F_0-flux) and O6-planes. This is an example of special interest as it provides a scale-separated AdS$_4$ solution, namely the size of the extra dimensions is smaller than the AdS size, so the vacuum is honestly four-dimensional. However, it is not clear whether this proposed supergravity solution can be uplifted to a UV-consistent string theory solution (see [113] for a review on potential issues). Recently, we have found [57] that this vacuum is in tension with the vanilla application of the Weak Gravity conjecture to domain walls, so either there is some inconsistency in the DGKT vacuum or the Weak Gravity Conjecture for domain walls must be reformulated, which would also have profound implications for the expected instability of non-SUSY vacua [40]. Any progress in constructing the CFT dual of this AdS solution could help to settle the debate and find out whether the vacuum is UV consistent or not. Notice that, at the moment, there is no single example of an AdS/CFT pair exhibiting scale separation; all AdS proposed scale-separated vacua do not have known CFT duals yet, and whether this is at all possible has been put into question [49, 51, 54, 55, 61]. Therefore, DGKT vacua (or similar constructions) have the potential to become the first scale-separated examples in the literature, if a CFT dual can be found.

A first step towards constructing the CFT dual is to find the brane picture of the AdS vacuum, which we will do in this section. The F_4-flux number N parametrizes the amount of control and scale separation, since g_s^{-1}, the overall volume and the ratio between the KK and the AdS energy scales grow with N. Hence, we will apply backtracking to this flux only, so the result will be some singularity in massive IIA where an $O6$-plane and H_3-flux end, as depicted in Figure 5.2. Probing this singularity with D4-branes should reproduce the AdS DGKT background as their near-horizon geometry.

When switching off the F_4-flux, we can study the problem using the explicit 4d $\mathcal{N}=1$ solution of [20]. Since the solution is scale-separated, the effective field theory captures in principle the full dynamics of all scalars. The remaining fluxes then generate the following potential

$$V = \frac{1}{s^3}\left[\frac{A_{F_0}u^3}{s} + \frac{A_{H_3}s}{u^3} - A_{O6}\right], \tag{5.42}$$

for which the potential P is

$$P = c_{F_0}u^{\frac{3}{2}}s^{-2} + c_{H_3}u^{-\frac{3}{2}}s^{-1} \tag{5.43}$$

and the flow equations (5.11) are solved by the following running solution

$$s(r) \sim r^{2/3}, \quad u(r) \sim r^{2/9}, \quad A(r) = \frac{13}{27}\ln r. \tag{5.44}$$

The geometry that the D4-branes should probe in the DGKT geometry is then an orientifold of

$$ds_{10}^2 = dy^2 + y^{-10/9}ds_3^2 + y^{2/3}ds_{CY}^2, \quad g_s(y) \sim y^{-1} \tag{5.45}$$

where $y = r^{1/3}$.

Result (5.45) is, to our knowledge, new; it unambiguously comes out from the flux backtracking procedure applied to DGKT. Unlike previous examples, this resulting

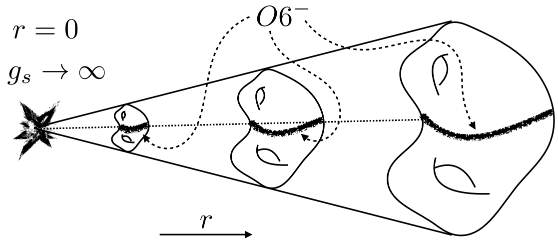

Figure 5.2: *Flux backtracking applied to DGKT produces a strongly coupled singularity in massive IIA, depicted in the figure. The CY_3 of DGKT is fibred along the r direction, reaching 0 at finite distance. Probing this singularity with D4 branes and taking the near-horizon limit should return the original DGKT vacuum. Notice that the $O6^-$ plane (depicted as a fuzzy line on each of the three illustrative slices) of DGKT ends at the singularity; this is possible because the geometry is also threaded with H_3 flux, cancelling the tadpole.*

singularity is neither conical nor deformed conical. This is to be expected, since massive IIA does not admit flat space as a solution; asymptotically conical geometries precisely asymptote to flat space. The interesting behavior however is the singularity at $y \to 0$. We observe that the string coupling diverges there. This is even more interesting, since the fact that the coupling becomes strong means that we will never be able to understand this singularity via D-brane techniques. In other examples (such as ABJM, discussed in Section 5.3.2), the reason why we understand the system is that there is a UV brane picture involving weakly coupled singularities, which can be probed via D-branes in string perturbation theory. Once the UV dynamics is known, it may flow to a strongly coupled system in the IR (like for ABJM for small k), but the QFT description provides a foothold from which we can understand the system explicitly, providing e.g. an upper bound for the central charge via the c-theorem. Since the DGKT singularity is strongly coupled, we cannot do any of the above, even in principle (see e.g. [67] for a proposal of increasing the number of degrees of freedom at strong coupling by considering multi-prong strings). We want to note, though, that the central charge is much larger than the $N^{5/3}$ scaling obtained in the massive IIA solution of [105], for which the scaling of the central charge is smaller than N^2 even though it is also strongly coupled.

Finally, reference [63] gave evidence that massive IIA cannot be strongly coupled. The precise bound they put forth was that the string coupling could not grow beyond the local curvature scale in string units, although most of the evidence from this statement was obtained in the supergravity approximation. Hence, the bound that needs to be satisfied is $g_s \lesssim l_s/R$ where R is the radius of curvature. Using (5.44) and (5.29), we obtain that $g_s \sim y^{-1} \lesssim l_s/R \sim y^{-1/3}$, where we have also used that the curvature radius in string units goes as $\mathcal{V}^{1/6} \sim u^{1/2}$. Hence, this bound holds as long as we are at a distance y from the singularity bigger than a string length (at smaller distances we cannot trust the solution anyway), so the solution is consistent with [63].

5.4.2 Scale-separated AdS_4 vacua in massless IIA/M-theory

The DGKT vacuum is not the only proposed scale-separated AdS vacuum in the literature. For instance, by performing two T-duality transformations on the DGKT solutions and doing a rescaling of fluxes, new scale-separated AdS_4 solutions were obtained in IIA without Romans mass [48]. These involve an unbounded F_6-flux, partly unbounded F_2-flux that generates the scale separation, and curvature of the internal Iwasawa manifold. The F_2-fluxes are distributed anisotropically over the internal manifold, resulting in an anisotropic scale separation. This setting has the advantage of allowing for strongly coupled solutions and hence an uplift to M-theory due to the absence of Romans mass. The backreaction of the O-planes is better understood in this context [48]. From an M-theory perspective, since all fluxes but F_6 are geometrized, this solution is of Freund-Rubin type, and should arise from compactifying M-theory on a 7-dimensional manifold that admits a weak G_2 structure [48]; equivalently, this means that the 7-manifold equipped with the scale-separated metric is the base of an eight-dimensional $Spin(7)$ cone. However, it is not clear that such a G_2 structure exists. Progress towards these questions can be found in [74].

In the following, we derive the singular geometry that should be probed by D2-branes in order to reproduce this AdS vacuum. When removing the F_6-flux and replacing it with D2-branes, the remaining core ingredients that generate the flow are the same as in the ABJM construction (Section 5.3.2): F_2-flux and curvature. While details and the precise flux configuration may differ, the overall dependence of the universal moduli on the radial coordinate will remain the same, yielding

$$ds^2_{10} = dy^2 + y^{2/3} ds^2_3 + y^2 ds^2_{\mathrm{Iwasawa}}, \quad g_s(y) \sim y \tag{5.46}$$

in ten dimensions, and

$$ds^2_{11} = d\tilde{y}^2 + ds^3_3 + \tilde{y}^2 ds^2_{\mathrm{Iwasawa}\times S^1}, \tag{5.47}$$

in eleven dimensions, where $\tilde{y} = y^{2/3}$. To derive this result, we have borrowed the result for the running solution in (5.32) and (5.33), but taking into account that the internal manifold is now the Iwasawa manifold. Here, the unbounded F_2-flux has been absorbed into the internal metric component. More details and the explicit solution including the flux dependence of each metric factor can be found in Appendix A.2.

Interestingly, the resulting metric is quite simple and corresponds to a conical singularity, unlike in the above DGKT setup; this is also what one would expect from the G_2 cone picture described above. Moreover, the singularity is weakly coupled as g_s vanishes as $y \to 0$, so one could aim to understand the growth of the central charge using worldsheet techniques. According to the bulk AdS solution, the central charge of the dual CFT should grow as $c \sim l_{AdS}^2 M_{p,4}^2 \sim N^{3/2} k_1^3 k_2^{-5/2}$, where N is the F_6-flux, dual to the D2-branes probing the weakly coupled singularity, and k_1, k_2 are different components of F_2-flux that lift to metric fluxes in M-theory. This is consistent with the expected $N^{3/2}$ scaling of M2-branes probing a conical singularity. The extra $k_1^3 k_2^{-5/2}$ factor should then arise from the particular singular geometry in which the D2-branes are placed. The flux k_1 is unbounded, while the component k_2 (corresponding to e_{16} in the notation of [48]) is fixed by the D6-brane tadpole. We keep it here to illustrate that for $k_1 \sim k_2 \sim k$ (which is an $O(1)$ quantity fixed by the tadpole), the above formula for the central charge is reminiscent of the small k/N ABJM result $c \sim N^{3/2} k^{1/2}$, including the leading k-dependence. In fact, if we restore the flux dependence on the result for the metric (5.47) (see Appendix A.2), we get

$$ds_{11}^2 = d\tilde{y}^2 + k_1^{4/3} k_2^{-10/9} ds_3^2 + (k_1^{-1} k_2)^{4/3} \tilde{y}^2 ds_{Iwasawa}^2 + k_2^2 k_1^{-4} \tilde{y}^2 dz^2 \tag{5.48}$$

with $g_s(y) = k_2 k_1^{-2} y$, which for $k_1 = k_2$ also resembles the k-dependence of the metric in ABJM[3]. We may understand this as arising from the fact that in both cases we have a IIA picture where the F_2 flux scales homogeneously with a single integer k.

The main difference with ABJM gets manifest when k_1 is taken parametrically large and independent of k_2, so that $l_{AdS}/l_{KK} \sim k_1^{1/2}$ and the solution becomes scale-separated. If $N \sim k_1$ then we also recover the same huge scaling of the central charge as in DGKT $c \sim N^{9/2}$. Hence, in any parametrically scale-separated solution within this family of AdS_4 vacua, the central charge must grow strictly faster than $N^{3/2}$.

In ABJM, when $k \sim N$, the supergravity solution breaks down since the t'Hooft coupling is given by $\lambda = l_c^4/l_s^4 = N/k$, so the internal curvature l_c becomes of string size. However, in this other family of AdS_4 vacua, the internal volume in string units scale as $N^{3/2} k_1^{-1}$, so that the supergravity regime remains valid as long as $k_1 \ll N^{3/2}$. Still, due to the similarities with ABJM, one could be tempted to imagine that the dual theory is described in the UV by a gauge theory (associated to the M2-branes probing the corresponding conical singularity). However, we do not know how to make this expectation consistent with the supposed growth of the central charge. For a gauge theory with a simple gauge factor, the central charge should grow as N^2 for fixed t'Hooft coupling[4]. If we assume the

[3] In ABJM, we have

$$ds_{10}^2 = dy^2 + k^{-4/9} y^{2/3} dx_n dx^n + y^2 d\tilde{s}_6^2, \tag{5.49}$$

with $y = k^{-1/3} r^{1/3}$, where we have restored the flux dependence in (5.32), (5.34). Using that the string coupling runs as $g_s = e^{\phi} = k^{-1} y$ and defining $\tilde{y} = k^{1/3} y^{2/3}$, the metric gets uplifted to 11d as follows,

$$ds_{11}^2 = e^{-2\phi/3} ds_{10}^2 + e^{4\phi/3} dz^2 = d\tilde{y}^2 + k^{2/9} dx_n dx^n + \tilde{y}^2 (d\tilde{s}_6^2 + k^{-2} dz^2). \tag{5.50}$$

[4] This is indeed satisfied in ABJM, where $c \sim N^{3/2} k^{1/2} \sim \frac{N^2}{\sqrt{\lambda}}$ for $\lambda = \frac{N}{k} \gg 1$.

same dependence as in ABJM, where $\lambda = l_c^4/l_s^4$ with l_c being the curvature of the internal space in string units, then we would get $c \sim \frac{N^6}{\lambda^{9/4}}$ for these scale-separated AdS vacua. This suggests that the dual CFT (if it exists) does not obviously flow from a gauge theory with a number of gauge factors not scaling with N in the UV, as far as we can see. This conclusions aligns with the observations in [126], which argued that DGKT might not come from a gauge theory in the UV (under certain assumptions on the cut-off) due to unusual and perhaps pathological moduli dependence of the one-loop coefficient to the sphere partition function.

5.4.3 Scale separated AdS$_3$ vacua in massive IIA

There have also been plenty of similar proposals of three dimensional –rather than four dimensional– scale-separated vacua in massive Type IIA using G2-holonomy manifolds instead of Calabi-Yau's. The construction presented in [22] is very similar to the DGKT vacua with similar ingredients such as an unbounded F_4-flux, Romans mass and an H_3-flux. In this way, they obtain scale-separated AdS$_3$-vacua, where the F_4-flux again parametrizes the amount of scale separation, and the internal manifold is instead a G2-holonomy manifold. After removing the unbounded F_4-flux from the action, the residual scalar potential and superpotential take the form

$$V = A_{F_0} u^{\frac{7}{2}} s^{-3} + A_{H_3} u^{-3} s^{-2} - A_{O6} u^{\frac{1}{4}} s^{-\frac{5}{2}}, \quad P = c_{H_3} u^{-\frac{3}{2}} s^{-1} + c_{F_0} u^{\frac{7}{4}} s^{-\frac{3}{2}}, \tag{5.51}$$

The flow generated by the Romans mass and the NSNS-flux in three dimensions is solved by

$$s(r) \sim r^{13/16}, \quad u(r) \sim r^{1/8}, \quad A(r) = \frac{11}{16} \ln r, \tag{5.52}$$

which results in the following ten-dimensional metric

$$ds_{10}^2 = [s(r)]^{-2} \left(dr^2 + e^{2A(r)} ds_2^2 \right) + [u(r)] ds_{G_2}^2 \quad = dy^2 + y^{-4/3} ds_2^2 + y^{2/3} ds_{G_2}^2, \tag{5.53}$$

with $y = r^{3/16}$ and $g_s(y) = y^{-1}$. This is the singularity that should be probed by domain walls consisting of D4-branes wrapped on 3-cycles in order to reproduce the above AdS$_3$ geometry as its near horizon limit. This geometry turns out to be very similar to the geometry found for the DGKT vacua (5.45) and is also strongly coupled. Again, we have a very large central charge $c \sim N^4$ [1], but we cannot say much in this regard since the singularity is strongly coupled. We have also checked that the flow is consistent with the criterium of [63] of not having strongly coupled supergravity configurations of masisve IIA, since here the curvature blows up faster than g_s^{-1} as $y \to 0$.

5.4.4 KKLT AdS_4 vacua

Another important proposed AdS vacuum whose CFT dual is unknown is the 4d $\mathcal{N}=1$ AdS vacuum in the KKLT scenario. This vacuum serves as the starting point before uplifting to a de Sitter solution, and recent advances have provided concrete progress in its construction within string theory [127–129]. Despite this progress, it remains unclear whether the vacuum is fully protected from all possible corrections, as it contains blow-up cycles of string size and the very little amount of supersymmetry makes its control very challenging. Unlike DGKT, this vacuum cannot exist at parametrically large values of the volume, and exhibits an even larger scale separation, as the internal space grows logarithmically with the AdS scale.[5] This seems to imply that the dual CFT, if it exists, should have a central charge that grows exponentially on N rather than polynomially (assuming that N is still related to the flux quanta).

The papers [56, 130] precisely attempted to construct brane picture of this vacuum, by dualizing all fluxes to branes. When these branes are placed in flat space, neglecting non-perturbative corrections, [56, 130] show that their worldvolume degrees of freedom are insufficient to explain the very large growth of the central charge of the putative dual CFT. While these branes in flat space alone are not expected to provide the CFT dual of KKLT, those papers aim to argue that the resulting worldvolume theory is a UV description that should flow to the actual CFT dual to KKLT in the IR, and therefore provide an upper bound on the central charge. Rather than entering into the discussion of the consistency, or even existence, of this putative RG flow, we are interested here in whether it is possible to get information directly about the singularity that should be probed by the branes in order to realized the putative CFT dual to KKLT. If a brane picture for KKLT exists, the branes will most likely have to probe some singular, complicated geometry in order to produce the KKLT AdS vacuum as their near horizon geometry. A natural question is, therefore, whether we can apply our flux backtracking procedure to determine such singular geometry where the branes should be placed to yield the dual field theory of KKLT.

Let us first describe the low energy effective theory for KKLT. The idea is to first introduce G_3 fluxes that stabilize the dilaton and all complex structure moduli of the Calabi-Yau threefold. The overall volume of the Calabi-Yau is then stabilized by considering non-perturbative corrections (e.g. from euclidean D3-branes), such that the superpotential has the following structure:

$$W = W_0 + ce^{-2\pi aT}, \tag{5.54}$$

[5] In particular, one has $l_{KK} \sim \sigma$ and $l_{AdS} \sim \sqrt{\sigma}e^{a\sigma}$ with σ parametrizing the overall volume.

where W_0 includes the flux dependence and it is evaluated at the flux stabilized values of the complex structure moduli and dilaton. The low energy scalar potential for the Kahler modulus T becomes

$$V = \frac{\pi ace^{-2\pi ac}}{\sigma^2}\left(\frac{2\pi ac\sigma e^{-2\pi ac}}{3} + W_0 + ce^{-2\pi ac}\right) \tag{5.55}$$

where $\sigma = Re(T)$ parametrizes the overall volume of the CY.

If we try to apply the flux backtracking method to the solution, we quickly see that there are key differences between KKLT and the other setups studied in this chapter. The main difference is that there is no flux that can be taken parametrically large while keeping a supersymmetric solution, since all involved fluxes are bounded by the D3-tadpole condition. Hence, the first step of the flux backtracking procedure may already be more difficult, since finite N corrections cannot be scaled away. Nevertheless, we may still try to switch off some flux, even if it enters in the tadpole, dualize it to a brane and see where this takes us.

When switching off a flux, the tadpole forces us to include additional D-branes in the compactification, which will change the low-energy effective field theory. In a generic position, each of the branes will contribute an open string sector with an $N = 4$ U(1) vector multiplet. At weak coupling, we can decouple the dynamics of these open string fields from the closed string modes, so that we can study the running solution considering only the latter. Moreover, the D3-brane tension will can cancel against the orientifold, as happens for the ISD fluxes in KKLT. This seems to suggest that we can still use (5.54) as a good approximation for the superpotential of the running solution. However, we should keep in mind that this approximation might fail close to the singularity where small volume or strong coupling effects can imply that open and closed string sectors do no longer decouple. Modulo this caveat, we can now check what the result would be for the singular geometry depending on which fluxes are dualized to branes.

In Appendix A.3, we study two particular cases: When all fluxes are removed (so $W_0 = 0$ classically), and the case where only F_3 flux is removed, so that the stack of branes would consist only of D5-branes. When all fluxes are removed, the remaining non-perturbative superpotential generates a running solution. In Appendix A.3, we study this solution and show that the running solution is singular both at $r \to \infty$ and $r \to 0$ towards small volumes in both cases. In one of the singularities, the volume goes to zero, as in the other examples studied in this paper; in the other, the internal volume decompactifies at a finite distance. A priori, either singularity could correspond to the 5-brane locus. Finally, if only F_3 flux is removed, we get similar behaviour if the remaining NS flux and the non-perturbative terms contribute with the same sign to the superpotential.

We wish to end this Section with a note of caution. The flux backtracking procedure only yields the crudest features of the would-be singularity, and cannot be used to conclusively establish its existence or properties. In other words, while we found no obstruction to the existence of a KKLT singularity with the right asymptotics, there might be other issues invisible to our analysis, which may however be taken as a first step towards a KKLT brane picture.

5.5 Conclusions

In this chapter we have explored the application of a "flux backtracking" procedure, allowing one to reverse-engineer a brane picture starting from a given AdS flux compactification. The main tool required is an off-shell scalar Lagrangian for the low-dimensional theory. Using this, one can construct a dynamical cobordism (in the sense of [107–109]) leading to the desired geometry. This Lagrangian could correspond either to a consistent truncation of the full system or to the low-energy effective field theory if the AdS vacuum is scale-separated.

We have checked this procedure in a number of known AdS/CFT pairs. In all these examples, the flux backtracking procedure reproduces the correct result for the possibly-singular geometry that should be probed by the stack of branes in order to generate the corresponding AdS background as its near-horizon geometry. More interestingly, we have applied it also to AdS flux vacua without known holographic dual, such as [105] or the scale-separated DGKT vacuum of [20]. Doing this, we have produced a candidate massive IIA singularity which we believe that, when probed with D4-branes, results in the DGKT solution as a near-horizon geometry. The question of whether this vacuum is UV consistent and has a CFT dual reduces then to whether the worldvolume theory of the D4-branes in this singular geometry flows to a suitable CFT. The singularity is strongly coupled and preserves 3d $\mathcal{N}=1$ supersymmetry only, which is not protected against quantum effects, so the system will be difficult to analyze explicitly. At the singularity, curvatures blow up; it would be interesting to elucidate its fate given the low supersymmetry of the system, but at present we do not know how to do this. However, we hope that our explicit derivation of the brane picture could help in the future towards either finding the CFT dual or showing that it does not exist.

We have also studied the scale-separated solution of [48], which is a massless IIA "cousin" of the DGKT solution. From the point of view of our procedure, this case is completely analogous to ABJM, and the resulting singularity is compatible with the natural picture that the system is obtained via a stack of M2 branes probing a $Spin(7)$ conical singularity in M-theory. However, whether an appropriate family of singularities exists is left for future work (see also [74]).

Finally, it is natural to try to flux backtrack KKLT vacua, particularly given the recent concrete progress and discussions on the topic [56, 127–130]. A key difference with the previous setups is that there is no unbounded flux, so the procedure is sensitive to finite N corrections. Ignoring this, we have found several singularities, which asymptote to Ricci flat geometries, and which if consistent (a question about which we have nothing to say) and if probed by appropriate stacks of branes, would yield the CFT dual to KKLT. In all cases studied, however, the singularity is either non-perturbative or must be probed by non-perturbative stacks of branes, which complicates using this perspective to learn something new about KKLT.

More generally, flux backtracking or other similar procedures can be applied to all sorts of vacua where the brane picture is not known, but nevertheless one has some control over the low energy effective bulk action. Hopefully, doing so will shed some light on these questions and illuminate new corners of the AdS/CFT Landscape.

6 Conclusions

In this thesis we have studied the potential holographic duals of scale-separated AdS vacua in string theory. The most phenomenologically interesting string vacua, such as the KKLT [18] and LVS [21] constructions, realise hierarchies in a very subtle way. As a first step, instead, we focus on a simpler class of vacua where hierarchies arise more transparently with an unbounded flux parameter N that can be dialled at will. These vacua are closer in structure to the holographically well-understood Freund-Rubin solutions, which also feature a large N parameter.

We have focused on the DGKT vacua, which are scale-separated AdS_4 vacua obtained by compactifying massive IIA string theory on a Calabi Yau manifold with fluxes (F_4, F_0, and H_3) and orientifold planes (O6). We also compared our results to the scale-separated AdS_3 vacua of [22], which arise from massive IIA compactified on a G2 holonomy manifold with analogous ingredients. It is interesting to investigate whether our findings depend on the dimensionality of AdS. Additionally, there is more technology to constrain 2d CFTs, potentially making scale separated AdS_3 vacua an easier target.

We found that the spectrum of low-lying operators dual to the stabilised moduli in DGKT consists entirely of operators with integer conformal dimensions. These dimensions were shown to be independent of the flux configuration and the intersection numbers of the internal manifold. The spectrum exhibits a high degeneracy: $h^{1,1}_-$ operators have conformal dimension 6, while the remaining one has dimension 10.

For the related AdS_3 vacua, irrational conformal dimensions appear. However, this does not appear to be a generic feature of 3 dimensions, as other recently constructed scale-separated AdS_3 vacua [23, 24, 86] feature integer conformal dimensions. This raises the question of whether such integer values are a necessary feature of parametric vacua, and whether they can serve as a kind of bootstrap constraint on large N CFTs that are dual to weakly coupled gravity.

The low amount of supersymmetry is not sufficient to enforce integer conformal dimensions. We have pointed out that the appearance of integer values in the large N limit, where the scalar fields become free, can only occur if the scalars possess a specific type of shift symmetry that depends polynomially on the spacetime coordinates. The origin of these polynomial shift symmetries remains mysterious. It would be very interesting to understand their holographic implications. In the case of a constant shift symmetry, the dual field theory admits a conformal manifold. Is there a broader notion of a conformal manifold associated with polynomial shift symmetries?

The central charge of the DGKT vacua scales with N as $c \sim N^{9/2}$, which exceeds the scaling found in known CFTs. If we turn the unbounded F_4 fluxes into stacks of N D4-branes, then these branes must somehow account for this large number of degrees of freedom. We have shown that, in the case where the internal manifold is taken to be toroidal, the supergravity harmonic superposition rules for intersecting branes reproduce the correct central charge. A similar procedure yields the correct large N scaling for other asymptotic AdS flux vacua as well. The only known exceptions are the AdS_3 vacua of [22], where the scaling cannot be recovered. However, this again does not appear to be related to dimensionality. Based on the examples studied, it appears that the harmonic superposition rules reproduce the correct scaling for asymptotic AdS_d vacua in which a specific combination of the dilaton and the volume modulus is dual to an operator with conformal dimension $\Delta = 2(d-1)$. We aim to formulate and prove a more precise statement in future work. In addition, a microscopic counting of degrees of freedom is needed to confirm the central charge scaling, and it would be important to extend this picture to non-toroidal internal manifolds.

One important question is: which background should the D4-branes probe? In the case of Freund-Rubin vacua, the branes are placed in flat spacetime. In Chapter 6, we presented a method to recover the probed background from the flux potential, and we apply it to several AdS flux vacua to check its consistency. We find that the DGKT background reconstructed in this way is strongly coupled, which makes a perturbative counting of the D4-brane degrees of freedom impossible.

In conclusion, our initial exploration of the holographic properties of DGKT vacua has already uncovered striking features. Understanding these CFTs would mark a significant departure from any known examples and open up a new corner of the holographic landscape. Conversely, if such CFTs can be ruled out, it would pose an important challenge for string phenomenology: how to construct vacua resembling our universe if the most straightforward compactifications fail to achieve small extra dimensions?

So far, we have found no inconsistencies in the holographic duals of the DGKT vacua. The appearance of integer operator dimensions together with the matching of all brane-dual scalings is suggestive of their existence. In contrast, the 3d counterparts seem problematic, as our analysis indicates a potential inconsistency: they do not exhibit integer dimensions or consistent brane-dual scalings, whereas this was the case for all other parametric AdS vacua we examined.

Scale separation is a crucial requirement for constructing phenomenologically viable string vacua, and so this work has taken a modest step toward understanding holography in a setting that could be relevant to our universe. It remains an open question whether other scale-separated AdS vacua can also be understood holographically, especially those that do not involve a large N limit, and what this might imply for de Sitter vacua obtained through uplift from AdS.

Appendices

A

More details on backtracking AdS flux vacua

Contents

A.1 Dimensional reduction of the ten-dimensional Type IIA action with fluxes

The ten-dimensional bosonic Type IIA action [31–33] is of the form

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g}\, e^{-2\phi} \left(R + 4(\partial_\mu \phi)^2 - \frac{1}{2}|H_3|^2 - e^{2\phi} \sum_p |F_p|^2 \right) + \text{local contributions}, \tag{A.1}$$

where local contributions arise from D-branes and/or O-planes. Consider a compactification of this theory to d dimensions. We are interested in determining the dependence of the d-dimensional potential on the universal moduli u and s. The internal volume in string frame is

$$\mathcal{V} = u^{\frac{10-d}{2}}, \tag{A.2}$$

and the ten-dimensional dilaton ϕ is related to these moduli by

$$e^{-2\phi} = s^{d-2} u^{\frac{d-10}{2}}. \tag{A.3}$$

The canonically normalised scalars in terms of these universal moduli are

$$\overline{u} \equiv \sqrt{\frac{10-d}{4}} \ln u, \quad \overline{s} \equiv \sqrt{d-2} \ln s. \tag{A.4}$$

The moduli dependence of the d-dimensional scalar potential is as follows

$$\begin{aligned} V = A_R u^{-1} s^{-2} + A_{H_3} u^{-3} s^{-2} + \sum A_p u^{\frac{10-d-2p}{2}} s^{-d} \\ + \text{local contributions}, \end{aligned} \tag{A.5}$$

where the A's are coefficients that we do not specify. The Ansatz 'superpotential' to reproduce this potential is

$$P = c_R u^{-\frac{1}{2}} s^{-1} + c_{H_3} u^{-\frac{3}{2}} s^{-1} + \sum c_p u^{\frac{10-d-2p}{4}} s^{-\frac{d}{2}}, \tag{A.6}$$

where we again do not specify constants as we only need the moduli dependence. The local sources do not contribute to the superpotential.

A.2 More details on scale-separated AdS$_4$ vacua in IIA with M-theory uplift

In this appendix, we provide additional details on the computation of the flow in the scale-separated AdS$_4$ vacua in massless IIA. In particular, we will give the flux dependence, illustrating the potentially non-isotropic scalings in this case. The EFT is described by the following 4d $\mathcal{N}=1$ superpotential and Kahler potential,

$$W = c_{F_6} - c_c T_1 S + c_{F_{2,1}} T_1 T_2 + c_{F_{2,2}} T_2^2, \quad K = -\ln s^4 u_1 u_2^2, \tag{A.7}$$

with contributions from unbounded F_6-flux, curvature and both unbounded and bounded F_2-flux. For the case of a toroidal internal manifold, u_1 parametrises the size of the first sub-torus, while u_2 corresponds to the size of the remaining two sub-tori. We label the flux quanta as follows,

$$c_{F_6} \sim N, \quad c_{F_{2,1}} \sim k_1, \quad c_{F_{2,2}} \sim k_2, \tag{A.8}$$

where both N and k_1 can in principle be arbitrarily large. The central charge scales with these fluxes as

$$c \sim N^{3/2} k_1^3 k_2^{-5/2}. \tag{A.9}$$

The unbounded F_2-flux is the one responsible for scale separation, as

$$\frac{L_{AdS}}{L_{KK}} \sim k_1^{1/2}. \tag{A.10}$$

We note that for $k_1 \sim N \gg 1$, we reproduce the DGKT scaling relations, while for $k_1 \sim k_2 \sim 1$, we obtain the ABJM scaling relations.

To find the geometry probed by D2-branes dual to the unbounded F_6-flux, we consider the flow generated by the following residual superpotential

$$W = -c_c T_1 S + c_{F_{2,1}} T_1 T_2 + c_{F_{2,2}} T_2^2, \quad K = -\ln s^4 u_1 u_2^2, \tag{A.11}$$

The flow equations are

$$\begin{aligned} s' &= \frac{s\left(2k_2 u_2^2 + u_1\left(s - 2k_1 u_2\right)\right)}{\sqrt{s^4 u_1 u_2^2}}, \quad u_1' = \frac{2u_1\left(k_2 u_2^2 + u_1\left(k_1 u_2 - s\right)\right)}{\sqrt{s^4 u_1 u_2^2}} \\ u_2' &= \frac{2\left(s u_1 u_2 - k_2 u_2^3\right)}{\sqrt{s^4 u_1 u_2^2}}, \quad A' = -\frac{u_1\left(k_1 u_2 - s\right) - k_2 u_2^2}{\sqrt{s^4 u_1 u_2^2}}, \end{aligned} \tag{A.12}$$

and a solution is given by

$$\begin{aligned} s(r) &= \frac{3}{2}\sqrt[3]{\frac{3}{5}}\sqrt[3]{k_2} r^{2/3} \sim k_2^{1/3} r^{2/3}, \\ u_1(r) &= \frac{27\sqrt[3]{\frac{3}{5}} k_2^{4/3} r^{2/3}}{20 k_1^2} \sim k_1^{-2} k_2^{4/3} r^{2/3}, \\ u_2(r) &= \frac{9\sqrt[3]{\frac{3}{5}}\sqrt[3]{k_2} r^{2/3}}{8k_1} \sim k_1^{-1} k_2^{1/3} r^{2/3}, \\ A(r) &= \frac{7}{9}\ln r. \end{aligned} \tag{A.13}$$

The resulting geometry is

$$\begin{aligned} ds_{10}^2 &= (k_2^{-2/3} r^{-4/3})[dr^2 + r^{14/9} ds_3^2] + k_2^{2/3} k_1^{-4/3} r^{2/3} ds^2_{Iwasawa} \\ &= dy^2 + k_2^{-4/9} y^{2/3} ds_3^2 + k_2^{4/3} k_1^{-4/3} y^2 ds^2_{Iwasawa}, \end{aligned} \tag{A.14}$$

where $y = k_2^{-1/3} r^{1/3}$. We stress that $k_2 \sim 1$ is bounded, while $k_1 \sim M$ is unbounded and we need $M \to \infty$ to achieve scale separation,

$$ds_{10}^2 = dy^2 + y^{2/3} ds_3^2 + M^{-4/3} y^2 ds^2_{Iwasawa}. \tag{A.15}$$

For the string coupling, we find

$$g_s(y) = k_2 k_1^{-2} y, \tag{A.16}$$

so that the uplifted metric is of the form

$$\begin{aligned} ds_{11}^2 &= d\tilde{y}^2 + k_1^{4/3} k_2^{-10/9} ds_3^2 + (k_1^{-1} k_2)^{4/3} \tilde{y}^2 ds^2_{Iwasawa} + k_2^2 k_1^{-4} \tilde{y}^2 dz^2 \\ &= d\tilde{y}^2 + M^{4/3} ds_3^2 + M^{-4/3} \tilde{y}^2 (ds^2_{Iwasawa} + M^{-8/3} dz^2). \end{aligned} \tag{A.17}$$

with $\tilde{y} = k_2^{-1/3} k_1^{2/3} y^{2/3}$, which means that the uplifted M2-branes probe a conical singularity.

A.3 KKLT vacua

A.3.1 Removing all fluxes

In KKLT, we have both fluxes and non-perturbative effects. The 3-form fluxes can be turned into D5- and NS5-branes, after which we only remain with non-perturbative effects. We will study what geometry these non-perturbative effects induce with our backtracking method. However, we need to be cautious as the fluxes we now extract are all bounded by tadpoles contrary to the previous examples we discussed.

We turn off all fluxes in the superpotential and consider

$$W = A_0 e^{-aT}, \tag{A.18}$$

where $t = \text{Re } T$ is the volume modulus related to the total internal volume in string frame as $\mathcal{V} \sim t^{3/2}$. We consider a Kahler potential of the form

$$K = -\ln\Big(T + \overline{T}\Big)^3 (S + \overline{S})(Z + \overline{Z})^p, \tag{A.19}$$

where S is the axio-dilaton and Z are the complex structure moduli, and $p > 0$ is a positive integer. The Kahler potential should take such a polynomial form in asymptotic regions of moduli space. The flow equations (with $s = \text{Re } S, z = \text{Re } Z$) for the moduli are given by

$$\begin{aligned} t' &= \frac{2A_0 e^{-at}(2at+3) z^{-p/2}}{3\sqrt{ts}}, & s' &= 2A_0 e^{-at} z^{-p/2} \sqrt{\frac{s}{t^3}}, \\ z' &= \frac{2A_0 e^{-at} z^{1-\frac{p}{2}}}{\sqrt{t^3 s}}, & A' &= \frac{A_0 e^{-at} z^{-p/2}}{\sqrt{t^3 s}}. \end{aligned} \tag{A.20}$$

Solving these equations in terms of the radial coordinate r leads to complicated expressions, but we can instead solve them in terms of the metric factor A,

$$t(A) = \frac{3c_1 e^{2A}}{1 - 2ac_1 e^{2A}}, \quad s(A) = c_2 e^{2A}, \quad z(A) = c_3 e^{2A}, \tag{A.21}$$

for constants c_1, c_2, c_3. As we approach the singularity $e^A \to 0$, the volume shrinks, and we flow towards strong coupling and small complex structure.

The residual potential is strictly positive and takes the form:

$$V(t, s, z) = \frac{A_0^2 e^{-2at} z^{-p} \left(4a^2 t^2 + 12at + 3p + 3\right)}{3t^3 s}. \tag{A.22}$$

Hence, the potential increases monotonically as the moduli decrease. The flow towards the singularity is therefore an uphill flow. This contrasts with all previous pure-flux examples, where the flow was always downhill.

To understand this better, let us explicitly see how this downhill flow is achieved in the Freund-Rubin case. The relevant equation is

$$\phi'' = \partial_\phi V - (d-1)\phi' A', \tag{A.23}$$

where the positive sign on the potential gradient reflects that the flow coordinate is spatial (unlike in cosmological evolution). The residual potential is negative and decays exponentially with the volume modulus,

$$V = -c^2 e^{-2\lambda\phi}. \tag{A.24}$$

This would lead to a positive (uphill) acceleration ($\partial_\phi V > 0$) for the volume modulus. However, the friction term $-(d-1)\phi' A'$ is negative and dominates, ensuring that $\phi'' < 0$. As a result, the flow remains downhill towards smaller volumes and vanishing metric factor $e^A \to 0$. A similar downhill behavior is found in all other pure flux vacua considered previously.

In contrast, for the KKLT scenario we find that $\partial_\phi V < 0$ for all moduli, and from the flow equations,

$$-(d-1)\phi' A' < 0, \tag{A.25}$$

so that

$$\phi'' < 0, \tag{A.26}$$

confirming that the flow is uphill towards smaller volumes and vanishing metric factor.

To understand better the flow near the singularity (and find the *local* dynamical cobordism), we approximate

$$t(A) = c_1 e^{2A}, \quad s(A) = c_2 e^{2A}, \quad z(A) = c_3 e^{2A}, \tag{A.27}$$

in which case the ten-dimensional metric is given by

$$\begin{aligned} ds_{10}^2 &= e^{2\phi}\mathcal{V}^{-1}\left(dr^2 + e^{2A(r)}ds_3^2\right) + \mathcal{V}^{1/3}ds_{CY}^2 \\ &= s^{-2}t^{-3/2}\left(dr^2 + e^{2A(r)}ds_3^2\right) + t^{1/2}ds_{CY}^2 \\ &= e^{-7A}dr^2 + e^{-5A}ds_3^2 + e^{A}ds_{CY}^2 \\ &= e^{(1+2p)A}dA^2 + e^{-5A}ds_3^2 + e^{A}ds_{CY}^2 \\ &= dy^2 + y^{-\frac{10}{1+2p}}ds_3^2 + y^{\frac{2}{1+2p}}ds_{CY}^2, \end{aligned} \tag{A.28}$$

where we used that $dr \approx e^{(4+p)A}dA$ in this limit and defined $y \sim e^{(1+2p)A/2}$. We however remark that near the singularity, there might be significant corrections to the Kahler potential that could change the flow solution.

We note that there is a second potential singularity when $e^{2A} \to (2ac_1)^{-1}$. In this limit, the volume blows up, while the complex structure moduli and dilaton remain constant.

From the 10d point of view, this corresponds to a singularity. Up to constants, we can solve the equation for the volume modulus t as

$$t(r) \approx \frac{1}{a} \ln r + \frac{1}{2a} \ln(\ln r) \,, \tag{A.29}$$

and then the 10d metric takes the approximate form

$$ds_{10}^2 = dy^2 + (\ln y)^{-3/2}\, ds_3^2 + (\ln y)^{1/2}\, ds_{CY}^2 . \tag{A.30}$$

The flow towards the second singularity is decelerated and downhill.

A.3.2 Removing the F_3 flux

As a toy model, the superpotential consists of

$$W = (f_0 + h_0 s) + A_0 e^{-at} . \tag{A.31}$$

We remove the flux f_0 contribution to the potential

$$W = h_0 s + A_0 e^{-at}, \tag{A.32}$$

The flow equations are given by

$$\begin{aligned} t' &= \frac{2e^{-at} z^{-p/2} \left(A_0(2at+3) + 3Bse^{at}\right)}{3\sqrt{ts}}, \qquad & s' &= \frac{2\sqrt{s} e^{-at} z^{-p/2} \left(A_0 - Bse^{at}\right)}{t^{3/2}}, \\ z' &= \frac{2e^{-at} z^{1-\frac{p}{2}} \left(Bse^{at} + A_0\right)}{\sqrt{t^3 s}}, & A' &= \frac{z^{-p/2} \left(A_0 e^{-at} + Bs\right)}{\sqrt{t^3 s}}. \end{aligned} \tag{A.33}$$

Now the evolution of the volume modulus and the dilaton as $A \to -\infty$ depends on the signs and magnitude of A_0 and h_0. When A_0 and h_0 have the same sign, the volume will be shrinking at the singularity (with small complex structure moduli as well) analogously to the case in subsection A.3.1. The evolution of the dilaton will depend on the magnitudes of A_0 and h_0. If they have different signs, we could not find an obvious running solution in which the volume shrinks.

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