Essays on Information and Incentives in Financial Markets



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Thesis submitted for the degree of

Doctor of Philosophy in Economics

Trinity Term 2019

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Acknowledgements

My deepest gratitude goes to my supervisors, Margaret Meyer and Joel Shapiro. This work would not have been possible without them. Special thanks also to Marco Pagano, who introduced me to finance and economics, for encouraging me to apply to and attend Oxford. As my mentors, these three have been an invaluable source of guidance, feedback, and support throughout my DPhil. I would also like to thank Merton College and the Economic and Social Research Council for their generous funding, which gave me the means and time to complete this project.

This dissertation is dedicated to my family. To my parents, Angelo and Luisa. To my brothers: Salvatore, who inspired me to become a researcher, Domenico, and Davide. To my partner, Jessie. I also dedicate it to my closest friends, from both Naples and Oxford. Thank you all for your encouragement, for your patience, and for the joy you have given me throughout these years.

Abstract

This thesis comprises three essays in financial economics. The common thread is the interaction between incentives and information in financial markets. Chapter 1 analyzes a model in which a credit rating agency's (CRA's) rating is followed by a market for credit risk that provides a public signal - the price. A more accurate rating decreases market informativeness, as it diminishes mispricing and, hence, incentives for investor information acquisition. On the other hand, more-informative trading increases CRA accuracy incentives by making rating inflation more transparent. We analyze implications for policy and the real economy. Chapter 2 develops a model of the interaction between executive compensation and stock market prices, and analyzes its implications for corporate short-termism. We show that inefficient short-termism can arise in equilibrium as a self-fulfilling prophecy, due to strategic complementarities between the firm's investment horizon and investors' decision to acquire information about short-term performance or long-term value. The model helps us assess evidence presented in the "myopia" debate and yields novel implications regarding ownership structure, executive compensation, and managerial horizon. Finally, Chapter 3 analyzes a model of competition between credit rating agencies (CRAs). In equilibrium, investors buy only assets that received high ratings from multiple CRAs. This has two contrasting effects on the quality of certification. On the one hand, the issuer needs to pass the screening of multiple CRAs; other things being equal, this improves certification. On the other hand, it has the perverse effect of incentivizing dishonest ratings, by introducing a team dimension that dilutes the CRAs' reputational concerns. When the perverse effect dominates, competition reduces the quality of certification. However, mandating disclosure of indicative ratings can restore the first-best outcome.

Chapter 1

Credit Ratings and Market Information

[1] How does market information affect credit ratings? How do credit ratings affect market information? In this chapter, we examine a model in which a credit rating agency's (CRA's) rating is followed by a market for credit risk that provides a public signal - the price. A more accurate rating decreases market informativeness, as it diminishes mispricing and, hence, incentives for investor information acquisition. On the other hand, more-informative trading increases CRA accuracy incentives by making rating inflation more transparent. We analyze implications for policy and the real economy.

1.1 Introduction

In recent years, a familiar pattern has arisen for credit ratings: market events prove them to be substantially inflated. Famous examples include the East Asian Financial Crisis (1997), Enron and Worldcom (2001, 2002), and the Financial Crisis (2007-2008), reflecting sovereign,

[1] This chapter derives from joint work with Joel Shapiro (Saïd Business School, University of Oxford). We thank Philip Bond, Itay Goldstein, Jens Josephson, Jakub Kastl, Xuewen Liu, Meg Meyer, Alan Morrison, Stephen Morris, Marco Pagano, Uday Rajan, Stefano Rossi, Günter Strobl, Sergio Vicente and seminar participants at Essex, Oxford, Banca d'Italia, the LBS FIT workshop, the Barcelona GSE Forum in Financial Intermediation and Risk, the Imperial College FTG conference, University of Naples Federico II, the Tilburg EBC Network Conference, the Carnegie Mellon Economics of Credit Ratings Conference, the Catalan Economic Society Conference, the Edinburgh Corporate Finance Conference, the Hong Kong FIRS Conference, the 13th CSEF-IGIER Symposium on Economics and Institutions, and the 1st Workshop on Advances in Industrial Organization for helpful comments.

corporate, and structured finance ratings, respectively.[2] At the same time, ratings do provide information to markets (e.g., Badoer and Demiroglu (forthcoming) and Chava, Ganduri, and Ornthanalai (2018)) and the popular media regularly reports on ratings rattling markets (e.g. China's downgrade in 2017[3]).

In this chapter, we examine the equilibrium interaction between credit ratings and the market for credit risk. We demonstrate that this has great relevance for policy, as it has implications for the regulation of credit rating agencies (CRAs), alternatives to credit ratings, and market governance. Moreover, as the information produced by CRAs and markets are used by real decision makers, we show that there are serious implications for the real economy.

In the model, a CRA receives information about the quality of a risky investment project. It decides how to rate this investment given that it will make more profits now from a higher rating, but may diminish its reputation if the investment proves to be of poor quality. This rating is released to the public, and investors may purchase the asset. A market for credit risk, which may represent the credit default swap market or a secondary market for the asset, then establishes a market price à la Kyle (1985): a speculator may acquire information to profit off of noise traders, and a market maker clears the market. After observing the rating and the market price, an investor, who may be new or the initial issuer, decides whether to undertake a project that is correlated with the initial issue. Lastly, the asset payoffs are realized, leading to monetary payoffs for investors and a reputational payoff for the CRA.

The interaction between the CRA and the market price has two contrasting effects. Ratings that are more inflated increase the informativeness of market trading since they increase the speculator's incentives to acquire information (by increasing mispricing). Therefore, from the speculator's perspective, rating inflation by the CRA and information acquisition by the speculator are strategic complements. On the other hand, more-informative trading decreases the CRA's incentives to inflate ratings by increasing transparency about whether the CRA inflated ratings and, thus, augmenting the CRA's reputational costs. The information coming from the

[2] More recent examples of ratings inflation are described in Baghai and Becker (2018) and Kedia, Rajgopal and Zhou (2017).

[3] "China's sovereign debt downgraded by Moody's," by Gabriel Wildau and Tom Mitchell, May 24, 2017.

market disciplines the CRA. This demonstrates that, from the CRA's perspective, rating inflation by the CRA and information acquisition by the speculator are strategic substitutes. This leads to a unique equilibrium.

We then examine the real effects of the interaction between ratings and market information. There are two real effects in the model: first, the information produced by the CRA affects whether the initial investment project is undertaken; second, the information from both the rating and the market for credit risk determine whether the second investment is made. We find that if strong enough, the negative effect of accurate ratings on information acquisition in the market can lead to a perverse result - policies such as reputational sanctions (e.g., increased liability standards) that make ratings more accurate can reduce total information and hence, investment efficiency. On the other hand, our result that the market disciplines CRAs suggests that policies to increase market informativeness (such as improving governance and trust in exchanges) reap informational benefits beyond the market itself and increase investment efficiency. Intriguingly, this also implies that when market informativeness is low, the CRA does not fill the informational gap, i.e., the CRA does not produce a lot of information either. Hence, a CRA fails to produce information at the time when it is most valuable.

We demonstrate that these results are likely to hold when the speculator's cost of information acquisition is low, i.e. for plain vanilla corporate bonds or when speculators are concentrated and have reached economies of scale.

We then look at the push by policymakers to replace rating-based regulation. Both the Dodd Frank act and EU legislation state that references to ratings must be removed from regulation. However, they don't state how ratings should be replaced.[4] Our model allows us to examine replacing rating-based regulation with a common proposal, CDS-based regulation. While this unambiguously lowers CRA incentives to inflate ratings, as regulators would expect, it may reduce efficiency in the model due to lower incentives of speculators to produce information. On the other hand, fostering a more active CDS market makes ratings-based regulation more reliable.

[4]Ratings play a significant role in assessing the risk of financial institutions (see European Commission, 2015).

There is substantial evidence that more reliable market information makes ratings more accurate (e.g. Gopolan, Gopolan, and Koharki (2017)).[5] There is also an important literature that connects information revealed through market prices to corporate investment policies (e.g. Chen, Goldstein, and Jiang (2007)). In this chapter, we connect the dots by providing an equilibrium analysis of different sources of investment information and their *impact* on real investment decisions.

The model views CRA reputational concerns as arising from reduced future profits. Do CRAs suffer reputational losses? In the case of the structured finance market, the market (and the need for ratings) dried up as the crisis hit, and stock market valuations for Moody's fell significantly.[6] One might view Standard and Poors' recent settlement with the U.S. government and states for $1.5 billion as a reputational sanction.[7] In addition, a regulatory environment that is more or less friendly to the CRAs may also affect the CRAs' reputational incentives. Lastly, Arthur Andersen's implosion represents a severe punishment to a certification intermediary in a similar line of business (auditing).

Next, we offer a summary of the related theoretical literature (Section 1.8 is dedicated to a discussion of the empirical literature and implications of the model).

Theoretical Literature

The link between ratings quality and reputation is key for our results. Mathis, McAndrews, and Rochet (2009) examine how a CRA's concern for its reputation affects its ratings quality. They present a dynamic model of reputation in which a monopolist CRA may mix between lying and truth-telling to build up/exploit its reputation. Strausz (2005) is similar in structure to Mathis et al. (2009), but examines information intermediaries in general.

Several papers have studied how a firm's disclosure policy affects information aggregation when there is a market composed of sophisticated investors and uninformed liquidity traders (e.g. Verrecchia (1982), Diamond (1985), and Kim and Verrecchia (1991)). Goldstein and Yang (2017)

[5] We discuss the evidence in this paragraph in detail in Section 1.8.

[6] Moody's is the only stand-alone public company of the three major CRAs, and thus the only one that has a public market price.

[7] http://www.bloomberg.com/news/articles/2015-02-03/s-p-ends-legal-woes-with-1-5-billion-penalty-with-u-s-states

review this literature in depth. Gao and Liang (2013) focus on the real impacts of this interaction. In these models, disclosure reduces gains to information acquisition by speculators, as in our paper.[8] However, we also examine information being produced by a strategic rating agency and the two-way interaction between market information and the rating agency's information.

The interaction between market information and the rating agency's information is a type of feedback effect. There is a substantial literature on feedback effects of market prices, which examines how markets guide real decisions and the feedback loop between the two that results - see Bond, Edmans, and Goldstein (2012) for a review. Our paper examines the feedback between two information providers, and their effects on real decisions. Bond and Goldstein (2015) look at both the real and informational feedback between government interventions and market information. When the intervention consists of disclosing information, they also find a crowding out effect of speculators' information acquisition. Goldstein and Yang (forthcoming) demonstrate that making an exogenous public signal more informative may reduce real efficiency. This is related to our result that more accurate ratings may decrease efficiency overall, but in our model ratings are endogenous. Goldstein and Huang (2017) have a feedback loop between investors, a firm undertaking an investment project, and a CRA. In their model, the CRA is the only source of information and a coordination problem among investors drives the feedback loop.

The rest of the chapter is organized as follows. Section 2 describes the basic model. We then proceed to characterize the market for credit risk and the rating process. In Section 3, we describe the equilibrium trading strategies in the market for risk, the speculator's information acquisition decision, and how this is affected by the CRA's rating strategy. In Section 4, we describe the equilibrium rating strategy and how this depends on the precision of the speculator's signal. This allows us to solve for the unique equilibrium and describe its properties in Section 1.5. In Section 6 we analyze the real effects of information. Section 7 explores extensions to the model. In Section 8, we discuss the empirical implications of the model and related evidence. Finally, Section 9 concludes. Detailed proofs are presented in the Appendix.

[8] Han, Tang, and Yang (2016) show that this effect may be mitigated by the information attracting more noise/liquidity traders.

1.2 The Model

Our model has two distinct elements, the ratings process and the market for credit risk. We first present them separately and then analyze the strategic interactions between them. The market for ratings takes place first, and is followed by the market for credit risk. Information has real effects in the model; initial investors are influenced by ratings, and a final investment decision uses the information produced by both the ratings and the market for credit risk.

All agents in the model are rational and risk neutral and, for simplicity, we assume that there is no discounting.

1.2.1 The Ratings Process

The ratings process takes place at time $t = 1$, and consists of three types of agents: an issuer, a monopoly credit-rating agency (CRA), and investors.

The issuer has access to a risky investment project but needs financing from investors. The project returns 1 in case of success and 0 in case of failure, and the cost of the project is I. We let $y_1 \in \{S, F, N\}$ denote the project's outcome, where S (F) signifies *Success* (*Failure*) and N represents the case in which the project is not undertaken. The quality of the project is denoted by $\theta \in \{B, G\}$, where $B(G)$ stands for $Bad(Good)$, and relates to its probability of failure: a bad project fails with probability $f_B \in (0, 1)$, and a good project fails with probability $f_G \in (0, f_B)$. Good and bad investments have ex-ante probability $\frac{1}{2}$ of occurring. The investment quality is *a priori* unknown, including to the issuer itself.

If the issuer wishes to undertake the project, it can sell a claim to the payoff of the project to investors. Let $V_\theta = 1 - f_\theta - I$ denote the NPV of a project of quality θ. We assume that good projects should be financed (they have positive NPV, i.e., $V_G > 0$) but that, without prior knowledge on the quality of the project, no financing takes place (ex-ante NPV is negative, i.e., $\frac{1}{2}V_G + \frac{1}{2}V_B < 0$). Therefore, the presence of a CRA can improve welfare by screening projects for investors.

The CRA observes the quality of the project and offers the issuer a credit report $\widetilde{r} \in \{H, L\}$,

where H signifies *high* and L signifies *low*.[9] The issuer either pays a rating fee and has the report publicized or refuses to purchase it. This allows for "rating shopping" by the issuer. We assume that the rating fee is an exogenously specified fraction ϕ of the issuer's surplus from selling the claim to investors and implementing the project. The outcome of the rating process, as observed by the investors, is thus $r \in \{\widetilde{r}, \varnothing\}$, where $r = \widetilde{r}$ signifies that the issuer had the credit report publicized by the CRA and $r = \varnothing$ signifies that there is no rating. If the issuer refuses to buy the CRA's report and goes on the market as unrated, that in itself is a signal to the investors.

Investors observe the rating and decide whether they wish to buy the claim, and if so at what price. Let p^r denote the price investors are willing to pay for a claim backed by the payoff of the project, for a given rating r. Investors are risk-neutral and perfectly competitive, so the market price p^r is equal to the expected value of the project's payoff given their posterior beliefs. We describe the expression for p^r in Section 1.2.4, after having introduced the CRA's reputation and its rating strategy. If the price p^r exceeds the investment cost, i.e., if $p^r > I$, the issuer sells the claim and undertakes the project; the investment outcome $y_1 \in \{S, F\}$ then realizes at time $t = 4$ and is observed by all players. Otherwise, the project is not implemented and all players observe $y_1 = N$ at the end of time $t = 1$.

1.2.2 The Market Price of Risk

At time $t = 2$, if the issuer implemented the investment project, a market for credit risk takes place. We will describe this market as the CDS market in what follows. The same setting can be used to model the secondary market for the asset when the speculator is endowed with some amount of the investment.[10]

Let p^{cds} denote the price for a CDS contract[11] and x the net volume of trades. A CDS

[9] The assumption that the CRA perfectly observes the project quality simplifies the exposition but does not affect any of the results; the results would hold if the CRA observes a noisy signal of the project quality.

[10] As will be seen below, it is necessary for the speculator to be able to hide her trades to make a profit. This means she must be able to take either long or short positions. The easiest way to model this in a secondary market is to endow the speculator with some of the asset. Allowing for shorting would be equally good, of course.

[11] In a CDS contract, a protection buyer pays a premium to the protection seller, in exchange for a payment from the latter if a credit event (usually bankruptcy) occurs on a given reference entity within a predetermined time period. The protection buyer does not need to hold the reference entity ("naked" CDS). The amount that

contract is formalized as follows: at time $t = 2$, the contract is signed, and the buyer of the swap pays an amount p^{cds} to the swap's seller. In return, the seller agrees that in the event of default at time $t = 4$, the seller will pay the buyer an amount 1.[12]

Trading occurs among liquidity/noise traders, one speculator and a competitive market maker, and p^{cds} is determined in a simplified model à la Kyle (1985). We now describe the agents in detail:

Speculator: Having observed the rating r, the speculator decides whether to acquire information about the investment. She privately chooses the precision of her signal $\iota \in [0, 1]$ at a cost $c(\iota)$. When the speculator chooses a level of precision ι, with probability ι she learns the project quality θ and with probability $1 - \iota$ she does not learn anything about θ. We assume:

$$c' \geq 0;\ c'' > 0;\ c(0) = c'(0) = 0. \tag{1.1}$$

The more she spends on ι, the more likely she learns the quality of the project. When the speculator learns θ, she tries to use her superior information to profit from mispricing in the market. Let x^s denote her demand. We use the convention $x^s < 0$ when she is selling protection and $x^s > 0$ when she is buying protection.

Noise traders: Aggregate demand from noise traders is $x^n \in \{-n; +n\}$, with both realizations equally likely.

Market maker: The market maker observes the trade orders - i.e., $\{x^s, x^n\}$, but not the identity of the trader submitting each order.[13] Having observed $\{x^s, x^n\}$ and r, he sets a price p^{cds} and clears the market. We assume that he makes zero profit, which implies $p^{cds} = E_\theta(f_\theta \mid r, \{x^s, x^n\})$, where the expectation takes into account equilibrium beliefs about the

the protection seller has to pay in case the credit event occurs is called the notional amount. The premium is quoted in basis points per year of the contract's notional amount and is called the CDS spread.

[12] We normalize the notional amount per contract to 1 and let p^{cds} represent the CDS spread.

[13] As in the discrete setup of Faure-Grimaud and Gromb (2000), to ensure existence of a Perfect Bayesian Equilibrium we allow the market maker to observe trade orders (but not the identity of those trade orders).

CRA's rating strategy and the speculator's choice of precision and trading strategy.[14]

Let $x = x^s + x^n$ denote the total order flow. The informativeness of market trading is defined by the speculator's choice of precision ι. The other agents in the model do not directly observe the speculator's choice of ι; we denote as ι^e their expectation about this choice.

1.2.3 Second Investment Stage

After the market for risk has realized, at time $t = 3$ a new investor observes the realizations of both the rating r and the information from the CDS market x and decides whether to invest or not in a new investment project of size χ she is endowed with.[15] For simplicity, we assume that this second investment project has the same characteristics as the project being implemented in the first period[16] and the quality of the projects in the two periods is perfectly correlated.[17]

Given that the quality of the two projects is perfectly correlated, the investors at the two investment stages would want to take the same decision about whether to invest or not. However, the initial investors only observe the rating r, while the investor at time $t = 3$ observes the realization of market trading x as well as the rating r before taking action. The second investment stage captures the real effects of the information produced in the CDS market, as is standard in the literature (see, e.g., Bond, Edmans, and Goldstein (2012)).

Let $y_2 \in \{S, F, N\}$ represent the outcome of the second investment project. If the project is implemented, at time $t = 4$ its outcome realizes and is observed by all players; otherwise, all players observe $y_2 = N$ at the end of time $t = 3$.

[14] We implicitly assume (i) that the speculator on the CDS market does not participate in the initial investment market and (ii) the investors who purchased in the initial investment market choose not to participate in the CDS market. Regarding (i), this is consistent with recent empirical work that shows that speculative trading concentrates in the CDS market, due to its relative liquidity advantage: see Oehmke and Zawadowski (2016). We discuss this further in Subsection 7.2. Regarding (ii), we show in Subsection 7.2 that risk-neutral initial investors would not want to participate in the CDS market and that while risk-averse initial investors may want to participate in the CDS market, our results still hold.

[15] The investor at the second investment stage could also be the issuer of the first investment project, as well as the investors who financed it.

[16] As with the initial project, a bad project fails with probability $f_B \in (0, 1)$, and a good project fails with probability $f_G \in (0, f_B)$. Both project types return 1 in case of success and 0 in case of failure; the cost of the project is I. Good and bad investments have an ex-ante probability $\frac{1}{2}$ of occurring. Conditional on both being implemented, the realizations of the two investment projects, i.e., y_1 and y_2, are independent.

[17] This is just to simplify the exposition. We could have less than perfect correlation, different NPVs, and correlated outcomes for the two projects.

1.2.4 The CRA's Reputation and Rating Strategy

The CRA can be of two different types: *strategic* or *committed*. Let $\tau \in \{\mathcal{S}, \mathcal{C}\}$ denote a realized type, where $\mathcal{S}$ ($\mathcal{C}$) stands for *Strategic* (*Committed*).[18] The realization of τ is the CRA's private information. Investors' prior beliefs about τ are given by

$$\Pr(\tau = \mathcal{C}) = q_0 \in (0,1); \quad \Pr(\tau = \mathcal{S}) = 1 - q_0. \tag{1.2}$$

The probability q_0 represents the CRA's initial reputation for being a committed type. A committed CRA is always honest in its assessment of credit risk. A strategic CRA maximizes the weighted sum of its profits from selling the rating and its expected reputational payoff; this captures the tension between reputational concerns and profits from selling high ratings. The reputational payoff is assumed to be the CRA's reputation for being a committed type. We represent this by the CRA's posterior probability of being a committed type $q^{(r,x,\mathbf{y})}$, given the rating r, the realization of the CDS market x,[19] and the observable realizations of the investments $\mathbf{y} \equiv (y_1, y_2)$. Since good (bad) projects fail (succeed) with positive probability, the realization of the projects' payoffs is not a perfect signal for their quality. Therefore, when a highly rated project defaults, investors cannot fully tell apart "bad luck" from "bad ratings". They use both $\mathbf{y}$ and the information coming from the CDS market to learn about the project's quality and, thus, about whether the rating was accurate.

We let γ denote the weighting factor, which represents the relative importance of reputational payoffs to time $t = 1$ profits. The weighting γ can be potentially larger than one (as, for example, in Laffont and Tirole (1993)), as future payoffs may arrive over a long time horizon after the game we describe has ended.

The CRA's private information refers to its type and the project quality. Hence, we can denote the CRA's overall type by a pair (τ, θ).

[18] This follows the approach of Fulghieri, Strobl, and Xia (2014) and Mathis, McAndrews, and Rochet (2009) (who, in turn, follow the classic approach of modeling reputation of Kreps and Wilson (1984) and Milgrom and Roberts (1984)).

[19] As we will see, it is not important for our analysis whether the investors directly observe the amount of trades or the corresponding price in the CDS market when updating their beliefs about the CRA's type, since these are observationally equivalent.

Rating Strategy The committed CRA always offers a high (low) rating for a good (bad) project. The strategic CRA chooses its report $\widetilde{r}$ to maximize a weighted sum of its current and future payoffs. The CRA knows that offering a low rating to the issuer is equivalent to making zero profit at $t = 1$, since the issuer will not purchase it: this creates the possibility of rating inflation. Let ε be the probability with which a strategic CRA chooses to inflate the rating for a bad project - i.e., offers a high rating after having observed a bad project. The CRA can also under-report the signal, offering a low rating after having observed a good project. This could allow it to build up reputation by appearing to be tough to investors. Let δ be the probability with which a strategic CRA chooses to deflate the rating for a good project. The rating strategy is characterized by the following probabilities:

$$\Pr\left(\widetilde{r} = H \mid \theta = G\right) = 1 - \delta; \;\; \Pr\left(\widetilde{r} = H \mid \theta = B\right) = \varepsilon. \tag{1.3}$$

Let ε^e and δ^e denote the investors' conjectures about ε and δ, respectively.

A committed CRA is restricted to truthful ratings, whereas a strategic CRA may lie and offer a high (low) rating to a bad (good) asset. Therefore, the issuance of a rating is informative about the CRA's type. The investors update their beliefs about the CRA's type first after the release of a rating and then, later on in the game, as more information about the project quality becomes available (i.e., after they observe the realizations of the CDS order flow x and asset payoff $\mathbf{y}$).

Price of the Initial Investment and the Rating Fee If the investors purchase the claim, they do so at the expected value of the project's payoff given their posterior beliefs; these beliefs depend on the rating as well as the credibility of the CRA that publicized it, which is given by the initial reputation q_0 and the conjectures about rating inflation ε^e and rating deflation δ^e.

We solve the model under the conjecture that the market price p^r exceeds the investment cost, i.e., $p^r > I$, only when the credit report is high. As a consequence, the issuer sells the claim and undertakes the project only when the report is high. This implies that the issuer chooses

to publicize only a high report, and the project goes unrated when the report is low - i.e., $r = \widetilde{r}$ when $\widetilde{r} = H$ and $r = \varnothing$ when $\widetilde{r} = L$. As we will see, this is consistent with the equilibrium of the game.[20] [21] Therefore, the investors either observe a project with a high rating ($r = H$) or an unrated project ($r = \varnothing$); in the latter case they infer that the issuer was offered a low report which went unpurchased.[22]

Given that the issuer buys the credit report only if it is high, the rating fee amounts to $\phi\left(p^H - I\right)$. Let μ^H denote the probability that the project quality is good after a high rating is publicized; the price p^H is characterized as follows:

$$p^H = \mu^H (1 - f_G) + \left(1 - \mu^H\right)(1 - f_B).$$

where

$$\mu^H = \Pr(\theta = G \mid r = H) = \frac{1}{1 + \frac{\Pr(r=H|\theta=B)}{\Pr(r=H|\theta=G)}} = \frac{1}{1 + \frac{(1-q_0)\varepsilon^e}{1-(1-q_0)\delta^e}}. \tag{1.4}$$

1.2.5 Timing of the Model

The timing of the model is written below and summarized in Figure 1.1:

Time $t = 0$:

- **Quality of the investment and CRA type**: Nature chooses the quality of the investment projects $\theta \in \{B, G\}$ and the CRA type $\tau \in \{\mathcal{S}, \mathcal{C}\}$.

Time $t = 1$ (Ratings Process):

- **Rating and Initial Investment Market:** The CRA observes its type τ and the project quality θ, and offers a report $\widetilde{r}$ to the issuer.

[20] In equilibrium, the strategic CRA never *deflates* a good signal - we prove this in Lemma 6. This means that a low report ($\widetilde{r} = L$) would conclusively reveal that the investment is bad and, thus, not worth being implemented. Therefore, the issuer never publicizes a low report.

[21] When the CRA's initial reputation q_0 is small, there always also exist *babbling equilibria* in which we never have $p^r > I$, even when the rating is high. We discuss this further in Appendix 1.10.1.

[22] The fact that investors observe when the project goes unrated simplifies the exposition but does not affect the results. We could have that, when the report is low and the project goes unrated, the issuer anticipates that $p^{\varnothing} < I$ and, hence, does not market the claim to investors. In this case, the investors would infer that the issuer was offered a low report and the analysis would be essentially the same.

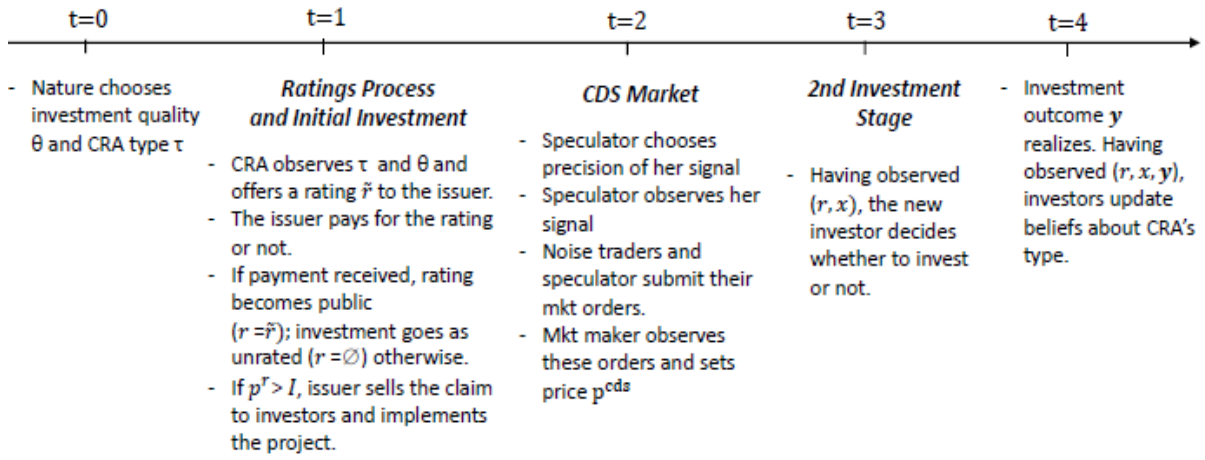


Figure 1.1: The timeline.

- The issuer either pays the rating fee and has the report publicized ($r = \widetilde{r}$) or refuses to purchase it and goes on the market as unrated ($r = \varnothing$).
- Given the rating r, the issuer sets a price p^r and markets a claim backed by the investment project to investors.
- If $p^r > I$, the issuer sells the claim and undertakes the project.

Time $t = 2$ (CDS Market): If the issuer undertakes the project, the CDS market takes place.

- **Information acquisition by the speculator**: The speculator chooses a level of precision ι and observes a signal about the project quality.
- **Market orders:** The noise traders and the speculator submit their orders $\{x^s, x^n\}$. Demand x realizes.
- **Market Clearing:** Having observed $(r, \{x^s, x^n\})$, the market maker sets a price p^{cds} and clears the market.

Time $t = 3$:

- **Second Investment Stage:** Having observed (r, x), a new investor decides whether to undertake or not the second investment project.

Time $t = 4$:

- **Investment Outcomes and Reputational Payoffs:** The investment outcomes $\mathbf{y} = (y_1, y_2)$ realize. Having observed $(r, x, \mathbf{y})$, the investors update their beliefs about the CRA's type.

We use *Perfect Bayesian Equilibrium* as the solution concept.

1.3 The CDS Market Equilibrium

We work our way backwards by first characterizing the CDS market equilibrium for given conjectures $(\varepsilon^e, \delta^e)$ about the CRA's rating strategy. The issuer does not undertake the project when the investment goes unrated and, thus, the CDS market does not take place in this case. Hence, we can focus on the case where the rating is high $(r = H)$ in what follows.

Proceeding by backward induction, we first solve for the equilibrium trading strategies and then, given these strategies, characterize the speculator's choice of precision.

1.3.1 Market Equilibrium

As in a standard Kyle-type setting, the speculator needs to camouflage her information-based trades with noisy trading. She is, therefore, constrained to trade an amount $x^s \in \{+n, -n\}$.[23] This implies that, when the speculator is trading, the total order flow x can take only three values - i.e., $\mathcal{X} = \{-2n, 0, +2n\}$. If $x \in \{-n, +n\}$, it must be instead that the speculator is not trading. The following Lemma characterizes the speculator's trading strategy and the market maker's inference from the order flow.

Lemma 1 *Given $r = H$ and for given ι^e and $(\varepsilon^e, \delta^e)$, the unique equilibrium of the CDS market is characterized as follows:*

1. *The speculator chooses not to trade when she does not learn the project quality θ. When she learns θ, her trading strategy is $x^s(B) = +n$ and $x^s(G) = -n$;*

[23] If x^s and x^n were different in absolute values, the market maker could always tell them apart, and so extract the speculator's private information. Expected profits from these trades would then be zero for the speculator.

2. *The market maker infers* $\theta = B$ *when* $x^s = x^n = +n$ *and* $x = +2n$, $\theta = G$ *when* $x^s = x^n = -n$ *and* $x = -2n$, *and nothing when* $x \in \{-n, 0, +n\}$.

Lemma 1 is quite intuitive. The speculator buys protection when receiving negative information about the investment, and sells otherwise. Given this trading strategy, the market maker's inference is also straightforward. When $x^s = x^n = +n$, both the speculator and noise traders are buying; when $x^s = x^n = -n$, both are selling. In these first two cases, the speculator's private information is revealed by the trade orders, so her expected profits from trading are zero; there is no mispricing. However, for $x^s = -x^n$ (i.e., $x = 0$), the market maker is unable to infer the direction of the speculator's order, and so her private signal; the speculator's expected profits are positive in this case. This occurs with positive probability and thus justifies costly information acquisition by the speculator. When $x \in \{-n, +n\}$, it must be that the speculator is not trading (only noise traders are trading), and therefore x is not informative about θ.

Notice that in equilibrium, all other agents also learn θ for any $x \in \{-2n, +2n\}$ and nothing about the investment quality when $x \in \{-n, 0, +n\}$. The equilibrium price is characterized as follows:

$$\begin{aligned} \widehat{p}^{cds}(x) &= \begin{cases} f_G & \text{if } x = -2n; \\ f_B & \text{if } x = +2n. \end{cases} \\ \widehat{p}^{cds}(x) &= \mu^H f_G + \left(1 - \mu^H\right) f_B \quad \text{for } x \in \{-n, 0, +n\}, \end{aligned} \tag{1.5}$$

where $\mu^H = \Pr(G \mid r = H)$ as given by equation (1.4).

Notice that the CDS price fully reveals the market maker's information about the project quality θ. Therefore, it is not important for our analysis whether the investors directly observe the total order flow x or the corresponding price in the CDS market when updating their beliefs about the CRA's type at time $t = 4$.

1.3.2 Informativeness of Market Trading

We need to evaluate the speculator's expected profits in order to examine her decision on how precise a signal to obtain. When the speculator is trading, her expected profits are zero con-

ditional on $x \in \{-2n, +2n\}$ since, in this case, the trade orders reveal the speculator's private information, and there is no informational advantage over the market maker. In what follows, let $\widehat{p}^{cds}$ be the equilibrium price when $x = 0$ and let Π^s denote the speculator's expected profits:

$$\Pi^s = \frac{n}{2}\left[\mu^H \iota \left(\widehat{p}^{cds} - f_G\right) + \left(1 - \mu^H\right) \iota \left(f_B - \widehat{p}^{cds}\right)\right] - c(\iota). \tag{1.6}$$

The ex-ante probability of observing $\theta = G$ is $\mu^H \iota$. When observing $\theta = G$, the speculator sells protection - i.e., $x^s = -n$. Trading profits are positive only if $x = 0$, which means that x^n has to be equal to $+n$; this occurs with probability $\frac{1}{2}$. She trades n units and the expected profit per unit is $\widehat{p}^{cds} - f_G$. This is because the speculator receives the premium $\widehat{p}^{cds}$ at $t = 2$, but with probability f_G the investment fails, meaning that she has to pay 1 at $t = 4$.

When the speculator learns $\theta = B$, she buys protection. To make profits, she needs $x^n = -n$ in this case, which occurs with probability $\frac{1}{2}$. Her profit is $f_B - \widehat{p}^{cds}$, since she is buying protection: she pays the premium $\widehat{p}^{cds}$ at $t = 2$, but at $t = 4$, with probability f_B the investment fails and she receives 1. This happens with ex-ante probability $\left(1 - \mu^H\right) \iota$.

The gross expected payoff is always non-negative for any strictly positive level of ι. Taking the derivative of the speculator's expected profits with respect to ι yields:

$$\frac{\partial \Pi^s}{\partial \iota} = \frac{n}{2}\left[\mu^H \left(\widehat{p}^{cds} - f_G\right) + \left(1 - \mu^H\right) \left(f_B - \widehat{p}^{cds}\right)\right] - c'(\iota). \tag{1.7}$$

Higher precision benefits the speculator by increasing the chances that she learns the project quality and can profit from mispricing in the market.

For a given level of expected rating inflation, the equilibrium level of precision $\widehat{\iota}\left(\varepsilon^e\right)$ sets equation (1.7) equal to zero. The existence and uniqueness of $\widehat{\iota}\left(\varepsilon^e\right) \in [0, 1]$ is guaranteed by the assumptions on the shape of the cost function. All the other agents in the model do not directly observe the speculator's choice of ι. However, they form consistent conjectures about it, given common knowledge of Π^s for any given level of expected rating inflation.

The effect of expected rating inflation on $\widehat{\iota}(\varepsilon^e)$ depends on how $\frac{\partial \Pi^s}{\partial \iota}$ changes with ε^e.

$$\frac{d}{d\varepsilon^e}\left(\frac{\partial \Pi^s}{\partial \iota}\right) = \frac{n}{2}\overset{<0}{\frac{d\mu^H}{d\varepsilon^e}}\left[\left(\widehat{p}^{cds} - f_G\right) - \left(f_B - \widehat{p}^{cds}\right)\right] + \frac{n}{2}\left(2\mu^H - 1\right)\overset{>0}{\frac{d\widehat{p}^{cds}}{d\varepsilon^e}}. \tag{1.8}$$

As expected rating inflation ε^e increases, the rating $r = H$ becomes a less reliable signal for $\theta = G$, and so it is relatively more likely that the investment is bad - i.e., μ^H decreases towards the prior of $\frac{1}{2}$.[24] This has two different effects on $\frac{\partial \Pi^s}{\partial \iota}$. On the one hand, since the speculator earns more from trading when the project is bad,[25] her incentives to acquire information increase when ε^e increases; this is the first term in equation (1.8), which is therefore positive. On the other hand, the market maker reacts to the rating being a less reliable signal of quality by increasing the price. This reduces the speculator's profits from trade when the project is bad (the difference $f_B - \widehat{p}^{cds}$ becomes smaller) and increases them when the project is good (the difference $\widehat{p}^{cds} - f_G$ becomes larger); this is the second term in equation (1.8). Given that the project is more likely to be good ($\mu^H > \frac{1}{2}$), the increase in expected profits dominates and this second effect is positive as well. Therefore, the marginal benefit of precision increases with expected rating inflation.

Lemma 2 *The equilibrium level of precision $\widehat{\iota}(\varepsilon^e)$ is increasing in the amount of expected rating inflation ε^e.*

Interestingly, from the speculator's point of view, information acquisition and rating inflation are strategic complements. That is, higher expected rating inflation increases the incentive to acquire information. This occurs through the mispricing channel, as the speculator can take advantage of wrong valuations due to more-opaque ratings.

[24] In the text, we consider the case where bad and good assets are ex-ante equally likely so that, after a good rating, the investors believe that the project is more likely to be good - i.e., $\mu^H > \frac{1}{2}$. This is true when the prior probability of the project being bad is close to $\frac{1}{2}$ or below. This is the relevant case to examine, since here a good rating is meaningful and, therefore, inflated ratings induce investors to mistakenly purchase bad assets.

[25] A high rating makes the market believe the project is more likely to be good and, thus, the price $\widehat{p}^{cds}$ is closer to f_G than to f_B, which implies that $f_B - \widehat{p}^{cds} > \widehat{p}^{cds} - f_G$. Hence, expected profits from trades are larger when the speculator learns that the project is bad and buys CDS protection.

1.4 The Rating Game

Having characterized the equilibrium trading strategies and level of precision in the CDS market, we can now characterize the equilibrium rating strategy for a given level of expected precision ι^e. The properties of the strategic interaction between credit ratings and the market for credit risk will then be used in the next section to characterize the unique equilibrium of the game.

Proposition 3 *For a given level of expected precision ι^e, the unique equilibrium rating strategy is:*

1. *For $\gamma \geq \frac{2\phi V_G}{\iota^e q_0} \equiv \overline{\gamma}$, a strategic CRA always truthfully reports the quality of the project;*
2. *For $\gamma < \frac{2\phi V_G}{\iota^e q_0}$, a strategic CRA inflates the rating for a bad project with positive probability $\widehat{\varepsilon}(\iota^e) \in (0,1]$ and never deflates the rating for a good project - i.e., $\widehat{\delta} = 0$.*

When the strategic CRA has sufficient reputation concerns, i.e., the weight on reputation γ is larger than a threshold $\overline{\gamma}$, it always offer honest ratings. Otherwise ($\gamma < \overline{\gamma}$), the CRA always inflates its ratings with positive probability.

Notice that truthful ratings depend on market monitoring to exist - the CRA must expect the speculator's level of precision to be positive ($\iota^e > 0$), so that $\overline{\gamma}$ is finite and $\gamma \geq \overline{\gamma}$ is feasible. This is because, when ratings are expected to be honest, there is no reputational loss from inflating the rating for the CRA unless market trading reveals that the project was bad (i.e., when $x = +2n$). [26] As we will see in Section 1.5, when ratings are expected to be always honest, the speculator has no incentives to acquire information and, thus, market trading is not informative about θ. Therefore, honest ratings are not actually possible in equilibrium and, thus, some degree of rating inflation is always an equilibrium outcome, while rating deflation never occurs. In the rest of this section, we analyze the building blocks that give us Proposition 3.

[26] When ratings are expected to be honest, the strategic and committed types are expected to play the same strategies. This implies that the rating is not informative about the CRA's type unless market trading reveals the the project was actually bad and, thus, that the conjecture of honest ratings was inaccurate.

1.4.1 Equilibrium Rating Inflation

We first characterize the equilibrium level of rating inflation for a given level of expected rating deflation δ^e and informativeness of market trading ι^e.

The choice of rating inflation is relevant only when, at $t = 1$, a strategic CRA observes a bad project (type $(\mathcal{S}, B)$) and decides whether to inflate or truthfully report it. Therefore, we focus on the strategic choice of this type in what follows.

We can write the total payoffs for the CRA as follows:

$$\Pi_B = \varepsilon \left\{ \phi \left(p^H - I \right) + \gamma E_{x,\mathbf{y}} \left[q^{(H,x,\mathbf{y})} \mid B \right] \right\} + (1 - \varepsilon) \gamma q^{\varnothing}. \tag{1.9}$$

With probability $1 - \varepsilon$, the rating is not inflated ($r = \varnothing$) and the rating fee is not collected. This conveys bad news about project quality to investors (i.e., $p^{\varnothing} < I$). Therefore, the initial investment project is not implemented and the CDS market does not take place, and so x does not realize. The investor at the second investment stage (time $t = 3$) also chooses not to invest, as she only observes the rating r and, thus, has the same information as the initial investors (the quality of the two projects is the same, i.e., θ). Given that both the initial and the second investment projects are not implemented, and that x does not realize, at time $t = 4$ investors update the CRA's reputation only based on the rating $r = \varnothing$; the continuation reputational payoff is therefore $q^{\varnothing}$ in this case.

With probability ε, the rating is inflated to $r = H$ and the fee $\phi \left(p^H - I \right)$ is collected; the continuation reputational payoff is $E_{x,\mathbf{y}} \left[q^{(H,x,\mathbf{y})} \mid B \right]$ in this case. At time $t = 1$, both the total order flow of the CDS market x and the investment outcomes $\mathbf{y}$ have not yet realized. However, the CRA knows that the project is bad and, thus, the expectation over realizations of x and $\mathbf{y}$ is conditional on $\theta = B$. [27] The amount of rating inflation ε is then chosen as a best response to the conjectures ε^e, δ^e and ι^e; let $\varepsilon^* \left(\varepsilon^e, \delta^e, \iota^e \right)$ denote this best response.

[27] Conditional on $r = H$, the first investment is always undertaken and so its outcome is always observed, i.e., $y_1 \in \{S, F\}$. The CDS market takes place in this case and, thus, x realizes. The distribution of y_2 depends then on the realization of x since, when x conveys negative information about project quality (i.e., when $x = +2n$), the second investment is not implemented and so $y_2 = N$. When $x \neq +2n$, the second investment is undertaken as well and we have $y_1 \in \{S, F\}$ and $y_2 \in \{S, F\}$.

Definition 4 *Any equilibrium level of rating inflation* $\widehat{\varepsilon}\left(\delta^e, \iota^e\right)$ *has to be a value of* ε^e *that satisfies the following fixed point condition:*

$$\varepsilon^* \left(\varepsilon^e, \delta^e, \iota^e\right) = \varepsilon^e. \tag{1.10}$$

Taking the derivative of Π_B with respect to ε yields:

$$MBI \equiv \frac{d\Pi_B}{d\varepsilon} = \phi\left(p^H - I\right) + \gamma E_{x,\mathbf{y}}\left[q^{(H,x,\mathbf{y})} \mid B\right] - \gamma q^{\varnothing}. \tag{1.11}$$

The derivative $\frac{d\Pi_B}{d\varepsilon}$ represents the marginal benefit of rating inflation for given ε^e, δ^e and ι^e.

In order to simplify equation 1.11, let us now describe the distribution of x conditional on the project being bad. With probability ι^e, the speculator learns the asset quality and, thus, buys CDS protection; with probability $\frac{1}{2}$ her trade is revealed to the market ($x = +2n$). In this case, rating inflation is unveiled, as the investors learn that the project is bad and the CRA inflated the rating. Therefore, the CRA loses all of its reputation, i.e., $q^{H,+2n,\mathbf{y}} = 0$ for any $\mathbf{y}$. When the speculator does not learn θ (and so does not trade) or when her trade is not revealed to the market, i.e., $x \in \{-n, 0, +n\}$, the realization of x is not informative about project quality and rating inflation is not revealed. This happens with probability $1 - \frac{\iota^e}{2}$; in this case, the CRA's posterior reputation depends only on the realization of $\mathbf{y}$, i.e., $q^{H,\mathbf{y}}$. We can rewrite the expression in equation (1.11) as follows:

$$MBI = \phi\left(p^H - I\right) + \gamma\left(1 - \frac{\iota^e}{2}\right) E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid B\right] - \gamma q^{\varnothing}. \tag{1.12}$$

Lemma 5 *The marginal benefit of rating inflation* ε *is always decreasing and continuous in* ε^e*. This implies that* $\widehat{\varepsilon}\left(\delta^e, \iota^e\right)$ *is unique; we have:*

1. $\widehat{\varepsilon}\left(\delta^e, \iota^e\right) = 0$*, whenever* $MBI < 0$ *at* $\left(\varepsilon^e = 0, \delta^e, \iota^e\right)$*;*

2. $\widehat{\varepsilon}\left(\delta^e, \iota^e\right) \in (0, 1]$ *otherwise.*

Figure 1.2 depicts the possible characterizations for the equilibrium level of rating inflation.

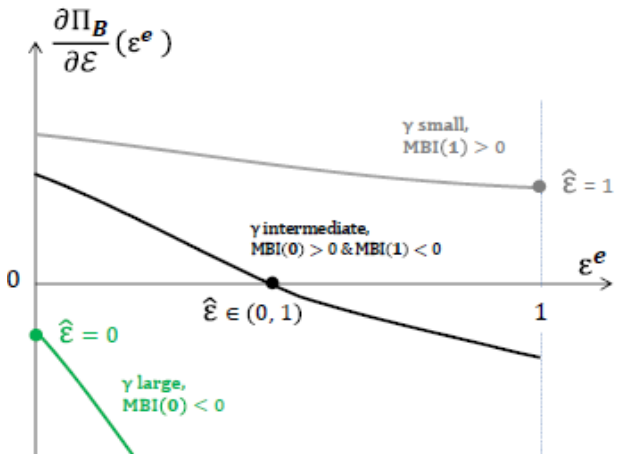


Figure 1.2: Characterization of MBI and equilibrium inflation.

The marginal benefit of inflation MBI is always decreasing in expected rating inflation ε^e. As ε^e increases, investors perceive it to be more likely that a strategic CRA inflated the rating, so reputation updating conditional on the rating $r = H$ is more severe. On the other hand, the rating $r = \varnothing$ becomes a stronger signal for a committed type, so forgone reputational payoffs when the CRA actually inflates are larger. Lastly, the rating $r = H$ is a less reliable signal for a good investment, which reduces the price the investors are willing to pay for the investment and, thus, the rating fee. Therefore, incentives to inflate are lower when expected rating inflation increases.

This implies that, for any given (δ^e, ι^e), we have a unique equilibrium level of rating inflation: no rating inflation when inflating is a strictly dominated action ($MBI < 0$), including at $\varepsilon^e = 0$; [28] or some positive level of rating inflation when inflating is a dominant action ($MBI \geq 0$) for some positive level of expected rating inflation ε^e.

Equilibrium Rating Deflation: We can now also characterize the equilibrium level of rating deflation. The choice of rating deflation is relevant only when, at $t = 1$, a strategic CRA observes a good project (type $(\mathcal{S}, G)$) and decides whether to deflate the rating. The analysis proceeds in the same way as the analysis of equilibrium rating inflation and, therefore, is placed with a discussion in Appendix A.5. We summarize the result in the following lemma:

[28] This equilibrium is supported by the off-equilibrium beliefs that when $r = H$ and $x = +2n$, the investors believe that the project is bad and that the CRA is strategic and inflated the rating.

Lemma 6 *The marginal benefit of deflation is always strictly negative in equilibrium; therefore, a strategic CRA never deflates the rating for a good project, setting* $\widehat{\delta} = 0$.

Given that the CRA never deflates the rating for a good project, an unrated asset reveals that project quality is bad and, thus, the initial investors never buy the claim when $r = \varnothing$.[29] When the rating is high, their willingness to pay for the project depends on the credibility of the CRA that publicized it, i.e., the initial reputation q_0 and the conjecture about rating inflation ε^e.

Lemma 7 *In equilibrium, the issuer always undertakes the initial investment project when the rating is high, i.e.,* $p^H > I$.

When ratings are honest, Lemma 7 is trivial, as a high rating is a perfect signal for a good project. In the equilibrium with positive rating inflation (i.e., when $\gamma < \overline{\gamma}$), Lemma 7 follows from the characterization of the marginal benefit of inflation, which must be non-negative in equilibrium. This means that the short-term gain from inflating the rating (i.e., the rating fee $\phi\left(p^H - I\right)$) is at least as large as the forgone reputational payoffs (i.e., the difference $\gamma q^{\varnothing} - \gamma E_{x,\mathbf{y}}\left[q^{(H,x,\mathbf{y})} \mid B\right]$). Since a committed CRA is restricted to truthful ratings, whereas a strategic CRA inflates its ratings with positive probability, a high rating is a signal for a strategic type, as it is relatively more likely to come from a strategic CRA. Therefore, reputation is updated downward following a high rating and upward when the asset is unrated. This means that the forgone reputational payoffs are always strictly positive in equilibrium and so is also the rating fee - i.e., $\phi\left(p^H - I\right) > 0$ which implies $p^H > I$. Hence, a high rating is always credible enough that the investors are willing to pay $p^r > I$ when $r = H$.

1.4.2 The Effect of Trading Informativeness on Rating Inflation

The effect of a change in the expected informativeness of market trading ι^e on equilibrium rating inflation depends on its effect on the marginal benefit of rating inflation. Taking the derivative

[29] When observing $r = \varnothing$, the investors infer that the issuer was offered a low report which went unpurchased and, thus, that project quality is bad.

of MBI with respect to ι^e in equation (1.12) yields:

$$\frac{dMBI}{d\iota^e} = -\frac{\gamma}{2} E_{\mathbf{y}} \left[q^{(H,\mathbf{y})} \mid B \right] < 0.$$

An increase in expected precision ι^e makes it more likely that rating inflation is revealed to the market through the speculator's trading activity.

Lemma 8 *Equilibrium rating inflation $\widehat{\varepsilon}(\iota^e)$ is decreasing in the expected level of informativeness ι^e.*

This indicates that from the point of view of the CRA, rating inflation and market trading informativeness are strategic substitutes. That is, more-informative market trading gives the CRA incentives to be more accurate. This arises because market transparency from informative trading makes reputational incentives more important.

1.5 Equilibrium and Comparative Statics

We have found that equilibrium rating inflation $\widehat{\varepsilon}(\iota^e)$ is *decreasing* in the expected level of the informativeness of market trading ι^e. On the other hand, the informativeness of market trading (choice of precision by the speculator) $\widehat{\iota}(\varepsilon^e)$ is *increasing* in the expected rating inflation, ε^e. The fact that these effects move in opposite directions implies that there is a unique equilibrium.

Proposition 9 *There exists a unique pair of rating inflation and market trading informativeness $(\widehat{\varepsilon}, \widehat{\iota})$ such that $\widehat{\varepsilon}(\iota^e = \widehat{\iota}) = \widehat{\varepsilon}$ and $\widehat{\iota}(\varepsilon^e = \widehat{\varepsilon}) = \widehat{\iota}$. In equilibrium, a strategic CRA inflates the rating for a bad project with positive probability (i.e., $\widehat{\varepsilon} > 0$) and the speculator produces information about project quality (i.e., $\widehat{\iota} > 0$).*

Figure 1.3 depicts the result in Proposition 9. The equilibrium values for market informativeness and rating inflation are $\widehat{\iota}(\varepsilon^e) \in [0,1]$, with $\widehat{\iota}(0) = 0$, and $\widehat{\varepsilon}(\iota^e) \in [0,1]$, with $\widehat{\varepsilon}(0) > 0$; this implies that the two functions in Figure 1.3 either cross (and do so only once) at some interior level $(\widehat{\varepsilon}, \widehat{\iota}) \in (0,1)^2$, or they do not cross, and we have a corner solution - i.e., $(\widehat{\varepsilon} = 1, \widehat{\iota} = \widehat{\iota}(1))$

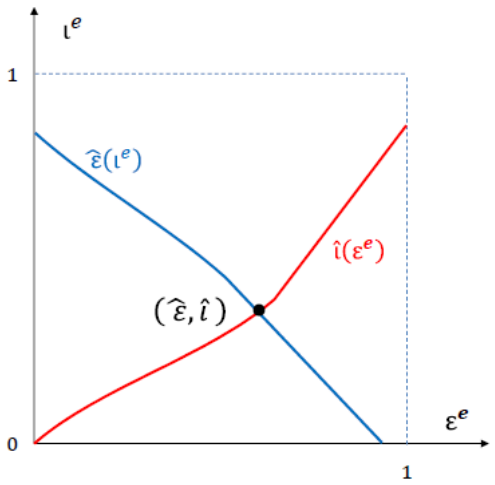


Figure 1.3: Rating Inflation and Trading Informativeness Equilibrium.

or $(\widehat{\varepsilon} = \widehat{\varepsilon}(1), \widehat{\iota} = 1)$. For any parameter constellation, we can always find γ large enough that the corner solution is ruled out.[30] Interior solutions are the most interesting for comparative statics, so we focus on this case in what follows.

Note that equilibria with honest ratings are not possible anymore when we consider the strategic interaction between the two markets. In order to have honest ratings, we would need some independent information production by the market (i.e., $\iota^e > 0$). However, when ratings are honest, there is no mispricing of risk and, thus, the speculator chooses not to acquire information about the project. The CRA anticipates this and, thus, finds it always optimal to inflate ratings with some positive probability. Therefore, the market fails to impose enough discipline on the CRA, making rating inflation endemic to the credit rating process.[31]

We now analyze the effect of varying the volume of trades by noise traders in the market for credit risk (n) and the weight on reputational payoffs in the CRA's objective function (γ). We first look at their effect on the equilibrium pair $(\widehat{\varepsilon}, \widehat{\iota})$. Then, in Section 1.6 we analyze their effects on the real investment decisions guided by the information produced in the two markets (the market for ratings and the CDS market). We find:

[30] Formally, we have an interior solution for any $\gamma > \widetilde{\gamma}$, where $\widetilde{\gamma}$ is such that $MBI(\iota^e = 1, \varepsilon^e = 0) = 0$.

[31] When the strategic interaction between the two markets is considered, the threshold value of γ in Proposition 3 becomes $\overline{\gamma} = \frac{2\phi V_G}{\widehat{\iota}(\varepsilon^e = 0) q_0}$. Given that $\widehat{\iota}(\varepsilon^e = 0) = 0$, $\overline{\gamma}$ goes to infinity and, thus, we always have $\gamma < \overline{\gamma}$ in equilibrium.

Lemma 10 *An increase in liquidity n increases equilibrium market informativeness $\widehat{\iota}$ and decreases equilibrium rating inflation $\widehat{\varepsilon}$. An increase in the CRA's reputation concern γ decreases $\widehat{\iota}$ and decreases $\widehat{\varepsilon}$.*

The comparative statics on n are straightforward. When n increases, the speculator can trade more and, thus, makes more profits. Therefore, she chooses a larger $\widehat{\iota}(\varepsilon^e)$ for any level of ε^e ($\widehat{\iota}(\varepsilon^e)$ rotates counter-clockwise). Note that the volume of noisy trading does not enter the CRA's objective function, so that a change in n affects the equilibrium only through its direct effect on $\widehat{\iota}(\varepsilon^e)$. This implies that the new equilibrium pair will feature a higher level of precision in the CDS market and lower rating inflation. Similar comparative statics results apply to a decrease in the cost of precision for the speculator.

This comparative static has an intriguing implication. When market liquidity is low (e.g., when there is a crisis or markets are fragmented) or the cost of precision is large (e.g., for complex assets), the informativeness of market trading is low. However, the CRA does not respond and act as a substitute for information production by the market, because of the lack of monitoring. In this sense, CRAs fail to produce information precisely when it would be most valuable.

An increase in γ moves only $\widehat{\varepsilon}(\iota^e)$, shifting it down. This is quite intuitive: the strategic CRA inflates less because it is more concerned about future reputation. Thus, the new equilibrium pair features a lower level of both rating inflation and precision. We will demonstrate in the next section that the decrease in information from the CDS market may have adverse real consequences that overwhelm the benefit of improved rating quality.

1.6 Real Effects of Information

In this section, we explore the real effects of the information produced in the economy. There are two real effects in the model: first, the information produced by the CRA affects whether the initial investment project is undertaken; second, the information from both the rating and the CDS market determine whether the second investment is undertaken. For simplicity, we

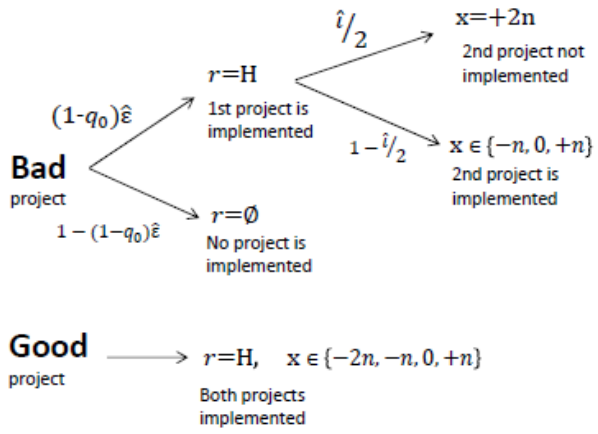


Figure 1.4: Sequence of events and investment decisions.

assume that the speculator's cost of precision has the following functional form: $c(\iota) = k\frac{\iota^2}{2}$. [32]

Given that the quality of the two projects is perfectly correlated, the investors at the two investment stages would want to take the same decision about whether to invest or not. However, the initial investors only observe the rating r, while the investor at time $t = 3$ observes the realization of market trading x as well as the rating r before taking action. Therefore, when $r = H$ and $x = +2n$, the first investment is implemented but the second one is not, as x reveals that the rating was inflated and the project quality is actually bad. Hence, the new investor chooses not to invest. Figure 1.4 describes the sequence of possible events and subsequent investment decisions.

As a measure of the real effects of information, we consider the surplus, which we denote by S. This consists of the NPV of the projects that are implemented minus the cost of the precision chosen by the speculator[33]. Since bad projects have negative NPV, surplus S is decreasing in the amount of bad projects that are implemented and increasing in the amount of good projects.

We can write surplus S as follows:

$$S = \frac{1}{2}(1+\chi)V_G + \frac{1}{2}(1-q_0)\widehat{\varepsilon}\left[1+\chi\left(1-\frac{\widehat{\iota}}{2}\right)\right]V_B - c(\widehat{\iota}). \tag{1.13}$$

[32] This simplifies the exposition but our qualitative results do not depend on the choice of the cost function; we could have any function satisfying the conditions in Equation 1.1.

[33] As prices in the model are transfers, they do not affect welfare. Therefore, we don't need to consider the prices in the market for the initial investment and the CDS market or the rating fee.

When project quality is good (ex-ante probability $\frac{1}{2}$), the projects are implemented in both periods and their expected NPV is V_G, since the rating is always high and x never conveys bad news about project quality θ (i.e., $x \in \{-2n, -n, 0, +n\}$). This is the first term in equation (1.13). The weight χ represents the size of the investment which is undertaken at the second investment stage, when both r and x have been observed by the new investor. When project quality is bad (ex-ante probability $\frac{1}{2}$), the first project is implemented only when the CRA is strategic and inflates the rating; this happens with probability $(1 - q_0)\widehat{\varepsilon}$. When the rating is high and the first project is implemented, the market for risk takes place and x realizes. With probability $\frac{\widehat{\iota}}{2}$ a bad project is revealed to the market (i.e., $x = +2n$) and, thus, the second project is not implemented. With probability $1 - \frac{\widehat{\iota}}{2}$, the realization of x is not informative about θ and the second project is implemented.

We make two remarks about the surplus before completing the analysis. First, notice that the surplus S is maximized when $\widehat{\varepsilon} = 0$ and $\widehat{\iota} = 0$, i.e., when the CRA always truthfully reports the project quality and the speculator does not spend resources in acquiring her own signal about it. However, as we have seen earlier in the analysis, this is never an equilibrium. Second, notice that when the size of the second project is small (χ small), surplus is determined by the first project, and therefore only the information from the rating is relevant for the surplus and market information has no real effect (aside from affecting equilibrium ratings). Both information from the rating and market information are relevant for the second project, so as its size grows, the aggregate amount of information becomes more important.

We now examine how the surplus is affected by the exogenous parameters.

Proposition 11 *There always exist values for the speculator's cost parameter (k^*, k^{**}), with $k^{**} > k^*$, and the size of the second investment project $\overline{\chi}$, such that:*

1. *For $k \geq k^{**}$, the surplus S is always increasing in the CRA's reputational concern γ and decreasing in market liquidity n;*

2. *For $k \leq k^*$, the surplus S is always decreasing in γ for $\chi \geq \overline{\chi}$, and always increasing in n.*

As we pointed out earlier, an increase in n leads to higher equilibrium precision $\widehat{\iota}$ and lower

equilibrium rating inflation $\widehat{\varepsilon}$. Both of these shifts increase the amount of information produced in the market, leading to more efficient investment decisions. However, when the cost of precision is large ($k \geq k^{**}$), the overall surplus diminishes because of the increase in the cost of precision that is associated with a larger $\widehat{\iota}$. As the speculator's payoffs don't enter our measure of surplus (since they represent a transfer), investment in making trading profits is wasteful in the model.

On the other hand, an increase in the CRA's reputational concern γ decreases rating inflation $\widehat{\varepsilon}$ but also decreases precision $\widehat{\iota}$. When precision is cheap ($k \leq k^{*}$), the speculator adjusts her choice of ι quite sharply to changes in the level of expected mispricing - i.e., rating inflation. This means that $\widehat{\iota}(\varepsilon^{e})$ is very sensitive to ε^{e} and, thus, the decrease in $\widehat{\iota}$ is much larger than the decrease in $\widehat{\varepsilon}$ when γ increases, reducing the efficiency of the second investment decision substantially.[34] When the size of the second project is large ($\chi \geq \overline{\chi}$), i.e., when future investment is more important than current investment, this leads to less efficient investment decisions overall. When instead the cost of precision is large ($k \geq k^{**}$), $\widehat{\iota}(\varepsilon^{e})$ is not very sensitive to ε^{e} and, thus, $\widehat{\iota}$ does not move much when $\widehat{\varepsilon}$ decreases. In this case, surplus S increases with γ.[35]

Figure 1.5 describes the result in Proposition 11. When reputational concern γ increases, rating inflation $\widehat{\varepsilon}(\iota^{e})$ shifts down to the dashed line. When $k \geq k^{**}$ (purple curve), precision $\widehat{\iota}(\varepsilon^{e})$ is not very sensitive to inflation ε^{e} and, thus, equilibrium market informativeness $\widehat{\iota}$ moves less than rating inflation $\widehat{\varepsilon}$ when γ increases. In contrast, when $k \leq k^{*}$ (red curve), $\widehat{\iota}(\varepsilon^{e})$ is very sensitive to ε^{e} and, as a consequence, the decrease in $\widehat{\iota}$ is much larger than the decrease in $\widehat{\varepsilon}$.

Proposition 11 has important implications. When the cost of speculator information acquisition is small ($k \leq k^{*}$) and the size of the second project is large ($\chi \geq \overline{\chi}$), the surplus S is decreasing in γ. This means that when the CRA becomes more concerned about future reputation and rating inflation decreases, the overall efficiency of the market declines. Therefore policies meant to discipline the CRA's incentives, such as larger reputational sanctions (e.g., increases in liability or regulatory scrutiny), may be undesirable, because of their perverse effect

[34] In a previous version of this paper (Piccolo and Shapiro 2017), we write down a measure of informativeness and demonstrate that this effect decreases the total amount of information transmitted.

[35] When $k \in (k^{*}, k^{**})$, the effects of both n and γ on S are ambiguous. For some parameters of the model, S is non-monotonic in γ, i.e., there exists a threshold γ^{*} such that: S is increasing in γ for $\gamma \geq \gamma^{*}$, and decreasing otherwise.

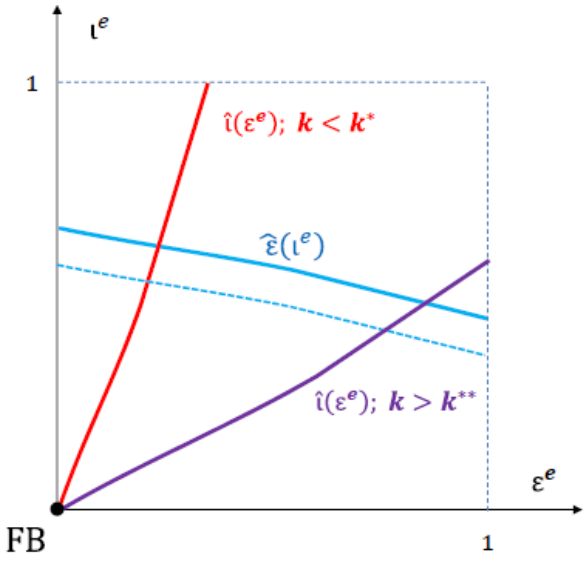


Figure 1.5: Comparative statics and Proposition 11.

on the market's incentives to acquire information and the resulting real investment decisions.

However, when the cost of speculator information acquisition is small ($k \leq k^*$), policies that foster market liquidity - for example, improved governance and regulation of exchanges that instills trust and attracts trade - increase market informativeness, which also provides indirect discipline on the CRA and reduces rating inflation, leading to more efficient outcomes overall.

The cost of speculator information acquisition is more likely to be small for non-complex assets/investment projects, where information production by the market is both reactive to the level of mispricing and not too expensive. This could be for more plain vanilla issuances, such as corporate bonds. Alternatively, if speculators have substantial size (either through natural growth or acquisitions) and have invested in research infrastructure, they may have reached economies of scale in information production.

On the other hand, when the cost of speculator information acquisition is large ($k \geq k^{**}$), the market fails to produce independent information about the asset and, thus, to both monitor the CRA and guide investment decisions. In this case, a planner would prefer not to spur speculators to acquire information due to the elevated cost. Furthermore, disciplining a CRA (e.g., by a regulator) would provide it with incentives to reveal more information about asset quality and improve surplus S. These large costs of speculator information acquisition may be due to the

complexity of the asset, such as in the case of structured finance products or corporates with not much of a track record. Alternatively, this may occur if the informed trading industry consisted of small players.

Our results also have some interesting implications for the limiting case where the CDS market or the ratings process does not exist. Ratings become more accurate when a secondary market on the asset is introduced - for example, when CDS start being actively traded on the asset. This is consistent with the empirical findings of Dilly and Mahlmann (2014). On the other hand, trading in the secondary market would be more informative if there were no ratings on the asset. Paradoxically, for some parameter configurations, efficiency would be larger if the market for ratings did not exist.[36] This is intriguing given the current regulatory debate on replacing ratings with market based measures. We discuss this particular aspect in more detail in the next section.

1.7 Adding to the Model

In this section, we examine replacing ratings in regulation with market measures and check the robustness of the model to alternative assumptions.

1.7.1 Moving away from Rating-based regulation

The massive downgrading and defaults of highly-rated securities during the 2008/2009 financial crisis have led regulators in the US and EU to consider replacing rating-based regulation with market-based measures (see, e.g., the Dodd Frank bill and European Commission (2015)).[37] In this section, we discuss the implications of our analysis for the welfare effects of this policy.

As in Opp, Opp, and Harris (2013), we assume that investors are willing to pay a premium $\mathcal{M}$ for a security that receives a favorable regulatory treatment. We then consider two different regulatory regimes: a regime with rating-contingent regulation, in which only a security with

[36] This is true when (i) the project has positive ex-ante NPV and, thus, the CRA is not needed to enable the initial financing by providing some screening of the projects and (ii) the speculator's choice of precision is quite reactive to ε^e (i.e., for k small).

[37] Rating-based regulations are used to ensure the safety of financial institutions (e.g. banks, pension funds, insurance companies).

a high rating ($r = H$) receives a favorable regulatory treatment; a regime with CDS-based regulation, where the regulatory treatment of a security depends on its CDS price, namely, whether the CDS price p^{cds} is strictly below a threshold $\overline{p} = f_B$. [38]

Lemma 12 *1. An increase in the regulatory benefit $\mathcal{M}$ increases both equilibrium rating inflation $\widehat{\varepsilon}$ and market informativeness $\widehat{\iota}$, in both regimes.*

2. Moving from rating-based regulation to CDS-based regulation reduces both $\widehat{\varepsilon}$ and $\widehat{\iota}$.

The premium $\mathcal{M}$ is reflected in the price of the asset and, as a consequence, in the rating fee. As $\mathcal{M}$ becomes larger, the rating fee increases, directly in the rating regime and indirectly in the CDS regime. This makes rating inflation more attractive for the CRA ($\widehat{\varepsilon}(\iota^e)$ in Figure 1.3 shifts upward). Therefore, when $\mathcal{M}$ increases, the new equilibrium features both higher rating inflation and market informativeness.

However, the link between a high rating and the premium $\mathcal{M}$ is less strong in the regime with CDS-based regulation. In this case, the security receives a favorable treatment only if the CDS price is below the threshold $\overline{p}$. When the investment project is bad, this occurs with probability lower than 1: with positive probability the speculator learns that the asset is bad and her signal is revealed by market trading ($x = +2n$); in this case, we have $p^{cds} = f_B$ and the security is not granted the favorable treatment. As a result, the rating fee is lower in the scenario with CDS-based regulation, because investors are willing to pay only a fraction of the premium $\mathcal{M}$ to the issuer. This makes rating inflation less attractive in the regime with CDS-based regulation. On the other hand, the speculator's incentives to acquire information don't change, as she does not hold the security. Therefore, both rating inflation $\widehat{\varepsilon}$ and market informativeness $\widehat{\iota}$ are lower in the regime with CDS-based regulation.[39]

The analysis in this section highlights that the welfare effects of moving from rating-based regulation to CDS-based regulation are ambiguous. On the one hand, moving to CDS-based

[38] In equilibrium, depending on market trading, we have three possible realizations of the CDS price, i.e., $p^{cds} \in \{f_G, \mu^H f_G + (1-\mu^H) f_B, f_B\}$. A high CDS price ($p^{cds} = f_B$) reveals negative information about the quality of the investment. Therefore, the regulator would want to grant a favourable regulatory treatment only to assets whose CDS price is strictly lower than f_B.

[39] As we discussed in Section 1.6, when $\widehat{\varepsilon}$ and $\widehat{\iota}$ move in the same direction, the effect on investment efficiency depends on the relative magnitude of the two variations and, thus, on the parameters of the model.

regulation reduces the regulatory arbitrage of the CRAs, diminishing their incentives to inflate ratings. On the other hand, the increased rating accuracy crowds out information production in the CDS market; as we discussed in Section 1.6, this can reduce welfare. Note that this a different downside than is discussed in Flannery, Houston, and Partnoy (2010), who point out that the volatility of CDS spreads make a CDS-based regulatory threshold impractical for market participants.

However, although market-based regulation can result in welfare losses, fostering an active CDS market makes it possible to make regulations depend on ratings, because of the discipline it provides on CRAs. This is an implication of the comparative statics in Lemma 10, where we showed that an increase in market liquidity n leads to both lower rating inflation and higher market informativeness in equilibrium.

1.7.2 The participation of the initial investors in the CDS market

The investors who purchase the investment in the initial investment market are risk neutral and therefore would choose not to participate in the CDS market. The reason for this is that they suffer an informational disadvantage with respect to the market maker,[40] and so cannot make profits (in expectation) from trading in this market. However, initial investors who are risk averse might want to participate in the CDS market, in order to buy protection for the investment they purchased. We now discuss a simple variation of the model which could allow for this possibility and demonstrate that, while risk-averse initial investors may want to participate in the CDS market, our results still hold.

Suppose risk averse initial investors buy an amount of protection Λ in the CDS market.[41] In this case, the speculator's trading strategy does not change with the respect to one specified in Lemma 1. The total order flow x then takes values in $\mathcal{X} = \{\Lambda - 2n,\ \Lambda,\ \Lambda + 2n\}$ when the speculator learns θ and trades the asset. If $x \in \{\Lambda - n,\ \Lambda + n\}$, it must be instead that the

[40] The signal observed by the speculator is not known to uninformed traders at the moment they submit their trade orders, while the market maker learns it with positive probability in equilibrium before setting $\widehat{p}^{cds}$.

[41] We could have that Λ depends on expected rating inflation - i.e., $\Lambda = \Lambda(\varepsilon^e)$, to allow for example the investors to demand more CDS protection when ε^e increases, making the rating $m = H$ a less reliable signal for a good investment. As long as the characterization of $\Lambda(\varepsilon^e)$ is common knowledge among the agents in the model, none of our results would change.

speculator is not trading. The market maker infers $\theta = G$ when $x = \Lambda - 2n$, $\theta = B$ when $x = \Lambda + 2n$, and nothing when $\in \{\Lambda - n,\ \Lambda,\ \Lambda + n\}$. The investors anticipate they are going to buy CDS protection when purchasing the investment at time $t = 1$. Their willingness to pay for the initial investment and therefore the rating fee $\phi\left(p^H - I\right)$ depend on ι^e in this case, as ι^e moves their expectations about the equilibrium CDS price. A sufficient condition for the results in the model to still hold in this framework would be that their willingness to pay for the initial investment is non-increasing in ι^e.[42] This is satisfied, since as we show in Appendix 1.11.5, the expected price of CDS protection does not change with ι^e, but its variance increases as ι^e increases. The increased variance of the price of CDS protection decreases risk averse investors' willingness to pay for the initial investment.

The speculator does not participate in the investment market: This is consistent with recent empirical work by Oehmke and Zawadowski (2016), who show that speculative trading concentrates in the CDS market, due to its relative liquidity advantage. If the speculator buys only a small amount of the investment initially, that will not affect her behavior in the CDS market. If, however, the speculator were to buy a large amount of the investment in the initial investment market, her incentives might change. Receiving a bad signal could make the purchase of insurance so valuable that the speculator would not want to mask her trades. Or if we were to model a further secondary market and liquidity shocks, the speculator with a bad signal might not want to reveal any information whatsoever, so as not to depress the price of the investment in the further secondary market.

1.8 Empirical Implications

This chapter examines how credit ratings and market information affect each other, and the real implications of this interaction. In this section we examine evidence surrounding testable implications of the model.

The key hypotheses the model offers for testing are:

[42] This is needed for Lemma 8, which proves that $\frac{dMBI}{d\iota^e} < 0$. If p^H depends on ι^e, $\frac{dMBI}{d\iota^e}$ will include the expression $\phi\frac{\partial p^H}{\partial \iota^e}$.

1. The model demonstrates that exogenous improvements in liquidity and reduction in information gathering costs make market trading more informative and ratings more informative. The effect on ratings arises due to the disciplining effect of market information.

2. The model shows that exogenous increases in the future profitability/reputation of a CRA leads to:

   (a) More-informative ratings.

   (b) Less-informative market trading.

   (c) Possible negative real effects. The negative real effects come from subsequent issuance and/or investment decisions whose quality is related to the initial issue. The negative effects are more likely when the costs of information acquisition are low for speculators. The costs may represent the complexity associated with the investment (e.g., structured finance vs. plain vanilla corporate bonds), the degree of information asymmetry (e.g., newer firms or ventures have less of a track record, internet firms often have low cash flows), or the fact that speculators are small and have not achieved economies of scale.

With regards to hypothesis (1), Badoer and Demiroglu (forthcoming) find that regulation that mandated disclosure about price and volume information on over-the-counter transactions of corporate debt makes ratings downgrades more sensitive to changes in credit spreads. Gopolan, Gopolan, and Koharki (2017) distinguish between listed and unlisted firms (listing makes information for market participants easier to find) and find that ratings of unlisted firms in India are higher, less sensitive to financial conditions, and contain less information about subsequent defaults than ratings of listed firms. Fong, Hong, Kacperczyk, and Kubik (2014) does not look at market information but instead shows that a plausibly exogenous decrease in information leads to less accurate ratings; when mergers reduce the number of equity analysts, ratings become less informative about defaults and future downgrades.

The first issue for testing the hypotheses (2a), (2b), and (2c) is establishing an exogenous shock to CRA reputation/future profitability that doesn't affect information gathering by spec-

ulators or real investments. In the finance literature, a few such exogenous events have been studied. They also provide evidence on hypothesis (2a). Baghai and Becker (2018) use S&P's exclusion from the CMBS business following a significant error. They find that when S&P returned, it issued higher ratings that Moody's and Fitch, particularly for larger issuers and larger deals. Kedia et al. (2014) demonstrate that Moody's IPO led Moody's to rate bonds significantly higher than Standard and Poors, particularly for large and frequent issuers and for firms right below a ratings threshold.

Given an exogenous shock to the CRA, in order to test hypothesis (2b) a measure of informed trading or mispricing is necessary. Informed trading is difficult to capture.[43] Mispricing is easier, as the econometrician can look for reversals in asset prices (as in Coval and Stafford, 2007). Such corrections can be verified to be information related if they take place around information-based events, such as earnings announcements. The mispricing channel might also arise indirectly; credit ratings may affect analyst forecasts which affect market prices (for this last point, see, e.g., Womack, 1996).

Lastly, real activity is easier to measure. Firm investment policies, capital structure, and R&D investment all fall under choices that may rely on information conveyed about other assets. Relating informativeness to investment is more difficult. Bakke and Whited (2010) show that the sensitivity of investment to Q depends on price informativeness when correcting for measurement error. Chen, Goldstein, and Jiang (2007) demonstrate that the sensitivity of investment to price is larger when several market microstructure measures indicate there is more private information in the market price. Durnev, Morck, and Yeung (2004) find a strong positive correlation between firm specific variation in stock returns and firm investment policies.

1.9 Conclusion

Information about credit risk has real effects. In this chapter, we examine two sources of information about credit risk; credit ratings and a market for credit risk (e.g., the CDS mar-

[43] Collin-Dufresne and Fos (2015) and others have recently demonstrated that a typical measure of informed trading such as PIN (Probability of informed trading) may not perform well when strategic choices by informed traders are a consideration.

ket). We look at both the information production and real effects from the interaction of these sources. More-informative ratings decrease the quality of information in market trading, but more-informative market trading increases the quality of ratings. It would be interesting to incorporate competition in the rating market and dynamics in market trading to create a richer environment.

Bibliography

[1] Badoer, D.C., and Demiroglu, C. Forthcoming. The Relevance of Credit Ratings in Transparent Bond Markets. Review of Financial Studies.

[2] Baghai, R., and Becker, B. 2018. Reputations and credit ratings—evidence from commercial mortgage-backed securities. Working Paper, Stockholm School of Economics.

[3] Bakke TE, Whited TM. 2010. Which firms follow the market? An analysis of corporate investment decisions. Review of Financial Studies 23(5):1941–80

[4] Begley, T. 2015. The Real Costs of Corporate Credit Ratings. Working Paper.

[5] Bond, P., Edmans A., and Goldstein I., 2012, The Real Effects of Financial Markets, Annual Review of Financial Economics, 4, 339-360.

[6] Bond, P. and Goldstein, I. 2015, Government Intervention and Information Aggregation by Prices, Journal of Finance, 70, 2777-2811.

[7] Chava, S., Ganduri, R., and Ornthanalai, C. 2018. Are Credit Ratings Still Relevant? Working Paper.

[8] Chen Q, Goldstein I, Jiang W. 2007. Price informativeness and investment sensitivity to stock price. Review of Financial Studies 20(3):619–50.

[9] Collin-Dufresne, P. and V. Fos, 2015, Do prices reveal the presence of informed trading? Journal of Finance 70, 1555-1582.

[10] Coval, J., and E. Stafford. 2007. Asset fire sales (and purchases) in equity markets. Journal of Financial Economics 86:479–512.

[11] Diamond, D.W., 1985. Optimal release of information by firms. Journal of Finance 40, 1071–1094.

[12] Dilly, M., and Mählmann, T., 2014. The disciplinary role of CDS markets: Evidence from agency ratings. Working paper.

[13] Durnev A, Morck R, Yeung B. 2004. Value-enhancing capital budgeting and firm-specific return variation. Journal of Finance 59(1):65–105.

[14] European Commission. 2015. Study on the Feasibility of Alternatives to Credit Ratings. Available at https://ec.europa.eu/info/system/files/alternatives-to-credit-rating-study-01122015_en.pdf

[15] Faure-Grimaud A. and Gromb D. 2004. Public Trading and Private Incentives. Review of Financial Studies 17, 985-1014.

[16] Flannery, M. J., J. F. Houston, and F. Partnoy. 2010. Credit default swap spreads as viable substitutes for Credit Ratings. University of Pennsylvania Law Review, 158(7), 2085-2123.

[17] Fong, K. Y., Hong, H. G., Kacperczyk, M. T., Kubik, J. D., 2014. Do security analysts discipline credit rating agencies? Working paper, University of New South Wales.

[18] Fulghieri, P., G. Strobl, and H. Xia, 2014. The Economics of Solicited and Unsolicited Credit Ratings, Review of Financial Studies, 27:2, 484-518.

[19] Gao P. and Liang P.J. 2013. Informational feedback, adverse selection, and optimal disclosure policy. Journal of Accounting Research. 51:1133–1158.

[20] Goldstein, I., and C. Huang. 2017. Credit Rating Inflation and Firms' Investments. Working Paper, University of Pennsylvania.

[21] Goldstein, I., and L. Yang. 2017. Information Disclosure in Financial Markets. Annual Review of Financial Economics, vol. 9, pp. 101-125.

[22] Goldstein, I., and L. Yang. Forthcoming. Good Disclosure, Bad Disclosure. Journal of Financial Economics.

[23] Gopolan, R., Gopolan, Y. and K. Koharki., 2017. Market Information and Rating Agency Catering. Working Paper, Washington University in St. Louis.

[24] Han, B., Tang, Y. and L. Yang. Public Information and Uninformed Trading: Implications for Market Liquidity and Price Efficiency. Journal of Economic Theory. 163(5): 604-643.

[25] Kedia, S., Rajgopal, S., Zhou, X., 2014. Did going public impair Moody's credit rating? Journal of Financial Economics 114(2), 293-315.

[26] Kedia, S., Rajgopal, S., Zhou, X., 2017. Large shareholders and credit ratings. Journal of Financial Economics 124(3), 632-653.

[27] Kim, O., Verrecchia, R.E., 1991. Market reactions to anticipated announcements. Journal of Financial Economics 30, 273–309.

[28] Kliger, D. and O. Sarig. 2000. The information value of bond ratings. Journal of Finance 55(6), 2879–2902.

[29] Kreps, D. M., and R. Wilson. 1982. Reputation and Imperfect Information, Journal of Economic Theory, 27, 253-279.

[30] Kyle, A. 1985. "Continuous Auctions and Insider Trading." Econometrica 53, 1315-1335.

[31] Laffont, J.-J., and J. Tirole. 1993. A Theory of Incentives in Procurement and Regulation (MIT Press, Cambridge).

[32] Mathis, J., McAndrews, J., Rochet, J.C. 2009. Rating the raters: Are reputation concerns powerful enough to discipline rating agencies? Journal of Monetary Economics 56:5, 657-674.

[33] Milgrom, P., and J. Roberts. 1982. Predation, Reputation and Entry Deterrence, Journal of Economic Theory, 27(2), 280-312.

[34] Oehmke, M., Zawadowski, A. forthcoming. The Anatomy of the CDS Market. Review of Financial Studies.

[35] Opp, C., M. Opp, M. Harris, 2012. Rating Agencies in the Face of Regulation. Journal of Financial Economics 108, 64-81.

[36] Strausz, R. 2005. Honest Certification and the Threat of Capture. International Journal of Industrial Organization 23:1-2, 45-62.

[37] Verrecchia, R.E. 1982. The use of mathematical models in financial accounting. Journal of Accounting Research 20 (Suppl.) 1–42.

[38] Womack, K. 1996. Do Brokerage Analysts' Recommendations Have Investment Value? Journal of Finance 51 : 137–67.

1.10 Appendix A

1.10.1 Beliefs about project quality

Let z represent a realized information set for the investors; the posterior probability that the asset is good given z is as follows:

$$\begin{aligned} \mu^z &= \Pr(G|z) = \frac{\Pr(z|G)\,\frac{1}{2}}{\Pr(z|G)\,\frac{1}{2} + \Pr(z|B)\,\frac{1}{2}} \\ &= \begin{cases} \left(1 + \frac{\Pr(z|B)}{\Pr(z|G)}\right)^{-1} & \text{if } \Pr(z|G) \neq 0; \\ 0 \text{ otherwise.} \end{cases} \end{aligned}.$$

Consider the investors' posterior beliefs following a high rating, i.e., $\mu^H = \Pr(G \mid r = H)$. We have:

$$\mu^H = \left(1 + \frac{\Pr(H|B)}{\Pr(H|G)}\right)^{-1} = \left(1 + \frac{(1-q_0)\,\varepsilon^e}{q_0 + (1-q_0)\,(1-\delta^e)}\right)^{-1}. \tag{1.14}$$

When the project is unrated, the investors's posterior about project quality is:

$$\mu^{\varnothing} = \left(1 + \frac{q_0 + (1-q_0)(1-\varepsilon^e)}{(1-q_0)\,\delta^e}\right)^{-1} \text{ if } \delta^e > 0, \quad \mu^{\varnothing} = 0 \text{ otherwise.}$$

The issuer is willing to sell the claim and undertake the project as long as $p^r > I$. For a given rating r, this means:

$$\mu^r (1 - f_G) + (1 - \mu^r)(1 - f_B) > I \quad \Leftrightarrow \mu^r > \frac{|V_B|}{|V_B| + V_G}. \tag{1.15}$$

Notice that $\frac{|V_B|}{|V_B|+V_G} > \frac{1}{2}$, since the ex-ante NPV of the project is assumed to be negative - i.e., $\frac{1}{2}V_G + \frac{1}{2}V_B < 0$, which implies $V_G < |V_B|$. Therefore, whenever the project is implemented, the issuer and the investors believe this is more likely to be good, i.e., $\mu^r > \frac{1}{2}$.

When the CRA's initial reputation q_0 is small, expected rating inflation ε^e is high, and rating deflation δ^e is low, the inequality in (1.15) does not hold for both $r = H$ and $r = \varnothing$. This means that, when q_0 is small, there always exist conjectures $\left(\varepsilon', \delta'\right)$ such that we never have $p^r > I$, and, thus, the issuer never publicizes the rating. Therefore, the strategic CRA is indifferent between any rating strategy and, hence, is fine with choosing the strategy $\left(\varepsilon', \delta'\right)$: this implies that the conjecture $\left(\varepsilon', \delta'\right)$ represents indeed an equilibrium of the game. In the literature on Communication Games, this type of equilibrium is referred to as a *babbling equilibrium*. However, since ratings play no role in this type of equilibria, they are not of interest for our analysis.

The only possible equilibria of the game are the ones we characterize in the text and the babbling equilibria described above. However, when the CRA's initial reputation is sufficiently large (i.e., $q_0 > \overline{q} = 1 - \frac{V_G}{|V_B|}$), a high rating is always credible enough that the investors always pay $p^H > I$ for a highly rated project and, thus, any babbling equilibrium fails to exist. In this case (i.e., $q_0 > \overline{q}$), the equilibrium we characterize in the analysis is the unique equilibrium of the game.[44]

[44] The threshold $\overline{q}$ is derived by evaluating the inequality in equation (1.15) for $r = H$, $\varepsilon^e = 1$, and $\delta^e = 0$.

1.10.2 Proof of Lemma 1

We prove Lemma 1 in two steps. First, we prove that the described strategies are the unique possible strategies in equilibrium. Second, we verify that, given the speculator's strategy, the market maker's inference is consistent with Bayes rule.

Notice that we have:

$$f_G \leq \widehat{p}^{cds}(x) \leq f_B. \tag{1.16}$$

The inequalities in (1.16) set bounds for $\widehat{p}^{cds}(x)$: it cannot be greater than f_B - i.e., the probability of failure of a bad project - and it cannot be lower than f_G - i.e., the probability of failure of a good project. When the speculator learns $\theta = B$ and for a given realization of x, if she buys (sells) protection, expected profit from trade is $f_B - \widehat{p}^{cds}(x)$ $(\widehat{p}^{cds}(x) - f_B)$. By (1.16), this is strictly positive (negative) when x does not reveal the speculator's private information to the market maker, and 0 otherwise. Thus, buying protection is always a dominant strategy when learning $\theta = B$. When the speculator learns $\theta = G$ and for a given realization of x, if she sells (buys) protection, expected profit from trade is $\widehat{p}^{cds}(x) - f_G$ $(f_G - \widehat{p}^{cds}(x))$, which is strictly positive (negative) when x does not reveal the speculator's private information to the market maker, and 0 otherwise. Hence, selling protection is always a dominant strategy when learning $\theta = G$. Notice that the described strategy is strictly dominant whenever the market maker does not always learn θ from x, which is the case in our setting (because of the camouflage from the noise traders).

Now, consider the case where the speculator does not learn anything about θ. In this case, her expectation about the probability of default of the project is $E(f_\theta \mid r = H)$. For a given realization of x, if she buys protection, her profit from trade is $E(f_\theta \mid r = H) - \widehat{p}^{cds}(x)$; this is: (i) strictly negative when $x = +2n$, since the market maker then sets $\widehat{p}^{cds}(+2n) = f_B > E(f_\theta \mid r = H)$; (ii) zero when $x = 0$, since the market maker then sets $\widehat{p}^{cds}(0) = E(f_\theta \mid r = H)$. If she sells protection, her profit from trade is $\widehat{p}^{cds}(x) - E(f_\theta \mid r = H)$; this is strictly negative when $x = -2n$, since the market maker then sets $\widehat{p}^{cds}(-2n) = f_G < E(f_\theta \mid r = H)$, and again zero when $x = 0$. Hence, when the speculator does not learn anything about θ, she is always

better off not trading.

Therefore, the strategies described in Lemma 1 represent an equilibrium of the game. It is straightforward to show that any other possible market maker's inference from the submitted orders admit profitable deviations by the speculator. Therefore, this is also the unique equilibrium.

When $x \neq 0$, the market maker's inference trivially satisfies Bayes rule. Here, we show that $\Pr(x = 0 \mid \theta) = \frac{1}{2} \forall \theta \in \{G, B\}$, and so no inference about θ is possible from $x = 0$.

When the speculator learns $\theta = G$, she is selling CDS protection, so $x^s = -n$. For x to be 0, we need to have $x^n = +n$, which has probability $\frac{1}{2}$. The opposite is true when she learns $\theta = B$. Hence, we have:

$$\Pr(x = 0 \mid G) = \Pr(x = 0 \mid B) = \frac{1}{2}.$$

Therefore, the market maker does not learn anything about θ when $x = 0$.

1.10.3 Proof of Lemma 2

We need to show that $\frac{d}{d\varepsilon^e}\left(\frac{\partial \Pi^s}{\partial \iota}\right) > 0$. We have

$$\frac{\partial \Pi^s}{\partial \iota} = \frac{n}{2}\left[\mu^H\left(\widehat{p}^{cds} - f_G\right) + \left(1 - \mu^H\right)\left(f_B - \widehat{p}^{cds}\right)\right] - c'(\iota). \tag{1.17}$$

The convexity of the cost function implies that Π^s is concave in ι and, thus, that $\widehat{\iota}(\varepsilon^e)$ exists and is unique. Given that $\widehat{p}^{cds} = \mu^H f_G + \left(1 - \mu^H\right) f_B$, we can write:

$$\widehat{p}^{cds} - f_G = \left(1 - \mu^H\right)(f_B - f_G); \quad f_B - \widehat{p}^{cds} = \mu^H (f_B - f_G).$$

Therefore, we have:

$$\begin{aligned} \frac{\partial \Pi^s}{\partial \iota} &= \frac{n}{2}\left[\mu^H\left(1 - \mu^H\right)(f_B - f_G) + \left(1 - \mu^H\right)\mu^H (f_B - f_G)\right] - c'(\iota) \\ &= n\mu^H\left(1 - \mu^H\right)(f_B - f_G) - c'(\iota). \end{aligned} \tag{1.18}$$

This implies that:

$$\frac{d}{d\varepsilon^e}\left(\frac{\partial \Pi^s}{\partial \iota}\right) = n\left(f_B - f_G\right)\frac{d\mu^H}{d\varepsilon^e}\left(1-2\mu^H\right).$$

We have:

$$\mu^H = \frac{1}{1+\frac{(1-q_0)\varepsilon^e}{q_0+(1-q_0)(1-\delta^e)}}. \tag{1.19}$$

Note that μ^H is always decreasing in ε^e. Notice also that, for the issuer to implement the project and, thus, for the CDS market to take place, we must have $\mu^H > \frac{1}{2}$. Therefore, we have $\frac{d\mu^H}{d\varepsilon^e}$ and $1-2\mu^H$ both negative and, thus, $\frac{d}{d\varepsilon^e}\left(\frac{\partial \Pi^s}{\partial \iota}\right) > 0$.

1.10.4 Proof of Lemma 5

In order to prove this Lemma, we need to evaluate how the terms p^H, $q^{(H,\mathbf{y})}$, and $q^{\varnothing}$ in equation 1.12 change with ε^e. First, we derive the expression $q^{(H,\mathbf{y})}$ and show that this is decreasing in ε^e for any given $\mathbf{y}$ and conjectures $(\varepsilon^e, \delta^e)$.

We have:

$$\frac{\Pr\left(H,\mathbf{y}|\ \tau=\mathcal{S}\right)}{\Pr\left(H,\mathbf{y}|\ \tau=\mathcal{C}\right)} = \frac{\overbrace{\left(1-\delta^e\right)\Pr\left(\mathbf{y}\mid G\right)}^{\text{prob. }(\mathcal{S},G)} + \overbrace{\varepsilon^e \Pr\left(\mathbf{y}\mid B\right)}^{\text{prob. }(\mathcal{S},B)}}{\underbrace{\Pr\left(\mathbf{y}\mid G\right)}_{\text{prob. }(\mathcal{C},G)}} = 1-\delta^e+\varepsilon^e\frac{\Pr\left(\mathbf{y}\mid B\right)}{\Pr\left(\mathbf{y}\mid G\right)}. \tag{1.20}$$

For given ε^e and δ^e, we can write $q^{(H,\mathbf{y})}$ as:

$$q^{(H,\mathbf{y})} = \Pr\left(\tau=\mathcal{C}\ |H,\mathbf{y}\right) = \left(1+\frac{1-q_0}{q_0}\frac{\Pr\left(H,\mathbf{y}|\ \mathcal{S}\right)}{\Pr\left(H,\mathbf{y}|\ \mathcal{C}\right)}\right)^{-1}. \tag{1.21}$$

Note that $q^{(H,\mathbf{y})}$ is decreasing in ε^e, since $\frac{\Pr(H,\mathbf{y}|\mathcal{S})}{\Pr(H,\mathbf{y}|\mathcal{C})}$ is increasing in ε^e.

The expression for $q^{\varnothing}$ is the following:

$$q^{\varnothing} = \Pr\left(\tau=\mathcal{C}\ |\varnothing\right) = \left(1+\frac{1-q_0}{q_0}\left(1-\varepsilon^e+\delta^e\right)\right)^{-1}. \tag{1.22}$$

Note that $q^{\varnothing}$ is, instead, increasing in ε^e.

Finally, the price p^H is increasing in μ^H which, as we discussed in Appendix 1.10.3, is decreasing in ε^e; this implies that p^H is decreasing in ε^e as well. Therefore, we have:

$$\frac{dMBI}{d\varepsilon^e} = \phi \left(\underbrace{\frac{\partial p^H}{\partial \varepsilon^e}}_{<0} - I \right) + \gamma E_{\mathbf{y}} \left[\underbrace{\frac{dq^{(H,\mathbf{y})}}{d\varepsilon^e}}_{<0} \mid B \right] - \underbrace{\frac{dq^{\varnothing}}{d\varepsilon^e}}_{>0} < 0.$$

1.10.5 Proof of Lemma 6

The choice of rating deflation is relevant when a strategic CRA observes a good project (type $(\mathcal{S}, G)$) and decides whether to deflate or truthfully report it. In what follows, we show that deflating a good signal is always a strictly dominated action for type $(\mathcal{S}, G)$ in equilibrium. We first discuss the intuition behind this result and then proceed with the proof.

In equilibria with no rating inflation - i.e., $\widehat{\varepsilon}\,(\delta^e, \iota^e) = 0$, there is no reputational loss from offering a high rating to a good project, since a high rating is not interpreted as a signal for a strategic CRA. Therefore, the CRA has no incentive to deflate a good signal. In equilibria with positive rating inflation, i.e., $\widehat{\varepsilon}\,(\delta^e, \iota^e) \in (0, 1]$, type $(\mathcal{S}, B)$ inflates the rating for a bad project with positive probability and, thus, must be at least indifferent between offering a high or a low rating to an issuer with a bad project. The reputational loss from offering $r = H$ is larger for type $(\mathcal{S}, B)$ than for type $(\mathcal{S}, G)$ - we show this in Step Two of the proof, as the former knows that the project is bad and that this will be (at least partly) revealed to the market via the realizations of $\mathbf{y}$ and x. This implies that type $(\mathcal{S}, G)$ must be always *strictly* better-off by selling the rating than deflating a good signal. Therefore, selling the rating is always attractive enough in equilibrium that there is no incentive to deflate.

We now proceed with the formal proof of the Lemma. First, we describe the expected payoffs for type $(\mathcal{S}, G)$ and characterize its incentives to deflate ratings, i.e., the marginal benefit of rating deflation. We then show that the marginal benefit of rating deflation is always negative, both in equilibria with no rating inflation (Step One of the proof) and in equilibria with positive rating inflation (Step Two).

The expected payoffs for type $(\mathcal{S}, G)$ are:

$$\Pi_G = (1-\delta)\left\{\phi\left(p^H - I\right) + \gamma E_{x,\mathbf{y}}\left[q^{(H,x,\mathbf{y})} \mid G\right]\right\} + \delta\gamma q^{\varnothing}.$$

The rating is deflated with probability δ, in which case $r = \varnothing$ and the rating fee is not collected; continuation payoff is $\gamma q^{\varnothing}$ in this case. If the rating is not deflated (probability $1-\delta$), the rating fee is collected and continuation payoff is $\gamma E_{x,\mathbf{y}}\left[q^{(H,x,\mathbf{y})} \mid G\right]$.

Type $(\mathcal{S}, g)$ chooses δ as the best response to the conjectures $(\varepsilon^e, \delta^e, \iota^e)$. Taking the derivative of Π_g with respect to δ yields:

$$\frac{d\Pi_G}{d\delta} = -\phi\left(p^H - I\right) - \gamma E_{x,\mathbf{y}}\left[q^{(H,x,\mathbf{y})} \mid G\right] + \gamma q^{\varnothing}. \tag{1.23}$$

Let us now describe the distribution of x conditional on the project being good. With probability ι^e, the speculator learns the asset quality and, thus, sells CDS protection; with probability $\frac{1}{2}$, her trade is revealed to the market ($x = -2n$). In this case, the investors learn that the asset is good and posterior reputation is $q^{H,G} = \Pr(C \mid r = H, \theta = G)$. When the speculator does not learn θ - and so does not trade - or when her trade is not revealed to the market, i.e., when $x \in \{-n, 0, +n\}$, the realization of x is not informative about project quality. This happens with probability $1 - \frac{\iota^e}{2}$; in this case, the CRA's posterior reputation depends only on the realization of $\mathbf{y}$, i.e., $q^{H,\mathbf{y}}$.

We can rewrite the expression in equation (1.23) as follows:

$$\frac{d\Pi_G}{d\delta} = -\phi\left(p^H - I\right) - \gamma\frac{\iota^e}{2}q^{(H,G)} - \gamma\left(1 - \frac{\iota^e}{2}\right)E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid G\right] + \gamma q^{\varnothing}. \tag{1.24}$$

The derivative $\frac{d\Pi_G}{d\delta}$ represents the marginal benefit of rating deflation. We want to show that this is always negative in equilibrium, so that deflating is strictly dominated (by truthfully reporting the signal) and $\widehat{\delta} = 0$. We prove this in two steps. First, we show that deflating is strictly dominated whenever the CRA is not expected to inflate its rating, i.e., $\frac{d\Pi_G}{d\delta} < 0$ for every conjecture $(\varepsilon^e = 0, \delta^e \geq 0, \iota^e \geq 0)$. Then, we show that deflating is also dominated in equilibria

with positive rating inflation, i.e., $\frac{d\Pi_G}{d\delta} < 0$ for every conjecture $(\varepsilon^e > 0, \delta^e \geq 0, \iota^e \geq 0)$, where $\varepsilon^e = \widehat{\varepsilon}\left(\delta^e, \iota^e\right)$.

Step One.

When $\varepsilon^e = 0$ and $\delta^e > 0$, reputation is updated downward following an unrated asset: since the strategic type is expected to deflate but not inflate ratings, a low rating is relatively more likely to come from a strategic type, i.e., $q^{\varnothing} < q_0$ - see equation (1.22). Reputation is instead updated upward following a high rating, i.e., $q^{(H,G)} > q_0$ and $q^{(H,\mathbf{y})} > q_0$ for any $\mathbf{y}$. Therefore, we have:

$$\gamma \underbrace{q^{\varnothing}}_{<q_0} < \phi\left(p^H - I\right) + \gamma\frac{\iota^e}{2}\underbrace{q^{(H,G)}}_{>q_0} + \gamma\left(1 - \frac{\iota^e}{2}\right)\underbrace{E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid G\right]}_{>q_0} \quad \Leftrightarrow \frac{d\Pi_G}{d\delta} < 0.$$

When the strategic CRA is expected to always offer honest ratings (i.e., $\varepsilon^e = \delta^e = 0$), the rating itself is not informative about the CRA's type, as both the strategic and the committed type are expected to truthfully report the project quality. Therefore, we have $q^{(H,G)} = q^{(H,\mathbf{y})} = q^{\varnothing} = q_0$ for any $\mathbf{y}$, which implies:

$$\gamma \underbrace{q^{\varnothing}}_{=q_0} < \phi\left(p^H - I\right) + \gamma\frac{\iota^e}{2}\underbrace{q^{(H,G)}}_{=q_0} + \gamma\left(1 - \frac{\iota^e}{2}\right)\underbrace{E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid G\right]}_{=q_0} \quad \Leftrightarrow \frac{d\Pi_G}{d\delta} < 0.$$

Step Two.

From the analysis in Section 1.4.1, we know that in any equilibrium with positive rating inflation we have:

$$\frac{d\Pi_B}{d\varepsilon} = \phi\left(p^H - I\right) + \gamma\left(1 - \frac{\iota^e}{2}\right)E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid B\right] - \gamma q^{\varnothing} \geq 0. \tag{1.25}$$

In order to have positive rating deflation in equilibrium, we would need to have $\frac{d\Pi_G}{d\delta} \geq 0$. By equation (1.24), this implies:

$$\phi\left(p^H - I\right) + \gamma\frac{\iota^e}{2}q^{(H,G)} + \gamma\left(1 - \frac{\iota^e}{2}\right)E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid G\right] - \gamma q^{\varnothing} \leq 0. \tag{1.26}$$

We can write:

$$E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid G\right] = f_G\Lambda + (1-f_G)\,\Phi,$$
$$\text{where}\;\; \Lambda \equiv f_G q^{(H,FF)} + (1-f_G)\,q^{(H,FS)};\;\; \Phi \equiv f_G q^{(H,SF)} + (1-f_G)\,q^{(H,SS)}.$$

$$E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid B\right] = f'_B\Lambda' + (1-f_B)\,\Phi',$$
$$\text{where}\;\; \Lambda' \equiv f_B q^{(H,FF)} + (1-f_B)\,q^{(H,FS)};\;\; \Phi' \equiv f_B q^{(H,SF)} + (1-f_B)\,q^{(H,SS)}.$$

Notice that we have $q^{(H,SS)} > q^{(H,SF)} = q^{(H,FS)} > q^{(H,FF)}$ and $f_B > f_G$, [45] which implies the following: $\Lambda > \Lambda'$, $\Phi > \Phi'$, and $\Phi > \Lambda'$. This means that:

$$\begin{aligned} & f_G\Lambda + (1-f_G)\,\Phi > f'_B\Lambda' + (1-f_B)\,\Phi' \\ & \Rightarrow E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid G\right] > E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid B\right]. \end{aligned} \tag{1.27}$$

[46] The above inequality, together with the one in equation (1.25), imply that the following always holds in equilibrium:

$$\begin{aligned} & \phi\left(p^H - I\right) + \gamma\frac{\iota^e}{2}q^{(H,G)} + \gamma\left(1-\frac{\iota^e}{2}\right)E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid G\right] - \gamma q^{\varnothing} > \\ & > \phi\left(p^H - I\right) + \gamma\left(1-\frac{\iota^e}{2}\right)E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid B\right] - \gamma q^{\varnothing} \geq 0. \end{aligned}$$

Therefore, the inequality in equation (1.26) is never satisfied and, thus, deflating the rating is always a dominated action.

[45] This is because we have $\frac{\Pr(SS|B)}{\Pr(SS|G)} < \frac{\Pr(SF|B)}{\Pr(SF|G)} = \frac{\Pr(FS|B)}{\Pr(FS|G)} < \frac{\Pr(FF|B)}{\Pr(FF|G)}$, and $q^{(H,\mathbf{y})}$ is decreasing in the ratio $\frac{\Pr(\mathbf{y}|B)}{\Pr(\mathbf{y}|G)}$ - see equations (1.20) and (1.21).

[46] Notice that the inequality in equation (1.27) implies that the reputational payoff following a high rating is larger for type $(\mathcal{S}, G)$ than for type $(\mathcal{S}, B)$ and, thus, that the reputational loss from offering $r = H$ is larger for type $(\mathcal{S}, B)$.

1.10.6 Proof of Proposition 3

Given that the strategic CRA never deflates a good signal, ratings are honest whenever the marginal benefit of inflation is negative at the conjecture $(\varepsilon^e = 0, \delta^e = 0, \iota^e)$, so that inflating the rating is a strictly dominated action and, thus, the conjecture $\varepsilon^e = 0$ is consistent with the play of the game. We have:

$$\frac{d\Pi_B}{d\varepsilon}\left(\varepsilon^e, \delta^e = 0, \iota^e\right) = \phi\left(p^H - I\right) + \gamma\left(1 - \frac{\iota^e}{2}\right) E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid B\right] - \gamma q^{\varnothing}$$

where

$$q^{\varnothing} = \frac{q_0}{q_0 + (1-q_0)(1-\varepsilon^e)}; \quad q^{H,\mathbf{y}} = \frac{q_0}{q_0 + (1-q_0)\left(1 + \frac{\Pr(\mathbf{y}|B)}{\Pr(\mathbf{y}|G)}\varepsilon^e\right)}.$$

When ratings are honest, i.e., $\varepsilon^e = \delta^e = 0$, we have two main implications: (i) $q^{\varnothing} = q^{H,\mathbf{y}} = q_0$ for any $\mathbf{y}$; [47] (ii) a high rating is a perfect signal for a good project, which implies $p^H = (1 - f_G)$. Therefore, we can write:

$$\begin{aligned}\frac{d\Pi_B}{d\varepsilon}\left(\varepsilon^e = \delta^e = 0\right) &= \phi\left(1 - f_G - I\right) + \gamma\left(1 - \frac{\iota^e}{2}\right) q_0 - \gamma q_0 \\ &= \phi V_G - \gamma\frac{\iota^e}{2} q_0.\end{aligned}$$

This implies:

$$\begin{aligned}\frac{d\Pi_B}{d\varepsilon}\left(\varepsilon^e = \delta^e = 0\right) \leq 0 \quad &\Leftrightarrow \quad \phi V_G - \gamma\frac{\iota^e}{2} q_0 \leq 0 \\ &\Leftrightarrow \quad \gamma \geq \frac{2\phi V_G}{\iota^e q_0} = \overline{\gamma}.\end{aligned}$$

Notice that, when $\gamma < \overline{\gamma}$, we have $MBI\left(\varepsilon^e = \delta^e = 0, \iota^e\right) > 0$. By Lemma 5, this implies that rating inflation is positive in equilibrium when $\gamma < \overline{\gamma}$.

[47] When the strategic CRA is expected to always offer honest ratings, the rating itself is not informative about the CRA's type, as both the strategic and the committed type are expected to truthfully report the project quality.

1.10.7 Proof of Lemma 7

Here, we show that the investors always buy when $r = H$ - i.e., p^H is strictly positive in equilibrium for every value of the initial reputation q_0.

In the equilibrium with honest ratings ($\gamma \geq \overline{\gamma}$), a high rating is a perfect signal for a good project and, thus, the market price of the claim is equal to the expected payoff of a good project, i.e., $p^H = 1 - f_G > 0$.

From the analysis in Section 1.4.1, we know that in any equilibrium with positive rating inflation we must have:

$$\phi\left(p^H - I\right) \geq \gamma q^{\varnothing} - \gamma\left(1 - \frac{\iota^e}{2}\right) E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid B\right]. \tag{1.28}$$

Notice that, for any given conjectures ($\varepsilon^e > 0, \delta^e = 0$), reputation is updated upward following $r = \varnothing$ and downward following $r = H$. We have:

$$\begin{aligned} q^{\varnothing} &= \underbrace{\frac{q_0}{q_0 + (1 - q_0)\left(1 - \varepsilon^e\right)}}_{<1} > q_0; \\ q^{H,\mathbf{y}} &= \underbrace{\frac{q_0}{q_0 + (1 - q_0)\left(1 + \dfrac{\Pr(\mathbf{y} \mid B)}{\Pr(\mathbf{y} \mid G)}\varepsilon^e\right)}}_{>1} < q_0. \end{aligned} \tag{1.29}$$

This implies that the right-hand side of the inequality in (1.28) must be strictly positive. Therefore, $\phi\left(p^H - I\right) > 0$ which implies $p^H > I$.

1.11 Appendix B

1.11.1 Proof of Proposition 9

The first order condition for the choice of precision by the speculator ($\frac{\partial \Pi^s}{\partial \iota} = 0$; equation 1.18) gives us the speculator's reaction function, i.e., $\widehat{\iota}\left(\varepsilon^e\right)$. We have:

$$\frac{\partial \Pi^s}{\partial \iota} = n\mu^H\left(1 - \mu^H\right)\left(f_B - f_G\right) - c'(\iota). \tag{1.30}$$

When $\varepsilon^e = 0$, a high rating is interpreted as a perfect signal for a good asset (i.e., $\mu^H = 1$) and, thus, $\frac{\partial \Pi^s}{\partial \iota}$ is negative for any $\iota > 0$. Therefore, the speculator chooses not to acquire information about the project when $\varepsilon^e = 0$, i.e., $\widehat{\iota}(0) = 0$.

Conversely, when $\iota^e = 0$, the marginal benefit of inflation (equation 1.12) is always positive at $\varepsilon^e = 0$, as the CRA can collect the fee by inflating the rating for a bad asset without incurring reputational losses. Therefore, the strategic CRA always inflates with positive probability when $\iota^e = 0$, i.e., $\widehat{\varepsilon}(0) > 0$. We have:

$$MBI\left(\iota^e, \varepsilon^e\right) = \phi\left(p^H - I\right) + \gamma\left(1 - \frac{\iota^e}{2}\right) E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid B\right] - \gamma q^{\varnothing}$$

$$\Leftrightarrow MBI\left(\iota^e = 0, \varepsilon^e = 0\right) = \phi\left(p^H - I\right) + \gamma E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid B\right] - \gamma q^{\varnothing} = \phi V_G > 0,$$

since, at $\varepsilon^e = 0$, $q^{(H,\mathbf{y})} = q^{\varnothing} = q_0$ for any $\mathbf{y}$ and $p^H = 1 - f_G$.

Given the two remarks above, the proof for Proposition 9 is straightforward. We have $\widehat{\iota}(\varepsilon^e) \in [0,1]$, with $\widehat{\iota}(0) = 0$, and $\widehat{\varepsilon}(\iota^e) \in [0,1]$, with $\widehat{\varepsilon}(0) > 0$; this implies that the two functions in Figure 1.3 either do cross (and do so only once) at some interior level $(\widehat{\varepsilon}, \widehat{\iota}) \in (0,1)^2$, or they do not cross, and we have a corner solution - i.e., $(\widehat{\varepsilon} = 1, \widehat{\iota} = \widehat{\iota}(1))$ or $(\widehat{\varepsilon} = \widehat{\varepsilon}(1), \widehat{\iota} = 1)$. This means that rating inflation and market informativeness are both always strictly positive in equilibrium.

Note that equilibria with honest ratings are not possible anymore when we consider the strategic interaction between the two markets. When ratings are honest there is no mispricing of risk and, thus, the speculator chooses not to acquire information about the project (i.e., $\widehat{\iota}(0) = 0$). However, inflating the rating is always a strictly dominant action when $\iota^e = 0$ and $\varepsilon^e = 0$ - i.e., $MBI\left(\iota^e = 0, \varepsilon^e = 0\right) > 0$. Therefore, the conjecture $\varepsilon^e = 0$ can never be consistent with the equilibrium play of the game and we always have positive rating inflation in equilibrium.

1.11.2 Proof of Lemma 10

The equilibrium pair $(\widehat{\varepsilon}, \widehat{\iota})$ is defined by the system of equations given by: (i) the first order condition for the choice of precision by the speculator ($\frac{\partial \Pi^s}{\partial \iota} = 0$; equation 1.18); (ii) the equilibrium

level of rating inflation ($MBI = 0$; equation 1.12).[48]

$$n\mu^H \left(1 - \mu^H\right) \left(f_B - f_G\right) - c'\left(\widehat{\iota}\right) = 0; \tag{1.31}$$

$$\phi\left(p^H - I\right) + \gamma\left(1 - \frac{\widehat{\iota}}{2}\right) E_{\mathbf{y}}\left[q^{(H,\mathbf{y})} \mid B\right] - \gamma q^{\varnothing} = 0, \tag{1.32}$$

where $\left(\mu^H, p^H, q^{(H,\mathbf{y})}, q^{\varnothing}\right)$ are functions of $\widehat{\varepsilon}$.

When we vary a parameter in one of the two equations, say γ in equation (1.32), we have a direct effect on rating inflation $\frac{d\widehat{\varepsilon}}{d\gamma}$ and an indirect effect on precision $\frac{d\widehat{\varepsilon}}{d\gamma}\frac{\partial\widehat{\iota}}{\partial\widehat{\varepsilon}}$.

Let us evaluate the sign of these two effects. From Lemma 2, we know that $\frac{\partial\widehat{\iota}}{\partial\widehat{\varepsilon}}$ is positive. The left-hand side in equation (1.32) is the marginal benefit of inflation MBI. We can apply the implicit function theorem to find $\frac{d\widehat{\varepsilon}}{d\gamma}$; by totally differentiating equation (1.32), we obtain:

$$\frac{\partial MBI}{\partial\gamma}d\gamma + \frac{\partial MBI}{\partial\widehat{\varepsilon}}d\widehat{\varepsilon} + \frac{\partial MBI}{\partial\widehat{\iota}}\frac{\partial\widehat{\iota}}{\partial\widehat{\varepsilon}}d\widehat{\varepsilon} = 0$$

$$\Leftrightarrow \frac{d\widehat{\varepsilon}}{d\gamma} = -\frac{\overset{<0}{\overbrace{\frac{\partial MBI}{\partial\gamma}}}}{\underbrace{\frac{\partial MBI}{\partial\widehat{\varepsilon}}}_{<0} + \underbrace{\frac{\partial MBI}{\partial\widehat{\iota}}}_{<0}\frac{\partial\widehat{\iota}}{\partial\widehat{\varepsilon}}} < 0.$$

Therefore, both the direct effect on rating inflation and the indirect effect on precision are negative.

The results on the comparative statics for n are similar. When we vary n in equation (1.31), we have a direct effect on precision $\frac{d\widehat{\iota}}{dn}$ and an indirect effect on rating inflation $\frac{d\widehat{\iota}}{dn}\frac{\partial\widehat{\varepsilon}}{\partial\widehat{\iota}}$. From Lemma 8, we know that $\frac{\partial\widehat{\varepsilon}}{\partial\widehat{\iota}}$ is negative. Let $h\left(\widehat{\varepsilon}, n\right)$ denote the first term in equation (1.31); by

[48] As we focus on interior solutions for rating inflation, we have MBI strictly equal to zero in equilibrium.

totally differentiating equation (1.31), we obtain:

$$\frac{\partial h}{\partial n}dn + \underset{>0}{\frac{\partial h}{\partial \widehat{\varepsilon}}}\frac{\partial \widehat{\varepsilon}}{\partial \widehat{\iota}}d\widehat{\iota} - c''(\iota)\,d\widehat{\iota} = 0$$

$$\Leftrightarrow \frac{d\widehat{\iota}}{dn} = \frac{\overset{>0}{\frac{\partial h}{\partial n}}}{c''(\iota) - \underbrace{\frac{\partial h}{\partial \widehat{\varepsilon}}\frac{\partial \widehat{\varepsilon}}{\partial \widehat{\iota}}}_{<0}} > 0.$$

In this case, the direct effect on precision is positive and, thus, the indirect effect on rating inflation is negative.

1.11.3 Proof of Proposition 11

Taking the derivative of S with respect to γ yields:

$$\frac{dS}{d\gamma} = \frac{d\widehat{\varepsilon}}{d\gamma}\left\{(1-q_0)\left[1+\chi\left(1-\frac{\widehat{\iota}}{2}\right)\right]\frac{V_B}{2} - \chi(1-q_0)\widehat{\varepsilon}\frac{\partial \widehat{\iota}}{\partial \widehat{\varepsilon}}\frac{V_B}{4} - k\widehat{\iota}\frac{\partial \widehat{\iota}}{\partial \widehat{\varepsilon}}\right\}.$$

Given that $\frac{d\widehat{\varepsilon}}{d\gamma} < 0$ from Lemma 10, $\frac{dS}{d\gamma}$ is negative when the following condition is satisfied:

$$\frac{\partial \widehat{\iota}}{\partial \widehat{\varepsilon}}\left(\chi(1-q_0)\widehat{\varepsilon}\frac{|V_B|}{4} - k\widehat{\iota}\right) \geq (1-q_0)\left[1+\chi\left(1-\frac{\widehat{\iota}}{2}\right)\right]\frac{|V_B|}{2}. \tag{1.33}$$

The inequality in equation (1.33) is never satisfied when k is large: as k goes to infinity, the term in brackets on the left-hand side becomes negative. In what follows, we show that this inequality is instead always satisfied when k is small and χ is large.

The equilibrium level of precision is given by:

$$\widehat{\iota} = \frac{n}{k}\mu^H\left(1-\mu^H\right)(f_B - f_G).$$

Taking the derivative of $\widehat{\iota}$ with respect to $\widehat{\varepsilon}$ yields:

$$\frac{\partial \widehat{\iota}}{\partial \widehat{\varepsilon}} = \frac{n}{k}(f_B - f_G)\underset{<0}{\left(1-2\mu^H\right)}\underset{<0}{\frac{\partial \mu^H}{\partial \widehat{\varepsilon}}} > 0.$$

Notice that $\frac{\partial \widehat{\iota}}{\partial \widehat{\varepsilon}}$ goes to infinity as k goes to zero. This is because, when precision is cheap (k small), the speculator adjusts her choice of ι quite sharply to changes in the level of expected mispricing - i.e., rating inflation. Conversely, when the cost of precision is large (k large), $\frac{\partial \widehat{\iota}}{\partial \widehat{\varepsilon}}$ is small.

As χ becomes large, the inequality in equation (1.33) becomes:

$$\begin{aligned} \frac{\partial \widehat{\iota}}{\partial \widehat{\varepsilon}}(1-q_0)\widehat{\varepsilon}\frac{|V_B|}{4} &\geq (1-q_0)\left(1-\frac{\widehat{\iota}}{2}\right)\frac{|V_B|}{2} \\ \Leftrightarrow \frac{\partial \widehat{\iota}}{\partial \widehat{\varepsilon}}\frac{\widehat{\varepsilon}}{2} &\geq 1-\frac{\widehat{\iota}}{2}. \end{aligned}$$

We can rearrange the inequality above as follows:

$$\frac{\partial \widehat{\iota}}{\partial \widehat{\varepsilon}}\frac{\widehat{\varepsilon}}{\widehat{\iota}} \geq \frac{2-\widehat{\iota}}{\widehat{\iota}}. \tag{1.34}$$

The term on the left-hand side of the inequality in equation (1.34) is the elasticity of the equilibrium level of precision with respect to changes in rating inflation, i.e., the percentage change in $\widehat{\iota} = \widehat{\iota}(\widehat{\varepsilon})$ in response to a one percent change in $\widehat{\varepsilon}$. This means that, for $\widehat{\varepsilon}'$ close to $\widehat{\varepsilon}$ and $\widehat{\varepsilon}' > \widehat{\varepsilon}$, we can rewrite the inequality in equation (1.34) as follows:

$$\begin{aligned} \frac{\frac{\widehat{\iota}(\widehat{\varepsilon}')-\widehat{\iota}(\widehat{\varepsilon})}{\widehat{\iota}(\widehat{\varepsilon})}}{\frac{\widehat{\varepsilon}'-\widehat{\varepsilon}}{\widehat{\varepsilon}}} &\geq \frac{2-\widehat{\iota}(\widehat{\varepsilon})}{\widehat{\iota}(\widehat{\varepsilon})} \\ \Leftrightarrow \frac{\widehat{\iota}(\widehat{\varepsilon}')-\widehat{\iota}(\widehat{\varepsilon})}{2-\widehat{\iota}(\widehat{\varepsilon})} &\geq \frac{\widehat{\varepsilon}'}{\widehat{\varepsilon}}-1. \end{aligned} \tag{1.35}$$

Notice that, for $\widehat{\varepsilon}'$ close to $\widehat{\varepsilon}$, we have $\widehat{\iota}(\widehat{\varepsilon}')-\widehat{\iota}(\widehat{\varepsilon}) \approx \frac{\partial \widehat{\iota}}{\partial \widehat{\varepsilon}}(\widehat{\varepsilon}'-\widehat{\varepsilon})$. As we discussed earlier, $\frac{\partial \widehat{\iota}}{\partial \widehat{\varepsilon}}$ goes to infinity as k goes to zero: this implies that we can always find k close enough to zero such that the inequality in equation (1.35) is satisfied.

Therefore, the inequality in equation (1.33) is always satisfied for k small and χ large enough.

The results on the comparative statics for n are similar. Taking the derivative of S with

respect to n yields:

$$\frac{dS}{dn} = \underbrace{\frac{d\widehat{\iota}}{dn}}_{>0} \left\{ (1-q_0)\frac{\partial\widehat{\varepsilon}}{\partial\widehat{\iota}}\left[1+\chi\left(1-\frac{\widehat{\iota}}{2}\right)\right]\frac{V_B}{2} - \chi(1-q_0)\widehat{\varepsilon}\frac{V_B}{4} - k\widehat{\iota}\right\}.$$

Hence, $\frac{dS}{dn}$ is positive when the following condition is satisfied:

$$(1-q_0)\underbrace{\frac{\partial\widehat{\varepsilon}}{\partial\widehat{\iota}}\frac{V_B}{2}}_{>0}\left[1+\chi\left(1-\frac{\widehat{\iota}}{2}\right)\right] \geq k\widehat{\iota} - \chi(1-q_0)\widehat{\varepsilon}\frac{|V_B|}{4}. \tag{1.36}$$

The right-hand side in the above inequality is always negative as k goes to zero. Therefore, the inequality is always satisfied for k that goes to zero. Conversely, as k goes to infinity, the right-hand side of the inequality goes to infinite as well and, thus, the inequality is never satisfied when k is large.

1.11.4 Proof of Lemma 12

Let $1_{\mathcal{R}}$ denote an indicator variable for the regulatory regime, where $1_{\mathcal{R}} = 1$ if the regulatory regime is the rating-based regime and $1_{\mathcal{R}} = 0$ if the regime is the CDS-based one. The price investors are willing to pay for the claim on the payoff of a high-rated project is:

$$\widetilde{p}^H = \mu^H(1-f_G) + \left(1-\mu^H\right)(1-f_B) + \left[1_{\mathcal{R}} + (1-1_{\mathcal{R}})\Pr\left(p^{cds} < f_B\right)\right]\mathcal{M},$$

where $\Pr\left(p^{cds} < f_B\right) = \Pr\left(x \neq +2n\right) = \frac{1}{2} + \frac{1}{2}\left(1-\frac{\widehat{\iota}}{2}\right) < 1$.

The first term in the expression for $\widetilde{p}^H$ is the expected payoff of a high-rated project; the second term reflects the premium $\mathcal{M}$ weighted by the probability that the security receives the favorable treatment. In the rating-based regime ($1_{\mathcal{R}} = 1$), a security with a high rating is always granted a favorable treatment. In the CDS-based regime ($1_{\mathcal{R}} = 0$), the security receives a favorable treatment only if p^{cds} is below the threshold f_B, i.e., when $x \neq +2n$. This happens with probability 1 when the quality of the project is good, and with probability $1 - \frac{\widehat{\iota}}{2}$ when it is bad.

The marginal benefit of rating inflation in equation (1.11) becomes then:

$$MBI = \phi\left(\widetilde{p}^{H} - I\right) + \gamma E_{x,\mathbf{y}}\left[q^{(H,x,\mathbf{y})} \mid B\right] - \gamma q^{\varnothing}.$$

The characterization of the equilibrium and the strategic interaction between the CRA and the CDS market do not change in this setting. Therefore, the equilibrium of the game is still as described in Proposition 9. [49] Notice that MBI increases with $\mathcal{M}$ and it is larger in the rating-based regime - i.e., when $1_{\mathcal{R}} = 1$. When MBI increases (decreases), $\widehat{\varepsilon}\left(\iota^{e}\right)$ in Figure 1.3 shifts upward (downward). Therefore, when $\mathcal{M}$ increases, the new equilibrium features both higher $\widehat{\varepsilon}$ and $\widehat{\iota}$. In contrast, when we move from a rating-based to a CDS-based regime, both $\widehat{\varepsilon}$ and $\widehat{\iota}$ diminish.

1.11.5 Effect of ι^{e} on expected CDS price

Given the assertion in Subsection 1.7.1 that initial investors don't participate in the CDS market, we demonstrate here that (i) the expected price of CDS protection does not depend on ι^{e}, but (ii) its variance increases as ι^{e} increases. Therefore, even if the investors who purchased the initial investment were risk averse and chose to buy CDS protection, their willingness to pay for the initial investment would be non-increasing in ι^{e}.

We can write the expected CDS price as:

$$E_{x}\left\{\widehat{p}^{cds}\left(x\right)\right\} = \mu^{H}\frac{\iota^{e}}{2}f_{G} + \left(1-\mu^{H}\right)\frac{\iota^{e}}{2}f_{B} + \left(1-\frac{\iota^{e}}{2}\right)E\left(f_{\theta} \mid r = H\right), \tag{1.37}$$

where $E\left(f_{\theta} \mid r = H\right) = \mu^{H}f_{G} + \left(1-\mu^{H}\right)f_{B}$ is the CDS price when x is not informative about project quality, i.e., when $x \in \{-n, 0, +n\}$.

We can rearrange equation (1.37) as follows:

$$\begin{aligned} E_{x}\left\{\widehat{p}^{cds}\left(x\right)\right\} &= \frac{\iota^{e}}{2}\left[\mu^{H}f_{G} + \left(1-\mu^{H}\right)f_{B}\right] + \left(1-\frac{\iota^{e}}{2}\right)\left[\mu^{H}f_{G} + \left(1-\mu^{H}\right)f_{B}\right] \\ &= \mu^{H}f_{G} + \left(1-\mu^{H}\right)f_{B}. \end{aligned}$$

[49] Notice that $\widetilde{p}^{H}$ is decreasing in $\widehat{\iota}$. This is needed for Lemma 8, which proves that $\frac{dMBI}{d\iota^{e}} < 0$.

Notice that $E_x\left\{\widehat{p}^{cds}(x)\right\}$ does not depend on ι^e.[50]

Given that $E_x\left\{\widehat{p}^{cds}(x)\right\}$ does not depend on ι^e, showing that $VAR_x\left\{\widehat{p}^{cds}(x)\right\}$ is increasing in ι^e is equivalent to showing that $E_x\left\{\left(\widehat{p}^{cds}(x)\right)^2\right\}$ increases with ι^e. Taking the derivative of $E_x\left\{\left(\widehat{p}^{cds}(x)\right)^2\right\}$ with respect to ι^e yields:

$$\frac{\partial E_x\left\{\left(\widehat{p}^{cds}(x)\right)^2\right\}}{\partial \iota^e} = \frac{1}{2}\left[\mu^H f_G^2 + \left(1-\mu^H\right) f_B^2 - \left(\mu^H f_G + \left(1-\mu^H\right) f_B\right)^2\right].$$

We can rearrange the term in brackets in the equation above as follows:

$$\mu^H\left(1-\mu^H\right) f_G^2 + \mu^H\left(1-\mu^H\right) f_B^2 - 2\mu^H\left(1-\mu^H\right) f_B f_G =$$
$$\mu^H\left(1-\mu^H\right)\left(f_G - f_B\right)^2 > 0.$$

Therefore, $\frac{\partial E_x\left\{\left(\widehat{p}^{cds}(x)\right)^2\right\}}{\partial \iota^e}$ is positive and, thus, $VAR_x\left\{\widehat{p}^{cds}(x)\right\}$ is increasing in ι^e.

[50] Point (i) is implied by the *Law of Iterated Expectations*. By the equilibrium of the CDS market, we have $\widehat{p}^{cds}(x) = E_\theta\left(f_\theta \mid H, x\right)$; therefore:

$$E_x\left\{\widehat{p}^{cds}(x)\right\} = E_x\left\{E_\theta\left(f_\theta \mid H, x\right)\right\} = E_\theta\left(f_\theta \mid H\right),$$

where the last equality comes from the *Law of Iterated Expectations* applied on the expectation over the realization of x. This means that the expected CDS price does not depend on the realization of x, and so on its distribution - i.e., on ι^e.

Chapter 2

Executive Compensation and Short-Termism

[1] The stock market is widely believed to pressure executives to deliver short-term earnings at the expense of long-term value. This chapter develops a model of the interaction between executive compensation and stock market prices, and analyzes its implications for corporate short-termism. I show that inefficient short-termism can arise in equilibrium as a self-fulfilling prophecy, due to strategic complementarities between the firm's investment horizon and investors' decision to acquire information about short-term performance or long-term value. However, the severity of the underlying agency problem between the manager and shareholders fully determines whether short-termism is an equilibrium outcome. This implies both that the stock-market cannot be identified as the cause of corporate short-termism and that it actually has the potential to alleviate the problem. The model helps us assess evidence presented in the "myopia" debate and yields novel implications regarding ownership structure, executive compensation, and managerial horizon.

[1] I would like to thank Charles Angelucci, Alessandro Bonatti, Marco Di Maggio, Denis Gromb, Tim Jenkinson, Jakub Kastl, Meg Meyer, Stephen Morris, Alan Morrison, Thomas Noe, Marco Pagano, Nicola Persico, Joel Shapiro and seminar participants at HEC Paris, Oxford, Naples, SMU Cox, Toronto (Rotman and Economics), IESE, Insead, Vienna, Cass, Queen Mary, Stockholm, UCL, Indiana (Kelley), and Pompeu Fabra for helpful comments.

2.1 Introduction

Managerial short-termism is a hotly debated issue in corporate, policy-making, and academic circles. Within the debate, two general and opposing views have taken shape. The most widely-held view, argued since the late 1970s (Lipton 1979), is that short-termism is a significant obstacle for firms in sustaining long-term value and the stock market is the primary culprit. The stock market pressures executives to deliver short-term earnings at the expense of long-term value; this encourages executives to hold back long-term investments and harms the firm and the economy. In support of this view, empirical evidence is often cited confirming the existence of short-termism (Graham et al. 2005, Budish et al. 2015, Edmans et al. 2017a,b) and its detrimental effects.[2] Recently, however, some have questioned the widespread concern about corporate short-termism and have cast doubt on its popular diagnosis. Instead, they claim that firms choose their investment horizon optimally; the stock market simply reflects these choices and does not drive inefficient short-termism. Some of these dissenters point to a lack of long-term evidence that is consistent with the predictions of the short-term critics (Kaplan 2017). Others have gone so far to say that corporate short-termism is an imaginary problem (Roe 2018).

The high stakes in this debate have naturally led to a substantial academic literature. Yet, previous work on short-termism either takes as exogenous the dependence of the manager's contract on the stock price (e.g., Stein 1989, Bebchuk and Stole 1993, Edmans 2009) or ignores the stock market and focuses on agency conflicts within the firm that make short-termism a second best (e.g., Narayanan 1985, Von Thadden 1995, Thakor 2018).[3] This is surprising, given that short-termism is about *both* markets and compensation: stock prices can pressure managers to deliver short-term earnings at the expense of long-term value, but whether managers care about this pressure depends on the structure of their compensation. The objective of this chapter is to explore the causes and consequences of corporate short-termism within a formal model in which both the optimal design of executive compensation and the stock market price

[2] Graham finds that 78% of surveyed executives would destroy economic value to boost earnings. This year, prominent business leaders have spoken out about the excessive focus on short-term performance, warning about its potential effects on the overall economy (Dimon and Bufett, 2018).

[3] I survey the literature on executive compensation in the next subsection.

are endogenously determined.

In the model, shareholders provide a manager with incentives to take a risky project. They can chooses either a short-term project that boosts current earnings or a long-term project that pays out in the future. The long-term project has higher returns but is costlier to incentivize, because the manager is risk-averse and the realization of the firm's long-term value is more volatile. The stock price can be used in the contract, but its informativeness is endogenous (as in Kyle 1985): a speculator acquires information to profit off of liquidity traders, and a market-maker clears the market. An important and novel feature of the model is that the speculator chooses whether to acquire information about the firm's short-term performance and/or the value of its long-term projects.[4]

I show that the strategic interaction between executive compensation and the informativeness of the stock price is characterized by a two-way feedback. One way goes from the firm to the stock market: when the firm is expected to invest in the long-term project, the speculator acquires information about it and this information is partly incorporated into the price through her trading. However, if the firm is expected to undertake the short-term project, the speculator only acquires information about the firm's short-term performance: there is no gain in learning about the long-term project, since the firm is not expected to invest in it. The second direction of the feedback runs from the stock market to the firm: if the speculator acquires information about the firm's long-term project, the stock price can be used to incentivize the manager to undertake a long-term project, enabling the shareholders to design a more efficient contract.[5] In turn, implementing a long-term project becomes more attractive for the firm.

This two-way feedback generates a strategic complementarity in the choice of horizons between the shareholders and the speculator. This strategic complementarity can lead to multiple equilibria, where *inefficient* short-termism can arise in equilibrium as a self-fulfilling prophecy, due to coordination failure between the speculator and the firm. When both *long-termism* - i.e.,

[4]This is different from other papers on market monitoring, like Holmstrom and Tirole (1993) and Edmans (2009), where the informed-trader can only acquire information about the firm's long-term value.

[5]It is worth emphasizing that an improvement in contracting is achieved even though I let the manager's contract be contingent on the firm's present and future return streams. The reason is that the speculator has information about the executive's choice that is not in the return realization. Therefore, the price contains unique information about managerial performance.

the firm investing in the long-term project and the speculator acquiring information about it - and *short-termism* - i.e., the firm investing in the short-term project and the speculator only acquiring information about short-term performance - are equilibria of the game, firm value is strictly larger under long-termism. The speculator, however, might be better off under short-termism, when the cost of acquiring information about the long-term project is higher or when she is looking for a quick profit from trades. In this case, the shareholders and the speculator's preferences over equilibria are not aligned, and coordination failure is a serious issue.

The modeled interaction and the resulting strategic complementarity uncovers a new mechanism by which the stock market can feed corporate short-termism through an excessive focus on short-term performance. This is the first main result of this chapter. Whereas previous work on short-termism takes as exogenous the dependence of the manager's contract on the stock price, here shareholders are free to choose the structure of executive compensation. Yet, despite this freedom, inefficient short-termism can still arise in equilibrium as a self-fulfilling prophecy. But does this support the claim that the stock market is the primary culprit of corporate short-termism?

To address this question, I analyze a benchmark where the stock price cannot be part of the manager's contract; this is the case, for example, when the firm shares are not publicly traded. Comparing the equilibrium outcomes in the benchmark with those in the model with stock prices yields two important implications. First, firms that were long-termist in the benchmark model continue to be long-termist when the stock price can be included in the contract. Therefore, the stock market does not increase the mass of short-termist firms in the economy. Second, a mass of firms that were short-termist in the benchmark can sustain efficient long-termism when the stock price can be included in the contract and is informative about the firm's long-term value.

Together, these two observations suggest that the role of the stock market in relation to corporate short-termism may be fundamentally misunderstood. The real cause of corporate short-termism is the underlying agency problem between the shareholders and the manager, which makes it more costly for shareholders to incentivize long-term projects. Far from being the primary culprit of corporate short-termism, the stock market can be a (potentially) alleviating

force: when the stock price is informative about long-term value, it enables a more efficient contract design that sustains efficient long-termism. However, an excessive focus on short-term performance in the stock market fails to alleviate the existing agency problem and leads to inefficient short-termism.

The analysis discussed so far naturally raises a question: which factors make coordination failure less likely to occur? If the speculator has a preexisting stake in the firm (i.e., if the speculator is a blockholder), her preferences over equilibria are closer to the one of the shareholders. Having a preexisting stake in the firm does not affect the speculator's trading strategies and, hence, does not affect her profits from trading. However, it creates a link between the speculator's expected payoff and the firm ex-ante value, aligning shareholders' and speculator's preferences over different equilibria: if the stake is sufficiently large, the equilibrium with long-termism Pareto dominates the one with short-termism. Therefore, coordination failure less likely.[6] This result uncovers a new strategic complementarity between inside and outside (the speculator in the model) shareholders. Compared to other informed-traders in the market, outside shareholders have a stronger incentive to trade on information about the long-term prospect of a firm, as this enables inside shareholders to design more efficient managerial contracts and increase firm value. These findings contribute to a new literature that tries to explain the predominance in the U.S. of small transient blockholders, "who typically lack control rights and instead follow the 'Wall Street Rule' of 'voting with their feet' - selling their stock if dissatisfied" (Edmans 2009).

The strategic complementarity between the firm's and informed-traders' investment horizons has important empirical implications. For example, the model predicts that, in firms with high growth opportunities, (i) executive pay should be more linked to stock prices and (ii) stock prices should be more informative about long-term value. The reason is that investing in long-term projects is always a dominant strategy for these firms. The speculator anticipates this and acquires information about the firm's long-term projects. As a result, the stock price

[6] The strategic interaction in the model boils down to two players, the principal (shareholders) and the speculator. Especially if the speculator has a stake in the firm and so has a way to communicate with the controlling shareholders, it is natural for players to coordinate on the payoff-dominant equilibrium.

is informative about long-term value and can be used to incentivize the manager. This is consistent with the evidence that stock-options are more prevalent in high-tech, "new economy" firms and more generally in growth industries, such as computer, software, and pharmaceutical firms (Murphy 1999, Core and Guay 2001, Ittner et al. 2003). At the same time, Price/Earnings ratios are higher in these industries, which implies that the stock market is taking into account the potential for future profits (Kaplan, 2017).

A second set of empirical implications relates to the importance blockholders have in the equilibrium selection. While the role of blockholders in encouraging long-term investments (Cronqvist and Fahlenbrach (2009)) and deterring myopia (Dechow et al. (1996), Farber (2005), Burns et al. (2008)) is well documented, there is less evidence about the specific channel that leads to this effect - see Edmans and Holderness (2017) for a review of the literature on blockholders. Blockholders can intervene directly into a firm's operations (*voice*) or simply trade on information about the firm (*exit*); if this information is impounded into the stock-price, this also disciplines management. The second channel works through the stock price and, thus, relies on prices being used in the manager's compensation. Because both compensation and price informativeness are endogenous in my model, my results offer new insights into how to empirically distinguish the two channels. The model predicts that, if the channel is *exit*, the increase in long-term investments associated with the presence of outside blockholders will be accompanied with (i) executive pay being linked more to stock prices and (ii) prices being more informative about the firm's long-term value. More broadly, while previous work has focused on the role of inside or outside shareholders taken alone, the model suggests that the study of their interaction may motivate new interesting avenues for empirical research.

Next, I offer a summary of the related theoretical literature.

Theoretical Literature

This chapter contributes to the literature on managerial short-termism. The earlier work on the topic focused on the distortions that result from exogenous short-term concerns of the managers (e.g., Narayanan 1985, Stein 1989, Bebchuk and Stole 1993). More recent work instead

analyzes short-termism in an optimal contracting setting. Bolton et al. (2006) show that optimal compensation contracts may emphasize short-term stock performance at the expense of long-run value, when current shareholders can sell the stock in the future to potentially overoptimistic investors. Peng and Roell (2014) analyze the trade-off between short-term incentives, that expose shareholders to the risk of manipulation by the manager, and long-term, which expose the manager to the risk of future contingencies. In both papers, the investors' incentives to acquire information are not examined and the informativeness of the stock price is exogenous. My paper contributes to this literature by examining a model in which both the structure of compensation and the informativeness of the stock price are endogenous.

The interaction between executive compensation and the informativeness of the stock price is a type of feedback effect. There is a substantial literature on feedback effects of market prices, which examines how markets affect real decisions and the resulting feedback loop between the two - see Bond, Edmans, and Goldstein (2012) for a review. Within this literature, the paper closest to mine is Holmstrom and Tirole (1993), which examines the value of the stock market as a monitor of managerial performance. The speculator in their model can only acquire information about the firm's long-term value, while she chooses which type of information (about short-term and/or long-term value) to acquire in my model. Moreover, in my paper the structure of compensation affects the speculator's incentives to acquire information. Therefore, the interaction between compensation and the stock market is two-way. This feature is absent in Holmstrom and Tirole, as the manager there only chooses effort, which does not affect the ex-ante uncertainty about firm value. Edmans (2009) also connects feedback effects and short-termism, focusing on the role of blockholders as a solution to managerial myopia: by gathering information about a firm's fundamental value and impounding it into prices, blockholders prevent managers from discarding efficient long-term investments that reduce short-term profits. The dependence of the manager's contract on current stock prices and the fact that the blockholders trade on long-term information is taken as given in his model, while both are endogenous choices in mine; this allows me to examine the interaction between inside and outside shareholders and its implications for executive compensation and the firm's optimal investment horizon.

A strategic complementarity between the informativeness of the stock price and the firm's investment decisions arises also in Dow, Goldstein, and Guembel (2017). In their model, the manager learns from the stock price about the quality of a firm's investment opportunity. In my paper, the manager does not learn from the price but the shareholders can use stock prices in the design of the manager's compensation. Similarly, Aghion and Stein (2008) also find a strategic complementarity between market expectations and firms' investment decisions, but their analysis does not feature information acquisition by the investors nor the optimal design of compensation.

The rest of the chapter is organized as follows. Section 3.2 describes the basic model. In Section 2.3, I describe the equilibrium trading strategies and information acquisition choice in the stock market, and how these are affected by the firm's investment horizon and the managerial contract. Section 2.4 describes the optimal contracts and how these depend on the informativeness of the stock market. This allows me to solve for the equilibrium and describe its properties in Section 2.5. In Section 3.7, I discuss the empirical implications of the model and related evidence. Finally, Section 3.8 concludes. Detailed proofs are presented in the Appendix.

2.2 The Model

I consider a publicly traded firm, run by a risk-averse manager and owned by different categories of risk-neutral investors. These categories are (i) inside owners, who hold a constant fraction of shares in each period; (ii) liquidity traders, who buy shares for investment purposes but may have to sell shares when unexpected events occur; and (iii) speculators (a single one in the model), who can collect information about the future value of the firm and make money by trading on that information.

The model has two periods, indexed $t = 1, 2$. At time $t = 1$, the insiders hire a manager to run the firm and a market for the shares of the firm takes place. The firm's short-term earnings then realize at the end of the period. Finally, at time $t = 2$ the firm is liquidated to shareholders. All agents in the model are rational. For simplicity, I assume that there is no discounting and,

therefore, the timing of payments is immaterial.

A. The Firm

At time $t = 1$, the shareholders (through the board of directors) choose the firm's investment horizon. They can choose either a short-term project that boosts the firm's earnings in the first period or a long-term project that pays out only in the second period. The expected return of a project increases with managerial effort. The interpretation is that, for a given investment horizon, the manager screens among different investment opportunities with the same horizon: the more effort he exerts in screening projects, the higher the expected return of the project that ends up being implemented.

At time $t = 1$, the firm produces earnings (gross of payments to the manager) in the amount

$$\pi_1 = \theta_1 + \eta_1. \tag{2.1}$$

The random variable θ_1 represents the return on the short-term project. If the manager undertakes the short-term project, θ_1 is normally distributed with mean e and variance σ_θ^2, where e is the manager's effort. If the manager does not undertake the short-term project, $\theta_1 = 0$. The random variable η_1 is a noise term, representing other factors outside the manager's control that affect the firm's short-term performance, and is normally distributed with mean 0 and variance σ_1^2; without loss of generality, I normalize σ_1^2 to 1.

At time $t = 2$, the firm is liquidated to shareholders. The resulting liquidation proceeds (gross of payments to the manager) are

$$\pi_2 = \theta_2 + \eta_2. \tag{2.2}$$

The random variable θ_2 represents the return on the long-term project. If the manager undertakes the long-term project, θ_2 is normally distributed with mean μe and variance σ_θ^2, where e is the manager's effort. If the manager does not undertake the long-term project, $\theta_2 = 0$. The random variable η_2 is a noise term, representing other factors outside the manager's control

that occur during the second period and affect the firm's liquidation value. I assume that η_2 is normally distributed with mean 0 and variance σ_2^2, and is independent of η_1.

For simplicity, I assume that the manager's effort can only take two values, i.e., $e \in \{0, 1\}$. I let $C(e)$ denote the manager's private cost of effort, where $C(1) = c$ and $C(0) = 0$. The shareholders cannot observe the manager's choice of effort. For a given investment horizon (short-term or long-term), they will have to write a compensation contract that incentivizes the manager to choose the desired level of effort. Notice that when $e = 0$, neither project creates value in expectation for shareholders. Therefore, a project is worth being implemented only if $e = 1$, regardless of its horizon. I make the following assumption regarding the cost of the manager's effort.

Assumption 1: $c + \frac{r}{2}c^2\left(\sigma_\theta^2 + 1\right) < 1$.

Assumption 1 ensures that incentivizing the manager to exert $e = 1$ creates value for shareholders (as c is not too large), at least for the short-term project. Therefore, shareholders will always want to hire the manager in equilibrium.[7]

Of course, providing incentives on a short-term or a long-term project requires different contracts and, therefore, implies different agency costs. Shareholders take into account both the expected return on the project and the relative agency costs when they choose which type of projects to pursue. The following assumption characterizes the key trade-off in their choice of the firm's investment horizon:

Assumption 2: $\mu > 1$; $\sigma_2 > 1$.

In a scenario where the stock price cannot be part of the contract (or is not informative about the manager's actions), shareholders have two options. They can link the manager's pay to $\pi_1 = \theta_1 + \eta_1$ and have the manager exert effort on a short-term project. Otherwise, they can link the manager's pay to $\pi_2 = \theta_2 + \eta_2$ and incentivize the long-term project. On the one

[7] The left-hand side of the inequality in Assumption 1 describes the total cost of incentivizing a short-term project, under an optimal contract. The first term (c) is the manager's cost of effort; the second term is the manager's risk-premium, since the contract will link his pay to π_1, which has volatility $\sigma_\theta^2 + 1$ when the short-term project is implemented. This total cost has to be lower than 1, which is the expected return on a short-term project.

hand, the long-term project has higher returns ($\mu > 1$). On the other hand, since the manager is risk-averse and η_2 is more volatile than η_1 ($\sigma_2 > 1$), incentivizing effort on the long-term project is more costly for shareholders. The rationale behind this assumption is the idea that, since the liquidation value π_2 realizes much later in the game, many factors that are outside the manager's control can affect its realization and contribute to the volatility of the future contingencies η_2. Therefore, shareholders face a trade-off between higher-returns and lower risk-premium to the manager when they choose the firm's investment horizon. The analysis in this chapter explores the effect that the information contained in the stock price has on this trade-off. [8]

B. The Stock Market

At time $t = 1$, after the manager undertakes the investment project, a market for the shares of the firm takes place. Trading occurs among liquidity/noise traders, one speculator and a competitive market maker, and the share price p is determined in a model à la Kyle (1985). In this model, market participants first submit their demands, and then prices are set such that expected trading profits are zero conditional on aggregate demand.

Let u denote the aggregate demand of the liquidity traders. This variable is assumed normally distributed with mean zero and variance σ_u^2, and is independent of η_1 and η_2. As usual, liquidity traders serve the purpose of disguising the trades of the informed; otherwise, prices would fully reveal the speculator's information and there would be no returns to collecting information for the speculator.

Before submitting her demand, the speculator can gather information about the firm's value. She can learn the firm's short-term earnings π_1, at a cost g_1; she can also learn the return/quality of the firm's long-term project θ_2, at a cost g_2. [9] Let $\mathbf{s} = (s_1, s_2)$ denote the signals observed by the speculator; $\mathbf{s}$ takes three possible values, i.e., $\mathbf{s} \in \{(\pi_1, \varnothing), (\varnothing, \theta_2), (\pi_1, \theta_2)\}$, where $\varnothing$ signifies that a signal was not acquired, since the speculator can decide to learn π_1 only, θ_2 only, or both

[8] The assumption that $\sigma_2 > 1$ makes the analysis interesting, otherwise shareholders would always choose the long-term project regardless of the information contained in the stock price. Similarly, if $\mu \leq 1$ and $\sigma_2 > 1$, they would always choose the short-term project.

[9] The assumption that the speculator perfectly learns π_1 and θ_2 simplifies the analysis but does not affect any of the results. I could have that the speculator observes imperfect signals $s_1 = \pi_1 + \epsilon_1$ and $s_2 = \theta_2 + \epsilon_2$, where the error terms ϵ_1 and ϵ_2 are both normally distributed.

π_1 and θ_2. After observing her signals, the speculator submits a demand x. The other agents in the model cannot observe the speculator's signals $\mathbf{s}$ or her demand x.

Notice that the speculator cannot learn the firm's liquidation value π_2, but just the component of this that depends on the firm's long-term project. The rationale for this assumption is that the firm's liquidation value at time $t = 2$, i.e., π_2, depends on future contingencies (η_2) that cannot be predicted (or are too costly to predict) by the speculator at time $t = 1$. This is similar to the specification of the speculator's signal in Holmstrom and Tirole (1993). [10]

Stock-market participants do not observe the type of project the firm implements or the manager's choice of effort, but form conjectures about them. The market's conjecture about the return on the short-term project is denoted by $\overline{\theta}_1$. Similarly, $\overline{\theta}_2$ denotes the conjecture about the return on the long-term project. The speculator and the market maker know that a project adds value to the firm only if $e = 1$ and, therefore, their conjecture about effort is always $\overline{e} = 1$. Given that $\overline{e} = 1$ and that the firm either invests in the short-term project or in the long-term project, the pair $\left(\overline{\theta}_1, \overline{\theta}_2\right)$ takes only two values: if the speculator and the market-maker expect the manager to undertake the short-term project, then $\left(\overline{\theta}_1, \overline{\theta}_2\right) = \left(\overline{\theta}_1 \sim N\left(1, \sigma_\theta^2\right), 0\right)$; if they expect the manager to undertake the long-term project, then $\left(\overline{\theta}_1, \overline{\theta}_2\right) = \left(0, \overline{\theta}_2 \sim N\left(\mu, \sigma_\theta^2\right)\right)$.

For simplicity, I assume that the cost of acquiring information (g_1 and g_2) is small compared to the volatility of liquidity trading:

Assumption 3: $g_1 + g_2 < \frac{1}{2}\sigma_u \left(1 + \sigma_\theta^2\right)^{\frac{1}{2}}$.

This assumption makes sure that the speculator acquires a signal whenever this grants her an informational advantage over the market maker. This simplifies the exposition and allows us to focus on the type of information the speculator chooses to acquire rather than whether she acquires information or not.

C. Managerial Contract

There are three sources of performance information in the model: the share price p, the firm's short-term performance π_1, and the firm's liquidation value π_2. As is standard in the

[10] In Section 2.5.4, I show that the key results of the paper are robust to a symmetric specification where $\mathbf{s} \in \{(\theta_1, \varnothing), (\varnothing, \theta_2), (\theta_1, \theta_2)\}$.

theoretical literature on executive compensation, I will only consider contracts that are linear in these three performance measures, i.e., of the form:

$$w = \alpha + \beta p + \gamma_1 \pi_1 + \gamma_2 \pi_2. \tag{2.3}$$

The manager's preference over income w is represented by an exponential utility function. The manager's private cost of exerting effort $c(e)$ is independent of his wealth. This implies that the manager's evaluation of the normally distributed (in equilibrium)[11] income lottery w can be represented by the certain equivalent measure

$$U(w, e) = E(w) - \frac{r}{2} Var(w) - C(e), \tag{2.4}$$

where r denotes the manager's coefficient of absolute risk aversion.

For simplicity, I set the manager's reservation utility to zero. The initial shareholders' problem is then to choose (through the board of directors) the firm's investment horizon and the contract $(\alpha, \beta, \gamma_1, \gamma_2)$ in order to maximize the firm's expected value at the beginning of time $t = 1$, i.e., the expected value of $\mathcal{V} = \pi_1 + \pi_2 - w$. [12] The contract must satisfy both the manager's participation and incentive constraints. Stock-market participants do not observe the managerial contract; I let $\left(\overline{\alpha}, \overline{\beta}, \overline{\gamma}_1, \overline{\gamma}_2\right)$ denote their conjectures about $(\alpha, \beta, \gamma_1, \gamma_2)$, respectively.

D. Sequence of Events

The timing of the model is summarized in what follows.

Time $t = 1$:

(i) Shareholders privately choose the firm's investment horizon and the managerial contract $(\alpha, \beta, \gamma_1, \gamma_2)$.

[11] I show in Proposition 13 that the stock price (p) follows a normal distribution in equilibrium. As a consequence, the wage w is normally distributed as well in equilibrium.

[12] The assumption that shareholders directly choose the investment horizon simplifies the analysis but does not change the key results in the paper. The shareholders could incentivize the manager to invest in a given project *indirectly*, via the design of the contract. This would imply an additional incentive constraint that can be binding in equilibrium and, therefore, the possibility of a positive rent to the manager.

(ii) The manager chooses whether to accept the contract or not. If the contract is accepted, the manager privately chooses the effort level e.

(iii) The speculator privately chooses her signal $\mathbf{s}$. Having observed $\mathbf{s}$, she privately chooses demand x.

(iv) Liquidity traders, the speculator, and the market maker trade shares at the market-clearing price p.

(v) The firm's short-term earnings π_1 publicly realize.

Time $t = 2$:

(vi) The firm's liquidation value π_2 publicly realize. The firm's total value $\pi_1 + \pi_2$ is divided among shareholders after deducting the manager's pay w.

I use *Perfect Bayesian Equilibrium* as the solution concept.

2.3 The Stock Market

Proceeding by backward induction, I first characterize the equilibrium in the stock market for given conjectures about the managerial contract and the firm's investment horizon, i.e., for given $\left(\overline{\alpha}, \overline{\beta}, \overline{\gamma}_1, \overline{\gamma}_2\right)$ and $\left(\overline{\theta}_1, \overline{\theta}_2\right)$.

For given conjectures about the managerial contract and the firm's investment horizon, the equilibrium price will depend on the speculator's trading and information acquisition strategies. Conversely, the speculator's optimal strategies will depend on how his trading affects the price. I am looking for a rational expectations equilibrium in which, for given conjectures $\left(\overline{\alpha}, \overline{\beta}, \overline{\gamma}_1, \overline{\gamma}_2\right)$ and $\left(\overline{\theta}_1, \overline{\theta}_2\right)$, the market-maker's beliefs about the speculator's behavior coincide with his actual behavior.

2.3.1 Preliminaries

Let $\overline{x}(\mathbf{s})$ denote the market-maker's conjecture about the speculator's demand as a function of her private signals $\mathbf{s}$. I posit that $\overline{x}(\mathbf{s})$ takes the following linear form:

$$\overline{x}(\mathbf{s}) = \phi_1^{\mathbf{s}} s_1 + \phi_2^{\mathbf{s}} s_2 + k^{\mathbf{s}}. \tag{2.5}$$

The coefficients $(\phi_1^{\mathbf{s}}, \phi_2^{\mathbf{s}})$ determine how aggressively the speculator trades on each signal, depending on which signals she decided to acquire. For a given choice of signals $\mathbf{s}$, $k^{\mathbf{s}}$ represents a constant term in $\overline{x}(\mathbf{s})$. I emphasize that the linear specification implies that the coefficients $(\phi_1^{\mathbf{s}}, \phi_2^{\mathbf{s}}, k^{\mathbf{s}})$ are free to change depending on which information the speculator decided to acquire, i.e., whether she observed π_1 only, θ_2 only, or both π_1 and θ_2, but do not depend on the exact realization of π_1 and θ_2. The market maker does not observe the speculator's choice of the signals; let $\overline{\mathbf{s}}$ denote his conjecture about $\mathbf{s}$.

The market maker observes total demand $q = x + u$ and sets a price

$$p = E\left[\pi_1 + \pi_2 - w \mid \overline{x}(\overline{\mathbf{s}}) + u = q\right], \tag{2.6}$$

where the expectation in (2.6) is taken with respect to u, π_1, and π_2, conditional on the observed q and the conjectured speculator's strategies $(\overline{x}, \overline{\mathbf{s}})$.

The speculator takes as given the market-maker's conjectures $\overline{x}$ and $\overline{\mathbf{s}}$. She then chooses her demand x, knowing the true signals $\mathbf{s}$ but unaware of u and, thus, of q. Therefore, her optimal demand x solves:

$$\overline{x}(\mathbf{s}) = \arg\max_x x\left\{E\left[\pi_1 + \pi_2 - w \mid x, \overline{\mathbf{s}}\right] - E\left[p \mid x\right]\right\}, \tag{2.7}$$

where the expectation in (2.7) takes into account that the price p is a function of both x and u, as described by equation (2.6).

2.3.2 Stock Market Equilibrium

The market equilibrium is determined by the linearity restriction (2.5), the pricing rule (2.6), and the rationality condition on the speculator's trading strategy (2.7). The following proposition characterizes the equilibrium.

Proposition 13 *Fix the market-maker's and speculator's conjectures about the manager's contract and the firm's investment horizon, i.e.,* $\left(\overline{\alpha}, \overline{\beta}, \overline{\gamma}_1, \overline{\gamma}_2\right)$ *and* $\left(\overline{\theta}_1, \overline{\theta}_2\right)$*. There exists a unique equilibrium satisfying conditions (2.5) to (2.7). In this equilibrium, we have:*

1. *If the market-maker and speculator expect the manager to undertake the short-term project (i.e.,* $\overline{\theta}_1 \sim N\left(1, \sigma_\theta^2\right)$*,* $\overline{\theta}_2 = 0$*), the speculator acquires information only about short-term earnings* π_1*.*

   (a) *The speculator's demand strategy is:*

$$x^* = \frac{1 - \overline{\gamma}_1}{2\lambda^*} \left(\pi_1 - 1\right), \tag{2.8}$$

   where $\lambda^* = \frac{1}{2\sigma_u} \left[\left(1 - \overline{\gamma}_1\right)^2 \left(\sigma_\theta^2 + 1\right)\right]^{\frac{1}{2}}$;

   (b) *The equilibrium price* p^* *is*

$$p^* = \frac{1}{1 + \overline{\beta}} \left[1 - \overline{\gamma}_1 - \alpha + \frac{1 - \overline{\gamma}_1}{2} \pi_1 + \lambda^* u\right]. \tag{2.9}$$

2. *If the market-maker and speculator expect the manager to undertake the long-term project (i.e.,* $\overline{\theta}_1 = 0$*,* $\overline{\theta}_2 \sim N\left(\mu, \sigma_\theta^2\right)$*), the speculator acquires information about both short-term earnings* π_1 *and the long-term project* θ_2*.*

   (a) *The speculator's demand strategy is*

$$x^{**} = \frac{1 - \overline{\gamma}_1}{2\lambda^{**}} \pi_1 + \frac{1 - \overline{\gamma}_2}{2\lambda^{**}} \left(\theta_2 - \mu\right), \tag{2.10}$$

   where $\lambda^{**} = \frac{1}{2\sigma_u} \left[\left(1 - \overline{\gamma}_1\right)^2 \cdot 1 + \left(1 - \overline{\gamma}_2\right)^2 \sigma_\theta^2\right]^{\frac{1}{2}}$;

(b) The equilibrium price p^{**} is

$$p^{**} = \frac{1}{1+\overline{\beta}}\left[(1-\overline{\gamma}_2)\,\mu - \overline{\alpha} + \frac{1-\overline{\gamma}_1}{2}\pi_1 + \frac{1-\overline{\gamma}_2}{2}\,(\theta_2-\mu) + \lambda^{**}u\right]. \tag{2.11}$$

The characterization in Proposition 13 calls for several comments. First, the firm's investment choice affects the uncertainty about the firm's value and, thus, the speculator's incentives to acquire information about it. The speculator's information is partly incorporated into the stock price via her trading activity. Therefore, through the effect on the speculator's equilibrium strategies, the firm's investment horizon affects the informativeness of the stock price: this is the first direction of the two-way feedback in the model. The intuition behind this result is the following. When the manager is expected to undertake the short-term project, there is no value for the speculator in acquiring information about the firm's long-term project, as this is not expected to be implemented ($\overline{\theta}_2 = 0$). Therefore, the speculator will only acquire information about the firm's short-term earnings in this case. As a consequence, the stock price will only incorporate information about the firm's short-term performance.

On the contrary, when the manager is expected to undertake the long-term project, the speculator can acquire information about it and profit off from the uninformed (liquidity) traders in the market. Therefore, the speculator will acquire information about *both* π_1 and θ_2. Notice that, when the manager undertakes the long-term project (i.e., $\theta_1 = 0$ and $\theta_2 \sim N\left(\mu, \sigma_\theta^2\right)$), the firm's short-term performance is fully determined by the first-period contingencies η_1, since $\pi_1 = \eta_1$ in this case. This implies that there is still uncertainty about the firm's short-term performance and, thus, incentives for the speculator to acquire information about it. As a result, the stock price will incorporate information about both the firm's short-term performance and its long-term value in this case.

Second, given the conjecture of a linear trading strategy for the speculator, the expected firm value conditional on aggregate demand q depends linearly on q. The coefficients λ^* and λ^{**} in Proposition 13 measure the sensitivity of expectations to the order flow; that is, λ^* and λ^{**} measure how informative aggregate demand is. Of course, this depends on the type

of information the speculator is trading on. Therefore, the sensitivity of the stock price to the order flow changes depending on the conjecture about the firm's investment horizon which, in turn, determines the speculator's information acquisition decision.

Finally, how aggressively the speculator trades on each signal depends on the conjectures about the parameters $\overline{\gamma}_1$ and $\overline{\gamma}_2$ of the manager's contract. The intuition for this result is as follows. Suppose γ_1 increases: a smaller share of the firm's short-term earnings π_1 goes to shareholders (a larger fraction goes to the manager) and, thus, affect the value of the shares. This has a negative externality on the speculator, who knows the realization of π_1 and profits from trading on it. The speculator reacts by trading less aggressively on π_1, so that a smaller fraction of π_1 is revealed to the market-maker. As a consequence, less of her signal is incorporated into the price. Therefore, the sensitivity of the stock price to the realization of π_1 (θ_2) decreases with $\overline{\gamma}_1$ ($\overline{\gamma}_2$).

Summing up, the informativeness of the stock price depends both directly and indirectly on managerial incentives. Directly, through the effect that the contract has on how aggressively the speculator trades on each signal. Indirectly, through the effect that the firm's choice of projects has on the speculator's incentives to acquire information. Since the stock price can be part of the contract, this has important implications for the structuring of managerial incentives themselves.

2.4 Optimal Contracting

Having characterized the equilibrium trading and information acquisition strategies in the stock market, I can now characterize the optimal contracts for a given project type and given conjectures about the informativeness of the stock price.

Both the manager and shareholders take the market's conjectures about the contract $\left(\overline{\alpha}, \overline{\beta}, \overline{\gamma}_1, \overline{\gamma}_2\right)$ and the firm's investment horizon $\left(\overline{\theta}_1, \overline{\theta}_2\right)$ as given. However, they understand the structure of the equilibrium in the stock market and, therefore, how these conjectures affect the informativeness of the stock price. I begin by describing the manager's choice of effort and how this depend

on the contract and on the informativeness of the stock price.

2.4.1 Managerial Incentives

Given the contract $(\alpha, \beta, \gamma_1, \gamma_2)$ and the type of the project (short-term or long-term), the manager chooses the effort level ($e \in \{0, 1\}$) that maximizes the certainty equivalent measure of his utility (equation (2.4)).

Lemma 14 *The manager's choice of effort is characterized as follows:*

1. *When the market-maker and speculator expect the manager to undertake the short-term project (i.e., $\overline{\theta}_1 \sim N\left(1, \sigma_\theta^2\right)$, $\overline{\theta}_2 = 0$):*

   (a) *If the manager were to undertake the short-term project, he would choose $e = 1$ if $\beta \frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)} + \gamma_1 \geq c$;*

   (b) *If the manager were to undertake the long-term project, he would choose $e = 1$ if $\gamma_2 \mu \geq c$.*

2. *When the market-maker and speculator expect the manager to undertake the long-term project (i.e., $\overline{\theta}_1 = 0$, $\overline{\theta}_2 \sim N\left(\mu, \sigma_\theta^2\right)$):*

   (a) *If the manager were to undertake the short-term project, he would choose $e = 1$ if $\beta \frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)} + \gamma_1 \geq c$;*

   (b) *If the manager were to undertake the long-term project, he would choose $e = 1$ if $\beta \frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)} \mu + \gamma_2 \mu \geq c$.*

Lemma 14 characterizes the manager's optimal effort choice for a given contract $(\alpha, \beta, \gamma_1, \gamma_2)$ and given market's conjectures about both the contract and the choice of the project. First, consider the case when the stock-market expects the firm to implement the short-term project. Suppose also this conjecture is consistent: shareholders have asked the manager to invest in the short-term project, and the manager is contemplating whether to exert effort in screening the project or not. Effort increases the expected return on the project ($\theta_1 \sim N\left(e, \sigma_\theta^2\right)$). When the

manager shirks and chooses $e = 0$, the expected short-term earnings go down, since $\pi_1 = \theta_1 + \eta_1$. This reduces the manager's expected pay in two ways. First, via its short-term incentives γ_1: the manager loses $\gamma_1 \cdot 1$. Second, via the expected stock price, which goes down by a factor $\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}$. This is because the speculator observes the true realization of π_1, which is distributed as $\pi_1 \sim N\left(0, \sigma_\theta^2 + 1\right)$ when $\theta_1 \sim N\left(0, \sigma_\theta^2\right)$. Therefore, in expectation, the speculator finds out that the firm is overvalued (the market maker expects $e = 1$ and, thus, $\pi_1 \sim N\left(1, \sigma_\theta^2 + 1\right)$) and sells shares, driving down the expected stock price. The speculator acts as a monitor for the manager and contributes to incentivizing high effort. If the losses from shirking are greater than the cost of effort, the manager is better off by choosing $e = 1$. This describes the inequality in Part 1.a of the Lemma.

Even if the stock market expects the short-term project, shareholders might prefer to incentivize the manager to take a long-term project instead. In this case, since the speculator has not acquired information about θ_2 (the conjecture is $\overline{\theta}_1 \sim N\left(1, \sigma_\theta^2\right)$, $\overline{\theta}_2 = 0$), the stock price will not reflect that the manager did not exert effort in screening the project, i.e., that $e = 0$. Therefore, if the stock market expects the short-term project will be implemented, the stock price is of no use in incentivizing effort on the long-term project. This describes the inequality in Part 1.b of the Lemma.

A similar logic yields the incentive constraints in Part 2 of the Lemma. Notice that the constraint in Part 2.a is the same as the one in Part 1.a. This is because regardless of the conjectures about project choice, the speculator always acquires information about π_1. Therefore, when the market expects the long-term project and the manager invests instead in a short-term project, he can boost the expected stock price by exerting effort on the project: π_1 would be then distributed as $\pi_1 \sim N\left(1, \sigma_\theta^2 + 1\right)$, while the market-maker and the speculator expect $\pi_1 \sim N\left(0, \sigma_\theta^2 + 1\right)$. [13]

[13] The sensitivity of the stock-price to short-term performance is also the same in both cases, i.e., $\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}$ (see Proposition 13).

2.4.2 Optimal Contracts

Like the manager, shareholders take the market's conjectures about the contract $\left(\overline{\alpha}, \overline{\beta}, \overline{\gamma}_1, \overline{\gamma}_2\right)$ and the firm's investment horizon $\left(\overline{\theta}_1, \overline{\theta}_2\right)$ as given when choosing the project type and the contract that maximize the firm's value. Following Grossman and Hart (1983), it is useful to think about this choice as a two-stage process. First, for a given project (short-term or long-term project), the shareholders find the optimal contract to incentivize the manager to exert high effort $e = 1$. This contract determines the firm's optimal value for a given project choice. Second, they compare the firm's value in the two scenarios and choose the investment horizon (and the associated contract) that leads to higher value. The analysis of contracting in this section describes the first step of this two-stage process. The properties of the strategic interaction between managerial incentives and the informativeness of the stock price will then be used in the next section to characterize the firm's optimal investment horizon and the equilibrium of the game.

The manager's participation constraint is always binding under an optimal contract, so that the following inequality is always satisfied with equality:

$$\alpha + E\left(\beta p + \gamma_1 \pi_1 + \gamma_2 \pi_2\right) - \frac{r}{2} Var\left(\beta p + \gamma_1 \pi_1 + \gamma_2 \pi_2\right) - c \geq 0 \tag{2.12}$$

Equation (2.12) describes the manager's participation constraint. The distribution of π_1 and π_2 in the equation is conditional on $e = 1$ and on the choice of the project, which affects the distribution of returns (θ_1, θ_2). The distribution of p depends on the true distribution of (θ_1, θ_2) and the market's conjectures (about (θ_1, θ_2) and the contract), via the effect on the stock market equilibrium.

Given that the participation constraint is always binding under an optimal contract, the optimal contract for a given investment horizon simply minimizes the manager's risk-premium under the respective incentive constraints. Therefore, for a given investment horizon, the optimal contract solves:

$$\min_{(\alpha, \beta, \gamma_1, \gamma_2)} \frac{r}{2} Var\left(\beta p + \gamma_1 \pi_1 + \gamma_2 \pi_2\right) \tag{2.13}$$

subject to the respective incentive constraint in Lemma 14.

As before, the distribution of π_1 and π_2 in program (2.13) is conditional on $e = 1$ and on the choice of the project, which affects the distribution of returns (θ_1, θ_2). The distribution of p depends on both the true distribution of (θ_1, θ_2) and the market's conjectures (about (θ_1, θ_2) and the contract), via the effect on the stock market equilibrium.[14]

The following proposition characterizes the optimal contracts for the short-term and the long-term projects.

Proposition 15 *Fix the market-maker's and speculator's conjectures about the manager's contract and the firm's investment horizon, i.e.,* $\left(\overline{\alpha}, \overline{\beta}, \overline{\gamma}_1, \overline{\gamma}_2\right)$ *and* $\left(\overline{\theta}_1, \overline{\theta}_2\right)$*. We have:*

1. *The optimal contract that incentivizes the manager to exert effort on a short-term project features:* $\gamma_1^* = c$, $\beta^* = \gamma_2^* = 0$*;*

2. *The optimal contract that incentivizes the manager to exert effort on a long-term project features:*

   (a) *If the market-maker and the speculator expect the manager to undertake the short-term project (and, thus, p is not informative about the long-term project θ_2), the contract features* $\gamma_1^\dagger = \beta^\dagger = 0, \gamma_2^\dagger = \frac{c}{\mu}$*;*

   (b) *If the market-maker and the speculator expect the manager to undertake the long-term project (and, thus, p is informative about θ_2), the contract features* $\gamma_1^{\dagger\dagger} = 0$*, and both* $\beta^{\dagger\dagger} > 0$ *and* $\gamma_2^{\dagger\dagger} > 0$*. The exact value of* $\beta^{\dagger\dagger}$ *and* $\gamma_2^{\dagger\dagger}$ *is characterized by the following system of equations:*

$$\gamma_2^{\dagger\dagger} = \frac{c}{\mu} \frac{2\left(\frac{1-\overline{\gamma}_1}{1-\overline{\gamma}_2}\right)^2 + \sigma_\theta^2}{2\left(\frac{1-\overline{\gamma}_1}{1-\overline{\gamma}_2}\right)^2 + \sigma_2^2 + \sigma_\theta^2}; \tag{2.14}$$

$$\gamma_2^{\dagger\dagger} + \beta^{\dagger\dagger} \frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)} = \frac{c}{\mu}. \tag{2.15}$$

[14] The exact expression for the variance in (2.13), for a given investment horizon and conjectures $\left(\overline{\alpha}, \overline{\beta}, \overline{\gamma}_1, \overline{\gamma}_2\right)$ and $\left(\overline{\theta}_1, \overline{\theta}_2\right)$, is described in Appendix 2.9.2.

For all three cases, the fixed component of pay α is chosen so that the manager's participation constraint in equation (2.12) is binding.

Notice that $\gamma_2^{\dagger\dagger}$ is always lower than $\frac{c}{\mu}$. Therefore, the optimal contract for the long-term project always links the manager's pay to both p and π_2. Moreover, as the volatility of future contingencies (σ_2^2) increases, the contract puts less weight on π_2 and more on p, i.e., $\gamma_2^{\dagger\dagger}$ decreases and, thus, $\beta^{\dagger\dagger}$ increases with σ_2^2. [15]

Regardless of the market's conjecture about the firm's investment horizon, the optimal contract for the short-term project links the manager's pay to the realization of short-term earnings only. The intuition for this result is the following. Depending on the market's conjectures, the stock price p can be informative about (i) π_1 only or about (ii) both π_1 and θ_2. In case (i), p and π_1 contain the same type of information.[16] However, the speculator does not fully reveal his information about π_1 to the market. Therefore, only a fraction of the realization of π_1 is incorporated into p. This means that the incentive power of p is lower and, thus, a larger β is required to provide incentives via p. This translates into a larger risk-premium, since a larger part of the manager's wealth is subject to risk. As a consequence, it is optimal to use only π_1 in the contract. In case (ii), p contains information about θ_2 as well, which is not useful to incentivize the short-term project. Of course, using only π_1 in the contract is optimal also in this case.

However, when it comes to incentivizing the manager to take the long-term project, the speculator's information acquisition becomes crucial. If the speculator acquires information about both π_1 and θ_2, p provides additional information about the manager's effort. This is because p does not include future contingencies (η_2) that will affect the firm's liquidation value π_2 but do not depend on the manager's effort. On the other hand, p includes current contingencies (η_1) that affect the firm's short-term performance π_1 but do not depend on the manager's effort

[15] The firm's liquidation value is $\pi_2 = \theta_2 + \eta_2$. As the volatility of η_2 (i.e., σ_2^2) increases, linking the manager's pay to π_2 becomes more expensive to shareholders, since the manager will require a higher risk-premium. In the limit as σ_2 tends to infinity, $\gamma_2^{\dagger\dagger}$ vanishes and $\beta^{\dagger\dagger} = \frac{c}{\mu}$.

[16] It is worth noticing that, while the exact realization of p also depends on the realization of liquidity trading u, the volatility of u has no direct effect on the equilibrium distribution of p. This is because the speculator's optimal trading strategy is such that $x(\mathbf{s})$ will adjust in response to changes in σ_u precisely so that the distribution of price is independent of the level of liquidity trading. Therefore, liquidity trading has only an indirect effect on the distribution of p, by incentivizing information collection by the speculator.

in screening among long-term projects. Therefore, shareholders will want to use a mix of both p and π_2 in the contract.[17] However, when the speculator acquires information about π_1 only, p is of no use in incentivizing the manager. Therefore, shareholders can use only π_2 in the contract.

2.5 Equilibrium

I have found that the strategic interaction between executive compensation and the informativeness of the stock price is characterized by a two-way feedback. One way goes from the firm to the stock market: when the firm is expected to invest in the long-term project, the speculator acquires information about it and this information is partly incorporated into the price through her trading. The other direction of the feedback goes from the stock market to the firm: if the speculator acquires information about the firm's long-term project, the stock price can be used to incentivize effort on a long-term project; therefore, implementing a long-term project becomes more attractive for the firm.

This two-way feedback generates a strategic complementarity in the choice of horizons between the shareholders and the speculator. As is typical in games with strategic complementarities, this can lead to multiple equilibria.

Definition 16 *Depending on the parameters of the model, there exist two types of equilibria:*

1. *An equilibrium with short-termism, where: the manager undertakes the short-term project; the speculator acquires information only about short-term earnings (π_1); shareholders set a contract $\gamma_1^* = c$, $\beta^* = \gamma_2^* = 0$;*

2. *An equilibrium with long-termism, where: the manager undertakes the long-term project; the speculator acquires information about both the long-term project (θ_1) and π_1; shareholders set a contract $\gamma_1^{**} = 0$, $\beta^{**} > 0, \gamma_2^{**} > 0$. The exact value of β^{**} and γ_2^{**} is*

[17] In Section 2.5.4, I consider a setting with a symmetric specification of the speculators' signals, i.e., $\mathbf{s} \in \{(\theta_1, \varnothing), (\varnothing, \theta_2), (\theta_1, \theta_2)\}$. In this setting, when the speculator learns about θ_1, p provides additional information about short-term performance as well, as it has information about θ_1 but not about η_1. Therefore, the optimal contract for the short-term project includes p in this case.

characterized by the following system of equations:

$$\gamma_2^{**} = \frac{c}{\mu} \frac{2\left(\frac{1}{1-\gamma_2^{**}}\right)^2 + \sigma_\theta^2}{2\left(\frac{1}{1-\gamma_2^{**}}\right)^2 + \sigma_2^2 + \sigma_\theta^2}; \tag{2.16}$$

$$\gamma_2^{**} + \beta^{**} \frac{1-\gamma_2^{**}}{2\left(1+\beta^{**}\right)} = \frac{c}{\mu}. \tag{2.17}$$

In both equilibria, the manager chooses $e = 1$ and the fixed component of pay α is chosen so that the manager's participation constraint in equation (2.12) is binding.

When the market conjectures about the contract are consistent with the actual contract, i.e., when $\overline{\beta} = \beta^{**}$, $\overline{\gamma}_1 = 0$, $\overline{\gamma}_2 = \gamma_2^{**}$, the system of equations that characterizes the optimal contract for the long-term project (i.e., equations (2.16) and (2.17)) is the same as the one in Proposition 15. The exact value of γ_2^{**} is a fixed point of equation (2.16). In Appendix 2.9.2, I show that this equation has a unique fixed point in $\gamma_2^{**} \in [0, \frac{c}{\mu}]$ and, thus, that γ_2^{**} is unique. Given γ_2^{**}, the value of β^{**} is uniquely pinned down by the incentive constraint in equation (2.17).

The analysis of the equilibrium becomes easier if we first analyze a benchmark where the stock price is not contractible. Therefore, I will first characterize the equilibrium in such a benchmark. This will help us characterize the equilibrium of the game in Proposition 18.

2.5.1 Benchmark without Stock Market

Consider a benchmark where the stock price p is not contractible. This is the case, for example, when the firm's shares are not publicly traded.

Lemma 17 *When the stock price cannot be part of the contract, we have:*

1. *The optimal contracts for the short-term and long-term projects are as described in Part 1 and Part 2.a of Proposition 15, respectively: the optimal contract for the short-term project is $\gamma_1^* = c$, $\beta^* = \gamma_2^* = 0$; the optimal contract for the long-term project is $\gamma_1^\dagger = \beta^\dagger = 0, \gamma_2^\dagger = \frac{c}{\mu}$;*

2. *There exists a (unique) threshold value $\overline{\mu}$, with $\overline{\mu} > 1$, such that:*

(a) when $\mu \leq \overline{\mu}$, shareholders choose the short-term project;

(b) when $\mu > \overline{\mu}$, shareholders choose the long-term project.

Lemma 17 is quite intuitive. Even when the stock price can be part of the contract, the optimal contract for the short-term project only uses the realization of short-term earnings π_1. Therefore, this contract does not change in the benchmark where the stock price is not contractible. When p is not informative about θ_2, the optimal contract for the long-term project only uses the realization of the firm's liquidation value π_2, as the stock price is not useful in incentivizing the manager. Since the stock price is not used in this case, this contract is the same as in the benchmark. This equivalence plays an important role in the characterization of the equilibrium.

Shareholders compare the firm's value under the two investment horizons and choose the one that leads to higher value. They choose to have the long-term project implemented, if the following condition is satisfied:

$$\mu - c - \frac{r}{2}\left(\frac{c}{\mu}\right)^2 \left(\sigma_\theta^2 + \sigma_2^2\right) \geq 1 - c - \frac{r}{2}c^2\left(\sigma_\theta^2 + 1\right). \tag{2.18}$$

The inequality above determines the (unique) threshold value $\overline{\mu}$. The right-hand side of the inequality is the firm's expected value when the short-term project is implemented; Assumption 1 ensures that this is non-negative. The left-hand side is instead the firm's expected value when the long-term project is implemented. This expected value increases with μ for two reasons. First, the long-term project becomes more profitable when μ goes up. Second, it becomes easier to incentivize the manager (the incentive constraint in Part 2.a of Lemma 14 becomes slacker), as now the manager loses more from shirking. When $\mu = \overline{\mu}$, these two effects perfectly compensate for the fact that η_2 is more volatile than η_1 ($\sigma_2 > 1$). Therefore, the long-term project is, all else equal, more costly to incentivize: shareholders are indifferent between the two investment horizons. As $\mu > \overline{\mu}$, they are strictly better off when the long-term project is implemented.

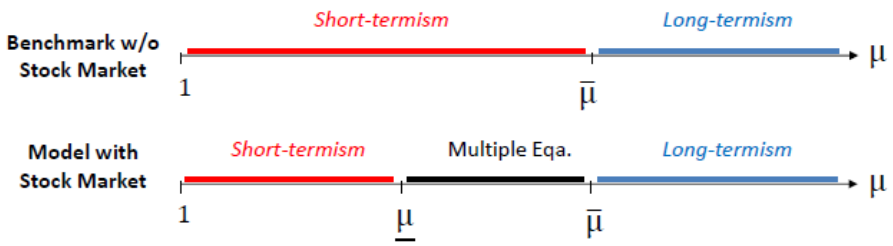


Figure 2.1: Equilibrium Characterization.

2.5.2 The Equilibrium

Having characterized the optimal investment horizon in a benchmark model where the stock price was not contractible, I can now describe the equilibrium of the game in the full model. The following proposition characterizes the equilibria as a function of the expected return of the long-term project (μ).

Proposition 18 *There exist two thresholds $\overline{\mu} > 1$ and $\underline{\mu} \in [1, \overline{\mu})$, where $\overline{\mu}$ is the same as in Lemma 17, such that:*

1. *If $\mu \leq \underline{\mu}$, short-termism is the unique equilibrium of the game;*
2. *If $\underline{\mu} < \mu < \overline{\mu}$, both short-termism and long-termism are equilibria of the game;*
3. *If $\mu > \overline{\mu}$, long-termism is the unique equilibrium of the game.*

Figure 2.1 describes the results in Proposition 18. First, consider the case when the market's conjecture is that the manager undertakes the short-term project. In this case, the speculator does not acquire information about the long-term project and, thus, p cannot be used to incentivize the long-term project. Therefore, the optimal contracts are as described in Part 1 and Part 2.a of Proposition 15. As discussed earlier, these optimal contracts are the same as in the benchmark without the stock market and, thus, the firm's optimal value for a given investment horizon is the same as in the benchmark. Shareholders then choose the firm's investment horizon according to the inequality (2.18). If $\mu \leq \overline{\mu}$, they choose the short-term project; if $\mu > \overline{\mu}$, they choose the long-term project. This has two important implications. First, it implies that the initial conjecture we started with, i.e., that the manager undertakes the short-term project, can

be consistent only when $\mu \leq \overline{\mu}$. Therefore, short-termism can be an equilibrium of the game if and only if $\mu \leq \overline{\mu}$. Second, it means that, when $\mu > \overline{\mu}$, the unique possible equilibrium is long-termism.

Now, consider the opposite market conjecture, i.e., that the manager undertakes the long-term project. In this case, the speculator acquires information about the long-term project and so p can be used to incentivize effort on the long-term project. What happens to the shareholders' optimal investment horizon in this case? Shareholders can now use a (strictly) better contract: the marginal firm $\overline{\mu}$, that was indifferent under the previous conjecture (and in the benchmark model), is strictly better off with the long-term project now. Therefore, the indifference condition obtains for a strictly lower threshold $\underline{\mu}$.

The fact that $\underline{\mu} < \overline{\mu}$ induces the multiplicity of equilibria, since when $\underline{\mu} < \mu < \overline{\mu}$, both short-termism and long-termism are equilibria. If the market conjectures that the manager implements the short-term project, shareholders are strictly better off incentivizing the short-term project $(\mu < \overline{\mu})$, since the stock price cannot be used in the contract: short-termism is an equilibrium. If the market conjectures that the manager implements the long-term project, shareholders are strictly better off choosing the long-term project $(\mu > \underline{\mu})$: long-termism is an equilibrium as well.

The interval $\left(\underline{\mu}, \overline{\mu}\right)$ becomes arbitrarily large when the volatility of future contingencies η_2 is large. In the limit as σ_2 tends to infinity, we have $\lim_{\sigma_2 \to \infty} \underline{\mu} < \infty$ and $\lim_{\sigma_2 \to \infty} \overline{\mu} = \infty$. This is because, as σ_2 becomes large, the optimal contract in the equilibrium with long-termism only depends on the price p ($\lim_{\sigma_2 \to \infty} \gamma_2^{**} = 0$ and $\lim_{\sigma_2 \to \infty} \beta^{**} = \frac{c}{\mu}$). Therefore, the contract does not expose the manager to the volatility of π_2 and, thus, the threshold $\underline{\mu}$ is finite even when $\sigma_2 \to \infty$. However, if the price does not incorporate information about θ_2, the contract for the long-term project needs to depend on π_2: $\overline{\mu}$ goes to infinity in the limit as σ_2 tends to infinity.

It is worth noticing that the multiplicity of equilibria does not depend on the assumption of private contracts. This multiplicity would survive in a setting in which contracts are publicly observed but the investment horizon is still private information. This is easy to see when σ_2 is very large: in this case, the optimal contract for the long-term project only includes the stock

price p. However, if p does not include information about long-term value, the manager cannot be incentivized to exert effort on a long-term project. Therefore, the shareholders would still choose the short-term project when they expect the market to be short-termist, even if the contract that was set is the one for the long-term project. This effect leads to the existence of multiple equilibria also in a setting with public contracts.

2.5.3 Equilibrium Selection

When both long-termism and short-termism are equilibria of the game, shareholders prefer long-termism, as firm value is strictly larger under long-termism. But what about the speculator? The following inequality characterizes her preference over the two equilibria.

$$\frac{\sigma_u}{2}\left[(1-\gamma_1^{**})^2\left(\sigma_\theta^2+1\right)\right]^{\frac{1}{2}}-g_1>\frac{\sigma_u}{2}\left[1+(1-\gamma_2^{**})^2\,\sigma_\theta^2\right]^{\frac{1}{2}}-(g_1+g_2)\,. \tag{2.19}$$

The left-hand side of the inequality in (2.19) describes the speculator's expected payoff under short-termism; the right-hand side instead describes her payoff under long-termism. When the inequality is satisfied, the speculator prefers short-termism.

Notice that the net profit from trading is always larger under long-termism (as we have $\gamma_1^{**}>\gamma_2^{**}$ and $\gamma_1^{**}<1$). However, information acquisition costs are also larger under long-termism, since the speculator learns about both short-term performance and long-term value. Therefore, if g_2 is sufficiently large, the speculator prefers short-termism. In this case, the shareholders' and the speculator's preferences over equilibria are not aligned, and coordination failure is a serious issue.[18]

The misalignment of preferences is (partly) due to the speculator having no direct stake in the firm at the information acquisition stage, which implies that she does not care if firm value is

[18] The notion of Risk-dominance (Harsanyi and Selten (1988)) does not apply to games with incomplete information. Therefore, I focus only on Payoff-dominance as an equilibrium selection criterion. It is often argued that players will coordinate on the Pareto-dominant equilibrium (provided one exists) if they are able to talk to one another before the game is played. The intuition for this is that, even though the players cannot commit themselves to play the way they claim they will, the preplay communication lets the players reassure one another about the low risk of playing the strategy of the Pareto-dominant equilibrium (Fudenberg and Tirole (1991)). This is a reasonable assumption in my setting, especially when the speculator is a blockholder and has a seat in the board of directors.

lower under short-termism. Recent empirical work documents the predominance in U.S. public companies of small transient blockholders, who typically lack control rights to directly intervene into a firm's operation but can sell their shares if the firm underperforms (or possibly consolidate their position if the firm overperforms).[19] Motivated by this stylized fact, the rest of this section explores the implications of having a pre-existing stake in the firm for the speculator's incentives.

Let Δ denote the speculator's endowment of shares at the information acquisition stage. The following Lemma describes the effect of Δ on the equilibrium.

Lemma 19 *A speculator's preexisting stake in the firm (Δ) has no direct effect on the equilibrium strategies. However, if Δ is sufficiently large, i.e., $\Delta \geq \overline{\Delta}$, the equilibrium with long-termism Pareto dominates the one with short-termism; otherwise, the two equilibria cannot be Pareto-ranked.*

The result in Lemma 19 is quite intuitive. Having a pre-existing stake in the firm does not affect the speculator's trading strategies and, as a consequence, does not affect her profits from trading.[20] However, Δ creates a link between the speculator's expected payoff and the firm's ex-ante value. In expectation, the speculator's demand is null (the price is correct in expectation). Therefore, she expects to keep the same stake Δ in the firm after trading. This means that, if firm value increases, the value of her expected stake in the firm increases as well. As a consequence, Δ aligns shareholders' and speculator's preferences over different equilibria: if the stake is sufficiently large, the equilibrium with long-termism Pareto dominates. Therefore, coordination on the equilibrium with long-termism is easier to achieve.[21]

This result uncovers a new strategic complementarity between inside and outside shareholders (the speculator in the model). Compared to other informed-traders in the market, outside shareholders have a stronger incentive to trade on information about the long-term prospect of

[19] Holderness (2009) documents that, when blockholders are defined as 5% shareholders, 96% of U.S. firms contain a blockholder. However, when the minimum ownership is defined as 20%, La Porta et al. (1999) find that only 20% (10%) of large (medium) U.S. firms contain a blockholder. They also estimate that a 20% stake gives effective control if the shareholder is an insider, while the threshold is likely to be higher for outside shareholders.

[20] This is because the speculator is allowed to short-sell, so Δ does not affect how much she can trade.

[21] The strategic interaction in the model boils down to two players, the principal (shareholders) and the speculator. Especially if the speculator has a stake in the firm and so has a way to communicate with the controlling shareholders, it is natural for players to coordinate on the payoff-dominant equilibrium.

a firm, because this enables inside shareholders to design more efficient managerial contracts and increase firm value.

2.5.4 Alternative Specification for the Speculator's Signal

Here, I show that the key results of the chapter are robust to a setting where the speculator can only acquire signals about the firm's projects θ_1 and θ_2, but not about the contingencies that affect its current or future profits, i.e., η_1 or η_2. This implies that the speculator's signal $\mathbf{s}$ takes values in $\{(\theta_1, \varnothing),(\varnothing, \theta_2),(\theta_1, \theta_2)\}$ as opposed to $\{(\pi_1, \varnothing),(\varnothing, \theta_2),(\pi_1, \theta_2)\}$, which is the case in the baseline specification of the model.

In this modified setting, the speculator will only acquire information about *either* θ_1 *or* θ_2, i.e., in equilibrium we have $\mathbf{s} \in \{(\theta_1, \varnothing),(\varnothing, \theta_2)\}$. The intuition is quite straightforward. The firm either implements the short-term project, in which case $\left(\overline{\theta}_1, \overline{\theta}_2\right) = \left(\overline{\theta}_1 \sim N\left(1, \sigma_\theta^2\right), 0\right)$, or the long-term project, i.e., $\left(\overline{\theta}_1, \overline{\theta}_2\right) = \left(0, \overline{\theta}_2 \sim N\left(\mu, \sigma_\theta^2\right)\right)$. Therefore, the speculator can never profit from acquiring information about both. This has an important implication on the informativeness of the stock price: when the speculator acquires information about θ_1, the price is useful to incentivize effort on the short-term project.[22] This result strengthens the strategic complementarity between the firm's investment horizon and the speculator information acquisition choices.

Definition 20 *Depending on the parameters of the model, there exist two types of equilibria:*

1. *An equilibrium with short-termism, where: the manager undertakes the short-term project; the speculator acquires information only about θ_1; shareholders set a contract $\gamma_1' > 0$, $\beta' > 0$, $\gamma_2' = 0$;*

2. *An equilibrium with long-termism, where: the manager undertakes the long-term project; the speculator acquires information only about θ_2; shareholders set a contract $\gamma_1'' = 0$, $\beta'' > 0$, $\gamma_2'' > 0$.*

[22] The logic is the same as for the long-term project in the baseline model. When the speculator observes θ_1, the price does not incorporate the realization of contingencies η_1, which does not depend on the manager's effort but affects the realization of the firm's profits π_1. Therefore, the price incorporates unique information about the manager's effort on the short-term project. This partly compensates for the fact that the speculator does not reveal all of his information about θ_1 and, thus, shareholders use both p and π_1 in the contract.

In both equilibria, the manager chooses $e = 1$ and the fixed component of pay α is chosen so that the manager's participation constraint in equation (2.12) is binding.

Like in the baseline model, I characterize the equilibria as a function of the expected return of the long-term project (μ).

Proposition 21 *There exist two thresholds $\mu'' > 1$ and $\mu' \in [1, \mu'')$, and $\overline{\mu} \in (\mu', \mu'')$, such that:*

1. *If $\mu \leq \mu'$, short-termism is the unique equilibrium of the game;*
2. *If $\mu' < \mu < \mu''$, both short-termism and long-termism are equilibria of the game;*
3. *If $\mu > \mu''$, long-termism is the unique equilibrium of the game.*

The key difference with respect to the baseline model is that now the interval with multiple equilibria extends to firms that were long-termist in the model where the stock price cannot be included in the contract (i.e., $\mu'' > \overline{\mu}$). To understand this result, consider the case when the market's conjecture is that the manager undertakes the short-term project. In the new setting, the information in p makes also short-termism more attractive for the firm: if the price incorporates information about θ_1, it enables a more efficient contract to incentivize effort on the short-term project. This moves the firm's indifference (between the two projects type) condition to some μ'' strictly larger then $\overline{\mu}$.

Intriguingly, although the possibility to include the stock price in the design of compensation enlarges the mass of firms that could be short-termist in equilibrium (depending on which equilibrium is played), it never makes the firms worse off. Therefore, in line with the results in the baseline model, the stock market may fail to alleviate the problem of corporate short-termism, but never makes it worse.

2.6 Implications

This chapter examines how the structure of managerial incentives and the informativeness of the stock price affect each other and the resulting consequences for corporate short-termism. This section discusses some implications of the model and the related empirical evidence.

The broad objective of this chapter is to show that the role of the stock market in relation to corporate short-termism may be fundamentally misunderstood. Although regarded as the primary culprit of short-termism, the stock market should instead be seen as a potentially alleviating force. This has important implications for the myopia debate. For example, Kaplan (2018) points to a lack of long-term evidence consistent with the predictions of the short-term critics. One of his main arguments relates to the role of Venture Capital (VC) or Private Equity (PE) investments over time. VC and PE firms can make long-term investments in innovative companies with time horizons of five to ten years or more. Their long-term commitment shelters management from short-term market pressures. Therefore, if (i) short-termism has increased over time, the scope for VC and PE funds has increased over time, and (ii) if short-termism is a problem, then VC and PE funds should be highly profitable. Kaplan shows that neither hypothesis is consistent with the evidence. My results suggest that a key factor might be missing in this argument: the underlying agency problem between management and shareholders. Even if VC or PE funds take a firm private, this problem remains. Therefore, the evidence in Kaplan (2017) is not necessarily indicative that corporate short-termism is not a problem.

Both Kaplan and Roe (2018) also point to high Price/Earnings (P/E) ratio as evidence against the short-term critics. A high P/E ratio suggests that the stock market is valuing a firm much more than can be justified by its current earnings, because it is taking into account the potential for future profits. The current P/E ratio of the S&P 500 is 25, versus a historical median of 15. The high valuations of unicorns, despite them making little or even negative earnings, indicates that the stock market must be valuing something other than current profits. In the model, firms with high growth opportunities ($\mu > \overline{\mu}$) benefit the most from being public. This is because investors anticipate that they will always invest in long-term projects and, thus, acquire information about them. This enables the firms to design a more efficient long-term contract for their managers. However, in firms with intermediate growth opportunities ($\underline{\mu} < \mu < \overline{\mu}$), the investment horizon depends on whether the stock market is focused on short-term performance or not. Therefore, the high valuations of unicorns cannot be considered as evidence that stock-market-driven short-termism is not a problem at all.

A second set of implications relates to the strategic complementarity between the firm's and informed-traders' investment horizons. A cross-sectional interpretation of the results in Proposition 18 is that, in firms with high growth opportunities ($\mu > \overline{\mu}$), (i) executive pay should be more linked to stock prices and (ii) stock prices should be more informative about long-term value. This is consistent with the evidence that stock-options are more prevalent in high-tech, "new economy" firms and more generally in growth industries, such as computer, software, and pharmaceutical firms (Murphy 1999, Core and Guay 2001, Ittner et al. 2003). At the same time, P/E ratios are higher in these industries, which implies that the stock market is taking into account the potential for future profits (Kaplan, 2017). The results in Proposition 18 also imply that, in firms with intermediate growth opportunities ($\underline{\mu} < \mu < \overline{\mu}$), the investment horizon depends crucially on the horizon of informed-trading. This is supported by the findings in Hansen and Hill (1991), Bushee (1998), and Wahal and McConnell (2000), which show a positive association between R&D and institutional ownership; the latter is typically highly correlated with blockholdings. Bushee also finds that myopia is driven by momentum investors who trade on current earnings and have small holdings.

Finally, the results in Lemma 19 relate to the importance blockholders have in the equilibrium selection. While the role of blockholders in encouraging long-term investments (Cronqvist and Fahlenbrach (2009)) and deterring myopia (Dechow, Sloan, and Sweeney (1996), Farber (2005), Burns, Kedia, and Lipson (2008)) is well documented, there is less evidence about the specific channel that leads to this effect. Blockholders can intervene directly into a firm's operations (*voice*) or simply trade on information about the firm (*exit*); if this information is impounded into the stock-price, this also disciplines management. The second channel works through the stock price and, thus, relies on prices being used in the manager's compensation. Because both compensation and price informativeness are endogenous in my model, the results in this paper offer new insights into how to empirically distinguish the two channels. Lemma 19 shows that, if the block is sufficiently large, the equilibrium with long-termism Pareto dominates and, thus, coordination failure is less likely. Therefore, the model predicts that, if the channel is *exit*, the increase in long-term investments associated with the presence of outside blockholders will be

accompanied with (i) executive pay being linked more to stock prices and (ii) and prices being more informative about the firm's long-term value.

2.7 Conclusion

This chapter studies the interaction between executive compensation and stock market prices, analyzing its implications for corporate short-termism. I show that inefficient short-termism can arise in equilibrium as a self-fulfilling prophecy, due to strategic complementarities between the firm's investment horizon and investors' decision to acquire information about short-term performance or long-term value. This uncovers a new mechanism by which the stock market can feed corporate short-termism through an excessive focus on short-term performance. At the same time, the analysis suggests that the real cause of corporate short-termism is the underlying agency problem between shareholders and management. Moreover, far from being the primary culprit, the stock market can be an alleviating force.

Bibliography

[1] Bebchuk, L. A. and Stole, L. A., 1993, Do short-term objectives lead to under or overinvestment in long-term projects? Journal of Finance, 48, 719–729.

[2] Biden, J. (2016). How short-termism saps the economy. Wall Street Journal, 27 Sep 2016.

[3] Bolton, P., Scheinkman, J., and Xiong, W., 2006, Executive compensation and short-termist behaviour in speculative markets, Review of Economic Studies, 73, 577–610.

[4] Bond, P., Edmans A., and Goldstein I., 2012, The Real Effects of Financial Markets, Annual Review of Financial Economics, 4, 339-360.

[5] Bushee, Brian, 1998, The influence of institutional investors on myopic R&D investment behavior, The Accounting Review 73, 305–333.

[6] Budish, E., Roin, B. N., and Williams, H., 2015. Do firms underinvest in long-term research? Evidence from cancer clinical trials, American Economic Review, 105, 2044–85.

[7] Burns, Natasha, Simi Kedia, and Marc Lipson, 2010, Institutional ownership and monitoring: Evidence from financial reporting practices, Journal of Corporate Finance, 16, 443-455.

[8] Core, John E., and Wayne R. Guay, 2001, Stock option plans for non-executive employees, Journal of Financial Economics 61, 253–287.

[9] Cronqvist, Henrik, and Rudiger Fahlenbrach, 2009, Large shareholders and corporate policies, Review of Financial Studies, 22, 3941-3976.

[10] Dechow, Patricia, Richard Sloan, and Amy Sweeney, 1996, Causes and consequences of earnings manipulation: An analysis of firms subject to enforcement actions by the SEC, Contemporary Accounting Research 13, 1–36.

[11] Dimon, J. and Buffet, W. E. 2018. Short-termism is harming the economy. Wall Street Journal, 07 June 2018.

[12] Dow, J., Goldstein, I., Guembel, A., 2017, Incentives for Information Production in Markets where Prices Affect Real Investment, Journal of the European Economic Association, Volume 15(4), 877–909.

[13] Edmans, A., 2009, Blockholder Trading, Market Efficiency, and Managerial Myopia. The Journal of Finance, 64, 2481-2513

[14] Edmans, A., Fang, V. W., and Huang, A., 2017a, The long-term consequences of short-term incentives, Working Paper.

[15] Edmans, A., Fang, V. W., and Lewellen, K. A., 2017b, Equity vesting and investment, Review of Financial Studies, 30, 2229–2271.

[16] Edmans, A., Holderness, C. G., 2017, Blockholders: A Survey of Theory and Evidence, The Handbook of the Economics of Corporate Governance, 1, 541-636.

[17] Farber, David, 2005, Restoring trust after fraud: Does corporate governance matter? The Accounting Review 80, 539–561.

[18] Fudenberg D, Tirole J. Game Theory. Cambridge, MA: MIT Press; 1991.

[19] Gopalan, R., Milbourn, T., Song, F., and Thakor, A. V. (2014). Duration of executive compensation. Journal of Finance, 69(6):2777–2817.

[20] Graham, John, Campbell Harvey, and Shivaram Rajgopal, 2005, The economic implications of corporate financial reporting, Journal of Accounting and Economics 40, 3–73.

[21] Grossman, Sanford J., and Oliver D. Hart, 1983. An Analysis of the Principal-Agent Problem. Econometrica, vol. 51(1):7–45.

[22] Hansen, Gary, and Charles Hill, 1991, Are institutional investors myopic? A time-series study of four technology-driven industries, Strategic Management Journal 12, 1–16.

[23] Harsanyi, J. C., and Selten, R., 1988, A General Theory of Equilibrium Selection in Games, MIT Press.

[24] Holderness, Clifford, 2009, The myth of diffuse ownership in the United States, Review of Financial Studies 22, 1377–1408.

[25] Holmstrom, B., and Tirole, J., 1993, Market liquidity and performance monitoring, Journal of Political Economy 101, 678–709.

[26] Ittner, Christopher D., Richard A. Lambert, and David F. Larcker, 2003, The structure and performance consequences of equity grants to employees of new economy firms, Journal of Accounting and Economics 34, 89–127.

[27] Kaplan, Steven, N., 2018, Are US companies too short-term oriented? Some thoughts, Innovation Policy and the Economy, 18, 107–124.

[28] Kyle, Albert, 1985, Continuous auctions and insider trading, Econometrica 53, 1315–1335.

[29] La Porta, Rafael, Florencio Lopez-de-Silanes, and Andrei Shleifer, 1999, Corporate ownership around the world, Journal of Finance 54, 471–517.

[30] Lipton, M., 1979, Takeover Bids in the Target's Boardroom, The Business Lawyer, 35, 101-134.

[31] Murphy, Kevin J., 1999, Executive compensation, in David Card, and Orley Ashenfelter, eds.: Handbook of Labor Economics, (Elsevier, Amsterdam).

[32] Narayanan, M. P., 1985, Managerial incentives for short-term results, Journal of Finance 40, 1469–1484.

[33] Peng, L. and Roell, A. (2014). Managerial incentives and stock price manipulation. Journal of Finance, 69(2):487–526.

[34] Roe, Mark J., 2018, Stock Market Short-Termism's Impact, Working Paper.

[35] Stein, Jeremy, 1989, Efficient capital markets, inefficient firms: A model of myopic corporate behavior, Quarterly Journal of Economics 104, 655–669.

[36] Thakor, Richard, 2018, A theory of efficient short-termism, Working Paper.

[37] von Thadden, Ernst-Ludwig, 1995, Long-Term Contracts, Short-Term Investment and Monitoring, The Review of Economic Studies, 62, 557–575.

[38] Wahal, Sunil, and John McConnell, 2000, Do institutional investors exacerbate managerial myopia? Journal of Corporate Finance 6, 307–329.

2.8 Appendix A

2.8.1 Proof of Proposition 13

Equilibrium with short-termism

Trading Strategies. Consider the case when the market-maker and speculator expect the manager to undertake the short-term project (i.e., $\overline{\theta}_1 \sim N\left(\mu, \sigma_\theta^2\right)$, $\overline{\theta}_2 = 0$; Part 1 of Proposition 13). As a consequence, the speculator acquires information only about short-term earnings π_1. The market-maker posits the following demand strategy for the speculator:

$$\overline{x}^* = \phi_1^* \pi_1 + k^*. \tag{2.20}$$

Let $\widehat{\pi}_1, \widehat{\pi}_2$, and $\widehat{\theta}_1$ represent the expected values of π_1, π_2, and θ_1, respectively, according to the conjectures about project choice. The market maker observes total demand q and sets a price

$$p = E\left[\pi_1 + \pi_2 - w \mid \overline{x}^* + u = q\right], \tag{2.21}$$

which implies

$$\begin{aligned} p &= \widehat{\pi}_1 + \widehat{\pi}_2 - \left(\overline{\alpha} + \overline{\beta}p + \overline{\gamma}_1\widehat{\pi}_1 + \overline{\gamma}_2\widehat{\pi}_2\right) + \frac{cov\left(\mathcal{V}, q\right)}{var\left(q\right)}q \\ &\Leftrightarrow \quad \left(1+\overline{\beta}\right)p = \left(1-\overline{\gamma}_1\right)\widehat{\pi}_1 + \left(1-\overline{\gamma}_2\right)\widehat{\pi}_2 - \overline{\alpha} + \underbrace{\frac{cov\left(\left(1-\overline{\gamma}_1\right)\pi_1 + \left(1-\overline{\gamma}_2\right)\pi_2, q\right)}{var\left(q\right)}}_{\equiv\lambda^*}q \\ &\Leftrightarrow \quad p = \frac{1}{1+\overline{\beta}}\left[\left(1-\overline{\gamma}_1\right)\widehat{\pi}_1 + \left(1-\overline{\gamma}_2\right)\widehat{\pi}_2 - \overline{\alpha} + \lambda^*\left(x+u\right)\right]. \end{aligned}$$

Equation (2.20) implies that

$$\lambda^* = \frac{\phi_1^*\left(1-\overline{\gamma}_1\right)\left(\sigma_\theta^2+1\right)}{\left(\phi_1^*\right)^2\left(\sigma_\theta^2+1\right)+\sigma_u^2}$$

The speculator's optimal trading strategy x^* solves

$$\max_x xE\left\{\mathcal{V} - p \mid \pi_1\right\} \tag{2.22}$$

We have:

$$\begin{aligned} \mathcal{V} - p &= \pi_1 + \pi_2 - \underbrace{\left(\overline{\alpha} + \overline{\beta}p + \overline{\gamma}_1\pi_1 + \overline{\gamma}_2\pi_2\right)}_{w} - p \\ &= \begin{array}{c} \left(1-\overline{\gamma}_1\right)\pi_1 + \left(1-\overline{\gamma}_2\right)\pi_2 - \overline{\alpha} \\ \underbrace{-\frac{1+\overline{\beta}}{1+\overline{\beta}}\left[\left(1-\overline{\gamma}_1\right)\widehat{\pi}_1 + \left(1-\overline{\gamma}_2\right)\widehat{\pi}_2 - \overline{\alpha} + \lambda^*\left(x+u\right)\right]}_{\left(1+\overline{\beta}\right)p} \end{array} \\ &= \left(1-\overline{\gamma}_1\right)\left(\pi_1 - \widehat{\pi}_1\right) + \left(1-\overline{\gamma}_2\right)\left(\pi_2 - \widehat{\pi}_2\right) - \lambda^*\left(x+u\right). \end{aligned} \tag{2.23}$$

This implies

$$E\left\{\mathcal{V} - p \mid \pi_1\right\} = \left(1-\overline{\gamma}_1\right)\left(\pi_1 - \widehat{\pi}_1\right) - \lambda^* x,$$

since $E\left(\pi_2\right) = \widehat{\pi}_2$ if the speculator doesn't have information about θ_2.

Therefore, the speculator's problem simplifies to

$$\max_x x\left[\left(1-\overline{\gamma}_1\right)\left(\pi_1 - \widehat{\pi}_1\right) - \lambda^* x\right]$$

The first-order condition gives us

$$(1-\overline{\gamma}_1)(\pi_1-\widehat{\pi}_1)-2\lambda^* x=0.$$

Thus, we have

$$x^*=\underbrace{\frac{1-\overline{\gamma}_1}{2\lambda^*}}_{\phi_1^*}(\pi_1-1),$$

since $\widehat{\pi}_1=1$ when $\overline{\theta}_1\sim N\left(1,\sigma_\theta^2\right)$.

The equilibrium values $\{\phi_1^*,k^*,\lambda^*\}$ solve the following system of equations:

$$\begin{cases}\lambda^*=\frac{\phi_1^*(1-\overline{\gamma}_1)\left(\sigma_\theta^2+1\right)}{(\phi_1^*)^2\left(\sigma_\theta^2+1\right)+\sigma_u^2};\\ \phi_1^*=\frac{1-\overline{\gamma}_1}{2\lambda^*};\quad k^*=-\frac{1-\overline{\gamma}_1}{2\lambda^*}.\end{cases}$$

We have:

$$\lambda^*=\frac{\frac{1}{2\lambda^*}(1-\overline{\gamma}_1)^2\left(\sigma_\theta^2+1\right)}{\left(\frac{1}{2\lambda^*}\right)^2\underbrace{(1-\overline{\gamma}_1)^2\left(\sigma_\theta^2+1\right)}_{\equiv\chi^*}+\sigma_u^2}$$

$$\Leftrightarrow\lambda^*=\left(\frac{2\lambda^*}{2\lambda^*}\right)^2\frac{\frac{1}{2\lambda^*}\chi^*}{\left(\frac{1}{2\lambda^*}\right)^2\chi^*+\sigma_u^2}\Leftrightarrow\lambda^*=\frac{2\lambda^*\chi^*}{\chi^*+(2\lambda^*)^2\sigma_u^2}$$

$$\Leftrightarrow\chi^*+(2\lambda^*)^2\sigma_u^2=2\chi^*.$$

This implies that

$$\lambda^* = \frac{1}{2\sigma_u}\left[(1-\overline{\gamma}_1)^2\left(\sigma_\theta^2+1\right)\right]^{\frac{1}{2}}; \tag{2.24}$$

$$x^* = \frac{1-\overline{\gamma}_1}{2\lambda^*}(\pi_1-1). \tag{2.25}$$

Price distribution. The equilibrium trading strategies described in equations (2.24) and (2.25) imply that, for given realizations of π_1 and u, the price is

$$\begin{aligned} p^* &= \frac{1}{1+\overline{\beta}}\left[(1-\overline{\gamma}_1)\underbrace{\widehat{\pi}_1}_{=1}+(1-\overline{\gamma}_2)\underbrace{\widehat{\pi}_2}_{=0}-\overline{\alpha}+\lambda\left(x+u\right)\right] \\ &= \frac{1}{1+\overline{\beta}}\left[1-\overline{\gamma}_1-\overline{\alpha}+\frac{1-\overline{\gamma}_1}{2}\left(\pi_1-1\right)+\lambda^* u\right]. \end{aligned}$$

The expected price is then

$$E\left(p^*\right)=\frac{1}{1+\overline{\beta}}\left[1-\overline{\gamma}_1-\overline{\alpha}+\frac{1-\overline{\gamma}_1}{2}\left[E\left(\pi_1\right)-1\right]+\lambda^* u\right].$$

If the market's conjecture that the firm implements the short-term project is consistent with the play of the game, we have $E\left(\pi_1\right)=1$ and, thus, the speculator expected demand is zero, i.e., $E\left(x^*\right)=0$. This implies

$$E\left(p^*\right)=\frac{1}{1+\overline{\beta}}\left[1-\overline{\gamma}_1-\overline{\alpha}\right].$$

Price volatility is instead

$$\begin{aligned} Var\left(p^*\right) &= \frac{1}{\left(1+\overline{\beta}\right)^2}\left[\left(\frac{1-\overline{\gamma}_1}{2}\right)^2 Var\left(\pi_1\right)+\left(\lambda^*\sigma_u\right)^2\right] \\ &= \frac{1}{\left(1+\overline{\beta}\right)^2}\left[\left(\frac{1-\overline{\gamma}_1}{2}\right)^2 Var\left(\pi_1\right)+\left(\frac{1-\overline{\gamma}_1}{2}\right)^2\left(\sigma_\theta^2+1\right)\right]. \end{aligned}$$

If the market's conjecture that the firm implements the short-term project is consistent with the play of the game, we have $Var\left(\pi_1\right)=Var\left(\theta_1+\eta_1\right)=\sigma_\theta^2+1$. This implies

$$Var\left(p^*\right)=\frac{1}{\left(1+\overline{\beta}\right)^2}\underbrace{\frac{1}{2}}_{\substack{\text{Monopoly}\\ \text{Distortion}}}\left(1-\overline{\gamma}_1\right)^2\left(\sigma_\theta^2+1\right);$$

$$Cov\left(\pi_1,p^*\right)=\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}\left(\sigma_\theta^2+1\right).$$

Speculator's expected profit. Let ER^* denote the speculator's ex-ante profit from trading in the equilibrium with short-termism. We have

$$ER^* = E\left\{x^*\left(\mathcal{V} - p^*\right)\right\} - g_1.$$

Equations (2.23) and (2.25) imply that

$$\begin{aligned}
\mathcal{V} - p^* &= \left(1-\overline{\gamma}_1\right)\left(\pi_1 - 1\right) + \left(1-\overline{\gamma}_2\right)\pi_2 - \lambda^*\left(x^* + u\right). \\
&= \left(1-\overline{\gamma}_1\right)\left(\pi_1 - 1\right) + \left(1-\overline{\gamma}_2\right)\pi_2 - \frac{1-\overline{\gamma}_1}{2}\left(\pi_1 - 1\right) - \lambda^* u \\
&= \left(1-\overline{\gamma}_2\right)\pi_2 + \underbrace{\frac{1-\overline{\gamma}_1}{2}\left(\pi_1 - 1\right)}_{x^*\lambda^*} - \lambda^* u.
\end{aligned}$$

Therefore, we can write:

$$\begin{aligned}
E\left\{x^*\left(\mathcal{V} - p^*\right)\right\} &= E\left\{x^*\left[\left(1-\overline{\gamma}_2\right)\pi_2 + x^*\lambda^* - \lambda^* u\right]\right\} \\
&= \lambda^* Var\left(x^*\right),
\end{aligned}$$

since $E\left(x^*\right) = E\left(u\right) = 0$ and $E\left\{x^*\pi_2\right\} = 0$.

This implies

$$\begin{aligned}
ER^* &= \lambda^*\left(\frac{1-\overline{\gamma}_1}{2\lambda^*}\right)^2\left(\sigma_\theta^2 + 1\right) - g_1 \\
&= \frac{1}{4}\underbrace{\frac{\left(1-\overline{\gamma}_1\right)^2\left(\sigma_\theta^2+1\right)}{\frac{1}{2\sigma_u}\left[\left(1-\overline{\gamma}_1\right)^2\left(\sigma_\theta^2+1\right)\right]^{\frac{1}{2}}}}_{\lambda^*} - g_1 \\
&= \frac{\sigma_u}{2}\left[\left(1-\overline{\gamma}_1\right)^2\left(\sigma_\theta^2+1\right)\right]^{\frac{1}{2}} - g_1.
\end{aligned}$$

As we will see, in an equilibrium with short-termism we have $\overline{\gamma}_1 = c$. For ER^* to be positive,

we need

$$\frac{\sigma_u}{2}\left[(1-c)^2\left(\sigma_\theta^2+1\right)\right]^{\frac{1}{2}} > g_1$$
$$\sigma_u^2 > 4\frac{g_1^2}{(1-c)^2\left(\sigma_\theta^2+1\right)}.$$

Assumption 3 ensures that the above inequality is satisfied.

Equilibrium with long-termism

Trading Strategies. Consider now the case when the market-maker and speculator expect the manager to undertake the long-term project (i.e., $\overline{\theta}_1 = 0$, $\overline{\theta}_2 \sim N\left(\mu, \sigma_\theta^2\right)$; Part 2 of Proposition 13). As a consequence, the speculator acquires information about both short-term earnings π_1 and the long-term project θ_2. The market-maker posits the following demand strategy for the speculator:

$$\overline{x}^{**} = \phi_1^{**}\pi_1 + \phi_2^{**}\theta_2 + k^{**}. \tag{2.26}$$

Let $\widehat{\pi}_1, \widehat{\pi}_2$, and $\widehat{\theta}_2$ represent the expected values of π_1, π_2, and θ_2, respectively, according to the conjectures about project choice. The market maker observes total demand q and sets a price

$$p = E\left[\pi_1 + \pi_2 - w \mid \overline{x}^{**} + u = q\right], \tag{2.27}$$

which implies

$$\begin{aligned}
p &= \widehat{\pi}_1 + \widehat{\pi}_2 - \left(\overline{\alpha} + \overline{\beta}p + \overline{\gamma}_1\widehat{\pi}_1 + \overline{\gamma}_2\widehat{\pi}_2\right) + \frac{cov\left(\mathcal{V}, q\right)}{var\left(q\right)}q \\
&\Leftrightarrow \quad \left(1+\overline{\beta}\right)p = \left(1-\overline{\gamma}_1\right)\widehat{\pi}_1 + \left(1-\overline{\gamma}_2\right)\widehat{\pi}_2 - \overline{\alpha} + \underbrace{\frac{cov\left(\left(1-\overline{\gamma}_1\right)\pi_1 + \left(1-\overline{\gamma}_2\right)\pi_2, q\right)}{var\left(q\right)}}_{\equiv\lambda^{**}}q \\
&\Leftrightarrow \quad p = \frac{1}{1+\overline{\beta}}\left[\left(1-\overline{\gamma}_1\right)\widehat{\pi}_1 + \left(1-\overline{\gamma}_2\right)\widehat{\pi}_2 - \overline{\alpha} + \lambda^{**}\left(x+u\right)\right].
\end{aligned}$$

Equation (2.26) implies that

$$\lambda^{**} = \frac{\phi_1^{**}\left(1-\overline{\gamma}_1\right) + \phi_2^{**}\left(1-\overline{\gamma}_2\right)\sigma_\theta^2}{\left(\phi_1^{**}\right)^2 + \left(\phi_2^{**}\right)^2\sigma_\theta^2 + \sigma_u^2}$$

The speculator solves $\max_x xE\left\{\mathcal{V}-p \mid \pi_1, \theta_2\right\}$; we have

$$\begin{aligned} \mathcal{V}-p &= \pi_1+\pi_2-\left(\alpha+\overline{\beta}p+\overline{\gamma}_1\pi_1+\overline{\gamma}_2\pi_2\right)-p \\ &= \underbrace{\begin{array}{c}\left(1-\overline{\gamma}_1\right)\pi_1+\left(1-\overline{\gamma}_2\right)\pi_2-\alpha \\ -\dfrac{1+\overline{\beta}}{1+\overline{\beta}}\left[\left(1-\overline{\gamma}_1\right)\widehat{\pi}_1+\left(1-\overline{\gamma}_2\right)\widehat{\pi}_2-\alpha+\lambda^{**}\left(x+u\right)\right]\end{array}}_{(1+\overline{\beta})p} \\ &= \left(1-\overline{\gamma}_1\right)\left(\pi_1-\widehat{\pi}_1\right)+\left(1-\overline{\gamma}_2\right)\left(\pi_2-\widehat{\pi}_2\right)-\lambda^{**}\left(x+u\right) \end{aligned} \tag{2.28}$$

This implies

$$E\left\{\mathcal{V}-p \mid \pi_1, \theta_2\right\} = \left(1-\overline{\gamma}_1\right)\left(\pi_1-\widehat{\pi}_1\right)+\left(1-\overline{\gamma}_2\right)\left(\theta_2-\widehat{\pi}_2\right)-\lambda^{**}x.$$

Therefore, the speculator's problem is

$$\max_x x\left[\left(1-\overline{\gamma}_1\right)\left(\pi_1-\widehat{\pi}_1\right)+\left(1-\overline{\gamma}_2\right)\left(\theta_2-\widehat{\pi}_2\right)-\lambda^{**}x\right]$$

The first-order condition gives us

$$\left(1-\overline{\gamma}_1\right)\left(\pi_1-\widehat{\pi}_1\right)+\left(1-\overline{\gamma}_2\right)\left(\theta_2-\widehat{\pi}_2\right)-2\lambda^{**}x = 0.$$

Thus, we have

$$x^{**} = \underbrace{\frac{1-\overline{\gamma}_1}{2\lambda^{**}}}_{\phi_1^{**}}\pi_1 + \underbrace{\frac{1-\overline{\gamma}_2}{2\lambda^{**}}}_{\phi_2^{**}}\left(\theta_2-\mu\right),$$

since $\widehat{\pi}_1 = 0$ and $\widehat{\pi}_2 = \mu$ when $\overline{\theta}_1 = 0$ and $\overline{\theta}_2 \sim N\left(\mu, \sigma_\theta^2\right)$.

The equilibrium values $\{\phi_1^{**}, \phi_2^{**}, k^{**}, \lambda^{**}\}$ solve:

$$\begin{cases} \lambda^{**} = \frac{\phi_1^{**}(1-\overline{\gamma}_1)+\phi_2^{**}(1-\overline{\gamma}_2)\sigma_\theta^2}{(\phi_1^{**})^2+(\phi_2^{**})^2\sigma_\theta^2+\sigma_u^2}; \\ \phi_1^{**} = \frac{1-\overline{\gamma}_1}{2\lambda^{**}}; \quad \phi_2^{**} = \frac{1-\overline{\gamma}_2}{2\lambda^{**}}; \quad k^{**} = -\frac{1-\overline{\gamma}_2}{2\lambda^{**}}\mu. \end{cases}$$

We have:

$$\lambda^{**} = \frac{\frac{1}{2\lambda^{**}}\left[(1-\overline{\gamma}_1)^2 + (1-\overline{\gamma}_2)^2\sigma_2^2\right]}{\left(\frac{1}{2\lambda^{**}}\right)^2 \underbrace{\left[(1-\overline{\gamma}_1)^2 + (1-\overline{\gamma}_2)^2\sigma_2^2\right]}_{\equiv\chi^{**}} + \sigma_u^2}$$

$$\Leftrightarrow \lambda = \left(\frac{2\lambda^{**}}{2\lambda^{**}}\right)^2 \frac{\frac{1}{2\lambda^{**}}\chi^{**}}{\left(\frac{1}{2\lambda^{**}}\right)^2\chi^{**} + \sigma_u^2} \Leftrightarrow \lambda^{**} = \frac{2\lambda^{**}\chi^{**}}{\chi^{**} + (2\lambda^{**})^2\sigma_u^2}$$

$$\Leftrightarrow \chi^{**} + (2\lambda^{**})^2\sigma_u^2 = 2\chi^{**}.$$

This implies that

$$\lambda^{**} = \frac{1}{2\sigma_u}\left[(1-\overline{\gamma}_1)^2 + (1-\overline{\gamma}_2)^2\sigma_\theta^2\right]^{\frac{1}{2}}; \tag{2.29}$$

$$x^{**} = \frac{1-\overline{\gamma}_1}{2\lambda^{**}}\pi_1 + \frac{1-\overline{\gamma}_2}{2\lambda^{**}}(\theta_2 - \mu). \tag{2.30}$$

Price distribution. The equilibrium trading strategies described in equations (2.29) and (2.30) imply that, for given realizations of π_1,θ_2, and u, the price is

$$\begin{aligned} p^{**} &= \frac{1}{1+\overline{\beta}}\left[(1-\overline{\gamma}_1)\underbrace{\widehat{\pi}_1}_{=0} + (1-\overline{\gamma}_2)\underbrace{\widehat{\pi}_2}_{=\mu} - \overline{\alpha} + \lambda(x+u)\right] \\ &= \frac{1}{1+\overline{\beta}}\left[(1-\overline{\gamma}_2)\mu - \overline{\alpha} + \frac{1-\overline{\gamma}_1}{2}\pi_1 + \frac{1-\overline{\gamma}_2}{2}(\theta_2-\mu) + \lambda^{**}u\right]. \end{aligned}$$

The expected price is then

$$E(p^{**}) = \frac{1}{1+\overline{\beta}}\left[(1-\overline{\gamma}_2)\mu - \overline{\alpha} + \frac{1-\overline{\gamma}_1}{2}E(\pi_1) + \frac{1-\overline{\gamma}_2}{2}[E(\theta_2)-\mu]\right].$$

If the market's conjecture that the firm implements the long-term project is consistent with the

play of the game, we have $E(\pi_1) = 0$ and $E(\theta_2) = \mu$. This implies

$$E(p^{**}) = \frac{1}{1+\overline{\beta}}\left[(1-\overline{\gamma}_2)\mu - \overline{\alpha}\right].$$

Price volatility is instead

$$\begin{aligned} Var(p^{**}) &= \frac{1}{\left(1+\overline{\beta}\right)^2}\left[\left(\frac{1-\overline{\gamma}_1}{2}\right)^2 Var(\pi_1) + \left(\frac{1-\overline{\gamma}_2}{2}\right)^2 Var(\theta_2) + (\lambda^{**})^2\sigma_u^2\right] \\ &= \frac{1}{\left(1+\overline{\beta}\right)^2}\left[\begin{array}{c}\left(\frac{1-\overline{\gamma}_1}{2}\right)^2 Var(\pi_1) + \left(\frac{1-\overline{\gamma}_2}{2}\right)^2 Var(\theta_2) \\ + \left(\frac{1-\overline{\gamma}_1}{2}\right)^2\cdot 1 + \left(\frac{1-\overline{\gamma}_2}{2}\right)^2\sigma_\theta^2\end{array}\right]. \end{aligned}$$

If the market's conjecture that the firm implements the long-term project is consistent with the play of the game, we have $Var(\pi_1) = Var(\eta_1) = 1$ and $Var(\theta_2) = \sigma_\theta^2$. This implies

$$Var(p^{**}) = \frac{1}{\left(1+\overline{\beta}\right)^2}\underbrace{\frac{1}{2}}_{\substack{\text{Monopoly}\\ \text{Distortion}}}\left[(1-\overline{\gamma}_1)^2 + (1-\overline{\gamma}_2)^2\sigma_\theta^2\right];$$

$$Cov(\pi_2, p^{**}) = \frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)}\sigma_\theta^2.$$

Speculator's expected profit. Let ER^{**} denote the speculator's ex-ante profit from trading in the equilibrium with long-termism. We have

$$ER^{**} = E\left\{x^{**}\left(\mathcal{V} - p^{**}\right)\right\} - (g_1 + g_2)$$

Equations (2.28) and (2.30) imply that

$$\begin{aligned} \mathcal{V} - p^{**} &= (1-\overline{\gamma}_1)\pi_1 + (1-\overline{\gamma}_2)(\theta_2 - \mu) - \lambda^{**}(x^{**} + u). \\ &= (1-\overline{\gamma}_1)\pi_1 + (1-\overline{\gamma}_2)(\theta_2 - \mu) - \frac{1-\overline{\gamma}_1}{2}\pi_1 + \frac{1-\overline{\gamma}_2}{2}(\theta_2 - \mu) - \lambda^{**}u \\ &= \underbrace{\frac{1-\overline{\gamma}_1}{2}\pi_1 + \frac{1-\overline{\gamma}_2}{2}(\theta_2 - \mu)}_{x^{**}\lambda^{**}} - \lambda^{**}u. \end{aligned}$$

Therefore, we can write:

$$\begin{aligned} E\left\{x^{**}\left(\mathcal{V}-p^{**}\right)\right\} &= E\left\{x^{**}\left(x^{**}\lambda^{**}-\lambda^{**}u\right)\right\} \\ &= \lambda^{**}Var\left(x^{**}\right), \end{aligned}$$

since both $E\left(x^{**}\right)$ and $E\left(u\right)$ are equal to zero.

This implies

$$\begin{aligned} ER^{**} &= \lambda^{**}\left[\left(\frac{1-\overline{\gamma}_1}{2\lambda^{**}}\right)^2+\left(\frac{1-\overline{\gamma}_2}{2\lambda^{**}}\right)^2\sigma_\theta^2\right]-\left(g_1+g_2\right) \\ &= \underbrace{\frac{1}{2\sigma_u}\left[\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2\right]^{\frac{1}{2}}}_{\lambda^{**}}\frac{\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2}{\frac{1}{2\sigma_u}\left[\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2\right]^{\frac{1}{2}}}-\left(g_1+g_2\right) \\ &= \frac{\sigma_u}{2}\left[\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2\right]^{\frac{1}{2}}-\left(g_1+g_2\right). \end{aligned}$$

As we will see, in an equilibrium with long-termism we have $\overline{\gamma}_1=0$ and $\overline{\gamma}_2<\frac{c}{\mu}$. For ER^{**} to be positive, we need

$$\begin{aligned} \frac{\sigma_u}{2}\left[1+\left(1-\frac{c}{\mu}\right)^2\sigma_\theta^2\right]^{\frac{1}{2}} &> g_1+g_2 \\ \sigma_u^2 &> 4\frac{\left(g_1+g_2\right)^2}{1+\left(1-\frac{c}{\mu}\right)^2\sigma_\theta^2}. \end{aligned}$$

Assumption 3 ensures that the above inequality is satisfied.

2.9 Appendix B

2.9.1 Proof of Lemma 14

Part 1 of Lemma 14

Suppose the market expects the manager to undertake the short-term project *and* exert high effort, i.e., $\overline{\theta}_1 \sim N\left(\overline{e}, \sigma_\theta^2\right)$, $\overline{\theta}_2 = 0$, and $\overline{e} = 1$. From Part 1 of Proposition 13, we have

$$p^* = \frac{1}{1+\overline{\beta}}\left[1 - \overline{\gamma}_1 - \overline{\alpha} + \underbrace{\frac{1-\overline{\gamma}_1}{2}(\pi_1 - 1)}_{x^*\lambda^*} + \lambda^* u\right].$$

First, suppose the market conjecture is correct and the manager undertakes the short-term project. This implies $\theta_1 \sim N\left(e, \sigma_\theta^2\right)$, $\theta_2 = 0$ and, thus, $\pi_1 \sim N\left(e, \sigma_\theta^2 + 1\right)$, $\pi_2 \sim N\left(0, \sigma_2^2\right)$ (since $\pi_1 = \theta_1 + \eta_1$ and $\pi_2 = \theta_2 + \eta_2$). In this case, the manager chooses to exert effort ($e = 1$) if the following condition is satisfied:

$$\begin{aligned}
&\beta \underbrace{\frac{1-\overline{\gamma}_1 - \overline{\alpha}}{1+\overline{\beta}}}_{E(p^*) \text{ when } e=1} - \frac{r}{2}\beta^2 \underbrace{\frac{1}{\left(1+\overline{\beta}\right)^2}\left[\left(\frac{1-\overline{\gamma}_1}{2}\right)^2 Var\left(\pi_1 \mid e=1\right) + \left(\lambda^*\sigma_u\right)^2\right]}_{Var(p^*)} + \\
&\qquad + \gamma_1 \cdot 1 - \frac{r}{2}\gamma_1^2\left(\sigma_\theta^2 + 1\right) + \gamma_2 \cdot 0 - \frac{r}{2}\gamma_2^2\sigma_2^2 - c \\
&\geq \beta \underbrace{\frac{1-\overline{\gamma}_1 - \overline{\alpha} - \overbrace{\frac{1-\overline{\gamma}_1}{2}}^{E[x^*\lambda^* \mid e=0]}}{1+\overline{\beta}}}_{E(p^*) \text{ when } e=0} - \frac{r}{2}\beta^2 \underbrace{\frac{1}{\left(1+\overline{\beta}\right)^2}\left[\left(\frac{1-\overline{\gamma}_1}{2}\right)^2 Var\left(\pi_1 \mid e=0\right) + \left(\lambda^*\sigma_u\right)^2\right]}_{Var(\widetilde{p})} \\
&\qquad + \gamma_1 \cdot 0 - \frac{r}{2}\gamma_1^2\left(\sigma_\theta^2 + 1\right) + \gamma_2 \cdot 0 - \frac{r}{2}\gamma_2^2\sigma_2^2.
\end{aligned}$$

The left-hand (right-hand) side of the inequality above represents the manager's payoff when he exerts $e = 1$ ($e = 0$). Notice that $Var\left(\pi_1 \mid e = 1\right) = Var\left(\pi_1 \mid e = 0\right)$, since effort only affects the mean of θ_1 but not its volatility. Therefore, deviating to $e = 0$ reduces the manager's expected

pay but not its volatility. The inequality above simplifies to

$$\gamma_1 - c \geq -\beta \frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}. \tag{2.31}$$

The inequality in (2.31) describes the incentive constraint in Part 1.a of Lemma 14.

Suppose now the market conjecture is wrong and the manager undertakes the long-term project instead. This implies $\theta_1 = 0$, $\theta_2 \sim N\left(\mu e, \sigma_\theta^2\right)$ and, thus, $\pi_1 \sim N(0,1)$, $\pi_2 \sim N\left(\mu e, \sigma_\theta^2 + \sigma_2^2\right)$. Notice that the stock price p^* does not incorporate any information about the true distribution of θ_2, since the speculator expects that the firm takes the short-term project and, thus, only acquires information about short-term earnings π_1. In this case, the stock price does not have any effect on the manager's incentives to exert effort. The manager chooses high effort if the following condition is satisfied:

$$\begin{aligned}
&\beta E\left(p^*\right) - \frac{r}{2}\beta^2 Var\left(p^*\right) + \gamma_1 \cdot 0 - \frac{r}{2}\gamma_1^2 + \gamma_2\mu - \frac{r}{2}\gamma_2^2\left(\sigma_\theta^2 + \sigma_2^2\right) - c \\
&\geq \beta E\left(p^*\right) - \frac{r}{2}\beta^2 Var\left(p^*\right) + \gamma_1 \cdot 0 - \frac{r}{2}\gamma_1^2 + \gamma_2 \cdot 0 - \frac{r}{2}\gamma_2^2\left(\sigma_\theta^2 + \sigma_2^2\right).
\end{aligned}$$

The inequality above simplifies to $\gamma_2\mu - c \geq 0$, which describes the incentive constraint in Part 1.b of Lemma 14.

Part 2 of Lemma 14

Suppose the market expects the manager to undertake the long-term project *and* exert high effort, i.e., $\overline{\theta}_1 = 0$, $\overline{\theta}_2 \sim N\left(\mu\overline{e}, \sigma_\theta^2\right)$, and $\overline{e} = 1$. From Part 2 of Proposition 13, we have

$$p^{**} = \frac{1}{1+\overline{\beta}}\left[\left(1-\overline{\gamma}_2\right)\mu - \overline{\alpha} + \underbrace{\frac{1-\overline{\gamma}_1}{2}\pi_1 + \frac{1-\overline{\gamma}_2}{2}\left(\theta_2 - \mu\right)}_{x^{**}\lambda^{**}} + \lambda^{**}u\right].$$

First, suppose the market conjecture is correct and the manager undertakes indeed the long-term project. This implies $\theta_1 = 0$, $\theta_2 \sim N\left(\mu e, \sigma_\theta^2\right)$ and, thus, $\pi_1 \sim N(0,1)$, $\pi_2 \sim N\left(\mu e, \sigma_\theta^2 + \sigma_2^2\right)$ (since $\pi_1 = \theta_1 + \eta_1$ and $\pi_2 = \theta_2 + \eta_2$). In this case, the manager chooses to

exert effort ($e = 1$) if the following condition is satisfied:

$$\beta \underbrace{\frac{(1-\overline{\gamma}_2)\,\mu - \overline{\alpha} + \overbrace{0}^{E[x^{**}\lambda^{**}|e=1]}}{1+\overline{\beta}}}_{E(p^{**}) \text{ when } e=1} - \frac{r}{2}\beta^2 \frac{1}{\left(1+\overline{\beta}\right)^2} Var\left(p^{**}\right) +$$
$$+\gamma_1 \cdot 0 - \frac{r}{2}\gamma_1^2 + \gamma_2\mu - \frac{r}{2}\gamma_2^2\left(\sigma_\theta^2 + \sigma_2^2\right) - c$$
$$\geq \beta \underbrace{\frac{(1-\overline{\gamma}_2)\,\mu - \overline{\alpha} - \overbrace{\frac{1-\overline{\gamma}_2}{2}\mu}^{E[x^{**}\lambda^{**}|e=0]}}{1+\overline{\beta}}}_{E(p^{**}) \text{ when } e=0} - \frac{r}{2}\beta^2 \frac{1}{\left(1+\overline{\beta}\right)^2} Var\left(p^{**}\right)$$
$$+\gamma_1 \cdot 0 - \frac{r}{2}\gamma_1^2 + \gamma_2 \cdot 0 - \frac{r}{2}\gamma_2^2\left(\sigma_\theta^2 + \sigma_2^2\right).$$

The left-hand (right-hand) side of the inequality above represents the manager's payoff when he exerts $e = 1$ ($e = 0$). Like before, effort only affects the mean of θ_2 but not its volatility and, thus, $Var\left(p^{**}\right)$ is the same on both sides of the inequality. The inequality above simplifies to

$$\gamma_2\mu - c \geq -\beta \frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)}\mu. \tag{2.32}$$

The inequality in (2.32) describes the incentive constraint in Part 2.b of Lemma 14.

Suppose now the market conjecture is wrong and the manager undertakes the short-term project instead. This implies $\theta_1 \sim N\left(e, \sigma_\theta^2\right)$, $\theta_2 = 0$ and, thus, $\pi_1 \sim N\left(e, \sigma_\theta^2 + 1\right)$, $\pi_2 \sim N\left(0, \sigma_2^2\right)$. The stock price p^{**} incorporates information about both π_1 and θ_2. Therefore, the speculator's expected demand is

$$E\left(x^{**}\right) = \frac{1-\overline{\gamma}_1}{2\lambda^{**}} \underbrace{e}_{E(\pi_1|e)} + \frac{1-\overline{\gamma}_2}{2\lambda^{**}} \underbrace{(-\mu)}_{E(\theta_2-\mu|\theta_2=0)}.$$

The manager chooses $e = 1$ if the following condition is satisfied:

$$\beta \underbrace{\frac{(1-\overline{\gamma}_2)\,\mu - \overline{\alpha} + \overbrace{\frac{1-\overline{\gamma}_1}{2}1 - \frac{1-\overline{\gamma}_2}{2}\mu}^{E[x^{**}\lambda^{**}|e=1]}}{1+\overline{\beta}}}_{E(p^{**}) \text{ when } e=1} - \frac{r}{2}\beta^2 \frac{1}{\left(1+\overline{\beta}\right)^2} Var\left(p^{**}\right) +$$

$$+\gamma_1 \cdot 1 - \frac{r}{2}\gamma_1^2\left(\sigma_\theta^2+1\right) + \gamma_2 \cdot 0 - \frac{r}{2}\gamma_2^2\sigma_2^2 - c$$

$$\geq \beta \underbrace{\frac{(1-\overline{\gamma}_2)\,\mu - \overline{\alpha} - \overbrace{\frac{1-\overline{\gamma}_2}{2}\mu}^{E[x^{**}\lambda^{**}|e=0]}}{1+\overline{\beta}}}_{E(p^{**}) \text{ when } e=0} - \frac{r}{2}\beta^2 \frac{1}{\left(1+\overline{\beta}\right)^2} Var\left(p^{**}\right)$$

$$+\gamma_1 \cdot 0 - \frac{r}{2}\gamma_1^2\left(\sigma_\theta^2+1\right) + \gamma_2 \cdot 0 - \frac{r}{2}\gamma_2^2\sigma_2^2.$$

The inequality above simplifies to

$$\gamma_1 + \beta \frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)} - c \geq 0,$$

which describes the incentive constraint in Part 2.a of Lemma 14.

2.9.2 Proof of Proposition 15

Optimal contract for the short-term project (Part 1 of Proposition 15)

Shareholders take the market's conjectures about the contract as given when solving for the optimal contract. The optimal contract for the short-term project solves:

$$\min_{\beta\geq 0,\gamma_1\geq 0} \beta^2 Var\left(p^*\right) + \gamma_1^2 Var\left(\theta_1+\eta_1\right) + 2\beta\gamma_1 Cov\left(\theta_1+\eta_1, p^*\right) \tag{2.33}$$

$$\text{subject to} \quad \beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}} + \gamma_1 \geq c.$$

First, suppose that the market correctly anticipates the choice of the project, i.e., $\overline{\theta}_1 \sim N\left(1, \sigma_\theta^2\right)$, $\overline{\theta}_2 = 0$. Given the distribution of p^* described in Proposition (13), the problem in

(2.33) simplifies to:

$$\min_{\beta\geq 0,\gamma_1\geq 0} \gamma_1^2\left(\sigma_\theta^2+1\right)+\beta^2\frac{\left(1-\overline{\gamma}_1\right)^2}{2\left(1+\overline{\beta}\right)^2}\left(\sigma_\theta^2+1\right)+2\gamma_1\beta\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}\left(\sigma_\theta^2+1\right) \tag{2.34}$$

$$\text{subject to} \quad \beta\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}+\gamma_1\geq c. \tag{2.35}$$

The Lagrangian function for this problem is

$$\mathcal{L}^{st}=\gamma_1^2+\beta^2\frac{\left(1-\overline{\gamma}_1\right)^2}{2\left(1+\overline{\beta}\right)^2}+\gamma_1\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}-\Lambda^{st}\left[\beta\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}+\gamma_1-c\right],$$

where Λ^{st} denotes the Lagrangian multiplier.

The first-order conditions are

$$\begin{aligned} 2\gamma_1+\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}-\Lambda^{st}&=0;\\ \beta\left(\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\right)^2+\gamma_1\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}-\Lambda^{st}\frac{1}{2}\frac{1-\overline{\gamma}_1}{\left(1+\overline{\beta}\right)}&=0. \end{aligned} \tag{2.36}$$

Notice that equation (2.36) can be rearranged as

$$2\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}+2\gamma_1-\Lambda^{st}=0.$$

Therefore, we can rewrite the first order conditions as

$$2\gamma_1+\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}-\Lambda^{st} = 0; \tag{2.37}$$

$$2\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}+2\gamma_1-\Lambda^{st} = 0. \tag{2.38}$$

The first order conditions in (2.37) and (2.38) together imply:

$$2\gamma_1+\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}-\Lambda^{st}=2\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}+2\gamma_1-\Lambda^{st} \quad \Leftrightarrow \beta=0,$$

since $\overline{\gamma}_1\leq c<1$.

Therefore, when $\overline{\theta}_1 \sim N\left(1, \sigma_\theta^2\right)$, $\overline{\theta}_2 = 0$, the optimal contract for the short-term project features $\beta^* = \gamma_2^* = 0$; the value of γ_1^* is pinned down by the incentive constraint, i.e., $\gamma_1^* = c$, and the fixed component of pay α^* is chosen so that the manager's participation constraint in equation (2.12) is binding.[23]

Now, suppose the market (mistakenly) expects the firm to implement the long-term project and, thus, $\overline{\theta}_1 = 0$, $\overline{\theta}_2 \sim N\left(\mu, \sigma_\theta^2\right)$. The true distributions are instead $\theta_1 \sim N\left(1, \sigma_\theta^2\right)$, $\theta_2 = 0$. From Proposition 13, we have:

$$p^{**} = \frac{1}{1+\overline{\beta}}\left[\left(1-\overline{\gamma}_2\right)\mu - \overline{\alpha} + \frac{1-\overline{\gamma}_1}{2}\pi_1 + \frac{1-\overline{\gamma}_2}{2}\left(\theta_2 - \mu\right) + \lambda^{**}u\right],$$

and, thus,

$$Var\left(p^{**}\right) = \frac{1}{\left(1+\overline{\beta}\right)^2}\left[\left(\frac{1-\overline{\gamma}_1}{2}\right)^2 \underbrace{\left(\sigma_\theta^2+1\right)}_{Var(\pi_1)} + \underbrace{\left(\frac{1-\overline{\gamma}_1}{2}\right)^2 \cdot 1 + \left(\frac{1-\overline{\gamma}_2}{2}\right)^2 \sigma_\theta^2}_{(\lambda^{**}\sigma_u)^2}\right];$$

$$Cov\left(\pi_1, p^{**}\right) = \frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}\left(\sigma_\theta^2+1\right).$$

In this case, the problem in (2.33) simplifies to

$$\min_{\beta\geq 0, \gamma_1 \geq 0} \gamma_1^2\left(\sigma_\theta^2+1\right) + \beta^2 \frac{\left(1-\overline{\gamma}_1\right)^2}{2\left(1+\overline{\beta}\right)^2}\left[\sigma_\theta^2 \underbrace{\frac{1+\left(\frac{1-\overline{\gamma}_2}{1-\overline{\gamma}_1}\right)^2}{2}}_{\equiv\Sigma} + 1\right] + 2\gamma_1\beta\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}\left(\sigma_\theta^2+1\right) \quad (2.39)$$

$$\text{subject to} \quad \beta\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)} + \gamma_1 \geq c. \quad (2.40)$$

[23] The manager's risk premium under the optimal contract is $\frac{r}{2}c^2\left(\sigma_\theta^2+1\right)$, since $\gamma_1^* = c$. If the shareholders where to set a contract with $\gamma_1 = 0$ and $\beta = 2c\frac{1+\overline{\beta}}{1-\overline{\gamma}_1}$ (from the incentive constraint), the risk-premium would be instead

$$\frac{r}{2}\left(2c\frac{1+\overline{\beta}}{1-\overline{\gamma}_1}\right)^2 \frac{\left(1-\overline{\gamma}_1\right)^2}{2\left(1+\overline{\beta}\right)^2}\left(\sigma_\theta^2+1\right) = \frac{r}{2}2c^2\left(\sigma_\theta^2+1\right).$$

The Lagrangian function for this problem is

$$\widetilde{\mathcal{L}}^{st}=\gamma_1^2\left(\sigma_\theta^2+1\right)+\beta^2\frac{\left(1-\overline{\gamma}_1\right)^2}{2\left(1+\overline{\beta}\right)^2}\left(\sigma_\theta^2\Sigma+1\right)+\gamma_1\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\left(\sigma_\theta^2+1\right)-\widetilde{\Lambda}\left[\beta\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}+\gamma_1-c\right],$$

where $\widetilde{\Lambda}$ denotes the Lagrangian multiplier.

The first-order conditions are

$$\begin{aligned}
2\gamma_1\left(\sigma_\theta^2+1\right)+\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\left(\sigma_\theta^2+1\right)-\widetilde{\Lambda}&=0;\\
\beta\left(\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\right)^2\left(\sigma_\theta^2\Sigma+1\right)+\gamma_1\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\left(\sigma_\theta^2+1\right)-\widetilde{\Lambda}\frac{1}{2}\frac{1-\overline{\gamma}_1}{\left(1+\overline{\beta}\right)}&=0. \qquad (2.41)
\end{aligned}$$

Notice that equation (2.41) can be rearranged as

$$2\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\left(\sigma_\theta^2\Sigma+1\right)+2\gamma_1\left(\sigma_\theta^2+1\right)-\widetilde{\Lambda}=0.$$

Therefore, we can rewrite the first order conditions as

$$\begin{aligned}
2\gamma_1\left(\sigma_\theta^2+1\right)+\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\left(\sigma_\theta^2+1\right)-\widetilde{\Lambda} &= 0; \qquad (2.42)\\
2\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\left(\sigma_\theta^2\Sigma+1\right)+2\gamma_1\left(\sigma_\theta^2+1\right)-\widetilde{\Lambda} &= 0. \qquad (2.43)
\end{aligned}$$

The first order conditions in (2.42) and (2.43) together imply:

$$\begin{aligned}
2\gamma_1\left(\sigma_\theta^2+1\right)+\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\left(\sigma_\theta^2+1\right)-\widetilde{\Lambda}&=2\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\left(\sigma_\theta^2\Sigma+1\right)+2\gamma_1\left(\sigma_\theta^2+1\right)-\widetilde{\Lambda}\\
\Leftrightarrow\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\left(\sigma_\theta^2+1\right)-2\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\left(\sigma_\theta^2\Sigma+1\right)&=0\\
\Leftrightarrow\beta\left[-1+\sigma_\theta^2\left(1-2\Sigma\right)\right]&=0.
\end{aligned}$$

Since $\Sigma>\frac{1}{2}$ and, thus, $1-2\Sigma<0$, we must have $\beta=0$ for the above equation to be satisfied. Therefore, the optimal contract for the short-term project features $\gamma_1^*=c$, and $\beta^*=\gamma_2^*=0$ regardless of the market conjecture about project choice.

Optimal contract for the long-term project (Part 2 of Proposition 15)

The optimal contract for the long-term project depends on whether the stock price incorporates information about θ_2.

If the speculator does not acquire information about θ_2 (this occurs when the speculator expects that the short-term project is implemented; see Proposition 13), we have $\gamma_1^\dagger = \beta^\dagger = 0$; $\gamma_2^\dagger$ is then pinned down by the incentive constraint, i.e., $\gamma_2^\dagger = \frac{c}{\mu}$.

If the speculator acquires information about θ_2, the optimal contract solves:

$$\min_{\beta\geq 0,\gamma_2\geq 0} \gamma_2^2\left(\sigma_\theta^2+\sigma_2^2\right)+\beta^2\underbrace{\frac{\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2}{2\left(1+\overline{\beta}\right)^2}}_{Var(p^{**})}+2\gamma_1\beta\underbrace{\frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)}\sigma_\theta^2}_{Cov(\theta_2+\eta_2,p^{**})} \tag{2.44}$$

$$\text{subject to}\quad \beta\frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)}+\gamma_2\geq\frac{c}{\mu}. \tag{2.45}$$

The Lagrangian function for this problem is

$$\mathcal{L}^{lt}=\gamma_1^2\left(\sigma_\theta^2+\sigma_2^2\right)+\beta^2\frac{\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2}{2\left(1+\overline{\beta}\right)^2}+2\gamma_1\beta\frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)}\sigma_\theta^2-\Lambda^{lt}\left[\beta\frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)}+\gamma_1-\frac{c}{\mu}\right],$$

where Λ^{lt} denotes the Lagrangian multiplier.

The first-order conditions are

$$2\gamma_2\left(\sigma_\theta^2+\sigma_2^2\right)+\beta\frac{1-\overline{\gamma}_2}{1+\overline{\beta}}\sigma_\theta^2-\Lambda^{lt}=0; \tag{2.46}$$

$$2\beta\frac{\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2}{2\left(1+\overline{\beta}\right)^2}+2\gamma_2\frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)}\sigma_\theta^2-\Lambda^{lt}\frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)}=0. \tag{2.47}$$

Equation (2.47) simplifies to

$$\underbrace{2\frac{\beta}{1+\overline{\beta}}}_{\equiv\widetilde{\beta}}\left[\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2\right]+2\gamma_2\left(1-\overline{\gamma}_2\right)\sigma_\theta^2-\Lambda^{lt}\left(1-\overline{\gamma}_2\right)=0$$

$$\Leftrightarrow 2\widetilde{\beta}\left[\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2\right]+2\gamma_2\left(1-\overline{\gamma}_2\right)\sigma_\theta^2=\Lambda^{lt}\left(1-\overline{\gamma}_2\right). \tag{2.48}$$

Using equations (2.46) and (2.48), we have

$$2\gamma_2\left(1-\overline{\gamma}_2\right)\left(\sigma_\theta^2+\sigma_2^2\right)+\widetilde{\beta}\left(1-\overline{\gamma}_2\right)\sigma_\theta^2=2\widetilde{\beta}\left[\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2\right]+2\gamma_2\left(1-\overline{\gamma}_2\right)\sigma_\theta^2$$
$$\Leftrightarrow 2\gamma_2\left(1-\overline{\gamma}_2\right)\sigma_2^2=\widetilde{\beta}\left[2\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2\right]. \tag{2.49}$$

The incentive constraint in (2.45) is binding and simplifies to

$$\beta\frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)}+\gamma_1=\frac{c}{\mu}\Leftrightarrow\widetilde{\beta}\left(1-\overline{\gamma}_2\right)=2\left(\frac{c}{\mu}-\gamma_2\right). \tag{2.50}$$

Combining equations (2.49) and (2.50), we have

$$2\gamma_2\left(1-\overline{\gamma}_2\right)\sigma_2^2=\underbrace{\frac{2\left(\frac{c}{\mu}-\gamma_2\right)}{1-\overline{\gamma}_2}}_{\widetilde{\beta}\text{ by }IC}\left[2\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2\right]$$
$$\Leftrightarrow\ \underbrace{\gamma_2\sigma_2^2-\left(\frac{c}{\mu}-\gamma_2\right)\left[2\left(\frac{1-\overline{\gamma}_1}{1-\overline{\gamma}_2}\right)^2+\sigma_\theta^2\right]}_{\equiv\Gamma(\gamma_2)}=0. \tag{2.51}$$

An optimal value of γ_2 must then satisfy $\Gamma\left(\gamma_2\right)=0$.

We have $\Gamma\left(0\right)<0$ and $\Gamma\left(\frac{c}{\mu}\right)>0$; this implies that $\gamma_2^{\dagger\dagger}\in\left(0,\frac{c}{\mu}\right)$ and, thus, that $\beta^{\dagger\dagger}>0$. Given $\gamma_2^{\dagger\dagger}$, $\beta^{\dagger\dagger}$ is pinned down by the incentive constraint (2.50); the fixed component of pay $\alpha^{\dagger\dagger}$ is then chosen so that the manager's participation constraint in equation (2.12) is binding. Notice that Γ is always increasing in γ_2: this implies that γ_2 such that $\Gamma\left(\gamma_2\right)=0$ is unique and, therefore, the optimal contract $(\alpha^{\dagger\dagger},\beta^{\dagger\dagger},\gamma_2^{\dagger\dagger})$ is unique.

Equation (2.51) can be rearranged as

$$\gamma_2^{\dagger\dagger}=\frac{c}{\mu}\frac{2\left(\frac{1-\overline{\gamma}_1}{1-\overline{\gamma}_2}\right)^2+\sigma_\theta^2}{2\left(\frac{1-\overline{\gamma}_1}{1-\overline{\gamma}_2}\right)^2+\sigma_2^2+\sigma_\theta^2}.$$

Notice that $\gamma_2^{\dagger\dagger}$ is decreasing in σ_2^2 and we have $\gamma_2^{\dagger\dagger}<\frac{c}{\mu}$.

Optimal Contract with Consistent Conjectures. Equation (2.51) characterizes the optimal value of γ_2 for given market's conjectures about the contract, i.e., for given $\left(\overline{\beta}, \overline{\gamma}_1, \overline{\gamma}_2\right)$. In equilibrium, these conjectures need to be consistent. Therefore, we need to evaluate equation (2.51) and the incentive constraint (2.50) at $\overline{\beta} = \beta^{**}$, $\overline{\gamma}_1 = \gamma_1^{**} = 0$, and $\overline{\gamma}_2 = \gamma_2^{**}$.

When $\overline{\beta} = \beta$, $\overline{\gamma}_1 = 0$, and $\overline{\gamma}_2 = \gamma_2$, the two equations (2.51) and (2.50) become

$$\underbrace{\gamma_2 \sigma_2^2 - \left(\frac{c}{\mu} - \gamma_2\right)\left[\frac{2}{(1-\gamma_2)^2} + \sigma_\theta^2\right]}_{\equiv \Gamma^{**}(\gamma_2)} = 0; \tag{2.52}$$

$$\frac{\beta}{1+\beta}(1-\gamma_2) = 2\left(\frac{c}{\mu} - \gamma_2\right). \tag{2.53}$$

As before, we have $\Gamma^{**}(0) < 0$ and $\Gamma^{**}\left(\frac{c}{\mu}\right) > 0$. In what follows, I show that Γ^{**} is increasing in γ_2, which implies that the equilibrium value γ_2^{**} is unique. We have

$$\begin{aligned}
\frac{\partial \Gamma^{**}}{\partial \gamma_2} &= \sigma_2^2 + \left[\frac{2}{(1-\gamma_2)^2} + \sigma_\theta^2\right] - \underbrace{\left(\frac{c}{\mu} - \gamma_2\right)}_{= \frac{\beta}{1+\beta}\frac{1-\gamma_2}{2} \text{ by eqn. (2.53)}} \frac{4}{(1-\gamma_2)^3} \\
&= \sigma_2^2 + \sigma_\theta^2 + \frac{2}{(1-\gamma_2)^2} - \frac{\beta}{1+\beta}\frac{2}{(1-\gamma_2)^2} \\
&= \sigma_2^2 + \sigma_\theta^2 + \frac{2}{(1-\gamma_2)^2}\left(1 - \frac{\beta}{1+\beta}\right) > 0.
\end{aligned}$$

Given γ_2^{**}, the value of β^{**} is pinned down by the incentive constraint in (2.53), i.e.,

$$\underbrace{\frac{\beta}{1+\beta}(1-\gamma_2^{**}) - 2\left(\frac{c}{\mu} - \gamma_2^{**}\right)}_{\equiv \Psi(\beta)} = 0.$$

An equilibrium value of β must satisfy $\Psi(\beta) = 0$.

Notice that we have $\Psi(0) < 0$ and Ψ always increasing in β. Therefore, the value β^{**} that satisfies $\Psi(\beta^{**}) = 0$ is unique.

Finally, γ_2^{**} decreases with μ. This is helpful in the proof of Proposition 18 in Appendix

2.10. By totally differentiating equation (2.52) we obtain

$$\frac{\partial \Gamma^{**}}{\partial \gamma_2} d\gamma_2^{**} + \frac{\partial \Gamma^{**}}{\partial \mu} d\mu = 0 \Leftrightarrow \frac{d\gamma_2^{**}}{d\mu} = -\frac{\frac{\partial \Gamma^{**}}{\partial \mu}}{\frac{\partial \Gamma^{**}}{\partial \gamma_2}}.$$

Given that Γ^{**} is increasing in both γ_2 and μ, $\frac{d\gamma_2^{**}}{d\mu} < 0$.

2.10 Appendix C

2.10.1 Proof of Lemma 17

The following equation characterizes the threshold $\overline{\mu}$:

$$\begin{aligned} & \mu - \frac{r}{2}\left(\frac{c}{\mu}\right)^2 \left(\sigma_\theta^2 + \sigma_2^2\right) = 1 - \frac{r}{2} c^2 \left(\sigma_\theta^2 + 1\right) \\ \Leftrightarrow \; & \mu - \frac{r}{2}\left(\frac{c}{\mu}\right)^2 \left(\sigma_\theta^2 + \sigma_2^2\right) - 1 + \frac{r}{2} c^2 \left(\sigma_\theta^2 + 1\right) = 0 \end{aligned} \tag{2.54}$$

Notice that the left-hand side of equation (2.54) is negative when $\mu = 1$, since $\sigma_2^2 > 1$ (Assumption 1). Therefore, we must have $\overline{\mu} > 1$. Moreover, it is always strictly increasing in μ. This implies that the threshold $\overline{\mu}$ is unique.

2.10.2 Proof of Proposition 18

I prove Proposition 18 in two steps. First, I show that firm value always increases with μ in the equilibrium with long-termism. Second, I characterize the new threshold value $\underline{\mu}$ and show that $\underline{\mu}$ is strictly lower than $\overline{\mu}$.

Step One: Monotonicity

As we discuss in Appendix 2.9.2, the optimal long-term contract solves the problem

$$\begin{aligned} \min_{\beta \geq 0, \gamma_2 \geq 0} \gamma_2^2 \left(\sigma_\theta^2 + \sigma_2^2\right) + \left(\frac{\beta}{1+\beta}\right)^2 \frac{1}{2}\left[(1-\overline{\gamma}_1)^2 + (1-\overline{\gamma}_2)^2 \sigma_\theta^2\right] + \gamma_2 \frac{\beta}{1+\overline{\beta}} (1-\overline{\gamma}_2)\sigma_\theta^2 \\ \text{subject to} \quad \beta \frac{1-\overline{\gamma}_2}{2\left(1+\overline{\beta}\right)} + \gamma_2 \geq \frac{c}{\mu}. \end{aligned} \tag{2.55}$$

The incentive constraint is always binding under an optimal contract, so we can use it to reduce the choice variables in the problem. The incentive constraint then implies

$$\frac{\beta}{1+\overline{\beta}} = 2\frac{\frac{c}{\mu}-\gamma_2}{1-\overline{\gamma}_2}.$$

The problem in (2.55) then simplifies to

$$\begin{aligned} & \min_{\gamma_2 \geq 0} \gamma_2^2\left(\sigma_\theta^2+\sigma_2^2\right) + 2\left(\frac{\frac{c}{\mu}-\gamma_2}{1-\overline{\gamma}_2}\right)^2\left[\left(1-\overline{\gamma}_1\right)^2+\left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2\right] + 2\gamma_2\left(\frac{\frac{c}{\mu}-\gamma_2}{1-\overline{\gamma}_2}\right)\left(1-\overline{\gamma}_2\right)\sigma_\theta^2 \\ \Leftrightarrow \quad & \min_{\gamma_2 \geq 0} \gamma_2^2\left(\sigma_\theta^2+\sigma_2^2\right) + 2\left(\frac{c}{\mu}-\gamma_2\right)^2\left[\left(\frac{1-\overline{\gamma}_1}{1-\overline{\gamma}_2}\right)^2+\sigma_\theta^2\right] + 2\gamma_2\left(\frac{c}{\mu}-\gamma_2\right)\sigma_\theta^2. \end{aligned} \tag{2.56}$$

The objective in (2.56) is the manager's risk-premium in the long-term contract.[24] Let me denote this risk-premium by $\mathcal{R}^{**}$. How does $\mathcal{R}^{**}$ change, in equilibrium, when μ changes? By totally differentiating $\mathcal{R}^{**}$ we obtain

$$\frac{d\mathcal{R}^{**}}{d\mu} = \frac{\partial\mathcal{R}^{**}}{\partial\gamma_2}\frac{\partial\gamma_2}{\partial\mu} + \frac{\partial\mathcal{R}^{**}}{\partial\overline{\gamma}_2}\frac{\partial\overline{\gamma}_2}{\partial\mu} + \frac{\partial\mathcal{R}^{**}}{\partial\mu}.$$

In equilibrium, γ_2 is chosen optimally; therefore, we have $\frac{\partial\mathcal{R}^{**}}{\partial\gamma_2}=0$. From equation (2.56), $\frac{\partial\mathcal{R}^{**}}{\partial\mu}$ is evidently negative. Notice also that, in equilibrium, $\frac{\partial\gamma_2^{**}}{\partial\mu}<0$ (see Appendix 2.9.2) and, since $\gamma_2=\overline{\gamma}_2$ in equilibrium, we have $\frac{\partial\overline{\gamma}_2}{\partial\mu}<0$. We have

$$\frac{\partial\mathcal{R}^{**}}{\partial\overline{\gamma}_2} = 2\left(\frac{c}{\mu}-\gamma_2\right)^2\frac{\left(1-\overline{\gamma}_1\right)^2}{\left(1-\overline{\gamma}_2\right)^3} > 0.$$

Therefore, we have $\frac{d\mathcal{R}^{**}}{d\mu}<0$ and, thus, firm value is always increasing in μ in an equilibrium with long-termism.

Step Two: Threshold Values

Let $\left(\mathcal{R}^*,\mathcal{R}^\dagger\right)$ denote the agent's risk-premium implied by, respectively, the optimal short-term and long-term contracts in the benchmark model. Notice that, while $\mathcal{R}^\dagger$ depends on the value

[24] Of course, differentiating the objective in (2.56) with respect to γ_2 yields the same condition for the optimal value of γ_2 in equation (2.51), as the two approaches are equivalent.

of μ through the incentive constraint, $\mathcal{R}^*$ does not. The threshold $\overline{\mu}$ is characterized by the following equality:

$$\overline{\mu} - c - \mathcal{R}^{\dagger}\left(\overline{\mu}\right) = 1 - c - \mathcal{R}^*$$
$$\Leftrightarrow \overline{\mu} = 1 - \mathcal{R}^* + \mathcal{R}^{\dagger}\left(\overline{\mu}\right).$$

The threshold $\underline{\mu}$ is instead characterized by the following equality:

$$\underline{\mu} - c - \mathcal{R}^{**}\left(\underline{\mu}\right) = 1 - c - \mathcal{R}^*$$
$$\Leftrightarrow \underline{\mu} = 1 - \mathcal{R}^* + \mathcal{R}^{**}\left(\underline{\mu}\right).$$

I want to show that $\underline{\mu} < \overline{\mu}$. I prove this by contradiction: suppose that instead $\underline{\mu} \geq \overline{\mu}$; this would imply

$$\underline{\mu} \geq \overline{\mu} \Leftrightarrow \mathcal{R}^{**}\left(\underline{\mu}\right) \geq \mathcal{R}^{\dagger}\left(\overline{\mu}\right). \tag{2.57}$$

The optimality of the contract implies that the above inequality cannot be satisfied. Given that the incentive constraint is relaxed when μ increases, and given that this constraint is binding at optimum, we cannot have that $\mathcal{R}^{**}\left(\underline{\mu}\right) > \mathcal{R}^{\dagger}\left(\overline{\mu}\right)$ when $\underline{\mu} > \overline{\mu}$. [25] Therefore, we are left with the case $\underline{\mu} = \overline{\mu}$, which implies $\mathcal{R}^{**}\left(\underline{\mu}\right) = \mathcal{R}^{\dagger}\left(\overline{\mu} = \underline{\mu}\right)$. This would imply that when $\mu = \underline{\mu}$, both contracts $(\gamma_2^{**}, \beta^{**})$ and $\gamma_2^{\dagger}$ are optimal, which contradicts the uniqueness of the contract $(\gamma_2^{**}, \beta^{**})$. Therefore, we must have $\underline{\mu} < \overline{\mu}$.

2.10.3 Proof of Lemma 19

I prove Lemma 19 in two steps. First, I show that Δ does not affect the speculator's optimal strategies. Therefore, it does not affect the characterization of the equilibrium. Second, I show that the speculator prefers the equilibrium with long-termism if Δ is sufficiently large.

The first part of the proof is trivial. Consider the case when the market-maker and the

[25] The shareholders can always ignore p in the contract, and, thus, the manager's risk-premium cannot be greater than $\mathcal{R}^{\dagger}\left(\underline{\mu}\right)$ when p is contractible, i.e., $\mathcal{R}^{**}\left(\underline{\mu}\right) \leq \mathcal{R}^{\dagger}\left(\underline{\mu}\right)$. Therefore, when $\underline{\mu} > \overline{\mu}$, we must have $\mathcal{R}^{**}\left(\underline{\mu}\right) \leq \mathcal{R}^{\dagger}\left(\underline{\mu}\right) < \mathcal{R}^{\dagger}\left(\overline{\mu}\right)$, which contradicts the inequality in 2.57.

speculator expect the manager to undertake the short-term project (i.e., $\overline{\theta}_1 \sim N\left(\mu, \sigma_\theta^2\right)$, $\overline{\theta}_2 = 0$; Part 1 of Proposition 13). The speculator's optimal trading strategy solves the following problem:

$$\max_x xE\left\{\mathcal{V} - p \mid \pi_1\right\} + \Delta E\left\{\mathcal{V} \mid \pi_1\right\} \tag{2.58}$$

Notice that the problem in (2.58) is equivalent to the problem $\max_x xE\left\{\mathcal{V} - p \mid \pi_1\right\}$, which characterizes the speculator's trading strategy in the baseline model where $\Delta = 0$. Therefore, the optimal trading strategy x^* does not change with Δ.

This implies that the equilibrium trading strategies (for both the market-maker and the speculator) and the speculator's profits from trades don't change as well. As a result, Δ does not affect her incentives to acquire information either. The same holds for the case when the firm is expected to invest in the long-term project. However, we have $E\left(x^*\right) = E\left(x^{**}\right) = 0$, which means that, after trading takes place, the speculator's *expected* net position in the firm remains Δ. As a consequence, Δ has an effect on the speculator's preferences over the two equilibria.

Let $\mathcal{V}^*$denote the firm's expected value under short-termism; $\mathcal{V}^{**}$instead denotes the firm's expected value under long-termism, with $\mathcal{V}^{**} > \mathcal{V}^*$. We can write the speculator's expected payoff in the two equilibria as follows:

$$\begin{aligned}
\Pi^* &= \Delta\mathcal{V}^* + \underbrace{\frac{\sigma_u}{2}\left[\left(1-\overline{\gamma}_1\right)^2\left(\sigma_\theta^2+1\right)\right]^{\frac{1}{2}} - g_1}_{ER^*}. \\
\Pi^{**} &= \Delta\mathcal{V}^{**} + \underbrace{\frac{\sigma_u}{2}\left[\left(1-\overline{\gamma}_1\right)^2 + \left(1-\overline{\gamma}_2\right)^2\sigma_\theta^2\right]^{\frac{1}{2}} - (g_1+g_2)}_{ER^{**}}.
\end{aligned}$$

Notice that since $\mathcal{V}^{**} > \mathcal{V}^*$, the difference $\Pi^* - \Pi^{**}$ decreases with Δ. Therefore, there always exist Δ large enough such that $\Pi^* < \Pi^{**}$.

2.10.4 Proof of Proposition 21

The proof of Proposition 21 follows the same steps as the analysis of the equilibrium in the baseline model.

I begin with characterizing the equilibrium with short-termism. In this case, the equilibrium

price function is

$$p' = \frac{1}{1+\overline{\beta}}\left[1-\overline{\gamma}_1-\overline{\alpha}+\underbrace{\frac{1-\overline{\gamma}_1}{2}(\theta_1-1)}_{x'\lambda'}+\lambda' u\right],$$

and, thus,

$$Var\left(p'\right) = \frac{1}{\left(1+\overline{\beta}\right)^2}\left[\left(\frac{1-\overline{\gamma}_1}{2}\right)^2 \underbrace{\sigma_\theta^2}_{Var(\theta_1)} + \underbrace{\left(\frac{1-\overline{\gamma}_1}{2}\right)^2 \sigma_\theta^2}_{(\lambda'\sigma_u)^2}\right] = \left(\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\right)^2 \frac{\sigma_\theta^2}{2};$$

$$Cov\left(\pi_1, p'\right) = \frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}\sigma_\theta^2.$$

The optimal contract for the short-term project then solves:

$$\min_{\beta\geq 0,\gamma_1\geq 0} \beta^2 Var\left(p'\right) + \gamma_1^2 Var\left(\pi_1\right) + 2\beta\gamma_1 Cov\left(\pi_1, p'\right) \tag{2.59}$$

$$\text{subject to} \quad \beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}} + \gamma_1 \geq c.$$

The Lagrangian function for this problem is

$$\mathcal{L}^{st} = \gamma_1^2\left(\sigma_\theta^2+1\right) + \beta^2\frac{\left(1-\overline{\gamma}_1\right)^2}{2\left(1+\overline{\beta}\right)^2}\sigma_\theta^2 + 2\gamma_1\beta\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}\sigma_\theta^2 - \Lambda\left[\beta\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)} + \gamma_1 - c\right].$$

The first-order conditions are

$$2\gamma_1\left(\sigma_\theta^2+1\right) + \beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\sigma_\theta^2 - \Lambda = 0;$$

$$2\beta\frac{\left(1-\overline{\gamma}_1\right)^2}{2\left(1+\overline{\beta}\right)^2}\sigma_\theta^2 + 2\gamma_1\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}\sigma_\theta^2 - \Lambda\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)} = 0$$

$$\Leftrightarrow 2\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\sigma_\theta^2 + 2\gamma_1\sigma_\theta^2 - \Lambda = 0.$$

The first order conditions above together imply:

$$2\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\sigma_\theta^2+2\gamma_1\sigma_\theta^2=2\gamma_1\left(\sigma_\theta^2+1\right)+\beta\frac{1-\overline{\gamma}_1}{1+\overline{\beta}}\sigma_\theta^2$$
$$\Leftrightarrow \beta\frac{1-\overline{\gamma}_1}{2\left(1+\overline{\beta}\right)}\sigma_\theta^2=2\gamma_1$$

Using the incentive constraint in program (2.59) gives

$$\left(c-\gamma_1\right)\sigma_\theta^2-2\gamma_1=0 \quad\Leftrightarrow\quad \gamma_1'=\frac{\sigma_\theta^2}{\sigma_\theta^2+2}c$$

Given γ_1', β' makes the incentive constraint - evaluated at $\overline{\gamma}_1=\gamma_1'$ and $\overline{\beta}=\beta'$ - binding. Therefore, we have both γ_1' and β' strictly positive in equilibrium.

The rest of the proof follows the same steps as in the baseline model and is thus omitted. The optimal contract in the equilibrium with long-termism features instead $\gamma_1''=0$, $\gamma_2''=\frac{\sigma_\theta^2}{\sigma_\theta^2+2\sigma_2^2}c$, and $\beta''>0$ such that $\beta\frac{1-\gamma_2''}{2(1+\beta)}+\gamma_2''-c=0$.

Chapter 3

Credit Ratings and Competition

[1] In this chapter, I analyze a model of competition between credit rating agencies (CRAs). In equilibrium, investors only buy assets that received high ratings from multiple CRAs. This has two contrasting effects on the quality of certification. On the one hand, the issuer needs to pass the screening of multiple CRAs; other things being equal, this improves certification. On the other hand, it has the perverse effect of incentivizing dishonest ratings, by introducing a team dimension that dilutes the CRAs' reputational concerns. When the perverse effect dominates, competition reduces the quality of certification. However, mandating disclosure of indicative ratings can restore the first-best outcome.

3.1 Introduction

This chapter is motivated by three key observations about credit ratings in structured finance markets. First, the vast majority of securities in these markets receive ratings from at least two rating agencies. Second, these ratings almost always agree, particularly in the AAA segment. Third, given that credit rating agencies (CRAs) are paid by the issuers of the securities they are asked to rate, they have a strong conflict of interests and, thus, high ratings do not always

[1] I would like to thank Alessandro Bonatti, Johannes Hörner, Meg Meyer, Alan Morrison, Stephen Morris, Volker Nocke, Martin Oehmke, Marco Pagano, Francesco Sangiorgi, Joel Shapiro, Jean Tirole, Juuso Välimäki, and seminar participants at Oxford, the Warwick Economics PhD Conference 2017, the Norwegian School of Economics, the Lindau Nobel Laureate Meetings 2017, the Belgrade Young Economists Conference, the 2nd Workshop on Corporate Debt Markets, the EARIE Conference 2018, and the Barcelona FIRS Conference for helpful comments.

translate into safe assets.

He, Qian and Strahan (2016) document that for AAA-rated MBS (90% of the sample), 93% receive two or three ratings, and those ratings agree 97% of the time. Griffin, Nickerson, and Tang (2013) find that, from 1997 to 2007, 85% of all AAA CDO capital (80% of the sample) with a rating from either Moody's or S&P also receives a rating from the other agency; these ratings agree for 96% of the capital. Holding highly rated securities lowers the burden of capital requirements for financial institutions (e.g., Acharya and Richardson 2009; Acharya, Schnabl, and Suarez 2013), while other institutional investors (e.g., pension funds) are constrained to hold fixed income assets that are certified as safe by multiple AAA ratings. When an issuer is offered a AAA rating by one agency only, it has a strong incentive to retain the asset (or restructure the deal) rather than going on the market with a single or split rating.[2] Therefore, disagreement among CRAs is rarely observed mainly because it rarely occurs for rated issues.

By the end of 2009, over half of the CDOs that were rated AAA at the end of the housing bubble (from 2005-2007) were "impaired" - that is either written-down to junk or suffered a principal loss. Griffin and Tang (2012) find that, from 1997 to 2007, one of the top three major CRAs frequently made positive adjustments beyond its main model that amounted to increasingly larger AAA tranche sizes. They show that accounting for adjustments and the criterion deviation indicates that, on average, AAA tranches were structured to BBB support levels.[3]

These observations raise important questions. Namely, how does the lack of disagreement among CRAs affect the quality of the certification process? What are the effects on the CRAs' incentives to offer honest ratings? What are the implications for the regulation of CRAs?

In this chapter, I analyze a dynamic model in which two rating agencies compete in selling ratings to issuers.[4] In each period, an issuer has a risky investment project but needs financing

[2] "There's going to be a three notch difference when we print the deal if it goes out as is. I'm already having agita about the investor calls I'm going to get." From an email to Moody's from a Chase investment banker complaining that a transaction would receive a significantly lower rating than the same product was slated to receive from another CRA. Wall Street and the Financial Crisis: Anatomy of a Financial Collapse. U.S. Senate (2011).

[3] More recent examples of ratings inflation are described in Baghai and Becker (2018) and Kedia, Rajgopal and Zhou (2017).

[4] I consider competition between two CRAs for simplicity, but the general logic of the model would hold with

from investors. The CRAs can improve welfare by screening projects for investors, but they have a conflict of interest because they are paid by the issuer. Each CRA offers an indicative rating to the issuer and the issuer then decides whether to pay a fee and have the rating released to investors or go as unrated. The CRAs rate the investment knowing both that they will make more profits now by issuing a higher rating, as this is more valuable to the issuer, and that issuing a high rating may diminish their reputation if the investment proves to be of poor quality.

In equilibrium, investors only finance investments that received high ratings from both CRAs, because they are skeptical when a project is rated by only one agency or has split ratings (i.e., one high and one low rating). Therefore, the issuer chooses to buy and publicize the ratings only when both CRAs offer a high rating. This has two main implications for the quality of the certification process. On the one hand, it implies that the project has to pass the screening of two different CRAs to reach the market. Keeping constant the probability with which the CRAs offer honest ratings, this improves the quality of the certification compared to the scenario with only one CRA, reducing the probability that a bad project is financed.[5] On the other hand, however, this *double screening* has the perverse effect of incentivizing dishonest ratings, as it introduces a team dimension that dilutes the CRAs' reputational concern. This perverse effect can be seen in two ways. First, when the issuer is offered split ratings and the project goes unrated, the investors cannot tell whether only one - and if so, which one - or both CRAs offered a low rating to the issuer. Therefore, a CRA cannot use an honest rating to look more honest than its competitor in the eyes of the investors. Hence, competition does not *per se* stimulate reputation-building behavior. Second, compared to a setting with a single agency, when a highly rated project performs poorly investors think it is relatively less likely that the project was bad and both ratings were inflated. This means that the reputational loss of *inflating* ratings is lower in a setting with competing agencies, making it more attractive to offer upward biased ratings. I show that when the CRAs' initial reputation is high, the perverse effect can

more than two CRAs.

[5] In equilibrium, the CRAs may offer a high rating to a bad project (*rating inflation*) but never offer a low rating to a good project. This implies that, keeping constant the probability of rating inflation, having a second screening of the project always improves certification.

be stronger; in this case, welfare is larger in a setting with a single agency.

The non-disclosure of indicative ratings (i.e., the rating each CRA offered to the issuer) when the issuer chooses to go as unrated is key in the worsening of the CRAs' incentives under competition, as it introduces the team dimension that dilutes their reputational concerns. Mandating disclosure of indicative ratings is currently a topic of regulatory discussion. In the U.S., the current framework based on the Dodd-Frank rules (Rule 17g–7(a), effective June 15, 2015) does not require disclosure of indicative ratings. The SEC formally proposed to require disclosure of indicative ratings in the fall of 2009, but the regulation was not adopted. In the E.U., Regulation (EU) No 462/2013 of the European Parliament requires CRAs to disclose any issuers' request of indicative ratings independently of rating publication.[6] However, this regulation has not been implemented yet and there is a discussion between the CRAs and the regulator regarding its implementation. In this model, I show that mandating disclosure of indicative ratings can restore the first-best outcome (i.e., honest ratings) in the scenario with competing agencies, by breaking the team dimension and restoring CRAs' concerns for relative reputation. This result introduces a new rationale for mandating disclosure of indicative ratings by CRAs.

Next, I offer a summary of the related theoretical literature (Section 3.7 discusses the empirical literature and implications of the model).

Theoretical Literature

This chapter is related to the literature on reputation building through ratings.[7] Mathis, McAndrews, and Rochet (2009) examine how a CRA's concern for its reputation affects its ratings quality. They present a dynamic model of reputation in which a monopolist CRA may mix between lying and truth-telling to build up or exploit its reputation. Strausz (2005) is similar in structure to Mathis et al. (2009) but examines information intermediaries in general. I contribute to this literature by investigating the effects of competition on the CRAs' reputational concerns, in a model where issuers can hire more than one rating agency to rate their assets.

[6] In accordance with the aforementioned regulation: "A CRA shall disclose on its website, and notify ESMA on an ongoing basis, information about all entities or debt instruments submitted to it for their initial review or for preliminary rating. Such disclosure shall be made whether or not issuers contract with the CRA for an official rating."

[7] Sangiorgi and Spatt (2017) review the literature on credit ratings in depth.

Despite the oligopolistic structure of the market for ratings,[8] very few papers in the literature have attempted to explicitly model competition between CRAs. Bar-Isaac and Shapiro (2013) find that CRAs' investments in rating accuracy can be either strategic substitutes or strategic complements, depending on how investors punish the CRAs when a highly rated project defaults. In their paper, rating fees are exogenous and they do not model the issuer's decision of which ratings to disclose, while both are endogenous in my model. In Farkas (2017), CRAs are tempted to inflate ratings but have to pay a penalty when a highly rated project defaults. He finds that, when a CRA can observe the indicative rating offered by its competitor, it selectively offers better ratings to issuers based on the more favorable assessment of the other agency. In my paper, the reputational loss from inaccurate ratings is endogenous, as it depends on the investors' learning about the CRAs' types, which affects the CRAs' continuation value in the next period. Bouvard and Levy (forthcoming) look at the threat of entry by a second rating agency and show that this may induce inaccurate ratings by the incumbent (and reduce welfare) due to the decreased value of reputation.

Another key effect of competition on ratings relates to the issuer's ability to "shop for ratings", i.e., to prevent low ratings from being disclosed. This induces a selection bias in the ratings that are observed by the investors. Skreta and Veldkamp (2009) and Bolton, Freixas, and Shapiro (2012) feature behavioral investors who do not fully internalize the winner's curse associated with this selective disclosure. In Skreta and Veldkamp (2009) rating bias is more likely to arise for complex assets, for which CRAs have more heterogenous views. Bolton et al. (2012) highlight that rating shopping benefits issuers at the expense of unsophisticated investors but, at the same time, harms sophisticated investors who are excluded from the market because of the high prices; competition among CRAs can reduce welfare by offering more opportunity for rating shopping. In Sangiorgi and Spatt (2016), CRAs are always honest and investors are rational, but they have uncertainty about which ratings were obtained and, as a result, the pricing of the assets is still systematically biased by the selective disclosure of ratings. In these

[8] The market is dominated by three rating agencies - Standard & Poor's (S&P), Moody's, and Fitch Group. The market shares (based on revenues or issues rated) are commonly estimated to be approximately 40, 40, and 15 percent for Moody's, S&P, and Fitch, respectively (White (2010)).

papers, forcing disclosure of indicative ratings improves welfare by uncovering the bias induced by rating shopping. In my paper, investors are rational and perfectly discount for rating shopping in equilibrium; however, mandating disclosure of indicative ratings still improves welfare by incentivizing CRAs to offer honest ratings.

More generally, my paper relates to the literature on communication (*cheap talk*) in dynamic settings, starting with Sobel (1985) and extended by Benabou and Laroque (1992). Morris (2001) and Ely and Välimäki (2003) show that, even when the sender's preferences are aligned with those of the receiver, the sender may still lie in an attempt to improve his or her reputation. Avery and Meyer (2012) analyze a repeated cheap-talk game and focus on the effects of the sender's reputational concerns on the quality of the receiver's decision process. Guembel and Rossetto (2009) consider a setting in which the sender has a reputational concern for an ability to predict a state of the world correctly, but the receivers may misunderstand the message sent. I contribute to this literature by analyzing the effects of competition on the senders' reputational concerns.

The rest of the chapter is organized as follows. Section 3.2 describes the model. In Section 3.3, I describe the equilibrium with a single CRA. In Section 3.4, I analyze the case of competition between two CRAs. Section 3.5 compares the CRAs' incentives and the equilibrium outcomes in the two settings. Section 3.6 investigates the effect on market efficiency of mandating the disclosure of indicative ratings by CRAs. Section 3.7 discusses some implications of the model and the related empirical evidence. Finally, Section 3.8 concludes. Detailed proofs are presented in the Appendix.

3.2 The Model

The game consists of three types of agents: issuers, investors, and two CRAs (i and j). All agents are risk-neutral and perfectly rational, and investors are competitive. I consider a simple two-period ($t \in \{1, 2\}$) communication model. In each period, an issuer has access to a risky investment project but needs financing from investors. If the issuer wishes to undertake the

project, it can sell a claim to the payoff of the project to investors. The project's quality is *a priori* unknown, including to the issuer himself, but the issuer can hire a CRA to rate the project.

A. The Asset

The project's quality can be *good* ($\theta = G$) with probability $\Pr(G)$, or *bad* ($\theta = B$) with probability $\Pr(B)$. The qualities of projects in different periods are uncorrelated. The cost of the project is I and V_θ denotes the expected payoff of a project of quality θ. I assume that good projects should be financed (they have positive NPV, i.e., $V_G > I$) but that, without prior knowledge on the quality of the project, no financing takes place (ex-ante NPV is negative, i.e., $V \equiv \Pr(G) V_G + \Pr(B) V_B \leq I$). Therefore, the presence of a CRA can improve welfare by screening projects for investors.

B. The CRAs

Each CRA observes the quality of the project and offers a rating and a fee at which the rating can be purchased to the issuer. The issuer either pays the fee and has the rating disclosed to investors or refuses to purchase it. A conflict of interest arises when the issuer is paying for certification: the CRA can charge more for a high rating, as a high rating is more valuable for the issuer. Each CRA can be of two different types ($\tau \in \{\mathcal{C}, \mathcal{O}\}$): with probability $q_0 \in (0,1)$, it is fully committed ($\tau = \mathcal{C}$) to always tell the truth; with probability $1 - q_0$, it is opportunistic ($\tau = \mathcal{O}$), in which case it chooses the rating that maximizes its continuation value.[9] The committed CRA is behavioral in the sense that it is restricted to truthful ratings about the project's quality, but is strategic in the way it sets the rating fee. The realizations of CRAs' types τ_i and τ_j are independent, and each CRA privately observes its own type.

Let ε_{it} be the probability with which, at time t, an opportunistic CRA-i chooses to inflate the rating for a bad project - i.e., offers a high rating after having observed a bad project. The CRA can also under-report the signal, offering a low rating after having observed a good project.

[9] This follows the approach of Fulghieri, Strobl, and Xia (2014) and Mathis, McAndrews, and Rochet (2009) (who, in turn, follow the classic approach of modeling reputation of Kreps and Wilson (1984) and Milgrom and Roberts (1984)).

This could allow it to build up reputation by appearing to be tough to investors. Let δ_{it} be the probability with which the opportunistic CRA-i chooses to deflate the rating for a good project. At each time t, the rating strategy of the opportunistic CRA-i is characterized by the following probabilities:

$$\Pr\left(\widetilde{r}_{it} = H \mid \theta = G\right) = 1 - \delta_{it}; \;\; \Pr\left(\widetilde{r}_{it} = H \mid \theta = B\right) = \varepsilon_{it}. \tag{3.1}$$

Let $\widehat{\varepsilon}_{it}$ and $\widehat{\delta}_{it}$ denote the investors' conjectures about ε_{it} and δ_{it}, respectively.

Let γ denote the payoff weight that both CRA types attach to their second-period profit. The weighting γ can be potentially larger than one (as, for example, in Laffont and Tirole (1993)), as future payoffs may arrive over a long time horizon. The realization of a CRA's type is its private information; I refer to the CRA's reputation as the probability the other agents assign to the CRA being the committed type. At the beginning of the game, both CRAs have initial reputation q_0. As the game evolves, the agents learn more about each CRA's type and update their beliefs.

C. Sequence of Events

Time $t = 1$. At the beginning of the period, each CRA observes the project quality θ_1 and privately offers a rating $\widetilde{r} \in \{H, L\}$ and a fee φ to the issuer. The issuer either pays the fee to the CRA and has the rating publicized or refuses to purchase it.[10] If a rating is publicized, it becomes public knowledge. The issuer observes both offers $\{\widetilde{r}_{i1}, \varphi_{i1}\}$ and $\{\widetilde{r}_{j1}, \varphi_{j1}\}$, and then decides which ratings to have publicized. Investors only observe the publicized ratings $\mathbf{r}_1 = (r_{i1}, r_{j1})$, namely one of four possible cases: a project with two ratings; a project only rated by i; a project only rated by j; or an unrated project. Therefore, we have $\mathbf{r}_1 \in \{(\widetilde{r}_{i1}\widetilde{r}_{j1}), (\widetilde{r}_{i1}\varnothing), (\varnothing\widetilde{r}_{j1}), (\varnothing\varnothing)\}$, where $\varnothing$ signifies *unrated*. Given the publicized ratings $\mathbf{r}_1$, the issuer markets a claim to the payoff of the project to investors. Let $p^{\mathbf{r}_1}$ denote the investors' willingness to pay for the claim, for given publicized ratings $\mathbf{r}_1$. If this price exceeds the investment cost, i.e., if $p^{\mathbf{r}_1} > I$, the

[10] This is in conformity with the industry practice. According to an SEC (2008) report "typically the rating agency is paid only if the credit rating is issued". Also, rating agencies do not publish the fees that they charge clients (Frenkel (2015) and Partnoy (2006)). If rating fees were paid even when the rating is not publicized (in which case the security is not issued) there would be no gain from lying for the CRA, and the conflict of interest would disappear.

issuer sells the claim and undertakes the project on behalf of investors.

At the end of the period, a signal $y_1 \in \mathcal{Y} \subseteq \mathbb{R}$ about the project quality realizes. Let $f_{\theta_1}(y_1)$ denote the distribution of y_1 conditional on the quality of the project θ_1; this distribution satisfies the *Monotone Likelihood Ratio Property*. Formally,

$$\forall y' > y, \quad \frac{f_G(y')}{f_B(y')} \geq \frac{f_G(y)}{f_B(y)}.$$

This implies that a large realization of y_1 is a (imperfect) signal for a good project.[11]

Time $t = 2$. A new issuer comes into play, and the stage game takes place again.

D. Timing

The timing of the model is summarized in what follows. At the beginning of the game, nature draws the CRAs' types (τ_i, τ_j), where $\Pr(\tau_i = \mathcal{C}) = \Pr(\tau_j = \mathcal{C}) = q_0$, and the realizations of τ_i and τ_j are independent. The game consists of two periods. In each period of the game, the following steps occur:

(i) Nature determines the project's quality $\theta_t \in \{B, G\}$

(iii) Each CRA observes θ_t and privately offers a rating $\widetilde{r}_t \in \{H, L\}$ and a fee φ_t to the issuer.

(iv) The issuer chooses which ratings to disclose. If a rating is disclosed, the issuer pays the fee to the respective CRA and the rating becomes public knowledge.

(v) Investors observe the disclosed ratings $\mathbf{r}_t = (r_{it}, r_{jt})$.

(vi) Given $\mathbf{r}_t$, the issuer sets a price $p^{\mathbf{r}_t}$ and markets a claim backed by the project to investors. If $p^{\mathbf{r}_t} > I$, the issuer sells the claim and undertakes the project.

(vii) The signal y_t realizes and is observed by all players.

[11] The assumption that the signal y_1 is observed regardless of whether the project is implemented or not simplifies the exposition but does not affect any of the results. We could have that y_1 is a vector of signals, where each component satisfies the monotone likehood ratio property, and some components (e.g., the realized payoff of the project) are observed only when the issuer undertakes the project and others (e.g., returns on similar assets, market prices, information provided by the issuer) are observed in any case.

(viii) Having observed $(\mathbf{r}_t, y_t)$, agents update their beliefs about each CRA's type.

In each period, a strategy for an opportunistic CRA is a fee at which a rating can be purchased and a rating strategy; the committed CRA only chooses the fee, as it is restricted to truthful ratings about the project's quality. A strategy for the issuer is a choice of the ratings to disclose, and whether to sell the claim (and, if so, at what price) and undertake the project. Given the disclosed ratings and their beliefs about the issuer's strategy, the CRAs' types, and the CRAs' strategies, investors form their willingness to pay for the claim on the project's payoff. In the following, I analyze a symmetric *Sequential Equilibrium* of the game (in which CRAs of the same type and with the same reputation play the same strategy).[12]

3.3 Monopolistic Rating Agency

I begin by examining the game with a single CRA. I conjecture (and verify) that, in equilibrium, the opportunistic CRA always offers a high rating after observing a good project. Therefore, while "rating inflation" (that is, bad projects receiving a high rating) will be part of the equilibrium, "rating deflation" (good projects receiving a low rating) will not.

The analysis of the equilibrium proceeds as follows. First, I describe the CRA's fee, the issuer's strategy, and the investors' inference, in a given period t, for any level of reputation q and expected rating inflation $\widehat{\varepsilon}$. Second, I describe the CRA's posterior reputation and continuation payoffs. Finally, I solve for the equilibrium level of rating inflation.

Market equilibrium. Investors observe the rating r and decide whether they wish to buy the claim and, if so, at what price. Investors are risk-neutral and perfectly competitive, so the market price p^r is equal to the expected value of the project's payoff given their posterior beliefs.

Let μ^H denote the probability investors assign to the project quality being good after a high rating ($r = H$) is observed, when the CRA's reputation at the beginning of the period is q and

[12] Sequential equilibrium is a refinement of Nash Equilibrium for extensive form games, introduced by Kreps and Wilson (1982a). I provide a formal definition of the concept in Appendix 3.9.1.

expected rating inflation is $\widehat{\varepsilon}$. The price $p^H(q,\widehat{\varepsilon})$ is characterized as follows:

$$p^H(q,\widehat{\varepsilon}) = \mu^H V_G + \left(1-\mu^H\right) V_B,$$

where

$$\mu^H = \frac{\Pr(G)}{\Pr(G) + \Pr(B)(1-q)\widehat{\varepsilon}}.$$

The higher reputation is and the lower rating inflation is, the more informative a high rating will be about the value of the project. Hence, μ^H and, as a consequence, p^H increase in the initial reputation q and decrease in rating inflation $\widehat{\varepsilon}$. In equilibrium, the issuer will want to publicize a high rating as long as $p^H > I$. For expositional convenience, I assume that the ex-ante NPV is zero (i.e., that $V = I$) and, thus, p^H is strictly larger than I unless both $q = 0$ and $\widehat{\varepsilon} = 1$. [13]

Lemma 22 *Consider a given time t, with the CRA's reputation at the beginning of time t being q and expected rating inflation being $\widehat{\varepsilon}$. The following holds in equilibrium:*

1. *The issuer publicizes the rating, sells the claim, and undertakes the project only when the rating is high ($r = H$); otherwise, he goes as unrated ($r = \varnothing$) and does not undertake the project;*

2. *Both CRA-types set a fee $\varphi(q,\widehat{\varepsilon}) = p^H(q,\widehat{\varepsilon}) - I$ for a high rating.*

A high rating conveys good news about project quality, i.e., $p^r > I$ when $r = H$. Therefore, the issuer always publicizes the rating and sells the claim when the rating is high. When the project is unrated, investors understand the issuer was offered a low rating which went unpurchased and, thus, that the CRA observed a bad project.[14] This conveys bad news about the project (i.e., $p^{\varnothing} < I$) and so the issuer does not sell the claim in this case. Given that the

[13] All of the results in the paper hold in full for lower values of V if we impose a lower bound on the CRA's initial reputation q_0.

[14] In equilibrium, the strategic CRA never *deflates* a good signal - I prove this in Proposition 23. This means that a low report ($\widetilde{r} = L$) would conclusively reveal that the investment is bad and, thus, not worth being implemented. Therefore, the issuer never publicizes a low report. However, investors understand this and still infer that $\widetilde{r} = L$ and, thus, that the project is of bad quality when $r = \varnothing$.

issuer cannot sell without a high rating, the CRA can extract all the issuer's surplus from selling the claim and implementing the project on behalf of the investors, i.e. $\varphi\left(q,\widehat{\varepsilon}\right)=p^{H}\left(q,\widehat{\varepsilon}\right)-I$.

Posterior Reputation. A committed CRA is restricted to truthful ratings, while an opportunistic CRA may lie and offer a high rating to a bad project. As a consequence, the issuance of a rating is informative about the CRA's type. Investors update their beliefs about the CRA's type first after the release of a rating and then, later on in the game, as more information about the project quality becomes available (i.e., after they observe the realization of the signal y_1). Since the rating fee increases with reputation, the CRA has an incentive to enter the second period with a strong reputation, in order to charge a higher fee at time $t=2$.

Let q^{H,y_1} denote, for a given realization of the signal y_1 and a given conjecture $\widehat{\varepsilon}_1$, the CRA's posterior reputation following a high rating at time $t=1$; $q^{\varnothing}$ denotes the posterior reputation following an unrated project. When the project goes unrated ($r=\varnothing$), investors understand that it was offered a low report which went unpurchased and, thus, that the CRA observed a bad project. Since they learn that project quality is bad, they don't use the signal y_1 when updating the CRA's reputation.

When the opportunistic CRA is expected to inflate its ratings with positive probability (i.e., $\widehat{\varepsilon}_1>0$), reputation decreases following a high rating, for any possible realization of y_1. This loss of reputation reflects the fact that a high rating is more likely to be offered by an opportunistic CRA than a committed one.[15] On the contrary, reputation increases when the project goes unrated. Therefore, we have $q^{H,y_1}\leq q_0\leq q^{\varnothing}$ for any $y_1\in\mathcal{Y}$, with strict inequalities when $\widehat{\varepsilon}_1>0$. The expressions for q^{H,y_1} and $q^{\varnothing}$ are described in Appendix 3.9.3.

Equilibrium Rating Strategy. Having characterized the issuer's and investors' strategies and the CRA's posterior reputation in equilibrium, we can now characterize the unique equilibrium rating strategy.

[15] It is useful to think about belief updating as a 2-stage process. First, investors use y_1 to set their beliefs about the distribution of project quality θ_1. Then, they use their conjecture $\widehat{\varepsilon}_1$ to see whether r_1 is more likely to come from a committed or a strategic CRA. A large y_1 is a signal for a good project. Therefore, q^{H,y_1} increases with y_1, as the likelihood that the project was bad (and so the rating is inflated) decreases with y_1.

Proposition 23 *In equilibrium, an opportunistic CRA:*

1. *Inflates the rating for a bad project with positive probability in the first period and always in the second period, i.e.,* $\varepsilon_1^m \in (0,1], \varepsilon_2^m = 1$;

2. *Never deflates the rating for a good project, i.e.,* $\delta_1^m = \delta_2^m = 0$.

At time $t = 2$, the market ends: since there is no reputational concern and the CRA cannot commit ex-ante to a given rating strategy, an opportunistic CRA always inflates the rating for a bad project in the last period (and never deflates the rating for a good project). Therefore, I will focus on the first-period rating strategy in what follows.

Consider an opportunistic CRA who, at time $t = 1$, has observed a bad project and decides whether to inflate its rating or not. The CRA's total payoffs, evaluated at the second-period equilibrium rating strategy, are as follows:

$$\Pi_B \equiv \varepsilon_1 \left\{ \varphi\left(q_0, \widehat{\varepsilon}_1\right) + \gamma \int_{y_1} \varphi\left(q^{H,y_1}, 1\right) f_B\left(y_1\right) dy_1 \right\} + \left(1 - \varepsilon_1\right) \gamma\varphi\left(q^{\varnothing}, 1\right),$$

where posterior reputations q^{H,y_1} and $q^{\varnothing}$ are evaluated at the investors' conjecture $\widehat{\varepsilon}_1$.

With probability $1 - \varepsilon_1$, the rating is not inflated, the first-period fee is not collected and $r_1 = \varnothing$. The CRA's continuation payoff is $\gamma\varphi\left(q^{\varnothing}, 1\right)$ in this case, because it always sells the rating and collects the fee in the second period.[16] With probability ε_1, the rating is inflated to $r_1 = H$ and the fee is collected; for a given realization of y_1, the continuation payoff is $\gamma\varphi\left(q^{H,y_1}, 1\right)$ in this case. At this stage, the signal y_1 has not yet realized. However, the CRA knows that the project is bad and, thus, the expectation over the realization of y_1 is conditional on $\theta = B$.

The amount of rating inflation ε_1 is then chosen as a best response to the conjectures $\widehat{\varepsilon}_1$; let $\varepsilon_1^*\left(\widehat{\varepsilon}_1\right)$ denote this best response.

[16] An opportunistic CRA always offers a high rating to the issuer in the second period: even when the project is bad, she always inflates the rating ($\varepsilon_2^m = 1$). Investors anticipate this and, as a consequence, the second-period fees are evaluated at the equilibrium rating inflation strategy, i.e., $\widehat{\varepsilon}_2 = \varepsilon_2^m = 1$.

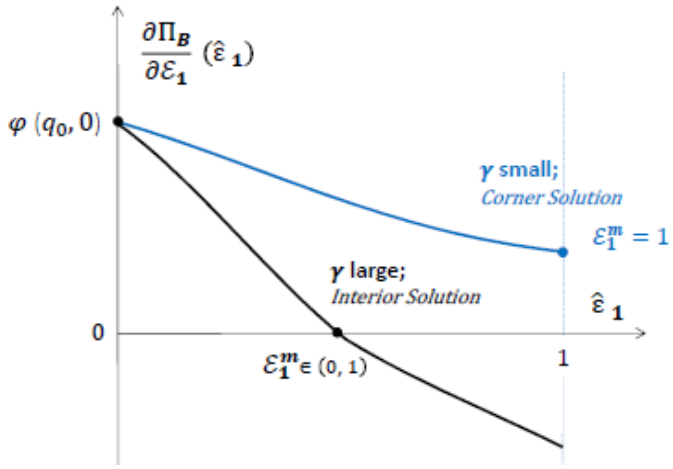


Figure 3.1: Characterization of MBI and equilibrium inflation.

Definition 24 *Any equilibrium level of rating inflation* ε_1^m *has to be a value of* $\widehat{\varepsilon}_1$ *that satisfies the following fixed point condition:* $\varepsilon_1^*(\widehat{\varepsilon}_1) = \widehat{\varepsilon}_1$.

Taking the derivative of Π with respect to ε_1 yields:

$$\frac{d\Pi}{d\varepsilon_1} = \varphi(q_0, \widehat{\varepsilon}_1) + \gamma \int_{y_1} \varphi\left(q^{H,y_1}, 1\right) f_B(y_1)\, dy_1 - \gamma\varphi\left(q^{\varnothing}, 1\right). \tag{3.2}$$

The derivative $\frac{d\Pi}{d\varepsilon_1}$ (henceforth MBI) represents the marginal benefit of rating inflation for a given $\widehat{\varepsilon}_1$. On the one hand, the CRA sells the rating and collects the first-period fee only if it inflates the rating, since the issuer only buys a high rating; this is the *Short-term Gain* of rating inflation, i.e., the first-period fee $\varphi(q_0, \widehat{\varepsilon}_1)$. On the other hand, each time the CRA sells the rating at time $t = 1$, it has a lower reputation ($q^{H,y_1} \leq q^{\varnothing}$) and, as a consequence, charges a lower fee in the second period; this is the *Reputational Loss* of rating inflation.

Figure 3.1 depicts the possible characterizations for the equilibrium level of rating inflation. The marginal benefit of inflation MBI is always positive at $\widehat{\varepsilon}_1 = 0$, as in this case the CRA can inflate the rating and collect the first-period fee without incurring any reputational losses in the second period.[17] This implies that honest ratings (i.e., $\widehat{\varepsilon}_1 = 0$) cannot be an equilibrium of the game because, when the CRA is never expected to inflate its rating, it finds it optimal to

[17] When $\widehat{\varepsilon}_1 = 0$, the opportunistic and committed types are expected to play the same strategies, i.e., both only offer honest ratings. This implies that both the rating and the signal about project quality y_1 are not informative about the CRA's type, i.e., $q^{H,y_1} = q_0 = q^{\varnothing}$ for any $y_1 \in \mathcal{Y}$.

always inflate it. As $\widehat{\varepsilon}_1$ increases, investors perceive it to be more likely that an opportunistic CRA inflated the rating, so reputation updating conditional on the rating $r = H$ is more severe. On the other hand, the rating $r = \varnothing$ becomes a stronger signal for a committed type, so forgone second-period profits when the CRA actually inflates are larger. Lastly, the rating $r = H$ is a less reliable signal for a good investment, which reduces the price investors are willing to pay for the claim and, thus, the rating fee. Therefore, incentives to inflate are lower when expected rating inflation increases.

In equilibrium, we have either a corner solution $\varepsilon_1^m = 1$ or an interior solution $\varepsilon_1^m \in (0, 1)$, where the CRA alternates between inflating and not. The corner solution occurs when the CRA does not care enough about its future profits (i.e., γ smaller than a threshold $\overline{\gamma}^m$) and, as a consequence, reputational concerns fail to impose any discipline on the opportunistic CRA. As the weight on future profits γ increases, reputational concerns become more important and the CRA inflates with probability lower than one.

Equation (3.2) implicitly defines the threshold value $\overline{\gamma}^m$. [18] This can be interpreted as an inverse measure of the CRA's reputational concerns: as $\overline{\gamma}^m$ approaches 0, the opportunistic CRA inflates its first-period rating with probability lower than 1 for any value of γ; as a result, it always provides the market with some information about project quality. We have:

$$\overline{\gamma}^m \equiv \frac{p^H(q_0, 1) - I}{p^H(q^{\varnothing}, 1) - \int_{y_1} p^H(q^{H,y_1}, 1) f_B(y_1) dy_1},$$

where $q^{\varnothing}$ and q^{H,y_1} are evaluated at the conjecture $\widehat{\varepsilon}_1 = 1$. [19]

[18] When $\gamma < \overline{\gamma}^m$, the opportunistic CRA always inflates the rating for a bad project also in the first period - i.e., $\varepsilon_1^m = \varepsilon_2^m = 1$.

[19] The opportunistic CRA never deflates the rating for a good project, as selling the rating is always attractive enough in equilibrium that there is no incentive to deflate. I discuss the intuition behind this result in Appendix 3.9.4.

3.4 Competition

I next examine the game where two CRAs compete in selling ratings to the issuer. Like in the setting with a single CRA, I conjecture (and verify) that, in equilibrium, the opportunistic CRAs always offer a high rating after observing a good project. The analysis of the game follows the same steps as the one with a single CRA.

Market equilibrium. Let μ^{HH} denote the probability investors assign to the project quality being good after two high ratings ($r= HH$) are observed, when the CRAs' reputation at the beginning of the period is q and the opportunistic CRAs i and j are expected to play the same rating inflation strategy $\widehat{\varepsilon}_i = \widehat{\varepsilon}_j = \widehat{\varepsilon}$. The price of the claim backed by the payoff of the project $p^{HH}(q, \widehat{\varepsilon})$ is characterized as follows:

$$p^{HH}(q, \widehat{\varepsilon}) = \mu^{HH} V_G + \left(1 - \mu^{HH}\right) V_B,$$

where

$$\mu^{HH} = \frac{\Pr(G)}{\Pr(G) + \Pr(B)\left[(1-q)\widehat{\varepsilon}\right]^2}.$$

The higher reputation is and the lower rating inflation is, the more informative is a high rating about the value of the project and, thus, the higher p^{HH} is. As for the scenario with a single CRA, in order to simplify the exposition, I assume that the ex-ante NPV is zero. Thus, p^{HH} is strictly positive unless both $q = 0$ and $\widehat{\varepsilon} = 1$.

Lemma 25 *Consider a given time t, where both CRAs have reputation q at the beginning of time t and expected rating inflation is $\widehat{\varepsilon}_{it} = \widehat{\varepsilon}_{jt} = \widehat{\varepsilon}$. The following holds in a symmetric equilibrium:*

1. *The issuer publicizes the ratings, sells the claim, and undertakes the project only when both ratings are high ($r= HH$); otherwise, he goes as unrated ($r= \varnothing\varnothing$) and does not undertake the project;*

2. *Both CRA-types set a fee $\varphi^c(q, \widehat{\varepsilon}) = \frac{1}{2}\left(p^{HH}(q, \widehat{\varepsilon}) - I\right)$ for a high rating.*

In equilibrium, investors must expect that an issuer with a good project discloses two high ratings.[20] Otherwise, the issuer would buy the cheapest of the two ratings and only one of the CRAs would be selling its rating. The CRAs would then start undercutting each others' fees, until they reach the fee at which the issuer chooses to disclose both ratings. This implies that an issuer with anything else than two high ratings must have a bad project and, thus, we have $p^{\mathbf{r}} < I$ when $\mathbf{r} \neq HH$. Therefore, the issuer publicizes the ratings and sells the claim only when both ratings are high.[21] Since the issuer needs two high ratings to sell the claim, the CRAs can together extract all the issuer's surplus. In a symmetric equilibrium, each CRA gets half of the issuer's surplus from selling the claim and implementing the project on behalf of the investors, i.e., $\varphi^c(q, \widehat{\varepsilon}) = \frac{1}{2}\left(p^{HH}(q, \widehat{\varepsilon}) - I\right)$.

Posterior Reputation. Lemma 25, has two main implications. First, reputation moves symmetrically across CRAs, as investors either observe two high ratings or an unrated project. In the latter case, they cannot tell whether only one - and, if so, which one - or both CRAs offered a low rating to the issuer. Since the CRAs have symmetric reputation at times $t = 1$ and $t = 2$, Lemma 25 describes the issuer's and investors' strategies, and the CRAs' fees, in both periods. Second, i's rating matters only when j is offering a high rating as well, as then i's rating is pivotal in determining whether the issuer chooses to publicize the ratings.

Let q^{HH,y_1} denote, for a given realization of y_1 and a given conjecture $\widehat{\varepsilon}_1$, the CRAs' posterior reputation when investors observe a project with two high ratings at time $t = 1$; $q^{\varnothing\varnothing}$ denotes the posterior reputation following an unrated project. When $r = \varnothing\varnothing$ investors learn that the project was bad and, thus, $q^{\varnothing\varnothing}$ does not depend on y_1. As for the monopolistic CRA, reputation decreases (increases) following a project with two high ratings (an unrated project), i.e., $q^{HH,y_1} \leq q_0 \leq q^{\varnothing\varnothing}$ for any $y_1 \in \mathcal{Y}$ (with strict inequalities for $\widehat{\varepsilon}_1 > 0$). The expressions for q^{HH,y_1} and $q^{\varnothing\varnothing}$ are described in Appendix 3.10.2.

[20] In equilibrium, an opportunistic CRA never *deflates* a good signal. This implies that an issuer with a good project is always offered two high ratings, i.e., $\widetilde{\mathbf{r}} = HH$.

[21] This implies that a project with only one high rating is off-the-equilibrium path. The following off-equilibrium beliefs support the equilibrium: when investors observe a project with only one high rating (i.e., $\mathbf{r} = H\varnothing$ or $\mathbf{r} = \varnothing H$), they believe that the project is bad and, thus, $p^{\mathbf{r}} < I$.

Equilibrium Rating Strategy. The following Proposition describes the unique equilibrium rating strategy.

Proposition 26 *In equilibrium, an opportunistic CRA:*

1. *Inflates the rating for a bad project with positive probability in the first period and always in the second period, i.e.,* $\varepsilon_1^c \in (0,1]$, $\varepsilon_2^c = 1$;
2. *Never deflates the rating for a good project, i.e.,* $\delta_1^c = \delta_2^c = 0$.

In equilibrium, an opportunistic CRA always inflates with positive probability the rating for a bad project and never deflates the rating for a good project. Compared to monopoly, competition does not affect the characterization of the equilibrium. However, as we will see in Section 3.5, it drastically changes the strength of the CRAs' reputational concerns. In the rest of this section, I analyze the building blocks that give us Proposition 26.

Consider an opportunistic CRA-i who, at time $t = 1$, has observed a bad project and decides whether to inflate its rating or not. Let $\widehat{\varepsilon}_{j1}$ be i's conjecture about an opportunistic CRA-j's rating inflation strategy ε_{j1}. Investors conjecture a symmetric strategy $\widehat{\varepsilon}_1$ for the two CRAs, which enters the first-period fee $\varphi^c(q_0, \widehat{\varepsilon}_1)$ and posterior reputations q^{HH,y_1} and $q^{\varnothing\varnothing}$. Notice that i's rating matters only when j is offering a high rating as well, i.e., when j is opportunistic and is inflating the rating too. In this case, i's rating is pivotal in determining whether the issuer publicizes the ratings or not. This means that, when i chooses whether to inflate its rating or not, the CRA considers itself pivotal and, so, faces a trade-off similar to the one faced by the monopolistic CRA: if i inflates its rating, both CRAs sell the rating at time $t = 1$, but then enter time $t = 2$ with lower reputation. The CRA's incentives to inflate the rating are determined by the following expression:

$$\frac{d\Pi_{i,B}}{d\varepsilon_{i1}} = (1 - q_0)\,\widehat{\varepsilon}_{j1} \left\{ \varphi^c(q_0, \widehat{\varepsilon}_1) + \gamma \int_{y_1} \varphi^c\left(q^{HH,y_1}, 1\right) f_B(y_1)\, dy_1 - \gamma \varphi^c\left(q^{\varnothing\varnothing}, 1\right) \right\}, \tag{3.3}$$

where $\Pi_{i,B}$ denotes i's total expected payoffs, evaluated at the second-period equilibrium rating strategy and where posterior reputations q^{HH,y_1} and $q^{\varnothing\varnothing}$ are evaluated at the investors'

conjecture $\widehat{\varepsilon}_1$ (see equation (3.25) in Appendix 3.10.3).

Equation (3.3) describes i's marginal benefit of rating inflation (henceforth MBI_i). The CRA's rating is pivotal only when j is opportunistic and is inflating the rating too; this happens with probability $(1-q_0)\widehat{\varepsilon}_{j1}$, because i expects an opportunistic CRA-j to inflate its rating with probability $\widehat{\varepsilon}_{j1}$. In this case, both CRAs sell the rating in $t=1$, collecting the first-period fee $\varphi^c(q_0, \widehat{\varepsilon}_1)$. However, they then enter time $t=2$ with low reputation and, for a given realization of y_1, collect the low fee, i.e., $\varphi^c\left(q^{HH,y_1}, 1\right)$, in the second period.[22] If i chooses not to inflate, the first-period fee is not collected but the CRAs enter time $t=2$ with high reputation and so collect the high fee $\varphi^c\left(q^{\varnothing\varnothing}, 1\right)$. Whether i chooses to inflate the rating or not depends on the sign of MBI_i.

The amount of rating inflation ε_1 is chosen as a best response to the investors' conjectures $\widehat{\varepsilon}_1$ and to i's conjecture about ε_{j1}, i.e., $\widehat{\varepsilon}_{j1}$. Let $\varepsilon^*_{i1}(\widehat{\varepsilon}_1, \widehat{\varepsilon}_{j1})$ denote this best response. Both the investors' and i's expectations need to be consistent with the play of the game in equilibrium. This implies that i and the investors have the same conjecture about ε_{j1}, i.e., $\widehat{\varepsilon}_{j1} = \widehat{\varepsilon}_1$. Therefore, when characterizing the equilibrium rating inflation strategies, we need to evaluate ε^*_{i1} at $\widehat{\varepsilon}_{j1} = \widehat{\varepsilon}_1$.

Definition 27 *Any equilibrium level of rating inflation ε^c_1 satisfies the following fixed point condition: $\varepsilon^*_{i1}(\widehat{\varepsilon}_1 = \varepsilon^c_1, \widehat{\varepsilon}_{j1} = \varepsilon^c_1) = \varepsilon^c_1$.*

We can thus rewrite equation (3.3) as follows:

$$\frac{d\Pi_{i,B}}{d\varepsilon_{i1}} = (1-q_0)\widehat{\varepsilon}_1 \left\{ \varphi^c(q_0, \widehat{\varepsilon}_1) + \gamma \int_{y_1} \varphi^c\left(q^{HH,y_1}, 1\right) f_B(y_1)\, dy_1 - \gamma \varphi^c\left(q^{\varnothing\varnothing}, 1\right) \right\}. \tag{3.4}$$

Figure 3.2 depicts the possible characterizations for the equilibrium level of rating inflation. Notice that $MBI_i = 0$ at $\widehat{\varepsilon}_1 = 0$, as j is not expected to inflate its rating and, thus, i believes it will never be pivotal in this case. Since i thinks it is never pivotal, the CRA is indifferent

[22] As both CRAs are opportunistic in this scenario, they always offer a high rating in the second period (if the project is bad, both always inflate the rating). Hence, the second-period fee is collected with probability 1.

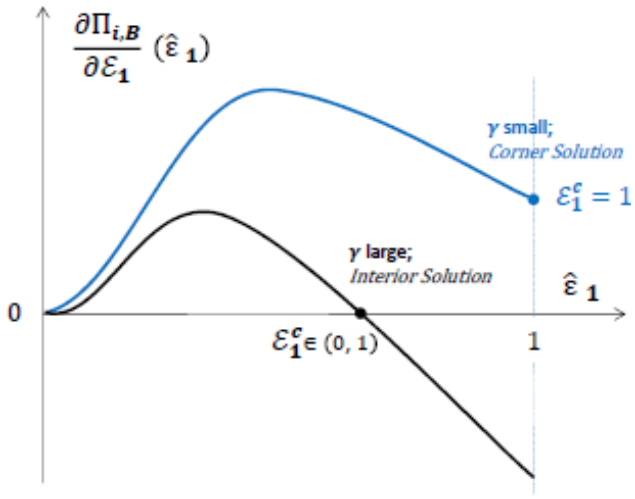


Figure 3.2: Characterization of MBI_i and equilibrium inflation.

among any possible inflation strategy ε_{i1}. However, i's belief of never being pivotal is not robust to trembles and, therefore, cannot be part of a Sequential Equilibrium.[23]

When $\widehat{\varepsilon}_1 > 0$, i is always pivotal with positive probability. In this case, the trade-off is similar to the one faced by the monopolistic CRA, and the equilibrium depends only on the sign of the expression in brackets in equation (3.4). [24] If the *Short-term Gain* of inflation, i.e., the first-period fee $\varphi^c(q_0, \widehat{\varepsilon}_1)$, is always larger than the *Reputational Loss, i.e.,* the difference in the second-period fees, the opportunistic CRAs always inflate their ratings in the first period. This occurs when γ is smaller than a threshold value $\overline{\gamma}^c$. Otherwise, short-term gain and reputational loss equate each other at some interior point $\varepsilon_1^c \in (0, 1)$, and the CRAs mix inflating and not in equilibrium.

[23] At $\widehat{\varepsilon}_1 = 0$, both CRAs are expected to never inflate their ratings. Hence, inflated ratings are off-the-equilibrium path. However, if we perturb these beliefs - e.g., j inflates its rating with some small probability, it is then always optimal for i to inflate its rating as well, and vice versa. This is because, like in the monopolistic setting, when $\widehat{\varepsilon}_1$ is close to 0, the short-term gain of rating inflation is large and the reputational loss almost null. Therefore, the conjecture $\widehat{\varepsilon}_1 = 0$ fails to satisfy the consistency requirement of a Sequential Equilibrium, as it is not robust to trembles.

[24] The probability of i being pivotal determines the shape of MBI_i - see Appendix 3.10.3 - but does not play any role in the characterization of the equilibrium. Once i is pivotal, what matters is only the trade-off between short-term gain and reputational loss of inflating the rating.

The value of the threshold $\overline{\gamma}^c$ is defined as follows:

$$\overline{\gamma}^c \equiv \frac{p^{HH}(q_0, 1) - I}{p^{HH}(q^{\varnothing\varnothing}, 1) - \int_{y_1} p^{HH}(q^{HH,y_1}, 1) f_B(y_1) dy_1},$$

where $q^{\varnothing\varnothing}$ and q^{HH,y_1} are evaluated at the conjecture $\widehat{\varepsilon}_1 = 1$.

When the CRAs compete in selling ratings, they equally split the issuer's surplus from selling the claim in both periods. Therefore, the sharing rule does not affect the CRAs' trade-off in equation (3.4). As a consequence, it does not enter the characterization of $\overline{\gamma}^c$. Like in monopoly, when $\gamma < \overline{\gamma}^c$, the opportunistic CRAs always inflate the rating for a bad project also in the first period (i.e., $\varepsilon_1^c = \varepsilon_2^c = 1$).

3.5 Market Efficiency

In this section, I compare the efficiency of the market under monopoly and competition. The total surplus in each regime is given by the expected return of the projects that end up being implemented, i.e., that receive high ratings, in equilibrium.[25] I consider only the return on the first-period project, which I denote by $\mathcal{V}^m$ in monopoly and $\mathcal{V}^c$ in duopoly. The second-period project is not considered, as the last period only serves as a modeling device to account for reputational concerns of the rating agencies. Since bad (good) projects have negative (positive) NPV, both $\mathcal{V}^m$ and $\mathcal{V}^c$ are decreasing (increasing) in the probability that a bad (good) project is implemented.

$$\mathcal{V}^m \equiv \Pr(G)(V_G - I) + \Pr(B)(1 - q_0)\varepsilon_1^m (V_B - I);$$
$$\mathcal{V}^c \equiv \Pr(G)(V_G - I) + \Pr(B)((1 - q_0)\varepsilon_1^c)^2 (V_B - I).$$

A good project is always implemented in equilibrium, since in both settings the opportunistic CRA never deflates the rating for a good project. In monopoly, the probability that the issuer

[25] As prices in the model are transfers, they do not affect welfare. Therefore, we don't need to consider the price of the claim or the rating fees.

sells the claim and implements a bad project in the first period is equal to $(1-q_0)\,\varepsilon_1^m$ - i.e., the probability that the CRA is opportunistic and chooses to inflate the rating. In duopoly, this happens with probability $[(1-q_0)\,\varepsilon_1^c]^2$ - i.e., the probability that both CRAs are opportunistic and are both inflating their ratings. Therefore, welfare is greater in monopoly when the probability that a bad project ends up being implemented is lower, i.e., when:

$$\varepsilon_1^m < (1-q_0)\,(\varepsilon_1^c)^2\,. \tag{3.5}$$

Proposition 28 *Competition has two contrasting effects on the quality of certification:*

1. *Other things being equal (initial reputation and rating inflation), it improves certification, as the project needs to pass the screening of two different CRAs;*

2. *Incentives to offer upward-biased ratings are stronger in duopoly, i.e.,* $\overline{\gamma}^m < \overline{\gamma}^c$.

Part 1 of Proposition 28 is straightforward. In the scenario with two CRAs, the issuer undertakes the project only when it receives two high ratings, which means the project has to pass the screening of two different CRAs. Therefore, other things being equal (initial reputation and rating inflation), the probability that a bad project is implemented is lower in duopoly.

However, this double screening has the perverse effect of increasing the CRAs' incentives to offer inflated ratings, which can be seen in two ways. First, the double screening implies that, other things being equal, the price of the claim backed by the project is higher under competition: investors are more confident that the project is of good quality, since it passed the screening of two CRAs. When the CRAs compete in selling ratings, they equally split the issuer's surplus in both periods, i.e., they each get a half of $p^{HH} - I$. Therefore, the sharing rule does not affect the CRAs' trade-off in equation (3.4) and, as a consequence, the *Short-term Gain* of rating inflation in the trade-off is actually larger in duopoly.

Second, other things being equal, investors' learn less about the CRAs' types in duopoly. When investors observe an unrated project, they cannot tell whether only one - and, if so, which one - or both CRAs offered a low rating to the issuer. Therefore, reputation goes up less than in monopoly ($q^{\varnothing\varnothing} \leq q^{\varnothing}$). When they instead observe a project with two high ratings, they think

it is relatively less likely - compared to the setting with a single CRA - that the project is bad and, thus, that both ratings are inflated. As a consequence, reputation goes down less in this case ($q^{HH,y_1} \geq q^{H,y_1}$). Therefore, the *Reputational Loss* of rating inflation is lower in a setting with competing CRAs.

Together, these two observations imply that incentives to inflate ratings are larger in duopoly, showing us the unexpected perverse effect of the double screening. The relative magnitude of the two effects - i.e., the double screening versus the perverse effect on the CRAs' incentives, determines whether competition improves certification and, thus, welfare compared to the scenario with a single agency.

Lemma 29 *Welfare can be larger either in monopoly or in duopoly, depending on the parameters of the model. In particular:*

1. *When $\gamma \leq \overline{\gamma}^m$, welfare is always larger in duopoly;*
2. *When the CRAs' initial reputation (q_0) is large enough, there always exist values of $\gamma > \overline{\gamma}^m$ for which welfare is larger in monopoly.*

When $\gamma \leq \overline{\gamma}^m$, the opportunistic CRAs always inflate their ratings in both settings (i.e., $\varepsilon_1^m = \varepsilon_1^c = 1$). Therefore, having two CRAs that screen the project always improves welfare, as each of them is committed (and offers only honest ratings) with positive probability q_0. However, there also exist sets of parameters for which welfare is larger under monopoly. In particular, when the CRAs' initial reputation is large enough, there always exist values of the weight on future payoffs (γ) for which competition reduces welfare. In what follows, I discuss the intuition behind this result.

The perverse effect of competition on the CRAs' incentives is quite strong when q_0 is large. This result, even though surprising at first, is very intuitive.[26] As q_0 approaches 1, it becomes very unlikely that a bad project receives high ratings and, as a consequence, investors strongly believe that a project with high ratings is a good project. This means that the short-term gain of

[26] The fact that the other CRA is highly likely to be honest does not reduce the opportunistic CRA's incentives to inflate ratings. The opportunistic CRA anticipates that, if the ratings disagree, the issuer does not disclose them and so the investors don't observe the indicative ratings that were offered by the CRAs.

rating inflation (the first-period fee) becomes very large while the reputational loss becomes very small, making the CRAs' incentives to inflate their ratings strong in both settings. However, this happens at a much faster rate in the scenario with competing agencies because of the double screening.[27] Therefore, when q_0 becomes large, the CRAs' incentives to be honest fade away at a much faster rate in duopoly (i.e., $\overline{\gamma}^c$ goes to infinity faster than $\overline{\gamma}^m$). Intriguingly, this effect is strong enough that, when γ is close to $\overline{\gamma}^c$ and q_0 approaches 1, ε_1^m goes to zero faster than $1 - q_0$ and, thus, the inequality in equation (3.5) is satisfied.[28]

In Appendix 3.11.3, I provide numerical simulations of the model. The numerical analysis demonstrates that the set of parameter values for which welfare is larger in monopoly is significant. However, in the next Section I show that, when the CRAs are forced to disclose the indicative ratings they offer to issuers, competition can help restore the first best in the economy.

3.6 Disclosure of disagreement

The non-disclosure of indicative ratings (i.e., the rating each CRA offered to the issuer) when the issuer chooses to go as unrated is key in the perverse effect of competition on the CRAs' incentives to be honest, since it introduces the team dimension that dilutes their reputational concerns. In this section, I explore the effect of mandating disclosure of indicative ratings on the CRAs' incentives and the efficiency of the market. Like in the previous section, the focus will be on the quality of certification in the first period.

To do so, I introduce another step to the timing of the stage game. Now, when the issuer chooses not to publicize any ratings, each CRA is forced to disclose its indicative rating at the end of the period. Therefore, at the beginning of the second period, investors update the CRAs' reputation based on the ratings they offered to the first-period issuer - i.e., on $\widetilde{r}_1$ in monopoly and on $\widetilde{\mathbf{r}}_1 = (\widetilde{r}_{i1}, \widetilde{r}_{j1})$ in duopoly - and the signal about project quality y_1.

[27] The probability that a bad project receives high ratings is a linear function of q_0 in the scenario with a single CRA (i.e., $(1 - q_0)\,\varepsilon_1^m$) and a quadratic function of q_0 in the scenario with competing CRAs (i.e., $[(1 - q_0)\,\varepsilon_1^c]^2$). Therefore, the latter goes to zero faster when q_0 approaches 1.

[28] When $\gamma = \overline{\gamma}^c$, the inequality in equation (3.5) simplifies to $\varepsilon_1^m < 1 - q_0$, since we have $\varepsilon_1^c = 1$ at $\gamma = \overline{\gamma}^c$.

Analysis

In both monopoly and duopoly, the disclosure of indicative ratings has no direct effect on the issuer's strategy nor the CRAs' fees.[29] This is because, for a given expected rating inflation $\widehat{\varepsilon}_1$, the disclosure of $\widetilde{r}_1$ and $\widetilde{\mathbf{r}}_1$ does not affect the first-period issuer's payoff and, thus, the fees the CRAs can charge.

In the setting with a single CRA, investors always *infer* the rating that was offered to the issuer. Therefore, mandating disclosure of indicative ratings does not change how much they learn about the CRA's type.[30] This means that, in monopoly, the policy has no effect on the equilibrium rating strategy either and, as a consequence, the CRA still always inflates its rating with positive probability in both periods. However, in the scenario with two CRAs, this policy opens up the possibility of equilibria with honest ratings.

In what follows, I first describe the set of off-equilibrium beliefs that support these equilibria and then show that, given these beliefs, honest ratings represent indeed an equilibrium of the game.

Off-equilibrium Beliefs. Consider a candidate equilibrium where the CRAs offer honest ratings at time $t = 1$. If the CRAs are honest, they offer the same rating to the issuer, since they perfectly observe the quality of the project. This means that split indicative ratings, i.e., $\widetilde{\mathbf{r}}_1 = HL$ or LH, are off-the-equilibrium path. Therefore, we need to specify off-equilibrium beliefs when these split ratings are observed.

In a sequential equilibrium, strategies and beliefs can be derived as limits of *totally* mixed strategies and associated beliefs. A totally mixed strategy assigns strictly positive probability to every action at every information set, so that Bayes' rule pins down beliefs at each information set. In our example, we let the opportunistic CRAs "tremble" and offer a *dishonest* rating, i.e., a high (low) rating to a bad (good) project, with some small probability σ^n. When $\widetilde{\mathbf{r}}_1 = HL$, off-equilibrium reputations then represent the limits for σ^n converging to 0 of the conditional

[29] Therefore, Lemma 22 and Lemma 25 still describe the equilibrium play of the game, respectively, under monopoly and duopoly.

[30] When the issuer goes unrated, they infer he was offered a low rating that went unpurchased.

probabilities $\Pr(\tau_i = \mathcal{C} \mid (HL, y_1), \sigma^n)$ and $\Pr(\tau_j = \mathcal{C} \mid (HL, y_1), \sigma^n)$. This has the powerful implication that off-equilibrium beliefs are not arbitrary but, rather, uniquely defined via Bayes' rule on the *perturbed* game. [31]

Let q_i^{HL,y_1} and q_j^{HL,y_1} denote the CRAs' off-equilibrium reputations, when $\widetilde{\mathbf{r}}_1 = HL$ and for a given realization of y_1, and let $\Pr(\theta_1 \mid y_1)$ denote the probability that the quality of the project in the first period is θ_1 conditional on the realization of y_1. We have:

$$q_i^{HL,y_1} = \Pr(G \mid y_1)\, q_0; \quad q_j^{HL,y_1} = \Pr(B \mid y_1)\, q_0. \tag{3.6}$$

[32] When investors observe split ratings, they learn that their conjecture of honest ratings was wrong and that one of the CRAs lied - though they don't know which one. Since a committed CRA never lies, they also learn that at least one of the CRAs is an opportunistic type. Therefore, both CRAs lose reputation off-equilibrium; which one loses *more* reputation depends on the investors' beliefs about project quality and, thus, on the realization of the signal y_1. If y_1 signals a bad project, i.e., $\Pr(B \mid y_1) > \frac{1}{2}$, they believe that i - who gave the high rating - was most likely the one lying and, as a consequence, $q_i^{HL,y_1} < q_j^{HL,y_1}$. The opposite occurs when $\Pr(B \mid y_1) < \frac{1}{2}$. The case when $\widetilde{\mathbf{r}}_1 = LH$ is the mirror image of the one in which $\widetilde{\mathbf{r}}_1 = HL$.

Now that we have defined the CRAs' off-equilibrium reputation, we need to specify beliefs about the rating fees off-the-equilibrium path. Let $\widetilde{p}^{HH}(q_i, q_j)$ denote the price investors are willing to pay for a claim on the payoffs of a project with two high ratings, when the CRAs' reputations are q_i and q_j and expected rating inflation is $\widehat{\varepsilon}_2 = 1$.[33] Both *on* and *off* equilibrium, the CRAs extract all the issuer's surplus from selling the project, i.e.,

$$\varphi_i(q_i, q_j) + \varphi_j(q_i, q_j) = \widetilde{p}^{HH}(q_i, q_j) - I.$$

[31] Note that no *perturbed* strategies can generate a dishonest rating from a committed type, as this action is never taken by the committed CRAs.

[32] The reputations q_i^{HL,y_1} and q_j^{HL,y_1} in equation (3.6) represent, respectively, the limits for σ^n converging to 0 of the conditional probabilities $\Pr(\tau_i = \mathcal{C} \mid (HL, y_1), \sigma^n)$ and $\Pr(\tau_j = \mathcal{C} \mid (HL, y_1), \sigma^n)$ - see Appendix 3.12.

[33] The price $\widetilde{p}^{HH}(q_i, q_j)$ is described in Appendix 3.12. The opportunistic CRAs always inflate the ratings in the second period; therefore, the price is evaluated at the conjecture $\widehat{\varepsilon}_2 = 1$. Notice also that $\widetilde{p}^{HH}(q_i, q_j)$ is symmetric in the CRAs' reputation. This means that, for a given y_1, the price is the same when $\widetilde{\mathbf{r}}_1 = HL$ or $\widetilde{\mathbf{r}}_1 = LH$.

The fees $\{\varphi_i(q_i, q_j), \varphi_j(q_i, q_j)\}$ represent a *sharing rule* of the issuer's surplus between the CRAs, as a function of their reputations. I focus on the class of equilibria supported by off-equilibrium sharing rules that are (weakly) increasing in the CRAs' reputations, which means that, when $\widetilde{\mathbf{r}}_1 = HL$ or $\widetilde{\mathbf{r}}_1 = LH$, the CRA with higher reputation charges a higher fee, i.e., $\varphi_i \geq (\leq) \varphi_j$ if $q_i \geq (\leq) q_j$, and, if they have the same reputation, they equally split the issuer's surplus, i.e., $\varphi_i = \varphi_j = \frac{1}{2}\left(\widetilde{p}^{HH} - I\right)$ if $q_i = q_j$. [34]

Incentives to inflate the rating. Here, I discuss the incentives to inflate ratings in the conjectured equilibrium with honest ratings. Consider an opportunistic CRA-i who, at time $t = 1$, has observed a bad project and decides whether to inflate its rating or not. Let $\lambda \equiv \Pr(G) + \Pr(B)(1 - q_0)$; its total payoffs are as follows:

$$\Pi_{i,B} = \gamma\lambda \left\{ \varepsilon_{i1} \int_{y_1} \varphi_i\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) f_B(y_1)\, dy_1 + (1 - \varepsilon_{i1}) \frac{1}{2}\left(p^{HH}(q_0, 1) - I\right) \right\}.$$

Given the conjecture of honest ratings in the first period, i expects j to offer a low rating and, thus, expects that the issuer will choose not to publicize the ratings. Therefore, the decision of whether to inflate the rating or not has an effect only on the CRAs' second-period payoff, through the disclosure of the indicative ratings $\widetilde{\mathbf{r}}_1$. In the second period, the CRAs sell the rating and collect the fee if the project is good or, if it is bad, only when j is opportunistic - i.e., with probability $\lambda = \Pr(G) + \Pr(B)(1 - q_0)$. With probability ε_{i1}, the rating is inflated to $\widetilde{r}_{i1} = H$ and, thus, $\widetilde{\mathbf{r}}_1 = HL$. For a given realization of the signal y_1, the CRAs' reputation is updated to $\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right)$ and i's fee is $\varphi_i\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right)$. With probability $1 - \varepsilon_{i1}$, the rating is not inflated and, thus, $\widetilde{\mathbf{r}}_1 = LL$; the CRA's continuation payoff is $\gamma\frac{1}{2}\left(p^{HH}(q_0, 1) - I\right)$ in this case.[35]

The CRA's expected payoff decreases with ε_{i1} and, thus, inflating the rating is a *strictly*

[34] Even though the CRAs may have different reputations off-equilibrium, the issuer still needs two high ratings to sell the project and, thus, any sharing rule represent a Nash Equilibrium of the fee-setting subgame.

[35] On the equilibrium path, investors expect ratings to be honest and, thus, not informative about the CRAs' type. Therefore, reputation is not updated on the equilibrium path, i.e., $q^{HH,y_1} = q^{\varnothing\varnothing} = q_0$ for any $y_1 \in \mathcal{Y}$. Since both CRAs have the same reputation, they equally split the issuer's surplus from selling the project.

dominated strategy, if the following condition is satisfied:

$$\int_{y_1} \varphi_i \left(q_i^{HL,y_1}, q_j^{HL,y_1} \right) f_B \left(y_1 \right) dy_1 < \frac{1}{2} \left(p^{HH} \left(q_0, 1 \right) - I \right). \tag{3.7}$$

Incentives to deflate the rating. Now that split ratings are observable to investors, an opportunistic CRA may want to deflate its rating as well, possibly to look *more honest* than its competitor in the eyes of the investors. The analysis is similar to the one before, of the incentives to inflate ratings. Consider an opportunistic CRA-i who, at time $t = 1$, has observed a good project and decides whether to deflate its rating or not. The CRA's total payoffs are as follows:

$$\begin{aligned} \Pi_{i,G} = \gamma\lambda\delta_{i1} \int_{y_1} \varphi_i \left(q_i^{LH,y_1}, q_j^{LH,y_1} \right) f_G \left(y_1 \right) dy_1 \\ + \left(1 - \delta_{i1} \right) \frac{1}{2} \left(V_G - I \right) + \gamma\lambda \left(1 - \delta_{i1} \right) \frac{1}{2} \left(p^{HH} \left(q_0, 1 \right) - I \right). \end{aligned}$$

Now i expects that its rating is pivotal in the first period, as the CRA expects j to offer a high rating to the issuer. With probability δ_{i1}, the rating is deflated to $\widetilde{r}_{i1} = L$ and, thus, $\widetilde{\mathbf{r}}_1 = LH$; for a given realization of y_1, reputation is updated to $\left(q_i^{LH,y_1}, q_j^{LH,y_1} \right)$ and i's fee is $\varphi_i \left(q_i^{LH,y_1}, q_j^{LH,y_1} \right)$. With probability $1 - \delta_{i1}$, the rating is not deflated ($\widetilde{\mathbf{r}}_1 = HH$) and the CRAs collect the first-period fee $\frac{1}{2}(V_G - I)$; [36] the CRA's continuation payoff is then $\gamma\lambda\frac{1}{2}\left(p^{HH} \left(q_0, 1 \right) - I \right)$.

The CRA's expected payoff decreases with δ_{i1} and, thus, deflating the rating is a *strictly* dominated strategy, if the following condition is satisfied:

$$\int_{y_1} \varphi_i \left(q_i^{LH,y_1}, q_j^{LH,y_1} \right) f_G \left(y_1 \right) dy_1 < \frac{1}{2} \left(p^{HH} \left(q_0, 1 \right) - I \right) + \frac{1}{2\gamma\lambda} \left(V_G - I \right). \tag{3.8}$$

The following Proposition demonstrates that there always exist off-equilibrium sharing rules

[36] Investors expect ratings to be honest in the first period. Therefore, the price of the claim backed by a high-rated project is equal to the expected payoff of a good project V_G.

for which both the inequalities in (3.7) and (3.8) are satisfied. Therefore, for these sharing rules, honest ratings represent an equilibrium of the game.

Proposition 30 *Under mandatory disclosure of indicative ratings, we have:*

1. *In monopoly, the equilibrium outcome does not change (i.e., $\varepsilon_1^m > 0$);*

2. *In duopoly, there always exists an equilibrium where the CRAs offer honest ratings at time $t = 1$ (i.e., $\varepsilon_1^c = 0$).*

When the CRAs, even off-equilibrium, always equally split the issuer's surplus, the inequalities in (3.7) and (3.8) are trivially satisfied. This is because, for any realization of y_1, the off-equilibrium fee is $\varphi_i = \frac{1}{2}\left[\widetilde{p}^{HH}\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) - I\right] = \frac{1}{2}\left[\widetilde{p}^{HH}\left(q_i^{LH,y_1}, q_j^{LH,y_1}\right) - I\right]$, which is always lower than the rating fee on the equilibrium path.[37] Therefore, honest ratings are always an equilibrium of the game when the off-equilibrium belief is that $\varphi_i = \varphi_j$ when $\widetilde{\mathbf{r}}_1 = HL$ or $\widetilde{\mathbf{r}}_1 = LH$ and for any realization of y_1. As the signal y_1 becomes more informative about the true project quality, the set of off-equilibrium sharing rules that support this equilibrium becomes larger. This is because, when y_1 is more informative, investors tend to be right more often about which CRA was most likely to be lying. Therefore, when a CRA *lies*, either by inflating or by deflating the rating, it is more likely to be the one with lower reputation and, thus, to charge the low fee in the second period, i.e., $\varphi \leq \frac{1}{2}\left(\widetilde{p}^{HH} - I\right)$. If the signal is informative *enough*, any increasing sharing rule of the kind described earlier supports the equilibrium.[38]

Example 31 *Consider a simple sharing rule where, given reputations q_i and q_j, i gets a fraction $\frac{q_i}{q_i+q_j}$ and j a fraction $\frac{q_j}{q_i+q_j}$ of the issuer's surplus $\widetilde{p}^{HH}(q_i, q_j) - I$. Assume also that $\Pr(G) = \Pr(B)$, and that there are two possible signals, i.e., $y_1 \in \{g, b\}$, where $\Pr(G \mid g) = \Pr(B \mid b) = \alpha \in [\frac{1}{2}, 1)$. If y_1 is informative about the true project quality, i.e., if $\alpha > \frac{1}{2}$, this sharing rule always supports the equilibrium with honest ratings.*[39]

[37] Both CRAs lose reputation off-equilibrium and so $\widetilde{p}^{HH}\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) < p^{HH}(q_0, 1)$, for any $y_1 \in \mathcal{Y}$.

[38] Of course, when realizations of y_1 for which $\Pr(B \mid y_1) >(<) \frac{1}{2}$ are almost sure when $\theta_1 = B$ ($\theta_1 = G$), a CRA that lies is almost sure to get the lower fee $\varphi \leq \frac{1}{2}\left(\widetilde{p}^{HH} - I\right)$ in the second period. Therefore, CRAs are always better off by offering honest ratings in this case, regardless of the specific form of the sharing rule.

[39] See Appendix 3.12 for a proof of Example 1.

In the setting without disclosure of indicative ratings (Section 3.4), inflating the rating is always a dominant strategy for an opportunistic CRA, even when the probability that the other CRA is inflating the rating is small. This is because investors never observe the indicative ratings and, thus, the CRA is never "caught" lying. Therefore, honest ratings are not possible in equilibrium. On the contrary, in the setting with mandatory disclosure that I analyzed in this section, investors observe the indicative ratings offered to the issuer. When they observe split ratings, they learn that at least one of the CRAs is opportunistic and, as a consequence, both CRAs lose reputation. This reputational loss, together with the restored CRAs' concerns for relative reputation, makes deviations from an equilibrium with honest ratings strictly unprofitable for a large class of off-equilibrium beliefs.

3.7 Implications

This chapter demonstrates that competition among CRAs can increase or decrease their incentives to offer honest ratings, depending on whether investors can observe disagreement among CRAs in the form of split ratings. This section discusses some implications of the model and the related empirical evidence.

A cross-sectional interpretation of the results in Propositions 28 and 30 is that (i) rating inflation should be lower and (ii) competition should improve ratings quality in markets where investors can observe disagreement among CRAs. There is consistent empirical evidence that rating inflation was larger in structured finance markets - see, e.g., Cornaggia, Cornaggia and Hund (2017). Intriguingly, in corporate bond markets disclosed ratings tend to disagree more often than in structured finance markets.[40] Furthermore, CRAs issue unsolicited ratings, i.e., ratings that are not paid for by the issuer, in corporate bond markets but not in the structured finance segment.[41] Similarly to the setting with mandatory disclosure of indicative ratings ana-

[40] S&P and Moody's both cover virtually all corporate bonds in the US. Several studies in the literature document that approximately 13% to 17% of U.S. corporate debt issues receive different letter ratings from Moody's and S&P - see Jewell and Livingston (1999) and Bongaerts, Cremers, and Goetzmann (2012).

[41] Given the scarcity of public information about structured finance products, unsolicited ratings in the structured finance segments are extremely rare, and even a CRA that is provided with information to form a prospective assessment is unlikely to issue a public rating of the product if it is not hired by the investment bank, since the final composition of the tranches may vary by the time the security is issued. Fulghieri, Strobl, and Xia (2014)

lyzed in Section 3.6, the possibility of issuing unsolicited ratings can break the team dimension and lead to equilibria with honest ratings.[42]

The empirical work on the effects of competition on ratings quality provides mixed predictions - see Bae, Driss and Roberts (2017) for a review. Interestingly, increased competition among CRAs is strongly associated with a positive bias in ratings in the structured products market. For corporate bonds, there are results on both sides, and the issue remains open for future research. The theoretical results in my paper highlight how the effect of competition on ratings quality crucially depends on certain features of the environment, namely, whether investors can observe split ratings and, if they can, the amount of information they have about the rated products. This suggests that in order to find a robust relation between competition and ratings quality, it may be necessary to condition on the appropriate features of the environment.

The result in Proposition 30 highlights a new rationale for requiring disclosure of indicative ratings by CRAs. In the model, this policy allows investors to learn more about the CRAs' type, making reputational concerns more effective in disciplining the CRAs' incentives. The implementation of the EU Regulation No 462/2013, that requires disclosure of indicative ratings by CRAs, would be a unique opportunity to test this prediction of the model.[43] From a different angle, the result in Proposition 30 also unveils a potential negative effect of policies that constrain institutional investors to hold fixed-income assets that are certified as safe by multiple AAA ratings. By reducing the issuers' incentives to go on the market when they receive split ratings, these policies might reduce the possibility of observing disagreement among CRAs. This could introduce the team dimension that dilutes the CRAs' reputational concerns and leads to inflated ratings, moving away the economy from its first best. It would be interesting to model the issuer's incentives to go on the market to create a richer environment and investigate in which cases this policy can reduce welfare.

analyze the incentives to issue unsolicited ratings in corporate bond markets for a monopolistic CRA.

[42] One simple way to introduce unsolicited ratings in the model is by allowing the CRAs to disclose their indicative ratings $(\widetilde{r}_{i1}, \widetilde{r}_{j1})$, when the issuer chose not to publicize them, as unsolicited ratings at the end of the first period, and assume that a committed CRA always discloses its indicative rating. Such a model is isomorphic to the one presented in Section 3.6, since, then, an opportunistic CRA is forced to disclose the indicative rating too, otherwise it would reveal its type to the investors.

[43] This regulation has not been implemented yet and there is discussion among the CRAs and the regulator surrounding its implementation - https://www.esma.europa.eu/sites/default/files/library/esma33-9-235_consultation_paper_on_draft_guidelines_under_the_cra_regulation.pdf

Finally, even when competition reduces the incentives to offer honest ratings, it can still increase welfare due to the positive effect of multiple signals. Lemma 29 contrasts the relative magnitude of the two effects, i.e., the double screening and the perverse effect on incentives, and shows that the perverse effect is more likely to dominate when the initial reputation of the CRAs is large. This implies that competition is more likely to reduce ratings quality during boom times, when the trust in the financial industry and the CRAs is high.

3.8 Conclusion

Credit ratings have real effects and, thus, an understanding of their dynamics is crucial. In this chapter, I analyzed the effect of competition among credit rating agencies on the quality of certification. I have shown that competition has two contrasting effects. On the one hand, the issuer needs to pass the screening of two CRAs; other things being equal, this improves the quality of certification. On the other hand, since it introduces a team dimension that dilutes the CRAs' reputational concerns, it has the perverse effect of incentivizing dishonest ratings. When the perverse effect dominates, competition reduces the quality of certification. However, mandating disclosure of indicative ratings can restore the first-best outcome, by inducing always honest ratings. These results provide new important insights into the effects of competition among information intermediaries, which is a central question in the regulatory debate.

Bibliography

[1] Acharya, V., and M. Richardson. 2009. Causes of the financial crisis. Critical Review, 21:195–210.

[2] Acharya, V., P. Schnabl, and G. Suarez. 2013. Securitization without risk transfer. Journal of Financial Economics, 107:515–36.

[3] Avery, C., and Meyer, M. Reputational Incentives for Biased Evaluators. Unpublished Manuscript.

[4] Bae, K-H., H. Driss, G. S. Roberts. 2017. Do Credit Rating Agencies Inflate Their Ratings? A Review. Journal of Financial Transformation, 45: 90-100.

[5] Baghai, R., and Becker, B. 2018. Reputations and credit ratings—evidence from commercial mortgage-backed securities. Working Paper, Stockholm School of Economics.

[6] Bar-Isaac, H. and J. Shapiro, 2013. Rating Quality over the Business Cycle. Journal of Financial Economics 108, 62-78.

[7] Benabou, R., and Laroque, G. 1992. Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility. Quarterly Journal of Economics 107 (3): 921–58.

[8] Bolton, P., Freixas, X. and Shapiro, J. 2012. The Credit Ratings Game. Journal of Finance, 67: 85–111.

[9] Bongaerts, D., Cremers, M., Goetzmann, W. 2012. Tiebreaker: Certification and Multiple Credit Ratings. Journal of Finance, 67: 113-152.

[10] Bouvard, M. and Levy, R. Forthcoming. Two-Sided Reputation in Certification Markets. Management Science.

[11] Cho, I.-K., and D. M. Kreps. 1987. Signaling games and stable equilibria. Quarterly Journal of Economics, 102:179–221.

[12] Cornaggia, J., Cornaggia, K., Hund, J. 2017. Credit Ratings across Asset Classes: A Long-Term Perspective. Review of Finance, 21 (2) 465-509.

[13] Ely, J. C., and Välimäki, J. 2003. Bad Reputation. Quarterly Journal of Economics 118 (3): 785–814.

[14] Farkas, M. 2017. Credit Rating Catering. Working Paper.

[15] Frenkel, S., 2015. Repeated Interaction and Rating Inflation: A Model of Double Reputation. American Economic Journal: Microeconomics 7, 250-280.

[16] Fudenberg D, Tirole J. Game Theory. Cambridge, MA: MIT Press; 1991.

[17] Fulghieri, P., G. Strobl, and H. Xia, 2014. The Economics of Solicited and Unsolicited Credit Ratings. Review of Financial Studies, 27:2, 484-518.

[18] Griffin, J. M., and Tang, D. Y. 2012. Did subjectivity play a role in CDO credit ratings? Journal of Finance, 67: 1293-1328.

[19] Griffin, J., Nickerson, J., and Tang, D. Y. 2013. Rating Shopping or Catering? An Examination of the Response to Competitive Pressure for CDO Credit Ratings. Review of Financial Studies, 26 (9): 2270-2310.

[20] Guembel, A., and Rossetto, S., 2009, Reputational Cheap Talk with Misunderstanding, Games and Economic Behavior, vol. 67(2), 736–744.

[21] He, J., Qian, J., Strahan, P.. 2016. Does the Market Understand Rating Shopping? Predicting MBS Losses with Initial Yields. Review of Financial Studies, 29:2, 457-485.

[22] Jewell, J. and Livingston, M. (1999), A Comparison of Bond Ratings from Moody's S&P and Fitch IBCA. Financial Markets, Institutions & Instruments, 8: 1-45.

[23] Kedia, S., Rajgopal, S., Zhou, X., 2017. Large shareholders and credit ratings. Journal of Financial Economics, 124(3), 632-653.

[24] Kreps, D. M., and R. Wilson. 1982a. Sequential Equilibria, Econometrica, 50, 4: 863-94.

[25] Kreps, D. M., and R. Wilson. 1982b. Reputation and Imperfect Information, Journal of Economic Theory, 27, 253-279.

[26] Laffont, J.-J., and J. Tirole. 1993. A Theory of Incentives in Procurement and Regulation (MIT Press, Cambridge).

[27] Mathis, J., McAndrews, J., Rochet, J.C. 2009. Rating the raters: Are reputation concerns powerful enough to discipline rating agencies? Journal of Monetary Economics 56, 657-674.

[28] Milgrom, P., and J. Roberts, 1982, Predation, Reputation and Entry Deterrence, Journal of Economic Theory, 27(2), 280-312.

[29] Morris, S. 2001. Political Correctness. Journal of Political Economy 109 (2): 231–65.

[30] Partnoy, F. 2006. How and Why Credit Rating Agencies are Not Like Other Gatekeepers. In Financial Gatekeepers: Can They Protect Investors? edited by Yasuyuki Fuchita and Robert E. Litan, 59–99. Washington, DC: Brookings Institution Press.

[31] Sangiorgi, and C. Spatt, 2016. Opacity, credit rating shopping, and bias. Management Science, 4016 - 4036.

[32] Sangiorgi, and C. Spatt, 2017. The Economics of Credit Rating Agencies. Foundations and Trends in Finance, 12, 1-116.

[33] Securities and Exchange Commission (SEC), 2008, Summary rating of Issues Identified in the Commission Staff's Examinations of Select Credit Rating Agencies. Available at http://sec.gov/news/studies/2008craexamination070808.pdf.

[34] Skreta, V., and L. Veldkamp. 2009. Ratings shopping and asset complexity: A theory of ratings inflation. Journal of Monetary Economics 56, 678–695.

[35] Sobel, J. 1985. A Theory of Credibility. Review of Economic Studies 52 (4): 557–73.

[36] Strausz, R. 2005. Honest Certification and the Threat of Capture. International Journal of Industrial Organization, 23:1-2, 45-62.

[37] White, L. J. 2010. Markets: The Credit Rating Agencies. Journal of Economic Perspectives, 24 (2): 211-26.

3.9 Appendix A

3.9.1 Sequential Equilibrium: Definition

In this subsection, I define the concept of Sequential Equilibria, as put forth by Kreps and Wilson (1982a). The exposition follows Fudenberg and Tirole (1991).

Let $u_{i(h)}(\sigma \mid h, \psi(h))$ be the expected utility of player $i(h)$ given that information set h is reached, that the player's beliefs are given by $\psi(h)$, and that the strategies are σ. An *assessment* (σ, ψ) specifies a strategy profile σ and a system of beliefs ψ. The set of all possible assessments is denoted by Ψ. Now, we need to relate σ and ψ to one another. There are two aspects to this:

First, the strategy profile σ must be "rational" given beliefs ψ; this is the idea of *sequential rationality*. It simply states that, given the system of beliefs ψ, no player can gain by deviating at any information set:

(S) An assessment (σ, ψ) is *sequentially rational* if, for any information set h and alternative strategy $\sigma'_{i(h)}$,

$$u_{i(h)}(\sigma \mid h, \psi(h)) \geq u_{i(h)}\left(\sigma'_{i(h)}, \sigma_{-i(h)} \mid h, \psi(h)\right).$$

Note that players believe that their opponents will adhere to the equilibrium profile σ at every information set, including ones that should not be reached if all players adhere to σ.

Second, beliefs ψ must be "rational" given the strategy profile σ; this is the notion of *consistency*. Consistency imposes two kinds of restrictions. One is for information sets that are "reached" under the strategy profile: for these sets, ψ must be given by Bayes' Rule, given σ. The second is for information sets which are not reached at all. Bayes' Rule cannot be applied to sets which are reached with probability zero; yet, we do need some specification. This is where the full definition of consistency is needed. Let Σ^0 denote the set of all completely mixed (behavioral) strategies, i.e., that assign strictly positive probability to every action at every information set. If $\sigma \in \Sigma^0$, then $\Pr^\sigma(v) > 0$ for all nodes v, so that Bayes' rule pins down beliefs at each information set: $\mu(v) = \Pr^\sigma(v) / \Pr^\sigma(h(v))$. Let Ψ^0 denote the set of all assessment (σ, ψ) such that $\sigma \in \Sigma^0$ and ψ is (uniquely) defined from σ by Bayes' rule.

(C) An assessment (σ, ψ) is *consistent* if

$$(\sigma, \psi) = \lim_{n \to \infty} (\sigma^n, \psi^n)$$

for some sequence (σ^n, ψ^n) in Ψ^0.

Note that the strategies need not be totally mixed; however, they and the beliefs can be regarded as limits of totally mixed strategies and associated beliefs. Thus, we require μ to be "the limit of a system of consistent beliefs" given a sequence of completely mixed strategy profiles that converge to the strategy profile in question.

Definition 32 *A sequential equilibrium is an assessment (σ, ψ) that satisfies conditions S and C.*

3.9.2 Proof of Lemma 22

Part 1 of Lemma 22 is straightforward and, thus, its proof is omitted. I focus on Part 2 of the Lemma in what follows.

Notice that, since investors don't observe the rating fee, the CRA cannot use it to signal its type to investors. Of course, the first-period issuer does observe the fee but (i) he does not take

action in the second period and (ii) he cannot credibly communicate the CRA's type (even if he knew it) to the investors. Therefore, signalling its type to the issuer is inconsequential for the CRA. Given that there is no signalling scope in the fee setting sub-game, and that the issuer cannot sell without a high rating, both CRA-types extract all the issuer's surplus from selling the project, i.e. $\varphi\left(q,\widehat{\varepsilon}\right)=p^{H}\left(q,\widehat{\varepsilon}\right)-I$.

The committed CRA-type is restricted to truthful ratings about the project's quality, but is strategic in the way it sets the fee. In principle, after observing a good project in the first period, a committed CRA may want to offer a fee which is too high for the issuer, i.e., $\varphi_1 > p^{H}\left(q_0,\widehat{\varepsilon}_1\right)-I$. This way the issuer would choose not to publicize the rating and the CRA could maintain its reputation (as then investors believe the CRA offered a low rating which went unpurchased).[44] I show that this never happens in equilibrium in Appendix 3.9.5, since I first need to characterize the equilibrium rating strategy. The opportunistic CRA could instead simply offer a low rating to the issuer, i.e., *deflate* the rating; as I show in Proposition 23, this also never happens in equilibrium. Therefore, both CRA-types find it optimal to set a fee $\varphi\left(q,\widehat{\varepsilon}\right)=p^{H}\left(q,\widehat{\varepsilon}\right)-I$ in each period.

3.9.3 Posterior Reputation

In what follows, all the probability measures are for a given conjecture $\widehat{\varepsilon}_1$ about the opportunistic CRA's rating inflation strategy at time $t=1$. At time $t=1$, an opportunistic CRA is concerned with the reputation it has when it enters time $t=2$. At the end of time $t=1$, for a given rating r_1 and a realization of the signal y_1, the CRA's posterior reputation is:

$$\Pr\left(\mathcal{C}\mid r_1,y_1\right)=\left\{1+\frac{1-q_0}{q_0}\frac{\Pr\left(r_1,y_1\right|\mathcal{O}\right)}{\Pr\left(r_1,y_1\right|\mathcal{C}\right)}\right\}^{-1}\equiv q^{r_1,y_1}.$$

At time $t=1$, the CRA's private information refers to the project quality θ_1 and its type $\tau\in\{\mathcal{O},\mathcal{C}\}$. We can then denote the CRA's overall type at time $t=1$ by a pair (τ,θ_1). We

[44] The CRA does not care about maintaining its reputation in the second period, as the game then ends. Therefore, both CRA-types always set $\varphi_2=p^{H}\left(q,\widehat{\varepsilon}_2\right)-I$ in the second period, for any level of reputation q.

have:

$$\begin{aligned}\Pr(r_1, y_1|\,\tau) &= \sum_{\theta_1\in\Theta}\Pr(\theta_1)\Pr(r_1, y_1|\,\tau,\theta_1)\\ &= \sum_{\theta_1\in\Theta}\Pr(\theta_1)\underbrace{\Pr(y_1\mid\theta_1)}_{\substack{y_1,r_1\text{ independent}\\ \text{conditional on }\theta_1}}\Pr(r_1|\,\tau,\theta_1)\\ &= \sum_{\theta_1\in\Theta}\Pr(y_1\cap\theta_1)\Pr(r_1|\,\tau,\theta_1).\end{aligned}$$

When $r_1 = H$, we have:

$$\frac{\Pr(H, y_1|\,\mathcal{O})}{\Pr(H, y_1|\,\mathcal{C})} = \frac{\overbrace{\Pr(y_1\cap G)}^{\text{type }(\mathcal{O},G)} + \overbrace{\Pr(y_1\cap B)\widehat{\varepsilon}_1}^{\text{type }(\mathcal{O},B)}}{\underbrace{\Pr(y_1\cap G)}_{\text{type }(\mathcal{C},G)}} = 1 + \frac{\Pr(y_1\cap B)}{\Pr(y_1\cap G)}\widehat{\varepsilon}_1.$$

Hence,

$$q^{H,y_1} = \left\{1 + \frac{1-q_0}{q_0}\left(1 + \frac{\Pr(y_1\cap B)}{\Pr(y_1\cap G)}\widehat{\varepsilon}_1\right)\right\}^{-1}.$$

First, notice that q^{H,y_1} is decreasing in $\widehat{\varepsilon}_1$. As $\widehat{\varepsilon}_1$ increases, investors expect that an opportunistic CRA inflates its rating more often at time $t = 1$. Therefore, when they observe a high rating, they think it is (relatively) more likely it came from an opportunistic CRA. Second, q^{H,y_1} is increasing in y_1. As y_1 increases, it is (relatively) less likely that the project was bad and the rating was inflated.

When $r_1 = \varnothing$, we have:

$$\frac{\Pr(\varnothing, y_1|\,\mathcal{O})}{\Pr(\varnothing, y_1|\,\mathcal{C})} = \frac{\overbrace{\Pr(y_1\cap B)(1-\widehat{\varepsilon}_1)}^{\text{type }(\mathcal{O},B)}}{\underbrace{\Pr(y_1\cap B)}_{\text{type }(\mathcal{C},B)}} = 1 - \widehat{\varepsilon}_1.$$

Hence,

$$q^{\varnothing} = \left\{1 + \frac{1-q_0}{q_0}(1-\widehat{\varepsilon}_1)\right\}^{-1}. \tag{3.9}$$

Note that $q^{\varnothing}$ does not depend on y_1. When the project is unrated, investors understand the CRA observed a bad project and chose not to inflate it. As a consequence, they learn the project is bad and, thus, don't update their beliefs about project quality based on the realization of y_1. Notice also that $q^{\varnothing}$ increases with ε_1^e: as ε_1^e increases, an opportunistic CRA is expected to inflate its ratings more often at time $t = 1$, which means that $r_1 = \varnothing$ is less likely to come from an opportunistic type.

3.9.4 Proof of Proposition 23

I prove Proposition 23 in two steps. In the first step, I prove that the marginal benefit of inflation MBI is always positive at $\widehat{\varepsilon}_1 = 0$ and decreasing (and continuous) in $\widehat{\varepsilon}_1$, which implies that the characterization of the equilibrium rating inflation ε_1^m is as described in Proposition 23. In the second step, given the characterization of ε_1^m, I prove that deflating the rating for a good project is always a dominated action in equilibrium.

Step one: Equilibrium Rating Inflation

We have:

$$\frac{d\Pi_B}{d\varepsilon_1} = \varphi\left(q_0, \widehat{\varepsilon}_1\right) + \gamma \int_{y_1} \varphi\left(q^{H,y_1}, 1\right) f_B\left(y_1\right) dy_1 - \gamma\varphi\left(q^{\varnothing}, 1\right). \tag{3.10}$$

Notice that the reputational loss of rating inflation is null at $\widehat{\varepsilon}_1 = 0$, as we have $q^{H,y_1} = q_0 = q^{\varnothing}$ for any $y_1 \in \mathcal{Y}$ when $\widehat{\varepsilon}_1 = 0$. Hence, MBI is always positive at $\widehat{\varepsilon}_1 = 0$, since $\frac{d\Pi_B}{d\varepsilon_1}\left(\widehat{\varepsilon}_1 = 0\right) = \varphi\left(q_0, 0\right) = V_G - I > 0$. [45]

$\frac{d\Pi_B}{d\varepsilon_1}$ is also always decreasing in $\widehat{\varepsilon}_1$; we have:

$$\frac{d\frac{d\Pi_B}{d\varepsilon_1}}{d\widehat{\varepsilon}_1} = \underbrace{\frac{\partial\varphi\left(q_0, \widehat{\varepsilon}_1\right)}{\partial\widehat{\varepsilon}_1}}_{<0} + \gamma \int_{y_1} \underbrace{\frac{\partial\varphi\left(q^{H,y_1}, 1\right)}{\partial q^{H,y_1}}}_{>0} \underbrace{\frac{\partial q^{H,y_1}}{\partial\widehat{\varepsilon}_1}}_{<0} f_B\left(y_1\right) dy_1 - \underbrace{\frac{\partial\varphi\left(q^{\varnothing}, 1\right)}{\partial q^{\varnothing}}}_{>0} \underbrace{\frac{\partial q^{\varnothing}}{\partial\widehat{\varepsilon}_1}}_{>0} < 0,$$

where the signs of the derivatives above come from the fact that, in equilibrium, we have $\varphi\left(q, \widehat{\varepsilon}\right) = p^H\left(q, \widehat{\varepsilon}\right) - I$ for any values of q and $\widehat{\varepsilon}$.

[45] When $\widehat{\varepsilon}_1 = 0$, a high rating is perceived by investors as a perfect signal for a good project and, thus, we have $p^H = V_G$.

Step two: No Rating Deflation in equilibrium

Deflating the rating for a good project is trivially a dominated strategy in the second period;[46] here, I show that it is also a dominated strategy in the first period. I first discuss the intuition behind this result and then proceed with the proof.

As we have seen in Section 3.3, an opportunistic CRA always inflates the rating for a bad project with positive probability. Therefore, it must be at least indifferent between offering a high or a low rating to an issuer with a bad project. The reputational loss from offering a *dishonest* high rating to a bad project is larger than the one from offering an *honest* high rating to a good project, as the quality of the project will be partly revealed to the market via the realizations of y_1. This implies that the opportunistic CRA must be always *strictly* better-off by selling the rating to a good project. Therefore, selling the rating is always attractive enough in equilibrium that there is no incentive to deflate.

I now proceed with the formal proof the proposition. Consider an opportunistic CRA who, at time $t = 1$, has observed a good project (i.e., type $(\mathcal{O}, G)$) and decides whether to deflate its rating or not. The following expression characterizes the CRA's trade-off:

$$\gamma\varphi\left(q^{\varnothing}, 1\right) - \varphi\left(q_0, \varepsilon_1^m\right) - \gamma \int_{y_1} \varphi\left(q^{H,y_1}, 1\right) f_G\left(y_1\right) dy_1. \tag{3.11}$$

If the CRA deflates the rating, the issuer goes unrated and the first-period fee is not collected. The CRA enters the second period with high reputation, and the continuation payoff is $\gamma\varphi\left(q^{\varnothing}, 1\right)$. If the CRA does not deflate, it collects the first-period fee $\varphi\left(q_0, \varepsilon_1^m\right)$ but enters the second period with low reputation and, thus, the continuation payoff is $\gamma\varphi\left(q^{H,y_1}, 1\right)$. Since the CRA knows that the project is good, the expectation over the realization of y_1 is conditional on $\theta_1 = G$.

If the sign of the expression in equation (3.11) is negative, the opportunistic CRA is always

[46] Suppose the CRA has reputation q in the second period. If the CRA deflates the report, the continuation payoff is 0; if it doesn't, the continuation payoff is $\varphi(q, 1)$. Hence, deflating is always a dominated strategy in the second period.

better off by not deflating the rating. Therefore, we want to show that:

$$\varphi\left(q_0, \varepsilon_1^m\right) > \gamma \left[\varphi\left(q^{\varnothing}, 1\right) - \int_{y_1} \varphi\left(q^{H,y_1}, 1\right) f_G\left(y_1\right) dy_1\right]. \tag{3.12}$$

Note that the equilibrium level of rating inflation ε_1^m satisfies the following condition:

$$\varphi\left(q_0, \varepsilon_1^m\right) \geq \gamma \left[\varphi\left(q^{\varnothing}, 1\right) - \int_{y_1} \varphi\left(q^{H,y_1}, 1\right) f_B\left(y_1\right) dy_1\right]. \tag{3.13}$$

Notice also that q^{H,y_1} is increasing in y_1 - see Appendix 3.9.3. The assumption that $f_\theta\left(y\right)$ satisfies the *Monotone Likelihood Ratio Property* also implies that the distribution of y_1 conditional on $\theta = G$ represents a *First-Order Stochastic Dominance* shift of the distribution of y_1 conditional on $\theta = B$. As a result, we have:

$$\int_{y_1} \varphi\left(q^{H,y_1}, 1\right) f_G\left(y_1\right) dy_1 \geq \int_{y_1} \varphi\left(q^{H,y_1}, 1\right) f_B\left(y_1\right) dy_1. \tag{3.14}$$

Therefore, the inequalities in equation (3.13) and (3.14) imply that the inequality in (3.12) is always satisfied. This means that deflating the rating for a good project is always a dominated action in equilibrium.

3.9.5 A committed CRA prefers to be hired in equilibrium

In the first period, if the CRA is a committed type and offers a fee $\varphi_1 > p^H\left(q_0, \varepsilon_1^m\right) - I$, the issuer chooses not to publicize the rating and the CRA enters the second period with reputation $q^{\varnothing}$. If the second-period project is good (the CRA is committed and, thus, will not inflate the rating if project quality is bad), the CRA then sells the rating and collects a fee $\varphi\left(q^{\varnothing}, 1\right)$.

If the CRA offers a fee $\varphi_1 = p^H\left(q_0, \varepsilon_1^m\right) - I$, the issuer publicizes the rating and, for a given realization of the signal y_1, the CRA enters the second period with reputation q^{H,y_1}; if the second-period project is good, it then sells the rating and collects a fee $\varphi\left(q^{H,y_1}, 1\right)$.

Therefore, a committed CRA prefers to be hired to publicize the rating in the first period

(i.e., sets a fee $\varphi_1 = p^H(q_0, \varepsilon_1^m) - I$) if the following condition is satisfied:

$$\Pr(G)\gamma\varphi\left(q^{\varnothing}, 1\right) < \varphi(q_0, \varepsilon_1^m) + \Pr(G)\gamma\int_{y_1}\varphi\left(q^{H,y_1}, 1\right) f_G(y_1)\, dy_1 \tag{3.15}$$

$$\Leftrightarrow \varphi(q_0, \varepsilon_1^m) > \Pr(G)\gamma\left[\varphi\left(q^{\varnothing}, 1\right) - \int_{y_1}\varphi\left(q^{H,y_1}, 1\right) f_G(y_1)\, dy_1\right]. \tag{3.16}$$

Equations (3.13) and (3.14) imply that the inequality above is always satisfied. This means that it is always optimal for a committed CRA to set $\varphi_1 = p^H(q_0, \widehat{\varepsilon}_1) - I$ and sell the rating for a good project in both periods.

3.10 Appendix B

3.10.1 Proof of Lemma 25

I prove Lemma 25 in two steps. First (Step One), I show that the conjecture that an issuer who received two high ratings discloses only one cannot be part of an equilibrium of the game. This result will then be used in Step Two to characterize the equilibrium strategies.

Step One.

A project with only one high rating must come from one of two cases: (1) an issuer who received two high ratings (i.e., $\widetilde{\mathbf{r}} = HH$) but disclosed only one; (2) an issuer who received a low rating and a high rating (i.e., $\widetilde{\mathbf{r}} = HL$ or $\widetilde{\mathbf{r}} = LH$), and disclosed only the high rating. Let $\widetilde{p}$ denote the price investors are willing to pay for the claim to the payoff of the project when the project has only one high rating (i.e., $\mathbf{r} = H\varnothing$ or $\mathbf{r} = \varnothing H$); the price $\widetilde{p}$ must be a convex combination of $p^{HH}(q, \widehat{\varepsilon})$, i.e., the value of the project's payoff in case (1) and the value investors assign to the project's payoff in case (2). [47]

[47] When $\widehat{\varepsilon} > 0$, investors believe the high rating was inflated and, thus, the value of the claim in case (2) is V_B, as they believe that the project is of bad quality. Hence, $p^{HH}(q, \widehat{\varepsilon}) > \widetilde{p} = V_B$ if $\widehat{\varepsilon} > 0$. When $\widehat{\varepsilon} = 0$, we have $p^{HH}(q, \widehat{\varepsilon}) = V_G$, while the exact value of $\widetilde{p}$ depends on the investors' beliefs in this case since, when $\widehat{\varepsilon} = 0$, split ratings are off-the-equilibrium path (CRAs perfectly observe project quality and, thus, should agree in their ratings if these are honest); however, we still must have $\widetilde{p} \leq V_G$ and, thus, $p^{HH}(q, \widehat{\varepsilon} = 0) \geq \widetilde{p}$. Therefore, we have $p^{HH}(q, \widehat{\varepsilon}) \geq \widetilde{p}$ for any value of $\widehat{\varepsilon}$.

If an issuer who received two high ratings is expected to disclose only one, then a project with two high ratings is off-the-equilibrium path. However, only type $\widetilde{\mathbf{r}} = HH$ can disclose two high ratings, as types $\widetilde{\mathbf{r}} = HL$ or $\widetilde{\mathbf{r}} = LH$ have been offered only one. Therefore, when observing $\mathbf{r} = HH$, investors are always willing to pay $p^{HH}(q, \widehat{\varepsilon})$ for the project. This implies that, for any fee $\varphi \leq p^{HH}(q, \widehat{\varepsilon}) - \widetilde{p}$, disclosing a second high rating represents a profitable deviation for type $\widetilde{\mathbf{r}} = HH$. [48] Notice also that, in this candidate equilibrium, type $\widetilde{\mathbf{r}} = HH$ publicizes the cheapest of the two ratings.[49]

Now, suppose that the project's quality is good. In this case, both CRAs offer a high rating to the issuer.[50] This means that, for any fee φ_j charged by j, it is optimal for i to slightly undercut φ_j, so to make sure that the issuer buys i's rating. The CRAs then start undercutting each others' fees until they reach a fee at which the issuer is willing to buy both ratings, i.e., $\varphi = p^{HH}(q, \widehat{\varepsilon}) - \widetilde{p}$. At this fee, however, type $\widetilde{\mathbf{r}} = HH$ chooses to disclose both ratings. Therefore, when the project is good, the issuer must disclose two high ratings in equilibrium.

As a consequence, when the project does not receive two high ratings ($\mathbf{r} \neq HH$), investors infer that the project is bad and, thus, $p^{\mathbf{r} \neq HH} = V_B$ in equilibrium. Therefore, the conjecture that an issuer who received two high ratings discloses only one cannot be part of an equilibrium.

Step Two.

In Step One of the proof, I showed that an issuer who received two high ratings must disclose both in equilibrium (i.e., $\mathbf{r} = HH$ when $\widetilde{\mathbf{r}} = HH$). This has two important implications. First, an issuer who did not receive two high ratings does not disclose any in equilibrium (i.e., $\mathbf{r} = \varnothing\varnothing$ when $\widetilde{\mathbf{r}} = HL$, $\widetilde{\mathbf{r}} = LH$, and $\widetilde{\mathbf{r}} = LL$). Second, given that the issuer always needs two high ratings to sell the claim and implement the project, the CRAs can extract all the issuer's surplus, i.e., $\varphi_i + \varphi_j = p^{HH}(q, \widehat{\varepsilon}) - I$. In a symmetric equilibrium, we have $\varphi_i = \varphi_j = \frac{1}{2}\big(p^{HH}(q, \widehat{\varepsilon}) - I\big)$.

[48] His net payoffs are: $\widetilde{p} - I - \varphi$ if he discloses only one rating; $p^{HH}(q, \widehat{\varepsilon}) - I - 2\varphi$ if he chooses to disclose both. Hence, if $p^{HH}(q, \widehat{\varepsilon}) - 2\varphi \geq \widetilde{p} - \varphi$, disclosing two high ratings is a profitable deviation.

[49] Since the CRAs are symmetric in the eyes of the buyers (same reputation and same rating inflation strategy), investors' willigness to pay for the claim is the same when the seller publicizes only i's rating or only j's rating. Therefore, the issuer always chooses to publicize the cheapest of the two.

[50] I solve the model under the conjecture that the opportunistic CRAs always offer a high rating when they observe a good project. As we will see with Proposition 26, this conjecture is satisfied in equilibrium.

[51]

Like in the monopoly setting, when the project quality is good, a committed CRA may want to offer a fee which is too high for the issuer, i.e., $\varphi_1 > p^{HH}(q_0, \widehat{\varepsilon}_1) - I$, so to force the issuer not to publicize the rating and, thus, enter the next period with higher reputation. I show that this never happens in equilibrium in Appendix 3.10.4, since I first need to characterize the equilibrium rating strategy.

Finally, note that, since the issuer only discloses two high ratings in equilibrium, a project with only one high rating is off-the-equilibrium path. The following off-equilibrium beliefs support the equilibrium: when investors observe a project with only one high rating (i.e., $\mathbf{r} = H\varnothing$ or $\mathbf{r} = \varnothing H$), they believe this comes from an issuer who was offered one high and one low rating ($\widetilde{\mathbf{r}} = HL$ or $\widetilde{\mathbf{r}} = LH$) and, thus, that the project's quality is bad. In what follows, I show that these off-equilibrium beliefs satisfy the consistency requirement of Sequential Equilibria, which is defined in Appendix 3.9.1.

I focus on the case when $\mathbf{r} = H\varnothing$, since the case when $\mathbf{r} = \varnothing H$ is the mirror image. Consider an arbitrary sequence ω^n converging to 0, and interpret ω^n as the probability that an issuer who received a split rating ($\widetilde{\mathbf{r}} = HL$) "trembles" and discloses $\mathbf{r} = H\varnothing$, and ω^{2n} as the probability that an issuer who received two high ratings ($\widetilde{\mathbf{r}} = HH$) discloses $\mathbf{r} = H\varnothing$. [52] Finally, let q and $\widehat{\varepsilon}$ be, respectively, the CRAs' reputation and the conjectured rating inflation strategy. These trembles generate the following posterior beliefs about project quality:

$$\Pr\left(G \mid \mathbf{r} = H\varnothing, \left(\omega^n, \omega^{2n}\right)\right) = \frac{1}{1 + \frac{\Pr(\mathbf{r}=H\varnothing|B,(\omega^n,\omega^{2n}))}{\Pr(\mathbf{r}=H\varnothing|G,(\omega^n,\omega^{2n}))}\frac{\Pr(B)}{\Pr(G)}},$$

where

$$\Pr\left(\mathbf{r} = H\varnothing \mid B, \left(\omega^n, \omega^{2n}\right)\right) = \overbrace{\left((1-q)\,\widehat{\varepsilon}\right)^2}^{\Pr(\widetilde{\mathbf{r}}=HH|B)}\omega^{2n} + \overbrace{(1-q)\,\widehat{\varepsilon}\left[q + (1-q)\,(1-\widehat{\varepsilon})\right]}^{\Pr(\widetilde{\mathbf{r}}=HL|B)}\omega^n;$$

$$\Pr\left(\mathbf{r} = H\varnothing \mid G, \left(\omega^n, \omega^{2n}\right)\right) = \underbrace{1}_{\Pr(\widetilde{\mathbf{r}}=HH|G)}\omega^{2n}.$$

[51] Likewise the monopoly scenario, there is no signalling scope for the CRAs in the fee-setting subgame.

[52] In words, this means that type $\widetilde{\mathbf{r}} = HH$ is less likely than $\widetilde{\mathbf{r}} = HL$ to make a mistake and disclose $\mathbf{r} = H\varnothing$.

Therefore, we have:

$$\Pr\left(G \mid \mathbf{r} = H\varnothing, \left(\omega^n, \omega^{2n}\right)\right) = \frac{1}{1 + \left(\Pr\left(\widetilde{\mathbf{r}} = HH \mid B\right) + \frac{\Pr(\widetilde{\mathbf{r}}=HL|B)}{\omega^n}\right)\frac{\Pr(B)}{\Pr(G)}}.$$

Notice that $\Pr\left(G \mid \mathbf{r} = H\varnothing, \left(\omega^n, \omega^{2n}\right)\right)$ goes to 0 as $\omega^n \to 0$ and, thus, the off-equilibrium beliefs specified above satisfy the consistency requirement of Sequential Equilibria.

3.10.2 Posterior Reputation

In what follows, all the probability measures are for given investors' conjectures $(\widehat{\varepsilon}_{i1}, \widehat{\varepsilon}_{j1})$ about the CRAs' rating strategies at time $t = 1$. Since I focus on symmetric equilibria, I consider the case $\widehat{\varepsilon}_{i1} = \widehat{\varepsilon}_{j1} = \widehat{\varepsilon}_1$. At time $t = 1$, an opportunistic CRA is concerned with the reputation it has when it enters time $t = 2$. For given observed ratings $\mathbf{r}_1$ and realization of the signal y_1, i's posterior reputation is:

$$\begin{aligned}\Pr\left(\mathcal{C}_i|\mathbf{r}_1, y_1\right) &= \frac{\Pr\left(\mathbf{r}_1, y_1|\, \mathcal{C}_i\right) q_0}{\Pr\left(\mathbf{r}_1, y_1|\, \mathcal{C}_i\right) q_0 + \Pr\left(\mathbf{r}_1, y_1|\, \mathcal{O}_i\right)(1 - q_0)} \\ &= \left(1 + \frac{1 - q_0}{q_0}\frac{\Pr\left(\mathbf{r}_1, y_1|\, \mathcal{O}_i\right)}{\Pr\left(\mathbf{r}_1, y_1|\, \mathcal{C}_i\right)}\right)^{-1} \equiv q_i^{\mathbf{r}_1, y_1}.\end{aligned} \tag{3.17}$$

$$\begin{aligned}\Pr\left(\mathbf{r}_1, y_1 \mid \tau_i\right) &= \sum_{\theta_1 \in \Theta} \Pr\left(\theta_1\right) \Pr\left(\mathbf{r}_1, y_1 \mid \tau_i, \theta\right) \\ &= \sum_{\theta_1 \in \Theta} \underbrace{\Pr\left(\theta_1\right) \Pr\left(y_1 \mid \theta_1\right)}_{\Pr(y_1 \cap \theta_1)} \underbrace{\Pr\left(\mathbf{r}_1 \mid \tau_i, \theta_1\right)}_{\substack{y_1, \mathbf{r}_1 \text{ independent} \\ \text{conditional on } \theta_1}} \\ &= \sum_{\theta_1 \in \Theta} \Pr\left(y_1 \cap \theta_1\right) \Pr\left(\mathbf{r}_1 \mid \tau_i, \theta_1\right).\end{aligned} \tag{3.18}$$

Notice that $\Pr\left(\mathbf{r}_1 \mid \tau_i, \theta_1\right)$ depends on the investors' beliefs about τ_j as well, since the realization of r_{j1} depends on j's type. Investors learn at the same time about both i and j's types. However, since both the realization of the CRAs' types and their ratings are independent, the investors' learning about i is independent from their learning about j.

Lemma 25 implies that the CRAs always end up with the same reputation at time $t = 2$.

Therefore, we have $q_i^{r_1,y_1} = q_j^{r_1,y_1} \equiv q^{r_1,y_1}$.

When $r_1 = HH$, we have:

$$\frac{\Pr(HH, y_1 \mid \mathcal{O}_i)}{\Pr(HH, y_1 \mid \mathcal{C}_i)} = \frac{\Pr(y_1 \cap G) + \Pr(y_1 \cap B) \overbrace{(1-q_0)\widehat{\varepsilon}_1}^{\Pr(H_j|B)} \overbrace{\widehat{\varepsilon}_1}^{\Pr(H_i|B,\mathcal{O}_i)}}{\Pr(y_1 \cap G)} = \tag{3.19}$$

$$= 1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)} (1-q_0) (\widehat{\varepsilon}_1)^2 . \tag{3.20}$$

The expression in equation (3.17) becomes:

$$q^{HH,y_1} = \left\{ 1 + \frac{1-q_0}{q_0} \left(1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)} (1-q_0) (\widehat{\varepsilon}_1)^2 \right) \right\}^{-1} . \tag{3.21}$$

Note that the right-hand side in equation (3.21) is always decreasing in $\widehat{\varepsilon}_1$ and simplifies to q_0 when $\widehat{\varepsilon}_1 = 0$. This implies that $q^{HH,y_1} \leq q_0$, with strict inequality for any $\widehat{\varepsilon}_1 > 0$.

When $r_1 = \varnothing\varnothing$, we have:

$$\frac{\Pr(\varnothing\varnothing, y_1 \mid \mathcal{O}_i)}{\Pr(\varnothing\varnothing, y_1 \mid \mathcal{C}_i)} = \frac{\Pr(y_1 \cap B) \left[1 - (1-q_0) (\widehat{\varepsilon}_1)^2 \right]}{\Pr(y_1 \cap B)} = \tag{3.22}$$

$$= 1 - (1-q_0) (\widehat{\varepsilon}_1)^2 . \tag{3.23}$$

Like in monopoly, when investors observe an unrated project, they understand that project quality is bad. Hence, posterior reputation does not depend on the realization of y_1. The expression in equation (3.17) becomes:

$$q^{\varnothing\varnothing} = \left\{ 1 + \frac{1-q_0}{q_0} \left[1 - (1-q_0) (\widehat{\varepsilon}_1)^2 \right] \right\}^{-1} \tag{3.24}$$

The right-hand side in equation (3.24) is always increasing in $\widehat{\varepsilon}_1$ and simplifies to q_0 when $\widehat{\varepsilon}_1 = 0$. This means that $q^{\varnothing\varnothing} \geq q_0$, with strict inequality for any $\widehat{\varepsilon}_1 > 0$.

3.10.3 Proof of Proposition 26

I prove Proposition 23 in two steps. First, I derive the expression for the marginal benefit of inflation MBI and show that this is always positive at $\widehat{\varepsilon}_1 = 0$, and decreasing and continuous in $\widehat{\varepsilon}_1$. This implies that the characterization of the equilibrium rating inflation ε_1^c is as described in Proposition 26. Second, given the characterization of ε_1^c, I prove that deflating the rating for a good project is always a dominated action in equilibrium.

Step one: Equilibrium Rating Inflation

Consider an opportunistic CRA-i who, at time $t = 1$, has observed a bad project and decides whether to inflate its rating or not. The CRA's total payoffs, evaluated at the second-period equilibrium rating strategies, are as follows:

$$\begin{aligned}\Pi_{i,B} &= (1-q_0)\widehat{\varepsilon}_{j1}\left\{\varepsilon_{i1}\left[\varphi^c(q_0,\widehat{\varepsilon}_1)+\gamma\int_{y_1}\varphi^c\left(q^{HH,y_1},1\right)f_B(y_1)\,dy_1\right]+(1-\varepsilon_{i1})\gamma\varphi^c\left(q^{\varnothing\varnothing},1\right)\right\}\\ &\quad +q_0\gamma\varphi^c\left(q^{\varnothing\varnothing},1\right)\Pr(G)+(1-q_0)(1-\widehat{\varepsilon}_{j1})\gamma\varphi^c\left(q^{\varnothing\varnothing},1\right). \qquad (3.25)\end{aligned}$$

The first line in equation (3.25) describes i's continuation payoff in the case in which j inflates its rating. This happens with probability $(1-q_0)\widehat{\varepsilon}_{j1}$, i.e., when j is opportunistic and inflates its rating - i has a conjecture $\widehat{\varepsilon}_{j1}$ about ε_{j1}. With probability ε_{i1}, i inflates its rating too. In this case, the issuer discloses both ratings and so, for a given realization of y_1, the CRAs' reputation in the second period is q^{HH,y_1}. Since both CRAs are opportunistic in this scenario, they always sell their ratings in the second period; continuation payoff is thus $\gamma\varphi^c\left(q^{HH,y_1},1\right)$ in this case. With probability $1-\varepsilon_{i1}$, i does not inflate its rating and both CRAs enter the second period with high reputation; continuation payoff is $\gamma\varphi^c\left(q^{\varnothing\varnothing},1\right)$.

The second line in equation (3.25) describes the continuation payoff in the cases in which j does not inflate its rating. This occurs when: (i) j is a committed type - probability q_0; (ii) j is opportunistic but chooses not to inflate its rating - probability $(1-q_0)(1-\widehat{\varepsilon}_{j1})$. In both cases, the first-period project goes unrated and so the CRAs enter the second period with reputation $q^{\varnothing\varnothing}$. In case (i), the CRAs sell the rating in the second period only if the project is good, as j

is committed and so will not inflate the rating if project quality is bad; the continuation payoff is thus $\gamma\varphi^c\left(q^{\varnothing\varnothing},1\right)\Pr\left(G\right)$. In case (ii), the CRAs always sell the rating in the second period and so the continuation payoff is $\gamma\varphi^c\left(q^{\varnothing\varnothing},1\right)$.

Taking the derivative of $\Pi_{i,B}$ with respect to ε_{i1} yields:

$$\frac{d\Pi_{i,B}}{d\varepsilon_{i1}}=\left(1-q_0\right)\widehat{\varepsilon}_{j1}\left\{\varphi^c\left(q_0,\widehat{\varepsilon}_1\right)+\gamma\int_{y_1}\varphi^c\left(q^{HH,y_1},1\right)f_B\left(y_1\right)dy_1-\gamma\varphi^c\left(q^{\varnothing\varnothing},1\right)\right\}. \tag{3.26}$$

In equilibrium, i's and investors' conjectures must be the same, i.e., $\widehat{\varepsilon}_{j1}=\widehat{\varepsilon}_1$. We can thus rewrite equation (3.26) as follows:

$$\begin{aligned}\frac{d\Pi_{i,B}}{d\varepsilon_{i1}} &= \left(1-q_0\right)\widehat{\varepsilon}_1\underbrace{\left\{\varphi^c\left(q_0,\widehat{\varepsilon}_1\right)+\gamma\int_{y_1}\varphi^c\left(q^{HH,y_1},1\right)f_B\left(y_1\right)dy_1-\gamma\varphi^c\left(q^{\varnothing\varnothing},1\right)\right\}}_{\equiv A(\widehat{\varepsilon}_1)} \\ &= \left(1-q_0\right)\widehat{\varepsilon}_1 A\left(\widehat{\varepsilon}_1\right).\end{aligned} \tag{3.27}$$

Taking the derivative of $\frac{d\Pi_{i,B}}{d\varepsilon_{i1}}$ with respect to $\widehat{\varepsilon}_1$ yields:

$$\frac{\frac{d\Pi_{i,B}}{d\varepsilon_{i1}}}{d\widehat{\varepsilon}_1}\left(\widehat{\varepsilon}_1\right)=\left(1-q_0\right)A\left(\widehat{\varepsilon}_1\right)+\left(1-q_0\right)\widehat{\varepsilon}_1\frac{dA\left(\widehat{\varepsilon}_1\right)}{d\widehat{\varepsilon}_1}.$$

Notice that $A\left(\widehat{\varepsilon}_1\right)$ is always positive at $\widehat{\varepsilon}_1=0$, since $\varphi^c\left(q_0,\widehat{\varepsilon}_1=0\right)=\frac{1}{2}\left(V_G-I\right)$ and $q^{HH,y_1}=q_0=q^{\varnothing\varnothing}$ at $\widehat{\varepsilon}_1=0$ for any $y_1\in\mathcal{Y}$ (see Appendix 3.10.2). We have:

$$\begin{aligned}A\left(\widehat{\varepsilon}_1=0\right) &= \frac{1}{2}\left(V_G-I\right)+\gamma\int_{y_1}\varphi^c\left(q_0,1\right)f_B\left(y_1\right)dy_1-\gamma\varphi^c\left(q_0,1\right) \\ &= \frac{1}{2}\left(V_G-I\right)+\gamma\varphi^c\left(q_0,1\right)-\gamma\varphi^c\left(q_0,1\right) \\ &= \frac{1}{2}\left(V_G-I\right)>0,\end{aligned}$$

and, thus,

$$\frac{\frac{d\Pi_i}{d\varepsilon_{i1}}}{d\widehat{\varepsilon}_1}\left(\widehat{\varepsilon}_1=0\right)=\left(1-q_0\right)A\left(\widehat{\varepsilon}_1=0\right)>0.$$

The above inequality implies that the marginal benefit of rating inflation is positive and so inflating the rating is always a strictly dominant strategy when $\widehat{\varepsilon}_1$ is close to 0. As a consequence, the candidate equilibrium at $\widehat{\varepsilon}_1 = 0$ is not robust to trembles and, therefore, cannot be a sequential equilibrium of the game.

$A(\widehat{\varepsilon}_1)$ is also always decreasing (and continuous) in $\widehat{\varepsilon}_1$, since we have:

$$\frac{dA(\widehat{\varepsilon}_1)}{d\widehat{\varepsilon}_1} = \underset{<0}{\frac{\partial \varphi^c(q_0, \widehat{\varepsilon}_1)}{\partial \widehat{\varepsilon}_1}} + \gamma \int_{y_1} \underset{>0}{\frac{\partial \varphi^c\left(q^{HH,y_1}, 1\right)}{\partial q^{HH,y_1}}} \underset{<0}{\frac{\partial q^{HH,y_1}}{\partial \widehat{\varepsilon}_1}} f_B(y_1)\, dy_1 - \underset{>0}{\frac{\partial \varphi^c\left(q^{\varnothing\varnothing}, 1\right)}{\partial q^{\varnothing\varnothing}}} \underset{>0}{\frac{\partial q^{\varnothing\varnothing}}{\partial \widehat{\varepsilon}_1}} < 0,$$

where the signs of the derivatives above come from the fact that, in equilibrium, we have $\varphi^c(q, \widehat{\varepsilon}) = \frac{1}{2}\left(p^{HH}(q, \widehat{\varepsilon}) - I\right)$ for any values of q and $\widehat{\varepsilon}$.

Step two: No Rating Deflation in equilibrium

Deflating the rating for a good project is trivially a dominated strategy in the second period;[53] here, I show that it is also a dominated strategy in the first period. The logic of the proof is similar to the scenario with a single CRA.

Consider an opportunistic CRA-i who, at time $t = 1$, has observed a good project (i.e., type $(\mathcal{O}, G)$) and decides whether to deflate its rating or not. The following expression characterizes the CRA's trade-off:

$$\gamma\lambda\varphi^c\left(q^{\varnothing\varnothing}, 1\right) - \varphi^c(q_0, \varepsilon_1^c) - \gamma\lambda \int_{y_1} \varphi^c\left(q^{HH,y_1}, 1\right) f_G(y_1)\, dy_1, \tag{3.28}$$

where $\lambda \equiv \Pr(G) + \Pr(B)(1 - q_0)$.

CRA-i expects that j offers a high rating to the issuer (the equilibrium conjecture is that the CRAs never deflate the rating for a good project). Hence, i's rating is always pivotal in whether the issuer buys the ratings or not. If the CRA deflates the signal, the issuer does not disclose any rating and the first-period fee is not collected. The CRAs enter the second period with high reputation, and continuation payoffs are $\gamma\lambda\varphi^c\left(q^{\varnothing\varnothing}, 1\right)$. This is because the CRA

[53] Suppose the CRA has reputation q in the second period. If the CRA deflates the report, the continuation payoff is 0; if it doesn't, the continuation payoff is $\varphi^c(q, 1)$, as the other CRA won't deflate the signal either. Hence, deflating is always a dominated strategy in the second period.

always offers a high rating in the second period but sells it only when j offers a high rating as well - i.e., when the project is good or, if the project is bad, when j is opportunistic, which occurs with probability $\Pr(G) + \Pr(B)(1 - q_0)$. If the CRA does not deflate, it collects the first-period fee $\varphi^c(q_0, \varepsilon_1^c)$ but enters the second period with low reputation; continuation payoff is $\gamma\lambda\varphi^c\left(q^{HH,y_1}, 1\right)$ in this case.

If the sign of the expression in equation (3.28) is negative, i is always better off by not deflating the rating. Therefore, we want to show that:

$$\varphi^c(q_0, \varepsilon_1^c) > \gamma\lambda\left[\varphi^c\left(q^{\varnothing\varnothing}, 1\right) - \int_{y_1} \varphi^c\left(q^{HH,y_1}, 1\right) f_G(y_1)\, dy_1\right]. \tag{3.29}$$

Notice that the equilibrium level of rating inflation ε_1^c satisfies the following condition:

$$\varphi^c(q_0, \varepsilon_1^c) \geq \gamma\left[\varphi^c\left(q^{\varnothing\varnothing}, 1\right) - \int_{y_1} \varphi^c\left(q^{HH,y_1}, 1\right) f_B(y_1)\, dy_1\right]. \tag{3.30}$$

Note also that q^{HH,y_1} is increasing in y_1 (see Appendix 3.10.2). The assumption that $f_\theta(y)$ satisfies the *Monotone Likelihood Ratio Property* also implies that the distribution of y_1 conditional on $\theta = G$ represents a *First-Order Stochastic Dominance* shift of the distribution of y_1 conditional on $\theta = B$. As a result, we have:

$$\int_{y_1} \varphi^c\left(q^{HH,y_1}, 1\right) f_G(y_1)\, dy_1 \geq \int_{y_1} \varphi^c\left(q^{HH,y_1}, 1\right) f_B(y_1)\, dy_1. \tag{3.31}$$

The inequalities in equation (3.30) and (3.31) together imply that the inequality in (3.29) is always satisfied. Therefore, deflating the rating for a good project is always a dominated action in equilibrium.

3.10.4 A committed CRA prefers to be hired in equilibrium

In the first period, if i is a committed type and offers a fee $\varphi_1 > \frac{1}{2}\left(p^{HH}(q_0, \varepsilon_1^c) - I\right) = \varphi^c(q_0, \varepsilon_1^c)$, the issuer chooses not to publicize the ratings and both CRAs enter the second period with

reputation $q^{\varnothing\varnothing}$. If the second-period project is good (the CRA is committed and, thus, will not inflate the rating if project quality is bad), they then sell the rating and collect a fee $\varphi^c\left(q^{\varnothing\varnothing},1\right)$.

If the CRA offers a fee $\varphi_1 = \frac{1}{2}\left(p^{HH}\left(q_0,\varepsilon_1^c\right) - I\right)$, the issuer publicizes both ratings and, for a given realization of the signal y_1, the CRAs enter the second period with reputation q^{HH,y_1}; if the second-period project is good, they then sell the rating and collect a fee $\varphi^c\left(q^{HH,y_1},1\right)$.

Therefore, a committed CRA-i prefers to be hired to publicize the rating in the first period (i.e., sets a fee $\varphi_1 = \frac{1}{2}\left(p^{HH}\left(q_0,\varepsilon_1^c\right) - I\right)$) if the following condition is satisfied:

$$\Pr(G)\,\gamma\varphi^c\left(q^{\varnothing\varnothing},1\right) < \varphi^c\left(q_0,\varepsilon_1^c\right) + \Pr(G)\,\gamma\int_{y_1}\varphi^c\left(q^{HH,y_1},1\right)f_G(y_1)\,dy_1$$

$$\Leftrightarrow \varphi^c\left(q_0,\varepsilon_1^c\right) > \Pr(G)\,\gamma\left[\varphi^c\left(q^{\varnothing\varnothing},1\right) - \int_{y_1}\varphi^c\left(q^{HH,y_1},1\right)f_G(y_1)\,dy_1\right]$$

Equations (3.30) and (3.31) imply that the inequality above is always satisfied. This means that it is always optimal for a committed CRA to set $\varphi_1 = \varphi^c\left(q_0,\varepsilon_1^c\right)$ and sell the rating for a good project in both periods.

3.11 Appendix C

3.11.1 Proof of Proposition 28

I prove Proposition 28 in two steps. First, I state and prove Lemma 33, which is useful for ordering the threshold values $\overline{\gamma}^m$ and $\overline{\gamma}^c$. Second, given the results in the lemma, I show that $\overline{\gamma}^m < \overline{\gamma}^c$.

Step One: Preliminaries

Lemma 33 *The following is true:*

1. *Other things being equal (reputation q and expected rating inflation $\widehat{\varepsilon}$), the price investors pay for the claim to the payoff of the project is larger in duopoly, i.e., $p^{HH}\left(q,\widehat{\varepsilon}\right) \geq p^{H}\left(q,\widehat{\varepsilon}\right)$ (strict inequality for $\widehat{\varepsilon} > 0$);*

2. *Other things being equal (initial reputation q_0 and expected rating inflation $\widehat{\varepsilon}_1$), investors' learn less about the CRAs' types in duopoly, i.e., $q^{H,y_1} \leq q^{HH,y_1} \leq q_0 \leq q^{\varnothing\varnothing} \leq q^{\varnothing}$ for any $y_1 \in \mathcal{Y}$ (strict inequalities for $\widehat{\varepsilon}_1 > 0$). Moreover, when $\widehat{\varepsilon}_1 = 1$, we have $q^{\varnothing} = 1$ and $q^{\varnothing\varnothing} = \frac{1}{2-q_0} < 1$.*

In what follows, I prove Lemma 33. We can rearrange $p^H(q, \widehat{\varepsilon})$ as:

$$p^H(q, \widehat{\varepsilon}) = \mu^H \underbrace{\left[V_G - V_B\right]}_{<0} + V_B; \quad \mu^H = \frac{\Pr(G)}{\Pr(G) + \Pr(B)(1-q)\widehat{\varepsilon}}.$$

Similarly, we can rewrite $p^{HH}(q, \widehat{\varepsilon})$ as follows:

$$p^{HH}(q, \widehat{\varepsilon}) = \mu^{HH}\left[V_G - V_B\right] + V_B; \quad \mu^{HH} = \frac{\Pr(G)}{\Pr(G) + \Pr(B)\left[(1-q)\widehat{\varepsilon}\right]^2}.$$

Therefore, showing that $p^{HH}(q, \widehat{\varepsilon}) \geq p^H(q, \widehat{\varepsilon})$ is equivalent to showing that $\mu^{HH} \geq \mu^H$. We have:

$$\mu^{HH} \geq \mu^H \Leftrightarrow (1-q)\widehat{\varepsilon} \geq \left[(1-q)\widehat{\varepsilon}\right]^2.$$

The above inequality is always satisfied and, thus, we have $p^{HH}(q, \widehat{\varepsilon}) \geq p^H(q, \widehat{\varepsilon})$ (strict inequality for $\widehat{\varepsilon} > 0$).

In what follows, I show that $q^{\varnothing\varnothing} \leq q^{\varnothing}$. From equations (3.9) and (3.24), we have:

$$q^{\varnothing} = \frac{1}{1 + \frac{1-q_0}{q_0}(1 - \widehat{\varepsilon}_1)}, \quad q^{\varnothing\varnothing} = \frac{1}{1 + \frac{1-q_0}{q_0}\left[1 - (1-q_0)(\widehat{\varepsilon}_1)^2\right]}.$$

We need to have:

$$1 - (1-q_0)(\widehat{\varepsilon}_1)^2 \geq 1 - \widehat{\varepsilon}_1.$$

Since $q_0 \in (0,1)$, this is always satisfied.

The same steps apply for the proof of $q^{HH,y_1} \geq q^{H,y_1}$, which is thus omitted.

Notice also that, when $\widehat{\varepsilon}_1 = 1$, posterior reputation $q^{\varnothing}$ is equal to 1. This is because investors don't expect a low rating to come from an opportunistic CRA and, as a consequence, they believe

the CRA is committed after observing an unrated project. This is not true in the setting with competing agencies. In this case, investors cannot tell whether only one - and, if so, which one - or both CRAs offered a low rating to the issuer. Therefore, they only learn that at least one of the two CRAs is committed.

Hence, when $\widehat{\varepsilon}_1 = 1$, we have $q^{\varnothing} = 1$ and $q^{\varnothing\varnothing} = \frac{1}{2-q_0} < 1$.

Step 2: Ordering the threshold values

We want to show that $\overline{\gamma}^m < \overline{\gamma}^c$, which is:

$$\frac{p^H(q_0,1) - I}{p^H(q^{\varnothing},1) - \int_{y_1} p^H(q^{H,y_1},1) f_B(y_1)\, dy_1} < \frac{p^{HH}(q_0,1) - I}{p^{HH}(q^{\varnothing\varnothing},1) - \int_{y_1} p^{HH}(q^{HH,y_1},1) f_B(y_1)\, dy_1},$$

where the posterior reputations are evaluated at the conjecture $\widehat{\varepsilon}_1 = 1$.

Given that, by Part 1 of Lemma 33, we have $p^{HH}(q_0,1) > p^H(q_0,1)$, the above inequality is satisfied if:

$$p^H\left(q^{\varnothing},1\right) - \int_{y_1} p^H\left(q^{H,y_1},1\right) f_B(y_1)\, dy_1 \geq p^{HH}\left(q^{\varnothing\varnothing},1\right) - \int_{y_1} p^{HH}\left(q^{HH,y_1},1\right) f_B(y_1)\, dy_1. \quad (3.32)$$

By Part 2 of Lemma 33, we have $q^{H,y_1} < q^{HH,y_1}$ for any $y_1 \in \mathcal{Y}$. This implies that $p^H\left(q^{HH,y_1},1\right) > p^H\left(q^{H,y_1},1\right)$ and, thus, that:

$$p^H\left(q^{\varnothing},1\right) - \int_{y_1} p^H\left(q^{H,y_1},1\right) f_B(y_1)\, dy_1 \geq p^H\left(q^{\varnothing},1\right) - \int_{y_1} p^H\left(q^{HH,y_1},1\right) f_B(y_1)\, dy_1.$$

Therefore, in order to show that the inequality in equation (3.32) holds, it is sufficient to show that:

$$p^H\left(q^{\varnothing},1\right) - \int_{y_1} p^H\left(q^{HH,y_1},1\right) f_B(y_1)\, dy_1 \geq p^{HH}\left(q^{\varnothing\varnothing},1\right) - \int_{y_1} p^{HH}\left(q^{HH,y_1},1\right) f_B(y_1)\, dy_1.$$

Notice that, at the conjecture $\widehat{\varepsilon}_1 = 1$, we have $q^{\varnothing} = 1$ and so $p^H\left(q^{\varnothing},1\right) = p^H(1,1) = V_G$:

when the CRA is believed to be committed with probability 1, a high rating is a perfect signal for a good project. This implies that $p^H\left(q^{\varnothing},1\right) > p^{HH}\left(q^{\varnothing\varnothing},1\right)$. Therefore, we just need to show that:

$$\int_{y_1} p^{HH}\left(q^{HH,y_1},1\right) f_B\left(y_1\right) dy_1 \geq \int_{y_1} p^H\left(q^{HH,y_1},1\right) f_B\left(y_1\right) dy_1.$$

The above inequality holds, as it is satisfied pointwise - i.e., for any realization of y_1, since $p^{HH}\left(q^{HH,y_1},1\right) \geq p^H\left(q^{HH,y_1},1\right)$ for any q^{HH,y_1}. Therefore, we have $\overline{\gamma}^m < \overline{\gamma}^c$ for any parameter constellation.

3.11.2 Proof of Lemma 29

I prove Lemma 29 in two steps. First, I show that, as the initial reputation q_0 approaches 1, the threshold $\overline{\gamma}^c$ goes to infinity faster than $\overline{\gamma}^m$. This will be used in Step Two, in which I show that, when $\gamma = \overline{\gamma}^c$ and q_0 approaches 1, ε_1^m goes to zero faster than $1 - q_0$. This implies that, when γ is close to $\overline{\gamma}^c$ and q_0 is close to 1, the inequality in equation (3.5) is satisfied and, thus, the probability that a bad project is implemented is larger in duopoly. Finally, I perform numerical simulations of the model.

Step One

I want to prove that $\lim_{q_0 \to 1} \frac{\overline{\gamma}^c}{\overline{\gamma}^m} = \infty$; this is equivalent to showing that:

$$\lim_{q_0 \to 1} \frac{p^{HH}\left(q_0,1\right) - I}{p^H\left(q_0,1\right) - I} \frac{V_G - \int_{y_1} p^H\left(q^{H,y_1},1\right) f_B\left(y_1\right) dy_1}{p^{HH}\left(q^{\varnothing\varnothing},1\right) - \int_{y_1} p^{HH}\left(q^{HH,y_1},1\right) f_B\left(y_1\right) dy_1} = \infty.$$

Both $p^{HH}\left(q_0,1\right)$ and $p^H\left(q_0,1\right)$ converge to V_G when q_0 approaches 1. This implies that the limit above is true when:

$$\lim_{q_0 \to 1} \frac{\int_{y_1} m\left(q_0,y_1\right) f_B\left(y_1\right) dy_1}{\int_{y_1} c\left(q_0,y_1\right) f_B\left(y_1\right) dy_1} = \infty, \tag{3.33}$$

where

$$m\left(q_0, y_1\right) \equiv V_G - p^H\left(q^{H,y_1}, 1\right); \quad c\left(q_0, y_1\right) \equiv p^{HH}\left(q^{\varnothing\varnothing}, 1\right) - p^{HH}\left(q^{HH,y_1}, 1\right).$$

The proof proceeds as follows. First, I show that we can write $m\left(q_0, y_1\right) = (1-q_0)\widetilde{m}(q_0, y_1)$, where, for any y_1, $\widetilde{m}(q_0, y_1)$ is strictly positive and continuous in q_0, for any $q_0 \in (0, 1]$. Second, I show that we can write $c\left(q_0, y_1\right) = (1-q_0)^2 k\left(q_0, y_1\right) \widetilde{c}(q_0, y_1)$, where, for any y_1, $\widetilde{c}(q_0, y_1)$ and $k\left(q_0, y_1\right)$ are both finite and continuous in q_0 for any $q_0 \in (0, 1]$, and $k\left(1, y_1\right) = 0$. This implies that we can rearrange the limit in equation (3.33) as:

$$\lim_{q_0 \to 1} \frac{\int_{y_1} \widetilde{m}(q_0, y_1) f_B\left(y_1\right) dy_1}{(1-q_0)\int_{y_1} k\left(q_0, y_1\right) \widetilde{c}(q_0, y_1) f_B\left(y_1\right) dy_1} = \infty.$$

Since $\widetilde{m}(1, y_1) > 0$ for any y_1, and $\widetilde{m}(q_0, y_1)$ is continuous in q_0, the numerator is always a strictly positive number, even when q_0 approaches 1. On the other hand, both $1 - q_0$ and $k\left(q_0, y_1\right)$ converge to 0 as q_0 approaches 1, while $\widetilde{c}(q_0, y_1)$ is bounded away from ∞; this implies that the denominator goes to zero as q_0 approaches 1. Therefore, the limit above is always satisfied.

I now proceed to the first part of the proof. Let $\eta \equiv \frac{\Pr(B)}{\Pr(G)}$; we have:

$$\begin{aligned} m\left(q_0, y_1\right) &= V_G - \underbrace{\left\{\frac{(V_G - V_B)}{1 + \eta\left[1 - q^{H,y_1}\right]} + V_B\right\}}_{p^H\left(q^{H,y_1}, 1\right)} \\ &= \left(1 - q^{H,y_1}\right) \frac{\eta\left(V_G - V_B\right)}{1 + \eta\left[1 - q^{H,y_1}\right]}. \end{aligned} \tag{3.34}$$

When $\widehat{\varepsilon}_1 = 1$, we have:

$$q^{H,y_1} = \frac{1}{1 + \frac{1-q_0}{q_0}\left(1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)}\right)}$$
$$\Leftrightarrow 1 - q^{H,y_1} = (1-q_0)\frac{\frac{1}{q_0}\left(1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)}\right)}{1 + \frac{1-q_0}{q_0}\left(1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)}\right)}. \tag{3.35}$$

Using equations (3.34) and (3.35), we can then write:

$$m(q_0, y_1) = (1-q_0)\underbrace{\frac{\frac{1}{q_0}\left(1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)}\right)}{1 + \frac{1-q_0}{q_0}\left(1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)}\right)}\frac{\eta(V_G - V_B)}{1 + \eta\left[1 - q^{H,y_1}\right]}}_{\equiv \widetilde{m}(q_0, y_1)}.$$

Note that $\widetilde{m}(q_0, y_1)$ is always continuous in q_0 and $\widetilde{m}(q_0, y_1) > 0$, for any $q_0 \in (0, 1]$. At $q_0 = 1$, we have $\widetilde{m}(1, y_1) = \left(1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)}\right)\eta(V_G - V_B) > 0$. [54]

We can apply similar steps to the denominator $c(q_0, y_1)$; we have:

$$c(q_0, y_1) = \underbrace{\frac{(V_G - V_B)}{1 + \eta\left[1 - q^{\varnothing\varnothing}\right]^2} + V_B}_{p^{HH}(q^{\varnothing\varnothing}, 1)} - \underbrace{\left\{\frac{(V_G - V_B)}{1 + \eta\left[1 - q^{HH,y_1}\right]^2} + V_B\right\}}_{p^{HH}\left(q^{HH,y_1}, 1\right)}$$
$$= \left\{\left[1 - q^{HH,y_1}\right]^2 - \left[1 - q^{\varnothing\varnothing}\right]^2\right\}\underbrace{\frac{(V_G - V_B)\eta}{\left\{1 + \eta\left[1 - q^{\varnothing\varnothing}\right]^2\right\}\left\{1 + \eta\left[1 - q^{HH,y_1}\right]^2\right\}}}_{\equiv \widetilde{c}(q_0, y_1)}. \tag{3.36}$$

Note that $\widetilde{c}(q_0, y_1)$ is always continuous in q_0 and $\widetilde{c}(q_0, y_1) < \infty$ for any $q_0 \in (0, 1]$. At $q_0 = 1$, we have $\widetilde{c}(1, y_1) = \eta(V_G - V_B) < \infty$. [55] When $\widehat{\varepsilon}_1 = 1$, we have:

$$q^{HH,y_1} = \frac{1}{1 + \frac{1-q_0}{q_0}\left(1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)}(1 - q_0)\right)}$$
$$\Leftrightarrow 1 - q^{HH,y_1} = (1-q_0)\frac{\frac{1}{q_0}\left(1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)}(1 - q_0)\right)}{1 + \frac{1-q_0}{q_0}\left(1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)}(1 - q_0)\right)}.$$

[54] When $q_0 = 1$, we have $q^{H,y_1} = 1$ as well. This implies that $\frac{\eta(V_G - V_B)}{1 + \eta\left[1 - q^{H,y_1}\right]}$ converges to $\eta(V_G - V_B)$ when q_0 approaches 1.

[55] When $q_0 = 1$, we have $q^{HH,y_1} = q^{\varnothing\varnothing} = 1$ as well.

We can write:

$$1 - q^{HH,y_1} = (1 - q_0) K' \left(q^{HH,y_1}\right), \tag{3.37}$$

where $K' \left(q^{HH,y_1}\right)$ converges to 1 when q_0 approaches 1.

Similarly, for reputation $q^{\varnothing\varnothing}$, we have:

$$\begin{aligned} q^{\varnothing\varnothing} &= \frac{1}{2 - q_0} \quad \Leftrightarrow \quad 1 - q^{\varnothing\varnothing} = (1 - q_0) \frac{1}{2 - q_0} \\ &\Leftrightarrow \quad 1 - q^{\varnothing\varnothing} = (1 - q_0) K'' \left(q^{\varnothing\varnothing}\right), \end{aligned} \tag{3.38}$$

where $K'' \left(q^{\varnothing\varnothing}\right)$ also converges to 1 when q_0 approaches 1.

Using equations from (3.36) to (3.38), we can write:

$$c\left(q_0, y_1\right) = (1 - q_0)^2 \underbrace{\left(K' \left(q^{HH,y_1}\right) - K'' \left(q^{\varnothing\varnothing}\right)\right)^2}_{\equiv k(q_0, y_1)} \widetilde{c}(q_0, y_1).$$

Finally, note that $k\left(q_0, y_1\right)$ is always continuous in q_0 and that $k\left(1, y_1\right) = 0$, since $K' \left(q^{HH,y_1}\right) = K'' \left(q^{\varnothing\varnothing}\right)$ when $q_0 = 1$, for any $y_1 \in \mathcal{Y}$.

Therefore, we can rearrange the limit in equation (3.33) as follows:

$$\lim_{q_0 \to 1} \frac{\int\limits_{y_1} \widetilde{m}(q_0, y_1) f_B\left(y_1\right) dy_1}{(1 - q_0) \int\limits_{y_1} k\left(q_0, y_1\right) \widetilde{c}(q_0, y_1) f_B\left(y_1\right) dy_1} = \infty.$$

As we discussed earlier, this limit is always satisfied since the numerator is strictly positive while the denominator goes to zero when q_0 approaches 1.

Step Two

When $\gamma \in (\overline{\gamma}^m, \overline{\gamma}^c]$, we have $\varepsilon_1^c = 1$ and $\varepsilon_1^m < 1$. Hence, the probabilities that a bad project is implemented at time $t = 1$ are: $(1 - q_0) \varepsilon_1^m$ in the setting with a single CRA; $(1 - q_0)^2$ in the setting with competing agencies. Therefore, when $\gamma \in (\overline{\gamma}^m, \overline{\gamma}^c]$, a bad project is less likely to be

implemented and, thus, welfare is larger under monopoly when:

$$(1-q_0)\,\varepsilon_1^m < (1-q_0)^2 \quad \Leftrightarrow \quad \varepsilon_1^m < 1-q_0. \tag{3.39}$$

The equilibrium level of rating inflation ε_1^m approaches zero when the derivative of the marginal benefit of inflation evaluated at $\widehat{\varepsilon}_1 = 0$ (i.e., $\frac{dMBI}{d\widehat{\varepsilon}_1} \mid_{\widehat{\varepsilon}_1=0}$) goes to $-\infty$ - see figure 3.1. This occurs when the reputational loss of rating inflation is large even for small values of expected rating inflation and, thus, the CRA almost never inflates its rating.

In what follows, I show that, when q_0 approaches 1, $\frac{dMBI}{d\widehat{\varepsilon}_1} \mid_{\widehat{\varepsilon}_1=0,\gamma=\overline{\gamma}^c}$ converges to $-\infty$ faster than $\frac{1}{1-q_0}$ converges to $+\infty$. This implies that, when $\gamma = \overline{\gamma}^c$ and q_0 approaches 1, ε_1^m vanishes faster than $1-q_0$. Therefore, when q_0 is close enough to 1 and γ is close enough to $\overline{\gamma}^c$, the inequality in equation (3.39) is satisfied.

We have:

$$\frac{dMBI}{d\widehat{\varepsilon}_1} = \frac{\partial p^H\left(q_0,\widehat{\varepsilon}_1\right)}{\partial\widehat{\varepsilon}_1} + \gamma\int_{y_1}\frac{\partial p^H\left(q^{H,y_1},1\right)}{\partial q^{H,y_1}}\frac{\partial q^{H,y_1}}{\partial\widehat{\varepsilon}_1}f_B\left(y_1\right)dy_1 - \gamma\frac{\partial p^H\left(q^{\varnothing},1\right)}{\partial q^{\varnothing}}\frac{\partial q^{\varnothing}}{\partial\widehat{\varepsilon}_1}.$$

We have:

$$q^{H,y_1} = \frac{1}{1+\frac{1-q_0}{q_0}\left(1+\frac{\Pr(y_1\cap B)}{\Pr(y_1\cap G)}\widehat{\varepsilon}_1\right)}; \quad \frac{\partial q^{H,y_1}}{\partial\widehat{\varepsilon}_1} = \frac{-\frac{1-q_0}{q_0}\frac{\Pr(y_1\cap B)}{\Pr(y_1\cap G)}}{\left[1+\frac{1-q_0}{q_0}\left(1+\frac{\Pr(y_1\cap B)}{\Pr(y_1\cap G)}\widehat{\varepsilon}_1\right)\right]^2};$$

$$q^{\varnothing} = \frac{1}{1+\frac{1-q_0}{q_0}\left(1-\widehat{\varepsilon}_1\right)}; \quad \frac{\partial q^{\varnothing}}{\partial\widehat{\varepsilon}_1} = \frac{\frac{1-q_0}{q_0}}{\left[1+\frac{1-q_0}{q_0}\left(1-\widehat{\varepsilon}_1\right)\right]^2}.$$

This implies that:

$$\frac{\partial q^{H,y_1}}{\partial\widehat{\varepsilon}_1} \mid_{\widehat{\varepsilon}_1=0} = -\left(1-q_0\right)q_0\frac{\Pr\left(y_1\cap B\right)}{\Pr\left(y_1\cap G\right)}; \quad \frac{\partial q^{\varnothing}}{\partial\widehat{\varepsilon}_1} \mid_{\widehat{\varepsilon}_1=0} = \left(1-q_0\right)q_0.$$

Let $\eta \equiv \frac{\Pr(B)}{\Pr(G)}$; for a given reputation q, we have:

$$p^H\left(q,1\right) = \mu^H\left[V_G - V_B\right] + V_B; \quad \mu^H = \frac{1}{1+\eta\left(1-q\right)}.$$

Hence, we have:

$$\frac{\partial p^H(q,1)}{\partial q} = \frac{\eta}{[1+\eta(1-q)]^2} > 0.$$

Notice that at $\widehat{\varepsilon}_1 = 0$, we have $q^{H,y_1} = q^{\varnothing} = q_0$; this implies that $\frac{\partial p^H\left(q^{H,y_1},1\right)}{\partial q^{H,y_1}} = \frac{\partial p^H\left(q^{\varnothing},1\right)}{\partial q^{\varnothing}} = \frac{\partial p^H(q_0,1)}{\partial q_0}$. Therefore, we have:

$$\frac{dMBI}{d\widehat{\varepsilon}_1}\mid_{\widehat{\varepsilon}_1=0} = \underbrace{\frac{\partial p^H(q_0,\widehat{\varepsilon}_1)}{\partial \widehat{\varepsilon}_1}\mid_{\widehat{\varepsilon}_1=0}}_{\equiv D} - \gamma(1-q_0)\,q_0 \underbrace{\frac{\partial p^H(q_0,1)}{\partial q_0}\left\{1+\int_{y_1}\frac{\Pr(y_1\cap B)}{\Pr(y_1\cap G)}f_B(y_1)\,dy_1\right\}}_{\equiv \widetilde{D}}. \tag{3.40}$$

We want to evaluate the above equation at $\gamma = \overline{\gamma}^c$. Following the analysis in Step One, we can rearrange $\overline{\gamma}^c$ as follows:

$$\begin{aligned}\overline{\gamma}^c &= \frac{p^{HH}(q_0,1)-I}{\int_{y_1} c(q_0,y_1)\,f_B(y_1)\,dy_1} \\ &= \frac{p^{HH}(q_0,1)-I}{(1-q_0)^2\int_{y_1} k(q_0,y_1)\,\widetilde{c}(q_0,y_1)f_B(y_1)\,dy_1}.\end{aligned}$$

Hence, we have:

$$\begin{aligned}\frac{dMBI}{d\widehat{\varepsilon}_1} \mid_{\widehat{\varepsilon}_1=0,\gamma=\overline{\gamma}^c} &= D - \frac{p^{HH}(q_0,1)-I}{(1-q_0)^2\int_{y_1} k(q_0,y_1)\,\widetilde{c}(q_0,y_1)f_B(y_1)\,dy_1}(1-q_0)\,q_0\widetilde{D} \\ &= D - \frac{p^{HH}(q_0,1)-I}{(1-q_0)\int_{y_1} k(q_0,y_1)\,\widetilde{c}(q_0,y_1)f_B(y_1)\,dy_1}q_0\widetilde{D}.\end{aligned}$$

q_0	$f=1$				$f=0.5$			
	$\overline{\gamma}^m$	$\overline{\gamma}^c$	ε_1^m	$1-q_0$	$\overline{\gamma}^m$	$\overline{\gamma}^c$	ε_1^m	$1-q_0$
0.25	0.15	0.54	0.84	0.75	0.15	0.49	0.84	0.75
0.5	0.42	1.82	0.57	0.5	0.38	1.44	0.6	0.5
0.6	0.59	3.19	0.42	0.4	0.53	2.35	0.45	0.4
0.65	0.7	4.46	0.34	0.35	0.63	3.18	0.36	0.35
0.7	0.85	6.63	0.25	0.3	0.75	4.6	0.27	0.3
0.8	1.33	19.6	0.1	0.2	1.11	12.8	0.11	0.2
0.9	2.65	139	0.02	0.1	2.04	89.8	0.02	0.1

Figure 3.3: Numerical Simulations.

Therefore, we have:

$$\frac{\frac{dMBI}{d\widehat{\varepsilon}_1}\mid_{\widehat{\varepsilon}_1=0,\gamma=\overline{\gamma}^c}}{\frac{1}{1-q_0}} = D\left(1-q_0\right) - \frac{p^{HH}\left(q_0,1\right)-I}{\int\limits_{y_1} k\left(q_0,y_1\right)\widetilde{c}(q_0,y_1)f_B\left(y_1\right)dy_1} q_0\widetilde{D}.$$

Notice that neither $q_0\widetilde{D}$ nor $p^{HH}\left(q_0,1\right)-I$ vanish as q_0 goes to 1 while, as we discussed earlier, $k\left(q_0,y_1\right)$ vanishes when $q_0 \to 1$. This implies that we have:

$$\lim_{q_0\to 1} \frac{\left|\frac{dMBI}{d\widehat{\varepsilon}_1}\mid_{\widehat{\varepsilon}_1=0,\gamma=\overline{\gamma}^c}\right|}{\frac{1}{1-q_0}} = \infty.$$

3.11.3 Numerical Simulations

Figure 3.3 reports the results of numerical simulations of the model. The baseline configuration of the parameters is $V_G=1, V_B=0, I=\frac{1}{2}$ and $\Pr(G)=Pr(B)=\frac{1}{2}$. I also assume that y_1 takes only two values, *success* and *failure*, and that bad projects always fail while good projects fail with probability f. I then vary f, the CRAs' initial reputation q_0, and the weight on future profits γ. In all the simulations, ε_1^m is evaluated at $\gamma=\overline{\gamma}^c$.

First, I assume $f=1$, which is the case when the signal y_1 is not informative about the project quality. This occurs, for example, when information about the quality of the first-period project is not observed before the second-period project is marketed to investors at time $t=2$. Notice that, starting from $q_0=0.65$, welfare is larger in monopoly, since $\varepsilon_1^m < 1-q_0$. For values of q_0 lower than 0.60, I could not find any γ for which monopoly is preferred. The simulations

for $f = 0.5$ show a similar pattern. Like in the previous set of simulations, for low values of q_0 ($q_0 < 0.7$ in this case) I could not find any γ for which monopoly is preferred. This highlights that welfare tends to be larger in monopoly only for high values of q_0.

3.12 Appendix D

I prove Proposition 30 in two steps. First, I derive the CRAs' off-equilibrium reputation, i.e., their reputation when investors observe split indicative ratings in the first period. Second, I show that there always exist off-equilibrium sharing rules for which the inequalities in equations (3.7) and (3.8) are satisfied. Therefore, for these sharing rules, offering honest ratings in the first period is a strictly dominant strategy for an opportunistic CRA. Finally, I provide a proof of the result in Example 31.

Step One: Off-equilibrium reputations.

In what follows, I focus on the case when $\widetilde{\mathbf{r}}_1 = HL$, i.e., i offered a high rating and j offered a low rating to the issuer; the case when $\widetilde{\mathbf{r}}_1 = LH$ is the mirror image of the case in which $\widetilde{\mathbf{r}}_1 = HL$.

When ratings are expected to be honest, split ratings are off-the-equilibrium path. Therefore, off-equilibrium beliefs need to satisfy the consistency requirement of Sequential Equilibria (defined in Appendix 3.9.1). Consider an arbitrary sequence σ^n converging to 0, and interpret σ^n as the probability that an opportunistic CRA "trembles" and offers a *dishonest* rating, i.e., a high (low) rating to a bad (good) project.[56] These trembles generate the following distribution of offered ratings conditional on CRA-i's type at time $t = 1$, i.e., (τ_i, θ_1), where θ_1 is the quality

[56] Note that no perturbed strategies can generate a dishonest rating from the committed type, as this action is never taken by the committed CRAs.

of the first-period project and τ_i is the behavioral type:

$$\begin{aligned}
&\Pr\left(HL \mid \tau_i = \mathcal{O}, \theta = B\right) = \sigma^n \left[q_0 + (1 - q_0)\left(1 - \sigma^n\right)\right]; \\
&\Pr\left(HL \mid \tau_i = \mathcal{C}, \theta = B\right) = 0; \\
&\Pr\left(HL \mid \tau_i = \mathcal{O}, \theta = G\right) = \left(1 - \sigma^n\right)\left(1 - q_0\right)\sigma^n; \\
&\Pr\left(HL \mid \tau_i = \mathcal{C}, \theta = G\right) = \left(1 - q_0\right)\sigma^n. \qquad (3.41)
\end{aligned}$$

When i is opportunistic, it offers a high (dishonest) rating to a bad project with probability σ^n. If j is a committed type (probab. q_0), it always offers a low rating to a bad project; if it is an opportunistic type, it offers a low rating with probability $1 - \sigma^n$. This describes the first conditional probability in equation (3.41). The remaining probabilities are derived following the same logic.

The conditional distribution above implies, for a given realization of the signal y_1, the following updated beliefs about the CRAs' types:

$$\begin{aligned}
q_i^{HL,y_1} &= \left(1 + \frac{1 - q_0}{q_0}\frac{\Pr\left(HL|\ \mathcal{O}_i, y_1\right)}{\Pr\left(HL|\ \mathcal{C}_i, y_1\right)}\right)^{-1} \\
&= \frac{1}{1 + \frac{1-q_0}{q_0}\left(1 - \sigma^n + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)}\frac{q_0 + (1-q_0)(1-\sigma^n)}{1-q_0}\right)}; \\
q_j^{HL,y_1} &= \left(1 + \frac{1 - q_0}{q_0}\frac{\Pr\left(HL|\ \mathcal{O}_j, y_1\right)}{\Pr\left(HL|\ \mathcal{C}_j, y_1\right)}\right)^{-1} \\
&= \frac{1}{1 + \frac{1-q_0}{q_0}\left(1 - \sigma^n + \frac{\Pr(y_1 \cap G)}{\Pr(y_1 \cap B)}\frac{q_0 + (1-q_0)(1-\sigma^n)}{1-q_0}\right)}.
\end{aligned}$$

The CRAs' reputation is then derived by taking the limit as $\sigma^n \to 0$ of the above expressions. We have:

$$\begin{aligned}
q_i^{HL,y_1} &= \frac{1}{1 + \frac{1-q_0}{q_0}\left(1 + \frac{\Pr(y_1 \cap B)}{\Pr(y_1 \cap G)}\frac{1}{1-q_0}\right)} = \Pr\left(G \mid y_1\right) q_0; \\
q_j^{HL,y_1} &= \frac{1}{1 + \frac{1-q_0}{q_0}\left(1 + \frac{\Pr(y_1 \cap G)}{\Pr(y_1 \cap B)}\frac{1}{1-q_0}\right)} = \Pr\left(B \mid y_1\right) q_0. \qquad (3.42)
\end{aligned}$$

[57] Given CRAs' reputations q_i^{HL,y_1} and q_j^{HL,y_1}, the off-equilibrium price $\widetilde{p}^{HH}\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right)$ is characterized as follows:

$$\widetilde{p}^{HH}\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) = \widetilde{\mu}^{HH}\left[V_G - V_B\right] + V_B,$$

where

$$\widetilde{\mu}^{HH} = \frac{1}{1 + \frac{\Pr(B)}{\Pr(G)}\left(1 - q_i^{HL,y_1}\right)\left(1 - q_j^{HL,y_1}\right)}.$$

Note that, since $\widetilde{\mu}^{HH}$ is symmetric in the CRAs' reputation, so is $\widetilde{p}^{HH}$. Therefore, we have $\widetilde{p}^{HH}\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) = \widetilde{p}^{HH}\left(q_i^{LH,y_1}, q_j^{LH,y_1}\right)$ for any y_1.

Given that, off-the-equilibrium path we always have $q_0 > q_i^{HL,y_1}$ and $q_0 > q_j^{HL,y_1}$, then $\widetilde{p}^{HH}\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) < p^{HH}(q_0, 1)$.

Step Two: Off-equilibrium fees.

We want to find an off-equilibrium sharing rule $\left\{\varphi_i(q_i, q_j), \varphi_j(q_i, q_j)\right\}$ such that the following inequalities are both satisfied:

$$\int_{y_1} \varphi_i\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) f_B(y_1)\, dy_1 \quad < \quad \frac{1}{2}\left(p^{HH}(q_0, 1) - I\right). \tag{3.43}$$

$$\int_{y_1} \varphi_i\left(q_i^{LH,y_1}, q_j^{LH,y_1}\right) f_G(y_1)\, dy_1 \quad < \quad \frac{1}{2}\left(p^{HH}(q_0, 1) - I\right) + \frac{1}{2\gamma\lambda}\left(V_G - I\right). \tag{3.44}$$

When the CRAs equally split the issuer's surplus, i.e., $\varphi_i = \varphi_j$ when $\widetilde{\mathbf{r}}_1 = HL$ or $\widetilde{\mathbf{r}}_1 = LH$ and for any realized y_1, the left-hand side of the inequalities above becomes, respectively, $\int_{y_1} \frac{1}{2}\left(\widetilde{p}^{HH}\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) - I\right) f_B(y_1)\, dy_1$ and $\int_{y_1} \frac{1}{2}\left(\widetilde{p}^{HH}\left(q_i^{LH,y_1}, q_j^{LH,y_1}\right) - I\right) f_G(y_1)\, dy_1$. Given that $\widetilde{p}^{HH}\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) = \widetilde{p}^{HH}\left(q_i^{LH,y_1}, q_j^{LH,y_1}\right) < p^{HH}(q_0, 1)$ for any realization of y_1, both the inequalities are trivially satisfied in this case.

[57] The derivations of the expressions for q_i^{HL,y_1} and q_j^{HL,y_1} in equation (3.42) use the fact that $\Pr(y_1 \cap \theta_1) = \Pr(\theta_1 \mid y_1)\Pr(y_1)$.

Let y_1^* denote the realization of y_1 for which $\Pr(G \mid y_1^*) = \Pr(B \mid y_1^*) = \frac{1}{2}$ and, thus, $\Pr(G \mid y_1^*) > \Pr(B \mid y_1^*)$ if $y_1 > y_1^*$, and the opposite when $y_1 < y_1^*$. Notice that this implies $q_i^{HL,y_1} \geq q_j^{HL,y_1}$ if $y_1 \geq y_1^*$, and $q_i^{HL,y_1} < q_j^{HL,y_1}$ when $y_1 < y_1^*$. If we restrict the attention to off-equilibrium sharing rules that are increasing in the CRAs' reputations, this also implies that:

$$\begin{aligned}
\varphi_i\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) &\geq \frac{1}{2}\left(\widetilde{p}^{HH}\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) - I\right) \quad \text{if} \quad y_1 \geq y_1^*; \\
\varphi_i\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) &\leq \frac{1}{2}\left(\widetilde{p}^{HH}\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) - I\right) \quad \text{if } y_1 \leq y_1^*.
\end{aligned}$$

and the opposite for $\varphi_i\left(q_i^{LH,y_1}, q_j^{LH,y_1}\right)$.

We can then rearrange the inequalities in equations (3.43) and (3.44) as follows:

$$\begin{aligned}
&\int_{y_1 \leq y_1^*} \varphi_i\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) f_B(y_1)\, dy_1 + \int_{y_1 \geq y_1^*} \varphi_i\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) f_B(y_1)\, dy_1 \\
&\qquad < \frac{1}{2}\left(p^{HH}(q_0, 1) - I\right). \qquad (3.45) \\
&\int_{y_1 \leq y_1^*} \varphi_i\left(q_i^{LH,y_1}, q_j^{LH,y_1}\right) f_G(y_1)\, dy_1 + \int_{y_1 \geq y_1^*} \varphi_i\left(q_i^{LH,y_1}, q_j^{LH,y_1}\right) f_G(y_1)\, dy_1 < \\
&\qquad < \frac{1}{2}\left(p^{HH}(q_0, 1) - I\right) + \frac{1}{2\gamma\lambda}\left(V_G - I\right). \qquad (3.46)
\end{aligned}$$

For a given increasing sharing rule $\left\{\varphi_i(q_i, q_j), \varphi_j(q_i, q_j)\right\}$, when $f_B(y_1)$ puts enough probability mass on realizations of y_1 such that $y_1 \leq y_1^*$, the inequality in equation (3.45) is satisfied since, when $y_1 \leq y_1^*$, we have $\varphi_i\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) \leq \frac{1}{2}\left(\widetilde{p}^{HH}\left(q_i^{HL,y_1}, q_j^{HL,y_1}\right) - I\right) < \frac{1}{2}\left(p^{HH}(q_0, 1) - I\right)$. For example, in the limit when $\Pr_{f_B}(y_1 \leq y_1^*) = 1$, this inequality is always satisfied, as φ_i is always lower than $\frac{1}{2}\left(\widetilde{p}^{HH} - I\right)$. Similarly, when $f_G(y_1)$ puts enough probability mass on realizations of y_1 such that $y_1 \geq y_1^*$, we have $\varphi_i \leq \frac{1}{2}\left(\widetilde{p}^{HH} - I\right)$ and, thus, the inequality in equation (3.46) is satisfied.

Therefore, a sufficient condition under which any increasing sharing rule supports the equilibrium with honest ratings is that the signal y_1 is informative enough about the true project

quality, i.e., that both $\Pr_{f_B}(y_1 \leq y_1^*)$ and $\Pr_{f_G}(y_1 \geq y_1^*)$ are close to one.

Proof of Example 31.

We have $\Pr(G) = \Pr(B)$, $\varphi_i(q_i, q_j) = \frac{q_i}{q_i+q_j}\left(\widetilde{p}^{HH}(q_i, q_j) - I\right)$, and $y_1 \in \{g, b\}$ where $\Pr(G \mid g) = \Pr(B \mid b) = \alpha \in [\frac{1}{2}, 1)$. This also means that $\Pr(g \mid G) = \Pr(b \mid B) = \alpha$. Notice that, given $q_i^{HL,y_1} = \Pr(G \mid y_1) q_0$ and $q_j^{HL,y_1} = \Pr(B \mid y_1) q_0$, we have:

$$\frac{q_i^{HL,y_1}}{q_i^{HL,y_1} + q_j^{HL,y_1}} = \frac{\Pr(G \mid y_1) q_0}{\Pr(G \mid y_1) q_0 + \underbrace{\Pr(B \mid y_1)}_{1-\Pr(G|y_1)} q_0} = \Pr(G \mid y_1).$$

Therefore, the inequality in equation (3.43) becomes:

$$\overbrace{\alpha(1-\alpha)}^{\Pr(G|g)\Pr(g|B)} \left[\widetilde{p}^{HH}\left(q_i^{HL,g}, q_j^{HL,g}\right) - I\right] + \\ + \underbrace{(1-\alpha)\alpha}_{\Pr(G|b)\Pr(b|B)} \left[\widetilde{p}^{HH}\left(q_i^{HL,b}, q_j^{HL,b}\right) - I\right] < \frac{1}{2}\left(p^{HH}(q_0, 1) - I\right).$$

Since both $\widetilde{p}^{HH}\left(q_i^{HL,g}, q_j^{HL,g}\right)$ and $\widetilde{p}^{HH}\left(q_i^{HL,b}, q_j^{HL,b}\right)$ are lower than $p^{HH}(q_0, 1)$, the inequality above is always satisfied if $2\alpha(1-\alpha) < \frac{1}{2}$, which implies $\alpha > \frac{1}{2}$.

Similarly, using the fact that $q_i^{LH,y_1} = \Pr(B \mid y_1) q_0$ and, thus, $\frac{q_i^{LH,y_1}}{q_i^{LH,y_1} + q_j^{LH,y_1}} = \Pr(B \mid y_1)$, we can rewrite the inequality in equation (3.44) as follows:

$$\overbrace{(1-\alpha)\alpha}^{\Pr(B|g)\Pr(g|G)} \left[\widetilde{p}^{HH}\left(q_i^{HL,g}, q_j^{HL,g}\right) - I\right] + \\ + \underbrace{\alpha(1-\alpha)}_{\Pr(B|b)\Pr(b|G)} \left[\widetilde{p}^{HH}\left(q_i^{HL,b}, q_j^{HL,b}\right) - I\right] < \frac{1}{2}\left(p^{HH}(q_0, 1) - I\right) + \frac{1}{2\gamma\lambda}(V_G - I).$$

Again, since $\widetilde{p}^{HH}\left(q_i^{HL,g}, q_j^{HL,g}\right)$ and $\widetilde{p}^{HH}\left(q_i^{HL,b}, q_j^{HL,b}\right)$ are lower than $p^{HH}(q_0, 1)$, the inequality above is always satisfied if $2\alpha(1-\alpha) < \frac{1}{2}$.