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The Implications of Heterogeneous Resource Intensities on Technical Change and Growth

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The implications of heterogeneous resource intensities on technical change and growth

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Abstract. We analyze the long-term dynamics of an economy in which sectors are heterogeneous with respect to the intensity of natural resource use. It is shown that heterogeneity induces technical change to be biased towards resource-intensive sectors. Along the balanced growth path, the sectoral structure of the economy is constant as the higher resource dependency in resource-intensive sectors is compensated by enhanced research activities. Resource taxes have no impact on dynamics except when the tax rate varies over time. Research subsidies and the sectoral provision of productivity-enhancing public goods raise growth and provide an effective tool for structural policy. JEL classification: O4, O41, Q01, Q3

Les implications de l'hétérogénéité des intensités dans l'usage des ressources naturelles sur le changement technique et la croissance. On analyse la dynamique à long terme d'une économie dans laquelle les secteurs sont hétérogènes pour ce qui est de l'intensité dans l'usage des ressources naturelles. On montre que cette hétérogénéité engendre un changement technique biaisé dans la direction des secteurs à forte intensité dans l'usage de la ressource naturelle. Le long d'un sentier de croissance balancée, la structure sectorielle de l'économie est constante à proportion que la dépendance plus grande de la ressource dans les secteurs à forte intensité d'utilisation de la ressource est compensée par des activités de recherche plus importantes. Les taxes sur la ressource n'ont pas d'effet sur la dynamique si ce n'est quand les taux de taxation varient dans le temps. Les subventions à la recherche et la production de biens publics qui renforcent la productivité sectorielle augmentent la croissance et sont un outil efficace de politique structurelle.

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1. Introduction

The last years have witnessed a remarkable rise in interest in natural resource scarcity and its long-run economic consequences. There are widespread concerns that diminishing resource input will have negative effects on our long-run living-standards. If economic growth were indeed impaired by lower resource use, this would particularly affect recently developed economies, which have grown heavily in energy-intensive sectors. Moreover, resource-intensive sectors in the leading economies would be expected to suffer most and possibly to vanish in the long run.

To counteract resource scarcity, improvements in technology are the most powerful mechanism. Several recent papers have shown that long-run growth may be compatible with the essential use of non-renewable resources, once we assume endogenous technological change, see Barbier (1999), Scholz and Ziemes (1999), and Grimaud and Rougé (2003, 2005). This literature uses a single final output framework, which is convenient but sidesteps the relationship between sectoral structure and aggregate growth. This is a limitation, since the economies are specialized in different sectors and aggregate development depends on sectoral growth. In particular, one could expect that technological progress is faster in resource-extensive sectors, that sectors exhibit diverging growth rates, and that economies specialized in resource-intensive sectors achieve lower development paths.

The paper shows that, contrary to common beliefs, resource-intensive sectors conduct more research and increase efficiency faster than the rest of the economy. Second, we demonstrate that the resource-intensive sectors are able to sustain output in the long run, because the declining input of natural resources is compensated for by sufficiently rising efficiency. We show the existence of a balanced growth path and provide conditions for saddle-path stability of the system. This holds true even when learning externalities in the research sector are purely sector specific. In addition, we demonstrate that the share of resource-intensive sectors can be constant in the long run, as profit incentives induce a more-than-proportional research effort in these sectors. Finally, we confirm that increasing resource scarcity need not hamper economic growth, even when sectors have large differences in resource use.

The model assumptions are based on empirical regularities. In reality, sectors differ substantially in terms of input intensities, specifically with regard to knowledge intensity and natural resource use, which is crucial for the kind of disaggregation used. Second, sectors offer significantly different investment opportunities and innovations are often sector specific. Third, policies directed at specific sectors are very popular and are often implemented in practice.

We show that under the these empirically relevant assumptions, incentives arise from the rising scarcity of resource-intensive goods to invest relatively more in R&D in the resource-intensive sector. Consequently, the composition of consumption in terms of productivity-weighted sectoral goods remains constant along a balanced growth path. Our results are in line with predictions of international organizations and recent empirical observations. The International Energy Agency (IEA) emphasizes that the largest potential for improving future energy efficiency lies in the energy-intensive sectors (IEA 2008, 112). Moreover, it sees good development perspectives for emerging economies despite their increasing shares of energy-intensive sectors (IEA 2008, 115). Along the same lines, Demailly and Quirion (2008) find that energy-intensive industries have performed much better economically under strict climate policies than was previously expected.[1]

We study the implications of various policies such as resource taxes, labour and research subsidies and the sectoral, productivity enhancing provision of public goods. We look at the effectiveness of these policies in lowering resource use and raising growth, that is, promoting 'sustainable' development. Interestingly, the results are mixed. In particular, research subsidies have positive growth effects in both sectors, but resource taxes affect dynamics only when the tax rate varies over time. It is also shown that sectoral policies might not induce the desired effects, and that the sectors targeted by policies may not matter for the actual policy effects. For example, a sector-specific provision of public goods that aims at increasing the share of the targeted sector will not raise but will lower the share of this sector. And, although the growth effect of public good provision is positive, this result is independent of the sector in which the goods are provided.

The paper is related to recent literature on innovation, growth, and resource use. The basic technology assumptions for the different sectors in the model are based on Romer (1990).[2] By stressing the role of sectoral research activities and directed technological change, we apply the theory of factor-induced technical change, as introduced by Hicks (1932) and applied by Acemoglu (2002), to economic sectors and determine the conditions for sector-induced research. Smulders and de Nooij (2003) and Di Maria and Valente (2008) are closest to our approach. However, these papers do not assume that natural resources and labour are employed in all the different sectors of the economy. Moreover, we introduce labour reallocation between the different production and research sectors, which realistically allows for more flexible adjustments in the economy. Owing to this endogeneity, policies can affect the speed of resource extraction as well as aggregate research activities – both of which are crucial for the dynamics of the economy.[3] Our approach is also close to papers in which heterogeneous sectors

1 For an extensive overview of the literature dealing with applied approaches to technological change and environmental policy, see, for example, Jaffe, Newell, and Stavins (2002) and references within.

2 This is similar to Bretschger and Pittel (2005) who consider a multi-sector economy with sector-specific natural resource use but without directed technological change, as the substitution elasticity between sectoral outputs is assumed to be unity. Withagen (1999), Pittel (2002), and Xepapadeas (2002) provide surveys on the impact of natural resource use on economic growth. The impact of natural resource use in dynamic multi-sector models is also treated by Peretto (2009) and Bretschger (2008).

3 Smulders and de Nooij (2003) take the supply of energy as exogenously given. We extend their approach by endogenizing the dynamics associated with the input of non-renewable natural

cause ongoing structural change; see Kuznets (1957), Kongsamut, Rebelo, and Xie (2001), López, Anriquez, and Gulati (2007), and Acemoglu and Guerrieri (2008). In contrast to this literature, we introduce a new kind of multi-sector economy suited to discussing the direction of technical change and development under natural resource constraints. We show that despite the heterogeneity of sectors, long-run growth might not be accompanied by structural change. In an extension of the literature, we also study implications of different types of policies that aim at supporting sustainability.

The remainder of the paper is organized as follows. Section 2 describes the model in detail. The short- and long-run dynamics of the model are analyzed in section 3. Section 4 deals with the effects of policies striving at increasing the share of resource extensive sectors and fostering sustainability, that is, raising growth and lowering resource extraction. Section 5 concludes.

2. The model

In our economy, horizontally differentiated goods are produced in two sectors: a resource-intensive and a resource-extensive sector. In each sector, the differentiated goods are assembled to sectoral outputs that are consumed by the households. Blueprints for new products are developed by sector-specific research activities and sold to monopolistic producers in each sector. Besides natural resources, labour constitutes the second primary input, which is employed in research as well as in intermediate production. Sectors differ with respect to resource intensity of production. We consider infinitely living households that maximize lifetime utility. Savings are in the form of investment either in bonds or in R&D.

2.1. Production

2.1.1. Sectoral output

The outputs of each of the two sectors, $\tilde{X}$ and $\tilde{Z}$, consist of a continuum of horizontally differentiated intermediate goods, x_i, $i \in [0, n]$, and z_j, $j \in [0, m]$, where n and m denote the number of varieties in the respective sectors.[4] Gains from

resources. Di Maria and Valente (2008) endogenize the supply of inputs (capital and resources), but again they do not assume that all inputs are used in the different sectors. They conclude that long-run development is characterized by resource-augmenting technological progress only. For the case in which the economic sectors employ all the inputs and differ only with respect to input intensities, we are able to show that every sector conducts R&D in the long run. The direction of technological change is endogenous and depends on the degree of heterogeneity with respect to resource intensities.

4 For notational convenience the time index will be suppressed whenever no ambiguity arises.

specialization arise; that is, the larger the variety of goods, the more productive the aggregate:[5]

$$\tilde{X} = \left(\int_0^n x_i^\beta di \right)^{1/\beta} \quad \text{and} \quad \tilde{Z} = \left(\int_0^m z_j^\beta dj \right)^{1/\beta}. \tag{1}$$

where $0 < \beta < 1$.

The index of consumption, C, reflects households' preferences for sectoral output. Alternatively, C could be interpreted as aggregate output of the economy; see, for example, section 4.2. C depends on $\tilde{X}$ and $\tilde{Z}$, according to the following CES function:

$$C_t = \left(\tilde{X}_t^{\frac{\nu-1}{\nu}} + \tilde{Z}_t^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}}, \qquad \nu < 1, \tag{2}$$

where ν denotes the elasticity of substitution between $\tilde{X}$ and $\tilde{Z}$. The analysis is limited to the case $\nu < 1$, in which $\tilde{X}$ and $\tilde{Z}$ are complements and both sectors are essential.[6]

To facilitate calculations without loss of generality, we choose the consumption good to be the numeraire of the system so that its price is unity; that is, $p_C \equiv 1$. At each point in time, utility maximization results in

$$\frac{\tilde{X}}{\tilde{Z}} = \left(\frac{p_{\tilde{X}}}{p_{\tilde{Z}}} \right)^{-\nu} \quad \Leftrightarrow \quad \frac{p_{\tilde{X}} \tilde{X}}{p_{\tilde{Z}} \tilde{Z}} = \left(\frac{\tilde{X}}{\tilde{Z}} \right)^{-\frac{1-\nu}{\nu}} = \frac{\phi}{1-\phi} = \tilde{\phi}, \tag{3}$$

where $\phi = (p_{\tilde{X}} \tilde{X})/C$ and $1 - \phi = (p_{\tilde{Z}} \tilde{Z})/C$ denote the expenditure shares of $\tilde{X}$ and $\tilde{Z}$, such that the relative sector share of x-goods is given by $\tilde{\phi}$, which will prove to be a very useful variable below.

Competition in x- and z-production is monopolistic. Each type of good is produced by only one firm, which has to acquire the according patent first. x- as well as z-intermediates are produced from labour, L, and non-renewable resources, R, using the following Cobb-Douglas production technologies:

$$x_i = (L_{x_i})^\alpha (R_{x_i})^{1-\alpha} \quad \text{and} \quad z_j = (L_{z_j})^\delta (R_{z_j})^{1-\delta}, \tag{4}$$

5 In contrast to the productivity-adjusted aggregates, $\tilde{X}$ and $\tilde{Z}$, we denote aggregate physical amounts of x_i and z_i by $X = \int_0^n x_i di$ and $Z = \int_0^m z_j dj$. The prices for $\tilde{X}$ and $\tilde{Z}$ are $p_{\tilde{X}}$ and $p_{\tilde{Z}}$. Variables p_{x_i} and p_{z_i}, on the other hand, denote prices for individual goods.

6 We abstract from the case of substitution, that is, $\nu > 1$, as well as from Cobb-Douglas preferences, that is, $\nu = 1$, for the following reasons: The substitution case is of little interest to a paper focusing on sustainable growth and sectoral structure, since for $\nu > 1$ one sector could vanish without adverse long-run consequences for growth. For a thorough discussion of $\nu > 1$, we therefore refer to Acemoglu (2002) and especially Peretto (2008), who also considers implications in the context of natural resources. Cobb-Douglas preferences, on the other hand, are discarded, as sector shares would not be endogenously determined in this case.

where $0 < \alpha, \delta < 1$. L_k and R_k, $k = x_i, z_j$, denote the input of labour and resources in the production of x_i and z_j. It is assumed that sectors differ with respect to their resource intensities; that is, $\alpha \neq \delta$.[7]

Maximization of profits gives the first-order conditions for the input of labour and resources in the two sectors. Considering that $x_i = x$ and $z_j = z$ in the symmetric equilibrium gives the sectoral demands for labour and resources in terms of $\tilde{\phi}$ and C:

$$\begin{aligned} L_X &= \alpha\beta\frac{\tilde{\phi}}{1+\tilde{\phi}}\frac{C}{w} \quad \text{and} \quad L_Z = \delta\beta\frac{1}{1+\tilde{\phi}}\frac{C}{w} \\ R_X &= (1-\alpha)\beta\frac{\tilde{\phi}}{1+\tilde{\phi}}\frac{C}{p_R} \quad \text{and} \quad R_Z = (1-\delta)\beta\frac{1}{1+\tilde{\phi}}\frac{C}{p_R}, \end{aligned} \tag{5}$$

where $R_X = \int_0^n R_{x_i}di$, $R_Z = \int_0^m R_{z_j}dj$, $L_X = \int_0^n L_{x_i}di$, and $L_Z = \int_0^m L_{z_j}dj$. Variables w and p_R denote the wage rate and the price of resources. Individual firms' demands are obtained by dividing the respective sectoral demands by the 'number' of intermediates in each sector, that is, n and m, respectively. Summing up the resource demands of the two sectors in (5) gives the aggregate extraction of resources at each point in time:

$$R = R_X + R_Z = ((1-\alpha)\tilde{\phi} + (1-\delta))\frac{\beta}{1+\tilde{\phi}}\frac{C}{p_R}. \tag{6}$$

From (5) and the production functions for x and z, (4), sectoral equilibrium profits from intermediates production can be derived:

$$\Pi_X = (1-\beta)\frac{\tilde{\phi}}{1+\tilde{\phi}}C \quad \text{and} \quad \Pi_Z = (1-\beta)\frac{1}{1+\tilde{\phi}}C. \tag{7}$$

2.1.2. R&D

Blueprints for new types of goods are generated in two separate R&D sectors. The only rival input to research is labour, yet production also profits from past research activities that give rise to positive sector specific spill-overs. Production is linear in labour as well as in research experience, which equals the 'number' of blueprints generated in the respective sector in the past (n for the x-sector and m for the z-sector):

7 We abstract from $\alpha = \delta$, as this case does not not provide much insight beyond the existing literature. In the long run, the relative sector share, $\tilde{\phi}$, is equal to unity and the model collapses to a model with only one intermediates sector (e.g., as analyzed by Scholz and Ziemes 1999; Schou 2002; and Grimaud, Magné, and Rougé 2009). The only difference to this type of model arises with respect to transitional dynamics as, in case the initial number of intermediate goods differs in the two sectors, sectoral R&D activities are not identical outside the balanced growth path.

$$\dot{n} = \frac{dn}{dt} = \frac{L_n}{a}n \qquad \text{and} \qquad \dot{m} = \frac{dm}{dt} = \frac{L_m}{a}m, \tag{8}$$

where L_n and L_m denote the input of labour to sectoral research. Parameter a represents the unit input coefficient of labour in research, which is assumed to be equal in the two sectors. In order not to build in sectoral convergence by the model assumptions, we exclude spillovers between the sectors.[8]

Given the four different uses of labour, equilibrium in the labour market requires

$$L_X + L_Z + L_n + L_m = 1, \tag{9}$$

where, for simplicity, the size of the labour force is set equal to unity.

Research markets are assumed to be perfectly competitive, such that in equilibrium patent values V_n, respectively V_m, are equal to marginal production costs:

$$V_n = \frac{aw}{n} \qquad \text{and} \qquad V_m = \frac{aw}{m}. \tag{10}$$

Furthermore, equilibrium on the patent market requires the value of a patent to be equal to the discounted stream of profits generated by the production of the respective intermediate good, which implies that the following no-arbitrage conditions hold:

$$\dot{V}_n = rV_n - \Pi_x \qquad \text{and} \qquad \dot{V}_m = rV_m - \Pi_z, \tag{11}$$

where r is the interest rate on all assets. $\Pi_x = \Pi_X/n$ and $\Pi_z = \Pi_Z/m$ stand for individual intermediate firms' profits; Π_X and Π_Z are given in (7).

2.1.3. Resources

Natural resources are non-renewable. The resource stock S is depleted by the extraction of resources R for production, such that the dynamics of the resource stock are

$$\dot{S} = -R. \tag{12}$$

8 Labour is taken to be the only rival input in R&D, as, due to the decreasing input of resources, integrating resources would imply growth to decrease over time. Long-run growth would thus require an additional growth engine, for example, population growth (see the literature on semi-endogenous growth in resource models, e.g., Groth and Schou 2007). However, as the inclusion of resources would not change our results regarding the sectoral behaviour, we abstain from integrating resources for the sake of simplicity.

It is assumed that resources are extracted at no cost. Resource firms maximize intertemporal profits

$$\max_{R} \int_0^\infty p_R(t)R(t)e^{-\int_0^t r(s)ds}dt, \tag{13}$$

subject to (12), which yields the familiar Hotelling rule:[9]

$$g_{p_R} = \frac{\dot{p}_R}{p_R} = r. \tag{14}$$

2.2. Consumers

Households derive utility from consumption C. The representative household maximizes discounted lifetime utility with respect to its intertemporal budget constraint:

$$\begin{aligned} \max_{c} \quad & \int_0^\infty \log C(t)e^{-\rho t}dt \\ \text{s.t.} \quad & \dot{W} = rW + w - C, \end{aligned} \tag{15}$$

where $W = nV_n + mV_m + p_R S$ denotes total household asset holdings that include the value of the resources extractable at time t in current prices ($p_R S$). We assume utility to be logarithmic in order to simplify the exposition (see Smulders and de Nooij 2003 and Di Maria and Valente 2008 for similar set-ups). Households supply labour inelastically. From the first-order conditions of household maximization we get the familiar Keynes-Ramsey rule;

$$g_C = r - \rho. \tag{16}$$

3. System dynamics

To analyze the dynamics of the economy we reduce the system to two first-order differential equations, which are functions of the relative sector share, $\tilde{\phi}$, and the input of labour in intermediates production of sector z, that is, L_Z.

LEMMA 1. *The dynamics of the economic system are given by*

$$\dot{L}_Z = \left(\frac{1}{2a}\left(\delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta}(1+\tilde{\phi})\right)L_Z - \rho - \frac{1}{2a} - \frac{\tilde{\phi}}{1+\tilde{\phi}}g_{\tilde{\phi}}\right)L_Z \tag{17}$$

9 Variable g_b denotes the growth rate of variable b; that is, $g_b = \dot{b}/b$, where $\dot{b}$ is the time derivative of b.

$$\dot{\tilde{\phi}} = \left(\frac{\left[\frac{1}{2a} \frac{\alpha - \delta}{\delta} \left(\delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta}(1+\tilde{\phi}) \right) - \frac{1}{\delta a} \left(\frac{1-\beta}{\beta} \right)^2 (1-\tilde{\phi}) \right] L_Z - \frac{1}{2a}(\alpha - \delta)}{\frac{1}{\nu - 1}} \right) \tilde{\phi}. \tag{18}$$

Proof. See appendix A.1.

These two equations describe the dynamics of the system along the balanced growth path (BGP) as well as during the transition to the BGP.

With respect to long-term growth, a path will be called a BGP if all variables grow at constant – possibly zero or negative – rates. This implies that along a BGP (i) aggregate production and production in both intermediate sectors grow at the same rate and (ii) expenditure shares, sectoral labour inputs, and the interest rate are constant over time ($\dot{\tilde{\phi}} = \dot{L}_Z = \dot{r} = 0$).

For the BGP values of L_Z and $\tilde{\phi}$ we get (see appendix A.2):

$$L_Z^* = \frac{\delta(1+2a\rho)}{\delta + \tilde{\phi}^*\alpha + \frac{1-\beta}{\beta}(1+\tilde{\phi}^*)} \tag{19}$$

$$\tilde{\phi}^* = \frac{A}{B}, \tag{20}$$

where

$$A = ((1-\beta)^2(1+2a\rho) - a\alpha\beta^2\delta\rho) - a(\alpha-\delta)\beta(1-\beta)\rho + a\delta^2\beta^2\rho \tag{21}$$

$$B = ((1-\beta)^2(1+2a\rho) - a\alpha\beta^2\delta\rho) + a(\alpha-\delta)\beta(1-\beta)\rho + a\alpha^2\beta^2\rho, \tag{22}$$

and asterisks indicate variable values or growth rates along the BGP.[10]

As is to be expected, the direct effect of labour productivity in the z-sector, δ, on L_Z^* is positive, while the direct effect of α is negative. Also the discount rate, ρ, exerts a positive direct effect, as higher impatience induces households to allocate less labour to research and more to today's goods production. Equally, the reaction of L_Z^* to β and $\tilde{\phi}^*$ follows intuition: higher gains of specialization:

10 In contrast to the findings of Acemoglu and Guerrieri (2006), optimization on markets leads to long-run balanced growth in our model. Acemoglu and Guerrieri assume R&D to be subject to decreasing or negative spillovers from knowledge, in which case non-balanced growth arises. In our paper, however, R&D is linear in knowledge, in which case the two sectors grow at the same rate.

that is, a lower β, and a higher relative sector share of x-products, $\tilde{\phi}^*$, lead to a lower input of labour into z-production.

With respect to $\tilde{\phi}^*$, (20) reflects that the gains of specialization, the productivity of R&D, as well as the discount rate affect both sectors symmetrically. Variable $\tilde{\phi}^*$ deviates from unity only because $\alpha \neq \delta$. Yet whether $\tilde{\phi}^*$ reacts positively or negatively to a rise in either α or δ depends crucially on the parametrization of the model. A rise in α, for example, on the one hand increases labour productivity in the x-sector, which, ceteris paribus, affects $\tilde{\phi}^*$ positively. On the other hand the rise in α decreases the productivity of resources, which lowers $\tilde{\phi}^*$. Yet less productive resources also render z-products scarcer and therefore increase z-prices and induce higher incentives to invest in n-R&D, both of which increase $\tilde{\phi}^*$. Which effect dominates depends crucially on the parametrization.

The equilibrium input of labour into x-intermediates can be derived from (5):

$$L_X^* = \tilde{\phi}^* \frac{\alpha}{\delta} L_Z^*. \tag{23}$$

Equation (23) shows that the share of labour allocated to x- rather than z-intermediates rises with the relative sector share and labour productivity in the x-sector.

From the no-arbitrage conditions for the patent market it follows that along the BGP the following relations hold (see appendix A.2):

$$L_n^* = \tilde{\phi}^* \frac{1-\beta}{\beta} \frac{L_Z^*}{\delta} - a\rho \tag{24}$$

$$L_m^* = \frac{1-\beta}{\beta} \frac{L_Z^*}{\delta} - a\rho. \tag{25}$$

Complementing the results on L_X^* and L_Z^*, we see that a ceteris paribus higher $\tilde{\phi}^*$ induces an allocation of labour towards R&D that develops new patents for the x-sector, while the input of labour into both research sectors rises with higher gains of specialization.

The growth rate of resource extraction along the BGP can be determined by expressing the aggregate demand for resources, (6), in growth rates and considering the Keynes-Ramsey rule, (16). This gives

$$g_R = -\rho. \tag{26}$$

Differentiation of (2) confirms that balanced growth requires consumption and the production of x- and z-aggregates to grow at the same rate; that is,

$$g_C^* = g_{\tilde{X}}^* = g_{\tilde{Z}}^*. \tag{27}$$

Identical constant growth rates of $\tilde{X}$ and $\tilde{Z}$ together with (3) imply that the sectoral expenditure shares, ϕ and $1-\phi$, as well as the relative expenditure share, $\tilde{\phi}$, are constant over time.

The condition that along the BGP aggregate production in both sectors has to grow at the same rate carries important implications for research efforts in equilibrium. When we consider the production technologies for x and z, (4), as well as (26), the growth rates of $\tilde{X}$ and $\tilde{Z}$ along the BGP are given by

$$g^*_{\tilde{X}} = \frac{1-\beta}{\beta} g^*_n - (1-\alpha)\rho \tag{28}$$

$$g^*_{\tilde{Z}} = \frac{1-\beta}{\beta} g^*_m - (1-\delta)\rho. \tag{29}$$

PROPOSITION 1. *Along the balanced growth path, research is biased towards the resource-intensive sector. If, for example, the z-sector is more resource intensive ($\alpha > \delta$), $g^*_m > g^*_n$ holds.*

Proof. From (28), (29), and $g^*_{\tilde{X}} = g^*_{\tilde{Z}}$ along the BGP it follows straightforwardly that the following relation holds:

$$g^*_m = (\alpha - \delta)\rho \frac{\beta}{1-\beta} + g^*_n. \tag{30}$$

□

This condition states that for balanced growth to be feasible, differences in resource intensities between sectors have to be compensated for by research. It can also easily be seen that in the case where sectors are identical, innovation rates along the BGP are the same in the two sectors. If, however, sectors differ with respect to resource intensities, more research will be conducted in the sector that is more resource intensive and thus subject to a stronger drag from declining resource inputs. The intuition behind this result is that, owing to the decrease of resource inputs over time, output of the more resource-intensive sector will become increasingly scarce, which – without rising research effort – would lead to an increase in prices, which makes it more profitable to invest in research in this sector. The induced increase in productivity and therefore the size of the sector induces a pressure on prices that exactly offsets the price increase from rising scarcity.

While the aggregate productivity-weighted amounts of goods produced in both sectors, $\tilde{X}$ and $\tilde{Z}$, grow at the same, potentially positive rate in equilibrium, the physical amounts of individual intermediates produced in either sector, x and z, decrease over time. Taking into account that labour shares are constant along the BGP, it follows from (4) and $g_{R_i} = g_R = -\rho$ that

$$g^*_x = -(1-\alpha)\rho < 0 \tag{31}$$

$$g_z^* = -(1-\delta)\rho < 0. \tag{32}$$

The reduction in the produced amounts is due to the decreasing input of natural resources. If the z-sector is more resource intensive than the x-sector, z falls faster than x. As economic intuition suggests, it follows from (3) that the price ratio follows a time path that is inverse to the development of quantities; that is, prices in the more resource-intensive sector rise faster because of increasing resource prices.

From (8), (28), and (29) we can express the equilibrium growth rate of consumption as

$$\begin{aligned} g_C^* &= \frac{1-\beta}{\beta}\frac{L_n^*}{a} - (1-\alpha)\rho \\ &= \frac{1-\beta}{\beta}\frac{L_m^*}{a} - (1-\delta)\rho, \end{aligned} \tag{33}$$

where L_n^* and L_m^* are specified in (24) and (25). Overall, the sign of g_C in (33) is ambiguous. Two forces determine whether long-term development is sustainable ($g_C > 0$): $-(1-\alpha)\rho$ and $-(1-\delta)\rho$ represent the negative growth effects stemming from the declining input of natural resources and impatience, while $((1-\beta)L_n^*)/(a\beta)$ and $((1-\beta)L_m^*)/(a\beta)$ reflect the growth-stimulating effects of research.

After we substitute (19), (24), and (25), g_C^* can be rewritten in terms of the model parameters only:

$$g_C^* = \frac{\dfrac{(1-\beta)^2}{a\beta} - \beta(\alpha^2+\delta^2)\rho}{2-\beta(2-\alpha-\delta)} - \rho. \tag{34}$$

The resulting expression shows that consumption growth along the BGP can be negative if research is not sufficiently productive (high a), such that the drag on growth from resources overcompensates for improvements in productivity from research. Similarly, growth might be negative if agents are highly impatient. With respect to changes in resource intensities, effects on growth are ambiguous. A decrease in resource intensity (increase in α, resp. δ) on the one hand induces a less severe drag on growth, but on the other hand causes a reallocation of labour away from research.

Let us finally consider the transitional dynamics of this economy.

LEMMA 2. *The system given by (17) and (18) is locally saddle-path stable.*

Proof. See appendix A.3.

The intuition behind the stability of the system is that both $\tilde{X}$ and $\tilde{Z}$ are essential in our economy such that R&D is biased towards the scarcer, that is, more expensive, product. Consequently, if the initial share of a product is lower than its equilibrium share, it will rise over time towards the BGP.[11]

4. Policy analysis

In section 3 we derived that growth depends on research effort and resource use. Growth effects of policy can therefore stem from either higher innovation rates or slower resource extraction or both. Yet alternative policies differ not only with respect to the channels through which they affect growth, but also with respect to their impact on the sectoral structure.

In the following we consider different types of policies that might constitute alternatives for a policy maker.[12] In this section we assume throughout that $\alpha > \delta$, that is, that the x-sector is less resource intensive. We check different policies with respect to their ability to foster growth, to slow down resource extraction, and to affect the sectoral structure of the economy. For those policy variables for which the tax or subsidy level has no impact on the dynamics of the economy, we check for possible effects of time-varying tax/subsidy schedules. Specifically, we focus on

- resource taxation (tax rate τ)
- labour subsidization (subsidy rate s_w)
- differentiated research subsidization (s_n and s_m)
- differentiated provision of productive public goods (shares μ_x, resp. μ_z, of consumption).

In the case of time-varying policy instruments, g_i denotes the growth rate of the respective policy variable i.

The analysis of the policy instruments is conducted in two steps. First, the traditional instruments, that is, taxes and subsidies, are analyzed in subsection 4.1, while the provision of public goods is treated in subsection 4.2.

11 It can be shown that our results regarding the existence of a BGP as well as the stability properties can be extended to economies in which sectors differ with respect to research productivity ($a_n \neq a_m$) and/or gains of specialization ($\beta_x \neq \beta_z$). Higher gains of specialization in sector x (i.e., $\beta_z > \beta_x$) would imply, for example, an even stronger bias towards z-research. For more details, see Pittel and Bretschger (2009).

12 We focus on policy instruments and targets that are currently discussed in the field of energy and climate policies rather than on optimal policies. The optimum is quite simple to detect because the only market failures actually included in the model are monopolistic competition in the intermediate sector and the externality from the research sectors, which are well studied in the endogenous growth literature (see Romer 1990). Here we concentrate on the question of whether the measures designed to save on resource use and to support resource-extensive sectors work, as is commonly assumed, or are ineffective or even counter-productive.

4.1. Policy analysis 1: taxes and subsidies

In the following we consider ad valorem taxes on the input of resources as well as uniform subsidies on labour and differentiated subsidies on research. Research subsidies are in the form of wage subsidies.

To clearly distinguish the effects of each instrument, we assume in the following that the government can balance its budget via lump-sum taxation or subsidization of households. In this case, policies are not tied together by budgetary requirements and each instrument can be analyzed independently.

Owing to the policy interventions, the BGP values of $\tilde{\phi}$ and L_Z modify to[13]

$$L_Z^{p1*} = \frac{\delta(1+2a\rho)}{\delta + \tilde{\phi}^{p1*}\alpha + \frac{1-\beta}{\beta}\left(\frac{1}{\bar{s}_m} + \frac{\tilde{\phi}^{p1*}}{\bar{s}_n}\right)} \tag{35}$$

$$\tilde{\phi}^{p1*} = \frac{\bar{s}_n}{\bar{s}_m}\frac{A - aD_1}{B - aD_2}, \tag{36}$$

where

$$D_1 = \beta(\alpha-\delta)(\rho\delta\beta(\bar{s}_m - 1) + g_{\bar{\tau}}(1-\beta+\bar{s}_m\beta\delta)) \tag{37}$$

$$D_2 = -\beta(\alpha-\delta)(\rho\alpha\beta(\bar{s}_n - 1) + g_{\bar{\tau}}(1-\beta+\bar{s}_n\beta\alpha)). \tag{38}$$

For notational convenience we denote $\bar{\tau} = 1+\tau$, $\bar{s}_m = 1 - s_m$ and $\bar{s}_n = 1 - s_{r_n}$. We retrieve the no-policy BGP values of the two variables by setting $\bar{s}_m = \bar{s}_n = 1$ and $g_{\bar{\tau}} = 0$.

By proceeding as in the no-policy section, we can show, furthermore, that

$$g_C^{p1*} = \frac{1-\beta}{\beta}\frac{L_m^{p1*}}{a} - (1-\delta)(\rho + g_{\bar{\tau}}) \tag{39}$$

$$L_m^{p1*} = \frac{1}{s_m\delta}\frac{1-\beta}{\beta}L_Z^{p1*} - a\rho. \tag{40}$$

For the BGP rate of resource extraction we get

$$g_R^{p1*} = -(\rho + g_{\bar{\tau}}). \tag{41}$$

By employing the above BGP relations we can now derive the comparative statics of the different policy instruments.

13 For the derivation of the underlying dynamic system as well as for analytical expressions of the comparative statics in proposition 2, see Pittel and Bretschger (2009).

PROPOSITION 2. *If resource tax rates, τ, and labour subsidies, s_w, are constant over time, they have no impact on long-run growth, resource extraction, and the relative sector share. Research subsidies, s_m and s_n, affect growth as well as the relative sector share even if their rates are constant over time:*

$$\begin{aligned} &\frac{d\tilde{\phi}^{p1*}}{ds_n} < 0, \quad \frac{dg_C^{p1*}}{ds_n} > 0, \\ &\frac{d\tilde{\phi}^{p1*}}{ds_m} > 0, \quad \frac{dg_C^{p1*}}{ds_m} > 0. \end{aligned} \tag{42}$$

Tax rates that change over time affect long-run growth, resource extraction, and the relative sector share as follows:[14]

$$\frac{d\tilde{\phi}^{p1*}}{dg_\tau} < 0, \quad \frac{dg_R^{p1*}}{dg_\tau} < 0, \quad \frac{dg_C^{p1*}}{dg_\tau} < 0. \tag{43}$$

Labour subsidies do not affect long-run growth, resource extraction, and the relative sector share even if they are time-varying:

$$\frac{d\tilde{\phi}^{p1*}}{dg_{s_w}} = \frac{dg_R^{p1*}}{dg_{s_w}} = \frac{dg_C^{p1*}}{dg_{s_w}} = 0. \tag{44}$$

Proof. Taking the derivatives of $\tilde{\phi}^{p1*}$, g_R^{p1*} and g_C^{p1*} as determined by (35), (36), (39), (40), and (41) with respect to the policy variables yields either zero or the above signs. □

Resource taxation affects long-run growth and the sectoral structure only if the rate of taxation changes over time. If τ is constant, resource taxation lowers resource demand permanently by a constant factor, while intertemporal arbitrage of resource owners remains unaffected. As a consequence, the producer price of resources declines, leaving the price that intermediates producers have to pay unchanged. Thus, constant taxation exerts an effect neither on resource and labour allocation nor on the speed of resource extraction. A rising rate of resource taxation, however, alters growth via two channels. An increase in taxation induces the speed of resource extraction to rise, as resource owners foresee the future decrease in the non-taxed share of resource revenues. The resulting negative effect on growth is naturally stronger in the more resource-intensive sector. To compensate for this stronger resource drag, labour is allocated to research in this sector. However, the resource extraction effect dominates such that the overall effect remains negative. These findings are in line with resource models that comprise single final goods sectors; see, for example, Groth and Schou (2007),

14 As resource taxes are assumed to be ad valorem taxes on the input of resources, tax rates that are continuously increasing and at some point exceed unity are feasible.

who reveal that they continue to hold in the case of heterogeneous sectors. We also show how the adjustment mechanisms work with multiple sectors. In intermediates production, labour is reallocated to the more resource-intensive sector. Owing to the tax-induced faster increase of intermediates' prices in z-production, the value share of the z-sector rises, which raises profitability and thereby attracts labour from the x-sector and lowers $\tilde{\phi}$.[15]

Labour subsidization has no effect on growth and sector structure – for either constant or rising subsidy rates. The intuition is that, as labour inputs in all sectors are equally affected by the subsidy, no labour reallocation is induced.[16] The level of research subsidy rates affects the allocation of labour in our model, as it distorts the production cost ratio between intermediates production and research. This effect corresponds to the standard results of endogenous growth theory (see Romer 1990). We add to the literature by analysing the direction in which research subsidies change the relative market shares in the multi-sector economy. This depends on whether the more or the less resource-intensive sector is subsidized. Subsidies to research in the less resource-intensive sector (s_n) induce the relative sector size of this sector to decrease, and vice versa for the more resource-intensive sector. The line of reasoning is equivalent to the case of resource taxation presented above. Research subsidization exerts no effect on resource extraction in our model. Although the interest rate and therefore the growth rate of the resource price change because of subsidization, the rate of extraction remains unaltered, as income and substitution effects of interest rate changes on the savings decision of households cancel.

4.2. Policy analysis 2: productive public goods

The present framework is especially suited to study sector-specific policies. As a second policy option we thus consider financing activities that enhance the productivity of resources in either one sector or both sectors. The productivity improvement is assumed to result from investment in the public provision of sector-specific infrastructure, which requires a sector-specific formulation of the approach of Barro (1990) and Barro and Sala-i-Martin (1992).

For simplicity we again assume that the financial revenues necessary are generated via lump-sum taxation. It is further assumed that the share of consumption (resp., aggregate output) used for public good provision is equal to the amount of public goods, G_k, $k = x, z$, produced from this share; that is, $G_k = \mu_k C$, $\mu_x + \mu_z < 1$. In this case, the production functions for x_i and z_j modify to[17]

15 Empirical evidence that taxes on oil have been rising considerably during the last decades is presented by Daubanes (2009). The current Swiss announcement of a rise in CO_2-taxes is an example of anticipated tax increases.

16 This neutrality result depends, of course, on the assumption that there is no labour-leisure choice in our model.

17 For (45) to be compatible with (4), one may think of (4) as a specific case of (45), where $G_k = 1$, that is, with a constant provision of public goods that is set equal to unity.

$$x_i = L_{x_i}^{\alpha}(G_x R_{x_i})^{1-\alpha} \qquad \text{and} \qquad z_j = L_{z_j}^{\delta}(G_z R_{z_j})^{1-\delta}. \tag{45}$$

such that in equilibrium $\tilde{X}$ and $\tilde{Z}$ read

$$\tilde{X} = n^{\frac{1-\beta}{\beta}} L_X^{\alpha}(\mu_x C R_X)^{1-\alpha} \qquad \text{and} \qquad \tilde{Z} = m^{\frac{1-\beta}{\beta}} L_Z^{\delta}(\mu_z C R_Z)^{1-\delta}.$$

The new equilibrium values of the relative sector share and labour input in z-intermediates are given by[18]

$$L_Z^{p2*} = \frac{\delta(1+2a\rho)}{\delta + \tilde{\phi}^{p2*}\alpha + \frac{1-\beta}{\beta}(1+\tilde{\phi}^{p2*})} \tag{46}$$

$$\tilde{\phi}^{p2*} = \frac{\alpha A + aE_1}{\delta B - aE_2} \tag{47}$$

with

$$E_1 = (1-\beta(1-\delta))[\beta((1-\delta)\alpha g_{\mu_z} - (1-\alpha)\delta g_{\mu_z}) + (\alpha-\delta)(\alpha\beta-1)\rho]$$
$$E_2 = (1-\beta(1-\alpha))[\beta((1-\delta)\alpha g_{\mu_z} - (1-\alpha)\delta g_{\mu_z}) + (\alpha-\delta)(\delta\beta-1)\rho].$$

Note that setting $g_\mu = 0$ does not replicate the no-policy equilibrium in this case. Furthermore, it can be shown, by proceeding as in the no-policy section, that

$$g_C^{p2*} = \frac{1}{\delta}\left(\frac{1-\beta}{\beta}\frac{L_m^{p2*}}{a} - (1-\delta)\rho\right) + \frac{1-\delta}{\delta} g_{\mu_z} \tag{48}$$

$$L_m^{p2*} = \frac{1}{\delta}\frac{1-\beta}{\beta} L_Z^{p2*} - a\rho, \tag{49}$$

where the functional forms of (48) and (49) are identical to the equilibrium conditions for g_C^* and L_m^* in the no-policy scenario. For the BGP rate of resource extraction we get

$$g_R^{p2*} = -\rho. \tag{50}$$

PROPOSITION 3. *The provision of public goods raises growth independently of the level of the consumption share devoted to productive public spending, μ_k, $k = x, z$.*

18 For the derivation of the underlying dynamic system as well as for the analytical expressions of the comparative statics in proposition 4, see Pittel and Bretschger (2009).

Proof. For the positive effect of public good provision on g_C^{p2*} see appendix B. This positive effect is independent of the level of μ_k for $g_{\mu_k} = 0$, as follows from (46) to (49). □

For the economic intuition behind this result, consider the case in which the policy maker provides public goods to the less resource-intensive sector only. In this case, the feedback effect of the provision of public goods on x-production is equivalent to a rise in x-sector productivity. This increase in productivity induces a slower increase of intermediates prices in x-production, which lowers profitability and leads to a reallocation of labour from x- to z-sector research. Owing to the increase in z-sector research, growth rises. In the x-sector, the reallocation of labour reduces research, which affects growth negatively. But in the aggregate this negative effect is overcompensated for by the productivity increase, as a result of public good provision.

For no-policy balanced growth (section 3) we showed that in equilibrium the difference in research activities between the two sectors is determined by (30). This relation remained unperturbed by the taxes and subsidies considered in the previous subsection, as neither directly affect production technologies. In the case of public good provision, however, the productivity of intermediate goods' production increases, owing to policy. The new equilibrium allocation of research efforts is determined by

$$\frac{1-\beta}{\beta}\left[\frac{g_m^{p2*}}{\delta} - \frac{g_n^{p2*}}{\alpha}\right] = \left(\frac{1-\delta}{\delta} - \frac{1-\alpha}{\alpha}\right)\rho - \frac{1-\alpha}{\alpha} g_{\mu_x} + \frac{1-\delta}{\delta} g_{\mu_z}. \tag{51}$$

A comparison of (30) and (51) shows that the gap between research in the two sectors might increase or decrease as a result of productive public spending, depending on the model calibration and policy rule.

Employing the BGP relations, (46) to (50), we get the comparative statics of the different policy instruments.

PROPOSITION 4. *As the growth effect of productive public spending is independent of the level of μ_k, $k = x, z$ (see proposition 3), a one-time increase in μ_k has no impact on long-run growth, resource extraction, and the relative sector share. If, however, the growth rate of μ_k changes, the BGP values of $\tilde{\phi}$, g_R, and g_C are affected as follows:*

$$\begin{aligned}
&\frac{d\tilde{\phi}^{p2*}}{dg_{\mu_x}} < 0, \quad \frac{dg_R^{p2*}}{dg_{\mu_x}} = 0, \quad \frac{dg_C^{p2*}}{dg_{\mu_x}} > 0,\\
&\frac{d\tilde{\phi}^{p2*}}{dg_{\mu_z}} > 0, \quad \frac{dg_R^{p2*}}{dg_{\mu_z}} = 0, \quad \frac{dg_C^{p2*}}{dg_{\mu_z}} > 0.
\end{aligned} \tag{52}$$

Proof. Taking the derivatives of $\tilde{\phi}^{p2*}$, g_R^{p2*}, and g_C^{p2*}, as given by (46) to (50), with respect to μ_k and g_{μ_k} yields either zero or the signs above. □

A rising share of public goods provision lowers profitability in the respective sector, which leads to a reallocation of labour to the other sector.[19] Owing to the increase in the research of this sector, growth rises. In the other sector research efforts decline, but in the aggregate the induced negative growth effect is again overcompensated for by continuing productivity increases.

5. Conclusions

The paper derives the long-run consequences of sectoral heterogeneity when sectors differ with respect to resource use. We have shown that sector-specific research activities and induced innovations are crucial for the dynamic behaviour of the economy. Research has to overcome the drag on growth that arises from rising resource scarcity. Moreover, resource-intensive sectors can stay competitive only if they succeed in achieving faster research growth. According to our results, markets provide sufficient incentives to investors that this indeed happens. Most important, we find that research is biased towards resource-intensive sectors and that an economy develops along a balanced growth path despite sectoral heterogeneity. By focusing on input substitution in a multi-sector setting, the paper adds to recent advances in growth economics. Given the empirical fact of large differences in resource intensity between the sectors and taking into account the predictions of increasing scarcities of natural resources, these results are relevant to predictions of the further development of living standards. Our findings are in line with empirical results on the competitiveness of energy-intensive industries under tight carbon policies, (see Demailly and Quirion 2008).

In the second part of the paper we analyzed the consequences of different policies aimed at fostering sectoral change and sustainability, that is, raising growth and lowering resource extraction. First, we considered the implications of traditional policy instruments: subsidies and taxes. It was shown that resource taxes raise growth and lower resource extraction only if the tax rate decreases over time. Labour subsidies, however, are allocation neutral in our model and do not generate any real effects. Subsidies on research activities proved to be more effective, the level of subsidy rates affecting growth positively, regardless of which sector receives the subsidies. Structural effects of policy arise, as the effect of research subsidization on market shares depends on which sector is subsidized. Subsidies to research in one sector induce the relative sector share of this sector to decrease.

Secondly, we considered the provision of productive public goods as a possible means to raise sectoral and overall growth. We showed that the introduction

19 The increase of the share is limited, of course, as $\mu_x + \mu_z < 1$ has to hold. Positive growth effects that are triggered by a rising share can therefore only be temporary.

of public goods affects growth directly when public good provision is tied to overall consumption. In this case, productivity in the sector in which public goods are provided rises and thereby affects growth as well as sector shares. Increasing the share of consumption devoted to public goods over time induces a further positive effect on growth. The rising share lowers profitability in the respective sector, which leads to a reallocation of labour towards the other sector. Owing to the increase in the research of this sector, growth rises. In the other sector, research efforts decline, but in the aggregate the induced negative growth effect is again overcompensated for by continuing productivity increases. The provision of public goods proves to be an effective tool for enhancing growth and simultaneously inducing sectoral change.

The present analysis could be extended by realistically assuming that research in the two sectors is subject to different technology risks. In this case the asymmetry in research returns could explain and justify the disproportionate investment in resource-extensive sectors, as it is required from institutional investors in some countries (see also Bretschger and Pittel 2005 on this topic). In the present set-up, the overproportional investment in the resource-extensive sector would simply be crowded out by an adjustment of non-regulated investment towards the resource-intensive sector. However, this analysis is left for future research.

Appendix A: No policy scenario

A.1. Derivation of dynamic system

To derive the equation of motion for L_Z, (17), substitute intermediates profits, (7), into the no-arbitrage condition for the patent market, (11), which gives

$$g_{V_n} = r - (1-\beta)\frac{\tilde{\phi}}{1+\tilde{\phi}}\frac{C}{V_n n} \tag{A1}$$

for sector x.

The equilibrium condition for the research sector, (10), implies $g_{V_n} = g_w - g_n$. Substituting the latter as well as (10) into (A1) and considering furthermore that from (5) we know that $((1+\tilde{\phi})/\tilde{\phi})C/w = L_X/(\alpha\beta)$ gives

$$g_w - g_n = r - \frac{1-\beta}{a}\frac{\tilde{\phi}}{1+\tilde{\phi}}\frac{C}{w} = r - \frac{1-\beta}{a\beta}\frac{L_X}{\alpha}. \tag{A2}$$

As (5) implies $g_w = g_C - g_{L_X} + (1+\tilde{\phi})^{-1} g_{\tilde{\phi}}$ and we have (16) from consumer optimization, (A2) can be rewritten as

$$g_C - g_{L_X} + \frac{1}{1+\tilde{\phi}} g_{\tilde{\phi}} - g_n = g_C + \rho - \frac{1-\beta}{a\beta}\frac{L_X}{\alpha}. \tag{A3}$$

From (5) we also know that $L_X = \alpha\tilde{\phi}L_Z/\delta$, which implies $g_{L_X} = g_{\tilde{\phi}} + g_{L_Z}$. Employing these relations as well as (8) gives, for (A3),

$$\frac{L_n}{a} = \tilde{\phi}\frac{1-\beta}{a\beta}\frac{L_Z}{\delta} - \rho - \frac{\tilde{\phi}}{1+\tilde{\phi}}g_{\tilde{\phi}} - g_{L_Z}. \tag{A4}$$

Proceeding equivalently, we get from the no-arbitrage condition of sector z, (11), that

$$\frac{L_m}{a} = \frac{1-\beta}{a\beta}\frac{L_Z}{\delta} - \rho - \frac{\tilde{\phi}}{1+\tilde{\phi}}g_{\tilde{\phi}} - g_{L_Z}. \tag{A5}$$

Adding (A4) to (A5) and rearranging gives

$$2g_{L_Z} = (1+\tilde{\phi})\frac{1-\beta}{a\beta}\frac{L_Z}{\delta} - \frac{L_m + L_n}{a} - 2\frac{\tilde{\phi}}{1+\tilde{\phi}}g_{\tilde{\phi}} - 2\rho. \tag{A6}$$

By considering that from the equilibrium condition for the labour market, (9), it follows that $L_n + L_m = 1 - (1 + \alpha\tilde{\phi}/\delta)L_Z$, we finally get (17):

$$\dot{L_Z} = \left[\frac{1}{2a\delta}\left(\alpha + \delta\tilde{\phi} + (1+\tilde{\phi})\frac{1-\beta}{\beta}\right)L_Z - \frac{1}{2a}(1+2a\rho) - \frac{\tilde{\phi}}{1+\tilde{\phi}}g_{\tilde{\phi}}\right]L_Z. \tag{A7}$$

To get an expression for $\dot{\tilde{\phi}}$ first consider that, from (3) and the production technologies in the two sectors, follows

$$\tilde{\phi}^{\frac{\nu}{\nu-1}} = \left(\frac{n}{m}\right)^{\frac{1-\beta}{\beta}}\frac{L_X^{\alpha}}{L_Z^{\delta}}\frac{R_X^{1-\alpha}}{R_Z^{1-\delta}}. \tag{A8}$$

Consideration of $L_X = (\alpha/\delta)\tilde{\phi}L_Z$ and $R_X = ((1-\alpha)/(1-\delta))\tilde{\phi}R_Z$ gives

$$\tilde{\phi}^{\frac{1}{\nu-1}} = \left(\frac{n}{m}\right)^{\frac{1-\beta}{\beta}}\left[\frac{\alpha^{\delta}}{\delta^{\delta}}\frac{(1-\alpha)^{1-\delta}}{(1-\delta)^{1-\delta}}\right]L_X^{\alpha-\delta}R_X^{\delta-\alpha}. \tag{A9}$$

Differentiating (A9) with respect to time and expressing the resulting expression in growth rates give, after substituting $g_n = L_n/a$ and $g_m = L_m/a$,

$$\frac{1}{\nu-1}g_{\tilde{\phi}} = \frac{1-\beta}{a\beta}(L_n - L_m) + (\alpha-\delta)(g_{L_X} - g_{R_X}). \tag{A10}$$

For the difference in the input of labour in the two types of R&D it follows from (A4) and (A5) that

$$L_n - L_m = (\tilde{\phi} - 1)\frac{1-\beta}{\beta}\frac{L_Z}{\delta}. \tag{A11}$$

Furthermore, (5), (14), and (16) imply that $g_{R_X} = (1+\tilde{\phi})^{-1} g_{\tilde{\phi}} - \rho$. By substituting this relation as well as (A11) into (A10), we get

$$\frac{1}{\nu-1} g_{\tilde{\phi}} = -(1-\tilde{\phi})\frac{1}{\delta a}\left(\frac{1-\beta}{\beta}\right)^2 L_Z + (\alpha-\delta)\left(g_{L_X} - \frac{\tilde{\phi}}{1+\tilde{\phi}} g_{\tilde{\phi}} + \rho\right). \quad \text{(A12)}$$

Recalling $g_{L_X} = g\tilde{\phi} + g_{L_Z}$ and (17) finally gives (18):

$$\dot{\tilde{\phi}} = \left[\frac{\left(\frac{1}{2a\delta}(\alpha-\delta)\left(\alpha+\delta\tilde{\phi}+(1+\tilde{\phi})\frac{1-\beta}{\beta}\right) - (1-\tilde{\phi})\frac{1}{\delta a}\left(\frac{1-\beta}{\beta}\right)^2\right) L_Z - \frac{1}{2a}(\alpha-\delta)}{\frac{1}{\nu-1}}\right]\tilde{\phi}. \quad \text{(A13)}$$

A.2. Balanced growth path

From (A7) and (A13) the BGP values of L_Z and $\tilde{\phi}$ can be obtained by setting $\dot{L}_Z = \dot{\tilde{\phi}} = 0$, which gives (19) and (20). Considering, furthermore, that $\dot{L}_Z = 0$, we get the BGP labour shares in the two research sectors, (24) and (25), from (A4) and (A5).

A.3. Stability

To check for the stability properties of the system, we derive the Jacobian of (17) and (18) in the proximity of the steady state, $\tilde{\phi}^*$ and L_Z^*:

$$D = \begin{pmatrix} \left.\frac{\partial\dot{\tilde{\phi}}}{\partial\tilde{\phi}}\right|_{\tilde{\phi}^*,L_Z^*} & \left.\frac{\partial\dot{\tilde{\phi}}}{\partial L_Z}\right|_{\tilde{\phi}^*,L_Z^*} \\ \left.\frac{\partial\dot{L}_Z}{\partial\tilde{\phi}}\right|_{\tilde{\phi}^*,L_Z^*} & \left.\frac{\partial\dot{L}_Z}{\partial L_Z}\right|_{\tilde{\phi}^*,L_Z^*} \end{pmatrix}. \quad \text{(A14)}$$

It can be shown that

$$\det D = -(1-\nu)\frac{\tilde{\phi}^{*2}L_Z^*}{2a^2\delta^2}\left(\frac{1-\beta}{\beta}\right)^2\left(2\frac{1-\beta}{\beta}+\alpha+\delta\right)$$

$$\operatorname{tr} D = \frac{1}{2a\delta}\frac{L_Z^*\left[(1+\tilde{\phi}^*)\left(\alpha\tilde{\phi}^*+\delta+(1+\tilde{\phi}^*)\frac{1-\beta}{\beta}\right)+(\nu-1)\tilde{\phi}^*\left(4\left(\frac{1-\beta}{\beta}\right)^2+(\alpha-\delta)^2\right)\right]}{1+\tilde{\phi}^*}, \quad \text{(A15)}$$

where $\det D < 0$. As $\det = EV_1 \cdot EV_2$ (where EV_1 and EV_2 are the eigenvalues of the system), one eigenvalue is negative for $\det < 0$. Since our system contains one jump variable and one predetermined variable, the economy is saddle-path stable.

Appendix B: Proof of proposition 3

Due to the productivity effect of public goods, labour inputs in x- and z-sector research change as follows compared to the no-policy scenario (assuming that $g_{\mu_k} = 0, k = x, z$):

$$L_n^{p2*} - L_n^* = \quad (1 - \beta + \delta\beta)\Omega \tag{B1}$$

$$L_m^{p2*} - L_m^* = -(1 - \beta + \alpha\beta)\Omega, \tag{B2}$$

where $\Omega = f(a, \alpha, \beta, \delta, \rho, g_{\mu_z}, g_{\mu_x})$. From (B1) and (B2) it follows that

$$sgn(L_n^{p2*} - L_n^*) = -sgn(L_m^{p2*} - L_m^*); \tag{B3}$$

that is, a policy-induced rise in L_n (resp. L_m) has to be accompanied by a decline of L_m (resp. L_n).

Furthermore, public good provision modifies the sectoral equilibrium growth rates (28) and (29) to

$$g_{\tilde{X}}^{p2*} = \frac{1}{\alpha}\left(\frac{1-\beta}{\beta}\frac{L_n^{p2*}}{a} - (1-\alpha)\rho\right) \tag{B4}$$

$$g_{\tilde{Z}}^{p2*} = \frac{1}{\delta}\left(\frac{1-\beta}{\beta}\frac{L_m^{p2*}}{a} - (1-\delta)\rho\right). \tag{B5}$$

If L_n^{p2*} and L_m^{p2*} were unchanged compared with the no-policy scenario, this would imply $g_{\tilde{X}}^* < g_{\tilde{X}}^{p2} < g_{\tilde{Z}}^{p2}$, where the relation $g_{\tilde{X}}^{p2} < g_{\tilde{Z}}^{p2}$ is not compatible with BGP growth (see (27)). Therefore, (B4) and (B5) together with (70) imply that a post-policy BGP with $g_{\tilde{X}} = g_{\tilde{Z}} = g_C$ can be compatible only with $L_n^{p2*} - L_n^* > 0$ and $L_m^{p2*} - L_m^* < 0$. From (B4) we see that this increase in L_n raises growth.

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