Essays on Industrial Organisation and Behavioural Economics

Rustu Duran

Hertford College

University of Oxford

A thesis submitted for the degree of

Doctor of Philosophy

Hilary 2017

I dedicate this study to the big-dreaming children with small budgets.

Acknowledgements

I want to thank my supervisors David Gill and Mark Armstrong for their generous supervision. Additionally, I thank Vincent Crawford for his excellent support. This gratitude extends to many faculty members and researchers I met during my degree.

I want to also thank Collin Raymond, Andrew Rhodes, Margaret Meyer, Christopher Wilson, Peter Eso, Yuval Heller, Miguel Ballester, Jacub Redlicki, David Ronayne, Murat Unur, Semih Uslu. Additionally, I thank the anonymous examiners for my first chapter for their fruitful comments.

Furthermore, I want to thank Turkish Ministry of National Education for their support for my education.

I would like to also thank my family and my friends for being there for me.

Abstract

This thesis is composed of three chapters each of which deals with markets with agents who are not fully rational.

The first chapter aims to understand the relationship between Word-of-Mouth marketing and traditional advertisement. With the advent of internet and social network web-sites, the cost of Word-of-Mouth marketing reduced dramatically. This chapter mainly analyses the effect of this change on the traditional advertisement efforts.

The second chapter analyses the market implications of manipulable consumers. Manipulable consumers are defined as the consumers whose beliefs about the future can be altered by advertisement decisions of firms. The equilibrium outcome is given and extensions are discussed.

The last chapter of the thesis studies the consumers who decide which firm to approach by sampling the price information offered by firms. Consumers do not conduct a rational evaluation but use one of the procedures which are motivated by axioms reflecting their limited understanding of the strategic environment. I analyse the market implications of such procedures and conduct several comparative statics.

Contents

List of Figures

Chapter 1

The Social Network Revolution and Advertisement

Abstract

The last decade was marked by an extensive use of online social marketing. This has caused a substantial decrease in the cost of Word of Mouth Marketing (WoM) effort. This reduction is expected to change the optimal mix of persuasive advertisement (advertisement efforts aiming to increase willingness to pay of consumers) and WoM for firms. This study aims to understand the nature of the optimal choice of these marketing efforts; whether they are substitutes or complements, in a differentiated goods market. The situation is modelled as a two-stage Hotelling line model. Studies in the literature, mostly empirical marketing papers, generally support the complementarity argument. My results suggest that for a monopolist these are complements to each other. For a symmetric duopoly, the optimal level of persuasive advertising does not depend on the cost of WoM. Additionally, I analyse how welfare, prices and profits are affected when the cost of WoM, the cost of persuasive advertising and the transportation cost parameter changes. Furthermore, I discuss the robustness of my main result - persuasive advertising level is independent of WoM cost (in the duopoly case).

1.1 Introduction

This study investigates whether firms increase their persuasive advertisement level when the cost of Word of Mouth Marketing (WoM) decreases. In general, I investigate the substitution or complementarity relation of persuasive advertisement and WoM efforts in horizontally differentiated good markets.[1] This question is especially important due to the explosion of the social network revolution which made WoM effort easier for firms.

The widely accepted definition of WoM is "[redacted]"(Liu, 2006, p.74). It is an indirect way of attracting new consumers compared to traditional advertising. However, it is different from traditional advertising in several aspects. It has limited reach, more credibility and less controllability (Armelini et al., 2010). Because of the modelling concerns, I disregard these differences and focus on the functional feature of positive WoM, that is WoM that increases the number of people who are aware of the existence of a product via referral.

It is not surprising to see a connection between the recent social network revolution (powered by the use of mobile devices) and WoM. On the most popular social networking website, facebook.com, over one billion people Like and Comment billions of times every day. This makes it possible for firms to use it as an important source of finding new consumers by using customised advertisements and WoM. On the Facebook website, this point is clearly stated: '[redacted] (facebook.com, nda). Of course, Facebook is not the only example, many firms

[1] When I say X is a complementary (substitute) effort to Y, I mean that when the cost of Y decreases, the use of X increases (decreases).

use Twitter, Badoo, Instagram, etc. in order to exploit this opportunity. You can see an example of a costly WoM effort conducted by a firm via an online social network in Figure 1.1 (BOOMZ, nd). It is an announcement of a firm which aims to convince people to disseminate their message by offering gifts.

One striking example of how WoM via online social networks is crucial is as follows. The English Cheesecake Factory, a company selling different desserts in various geographic areas, reported that about one-third of their new customers were, in fact, those who learnt about the firm via their Facebook friends (facebook.com, ndb). So, what makes online social networking that fast and effective?

[redacted]

Figure 1.1: An Example of a WoM via a Social Networking Website

A plausible explanation is given by social media expert, Cynthia Boris: “[redacted]

[redacted]"(Boris, 2012). The issue highlighted here is important because it shows why Facebook is a very popular and effective advertising platform for firms. It allows firms to increase the number of referrals drastically. This is the WoM side of marketing; what follows is a brief explanation of persuasive advertising.

There are two main kinds of advertising, persuasive and informative: Informative advertising refers to advertising efforts which aim to inform customers about the features or existence of a product, persuasive advertising, alternatively, aims to attract consumers via manipulating their preference (Bagwell, 2007). There are different kinds of persuasive advertising, but the generally accepted version has an upward shifting effect on the demand in the market. For example, TV commercials for sports equipment that use sports celebrities to persuade the customers that their product is the best because even the champions use them.

At this point, an important question arises: are persuasive advertisement and WoM substitutes or complements for each other? If they are substitutes (complements), we expect firms to decrease (increase) their level of persuasive advertisements after the social network revolution. This question is important because firms want to know what is the best combination of marketing effort that maximises their profits.

Assume that the cost of WoM decreases with the social network revolution. Intuitively, if the cost of WoM decreases, we may expect the use of persuasive advertisements to decrease because persuasive advertising and WoM are different ways of attracting consumers. Thus, the firm may want to give more weight to WoM instead of persuasive advertising. Conversely, the firm may also want to increase its persuasive advertisement level as people who were attracted via WoM are those who are referred to the product by another friend who may have come to know the product through persuasive advertising in the first place. For this reason, every additional consumer who is attracted via a persuasive advertisement is more valuable as these consumers' capability to disseminate information regarding the product has increased. Therefore, the firm may want to increase the level of its persuasive advertisement as well. It follows that the direction of the use of persuasive advertisements is not obvious. The study aims to understand whether persuasive and informative advertisings are complements or substitutes and how their relation depends on the competitive setting via using a Hotelling (1931) framework.

In order to answer this question, I use the Hotelling line model with monopoly and duopoly. There are two periods. In the first period, the firm(s) decide(s) on the first-period price, the level of persuasive advertisement effort and the WoM effort level. Both of these efforts are costly. The willingness to pay of the consumers increases with the use of persuasive advertisement effort. Consumers decide to buy the product if their perceived quality less price, less transportation cost is high enough. I say "perceived" as the valuation of the good is distorted by persuasive advertisement. In the second period, the firm(s) decide(s) on the second period price. The first-period consumers inform their friends about the product. The number of the friends informed is determined by the WoM effort of the firm. The second-period consumers buy the product if the price and transportation cost is not prohibitively high and the game ends.

The study has some interesting empirical implications to be tested in further studies. Firstly, the monopoly, which is located at one of the extremes of the Hotelling line, would increase its persuasive advertisement level and decrease the goods' price if the cost of WoM decreases. On the other hand, duopolists which are located at the extremes do not change the level of their persuasive advertisement, and again, the prices decrease. In this study, I also check the welfare implications and the effect of the changes in the substitutability of the products and the cost of persuasive advertisement. Furthermore, I discuss the robustness of my main result - persuasive advertising level is independent of WoM cost (in the duopoly case).

The organisation of the study is as follows. The next section is the literature review on this subject. The third section introduces the model. The fourth section deals with the monopoly case and the fifth section with the duopoly case. The sixth section is the robustness check. The last section concludes. Most of the proofs can be found in the appendix.

1.2 Literature Review

There are only a few studies that focus on the substitutability and complementarity relation of WoM and persuasive advertisement. They are mostly related to the marketing literature. The examples in the literature are generally empirical or based on arguments which do not differentiate advertisement types. Unfortunately, to my best knowledge, there is no previous theoretical study close to mine in the literature. I briefly review the studies on WoM and persuasive advertisement in differentiated goods markets. Later, I move on to the studies which deal with their substitutability-complementarity relation; the main question of this study.

The advertisement efforts are generally classified by informative and persuasive advertisement (Bagwell, 2007). The former is conducted in order to increase a particular product's familiarity. For instance, newspaper advertisements about a new restaurant in a city can be considered in this group. The latter manipulates the preferences of the consumers, which makes them choose the advertised products. The TV commercials, in which a celebrity plays a role, might imply their success is related to that product. These kinds of advertisements are examples to persuasive advertisements.

One of the first studies on persuasive advertisements in a Hotelling framework is Bloch & Manceau (1999). They take persuasive advertisement as a means of manipulating the distribution of types on the Hotelling line. They aim to observe how persuasive advertisement affects the market prices in different competitive environments and welfare and conclude that it decreases prices if there is competition.

Persuasive advertisement can also be interpreted as false advertisement: the advertisements that affect the perceived value of the product. Hattori & Higashida (2012), examine a duopoly where firms can distort the valuation of the consumers in a horizontally differentiated goods market. Rhodes & Wilson (2016) study a similar environment where the policy maker has tools to discipline the false advertisement. They determined the conditions where false advertising is an equilibrium outcome. These interpretations of false advertisement are functionally what I use in the present paper when formulating persuasive advertisement.

Another paper by Von der Fehr & Stevik (1998) deals with the puzzle of high advertisement levels when the competing products are highly substitutable. In order to solve the puzzle, they differentiate three kinds of persuasive advertisement in terms of how they affect the preferences of consumers. They show that markets marked by the kind of persuasive advertisement which emphasises product differences can experience such phenomenon (Von der Fehr & Stevik, 1998). The persuasive advertising type I focus on is the most well-known type: increasing the willingness to pay of consumers. This persuasive advertisement shifts the demand curve up. The additive formulation of persuasive advertisement I use is by Von der Fehr & Stevik (1998).

On the WoM side, Chevalier & Mayzlin (2006) show how WoM has a substantial effect on sales. They conduct an empirical study on two main online bookstores. Their study confirms that positive WoM has a significant effect on sales. Additionally, they show that negative WoM is more powerful than positive WoM. This paper elegantly shows why Internet should be taken seriously by firms when considering WoM.

A stimulating work by Galeotti (2010) studies WoM as a strategic decision. Consumers have the option to search or they can alternatively obtain information through referrals. They analyse how the equilibrium behaves with the cost of WoM or the cost of search. The interesting result they have is when the cost of the WoM reduces, the competitive pressure within the market reduces too. The reason is the incentive to search will be overwhelmed by the temptation of WoM, therefore there will be fewer consumers who collect information directly from firms. In the present paper, as the consumers who are exposed to WoM do not have access to information directly, such concern does not exist.

The first paper on the issue of the relation between traditional advertising and WoM is by Hogan et al. (2004). They measure the effect of WoM (they call it ripple effect) empirically and show that WoM may be complementary to rather than a substitute for persuasive advertisement, supporting the results of this study. However, they do not analyse different forms of advertisements and competitive environments.

Monahan (1984) shows that an increase in the power of WoM results in a decrease in the advertisement effort via a single product monopoly model. However there are two differences between this study and Monahan (1984). Firstly, Monahan (1984) does not differentiate the advertisement efforts. His formulation mainly refers to informative advertisement. Secondly and more importantly, there is no cost to WoM in his model. The power of WoM is given exogenously. In my model WoM is a costly effort and choice variable of the firm. This is more realistic since a firm can increase the level of WoM by different means such as buddy deals and group discounts. Furthermore, these efforts are costly as the firm needs to convince the buyers to disseminate the message.

An argument based study by Armelini et al. (2010) tries to understand in which circumstances the WoM and persuasive advertisement are substitutes and complements to each other. The paper is a marketing literature survey and includes many different situations. However, they do not differentiate the kinds of advertisements and competitive environments in their study. Overall they conclude that the WoM is complementary to advertising.

Similarly, Awad et al. (2004) conduct a survey study on movie consumption decisions. They investigate the complementarity and substitutability of WoM and advertising. Their study indicates that these are substitutes for each other. Another study by Dichter (1966) similarly suggests that increased advertising can reduce consumers' interest in providing WoM, and hence they may serve as substitutes. Note that my research question does not focus on consumer motives of message dissemination. Feng & Papatla (2011) investigate the reasons deeper by examining car industry and online referrals. They determine under which conditions the substitutability is a better approach. However, the possible explanations they came up with include selective consumer response and self-involvement upon which I do not focus on in this study.

The present paper also relates to the studies regarding the networks. One important result obtained in the present study is that it is possible to observe very low prices (maybe even negative prices) in the first period in order to attract more second-period consumers. This is related to the concept of "bargains followed by ripoffs" within the literature e.g. Farrell & Klemperer (2007). However, note that, bargains followed by ripoffs are generally exercised on the same consumers in the literature, while in the present study it is on different consumers.

Lastly, it is pertinent to note that persuasive advertisement manipulates preferences. At this point, one may argue that the consumer welfare should be evaluated with respect to pre-advertising valuation level, while one may also argue that it should be evaluated with respect to post-advertising level. In the literature, some authors use the first criterion (Braithwaite, 1928) while others use both, (Dixit & Norman, 1978). In this study, I calculate both ways of evaluation.

1.3 The Model

I study a Hotelling line model with the features of persuasive advertising and WoM in monopoly and duopoly. I first explain how the monopoly model works and then discuss how the model can be extended in duopoly case.

For the first period, a unit mass of consumers is located on the unit line. For the second period, the density of the consumers depends on the number of first-period consumers and the WoM effort of the firm. The monopolist is located at the left extreme.

The timing of the game is as follows:

- At the beginning of the first period, the firm chooses the first-period price (p_1), a non-negative persuasive advertising level (α) and a non-negative WoM level (φ). The persuasive advertisement effort increases the willingness to pay of consumers. The price, the intrinsic value of the product (v), transportation cost (t) and persuasive advertising level determine the demand of the first period.
- Each first-period consumer informs φ of their friends, who are lined up on the Hotelling line uniformly, about the product. That is, φ increases the density of informed friends.
- At the beginning of the second period, the firm decides on the second-period price (p_2). The second-period price and transportation cost determine the

second-period demand. The second-period consumers consist of the friends informed about the product at the beginning of the second period.

The problem of the monopolist is,

$$\max_{\{p_1,p_2,\alpha,\varphi\}} \frac{v+\alpha-p_1}{t}p_1 + \frac{v+\alpha-p_1}{t}\varphi\frac{v-p_2}{t}p_2 - \frac{\lambda\alpha^2}{8} - \frac{\mu\varphi^2}{8} \tag{1.3.1}$$

The solution method is backward maximisation. I use interior demands as constraints.

Let us see how is the problem in equation 1.3.1 set up. Recall that at the beginning of the first period the firm decides on price (p_1), persuasive advertising (α) and WoM effort level (φ). The index of the furthest consumer who buys the product (x) satisfies $v+\alpha-p_1-tx=0$, where v is the value of the product. Note how persuasive advertisement shifts the index to the right, hence increases the demand. Since all of the consumers and only those consumers between 0 and x buy the product, this equation gives the demand of the monopoly at the first period: $\frac{v+\alpha-p_1}{t}$. Therefore the first-period profit is $\frac{v+\alpha-p_1}{t}p_1$ as marginal cost is zero. This is the first element in the maximisation problem.

At the beginning of the second period, the firm decides on second-period price p_2 and each consumer of the first period informs φ of their friends about the product on the same Hotelling line. Hence, the density of the consumers on the Hotelling line is now the number of the previous period's consumers multiplied by φ, that is $\frac{v+\alpha-p_1}{t}\varphi$. Remember that these consumers value the product by v rather than $v+\alpha$ as they were not exposed to persuasive advertising. Thus, the index of the furthest consumer who buys the product (x) satisfies $v-p_2-tx=0$. Therefore the second period demand is $\frac{v+\alpha-p_1}{t}\varphi\frac{v-p_2}{t}$ which gives the second period profit of $\frac{v+\alpha-p_1}{t}\varphi\frac{v-p_2}{t}p_2$. This is the second element in the maximisation problem.

The last two elements represent the costs of persuasive advertisement and WoM efforts.

I now scrutinise my assumptions.

The persuasive advertising works as follows. In the usual Hotelling line framework, the willingness to pay of the consumer located at x is (for the firm located at the left extreme) $max(v - tx, 0)$ where v is the value of the product and t is the transportation cost (consumers have an outside option of value zero). Persuasive advertising is defined as the advertising efforts aiming to increase the willingness to pay. I take a position for an additive function for that (in the robustness discussion section, I also employ another alternative), that is after α level of persuasive advertising, the willingness to pay of the consumer located at x becomes $max(v + \alpha - tx, 0)$ only for the consumers at that period. I assume that there is no budget constraint of consumers. Note that persuasive advertising effort is different from quality-enhancing activities, whose effect would be permanent and last even at the second period, rather than being ephemeral like persuasive advertisements.

The WoM works as follows: WoM effort is the effort aiming to persuade first period buyers to tell their friends about the product. Each consumer who bought the product at the first period informs φ of her friends about the product who are uniformly lined up on the unit Hotelling line of the second period. Note that its function is increasing the density of consumers. These friends were not aware of the product in the first period. Here the underlying assumption is that the first-period consumers have enough friends: φ is never greater than the number of the friends one has. Note that the friends were not exposed to persuasive advertising. Hence their willingness to pay is $max(v - tx, 0)$. Another important assumption is that the first-period consumers have no mutual friends. This is a restrictive assumption, and the main reason is the tractability.[2]

[2]One function of this assumption is that it helps us gathering all WoM effects on only one

Finally, assume that the set of first-period consumers and the set of friends referred at the second period are mutually exclusive. This assumption removes many complications regarding mixed valuations at the second period. Otherwise, at the second period, there would be some consumers valuing the product as $max(v + \alpha - tx, 0)$ and some others $max(v - tx, 0)$.

Both persuasive advertising and WoM are the efforts which increase the revenue of the firm. However, they are not costless. The persuasive advertising is costly because the widely known examples such as TV advertisements featuring celebrities are all costly. The WoM is also costly. Remember WoM effort is the effort aiming to persuade first-period buyers to tell their friends about the product. Widely known examples such as buddy deals and friend-subscription special offers are all costly attempts that try to convince the consumers to spread the information about the product. I assumed their costs are separable and quadratic.[3] I solve the model for interior solutions and check some possibly interesting corner solutions.

For the duopoly, the model extends naturally. The duopolists are located at the extremes of the Hotelling line. The solution method is backward induction, and the equilibrium notion is Subgame Perfect Nash Equilibrium. The details can be seen in the duopoly section.

---

parameter. Otherwise, one needs to define a parameter for the proportion of mutual friends. This assumption also removes the strategic considerations at the second round for duopolists which constitute the main tractability easing. Otherwise, there would be three different sets of consumers in the second period: the ones who only know only one of the firms and the ones who know both firms. The latter group would cause firms to compete at the second round which in turn gives high degree polynomials in the solution. It is not a realistic assumption if the two products are already very popular like Adidas and Nike. However, if you consider two small confectionery shops in a city, the proportion of second-period consumers who know both products by referral would be negligible.

[3]This is a technical assumption for the second order conditions but also intuitive. For the persuasive advertising, if the cost was linear and unit cost is low enough the profit of the firm would be unbounded. If the unit cost were high, then there would not be any persuasive advertising. For the WoM effort, it is plausible to think that the firm would need to exert more than twice the effort, in order to convince the first-period consumers to inform their 10 friends, compared with making the first-period consumers inform only five friends. Lastly, I assumed the marginal cost of producing one unit of product is zero.

Lastly, I explain the welfare notions I use.

There are two possible points of view to the first-period consumer's surplus. The first one assumes that the consumer enjoys $v + \alpha - p_1 - tx$ which is a usual understanding of welfare. I call the welfare calculations which take this position for the first-period consumers as the primary welfare criterion. The second view is that the first-period consumers enjoy $v - p_1 - tx$ of consumer surplus. Note that in this second view, even though the agents decide whether or not to buy the product according to the following inequality $v + \alpha - p_1 - tx \geq 0$ they may experience a loss. I call the welfare calculations which take this position for the first-period consumers as the secondary welfare criterion. Note that, these welfare notions give the same results for the second-period consumers as there is no persuasive advertisement for them. Both views are defensible. Thus, I evaluate changes in environments according to both criteria.

Let W_i^P and W_i^S denote the welfare of the consumers at the i'th period for primary and secondary welfare criteria respectively. Also, let W^P and W^S denote the welfare of the consumers for primary and secondary welfare criteria respectively. Note that $W_2^P$and W_2^S are equal as the only difference between these two criteria is how they define the first-period consumer welfare. Hence, for the monopoly case,

$$
\begin{aligned}
W_1^P &= \int_0^{\frac{v+\alpha-p_1}{t}} (v + \alpha - p_1 - tx)dx \\
W_1^S &= \int_0^{\frac{v+\alpha-p_1}{t}} (v - p_1 - tx)dx \\
W_2^S &= W_2^P = \frac{v + \alpha - p_1}{t}\varphi \int_0^{\frac{v-p_2}{t}} (v - p_2 - tx)dx
\end{aligned}
$$

1.4 Monopoly

Recall the problem of the monopolist is as follows:

$$max_{\{p_1,p_2,\alpha,\varphi\}}\frac{v+\alpha-p_1}{t}p_1+\frac{v+\alpha-p_1}{t}\varphi\frac{v-p_2}{t}p_2-\frac{\lambda\alpha^2}{8}-\frac{\mu\varphi^2}{8}$$

$$subject\ to, 0\leq\frac{v+\alpha-p_1}{t}\leq 1, 0\leq\frac{v-p_2}{t}\leq 1$$

These constraints make sure the demands do not exceed the unit line. The reason for these two constraints is to have suitable analogies when discussing the implications at monopoly and duopoly.

The solution to this problem reveals that the interior solution is

$$p_1=\frac{v\lambda(4\mu t^3-v^4)}{8\lambda\mu t^3-\lambda v^4-16\mu t^2}, \alpha=\frac{16v\mu t^2}{8\lambda\mu t^3-\lambda v^4-16\mu t^2}, \varphi=\frac{4\lambda t v^3}{8\lambda\mu t^3-\lambda v^4-16\mu t^2}$$

given $8\mu t^2(\lambda t-2)-\lambda v^4>4\lambda v\mu t^2$; $v<2t$ and $8\mu t^2(\lambda t-2)>\lambda v^4$ (See Appendix for a complete proof).

The first and second inequalities ensure that the first and second-period demands, respectively, are in the interior. The last is due to Hessian Matrix. The first inequality above implies the last one. The reason I give them separately is to state them explicitly when I have corner solutions.

One interesting implication is that for some parameters, especially when the cost of WoM is very low, the first-period price can be negative. This is because, the return to the second period becomes very large. The firm wants to exploit this high return by reducing the first-period prices in order to attract more consumers in the second period.

Below, I present the results with explanations. The first proposition examines how a decrease in WoM cost affects the optimal solution and welfare. The second proposition investigates how a decrease in persuasive advertisement cost would affect the optimal solution and welfare. Lastly, I show that even for the corner solutions, the main results continue to hold.

1.4.1 The effects of a decrease in WoM effort cost

Proposition 1.4.1. *A decrease in the cost of WoM*

- Decreases optimal level of the first-period price;
- Increases both persuasive advertisement and WoM effort level;
- Increases consumer welfare unambiguously for the primary welfare criterion. For the secondary criterion, it increases if the cost of persuasive advertisement and/or the cost of transportation is large enough (see Appendix for the proof).

It is not surprising that the WoM level increases. Knowing this, the firm understands each additional customer at the first period is more valuable than before in the sense that they will inform more friends about the product. This creates a strong incentive to increase the level of persuasive advertisement because an increase in persuasive advertisement means an increase in demand. Increase in demand means there will be more first-period consumers reporting to their friends. This incentive is so strong that the firm does not exploit the increase in the persuasive advertisement by increasing the first-period prices. Otherwise, increase in prices would crowd out the effect, at least partially, of the persuasive advertisement's attraction of the consumers. What is observed is that the firm, on the contrary, decrease price in order to attract even more consumers in the second period.

It is useful to illustrate the trade-offs regarding the effect of WoM cost reduction on the optimal level of advertising by conducting comparative statics. I check the total derivative of persuasive advertising with respect to the cost of WoM effort. The optimal persuasive advertising level is an implicit function of optimal first-period price (I make a slight abuse of notation; I use p instead of p_1 for the monopoly section) and WoM effort level (φ). Hence:

$$\frac{d\alpha(\varphi, p)}{d\mu} = \frac{\partial \alpha}{\partial \varphi}\frac{d\varphi}{d\mu} + \frac{\partial \alpha}{\partial p}\frac{dp}{d\mu}.$$

A decrease in the cost of WoM effort increases the optimal level of WoM effort and decreases the prices: -$\frac{d\varphi}{d\mu} > 0, -\frac{dp}{d\mu} < 0$. For the partial derivatives I can employ one of the first order conditions. Take the one with respect to price: $4t(v + \alpha - p) - 4tp - \varphi v^2 = 0$. Therefore, $\frac{\partial \alpha}{\partial \varphi} > 0$, $\frac{\partial \alpha}{\partial p} > 0$.

These calculations imply the following interpretation. When the cost of WoM decreases, there are two opposing effects on the persuasive advertising: The first is via the WoM channel. The increase in WoM makes an increase in persuasive advertising tempting. On the other hand, the decrease in prices makes the increase in persuasive advertising less useful at the first period, and this drags the persuasive advertising level down. What I observe is that the first effect dominates and the level of advertising increases. Note that the first part of the total derivative works as a propagation effect. It drives the level of persuasive advertising up because each additional consumer at the first period is more powerful. However, the second part works as a substitution effect. It drives the persuasive advertising level down because the firm is tempted to give more weight on the second round. This effect shows itself via price.

For the primary welfare criterion, as prices decrease and persuasive advertisement effort increases the welfare of each existing customer increases, and new customers decide to buy. The second-period consumers' welfare also increases just because of the increased density of the Hotelling line. Therefore, the total consumer welfare will increase unambiguously.

For the secondary welfare criterion, the increase in persuasive advertisement is harmful. It is because there are even more people coming from far while they should not come. This is because the consumers decide on buying according to $v + \alpha - p_1 - tx \geq 0$ but they enjoy only $v - p_1 - tx \geq 0$, and α is increased. Here I understand that even though there is a price decrease which is supposed to increase welfare to some extent, it may not be enough because the fooled consumers' welfare reduction may be enormous. What determines whether or not the welfare increase resulting from a price decrease outweighs the welfare decrease because of fooled consumers is λt (recall λ is the cost of persuasive advertisement). If λt is large enough, the first-period consumers are happier in total, but of course not each individual. Why is λt that important? For a given t, if λ is high enough the firm will not be able to increase its advertisement level considerably. Hence there will be not many new fooled consumers.[4] Also for a given λ, if t is very large then the firm's effort to attract new consumers will be very limited. As a result, there will not be many new fooled consumers. The evaluation of the second period is the same as the primary welfare criterion because both these criteria differ only when evaluating the first-period consumers. Therefore, the first-period consumers' welfare increases if λt is above a benchmark. The second-period consumers' welfare increases unambiguously. Hence the total consumer welfare increases if λt is above some other lower benchmark.

Now I examine how a decrease in persuasive advertisement affects the optimal

[4] When I say a consumer with the index of x is fooled if $v + \alpha - p_1 - tx \geq 0$ *but* $v - p_1 - tx < 0$.

solution and the welfare. Then I will briefly compare the results.

1.4.2 The effects of a decrease in persuasive advertising effort cost

Proposition 1.4.2. *A decrease in the cost of persuasive advertising*

- Increases optimal level of first-period price, WoM effort and persuasive advertisement effort;
- Increases consumer welfare unambiguously for the primary welfare criterion. For the secondary criterion, it increases if v is large, t and μ (cost of WoM) is low enough (See Appendix for the proof).

It is not unreasonable to observe an increase in persuasive advertisement effort. This creates an incentive to exploit the increase in demand due to the increase in persuasive advertisement. In fact, it does increase the price tempted by this incentive. However, the price does not increase that much so that the total number of the first-period consumers increases. This means an increase in WoM effort is much more valuable now because there are more people to propagate the information about the product than before. For this reason, the WoM effort also increases.

It is useful to illustrate the trade-offs regarding the effect of persuasive advertisement cost reduction on the optimal level of WoM by conducting comparative statics. I check the total derivative of WoM with respect to the cost of persuasive advertising. The optimal WoM level is an implicit function of optimal first-period price (p) and persuasive advertising level (α). Hence:

$$\frac{d\varphi(\alpha,p)}{d\lambda} = \frac{\partial\varphi}{\partial\alpha}\frac{d\alpha}{d\lambda} + \frac{\partial\varphi}{\partial p}\frac{dp}{d\lambda}$$

A decrease in the cost of WoM increases both the optimal level of persuasive advertising effort and the first-period price: -$\frac{d\alpha}{d\lambda} > 0, -\frac{dp}{d\lambda} > 0$. For the partial derivatives I can employ one of the first order conditions and obtain $\frac{\partial \varphi}{\partial \alpha} > 0$, $\frac{\partial \varphi}{\partial p} < 0$.

These calculations imply the following interpretation. When the cost of persuasive advertising decreases, there are two opposing effects on the WoM: the first is via the persuasive advertising channel. The increase in persuasive advertising makes an increase in WoM advertising attractive because the density of second-period consumers increases. Conversely, the increase in prices makes the increase in WoM less useful at the first period because higher price drags the second-round consumer density down. Hence this effect drags the WoM level down. What is observed is the first effect dominates and the level of WoM increases. Note that the first part of the total derivative works as a propagation effect: it drives the level of WoM up because an increase in persuasive advertising increases the return to WoM. However, the second part works as a substitution effect. It drives the WoM level down because the firm is tempted to give more weight to the first period. This effect shows itself via price.

For the primary welfare criterion, there are two opposing effects in the first period. Bad news due to higher prices and good news due to increased persuasive advertisement. Good news outweighs the bad news. This is because even though prices increase, it is not drastic because the firm also wants to attract more consumers at the second round. The density of consumers at the second round increases, therefore their welfare also increases. Hence the total consumer welfare increases.

For the secondary welfare criterion, bad news outweighs good because there is no good news at all. Remember that the persuasive advertisement level does not enter as pleasure in their welfare assessment. However, the second round consumers are better off as it is the same as the primary welfare criterion. What is obtained is that if μ (the cost of WoM effort) is low given other parameters, the total consumer welfare increases. It is quite clear that if μ is very low, then the second round consumers are more crowded and as a result, the increase in their welfare outweighs the decrease in the welfare of the first round consumers. Additionally, if v is high enough the utility per consumer increases hence when it is higher, it may suppress the welfare decrease of the first period. If t is low enough the welfare per individual at the second period increases. On the other hand, the lower the t, the more people suffering from being fooled at the first period. Note that, a smaller t would harm some of the first round potential consumers but it would benefit to the second-period consumers. What is obtained is that if t is low enough, the total welfare increases.

Comparing these two results reveals that the effect of reducing the costs of these two different means of marketing have different implications on prices and also on welfare. If WoM gets cheaper, the first-period price decreases but it increases when persuasive advertising becomes cheaper. With respect to the primary welfare criterion the total welfare increases in both cases. However, with respect to the secondary welfare criterion, it may decrease depending on the parameter values. The second-period consumer surplus always increases for both of the criteria.

1.4.3 Corner Solutions

The analysis above is conducted for interior solutions. One may wonder whether these results, particularly what happens if the cost of WoM decreases, hold even though I have corner solutions. When I say corner solutions I mean for at least one period, the furthest consumer strictly wants to buy the product. Note that the corner solutions concerning the case that the closest consumer does not want to buy the product are not realistic. I focus on only the effect of WoM cost reduction on the persuasive advertisement level. Therefore we have three possible corner solutions.

The first one is that at the first period the problem has an interior solution, but for the second period it has a corner solution.[5] In this case, one may think that when the cost of WoM decreases, the firm, as it can already attract all consumers at the second period may want to decrease the spending on persuasive advertising. However, the incentives for the firm for increasing the persuasive advertising level still exist. The reason is that for a given second-period profit the firm still faces the following trade-off: it can increase the level of persuasive advertisement and attract more consumers in the first period. These consumers will inform more of their friends about the product as the WoM level increases. The power of WoM, in my model, is not about informing friends who are far from the store. It is about increasing the density of the people who are aware of the product and who are lined up on the Hotelling line uniformly. This effect is powerful enough to suppress the temptation of the firm to decrease the persuasive advertisement level as a substitution effort. Hence, my result still holds in this framework.

The second one is that at the first period the problem has a corner solution, but for the second period it has an interior solution.[6] Assume a decrease in the cost of WoM. Noting that the first-period price decreases, analyse the situation. Does the firm have an incentive to increase the level of persuasive advertisement? No, because all consumers on the Hotelling line are already informed, so no one is left. Thus, there is no incentive to increase it. Does the firm have an incentive to decrease the level of persuasive advertisement? No again, because the second period is now even more profitable as the level of WoM is increased. Therefore, the change in cost of the WoM has no effect on the persuasive advertisement level.

The last possible scenario is that the problem has corner solutions for each period.[7] The same reasoning as the previous case about incentives on both sides works here

[5]This happens when $v > 2t$ and $8\mu t^2(\lambda t - 2) - \lambda v^4 \geq 4\lambda v \mu t^2$.
[6]This happens when $0 \leq v \leq 2t$ and $8\mu t^2(\lambda t - 2) - \lambda v^4 < 4\lambda v \mu t^2$.
[7]This happens when $v > 2t$ and $8\mu t^2(\lambda t - 2) - \lambda v^4 < 4\lambda v \mu t^2$.

as well. Hence the level of persuasive advertisement does not change.

This corner solution analysis indicates that the result which says the level of persuasive advertisement does not decrease when the cost of WoM decreases holds in all cases.

1.5 Duopoly

The second case I examine is the competitive environment with symmetric duopolists. Note that, these duopolists compete in the first period which then gives each firm a local monopoly in the second period. At the left/right extreme, the firm 1/2 is located respectively. The decisions of firms are named with their indices: $1, 2$. That is: first period decisions are α_i, p_i, and φ_i . The second period price of firm i is p_2^i.

At the beginning of the first period, firms decide on prices, persuasive advertising levels and WoM effort levels. Assuming the market is covered, the index of the indifferent consumer on the line is determined by the following equation: $v + \alpha_1 - p_1 - tx = v + \alpha_2 - p_2 - t(1 - x)$. When I say the market is covered, in a competitive market, I mean every consumer is served. In the second period, if all consumers are served I call it the corner solution. As all of the consumers and only those consumers between the left extreme and x will buy from 1, x denotes the demand of firm 1. By the same token $1 - x$ denotes the demand of firm 2. The solution of the above equality gives the following: the demand of firm i is $\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}$. Therefore, the first period profit of the firm i is: $\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}p_i$ as marginal cost is zero.

At the beginning of the second period the firms decide on the second period prices (p_i^2) and each consumer of the first period informs φ_i of their friends about the product where i represents the firm from which they bought their product at the first period. Note that the no mutual friends assumption is on work: the second period decisions are not strategic per se. The density of the consumers on the Hotelling line now is the number of the previous period's consumers multiplied by φ_i : $\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}\varphi_i$. Remember that these consumers value the product by v rather than $v+\alpha$ as they were not exposed to persuasive advertising. Hence the index of the furthest consumer who buys the product (x) satisfies $v-p_2^1-tx=0$ and $v-p_2^2-t(1-x)=0$ for firm 1 and 2 respectively. Therefore the second-period demand is $\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}\varphi_i\frac{v-p_2^i}{t}$ which gives the second-period profit of $\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}\varphi_i\frac{v-p_2^i}{t}p_2^i$.

The cost of the persuasive advertising effort is $\frac{\lambda\alpha_i^2}{8}$ and the WoM effort is $\frac{\mu\varphi_i^2}{8}$. This discussion gives the following constrained maximisation problem of the firm:

$$\max_{(p_i,\alpha_i,\varphi_i)}\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}(p_i+\varphi_i\frac{v-p_2^i}{t}p_2^i)-\frac{\mu\varphi_i^2}{8}-\frac{\lambda\alpha_i^2}{8} \quad (1.5.1)$$

where $0\leq\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}\leq 1$ and $0\leq\frac{v-p_2^i}{t}\leq 1$.

The conditions make sure that the solution is interior and the formulation of the maximisation problem assumes the market being covered. The interior, market covering symmetric equilibrium solution is

$$p_i=t-\frac{v^4}{8\mu t^2},\alpha_i=\frac{2}{\lambda}\text{ and }\varphi_i=\frac{v^2}{2\mu t}$$

If the market was not covered, I would obtain the same result with the monopoly case, therefore, it is not an interesting case. In the event that the problem has a corner solution in the second period, the results do not change (See Appendix for a complete proof).

1.5.1 The effects of a decrease in WoM effort cost

Proposition 1.5.1. *A decrease in the cost of WoM*

- Decreases equilibrium level of the first-period price;
- Increases WoM effort level but does not affect the persuasive advertising level;
- Decreases the profit. The consumer welfare increases unambiguously for both primary and secondary welfare criteria (See Appendix for a proof).

The WoM effort level increases unsurprisingly. Hence, the return to each additional consumer at the first period is more valuable right now as they will talk to more friends about the product. This creates an incentive to reduce the price. On the other hand, the same incentive to increase the persuasive advertising level does not work. This is because, in contrast to the monopoly case, for a given price there is a competitive pressure. Increasing persuasive advertisement level does not increase the number of first-period consumers as much as the monopoly case.

It is useful to illustrate the trade-offs of the effect of WoM cost reduction on the optimal level of advertising by conducting comparative statics. I check the total derivative of persuasive advertising with respect to the cost of WoM effort. The optimal persuasive advertising level is an implicit function of optimal first-period prices (p_1, p_2), WoM effort levels (φ_1, φ_2) and the opponent's persuasive advertising level decision α_j. Without loss of generality let us check for the first firm. That is:

$$\frac{d\alpha_1(\varphi_1, \varphi_2, p_1, p_2, \alpha_2)}{d\mu} = \frac{\partial\alpha_1}{\partial\varphi_1}\frac{d\varphi_1}{d\mu} + \frac{\partial\alpha_1}{\partial\varphi_2}\frac{d\varphi_2}{d\mu} + \frac{\partial\alpha_1}{\partial p_1}\frac{dp_1}{d\mu} + \frac{\partial\alpha_1}{\partial p_2}\frac{dp_2}{d\mu} + \frac{\partial\alpha_1}{\partial\alpha_2}\frac{d\alpha_2}{d\mu}$$

Using first order conditions and the proposition's results we know -$\frac{d\varphi_1}{d\mu} > 0$,-$\frac{d\varphi_2}{d\mu} > 0$,-

$\frac{dp_1}{d\mu} < 0$,-$\frac{dp_2}{d\mu} < 0$,-$\frac{d\alpha_2}{d\mu} = 0$, $\frac{\partial\alpha_1}{\partial\varphi_1} > 0$, $\frac{\partial\alpha_1}{\partial\varphi_2} = 0$, $\frac{\partial\alpha_1}{\partial p_1} > 0$, $\frac{\partial\alpha_1}{\partial p_2} < 0$, $\frac{\partial\alpha_1}{\partial\alpha_2} > 0$. The first differentiation is positive, the second one is zero, the third one is negative, the fourth one is positive, the last one is zero.

So we effectively have this:

$$\frac{d\alpha_1(\varphi_1, \varphi_2, p_1, p_2, \alpha_2)}{d\mu} = \frac{\partial\alpha_1}{\partial\varphi_1}\frac{d\varphi_1}{d\mu} + \frac{\partial\alpha_1}{\partial p_1}\frac{dp_1}{d\mu} + \frac{\partial\alpha_1}{\partial p_2}\frac{dp_2}{d\mu}$$

The first element of the right-hand side represents the effect via WoM. With a reduction in the cost of WoM, WoM effort increases and hence the return to persuasive advertising increases as each additional first-period consumer is more important. The second one is as prices decrease, the use of persuasive advertising at the first round decreases and this drags its level down. As you see these first two effects are the same as the monopoly case. Here, additionally, there is the last component which reflects the strategic concerns. As the price of your opponent decreases, the firm reacts by decreasing its persuasive advertising level. This is because as the opponent decreases its price, the return to the first period decreases as the market gets more competitive. These three effects cancel each other, and as a result, there is no change in the symmetric equilibrium level of persuasive advertising after a reduction in WoM cost.

What happens to the profit? The profit extracted from the second-period consumers increases due to the increase in WoM level keeping in mind that the demand at the first period has not changed for a certain firm. The profit of the first period decreases as the prices decrease, but demand per firm does not change. This second effect dominates the first one. This result has a spirit of prisoner's dilemma. The competitors would be better of if both would keep their first-period prices higher, however there is a unilateral incentive to reduce it drastically. Therefore they both end up decreasing their first-period prices and hence reducing their profit.

As the persuasive advertising level does not change, the welfare notions have the same direction here. The second-period consumers are better off for sure as the WoM effort increased while the demand per firm does not change. For the first-period consumers, the only change is the decrease in prices which increases their welfare as well. As a result, the total consumer welfare increases.

1.5.2 The effects of a decrease in persuasive advertising effort cost

Proposition 1.5.2. *A decrease in the cost of persuasive advertising*

- Increases optimal level of persuasive advertisement effort and welfare with respect to primary welfare criterion;
- The profit decreases;
- Everything else stays the same (See Appendix for a proof.)

When the cost of persuasive advertisement decreases, the firm increases the level of persuasive advertisement to attract more consumers. However, the other firm reacts the same way, and hence for given symmetric prices, the demand does not change. Therefore, the WoM's return does not change as well, so WoM does not change. Given this, decreasing/increasing price has no additional advantage compared to status quo. Therefore prices do not change as well.

It is useful to illustrate the trade-offs of the effect of persuasive advertising cost reduction on the optimal level of advertising by conducting comparative statics. I check the total derivative of WoM with respect to the cost of persuasive advertising. The optimal WoM level is an implicit function of optimal first-period prices (p_1, p_2), WoM effort levels (α_1, α_2) and the opponent's WoM level decision φ_2. Without loss of generality let us check for the first firm. Hence:

$$\frac{d\varphi_1(p_1, p_2, \alpha_1, \alpha_2, \varphi_2)}{d\lambda} = \frac{\partial\varphi_1}{\partial p_1}\frac{dp_1}{d\lambda} + \frac{\partial\varphi_1}{\partial p_2}\frac{dp_2}{d\lambda} + \frac{\partial\varphi_1}{\partial\alpha_1}\frac{d\alpha_1}{d\lambda} + \frac{\partial\varphi_1}{\partial\alpha_2}\frac{d\alpha_2}{d\lambda} + \frac{\partial\varphi_1}{\partial\varphi_2}\frac{d\varphi_2}{d\lambda}$$

Using first order conditions and proposition's results I know -$\frac{dp_1}{d\lambda} = 0$, -$\frac{dp_2}{d\lambda} = 0$, -$\frac{d\alpha_1}{d\lambda} > 0$, -$\frac{d\alpha_2}{d\lambda} > 0$, -$\frac{d\varphi_2}{d\lambda} = 0$, $\frac{\partial\varphi_1}{\partial p_1} < 0$, $\frac{\partial\varphi_1}{\partial p_2} > 0$, $\frac{\partial\varphi_1}{\partial\alpha_1} > 0$, $\frac{\partial\varphi_1}{\partial\alpha_2} < 0$, $\frac{\partial\varphi_1}{\partial\varphi_2} = 0$. The first, the second and the last one is zero. So I effectively have this:

$$\frac{d\varphi_1(p_1, p_2, \alpha_1, \alpha_2, \varphi_2)}{d\lambda} = \frac{\partial\varphi_1}{\partial\alpha_1}\frac{d\alpha_1}{d\lambda} + \frac{\partial\varphi_1}{\partial\alpha_2}\frac{d\alpha_2}{d\lambda}$$

The first item here drags the WoM up. When the cost of persuasive advertisement decreases, the firm increases its persuasive advertisement level. Thus, the density of the second-period Hotelling line increases. For this reason, the firm is attracted to increase its WoM level to take the advantage of this increase. On the other hand, the opponents' persuasive advertising level also increases. This makes the first period more competitive and reduces the density of the second period. What I observe here is that these effects cancel each other.

The profit, contrarily, decreases. The increased effort on persuasive advertisement causes exerting more effort to the extent that cost of effort is also higher than before, but there is no return at all.

With respect to primary welfare criterion, the welfare of the first-period consumers increases as price does not change but the level of advertising increases. For the second-period consumers, there is no change as WoM effort and the first round individual demands do not change. Hence the total consumer welfare increases.

For the secondary welfare criterion, the welfare of the first round consumers does not change as the price does not change as well. The number of the fooled consumers also does not change. For the second round consumers one can repeat the same argument for primary welfare criterion. As a result, the total consumer welfare does not change.

1.5.3 The effects of an increase in substitutability

Proposition 1.5.3. *If the horizontal differentiation reduces, that is if t decreases, then*

- The first-period prices and profits of the firms decrease;
- WoM effort and welfare increases unambiguously for both welfare criteria;
- The level of persuasive advertisements does not change (See Appendix for a proof).

At the second period, a decrease in t means the profit from each friend increases as their willingness to pay increases. Hence the return to WoM increases which causes WoM effort increases, given symmetric demands. As the second period's return increases, the firms will try to take an advantage by decreasing their first-period prices hoping that their individual demands at the first period increase.

The return to change in persuasive advertising does not change. The reason is that two forces balance each other. The first one is the increase in the return to each additional consumer due to second-period WoM effort. This drags persuasive advertisement up. Contrarily, the decrease in prices drags it down as there will be less gain in increasing persuasive advertisement in the first period.

The second-period profit increases. On the other hand, first-period prices decrease while persuasive advertisement levels do not change. The second effect outweighs the first. Hence the first-period profit decreases.

As the persuasive advertisement levels do not change, the direction of each welfare notion is the same. For the second-period consumers the surplus increases because for each referral, more friend-consumers will buy the product due to the decrease in t. For the first-period consumers, the prices decrease, and thus the welfare increases as there is no other change. So the consumer welfare increases unambiguously.

1.5.4 Comparing Monopoly and Duopoly Prices and Profits

Remember that for interior solutions, the prices for monopolist and duopolists respectively $\frac{v\lambda(4\mu t^3 - v^4)}{8\lambda\mu t^3 - \lambda v^4 - 16\mu t^2}$ and $t - \frac{v^4}{8\mu t^2}$. The very first thing to notice is that, in interior solutions it is possible to see negative prices. This is because the firm may want to use lower prices to attract more consumers at the first period and use it to attract many more consumers at the second period. Additionally, for different parametrisations, monopoly might have higher or lower prices than the duopoly model just like regular Hotelling models.

For the profit comparison, the monopoly does not earn less for the same parameter values. This is because the monopoly is not affected by the competitive pressure. On the other hand, if there is a corner solution for duopolists for the first-period demands then, (for the parameters satisfying the interior solution for both frameworks) both frameworks generate the same profit per firm as the duopolists will be monopolies of their niche.

1.5.5 Summary of the Results

The following chart summarises the results (See Figure 1.2). The chart is read as follows. The first column is the list of the criteria we want to observe what happens if a parameter changes. The second and third columns indicate which parameter changes and what happens to the corresponding criteria indicated in the first column. The first table summarises the results for a decrease in WoM cost and the second table does the same thing for a decrease in persuasive advertisement cost and transportation cost. * means there is no change.

For instance, assume we want to see how a decrease in WoM cost affects the welfare of the consumers at the first period with respect to secondary welfare criterion in the monopoly case. We look at the second column of the first table with the heading of WoM cost. Then we pick the row of "Pd 1 Consumer Welfare (Secondary)". Next, we look at the intersection of that column and row. As we look at the case of monopoly, we check the first entry. What we see is $(if\ \lambda\ is\ high)+$.Therefore the welfare of the first-period consumers "increases" with respect to secondary welfare criterion when we decrease WoM Cost if λt is high.

1.6 Robustness Discussion

This section consists of a discussion of my main results' robustness, in particular, those of the duopoly solution. My result says the change in the cost of WoM does not affect the persuasive advertisement level. I try to understand how changes in some particular assumptions reflect the market outcome.

- *No price discrimination*

One may argue that price discrimination may not be possible to exercise for most of the markets. This is a fair criticism. The present model reflects the early-adapter and late-adapter consumer framework. It is hard to solve the model in closed form.

	When WoM Cost Decreases	
	Monopoly	*Symmetric Duopoly*
Persuasive ad level	+	*
Pd1 Price	−	−
WoM level	+	+
Profit	+	−
Pd 1 Consumer Welfare (Primary)	+	+
Pd 2 Consumer Welfare (Primary)	+	+
Total Consumer Welfare (Primary)	+	+
Pd 1 Consumer Welfare (Secondary)	(if λt high) +	+
Pd 2 Consumer Welfare (Secondary)	+	+
Total Consumer Welfare (Secondary)	(if λt is high) +	+

	When Persuasive Ads Cost Decreaes		When Transportation Cost Decreases
	Monopoly	Symmetric Duopoly	Symmetric Duopoly
Persuasive ad level	+	+	*
Pd1 Price	+	*	−
WoM level	+	*	+
Profit	+	−	−
Pd 1 Consumer Welfare (Primary)	+	−	+
Pd 2 Consumer Welfare (Primary)	+	−	+
Total Consumer Welfare (Primary)	+	−	+
Pd 1 Consumer Welfare (Secondary)	−	*	+
Pd 2 Consumer Welfare (Secondary)	+	*	+
Total Consumer Welfare (Secondary)	(if v is high) +	*	+

Figure 1.2: Summary of the Results

However, it is possible to make an informed guess about this situation.

If there is no room for price discrimination, and assuming the market is covered at the first period, the surplus which can be extracted from consumers coming via WoM will be reduced. This will affect the model in two ways. Firstly, the use of persuasive advertisement will decrease. Secondly, the use of WoM will decrease. These effects reinforce each other.

What will happen if the cost of WoM decreases? If the cost of WoM decreases, there will be an incentive to increase the level of WoM. The increase in the level of WoM will create an incentive to increase the level of persuasive advertisement. On the other hand, as usual, there will be incentive to decrease the level of persuasive advertisement as well because the firm might want to devote most of its effort to

WoM. In the main model, these two effects cancel each other.

In the case of no-price discrimination, both of these affects are still there, but they are now weakened. However note that, the first period-price, which is affected by the persuasive advertisement decision, will be reflected in the second-period price. So then, the second-period price will be a function of the persuasive advertisement level. In the main model, this was not the case. Therefore, there is one more incentive to increase the persuasive advertisement level. I conjecture, these effects together might dominate and hence the persuasive advertisement level might increase. However, keep in mind that if some important parameters, t or μ, are high enough, then the WoM market in the second period might be very small in size, and hence the additional effect might be very weak.

- *Perfect split of consumers*

In the main model, it is assumed that there is a perfect split of consumers in terms of the advertisement they are exposed to. It is alternatively possible to think that some consumers might be exposed to both kinds of advertisements. It is, again, a little hard to solve the model in closed form. Think of a market in which consumers in both periods are affected by persuasive advertisement.

Remember that there are two opposing forces on the persuasive advertisement level in the case of a reduction in the cost of WoM, in the original duopoly model. Apart from those opposing forces, which cancel each other, there is one more force in the positive direction. This force is positive because not only in the first period but also in the second period, unit demand is affected by the persuasive advertisement level.

Therefore, it would be a strong conjecture to say that persuasive advertisement level increases.

The existence of consumers who are affected by two different kinds of advertising gives rise to the question of limited attention. Limited attention can be defined as demand which is lower than that which the fundamental functional form of advertisement suggests. For instance, the effective persuasive advertisement for the second-period consumer (α_i^L) would be α_i if $\varphi_i + \alpha_i \leq k$ and $k - \varphi_i$ otherwise. This formulation suggests limited attention towards persuasive advertisement. It is not easy to obtain a closed form solution of the model. However, the existence of consumers who have limited attention weakens the conjecture stating that the level of persuasive advertisement increases. If the cost of persuasive advertisement is low enough, the intensity of second-period consumers will decrease dramatically.

One more extension which can be applied might be having negative WoM. Negative WoM can be defined as the reduced incentive of first-period consumers to propagate the message. This reduced incentive stems from the lower benefit they get from the product. The concept of negative WoM is possible only in the case of secondary welfare notion. If it is assumed that only the first-period consumers who get negative utility in the first period stop propagating the message, then the second-period demand would be $\varphi_i(\frac{t-\alpha_i}{t})\frac{v-p_2^i}{t}p_2^i$. I could not derive a simple closed form solution for this case but there is a strong incentive to think that the persuasive advertisement level would fall when the cost of WoM decreases. The reason is that the second-period consumer density will be reduced and furthermore this reduction increases with the size of the advertisement level.

- *Homogenous firms*

Note that in the original model, both firms have an intrinsic value of v. It is natural to wonder about what would happen if they differ in this respect, i.e. they were differentiated vertically as well.

With vertical differentiation the problem of the duopolist would be as follows,

$$max_{(p_i,\alpha_i,\varphi_i)} \quad \frac{t+v_i+\alpha_i-(v_j+a_j)+p_j-p_i}{2t}p_i + \frac{t+v_i+\alpha_i-(v_j+a_j)+p_j-p_i}{2t}\varphi_i\frac{v_i-p_2^i}{t}p_2^i - \frac{\mu\varphi_i^2}{8} - \frac{\lambda\alpha_i^2}{8}$$

The first intuition is that the high quality good will have a higher return on its persuasive advertising effort. It has this advantage because it can charge more in the first period as consumers value it more. Therefore, as I assume that the marginal costs of both firms are equal to zero, a small increase in persuasive advertising pays more to the high-quality good producer. However, there is another effect which also drags the level of advertising up. This happens via the second-period profit. A decrease in WoM effort cost increases the WoM effort. The second-period profit per consumer is higher for the high-quality good. Hence, attracting more consumers in the first period is more tempting to both firms. For the low-quality firm, the first of these effects is in the opposite direction, because the competition in the first round is now more against it. What is observed is that, generally, the first drag down effect for the low-quality good firm dominates. Thus, it reduces the persuasive advertising level. This result is stated below.

Proposition 1.6.1. *A decrease in cost of WoM decreases the persuasive advertising level for the low-quality firm but increases it for the high-quality firm under quite general conditions (See Appendix for a proof).*

When I say, under general conditions I mean $\lambda t > 4/3$. I already assumed $\lambda t > 1$ for the interior solution. Hence my conclusion holds except for the thin band on λ, t space. Note that this is the first time where there is a reduction in the persuasive advertising level when the cost of WoM decreases. In the monopoly case, the firm is increasing its persuasive advertising level and in the symmetric duopoly case, there is no change. However here, the low-quality product firm decreases it. This may be an explanation for why some firms in more or less competitive environments put a lot of effort into social networking when it becomes cheaper and reduces the effort of persuasive advertisement. A test can be conducted relating the efforts in social networking effort, persuasive advertising effort with the quality of the product.

- *No competition at the second period*

One other assumption is that in the duopoly model, the firms are monopolists in the second period. This assumption can be altered in a particular way in order to see the implications. Assume that the second-period demand is determined by the WoM efforts of both firms. That is the second-period duopoly for the fully covered market is $(\varphi_i+\varphi_j)\frac{t+p_2^j-p_2^i}{2t}p_2^i$. The symmetric solution is then $\frac{2}{\lambda}=\alpha, \frac{t}{\mu}=\varphi, p=t-\frac{t^2}{\mu}$. It is seen that the persuasive advertisement level is independent of the WoM cost (See Appendix).

- *Particular specification of functional forms for costs and advertisements*

The functional forms assumed are standard quadratic cost functions and additive persuasive advertisement.

For additive persuasive advertisement assume alternatively, it works in a multiplicative manner. That is it increases the willingness to pay (before price) from $v-t$ to $v(1+\alpha)-t$. The symmetric solution is then $\frac{2v}{\lambda}=\alpha, \;\; t-\frac{v^4}{8t^2\mu}=p, \; \frac{v^2}{2t\mu}=\varphi$ (See Appendix).

Again the main result is robust to this check.

The cost functions of the advertisements also play an important role. The features of being continuous, increasing and convex are crucial. If a more convex cost function for WoM would assumed, then the result might change. That is the incentive for increasing the persuasive advertisement level would be more powerful than the incentive for decreasing the persuasive advertisement level. This is because a much steeper cost function would prohibit the benefits of shifting the effort to WoM. Similarly, assuming a persuasive advertisement cost which is more convex would weaken the benefit of increasing its level.

Another exercise would be employing a less convex cost function to both advertisement efforts. If linear cost functions were assumed, the main result will be such that if the cost of WoM is lower than some threshold value, the firm would put all its effort to WoM; otherwise to persuasive advertisement. Therefore, decreasing WoM cost might decrease but never increase the persuasive advertisement effort.

Therefore, the form of the cost function is quite crucial for our purpose. On the other hand, quadratic cost functions are standard, tractable and also realistic for most of the cases.

1.7 Conclusion

In the last decade, the most important phenomenon in communication has been the explosion of online social networking. This innovation made it possible for firms to reach larger audiences via WoM. This means the cost of WoM effort for firms decreased.

This study seeks mainly the answer to the following question: Does the level of persuasive advertisement increase or decrease when the cost of Word of Mouth marketing effort decreases in the differentiated goods market? The answer is that the level of persuasive advertisement does not decrease, hence it is a complementary effort for WoM. If the firm is a monopoly then the level of persuasive advertisement increases. If there are two symmetric duopolies it does not change.

In this framework I also checked what happens to the WoM level when the costs of persuasive advertisement and transportation change. Specifically, when the cost of persuasive advertisement decreases WoM effort increases for a monopoly but there is no change for symmetric duopolists. Symmetric duopolists increase their WoM and do not change their persuasive advertisement level when the transportation cost decreases. These results stem from the fact that the reduction in WoM cost increases the second period profit per first-period consumer and the firms want to exploit this fact.

I defined two welfare notions, one of them includes the persuasive advertisement level into welfare and the other does not. It is observed that that a decrease in cost of WoM generally increases consumer welfare and profit. This result is also valid for a decrease in persuasive advertisement cost and substitutability. Furthermore, the prices move in opposite directions when the cost of WoM and the cost of persuasive advertisement decrease. A decrease in WoM cost reduces the first-period price of the firms. The reason is that the firms want to exploit the increase in consumer density at the second period. However the decrease in cost of persuasive advertisement does not create this second-period advantage.

Furthermore, I conducted robustness discussion. The results suggest that the finding on the change of persuasive advertisement when the cost of WoM decreases is generally robust to these alternative cases as well.

These results directly call for empirical validation. These results can be evaluated using the data from the markets which broadly obey my assumptions. Of course, the introduction of online social networks also affected the reach of persuasive advertisement, as a result its cost has also changed. The parameters should be calibrated and the distribution of types may not be uniform in many cases. Taking these into consideration as well, a suitable test is possible.

Additionally, it is also possible to set up a model for the informative advertisement. Note that in my model, WoM's function is to inform people who do not know about a product before via referral. Hence, it is functionally an informative marketing effort. Therefore, a model with informative advertisement is to be a general WoM propagation model. The literature is not poor at this point. There are many models of dissemination of information using graph theory.

1.8 Appendix

Solution for the Monopoly Case (Main Model)

I use first order conditions and Hessian matrices as solution tools for constrained maximisation which can be found in many textbooks such as Mas-Colell et al. (1995).

The solution method is backward induction. Also note that the firm does not suffer from time inconsistency. At the second period the firm is again the monopoly on the Hotelling line where consumers' willingness to pay is $v - tx$. The density of Hotelling line is $\frac{v+\alpha-p_1}{t}\varphi$ where φ, p_1 and α are already determined at the first period. Hence, the second period maximisation problem will be, $max_{\{p_2\}}\frac{v+\alpha-p_1}{t}\varphi\frac{v-p_2}{t}p_2$. Therefore the FOC implies $v - p_2 = p_2 \Rightarrow p_2 = v/2$ and second derivative is $\frac{v+\alpha-p_1}{t^2}\varphi(-2) < 0$. Hence $p_2 = v/2$ is optimal if $0 \leq v \leq 2t$.

See the solution is interior. This helps us to rewrite the objective function as follows:

$$max_{\{p_1,p_2,\alpha,\varphi\}}\frac{v+\alpha-p_1}{t}(p_1+\varphi\frac{v^2}{4t})-\frac{\lambda\alpha^2}{8}-\frac{\mu\varphi^2}{8}$$

subject to

$$0 \leq \frac{v+\alpha-p_1}{t} \leq 1, 0 \leq \frac{v-p_2}{t} \leq 1$$

or alternatively (by denoting the first-period price as p)

$$max_{(p,\alpha,\varphi)}(v+\alpha-p)(4tp+\varphi v^2)-\frac{\mu\varphi^2t^2}{2}-\frac{\lambda\alpha^2t^2}{2}$$

How I solve this problem is as follows. First I check the unconstrained maximisation problem. For this, the first order necessary conditions (FOCs) are checked, the set of local maxima/minima is a subset of the vectors satisfying FOC. For the second order sufficient conditions, I examine the Hessian matrix. I look for the conditions which make the Hessian matrix negative definite which ensures the only solution I derive in FOC is strict global maximum. Secondly I examine the constraints for the solution and define inequalities which characterise the interior solution for the problem.

The first order conditions (FOC) gives the following,

$$p \;:\; 4t(v+\alpha-p) - 4tp - \varphi v^2 = 0 \tag{1.8.1}$$

$$\varphi \;:\; v^2(v+\alpha-p) - \mu\varphi t^2 = 0 \tag{1.8.2}$$

$$\alpha \;:\; 4tp + \varphi v^2 - \lambda\alpha t^2 = 0 \tag{1.8.3}$$

And the Hessian matrix is as follows:

	p	α	φ
p	$-8t$	$4t$	$-v^2$
α	$4t$	$-\lambda t^2$	v^2
φ	$-v^2$	v^2	$-\mu t^2$

Note that it does not include any choice variable, hence it is possible to check negative definiteness even before solving FOC. I need this matrix to be negative definite the solution I find in FOC to be local max. I conduct determinant test to characterise negative definiteness of the Hessian Matrix. Some algebraic manipulation gives the following inequalities.

$$
\begin{aligned}
-(-8t) &> 0 \text{ which is true always.} && (1.8.4)\\
\lambda t &> 2 && (1.8.5)\\
8\mu t^2(\lambda t-2) &> \lambda v^4 && (1.8.6)
\end{aligned}
$$

See that the requirement in equation 6 implies 5.

Now I am ready to solve FOCs

$$
\begin{aligned}
2,3 \text{ gives } v^2(v+\alpha-p) &= \mu t^2 \frac{\lambda\alpha t^2-4tp}{v^2} && (1.8.7)\\
1,3 \text{ gives } \ 4t(v+\alpha-p) &= \lambda\alpha t^2 \text{ equivalently } \alpha = \frac{4(v-p)}{\lambda t-4} && (1.8.8)\\
7,8, \text{ gives} \qquad p &= \frac{v\lambda(4\mu t^3-v^4)}{8\lambda\mu t^3-\lambda v^4-16\mu t^2} && (1.8.9)\\
8,10 \text{ gives} \qquad \alpha &= \frac{16v\mu t^2}{8\lambda\mu t^3-\lambda v^4-16\mu t^2} && (1.8.10)\\
2,10,11 \text{ gives} \qquad \varphi &= \frac{4\lambda t v^3}{8\lambda\mu t^3-\lambda v^4-16\mu t^2} && (1.8.11)
\end{aligned}
$$

Now I check the conditions needed for interior solution.

In order to the demand to be in the interior ($0 \le \frac{v+\alpha-p}{t} \le 1$) of the Hotelling line, I need

$$8\mu t^2(\lambda t-2)-\lambda v^4 \ge 4\lambda v\mu t^2$$

Additionally, because of the same requirement at the second period, we need $\frac{v-p_2^i}{t} = \frac{v}{2t}$ to be in the interior that is $0 \le v \le 2t$. Note that no other requirement implies this.

As a summary if

$$\begin{aligned} 0 &\leq v \leq 2t \\ 8\mu t^2(\lambda t - 2) - \lambda v^4 &\geq 4\lambda v\mu t^2 \end{aligned}$$

then the interior solution is

$$p_1 = \frac{v\lambda(4\mu t^3 - v^4)}{8\lambda\mu t^3 - \lambda v^4 - 16\mu t^2}, \alpha = \frac{16v\mu t^2}{8\lambda\mu t^3 - \lambda v^4 - 16\mu t^2} \; and \; \varphi = \frac{4\lambda t v^3}{8\lambda\mu t^3 - \lambda v^4 - 16\mu t^2}.$$

And the monopoly profit is

$$\frac{2\mu\lambda t^2 v^2}{(8\lambda\mu t^3 - \lambda v^4 - 16\mu t^2)}$$

Welfare Calculation

$$\begin{aligned} W_1^P &= \int_0^{\frac{v+\alpha-p_1}{t}} (v + \alpha - p_1 - tx)dx = \frac{1}{2t}(v+\alpha-p_1)^2 == \frac{1}{2t}\left(\frac{4\lambda\mu v t^3}{8\lambda\mu t^3 - \lambda v^4 - 16\mu t^2}\right)^2 \\ W_1^S &= \int_0^{\frac{v+\alpha-p_1}{t}} (v - p_1 - tx)dx == \frac{1}{2t}\left(\alpha^2 - (v-p_1)^2\right) = \frac{8\lambda\mu^2 v^2 t^4(8-\lambda t)}{(8\lambda\mu t^3 - \lambda v^4 - 16\mu t^2)^2} \\ W_2^S &= W_2^P = \frac{v+\alpha-p_1}{t}\varphi\int_0^{\frac{v-p_2}{t}} (v - p_2 - tx)dx = \frac{2\lambda^2 t^2 v^6 \mu}{(8\lambda\mu t^3 - \lambda v^4 - 16\mu t^2)^2} \end{aligned}$$

Corner Solutions

Recall interior solution requires $0 \leq v \leq 2t$ and $8\mu t^2(\lambda t - 2) - \lambda v^4 \geq 4\lambda v\mu t^2$

Case1: $v > 2t$ *and* $8\mu t^2(\lambda t - 2) - \lambda v^4 \geq 4\lambda v\mu t^2$

Then the second-period demand is 1 that is $\frac{v-p}{t} = 1$. The profit therefore is, p, which is equal to $v-t$. Hence the problem of the monopoly is

$$max_{\{p_1,p_2,\alpha,\varphi\}}\frac{v+\alpha-p_1}{t}(p_1+\varphi(v-t))-\frac{\lambda\alpha^2}{8}-\frac{\mu\varphi^2}{8}$$

FoCs are

$$\begin{aligned}\alpha &: \quad 4(p_1+\varphi(v-t))=\lambda\alpha t\\ \varphi &: \quad 4(v+\alpha-p_1)(v-t)=\mu\varphi t\\ p_1 &: \quad (p_1+\varphi(v-t))=v+\alpha-p_1\end{aligned}$$

SoCs

	p	α	φ
p	$\frac{-2}{t}$	$\frac{1}{t}$	$-\frac{(v-t)}{t}$
α	$\frac{1}{t}$	$-\frac{\lambda}{4}$	$\frac{(v-t)}{t}$
φ	$-\frac{(v-t)}{t}$	$\frac{(v-t)}{t}$	$-\frac{\mu}{4}$

We need $2\lambda(t-v)^2<\mu(\lambda t-2)$ to solution to be maximum.

Some algebraic manipulation gives

$$\begin{aligned}\alpha &= \frac{2v\mu}{(\mu t-2(v-t)^2)\lambda-2\mu}\\ \frac{d(\alpha)}{d\mu} &= \frac{-4v\lambda\left(t-v\right)^2}{\left(2\lambda t^2-4\lambda tv-\lambda\mu t+2\lambda v^2+2\mu\right)^2}<0\end{aligned}$$

Case 2: $0\le v\le 2t$ *and* $8\mu t^2(\lambda t-2)-\lambda v^4<4\lambda v\mu t^2$

As the first-period demand is 1.: $\frac{v+\alpha-p_1}{t}=1$, the problem becomes

$$max_{\{p_1,p_2,\alpha,\varphi\}}(v+\alpha-t+\varphi\frac{v^2}{4t})-\frac{\lambda\alpha^2}{8}-\frac{\mu\varphi^2}{8}$$

FOCs $\alpha: 1 = \frac{\lambda\alpha}{4}$ and $\varphi: \frac{v^2}{4t} = \frac{\mu\varphi}{4}$ SoC:

$$\begin{array}{ccc} & \alpha & \varphi \\ \alpha & -\frac{\lambda}{4} & 0 \\ \varphi & 0 & -\frac{\mu}{4} \end{array}$$

always negative definite. Hence $\alpha = \frac{4}{\lambda}$ does not depend on μ.

Case 3 $v > 2t$ *and* $8\mu t^2(\lambda t - 2) - \lambda v^4 < 4\lambda v\mu t^2$

$$max_{\{p_1,p_2,\alpha,\varphi\}}(v + \alpha - t + \varphi(v - t)) - \frac{\lambda\alpha^2}{8} - \frac{\mu\varphi^2}{8}$$

FOCs: $\alpha : 1 = \frac{\lambda\alpha}{4}$ *and* $\varphi: (v - t) = \frac{\mu\varphi}{4}$ SoC

$$\begin{array}{ccc} & \alpha & \varphi \\ \alpha & -\frac{\lambda}{4} & 0 \\ \varphi & 0 & -\frac{\mu}{4} \end{array}$$

always negative definite: Hence $\alpha = \frac{4}{\lambda}$ does not depend on μ.

Proof of the Proposition 1.4.1

Here we give the results of total derivatives and their signs. We conduct it for the first-period price, persuasive advertising level, WoM effort level .

$$\begin{aligned} \frac{dp}{d\mu} &= \frac{4t^2v^4(\lambda t - 4)}{(8\lambda\mu t^3 - \lambda v^4 - 16\mu t^2)^2} > 0 \\ \frac{d\alpha}{d\mu} &= \frac{16vt^2(-\lambda v^4)}{(8\lambda\mu t^3 - \lambda v^4 - 16\mu t^2)^2} < 0 \\ \frac{d\varphi}{d\mu} &= -32t^3v^3\lambda\frac{t\lambda - 2}{(8\lambda\mu t^3 - \lambda v^4 - 16\mu t^2)^2} < 0. \end{aligned}$$

Now the first-period consumer welfares,

Hence,

$$\frac{dW_1^P}{d\mu} = 16t^5v^6\lambda^3\frac{\mu}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^3} < 0$$
$$\frac{dW_1^S}{d\mu} = 16t^4v^6\lambda^2\mu\frac{t\lambda-8}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^3}$$

the latter one is positive if and only if $t\lambda < 8$.

The second-period consumer welfare,

Hence

$$\frac{dW_2^S}{d\mu} = 2t^2v^6\lambda^2\frac{8\lambda\mu t^3-16\mu t^2+\lambda v^4}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^3} < 0$$

The change in total consumer welfare with respect to primary welfare notion

is, therefore,

$$16t^5v^6\lambda^3\frac{\mu}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^3} + 2t^2v^6\lambda^2\frac{8\lambda\mu t^3-16\mu t^2+\lambda v^4}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^3} < 0$$

The change in total consumer welfare with respect to secondary welfare

notion is, therefore,

$$= \frac{2t^2v^6\lambda^2[16\lambda\mu t^3-80\mu t^2+\lambda v^4]}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^3}$$

ambigious.

The denominator is always negative. The nominator has the same sign with $16\lambda\mu t^3 - 80\mu t^2 + \lambda v^4 = 16\mu t^2(\lambda t - 5) + \lambda v^4$. See if $\lambda t > 5$ it is positive.

The change in profit

$$\frac{d((profit)}{d\mu} = -2t^2v^6\frac{\lambda^2}{(-8\lambda\mu t^3 + 16\mu t^2 + \lambda v^4)^2}$$

Hence the change in total welfare with respect to primary welfare notion is

$$\text{-2t}^2 v^6\frac{\lambda^2}{(-8\lambda\mu t^3+16\mu t^2+\lambda v^4)^2} + 16t^5v^6\lambda^3\frac{\mu}{(-8\lambda\mu t^3+16\mu t^2+\lambda v^4)^3} + 2t^2v^6\lambda^2\frac{8\lambda\mu t^3-16\mu t^2+\lambda v^4}{(-8\lambda\mu t^3+16\mu t^2+\lambda v^4)^3}$$

The change in total welfare with respect to secondary welfare notion is

$$= \frac{2t^2v^6\lambda^2}{(-8\lambda\mu t^3 + 16\mu t^2 + \lambda v^4)^3}(32\lambda\mu t^3 - 112\mu t^2 - \lambda v^4).$$

which is negative only if

$$32\lambda\mu t^3 - 112\mu t^2 - \lambda v^4 > 0$$

Proof of the Proposition 1.4.2

Now we conduct the same procedure for a change in persuasive advertising cost.

$$\begin{aligned}
\frac{dp}{d\lambda} &= 16t^2v\mu\frac{v^4-4t^3\mu}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^2}<0\\
\frac{d\varphi}{d\lambda} &= -64t^3v^3\frac{\mu}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^2}<0\\
\frac{d\alpha}{d\lambda} &= 16t^2v\mu\frac{v^4-8t^3\mu}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^2}<0\\
\frac{dW_1^P}{d\lambda} &= -32t^4v\frac{\mu^2}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^2}<0\\
\frac{dW_1^S}{d\lambda} &= -\frac{64t^4v^2\mu^2}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^3}\left(4\lambda t^3\mu+16t^2\mu-\lambda v^4\right)>0\\
\frac{dW_2^S}{d\lambda} &= 64t^4v^6\lambda\frac{\mu^2}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^3}<0
\end{aligned}$$

The change in total consumer welfare with respect to primary welfare notion is therefore,

$$-32t^4v\frac{\mu^2}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^2}+64t^4v^6\lambda\frac{\mu^2}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^3}<0$$

The change in total consumer welfare with respect to secondary welfare notion is therefore,

$$=\frac{64t^4v^2\mu^2(-4\lambda t^3\mu-16t^2\mu+\lambda v^4+v^4\lambda)}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^3}$$

The denominator is negative. The nominator′s sign depends on $-2\lambda t^3\mu-8t^2\mu+\lambda v^4$. Hence the welfare increases iff $\lambda v^4>2\lambda t^3\mu+8t^2\mu$. Note that this is neither implied nor negated by interior solution conditions.

The change in profit

$$\frac{d((\text{profit})}{d\lambda}=-32t^4v^2\frac{\mu^2}{\left(-8\lambda\mu t^3+16\mu t^2+\lambda v^4\right)^2}$$

Hence the change in total welfare with respect to primary welfare notion is

$$-32t^4v^2\frac{\mu^2}{(-8\lambda\mu t^3+16\mu t^2+\lambda v^4)^2}-32t^4v\frac{\mu^2}{(-8\lambda\mu t^3+16\mu t^2+\lambda v^4)^2}$$
$$+64t^4v^6\lambda\frac{\mu^2}{(-8\lambda\mu t^3+16\mu t^2+\lambda v^4)^3}$$

The change in total welfare with respect to secondary welfare notion is

$$=-32t^4\mu^2v^2(=(\frac{-1536t^6\mu^3v^2}{(-8\lambda\mu t^3+16\mu t^2+\lambda v^4)^3})>0$$

Solution for the Duopoly Case (Main Model)

The problem we examine here is as follows:

The first period is a full information simultaneous finite game. The second period is a decision problem for each firm. Hence, firms will maximise their total profit taking into account the first-period strategic concerns. The symmetric pure Nash equilibrium for this reason is a suitable equilibrium concept. Also note that the firms do not suffer from time inconsistency. At the second period the representative firm is again the monopoly on the Hotelling line where consumers′ willingness to pay is $v-tx$. The density of Hotelling line is $\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}\varphi_i$ where $\varphi_i, p_i, p_j, \alpha_i, a_j$ are already determined at the first period. Hence, the second-period maximisation problem will be,

$$max_{\{p_2^i\}}\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}\varphi_i\frac{v-p_2^i}{t}p_2^i.$$

Therefore the optimal second-period price is $v/2$ given $v\leq 2t$ as we did for the monopoly case, as an interior solution. This helps us to rewrite the objective function as follows:

$$max_{(p_i,\alpha_i,\varphi_i)}(\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t})(p_i+\varphi_i\frac{v^2}{4t})-\frac{\mu\varphi_i^2}{8}-\frac{\lambda\alpha_i^2}{8}$$

subject to $0 \leq \frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t} \leq 1$ and $0 \leq \frac{v-p_2^i}{t} \leq 1$

How I solve this problem is as follows. I check the unconstrained maximisation problem, given the competitor′s strategy. The first order necessary conditions (FOC) give the set of local maxima/minima is a subset of the vectors satisfying FOC. For the second order sufficient conditions I examine the Hessian matrix. I look for the conditions which make the Hessian matrix negative definite which ensures the only solution we derive in FOC is the strict global maximum. Secondly I examine the constraints for the solution and define inequalities which characterise the interior solution for the problem. Lastly, in order to find symmetric equilibrium I solve the first order conditions of both firms together.

As the boundary solutions are not interesting, we focus on the interior solutions. Our result is as follows:

The maximisation problem can be rewritten as follows,

$$max_{(p_i,\alpha_i,\varphi_i)}(t+v+\alpha_i-(v+a_j)+p_j-p_i)(4tp_i+\varphi v^2)-t^2\mu\varphi_i^2-t^2\lambda\alpha_i^2$$

$$\text{subject to } 0 \leq \frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t} \leq 1, 0 \leq \frac{v-p_2^i}{t} \leq 1$$

First order conditions,

$$p_i \quad : \quad 4t(t+v+\alpha_i-(v+a_j)+p_j-p_i)-(4tp_i+\varphi v^2)=0 \tag{1.8.12}$$

$$\varphi_i \quad : \quad v_i^2(t+v+\alpha_i-(v+a_j)+p_j-p_i)-2t^2\mu\varphi_i=0 \tag{1.8.13}$$

$$\alpha_i \quad : \quad 4tp_i+\varphi v^2-2\lambda\alpha_i t^2=0 \tag{1.8.14}$$

Note that the Hessian matrix does not include any choice variable, hence it is possible to check negative definiteness even before solving FOC. We need this matrix to be negative definite to solution we find in FOC to be local max. I conduct determinant test to characterise negative defineteness of the Hessian Matrix.

SOCs are on the other hand,

	p	α	φ
p	$-8t$	$4t$	$-v^2$
α	$4t$	$-2\lambda t^2$	v^2
φ	$-v^2$	v^2	$-2\mu t^2$

For an interior maximisation we need the following to hold, $16\mu t^2(\lambda t-1)>\lambda v^4$.

Now we are ready to find the explicit solution.

12,14 gives	$2(t+v+\alpha_i-(v+a_j)+p_j-p_i)$	$=$	$\lambda\alpha_i t$	(1.8.15)
13,14 gives	$v_i^4(t+v+\alpha_i-(v+a_j)+p_j-p_i)$	$=$	$2t^2\mu(2\lambda\alpha_i t^2-4tp_i)$	(1.8.16)
15,16 gives	p_i	$=$	$\alpha_i\dfrac{\lambda(8\mu t^3-v^4)}{16\mu t^2}=\alpha_i X$	
where	X	$=$	$\dfrac{\lambda(8\mu t^3-v^4)}{16\mu t^2}$	(1.8.17)
15,17 gives	$2(t+v+\alpha_i-(v+a_j)+\alpha_j X-\alpha_i X)$	$=$	$\lambda\alpha_i t$	(1.8.18)
and	$2(t+v+\alpha_j-(v+a_\imath)+\alpha_\imath X-\alpha_j X)$	$=$	$\lambda\alpha t$	(1.8.19)
18,19 gives	$\alpha_i+\alpha_j$	$=$	$\dfrac{4}{\lambda}$	(1.8.20)
18,20 gives	$2(t+v+\alpha_i-(v+\dfrac{4}{\lambda}-\alpha_i)$	$=$	$\lambda\alpha_i t$ -$(\dfrac{4}{\lambda}-\alpha_i)X-\alpha_i X)$	
or equivalently	α_i	$=$	$\dfrac{2(t-\frac{4}{\lambda}(1-X))}{\lambda t-4+2X+2X}$	(1.8.21)
14,17 gives	φ_i	$=$	$\dfrac{2t\alpha_i(\lambda t-2X)}{v^2}$	(1.8.22)

Therefore, algebraic manipulation reveals

$$\alpha_i=\frac{2}{\lambda}\quad p_i=t-\frac{v^4}{8\mu t^2}\quad \varphi_i=\frac{v^2}{2\mu t}$$

Note that the first constraint is not binding as the demand per firm is exactly 1/2. The second constraint holds if $0\leq v\leq 2t$.

The demand of the representative firm is 1/2. The surplus that the indifferent consumer gets (with respect to first welfare criterion as consumers′ decision takes the persuasive advertisement into account) is

$$v+\alpha_i-tx-p=v+\frac{2}{\lambda}-\frac{3t}{2}+\frac{v^4}{8\mu t^2}\geq 0$$

We need to assume this holds in order to the market be covered.

Welfare Calculation

Define W_{ij}^{P} (W_{ij}^{S}) as the firm j's consumers$'$ welfare at period i with respect to primary (secondary) welfare criteron. We calculate the values at the equilibrium.

Therefore the welfare of the first (second) period consumers is $W_{i1}^{P}+W_{i2}^{P}$ $(W_{i1}^{S}+W_{i2}^{S})$ Using symmetricity;

$$\begin{aligned}
W_{11}^{P} &= \frac{1}{16t^2\lambda\mu}\left(-10\lambda\mu t^3+8\lambda\mu t^2 v+16\mu t^2+\lambda v^4\right)=W_{12}^{P}\\
W_{11}^{S} &= \frac{1}{16t^2\mu}\left(-10\mu t^3+8\mu t^2 v+v^4\right)=W_{12}^{S}\\
W_{21}^{P} &= \frac{1}{32t^2}\frac{v^4}{\mu}=W_{22}^{P}
\end{aligned}$$

Also note that $W_{21}^{P}=W_{21}^{S}$as only the first period differs for the welfare notions, and furthermore $W_{21}^{P}=W_{22}^{S}$ by symmetricity again.

Lastly the equilibrium level profit of the representative firm is

$$\pi_i=\frac{t}{2}-\frac{v^4}{32\mu t^2}-\frac{1}{2\lambda}$$

Total welfare with respect to primary welfare notion is:

$$=-\frac{11t}{4}+2v+\frac{4}{\lambda}+\frac{5v^4}{16\mu t^2}$$

Total welfare with respect to secondary welfare notion is

$$=\frac{-11t}{4}+2v+\frac{2}{\lambda}+\frac{5v^4}{16\mu t^2}$$

As a summary if

$$\begin{aligned} 16\mu t^2(\lambda t-1) &> \lambda v^4 \\ v+\frac{2}{\lambda}-\frac{3t}{2}+\frac{v^4}{8\mu t^2} &\geq 0 \\ 0 &\leq v \leq 2t \end{aligned}$$

the interior, market covering solution is

$$p_i = t - \frac{v^4}{8\mu t^2}, \alpha_i = \frac{2}{\lambda} \text{ and } \varphi_i = \frac{v^2}{2\mu t}$$

Corner Solutions

Corner solution:1 $v+\frac{2}{\lambda}-\frac{3t}{2}+\frac{v^4}{8\mu t^2} < 0$ Then we have the same situation with monopoly.

Corner soluton 2: $v+\frac{2}{\lambda}-\frac{3t}{2}+\frac{v^4}{8\mu t^2} \geq 0$ and $v > 2t$.

The equivalent maximisation problem is

$$max_{(p_i,\alpha_i,\varphi_i)} 2(t+v+\alpha_i-(v+a_j)+p_j-p_i)(p_i+\varphi(v-t)) - \frac{\mu t\varphi_i^2}{2} - \frac{\lambda t\alpha_i^2}{2}$$

FOC:

$$\begin{aligned} p &: \quad p_i+\varphi(v-t) = t+v+\alpha_i-(v+a_j)+p_j-p_i \\ \alpha &: \quad 2(p_i+\varphi(v-t)) = t\lambda\alpha \\ \varphi &: \quad 2(v-t)(t+v+\alpha_i-(v+a_j)+p_j-p_i) = \mu t\varphi \end{aligned}$$

in the symmetric eqm.

$$\begin{aligned} p_i + \varphi(v-t) &= t \\ 2(p_i + \varphi(v-t)) &= t\lambda\alpha \\ 2(v-t) &= \mu\varphi \end{aligned}$$

This shows that $\alpha = \frac{2}{\lambda}$

SoC:

	p	α	φ
p	-4	2	$-2(v-t)$
α	2	$-\lambda t$	$2(v-t)$
φ	$-2(v-t)$	$2(v-t$	$-\mu t$

We need $\lambda t > 1$ and $\lambda t^2 - 2\lambda tv - \lambda\mu t + \lambda v^2 + \mu < 0$ to have negative definiteness.

Proof of the Proposition 1.5.1

The total derivatives for the 1st period price, WoM, persuasive advertisement level and welfares:

$$\begin{aligned}
\frac{dp_i}{d\mu} &= \frac{1}{8t^2}\frac{v^4}{\mu^2} > 0 \\
\frac{d\varphi_i}{d\mu} &= -\frac{1}{2t}\frac{v^2}{\mu^2} < 0 \\
\frac{d\alpha_i}{d\mu} &= 0 \\
\frac{d\pi_i}{d\mu} &= \frac{1}{32t^2}\frac{v^4}{\mu^2} > 0 \\
\frac{dW_{11}^P}{d\mu} &= -\frac{1}{16t^2}\frac{v^4}{\mu^2} < 0 \\
\frac{dW_{11}^S}{d\mu} &= -\frac{1}{16t^2}\frac{v^4}{\mu^2} < 0 \\
\frac{dW_{21}^P}{d\mu} &= -\frac{1}{32t^2}\frac{v^4}{\mu^2} < 0
\end{aligned}$$

So the consumer welfare changes in the first and second period are

$$\begin{aligned}
\frac{d(W_{11}^P + W_{12}^P)}{d\mu} &= 2\frac{dW_{11}^P}{d\mu} = -\frac{1}{8t^2}\frac{v^4}{\mu^2} < 0 \\
\frac{d(W_{11}^S + W_{12}^S)}{d\mu} &= 2\frac{dW_{11}^S}{d\mu} = -\frac{1}{8t^2}\frac{v^4}{\mu^2} < 0 \\
\frac{d(W_{21}^P + W_{22}^P)}{d\mu} &= \frac{d(W_{21}^S + W_{22}^S)}{d\mu} = 2\frac{dW_{21}^P}{d\mu} = -\frac{1}{16t^2}\frac{v^4}{\mu^2} < 0
\end{aligned}$$

Lastly the total consumer welfare change, $W^P(W^S)$ denoting with respect to primary and secondary respectively, is:

$$\frac{dW^P}{d\mu} = \frac{dW^S}{d\mu} = -\frac{1}{8t^2}\frac{v^4}{\mu^2} - \frac{1}{16t^2}\frac{v^4}{\mu^2} = -\frac{3}{16t^2}\frac{v^4}{\mu^2} < 0$$

Total welfare change with respect to primary welfare notion is:

$$\frac{dW}{d\mu} = \frac{d(-\frac{11t}{4} + 2v + \frac{4}{\lambda} + \frac{5v^4}{16\mu t^2})}{d\mu} = -\frac{5}{16t^2}\frac{v^4}{\mu^2} < 0$$

Total welfare with respect to secondary welfare notion is

$$\frac{dW}{d\mu} = \frac{d(\frac{-11t}{4} + 2v + \frac{2}{\lambda} + \frac{5v^4}{16\mu t^2})}{d\mu} = -\frac{5}{16t^2}\frac{v^4}{\mu^2} < 0$$

Proof of the Proposition 1.5.2

The total derivatives for the first-period price, WoM and persuasive advertisement level, and welfares are

$$\begin{aligned}
\frac{dp_i}{d\lambda} &= \frac{d\varphi_i}{d\lambda} = \frac{d\alpha_i}{d\lambda} = 0 \\
\frac{d\pi_i}{d\lambda} &= \frac{1}{2\lambda^2} > 0 \\
\frac{dW_{11}^P}{d\lambda} &= -\frac{1}{\lambda^2} < 0 \\
\frac{dW_{11}^S}{d\lambda} &= 0 \\
\frac{dW_{21}^P}{d\lambda} &= 0
\end{aligned}$$

So the consumer welfare changes in the first and second period are

$$\begin{aligned}
\frac{d(W_{11}^P + W_{12}^P)}{d\lambda} &= 2\frac{dW_{11}^P}{d\lambda} = -\frac{2}{\lambda^2} < 0 \\
\frac{d(W_{11}^S + W_{12}^S)}{d\lambda} &= 2\frac{dW_{11}^S}{d\lambda} = 0 \\
\frac{d(W_{21}^P + W_{22}^P)}{d\lambda} &= \frac{d(W_{21}^S + W_{22}^S)}{d\lambda} = 0
\end{aligned}$$

Lastly, the total consumer welfare change, $W^P(W^S)$ denoting with respect to primary and secondary respectively, is: $\frac{dW^P}{d\lambda} = -\frac{2}{\lambda^2}$ and $\frac{dW^S}{d\lambda} = 0$

Total welfare change with respect to primary welfare notion is

$$\frac{dW}{d\lambda} =: -\frac{4}{\lambda^2} < 0$$

Total welfare with respect to secondary welfare notion is

$$\frac{dW}{d\lambda} = -\frac{2}{\lambda^2} < 0$$

Proof of the Proposition 1.5.3

The total derivatives for the first-period price, WoM and persuasive advertisement level, and welfares are

$$\begin{aligned}
\frac{dp_i}{dt} &= \frac{1}{4t^3\mu}\left(4\mu t^3 + v^4\right) > 0 \\
\frac{d\varphi_i}{dt} &= -\frac{1}{2t^2}\frac{v^2}{\mu} < 0 \\
\frac{d\alpha_i}{dt} &= 0 \\
\frac{d\pi_i}{dt} &= \frac{1}{16t^3\mu}\left(8\mu t^3 + v^4\right) > 0
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{dW_{11}^P}{dt} &= -\frac{1}{8t^3\mu}\left(5\mu t^3 + v^4\right) < 0 \\
\frac{dW_{11}^S}{dt} &= -\frac{1}{8t^3\mu}\left(5\mu t^3 + v^4\right) < 0 \\
\frac{dW_{21}^P}{dt} &= -\frac{1}{16t^3}\frac{v^4}{\mu} < 0
\end{aligned}$$

So the welfare changes in the first and second period are

$$
\begin{aligned}
\frac{d(W_{11}^P + W_{12}^P)}{dt} &= 2\frac{dW_{11}^P}{dt} = -\frac{1}{4t^3\mu}\left(5\mu t^3 + v^4\right) < 0 \\
\frac{d(W_{11}^S + W_{12}^S)}{dt} &= 2\frac{dW_{11}^S}{dt} = -\frac{1}{4t^3\mu}\left(5\mu t^3 + v^4\right) < 0 \\
\frac{d(W_{21}^P + W_{22}^P)}{dt} &= \frac{d(W_{21}^S + W_{22}^S)}{dt} \\
&= 2\frac{dW_{21}^P}{dt} = -\frac{1}{8t^3}\frac{v^4}{\mu} < 0
\end{aligned}
$$

Lastly the total consumer welfare change, $W^P(W^S)$ denoting with respect to primary and secondary respectively, is:

$$
\frac{dW^P}{dt} = \frac{dW^S}{dt} = -\frac{1}{4t^3\mu}\left(5\mu t^3 + v^4\right) - \frac{1}{8t^3}\frac{v^4}{\mu} < 0
$$

Total welfare change with respect to primary welfare notion is:

$$
\frac{dW}{dt} = \frac{d(-\frac{11t}{4} + 2v + \frac{4}{\lambda} + \frac{5v^4}{16\mu t^2})}{dt} : -\frac{1}{8t^3\mu}\left(22\mu t^3 + 5v^4\right) < 0
$$

Total welfare with respect to secondary welfare notion is

$$
\frac{dW}{dt} = \frac{d(\frac{-11t}{4} + 2v + \frac{2}{\lambda} + \frac{5v^4}{16\mu t^2})}{dt} = -\frac{1}{8t^3\mu}\left(22\mu t^3 + 5v^4\right) < 0
$$

Proof of Proposition 1.6.1

$$
max_{(p_i,\alpha_i,\varphi_i)}(\frac{t + v_i + \alpha_i - (v_j + a_j) + p_j - p_i}{2t})(p_i + \varphi\frac{v_i^2}{4t}) - \frac{\mu\varphi_i^2}{8} - \frac{\lambda\alpha_i^2}{8}
$$

subject to $0 \le v \le 2t$ and market is covered.

or equivalently,

$$max_{(p_i,\alpha_i,\varphi_i)}(t + v_i + \alpha_i - (v_j + a_j) + p_j - p_i)(4tp_i + \varphi v_i^2) - t^2\mu\varphi_i^2 - t^2\lambda\alpha_i^2$$

FOCs are therefore:

$$p_i \quad : \quad 4t(t + v_i + \alpha_i - (v_j + a_j) + p_j - p_i) - (4tp_i + \varphi v_i^2) = 0 \qquad (1.8.23)$$

$$\varphi_i \quad : \quad v_i^2(t + v_i + \alpha_i - (v_j + a_j) + p_j - p_i) - 2t^2\mu\varphi_i = 0 \qquad (1.8.24)$$

$$\alpha_i \quad : \quad 4tp_i + \varphi v_i^2 - 2\lambda\alpha_i t^2 = 0 \qquad (1.8.25)$$

SOC′s are on the other hand,

	p	α	φ
p	$-8t$	$4t$	$-v_i^2$
α	$4t$	$-2\lambda t^2$	v_i^2
φ	$-v_i^2$	v_i^2	$-2\mu t^2$

For an interior maximisation we need the followings to hold, $16\mu t^2(\lambda t - 1) > \lambda v_i^4$ $v_i < 2t$.

The algebra is very complicated for this part, hence we check our result implicitly.

Under these conditions the solutions to FOC are local maximisers.

23,25 gives	$2(t+v_i+\alpha_i-(v_j+a_j)+p_j-p_i)$	$=$	$\lambda\alpha_i t$	(1.8.26)
14,25 gives	$v_i^4(t+v_i+\alpha_i-(v_j+a_j)+p_j-p_i)$	$=$	$2t^2\mu\varphi_i(2\lambda\alpha_i t^2-4tp_i)$	(1.8.27)
26,27 gives	p_i	$=$	$\alpha_i\frac{\lambda(8\mu t^3-v_i^4)}{16\mu t^2}=\alpha_i X_i$	
where	X_i	$=$	$\frac{\lambda(8\mu t^3-v_i^4)}{16\mu t^2})$	(1.8.28)
26,28 gives	$2(t+v_i+\alpha_i-(v_j+a_j)+\alpha_j X_j-\alpha_i X_i)$	$=$	$\lambda\alpha_i t$	(1.8.29)
and	$2(t+v_j+\alpha_j-(v_\imath+a_\imath)+\alpha_\imath X_\imath-\alpha_j X_j)$	$=$	$\lambda\alpha_j t$	(1.8.30)
29,30 gives	$\alpha_i+\alpha_j$	$=$	$\frac{4}{\lambda}$	(1.8.31)
29,31 gives	$2(t+v_i+\alpha_i-(v_j+\frac{4}{\lambda}-\alpha_i)$	$=$	$\lambda\alpha_i t-(\frac{4}{\lambda}-\alpha_i)X_j-\alpha_i X_i)$	
or equivalently	α_i	$=$	$\frac{2(t+v_i-v_j-\frac{4}{\lambda}(1-X_j)}{\lambda t-4+2X_j+2X_i}$	(1.8.32)
25,28 gives	φ_i	$=$	$\frac{2t\alpha_i(\lambda t-2X_i)}{v_i^2}$	(1.8.33)

Now we are ready to have comparative statistics,

$$\frac{d(\alpha_i)}{d\mu} = -16t^2\frac{v_i-v_j}{\left(-24\lambda\mu t^3+32\mu t^2+\lambda v_i^4+\lambda v_j^4\right)^2} * \left(\begin{array}{c}\lambda v_i^4-4v_i^2v_j-4v_i^3-4v_j^3-4v_iv_j^2+\lambda v_j^4\\ +3t\lambda v_i^3+3t\lambda v_j^3+3t\lambda v_iv_j^2+3t\lambda v_i^2v_j\end{array}\right)$$

The sign of the derivative is the same as,

$$(v_i-v_j)(-\lambda v_i^4+4v_i^2v_j+4v_i^3+4v_j^3+4v_iv_j^2-\lambda v_j^4-3t\lambda v_i^3-3t\lambda v_j^3-3t\lambda v_iv_j^2-3t\lambda v_i^2v_j)$$

$$(v_i-v_j)(4v_iv_j(v_i+v_j)-3t\lambda v_iv_j(v_i+v_j)-3t\lambda(v_i^3+v_j^3)-\lambda(v_i^4+v_j^4)+4(v_i^3+v_j^3))$$

$$(v_i-v_j)(v_iv_j(4-3t\lambda)(v_i+v_j)+(4-3t\lambda)(v_i^3+v_j^3)-\lambda(v_i^4+v_j^4))$$

Hence if $v_i > v_j$, then the level of advertising increases for firm i, and decreases for firm j provided $t\lambda > 4/3$.

It is also possible, to some extent, to analyse how price moves: Remember, $p_i = \alpha_i X_i$. Hence $\frac{d(p_i)}{d\mu} = \frac{d(\alpha_i)}{d\mu} X_i + \alpha_i \frac{d(X_i)}{d\mu}$. The component after the plus sign is always positive. The first component after the equal sign is negative for the better product but positive for the worse product. Hence, when the cost of WoM decreases, the price decreases (if X_i and X_j are negative) for the better product but what happens to the worse product?

First find sign

$$\begin{aligned}
p_i/p_j &= \alpha_i X_i/\alpha_j X_j \\
(p_i/p_j) &= \left(\frac{2(t+v_i-v_j-\frac{4}{\lambda}(1-\frac{\lambda(8\mu t^3-v_j^4)}{16\mu t^2})}{\lambda t-4+2\frac{\lambda(8\mu t^3-v_j^4)}{16\mu t^2}+2\frac{\lambda(8\mu t^3-v_i^4)}{16\mu t^2}}\frac{\lambda(8\mu t^3-v_i^4)}{16\mu t^2}\right)/ \\
&\quad \left(\frac{2(t+v_j-v_i-\frac{4}{\lambda}(1-\frac{\lambda(8\mu t^3-v_i^4)}{16\mu t^2})}{\lambda t-4+2\frac{\lambda(8\mu t^3-v_j^4)}{16\mu t^2}+2\frac{\lambda(8\mu t^3-v_i^4)}{16\mu t^2}}\frac{\lambda(8\mu t^3-v_j^4)}{16\mu t^2}\right) \\
&= \frac{v_i^4-8t^3\mu}{v_j^4-8t^3\mu}\frac{2t+2v_i-2v_j-\frac{8}{\lambda}\left(\frac{1}{16t^2}\frac{\lambda}{\mu}\left(v_j^4-8t^3\mu\right)+1\right)}{2t+2v_j-2v_i-\frac{8}{\lambda}\left(\frac{1}{16t^2}\frac{\lambda}{\mu}\left(v_i^4-8t^3\mu\right)+1\right)}
\end{aligned}$$

See that if $v_i > v_j$ then nominator is larger. Hence the price of the better firm′s product is larger. Additionally, if the cost of WoM decreases then the nominator decreases less than the denominator hence, the price gap increases. This implies the decrease in the price of the worse product is more than the decrease in the price of the better product.

Duopoly at the Second Period

The problem of the firm is then:

$$max_{(p_i,\alpha_i,\varphi_i)} \frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}(p_i+(\varphi_i+\varphi_j)\frac{t+p_2^j-p_2^i}{2t}p_2^i)-\frac{\mu\varphi_i^2}{8}-\frac{\lambda\alpha_i^2}{8}$$

$$subject\ to\ 0<\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}<1\ and\ 0<\frac{t+p_2^j-p_2^i}{2t}<1$$

The second period symmetric solution is then $p_2^j=p_2^i=t$ which makes the profit $(\varphi_i+\varphi_j)\frac{t}{2}$. Therefore we have the objective of $\frac{t+v+\alpha_i-(v+a_j)+p_j-p_i}{2t}(p_i+(\varphi_i+\varphi_j)\frac{t}{2})-\frac{\mu\varphi_i^2}{8}-\frac{\lambda\alpha_i^2}{8}$.

FOCs:

$$\begin{aligned}
\alpha_i &: \frac{1}{4t}(2p_i+t\varphi_i+t\varphi_j-t\lambda\alpha_i)=0\\
\varphi_i &: \frac{1}{4}t+\frac{1}{4}\alpha_i-\frac{1}{4}a_j-\frac{1}{4}p_i+\frac{1}{4}p_j-\frac{1}{4}\mu\varphi_i=0\\
p_i &: -\frac{1}{4t}(2a_j-2\alpha_i-2t+4p_i-2p_j+t\varphi_i+t\varphi_j)=0
\end{aligned}$$

The symmetric solution, by algebraic manipulation is then $\frac{2}{\lambda}=\alpha,\frac{t}{\mu}=\varphi,p=t-\frac{t^2}{\mu}$

SOCs

	p	α	φ
p	$-\frac{1}{t}$	$\frac{1}{2t}$	$-\frac{1}{4}$
α	$\frac{1}{2t}$	$-\frac{\lambda}{t}$	$\frac{1}{4}$
φ	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{\mu}{4}$

We need, $4\lambda-1>0$ and $2\mu\lambda-\mu+1-2\lambda<0$ for SOCs to be satisfied.

And also we need $2v\lambda\mu+4\mu-3\mu t\lambda+2t^2\lambda\geq 0$ for the first period market to be covered and $v-1.5*t+t^2/\mu\leq 0$ for the second market interior solution.

Multiplicative Advertisement

Similarly as before, solving the second period and inserting into the objective function.

$$max_{(p_i,\alpha_i,\varphi_i)}\frac{t+v(1+\alpha_i)-v(1+a_j)+p_j-p_i}{2t}(p_i+\varphi_i\frac{v^2}{4t})-\frac{\mu\varphi_i^2}{8}-\frac{\lambda\alpha_i^2}{8}$$

$$subject\ to\ 0<\frac{t+v(1+\alpha_i)-v(1+a_j)+p_j-p_i}{2t}<1\ and\ 0<\frac{v-p_2^i}{t}<1$$

FOCs:

$$\begin{aligned}
\alpha_i &: \frac{1}{8t^2}\left(-2\lambda\alpha_i t^2+4p_i tv+\varphi_i v^3\right)=0\\
\varphi_i &: \frac{1}{8t^2}\left(tv^2+v^3\alpha_i-v^3a_j-v^2p_i+v^2p_j-2t^2\mu\varphi_i\right)=0\\
p_i &: \frac{1}{8t^2}\left(4tp_j-8tp_i-v^2\varphi_i+4t^2+4tv\alpha_i-4tva_j\right)=0
\end{aligned}$$

The symmetric solution is then $\frac{2v}{\lambda}=\alpha,\quad t-\frac{v^4}{8t^2\mu}=p,\ \frac{v^2}{2t\mu}=\varphi.$

We need this $v-\frac{t}{2}-t+\frac{v^4}{8t^2\mu}+\frac{2v}{\lambda}\geq 0$, first period market to be covered and $v-2t\leq 0$ for second period interior solution.

SOCs

	p	α	φ
p	$\frac{-1}{t}$	$\frac{v}{2t}$	$\frac{-v^2}{8t^2}$
α	$\frac{v}{2t}$	$\frac{-\lambda}{4}$	$\frac{v^3}{8t^2}$
φ	$\frac{-v^2}{8t^2}$	$\frac{v^3}{8t^2}$	$-\frac{\mu}{4}$

We need, $v^2>\lambda t\ \ and\ 16t^2v^2-16t^3\lambda-8t^3v\mu+v^5+4tv^4-2t^2\lambda v^2<0.$

Chapter 2

Contracting with Manipulable Agents

Abstract

This study aims, given the manipulative advertising is costly, to (1) understand how manipulation choices are made by firms and (2) analyse the welfare implications of the naive agent's existence within different competitive settings in a behavioural contract environment. Naiveté is defined as having limited understanding of the strategic environment, that is, naive agents fail to understand that advertisements do not provide reliable information about the true state but are simply strategic decisions aiming to manipulate them. I fully characterise the advertisement choices and the shape of contracts along the equilibrium path. The main results for naive agents show that (a) firms manipulate the agent towards the improbable state if the cost of advertising is low, (b) firms manipulate the agent towards the probable state if the cost is high but not prohibitive, and the true probability of the improbable state is close to the probability of the probable state. (c) The expected surplus of the naive agent decreases if the size of the manipulation increases or competitiveness decreases.

2.1 Introduction

Let me start with a motivating example before going into details.

Assume there is a cable TV firm which is a monopolist (See Figure 2.1 (Comcast, nd). You want to buy a cable TV plan for your partner (e.g. your wife). Suppose the firm comes up with a package called HD Triple Play, which includes some TV channels, and there are some other channels available for additional payment.

The firm disseminates a manipulative advertisement which favours a particular state. The advertisement is manipulative in the sense that it affects the naive agent's belief on the probability of a state happening. Say this state is that your partner will like this package. So, if you receive a manipulative advertisement favoring this state, then you believe that this state happens for sure. The advertisement also includes, as seen in Figure 2.1, information about the firm; its name. Assume the other state is that your wife will like the other channels. Further assume that you have two viable actions after you sign the contract with Comcast; using HD Triple Play channels or using other channels.

The contract offered by Comcast is not contingent on your partner's taste but contingent on your action. I further assume a congruent relationship; your preferences and budgets are the same. In my model there is only one consumer; yourself. This is just in the case of this particular example. In my model, the partner buys the package for herself.

Assume that the probability that your wife will like the package is q (and she will like other channels with probability $1-q$), but you are naive such that after seeing this advertisement you believe that q is 1. If you believe that it is worth it, you sign the contract and buy the package. Later the state is realised, and you discover what your wife likes. You take action and make the payment accordingly.

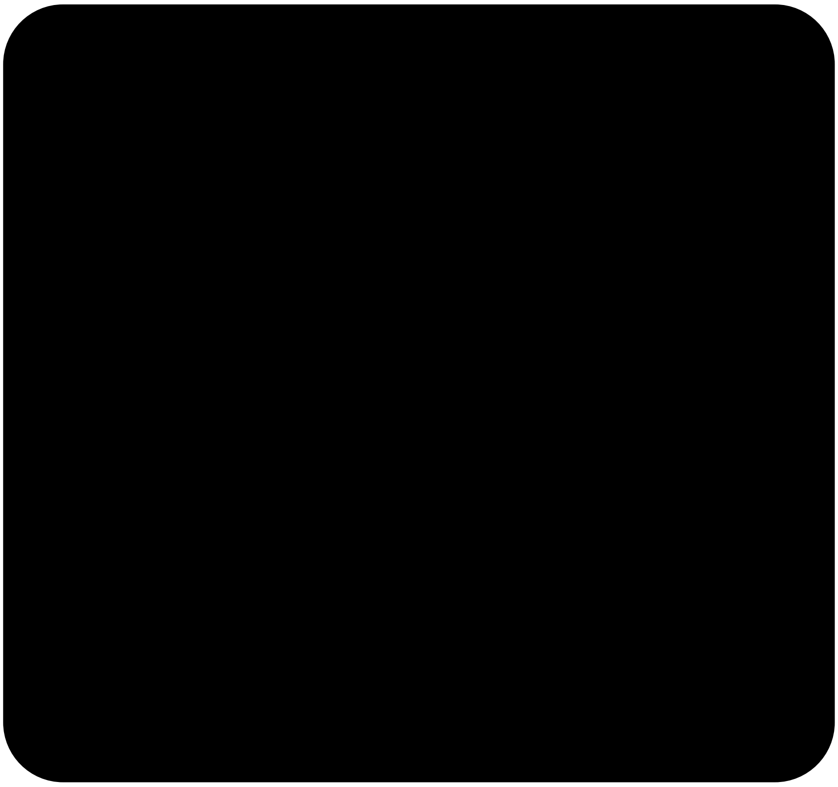


Figure 2.1: An Example of Manipulative Advertisement

Being motivated by these kinds of markets in which manipulation is a concern, this study aims to

1. Understand the nature of manipulation choice of firms;

2. Understand the welfare and competitive implications of manipulation;

using a model in which firms make costly manipulative advertisements in a market where there exists a naive agent.

The concept of manipulating naive agents is not a new question. Naiveté is defined as a limited understanding of strategic interactions. That is, the naive agent does not understand that the advertisements are intentional decisions made by firms and not impartial information about the state. There are some works on this subject of naiveté (e.g. Grubb (2009); Gabaix & Laibson (2006)). Conversely, to the best of my knowledge, the studies in this literature mostly assume either (1) the agents being exogenously manipulated - that is, the consumers already hold a belief different than the reality without any intervention by firms - or (2) the agents being manipulated without any cost or (3) the cost of manipulation is independent of the size of the manipulation - that is making a consumer believe in a contingency which is very improbable is not more costly than making her believe in a more probable contingency.

This paper contributes to the literature by incorporating these concerns into a contracting framework. Additionally, the model also controls for competitive pressure. Understanding the nature of costly manipulation better and how it reacts to the competitive pressure are the main considerations of this study.

The model consists of two states (S_1, S_2), where the probabilities to be realised are q and $1-q$ respectively. The state with higher (lower) probability is called the probable (improbable) state. There are two actions (A_1, A_2) and a utility function of a naive agent dependent on the state and the action taken. Each action is better than the other action in only one state. That is A_i gives utility of 1 at S_i and 0 at S_j where $i \neq j$ and $i, j = 1, 2$. At time $Z = 1$, a single firm makes a costly advertising decision from the set $\gamma = \{\gamma_1, \gamma_2, \varnothing\}$ where γ_i favours S_i and $\varnothing$ favours none (call this an empty advertisement). The game is common knowledge and none of the players know the state before the contract decision is made. I take a quadratic cost function increasing with the size of the manipulation. The advertisement reaches the naive agent with probability r and if reached, the naive agent believes that S_i happens for sure if γ_i is received (manipulated) and believes that each state happens with a probability of 0.5 if $\varnothing$ is received. If the advertisement does not reach, then the agent does not know the firm exists, and no transaction takes place. The rational agent and the firm know that the probability of S_1 happening is q. At $Z = 2$, the firm decides on the contract $T = (T_1, T_2)$ where T_i is the payment by the consumer to the firm if the consumer takes action A_i. At $Z = 3$, the agent who received the advertisement decides whether to accept or reject the contract. At $Z = 4$, the state is realised, and at $Z = 5$ the agent takes action (See Figure

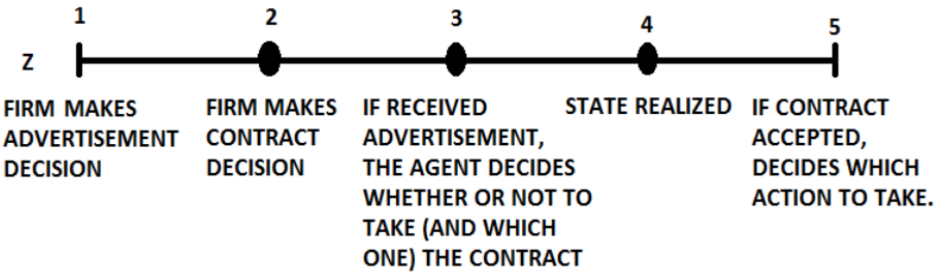


Figure 2.2: The Timing of the Game

2.2).

I solve the model both for a rational and a naive agent in a monopoly, and later I extend the model to a duopoly of symmetric firms and again solve the model both for a rational and a naive agent. Using the solution of the model, I conduct a welfare analysis for both rational and naive agents and compare them. Furthermore, I make observations on the effect of competitive pressure. The parameter r, the probability of an advertisement being received by the agent, is interpreted as a measure of competitiveness in the duopoly version of the model.

The results are as follows:

Monopoly issues an empty advertisement if the agent is rational. If the agent is naive, for lower values of manipulation cost, the firm manipulates towards the probable state if r is low enough and towards the improbable state if r is high enough. For prohibitively higher values of the manipulation cost, the firm issues an empty advertisement. The advertisement decisions can be seen in Figure 2.3. The manipulation might be exploitative for naive agents. That is, the agents would not accept the contract if they were not manipulated.

In the duopoly case, the advertisement decision is empty if the agent is rational. If the agent is naive, for higher values of the manipulation cost, the firms issue an empty advertisement. For lower values of the manipulation cost, firms would choose to manipulate towards the improbable state and towards the probable state if q is close to 0.5 and the cost is high but not prohibitive. The advertisement decisions can be seen in Figure 2.4. Further details on the nature of manipulation choice are discussed in the solution of the model below.

The contract decisions of the firms reveal a unique mixed Nash Equilibrium at which the firms compete on the prospective agent who receives advertisements from both firms. Again, contracts might be exploitative for the naive agent. On the other hand, the degree of exploitation decreases with competition, and it may even be possible that the contracts are not exploitative at all. Further discussion on the comparative statics can be found in Section 2.4.

Organisation of the study is as follows:

A literature review is conducted in the following section. The third section is the monopoly model and a discussion of the assumptions. The fourth section is the duopoly model, and the fifth is the conclusion. The sixth section is the Appendix.

2.2 Literature Review

The recently accelerated incorporation of psychology and economics has given pace to studies on the potentially boundedly-rational consumers and the market implications of these agents' existence. Bounded rationality can take different interpretations depending on the environment studied (e.g.Vigna & Malmendier (2006); Piccione & Spiegler (2012); Spiegler (2011)). The environments where there is an uncertainty about the state which affects the welfare of the consumer bring increased curiosity regarding the market implications of consumers who cannot understand the stochastic nature of the environment. This limited understanding might cause consumers to end up signing for exploitative contracts. That is agents might sign the contracts which hurt their real participation constraint - they might sign a contract which would give lower expected utility than their outside option. Not only theoretical but also experimental studies confirmed that exploitative contracts arise in many contexts (e.g. Heidhues & Koszegi (2010)).

One of the most important shortcomings of these studies is that they take the biased consumers as given, but these biases could result from firms exerting effort to manipulate agents. Additionally, the size of the bias might be determined by the size of this effort. This paper takes these concerns into consideration. To my best knowledge, there is no previous theoretical study close to mine in the literature.

Spiegler (2011) and Kőszegi (2014) are two of the comprehensive surveys on the behavioural contract theory in general. That is, market environments which allow contracts and consumers are exposed to behavioural biases. DellaVigna & Malmendier (2004) is one of the first examples of market analysis where consumers have self-control problems. Eliaz & Spiegler (2006) analyses the use of contracts on the screening the types of consumer - these types refer to the level of consumers' sophistication. Later in literature, the implications of different concepts of naivete are investigated by other studies. For instance, Eliaz & Spiegler (2008) deals with naive agents who have optimistic beliefs (they attach higher than real probabilities) on states which would provide large gains to them. Overconfidence, which is having optimistic beliefs not only on states but also the abilities one has, is studied by Grubb (2015). Another paper by Von Thadden & Zhao (2012) analyses contracts with agents who are not aware of some contingencies. The basic model, without the special features my model has, might find suitable interpretations in these studies. Additionally, the benchmark cases of the present study, where the agent is rational, can find themselves suitable interpretations in neo-classical contract theory in general such as Laffont & Martimort (2009); Mas-Colell et al. (1995); Shavell (1979).

Within the behavioural contract theory literature, the questions of whether and how competition suppresses the biases' effects within a market are central. There might be several reasons for this suppression. The most relevant reason might be the costs that firms might need to exert in order to create such biases. If these costs are undertaken before market prices settle down, high competitive pressure might prohibit such biasing. Ellison (2005) argues that the persistence of shrouding (hiding the existence of some contingencies that you will need to pay for, such as an internet connection at an hotel - naive agents are not aware of such contingencies) or biasing in a general sense might be because of costly advertisement or boundedly-rational agents.

This relationship between persistent shrouding and bounded-rationality is discussed by Gabaix & Laibson (2006) for unforeseen contingencies. In their paper, they have a competitive set up consisting of naive and rational agents. If the firm chooses to unshroud add-on prices, rational consumers and some of the boundedly-rational consumers understand the existence of add-on prices. Consumers can incur a cost to get rid of the add-on prices (e.g. bringing your own chocolate to the hotel rather than eating the ones in the hotel). What they have shown is, if the fraction of rational consumers is low enough, it might be possible to observe a shrouding equilibrium.

Even though this paper is, to the best of my knowledge, the closest study to mine, there are some major differences. First of all, the game in Gabaix & Laibson (2006) is not exactly a contracting game. Still, when naive consumers do not exert a costly effort of getting rid of the future cost, their game behaves as if it is a contracting game. This costly effort is like bringing something to drink to the hotel in order to make sure you will not use the mini bar. Secondly, the agents are heterogeneous in their model by three dimensions: naiveté, information and demand. In the present model, there is only one dimension but it has some additional features. These are the three kinds of advertisement decisions, and the parameter used for competitiveness.

Thirdly, they do not control for the probability of needing an add-on product. This exists in the present study.

I would like to emphasise that I did not assign any intrinsic meanings to the post-contractual actions in my model. It is possible to interpret it as an unforeseen contingencies model, but also for instance as a persuasive advertisement model.[1]

The studies aiming to understand this relationship between biases and competition have two main problems.

The first is that these models take exogenously biased agents as given. However, the question of how agents came to have such biases is a pre-emptive question. I do not mean how agents become naive from being rational as this question is a rather philosophical question. Rather, I mean how agents' naiveté has some particular direction.

This practice is especially important for biases whose directions are not self-evident. That is to say, we can argue that agents have present-biased preferences as the direction is mostly self-evident - there are not many people who do not have present-biased preferences (Laibson, 1997). However, if we argue that agents have reference-dependent preferences, we might need to explain how they determine this reference point (Kőszegi & Rabin, 2006; Crawford & Meng, 2011).

The direction of bias might result from different motivations. For instance, the direction of biases stemming from misinterpretation of signals (a particular mistake of Bayesian Update) might be the result of limited calculation capabilities (Tversky & Kahneman, 1975). This direction might also be generated by the effort of firms. For instance, firms might want to emphasise the status quo (in order to make the

[1] The model of this study can be interpreted as a persuasive advertisement and framing model as well. For instance, Bloch & Manceau (1999) used persuasive advertisement (advertisements increasing the willingness to pay of the consumer) in a Hotelling model. However, again to the best of my knowledge, there is not a study which is close to mine from this route as well.

status quo perceived as the reference by the consumers) as a competition suppressing effort.

Note that, when they say advertising, especially in shrouding exercises, they assume it is a debiasing effort. However, advertisements are generally used to distort beliefs, not to rectify them. The present study takes this into consideration. Additionally, there is a further advertisement option, which is an empty advertisement. It is possible to take empty advertisement as a debiasing tool. In this study, it is not because the understanding of the naive agent in the model is that they are so naive that, not being manipulated does not make the naive agent sophisticated. Indeed, the main results would not change, even if I would taken as a debiasing tool.

The second problem is that these models, to the best of my knowledge, do not take into account that the advertisement efforts which aim to affect (the size of) agent's bias are actually costly which increases with the size of the bias.

For instance, in a basic context assume that there is a gullible person you are talking to. You want to manipulate this person. Is there any cost to that? Apparently, yes. You might need to talk with this person in order to convince her, which is costly. Additionally, the size of the manipulation might be critical. You may need to talk to her for one minute in order to make her believe that you have an Aspirin in your pocket, but you might need to talk her half an hour to make her believe that you have two frogs in your pocket.

This study is also related to models of persuasion through controlling the informational environment. For instance, Kamenica & Gentzkow (2011), examine a strategic situation with a sender and receiver. The sender has a direct access to the information and having collected the information she sends a signal to the receiver. The receiver, after conducting a rational (Bayesian) evaluation takes an action which determines the payoff of both parties. They find the set of signals that can be used, successfully, by the sender. Another paper by Brocas & Carrillo (2007), with a similar environment, characterises the value of being the party who has access the information directly.

These studies are however inherently different from the present study. Firstly, these models have all rational players. They are looking for ways of persuading a rational agent. In my model, the ignorant party is also a naive one. Secondly, these models are structured for two-sided strategic interaction rather than a market situation with competition.

The present study has a basic and natural set up and attempts to understand how costly advertisement, bounded rationality and competition interact while extending the discussion in Ellison (2005).

2.3 Monopoly

2.3.1 Agents

There is a single, risk-neutral agent who is either rational or naive, whose type is known to the firm. The set of (mutually exclusive) states is represented by $S = \{S_1, S_2\}$ where S_1 occurs with probability of q (S_2 is with $1 - q$). Naiveté of the agent is defined such that his belief over the probabilities of states can be affected by the firm. The agent is offered a contract by the firm, before the state is realised, which is contingent upon his action. The action is taken after the state is realised. The set of actions is represented by $A = \{A_1, A_2\}$. I assume a particular utility function:

	A_1	A_2
S_1	1	0
S_2	0	1

That is the utility function can be written as $u(A_i|S_j) = I_{\{i=j\}}$ where $i, j = 1, 2$ and I is the indicator function. These are gross utility functions before price. The utility functions and the game itself are common knowledge to both the agent and the firm.

The contract is represented as $T = (T_1, T_2)$ where T_1 is the payment by the agent to the firm if the agent takes action A_1, and T_2 is for A_2. The contract is contingent on the action rather than the state as it is assumed that the state is not verifiable, but actions are. These payments might be negative, and in this case, it is a payment from the firm to the agent. If he signs the contract, call him a customer.

The rational agent and the naive agent differ in their understanding of states' probabilities. Assume q_b is the belief of the agent on q. For a rational agent $q_b = q$. On the other hand, the naive agent constitutes his belief after he sees the advertisement. The advertisement might have a manipulative nature in the sense that it affects the belief of the naive agent. That is to say, if the manipulative advertisement favours S_1, then the naive agent has $q_b = 1$. If it favours S_2, then $q_b = 0$.

Apart from manipulative advertisement favouring one of the states, there is one more advertisement which is empty. In this case the naive agent believes $q_b = 0.5$.

The advertisements also include information about the existence of the firm. For this reason, if the agent does not receive any advertisement from the firm, he does not know about the firm as well. In this case, there will be no transaction. The probability the advertisement reaches the consumer is r. The contract decision of the firm is made after the advertisement decision and then, being manipulated by the advertisement, the agent considers the contract offered.

The agent has an outside option of zero before he accepts any contract, but has only two actions after he decides to take the contract. I assume as a tie-breaking rule that, if the contract generates 0 surplus (from the agent's perspective), then the agent accepts the contract. If two actions generate the same non-negative surplus to the agent, he picks the one which generates more profit to the firm; if they are equal, he randomly picks one of them.

Note that, when I say "agents", I do not mean there are multiple agents. I mean there are multiple types of a single agent, in terms of the type of the advertisement he receives. In any subgame, there is only one type of agent.

2.3.2 Firm

The firm's objective is maximising its expected profit.

The marginal cost of each action, taken by the agent, to the firm is zero. The firm first decides on the type of the advertisement which also carries information about the existence of the firm. The set of advertisements is $\gamma = \{\gamma_1, \gamma_2, \varnothing\}$. γ_1 is manipulative effort towards S_1 and γ_2 is for S_2. Empty advertisement effort ($\varnothing$) carries only the information about the existence of the firm. The cost for each advertisement is $c_1, c_2, c_\varnothing$ respectively.

I assume that the cost function has the following structure. $c_\varnothing = 0$, $c_1 = k(1-q)^2$, $c_2 = kq^2$ where $k > 0$. This functional form has two merits. Firstly, it increases with the size of the manipulation. When I say size, for instance for the manipulative advertisement towards S_1, the manipulated belief diverges from reality by $1-q$. In other words, it is hard to create a convincing manipulative advertisement for improbable events. Secondly, it is quadratic. That reflects the thought that it is harder to change belief to the direction of the closer extreme than the further extreme and this hardness ratio is more than the ratio of the distances to the extremes.

Next, the firm decides on the contract offered. It is assumed that contract terms are contingent on the action taken. For the monopoly, the sequentiality of the manipulative advertisement and the contract decisions does not matter, although it does matter for the duopoly case which I will adapt the model later on.

Lastly, the manipulative advertisement is received by the agent with probability r. The agent can see the contracts if he receives the manipulative advertisement by that company. "r" is the reachability of the advertisement. If he does not receive any advertisement, the agent stays inactive. This is by Grossman & Shapiro (1984).

2.3.3 Timing of the Game

There are five periods (See Figure 2.2). At $Z = 1$, the firm decides on manipulative advertisement. At $Z = 2$, the firm decides on contract. At $Z = 3$, if the agent observes at least one advertisement, then he decides whether or not to accept the contract offered. If not, he stays inactive. At $Z = 4$, the state is realised. At $Z = 5$, the agent, if he is a customer (if he signed the contract), decides on which action to take and makes the payment. He stays inactive otherwise.

2.3.4 A Simple Example (with naive agent)

Let me examine the example I described in the introduction. Assume that there is a cable TV company. Assume that the company offers a package, HD Triple Play, with 120 channels. There are also some other channels available with some additional payment. There are two possible states: S_1 is the state in which the agent likes to watch HD Triple Play channels, and S_2 is where he likes to watch other channels. Assume that the agent will get fun from only one of the channels. A_1 is the post-contract action where the customer watches an HD Triple Play channel A_2 is the action where he watches a channel, not in HD Triple Play. The utility function is as follows:

	A_1(watches a HD Triple Play channel)	A_2(watches a channel not in HD Triple Play)
S_1(likes a HD Triple Play channel)	1	0
S_2(likes a channel not in HD Triple Play)	0	1

At $Z = 1$, firm issues the poster:

Note that this poster (like in Figure 2.1, I cut the phone number and a small script for the sake of simplicity) represents the manipulative advertisement. It has only the two parts required. The first one is the information regarding the firm's existence: its name. The second one is the effort of manipulation towards S_1; that is the state at which the agent will like HD Triple Play channels.

At $Z = 2$, the firm determines the contract offered. Assume that the agent is naive. The contract will be $T = \{T_1, T_2\}$ where T_1 is the payment when the agent takes the action A_1 and T_2 for $A_{2.}$ In words, T_1 is the amount of the payment when the customer watches HD Triple Play channels. T_2 on the other hand, is the payment for watching other channels. Assume that the firm set $T_1 = 1$ and $T_2 = 2$.

Assume that, the agent sees the poster (receives the manipulative advertisement). At $Z = 3$, he will decide whether or not to accept the offer. If he is naive, he thinks

S_1 will happen for sure, then his evaluation of the contract will be false. That is $1 - T_1$ rather than $(1 - (qT_1 + (1 - q)T_2))$. The naive agent will decide to accept the contract as $1 - T_1 = 0 \geq 0$.

At $Z = 4$ the state will be realised. Assume that S_2 turns out to be the prevailing state. In this case, the agent will be surprised as he would not expect S_2 to happen and at $Z = 5$ he will take A_2 as $2 - T_2 \geq 1 - T_1$. Note that he would not have wanted to sign the contract in the first place as $(1 - (qT_1 + (1 - q)T_2) = 1 - (q + (1 - q)2) = 1 - 2 + q = q - 1 < 0$. He signed an exploitative contract.

Note that T can be interpreted as follows. The agent makes a payment of 1 for sure, and if he takes A_1, he does not pay anything and if he takes A_2 then he pays 1 more. This is just a matter of interpretation; it does not affect any result.

2.3.5 Discussions of Assumptions

" I assume a particular utility function".

For the sake of simplicity, I take two post-contractual actions and state space of two mutually exclusive elements. There are, of course, an infinite number of utility functions which can be assumed. Two main kinds of utility functions appear in the analysis. Firstly, the ones at which one action is always better than the other. Analysis of this kind is straightforward as the firm will always make (and will be able to make) the agent take the dominating action. Secondly, the ones in which one action is better at one state but worse in the other. This case draws more attention as it naturally arises more often and additionally it makes the states and actions jointly relevant to the analysis. Again, for the sake of simplicity, I assume a very simple utility function of only zeros and ones.

Additionally, both cases can be interpreted as an alternative formulation to the persuasive advertisement (advertisement aimed to increase the willingness to pay of the consumer). However, there are some crucial differences. The most important difference comes from welfare analysis. For the persuasive advertisement, it is generally not obvious which welfare function to use. Think about a watch. In the case of persuasive advertisement, the advertisement increases your willingness to pay; say from 50 dollars to 100 dollars. Hence, you buy the watch. Now, what is the consumer surplus? Is it 100-price of the watch or 50-price of the watch? Both can be justified to some extent. However in the present study case, even if it can be formulated in a persuasive advertisement framework, there is not such issue.

"The manipulative advertisement includes information about the existence of the firm." " The manipulative advertisement is received by the agent with probability r."

These assumptions are two of the crucial assumptions. Poster analogy will lead us to the former part of the statement above. That is to say, advertisements also include information about the existence of the firm. It is not optimal for a firm not to append the information regarding its existence to the poster.

Regarding the reachability r of the manipulative advertisement, it is possible to make r a choice variable of the firm. This would increase the complexity of the problem. Tractability is the main reason why I refrain from doing so.

"The set of advertisements is $\gamma = \{\gamma_1, \gamma_2, \varnothing\}$. The costs for each advertisement are $c_1, c_2, c_\varnothing$ respectively."

The set of advertisements has only three elements: The one which attempts to persuade the agent S_1 will happen for sure, the one which attempts to persuade the agent S_2 will happen for sure and a silent one. It can be thought that there are some other manipulative advertisements which aim to some intermediate levels. This is a fair criticism. The naive agent in the model is so naive that, he cannot calculate and interpret messages easily. Additionally, our language is not that elaborate. Extreme messages are much easier stated. Because of these reasons and for the sake of simplicity γ is taken to have three elements.

It is assumed that when the manipulative advertisement is empty then $q_b = 0.5$. One might think that it is more suitable to assume $q_b = q$. This will be touched upon in the discussion section. Again, in the present study, the agent has a very simple mind: he cannot conduct any complicated evaluation about the state's probability.

Additionally, I assume the cost function has the following structure. $c_\varnothing = 0$, $c_1 = k(1-q)^2$, $c_2 = kq^2$. It is possible to think of some alternative cost structures, but because of the merits I have stated (the convex and increasing structure) and tractability issues, I will follow this particular functional form.

" There are five periods. At Z=1, the firm decides on advertisement. At Z=2, the firm decides on contracts. "

The most crucial assumption here is sequentiality of the manipulative advertisement decision and the contract decision. This assumption is rather realistic. Why? A manipulative advertisement is not something one may change easily. If a firm tries to persuade the agent to believe there will be a lot of rain this year, then it is very hard for it to change the decision over this advertisement. This is due to several reasons including reliability issues arising after an ambivalent behaviour and high costs of preparing and disseminating new advertisements. Conversely, a contract decision is rather flexible.

2.3.6 Solution of the Monopoly Model

2.3.6.1 Rational Agent

If the agent is rational, then the monopolist will choose an empty advertisement. This is because a rational agent, by definition, is not manipulable and the only thing that matters is the informative content of the advertisement.

Proposition 2.3.1. *The Monopoly Outcome in the Rational Agent Case*

The monopolist issues an empty advertisement. The optimal contract is $T = (T_1, T_2)$ *such that* $qT_1 + (1-q)T_2 = 1$, $|T_1 - T_2| \leq 1$ *(See Appendix for the proof).*

This is a straightforward result: the monopolist sets the price of the good equal to the willingness to pay of the consumer. The profit-maximising contract's expected revenue is equal to the willingness to pay of the agent ($qT_1 + (1-q)T_2 = 1$). The condition $|T_1 - T_2| \leq 1$ makes sure that the agent takes the action with the highest willingness to pay for the state realised. The entire surplus is extracted from the agent. The profit is equal to $r * 1 = r$. There is no exploitation of the agent, as he is not naive.

2.3.6.2 Naive Agent

As expected, costs are vital. As the evaluation is symmetric for $q \leq 0.5$ and $q \geq 0.5$, I conduct the analysis for $q \geq 0.5$.

Proposition 2.3.2. *The Monopoly Outcome in the Naive Agent Case*

- $\gamma = \gamma_1$ *and* $T_1 = 1, T_2 = 2$ *if* $\frac{k}{r} \in [1, \frac{3-4q}{2(1-q)^2}]$.
- $\gamma = \gamma_2$ *and* $T_1 = 2, T_2 = 1$ *if* $\frac{k}{r} \leq \frac{1}{2q^2}$ *and* $\frac{k}{r} \leq 1$.

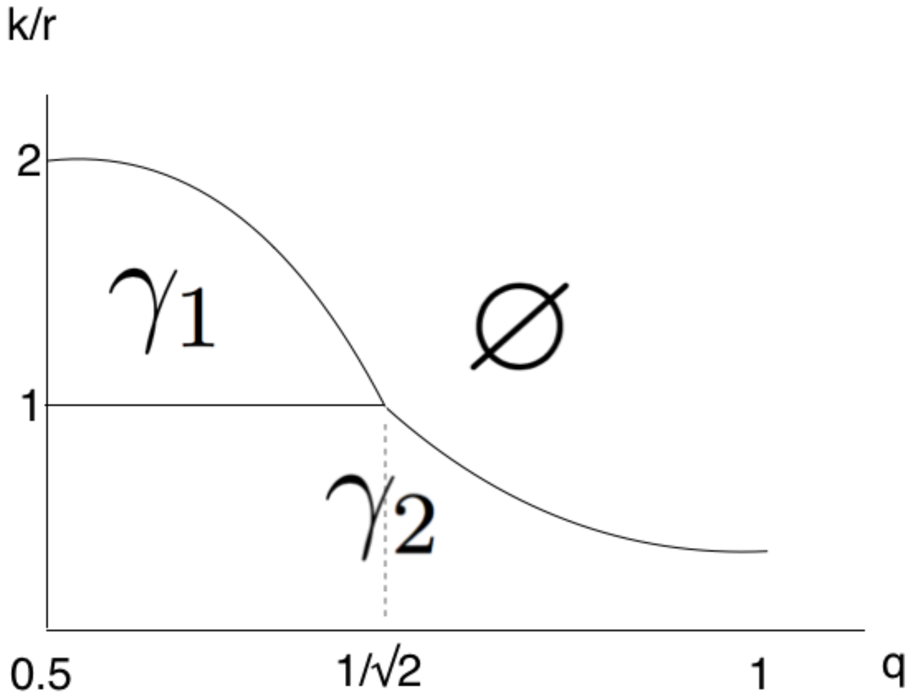


Figure 2.3: The Advertising Decision of a Monopolist, Naive Agent

- $\gamma = \varnothing$ *and* $T_1 = 3/2$ *and* $T_2 = 1/2$ *otherwise. (See Appendix for a proof.)*

In words, the statement above says a few main things:

1. If q is close to the extremes, and k/r is low enough, manipulate towards the other extreme.
2. If q is close to 0.5 and k/r is high but not prohibitive, then manipulate towards the probable state.
3. If q is close to 0.5 and k/r is low, then manipulate towards to other extreme.
4. If k/r is prohibitively high, issue an empty advertisement.
5. The requirement on k/r gets tighter -for the decision over manipulation or not- as q gets closer to the extremes. When I say tighter I mean that for a

given q, k/r needs to be lower for manipulation to be the outcome compared to lower levels q.

Note that, the pricing decision when manipulating towards the probable state (S_1) is such that $T_1 = 1$ and $T_2 = 2$. In this case the consumer will take only T_1 into account and hence he is indifferent between signing the contract and not. However, T_2 is chosen in extreme, 2, which is overlooked by the consumer who is manipulated. The symmetric evaluation can be made for S_2. In the case where the monopolist issues an empty advertisement, the prices are $T_1 = 3/2$ and $T_2 = 1/2$. This is because the monopolist chooses the prices where their mean to be 1, but T_1 is as high as possible. This is because, the probability of S_1 to happen is larger than S_2.

Note that k/r is crucial when determining the optimal manipulative advertisement. k/r (for not prohibitively high values) reflects the trade-off between manipulating towards the first and the second state. Higher k means the manipulation is hard. It is less costly to manipulate towards the probable state. Higher r means, the reach of the advertisement is high. Higher reach makes the manipulation towards the improbable state more profitable. This trade-off reflects itself to k/r. Therefore, for prohibitively high values of k/r, it is too costly to manipulate towards any state. For not prohibitively larger values of k/r, it reflects the trade-off between manipulating towards the first and the second state.

The advertisement choice part of the result is a shifted version of the duopoly solution I will examine in detail later. It is a shifted version because as you will see, the duopolists are the monopolist on some prospective fraction of the naive agent. For pricing see that there is a difference of one between terms of contracts because of the use of an incentive compatibility constraint (the constraint which makes sure the consumer take profit maximising action) in order to exploit the participation constraint.

An interesting implication is, crudely, as follows. If you have limited reach, and the cost of manipulation is high, try to direct the belief of people to the probable state. As a monopoly, if you have a large (r), a gullible audience (naive) and efficient manipulation devices (small k), tell a "big lie" (manipulate the agent towards the improbable state).

2.4 Duopoly Model

It is possible to extend the monopoly model for the duopoly case. Define T^i as the contract offered by firm i.

I extend the discussion on manipulation for the duopoly.

The timing is extended as follows. After the simultaneous advertisement decisions, firms observe the other firm's advertisement choice. Later, each firm observes the belief of the consumer. However, firms cannot observe which advertisements are observed by the consumer. Then next, firms simultaneously decide the contracts they will offer.

The agent forms her belief as follows:

- If an agent receives only one advertisement, then the belief is formed just like the monopoly case. That is if she receives either $\gamma_1, \gamma_2, \varnothing$, she believes $q = 1, 0, 0.5$ respectively.
- If an agent receives two identical advertisements, then the belief is formed just like she receives only one of them. For instance if she receives γ_1, γ_1 then she believes $q = 1$.
- If an agent receives two conflicting advertisements, then she forms her belief

according to an average rule. That is if she receives γ_1, γ_2, then she believes $q = \frac{1+0}{2} = 0.5$. If she receives $\gamma_1, \varnothing$, then she believes $q = \frac{1+0.5}{2} = 0.75$. If she receives $\gamma_2, \varnothing$, then she believes $q = \frac{0+0.5}{2} = 0.25$.

- If an agent does not receive any advertisement, we assume she is inactive within the market. It is known to firms if she is inactive.

Tie-breaking for two firms is as follows. If the firms offer the contracts generating the same non-negative utility under the agent's belief, then the agent chooses one of the firms with equal probability.

In the case of conflicting advertisements received by the consumer, each firm can infer if the advertisement of the other firm is received by the consumer. Let me analyse the information sets of each possible subgame before the pricing decision.

- The firms have chosen the same advertisements.

  In this case, if consumer received at least one advertisement, she holds the same belief what the firm advertises. However, firms do not know which advertisements the agent observed.

- The firms have chosen different advertisements.

  In this case, each firm can infer if the consumer received only its advertisement or both. For instance assume that firm 1 and firm 2 advertised γ_1 and γ_2 respectively. Take firm 1. It knows that the consumer received both advertisements if and only if her belief is $q = 0.5$. It also knows that the consumer received only firm $1's$ advertisement if and only if $q = 1$. A symmetric analysis is also true for firm 2. A similar analysis can be conducted for other combinations of advertisement choices.

One might disagree the q_b being 0.5 when they received empty advertisements only.

An alternative might be taking $q_b = q$. I discuss this alternative specification in the conclusion and discussion section.

Additionally the reachability parameter r gets an additional and crucial meaning in the duopoly case. The agent receives an advertisement with a probability of r for each firm. As r increases, the possibility that the agent receives both advertisements increases, hence the agent will have a better opportunity to compare the prices. That is r can also be interpreted as a measure of competitiveness.

The solution concept is Subgame Perfect Nash Equilibrium. This is because of the sequentiality of the decisions made by the firms. Secondly, the naive or the rational agents are only the decision makers.

2.4.1 Solution of the Duopoly Model

2.4.1.1 Rational Agent

Before stating the next proposition, define P^i. This is the expected payment the rational agent will pay if he accepts T^i. Therefore $P^i = qT_1^i + (1-q)T_2^i$ where the incentive compatibility constraint holds. This is a sufficient statistic for (T_1, T_2). The reason is that, the rational agent will base his decision on the expected payment he will make (which turns to be the correct belief). The firm will also only care about the expected payment it will receive from the rational consumer- the same T^i is evaluated by the firm and rational agent in exactly the same way. Note that, for $P^i = qT_1^i + (1-q)T_2^i$, I need $|T_1 - T_2| \leq 1$. This inequality makes sure the consumer chooses T_i in the state S_i. For each contract where this inequality does not hold (e.g. $T_1 - T_2 < -1$), there is another contract obeying this inequality (e.g. $T_2 = T_1$), and generating the same expected profit (T_1) for a firm and higher expected utility for the consumer ($1 - T_1$ rather than $q - T_1$). Additionally, for each contract where

$T_1 \neq T_2$ but obeying $|T_1 - T_2| \leq 1$, the contract $T_1^* = T_2^* = qT_1 + (1-q)T_2$ generates the same expected profit for the firm and the same willingness to pay for the consumer. P^i therefore, represents many equivalent contracts with the same expected payment without excluding best responses. For these reasons, without loss of generality, the firms compete on the expected payment where the contract exerts the same payment for each contingency.

Proposition 2.4.1. *The Unique Equilibrium Outcome in the Rational Agent Case*

The equilibrium outcome is that the firms conduct empty advertisement and compete on the expected payment P^i, which is distributed with CDF of $F(P)$, $P \in (1-r, 1]$ (See Appendix for the proof).

$$F(P^i) = 1 - \frac{(1-P^i)(1-r)}{P^i * r}$$

Before discussing Proposition 2.4.1, I would like to emphasise that this solution represents many equivalent solutions. The outcome is unique in the sense of P; any contract not violating the incentive compatibility constraint and the same expected payment as P can be used for any P.

Let us try to understand the contract setting strategy at $Z = 2$.

If the firms had maximum reach, that is $r = 1$, then $F(P) = 1$ for $P \in (0, 1]$. Hence $P = 0$ for both firms; this is the simple Bertrand solution as expected.

What happens if the reach of an advertisement (r) increases? The equilibrium distribution with a lower reach stochastically dominates (in the first sense) the higher reach. This means, in this context, as the reach increases the expected prices are lower. The reason is that, the higher reach increases the competitive pressures on prices.[2]

Lastly, I analyse the welfare implications.

Proposition 2.4.2. *The Welfare Implications of Pricing in the Rational Agent Case*

- *The expected surplus of the agent who receives only one advertisement is* $1 + \frac{1-r}{r}\ln(1-r)$.
- *The expected surplus of the agent who receives both advertisements is* $1 - \frac{2(1-r)}{r}(\frac{(1-r)\ln(1-r)}{r} + 1)$.
- *The expected profit of each firm is* $r(1-r)$ *(See Appendix for a proof).*

Note that the prospective agent who receives both advertisements has a larger expected surplus than the other prospective agent because he can compare two prices.

Additionally, the expected surplus of the agent increases with r, as competitive pressure shifts surplus towards the rational agent.[3]

2.4.1.2 Naive Agent

Proposition 2.4.3. *Advertisement Decisions in the Naive Agent Case*

---

[2]

$$d(1 - \frac{(1-P)(1-r)}{P*r})/dr = (1-P)/(Pr^2) > 0$$

[3]

$$\begin{aligned} d(1 + \frac{1-r}{r}\ln(1-r))/dr &= -\frac{1}{r^2}\left(r + \ln\left(1-r\right)\right) > 0 \\ d(1 - 2\frac{1-r}{r}(\frac{(1-r)\ln(1-r)}{r} + 1))/dr &= \frac{2}{r^3}\left(2r + 2\ln\left(1-r\right) - 2r\ln\left(1-r\right) - r^2\right) > 0 \end{aligned}$$

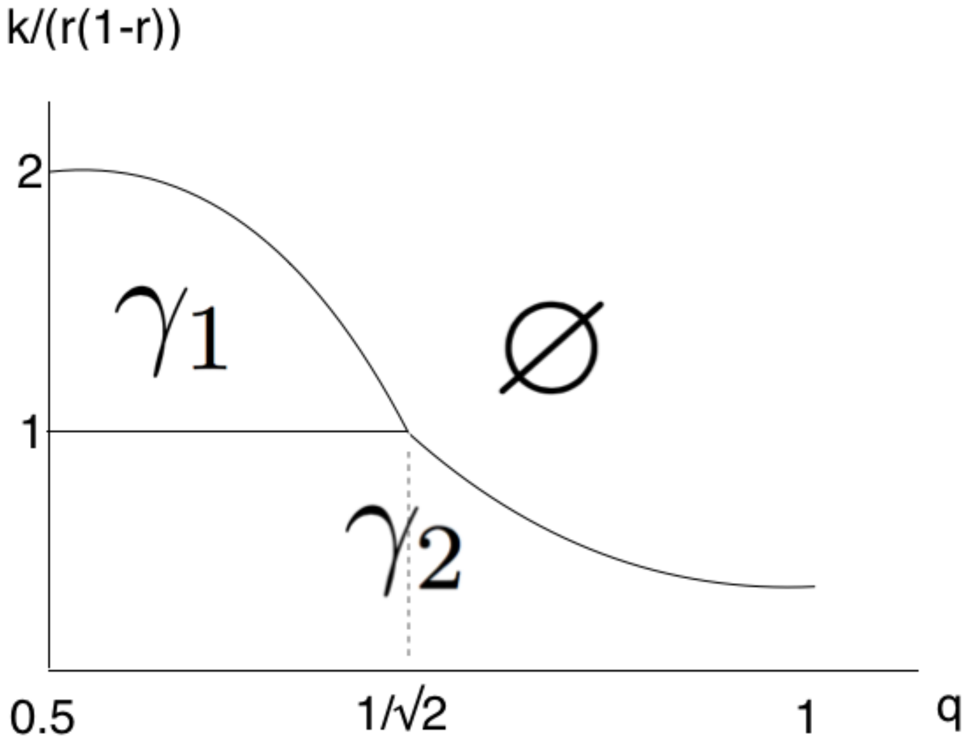


Figure 2.4: The Advertising Decision of a Duopolist, Naive Agent

The advertisement decisions at $Z = 1$ are as below:

- *The firms choose advertisements favouring S_1 if $\frac{k}{r(1-r)} \in [1, \frac{3-4q}{2(1-q)^2}]$ and S_2 if $\frac{k}{r(1-r)} \leq \frac{1}{2q^2}$ and $\frac{k}{r(1-r)} \leq 1$.*
- *The firms choose an empty advertisements otherwise (The proof is in the Appendix).*

The advertisement decisions for q and $k/(r(1-r))$ are depicted in Figure 2.4. I now analyse the graph:

Observation 1

If $k/(r(1-r))$ is prohibitively large, then the equilibrium outcome is empty advertisement. $k/(r(1-r))$ can be very large because when k is large which means the manipulative advertisement cost is large. Alternatively $r(1-r)$ can be very small. $r(1-r)$ is the probability that the agent only hears about a firm's advertisement. If this probability is very small, then the gain from those agents is very low. This is the reason why $k/(r(1-r))$ being large makes the empty advertisement the equilibrium outcome.

Observation 2

$k/(r(1-r))$ determines the trade-off between γ_1 and γ_2 in the equilibrium outcome when q is close to 0.5. First note that, if q is large, then manipulation towards the first state is not profitable because the agent's belief is very close to the real odds. If q is close to 0.5, then the gain from manipulating towards one of the states is more profitable because the agent's belief is diverging from the extremes largely. In this case, the outcome is towards manipulating to the improbable state (γ_2), becomes more profitable if the cost is very low or the probability the agent will hear about only a firm's advertisement is large. This trade-off represented by $k/(r(1-r)) > 1$ or not.

Observation 3

What happens as r changes? When $r < 0.5$ ($r > 0.5$) then increasing r shifts the curve between manipulative advertisements and empty advertisement in the graph up (down). This is because the profit is determined by the expected size of the agent who receives only one firm's advertisement, that is $r(1-r)*$expected profit from this agent. This profit increases if r increases (decreases) when $r < 0.5$ ($r > 0.5$). Thus, manipulation gets more profitable compared to $\gamma = \varnothing$.

Proposition 2.4.4. *The Equilibrium Pricing Decisions, Naive Agent*

The pricing decisions at $Z = 2$, uniquely, are given below ($i = 1, 2$).

$$if\ \gamma = \varnothing\ and\ then\ \ F^i(T_2^i) = 1 - (\frac{(1-r)}{r(q+T_2^i)}(\frac{1}{2}+q) - \frac{(1-r)}{r})\ where\ T_2^i + 1 = T_1^i$$

$$if\ \gamma = \gamma_1\ then\ F^i(T_1^i) = 1 - (\frac{(1-r)(2-q)}{r(T_1^i+1-q)} - \frac{(1-r)}{r})\ where\ T_1^i + 1 = T_2^i$$

$$if\ \ \gamma = \gamma_2\ then\ F^i(T_2^i) = 1 - (\frac{(1-r)(1+q)}{r(T_2^i+q)} - \frac{(1-r)}{r})\ where\ T_2^i + 1 = T_1^i$$

(See proof in the Appendix).

Now I analyse the pricing decisions.

I will start analysing these equilibrium distributions by comparing with the rational case. I will conduct the analysis for γ_1.

In order to be able to compare it, I write the equilibrium distribution rational case in a form $T_1 + 1 = T_2$ and $T_1 = T$. $F(T) = 1 - \frac{(q-T)(1-r)}{r(T+1-q)}$. For the equilibrium manipulating towards the first state $F(T) = 1 - \frac{(1-T)(1-r)}{r(T+1-q)}$. Note that, the latter distribution stochastically dominates the former one, in the first sense. This reflects the suppressed competitiveness in the case of manipulation. When I say suppressed competitiveness, I mean that the profit from selling to the prospective consumer who only sees that firm's advertisement is higher in the case of naive consumer. This fact reflects as a higher equilibrium payoff. This can be interpreted as suppressed competitiveness introduced by the presence of naive consumer compared to the rational consumer.

A similar exercise can be conducted for γ_2. Again a similar exercise would also show the empty advertisement equilibrium distribution stochastically dominates the rational case. This reflects how naivete is exposed to exploitation.

The next thing to discuss is the comparative statics of pricing.

Firstly, I conduct the comparative statics over the reach of the advertisement. For $\gamma = \gamma_1$. $dF^i(T_1^i)/dr > 0$ and for $\gamma = \varnothing$ and $dF^i(T_1^i)/dr > 0$. These calculations show that the pricing with lower reach of advertisement stochastically dominates, in the first sense, the higher one. That is as r decreases the expected price increases. However, it does not mean that the profit increases. The reason is if $r > 0.5$ higher reachability means lower probability that the agent will see only one firm's advertisement. So, even though the expected price increases, when $r > 0.5$ the agent will have higher probability, therefore higher chance to be able to compare prices.

Secondly, I conduct the comparative statics over q. For $\gamma = \gamma_1$ $dF^i(T_1^i)/dq > 0$ and for $\gamma = \varnothing$ and $dF^i(T_1^i)/dq > 0$. These calculations show that the pricing with lower q stochastically dominates, in the first sense, the higher one. That is as q decreases, the expected price increases. The reason is as q approaches 1, the naive beliefs of the agents approach to the true q. Therefore, the exploitability decreases and this reflects the prices.

Additionally, when I say the agent who receives one advertisement and the agent who receives both advertisements, I mean the prospective agent who receives one advertisement and two advertisements. It does not mean there are two agents in a subgame. I also call the prospective agent who receives one advertisement as the *less-informed agent* and the agent who receives two advertisements as the *better-informed agent.*

Proposition 2.4.5. *The Welfare Implications of Pricing for the Naive Agent Case*

For $\gamma = \gamma_1$,

- *The expected surplus of the less-informed agent is* $1 + \frac{(1-r)(2-q)}{r}\ln(1-r)$.
- *The expected surplus of the better-informed agent is* $1 - 2\frac{(1-r)(2-q)}{r^2}((1-r)\ln(1-r)+r)$.
- *The expected profit of each firm is* $r(1-r)(2-q)$.
- *The expected ex-ante surplus of the consumer is* $2r(1-r)(q-1)+r^2$.

  (Write $1-q$ *in these expressions instead of* q *for* $\gamma = \gamma_2$*).*

For $\gamma = \varnothing$

- *The expected surplus of the less-informed agent is* $1 + \frac{(1+2q)(1-r)}{2r}\ln(1-r)$.
- *The expected surplus of the better-informed agent is* $1 - \frac{(1+2q)(1-r)}{r^2}((1-r)\ln(1-r)+r)$.
- *The expected profit of each firm is* $r(1-r)\frac{1+2q}{2}$.
- *The expected ex-ante surplus of the consumer is* $2r(1-r)\frac{1-2q}{2}+r^2$. *(Write 1-q in these expressions for* $\gamma = \varnothing$ *and* $q \leq 0.5$*). (See Appendix for the proof.)*

Corollary 2.4.6. *A Comparative Statics for the Welfare of the Naive Agents*

- *The expected surplus of the consumer decreases as the size of the manipulation increases or competitiveness decreases.*
- *The expected difference between a better-informed agent and a less-informed agent surpluses decrease with the size of the manipulation.*
- *This difference increases with competitiveness up to some threshold and decreases after that threshold as r increases. (See Appendix for a proof.)*

Note that the expected surpluses might be negative. For instance for $r = 0.2$ and $q = 0.9$ all payoffs are negative for γ_2. This is different from rational agents. As there is no exploitation opportunity for rational agents, the surpluses are always positive in both cases.

Remember that I defined the size of manipulation as the size of the consumer's belief's divergence from truth. So if the consumer is manipulated towards S_1 (S_2), the size of manipulation is $1 - q$ (q.)

Without loss of generalisation, I conduct the qualitative interpretation here for equilibrium outcome of manipulating towards S_1. The expected payoffs of the agents increase if q increases. Another way of saying is that, the expected payoff of the consumer increases if the size of the manipulation decreases. Additionally, r reflects the competitiveness of the market. Higher r means higher probability of receiving price information from both firms (r^2). Increasing the size of the manipulation reflects differently on the damage on the expected surpluses of the agents receiving one or two advertisements. This damage is lower for the agent who receives both advertisements.

The surplus of the agent who receives both advertisements is higher than the one who receives only a single advertisement. Additionally, these differences decrease as q increases (for γ_1 in equilibrium) because the exploitation decreases as the beliefs come closer to the truth. This decrease of exploitation reflects on the less-informed agent better because the cumulative density function domains get narrower.

From lower values of r, the difference increases as r increases because the contracts are extremely exploitative, therefore comparison makes a crucial difference. However, after some threshold the difference decreases because the contracts will be very competitive, this will make the less-informed agent less relevant.

It is also possible to compare the surpluses of the naive agent with the rational agent. Payoff differences decrease in r and q (for γ_1 in equilibrium). They decrease when r increases because the competitive pressure decreases exploitation. They decrease when q increases because the gain of the firms from deception decreases as the beliefs of the naive agent comes closer to the reality.

Finally, these results imply some regulation advice. However, it is very hard to talk about any regulation policy (on advertisement, not on pricing) on this particular issue. It is not like any other exploitation of a bias. For instance, the inefficiency resulting from shrouding can be rectified via a transparency policy. Or the inefficiency resulting from time inconsistency can be rectified via compulsory self-control devices. On the other hand, the kind of bias this study deals with has no a mainstream regulation device. For instance, it is not very mainstream to stop a firm from advertising about a state's probability (true or not) or to force a firm to advertise about a state's probability. This is because, for most cases, the true probability will not be verifiable. Additionally, it is very hard objectively to distinguish the manipulative advertisements from other kinds of advertisements. Yet, the policy advice would be the following, if applicable:

If the policy maker values the profit of the firms and consumer surplus equally, then as this is a zero sum game, the advice would be issuing an empty advertisement, which is not costly. However, if the policy maker cares only about the total consumer welfare:

- Increasing r, that is for instance creating a poster (it can be also a different tool, for the example in this paper, it is poster) areas in most popular places.
- If $q > 0.75$ make sure firms advertise towards S_1, if $q < 0.25$ towards S_2 and empty advertisement if else. This policy retracts the wrong belief towards the truth and decreases exploitation.

2.5 Discussion & Conclusion

In this study, I analyse the relationship between competition, bounded rationality and manipulative advertisement. Monopoly and duopoly cases for both naive and rational agents are solved. The results are compared and contrasted; and the welfare implications and advertisement choices are derived.

In the monopoly case, the rational agent cannot be exploited, as expected. On the other hand, the naive agent is both manipulated and exploited. The direction of the manipulation depends on the relative size of the cost and reach of advertising. If the cost is large but not prohibitive, the manipulation is towards the probable state; otherwise, it is towards the improbable state.

In the duopoly case, there is a unique Subgame Perfect Nash Equilibrium for both the rational and the naive agent.

For the rational agent, the advertisement choice is empty. The pricing is not exploitative. For the naive agent, the advertisement choice depends on the relation between cost (k), competitiveness (r) and true belief (q). The main features are as follows. If the cost is large, the advertisement choice is empty. If the cost is high, but not prohibitive and q is around 0.5, the advertisement choice favours the probable state. If the cost is low, the advertisement choice favours the improbable state. The prices might be exploitative and they get less exploitative as competitive pressure increases and q approaches to the manipulated beliefs. I also made comparisons with the pricing and welfare of the rational agent case.

The expected surplus of the consumer decreases if the size of the manipulation increases or competitiveness decreases. The expected difference between the better-informed agent and the less-informed agent surpluses decrease with the size of the manipulation. This difference increases in competitiveness up to some threshold and decreases after that threshold as r increases.

Of course, the model I studied has limitations as well, mostly because I wanted to keep the model tractable. Assumptions are discussed in the subsection 2.3.5.

I first discuss the utility function used in the model. Changing the entries of the utility matrix, I conjecture, would give similar results unless the property of having one action being better only in one state is kept. There would be normalisations in the pricing and advertisement decisions, but the main results would still be valid. Increasing the size of the state or action set might make the model hard to track. In this case, one might need to have some reductive assumptions which might be suitable in some frameworks. For instance, for demand functions of telecommunication services, one might think that the agent has a linear, increasing utility function up to some satiation point (Grubb, 2009).

For the advertisement choices, one issue is the educative advertisement. When I say educative advertisements, I mean the advertisements which makes the recipient consumer have correct beliefs about the states. Sometimes, advertisements might be in the form of educating consumers. For instance, the advertisement might warn consumers about the contingencies the consumers are not aware of. If the empty advertisement is educative, the main results do not change and the general interpretation is still valid. The reason is, the only change is the profit expected from issuing educative advertisement would be $r(1-r)$.

Let me do the analysis for $q \geq 0.5$. In this case, in duopoly, at $Z = 1$ the firms would decide to educate consumers if $\frac{k}{r(1-r)} \geq \frac{1}{1-q}$ and $\frac{k}{r(1-r)} \geq \frac{1}{q}$ and manipulate towards S_1 if $\frac{k}{r(1-r)} \in [1, \frac{1}{(1-q)}]$ and manipulate towards S_2 if $\frac{k}{r(1-r)} \leq 1$ and $\frac{k}{r(1-r)} \leq \frac{1}{q}$. The proof is the same except we take empty advertisement profit as $r(1-r)$ rather than $\frac{3-2q}{2} r(1-r)$. The advertisement decisions are represented by Figure 2.5.

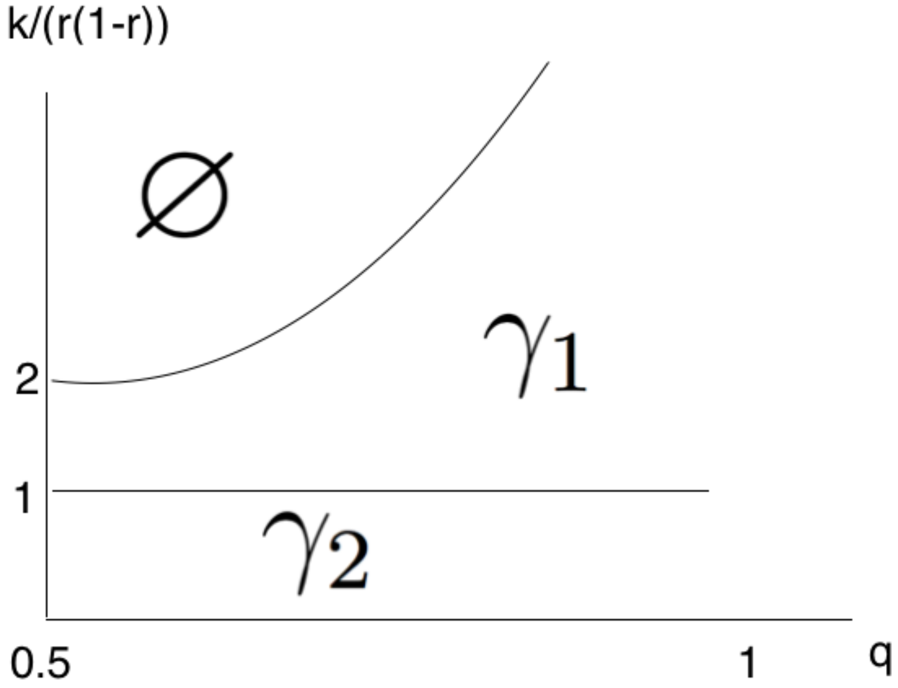


Figure 2.5: Advertisement Decisions, Empty Advertisement is Educative

It can be observed that the bell-shaped advertisement decision is somewhat preserved. However, the set of k,q's where the firms choose empty advertisement is narrower. The reason is once it is educative, there is no room for exploitation anymore. Note that, as q gets closer to 0.5 then the firm needs less k to do empty advertisement. The cost of manipulation increases as the size of the manipulation, to any direction increases. Furthermore, the advertisement decisions are not empty anymore for high k even if q is low. As the empty advertisement is not exploitative anymore, manipulation will be pervasive. I would like to emphasise one important point. It is hard to claim that the educative advertisement is costless. It might alternatively have a constant cost. In this case, the educative advertisement region would be, very similar to what you see in Figure 2.4, but even narrower as expected.

The analysis for the educative advertisements is also a suitable response to a possible criticism. One might think the cost of manipulation should be defined over the distance from the beliefs in the case the agent receives empty advertisements only. That is to say $c_1 = k(0.5-1)^2$ and $c_2 = k(0.5-0)^2$. In the duopoly model, this would create constant costs which means the cost of manipulation does not depend on the true q. Therefore the manipulation choices would only depend on whether $q > 0.5$ and $\frac{k}{4} > 1$. For educative advertisement, the implication of this kind of formulation is shown in Figure 2.5.

One other issue on advertisement choice is the continuous advertisement decision. That is firms have the option of not only manipulating towards extremes but any belief. This would be an alternative and complicated model. However the main results, I conjecture, will continue to hold. The model naturally extends. That is, the cost of manipulation towards q^b is $k(q-q^b)^2$. The reason for this argument is as follows:

When k is low enough, the firms will exploit the naive agent by manipulating to the further extreme. As the cost is so low that there is no interior solution and manipulation towards the further extreme is more profitable. When k is higher, the firm finds out that the only thing that matters is the size, not the direction of the manipulation. It is not optimal to manipulate agents to the extreme as the cost is high. On the other hand, for extreme values of q and intermediate values of k, manipulation towards the other extreme is more profitable as there is not enough room for manipulation towards the probable event, but also it is not optimal to manipulate the agent to the other extreme as well. When k is high but not prohibitive, it is possible to manipulate towards either extreme for the values around q. If q is around 0.5 and k is high enough, the direction of the manipulation does not matter as there is an interior solution for both directions.

The cost structure of the advertising is another important issue. Changing the cost structure would create changes in the model. However, I think the main results might continue to hold for quite general and suitable cost function assumptions. For lower values of the cost of manipulation, the manipulation towards the improbable state should still be optimal as the profit will mainly be determined by the revenue. For an intermediate level of costs of manipulation and for extreme values of q, the tension is mainly between $\gamma = \varnothing$ and γ_1 *or* between $\gamma = \varnothing$ and γ_2 . Manipulation towards the probable event will be much less profitable. This tension will favour $\gamma = \varnothing$ less and favour the manipulation towards the improbable event more as q approaches 0.5. When manipulation cost increases further, and q is around 0.5, the revenue coming from manipulating the agent towards the improbable event and the probable event will be close, but the cost might be higher for the improbable event. Here, the shape of the cost function plays an important role. If the cost function is convex, the difference in the costs will be higher. If this difference is high enough, there is enough convexity, then the results would be maintained.

Additionally, the model features a coordination incentive: firms choose to manipulate towards the same state in equilibrium. On the other hand, asymmetry among firms might result in an incentive for not coordinating. This might be due to asymmetric costs of each action taken by the consumer. For instance, one of the actions is costly for one firm while the other action is costly for the other firm. In this case, firms might have an incentive to advertise different states and the loss stemming from “not being able to manipulate the consumer who receives both advertisements” might be overweighted by the gain in the advantageous action exerted by the contract.

The pre-existing motivating studies, hopefully this study and ongoing research will help us understand the nature of exploitation via manipulation better. This study answers some important questions but also opens doors for further questions. These might be subjects to further research. Some of these are as follows. In an environment where manipulation is costly, and its cost increases with the size of the manipulation: What are the implications of having a mixed population of naive and rational agents? This creates 6 different kinds of prospective agents. In order to be able to deal with this complication, one may need some restricting assumptions. Additionally, What are the implications of the time inconsistency of agents' preferences? That is, agents' preferences might change after the contract is signed. This might create additional room for exploitation. Lastly, what are the implications of a repeated game where a naive agent reacts to being fooled? This might reduce the incentives of firms which offer exploitative contracts because of potential punishment by consumers. It is important to understand if an extended time frame is enough to completely remove a sustained manipulation.

2.6 Appendix

Proof. Proposition 2.3.1

The incentive compatibility form is $|T_1 - T_2| \leq 1$. This is because the surplus from T_1 at S_1 is $1 - T_1$ and the surplus from T_1 at S_2 is $0 - T_1$. On the other hand the surplus from T_2 at S_1 is $0 - T_2$ and the surplus from T_2 at S_2 is $1 - T$. Hence when S_1, T_1 the better choice if $1 - T_1 > 0 - T_2$. and then S_2, T_2 is the better choice if $1 - T_2 > 0 - T_1$.

If this constraint does not hold, the maximum profit to be obtained is q or $(1 - q)$. To see that assume T_1 is chosen in both state. In this case, the consumer will get the contract if $T_1 < q$. Therefore, the maximum profit $qT_1 + (1 - q)T_1 = q$. However, if it holds the monopolist can extract up to 1. Thus we need $|T_1 - T_2| \leq 1$.

Therefore, the participation constraint holds with equality as it also reflects the profit. Note that, the incentive compatibility constraint does not harm the participation constraint. All the T's obeying these both constraints will equivalently maximise the profit. The expected profit is $r * (qT_1 + (1 - q)T_2) = r$. Also note that $(T_1, T_2) = (1, 1)$ is a solution. We multiply with r as it is the reachability. □

Proof. Proposition 2.3.2

The problem of the firm, when the agent is naive, is as follows

$$max_{\{T_1, T_2, \gamma\}}\ r(qT_1 + (1 - q)T_2) - c_{\varnothing} I_{\gamma=\varnothing} - c_1 I_{\gamma=\gamma_1} - c_2 I_{\gamma=\gamma_2}$$

$$\begin{aligned} \text{subject to } \frac{1}{2}T_1 + \frac{1}{2}T_2 &\leq 1 \text{ if } \gamma = \varnothing \text{ and } T_1 \leq 1 \text{ if } \gamma = \gamma_1 \text{ and } T_2 \leq 1 \text{ if } \gamma = \gamma_2 \\ \textit{and } |T_1 - T_2| &\leq 1 \end{aligned}$$

The objective function is calculated by the expected gain and a certain cost of manipulative advertisement undertaken. The first three inequalities are the participation constraints for each manipulative advertisement and the last inequality is the incentive compatibility constraint.

Firstly, note that, the incentive compatibility constraint needs to hold for all cases below. Secondly, we do not need to worry about a negative profit because $\gamma = \varnothing$ is costless which generates a positive profit.

<u>Case 1</u> $\gamma = \varnothing$

In this case if $q \geq 1/2$ then $(T_1, T_2) = (3/2, 1/2)$ and if $q \leq 1/2$ then $(T_1, T_2) = (1/2, 3/2)$. Hence the profit is $r(0.5 + q)$ if $q \geq 1/2$ and $r(1.5 - q)$ if $q \leq 1/2$.

This is because, if $q \geq 1/2$ and as the participation constraint does not get hurt by the incentive compatibility constraint, the profit is higher if T_1 is higher. Thus $T_1 = 3/2$ and $T_2 = 1/2$. The case with $q \leq 1/2$ and other cases follow this logic.

<u>Case 2</u> $\gamma = \gamma_1$

In this case, the solution is $T_1 = 1, T_2 = 2$. The expected profit is $r(2-q) - k(1-q)^2$

<u>Case 3</u> $\gamma = \gamma_2$

In this case, the solution is $T_1 = 2, T_2 = 1$. The expected profit is $r(1+q) - kq^2$

Comparing the profits, when $q \geq 0.5$, $\gamma = \varnothing$ is optimal if $2k(1-q)^2 \geq r(3-4q)$ and $2kq^2 \geq r$. $\gamma = \gamma_1$ is optimal if $2k(1-q)^2 \leq r(3-4q)$ and $k \geq r.\gamma = \gamma_2$ is optimal if $2kq^2 \leq r$ and $r \geq k$.

When $q \leq 0.5$, $\gamma = \varnothing$ is optimal if $2k(1-q)^2 \geq r$ and $2kq^2 \geq r(4q-1)$. $\gamma = \gamma_1$ is optimal if $2k(1-q)^2 \leq r$ and $r \geq k$. $\gamma = \gamma_2$ is optimal if $2kq^2 \leq r(4q-1)$ and $k \geq r$. □

Proof. Proposition 2.4.1

First note that, as the consumer is rational, the firms will choose the empty advertisement.

Therefore, I will solve a market where, each firm setting prices (remember that P is a sufficient statistic for the purpose of equilibrium analysis, hence we can interpret this game as a price competition where the price of the good is P for which the willingness to pay of the consumer is 1) simultaneously, and where the consumer receives price information from each firm with probability r.

The case where the agent is rational will be solved as an application of a general case.

The general case is that the agent's belief might not be true and this belief is known by the firms. However, firms cannot observe if the agent receives both advertisements.

This general case is introduced and solved by Lemma 1 below.

Later, I show how this general case can be applied to this proposition as a special case where the agent's belief is correct.

The reason why I give a general case is that, I will use Lemma 1 in the proofs of subsequent propositions as well.

Lemma 1:

This lemma analyses a general pricing competition model. General in the sense that the price perceived (by consumer) and and the price offered (by firms) can differ. I will first provide a general setting which can be adopted for both rational and naive agent with a willingness to pay of 1 for a single unit.

Assume there is a homogeneous good offered by two identical firms with marginal cost of zero competing on prices simultaneously.

Assume with $\alpha > 0$ probability the agent can observe only the first firm's advertisement and with α only the second. Additionally, with $\beta > 0$ probability both advertisements are observed.

The relationship between perceived and the real price is established by $R(p)$ where $R(p)$ is the real, p is the perceived price. Further assume that $R(p)$ is a strictly increasing continuous function and $R(p) \geq p$. Define $\Pi(p)$ as the profit (as cost is already undertaken I will call this as profit) of the Firm i conditional on selling the consumer whose perceived price is p. Therefore, $\Pi(p) = R(p)$ for $p \leq 1$ and $\Pi(p) = 0$ for $p > 1$ (the consumer will not buy the product).

The consumer buys from the firm offering a lower perceived price. If they are equal, then she chooses each firm with 0.5 probability. Lastly, assume that p encompasses all the strategies of firms which are not dominated.

This lemma and the Lemma 2 analyse an environment where the price offered and perceived may differ. That is $R(p) \geq p$. The case where these do not differ, $R(p) = p$, is a price competition model with rational agents, therefore it is possible to find analogies from neo-classical models such as Rosenthal (1980); Baye et al. (2006); Varian (1980). For both lemmata, these steps below follow standard steps (used to find the mixed Nash equilibrium) which appears in various studies with the spirit of price competition with unit demand (e.g. Spiegler (2006b); Myerson (1993); Varian (1980)) - generalised for the present study's purpose.

Lemma 1 can be stated as follows: There is a unique Nash Equilibrium which is symmetric. This is where each agent plays with CDF of $F(p) = 1 - \frac{\alpha(\Pi(1)-\Pi(p))}{\beta\Pi(p)}$.

Define $\Pi(\underline{p})(\alpha + \beta) = \Pi(1)\alpha$ and $\Pi(L) = 0$.

Claim 1: There is no Pure Nash Equilibrium.

The best response of firm i is setting a slightly smaller value for the prices offered by firm $\underline{p} < p \leq 1$ and setting the $p = 1$ for $\underline{p} > p$ or $p > 1$. The best responses do not intersect; there is no Pure Nash Equilibrium.

Claim 2: In any Nash Equilibrium, there is no mass for $[L, 1)$.

Assume there is a mass at $1 > p > L$ in the equilibrium CDF of firm 1.

First note that it is not possible both firm has a mass at p. Assume there is. In this case the expected profit of firm 1 from playing p is $\Pi(p)(\alpha + \beta((1 - F_2(p)) + (m/2)))$ where m is the mass attached to p by firm 2. Now take a $\delta > 0$. The minimum profit expected from playing $p - \delta$ is $\Pi(p - \delta)(\alpha + \beta((1 - F_2(p)) + (3m/4)))$ For small enough δ, the latter is larger than the former.

If $\exists\epsilon > 0$ where the CDF of firm 2 is flat at $[p, p + \epsilon)$ then firm i has a profitable deviation to shift the mass to $p + \epsilon/2$ as playing p and $p + \epsilon/2$ does not change the expected demand but increases the price.

Therefore it is not flat for any ϵ. Then $\exists\delta > 0$ where the firm 2 has a profitable deviation shifting the weight $[p, p+\epsilon)$ to $p - \delta$ for some $\delta > 0$: The maximum profit expected playing $p* \in [p, p + \epsilon)$ is $\Pi(p + \epsilon)(\alpha + \beta(1 - F_1(p)))$. The minimum profit expected playing $p - \delta$ is $\Pi(p - \delta)(\alpha + \beta(1 - (F_1(p) - m^*/2)))$ where m^* is the mass attached to p by firm 1. For small enough ϵ and δ, the latter is larger than the former.

If the mass is at L, then that firm gets zero profit in the equilibrium. However, by playing 1 it could guarantee at least $\Pi(1)\alpha > 0$.

Claim 3: The maximum of the suprema of CDFs played is 1.

Assume not and call that supremum as p. By previous claim we know that no player can assign atom at the p. In this case the profit of the firm is $\Pi(p)\alpha < \Pi(1)\alpha$, hence it cannot be a best response.

<u>Claim 4</u>: In any Nash Equilibrium, firms get the same expected profit. This profit is equal to $\Pi(1)\alpha$.

We know that the equilibrium CDF's have no mass above L. All infima are larger than L as L generates a zero profit but playing $p = 1$ generates at least $\Pi(1)\alpha > 0$.

Assume that firm i gets more profit than firm j. Firm j can guarantee the same profit of firm j by playing firm i's infimum.

Additionally we know the maximum of the suprema is 1. It is not possible that both firms assign a mass to 1. Otherwise one of the firms would have a profitable deviation to shift the mass to a slightly lower value.

To see that, note that the expected profit of setting price 1 for firm j is, $\Pi(1)(\alpha + \beta m/2)$ where m is the probability firm i playing 1. The minimum expected profit of setting price $1-\epsilon$ is $\Pi(1-\epsilon)(\alpha+\beta m)$. As Π is continuous, then the latter dominates the former for small ϵ.

Therefore, the profit of the firm having the supremum of 1 is $\Pi(1)\alpha$ and this is what both firm expects.

<u>Claim 5</u>: No atom at 1.

We know that the maximum of suprema is 1 and assume firm i assigns an atom to 1. Assume firm j has a smaller supremum, say p. This means firm firm j' CDF is flat at $[p, 1]$. This in turn means firm i's CDF is flat at $[p, 1)$. In this case firm j can profitably deviate setting a price higher than its supremum as it's prospective demand will be the same.

Therefore, the suprema are the same. We know it is not possible both firms assign an atom to 1: only one of the firms assign atom to 1. However, in this case the firms earn different payoffs of setting the price 1. This contradicts with the same payoff at the equilibrium claim.

Claim 6: Infima are the same.

Remember that, there is no mass at the infimum.

Assume the infimum of a firm is higher than the other one. In this case, the firm with a lower infimum can deviate to playing the higher infimum. This does not change the expected demand but increases price.

Claim 7: In any Nash Equilibrium, CDFs are strictly increasing within the support.

Assume Firm i's CDF is flat at (p, p^*). In this case firm j cannot assign any weight to (p, p^*) as $p' + \gamma < p^*$ strictly dominates any $p' \in (p, p^*)$ for firm j. This means Firm j's CDF is also flat at (p, p^*).

However, in this case, any firm has a profitable deviation playing $p^* - \lambda > p$ rather than $p" \in (p - \epsilon, p)$ for an $\epsilon > 0$ such that $F(p - \epsilon) > 1 - (1 - F(p))\frac{\Pi(p^* - \lambda)}{\Pi(p)}$. (Such ϵ exists because of continuity.) It is because the maximum expected profit from $p"$ is $\Pi(p)(\alpha + \beta(1 - F(p - \epsilon)))$ but from $p^* - \lambda$ it is $\Pi(p^* - \lambda)(\alpha + \beta(1 - F(p)))$. The latter is larger.

Claim 8: Equilibrium is symmetric and $F(p)$ is given below.

From the claims above, we can write $\Pi(1)\alpha = \Pi(p)(\alpha + \beta(1 - F(p)))$ for all p's between the common infimum and the minimum of suprema, for both firms.

This implies $F(p) = 1 - \frac{\alpha(\Pi(1) - \Pi(p))}{\beta\Pi(p)}$ and common supremum of 1 in particular.

Now, we are ready to apply this lemma to the case where the agent is rational.

In this case, both firms will issue an empty advertisement. The reason is that the rational agent cannot be manipulated by definition and therefore both firms choose the cheapest advertisement. As the rational agent has the true belief about the states, define $\Pi(p) = pq + (1-q)p = p$.

Note that $\frac{\alpha}{\beta} = \frac{(1-r)}{r}$. Therefore the equilibrium distribution $F(p) = 1 - \frac{(1-r)(1-p)}{rp}$.

□

Proof. Proposition 2.4.2

Recipient of only one advertisement , $1-E(P) = 1-\int_{1-r}^{1} P(\frac{1-r}{r*P^2})dP = 1+\ln(1-r)\frac{1-r}{r}$ and the expected surplus of an agent who is the recipient of both advertisements is:

(Note that he expected value of the minimum of two independent random variables is the sum of the expected values minus expected value of the maximum of those random variables (Lewellen, 2013))

$$\begin{aligned}
1 - E(P^i &< P^j) = 1 - 2E(P) + 2\int_{1-r}^{1} P(1 - \frac{(1-P)(1-r)}{P*r})\frac{1-r}{rP^2}dP \\
&= 1 - 2E(P) + 2\int_{1-r}^{1} P(1 - \frac{(1-P)(1-r)}{P*r})\frac{1-r}{rP^2}dP \\
&= 1 - 2(-\ln(1-r)\frac{1-r}{r}) + 2\int_{1-r}^{1} (\frac{-1+P+r}{P*r})\frac{1-r}{rP}dP \\
&= 1 + 2\frac{1-r}{r}\ln(1-r) + 2(\frac{(1-r)^2}{r^2} - \frac{(1-r)}{r^2}(\ln(1-r)+1)) \\
&= 1 + 2\frac{1-r}{r}\ln(1-r) + 2\frac{1-r}{r^2}((1-r) - (\ln(1-r)+1)) \\
&= 1 + 2\frac{1-r}{r}\ln(1-r) + 2\frac{1-r}{r^2}(-r - \ln(1-r)) \\
&= 1 + 2\frac{1-r}{r}(\ln(1-r) - 1 - \frac{\ln(1-r)}{r}) \\
&= 1 - 2\frac{1-r}{r}(\frac{(1-r)\ln(1-r)}{r} + 1)
\end{aligned}$$

On the other hand the expected profit of one firm is, of course, $r(1-r)$. It can be confirmed as follows,

$$\begin{aligned}
& \int_{1-r}^{1} (P(r(1-r) + r^2(1-F(P))))f(P)dP \\
= & \int_{1-r}^{1} (P(r(1-r) + r^2(\frac{(1-P)(1-r)}{P*r})))(\frac{1-r}{r*P^2})dP \\
= & -(1-r)^2 - (-(1-r)) = r(1-r)
\end{aligned}$$

□

Proof. Proposition 2.4.3 and 2.4.4

We use backward induction to find the solution of Subgame Perfect Nash Equilibrium.

The structure of the proof is as follows. First, I give a description of subgames followed by the advertisement choices. Later, I present Lemma 2 and explain how Lemma 1 and Lemma 2 are used to analyse these subgames. Next, I apply these lemmata to the subgames. Finally, I conduct backward induction using the results I found throughout the proof.

There are four classes of possible subgames:

1)Both firms do empty advertisement.

In this case, firms cannot infer which advertisements the consumer received as the belief of the consumer is not different than what is advertised. The agent, if she received at least one advertisement, believes that $q_b = 0.5$, which is known to the firms too.

2)Both firms attempt to manipulate towards the same state.

In this case, firms cannot infer which advertisements the consumer received as the belief of the consumer is not different than what is advertised. The agent, if she received at least one advertisement, believes that $q_b = 1$ if γ_1 and $q_b = 0$ if γ_2 is chosen by firms. The belief is known to the firms.

3)Firms attempt to manipulate towards different states.

In this case, firms can infer they are competing or not. This is because the belief of the consumer will be $q_b = 0.5$ if and only if she received both advertisements. Additionally firms also know that the consumer received only their advertisement if and only if she believes what it advertised.

4)One firm sends empty advertisement and the other sends non-empty one.

In this case, firms can infer they are competing or not. This is because, the belief of the consumer will be $q_b = 0.75$ (for the case where advertisement choices are γ_1 and $\varnothing$) or $q_b = 0.25$ (for the case where advertisement choices are γ_2 and $\varnothing$) if and only if she received both advertisements. Additionally firms also know that the consumer received only their advertisement if and only if she believes what it advertised.

Remember that Lemma 1 is a general description of a market where the consumer holds a belief which is known by the firms but firms cannot observe if the consumer observed an advertisement from the other firm. Lemma 1, therefore can be used to analyse the classes of subgames 1 and 2.

Below, I give Lemma 2 where the setting is the same as Lemma 1, but firms know that the consumer observed advertisements from both firms. Therefore Lemma 2 can be used to analyse the classes of subgames 3 and 4 where it is possible that the consumer holds a different belief than what is advertised. Remember that, in these subgames each firm knows if the consumer got an advertisement from both firms. If the consumer got an advertisement only from a firm, then the firm will set the price equal to the willingness to pay of the consumer. If it is the case that the consumer received advertisements from both firms, then it will be a price competition between firms. Lemma 2 below, is used to analyse the cases in subgames 3 and 4 where the consumer received advertisements from both firms.

Lemma 2: In the same setting as Lemma 1 with $\alpha = 0$, there is a unique Nash equilibrium which is pure, generating zero profits for both firms.

Claim: There is a unique Pure Nash Equilibrium.

The best response of firm i is setting a slightly lower price for prices offered between $(L, 1]$. For the price L, the best response is choosing any value from $[L, 1]$. For prices higher than 1, the best response is 1. Therefore the only Pure Nash equilibrium i both firm choose L.

Claim : There is no mixed strategies Nash Equilibrium.

Assume there is.

First note that, the infima cannot be smaller than L. Assume one firm has an infimum of $I_i \leq I_j$. Firm i, playing I_i receives a negative profit which is dominated by setting a price 2 which generates a zero profit.

The no-mass argument at Lemma 1 is adopted. The only difference is the no-mass argument at L. Assume firm i assigns a mass at L. This means the firm assigning a mass to L makes zero profit. Assume it assigns a positive weight to some other interval, say at (p, p^*), too. This means the other firm should not assign any weight to $[L, p)$. It is because a p" $\in (p, p^*)$ would generate a positive profit. However, in this case choosing $(p + L)/2$ would be a profitable deviation for firm j. This means firm i plays L with probability of 1, which is the pure Nash equilibrium we found above. This also shows no firm has infimum L.

Now assume infima are larger than L. The firms should have the same infimum. Because of the continuity, the firm with lower infimum would have a profitable deviation shifting the weight around the infimum to slightly higher value. This means each firm makes a positive and the same profit in the equilibrium.

Now think about the supremum. If they both have a supremum smaller than 1, then each firm have the same supremum. Otherwise the firm with a larger supremum would make zero profit. If they both have the same supremum, then by continuity they both would make zero profit.

This means at least one of them has a supremum of 1. The other firm cannot have a smaller supremum because in that case the firm with supremum of 1 would get zero profit. This means both have a supremum of 1.

In this case, it is not possible that none of the firms assign atom to 1. Because in that case, playing the supremum would generate a zero profit. This means at least one of them assign mass to 1.

Note that it is not possible that only one of the firms assign atom to 1 because in that case they would get different payoffs. Therefore, both firms assign atom to 1. In this case both firms has a profitable deviation of shifting the atom to a slightly smaller value.

As a result, the infima are both L. Condradiction.

Now I am ready to analyse each classes (cases) of subgames below:

Case 1:

We can make use of Lemma 1.

Assume $q \geq 0/5$.

Define $p = (T_1^i + T_2^i)/2$ and setting $T_2^i = T_1^i - 1$ and $p = T_2^i + 0.5$ does not eliminate any profit maximising strategy. Therefore $\Pi(p) = (p+0.5)q + (p-0.5)(1-q) = p + q - 0.5$.

Thus, $F(p) = 1 - \frac{\alpha(\Pi(1) - \Pi(p))}{\beta\Pi(p)} = 1 - \frac{(1-r)(1-p)}{r(p+q-0.5)}$ with an expected profit of $(0.5 + q)r(1-r)$.

Case 2:

Subcase 1. If they manipulate towards the first state:

Define $p = T_1^i$ and setting $T_2^i - 1 = T_1^i = p$ does not eliminate any profit maximising strategy. Therefore $\Pi(p) = pq + (p+1)(1-q) = p + 1 - q$.

Thus, $F(p) = 1 - \frac{(1-r)(\Pi(1) - \Pi(p))}{r(p+1-q)} = 1 - \frac{(1-r)(1-p)}{r(p+q)}$ with an expected profit of $(2 - q)r(1-r)$.

Subcase 2. If they manipulate towards the second state:

Define $p = T_2^i$ and setting $T_2^i = T_1^i - 1 = p$ does not eliminate any profit maximising strategy. Therefore $\Pi(p) = (p+1)q + p(1-q) = p + q$.

Thus, $F(p) = 1 - \frac{(1-r)(\Pi(1) - \Pi(p))}{r(p+q)} = 1 - \frac{(1-r)(1-p)}{r(p+q)}$ with an expected profit of $(1 + q)r(1-r)$.

Case 3:

If the agent only hears about your advertisement the profit is $(2 - q)$ if the firm manipulated towards the first state and $(1 + q)$ if second. The probability of this happen is $r(1-r)$.

If the agent hears about both advertisements, then we can make use of the Lemma 2. There is a unique Nash Equilibrium. That is each firm offers a contract generating a profit of zero ($T_1 = 1 - q$ and $T_2 = -q$.)

Case 4:

If the agent only hears about your manipulative advertisement the profit is $(2 - q)$ if the firm manipulated towards the first state and $(1+q)$ if second. The probability of this happen is $r(1 - r)$.

If the agent only hears about your empty advertisement, the profit $(0.5 + q)$ for $q \geq 0.5$. The probability of this happen is $r(1 - r)$.

If the agent hears about both advertisements, then we can make use of the Lemma 2. There is a unique Nash Equilibrium. That is each firm offers a contract generating a profit of zero. ($T_1 = 1 - q$ and $T_2 = -q$ if the manipulating firm has chosen γ_2, or it has chosen γ_1 and $q \geq 0.75$; $T_1 = q - 1$ and $T_2 = q$ if it has chosen γ_1 and $q \leq 0.75$.)

As a summary, for $q \geq 0.5$ the expected profit of the firm manipulating towards the first state is $(2 - q)r(1 - r)$ at any subgame with a cost of $k(1 - q)^2$. The expected profit of the firm manipulating towards the second state is $(1 + q)r(1 - r)$ payoff at any subgame with a cost of kq^2. The expected profit of the firm with an empty advertisement is $(0.5 + q)r(1 - r)$ at any subgame with a cost of 0.

Therefore each firm manipulates towards the first state if $(2-q)r(1-r)-k(1-q)^2 \geq max\{(1 + q)r(1 - r) - kq^2, (0.5 + q)r(1 - r)\}$. Each firm manipulates towards the second state if $(1+q)r(1-r)-kq^2 \geq max\{(2-q)r(1-r)-k(1-q)^2, (0.5+q)r(1-r)\}$. Lastly, Each firm issues an empty advertisement if $(0.5 + q)r(1 - r) \geq max\{(2 - q)r(1 - r) - k(1 - q)^2, (1 + q)r(1 - r) - kq^2\}$. After simplifications we see:

A firm manipulates towards the first state if, $\frac{k}{r(1-r)} \geq 1$ and $\frac{(3-4q)}{2(1-q)^2} \geq \frac{k}{r(1-r)}$. A firm manipulates towards the second state if, $\frac{k}{r(1-r)} \leq 1$ and $\frac{k}{r(1-r)} \leq \frac{1}{2q^2}$. The firm plays empty advertisement at all other contingencies.

□

Proof. Proposition 2.4.5

Casel 1: γ_1

The expected surplus of the agent who receives only one advertisement is,

$$\begin{aligned} & 1 - E(PRICE) \\ = \; & 1 - \int_{1-r(2-q)}^{1} (Tq + (T+1)(1-q))(\frac{(1-r)(2-q)}{r\,(T-q+1)^2})dT \\ = \; & 1 - \frac{(1-r)(2-q)}{r} \ln(Tq + (T+1)|_{1-r(2-q)}^{1} \\ = \; & 1 + \frac{(1-r)(2-q)}{r} \ln(1-r) \end{aligned}$$

Note that the expected value of the minimum of two independent random variables is the sum of the expected values minus expected value of the maximum of those random variables.

The expected surplus of the agent who receives both advertisement is,

$$\begin{aligned}
& 1-E(lowerPRICE) \\
= \; & 1-2E(PRICE)+ \\
& 2\int_{1-r(2-q)}^{1}(Tq+(T+1)(1-q))(1-(\frac{(1-r)(2-q)}{r(T^i+1-q)}-\frac{(1-r)}{r}))(\frac{(1-r)(2-q)}{r\left(T-q+1\right)^2})dT \\
= \; & 1-2(-\frac{(1-r)(2-q)}{r}\ln(1-r)) \\
& +2\int_{1-r(2-q)}^{1}(T+1-q)(1-(\frac{(1-r)(2-q)}{r(T+1-q)}-\frac{(1-r)}{r}))(\frac{(1-r)(2-q)}{r\left(T-q+1\right)^2})dT \\
= \; & 1+2\frac{(1-r)(2-q)}{r^2}r\ln(1-r)+2\frac{(1-r)(2-q)}{r^2}((-r)-\ln(1-r)) \\
= \; & 1-2\frac{(1-r)(2-q)}{r^2}((1-r)\ln(1-r)+r)
\end{aligned}$$

Casel 2: $\varnothing$

The expected surplus of the agent who receives only one advertisement is,

$$\begin{aligned}
& 1-E(PRICE)=1-\int_{1-r(1+q)}^{1}((T+1)q+T(1-q))(\frac{1}{r}\left(r-1\right)\frac{1-q-\frac{3}{2}}{\left(T+q\right)^2})dT \\
= \; & 1-\left(\frac{1}{2r}\left(\ln\left(T+q\right)\right)\left(-2q-1\right)\left(r-1\right)\right)|_{\frac{(1-r)(1-q)-2(q)}{2}}^{1/2} \\
= \; & 1-\frac{\left(1-2q\right)\left(r-1\right)}{2r}\left(\ln\left(T-q\right)\right)|_{\frac{(1-r)(1+2q)-2(q)}{2}}^{1/2} \\
= \; & 1-\frac{\left(-2q-1\right)\left(r-1\right)}{2r}\left(\ln\left(0.5+q\right)-\ln\left(\frac{(1-r)(1+2q)-2(q)}{2}+q\right)\right) \\
= \; & 1-\frac{\left(-2q-1\right)\left(r-1\right)}{2r}\left(\ln\left(0.5+q\right)-\ln\left(\frac{(1-r)(1+2q)}{2}\right)\right) \\
= \; & 1+\frac{\left(1+2q\right)\left(1-r\right)}{2r}\ln(1-r)
\end{aligned}$$

The expected surplus of the agent who receives both advertisement is

$$
\begin{aligned}
& 1-E(lowerPRICE)=1-2E(PRICE) \\
& +2\int_{\frac{(1-r)(1+2q)-2(q)}{2}}^{1/2}((T+1)q+T(1-q)) \\
& (1-(\frac{(1-r)}{r(q+T)}(\frac{1+2q}{2})-\frac{(1-r)}{r}))(\frac{1}{r}(r-1)\frac{1-q-\frac{3}{2}}{(T+q)^2})dT \\
= & 1+2\frac{(1+2q)(1-r)}{2r}\ln(1-r)+2\int_{\frac{(1-r)(1+2q)-2q}{2}}^{1/2}((T+1)q+T(1-q)) \\
& (1-(\frac{(1-r)}{r(q+T)}(\frac{1+2q}{2})-\frac{(1-r)}{r}))(\frac{(1-r)}{2r}\frac{(1+2q)}{(T+q)^2})dT \\
= & 1+2\frac{(1+2q)(1-r)}{2r}\ln(1-r) \\
& +2\left(2\frac{(1-r)(1+2q)}{4r^2}\ln(T+q)+\frac{(1-r)^2(1+2q)^2}{4r^2}\frac{1}{T+q}\right)|_{\frac{(1-r)(1+2q)-2q}{2}}^{1/2} \\
= & 1+2\frac{(1+2q)(1-r)}{2r}\ln(1-r)+2(-2\frac{(1-r)(1+2q)}{4r^2}\ln(1-r)) \\
& +2(\frac{(1-r)^2(1+2q)^2}{4r^2}\frac{-2r}{(1-r)(1+2q)}) \\
= & 1+2\frac{(1+2q)(1-r)}{2r}\ln(1-r)+2(-2\frac{(1-r)(1+2q)}{4r^2}\ln(1-r))+2(\frac{-(1-r)(1+2q)}{2r}) \\
= & 1+2\frac{(1+2q)(1-r)}{2r}\ln(1-r)+(-\frac{(1-r)(1+2q)}{r^2}\ln(1-r))-(\frac{(1-r)(1+2q)}{r}) \\
= & 1+\frac{(1+2q)(1-r)}{r}(\ln(1-r)-\frac{1}{r}\ln(1-r)-1) \\
= & 1-\frac{(1+2q)(1-r)}{r}(\frac{1-r}{r}\ln(1-r)+1)
\end{aligned}
$$

The profit of each firm can be found by calculating the profit generated by the supremum of their support (1) and the total ex-ante consumer surplus can be found by averaging the expected surpluses found above with the weights equal to the probability of being less- or better- informed agent. □

Proof. Corollary 2.4.6

For $r\epsilon(0,1)$,

Observation 1: $(\ln(1-r))(r-1) > 0$ (Both entries are negative)

Observation 2: $(r + \ln(1-r) - r\ln(1-r)) = r + (1-r)\ln(1-r)$. Additionally, $\frac{d(r+(1-r)\ln(1-r))}{dr} = -\ln(1-r) > 0$ therefore it reaches to minimum when $r = 0$ where its value is also zero. Hence $r + (1-r)\ln(1-r) > 0$.

Observation 3: $(2r + 2\ln(1-r) - r\ln(1-r)) = (2r + (2-r)\ln(1-r))$. Additionally, $\frac{d(2r+(2-r)\ln(1-r))}{dr} = \frac{1}{r-1}(r + \ln(1-r) - r\ln(1-r))$, by observation 2 this is negative, therefore it reaches maximum when $r = 0$ where its value is also zero. Hence,$(2r + 2\ln(1-r) - r\ln(1-r)) < 0$.

Observation 4: $(r + \ln(1-r))$ and $\frac{d(r+\ln(1-r))}{dr} = \frac{r}{r-1} < 0$ Therefore it reaches to maximum then $r = 0$ where its value is also zero. Hence $(r + \ln(1-r)) < 0$

Observation 5: $2r+2\ln(1-r)-2r\ln(1-r)-r^2 = r(2-r)+2(1-r)\ln(1-r)$ and $\frac{d(r(2-r)+2(1-r)\ln(1-r))}{dr} = -2r - 2\ln(1-r) = -2(r + \ln(1-r)) > 0$, by observation 4. Therefore it reaches its minimum when $r = 0$ where its value is zero. Hence, $2r + 2\ln(1-r) - 2r\ln(1-r) - r^2 > 0$.

Observation 6 $\frac{1}{r^3}(4r + 4\ln(1-r) - 3r\ln(1-r) - r^2)$ has a root around 0.78.

Observation 7: The agent who receives both advertisement has always a larger surplus than the agent who receives only one advertisement by definition.

These observations, the previous proposition and the derivatives below straightforwardly leads to the corollary.

Casel 1a: γ_1 and change in q

The expected surplus of the agent who receives only one advertisement: (Observation 1)

$$\frac{d(1+\frac{(1-r)(2-q)}{r}\ln(1-r))}{dq}$$
$$= \frac{1}{r}\left(\ln\left(1-r\right)\right)\left(r-1\right) > 0$$

The expected surplus of the agent who receives both advertisement is (Observation 2)

$$\frac{d(\ 1-2\frac{(1-r)(2-q)}{r^2}((1-r)\ln(1-r)+r))}{dq}$$
$$= -\frac{2}{r^2}\left(r-1\right)\left(r+\ln\left(1-r\right)-r\ln\left(1-r\right)\right) > 0$$

The differences in surpluses (Observation 3)

$$\frac{d(1+\frac{(1-r)(2-q)}{r}\ln(1-r)-(1-2\frac{(1-r)(2-q)}{r^2}((1-r)\ln(1-r)+r)))}{dq} =$$
$$\frac{1}{r^2}\left(r-1\right)\left(2r+2\ln\left(1-r\right)-r\ln\left(1-r\right)\right) > 0$$

Casel 1b: γ_1 and change in r

The expected surplus of the agent who receives only one advertisement: (Observation 4)

$$\frac{d(1+\frac{(1-r)(2-q)}{r}\ln(1-r))}{dr}$$
$$= \frac{1}{r^2}\left(r+\ln\left(1-r\right)\right)\left(q-2\right) > 0$$

The expected surplus of the agent who receives both advertisement is (Observation

5)

$$\frac{d(\ 1-2\frac{(1-r)(2-q)}{r^2}((1-r)\ln(1-r)+r))}{dr} =$$
$$-\frac{2}{r^3}(q-2)\left(2r+2\ln(1-r)-2r\ln(1-r)-r^2\right) > 0$$

The differences in surpluses

$$\frac{d(1+\frac{(1-r)(2-q)}{r}\ln(1-r)-(1-2\frac{(1-r)(2-q)}{r^2}((1-r)\ln(1-r)+r)))}{dr}$$
$$= \frac{1}{r^3}(q-2)\left(4r+4\ln(1-r)-3r\ln(1-r)-r^2\right)$$

Casel 2a: $\varnothing$ and change in q

The expected surplus of the agent who receives only one advertisement: (Observation 1)

$$\frac{d(1+\frac{(1+2q)(1-r)}{2r}\ln(1-r))}{dq}$$
$$= -\frac{1}{r}(\ln(1-r))(r-1) < 0$$

The expected surplus of the agent who receives both advertisement is (Observation 2)

$$\frac{d(1-\frac{(1+2q)(1-r)}{r^2}((1-r)\ln(1-r)+r)\ )}{dq}$$
$$= \frac{2}{r^2}(r-1)(r+\ln(1-r)-r\ln(1-r)) < 0$$

The differences in surpluses (Observation 3)

$$\frac{d(1+\frac{(1+2q)(1-r)}{2r}\ln(1-r)-(1-\frac{(1+2q)(1-r)}{r^2}((1-r)\ln(1-r)+r)\ ))}{dq}$$
$$= \frac{1}{r^2}(r-1)(2r+2\ln(1-r)-r\ln(1-r))<0$$

Casel 2b: $\varnothing$ and change in r

The expected surplus of the agent who receives only one advertisement: (Observation 4)

$$\frac{d(1+\frac{(1+2q)(1-r)}{2r}\ln(1-r))}{dr}$$
$$= \frac{1}{2r^2}(-1=2q)(r+\ln(1-r))>0$$

The expected surplus of the agent who receives both advertisement is (Observation 5)

$$\frac{d(1-\frac{(1+2q)(1-r)}{r^2}((1-r)\ln(1-r)+r))}{dr}$$
$$= -\frac{1}{r^3}(-1-2q)\left(2r+2\ln(1-r)-2r\ln(1-r)-r^2\right)>0$$

The differences in surpluses

$$\frac{d(1+\frac{(1+2q)(1-r)}{2r}\ln(1-r)-(1-\frac{(1+2q)(1-r)}{r^2}((1-r)\ln(1-r)+r)\ ))}{dr}$$
$$= \frac{1}{2r^3}(-1-2q)\left(4r+4\ln(1-r)-3r\ln(1-r)-r^2\right)$$

The result for differences in surpluses is by Observation 6&7. $\square$

Chapter 3

Ordinal Sampling-Based Procedures

Abstract

Consumers may adopt simple heuristic rules to evaluate complex pricing schemes offered by firms. The heuristic rule they use when deciding where to buy from might be based on their sampling of the price information they collected. This paper analyses market implications of a large family of such heuristics characterised by some natural axioms. Additionally, how the number of firms and the number of observations per firm affect market prices are investigated.

3.1 Introduction

Consumers may adopt simple procedures to evaluate complex pricing schemes offered by firms. These procedures (which is used to determine which firm to approach) might be based on their sampling of the price information they collected. These procedures are called *sampling-based procedures.* Note that, the price sample they collected might not perfectly represent the pricing scheme firms offer. The consumer's bounded rationality might feature not only the limited understanding of this fact but also limited cognitive capabilities. This deviation from full rationality might cause them to be tricked by the aspects of the pricing scheme they overlook. This situation might grant firms a strategic advantage over consumers which might dampen the competitiveness of the market.

A particular kind of sampling-based procedure, that is sampling each alternative a number of times and choosing the one with the highest average payoff, is introduced by Osborne & Rubinstein (1998) as an equilibrium concept. Spiegler (2006a) studied this procedure in a market setting, for a single observation per firm. He shows that firms respond to a higher number of firms by dispersing their equilibrium distribution without changing the mean price. This is rather an unexpected result because the usual policy result is that a higher number of firms reduces the market prices. On the other hand, there are many other plausible procedures that a consumer may employ, for any number of observations per firm. The present study takes the richness of these procedures into account and provides a general, tractable setting by generalising the Spiegler (2006a) framework. The main technical difficulty of studying sampling-based procedures stems from the loss of tractability when calculating the expected demand. This study overcomes this limitation by focusing on procedures which are characterised by axioms reflecting consumer's limited cognitive capabilities.

The main contribution of this paper concerns the relationship between the number of firms and the expected price. I show that for a large set of procedures with reasonable features, the expected price decreases with the number of firms. For the procedure studied by Spiegler (2006a), the expected price does not decrease with the number of firms. Additionally, I analyse the relationship between the expected price and the number of observations per firm. Higher internet availability might result in a higher number of observations per firm. It is important to understand how this phenomenon would affect market prices. I show that the expected price is independent of the number of observations per firm.

Suppose that you observe the gas prices at Shell, BP and Exxon for two days. Each firm has determined a pricing cumulative distribution function (CDF) from which one price is independently drawn for each day (3×2 prices are observed. Henceforth, sample points and prices (drawn/observed) will be used interchangeably.). On the third day, you happen to be in need of gas and you decide which gas station you will go to without visiting all of them once again. Using the data you collected over the last two days, you will choose one of the gas stations by employing a sampling-based procedure.[1] You make a final, *independent* draw from the gas station you have chosen and buy gas for that price if it is not more than your willingness to pay, otherwise you do not buy.[2]

How would you process the data you collected? You might take the average price for each gas station and pick the one with the minimum average price. However, would you be able to recall all prices exactly? Additionally, would it be plausible to assume that a boundedly-rational agent is able to make such complex calculations?

Instead, you might give x points to the firm with the x^{th} highest price for each day. Then you might pick each firm with probability equal to the proportion of

[1] Assume marginal cost is 0, willingness to pay is 1. Assuming a general cost or willingness to pay do not alter the main lessons of the study. I provide a general result in the Appendix.

[2] Firms are committed to the CDFs they determined on the 1^{st} day.

points it has. Call this *the Focus Procedure* - which is a member of a large family of sampling-based procedures which are called **Generalised Borda Procedures** (GBPs). I use this naming by de Borda (1781).

A procedure is a GBP if there is a non-negative, non-decreasing sequence which is employed to determine firms' probabilities of being chosen in a particular way. Each day, the firm with the x^{th} highest observation is assigned the x^{th} lowest number (call GBP points) of the sequence. A firm's probability of being chosen (Pr) is determined by dividing the sum of GBP points the firm has by the total of the numbers assigned. Take the Focus Procedure with 3 firms and 2 days: Monday (Mon.) and Tuesday (Tue.) The sequence of the Focus Procedure is $1, 2, 3$. That is, for each day, the firm which has the highest price observed will be assigned 1 point, the firm which has the second highest price observed will be assigned 2 points, and the firm which has the lowest price observed will be assigned 3 points. (See Table 3.1. e.g. the price observed from Firm 1 on Mon. is 0.4.)[3]

Table 3.1: A GBP at Work

	Mon	GBP points for Mon.	Tue	GBP points for Tue.	Pr
Firm 1	0.4	3	0.8	1	$(3+1)/[(1+2+3)+(1+2+3)] = 1/3$
Firm 2	0.5	2	0.3	3	$(2+3)/[(1+2+3)+(1+2+3)] = 5/12$
Firm 3	0.7	1	0.5	2	$(1+2)/[(1+2+3)+(1+2+3)] = 1/4$

As a summary the timing is as follows: Firms determine their pricing CDFs simultaneously. Then the consumer samples observations from these CDFs which are used to determine the probability of each firm being chosen by a GBP. Next, one of these firms is chosen. The agent draws one more price from the firm she has picked and buys the good if it is not more than 1. Otherwise, she stays inactive.

Studying GBPs reveals the following results:

[3] The tie-breaking rule is as follows: If Firm 2 would have an observed price of 0.4 on Monday, then both Firm 1 and Firm 2 would be assigned $\frac{3+2}{2} = 2.5$ GBP points. Note that choosing a CDF is not the same as playing a mixed strategy. Take the Focus Procedure with two firms where one of the firms plays $F(p) = 2p - 1$. A best response is playing the same strategy which generates 3/8 payoff. The infimum of the support, 0.5, however, generates 1/3 payoff.

1: GBPs can be characterised by 4 properties: *Ordinality;* the procedure uses the arrangement of prices with respect to their magnitude. Some features of the data collected might be lost or neglected by the consumer due to the consumer's limited memory or calculation capability. For this model, this feature is cardinal values. *Monotonicity*; the procedure assigns higher probabilities of being chosen for lower prices observed. *Label-Blindness*; the procedure does not favour any firms or days. *Limited Trade-Off Efficacy;* the change in prices observed on a day (say, Mon.) alters the probabilities of being chosen to an extent independent of prices observed on a different day (Tue.) For the example above, changing Firm 2's observation on Mon. from 0.5 to 0.2 would increase Firm 2's Pr by 1/12 (it would get 3 points from Mon. rather than 2) which is independent of observations made on Tue. This property reflects the consumer's inability to conduct trade-offs across sources (days).[4] A detailed discussion and motivation regarding these properties can be found in the *Discussion of Assumptions* sub-section.

2: Given the number of firms, the symmetric equilibrium expected price (paid) is determined by L: the expected demand of the firm setting its price equal to the willingness to pay of the consumer.

This result says that L is the sufficient statistic for a sequence which characterises a GBP when calculating the equilibrium expected price. NL, on the other hand, is the sufficient statistic for the equilibrium expected price for a given GBP. Therefore, for instance, increasing the number of firms affects the equilibrium expected price via both N and L.

In a competitive market, L is zero. In a monopoly, as another extreme, L is one. The higher value of L, the firm will have a stronger incentive to increase its price. This is because, even if the price observed from a firm is the highest, it can still be chosen by the consumer. For this reason, given the number of firms, it is possible

[4] Henceforth, source and day will be used interchangeably.

to interpret L as the anti-competitiveness of the market. The importance of this result is that properties of a procedure other than L, which is also easy to calculate, have no effect on the expected price.

Assume the number of firms increases by one. Note that, in this case, increasing the number of firms affects both N and L. The key question is how L changes. If L decreases considerably, then the expected price of the market will decrease with the number of firms. A consumer who gets pickier as the number of firms increases would guarantee such behaviour. This result is the main update of the present study to the related literature which says the expected price is independent of the number of firms.

Alternatively, assume we increase the number of observations per firm. Again the key question is how L changes. I assume that the consumer's evaluation of the data she receives does not change with the number of observations per firm. That is, the sequence she uses is the same for any number of observations per firm. Therefore, L stays the same - the expected price is independent of the number of observations per firm.

Take the Focus Procedure with 3 firms and 1 observation per firm. The sequence of this procedure is $1, 2, 3$. Remember that, this means, the firm which has the highest price observed will be assigned 1 point; the firm which has the second highest price observed will be assigned 2 points; the firm which has the lowest price observed will be assigned 3 points. Therefore L is $\frac{1}{1+2+3} = \frac{1}{6}$.[5] If one more firm is introduced, the sequence the consumer uses is $1, 2, 3, 4$. Therefore $L = \frac{1}{1+2+3+4} = \frac{1}{10}$. The expected price is lower for four firms as the consumer gets considerably pickier when the number of firms is increased. Alternatively, if there are two observations per firm, as the same sequence $(1, 2, 3)$ will be used, L, and therefore the expected price, stays the same.

[5] The equilibrium CDF is shown to be continuous.

These are the main results of this study. Additionally, I provide a way of building a procedure to obtain any expected price. This method is important if the policy-maker is able to affect the procedure the consumer uses (e.g. via framing). Finally, I show how to calculate the equilibrium CDF and provide equilibrium outcomes of some examples.

It is possible to interpret this model in a multi-dimensional pricing setting (Spiegler (2006a)). Suppose that you want to open a bank account. The prices that a bank account holder may face are conditional on numerous contingencies. It is hard for a consumer to take all of the contingencies concerning herself into account. For this reason, she may sample some contingencies (i.e. dimensions) randomly such as ATM fees, electronic funds transfer fees and overdraft fees (there are three sources). Later she collects pricing data for these contingencies from each bank and decides which one to choose. The consumer's bounded rationality in the model is motivated by the sporadic evaluation of prices in the previous interpretation and by the limited understanding of relevant contingencies in this interpretation. This interpretation is equivalent to the previous one when the number of contingencies is infinite, and all contingencies are equally valuable (equally probable with the same marginal cost and willingness to pay).

3.1.1 Related Literature

The studies regarding market implications of boundedly-rational agents amassed a literature which might be called as behavioural industrial organisation. Bounded rationality, depending on the demand and appeal of the context studied, can reveal itself in the form of coarse reasoning (Jehiel, 2005), overconfidence (Grubb, 2009) or dynamically inconsistent preferences (Vigna & Malmendier, 2006), etc.

When markets are complicated, consumers may fail to choose the firm offering the lowest price (KalaycI & Potters, 2011). This might stem from making mistakes when ranking prices (Shilony, 1977) or costly attention (Matejka & McKay, 2011). Alternatively, firms might use the complex pricing schemes to obfuscate the consumer. This might be motivated by modelling how the consumer chooses. One way of modelling consumer behaviour in a complex pricing framework is assuming that they focus on a subset of the data available from which she over-deduces (sampling-based reasoning).

Sampling-based reasoning is investigated by Osborne & Rubinstein (1998) as an equilibrium concept in which agents observe the result of each action once and simultaneously decide which action to take. All agents in their framework conduct this procedure. They study the existence properties and the relationship with the Nash Equilibrium. Extending this idea to where agents sample each action for a number of times and take the action that gives the highest average payoff, they show that the unique mixed Nash Equilibrium is attained as the number of observations per alternative goes infinity.

Spiegler (2006a) applied this notion of decision making to a context where firms are rational, but consumers employ this sampling-based procedure for a single observation per firm. He shows that firms respond to higher competitive pressure by increasing the variance of their equilibrium CDF without changing the mean price. He does not provide a solution for larger numbers of observations per firm as the indifference principle - an implication of the expected probability of being chosen as a function of the observed price being linear - does not work anymore. However, there are many other plausible procedures than the one introduced by Osborne & Rubinstein (1998) which can be adopted by boundedly-rational consumers. The present study deals with a large family of such procedures by generalising Spiegler (2006a).[6]

[6] My model generalises Spiegler (2006a); e.g. The GBP with the sequence $0, 0, 1$ (for three firms and one observation per firm) is the procedure he studies. The present paper generalises his

Sampling-based procedures found other applications in various contexts. Spiegler (2006b), for instance, analysed a market where consumers act on the anecdotes they collected. He has shown that it is possible that quacks can survive within such markets. Szech (2011), shows how Spiegler (2006b) can be extended in order to endogenise the success rates of quacks within a market.

To the best of my knowledge, GBP procedures for this framework have not been characterised and studied before. Rank-scoring procedures (of a voting framework) (Myerson, 1993) and Luce rule (of an individual, probabilistic choice theory) (Luce, 2005) are close to GBPs (for $K = 1$) in terms of formulation. Rank-scoring choice mechanisms, however, choose only the alternative with the maximal sum of points assigned - violates Limited Trade-Off Efficacy. Additionally, Rank-scoring procedures forces $\alpha_1 = 0$ and $\alpha_N = 1$. Luce rule forces $\alpha_1 \neq 0$. In a wider context, apart from price competition and social choice, GBPs can be used to analyse consumers' evaluation of quantities, choices in an extended time horizon etc. These are potential research topics for future.

There are several works relating variance, which is different from the price dispersion, arised the market. This generally arise as a price discrimination device based on categorisation capability (Rubinstein, 1993) or search cost (Salop, 1977).

This literature also relates to the literature on price dispersion. However, the model and the motivation should not be confused with fixed-sample search models (MacMinn, 1980; Burdett & Judd, 1983). In these models, rational agents are limited to a fixed number of firms to evaluate and pick, which might be motivated by search cost. However, boundedly-rational agents of the present study go through all firms but are either limited to a subset of contingencies or evaluate prices sporadically depending on the interpretation of the model.

---

basic structure (also its jargon and interpretation) for any number of observations per firm and any non-decreasing sequence used by the consumer.

Sampling-based procedures I study might relate to the literature of computational psychology on "perceptron - a concept borrowed from computational geometry. A perceptron consists of sensors and a threshold. Each sensor takes an observed price vector and returns a real number: the decision maker sums all the outputs of sensors and returns affirmative (of a binary) output if it is above some threshold (Rubinstein, 1993). The present study can be interpreted as a generalised perceptron mechanism which has an ordinal nature.

The following section presents the model, and the third section discusses the equilibrium outcome. The fourth section concludes and is followed by the Appendix.

3.2 Model

There is a market with $N > 1$ expected-profit maximising firms and a boundedly-rational, risk-neutral consumer with an outside option of zero.

The strategy set $\mathbb{F}$ of a firm $i \in \{1, 2, ..., N\}$ consists of CDFs, with a typical element of $F_i(p)$, where p is the price. Firms produce a homogeneous good for which the consumer's willingness to pay is normalised to 1 and the marginal cost is zero. Firms decide on CDFs simultaneously with complete information.

Later, the consumer draws $K \in \mathbb{Z}_+$ independent sample points from each CDF. Denote the k-th sample point drawn from $F_i(p)$ as p_i^k. The set of all possible K sample points is $\mathbb{R}^K$ with a typical element of ω_i. The k-th entry of vector ω_i is p_i^k. Define $\mathbb{R}^{NK} = \prod_{i=1}^{N} \mathbb{R}^K$ with a typical element of ω ($\omega_{i,k} = p_i^k$.)

The consumer decides on the probability of choosing each firm by executing a sampling-based procedure (which is known by firms at the beginning of the game) $S : \mathbb{R}^{NK} \to \triangle^{N-1}$ where the range is the probability simplex on $\mathbb{R}^N$.[7] $S_i(\omega)$ is the

[7] The consumer is allowed to choose not to buy and go for her outside option at the last stage.

i-th entry of the N-sized vector of $S(\omega)$. This procedure is a GBP which calls for two notions for a formal definition:

Definition 3.2.1. *Counter Functions : Lower Counter Function* $c_{i,k}(\omega) = \sum_{n=1}^{N} I_\omega(p_i^k > p_n^k)$*, and Upper Counter Function* $C_{i,k}(\omega) = \sum_{n=1}^{N} I_\omega(p_i^k < p_n^k)$.[8]

Lower (Upper) Counter Function has entries which take an observation of a firm and counts how many lower (higher) prices are observed from the same source.

A GBP is characterised by a set of weights (α_j's in the definition below). These weights are the entries of a non-decreasing sequence. These weights, which are assigned to the firms according the observed prices' ordinal ranking, determine the probability of each firm to be chosen by the consumer.

Definition 3.2.2. *Generalised Borda Procedure : S is a Generalised Borda Procedure if* $\exists\ (\alpha_j)_{j=1}^N$ *where* $0 \leq \alpha_j \leq \alpha_{j+1}$ *for* $j \in 1, 2, ..., N-1$ *with* $\alpha_1 \neq \alpha_N$*, such that* $\forall i \in 1, 2, ..., N$*:*

$$S_i(\omega) = \left\{ \sum_{k=1}^{K} \left(\frac{\sum_{t=C_{i,k}(\omega)+1}^{N-c_{i,k}(\omega)} \alpha_t}{N - (c_{i,k}(\omega) + C_{i,k}(\omega))} \right) \right\} \Big/ (K * \sum_{j=1}^{N} \alpha_j) \tag{3.2.1}$$

This definition takes a non-decreasing sequence with N non-negative entries. The firm having the j^{th} highest price observed from a source k is assigned with the j^{th} lowest number in the sequence. This assignment is conducted for all firms and sources. The probability of a firm being picked by this procedure is determined by the sum of the numbers collected by that firm (numerator), divided by the sum of all numbers assigned (denominator). The firms which have the same observation for a source (there are $N - (c_{i,k}(\omega) + C_{i,k}(\omega))$ of them) are assigned the same number keeping the sum of numbers assigned and also keeping the numbers assigned to

Therefore, it is innocuous to assume such a range.

[8] I is an indicator function such that $I_A(a) = 1$ if $A \implies a$, zero otherwise given that A is a set of statements and a is any statement.

other firms (corresponding their orders) constant.

Let me explain how is equation 3.2.1 applied. The crowded bit within the formulae $(\frac{\sum_{t=C_{i,k}(\omega)+1}^{N-c_{i,k}(\omega)} \alpha_t}{N-(c_{i,k}(\omega)+C_{i,k}(\omega))})$ is due to cases of tie - that is some firms have the same prices observed for a source -. First, consider a case with no ties. Take the Focus procedure with a sequence of $1, 2, 3, 4, 5$ for 5 firms and $K = 1$. That is $\alpha_i = i$. Assume the prices observed for firm $1, 2, 3, 4, 5$ are $0.1, 0.3, 0.4, 0.6, 0.8$ respectively. In this case, note that, $C_{i,1}(\omega)+c_{i,1}(\omega) = N-1$ for all firms - there are no ties. Therefore, $S_i(\omega) = \frac{\alpha_{C_{i,1}(\omega)+1}}{\sum_{j=1}^{N} \alpha_j}$. For instance, for firm 2, $C_{2,1}(\omega) = 3$. Therefore, $S_2(\omega) = \frac{\alpha_4}{1+2+3+4+5} = 4/15$.

Now turn to the case where there is a tie. Take again the Focus procedure with a sequence of $1, 2, 3, 4, 5$ for 5 firms and $K = 1$. Assume this time, the prices observed for firm $1, 2, 3, 4, 5$ are $0.1, 0.4, 0.4, 0.6, 0.8$ respectively. In this case, $C_{2,1}(\omega) = 2$ and $c_{2,1}(\omega) = 1$. Therefore, $C_{2,1}(\omega) + c_{2,1}(\omega) = N - 2$. This means $(\frac{\sum_{t=C_{i,1}(\omega)+1}^{N-c_{i,1}(\omega)} \alpha_t}{N-(c_{i,1}(\omega)+C_{i,1}(\omega))}) = \frac{\sum_{t=2+1}^{5-1} \alpha_t}{N-(C_{2,1}(\omega)+c_{2,1}(\omega))} = \frac{\alpha_3+\alpha_4}{2} = (3+4)/2 = 7/2$. Therefore. $S_2(\omega) = \frac{7/2}{1+2+3+4+5} = 7/30$.

Once procedure S is executed, firm i is chosen with probability $S_i(\omega)$.

After a firm is chosen, say it is firm i, the consumer makes one more, independent draw from $F_i(p)$ and the good is exchanged for that price if it is not more than 1. If it is more than 1, the consumer goes for the outside option.

Figure 3.1 illustrates the timing of the model:

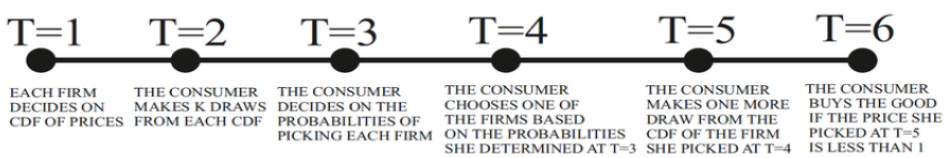


Figure 3.1: The Timing of the Model

3.2.1 An Axiomatic Characterization of GBPs

Definition 3.2.3. *Ordinality :* $c_i(\omega) = c_i(\omega')$ *and* $C_i(\omega) = C_i(\omega')$ *implies* $S_i(\omega) = S_i(\omega')$.

An ordinal procedure assigns the same probability of being chosen to a firm for observation profiles having the same orderings. The case where observed prices are (for firm $1, 2, 3$ respectively, $K = 1$) $0.2, 0.4, 0.5$ and the case $0.3, 0.6, 0.8$ are evaluated in the same manner by the procedure. If the number of firms and days observed are high, then it might be plausible to assume some features of the data collected can be neglected or lost by the consumer. In this setting, this feature is the cardinal value. This negligence might be motivated by consumers' limited calculation capabilities or limited memory. A detailed discussion on ordinal evaluation can be found in Stewart et al. (2006).

I use "order" for the arrangement of prices with respect to their magnitude. "Rank" refers to the enumeration of source's label, generically k such as $k = 1$ for Mon., $k = 2$ for Tue. etc.

Definition 3.2.4. *Monotonicity :* $\omega_i \leq \omega_i^*$ *and* $\omega_{-i} = \omega_{-i}^*$ *implies* $S_i(\omega) \geq S_i(\omega^*)$ *where* $\omega_{-i} = \omega \setminus \omega_i$. *Additionally,* $\omega_i < \omega_j < \omega_i^* \ \forall j \neq i$ *implies* $S_i(\omega) > S_i(\omega^*)$.

(1) Lower prices observed for a firm does not lower its probability of being picked by the consumer, given observed prices of other firms and (2) the firm would have a strictly higher probability of being chosen if its observations are always lower (than other firms' observations) for each source than the case where they were always higher. See the example below. Table 3.2 has two panels. The observations of the Firm 1 are lower (higher) than Firm 2 for each day on the left (right) panel. Monotonicity, the 2^{nd} part of the definition, says that the number on the upper right corner of the table (0.35) should be lower than 0.75.

Table 3.2: Monotonicity Example

	Mon	Tue	Pr	Mon	Tue	Pr
Firm 1	0.2	0.7	0.75	0.6	0.9	0.35
Firm 2	0.5	0.8	0.25	0.5	0.8	0.65

Definition 3.2.5. *Label-Blindness : A procedure is Label-Blind if and only if it is both Firm- and Source-Blind. (P is a permutation matrix).* $\forall P$:

- *Firm-Blindness :* $\omega = P\omega'$ *and* $\omega_i = \omega'_j$ *implies* $S_i(\omega) = S_j(\omega')$.
- *Source-Blindness :* $\omega = \omega' P$ *implies* $S_i(\omega) = S_i(\omega')$.

Remember that ω is an $N \times K$ matrix. For firm-blindness P is a $N \times N$ matrix with entries of zeros and ones where there is only one "one" at each row and column. This matrix permutates the rows of ω when multiplied from the left. For source-blindness, P is a $K \times K$ matrix with entries of zeros and ones where there is only one "one" at each row and column. This matrix permutates the columns of ω when multiplied from the right.

The procedure does not favour any subset of the firms (firm-blindness) and any subset of the days (source-blindness). Table 3.3 has three panels. The left and middle panels show how firm-blindness works: A permutation of observations over firms swaps the probabilities of being chosen. The left and right panels show how source-blindness works: A permutation over sources (days) does not alter the probabilities.

Table 3.3: Label-Blindness Example

	Mon	Tue	Pr	Mon	Tue	Pr	Mon	Tue	Pr
Firm 1	0.2	0.3	0.65	0.5	0.4	0.35	0.3	0.2	0.65
Firm 2	0.5	0.4	0.35	0.2	0.3	0.65	0.4	0.5	0.35

Definition 3.2.6. *Limited Trade-Off Efficacy :* $S_i(\omega) - S_i(\underline{\omega}) = S_i(\omega') - S_i(\overline{\omega})$, *where* $\omega, \omega', \underline{\omega}, \overline{\omega} \in \mathbb{R}^{NK}$ *and* $(\omega^T)_k = (\omega'^T)_k$, $(\underline{\omega}^T)_k = (\overline{\omega}^T)_k$, $(\omega^T)_{-k} = (\underline{\omega})^T_{-k}$, $(\omega'^T)_{-k} = (\overline{\omega}^T)_{-k}$.

The procedure is sensitive to a change in an observation from a source to an extent which is independent of the observations made from other sources. Take Case 1 below in Table 3.4. The only observation that changes from the left panel to the right panel of Case 1 is the observation of Firm 1 on Mon. (This is also true for Case 2.) This change decreases the probability of Firm 1 to be chosen by 0.25 (0.65 − 0.40). The Limited Trade-Off Efficacy states that, the probability of being chosen for Firm 1 for Case 2 should also decrease by 0.25 (0.75 − 0.50): the extent of the decrease is independent of observations from other sources (i.e. Tue.) This assumption is a kind of independence of irrelevant alternatives which arises in different choice contexts. The equal sensitivity assumption of Intriligator (1973) (a social choice model) is close to this assumption.

Table 3.4: An Example of Limited Trade-Off Efficacy

(a) Case 1

	Mon	Tue	Pr	Mon	Tue	Pr
Firm 1	0.2	0.3	0.65	0.6	0.3	0.40
Firm 2	0.5	0.4	0.35	0.5	0.4	0.60

(b) Case 2

	Mon	Tue	Pr	Mon	Tue	Pr
Firm 1	0.2	0.1	0.75	0.6	0.1	0.50
Firm 2	0.5	0.6	0.25	0.5	0.6	0.50

Note that the Focus Procedure obeys these four properties.

Proposition 3.2.7. *A procedure is a GBP if only if it obeys Ordinality, Monotonicity, Label-Blindness and Limited Trade-Off Efficacy. (See complete proof in the Appendix.)*

Take a GBP. As the assignment of sequence's numbers only concerns the order of prices, Ordinality is satisfied. Monotonicity comes from the increasing nature of the sequence. Label-Blindness follows from the sequence being source- and firm-neutral. Limited Trade-Off Efficacy is satisfied as the sequence does not vary with the observations from other sources.

Take a procedure which is Ordinal, Monotonic, Label-Blind and obeys Limited Trade-Off Efficacy. Limited Trade-Off Efficacy states that the value of an observation only matters for the observations from the same source, which implies additivity. Ordinality and Monotonicity imply an increasing sequence. Label-Blindness implies that this sequence is the same for all sources and firms. These all define a GBP.

3.2.2 Discussion of Assumptions

The assumption of buying the good at the last stage of the game even if the price drawn is 1 is more evident in the alternative interpretation. For instance, assume you wanted to send money to an overseas bank and you happened to see the fee for this service is high. It is plausible to assume that you do not visit the other banks, find the bank offering the best price for this service, open an account in that bank, wait for your account to be activated, transfer some money to your new account etc. The switching cost is generally higher for the examples represented by the alternative interpretation.

This assumption might be plausible for the sporadic evaluation interpretation as well. Assume you go to a gas station and see a higher price. The cost you will incur might be pretty high compared to the expected benefit of making an extensive search.

Remember the Limited Trade-Off Efficacy assumption. It says changing an observation from a source affects the firms' probability of being chosen independently of the observations from other sources. This can be interpreted as a limited capability of calculating trade-offs across contingencies.

For instance, take two firms and two sources. Say the first (contingency) source is electric funds transfer and the second source is ATM fees. Assume you observed the ATM fee for the first firm to be 0.4. How would your probability of choosing the 1^{st} firm change if this price was 0.7? It might be possible that your answer depends on the observations you made for electronic fund transfers. In that case, you need to have a highly complex decision function, especially if the number of sources sampled is high. Could a boundedly-rational agent be able to handle such complexity? She might not be able to do so and need some sort of simplification.

Limited Trade-Off Efficacy, here approaches the potential inefficacy of calculating this trade-off, by saying that the agent does not take the other sources into consideration, while calculating the effect of a change in observations obtained from a source. A similar reasoning might be motivated for the sporadic evaluation interpretation of the model. For both interpretations, one might take this assumption as a kind of "choice bracketing" (Read et al., 1999) which damages a proper trade-off across sources.

The present study better represents the environments with higher last stage search-switching costs and complicated-crowded contingencies.

3.2.3 Rational Agent

An agent who employs a sampling-based procedure and a rational agent differ. The premise of bounded rationality is that the agent has no obstacle to access the CDF but cannot process the CDF rationally and therefore acts based on her samples. However, a rational agent does not have this limitation and is able to process the CDFs rationally. Therefore, the equilibrium price is equal to the marginal cost for the rational agent.[9]

[9] Sometimes models with boundedly-rational agents might be nested in a rational framework. It might be possible to do so for my model by modifying the information structure of the game with a prohibitive switching cost.

3.3 Equilibrium

Before presenting the result remember that L is the probability of a firm, which has the highest price observed, being chosen. I show that the symmetric equilibrium which is unique consists of a continuous CDF. Therefore, the firm with the highest price observed will be assigned the lowest number of the sequence associated to the GBP employed by the consumer. Therefore, $L = \frac{\alpha_1}{\sum_{j=1}^{N} \alpha_j}$.

Proposition 3.3.1. *Within the class of symmetric Nash Equilibria, there is a unique such equilibrium, and the expected price is* $1/(2 - NL)$ *(See Appendix for a proof).*

This proposition describes how a procedure relates to the equilibrium expected price. Remember that $L = \frac{\alpha_1}{\sum_{j=1}^{N} \alpha_j}$: firm i's expected probability of being chosen if the observation drawn from firm i's CDF is the highest price, when $K = 1$.

The result above shows that the expected price is independent of K. Indeed, the equilibrium outcome, i.e. equilibrium CDFs as well, is independent of K.

I conduct the analysis for $K = 1$ and then show why it is true for all K.

The key observation in Proposition 3.3.1 is that, given N, a procedure is linked to the expected price with a very simple property of the procedure: L. Remember that L is the probability of the firm with the highest price observed being chosen. This can be interpreted as a measure of the anti-competitive advantage an individual firm has. Proposition 3.3.1 suggests that the sum of anti-competitive advantages of individual firms can be interpreted as the market's anti-competitiveness.

One might expect that there would be many other attributes of a procedure affecting the expected price via different parameters. However, given N, L is the only determinant of the expected price. I focus on the competitive environment created by the procedure in order to explain the nature of this rather unexpected result.

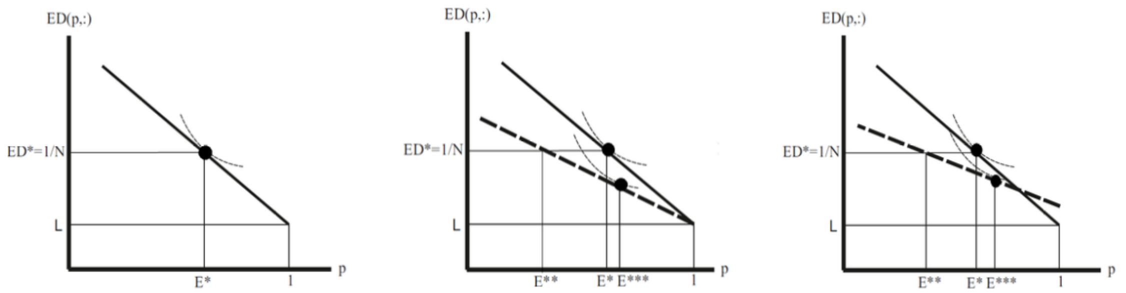
The supremum (sup) of the equilibrium distribution's support is 1. The sup of the support cannot be more than 1. A deviation which shifts probability weight assigned above 1 (the consumer's willingness to pay) to 1 would increase the expected (selling) price as willingness to pay is only 1. On the other hand, the expected probability of being chosen would not decrease because of monotonicity. Therefore, it would be a profitable deviation. Additionally, the sup cannot be less than 1 in the equilibrium. In that case, shifting the weight around the sup to 1 would increase the expected price but would not hurt the expected probability of being chosen by much. The reason why the expected probability of being chosen is not affected severely is that the procedure is ordinal. Such a deviation would affect only the orderings where there are at least two observations (one from the deviating firm and one from any other firm) in the proximity of the so-called sup. This is a quadratic effect dominated by the linear effect in the expected profit if the proximity is taken to be narrowly enough. Similar reasoning proves that there is no hole in the support.

Let me illustrate the problem of the firm in Figure 3.2a.

In order to follow the discussion further it is useful to introduce some notation. E_i is the expected value of the CDF the firm i uses. $ED_i(p, :)$ is the expected probability of firm i to be chosen where p is observed from the CDF of firm i, and given the CDFs of other firms. Ed_i is the probability of firm i to be chosen. That is Ed_i is the expected value of $ED_i(p, :)$ over the CDF of firm i. By definition $ED_i(1, :)$ is equal to L if the CDF of each firm is continuous. Noting that the price paid by the consumer is an independent, new draw from the CDF of the chosen firm, the ex-ante expected profit of firm i is the expected price (E_i) multiplied by the expected probability of it being chosen (Ed_i).

Note that the price is on the horizontal axis and $ED(p, :)$ (the probability Firm

i being chosen given that the observation drawn is p.) is on the vertical axis. The solid curve represents $ED(p, :)$. It is decreasing because of monotonicity. The firm chooses a CDF that maximises its expected profit given $ED(p, :)$. (Note that $L = ED(1, :)$.)



(a) The Maximisation Problem of the Firm

(b) The Slope of $ED(p, :)$ is Determined Only by L

(c) Higher the L, Higher the Expected Price

Figure 3.2: The Competitive Nature of Ordinal Procedures

A close inspection of $ED(p, :)$ reveals that it is linear. Assume there is a bump around a price.[10] Then any firm would find a profitable deviation which shifts the weights around the bump to the peak of the bump, preserving the expected value of the CDF. This deviation is profitable because even if it does not change the expected value of the CDF, it increases the expected probability of being chosen.

The reason is that decreasing one mass unit from two values with the same distance from the bump and adding those two mass units to the peak of the bump keeps the expected value constant, but as the bump pays higher than the linear relationship (by the definition of the bump), the expected probability of being chosen increases. For a pit, the reasoning is the reverse.

This linearity implies that the expected probability of being chosen becomes a linear function of the expected value of the observed price. Therefore, the expected profit of the firm is a quadratic function of the expected value of the observed price. This implies that I can draw the dotted isoprofit curves.

[10] When I say a bump, I mean that there exist $p_3 > p_2 > p_1$ where $ED(p_2,:)$ is above the line connecting $ED(p_1,:)$ and $ED(p_3,:)$. Assume $p_3 - p_2 = p_2 - p_1$ for simplicity.

Note that as the individual firm is indifferent between the equilibrium CDF and the equivalent expected price (by linearity), the Nash Equilibrium is not strict.

Additionally, in the symmetric equilibrium, the expected probability of being chosen is Ed^*=$ED(E^*, :) = 1/N$. Also note that $L < (1/N)$ by monotonicity.

As a summary; at the equilibrium, the dotted isoprofit curve is tangential to the linear, downward-sloping $ED(p, :)$ curve where $ED(p^*, :) = 1/N$ and the supremum is 1.

If $ED(p, :)$ is a linear function passing through $(1, L)$, then the other attributes of the procedure might reflect on the slope of $ED(p, :)$. However, another inspection reveals that two procedures with the same L have the same slope at the equilibrium.

Consider another procedure which has the same L but its $ED(p, :)$ has a flatter slope -dashed line- (See Figure 3.2b). By Cobb-Douglass (Cobb & Douglas, 1928) reasoning, [11] any firm can make a profitable deviation by choosing the price $E^{***} > E^{**}$.

The only remaining area where other attributes of procedures can show their effect is the infimum. Assume the infimum decreases, then as the slope does not change, it does not change the optimal expected price. If the infimum increases, then again the optimal price does not change because $ED(I, :) > (1/N)$, where I is the infimum, by monotonicity.

If the only link between the procedure and the expected price is L [12] , it is important to see how L affects the expected price. From the equation given in Proposition

[11] When I say Cobb-Douglass reasoning, I mean the study of profit maximising behaviour by employing the study of income and substitution effects for Cobb-Douglass utility functions that fits into our framework.

[12] Continuity is another space where other aspects of a procedure may reflect. Shifting the atom to a lower price would decrease the expected price which is dominated by the certain increase in the expected probability of being chosen -for the cases at which all observations made realised at the atom- when the shift is taken narrowly enough.

3.3.1, the expected price is increasing in L. Assume this is not the case (see Figure 3.2c). If for higher L the $ED(p,:)$ is flat enough such that it cuts the $ED = 1/N$ line from the left of E^*, then again by Cobb-Douglass reasoning, the optimal expected price would be E^{***}. This is true for all cases where $ED(p,:)$ cuts $ED = 1/N$ line from the left. Therefore, the higher the L, the higher the expected price.

The generalisation for $K > 1$ follows from the linearity of the expected value operator. Remember that the expected profit is the expected value of the CDF multiplied by the expected probability of being chosen. The expected probability of being chosen is the same for any K. The reason is that the expected probability of being chosen is just the average, over all sources, of the expected fraction of points collected from each source. Additionally, the expected fraction of points received from any given source is the same for all sources. This makes the equilibrium expected price (equilibrium CDF as well) independent of K.

This last observation concludes my investigation on the link between the expected price and L. To summarise,

1. NL is a measure of anti-competitiveness of the market exerted by the procedure.
2. The aspects of a procedure other than L have no effect on the expected price: they are neutralised by the nature of sampling-based reasoning and the competitive environment.
3. The expected price is independent of the number of sources.

Proposition 3.3.2 updates the main lesson of Spiegler (2006a) that increasing the number of the firms does not alter the expected price. Corollary 3.3.3 presents the calculative aspects of the equilibrium CDF and Proposition 3.3.6 gives the equilibrium CDFs of example procedures. Proposition 3.3.7 presents a family of procedures generating a given expected price.

3.3.1 Comparative Statics

Recall that the expected price is $(2 - NL)^{-1}$ by Proposition 3.3.1: NL serves as a measure of the anti-competitiveness of the market: the higher the NL, the higher the expected price.

Note that L might be a function of N. Denote L_N as the $ED(1,:)$ for N firms.

I need to take a position on how L_{N+1} is formed. For instance, if there are 3 firms where $\alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 3$, then what is the sequence if there are 4 firms?

I take the position that L_{N+1} is formed using for the same numbers used for L_N but with one more number is introduced. That is, for the example in the previous paragraph, the sequence will be formed by the numbers $1, 2, 3$ and $r \in \mathbb{R}_+$ when evaluating 4 firms. Taking this position reveals the following proposition.

Proposition 3.3.2. *Assume that the sequence of the GBP used when evaluating N firms is $\alpha_1, \alpha_2, ..., \alpha_N$. Assume further that the sequence of the GBP used when evaluating $N + 1$ consists of α_j's ($j = 1, 2, ..., N$) and α_{new}. If $\alpha_1 > 0$, the expected price decreases with the number of firms if $\alpha_{new} > \frac{\sum_{j=1}^{N} \alpha_j}{N}$. If $\alpha_1 = 0$, then the expected price does not change with the number of firms.*

Let me start with the case $\alpha_1 > 0$.

In this case, the expected price decreases if and only if $\frac{1}{2-NL_N} > \frac{1}{2-(N+1)L_{N+1}}$ (or alternatively $NL_N > (N + 1)L_{N+1}$.)

$\alpha_{new} > \frac{\sum_{j=1}^{N} \alpha_j}{N}$ guarantees this condition. See that, $\alpha_{new} > \frac{\sum_{j=1}^{N} \alpha_j}{N}$ implies $N\alpha_{new} >$

$\sum_{j=1}^{N}\alpha_j$, which implies $\alpha_1 N\sum_{j=1}^{N}\alpha_j + \alpha_1 N\alpha_{new} > (N+1)\alpha_1\sum_{j=1}^{N}\alpha_j \geq (N+1)min\{\alpha_1,\alpha_{new}\}\sum_{j=1}^{N}\alpha_j$. This inequality and using $NL_N = N\frac{\alpha_1}{\sum_{j=1}^{N}\alpha_j}$ and $(N+1)L_{N+1} = (N+1)\frac{min\{\alpha_1,\alpha_{new}\}}{(\sum_{j=1}^{N}\alpha_j)+\alpha_{new}}$ prove the result.

For the case $\alpha_1 = 0$ both L_N and L_{N+1} are zero. Therefore the expected price will be 0.5 for both cases.

For instance, using the general rule $\alpha_j = j\ \forall N$ would decrease the expected price as the number of firms increases.

For $L = 0$ the expected price does not change. This is the result obtained in Spiegler (2006a). In his model $L = 0$ because the firm which has the highest price observed gets zero expected demand. For the case where $L = 0$, the expected demand for the worst-performing firm is already the lowest, 0 - which cannot decrease further. Therefore, the expected price would not change.

The condition stating that the new number introduced to be high might be interpreted as the consumer getting pickier as the number of firms increases. Assume you observe 2 firms. Is your probability of picking the worst-performing one lower than the case of 3 firms? If yes, by how much? Proposition 3.3.2 says if it is low enough, then having more firms within the market reduces the expected price.

A plausible kind of limited attention might imply such behaviour. Assume after having observed all firms ($N > 3$), you focus on the two best-performing firms. The limited attention is defined as follows: The probability of picking one of these two best-performing firms is fixed (and > 0.5, one of them is chosen uniformly randomly) and if you do not pick one of these two best-performing firms, you uniformly randomly pick one of the others. This specification of ordinal procedures with a spirit of limited attention would satisfy the condition of Proposition 3.3.2.

Aside from the plausible conditions and frameworks which make sure the expected price decreases with the number of firms, it is also possible that the expected price to increase with the number of firms. For instance, for $N = 2$, assume the sequence used is $1, 2$ and for $N = 3$, it is $1, 1, 2$. For $N = 2$, the expected price is $3/4$ but for $N + 1$ it is $4/5$. Therefore, the expected price increases with the number of firms. The reason is the consumer does not punish the worst performing firms as severely as enough to make competition fiercer. Also note that the condition given in Proposition 3.3.2 is a sufficient condition. It is possible to find some other specification of GBPs that does not obey the conditions given in Proposition 3.3.2 but the expected price decreases with the number of firms. For instance, for $N = 2$, assume the sequence used is $1, 2$ (the expected price is $3/4$) and for $N = 3$, it is $0, 1, 2$ (the expected price is $1/2$.)

3.3.2 Equilibrium CDF

Proposition 3.3.1 gives the expected price for a given GBP. The agents in the model are risk-neutral. Nevertheless, it might be important to have the CDF explicitly which is given in the following corollary.

Corollary 3.3.3. *The equilibrium CDF satisfies:*

$$L + \frac{2 - NL}{N}(1 - p) = \frac{\sum_{i=1}^{N} \alpha_i * C(N, i-1) * F(p)^{N-i} * (1 - F(p))^{i-1}}{\sum_{j=1}^{N} \alpha_j}. \quad (3.3.1)$$

It is possible to specify the equilibrium CDF by employing the linearity of $ED(p, :)$, which satisfies the following: (see equation 3.5.1 in the Appendix for the following equality)

$$ED(p, :) = L + \tfrac{2-NL}{N}(1 - p)$$

This gives the left-hand side of the equation 3.3.1. On the other hand, $ED(p, :)$

is the expected probability of being chosen. It is equal to the expected value of numbers assigned divided by the sum of numbers assigned on a draw:

$$ED(p,:) = \frac{\sum_{i=1}^{N} \alpha_i * C(N,i-1) * F(p)^{N-i} * (1-F(p))^{i-1}}{\sum_{j=1}^{N} \alpha_j}.$$

where $C(x,y)$ is the number of y-combinations from a set with x elements. This gives the right-hand side of the equation 3.3.1.

Equating these two equations above gives another equation for which the equilibrium CDF satisfies.

It might be difficult to have a closed form solution for higher degree polynomials in some cases (e.g. the Focus Procedure when $N > 2$). Below, I provide the equilibrium expected price outcome and CDFs for the example procedures.

3.3.3 Examples Revisited

Definition 3.3.4. *The Basic Scoring Procedure : S is the Basic Scoring Procedure if* $\alpha_j = I_{T=N}(j = T)$.

Definition 3.3.5. *The Focus Procedure : S is the Focus Procedure if* $\alpha_j = j$.

The following Proposition 3.3.6 solves the example procedures.

Proposition 3.3.6. *For the two examples;*

- *The expected price of the Basic Scoring Procedure is* 0.5. *The equilibrium CDF is* $F(p) = 1 - \sqrt[N-1]{\frac{2(1-p)}{N}}$. *where* $p \in [\frac{2-N}{2}, 1]$.
- *The expected price for the Focus Procedure is* $(N+1)/(2N)$. *For* $N = 2$, *the equilibrium CDF is* $F(p) = 2p - 1$ *where* $p \in [0.5, 1]$.

The result for the Basic Scoring Procedure is the same as the result obtained in Spiegler (2006a). The number of sources does not alter the outcome. This is expected as it is shown that the outcome of the equilibrium is independent of the number of sources. Note that the lower bound of prices is negative when $N > 2$. That is, the equilibrium is highly dispersed in order to make use of the obfuscation. This might result in observing prices even below the marginal cost. On the other hand the expected equilibrium price is the same for all N. That is a higher number of firms reflects to the variance, not the expected price of the equilibrium distribution.

The result of the Focus Procedure is more interesting. In contrast with the Basic Scoring Procedure, the expected price decreases with the number of firms. Remember that LN is a measure of anti-competitiveness of the market. Increasing the number of firms by one decreases L drastically.

Note that the expected price is higher for the Focus Procedure. This stems from the fact that higher prices are not punished as severely as in the case of the Basic Scoring Procedure.

3.3.4 A GBP for Each Expected Price

Proposition 3.3.7. *There exists a GBP generating each expected price* $E \in [0.5, 1)$, *where* $\alpha_j = \frac{2E-1}{NE} + \frac{2(j-1)}{N-1}\frac{1-E}{NE}$.

A reverse engineering for Proposition 3.3.1 is also possible to some extent. It is possible to build a procedure for each expected price greater than 0.5.

This proposition becomes critical when it is possible to affect the procedure the consumer uses for evaluating prices via different means. Framing might be one of the tools that can be used in order to affect the evaluation process. For instance, assume there is a market where the policy maker or the information provider has power to affect sampling-based price evaluation. Further assume that this policy maker or the information provider wants the expected price to be 2/3 where there are 3 firms. In order to do so, the sequence to impose on the consumer can be built as $\alpha_i = i/6$ (By Proposition 3.3.7.) Furthermore, the equilibrium CDF obtained is $F(p) = \frac{3-\sqrt{4-3p}}{2}$ (By Corollary 3.3.3.)

The question on how a policy-maker or information-provider might affect sampling-based price evaluation is an open research question. Comparison web-sites, informative brochures and advertisements are some candidates which might be employed for this purpose. This corollary shows how this potential ability to affect evaluation might be exercised.

3.4 Discussion and Conclusion

This study investigates the market implications of sampling-based procedures. An axiomatic characterisation of GBPs was presented. The equilibrium outcome was given and the calculative aspects were explained. A tool for building procedures generating a given expected price was presented. The implications of higher N and K were investigated. A potential road map for further research is as follows:

Assume quality is a choice variable of firms. It is already shown that S(1) (the procedure introduced by Osborne & Rubinstein (1998)) might result in an inefficient market outcome (Szech, 2011; Spiegler, 2006a). It is an interesting research question to understand the relationship between a procedure (sampling-based procedures over sampled consumer surpluses) and the level of efficiency in a general setting. In particular, the procedure itself might affect the share of surplus received by the consumer. This sharing and the level of efficiency might move in the opposite or the same directions for alternative procedures.

Investigating comparison web-sites might give a rise to interesting questions. For example, assume there is a website where only the best comments about firms are revealed. Call this website as firms-lobby.com. Assume another website where only the worst comments are revealed. Call this website complainfirms.com. Assume the agents are indifferent between them - so they share the consumers evenly at period 1. Further assume firms-lobby.com is subsidised by firms and complainfirms.coms earnings are only based on the number of visits. At period 2, the agents visit the same website only if they experienced higher surplus than the surplus indicated in the comment they have read (no disappointment). The interesting research question is whether or not complainfirms.com can survive within the website market.

Market intervention by enforcing simple pricing schedules (e.g. degenerate CDFs) might have welfare enhancing effects. For $S(1)$, it is shown by Spiegler (2006a) that firms respond to such a policy by increasing the variance of the equilibrium CDF, which is similar to the response of the firms to an additional firm within the market. This finding might be updated for alternative procedures.

The existence of equilibrium, adapting $S(1)$ equilibrium concept for S, for agents both employing S and the convergence relationship with Nash Equilibrium are other possible, theoretical directions for further research.

The information gathering process of the study is not modelled. This process can be motivated via either Word-of-Mouth marketing, advertisements or experience. These alternative motivations have inherently different natures which might enrich our understanding of sampling-based reasoning.

The extent of the manipulation that firms, the policy maker or information provider can exert on the procedures that consumers might employ is a crucial research area. This is because the procedure employed has a direct effect on the expected market price and the distribution of the market price. Additionally, the sampling procedure might be affected extrinsically (e.g. by the firms (Gabaix & Laibson, 2006)), or even intrinsically (Grubb, 2009). This is currently a very vivid and productive research area.

The model of this paper includes only one type of consumer. However, the case of heterogeneous consumer population might have interesting and important implications. In particular, further research on whether savvy consumers help (or benefit from) non-savvy ones might reveal important policy lessons (Armstrong, 2015). For the original interpretation of the model, it is possible to introduce savvy consumers who are not exposed to sporadic evaluation; that is they can buy the good with an observed price. This might not necessarily create an incentive for lower prices - but maybe even reverse. That is because increasing the variance of the CDF will act as a competitive instrument for both types of consumers which might render decreasing the expected price void. As the savvy consumer is not exposed to sporadic evaluation, she might end up buying the good with lower, maybe lower than the marginal cost, prices in expectation. As a result, it is possible to have a market outcome where the presence of non-savvy consumers helps savvy ones.

Empirical evidence might be sought for evaluating the implications of the theoretical model. The relationship between the procedures employed and the expected price as well as the comparative statics over N or K can be investigated through empirical studies. This can be done both via laboratory experiments and through field evidence. Not only the implications but also the nature of procedures employed by the agents in different environments can be investigated via mouse- or eye-tracking tools and the data consumers look for (e.g. in the internet) before deciding the company they will interact with.

3.5 Appendix

Proof of The Proposition 3.2.7. Step 1 shows the "only if" part of the proposition and the Step 2 shows the "if" part.

Step 1

Take a GBP. It is ordinal and monotonic by definition. As the assignment of sequence entries is independent of the label of the rank and the firm, it is label-blind. As the sequence is independent of the values from different resources, it also obeys Limited Trade-Off Efficacy.

Step 2

Take a procedure which is Ordinal(**O**), Monotonic (**M**), Label-Blind (**LB** which is Firm-Blindness - **FB** - and Source-Blindness - **SB** -) and obeys Limited Trade-Off Efficacy (**LTE**).

Take firm i with observations ω_i. Its probability of being chosen:

$$
\begin{aligned}
S_i(\omega) &= S_i((\omega^T)_1, (\omega^T)_2, ..., (\omega^T)_K) \\
&= S_i'(C_i(\omega), c_i(\omega)) && \text{by } \mathbf{O} \\
&= S_i^*((c_{i,1}(\omega), N, N - c_{i,1}(\omega) - C_{i,1}(\omega)), \\
&\qquad (c_{i,2}(\omega), N, N - c_{i,2}(\omega) - C_{i,2}(\omega)), ..., \\
&\qquad (c_{i,K}(\omega), N, N - c_{i,K}(\omega) - C_{i,K}(\omega)))^{13} \\
&= \sum_{k=1}^{K} Co_i^k(c_{i,k}(\omega), N, N - c_{i,k}(\omega) - C_{i,k}(\omega)) && \text{by } \mathbf{LTE}\ ^{14} \\
&= \sum_{k=1}^{K} Co^k(c_{i,k}(\omega), N, N - c_{i,k}(\omega) - C_{i,k}(\omega)) && \text{by } \mathbf{FB} \\
&= \sum_{k=1}^{K} Co(c_{i,k}(\omega), N, N - c_{i,k}(\omega) - C_{i,k}(\omega)) && \text{by } \mathbf{SB}\ ^{15}
\end{aligned}
$$

Co can be interpreted as the contribution of an observation to the probability of being chosen. Co takes observations from a source and returns the contribution.

Now think about ω where all observations are distinct. Then,

$$Co(c_{i,k}(\omega), N, N - c_{i,k}(\omega) - C_{i,k}(\omega))) = Co(c_{i,k}(\omega), N, 1)$$

Now take ω^* where $W > 1$ firms' observations have the same ordering for some k and apart from those observations it is as same as ω.

By **O**,

[13] Note that $(C_{i,k}(\omega), c_{i,k}(\omega))$ and $(c_{i,k}(\omega), N, N - c_{i,k}(\omega) - C_{i,k}(\omega)))$ carry the same information. This representation on this step is used for the illustrative easiness in the further steps of the proof.

[14] Assume **LTE** holds for a function $f : \mathbb{R}^2 \to \mathbb{R}$. Then $\forall$x,y, $\Delta \in \mathbb{R}$, $f(x + \Delta, y) - f(x, y) = \tau(x, \Delta)$. For $\Delta = -x$ I obtain $f(0, y) - f(x, y) = \tau(x, -x)$. Taking $f(0, y) = h(y)$ and $\tau(x, -x) = -g(x)$ gives $f(x, y) = g(x) + h(y)$. A direct generalisation of this enables me to conduct the regarding step.

[15] $f, g : \mathbb{R} \to \mathbb{R}$. **SB** states $\forall$x,y$\in \mathbb{R}$, $f(x) + g(y) = f(y) + g(x) \implies \exists\ z \in \mathbb{R}$ such that $\forall$x$\in \mathbb{R}$, $f(x) = g(x) + z \implies$ defining $h(x) = f(x) - (z/2)$ gives $\forall$x,y$\in \mathbb{R}$, $f(x) + g(y) = h(x) + h(y)$. A direct generalisation of this reasoning enables me to conduct the regarding step.

$$Co(c_{i,k}(\omega*), N, W) = Co(c_{j,k}(\omega*), N, W)$$

where firms j, k are two of W firms.

Additionally, by **O** ,

$$Co(c_{x,k}(\omega), N, 1) = Co(c_{x,k}(\omega*), N, 1)$$

where firm x is not of those W firms.

Therefore, $(W = N - (c_{i,k}(\omega^*) + C_{i,k}(\omega^*)))$

$$Co(c_{i,k}(\omega*), N, W) = \frac{\sum_{t=C_{i,k}(\omega)+1}^{N-c_{i,k}(\omega)} Co(t,N,1)}{N-(c_{i,k}(\omega)+C_{i,k}(\omega))}$$

Defining $\alpha_j = Co(N-j, N, 1) * (K * \sum_{j=1}^{N} \alpha_j)$ produces the relevant formulae for the probability of being chosen defined by GBP.

The only remaining bit for the proof is showing that **M** implies the existence of a sequence obeying the requirements on α's in the definition of a GBP.

Note that, $\sum_{j=1}^{N} \alpha_j$ can be fixed to a positive value as not the exact values of $\alpha_j' s$, but the relative to sum matters. Therefore all α's are non-negative.

Remember the definition:

Definition 3.5.1. *Monotonicity :* $\omega_i \leq \omega_i^*$ *and* $\omega_{-i} = \omega_{-i}^*$ *implies* $S_i(\omega) \geq S_i(\omega^*)$ *where* $\omega_{-i} = \omega \setminus \omega_i$*. Additionally,* $\omega_i < \omega_j < \omega_i^*$ $\forall j \neq i$ *implies* $S_i(\omega) > S_i(\omega^*)$.

Take ω and ω^* with no ties. Assume that firm i has the x^{th} highest prices observed for ω and y^{th} highest prices observed for ω^* for each $k = 1, 2, ..., K$. For $x = y + 1$ where $y = 1, 2, ..., N-1$, by the first part of the definition, $S_i(\omega) - S_i(\omega^*) = K(Co(N-x, N, 1) - Co(N-y, N, 1)) \geq 0$, therefore $\alpha_{y+1} \geq \alpha_y$. For $x = N$ and $y = 1$, by the second part of the definition, $S_i(\omega) - S_i(\omega^*) = K(Co(0, N, 1) - Co(N-1, N, 1)) > 0$. Therefore, $\alpha_N > \alpha_1$, which concludes the proof.

□

Proof of the Proposition 3.3.1. *Firstly, I write the problem of the firm for* $K = 1$, *later I show, in the symmetric equilibrium, the CDFs are continuous, strictly increasing with a support of supremum 1. Next, I show the expected probability of being chosen is linear for the prices drawn at the first observation. Furthermore, I solve the problem of the firm. Lastly, I show the equilibrium CDF is also the symmetric equilibrium for* K *in general. This proof follows the steps of Spiegler (2006a) and generalises it in terms of the number of observations per firm and the probability assignment for the whole simplex (non-degenerate probability of being chosen for firms - alternatively, any non-decreasing sequence used by the consumer -).*

Step 1: The problem of the firm.

The firm maximises its expected profit over $\mathbb{F}$. $F_i(p)$ determines the expected probability of being chosen (Ed_i) given the CDFs of other firms. The expected value of prices: E_i. Note that the price paid by the consumer is an independent, new draw from the CDF of the chosen firm. Therefore the problem of the firm is:

$$max_{F_i(p)\in\mathbb{F}} E_i * Ed_i$$

Define $x = 1 - p$ which will be the domain of functions I will use for this proof and define $G_i(x)$=$1 - F_i(p)$ for all i [16]. This definition of x alters the formulae but it does not change the problem by any means other than notation. With this change, the expected price is now $1 - E_i$. $ED_i(x, :)$ is the expected probability of firm i to be chosen where x is observed from the CDF of firm i, and given the CDFs of other firms. Ed_i is the probability of firm i to be chosen. That is Ed_i is the expected value of $ED_i(x, :)$ over the CDF of firm i.

[16] $F(p) = P(p \geq P) = P(1 - p \leq 1 - P) = P(x \leq X) = 1 - G(x)$

Therefore it can also be written as

$$max_{G_i(x)\in\mathbb{F}}(1 - E_i) * Ed_i$$

The analysis will be conducted on a symmetric equilibrium.

Step 2: $F_i(p)$ is continuous on $(-\infty, 1)$.

Shifting an atom to a lower value of p (higher value of x) is a profitable deviation for a small shift. Assume there is a discontinuity (a mass) at x. Take deviation shifting the weight from $(x - \epsilon, x]$ to $x + \epsilon$. This deviation increases both E_i (which decreases the expected profit) and Ed_i (which increases the expected profit).

Claim: The maximum increase in E_i is $2 * \epsilon * (G(x) - G(x - \epsilon))$

Every values in $(x - \epsilon, x]$ shifts by maximum of $2 * \epsilon$. Therefore, the expected value increases at most by $2 * \epsilon * (G(x) - G(x - \epsilon))$.

Claim: The minimum increase in Ed_i is $(G(x) - \lim_{\epsilon\to 0} G(x - \epsilon))^N * t$ where $t \in \mathbb{R}_+$.

I spot some profiles which increases Ed_i. These are the observation profiles at which all the observations made are at x.

For small enough ϵ, the increase in E_i is dominated by the increase in Ed_i.

Thus, G is continuous at $(0, \infty)$ at equilibrium. G being continuous implies F is continuous.

Step 3: The supremum of the support of equilibrium F is 1.

First note that, it can't be more than 1 (the infimum of x can't be less than 0.) I use notation on p instead of x for this paragraph. The deviation shifting all the weight corresponding the values larger than 1 to 1 will result selling the good for any draw conditional on being chosen and the expected probability of being chosen does not decrease, by monotonicity. More explicitly assume the expected value is v below 1 and $F_i(1) = z$. Then the expected profit is vEd_i. After the deviation the expected profit is at least $(v + 1 - z) * Ed_i > v * Ed_i$.

Let me return to notation with G rather than F. Assume the infimum is $x > 0$. Take deviation shifting all weights in the interval $(x, x + \epsilon]$ to 0. This deviation reduces both E_i (which increases the expected profit) and Ed_i (which decreases the expected profit).

Claim: The minimum reduction in E_i is $x * G(x + \epsilon)$.

Note that, $G(x) = 0$ by Step 2. In this case E_i reduces by $x * G(x + \epsilon)$.

Claim: The maximum reduction in Ed_i is $G(x + \epsilon) * (1 - (1 - G(x + \epsilon))^{N-1}) * t$ where $t \in \mathbb{R}_+$.

Think about the cases which the deviating firm has one observation in the interval $(x, x+\epsilon]$ and there is at least one observation from other firms in the same interval.

For small enough ϵ, the quadratic reduction in Ed_i is dominated by the reduction in E_i.

Step 4: The $F_i(p)$ is strictly increasing on $[1 - M, 1]$. (M represents the supremum of x.)

First note that, there is an infimum of F because of Monotonicity. I use the same reasoning in Step 3. Assume G_i is flat over $[x_1, x_2]$ where $x_2 < M$. The deviation which shifts weight from $(x_2, x_2+\epsilon]$ to x_1. Both E_i and Ed_i reduces. The minimum decrease in E_i is $(x_2 - x_1) * (G(x_2 + \epsilon) - G(x_2))$. The maximum decrease in Ed_i is $((G(x_2 + \epsilon) - G(x_2))) * (1 - (1 - (G(x_2 + \epsilon) - G(x_2)))^{N-1}) * t$ where $t \in \mathbb{R}_+$. For small enough ϵ the decrease in E_i dominates the decrease in Ed_i.

Step 5: $F_i(p)$ is continuous over $[1 - M, 1]$

The analysis of this step is over x. Assume there is a mass at 0 valued m. Consider the deviation shifting the mass to $\epsilon > 0$. Both E_i and Ed_i increases. The increase in E_i is $\epsilon * m$. The minimum increase in Ed_i is when all the observations is 0, which is $m^N * t$ where $t \in \mathbb{R}_+$. For small enough ϵ, this is a profitable deviation.

Step 6: ED_i is linear in the support of $F_i(p)$.

This result is by Spiegler (2006a) which has a precedence at Myerson (1993).

This step also implies that for the values within the interval bounded by the infimum and supremum of the support, the ED_i can't have values above the linear direction.

Again, I conduct the inspection for x. I drop the subscript i in notation for convenience.

Assume it is not linear, therefore there exist $x_3 > x_2 > x_1$ either $ED(x_2, :)$ is below or above the line connecting $ED(x_3, :)$ and $ED(x_1, :)$. By monotonicity, $ED(x_3, :)$ $\geq ED(x_2, :) \geq ED(x_1, :)$. Assume G places an atom each of these points.

Assume it is above. Then,

$$(ED(x_1, :))(x_3 - x_2) + ED(x_2, :)(x_1 - x_3) + ED(x_3, :)(x_2 - x_1) < 0$$

Consider the deviation which does not change the expected value and shifts weight from x_1 and x_3 to x_2 in a way to increase Ed. That is find $\epsilon_1, \epsilon_2, \epsilon_3$ small enough which satisfies,

1. $x_1\epsilon_1 + x_2\epsilon_2 + x_3\epsilon_3 = 0$

2. $\epsilon_1(ED(x_1, :)) + \epsilon_2(ED(x_2, :)) + \epsilon_3(ED(x_3, :)) > 0$. Given

3. $(ED(x_1, :))(x_2 - x_3) + (ED(x_2, :))(x_3 - x_1) + (ED(x_3, :))(x_1 - x_2) > 0$.

There is small enough $t > 0$,

$$\epsilon_1 = \frac{t(x_2-x_3)}{x_3-x_1}, \epsilon_2 = t, \epsilon_3 = \frac{t(x_1-x_2)}{x_3-x_1}$$

solving this system of inequalities. (if it is below, use the negative of t.)

This is a profitable deviation. The reason is the expected price has not changed but the expected probability of being chosen increased.

Mass points assumed for the sake of easiness in illustration. The same reasoning for the continuous case by shifting weights in a small neighbourhood of x_1, x_2, x_3 constitutes a profitable deviation.

Step 7: Solution of the firms problem.

By Step 2-6, the expected probability of being chosen can be rewritten as

$$ED_i(x,:) = ED_i(0,:) + \frac{((ED_i(M,:)-ED_i(0,:))x}{M}$$

Remember that $ED_i(x,:)$ is linear for x being the first observation.

By definition

$$Ed_i = ED_i(0,:) + \frac{((ED_i(M,:)-ED_i(0,:))E_i}{M}$$

Therefore the problem of the firm becomes

$$max_{G_i}(1-E_i) * Ed_i = max_{G_i}(1-E_i) * (ED_i(0,:) + \frac{((ED_i(M,:)-ED_i(0,:))E_i}{M})$$

First order condition with respect to E_i gives

$$E_i^* = \frac{ED_i(M,:)-ED_i(0,:)-M*ED_i(0,:)}{2(ED_i(M,:)-ED_i(0,:))}$$

and the second order condition satisfies.

In equilibrium $Ed = 1/N$. Inserting E_i^* into Ed_i gives

$$M = \frac{(ED_i(M,:)-ED_i(0,:))N}{2-N*ED_i(0,:)}$$

and therefore

$$ED_i(x, :) = ED_i(0, :) + \frac{2 - N * ED_i(0, :)}{N} x \quad (3.5.1)$$

[17]

Inserting M into E_i^* gives

$$E_i^* = 0.5 - \frac{N * ED_i(0,:)}{2(2 - N * ED_i(0,:))}.$$

The expected price is then

$$1 - E_i^* = \frac{1}{2 - N * ED_i(0,:)}$$

L is defined as $ED_i(0, :)$.

Note that, the straightforward generalisation for willingness to pay is v and marginal cost is c produces the expected price of, if there is an interior solution, $\frac{v + c(1 - NL)}{2 - NL}$.

Step 8: Generalisation for any K.

I show here that the equilibrium CDF found for $K = 1$ also satisfies for any K. That is if $G(x)$ is an equilibrium strategy for $K = 1$, it is also an equilibrium strategy for any K.

Take a strategy profile which is all firms choose $G(x)$. Each firm maximises $E * Ed_i$ given strategies of other firms.

For any firm $E * Ed_i = E * \frac{\sum_{j=1}^{K} E(\alpha)}{K * \sum_{j=1}^{N} \alpha_j}$ by the linearity of expected value operator where $E(\alpha)$ is the expected value of the α's for each k, which is the same for all k. Therefore $E * Ed_i = \frac{E * E(\alpha)}{\sum_{j=1}^{N} \alpha_j}$ which is the same optimisation problem with $K = 1$. □

Proof of the Proposition 3.3.6. The equilibrium distributions and the expected prices

[17] Note that $ED_i(x, ;)$ is defined the probability of firm i being chosen, given the price observed is $1 - x$ throughout the proof.

are calculated as follows

The Basic Scoring Procedure

$L = 0$. Therefore the expected price is 0.5. Inserting α's gives $ED(p,:) = F(p)^{N-1}$ and using $ED(p,:) = L + \frac{2-NL}{N}(1-p)$ gives $ED(p,:) = \frac{2(1-p)}{N}$. Combining these gives the solution.

The Focus Procedure for $N = 2$

$L = \frac{2}{N(N+1)}$. Therefore the expected price is $\frac{1}{2-\frac{2}{N+1}} = \frac{N+1}{2N}$. Inserting α's gives $ED(p,:) = (1/3)*F(p)+(2/3)*(1-F(p))$ and using $ED(p,:) = (1/3)+\frac{2-(2/3)}{2}(1-p) = (1/3)+(2/3)(1-p)$. Therefore $F(p) = 2p-1$. $\square$

Proof of the Proposition 3.3.7. The procedure is a GBP. Therefore I can make use of Proposition 3.3.1.

$$
\begin{aligned}
L &= \frac{\alpha_1}{\sum_{j=1}^{N}\alpha_j}, \\
&= \frac{\frac{2E-1}{NE} + \frac{1-1}{N-1}\frac{1-E}{E}}{\sum_{j=1}^{N}\frac{2E-1}{NE} + \frac{2(j-1)}{N-1}\frac{1-E}{NE}}, \\
&= \frac{\frac{2E-1}{NE}}{\frac{2E-1}{E} + \sum_{j=1}^{N}\frac{2(j-1)}{N-1}\frac{1-E}{NE}}, \\
&= \frac{\frac{2E-1}{NE}}{\frac{2E-1}{E} + \frac{1-E}{EN(N-1)}\sum_{j=1}^{N}2(j-1)}, \\
&= \frac{\frac{2E-1}{NE}}{\frac{2E-1}{E} + \frac{1-E}{EN(N-1)}\frac{2(N-1)N}{2}}, \\
&= \frac{2E-1}{EN}
\end{aligned}
$$

Therefore, the expected price is $\frac{1}{2-NL} = E$ $\square$

References

Armelini, G., Villanueva, J., et al. (2010). Marketing expenditures and word-of-mouth communication: Complements or substitutes? *Foundations and Trends® in Marketing*, *5*(1), 1–53.

Armstrong, M. (2015). Search and Ripoff Externalities. *Review of Industrial Organization*, *47*(3), 273–302.

Awad, N. F., Dellarocas, C., & Zhang, X. (2004). Is online word of mouth a complement or substitute to traditional means of consumer conversion. In *Sixteenth Annual Workshop on Information Systems Economics (WISE), Washington, DC*. Citeseer.

Bagwell, K. (2007). The economic analysis of advertising. *Handbook of industrial organization*, *3*, 1701–1844.

Baye, M. R., Morgan, J., Scholten, P., et al. (2006). Information, search, and price dispersion. *Handbook on economics and information systems*, *1*, 323–77.

Bloch, F. & Manceau, D. (1999). Persuasive advertising in hotelling's model of product differentiation. *International Journal of Industrial Organization*, *17*(4), 557–574.

BOOMZ (n.d.). Untitled boomz and say-it wear advertising. http://media-cache-ec0.pinimg.com/736x/45/9a/66/459a667282db53a85e9a65e9b5642edc.jpg. [accessed 23-Feb-2015].

Boris, C. (2012). The art of social referral. http://www.marketingpilgrim.com/2012/04/the-art-of-the-social-referral.html. [accessed 22-February-2015].

Braithwaite, D. (1928). The economic effects of advertisement. *The Economic Journal*, *38*(149), 16–37.

Brocas, I. & Carrillo, J. D. (2007). Influence through ignorance. *The RAND Journal of Economics*, *38*(4), 931–947.

Burdett, K. & Judd, K. L. (1983). Equilibrium Price Dispersion. *Econometrica*, *51*(4), 955–69.

Chevalier, J. A. & Mayzlin, D. (2006). The effect of word of mouth on sales: Online book reviews. *Journal of marketing research*, *43*(3), 345–354.

Cobb, C. W. & Douglas, P. H. (1928). A theory of production. *The American Economic Review*, *18*(1), 139–165.

Comcast (n.d.). Untitled comcast advertisement. http://s3images.coroflot.com/user_files/individual_files/original285576KvvKixXYDCgeXi1G1TPN7CTgy.jpg. [accessed 22-Feb-2015].

Crawford, V. P. & Meng, J. (2011). New york city cab drivers' labor supply revisited: Reference-dependent preferences with rationalexpectations targets for hours and income. *The American Economic Review*, *101*(5), 1912–1932.

de Borda, M. (1781). Histoire de l'académie royale des sciences. *Mémoires sur les élections au scrutin*.

DellaVigna, S. & Malmendier, U. (2004). Contract design and self-control: Theory and evidence. *The Quarterly Journal of Economics*, *119*(2), 353–402.

Dichter, E. (1966). How word-of-mouth advertising works. *Harvard business review*, *44*(6), 147–160.

Dixit, A. & Norman, V. (1978). Advertising and welfare. *The Bell Journal of Economics*, 1–17.

Eliaz, K. & Spiegler, R. (2006). Contracting with diversely naive agents. *The Review of Economic Studies*, *73*(3), 689–714.

Eliaz, K. & Spiegler, R. (2008). Consumer optimism and price. *Theoretical Economics*, *3*(4), 459–497.

Ellison, G. (2005). A model of add-on pricing. *The Quarterly Journal of Economics*, *120*(2), 585–637.

facebook.com (n.d.a). Facebook for business. `https://www.facebook.com/business/overview`. [accessed 06-April-2013].

facebook.com (n.d.b). Facebook for business. `https://www.facebook.com/business/theenglishcheesecakecompany`. [accessed 06-April-2013].

Farrell, J. & Klemperer, P. (2007). Coordination and lock-in: Competition with switching costs and network effects. *Handbook of industrial organization*, *3*, 1967–2072.

Feng, J. & Papatla, P. (2011). Advertising: stimulant or suppressant of online word of mouth? *Journal of Interactive Marketing*, *25*(2), 75–84.

Gabaix, X. & Laibson, D. (2006). Shrouded attributes, consumer myopia, and information suppression in competitive markets. *The Quarterly Journal of Economics*, *121*(2), 505–540.

Galeotti, A. (2010). Talking, searching, and pricing. *International Economic Review*, *51*(4), 1159–1174.

Grossman, G. M. & Shapiro, C. (1984). Informative advertising with differentiated products. *The Review of Economic Studies*, *51*(1), 63–81.

Grubb, M. D. (2009). Selling to overconfident consumers. *The American Economic Review*, *99*(5), 1770–1807.

Grubb, M. D. (2015). Overconfident consumers in the marketplace. *The Journal of Economic Perspectives*, *29*(4), 9–35.

Hattori, K. & Higashida, K. (2012). Misleading advertising in duopoly. *Canadian Journal of Economics/Revue canadienne d'économique*, *45*(3), 1154–1187.

Heidhues, P. & Koszegi, B. (2010). Exploiting naivete about self-control in the credit market. *The American Economic Review*, *100*(5), 2279–2303.

Hogan, J. E., Lemon, K. N., & Libai, B. (2004). Quantifying the ripple: Word-of-mouth and advertising effectiveness. *Journal of Advertising Research*, *44*(3), 271–280.

Hotelling, H. (1931). The economics of exhaustible resources. *Journal of political*

Economy, *39*(2), 137–175.

Intriligator, M. D. (1973). A probabilistic model of social choice. *The Review of Economic Studies*, *40*(4), 553–560.

Jehiel, P. (2005). Analogy-based expectation equilibrium. *Journal of Economic Theory*, *123*(2), 81–104.

KalaycI, K. & Potters, J. (2011). Buyer confusion and market prices. *International Journal of Industrial Organization*, *29*(1), 14–22.

Kamenica, E. & Gentzkow, M. (2011). Bayesian persuasion. *The American Economic Review*, *101*(6), 2590–2615.

Kőszegi, B. (2014). Behavioral contract theory. *Journal of Economic Literature*, *52*(4), 1075–1118.

Kőszegi, B. & Rabin, M. (2006). A model of reference-dependent preferences. *The Quarterly Journal of Economics*, *121*(4), 1133–1165.

Laffont, J.-J. & Martimort, D. (2009). *The theory of incentives: the principal-agent model*. Princeton university press.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics*, *112*(2), 443–478.

Lewellen, G. (2013). Expected maximum and minimum of real-valued continuous random variables. `https://antimatroid.wordpress.com/2013/01/01/expected-maximum-and-minimum-of-real-valued-continuous-random-variables/`. [accessed 17-March-2017].

Liu, Y. (2006). Word of mouth for movies: Its dynamics and impact on box office revenue. *Journal of marketing*, *70*(3), 74–89.

Luce, R. D. (2005). *Individual choice behavior: A theoretical analysis*. Courier Corporation.

MacMinn, R. D. (1980). Search and market equilibrium. *Journal of Political Economy*, *88*(2), 308–27.

Mas-Colell, A., Whinston, M. D., Green, J. R., et al. (1995). *Microeconomic theory*, volume 1. Oxford university press New York.

Matejka, F. & McKay, A. (2011). Rational Inattention to Discrete Choices: A New Foundation for the Multinomial Logit Model. CERGE-EI Working Papers wp442, The Center for Economic Research and Graduate Education - Economics Institute, Prague.

Monahan, G. E. (1984). A pure birth model of optimal advertising with word-of-mouth. *Marketing Science*, *3*(2), 169–178.

Myerson, R. B. (1993). Incentives to cultivate favored minorities under alternative electoral systems. *The American Political Science Review*, *87*(4), 856–869.

Osborne, M. J. & Rubinstein, A. (1998). Games with Procedurally Rational Players. *American Economic Review*, *88*(4), 834–47.

Piccione, M. & Spiegler, R. (2012). Price competition under limited comparability. *The quarterly journal of economics*, qjr053.

Read, D., Loewenstein, G., & Rabin, M. (1999). Choice Bracketing. *Journal of*

Risk and Uncertainty, *19*(1-3), 171–97.

Rhodes, A. & Wilson, C. M. (2016). False advertising. Working paper.

Rosenthal, R. W. (1980). A model in which an increase in the number of sellers leads to a higher price. *Econometrica: Journal of the Econometric Society*, 1575–1579.

Rubinstein, A. (1993). On Price Recognition and Computational Complexity in a Monopolistic Model. *Journal of Political Economy*, *101*(3), 473–84.

Salop, S. (1977). The Noisy Monopolist: Imperfect Information, Price Dispersion and Price Discrimination. *Review of Economic Studies*, *44*(3), 393–406.

Shavell, S. (1979). Risk sharing and incentives in the principal and agent relationship. *The Bell Journal of Economics*, 55–73.

Shilony, Y. (1977). Mixed pricing in oligopoly. *Journal of Economic Theory*, *14*(2), 373–388.

Spiegler, R. (2006a). Competition over agents with boundedly rational expectations. *Theoretical Economics*, *1*(2), 207–231.

Spiegler, R. (2006b). The market for quacks. *The Review of Economic Studies*, *73*(4), 1113–1131.

Spiegler, R. (2011). *Bounded rationality and industrial organization*. Oxford University Press.

Stewart, N., Chater, N., & Brown, G. D. (2006). Decision by sampling. *Cognitive psychology*, *53*(1), 1–26.

Szech, N. (2011). Becoming a bad doctor. *Journal of Economic Behavior & Organization*, *80*(1), 244–257.

Tversky, A. & Kahneman, D. (1975). Judgment under uncertainty: Heuristics and biases. In *Utility, probability, and human decision making* (pp. 141–162). Springer.

Varian, H. R. (1980). A Model of Sales. *American Economic Review*, *70*(4), 651–59.

Vigna, S. D. & Malmendier, U. (2006). Paying not to go to the gym. *The American Economic Review*, *96*(3), 694–719.

Von der Fehr, N.-H. M. & Stevik, K. (1998). Persuasive advertising and product differentiation. *Southern Economic Journal*, 113–126.

Von Thadden, E.-L. & Zhao, X. (2012). Incentives for unaware agents. *The Review of Economic Studies*, rdr050.