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# An investigation of efficient muon production for use in muon catalyzed fusion

R Spencer Kelly<sup>\*</sup> , Lucy J F Hart and Steven J Rose

Imperial College London, London, United Kingdom

<sup>\*</sup> Author to whom any correspondence should be addressed.

E-mail: [r.kelly12@imperial.ac.uk](mailto:r.kelly12@imperial.ac.uk)

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Supplementary material for this article is available [online](#)

## Abstract

We model the energy cost of producing muons for use in muon catalyzed fusion and show that by careful design the cost can be reduced by a factor of 2.5 below current values. This is done by recapturing the kinetic energy of waste particles and generating heat through tritium breeding. When put together with the modeling of muon catalyzed fusion we estimate that electrical output/electrical input of 14% can be achieved currently.

## 1. Introduction

In this paper, we present the results of a model optimizing the creation of muons for use in muon catalyzed fusion ( $\mu\text{cf}$ ). After giving a brief overview of  $\mu\text{cf}$ , we present a short description of our methodology and a summary of our results. This is followed by an estimate of how close  $\mu\text{cf}$  is to being a viable energy source.

For a detailed description of our methodology and a more comprehensive reporting of our results, please see the supplementary information (available online at [stacks.iop.org/JPEN/3/035003/mmedia](https://stacks.iop.org/JPEN/3/035003/mmedia)) associated with this publication.

## 2. Overview of muon catalyzed fusion

The structure of the muonic diatomic molecule is used in  $\mu\text{cf}$  to bring nuclei close enough together for fusion to occur. The energy produced by the fusion reactions generates heat which boils water, turns turbines and generates electricity. This method of bringing the nuclei within the range of the strong force contrasts with the approaches used in magnetic and inertial confinement fusion where the Coulomb repulsion between the nuclei is overwhelmed by employing high temperature plasmas. If it could be made viable,  $\mu\text{cf}$  would lend itself to the design and construction of smaller and less expensive power plants.

The process of  $\mu\text{cf}$  involves injecting negative muons into a deuterium-tritium (dt) mixture. The muon becomes the binding particle in a dt molecule and the inter-nuclear distance decreases by a factor of about 200 (the mass ratio of the muon to the electron). Once the molecule is formed, the fusion reaction  $\text{d} + \text{t} \rightarrow \text{n} + \alpha$  ( $\text{n}$  = neutron,  $\alpha$  = alpha particle) quickly occurs releasing 17.6 MeV of kinetic energy in the  $\text{n}$  and  $\alpha$ . A free muon usually emerges and joins another molecule to catalyze the fusion reaction once again. However, occasionally (with probability  $p$ ) the negatively-charged muon sticks (and remains stuck<sup>1</sup>) to the positively charged alpha particle so the muon is unable to catalyze further fusion reactions. The number of fusion reactions is limited by this 'alpha sticking' and by the time it takes to form a muonic dt molecule ( $t_c$ ). With  $\tau = 2.2 \mu\text{s}$  as the mean lifetime of the muon, the number of reactions catalyzed by a single muon, to a good approximation, is  $1/(p + t_c/\tau)$ . For a detailed description of the  $\mu\text{cf}$  process, see [1–3] or chapter 2 of [4] for example.

<sup>1</sup> So  $p$  is the net sticking probability taking into account the likelihood of the muon being stripped off the alpha particle after initially sticking to it.

The viability of  $\mu cf$  is determined by two factors: the number of fusion reactions catalyzed per muon and the energy cost of producing muons. In 1986, Jones [5] achieved approximately 150 reactions per muon. It has been found using the model developed in chapter 3 of [4] that  $p \cong 0.47\%$  and  $t_c/\tau \cong 0.18\%$  for a dt mixture at solid hydrogen density, figures consistent with Jones's finding. The cost of producing a muon can be found in the current literature to be about 5–6 GeV [6]. As pointed out in [7], each fusion reaction not only produces 17.6 MeV of kinetic energy in the resultant particles but can also produce heat energy due to the exothermic tritium breeding reaction:



if the neutron is absorbed into a lithium-lead blanket. As determined in [4], each fusion neutron generates approximately 1.75 of the tritium breeding reactions (1). This amounts to 8.4 MeV of additional energy per fusion reaction for a total of 26 MeV of heat energy. Thus, 150 fusion reactions catalyzed per muon will produce 3.9 GeV of heat energy. So, defining  $Q = (\text{heat energy produced})/(\text{kinetic energy of the beam used to create muons})$ , we find that  $Q$  currently lies between 0.65 and 0.78 which is similar to the highest values ever obtained using magnetic or inertial confinement fusion [8].

### 3. Research plan

What is described in this paper is the first of four phases of research into the optimal design of a  $\mu cf$  reactor:

- (a) In phase one we will optimize a system where a beam of particles (either deuterons or tritons) impacts a target producing negative pions (which decay into the negative muons needed for fusion) and other particles from which we can recover kinetic energy and generate tritium breeding reactions.
- (b) In phase two we will optimize the shape and thickness of a lithium-lead shell in order to capture most of the energy of recoverable particles and fusion neutrons as well as generate extra heat through tritium breeding.
- (c) In phase three we will construct a system of magnetic mirrors (as suggested in [6, 9]) surrounding the target and inside the shell in order to contain the negative pions allowing them to decay into muons.
- (d) In phase four we will optimize the size, shape, wall thickness and position of deuterium-tritium pods in order to capture a high fraction of muons without capturing many pions. Fusion reactions will be catalyzed in these vessels.

We will also determine the maximum beam intensity by calculating the muon flux which will result in vessel failure. This will be similar to the calculation done in [4] but instead of assuming a perfect beam of muons will use the spatial and momentum distributions of the muons determined in phase three.

At this point the plan is to place the pods so that the circulating muons eventually hit them and enter the dt mixtures. However it remains to be seen whether this approach will result in the loss of an unacceptably high fraction of the negative pions prior to their decay to muons. If that is the case, we will need to introduce (as yet undetermined) design features which separate the pions from the muons.

The four phases of research are depicted in figure 1.

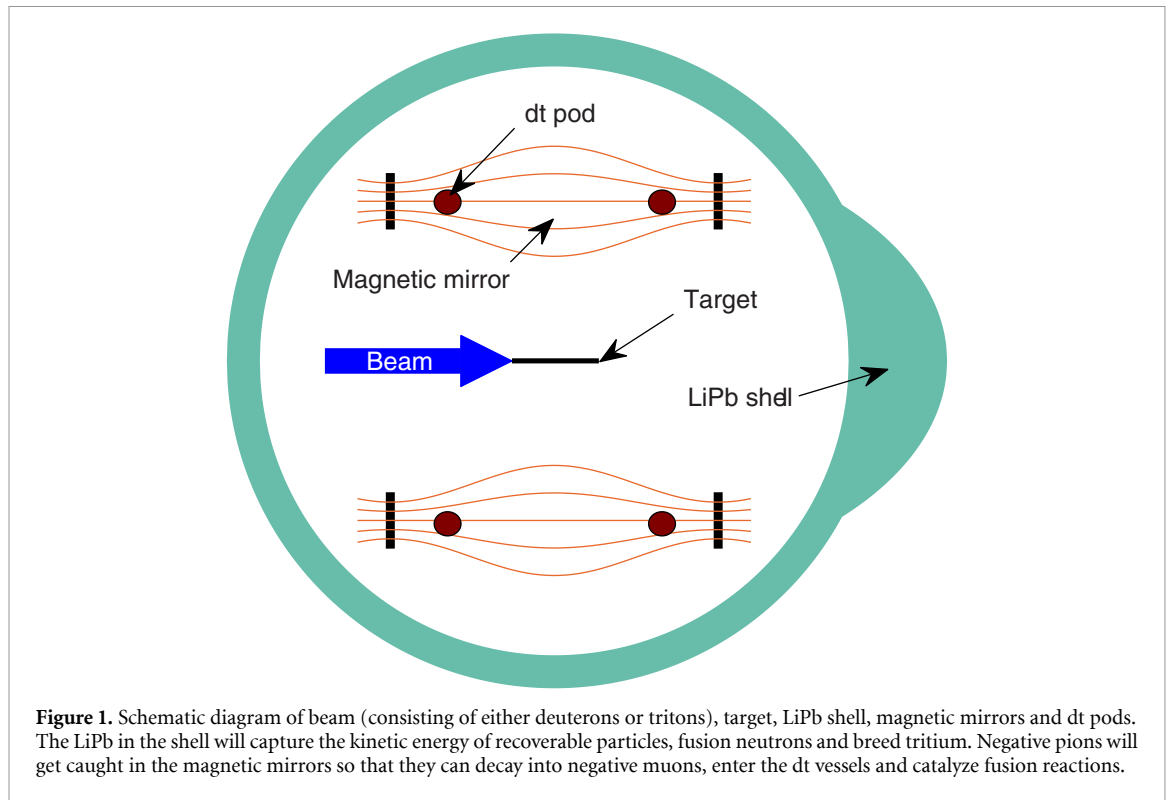
The overall goal is to determine  $Q_{\text{elec}}$ , the ratio of electricity produced to electricity consumed, for various fusion per muon assumptions.  $Q_{\text{elec}}$  will be estimated for each phase of research with the estimate becoming increasingly accurate in the later stages. This measurement will let us know how far  $\mu cf$  is from being a viable energy source and give us an idea of what technological improvements are necessary to make it so.

### 4. Description of model and methodology

What follows is a brief description. For the full details of the model and methodology, please see the supplementary information associated with this article.

In order to produce negative muons ( $\mu^-$ ), a beam of particles collides with a fixed target. Many particles emerge including negative pions ( $\pi^-$ ). These have an average life of 26 ns and decay via  $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ . The  $\mu^-$  are then delivered to vessels of dt where they catalyze fusion reactions.

To simulate the production of particles and heat by colliding beam particles with a target, we use the computational model G4Beamline [10]. G4Beamline uses the Geant4 toolkit [11] for the simulation of particle dynamics. Monte Carlo simulations are performed and, for each beam particle hitting the target, the particles emerging from the target, their momenta and trajectories are identified. In addition, the average heat deposited in the target per beam particle is calculated.



We consider beams of deuterons or tritons colliding with cylindrical targets made of carbon, a lithium-lead<sup>2</sup> mixture or tungsten. In addition to considering different materials for the beam and target, we vary the energy of the beam and the dimensions of the target. The combination of all of these variables is optimized in order to find the configuration with the highest  $Q$ . This optimization is conducted for assumed fusion reactions per negative muon varying from 100 to 600.

As Jändel [6] showed, beams of neutrons are more effective at producing negative pions than beams of protons. Deuterons and tritons are chosen as candidate beam particles instead of protons because the additional neutrons increase negative pion production. These negative pions decay into negative muons which catalyze fusion reactions. The beam particles are not directly involved in the dt fusion.

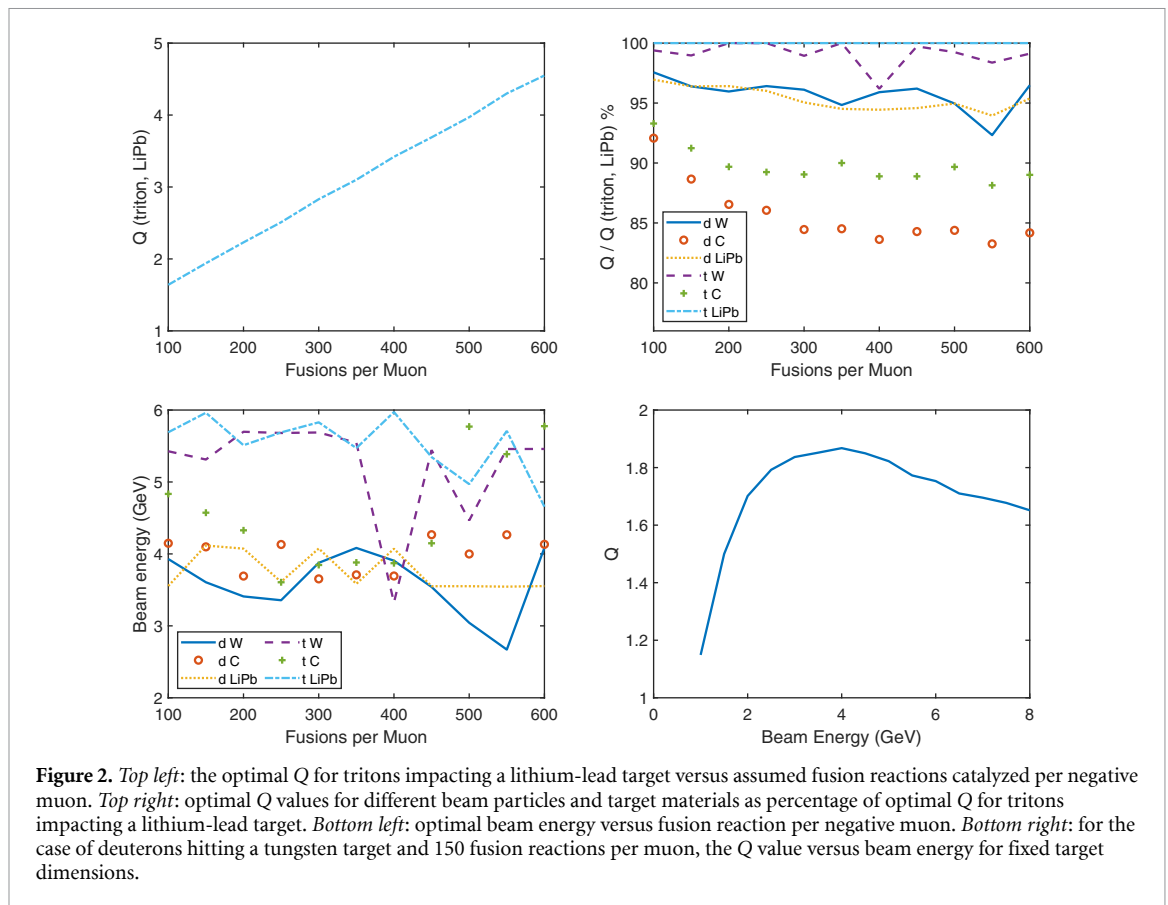
We use a genetic optimization algorithm to determine the configuration with the highest  $Q$ . 512 different configurations are randomly generated and the  $Q$  for each determined. The best performing configurations are kept and ‘bred’ in subsequent generations to converge to an optimized configuration. The optimization method is described in detail in the supplementary material.

The particles emerging from the target can be put into one of three categories:

- Required. These are the  $\pi^-$  and  $\mu^-$  that are needed to catalyze fusion reactions.
- Lost. These are the neutrinos emerging from the target.
- Recoverable. These are all the other particles emerging from the target. We aim to convert their kinetic energy to heat by absorbing them into a lithium-lead blanket that surrounds the target. In addition, some of the particles will cause the exothermic tritium breeding reaction (equation (1)) to occur in the LiPb blanket, generating additional heat.

For the purposes of the phase one optimization, we assume that 100% of the ‘required’ particles are utilized for fusion catalysis and 100% of the ‘recoverable’ particles are delivered to the lithium-lead shell. We expect the actual utilization of the required particles to be well below 100% but our approach should give us the correct relative rankings of configurations allowing us to determine the one which is optimal.

<sup>2</sup> The lithium-lead mixture is 17% lithium by molar density. The lithium is 90%  $^6\text{Li}$  and 10%  $^7\text{Li}$ . Because of its low melting point, the lithium-lead target is encased in a 1 mm thick titanium shell. This is one of the blanket types being considered for ITER [12].



## 5. Results

We ran the optimization for the six beam particle/target material combinations (each of deuteron and triton beams with each of carbon, LiPb and tungsten targets). The optimum  $Q$  values as a function of fusion reactions per muon are presented in figure 2.

We can see from the top two charts that the best performing combination is a triton beam impacting a lithium-lead target. However, as we can see from the top right chart, the  $Q$  values generated by a triton beam are only marginally ahead of those produced by a deuteron beam. Given the high cost of using a triton beam both due to the expense of purchasing the tritium<sup>3</sup> and building the necessary health and safety infrastructure relating to a triton beam, we believe that a deuteron beam is a better choice. This is further supported by the lower beam energies for deuterons (bottom left chart, figure 2) as particle accelerators with higher beam energies will be more expensive to build.

As for the target material, the  $Q$  values for LiPb and W are about the same for a deuteron beam. We opt for using the tungsten as it is a simpler design<sup>4</sup> but in future phases of this research, the LiPb should be kept as an option due to the benefits of tritium breeding.

In the lower left chart of figure 2, the optimal energy as a function of assumed fusions per muon is quite volatile. This is indicative of a relatively flat-topped  $Q$  versus beam energy curve (see the lower right chart). Changes in the other input parameters (beam particle, target material, target dimensions) can shift the peak value of  $Q$  to a materially different value of beam energy.

As an example of the results, for 150 fusion reactions per muon, we find the optimal configuration consists of a 3.61 GeV beam of deuterons impacting a tungsten cylinder 652 mm long and 5.1 mm in diameter. We find that  $Q = 1.87$ . The details of a simulation using these parameters are presented in table 1.

In the table, it is worth noting:

- (a) In (A), the number of negative pions (and therefore negative muons) created per beam particle is 0.77. If we simply divide the beam energy by this amount, we get a cost of 4.70 GeV per muon which is slightly lower than previous studies such as [6]. Using 26.0 MeV of heat energy for each of the 150 fusion reactions

<sup>3</sup> The midpoint estimate of Willms [13] is \$150 000 per gram or about 2300 times the price of gold as at 12 August 2020.

<sup>4</sup> As there is no titanium wall.

**Table 1.** The G4Beamline simulation assuming a deuteron beam, a tungsten target and 150 fusions per muon producing the highest  $Q$  has a beam energy of 3.61 GeV, a target length of 652 mm and target diameter of 5.1 mm. The average heat energy produced per beam particle by each of the five sources of heat energy is presented below. ‘Count’ for items (B), (C) and (E) is simply the number of each type of particle (recoverable,  $\pi^+$  or triton through breeding) produced per beam particle. For (A), ‘Count’ is the number of  $\pi^-/\mu^-$  produced per beam particle.  $Q = F/G = 1.87$ . Based on a simulation of 5000 beam particles.

Source	Count	Energy (MeV)
(A) Fusion	0.77	2991
(B) Recoverable particles	42.15	2664
(C) Positive pions	0.55	23
(D) Heat in target		526
(E) Tritons bred in shell	110.34	530
(F) Subtotal (sum (A)–(E))		6734
(G) Beam energy		3606

catalyzed per negative muon,  $Q = 0.83$ . To be clear, this is before taking into account the kinetic energy of recoverable particles being used to generate heat or capturing the heat due to the tritium breeding reaction (equation (1)).

- (b) In (B), most (2.32 GeV) of the recoverable particles’ energy was in the form of kinetic energy of protons, neutrons and deuterons.
- (c) In (E), additional heat is generated in the lithium-lead shell through the exothermic tritium breeding reaction (1). This occurs both from the neutrons emerging from the tungsten target and from neutrons from the lead nuclei generated due to the impact of the energetic protons and neutrons. Note that this is in addition to the energy created from fusion neutrons which is included in (A).

## 6. Electrical breakeven

In this work we get a  $Q > 1$  based on current fusion per muon capability of 150. However, this definition of  $Q$  (which is consistent with that used throughout the various fields of fusion research) ignores important inefficiencies. If these are included in our calculation we get lower  $Q$  values but a truer measure of how close we are to creating a viable energy source.

We should include efficiency factors for (a) accelerators converting electricity into the kinetic energy of the particle beam ( $\eta_{acc}$ ); (b) loss of recoverable particles ( $\eta_{rec}$ ); (c) failure to deliver negative muons to dt vessels ( $\eta_{\mu}$ ); and (d) conversion of heat to electricity ( $\eta_{heat}$ ).

Using these efficiency factors we can calculate the fraction of electricity produced to electricity used which we will call  $Q_{elec}$ . We can write:

$$Q_{elec} = \frac{(F\eta_{\mu} + H\eta_{rec})}{B} \eta_{acc}\eta_{heat}, \quad (2)$$

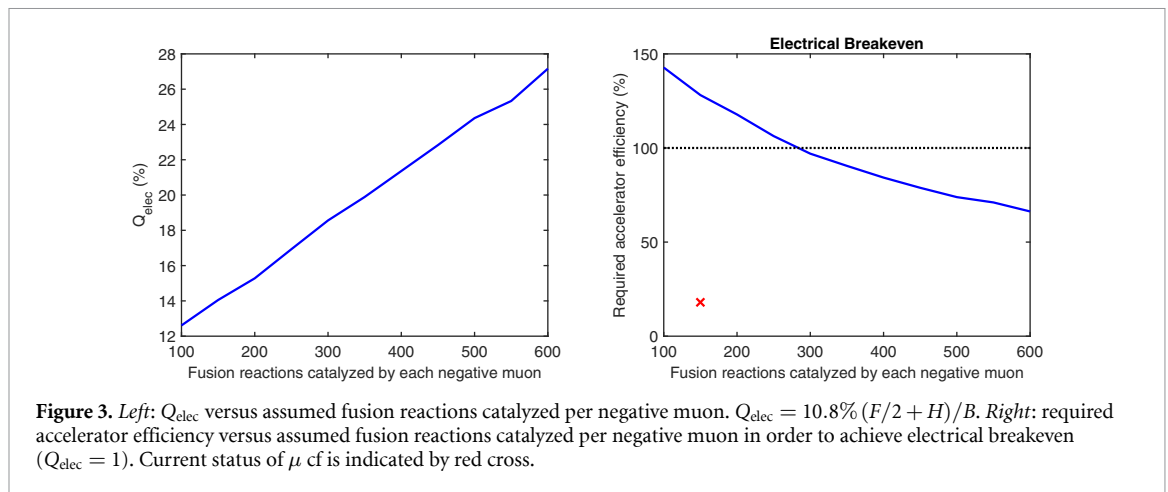
where  $F$ , the fusion heat energy, is the right-hand item in row (A) in table 1,  $H$ , the recoverable heat energy, is the sum of the right-hand items of rows (B) through (E) and  $B$  is the beam energy item (G). Note that up to this point we have been using  $Q = (F + H)/B$ .

We use a reasonable current estimate for  $\eta_{acc}$  and  $\eta_{heat}$ . The efficiency of the 590 MeV high intensity proton accelerator at the Paul Scherrer Institute is 18% [14] which we take for  $\eta_{acc}$ . The conversion of heat to electricity is a challenge for most electricity generation plants and has been found to be above 60% using the most modern equipment [15]. If one studies figure 1, one can conclude that 100% of the energy included in items (B)–(E) in table 1 will be captured (as long as the LiPb shell is thick enough) as heat. On the other hand, we do not know  $\eta_{\mu}$  and finding it will be the focus of phase three of our research. For the current phase we will use the arbitrary but reasonable assumption of  $\eta_{\mu} = 50\%$ . Thus  $Q_{elec} = 10.8\% (F/2 + H)/B$ .  $Q_{elec}$  as a function of fusions per muon is presented in the left-hand chart of figure 3. At our current assumed level of 150 fusion reactions per muon, we obtain  $Q_{elec} = 14\%$ .

In the lower right chart of figure 2 we see that the optimal  $Q$  as a function of beam energy steeply increases up to beam energy of about 2 GeV but has a relatively flat top over to about 5 GeV. In reality,  $\eta_{acc}$  is likely to decrease with higher energies so the actual optimal energies may be lower than the ones we are calculating.

## 7. Discussion

It is interesting to consider what level of accelerator efficiency would be necessary to bring  $Q_{elec}$  to 100%. This is presented in the right-hand chart of figure 3. Our current situation (150 fusions per muon, 18% accelerator efficiency) is denoted by the red cross on the chart.



In order to increase  $Q_{\text{elec}}$  to over 100% we need to investigate improvements in accelerator efficiency, muon production efficiency and fusion reactions catalyzed per muon.

As noted by Yakovlev *et al* 'all the considered accelerators have a lot of room for the power consumption improvements' [16]. It is expected that accelerator efficiency could improve considerably, especially since (1) the required beam characteristics (beam size and energy spread) for  $\mu$ cf are likely to be less difficult than those necessary for many other applications such as high energy physics and (2) the heat produced by the accelerator may be of use for the fusion reactor.

Our continued investigation into efficient muon production may yield further improvements moving the blue line in the right-hand chart of figure 3 down.

The improvement of the number of fusion reactions catalyzed by muons is a difficult challenge. As suggested by Nagamine<sup>5</sup> [17], perhaps electromagnetic fields could positively impact the molecular formation time,  $t_c$ . However, the more important factor in the number of fusions per muon,  $1/(p + t_c/\tau)$ , is  $p$ , the sticking probability. If dt mixtures with higher densities than solid hydrogen could be achieved,  $t_c$  would be reduced<sup>6</sup> and  $p$  may be reduced<sup>7</sup>. Currently, there is some research being undertaken to investigate creating high density hydrogen using laser-heated diamond anvil cells for uses in  $\mu$ cf [20]. If this effort can yield a usable high density dt mixture or if another approach is successful, a material increase in the number of fusion reactions per muon may be achieved.

We believe that our estimate of  $Q_{\text{elec}} = 14\%$  for  $\mu$ cf compares favorably to similar measures for magnetic and interial confinement fusion. However, experts in those fields would need to estimate this measure, taking into account all of the inefficiencies as we have done, in order for a proper comparison to be made.

Given the improvement in  $Q$  by a factor of  $\sim 2.5$  presented in this paper,  $\mu$ cf shows considerable potential to become a viable method for generating electrical energy. It is currently not a field of research receiving significant attention but it is the view of the authors that it should be.

## Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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## Conflict of interest

The authors declare that they have no conflicts of interest.

<sup>5</sup> Page 94.

<sup>6</sup> In 1986, Jones *et al* [18] observed that  $\lambda_c = 1/t_c$  increases faster than linearly with density.

<sup>7</sup> Higher densities will lead to higher collision rates for muonic alpha particles emerging from a catalyzed fusion reaction. This will increase the rate of stripping the muon from the alpha particle and the rate of de-excitation to the ground state which suppresses stripping. It is not clear which of these two effects will be dominant [19].

## ORCID iDs

R Spencer Kelly  <https://orcid.org/0000-0001-6354-0707>

Lucy J F Hart  <https://orcid.org/0000-0002-6269-4672>

Steven J Rose  <https://orcid.org/0000-0001-6808-6355>

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