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THE ROLE OF CONSUMER LEVERAGE IN GENERATING FINANCIAL CRISES

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The Role of Consumer Leverage in Generating Financial Crises

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Abstract

Consumer leverage can generate financial crises characterized by increased bankruptcy, tightened credit access and reduced demand for goods. This paper embeds financial frictions in the mortgage contracts of homeowners within a two-sector economy to show that, even at moderate initial levels, household indebtedness can create a lasting financial downturn such as the subprime mortgage crisis. Using two seemingly positive disturbances that triggered the subprime mortgage crisis - an increased housing supply and a relaxation of borrowing conditions - the model demonstrates that the subprime downturn was not a precedent but the natural consequence of financial frictions. The oversupply of houses lowers asset prices and reduces the value of the real estate collateral used in the mortgage. This worsens the leverage of indebted consumers and raises their bankruptcy prospects generating a pro-cyclical risk premium. A relaxation of borrowing conditions turns credit-constrained households into a potential source of disturbances themselves when market optimism allows them to overleverage with little downpayment. In both cases, the resulting excessive consumer leverage impairs household credit access for a lengthy after-shock period and diverts resources from their consumption. Their reduced demand for goods may propagate the downturn to the rest of the economy depressing output in other sectors. Adding credit constraints in the financial sector that provides housing mortgages deepens the negative impact of the shocks and makes recovery even more protracted.

JEL Classification: E21, E27, E44, G21, G33

Keywords: Financial Frictions,Consumer Leverage, Credit-Constrained Consumers, Subprime Mortgage Crisis, Pro-Cyclical Risk-Premium

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1 Introduction

In 2004, Edmund Andrews, an economic reporter for *The New York Times*, joined millions of American home-buyers in purchasing a house at the peak of the real estate price bubble (Andrews 2009). The fact that he had regularly reported from the Federal Reserve, covered the Asian financial crisis of 1997, the Russian meltdown of 1998 and the dot-com collapse of 2000, did not prepare him for what was in store at the subprime mortgage party. "I had just come up with almost a half-million dollars, and I had barely lifted a finger. It had been so easy and fast," Andrews said of obtaining his mortgage despite having modest disposable income and putting down very little downpayment. His mortgage was a classic subprime loan. The monthly payment first jumped from $2,500 to $3,700. If he kept the mortgage for two years, the interest rate could jump as high as 11.5 percent, and the monthly payments could ratchet up to $4,500. After his wife lost her job, he fell behind on all bills from the mortgage to the electricity payment. When he finally defaulted, he was far from being the only one. In fact, he had outlived two of his three mortgage lenders. The first one collapsed overnight when the financial markets first froze up in August 2007 and the second one was forced out of the mortgage business by federal regulators.

Andrews' story is just one of the hundreds of recent foreclosure experiences that we have collectively come to know as the subprime mortgage crisis. The subprime mortgage crisis was a seemingly unusual one since it created three precedents: it originated in only one sector of the economy, it was not precipitated by production firms or entrepreneurs, and it was not triggered by adverse circumstances. The unraveling of the mortgage market began when two seemingly positive factors, increased housing supply and easy refinancing conditions, allowed borrowers to overleverage with debt to disastrous consequences. At the heart of of the downturn were not production agents but heavily indebted consumers and the financial institutions that securitized their mortgages. Finally, the collapse of one single sector such as the highly leveraged mortgage market was sufficient to trigger an economy-wide downturn.

The asset bubble that preceded the subprime mortgage crisis increased housing inventories to record high numbers (Coleman, LaCour-Little and Vandell 2008). After the bubble burst, the oversupply of real estate reduced both the sale price of real estate and the value of the houses held by homeowners (Duca, Muellbauer and Murphy 2010; Ellis 2010). As a result, many mortgage-holders owed more on their housing loan than their residence was worth. Furthermore, a the peak of the asset bubble many investors eager to tap into mortgage profits underwrote housing loans secured with very little downpayment and a variable repayment rate, the so-called "subprime" loans. These lax borrowing conditions allowed consumers to overleverage with debt and to be at the mercy of the adjustable interest rate. Once the first signs of trouble raised the repayment rate, their risk premium rose worsening their indebtedness (Demirgüç-Kunt, Evanoff and Kaufman 2011; Laeven, Igan and Dell'Ariccia 2008; Mian and Sufi 2009). This caused a flurry of short sales and defaults as mortgage-buyers attempted to deleverage or declared bankruptcy. The result was a depressed housing market with even lower asset prices and sharply tightened credit access (Dennis 2010; Duke 2012; Madigan 2012). The housing market downturn also exported the worsened financial conditions beyond the affected sector and depressed demand for goods in the rest of the economy.

This paper develops a model of financial frictions in the mortgage market to study the role of excess housing supply and lax borrowing conditions on the leverage of mortgage-buying consumers. In doing so, it demonstrates that the three precedents observed in the subprime mortgage crisis are not a unique confluence of events but the natural consequence of financial frictions. For this purpose, this research extends the one-sector model of credit constraints of Bernanke, Gilchrist, Gertler (1999) to two sectors and shifts financial frictions from producers to homeowner consumers. A two-sector model can generate a pro-cyclical risk premium that widens following improvements in the housing market. One such improvement is increased housing supply that reduces the equilibrium price of housing. Since housing is also a collateral in the mortgage contract, this creates a Fisher-like effect in which the price decrease worsens the value of the housing equity of indebted households and increases their leverage. The excessive leverage triggers an increase in the adjustable mortgage interest rate and the risk premium widens. Hence the two-sector economy allows fluctuations in the equilibrium price of the collateral to generate a pro-cyclical external finance premium unlike the one sector version of Bernanke, Gilchrist and Gertler (1999) that has no price effects.

The two-sector model with credit frictions in the household sector also allows to demonstrate two other occurrences of the subprime mortgage crisis. Excessive leverage can export a downturn from the mortgage market to the rest of the economy since tighter refinancing possibilities for indebted consumers can reduce demand for other goods as well. The presence of financial frictions also means that production and financial sectors are not the only agents whose credit frictions can precipitate a recession, unlike Bernanke, Gilchrist, Gertler (1999) and Luk and Vines (2011). Although consumers do not generate output like firms and may not be initially as highly leveraged as financial institutions, they can affect other sectors of the economy though their purchasing decisions. When debt-servicing diverts resources from other economic activity, a recession can ensue regardless of who is the credit-constrained agent.

It order to describe the behavior of leveraged consumers, the model embeds consumer credit constraints in a general economic framework. Unlike Aoki, Proudman and Vlieghe (2004) which imposes an exogenously determined leverage limit, this paper develops a microfounded mortgage contract similar to the financial accelerator approach in Bernanke, Gilchrist, Gertler (1999). This setup allows unexpected disturbances to influence the level of indebtedness and the terms of the mortgage contract. In addition to that, the household credit frictions in this paper allow to study the role of leveraged consumers in the subprime crisis and so can go further than previous papers with microfounded credit constraints that focus primarily on financial institutions. Bernanke, Gilchrist and Gertler (1999) and Luk and Vines (2011) show that highly leveraged financial institutions can be instrumental in precipitating a financial downturn. Similarly, Fernandez-Villaverde (2010) and Gertler and Kiyotaki (2010) demonstrate that a disruption to financial intermediation can induce a crisis that affects real economic activity. These works, however, overlook the role of consumer leverage in the recession since households are generally not as leveraged as financial institutions. One of the few studies on consumer mortgage decisions, Mian and Sufi (2009), establishes that consumers can overleverage as a result of deteriorating lending practices but does not study the consequences of this overleverage for the economy. This paper augments our understanding of credit frictions by considering explicitly the role of household leverage in creating and propagating downturns. It also demonstrates that although households are not as indebted as financial institutions, changes in their leverage can have lasting consequences for their credit access and the economy as a whole.

At the heart of the financial frictions setup is the inability of credit-constrained consumers to fully finance their housing purchase so they need to borrow external funds from risk-neutral investors. This borrowing is complicated by the presence of an idiosyncratic risk of default on the part of mortgage-buyers that is known to them but is unknown to lenders (Townsend 1979). Unlike Curdia and Woodford (2009) where borrowers can choose whether to default, here bankruptcy is involuntary but depends on factors both endogenous (the size of the mortgage) and exogenous (adverse shocks) to consumer decisions. If credit-constrained households default, investors must pay an auditing fee and assume possession of any remaining assets. Since investors cannot fully diversify away this risk, they must charge borrowers a risk premium that would offset the expenses associated with eventual bankruptcy. Investors obtain their funds from Ricardian consumers who have significant non-wage income from firm profits and shares so they save money with investors. In contrast, credit-constrained consumers earn only wage income and have no savings. The economy is completed by two production sectors: one that produces a conventional consumption good and one that manufactures housing. Both are perfectly competitive and there is no aggregate uncertainty in their returns.

The model demonstrates the effects of two shocks that contributed to precipitating the subprime mortgage crisis: the oversupply of housing and the reduction in financial volatility. The overproduction of housing can depress the price of housing and worsen consumer leverage. Initially, credit-constrained consumers need to borrow less to purchase new housing since it becomes more affordable. However, the value of their collateral, which is primarily composed of their housing equity, is also affected by the lower housing price and the decrease is proportionally larger than the fall in the mortgage value so their leverage increases. The higher leverage signals increased risk of default so lenders that provide loans to credit-constrained consumers are exposed to bigger risks. To compensate for this, investors require a higher return from indebted households so the risk premium rises. The higher risk premium can depress credit access for a long time while consumers attempt to repair their debt position by diverting resources from their regular consumption. The oversupply of housing impacts the debt position of credit-constrained consumers via reduced house prices while the resulting depressed demand for other goods exports the downturn to the rest of the economy.

A lower housing price is not the only disturbance that can cause a rise in the leverage of indebted agents. A relaxation of credit access would improve borrowing conditions so mortgage loans would attract a lower risk premium and require a smaller downpayment to secure (Mian and Sufi 2009). The lax credit access prior to the crisis reduced interest rates and lured many potential home buyers into obtaining mortgages with little collateral. This lower collateral implies higher indebtedness for mortgage-buyers that can easily lead to higher probability of default. Borrowers reduce their purchase of regular goods in order to improve their mortgage position but since they cannot achieve that overnight, their increased bankruptcy prospects prompt investors to raise the repayment interest rate. As a result, households experience a lasting recession with diminished credit access and reduced consumption.

In an extension to the main model, the paper considers credit-constrained financial institutions that serve as intermediaries in the loan contract between investors and households. Such a setup borrows from the chained contract model of Hirakata, Sudo and Ueda (2009), but unlike it, the final borrowers are not entrepreneurs but credit-constrained households. The presence of credit frictions in both the consumer and financial sectors increases the duration and the magnitude of the downturn in the economy. Financial institutions are often more leveraged than consumers so they experience a proportionally bigger deterioration of their leverage ratio and default prospects. Hence owing to the high initial leverage of banks, the recession can be more prolonged and disproportionally tilted toward financial institutions. The scenario with a relaxation of borrowing conditions for banks comes closest to the consequences of the subprime mortgage crisis. When banks overleverage as a result of improved access to credit, they attempt to repair their balance sheets by squeezing the credit access of indebted households drawing consumers into a protracted recession. The extension with chained credit frictions offers additional insight into the subprime mortgage crisis by modeling the deterioration of the debt positions of both consumers and financial institutions and the resulting long credit crunch.

2 The Mortgage Contract

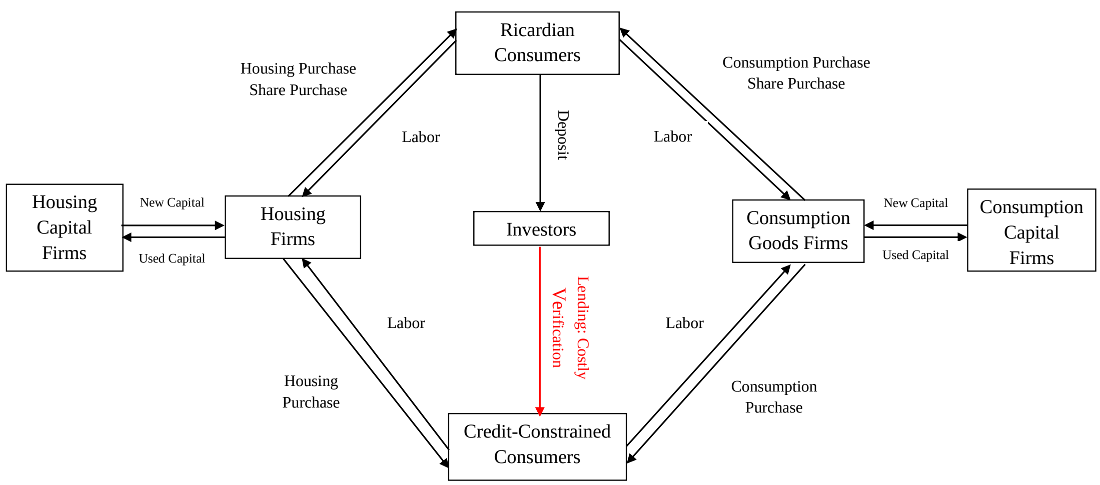
This section develops the consumer mortgage contact in a partial equilibrium framework, taking as given the price of the collateral and the risk-free rate of interest. The subsequent section endogenizes these variables as part of a general equilibrium solution.

In order to motivate credit frictions in the household sector, this model formally splits consumers into two types (Figure 1). Ricardian consumers posses no intrinsic risk of default and can borrow at the risk-free rate. They have significant non-wage income in the form of revenue from the shares of final goods firms and the profits of capital producers. Their wealth allows them to finance their own housing purchase. Ricardians can save any unused income by making deposits with investors. Credit-constrained consumers, on the other hand, receive only wage income so their net worth is not enough to finance their housing acquisition. Therefore, they must obtain an external mortgage from investors. However, their ability to repay the mortgage is subject to an idiosyncratic probability of default. This bankruptcy prospect is known to consumers but unknown to lenders. If investors want to learn the probability of default of a particular household, they must pay an auditing fee. The presence of this default risk also motivates the need for a loan (mortgage) contract between investors and credit-constrained consumers as opposed to a share contract that usually takes place in the absence of aggregate uncertainty.

Credit-constrained households obtain a loan from investors who pool the deposits of Ricardian consumers, paying them the risk-free rate, and lend to credit-constrained households at a markup interest rate. The difference between the borrowing and the lending rates, known as the external finance (risk) premium, exists in order to offset the expenses associated with a potential bankruptcy of mortgage-buyers. Investors do not make any profit on the loan to indebted households. The risk premium is exactly sufficient to offset the costs associated with a potential borrower default. Hence the risk premium does not generate returns but is a key indicator of the financial environment in the economy.

Financial frictions in the loan contract arise due to the existence of an agency problem that makes uncollateralized external finance more expensive than internal finance. The reason for this discrepancy is that borrowers posses an inherent risk of default that is known to them but unobservable to lenders. In this case, investors cannot perfectly observe the borrower's ability to repay and must pay an auditing cost in order to verify repayment ability. The optimal contract therefore is a loan contract, and when indebted consumers are unable to repay, investors pay the monitoring fee and take possession of all of the borrower's remaining assets. In the household context, these auditing costs can be interpreted as the costs of legal proceedings to the value the borrower's assets and the administrative costs of selling the house to realize its collateral value. The greater the default prospect, the more likely the lender will have to pay the auditing cost so the risk premium will be higher. Hence the external finance premium is a function of the default rate, the collateral (net worth) of credit-constrained consumers and the value of the mortgage.

Figure 1. Model with Financial Frictions in the Consumer Sector



The mortgage model is real and abstracts from nominal rigidities. In a one-sector model such as Fernandez-Villaverde (2010), a nominal setup allows for Fisher price effects on the net worth and leverage of indebted agents. The two-sector economy described here produces two goods whose relative prices allow for such price effects to arise naturally without the need to consider nominal rigidities.

2.1 The Loan Contract of Credit-Constrained Households

The mortgage loan begins when at time t, credit-constrained household i obtains a mortgage from investors to purchase housing at price p_t for use at $t+1$. The quantity of housing purchased is denoted $H_{i,t}^{C}$, with the subscript denoting the period in which the housing is purchased and the superscript indicating the type of consumer that purchases it (C denotes credit-constrained households and R refers to Ricardian consumers). Each credit-constrained household faces an idiosyncratic shock ω_i^C to their return to housing. In each period, homeowner i draws from a distribution of ω_i^C where ω_i^C follows a log-normal distribution with mean one and variance σ^{C^2}. The variance σ^{C^2} can also be thought as a measurement of the volatility of the default probability ω_i^C - a lower variance would imply safer loan returns. The ex-post gross return on housing is $\omega_i^C R^C$, where ω_i^C is an idiosyncratic disturbance to household i's return and R^C is the ex-post aggregate return to housing for credit-constrained consumers (the markup interest rate paid on the mortgage). The idiosyncratic shock ω_i^C is also independent across time and across credit-constrained households. Lenders can diversify away the idiosyncratic risks of the borrowers by investing in a lot of borrowers. Note that a contract agreed in period t is repaid back in period $t+1$ at the realized return R^C_{t+1} which incorporates all updated information about an unexpected shock concurring in $t+1$. Hence the consumer interest rate R^C_{t+1} at which the mortgage is repaid forms at $t+1$, whereas the risk-free rate R_t is predetermined at t.

This setup follows the financial accelerator approach, used by Bernanke, Gertler and Gilchrist (1999), which assumes that there is costly state verification of monitoring on the lender's part that motivate financial frictions. If investors want to observe the idiosyncratic shock ω^C_i for a specific homeowner, they need to pay a monitoring cost which is a fraction μ^C of the homeowner's total housing stock. Due to the monitoring cost, it would be too costly to monitor every contract. Therefore, there is a cutoff value of the shock ω^C above which investors do not monitor and below which they do. Above the cutoff, the credit-constrained homeowner gets a return sufficient to pay investors a fixed rate of interest Z^C and keeps what is left for themselves; investors have no incentive to monitor the homeowner' true return. Below the cutoff, credit-constrained households are bankrupt, and the lenders pay the monitoring cost and take their entire wealth.

The investors, who make loans to credit-constrained households, are risk-neutral and the opportunity cost of their funds is the risk-free interest rate. They will agree to lend to credit-constrained households only if they can break even on their investment. This implies that at time t, the interest rate R^C_t charged to borrowers must be such that the expected gross returns on the loan extended in the previous period must equal the opportunity cost of lending in that period:

$$\bar{\omega}^C_{i,t} R^C_t p_{t-1} H^C_{i,t-1} = Z^C_{i,t}(p_{t-1} H^C_{i,t-1} - N^C_{i,-1t}) \tag{1}$$

The left hand side is the gross revenue from housing to the credit-constrained homeowner i whose risk is just at the cutoff point. The right hand side is the amount the consumer needs to repay the investors: the total value of the loan $p_{t-1}H^C_{i,t-1}$ taken the previous period minus the net worth of the borrower $N^C_{i,t-1}$ at the time the contract was agreed. The household is just able to repay the loan at the contractual rate.

The idiosyncratic shock is continuous and has a cumulative distribution function $F(\bar{\omega}^C_i)$. To be perfectly insured against the idiosyncratic shocks of households, each investor signs contracts with an infinite number of households. The solution of the loan contract defines how the investment revenue is split between the lenders (investors) and the borrowers (credit-constrained households). Each lender gets a net share $\Gamma(\bar{\omega}^C_i)$ that is the sum of the gross share (before monitoring) in case the household defaults, and the fixed interest payment the lender gets if the debtor does not default with probability $(1 - F(\bar{\omega}^C_i))$:

$$\Gamma(\bar{\omega}^C_i) = \int_0^{\bar{\omega}^C_i} \bar{\omega}^C_i dF(\bar{\omega}^C_i) + (1 - F(\bar{\omega}^C_i))\bar{\omega}^C_i = G(\bar{\omega}^C_i) + (1 - F(\bar{\omega}^C_i))\bar{\omega}^C_i \tag{2}$$

The borrower gets the remaining share $(1-\Gamma(\bar{\omega}^C_i))$. After taking into account the monitoring cost $\mu^C G(\bar{\omega}^C_i)$, the net share that goes to the lender is:

$$\Gamma(\bar{\omega}^C_i) - \mu^C G(\bar{\omega}^C_i) \tag{3}$$

By assumption, the investors make a zero profit on their loans to credit-constrained households. As the loan risk is totally diversifiable for the lenders, the relevant opportunity cost to the investors is the risk-free rate R_t. Hence the zero-profit condition of lending to a particular borrower is:

$$\left(\Gamma(\bar{\omega}^C_{i,t}) - \mu^C G(\bar{\omega}^C_{i,t})\right) R^C_t p_{t-1} H^C_{i,t-1} = R_{t-1}\left(p_{t-1} H^C_{i,t-1} - N^C_{i,t-1}\right) \tag{4}$$

The left hand side is the net return on the loan to investors (the gross return $(1 - \Gamma(\bar{\omega}^C_{i,t}))$ minus the monitoring cost $\mu^C G(\bar{\omega}^C_{i,t})$) and the right side is the opportunity cost of lending their funds (the value of the housing purchase $p_{t-1} H^C_{i,t-1}$ minus the consumer's net worth $N^C_{i,t-1}$) valued at elsewhere at the risk-free rate R_{t-1} that prevailed at the time the loan was agreed.

Since the price of housing, the risk-free interest rate and the homeowner's net worth $N^C_{i,t}$ are predetermined, at time t the credit-constrained household gets to choose a pair of $(E_t \bar{\omega}^C_{i,t+1},\ H^C_{i,t})$ according to the zero-profit condition. Given the other variables, this is equivalent to the lenders offering a schedule of loans $B^C_{i,t+1}$ and non-default interest rate R_t to the household.

In this partial equilibrium setting, the credit-constrained homeowner i is faced with a menu of loan $B^C_{i,t+1}$ and interest rate R^C_{t+1} where both the loan and the interest rate are related by the participation constraint of the investors. Every period, borrowers choose the optimal pair of housing and the expected cutoff risk $(E_t \bar{\omega}^C_{i,t+1},\ H^C_{i,t})$ to maximize their expected share of the loan at the time of repayment at time $t + 1$:

$$max\ E_t\left(1 - \Gamma(\bar{\omega}^C_{i,t+1})\right) R^C_{t+1} p_t H^C_{i,t} \tag{5}$$

subject to the zero profit condition of investors (4). Solving the maximization problem in terms of the risk-free and the markup interest rate gives the relationship between the interest rates $\frac{R^C_{t+1}}{R_t}$ as a function of the cutoff default risk $\bar{\omega}^C_{i,t+1}$:

$$\frac{R^C_{t+1}}{R_t} = \frac{\frac{\Gamma_\omega(\bar{\omega}^C_{i,t+1})}{\left(\Gamma_\omega(\bar{\omega}^C_{i,t+1}) - \mu^C G_\omega(\bar{\omega}^C_{i,t+1})\right)}}{\left(1 - \Gamma(\bar{\omega}^C_{i,t+1})\right) + \frac{\Gamma_\omega(\bar{\omega}^C_{i,t+1})}{\left(\Gamma_\omega(\bar{\omega}^C_{i,t+1}) - \mu^C G_\omega(\bar{\omega}^C_{i,t+1})\right)}\left(\Gamma(\bar{\omega}^C_{i,t+1}) - \mu^C G(\bar{\omega}^C_{i,t+1})\right)} \tag{6}$$

This equation implies that unless the monitoring cost μ^C is zero, investors would require the aggregate return to the their investment R^C_{t+1} to be larger than the risk-free rate R_t.

An interesting feature of the interest rates ratio $\frac{R^C_{t+1}}{R_t}$ is that it does not depend on the amount of the loan; only on the cutoff risk $\bar{\omega}^C_{i,t+1}$. This implies that a higher cutoff risk will adjust the markup repayment rate R^C_{t+1} upward for a given risk-free rate. Similarly, the interest rates ratio is decreasing in the variance σ^{C^2} (the volatility parameter) of $\bar{\omega}^C_{i,t+1}$ so a lower risk volatility would reduce the gap between the markup rate and risk-free interest rate.

Since every credit-constrained consumer solves the same loan contract, they choose the same expected cutoff risk $E_t \bar{\omega}^C_{i,t+1}$ to maximize their share of the loan. Each household is essentially a representative home-buyer so equation (6) holds for every household that obtains a mortgage. Hence it is possible to aggregate across all credit-constrained households so that this equation is valid for the whole economy. The ratio of the realized return to housing to the expected return on the right hand side is determined macroeconomically, and given that every borrower chooses the same expected cutoff default risk, then the aggregate cutoff risk is $E_t\bar{\omega}^C_{i,t+1} = E_t\bar{\omega}^C_{t+1}$ Hence equation (6) holds on the aggregate level as equation (7) and determines the macroeconomic cutoff risk $\bar{\omega}^C_{t+1}$:

$$\frac{R^C_{t+1}}{R_t} = \frac{\frac{\Gamma_\omega(\bar{\omega}^C_{t+1})}{\left(\Gamma_\omega(\bar{\omega}^C_{t+1})-\mu^C G_\omega(\bar{\omega}^C_{t+1})\right)}}{(1-\Gamma(\bar{\omega}^C_{t+1})) + \frac{\Gamma_\omega(\bar{\omega}^C_{t+1})}{\left(\Gamma_\omega(\bar{\omega}^C_{t+1})-\mu^C G_\omega(\bar{\omega}^C_{t+1})\right)}\left(\Gamma(\bar{\omega}^C_{t+1}) - \mu^C G(\bar{\omega}^C_{t+1})\right)} \tag{7}$$

Similarly, the aggregate amount of net worth held by indebted households in the economy is $N^C_t = \sum N^C_{i,t}$ and the aggregate amount of housing purchased by credit-constrained consumers is $H^C_t = \sum H^C_{i,t}$.

Note that the the return rate at which credit-constrained consumers obtain their mortgage is state-contingent and depends on the realization of the ex-post return on housing R^C_{t+1}. It is at this rate that the loan contract is repaid, rather than the expected return on housing. The expected return on housing $E_tR^C_{t+1}$ is unknown at the time the contract is negotiated and it does not account for unexpected shocks that can affect the repayment ability of credit-constrained consumers at the mortgage maturity date. Hence to ensure a fair return, investors require repayment at the realized ex-post return R^C_{t+1} that is formed only after all disturbances have occurred.

When a positive productivity shock happens, the price of housing p_t falls. This causes the net worth of indebted households to decrease. To satisfy the household participation constraint, the investors will either reduce lending or raise the interest rate they charge to the households or both, which implies that the participation constraint contracts. However, in the general equilibrium context, the supply of housing is predetermined. To maintain an equilibrium with the original level of housing, the first-order condition moves to a higher level. This means that the ratio of the realized return to housing to the risk-free rate increases raising the external finance premium. Observing this, households choose a higher cutoff risk $\bar{\omega}^C$ and an unchanged profit-maximizing level of housing.

In order to complete the partial equilibrium setting, it is necessary to determine the evolution of the credit-constrained households' net worth. In any given period, the equity of the credit-constrained households, V^C_t, is the remaining share of the mortgage value after replaying back investors:

$$V^C_t = \left(1 - \Gamma(\bar{\omega}^C_t)\right) R^C_t p_{t-1} H^C_{t-1} \tag{8}$$

Consumers can spend this dividend income on new housing. When house prices fall—and therefore the equity of the households, V^C_t —the household faces the following decision problem. If it decreases housing demand today, current household utility would fall. But, if demand were kept constant, net worth would decrease, increasing the future external finance premium. Thus the household faces a trade-off between current housing purchase and future borrowing.

It is also necessary to make sure that credit-constrained consumers do not eventually grow out of their financial constraints. Therefore, this paper assumes that every period a constant fraction $1-\nu^C$ of households retire. When they retire, they spend their remaining equity on the consumption good. The retirement consumption C_t^{CR} of credit-constrained consumers is:

$$C_t^{CR} = \left(1-\nu^C\right)\left(1-\Gamma(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C \tag{9}$$

Credit-constrained consumers need to get started on their net worth with some income w_t^C not devoted to purchasing consumption goods and housing. This is equivalent to establishing a savings account dedicated to contributing to the mortgage downpayment. Hence the model assumes they provide one unit of labor inelastically to the production of housing that generates a wage w_t^C. This labor supply is solely for the purposes of accumulating net worth and is weighted heavily so that it does not distort the overall labor supply. Having determined the period equity and the startup net worth of credit-constrained consumers, is it easy to describe the evolution of their net worth. The evolution of the credit-constrained households' net worth is the sum of equity of non-retiring households plus their income from work:

$$N_t^C = \nu^C V_t^C + w_t^C \tag{10}$$

2.2 The Share Purchase of Ricardian Households

The contract between Ricardian consumers on one side and producers of consumption good and housing on the other hand is a shares contract. As equity holders of firms in both sectors, Ricardian households finance their capital purchase and absorb their profits and losses. No monitoring takes place. The manufacturers and the Ricardian households split the revenue according to the shares of their investments, regardless of the idiosyncratic shocks to consumption good producers ω^F and to housing producers ω^H. To diversify away from the firm-specific idiosyncratic risk, each consumer will invest in an infinite number of firms.

Ricardian households finance the capital purchase $q_{F,t}K_{F,t}$ of consumption good firms, which occurs one period prior to production. The aggregate return to capital purchased by the consumption firms is the risk-free rate R_t. In the following period, the share of consumption firms' revenue that goes to Ricardian households is Γ^F.

$$\Gamma^F R_{t-1} q_{F,t-1} K_{F,t-1} = q_{F,t-1} K_{F,t-1} - N_{t-1}^F \int_0^\infty \omega^F R_t q_{F,t-1} K_{F,t-1} dF(\omega^F)$$

$$\Gamma^F = \frac{q_{F,t-1}K_{F,t-1} - N_{t-1}^F}{q_{F,t-1}K_{F,t-1}} \tag{11}$$

The share Γ^F is independent of the idiosyncratic shock ω^F since Ricardians always receive the same share $\frac{q_{F,t-1}K_{F,t-1}-N_{t-1}^F}{q_{F,t-1}K_{F,t-1}}$ regardless of the shock.

Firms in the consumption good sector accrue profits which they split with Ricardian consumers according to their share contract. In any given period, the equity of consumption good firms is:

$$V_t^F = \left(1-\Gamma(\bar{\omega}_t^F)\right) R_{t-1} q_{F,t-1} K_{F,t-1} = R_{t-1} N_{t-1}^F \tag{12}$$

In order to prevent consumption good firms from growing out of the financial constraints, the model assumes that at the end of every period, a constant fraction $1-\nu^F$ exits the market. Exiting firms consume immediately their remaining equity. In this case consumption good producers' consumption C_t^F on exit is:

$$C_t^F = \left(1 - \nu^F\right) R_{t-1} N_{t-1}^F \tag{13}$$

Similarly to consumers, firms need to start off with some net worth so this setup assumes they provide inelastically one unit of labor to the production of their output for which they receive a wage w_t^F. The labor contribution of consumption good producers is significantly discounted to avoid crowding out regular household labor supply. The evolution of firms' net worth is the sum of equity of surviving firms plus their income from work:

$$N_t^F = \nu^F V_t^F + w_t^T \tag{14}$$

Ricardian households also finance the capital purchase $q_{H,t} K_{H,t}$ of housing firms. Similarly to the consumption good sector, there is no aggregate risk in the housing sector and firms can borrow at the risk free rate. The share purchase in the housing sector Γ^H equals:

$$\Gamma^H = \frac{q_{H,t-1} K_{H,t-1} - N_{t-1}^H}{q_{H,t-1} K_{H,t-1}} \tag{15}$$

Housing producers also accrue profits which they split with Ricardian consumers according to their share contract. In any given period, the equity of housing firms is:

$$V_t^H = \left(1 - \Gamma(\bar{\omega}_t^H)\right) R_{t-1} q_{H,t-1} K_{H,t-1} = R_{t-1} N_{t-1}^H \tag{16}$$

Housing producers' consumption C_t^H on exit is:

$$C_t^H = \left(1 - \nu^H\right) R_{t-1} N_{t-1}^H \tag{17}$$

Housing firms also provide inelastically one unit of labor to the production of their output for which they receive a wage w_t^H. The labor contribution of housing producers is also discounted. The evolution of firms' net worth is the sum of equity of surviving firms plus their income from work:

$$N_t^H = \nu^H V_t^H + w_t^H \tag{18}$$

3 The Complete Model

This section embeds the partial equilibrium of the loan contract derived in the previous section into a general equilibrium framework that endogenizes the risk-free rate R_t and the price of housing p_t. The economy consists of two production sectors: consumption good and housing. Capital producers supply sector-specific capital to both types of final good firms. Households consume both the consumption good and housing.

3.1 Consumption Capital Sector

Firms that produce capital $K_{F,t}$ for the consumption good sector own technology that converts goods into capital. They purchase depreciated capital from final goods firms and make investments to produce new capital. The investment $I_{F,t}$ is the consumption good. The newly produced capital is sold back to consumption good producing firms.

Following Gould (1968), there are standard quadratic adjustment costs to producing capital. These capital adjustment costs for the consumption good capital are:

$$K_{F,t} = (1-\delta)K_{F,t-1} + J\left(\frac{I_{F,t}}{K_{F,t-1}}\right)K_{F,t-1} \tag{19}$$

The function J is such that $J' > 0$ and $J'' < 0$. New capital is produced within the period and sold to final good producing firms at the price $q_{F,t}$. The profit maximization problem of capital producing firms gives the optimal condition for investment:

$$q_{F,t}J'\left(\frac{I_{F,t}}{K_{F,t-1}}\right) = 1 \tag{20}$$

3.2 Consumption Good Producers

Firms in the consumption sector use capital $K_{F,t}$, labor $L_{F,t}$ and sector-specific technology $A_{F,t}$ to produce their output. Consumption good producing firms have to buy capital one period earlier. They borrow funds for the purchase of capital at the risk-free rate R_t, which is equal to the expected return on capital. In order to do so, these firms issue claims to Ricardian consumers at the prevailing price of capital $q_{F,t}$. At the end of each production period, they sell the remaining capital back to capital producing firms. The production function of consumption firms is:

$$Y_{Ft} = A_{F,t}K_{F,t-1}^{\alpha_F}L_{F,t}^{1-\alpha_F} \tag{21}$$

According to the share purchase setup, consumption good firms supply inelastically one unit of labor in order to start the accumulation of their net worth. Factoring in this labor supply in the production function the total labor supply in the consumption good sector is:

$$L_{F,t} = L_{FH,t}^{1-\Omega_F}L_{FF,t}^{\Omega_F} = L_{FH,t}^{1-\Omega_F} \tag{22}$$

Where $L_{FH,t}$ is the regular labor supply by both types of consumers and $L_{FF,t}$ is the labor supply by consumption good producers.

Recasting the production function only in terms of household labor gives:

$$Y_{F,t} = A_{F,t}K_{F,t-1}^{\alpha_F}L_{FH,t}^{(1-\alpha_F)(1-\Omega_F)} \tag{23}$$

The firms in the sector are perfectly competitive so they maximize profits subject to input costs. The price of the consumption good is normalized to 1. The first-order conditions for capital and labor are:

$$w_t = (1-\alpha_F)(1-\Omega_F)\left(\frac{Y_{F,t}}{L_{FH,t}}\right) \tag{24}$$

$$R_t = \frac{\alpha_F \frac{Y_{F,t+1}}{K_{F,t}} + (1-\delta) q_{F,t+1}}{q_{F,t}} \tag{25}$$

3.3 Housing Capital Sector

Firms that produce capital $K_{H,t}$ for the housing good sector own technology that converts goods into capital. They purchase depreciated capital from final goods firms in the same sector and make investments to produce new capital. The investment $I_{H,t}$ is the consumption good. The newly produced capital is sold back to housing producers.

Housing capital is subject to the same adjustment costs as regular capital. These housing capital production equation is:

$$K_{H,t} = (1-\delta) K_{H,t-1} + J\left(\frac{I_{H,t}}{K_{H,t-1}}\right) K_{H,t-1} \tag{26}$$

New capital is produced within the period and sold to final goods producing firms at the price $q_{H,t}$. The profit maximization problem of capital producing firms gives the first-order condition for investment in housing capital:

$$q_{H,t} J'\left(\frac{I_{H,t}}{K_{H,t-1}}\right) = 1 \tag{27}$$

3.4 Housing Producers

Housing producers use capital $K_{H,t}$, labor $L_{H,t}$, land X_t and sector-specific technology $A_{H,t}$ to produce houses. Housing firms also buy capital one period earlier. They borrow funds for the purchase of capital at the risk-free rate R_t, which is equal to the expected return on capital. In order to do so, these firms issue claims to Ricardian consumers at the prevailing price of capital $q_{H,t}$. At the end of each production period, they sell the remaining capital back to housing firms. The production function of housing firms is:

$$Y_{H,t} = A_{H,t} K_{H,t-1}^{\alpha_H} X_{t-1}^{\varepsilon} L_{H,t}^{1-\alpha_H-\varepsilon} \tag{28}$$

The amount of land is fixed and normalized to one. Furthermore, both the share purchase setup and the loan contract of credit-constrained consumers assumed that housing producers and indebted households supply inelastically one unit of labor in order to start the accumulation of their net worth. Factoring in their supply, the total labor supply by origin in the production of housing is:

$$L_{H,t} = L_{HH,t}^{1-\Omega_H-\Omega_C} L_{HF,t}^{\Omega_H} L_{HCC,t}^{\Omega_C} = L_{HH,t}^{1-\Omega_H-\Omega_C} \tag{29}$$

Where $L_{HH,t}$ is the labor supply by both types of consumers for the purpose of financing their regular consumption, $L_{HCC,t}$ is the labor supply by credit-constrained consumers in order to start their net worth accumulation, and $L_{HF,t}$ is the labor supply by housing producers.

Recasting the production function only with household labor and factoring in that the amount of land is fixed gives:

$$Y_{H,t} = A_{H,t} K_{H,t-1}^{\alpha_H} L_{HH,t}^{(1-\Omega_H-\Omega_C)(1-\alpha_H-\varepsilon)} \tag{30}$$

The price of housing is p_t. The firms in the sector are perfectly competitive so they maximize profits subject to input costs obtaining the following optimal conditions for housing capital and labor:

$$w_t = (1-\Omega_H-\Omega_C)(1-\alpha_H-\varepsilon)\left(\frac{p_t Y_{H,t}}{L_{HH,t}}\right) \tag{31}$$

$$R_t = \frac{\alpha_H \frac{p_{t+1}Y_{H,t+1}}{K_{H,t}} + (1-\delta) q_{H,t+1}}{q_{H,t}} \tag{32}$$

3.5 Consumers

Both Ricardian and credit-constrained consumers have the same preferences. Households choose consumption C_t^i, housing H_t^i and labor L_t^i subject to their respective budget constraints. Here the superscript i denotes the type of consumers: R for Ricardian and C for credit-constrained. Each household seeks to maximize its lifetime expected utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^i, H_{t-1}^i, L_t^i) \tag{33}$$

The period-utility of each household is given by:

$$U(C_t^i,\ H_{t-1}^i,\ L_t^i) = log(C_t^i) + \kappa log(H_{t-1}^i) - \gamma \frac{L_t^{i(1+\varphi)}}{1+\varphi} \tag{34}$$

The period utility function is separable in consumption C_t^i, housing H_{t-1}^i and leisure L_t^i. Housing is purchased one period in advance and consumed the following period. At the end of the period, the remaining housing minus depreciation is sold back on the market. Housing enters the utility function additively, rather than as part of a consumption aggregation in order to highlight the role of consumer decisions on the the pass-through of shocks from one production sector to the other. There is a taste parameter κ that reflects the relative preference for the consumption good and housing.

3.5.1 Ricardian Consumers

Ricardian consumers purchase consumption goods and housing. Each period, they also lend the amount B_t to investors at the risk-free rate. Their lending essentially provides funds for the loan contract of credit-constrained households as well as finances the capital purchase of consumption good firms and housing producers though their respective share arrangements. Ricardian consumers also absorb the profits Π_t of both capital sectors. The budget constraint for Ricardian households is:

$$C_t^R + p_t H_t^R + B_t = w_t L_t^R + (1-\delta) p_t H_{t-1}^R + R_{t-1} B_{t-1} + \Pi_t \tag{35}$$

Ricardian consumers maximize their utility function subject to this budget constraint. The left hand side reflects their consumption and housing purchases as well as their lending, while the right hand side represents their income from wages and from reselling the non-depreciated housing from the previous period, returns from lending and capital firms profits. The optimization problem yields three first-order conditions for consumption, housing and leisure. The first-order condition for the consumption-labor tradeoff is fairly standard:

$$\gamma L_t^{R\varphi} C_t^R = w_t \tag{36}$$

The relationship between the consumption good and housing reflects the fact that housing is purchased one period in advance and the tradeoff between housing and consumption depends on both the current and future price of housing as well as the intertemporal consumption substitution and the depreciation rate of housing:

$$\left(\frac{p_t}{\beta} E_t\left(\frac{C_{t+1}^R}{C_t^R}\right) - (1-\delta)p_{t+1}\right) H_t^R = \kappa C_{t+1}^R \tag{37}$$

Ricardian consumers also have a standard Euler equation:

$$E_t\left(\frac{C_{t+1}^R}{C_t^R}\right) = \beta R_t \tag{38}$$

Combing the last two equations yields a simpler expression for the consumption-housing substitution:

$$\left(R_t p_t - (1-\delta)p_{t+1}\right) H_t^R = \kappa C_{t+1}^R \tag{39}$$

3.5.2 Credit-Constrained Consumers

Just like Ricardian households, credit-constrained households also consume both consumption good and housing. However, they cannot purchase housing directly but must obtain a loan from investors, which they repay at the markup interest rate R_t^C. As a result, buying a house is costly for them. They earn income only from labor and do not own any shares. They also cannot optimize intertemporally their consumption of the conventional good so their consumption needs must be met solely with their wages after their mortgage is repaid. The budget constraint for credit-constrained households is:

$$C_t^C + R_t^C p_{t-1} H_{t-1}^C = w_t L_t^C + (1-\delta) p_t H_{t-1}^C \tag{40}$$

The left hand side reflects their consumption purchase as well as their housing mortgage, while the right hand side represents their income from wages and returns from reselling the non-depreciated housing from the previous period. Credit-constrained consumers maximize their utility function subject to this budget constraint. The optimization problem yields two first-order conditions for housing and leisure. The first-order condition for consumption-labor tradeoff is identical to that of Ricardian households:

$$\gamma L_t^{C\varphi} C_t^C = w_t \tag{41}$$

The relationship between the consumption goods and housing however depends on the markup interest rate R_t^C instead of the risk-free rate R_t:

$$\left(R_t^C p_{t-1} - (1-\delta)p_t\right) H_{t-1}^C = \kappa C_t^C \tag{42}$$

The first-order condition for housing-consumption tradeoff for credit-constrained consumers is lagged, unlike that for Ricardian households which is forward-looking. This is due to the fact that the return rate at which credit-constrained consumers obtain their mortgage is state-contingent and depends on the realization of the ex-post return on housing R_t^C that incorporates all shocks at the time of the repayment. Hence the realized return on housing R_t^C depends on the past purchasing price p_{t-1} and the current selling price p_t of housing. It does not depend on the expected future price of housing which is unknown at the time of loan contract. Hence equation (42) depends on the ex-post return on housing R_t^C, while the corresponding equation for Ricardian households (39) is not lagged since Ricardians borrow at the risk-free rate which is not state-contingent.

Finally, credit-constrained consumers cannot optimize intertemporally their consumption good purchase since they cannot borrow at the risk-free rate R_t. Their demand for the consumption good must be met by their income once all housing loans are repaid:

$$C_t^C = w_t L_t^C + ((1-\delta)p_t - R_t^C p_{t-1})H_{t-1}^C \tag{43}$$

3.6 Market Clearing

Market clearing requires that the output of the consumption good must cover household consumption, consumption by conventional good producers on exit and consumption by credit-constrained consumers on retirement, as well as investment in the two sectors:

$$Y_{F,t} = C_t^R + C_t^C + C_t^F + C_t^{CR} + I_{F,t} + I_{H,t} \tag{44}$$

Housing is a multiperiod good and each period a fraction of the housing available on the market depreciates. The remaining non-depreciated housing along with new production constitutes the available housing in the subsequent period. In each period, the sum of new production and leftover housing must meet the housing needs of consumers as well as the consumption of housing firms on exit and the monitoring costs in the mortgage contract:

$$p_t Y_{H,t} + (1-\delta)p_t(H_{t-1}^R + H_{t-1}^C) = p_t H_t^R + p_t H_t^C + C_t^H + \mu^C G(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C \tag{45}$$

The labor that both types of households supply equals the demand by housing and consumption good firms:

$$L_{FH,t} + L_{HH,t} = L_t^R + L_t^C \tag{46}$$

Finally, Ricardian consumer lending must equal the loan to credit-constrained households and the share purchase of consumption good firms and housing firms:

$$B_t = p_t H_t^C - N_t^C + q_{F,t} K_{F,t} - N_t^F + q_{H,t} K_{H,t} - N_t^H \tag{47}$$

The complete model is solved for the deterministic steady state and then log-linearized around that steady state. The parameters chosen for the calibration of the model are discussed in Appendix A, while Appendix B outlines the log-linearization of the complete model.

4 Single Contract Model Results

The paper simulates the effects of two major shocks that contributed to the subprime mortgage crisis - the oversupply of housing and the reduction in financial volatility - on the leverage of indebted consumers and on the economy as a whole.

The oversupply of housing is modeled as a 10% technological innovation in the production of housing. This stylistically simple setup allows to generate excess supply of houses without unnecessarily complicating the model with discussions of the preceding housing bubble. Nevertheless, this approach successfully captures the excessive housing inventory that amassed during the boom that depressed housing prices and worsened the indebtedness of mortgage-holders. The innovation is simulated as a permanent shock since the housing oversupply began at the peak of the asset bubble and continued during the subsequent recession as the housing demand shrank (Duke 2012; Madigan 2012).

The relaxation of borrowing conditions is introduced as a 10% reduction in the volatility of the default risk, which permitted borrowers to secure loans with little collateral and as a result to overleverage with debt. The reduction in financial volatility is a one-time event corresponding to the brief period of lax lending practices that prevailed on the cusp of the subprime crisis. This disturbance is temporary not only because access to credit tightened with the economic downturn (Dennis 2010; Duke 2012), but also because a permanent shock would not be able to demonstrate the leverage worsening that follows a transient volatility reduction. Taken together, the two shocks demonstrate that the negative consequences of a positive shock, the outsize role of one single leveraged sector and the disturbances originating in non-production sectors are not precedents of the subprime crisis, but are the natural consequences of financial frictions.

4.1 The Oversupply of Housing

The oversupply of housing affects the leverage of indebted consumers by reducing the value of the mortgage collateral (Figure 3). A permanent positive technological shock in the production of housing raises the supply of houses (Figure 3.2) and creates excess real estate inventory for the unchanged price of housing. In the absence of concurrent changes in housing demand, the equilibrium price of housing decreases (Figure 3.17) to accommodate the extra supply. The lower price of housing, however, applies not only to newly produced houses, but also to the existing housing stock, including the housing equity held by credit-constrained consumers. The housing equity of borrowers consists of their share of the loan contract after repaying investors and depends both on the price of housing and the realized return on housing from the mortgage R_t^C. The unanticipated fall in housing price also reduces the return to housing in the period in which the shock occurs (Figure 3.20). Hence the lower price of housing affects the net worth of credit-constrained consumers both directly and via the ex-post return on housing R_t^C, whereas the value of the loan diminishes only by the drop in the housing price. As a result, the net worth of indebted households (Figure 3.21) falls proportionally more than the value of the mortgage so the leverage of borrowers increases (Figure 3.24). Higher leverage signals increased probability of default for credit-constrained consumers (Figure 3.23).

The aggregate net worth of credit-constrained consumers falls not only because the realized return of housing is low relative to the expected return evaluated one period before, but also because a higher cutoff default risk increases bankruptcy prospects and reduces the share of the mortgage returns that goes to credit-constrained consumers in the loan contract. The investors are now exposed to bigger risks of default on the part of credit-constrained consumers. To compensate for the rise in the monitoring costs, the investors require a higher return from borrowers, forcing up the external finance premium $R_{t+1}^{C} - R_t$ (Figure 3.29). Consequentially, the falling equilibrium price of housing generates a pro-cyclical risk premium that widens following a positive shock; quite different from the anti-cyclical risk premium typical of one-sector models such as Bernanke, Gertler and Gilchrist (1999) and Luk and Vines (2011). The two-sector model demonstrates that a positive disturbance such as a technological innovation can worsen the bankruptcy prospects of indebted agents since it triggers Fisher-type effects of the collateral price.

The purchasing decisions of credit-constrained consumers propagate the effects of a lower price of housing to the consumption good sector (Figure 3.1). Since indebted households initially enjoy a lower repayment rate on their existing loan and can buy houses more cheaply, they experience the equivalent of income effects. Their labor supply slightly diminishes (Figure 3.12) and they can afford more of the consumption good (Figure 3.5). However, their welfare improvement is short-lived. The negative effects of the worsening leverage and the widening risk premium soon catch up with them and credit-constrained consumers have to increase their labor supply (Figure 3.12) and reduce their demand for the consumption good (Figure 3.5) so that they can improve their net worth (Figure 3.21) and their borrowing abilities. This reduced demand for the consumption good contributes to a depressed output in that sector (Figure 3.1) for the duration of the net worth recovery beyond the substitution effect triggered by a lower housing price. Over time, indebted consumers gradually increase their housing stock (Figure 3.6), which translates into higher net worth (Figure 3.21) and decreasing default prospects (Figure 3.23).

The leverage and probability of default of credit-constrained consumers eventually returns to pre-shock levels thanks to their diverting resources away from the consumption good toward increasing their housing stock (Figures 3.21 and 3.23). This improvement, however, is relatively slow and takes about 100 quarters (25 years) to return to their pre-crisis position. Even with moderate initial indebtedness, the recovering of household leverage is a protracted process. The simulations demonstrate that the interim period is characterized by a subprime crisis with higher leverage on the part of credit-constrained households and increased bankruptcy prospects.

4.2 The Reduced Financial Volatility

Leading up to the subprime mortgage crisis, the housing market was characterized by exceptionally favorable borrowing conditions with low interest rates and little downpayment requirements. In the model, this lax lending atmosphere is captured by a negative temporary shock on σ^{C^2}, the variance of the cutoff risk $\bar{\omega}^{C}$. The reduction in volatility is modeled as an autoregressive temporary disturbance with moderate persistence to reflect that this phenomenon was relatively short-lived and to demonstrate the worsening of refinancing possibilities as the initial shock dies off. A negative shock implies less volatility in the default prospects of borrowers and thus safer loan contracts. Such relaxed borrowing conditions prompt mortgagers to overleverage leading to a deterioration of their debt position as credit access tighten subsequently.

Reduced financial volatility implies safer loan contracts that require less downpayment and are associated with a smaller risk premium (Figure 4.29). The lower risk premium is passed on to credit-constrained consumers as a decreased markup interest rate R_t^C (Figure 4.20). Faced with a more favorable borrowing rate, credit-constrained consumers increase their housing demand (Figure 4.6), driving marginally up the price of housing (Figure 4.17). The higher housing price raises the value of their housing stock and positively influences their net worth. However, the downward effect of the reduced markup rate is quantitatively larger than the upward influence of the housing price so the net worth of borrowers eventually diminishes below pre-shock levels (Figure 4.21). The falling net worth and rising value of the housing stock, coupled with the subsequent credit tightening as the temporary shock wears out, worsen the leverage of credit-constrained households (Figure 4.24). Higher leverage signals increased probability of bankruptcy despite the improved financial conditions (Figure 4.23).

Seeing their indebtedness increase, credit-constrained consumers attempt to deleverage. They reduce both their housing and consumption good demand in an attempt to improve their net worth (Figure 4.6). It takes 100 quarters (25 years) of gradual improvement for their debt position to recover and for their bankruptcy prospects to return to pre-shock levels (Figures 4.23 and 4.24). Overall, the effects of reduced default risk volatility are far from beneficial to indebted consumers. The initial benefits of relaxed borrowing conditions are quickly overshadowed by the tightening of credit access as the shock wears out. The result is rising consumer leverage and increased bankruptcy prospects. Even a temporary easing of lending has long-lasting consequences for the debt position of credit-constrained households. The volatility shock, just as the production shock, shows that positive disturbances can have negative effects on the indebtedness of households since it permits them to overleverage. It also demonstrates that non-production agents, such as consumers, can be a source of financial disturbances when they are leveraged.

4.3 Variations in the Degree of Consumer Leverage

The baseline model assumes credit-constrained households are leveraged twice the amount of their collateral i.e. the value of their housing stock is two times their net worth (Appendix A). The previous two sections concluded that even with moderate degrees of indebtedness, borrowers still can experience a lasting recession following either one of the discussed disturbances. This section focuses on the effects of raising the leverage ratio of credit-constrained consumers to five and ten times respectively. Figure 5 replicates the housing oversupply shock with indebtedness levels of two times (solid line), five times (dashed line) and ten times (dotted line). Higher indebtedness leads to a greater reduction in the housing demand of credit-constrained households (Figure 5.6) and a more significant deterioration of their net worth (Figure 5.21). Higher initial indebtedness also implies a steeper worsening of the leverage ratio of borrowers (Figure 5.24) and a slight increase in their default prospects (Figure 5.23). A starting leverage ratio of five times implies three times higher indebtedness after the unexpected shock occurs than a starting leverage ratio of two times. A ten times initial indebtedness makes subsequent leverage almost seven times more volatile than in the baseline setup. This implies that there are accelerator effects of high starting leverage - the volatility of the post-shock leverage increases more than proportionally to the starting consumer indebtedness.

The subsequent recovery, however, is faster in the cases with higher initial leverage. The more indebted households are at the beginning, the quicker they have to deleverage following the unexpected disturbance. The participation constraint of investors binds sooner in cases with higher leverage so the credit-constrained consumers have few options besides channeling all resources into repairing their debt positions. This forced recovery happens at the expense of a sharper drop in housing demand (Figure 5.6). The significant tightening of demand that occurs with higher initial indebtedness allows the leverage ratio to return to pre-shock levels faster than in the baseline scenario - an initial leverage ratio of ten times implies a subsequent indebtedness recovery in 40 quarters (10 years), whereas an initial leverage ratio of two times drags recovery for 100 quarters (25 years). Higher starting indebtedness results in a relatively shorter but much deeper recession that is more detrimental to consumer welfare while it lasts than the one induced by moderate leverage ratios.

Varying the initial leverage ratio in the case with reduced financial volatility results in a different impact on consumer leverage and bankruptcy prospects. Although higher initial leverage does result in increased indebtedness volatility (Figure 6.24), the difference is quantitatively small and the recovery process is fairly similar across all initial leverage variations. The rise in the probability of default (Figure 6.23) is smaller, the larger the starting indebtedness level. This does not imply, however, that higher initial leverage is more beneficial to borrowers. When the starting indebtedness level is relatively high, consumers are already overleveraged to begin with so they do not have much room to increase their leverage following the relaxation of lending conditions. Hence the effect of the reduction in financial volatility is very modest and often counter-intuitive. Put in other words, the higher the initial indebtedness, the less are borrowers able to benefit from the temporarily improved access to credit. This is also evident from the fact that Ricardian lending experiences a more moderate jump in cases with higher initial leverage (Figure 6.30).

Higher initial leverage can be detrimental to borrowers on several fronts. As evident in the housing oversupply scenario, it can make a recession deeper and can squeeze consumer demand for goods more than moderate indebtedness levels. Overleveraged mortgage-buyers also cannot benefit from a temporary relaxation of borrowing conditions. Both cases are characterized by a more unstable debt position whose volatility may accelerate with higher initial indebtedness levels.

Figure 3. Effect of Housing Oversupply in Economy with Credit-Constrained Consumers

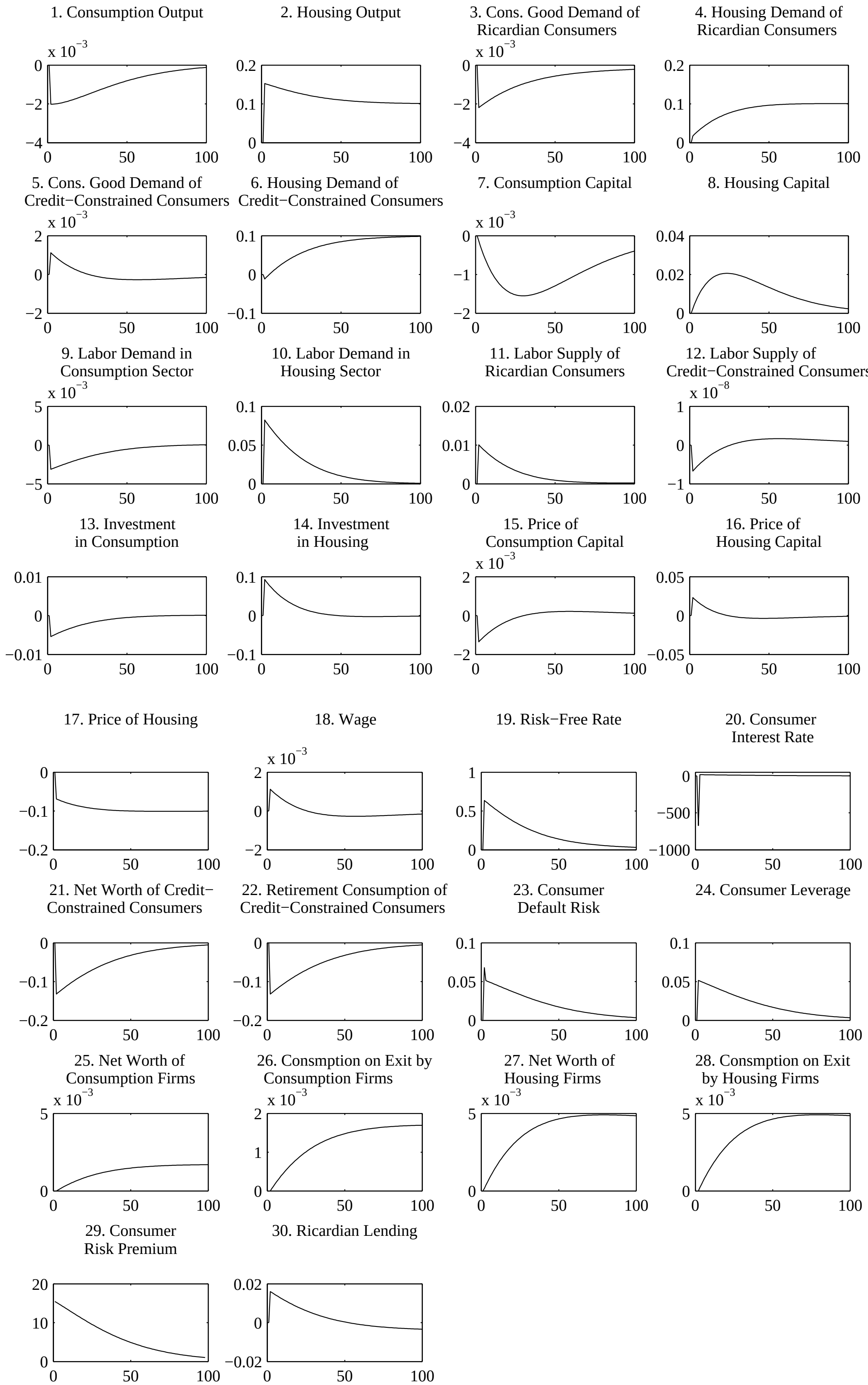


Figure 4. Effect of Reduced Volatility in Economy with Credit-Constrained Consumers

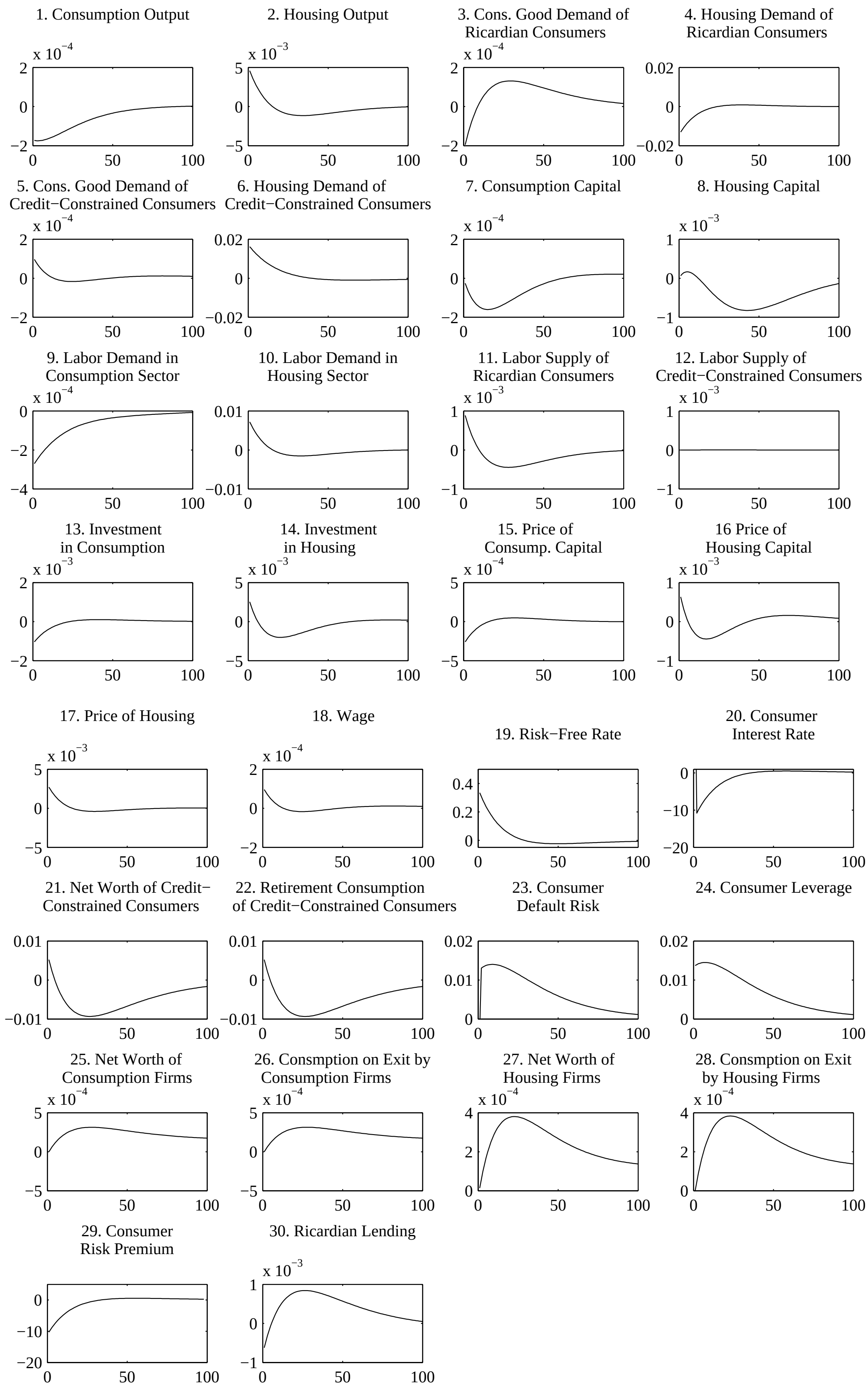


Figure 5. Varying the Degree of Consumer Leverage with Housing Oversupply

(solid line denotes a leverage ratio of two times, dashed line - five times and dotted line - ten times)

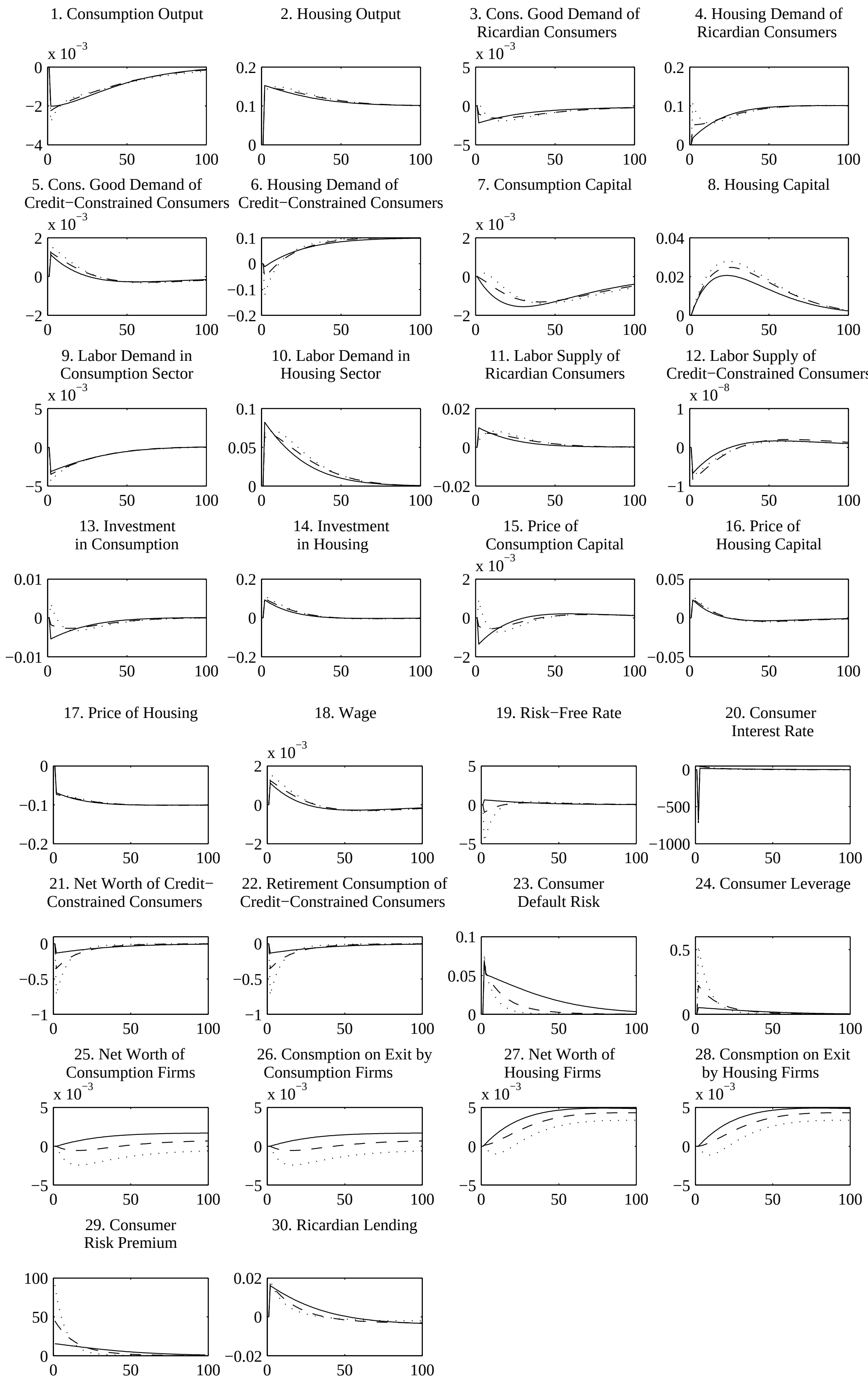
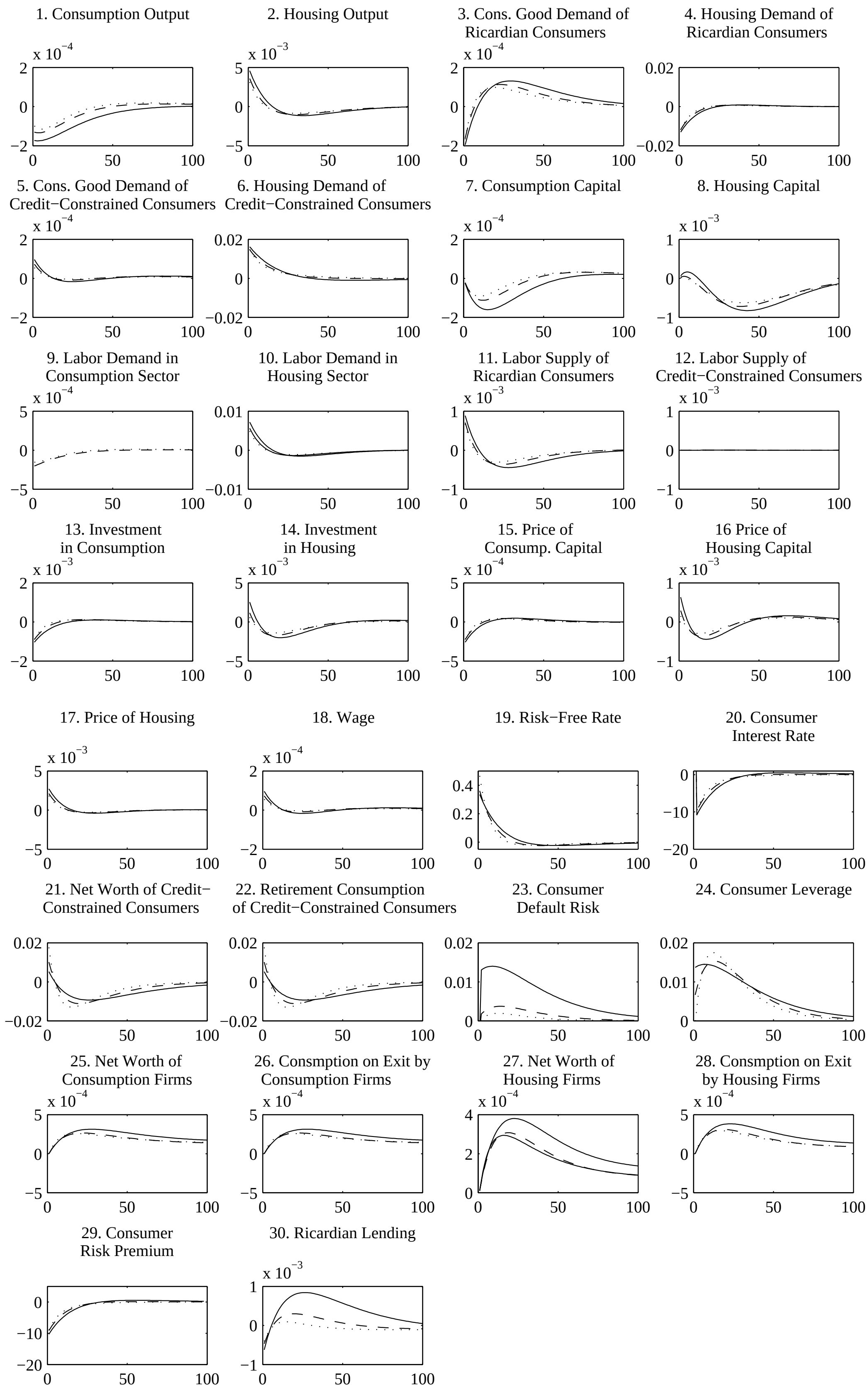


Figure 6. Varying the Degree of Consumer Leverage with Reduced Volatility

(solid line denotes a leverage ratio of two times, dashed line - five times and dotted line - ten times)

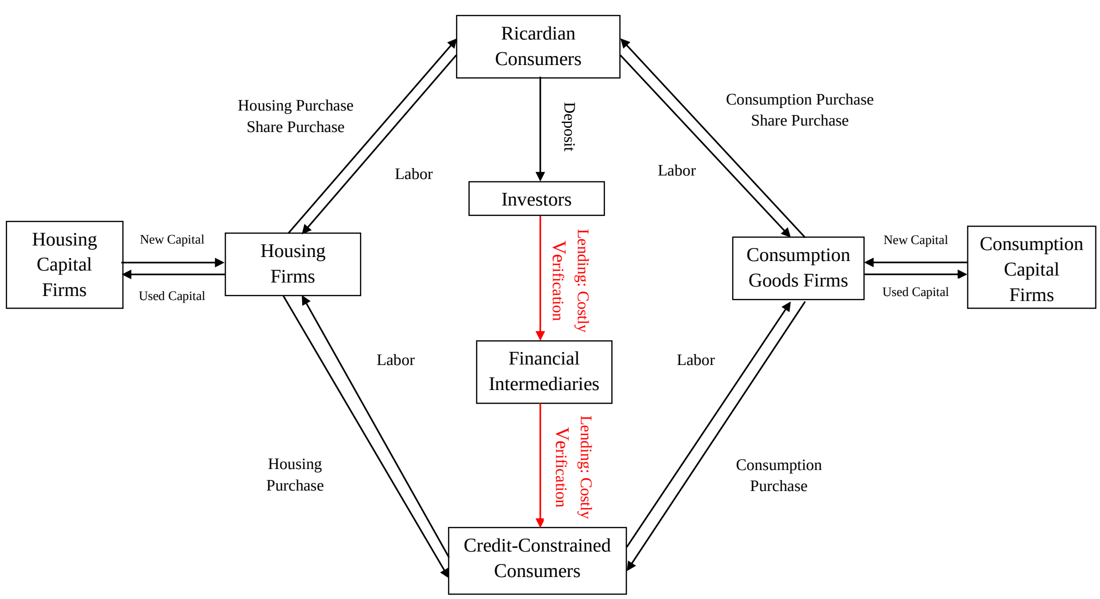


5 The Chained Loan Contract

The model in the previous sections considered credit frictions in the loan contract between investors and credit-constrained consumers. This extension adds financial institutions to the mortgage contract. The model assumes that financial institutions act as intermediaries that borrow funds from investors and in turn lend to credit-constrained households. Taken together, the two transactions constitute two chained mortgage contracts similar to Hirakata, Sudo and Ueda (2009).

In this extension, financial institutions (banks) have an active intermediary role in the loan contract. Unlike investors whose sole role in the model is to facilitate the lending of Ricardian consumers and who do not posses intrinsic risk, in this version banks posses inherent probability of default just as credit-constrained consumers do but their risk is separate from that of borrowers (Figure 2). The financial institutions in this setup are also leveraged and often much more than indebted households. The distinction between investors and banks is not purely a model complication and mirrors real life financial markets. Investors can be characterized as representing "safe" mutual funds that possess no aggregate risk, and financial institutions correspond to investment banks that are highly leveraged. The emergence of such highly leveraged banks may be traced back to the desire for higher profits from riskier investments, which safe financial agents such as investors would be unwilling to finance directly. The presence of financial institutions that possess aggregate risk on their own, can add further dimensions to financial crises beyond these observed in a single loan contract. The remainder of the paper explores the role of leveraged financial institutions in magnifying the effect of exogenous disturbances.

Figure 2. Model with Financial Frictions in the Consumer and Financial Sectors



5.1 The Chained Contract

Credit-constrained financial institutions (i.e banks) borrow from investors and lend to mortgage-buying consumers. There are financial frictions in the contract between investors and financial institutions motivated by bankruptcy prospects inherent to banks that are quite separate from the consumer default probability. This extra loan contract, between investors and financial institutions, is in addition to the loan contract between banks and credit-constrained households. Taken together, the two contracts create a chained contract, in which both credit-constrained consumers and banks have idiosyncratic risks of default.

In this version, there is an idiosyncratic shock ω^B associated with lending to financial institutions. Similar to the credit-constrained consumers' setup, investors have to pay an auditing fee to learn the realization of ω^B. This makes lending to banks risky so financial institutions have to pay a premium on external funds. The interest rate charged to financial institutions is R_t^B.

Since the individual optimization problem of each bank can be aggregated to hold for the whole economy in the same way as the loan problem of credit-constrained consumers in the previous model, the chained contract setup proceeds directly on the aggregate level. Banks borrow funds from investors and in turn lend to credit-constrained households. Every period, they choose the optimal pair of cutoff risk $\bar{\omega}^B$ and housing H^C to maximize their next period expected share of the loan $1-\Gamma(\bar{\omega}_{t+1}^B)$ of the total value of the contact that consists of the housing stock $p_t H_t^C$ minus the net worth of credit-constrained consumers N_t^C:

$$max\ E_t\left(1-\Gamma(\bar{\omega}_{t+1}^B)\right)R_{t+1}^B\left(p_t H_t^C - N_t^C\right) \tag{48}$$

The expected earnings of financial institutions from lending to credit-constrained consumers equal the share they receive from the loan $\Gamma(\bar{\omega}_t^C)$ made to households last period minus the auditing fee on insolvent consumer loans $\mu^C G(\bar{\omega}_t^C)$:

$$\left(\Gamma(\bar{\omega}_t^C) - \mu^C G(\bar{\omega}_t^C)\right)R_t^C p_{t-1} H_{t-1}^C$$

The earnings must equal the opportunity cost of lending to credit-constrained consumers, which is the value of the loan to banks at the markup interest rate R_t^B. Hence the gross return on the banks' loan to credit-constrained consumers is:

$$\left(\Gamma(\bar{\omega}_t^C) - \mu^C G(\bar{\omega}_t^C)\right)R_t^C p_{t-1} H_{t-1}^C = R_t^B\left(p_{t-1}H_{t-1}^C - N_{t-1}^C\right) \tag{49}$$

The left hand side is the banks' share of the loan after monitoring and the right hand side is the gross return (the value of the housing purchase $p_{t-1}H_{t-1}^C$ minus the consumers' net worth N_{t-1}^C valued at the bank interest rate R_t^B) from the housing purchase to financial institutions.

Credit-constrained households will participate in the chained loan contract only if their participation constraint is met. Instead of taking part in the loan contract, credit-constrained consumers can purchase housing using their own net worth N_{t-1}^C. In this alternative case, the ex-post return to their investments equals $R_t^C N_{t-1}^C$. Hence credit-constrained consumers will participate in the chained contract only if their share of the loan is at least equal to the value of their net worth:

$$\left(1-\Gamma(\bar{\omega}_t^C)\right)R_t^C p_{t-1}H_{t-1}^C \geq R_t^C N_{t-1}^C \tag{50}$$

The first part of the chained contracts consists of investors lending to banks. Financial institutions split their gross profit from their loan to credit-constrained consumers with investors. This contract has the same costly state verification structure as the single loan contract, but financial institutions now are the primary borrowers rather than credit-constrained consumers. Banks own the net worth N^B_{t-1} and invest in the loan to credit-constrained consumers at the amount of $p_{t-1}H^C_{t-1} - N^C_{t-1}$. They borrow the rest $p_{t-1}H^C_{t-1} - N^C_{t-1} - N^B_{t-1}$ from investors, and repay the loan using its profit from the loan contract with credit-constrained households. Financial institutions are subject to idiosyncratic productivity shock ω^B and their ex-post gross return on the loans to credit-constrained consumers is $\omega^B R^B$. Investors would participate in the chained contract only if they get a fair return on their lending. Hence their zero profit participation constraint must specify the amount of funds that banks borrow from investors, the cut-off value of the idiosyncratic shock $\bar{\omega}^B$, and the return rate of the loan for non-defaulting banks:

$$\left(\Gamma(\bar{\omega}^B_t) - \mu^B G(\bar{\omega}^B_t)\right) R^B_t \left(p_{t-1}H^C_{t-1} - N^C_{t-1}\right) \geq R_{t-1}\left(p_{t-1}H^C_{t-1} - N^C_{t-1} - N^B_{t-1}\right) \tag{51}$$

Like in the single contract model, the lender's share of the profit in the contract with financial institutions $\left(\Gamma(\bar{\omega}^B_t) - \mu^B G(\bar{\omega}^B_t)\right) R^B_t \left(p_{t-1}H^C_{t-1} - N^C_{t-1}\right)$ after paying the monitoring fee $\mu^B G(\bar{\omega}^B_t)$ must at least equal the opportunity cost of the investors' lending. Lenders sign contracts with a lot of banks, to diversify away the idiosyncratic risks of financial institutions.

Substituting equation (49) into (51) and noting that both equations (50) and (51) bind at the optimum, eliminates the bank interest rate R^B_t and reduces the conditions that financial institutions must satisfy to two:

$$\begin{aligned}\left(\Gamma(\bar{\omega}^B_t) - \mu^B G(\bar{\omega}^B_t)\right)\left(\Gamma(\bar{\omega}^C_t) - \mu^C G(\bar{\omega}^C_t)\right) R^C_t p_{t-1}H^C_{t-1} = \\ = R_{t-1}\left(p_{t-1}H^C_{t-1} - N^C_{t-1} - N^B_{t-1}\right)\end{aligned} \tag{52}$$

$$\left(1 - \Gamma(\bar{\omega}^C_t)\right) p_{t-1}H^C_{t-1} = N^C_{t-1} \tag{53}$$

The important difference between the credit-constrained consumers' loan contract and the chained loans contract is that the chained loan contract assumes that the intermediaries (financial institutions) maximize their profits subject to satisfying the participation constraints of both the credit-constrained consumers and investors. In this case banks, which are borrowers in one part of the contract and lenders in the other part optimize the two sides of the contract. This is in contrast with the previous version where the final borrowers (credit-constrained consumers) maximize their loan. The reason for banks maximizing their profits is that they are act as monopolies. This can be easily demonstrated by assuming that there are infinitely many towns and each of them has only one bank. Consumers in each town have access only to their local bank and not to financial institutions located in neighboring towns. The presence of a monopolist bank can be explained with the necessity of the bank to use monitoring technology to collect information on borrowers and lenders that would reduce the agency cost. This process is both location-specific and there are economies of scale to amassing it (i.e. it would not be profitable for two banks to operate the same technology and collect the same information). Hence banks can act as sole recipients of financing from investors and of distributors to consumers of loans and can maximize the two sides of the chained contract. The single loan contract model instead assumed that the credit-constrained consumers optimize their share of the loan subject to satisfying the zero profit condition of the competitive investors.

Banks choose the optimal level of $\bar{\omega}^C$, $\bar{\omega}^B$ and H^C by maximizing (48) subject to (52) and (53). Let for simplicity of expression $\Gamma(\bar{\omega}_t^B) - \mu^B G(\bar{\omega}_t^B) = \Psi(\bar{\omega}_t^B)$ and $\Gamma(\bar{\omega}_t^C) - \mu^C G(\bar{\omega}_t^C) = \Psi(\bar{\omega}_t^C)$. The optimization problem determines the ratio of the consumer interest rate to the risk-free rate $\frac{R_{t+1}^C}{R_t}$ as a function of the cutoff default risks $\bar{\omega}^C$ and $\bar{\omega}^B$:

$$\frac{R_{t+1}^C}{R_t} = \frac{\Gamma_\omega(\bar{\omega}_{t+1}^C)}{(1-\Gamma(\bar{\omega}_{t+1}^C))\,\Psi_\omega(\bar{\omega}_{t+1}^C) + \Gamma_\omega(\bar{\omega}_{t+1}^C)\Psi(\bar{\omega}_{t+1}^C)} \times \tag{54}$$

$$\frac{\Gamma_\omega(\bar{\omega}_{t+1}^B)}{(1-\Gamma(\bar{\omega}_{t+1}^B))\,\Psi_\omega(\bar{\omega}_{t+1}^B) + \Gamma_\omega(\bar{\omega}_{t+1}^B)\Psi(\bar{\omega}_{t+1}^B)}$$

Just like credit-constrained consumers, financial institutions also earn positive profits and amass net worth. The net worth of credit-constrained consumers remains the same while the net worth of banks is:

$$N_t^B = \nu^B \left(1 - \Gamma(\bar{\omega}_t^B)\right) R_t^B \left(p_{t-1} H_{t-1}^C - N_{t-1}^C\right) + w_t^B = \tag{55}$$

$$= \nu^B \left(1 - \Gamma(\bar{\omega}_t^B)\right) \Psi(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C + w_t^B$$

In a similar fashion to the single contract between investors and credit-constrained consumers, financial institutions supply inelastically one unit of labor to the housing sector that goes toward starting their net worth. They earn wages w_t^B.

Finally, every period a constant fraction $1 - \nu^B$ of financial institutions exit the market. When they exit they spend their remaining equity on the consumption good. In this case the consumption C_t^B on exit is:

$$C_t^B = \left(1 - \nu^B\right) \left(1 - \Gamma(\bar{\omega}_t^B)\right) \Psi(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C \tag{56}$$

5.2 Complete Model Changes

The introduction of the chained contracts necessitates some changes to the complete model. These involve the labor supply in the housing sector and the market clearing conditions for the consumption good and housing. The production functions in both sectors, the capital development and the goods demand by consumers remain unchanged.

In the chained contracts model, the labor supply equation in the housing sector must also account for the unit labor supply by financial institutions so the production function of housing becomes:

$$Y_{H,t} = A_{H,t} K_{H,t-1}^{\alpha_H} L_{HH,t}^{(1-\Omega_H-\Omega_C-\Omega_B)(1-\alpha_H-\varepsilon)} \tag{57}$$

The optimal condition for housing capital remains the same while this for labor becomes:

$$w_t = (1 - \Omega_H - \Omega_C - \Omega_B)(1 - \alpha_H - \varepsilon)\left(\frac{p_t Y_{H,t}}{L_{HH,t}}\right) \tag{58}$$

The market clearing condition for the consumption good now includes the consumption of financial institutions on exit:

$$Y_{F,t} = C_t^R + C_t^C + C_t^F + C_t^{CR} + C_t^B + I_{F,t} + I_{H,t} \tag{59}$$

The market clearing condition for housing is augmented by the monitoring fee paid by investors on their loan to banks, which is in addition to that paid by financial institutions on their lending to credit-constrained consumers:

$$p_t Y_{H,t} = p_t H_t^R + p_t H_t^C - (1-\delta) p_t (H_{t-1}^R + H_{t-1}^C) + C_t^H + \tag{60}$$

$$+ \left[\mu^C G(\bar{\omega}_t^C) + \mu^B G(\bar{\omega}_t^B)\Psi(\bar{\omega}_t^C)\right] R_t^C p_{t-1} H_{t-1}^C$$

The parameters governing the general equilibrium are the same as in the single mortgage version. The log-linearization of the additional equations is in Appendix C.

6 The Chained Contract Results

Just as in the single loan contract version, this paper simulates the role of an oversupply of housing and a reduction in financial volatility in an economy where both consumers and financial institutions are subject to credit frictions.

6.1 The Oversupply of Housing

Like the single contract model, this version simulates a permanent technological improvement in the production of housing. The presence of credit frictions in the financial sector does not change the directional impact of the housing innovation shock on households and producers. The oversupply of housing still triggers a reduction in the housing price that shifts demand away from the consumption good toward housing. However, the chained model version yields a qualitatively and quantitatively different effects on the leverage of indebted agents (Figure 7). Credit-constrained consumers experience a mild improvement in their debt position while leveraged financial institutions enter a considerable recession (Figures 7.25 and 7.26). The diverging experiences can be attributed to the relative position in the chained contract and the initial leverage ratios of both borrowers. As end participants in the extended mortgage setup, indebted households can expect banks to honor their participation constraint so they are relatively shielded from the adverse effects of the falling housing price on the housing equity. Their net worth still diminishes but by less than in the single loan contract when credit-constrained consumers had to meet the participation constraint of investors. Overall, the decrease in the net worth of credit-constrained consumers (Figure 7.19) is proportionally less than the fall in the value of their housing stock so households experience a small improvement in their leverage position.

Financial institutions, however, experience the full force of the devaluation of the housing equity. This is due to two features of the chained loan contract. On one hand, the initial leverage of banks is five times that of consumers so their debt position is more volatile. On the other hand, financial institutions are intermediaries in the mortgage contract so they must ensure the participation of both lenders (investors) and final borrowers (credit-constrained consumers). Hence when an adverse shock hits the economy, financial institutions must absorb the negative consequences of the disturbance in order to meet the participation constraints of investors and households. As a result, the net worth of banks bears most of the devaluation in the housing equity and their leverage ratio deteriorates accordingly (Figures 7.20 and 7.26). Their intermediary position, coupled with their high initial indebtedness, brings a proportionally larger decrease in their leverage than the decrease in the debt position of credit-constrained consumers in the single contract model (Figures 7.26 and 3.24).

Comparing the subsequent recovery of the optimizing agents in both versions of the model, consumers in the single loan contract and financial institutions in the chained contracts version, it is clear that financial institutions in the chained contract return to their pre-shock position faster than consumers in the single contract. This disparity is due to the fact that banks operate as monopolists facing no competition in their respective area so they can recover faster than mortgaged households, who have no market power (Figures 7.26 and 3.24). The leverage of financial institutions in the chained contracts version takes about 50 quarters (12 years) to return to pre-shock levels while that of credit-constrained consumers in the single contract remains above its steady state value for almost 100 quarters (25 years). The default risk mirrors the respective leverage evolution of the two credit-constrained agents (Figures 7.24 and 3.23). As a result, in a chained contract, monopolist banks may experience a proportionally larger hit but can recover faster, while indebted consumers in the single mortgage contract are faced by a protracted credit crunch.

The main loan contract and its chained contracts extension also offers insight into the contrasting experience of credit-constrained households. In the single mortgage version they maximize the loan contract and in the chained extension they are final borrowers whose participation constraint must be satisfied by financial institutions. As maximizers in the first version of the model, indebted households bear fully the negative consequences of the oversupply of houses, while in the extension they are relatively shielded by the binding of their participation condition. However, this does not imply that the chained contract is preferable. Although the chained contract model may be more favorable toward credit-constrained consumers, it worsens the debt position of banks quantitatively more and makes the economy more volatile. Its only redeeming quality is that it results in a relatively short-lived downturn compared to the single loan contract economy since financial institutions use their monopolist position to repair their leverage position relatively fast. What used to be a moderate but protracted credit crunch in the single loan setup, becomes a steep but short-lived financial downturn in the chained loans version. Nevertheless, in both scenarios the falling housing price can act as a vehicle of shock propagation that turns a positive innovation in the housing sector into the main cause of debt worsening for credit-constrained agents that characterizes a downturn such as the subprime mortgage crisis.

The models described so far, although, theoretical in nature, have important policy implications. Both versions of credit frictions, with only indebted consumers and with both credit-constrained households and leveraged banks, demonstrate the importance of sound financial regulation. Policymakers should consider regulating the maximum permissible leverage ratio for borrowers. However, mandating a debt-to-income cap only for indebted households may be of limited usefulness when banks are also leveraged. As intermediaries in the mortgage contract, financial institutions usually enter the loan arrangement with much less equity and experience a more volatile leverage than households. Their excessive indebtedness in the chained contracts version causes a far larger drag on economic performance than that of consumers in the single mortgage model so it is important that there are sufficient loan loss provisions and reserve requirements in place to guarantee the liquidity of the financial system in a downturn.

6.2 The Reduced Financial Volatility

Unlike the single contract model that demonstrates the consequences of a temporary improvement in credit access for indebted households, the chained contracts setup allows to study two versions of a relaxation of borrowing conditions - one that targets credit-constrained consumers and one that pertains to financial institutions. A reduction in the volatility of consumer borrowing in the chained contracts model yields qualitatively different results than the same disturbance in the single contract version (Figure 8). The decreased consumer volatility in the chained contracts version reduces not only the household risk premium but also the risk premium of banks since the loan contracts are linked in the same maximization problem (Figures 8.27 and 8.28). The improvements in consumer credit access are passed on to financial institutions, which as intermediaries in the loan contract, need less downpayment to secure funding from investors on behalf of consumers (Figure 8.20). However, since the initial leverage of banks is higher than that of consumers, the collateral reduction of financial institutions has a larger effect on their own debt position so their indebtedness rises more than that of households (Figures 8.25 and 8.26). The result is a spike in the default probabilities of both banks and households far larger than the spike in bankruptcy prospects of indebted consumers in the single mortgage contract version (Figures 8.23, 8.24 and 4.23).

Despite the severe downturn caused by a reduction of consumer volatility in the chained contracts model, the debt position of credit-constrained consumers recovers relatively fast and less than 10 quarters (2.5 years) after the shock, their leverage ratio and bankruptcy prospects return to pre-shock levels (Figure 8.25). Household recovery, however, is at the expense of that of financial institutions. As end participants in the chained contract, consumers can force banks to honor their participation constraint so most of the subsequent improvements in housing equity benefit credit-constrained consumers delaying the recouping of financial institutions. The net worth of households and their leverage position improve relatively fast, while banks are faced with a protracted recovery (Figure 8.26). Satisfying the participation constraints of both consumers and investors costs financial institutions lost equity and their debt position remains above pre-shock levels for close to 100 quarters (25 years).

While a temporary relaxation of consumer credit access in the chained contacts extension yields a deeper recession than the same disturbance in the single contract version, an easing of borrowing conditions for banks in the chained contracts setup causes the most severe worsening in the debt position of both credit-constrained agents out of the three scenarios (Figure 9). The direction of change in net worth, leverage and default prospects of both banks and indebted consumers is the same as in the case of a decrease in consumer volatility in the chained contracts model, however the magnitudes of deviation are much larger. Following a reduction in their own volatility, financial institutions decrease their net worth much more than when they are faced with a reduction in consumer volatility (Figure 9.20). Owing to their higher initial leverage, financial institutions are more sensitive to changes in their own riskiness, so the tightening after the temporary reduction in their own volatility has a larger effect on their net worth than a reduction in consumer volatility. Hence the leverage of banks experiences a proportionally larger increase (Figure 9.26) than in the previous case. The deterioration of bank indebtedness is so large that it spills out of the intermediary section of the chained contract and worsens consumer leverage despite the binding of the household participation constraint (Figure 9.25). Credit-constrained consumers suffer the negative consequences of a reduction in bank volatility to a greater extent than following a decrease in their own volatility. The financial troubles of banks also draw them into a prolonged downturn. Unlike the previous scenario where they recover relatively fast, here their leverage remains above its steady state value for more than 100 quarters (25 years).

The last scenario describing a reduction in bank volatility in a chained contracts setup implies the most austere recession for both banks and households and comes closest to the consequences of the subprime mortgage crisis. The balance sheets of banks were deemed sufficiently low risk and fairly stable, which allowed them to accumulate excessive debt. The resulting downturn was passed on to consumers who experienced significant worsening in their debt positions that led to a prolonged tightening of credit access. Nevertheless, all three scenarios in the single and chained contracts context demonstrate the perils of overleverage that a temporary relaxation in borrowing conditions can create.

The policy recommendations for the scenario with decreased financial volatility point in the same direction as these for housing oversupply. In order to safeguard against such changes in perceived financial riskiness that reflect animal spirits and are not supported by macroeconomic fundamentals, it is important to have loan-to-value requirements and debt-to-income caps in place for mortgage-buyers. Both would ensure that homeowners would not overleverage or secure loans with little downpayments. Like in the case with housing oversupply, macroprudential regulations should be aimed at financial institutions to ensure that they hold sufficient reserves to weather unexpected downturns. The scenario with a reduction in bank volatility also implies that in times of bank distress due to excessive leverage, consumers should not be forced to share the burden of repairing the balance sheets of financial institutions and may need alternative sources of credit.

Figure 7. Effect of Housing Oversupply in Economy with Chained Contracts

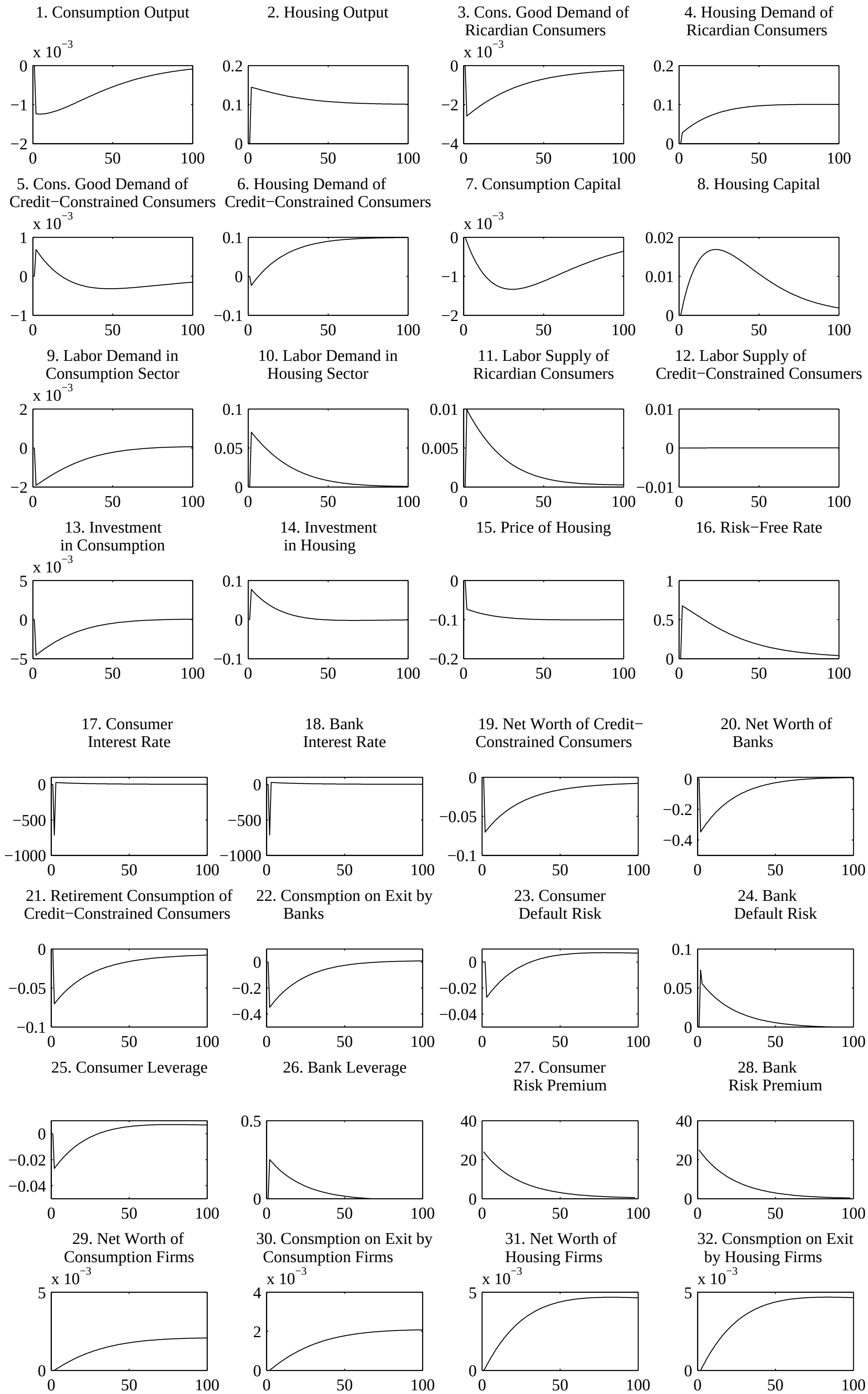


Figure 8. Effect of Reduced Consumer Volatility in Economy with Chained Contracts

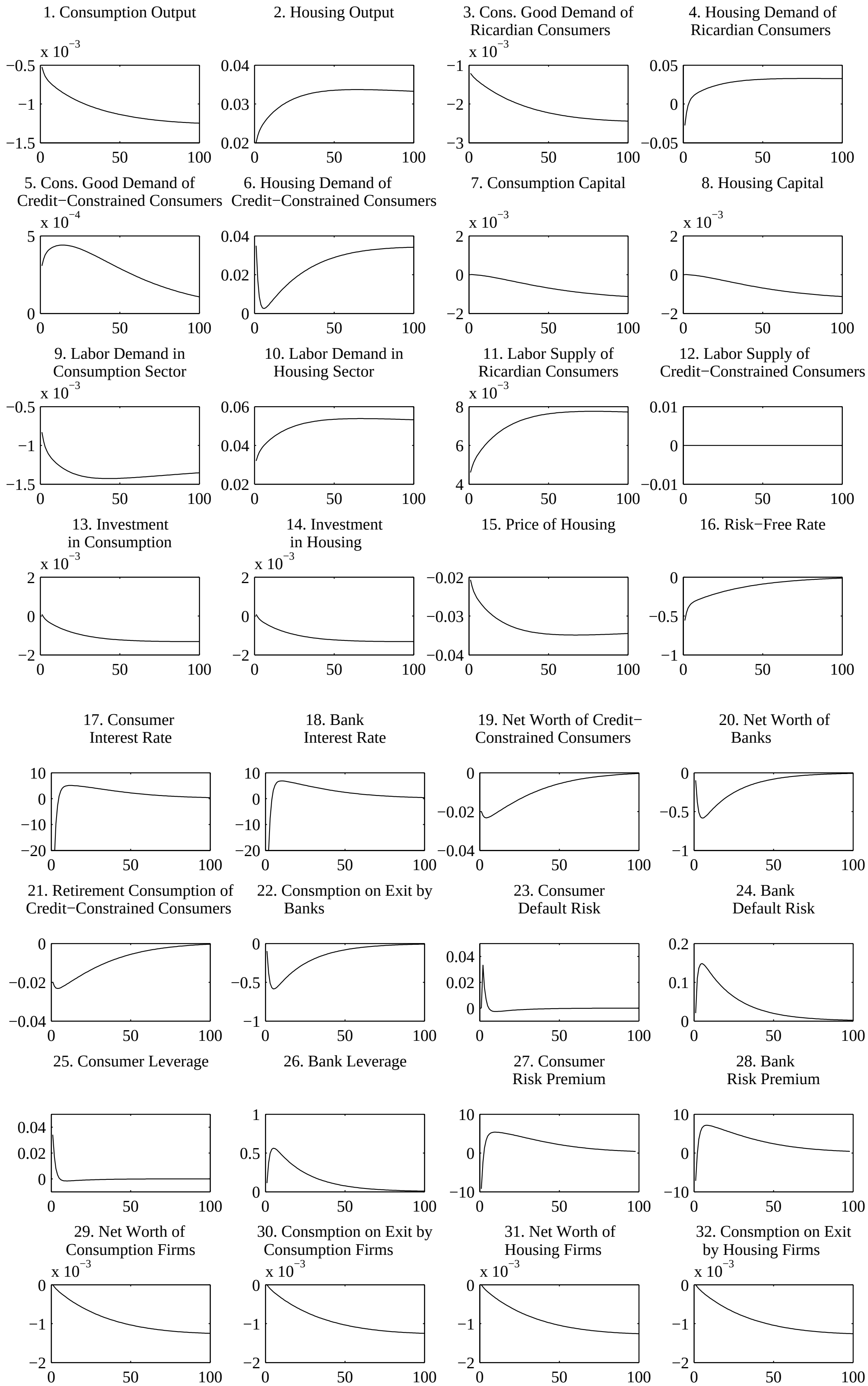
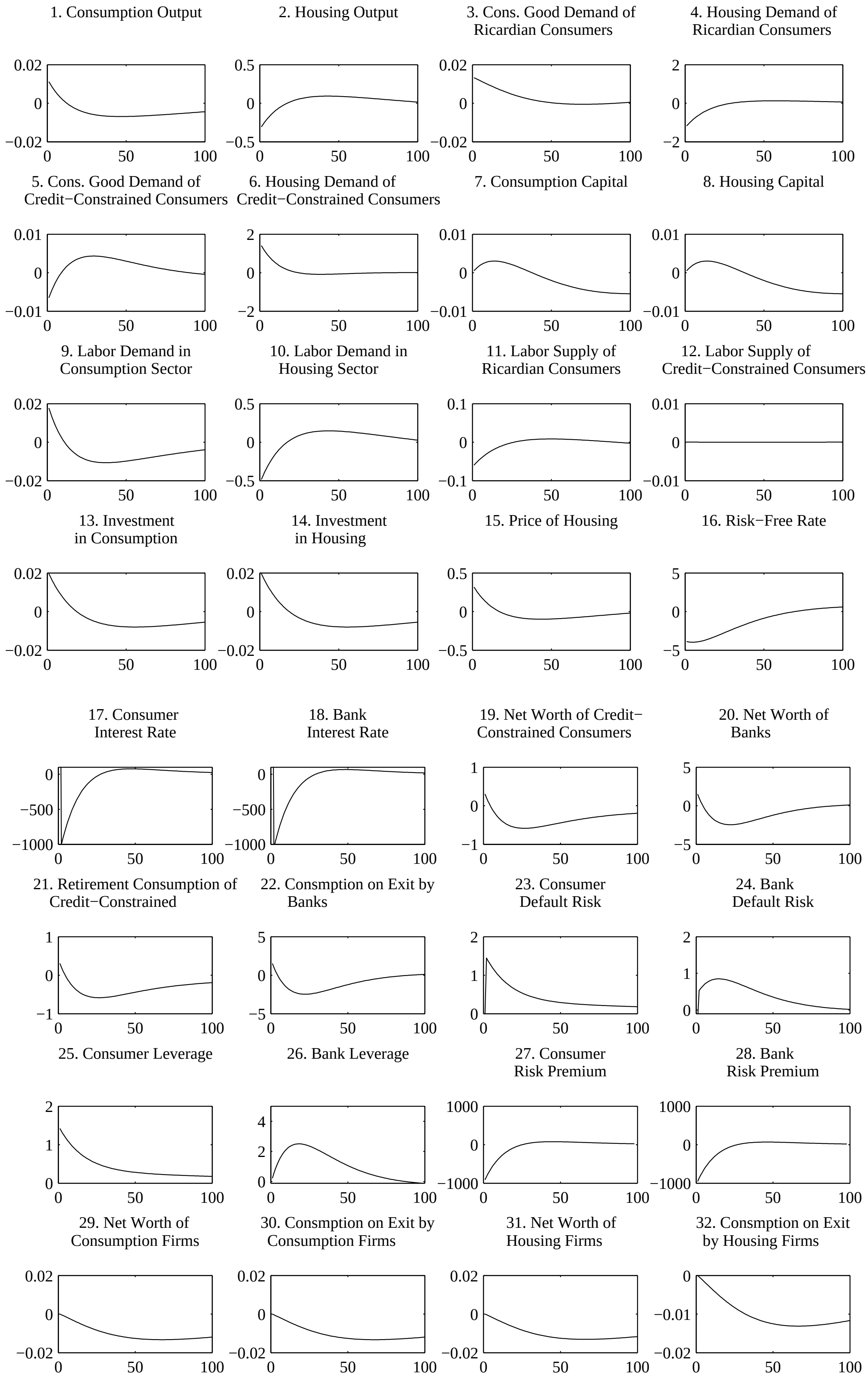


Figure 9. Effect of Reduced Bank Volatility in Economy with Chained Contracts



7 Conclusion

The subprime mortgage crisis dashed the hopes of many for home ownership and set off a deep recession. Prospective loan applicants saw their financing prospects reduced for years ahead as suddenly prudent banks struggled to improve their balance sheet positions. Many financial institutions were brought to the brink of collapse or saved by the "too big to fail" policy only to push through their recovery at the expense of credit-squeezed consumers. The consequences of excessive leverage and excessive lending as subprime loans had many calling for more stringent supervision of borrowing transactions. An improved regulation framework cannot emerge without understanding how the subprime mortgage crisis happened to be.

The straightforward setup of financial frictions in the household and financial sectors in this paper updates the financial accelerator approach of Bernanke, Gertler and Gilchrist (1999) to capture some of the causes and consequences of the subprime recession. The model establishes that the subprime crisis is not a unique occurrence, on the contrary, a financial downturn can be triggered relatively easily when there are credit frictions associated with lending. Using a two-sector economy, this model demonstrates that a positive housing supply shock can have negative repercussions for mortgage-buyers since it reduces the value of the good used as collateral in the mortgage contract. A lower value of the collateral implies higher leverage of indebted home-buyers, increased default prospects and a higher risk premium. Such a pro-cyclical risk premium that occurs in a two-sector economy is in stark contrast to Bernanke, Gilchrist and Gertler (1999) where one-sector model shocks can generate only an anti-cyclical external finance premium.

An improvement in the borrowing conditions also causes adverse consequences for leveraged agents and demonstrates that the credit-constrained sector can be the source of a crippling downturn even if it is not a production sector. A reduction in the riskiness of the financial market can impact indebted consumers directly by allowing them to excessively leverage using relatively little collateral. When as a result of the overleveraging, the lending rate adjusts upwards, these households end up with subprime mortgages that they cannot service and are more likely to become bankrupt. Furthermore, higher initial leverage can be detrimental to borrowers by inducing a shorter but steeper recession and squeezing their demand for goods more than moderate indebtedness levels. Finally, as the chained contacts extension demonstrated, the credit crunch can be more prolonged and disproportionally tilted toward financial institutions when they also participate in the mortgage contract. Banks may shift some of the consequences of the downturn onto credit-constrained consumers dragging them into a protracted recession.

While the presence of idiosyncratic default probabilities implies that financial frictions cannot be easily eliminated, the consequences of excessive leverage hint at the type of credit regulation needed to safeguard a sound financial market. Debt-to-income caps and loan-to-value requirements for consumers would ensure that households obtain sensible mortgages that they can service even in a downturn. Reserve requirements and loan loss provisions for financial institutions would guarantee sufficient liquidity in times of distress. Taken together, these measures would prevent a potential overleverage that can cripple credit access for a prolonged time and can spill out of the mortgage market and distress the whole economy.

The model, while aptly demonstrating the role of leverage in triggering a financial downturn, could be enriched further to offer a deeper understanding of credit mechanisms. When the crisis started, many banks attempted to salvage their debt position by recalling loans to other financial institutions, rather than merely bearing out the increasing bankruptcy risk as described in this paper. The model can be augmented by adding inter-bank relationships and loan networks that may trigger a domino-like effect of rising default risk. Furthermore, the potential role of a bailout policy could be explored by adding government to the existing setup. It may also be a cautionary tale to consider the possibility of endogenous steady state leverage ratio for both banks and consumers. As history leading to the crisis demonstrates, leverage limits were poorly regulated and enforced prior to the subprime mortgage crisis allowing instead financial institutions to reach dangerously high debt to equity ratios. Finally, an important contribution could be to model the role of heterogeneous consumer expectations. It may be especially interesting to consider to what extent departures from the representative agent theory could explain the collective failure of agents to foresee the subprime mortgage crisis.

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Appendix A
Model Calibration

The calibrated model is quarterly so four periods correspond to a year. The model parameters are chosen to be as close as possible to generally accepted values while reflecting a number of key model features. The parameters that govern the complete equilibrium are fairly standard and are described in Table 1. Housing production is one tenth relative to the conventional good market since housing is a multiperiod good and production only replaces depreciated housing (Iacovello and Neri 2010). The relative size of the two sectors is calibrated through the coefficient κ that determines the consumption-housing tradeoff taking as given the interest rate and the depreciation rate. The housing sector is less capital-intensive than the consumption good sector, but unlike the consumption good sector, it also utilizes land, which is in fixed supply (Iacovello and Neri 2010). The depreciation rate of both types of capital has its conventional value. Finally, the labor supply of credit-constrained consumers, consumption good producers, housing producers and financial institutions for the purpose of starting their net worth is heavily weighted by Ω_i where $i = C$, F, H, B so that it does not crowd out regular labor supply by consumers. For the temporary volatility shock, the persistence persistence parameter ρ has a value of 0.95, a conventional choice for the frequency of the model (Fernandez-Villaverde 2010).

Table 1. General Model Parameters

Parameter	α_F	α_H	ε	β	γ	φ	κ	J"	δ	Ω_i
Value	0.35	0.25	0.10	0.99	0.33	0.01	0.18	-10	0.025	0.01

The loan contract is calibrated separately and the parameters governing the loan contract are designed to satisfy several steady state conditions:

1. The steady state rate of the external risk premium $R^C - R$ is 0.5% (Bernanke, Gilchrist and Gertler 1999).

2. The steady state leverage of credit-constrained households, i.e. value of housing stock to net worth ratio is $\frac{pH^C}{N^C} = 2$ (Bernanke, Gilchrist and Gertler 1999).

3. The failure rate of credit-constrained consumers $F^C(\bar{\omega}^C)$ (i.e. the number of credit-constrained consumers who retire each period) is 2% (Bernanke, Gilchrist and Gertler 1999).

Solving the first-order condition of credit-constrained consumers, the participation constraint and the cumulative distribution function $F^C(\bar{\omega}^C)$ in the steady state gives the parameters μ^C, ν^C , ν^H, ν^F, the steady state cutoff value of the cutoff risk $\bar{\omega}^C$ and the variance σ^C. The credit contract parameters are given in Table 2.

Table 2. Loan Contract Parameters in the Single Contract Economy

Parameter	ν^F	ν^H	ν^C	μ^C	$\bar{\omega}^C$	σ^C
Value	0.99	0.99	0.98	0.06	0.50	0.31

The parameters that govern the general equilibrium for the chained contracts model are the same as those described in Table 1 to ensure comparability of the two models. The presence of financial institutions in the loan contract setup is calibrated to satisfy the following requirements in addition to the aforementioned ones for credit-constrained consumers:

1. The steady state leverage of financial institutions is $\frac{pH^C}{N^B} = 10$ (banks are five times more leveraged than credit-constrained consumers) (Hirakata, Sudo and Ueda 2009).

2. The failure rate of financial institutions $F^B(\bar{\omega}^B)$ is 2% (same as that of credit-constrained consumers).

The loan parameters for this extension of the model are in Table 3.

Table 3. Loan Contract Parameters in the Chained Contracts Economy

Parameter	ν^C	ν^B	μ^C	μ^B	$\bar{\omega}^C$	σ^C	$\bar{\omega}^B$	σ^B
Value	0.98	0.97	0.06	0.03	0.501	0.312	0.797	0.107

Appendix B
Log Linearization of Single Contract Model

This appendix derives the log-linearized form of the model equations as used in the simulations. The steady state value for each variable appears without time subscript and hats denote the percentage deviation of the variable from its steady state such that $\hat{Z}_t = \frac{Z_t - Z}{Z}$.

The participation constraint for investors:

$$\hat{R}^C_t = \hat{R}_{t-1} + \frac{N^C}{pH^C - N^C}\left(\hat{p}_{t-1} + \hat{H}^C_{t-1} - \hat{N}^C_{t-1}\right) - \frac{\Gamma^C_\omega(\bar{\omega}^C) - \mu^C G^C_\omega(\bar{\omega}^C)}{\Gamma^C(\bar{\omega}^C) - \mu^C G^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}^C_t \tag{61}$$

The first-order condition of credit-constrained households' maximization:

$$0 = \frac{\Gamma_{\omega}^{C}(\bar{\omega}^{C})}{\Gamma_{\omega}^{C}(\bar{\omega}^{C}) - \mu^{C} G_{\omega}^{C}(\bar{\omega}^{C})} (\hat{R}_{t+1}^{C} - \hat{R}_{t}) - \tag{62}$$

$$-(1 - \Gamma^{C}(\bar{\omega}^{C})) \frac{R^{C}}{R} \left(\frac{\Gamma_{\omega\omega}^{C}(\bar{\omega}^{C})}{\Gamma_{\omega}^{C}(\bar{\omega}^{C})} - \frac{\Gamma_{\omega\omega}^{C}(\bar{\omega}^{C}) - \mu^{C} G_{\omega\omega}^{C}(\bar{\omega}^{C})}{\Gamma_{\omega}^{C}(\bar{\omega}^{C}) - \mu^{C} G_{\omega}^{C}(\bar{\omega}^{C})} \right) \bar{\omega}^{C} \hat{\bar{\omega}}_{t+1}^{C}$$

The consumption of credit-constrained households on retirement:

$$\hat{C}_{t}^{CR} = \hat{R}_{t}^{C} + \hat{p}_{t-1} + \hat{H}_{t-1}^{C} - \frac{\Gamma_{\omega}^{C}(\bar{\omega}^{C})}{1 - \Gamma^{C}(\bar{\omega}^{C})} \bar{\omega}^{C} \hat{\bar{\omega}}_{t}^{C} \tag{63}$$

The law of motion of the net worth of credit-constrained households:

$$\hat{N}_{t}^{C} = \nu^{C}(1 - \Gamma^{C}(\bar{\omega}^{C})) \frac{R^{C} H^{C}}{N^{C}} (\hat{R}_{t}^{C} + \hat{p}_{t-1} + \hat{H}_{t-1}^{C}) - \nu^{C} \frac{R^{C} H^{C}}{N^{C}} \Gamma_{\omega}^{C}(\bar{\omega}^{C}) \bar{\omega}^{C} \hat{\bar{\omega}}_{t}^{C} + \tag{64}$$

$$+(1 - \alpha_{H} - \varepsilon) \Omega_{C} \frac{Y_{H}}{N^{C}} \left(\hat{p}_{t} + \hat{Y}_{H,t} \right)$$

The consumption on exit by consumption good firms:

$$\hat{C}_{t}^{F} = \hat{R}_{t-1} + \hat{N}_{t-1}^{F} \tag{65}$$

The law of motion of the net worth of consumption good firms:

$$\hat{N}_{t}^{F} = \nu^{F} R (\hat{R}_{t-1} + \hat{N}_{t-1}^{F}) + (1 - \alpha_{F}) \Omega_{F} \frac{Y_{F}}{N^{F}} \hat{Y}_{F,t} \tag{66}$$

The consumption on exit by housing firms:

$$\hat{C}_{t}^{H} = \hat{R}_{t-1} + \hat{N}_{t-1}^{H} \tag{67}$$

The law of motion of the net worth of housing firms:

$$\hat{N}_{t}^{H} = \nu^{H} R (\hat{R}_{t-1} + \hat{N}_{t-1}^{H}) + (1 - \alpha_{H} - \varepsilon) \Omega_{H} \frac{Y_{H}}{N^{H}} \left(\hat{p}_{t} + \hat{Y}_{H,t} \right) \tag{68}$$

The consumption capital accumulation equation:

$$\hat{K}_{F,t} = \hat{K}_{F,t-1} + \delta \left(\hat{I}_{F,t} - \hat{K}_{F,t-1} \right) \tag{69}$$

Investment demand by consumption capital firms:

$$\hat{q}_{F,t} = J'' \delta \left(\hat{K}_{F,t-1} - \hat{I}_{F,t} \right) \tag{70}$$

The consumption good production function:

$$\hat{Y}_{F,t} = \hat{A}_{F,t} + \alpha_{F} \hat{K}_{F,t-1} + (1 - \alpha_{F})(1 - \Omega_{F}) \hat{L}_{FH,t} \tag{71}$$

The labor demand by consumption good firms:

$$\hat{w}_t = \hat{Y}_{F,t} - \hat{L}_{FH,t} \tag{72}$$

The capital demand by consumption good firms:

$$\hat{R}_t + \hat{q}_{F,t} = \frac{R-(1-\delta)}{R}(\hat{Y}_{F,t+1} - \hat{K}_{F,t}) + \frac{(1-\delta)}{R}\hat{q}_{F,t+1} \tag{73}$$

The housing capital accumulation equation:

$$\hat{K}_{H,t} = \hat{K}_{H,t-1} + \delta\left(\hat{I}_{H,t} - \hat{K}_{H,t-1}\right) \tag{74}$$

Investment demand by housing capital firms:

$$\hat{q}_{H,t} = J\text{"}\delta\left(\hat{K}_{H,t-1} - \hat{I}_{H,t}\right) \tag{75}$$

The housing production function:

$$\hat{Y}_{H,t} = \hat{A}_{H,t} + \alpha_H\hat{K}_{H,t-1} + (1-\Omega_H-\Omega_C)(1-\alpha_H-\varepsilon)\hat{L}_{HH,t} \tag{76}$$

The labor demand by housing firms:

$$\hat{w}_t = \hat{p}_t + \hat{Y}_{H,t} - \hat{L}_{HH,t} \tag{77}$$

The capital demand by housing firms:

$$\hat{R}_t + \hat{q}_{H,t} = \frac{R-(1-\delta)}{R}(\hat{p}_{t+1} + \hat{Y}_{H,t+1} - \hat{K}_{H,t}) + \frac{(1-\delta)}{R}\hat{q}_{H,t+1} \tag{78}$$

The labor supply of Ricardian consumers:

$$\varphi\hat{L}_t^R + \hat{C}_t^R = \hat{w}_t \tag{79}$$

The consumption housing tradeoff of Ricardian consumers:

$$\hat{C}_{t+1}^R = \hat{H}_t^R + \hat{p}_{t+1} + \frac{R}{(R-1+\delta)}\left(\hat{C}_{t+1}^R - \hat{C}_t^R + \hat{p}_t - \hat{p}_{t+1}\right) \tag{80}$$

The Euler equation of Ricardian consumers:

$$\hat{C}_{t+1}^R - \hat{C}_t^R = \hat{R}_t \tag{81}$$

The labor supply of credit-constrained consumers:

$$\varphi\hat{L}_t^C + \hat{C}_t^C = \hat{w}_t \tag{82}$$

The consumption housing tradeoff of credit-constrained consumers:

$$\hat{C}_t^C = \hat{H}_{t-1}^C + \hat{p}_t + \frac{R^C}{(R^C-1+\delta)}\left(\hat{R}_t^C + \hat{p}_{t-1} - \hat{p}_t\right) \tag{83}$$

The consumption purchase equation of credit-constrained consumers:

$$\hat{C}_t^C = \frac{wL^C}{C^C}(\hat{w}_t + \hat{L}_t^C) + \frac{(1-\delta)H^C}{C^C}\hat{p}_t - \frac{R^CH^C}{C^C}(\hat{R}_t^C + \hat{p}_{t-1}) + \frac{(1-\delta-R^C)H^C}{C^C}\hat{H}_{t-1}^C \tag{84}$$

Market clearing in the consumption good sector:

$$\hat{Y}_{F,t} = \frac{C^R}{Y_F}\hat{C}_t^R + \frac{C^C}{Y_F}\hat{C}_t^C + \frac{C^F}{Y_F}\hat{C}_t^F + \frac{C^{CR}}{Y_F}\hat{C}_t^{CR} + \frac{I_F}{Y_F}\hat{I}_{F,t} + \frac{I_H}{Y_F}\hat{I}_{H,t} \tag{85}$$

Market clearing in the housing sector:

$$\hat{Y}_{H,t} = \frac{\delta(H^R + H^C) - Y_H}{Y_H}\hat{p}_t + \frac{H^R}{Y_H}\hat{H}_t^R + \frac{H^C}{Y_H}\hat{H}_t^C - \frac{(1-\delta)}{Y_H}(H^R\hat{H}_{t-1}^R + H^C\hat{H}_{t-1}^C) + \tag{86}$$

$$+\frac{C^H}{Y_H}\hat{C}_t^H + \mu^C G^C(\bar{\omega}^C)\frac{R^C H^C}{Y_H}\left(\hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C\right) + \mu^C G_\omega^C \bar{\omega}^C \hat{\bar{\omega}}_t^C \frac{R^C H^C}{Y_H}$$

Market clearing in the labor market:

$$L_{FH}\hat{L}_{FH,t} + L_{HH}\hat{L}_{HH,t} = L^R\hat{L}_t^R + L^C\hat{L}_t^C \tag{87}$$

Appendix C
Log Linearization of the Chained Contracts Model

This appendix lig-linearizes the additional equations for the chained contracts model.

The participation constraint for investors:

$$\hat{R}_t^C = \hat{R}_{t-1} + \frac{1}{pH^C - N^C - N^B}\left(\left(N^C + N^B\right)\left(\hat{p}_{t-1} + \hat{H}_{t-1}^C\right) - N^C\hat{N}_{t-1}^C - N^B\hat{N}_{t-1}^B\right) \tag{88}$$

$$-\frac{\Psi_\omega(\bar{\omega}^C)}{\Psi(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C - \frac{\Psi_\omega(\bar{\omega}^B)}{\Psi(\bar{\omega}^B)}\bar{\omega}^B\hat{\bar{\omega}}_t^B$$

The first-order condition of financial institutions' maximization of the chained contracts:

$$\hat{R}_{t+1}^C - \hat{R}_t = \frac{\left(1-\Gamma(\bar{\omega}^C)\right)\left(\Gamma_{\omega\omega}(\bar{\omega}^C)\Psi_\omega(\bar{\omega}^C) - \Gamma_\omega(\bar{\omega}^C)\Psi_{\omega\omega}(\bar{\omega}^C)\right)}{\left[\left(1-\Gamma(\bar{\omega}^C)\right)\Psi_\omega(\bar{\omega}^C) + \Gamma_\omega(\bar{\omega}^C)\Psi(\bar{\omega}^C)\right]\Gamma_\omega(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_{t+1}^C + \tag{89}$$

$$+\frac{\left(1-\Gamma(\bar{\omega}^B)\right)\left(\Gamma_{\omega\omega}(\bar{\omega}^B)\Psi_\omega(\bar{\omega}^B) - \Gamma_\omega(\bar{\omega}^B)\Psi_{\omega\omega}(\bar{\omega}^B)\right)}{\left[\left(1-\Gamma(\bar{\omega}^B)\right)\Psi_\omega(\bar{\omega}^B) + \Gamma_\omega(\bar{\omega}^B)\Psi(\bar{\omega}^B)\right]\Gamma_\omega(\bar{\omega}^B)}\bar{\omega}^B\hat{\bar{\omega}}_{t+1}^B$$

The financial institutions' interest rate:

$$\hat{R}_t^B = \hat{R}_t^C + \frac{N^C}{pH^C - N^C}\left(\hat{N}_{t-1}^C - \hat{p}_{t-1} - \hat{H}_{t-1}^C\right) + \frac{\Psi_\omega(\bar{\omega}^C)}{\Psi(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C \tag{90}$$

The participation constraint of credit-constrained consumers:

$$\hat{N}_{t-1}^C = \hat{p}_{t-1} + \hat{H}_{t-1}^C - \frac{\Gamma_\omega^C(\bar{\omega}^C)}{1-\Gamma^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C \tag{91}$$

The consumption on exit of financial institutions:

$$\hat{C}_t^B = \hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C - \frac{\Gamma_\omega^B(\bar{\omega}^B)}{1-\Gamma^B(\bar{\omega}^B)}\bar{\omega}^B\hat{\bar{\omega}}_t^B + \frac{\Psi_\omega(\bar{\omega}^C)}{\Psi(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C \tag{92}$$

The law of motion of the net worth of financial institutions:

$$\hat{N}_t^B = \nu^B(1-\Gamma^B(\bar{\omega}^B))\Psi(\bar{\omega}^C)\frac{R^C pH^C}{N^B}(\hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C) + (1-\alpha_H-\varepsilon)\Omega_H\frac{Y_H}{N^B}\left(\hat{p}_t + \hat{Y}_{H,t}\right) +$$

$$-\nu^B\frac{R^C pH^C}{N^B}\Psi(\bar{\omega}^C)\Gamma_\omega^B(\bar{\omega}^B)\bar{\omega}^B\hat{\bar{\omega}}_t^B + \nu^B\frac{R^C pH^C}{N^B}\left(1-\Gamma(\bar{\omega}^B)\right)\Psi_\omega^C\bar{\omega}^C\hat{\bar{\omega}}_t^C \tag{93}$$

The housing production function:

$$\hat{Y}_{H,t} = \hat{A}_{H,t} + \alpha_H\hat{X}_{t-1} + (1-\Omega_H-\Omega_C-\Omega_B)(1-\alpha_H-\varepsilon)\hat{L}_{HH,t} \tag{94}$$

The labor demand by housing firms:

$$\hat{w}_t = \hat{p}_t + \hat{Y}_{H,t} - \hat{L}_{HH,t} \tag{95}$$

Market clearing in the consumption good sector:

$$\hat{Y}_{F,t} = \frac{C^R}{Y_F}\hat{C}_t^R + \frac{C^C}{Y_F}\hat{C}_t^C + \frac{C^F}{Y_F}\hat{C}_t^F + \frac{C^{CR}}{Y_F}\hat{C}_t^{CR} + \frac{C^B}{Y_F}\hat{C}_t^B + \frac{I_F}{Y_F}\hat{I}_{F,t} + \frac{I_H}{Y_F}\hat{I}_{H,t} \tag{96}$$

Market clearing in the housing sector:

$$\hat{Y}_{H,t} = \frac{\delta(H^R+H^C)-Y_H}{Y_H}\hat{p}_t + \frac{H^R}{Y_H}\hat{H}_t^R + \frac{H^C}{Y_H}\hat{H}_t^C - \frac{(1-\delta)}{Y_H}(H^R\hat{H}_{t-1}^R + H^C\hat{H}_{t-1}^C) + \tag{97}$$

$$+\frac{C^H}{Y_H}\hat{C}_t^H + \left[\mu^C G^C(\bar{\omega}^C) + \mu^B G(\bar{\omega}^B)\Psi(\bar{\omega}^C)\right]\frac{R^C H^C}{Y_H}\left(\hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C\right) +$$

$$+\left[\mu^C G_\omega^C + \mu^B G(\bar{\omega}^B)\Psi_\omega^C\right]\frac{R^C H^C}{Y_H}\bar{\omega}^C\hat{\bar{\omega}}_t^C + \mu^B G_\omega^B\Psi(\bar{\omega}^C)\frac{R^C H^C}{Y_H}\bar{\omega}^B\hat{\bar{\omega}}_t^B$$