Distributed Optimization for Energy Management in Building Networks

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***Abstract*—We formulate and solve an energy management problem for a cooperative network of buildings sharing some resource and communicating over a time varying network, each of which desires to maintain its privacy. As the network is time-varying, we are able to accommodate communication constraints or failure between agents. We begin by introducing validated bilinear modelling techniques for individual buildings, and continue by showing how the energy management problem involving a network of buildings can be addressed using a distributed optimization algorithm recently proposed in the literature, with individual buildings treated as agents. To facilitate these assumptions, we linearize the bilinear models generated, and introduce linear local and coupling constraints to model a shared thermal energy storage device. We formulate a linear objective function to minimize based on the cost of energy used by the agent not taken from the storage. Finally, we demonstrate the efficacy of the distributed algorithm as applied to the energy management problem using an extensive simulation based study.**

***Index Terms*—Distributed optimization, building control, energy management, dual decomposition**

I. Introduction

Climate control within buildings accounts for up to 15% of US and European energy consumption [1], and hence is responsible for a significant proportion of total CO_2 emissions. This motivates "green" building paradigms and offers incentives to decrease energy usage, however, it does not constitute a viable solution from an economic point of view.

To this end, the development of algorithms for control and regulation of buildings sharing energy resources has attracted significant research attention. Among the different approaches, optimization based control offers a flexible framework that allows determining energy consumption strategies by solving of an energy-cost based optimization problem subject to various comfort-constraints of the building users. The success of such systems is heavily dependent on the choice of modelling techniques used. To this end, optimization based models of varying complexity have been proposed; the reader is referred to [2], [3], [4], [5].

Energy management problems referring to a single building have been extensively studied in the literature; however, there is an increasing need for coordinated management of networks of buildings that interact with each other either physically or via message passing to share resources like storage, which is in general expensive to operate at an individual level. These buildings, which shall be called agents in this paper, can cooperate with those in their community, commonly via means of energy sharing, in order to find network-wide optimal energy-management solutions. Examples of such collaborative energy management schemes appear in [6], [7] and [8].

A clear motivation in such building networks is for agents to maintain the privacy of information regarding their energy consumption profile [9], typically encoded via the choice of the constraint sets in an optimization context. Therefore, it is essential that any scheme used to find optimal energy management solutions over such networks is able to guarantee privacy of the agents within the network. This can be achieved by distributed optimization. In [6], a primal based distributed scheme based on [10] is proposed, where a network of buildings share a cooling resource; while privacy is maintained for the local decision variables of each agent, agents must share their global decisions with each other, thus raising privacy concerns. This problem could be avoided if a primal-dual algorithm is used instead, allowing the decisions of each agent to be entirely local, and restrict information sharing to exchange of dual variables related to the resource sharing constraints. To achieve this, we exploit algorithms that have recently appeared in the distributed optimization literature [11], [12]. However, the latter is limited to time-invariant networks, which poses a difficulty with regards to problem robustness to network communication constraint or failure, hence in the sequel we will be employing [11].

The remainder of the paper is organized as follows. Section II provides background on the modelling of each individual building and on the proposed distributed scheme, while Section III introduces the energy management for a time-varying network of buildings. Section IV illustrates the efficacy of the proposed methodology via an extensive simulation based study, while Section V concludes the paper and provides directions for future work.

Research was supported by EPSRC UK under the grant EP/P03277X/1.

II. Preliminaries

A. Building dynamics and constraints

The MATLAB Building Resistance Capacitance Toolbox (BCRM) [5], provides a framework to produce bilinear continuous or discrete time models of buildings suitable for use with Model Predictive Control. This is performed via the generation and declaration of thermal and External Heat Flux (EHF) models. In this paper we employed the BCRM toolbox to generate continuous time building models that were subsequently discretized at a sampling period of $T = 0.25$ hours, as suggested by [3], which gives the following system dynamics

$$x_{t+1} = Ax_t + B_u u_t + B_v v_t + \sum_{j=1}^{n_u} (B_{vu} v_t + B_{xu} x_t) u_{t,j}, \quad (1)$$

and constraint

$$F_{x,t} x_t + F_{u,t} u_t + F_{v,t} v_t \leq g_t. \quad (2)$$

Here, $x_t \in \mathbb{R}^{n_s}$ is the system state, which includes the temperature of zones and wall slices, $u_t \in \mathbb{R}^{n_u}$ is the control input, including radiator heating power, and air flow rate and heating power of an air handling unit, and $v_t \in \mathbb{R}^{n_v}$ is the disturbance vector, including solar radiation and ambient temperature, each at time-step t. The system is assumed to be deterministic and hence the disturbance vector v_t is known for all time-steps. The state matrix A, input matrix B_u, disturbance matrix B_v, disturbance-input matrix B_{vu} and state-input matrix B_{xu} are all of appropriate dimension. Furthermore, $F_{x,t}, F_{u,t}, F_{v,t}$ and g_t are time-varying vectors and matrices that provide the actuator and comfort constraints of the system.

B. Distributed Optimisation

The procedure presented in [11] provides a distributed methodology to solve optimization problems in the form of

$$\begin{aligned} \mathcal{P} : & \min_{\{u_i \in U_i\}_{i=1}^m} \sum_{i=1}^m f_i(u_i) \\ & \text{subject to:} \sum_{i=1}^m g_i(u_i) \leq 0. \end{aligned} \quad (3)$$

involving m agents communicating over a time-varying network. Each agent has a local vector $u_i \in \mathbb{R}^{n_u^i}$ of n_u^i decision variables, a local constraint set $U_i \subset \mathbb{R}^{n_u^i}$ and local objective function $f_i(\cdot) : \mathbb{R}^{n_u^i} \to \mathbb{R}$. Agents also have a function $g_i(\cdot) : \mathbb{R}^{n_u^i} \to \mathbb{R}^p$ which forms the contribution to p coupling constraints. Consider now the Lagrangian function $L(u, \lambda) : \mathbb{R}^n \times \mathbb{R}_+^p \to, \mathbb{R}$

$$L(u, \lambda) = \sum_{i=1}^m \{f_i(u_i) + \lambda^\top g_i(u_i)\}, \quad (4)$$

where $u = [u_1^\top \dots u_m^\top] \in U = U_1 \times \dots U_m \subset \mathbb{R}^n$, with $n = \sum_{i=1}^m n_u^i$, and $\lambda \in \mathbb{R}_+^p$ is the vector of Lagrange multipliers. The corresponding dual problem can then be defined as

$$\mathcal{D} : \max_{\lambda \geq 0} \min_{u \in U} L(u, \lambda). \quad (5)$$

Algorithm 1 Distributed algorithm

1: **Initialization**
2: $k = 0$.
3: Consider $\hat{u}_i(0) \in U_i$ for all $i =, \dots, m$
4: Consider $\hat{\lambda}_i(0) \in \mathbb{R}_+^p$ for all $i =, \dots, m$
5: **For** $i = 1, \dots, m$ **repeat until convergence**
6: $l_i(k) = \sum_{j=1}^m a_j^i(k) \lambda_j(k)$.
7: $u_i(k+1) \in \arg\min_{u_i \in U_i} f_i(u_i) + l_i(k)^\top g_i(u_i)$
8: $\lambda_i(k+1) = [l_i(k) + c(t) g_i(u_i(k+1))]_+$
9: $\hat{u}_i(k+1) = \hat{u}_i(k) + \frac{c(k)}{\sum_{r=0}^k c(r)} (u_i(k+1) - \hat{u}_i(k))$
10: $k \leftarrow k + 1$.

To facilitate the reader, we provide the main steps of the scheme in Algorithm 1. At each iteration, agents share their estimates l_i of the dual variables λ_i with their neighbours, while in turn receiving the dual estimates of these agents and construct a weighted estimate of these variables denoted by $l_i(k)$, where the weighting coefficients $a_j^i(k)$ denote how agent i weights the information received by agent j at iteration k of the algorithm. Agents then compute a tentative decision $u_i(k+1)$ as far as their local decisions are concerned (see step 7), and use this solution to update their dual estimates in step 8, where $[\cdot]_+$ is the projection of its argument onto the positive orthant. This repeats until consensus. Note that there is also an auxiliary update variable, $\hat{u}_i(k+1)$, which constitutes the running average of $u_i(k+1)$ and for which convergence results are proven [11].

The algorithm requires the following assumptions:

Assumption 1. *For each $i = 1, \dots, m$, the function $f_i(\cdot) : \mathbb{R}^{n_u^i} \to \mathbb{R}$ and each component of $g_i(\cdot) : \mathbb{R}^{n_u^i} \to \mathbb{R}^p$ are convex; for each $i = 1, \dots, m$, the set $U_i \subset \mathbb{R}^{n_u^i}$ is convex and compact.*

Assumption 2. *There exists $\tilde{u} = [\tilde{u}_1 \dots \tilde{u}_m] \in \mathrm{relint}(U)$, where $\mathrm{relint}(U)$ is the relative interior of the set U, such that $\sum_{i=1}^m g_i(\tilde{u}_i) \leq 0$ for those components of $\sum_{i=1}^m g_i(u_i)$ that are linear in u, if any, while $\sum_{i=1}^m g_i(\tilde{u}_i) < 0$ for all other components.*

Under assumptions 1 and 2 we have that strong duality holds, and hence an optimal prime-dual pair (u^*, λ^*) exists. Note that while the existence of a Slater point is required, it does not require agents to have knowledge of it.

Assumption 3. *$\{c(k)\}_{k \geq 0}$ is a non-increasing sequence of positive real numbers such that $c(k) \leq c(r)$ for all $k \geq r$, with $r \geq 0$. Moreover,*

1) $\sum_{k=0}^\infty c(k) = \infty$
2) $\sum_{k=0}^\infty c(k)^2 < \infty$

Assumption 4. *There exists $\eta \in (0, 1)$ such that for all $i, j \in 1, \dots, m$ and all $k \geq 0$, $a_j^i(k) \in [0, 1), a_j^i(k) \geq \eta$, and $a_j^i(k) > 0$ implies that $a_j^i(k) \geq \eta$. Moreover, for all $k \geq 0$,*

1) $\sum_{j=1}^m a_j^i(k) = 1$ *for all $i = 1, \dots, m$,*
2) $\sum_{i=1}^m a_j^i(k) = 1$ *for all $i = 1, \dots, m$.*

For fixed $k \geq 0$, the information exchanged between the m agents can be coded via the directed graph (V, E_k), where the nodes $V = \{1, \ldots, m\}$ represent the agents, and the set E_k of directed edges is defined as

$$E_k = \{(j, i) : a_j^i(k) \geq 0\}. \tag{6}$$

Let $E_k = \{(j, i) : (j, i) \in E_t\}$ denote the set of edges (i, j) representing pairs of agents that communicate directly infinitely often. This leads to the final assumption.

Assumption 5. *The graph (V, E_∞) is strongly connected. Moreover, there exists $T \geq 1$ such that for every $(i, j) \in E_\infty$, agent i receives information from a neighbouring agent j at least once every consecutive T iterations.*

Under these assumptions, we have from Theorem 1 from [11] that the dual variables in step 8 converge to some optimal dual variable of the dual problem corresponding to $\mathcal{P}$. Furthermore, from Theorem 2 we have that all limit points of the running average sequences in step 9 are optimal primal solutions.

III. Energy Management for Building Networks

In this section, we will formulate an energy management problem for a network of buildings in the form of $\mathcal{P}$. Our approach is capable of solving a more general class of setups where agents share common resources, but for concreteness of exposition we focus on a set-up where individual buildings share an energy storage device.

A. Abstraction by a linear model

To facilitate the convexity assumption we proceed with linearising the bilinear model obtained by the BCRM toolbox. We linearize the state-input and disturbance-input bi-linearities around an arbitrary input trajectory [13],[14]. This leads to the following system dynamics

$$x_{t+1} = Ax_t + B'_{u,t} u_t + B_v v_t + \sum_{j=1}^{n_u} (\gamma^j_{x,t} x_t + \gamma^j_{u,t} u_t + c^j_t), \tag{7}$$

where $B'_{u,t}, \gamma^j_{x,t}, \gamma^j_{u,t}$ and c^j_t are vectors and matrices of appropriate dimension which emanate from linearisation of the bilinear terms.

B. Resource sharing

We consider a scenario where a group of m separate buildings solving an optimization problem over a time horizon of N_T time-steps. Buildings share a thermal energy storage device, and at every time-step they can decide to either store or retrieve energy from this device. An autoregressive exogenous model similar to [6] is used as follows

$$E_{storage}(t+1) = \alpha E_{storage}(t) - \sum_{i=1}^{m} e_s^i(t). \tag{8}$$

Here $E_{storage}(t) \in \mathbb{R}$ is the thermal energy stored at time-step t, $e_s^i(t) \in \mathbb{R}$ is the amount of energy a building agent $i, i = 1, \ldots, m$ chooses to retrieve from storage at time-step $t, t = 1, \ldots, N_t$, and $\alpha \in [0, 1]$ represents the energy decay over time.

The following constraints pertaining the energy storage exchange are imposed. The amount of energy that a building can exchange with the storage over a given time-step is constrained by some upper and lower bounds, $e_{\mathrm{s,max}} \in \mathbb{R}$ and $e^i_{\mathrm{s,min}}(t) \in \mathbb{R}$, giving for all $t = 1, \ldots, N_T$,

$$e^i_{\mathrm{s}}(t) \leq e^i_{\mathrm{s,max}}(t), \tag{9}$$

$$e^i_{\mathrm{s}}(t) \geq e^i_{\mathrm{s,min}}(t). \tag{10}$$

In addition, it is assumed that the energy storage has some non-negative finite capacity limit $E_{s,max} \in \mathbb{R}$, such that for all $t = 1, \ldots, N_T$,

$$E_{\mathrm{storage}}(t) \geq 0, \tag{11}$$

$$E_{\mathrm{storage}}(t) \leq E_{\mathrm{storage,max}}. \tag{12}$$

It is clear from (8) that the storage constraints in (11) and (12) are functions of the decision variables of multiple agents, and hence are the coupling constraints of the problem. They can be expressed as a sum of separable functions local to each building agent.

C. Objective function

In energy management problems the objective function to be minimized is the cost of energy used to satisfy the constraints. As in [6], we consider the energy request $E^i_{req}(t) \in \mathbb{R}$ of each building at every time-step - i.e., we construct an energy balance between the energy a building requires to satisfy its constraints and the energy stored or retrieved from the storage. If the energy request from the building cannot be met by the storage then this deficit must instead be supplied from some external grid, which is where the cost is introduced.

The energy balance is evaluated for each agent i as

$$E^i_{\mathrm{req}}(t) = E^i_{\mathrm{walls}}(t) + E^i_{\mathrm{internal}}(t) + E^i_{\mathrm{inertia}}(t) + E^i_{\mathrm{act}}(t). \tag{13}$$

Here, $E^i_{\mathrm{walls}}(t) \in \mathbb{R}$ represents the energy transfer between walls and zones at time-step t, $E^i_{\mathrm{inertia}}(t) \in \mathbb{R}$ represents the thermal inertia of the building at time-step t, and $E^i_{\mathrm{act}}(t) \in \mathbb{R}$ represents the thermal energy a building receives from its local actuators - e.g., from radiators and air handling units. Note that these terms are all linear functions of the local decision variables u_i for each building agents. Meanwhile, $E^i_{\mathrm{internal}}(t) \in \mathbb{R}$ represents the energy contribution from the building occupants and internal gains, and is considered equivalent to the "internal gains" parameter of the system model generated by the BCRM toolbox. This means that $E^i_{\mathrm{req}}(t)$ is a linear function of the decision variables for each individual agent.

Agents are incentivized to store energy based on a two-tier pricing plan $\varphi_i(t) : \mathbb{R} \to \mathbb{R}$, in which energy costs more during typical "peak" hours of the day. We are now able to define a linear cost function for each building agent

$$f_i(u_i) = \sum_{t=1}^{N_T} \varphi_i(t) E^i_{\mathrm{req}}(t). \tag{14}$$

D. Overall Optimisation Problem

We are now able to form a suitable optimization problem. We consider a group of building agents, each with separate local heating systems. We form the local constraint set for each agent U_i using the linear constraints from (2), (7), (9) and (10), with local linear objective function given by (14). The coupling constraint equations for each agent i are given by (8), (11) and (12). This gives us an optimisation program in the form of $\mathcal{P}$ which satisfies Assumptions 1 and 2 of the proposed scheme.

IV. Simulation Results

We first solve the problem in a centralized fashion - where all information is shared between building agents - and the solution is extracted to allow for comparison with the distributed solver. Next, we apply Algorithm 1 to the problem stated over a network of 3 building agents with a time horizon of 6 hours, giving rise to 48 coupling constraints. Agents use a step coefficient $c(k) = \frac{\beta}{k+1}$, for some constant $\beta > 0$, satisfying Assumption 3, and treat information from other agents as trustworthy as their own information, such that at in step 6 agents compute the average of theirs and their neighbours dual variables. This satisfies Assumption 4. The minimization procedure in step 7 is achieved using the MATLAB interface YALMIP [15].

Three possible network structures are presented: a fully connected, time-invariant graph and two time-varying graphs, as shown in Figure 1. In Network A, at every iteration, each agent communicates its dual predictions with all other agents in the network.

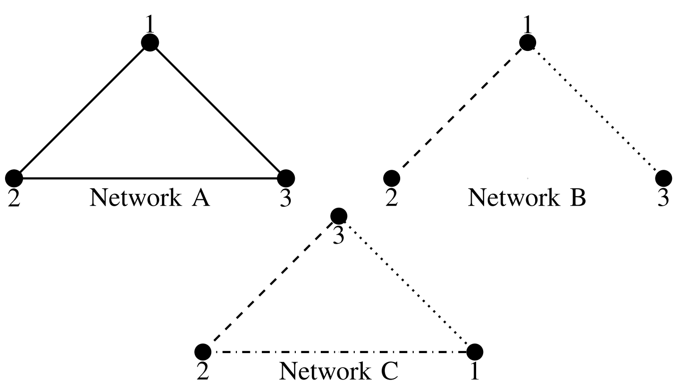

Fig. 1: The Simulated Building Networks: a fully connected graph(left), and two time-varying graphs, represented by non-solid lines (right and bottom).

In Network B, we alternate between two different topologies, where there is some agent which at every odd iteration communicates with one of the remaining agents, and at every even iteration communicates with the other remaining agent. The first agent can be interpreted as some "trusted" or decentralized agent which others in the network are willing to cooperate with, but may have some constraint on how many agents it can communicate with in a given time period. In Network C, agents communicate with each other in a time-varying cycle. Over three iterations, an agent will communicate with one of the other agents in the first, then the other agent in the second, and then will not communicate with either in the third. Note that Network A has a graph period $T = 1$, whereas for both Networks B and C the period $T = 2$.

A. Step Size Coefficient

An investigation into the effect of the parameter β in the step size sequence was completed. Simulations were ran for 30000 iterations over a three building agent problem over a time horizon of 6 hours (48 time-steps), with β ranging in powers of 10 from 0.1 to 10^{-5}. Next, at each iteration, the relative dual error for each agent was calculated as

$$\bar{\lambda}_i(k) = \frac{||\lambda_i(k) - \lambda^*||_2^2}{||\lambda^*||_2^2}, \tag{15}$$

where λ^* is the optimal dual solution of $\mathcal{D}$ as obtained from solving the centralized problem counterpart, and $||.||_2^2$ represents the vector 2-norm. Next, for each value of β, the maximum dual error across the three agents was taken. These results can be seen in Figure 2. It is worth noting the oscillatory nature of convergence, most evident for larger values of β, as well as the slow convergence rate; the latter is expected as the choice of harmonic series for the step-size $c(k)$ is known to lead to slow convergence; however, is necessary to meet the convergence requirements in [11]. It is clear that the fastest rate of convergence is achieved with $\beta = 10^{-3}$, which achieves a maximum error in the dual variables of less than 0.5% after fewer than 7000 iterations.

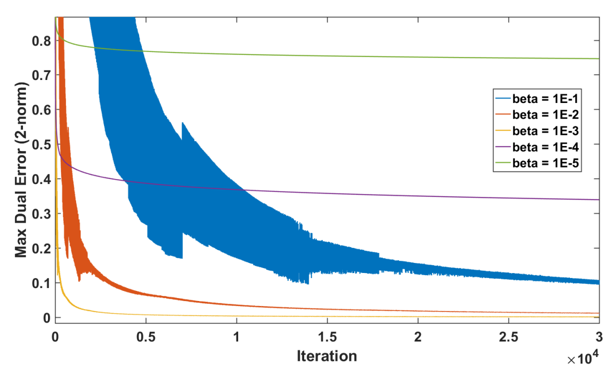

Fig. 2: Evolution on the maximum relative dual error $\frac{||\lambda_i(k)-\lambda^*||_2^2}{||\lambda^*||_2^2}$ across all agents for varying step-sizes β.

B. Algorithm Convergence

Before discussing the convergence of Algorithm 1, it is worth defining several variables calculated and used to quantify performance.

Firstly, recall the relative dual error, as defined in (15). Secondly, we also define the relative objective error at each iteration as

$$\bar{p}_i(k) = \frac{|p_i(k) - p^*|}{|p^*|}, \tag{16}$$

where p^* is the optimal value of $\mathcal{P}$ as determined by the centralized problem counterpart, $p_i(k)$ is the value of the objective function calculated using, $\hat{u}_i(k+1)$, across all agents at iteration k, and $|.|$ is the absolute value function. Thirdly, define the maximum residual at each iteration as

$$\bar{r}(k) = ||[\sum_{i=1}^{m} g_i(u_i(k))]_+||_\infty, \tag{17}$$

where $||.||_\infty$ represents the infinity norm of a vector. This metric is strictly positive if at least one constraint is violated, and serves as a worst-case scenario amount of violation.

In order to facilitate the convergence to the optimal solution of the primal problem, a "refresh" of the auxiliary decision vector is systematically calculated by resetting the step size upon a given iteration index, as described in [11]. This is achieved by redefining Step 9 of Algorithm 1 as

9. $\hat{u}_i(k+1) = \hat{u}_i(k) + \frac{c(k)}{\sum_{r=k'}^{k} c(r)}(u_i(k+1) - \hat{u}_i(k))$,

where k' is the iteration number at which an agent detects practical convergence. This is defined as the first iteration at which the statement $||\lambda_i(k+1) - l_i(k)||_2 \leq \varepsilon$, where $\varepsilon \in \mathbb{R}$ is some small threshold for N_r consecutive iterations. This refresh encourages convergence to the primal solution if the dual variables have sufficiently converged. Due to the difficultly in determining sufficient convergence of the dual variables with agents acting individually, an adaptive procedure was implemented on the threshold ε. This involves initialising ε for all agents, and whenever a refresh is done by an agent, its threshold ε is decreased by some factor.

Figure 3 shows the dual error $\bar{\lambda}_i(k)$ for each of the three building agents over the course of the iterative scheme. The figure demonstrates the convergence of the dual variables to within a numerical tolerance.

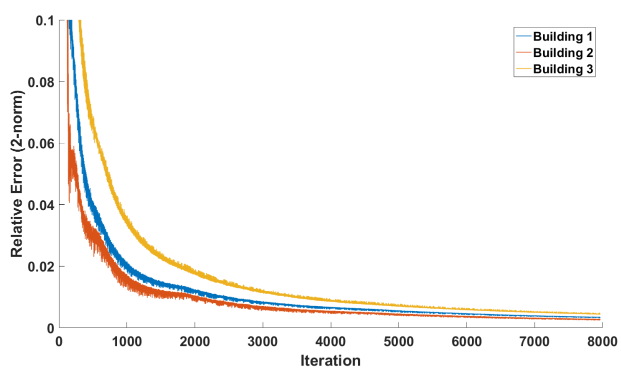

Fig. 3: Evolution on the relative dual error $\frac{||\lambda_i(k) - \lambda^*||_2^2}{||\lambda^*||_2^2}$ of each agent for Network A.

Figure 4 shows the evolution of the relative error in the global objective function $\bar{p}_i(k)$ as calculated using $\hat{u}_i(k+1)$, to the true global optimum from the centralized solution. This appears to converge very quickly. This is aided by the refresh procedure described above, as seen by the sudden changes in $\bar{p}_i(k)$ in each figure.

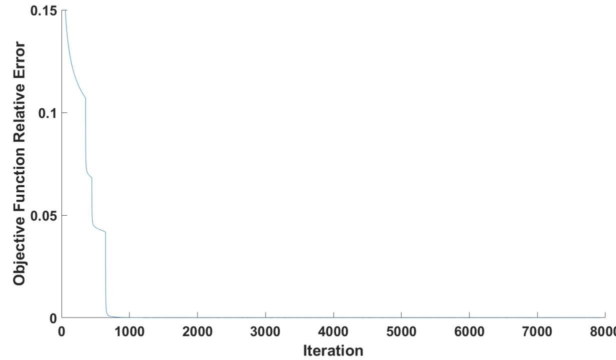

Fig. 4: Evolution of the relative error in the global objective function $\frac{|p_i(k) - p^*|}{|p^*|}$ for Network A.

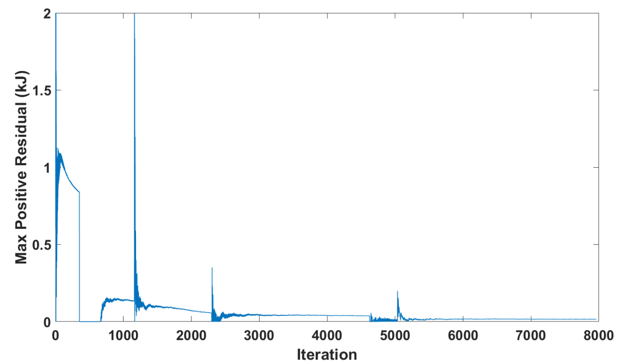

Fig. 5: The maximum residuals(constraint violation) $||[\sum_{i=1}^{m} g_i(u_i)]_+||_\infty$ for Network A.

Satisfaction of the coupling constraints was investigated by means of the residual metric defined in (17). The maximum residual of these was calculated as described, and is shown for Network A in Figure 5. Firstly, note the apparent spikes in the figures, caused by the refreshes of the primal auxiliary variables. This is an advantage of using a decreasing adaptive threshold, as it means these spikes become spaced further apart and the residuals are not dominated by them. Secondly, it can be seen for that as the iterative scheme continues, and more importantly as the dual variables converge, so does the maximum residual converge to zero - within some numerical tolerance.

It is clear then that for Network A application of Algorithm 1 to the energy management optimization program $\mathcal{P}$ problem leads to convergence of both the dual and primal problems, as expected.

Network Structure	Iterations to: 1% dual error	Iterations to: 0.5% dual error
A	3479	6894
B	10588	39618
C	4366	10589

TABLE I: Number of iterations to achieve a given dual error for different network structures.

We also applied Algorithm 1 to the both Network B and Network C. For both network structures that algorithm gave convergence to the optimal solution. Table I compares the rate to different levels of convergence of the dual variables for each network considered. In consideration of the oscillating convergence of the dual variables, we consider the number of iterations required for the relative dual error, as defined in (15), to be below a threshold for at least two consecutive iterations. Immediately it can be noted that Network A demonstrates the fastest rate of convergence, as expected since it has increased communication between agents. We then see that Network B and Network C require notably more iterations to convergence than A. The slower rate of convergence for the latter two networks is expected due to the larger period T of the graph, so information is exchanged less regularly between agents. Furthermore, the significantly higher number of iterations for Network B since it contains two agents which never communicate with each other, meaning that this information must be passed indirectly via the intermediate agent.

V. Concluding remarks

In this paper we construct a suitable energy management problem for a potentially time-varying network of buildings using validated modelling techniques, such that we are able to apply a novel distributed scheme to the problem. We demonstrate the capability of a group of building agents which cooperate over some shared resource to reach a globally optimum solution to the use of this resource without divulging information regarding their local decisions, or in fact the decisions they make with regards to the shared resource. Finally, we consider the influence of time-varying communication between the building agents on the convergence towards the optimal solution.

Future work will consider the sensitivity of convergence rate to the dynamics of the buildings modelled, as well as the robustness of the algorithm to larger networks of agents and their varying communication structures.

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