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**Implementable Fiscal Rules for an Oil-Exporting Small Open
Economy Facing Depletion**

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Implementable Fiscal Rules for an Oil-Exporting Small Open Economy Facing Depletion*

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Abstract

In this paper I compute implementable fiscal rules for a small open economy whose treasury is dependent on oil revenues and whose oil sector is shrinking. I model production in the oil and non oil sector and I analyze the effects of implementing different sustainable fiscal rules in the context of a deteriorating oil sector. I assess the policy's performance in terms of conditional and unconditional welfare. I show that rules that finance government purchases with structural revenue are preferred only if government purchases do not enter the utility function. Otherwise, when government purchases are complements with private consumption, depletion makes rules that finance government purchases with current revenue more attractive. Furthermore, the lower the sustainable level of oil extraction, the harder it is to reject a rule that finances government purchases with current oil revenue.

JEL Classification: F41, H30, H60, Q32, Q33, Q38, Q43

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1 Introduction

In this paper, I consider again a small open economy whose treasury is dependent on oil revenues, but now I model production in the oil and non oil sector and I analyze the effects of implementing different sustainable fiscal rules in the context of a deteriorating oil sector.

The motivation for this analysis is simple. Governments from oil-rich economies often rely heavily on oil revenue to finance their spending. When fiscal policy is designed without restriction on the use of oil revenue, persistent positive oil price shocks will increase oil revenue and governments may get used to potentially unsustainable levels of spending. When oil reserves are running out, sustainability becomes a relevant issue. Table 1 shows estimated proven oil reserves, production and reserve life for the twenty countries with the largest estimated proven oil reserves in the world as of January 2008 (in descending order).¹ Note that China, the United States, Norway and Mexico exhibit a surprisingly low estimated reserve life. The conditions for Mexico are particularly dire, given that PEMEX (the state-owned oil company) lacks the resources to invest in exploration and development of new oil fields and it is banned by law to engage in joint ventures for this purpose with international oil companies.²

Therefore, it is particularly relevant to analyze alternative sustainable fiscal rules in the context of a declining oil sector in countries like Mexico, which are oil-price-takers and have institutional restrictions that limit the possibility of expanding proven oil reserves. I show how rules that finance expenditure without the cyclical component of oil revenue are welfare improving compared to those that finance expenditure with overall current revenue.

I start the analysis by laying down a small open economy model with an oil and a non-oil sector. There is production in both sectors, assuming perfect competition in the non-oil sector. The objective of the oil-extracting firm is to maximize the expected net

¹See below for a detailed description of the different definitions of oil reserves.

²PEMEX cannot engage in (upstream) partnerships with any international company, domestically. Abroad, though, Shell and PEMEX established a (downstream) joint venture in Deer Park refinery, near Houston, in 1993. The incentive for PEMEX to join was to improve its refining capacity.

Table 1: Estimated proven oil reserves and production as of 2008

Country	Proved Oil Reserves (billions of barrels)	Production per day (millions of barrels per day)	Reserve life ¹ (years)
1. Saudi Arabia	266.75	10.66	69
2. Canada	178.59	3.29	149
3. Iran	138.40	4.15	91
4. Iraq	115.00	2.01	157
5. Kuwait	104.00	2.68	107
6. United Arab Emirates	97.80	2.95	91
7. Venezuela	87.04	2.80	85
8. Russia	60.00	9.68	17
9. Libya	41.46	1.81	63
10. Nigeria	36.22	2.44	41
11. Kazakhstan	30.00	1.39	59
12. United States	20.97	5.10	11
13. China	16.00	3.84	11
14. Qatar	15.21	1.14	37
15. Algeria	12.20	2.12	16
16. Brazil	12.18	2.17	15
17. Mexico	11.65	3.47	9
18. Angola	9.04	1.43	17
19. Azerbaijan	7.00	0.65	30
20. Norway	6.87	2.79	7

Source: Energy Information Administration

1. Reserve life in years, calculated as reserves/ annual production

present value of net profits.³ I model the extraction problem, specifying the production technology and the exploration, development and extraction costs. I assume that the oil extracting firm is no longer investing in exploration and development of new reserves, so it is depleting a finite stock that will likely run out in the non-distant future.

The treasury receives oil and non-oil revenue. I decompose the present value of oil revenue into its expected and its unexpected component to show that unanticipated surprises caused by an oil price shock generate a one-time adjustment in the stock price of oil revenue. Furthermore, the larger the stock of available proved oil reserves, the larger the expected net oil wealth and therefore the laxer the lifetime budget constraint of the treasury, all else equal. It turns out that the structure of the budget constraint of an oil-rich treasury is analogous to the one faced by a regular one. The difference can be expressed in terms of net debt. When there is oil revenue, debt can be defined in net terms, as gross debt minus the expected present value of oil revenue.

The non-oil goods producing firm and the households take oil prices, oil extraction and non-oil productivity as exogenous processes. I consider three alternative fiscal rules. Rule 1 finances government purchases with overall current revenue, Rule 2 finances them with overall non-oil revenue plus structural (non-cyclical) oil revenue, and Rule 3 finances them with structural oil and non-oil revenue.⁴

I calibrate the model to Mexico, whose oil reserves have an expected life of 9 years. I use second-order perturbation methods. Once I have obtained the policy functions, I account for oil depletion by imposing a specific path for the innovations to the extraction process and I perform multiple simulations in order to analyze, in this context, the dynamics generated by innovations to the exogenous processes, under each one of the fiscal rules. To make them truly comparable, I expose them to the exact same uncertainty in each simulation. I compute the associated first and second unconditional and conditional moments of the

³One could question whether the incentives of the managers of the state-owned firm align with a profit maximization strategy. For the moment I abstract from this consideration.

⁴Some resource-rich countries like Chile have implemented fiscal rules that target the structural rather than the actual balance. See Fiess (2004) for a detailed description.

endogenous variables under the different fiscal rules in different scenarios of sustainable oil extraction and I rank the policies based on the moments that they generate for welfare, consumption and labor.

I find that Rule 3 is conditionally and unconditionally preferred if government purchases do not yield utility. When they do, however, this rule is the least preferred and the ranking between Rules 1 and 2 depends on the sustainable oil extraction level, on whether the analysis is based on unconditional or conditional welfare and also on the timing of the policymaker's decision, when there is oil depletion.

The reason why Rules 1 or 2 are preferred when government purchases enter the utility function is a balance among discounting, complementarity and the volatility of consumption, labor and government purchases under each rule. Since government purchases are gross complements with private consumption, households want the former to be as high as possible, and more so early on, while there is higher oil revenue. The lower the sustainable oil extraction level, the stronger this argument. But households also care about the associated volatility of consumption, labor and government purchases under each rule. While higher means are welfare increasing, higher volatility is welfare decreasing due to risk aversion.

When government purchases do not yield utility, Rule 3 is preferred for all levels of sustainable oil extraction, as it induces the highest expected c_t , conditionally and unconditionally. Furthermore, the timing of the policymaker's decision is not relevant in this case. Depletion does not affect the policy ranking at date 0, and it does not affect it in future dates either.

In a broader context, there are mixed reviews in the literature on the effectiveness of fiscal rules. Some authors have pointed out the advantages of a well designed fiscal rule as an effective tool to reduce economic volatility. For instance, in a cross-section study including 51 countries, Fatás and Mihov (2003) find that rules-based fiscal policy helps to

stabilize the business cycle. Some other authors have found fiscal rules to be ineffective (in developed countries). Bayoumi and Eichengreen (1994), for instance, show that for the United States fiscal rules decrease the ability of state governments to use fiscal policy to smooth the business cycle and therefore can lead to higher output volatility.

Fiscal rules per se, however, do not guarantee fiscal discipline. According to Fiess (2004), in order for them to be efficient, they need to be transparent, credible and yet flexible. A rigid policy might be credible, but may be incapable of addressing the effects of unexpected shocks on the business cycle, and a rule that focuses solely on reducing output volatility may not be sustainable in the long run and hence may not be credible. Hence, a well-designed rule should facilitate the operation of automatic stabilizers while avoiding a deficit bias.⁵

Since the seminal work of Lucas and Stokey (1983), policy evaluation has been vastly analyzed in terms of optimality. Most of the literature on optimal policy has focused on the analysis of optimal monetary policy, in the context of stylized theoretical frameworks with nominal rigidities (For instance Chari and Kehoe (1999), Rotemberg and Woodford (1998a), Rotemberg and Woodford (1998b), Clarida, Galí, and Gertler (1999), Galí and Monacelli (2005), Schmitt-Grohé and Uribe (2004a) among many others).

Looking for optimality in the class of simple and implementable policy rules, Schmitt-Grohé and Uribe (2007) analyze—in the context of a closed economy—the stabilizing properties of monetary and fiscal rules that are designed as a function of observable aggregates. Their measure of stabilization is given by the level of private agent’s welfare. They estimate the coefficients of the feedback rule to maximize welfare and they find that the optimized monetary and fiscal rules attain virtually the same level of welfare as the Ramsey optimal

⁵Fiscal policy can be separated into two components, automatic stabilizers and discretionary fiscal policy. As its name suggests, the first one responds automatically to the business cycle, without any government action. Income tax revenue, for example, responds positively to the cycle. The second one, on the other hand, is active policy that is meant to stimulate the economy during recessions. Discretionary policy, however, is the one that can prevent macroeconomic stability. Many countries have implemented fiscal rules, in an attempt to allow automatic stabilizers to work freely while protecting the surplus generated during good times from the political process in normal discretionary budget decisions.

policy.

Deviating from the Ramsey approach, Schmitt-Grohé and Uribe (2001) compare—in the context of a small open economy—monetary policies by computing the welfare cost of implementing a particular policy as the fraction of non-stochastic steady-state consumption that households would be willing to forgo in order to be indifferent between the corresponding constant sequences of consumption and hours and the equilibrium stochastic processes for these two variables associated with the monetary policy under consideration. The welfare costs are comparable across policies because they are specified such that they yield the same non-stochastic steady state.

In contrast with the existing literature, in this paper I compare implementable fiscal policies in a small open economy and I evaluate them by the level of conditional and unconditional welfare that each one of them yields, without setting the Ramsey welfare as a benchmark and without comparing them to the welfare attained in the non-stochastic steady state. Here, a fiscal rule is considered preferred over the other ones if it attains a higher level of conditional and unconditional welfare (to be described in detail below).

The paper is organized as follows. Section 2 describes the small open economy model. Section 3 describes the solution method accounting for oil depletion. Section 4 shows the simulation results and ranks the different fiscal rules in terms of conditional and unconditional welfare. Finally, section 5 concludes.

2 A small open economy model

Consider a small open economy with two sectors, oil (O) and non-oil (NO). Without loss of generality I assume that all the oil (Y_t^O) is sold abroad, at the world oil dollar price (p_t^O), which is exogenous. Households work only in the non-oil sector, producing tradable goods (Y_t^{NO}). Assume PPP holds for non-oil goods. Normalizing the U.S. price level (P_t^*) to 1

yields

$$P_t^{NO} = S_t,$$

where S_t denotes the nominal exchange rate. Hence, GDP in units of dollars is given by

$$GDP_t = Y_t^{NO} + p_t^O Y_t^O. \quad (1)$$

Oil prices are exogenous, and their evolution obeys the following process

$$\ln p_t^O = (1 - \rho_{p^O}) \ln p^O + \rho_{p^O} \ln p_{t-1}^O + \epsilon_t^{p^O}; \quad \epsilon_t^{p^O} \sim i.i.d(0, \sigma_{p^O}^2). \quad (2)$$

I assume that the world interest rate (r) is constant and taken as given. In order to induce stationarity, I assume that the country faces a risk premium, which is a function of the net foreign asset holdings at each date (f_t). Following Schmitt-Grohé and Uribe (2003), the domestic interest rate r_t is given by

$$r_t = r + \zeta (\exp\{f - f_t\} - 1), \quad (3)$$

where f denotes the steady state level of net foreign asset holdings. This function, therefore, implies that there is no country premium in steady state.

2.1 Oil extracting firm

2.1.1 Industry-specific background

It seems useful to provide some industry-specific background before describing the problem of the oil extracting firm. Oil in the ground is not a “reserve” unless it is claimed to be economically recoverable. The three most common categories of reserves used in the oil industry are proven (or proved), probable and possible. They indicate the probability that a reserve exists based on the geologic and engineering data for a particular location.

According to the Society of Petroleum Engineers, proven reserves are defined as “reasonably certain” (90% certainty) to be producible using current technology at current prices, with current commercial terms and government consent. They are known in the industry as 1P. They can be divided into developed and undeveloped. Proven developed reserves can be produced with existing wells and perforations, or from additional reservoirs where minimal additional investment or operating costs are required. Proven undeveloped reserves, on the other hand, require additional capital investment to bring the oil and gas to the surface, such as drilling new wells, installing gas compression, etc.. Probable reserves are defined as “reasonably probable” (50% certainty) of being produced using current or likely technology at current prices, commercial terms and government consent. These are known in the industry as 2P. Finally, possible reserves are defined as “having a chance of being developed under favorable circumstances”, implying ideally having a 10% certainty of being produced in the foreseeable future. These are known in the industry as 3P.⁶

In what follows, I borrow some but certainly not all of the technology features described in Hartley and Medlock (2006).

2.1.2 Technology

In this model the country is endowed with a stock \bar{R}^O of oil that may or may never be fully discovered. Investment in exploration and development is required in order to have extractable proven reserves. Denote by D_t the stock of oil discoveries, which I assume is equal to the cumulative amount of resources used up to date t in investment (I_t^O). This implies assuming an exploration success rate of 100%, which is a useful simplification, but of course in reality exploration efforts may not always yield fruitful results.

Thus, every time there is additional investment in new proven reserves, the discovered stock will increase

$$D_{t+1} = D_t + I_t^O. \tag{4}$$

⁶For more information see: <http://www.spe.org>.

Extractable proven reserves are denoted by R_t^O . At any date t , they are equal to the discovered stock minus the stock that has already been extracted (X_{t-1})

$$R_t^O = D_t - X_{t-1}. \quad (5)$$

Thus, X_t is defined as cumulative exploitation

$$X_t \equiv \sum_{s=0}^t Y_s^O$$

which evolves according to

$$X_t = X_{t-1} + Y_t^O, \quad (6)$$

where Y_t^O is the flow of oil extracted at date t . Oil extraction at date t is proportional to extractable reserves

$$Y_t^O = \kappa_t R_t^O, \quad (7)$$

where κ_t is the extraction rate. I assume that κ_t follows a standard stationary process of the form

$$\ln \kappa_t = (1 - \rho_\kappa) \ln \kappa + \rho_\kappa \ln \kappa_{t-1} + \epsilon_t^\kappa; \quad \rho_\kappa \in (0, 1); \quad \epsilon_t^\kappa \sim i.i.d. (0, \sigma_\kappa^2) \quad (8)$$

where $0 < \kappa < 1$ is the steady state rate, ρ_κ is the serial correlation coefficient, ϵ_t^κ is an i.i.d shock with zero mean and standard deviation σ_κ . Notice that alternatively I could allow for some correlation between the extraction rate and other factors; such that $\epsilon_t^\kappa = \rho_\epsilon \epsilon_t^O + u_t$; $u_t \sim i.i.d. (0, \sigma_u^2)$, so ϵ_t^κ is a shock that is correlated with the innovation to the oil price (with a correlation coefficient ρ_ϵ), and u_t is an orthogonal error.

Notice that (7) implies that maintaining a given production level becomes increasingly difficult as the extractable proved reserves decrease (or equivalently as X_t increases). An empirical fact consistent with this specification is that more water and/or gas injections

are usually required to keep older reservoirs producing.⁷ It is important to point out that this rising difficulty may likely make the firm decide to stop production before the available reserves are completely exhausted.

It is easy to see that ultimately Y_t^O is only a function of the extraction rate and the flow of investment in exploration and development. Substituting (4), (5) and (6) into (7) yields

$$Y_t^O = \kappa_t I_{t-1}^O + \frac{\kappa_t}{\kappa_{t-1}} (1 - \kappa_{t-1}) Y_{t-1}^O. \quad (9)$$

2.1.3 Exploration and development costs

As explained by Hartley and Medlock (2006), development occurs first in areas with natural oil seeps, or with geological structures that are less tightly folded, are closer to the surface, or are on land rather than offshore. Therefore, I make two assumptions regarding the costs faced by the firm. First, investment faces convex adjustment costs, so I introduce a cost function $F(I_t^O)$, such that $F(0) = 0$; $F' > 0$; $F'' > 0$. Second, as a byproduct, the stock of investment D also becomes more expensive as the firm moves on to less attractive prospects, so I introduce $H(D_t)$, such that $H' > 0$; $H'' > 0$; $H(D) \rightarrow \infty$ as $D \rightarrow \bar{R}^O$. This last property implies that the costs become extremely high when overall demonstrated reserves approach the limit of existing reserves. These increasing costs will also play an important role in stopping production before actual existing reserves are exhausted.

Thus, I assume that the overall reserves replenishment cost is given by

$$C(I_t^O, D_t) = F(I_t^O) H(D_t) \quad (10)$$

where $C(0, D) = 0$.

⁷(Solow and Wan 1976) have a similar assumption. They investigate resource extraction in a general equilibrium growth model where output is produced using reproducible capital, an exhaustible resource and exogenously supplied labor. They assume that the number of units of the composite output good needed to extract one unit of the exhaustible resource increases with cumulative extraction of the resource.

2.1.4 Objective of the oil extracting firm

The firm's instantaneous net profit function is given by

$$\Pi_t^O = (1 - \tau_t^O) [(1 - \xi) p_t^O Y_t^O - C(I_t^O, D_t)], \quad (11)$$

I assume that the objective of the firm is to maximize the expected net present value of its lifetime profits

$$\max_{\{I_t^O, D_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{M_t}{M_0} \Pi_t^O \quad (12)$$

where $M_t \equiv \prod_{j=0}^t (1 + r_j)^{-1}$, subject to (4), (9), (10). Note that the firm will optimally choose how much to extract, taking into account that sometimes there may be some gain in waiting to extract in the future, when and if prices are expected to rise. It also takes into consideration that the faster it extracts, the harder it becomes to extract and the faster investment costs grow.

The following is the associated Lagrangian

$$\begin{aligned} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \frac{M_t}{M_0} \{ & (1 - \tau_t^O) [(1 - \xi) p_t^O Y_t^O - F(I_t^O) H(D_t)] + \mu_t^1 (D_t + I_t^O - D_{t+1}) \\ & + \mu_t^2 \left(Y_t^O - \kappa_t I_{t-1}^O - \frac{\kappa_t}{\kappa_{t-1}} (1 - \kappa_{t-1}) Y_{t-1}^O \right) \} \end{aligned}$$

and these are the corresponding first order conditions:

$$[I_t^O] : \mu_t^1 = (1 - \tau_t^O) F'(I_t^O) H(D_t) + E_t \left(\frac{\mu_{t+1}^2 \kappa_{t+1}}{1 + r_{t+1}} \right) \quad (13)$$

$$[D_{t+1}] : \mu_t^1 = E_t \left(\frac{\mu_{t+1}^1 - (1 - \tau_{t+1}^O) F(I_{t+1}^O) H'(D_{t+1})}{1 + r_{t+1}} \right) \quad (14)$$

$$[Y_t^O] : (1 - \tau_t^O) (1 - \xi) p_t^O + \mu_t^2 = \frac{1 - \kappa_t}{\kappa_t} E_t \left(\frac{\mu_{t+1}^2 \kappa_{t+1}}{1 + r_{t+1}} \right) \quad (15)$$

The intuition is standard. The marginal cost of oil investment (RHS in (13)) is equal to its marginal benefit (μ_t^1). Consequently, the marginal cost of an additional unit of discovery

stock (right hand side in (14)) is equal to the marginal benefit of investing (μ_t^1).

2.1.5 Deteriorating oil sector

If the firm does not invest in new exploration and development, Y_t^O becomes an exogenous process, and a declining one unless κ_t is hit by a positive shock. If there is no investment, there is nothing that the firm can choose at date t that can affect the level of Y_t^O , as it becomes only a function of κ_t , κ_{t-1} —which are exogenous—and of past extraction, Y_{t-1}^O . To see this, rewrite (9) setting $I_t^O = 0 \forall t$:

$$Y_t^O = \frac{\kappa_t}{\kappa_{t-1}} (1 - \kappa_{t-1}) Y_{t-1}^O. \quad (16)$$

Note that no investment also implies $D_{t+1} = D_t = D$, $C(0, D) = 0$, and the disappearance of (13) and (14).

To analyze the relationship between κ_t and Y_t^O , suppose momentarily that κ_t is constant (i.e. $\epsilon_t^\kappa = 0 \forall t$). Thus, (16) collapses into

$$Y_t^O = (1 - \kappa) Y_{t-1}^O \quad (17)$$

or equivalently,

$$Y_t^O = \kappa (1 - \kappa)^t D. \quad (18)$$

For illustration purposes and without loss of generality, suppose $D = 1$. Figure 1 shows how Y_t^O evolves through time (t) for different values of κ . Notice that there is a nonlinear relationship between the size of κ and the initial level of Y_t^O , and a constant depletion rate given by $-\kappa$.

Note that the deterministic process 18 is obviously non-stationary. This is problematic for the solution of the model. Additionally, if one uses perturbation methods, as long as the shocks considered are normally distributed, the policy function coefficients are computed

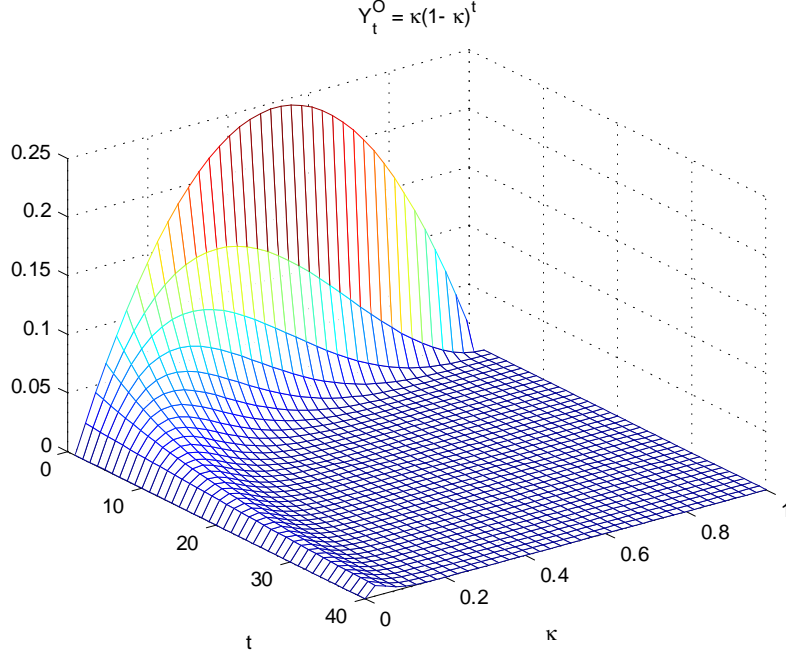


Figure 1: Oil extraction as a function of the extraction rate

perturbing the steady state in all directions, so it considers positive and negative deviations from steady state. This is also problematic for the computation of the policy function coefficients as $Y_t^O < 0$ is inconsistent with the model.⁸

Therefore, I assume instead that there is a very low sustainable level of oil extraction which is near depletion but not zero. Additionally, I assume that (16) can be approximated with the following reduced form representation

$$\ln Y_t^O = (1 - \rho_{Y^O}) \ln Y^O + \rho_{Y^O} \ln Y_{t-1}^O + \epsilon_t^{Y^O}; \quad \rho_{Y^O} \in (0, 1); \quad \epsilon_t^{Y^O} \sim i.i.d(0, \sigma_{Y^O}^2). \quad (19)$$

Note that (19) implies that Y_t^O has a steady state value (Y^O) and it can only approach zero from above. I calibrate the parameter ρ_{Y^O} in a way consistent with the persistence parameter in the extraction rate process, ρ_κ . Note that the shocks to this autoregressive

⁸Note that even if the firm was investing in exploration and development, it is easy to see from (4) that the steady state level for I^O is zero. Perturbing around $I^O = 0$ would imply allowing for $I_t^O < 0$, which is also inconsistent with the model.

process $(\epsilon_t^{Y^O})$ can be mapped into the innovations to the extraction rate ϵ_t^κ .

2.2 Non-oil sector

Non-oil goods are produced competitively according to the following technology

$$Y_t^{NO} = A_t^{NO} L_t^\alpha, \quad (20)$$

where L_t denotes labor and A_t^{NO} is an exogenous stationary productivity process that evolves according to

$$\ln A_t^{NO} = (1 - \rho_A) \ln A^{NO} + \rho_A \ln A_{t-1}^{NO} + \epsilon_t^A; \quad \epsilon_t^A \sim i.i.d(0, \sigma_A^2). \quad (21)$$

Profits are defined by

$$\Pi_t^{NO} = Y_t^{NO} - w_t L_t, \quad (22)$$

where w_t denotes wages. Non-oil firms maximize profits by choosing labor. The associated first order condition is

$$[L_t] : \alpha \frac{Y_t^{NO}}{L_t} = w_t \quad (23)$$

which implies (as usual in a competitive framework) that the wage is equal to the marginal product of labor.

2.3 Households

Households enjoy leisure, so they choose sequences of labor (L_t) , non-oil consumption (c_t) , and real assets (a_{t+1}) in order to maximize their lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{[\tilde{c}_t - \psi L_t^\varpi]^{1-\sigma} - 1}{1-\sigma}; \quad \psi > 0; \varpi > 1; \sigma > 0; 0 < \beta < 1, \quad (24)$$

where \tilde{c}_t is effective consumption, defined as

$$\tilde{c}_t = \left[\phi c_t^{\frac{\varkappa-1}{\varkappa}} + (1-\phi) (g_t^{NO})^{\frac{\varkappa-1}{\varkappa}} \right]^{\frac{\varkappa}{\varkappa-1}}; \quad 0 \leq \phi \leq 1; \quad 0 \leq \varkappa, \quad (25)$$

where ϕ is the share of private consumption in effective consumption and \varkappa is the elasticity of substitution between private consumption and government purchases.^{9,10}

Households' disposable income consists of after-tax labor income, after-tax oil profits, returns from real assets acquired in the previous period and net treasury transfers. Income is used to consume and save in real and nominal assets. Thus, the flow budget constraint of households is given by

$$(1 - \tau_t^{NO}) w_t L_t + \Pi_t^O + r_t a_t + v_t \geq c_t + (a_{t+1} - a_t) \quad . \quad (26)$$

Households are subject to a no-Ponzi game constraint of the form

$$\lim_{j \rightarrow \infty} E_t M_{t+j} a_{t+j} \leq 0. \quad (27)$$

The associated first order conditions are:

$$[c_t] : \quad \phi \left(\frac{\tilde{c}_t}{c_t} \right)^{\frac{1}{\varkappa}} [\tilde{c}_t - \psi L_t^{\varpi}]^{-\sigma} = \lambda_t \quad (28)$$

$$[L_t] : \quad [\tilde{c}_t - \psi L_t^{\varpi}]^{-\sigma} \varpi \psi L_t^{\varpi-1} = \lambda_t (1 - \tau_t^{NO}) w_t \quad (29)$$

$$[a_{t+1}] : \quad \beta (1 + r_t) E_t \lambda_{t+1} = \lambda_t \quad (30)$$

$$[\lambda_t] : \quad (1 - \tau_t^{NO}) w_t L_t + \Pi_t^O + (1 + r_t) a_t + v_t = c_t + a_{t+1} \quad (31)$$

⁹When there is zero elasticity of substitution, private consumption and government purchases become perfect complements. In the other extreme, when the elasticity approaches positive infinity, private consumption and government purchases become perfect substitutes. The Cobb-Douglas case is obtained with unit elasticity.

¹⁰Bouakez and Rebei (2007) consider a similar framework, with complementarity between private consumption and government purchases in the context of a one-sector closed economy. Their work assumes separable preferences between effective consumption and labor, so the complementarity between government purchases and private consumption also implies Edgeworth complementarity (i.e. $U_{cg} > 0$) which is too strong a condition when preferences are not separable.

2.4 Treasury

The treasury's period by period budget constraint is given by

$$\tau_t^{NO} w_t L_t + T_t^O + (b_{t+1} - b_t) = g_t^{NO} + v_t + r_t b_t \quad . \quad (32)$$

The left hand side of (32) shows the sources of income, namely labor income tax, oil revenue

$$T_t^O \equiv \tau_t^O [(1 - \xi) p_t^O Y_t^O - C(I_t^O, D_t)] , \quad (33)$$

and newly issued dollar-denominated real non-contingent debt. The treasury uses its resources to purchase non-oil goods (g_t^{NO}) to give net lump-sum transfers to households (v_t) and to service outstanding debt.

Given that in an open economy net foreign assets can be different from zero, lump-sum transfers adjust in order to satisfy the lifetime budget constraint,

$$b_0 - \sum_{t=0}^{\infty} M_t T_t^O = \sum_{t=0}^{\infty} M_t (\tau_t^{NO} w_t L_t - g_t^{NO} - v_t) \quad , \quad (34)$$

and a no-Ponzi game constraint of the form

$$\lim_{j \rightarrow \infty} E_t M_{t+j} b_{t+j} \leq 0. \quad (35)$$

Oil income introduces the following features to the treasury's lifetime budget constraint:

1. The expected stock price of oil revenue effectively improves the net position of the treasury.

Defining expected net debt (b_t^*) as the real debt net of the expected stock price of oil revenue at date t ,

$$b_t^* \equiv b_t - E_t \sum_{j=0}^{\infty} \frac{M_{t+j}}{M_t} T_{t+j}^O. \quad (36)$$

The second term on the right hand side is the expected stock price of oil revenue. Note that the higher the expected stock price, the lower the treasury's net debt.

2. Given the definitions of M_t and T_t^O , the stock price of oil revenue can be higher due to either higher oil tax rates, oil prices and oil flows, or to lower world interest rates.

To see this, denote by $E_t P_t^{T^O}$ the expected stock price of oil revenue

$$P_t^{T^O} \equiv E_t \sum_{j=0}^{\infty} \frac{M_{t+j}}{M_t} T_{t+j}^O.$$

Thus,

$$E_t P_t^{T^O} = E_t \sum_{j=0}^{\infty} \frac{M_{t+j}}{M_t} I(s) \tau_{t+j}^O [(1 - \xi) p_{t+j}^O Y_{t+j}^O - C(I_{t+j}^O, D_{t+j})], \text{ and}$$

$$\frac{\partial E_t P_t^{T^O}}{\partial \tau_{t+j}^O} > 0; \frac{\partial E_t P_t^{T^O}}{\partial p_{t+j}^O} > 0; \frac{\partial E_t P_t^{T^O}}{\partial Y_{t+j}^O} > 0; \frac{\partial E_t P_t^{T^O}}{\partial r_{t+j}} < 0 \quad .$$

3. Given an initial stock of government net debt, b_0^* , the treasury needs to adjust at least one of these paths $\{\tau_t^{NO}, v_t\}_{t=0}^{\infty}$, in order to account for unexpected changes in the stock price of oil revenue, $\epsilon_t^{T^O}$, and still attain a sustainable fiscal policy.

To see this, rewrite (34) in terms of the expected net debt

$$b_0^* = \sum_{t=0}^{\infty} M_t \left(\tau_t^{NO} w_t L_t - g_t^{NO} - v_t + \epsilon_t^{T^O} \right) \quad (37)$$

where $\epsilon_t^{T^O}$ denotes innovations to the stock price of oil revenue

$$\epsilon_t^{T^O} \equiv (1 + r_t) E_t \sum_{j=0}^{\infty} \frac{M_{t+j}}{M_t} T_{t+j}^O - E_{t+1} \sum_{j=0}^{\infty} \frac{M_{t+1+j}}{M_{t+1}} T_{t+1+j}^O$$

$$\epsilon_t^{T^O} \equiv (1 + r_t) E_t P_t^{T^O} - E_{t+1} P_{t+1}^{T^O}.$$

Note that the first term on the right hand side is the expected stock price of oil

revenue tomorrow (expected to grow at a net rate of r_t), and the second term is the realized price of tomorrow.

In order to maintain a sustainable fiscal policy, net debt must satisfy (37), or equivalently

$$b_{t+1}^* = (1 + r_t) b_t^* + g_t^{NO} + v_t - \tau_t^{NO} w_t L_t - \epsilon_t^{TO}. \quad (38)$$

Eq. (38) has the same structure as the flow budget constraint of a treasury without oil revenue, except for the fact that this treasury has to adjust the flow if and only if there are any innovations to the stock price of oil revenue.

2.4.1 Introducing fiscal rules

If government purchases are assumed to be exogenous, then for a given pair of tax rate sequences $\{\tau_t^O, \tau_t^{NO}\}_{t=0}^\infty$, the lifetime sequence of net transfers $\{v_t\}_{t=0}^\infty$ has to adjust such that (34) is satisfied.

Alternatively, g_t^{NO} can be endogenized by implementing a fiscal rule. For instance, I can pick tax rate sequences $\{\tau_t^O, \tau_t^{NO}\}_{t=0}^\infty$ and a fiscal rule of choice, and then $\{g_t^{NO}\}_{t=0}^\infty$ and $\{v_t\}_{t=0}^\infty$ will be pinned down by satisfying the rule and (34). Note that oil and non-oil revenue are still exogenous, given that Y^O , P^O and A^{NO} are subject to stochastic shocks. This approach is somewhat unorthodox, because the literature usually endogenizes revenue and assumes that g_t is an exogenous process. In this case I am interested in using g_t^{NO} as the instrument because it has key policy implications as it enters the households' utility function.

In the remainder of the paper, I consider the general equilibrium implications in terms of output and consumption volatility and welfare of implementing three different Ricardian fiscal rules:

Rule 1: Balanced budget rule Government purchases are pinned down by the following rule

$$g_t^{NO} = \tau^{NO} w_t L_t + T_t^O - v - r_t b_t \quad (39)$$

$$v_t = v \quad (40)$$

where v denotes the steady state value of net transfers.

Rule 2: Non-oil revenue and structural oil revenue rule Government purchases are financed by current non-oil revenue and structural oil revenue net of debt services and transfers. The structural oil revenue is equal to the one obtained in steady state, given that there is no growth path in the economy.

$$g_t^{NO} = \tau^{NO} w_t L_t + T^O - v - r_t b_t \quad (41)$$

$$v_t = (T_t^O - T^O) + v \quad (42)$$

In this case households receive additional rebates from the difference between the actual and the structural oil revenue. Note that this rule yields the same steady state value for government purchases as the one in Rule 1. Furthermore, it also yields a constant level of debt.

Rule 3: Structural balance rule In this case, government purchases can only be financed with the structural balance

$$g_t^{NO} = \tau^{NO} wL + T^O - v - rb \quad (43)$$

$$v_t = (T_t^O - T^O) + \tau^{NO} (w_t L_t - wL) + v - (r_t b_t - rb). \quad (44)$$

In order to guarantee the same steady state values and a constant level of debt, net transfers adjust so that households receive rebates from the difference between the actual and the structural revenue, net of debt services.

2.5 Goods market clearing condition

The goods market clearing condition is given by

$$Y_t^{NO} + r_t f_t + p_t^O Y_t^O = c_t + g_t^{NO} + (f_{t+1} - f_t), \quad (45)$$

where f_t are net foreign assets, defined as

$$f_t \equiv a_t - b_t. \quad (46)$$

Equation (45) implies that the country's income—constituted by tradable output, the return on last period's net foreign assets and oil income—can be used for private and public consumption, for public oil investment and to adjust net foreign asset holdings.

2.6 Equilibrium

In the context of a deteriorating oil sector, a competitive equilibrium consists of sequences of allocations $\{c_t, L_t, a_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ and prices $\{r_t, w_t\}_{t=0}^{\infty}$ such that, taking as given b_0, a_0, D and the exogenous processes $\{p_t^O, Y_t^O, A_t^{NO}\}_{t=0}^{\infty}$:

- $\{c_t, L_t, a_{t+1}\}_{t=0}^{\infty}$ solve the households' problem
- $\{L_t\}_{t=0}^{\infty}$ solves the non-oil goods producing firm's problem
- Given $\{\tau^O, \tau^{NO}\}_{t=0}^{\infty}$, the treasury chooses a sequence of government purchases $\{g_t^{NO}\}_{t=0}^{\infty}$ and a sequence of lump-sum transfers $\{v_t\}_{t=0}^{\infty}$ so that the fiscal rule and the treasury's lifetime budget constraint are satisfied

- Eq. (45) is satisfied
- $\{w_t\}_{t=0}^{\infty}$ is such that the labor market clears
- $\{r_t\}_{t=0}^{\infty}$ is such that (3) is satisfied

2.7 Deterministic steady state and calibration: deteriorating oil sector

The following are the calibration choices and the deterministic steady state of the model with a deteriorating oil sector. Note that all the fiscal rules considered yield the same deterministic steady state. I calibrate the model to match some features of the Mexican data.

There are five parameters of preferences to set: β , ϖ , σ , ϕ and \varkappa . Since the frequency of the model is quarterly, I follow Schmitt-Grohé and Uribe (2001) and calibrate β so that the net quarterly interest rate in the steady state is $r = 0.015$. Since $\beta = 1/(1+r)$ I set $\beta = 0.98$. Following Mendoza and Uribe (2000), I assume that the coefficient of relative risk aversion is $\sigma = 5$. I set $\psi = 6.359$ and $\varpi = 5$ so that in the deterministic steady state of the competitive equilibrium households work 20 percent of their time ($L = 0.2$). I set $\alpha = 0.4$, $\phi = 0.81$ and $\varkappa = 0.17$.

The units of the nominal exchange rate, the non-oil production and the oil endowment are arbitrary. Hence, without loss of generality, I set $S = Y^{NO} = Y^O = 1$. From (20) and since $L = 0.2$, then $A^{NO} = 1.9037$. Similarly, from (23), $w = 2$.

I calibrate the exogenous processes as follows. For (2), I regress Mexican oil export prices on a constant and oil prices lagged one period. That regression implies $\rho_{p^O} = 0.9303$ and $\sigma_{p^O} = 0.0125$. For (19), I compute ρ_{Y^O} such that at half life, 50% of the oil reserves are still left. As seen in 1, Mexican proven oil reserves are expected to last 9 years. Thus, half-life is 18 quarters, so $\rho_{Y^O} = 0.9622$ satisfies $(\rho_{Y^O})^{18} = 0.5$. In order to pin down σ_{Y^O} , I regress Mexican oil production (actual barrels) on a constant and production lagged one period and I obtain $\sigma_{Y^O} = 0.062$. Note that I could use the estimated coefficient of this

Table 2: Calibrated parameters

Symbol	Description	Value
β	Discount factor	0.98
r	World interest rate	0.015
σ	Coefficient of relative risk aversion	5
\varkappa	Elasticity of substitution between c and g^{NO}	0.17
ϕ	Share of private consumption in effective consumption	0.81
ϖ	Preference parameter	5
ψ	Preference parameter	6.359
L	Steady state share of time spent working	0.2
A^{NO}	Steady state level of technology parameter in Y^{NO}	1.9037
w	Wage	2
ρ^A	Autocorrelation coefficient of technology process	0.95
σ_A	Standard deviation of technology process	0.002
Y^{NO}	Steady state non-oil production	1
Y^O	Steady state oil extraction	1
ρ_{Y^O}	Autocorrelation coefficient of oil flow process	0.6
σ_{Y^O}	Standard deviation of oil flow process	0.062
ρ_{p^O}	Autocorrelation coefficient of oil flow process	0.9303
σ_{p^O}	Standard deviation of oil flow process	0.0125

regression ($\rho_{Y^O} = 0.8363$) to pin down ρ_{Y^O} , but I choose not to do so because it implies that at half-life there is only 4% of oil reserves left. In both cases, I use quarterly data starting on 1980:1 and ending in 2006:4. Finally, for (21), I follow Cooley and Quadrini (2001) and set $\rho_A = 0.95$ and $\sigma_A = 0.002$. Table 2 summarizes the calibrated parameters.

2.7.1 Oil Share-dependent steady state

The remaining steady state values depend on the value assigned to the share of oil production in GDP, $s^O \equiv p^O Y^O / GDP$. I compute the deterministic steady state consistent with three different levels of s^O : 1) the average seen in the data, $\bar{s}^O = 0.071$, 2) a share consistent with a lower sustainable extraction level, $s^O = 0.03$, 3) a share consistent with an even lower sustainable level, $s^O = 0.01$. That way, I can analyze a deteriorating oil sector where oil extraction at date zero is consistent with the level implied by the data (\bar{s}^O)

Table 3: Implied steady state values

Symbol	Description	$s^O = 0.071$	$s^O = 0.03$	$s^O = 0.01$
p^O	Oil price	0.0765	0.031	0.0101
c	Consumption	1.345	1.293	1.270
\tilde{c}	Effective consumption	0.376	0.360	0.353
a	Household's assets	62.522	61.951	61.605
b	Government debt	-4.736	-4.536	-4.444

and slowly converges to the actual sustainable steady state consistent with $s^O = 0.03$ or $s^O = 0.01$.

The remaining variables are obtained as a function of s^O as follows. Given $Y^{NO} = Y^O = 1$, then $p^O = (s^O/1 - s^O)$. From (1) and given that $Y^{NO} = 1$ and $Y^O = 1$, GDP in the steady state is equal to $Y^{NO} + p^O Y^O$.

Government purchases of goods and services represent, on average, 13 percent of GDP in Mexico. Hence, $g^{NO} = 0.13 \times (Y^{NO} + p^O Y^O)$. Government non-oil revenue represents, on average, 16 percent of GDP in Mexico. Thus, in steady state, $\tau^{NO} wL = \tau^{NO} = 0.16$. Similarly, transfers represent, on average, 9 percent of GDP, so $v = 0.09 \times (Y^{NO} + p^O Y^O)$. Oil-based revenue represents, on average, 9 percent of GDP. Thus, I set τ^O so that $\tau^O = 0.09 \times (Y^{NO} + p^O Y^O) / p^O Y^O$.

Finally, the only remaining unknowns are c , \tilde{c} , a , b and λ , which I find by solving the system of equations formed by (28) (29), (25), (31) and (32) in steady state. The resulting steady state values are summarized in Table 3.

3 Solution Method: Perturbation and simulation under depletion

Using perturbation methods, I obtain a set of policy functions expressing the current value of the endogenous variables of the model in terms of the previous state and the shocks observed at the current period. In the derivations below I follow the notation of Schmitt-

Grohé and Uribe (2004b), which implies a reassignment of the symbols y_t and x_t to represent the whole vector of control variables and state variables, respectively. The set of equations that solve the model can be summarized as:

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \quad (47)$$

where x_t has dimensions $n_x \times 1$ and denotes the vector with the predetermined (or state) variables. The state variables can in turn be partitioned into the endogenous state variables, in this case households assets and government debt (a_t and b_t), and the exogenous state variables z_t , which include the oil extraction (Y_t^O), the oil price (p_t^O), and the non-oil productivity (A_t^{NO}). The vector y_{t+1} , with dimension $n_y \times 1$, contains the control variables which includes private consumption (c_t), effective consumption (\tilde{c}_t), labor (L_t), government purchases (g_t^{NO}), government transfers (v_t), wages (w_t), domestic interest rate (r_t) and non-oil production (Y_t^{NO}).

Assume that the exogenous state variables follow the law of motion:

$$z_{t+1} = \Gamma z_t + \sigma \Sigma \epsilon_{t+1} \quad (48)$$

where σ is the so-called perturbation parameter which equals 0 when determining the deterministic steady state of the model and which equals 1 in the stochastic version of the model. The vector ϵ_{t+1} is assumed to be independently and identically distributed, with mean zero and variance/covariance matrix I . The matrix Γ is assumed to satisfy the usual stationarity conditions.

The solution to the model is of the form:

$$y_t = \hat{g}(x_t)$$

and

$$x_{t+1} = \hat{h}(x_t) + \eta\sigma\epsilon_{t+1}$$

Define $n = n_x + n_y$. The function f then maps $R^{n_y} \times R^{n_y} \times R^{n_x} \times R^{n_x}$ into R^n , because there are n equations and $2n_y + 2n_x$ variables, that is y_{t+1}, y_t, x_{t+1} and x_t . The matrix η is given by:

$$\eta = \begin{bmatrix} \emptyset \\ \Sigma \end{bmatrix}$$

The key idea of perturbation is to interpret the solution to the model as a function of the state vector x_t and of the perturbation parameter σ which scales the amount of uncertainty in the economy, leading to:

$$y_t = g(x_t, \sigma) \tag{49}$$

and

$$x_{t+1} = h(x_t, \sigma) + \eta\sigma\epsilon_{t+1} \tag{50}$$

where the function g maps $R^{n_x} \times R^+$ into R^{n_y} and the function h maps $R^{n_x} \times R^+$ into R^{n_x} .

Given this interpretation, perturbation methods find a local approximation of the functions g and h . This local approximation is valid in the neighborhood of a certain point $(\bar{x}, \bar{\sigma})$.

I solve a system of 15 variables: $\{a_t, A_t^{NO}, b_t, c_t, \tilde{c}_t, g_t^{NO}, \lambda_t, L_t, p_t^O, r_t, v_t, w_t, \text{Welfare } (W_t), Y_t^{NO}, Y_t^O\}$, and 15 equations: (2), (3) combined with (46), (19), (20), (21), (23), (25), (28), (29), (30), (31), (32), one of these pairs: Rule 1: (39), (40); Rule 2: (41),(42); Rule 3: (43), (44), and (52). I compute a second-order approximation around the nonstochastic steady state and I perform simulations to analyze the behavior of the model in the case of a deteriorating oil sector.

3.1 Accounting for depletion

Once I have computed the policy functions using a second order approximation, I account for the shrinking oil sector. Because the certainty equivalence principle no longer holds under a second-order approximation, it is quite different to make Y_t^O a deterministic process or to let it follow a stochastic process as the one defined in (19). If I set Y_t^O to be a deterministic process, then the coefficients of the policy functions of the endogenous variables will omit the impact of σ_{Y^O} on all the endogenous variables. Therefore, making Y_t^O follow a deterministic process would actually be a special case in which $\sigma_{Y^O} \rightarrow 0$.

To account for depletion, I assume that at date 0 the economy is hit by a one-time-only positive shock to extraction ($\epsilon_0^{Y^O}$). Due to the autoregressive nature of (19), oil extraction will decline steadily converging to $Y^O(s^O)$, the steady state consistent with the oil sector's sustainable share in GDP. To make the scenarios comparable with each other, I impose the same $\epsilon_0^{Y^O}$ across all rules. More specifically, I assume that in all cases $\epsilon_0^{Y^O}$ is such that at date 0 oil extraction (Y_0^O) is above steady state, at the level consistent with the average share of oil output in total output seen in the data (\bar{s}^O):

$$s_0^O = \frac{Y_0^O p^O(s^O)}{Y^{NO}(s^O) + p^O(s^O) Y_0^O} = \bar{s}^O \quad \implies \quad Y_0^O = \frac{\bar{s}^O}{1 - \bar{s}^O} \frac{Y^{NO}(s^O)}{p^O(s^O)},$$

where $p^O(s^O)$ and $Y^{NO}(s^O)$ denote the steady state levels consistent with s^O .

Due to the fact that the timing of the path of Y_t^O matters, I perform multiple simulations to compute the unconditional and conditional moments of the model, as opposed to a single simulation with a very large time dimension.

For the unconditional moments, I randomly draw the starting value of the states from their unconditional distribution. To do so, I choose a lead-in of length \tilde{t} , and for all dates $t \in [-\tilde{t}, 0)$, I randomly draw $\epsilon_t^{Y^O}$, $\epsilon_t^{p^O}$ and ϵ_t^A and I let the model run so that by date 0 I have values for the predetermined variables that are virtually drawn from the unconditional distribution.

For the conditional analysis, the usual start-off point used in the literature is a deterministic steady state (either competitive or Ramsey). Given that the certainty equivalence principle does not hold for higher order approximations, it is extremely unlikely that the economy would ever be at either one of these deterministic equilibria.¹¹ In principle, it would be ideal for the conditional analysis to have specific information about a known stochastic competitive equilibrium to set the states off at date $t = -1$. Due to the lack of a better candidate, I choose the unconditional mean as the starting value of the states ($\tilde{t} = 1$).

In both types of analyses, I pin down the path of $\epsilon_t^{Y^O}$ for all $t \geq 0$, while still randomly drawing $\epsilon_t^{p^O}$ and ϵ_t^A . Hence, I impose

$$\epsilon_t^{Y^O} = \begin{cases} \sim i.i.d(0, \sigma_{Y^O}^2) & , \quad \forall t \in [-\tilde{t}, 0) \\ Y_0^O - Y^O(s^O) & , \quad t = 0 \\ 0 & , \quad t > 0 \end{cases} \quad (51)$$

while $\epsilon_t^{p^O} \sim i.i.d(0, \sigma_{p^O}^2)$, $\forall t \in [-\tilde{t}, \infty)$; $\epsilon_t^A \sim i.i.d(0, \sigma_A^2)$, $\forall t \in [-\tilde{t}, \infty)$.

It is extremely important to emphasize that in order to make the rules truly comparable with each other at each date and state, in each simulation I use the same realization of $\epsilon_t^{Y^O}$, $\epsilon_t^{p^O}$ and ϵ_t^A across rules in each date and for all s^O scenarios. This way I guarantee that the policy ranking is robust. Otherwise, there could be more favorable draws at some dates for some rule that could mistakenly suggest that it should be preferred over another. Moreover, choosing the same paths of uncertainty for all s^O scenarios also allows me to assess whether different levels of sustainable oil extraction matter or not for policy ranking.

¹¹Computing the Ramsey equilibrium in the context of an resource-rich small open economy with incomplete markets and distortionary taxes is all but trivial, so I am not considering it here. It is, however, work in progress in my research agenda.

4 Welfare measure and simulations results with oil depletion

A rule is considered preferred over the other ones if the contingent plans for consumption, government purchases and labor associated with that rule yield the highest level of expected welfare (unconditional $E(\cdot)$, conditional $E_t(\cdot)$), where welfare (W_t) is defined as

$$W_t \equiv E_t \sum_{j=0}^{\infty} \beta^j \frac{[\tilde{c}_t - \psi L_t^{\varpi}]^{1-\sigma} - 1}{1-\sigma},$$

or recursively,

$$W_t = \frac{[\tilde{c}_t - \psi L_t^{\varpi}]^{1-\sigma} - 1}{1-\sigma} + \beta E_t W_{t+1}. \quad (52)$$

For both the unconditional and the conditional analyses, I compare the performance of Rules 1, 2 and 3 for different parameter combinations of ϕ and s^O as described in Table 4. I consider the case in which g_t^{NO} does not enter the utility function ($\phi = 1$), to assess the dependance of the policy ranking on government purchases yielding utility.

Table 4: Policy evaluation scenarios

	$\phi = 0.81$	$\phi = 1$
$s^O = \bar{s}^O$	Scenario 1	Scenario 4
$s^O = 0.03$	Scenario 2	Scenario 5
$s^O = 0.01$	Scenario 3	Scenario 6

4.1 Results

Table 5 reports conditional and unconditional expected welfare levels at date 0 and the corresponding policy ranking in each scenario. Notice that the ranking depends on the sustainable oil extraction level (s^O), only when $\phi = 0.81$, showing that depletion matters when g_t^{NO} yields utility. While Rule 2 is preferred unconditionally for high and medium levels of s^O , it is only second best when the sustainable level is low (Rule 1 is preferred then). Conditionally, Rule 1 is preferred not only for low but also for medium levels of s^O ,

while Rule 2 is preferred for high sustainable levels.

Table 5: Unconditional and conditional welfare across rules

		$\phi = 0.81$		$\phi = 1$	
Expected Value		Unconditional	Conditional	Unconditional	Conditional
		$E(W_0)$	$E_0(W_0)$	$E(W_0)$	$E_0(W_0)$
$s^O = 0.01$	Rule 1	-1089.27	-1078.29	-0.0001	-0.0001
	Rule 2	-1119.88	-1119.87	-0.0001	-0.0001
	Rule 3	-1218.26	-1218.25	-0.0001	-0.0001
$s^O = 0.03$	Rule 1	-1111.65	-1102.08	-5.994	-5.996
	Rule 2	-1111.01	-1111.20	-5.989	-5.991
	Rule 3	-1196.62	-1196.60	-5.522	-5.524
$s^O = 0.07$	Rule 1	-862.57	-856.65	-8.1833	-8.1855
	Rule 2	-844.74	-845.73	-4.8198	-4.8197
	Rule 3	-926.40	-926.38	-4.4486	-4.4486
Ranking		Unconditional	Conditional	Unconditional	Conditional
		$E(W_0)$	$E_0(W_0)$	$E(W_0)$	$E_0(W_0)$
$s^O = 0.01$	Rule 1	1	1	3	3
	Rule 2	2	2	2	2
	Rule 3	3	3	1	1
$s^O = 0.03$	Rule 1	2	1	3	3
	Rule 2	1	2	2	2
	Rule 3	3	3	1	1
$s^O = 0.07$	Rule 1	2	2	3	3
	Rule 2	1	1	2	2
	Rule 3	3	3	1	1

Preferring Rule 1 or 2 when $\phi = 0.81$ has to do with discounting, complementarity and the volatility of c_t , L_t and g_t^{NO} under each rule. Since government purchases are gross complements with private consumption ($\varkappa < 1$), households want to have as high g_t^{NO} as possible, and more so early on, while there is higher oil revenue. The lower s^O the stronger this argument. That is why they prefer Rule 1 when $s^O = 0.01$. But households also care about the associated volatility of c_t , L_t and g_t^{NO} under each rule. While higher means are welfare increasing, higher volatility is welfare decreasing due to risk aversion.

Tables 6 and 7 report, respectively, the unconditional and conditional means and standard deviations obtained under each rule in all scenarios. Since Rule 1 allows government

purchases to be financed with oil and non-oil current revenue, g_t^{NO} is fully exposed to the volatility of the three exogenous shocks. Rule 2, on the other hand, isolates g_t^{NO} from the volatility of the oil price and oil extraction, by financing government purchases with current non-oil revenue and structural oil revenue. Rule 3 fully disconnects g_t^{NO} from any fluctuation by financing it with structural revenue. Hence, Rule 2 constitutes a compromise between highest expected level of c_t and g_t^{NO} (under Rule 3 and Rule 1, respectively) and the lowest expected volatility (under Rule 1 and Rule 3, respectively).

Rule 3 is the least preferred when g_t^{NO} yields utility, but the opposite is true when $\phi = 1$. Then, it yields the highest conditional and unconditional expected welfare, and the highest expected c_t , while Rule 2 is always second best. This makes sense, when households do not attain utility from higher government purchases, they prefer the lowest possible g_t^{NO} , which given the available rules means financing them with the structural balance alone.

While the unconditional means of consumption, labor and government purchases are very similar across rules across s^O when $\phi = 0.81$, the unconditional standard deviations differ greatly in size. The volatility under Rule 1 is one order of magnitude larger than that under Rule 2, and the latter is one order of magnitude larger than that under Rule 3. When government purchases do not enter the utility function, the volatility of government purchases depicts the same order of magnitude features described above, but now it does not affect consumption nor labor directly, so the volatility of c_t and L_t is very similar across rules (slightly higher for c_t the higher s^O). For all levels of s^O , labor exhibits the same volatility across policies. This has to do with the GHH preference specification, as the marginal rate of substitution is independent of labor. Therefore, there is no wealth effect on labor, so labor does not adjust to smooth consumption.

Table 6: Unconditional moments

		$\phi = 0.81$			$\phi = 1$		
$s^O = 0.01$		Rule 1	Rule 2	Rule 3	Rule 1	Rule 2	Rule 3
c	Mean	1.2698	1.2696	1.2707	18.4548	18.4547	18.4806
	Std	0.0199	0.0020	0.0006	0.0008	0.0008	0.0008
\tilde{c}	Mean	0.3528	0.3526	0.3452	–	–	–
	Std	0.0069	0.0008	0.0000	–	–	–
g^{NO}	Mean	0.1314	0.1313	0.1282	0.1491	0.1492	0.1233
	Std	0.0031	0.0004	0.0000	0.0033	0.0006	0.0000
L	Mean	0.2001	0.2000	0.1975	0.4233	0.4233	0.4233
	Std	0.0040	0.0007	0.0002	0.0006	0.0006	0.0006
$s^O = 0.03$		Rule 1	Rule 2	Rule 3	Rule 1	Rule 2	Rule 3
c	Mean	1.2932	1.2927	1.2946	1.3728	1.3729	1.3993
	Std	0.0205	0.0021	0.0007	0.0010	0.0010	0.0010
\tilde{c}	Mean	0.3532	0.3532	0.3466	–	–	–
	Std	0.0070	0.0008	0.0000	–	–	–
g^{NO}	Mean	0.1313	0.1313	0.1285	0.1793	0.1791	0.1528
	Std	0.0032	0.0004	0.0000	0.0033	0.0005	0.0000
L	Mean	0.1980	0.1980	0.1958	0.4230	0.4230	0.4230
	Std	0.0040	0.0007	0.0002	0.0006	0.0006	0.0006
$s^O = 0.07$		Rule 1	Rule 2	Rule 3	Rule 1	Rule 2	Rule 3
c	Mean	1.3307	1.3276	1.3291	1.2887	1.4584	1.4861
	Std	0.0279	0.0026	0.0010	0.0011	0.0012	0.0013
\tilde{c}	Mean	0.3775	0.3783	0.3696	–	–	–
	Std	0.0103	0.0009	0.0000	–	–	–
g^{NO}	Mean	0.1410	0.1413	0.1377	0.3202	0.1504	0.1227
	Std	0.0047	0.0004	0.0000	0.0052	0.0006	0.0000
L	Mean	0.2019	0.2023	0.1996	0.4264	0.4264	0.4264
	Std	0.0054	0.0007	0.0002	0.0006	0.0006	0.0006

Table 7: Conditional moments

		$\phi = 0.81$			$\phi = 1$		
$s^O = 0.01$		Rule 1	Rule 2	Rule 3	Rule 1	Rule 2	Rule 3
c	Mean	1.2697	1.2696	1.2707	18.6412	18.6411	18.6672
	Std	0.0201	0.0020	0.0006	0.0008	0.0008	0.0008
\tilde{c}	Mean	0.3528	0.3526	0.3452	–	–	–
	Std	0.0069	0.0008	0.0000	–	–	–
g^{NO}	Mean	0.1314	0.1313	0.1282	0.1505	0.1507	0.1245
	Std	0.0032	0.0004	0.0000	0.0032	0.0006	0.0000
L	Mean	0.2001	0.2000	0.1975	0.4276	0.4276	0.4276
	Std	0.0040	0.0007	0.0002	0.0006	0.0006	0.0006
$s^O = 0.03$		Rule 1	Rule 2	Rule 3	Rule 1	Rule 2	Rule 3
c	Mean	1.2931	1.2927	1.2946	1.3875	1.3876	1.4133
	Std	0.0206	0.0021	0.0007	0.0011	0.0011	0.0011
\tilde{c}	Mean	0.3533	0.3532	0.3466	–	–	–
	Std	0.0070	0.0008	0.0000	–	–	–
g^{NO}	Mean	0.1313	0.1313	0.1285	0.1801	0.1801	0.1544
	Std	0.0032	0.0004	0.0000	0.0033	0.0006	0.0000
L	Mean	0.1980	0.1980	0.1958	0.4272	0.4272	0.4272
	Std	0.0040	0.0007	0.0002	0.0006	0.0006	0.0006
$s^O = 0.07$		Rule 1	Rule 2	Rule 3	Rule 1	Rule 2	Rule 3
c	Mean	1.3309	1.3276	1.3291	1.2887	1.4584	1.4861
	Std	0.0278	0.0026	0.0011	0.0011	0.0012	0.0012
\tilde{c}	Mean	0.3775	0.3783	0.3696	–	–	–
	Std	0.0103	0.0009	0.0000	–	–	–
g^{NO}	Mean	0.1410	0.1413	0.1377	0.3201	0.1504	0.1227
	Std	0.0047	0.0004	0.0000	0.0053	0.0006	0.0000
L	Mean	0.2018	0.2023	0.1996	0.4264	0.4264	0.4264
	Std	0.0132	0.0132	0.0132	0.0092	0.0092	0.0092

4.2 Welfare Outlook

Up until now I have evaluated policies from the perspective of the policymaker who has to decide at date 0 which rule to implement. It is interesting, however, to see whether he/she would make the same choice if he/she had to choose at a date $t > 0$. Since (52) is one of the equations in (47), I can simulate time series of conditional and unconditional welfare under each rule in each scenario. Figure 2 depicts the average realization of unconditional welfare at each date for the first 40 quarters in each scenario.¹²

The policy ranking seen in Figure 2 at date 0 is entirely consistent with the unconditional results reported in Table 5. Rule 1 is only preferred over Rule 2 when $s^O = 0.01$ and $\phi = 0.81$. But notice that the bottom left panel in Figure 2 also shows that as time goes by, Rule 1 is relatively less appealing as it still implies higher volatility of c_t , g_t^{NO} and L_t than under Rule 2, but oil extraction is not so far above its sustainable level anymore. By the 28th quarter, the policymaker would be indifferent between implementing Rules 1 and 2, and if he/she had to decide after $t = 28$, then he/she would choose to implement Rule 2.

When $s^O = 0.03$ and $\phi = 0.81$, the policymaker would always choose to implement Rule 2, and even more so if he/she had to decide at a later date, since the gap between the welfare level attained under Rule 2 and Rule 1 widens over time.

Figure 2 also shows that the timing of the policymaker's decision is not relevant when $\phi = 1$. Depletion does not affect the policy ranking at date 0 as seen in Table 5, and it does not affect it in future dates either.

I can perform the same analysis with conditional welfare. Figure 3 depicts the average realization of conditional welfare at each date for the first 40 quarters in each scenario. Note that the policy ranking seen in Figure 3 at date 0 is also entirely consistent with the conditional results reported in Table 5. As in the unconditional case, the timing of the policymaker's decision is not relevant when $\phi = 1$, but now it is important not only for low sustainable levels of oil extraction but also for intermediate ($s^O = 0.03$). The bottom

¹²Recall that by the 36th quarter oil extraction will have converged to its sustainable level.

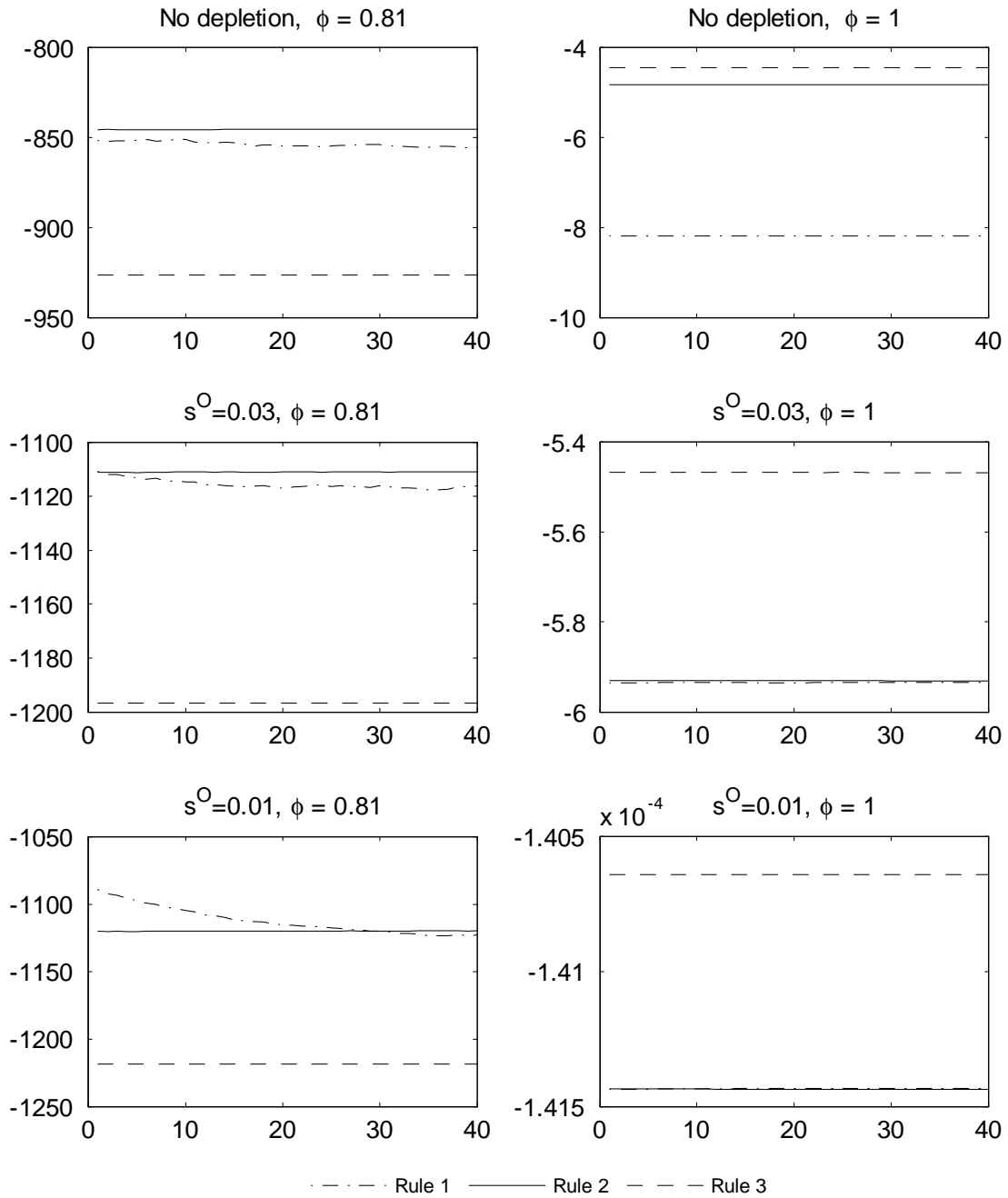


Figure 2: Average unconditional welfare realization under each rule

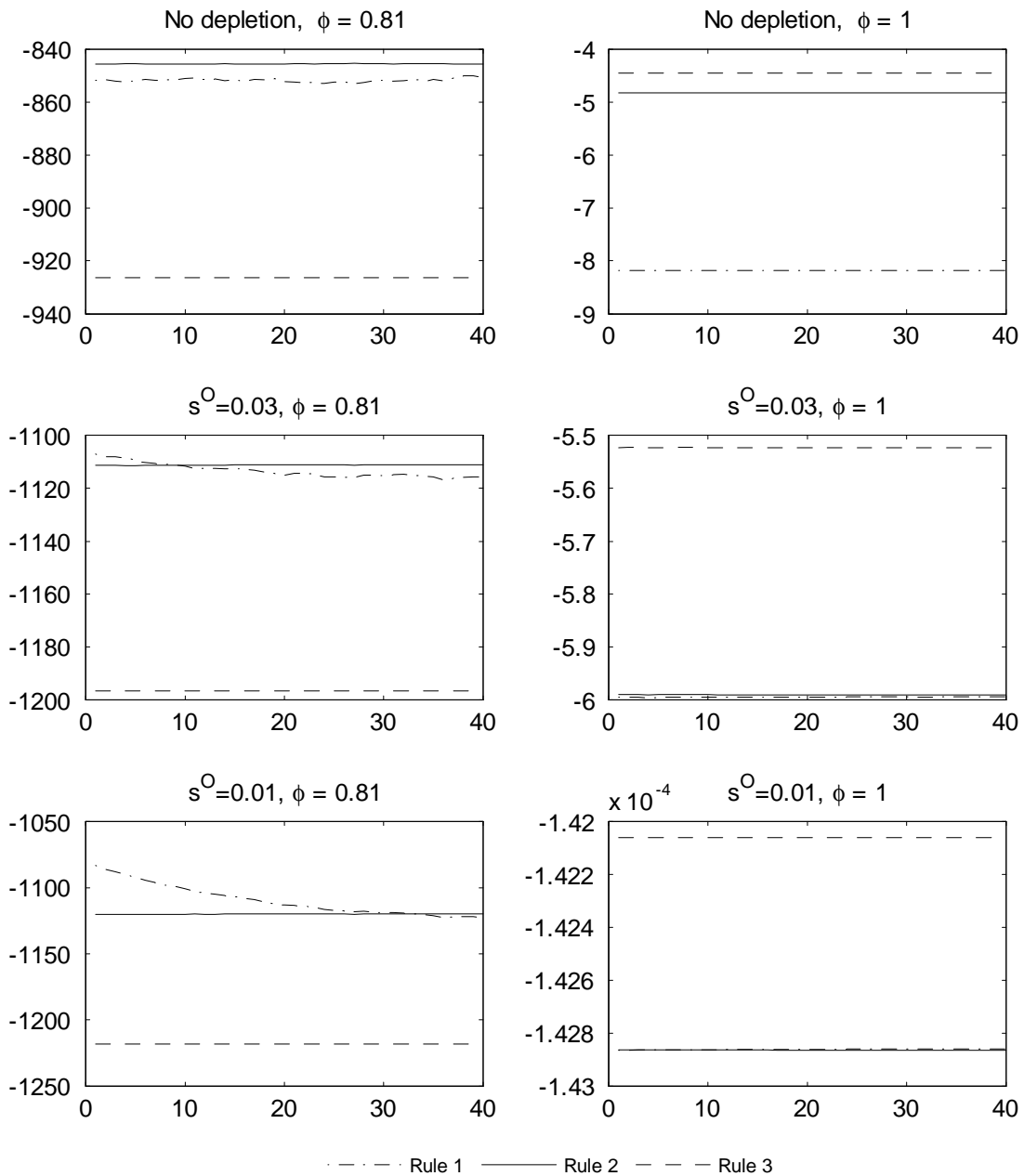


Figure 3: Average conditional welfare realization under each rule

and middle left panels show that, conditionally, the policymaker would implement Rule 1 if he/she had to choose at date 0, but if the decision were to happen at the ninth quarter, then he/she would no longer implement Rule 1 for $s^O = 0.03$ but he/she would rather implement Rule 2. For $s^O = 0.01$ the ranking switches about at the same date as in Figure 2, implying that the policymaker would make the same choice regardless of whether he/she relies in conditional or unconditional expectations.

5 Concluding remarks

In this paper I developed a dynamic stochastic general equilibrium model to analyze the dynamics generated by implementing different fiscal rules in the context of a small open economy that has a deteriorating oil sector. Under each rule, I studied the implications of a shrinking oil sector for the contingent paths of consumption, labor and government purchases, when facing stochastic oil price and technology shocks.

I compared three alternative fiscal rules in terms of unconditional and conditional welfare. I find that Rule 3 is conditionally and unconditionally preferred if government purchases do not yield utility. When they do, however, this rule is the least preferred and the ranking between Rules 1 and 2 depends on the sustainable oil extraction level, on whether the analysis is based on unconditional or conditional welfare and also on the timing of the policymaker's decision, if there is depletion.

When $\phi = 0.81$ and there is no depletion ($s^O = 0.07$), Rule 2 is preferred conditionally and unconditionally. When $s^O = 0.03$, Rule 2 is unconditionally preferred at date 0, while Rule 1 yields the highest conditional welfare. Even though conditionally Rule 1 is preferred at date 0, if the policymaker were to choose at date $t = 9$ or later, then he/she would also choose Rule 2, conditionally. Finally, when $s^O = 0.01$, Rule 1 is preferred conditionally and unconditionally at date 0, but if the policymaker had to choose a rule at date $t \geq 32$, then he/she would choose Rule 2, conditionally and unconditionally.

The reason why Rules 1 or 2 are preferred when $\phi = 0.81$ is a balance among discounting, complementarity and the volatility of c_t , L_t and g_t^{NO} under each rule. Since government purchases are gross complements with private consumption ($\varkappa < 1$), households want to have as high g_t^{NO} as possible, and more so early on, while there is higher oil revenue. The lower s^O the stronger this argument. But households also care about the associated volatility of c_t , L_t and g_t^{NO} under each rule. While higher means are welfare increasing, higher volatility is welfare decreasing due to risk aversion.

On the other hand, if $\phi = 1$, then Rule 3 is always preferred as it induces the highest expected c_t , conditionally and unconditionally. Furthermore, the timing of the policymaker's decision is not relevant when $\phi = 1$. Depletion does not affect the policy ranking at date 0, and it does not affect it in future dates either.

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A Taylor expansion

Perturbation methods find a local approximation of the functions g and h . This local approximation is valid in the neighborhood of a certain point $(\bar{x}, \bar{\sigma})$. Taking a Taylor series approximation of the functions g and h around the point $(x, \sigma) = (\bar{x}, \bar{\sigma})$ yields

$$\begin{aligned}
 g(x_1, x_2, \sigma) &= g(\bar{x}_1, \bar{x}_2, \bar{\sigma}) + \sum_{i=1}^2 g_{x_i}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(x_i - \bar{x}_i) + g_{\sigma}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(\sigma - \bar{\sigma}) \quad (53) \\
 &+ \sum_{i=1}^2 g_{x_i x_i}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(x_i - \bar{x}_i)^2 + g_{x_1 x_2}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(x_1 - \bar{x}_1)(x_2 - \bar{x}_2) \\
 &+ \sum_{i=1}^2 g_{x_i \sigma}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(x_i - \bar{x}_i)(\sigma - \bar{\sigma}) + g_{\sigma \sigma}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(\sigma - \bar{\sigma})^2 + \dots
 \end{aligned}$$

and:

$$\begin{aligned}
 h(x_1, x_2, \sigma) &= h(\bar{x}_1, \bar{x}_2, \bar{\sigma}) + \sum_{i=1}^2 h_{x_i}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(x_i - \bar{x}_i) + h_{\sigma}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(\sigma - \bar{\sigma}) \quad (54) \\
 &+ \sum_{i=1}^2 h_{x_i x_i}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(x_i - \bar{x}_i)^2 + h_{x_1 x_2}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(x_1 - \bar{x}_1)(x_2 - \bar{x}_2) \\
 &+ \sum_{i=1}^2 h_{x_i \sigma}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(x_i - \bar{x}_i)(\sigma - \bar{\sigma}) + h_{\sigma \sigma}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(\sigma - \bar{\sigma})^2 + \dots
 \end{aligned}$$

where for ease of exposition I adapt the notation such that there is only one control variable and only one state variable of each type, that is, one endogenous state variable x_1 and one exogenous state variable x_2 . The terms $g_{\sigma \sigma}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(\sigma - \bar{\sigma})^2$ and $h_{\sigma \sigma}(\bar{x}_1, \bar{x}_2, \bar{\sigma})(\sigma - \bar{\sigma})^2$ measure the influence of the uncertainty in the model on the control and the state variables, respectively.