ISSN 1471-0498

DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

A FABLE OF BEES AND GRAVITY

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Number 716
July 2014

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July 31, 2014

Abstract

Gravity equations in trade imply that trade flows are proportional to the size of a country and inversely proportional to distance. This paper develops an analogy of these observations with gravity in physics, and provides geometric intuition for a large class of mathematical processes in two dimensional space for which these relationships would be expected. It then gives examples of trade processes that conform to gravity, including foraging patterns of various animal species.

JEL classification: F00

Keywords: Foraging, Gravity, International Trade

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1 Introduction

Jan Tinbergen (1962) introduces gravity equations in economics as

$$E_{ij} = \alpha_0 Y_i^{\alpha_1} Y_j^{\alpha_2} D_{ij}^{\alpha_3}, \tag{1}$$

where E_{ij} denotes exports of country i to country j, Y_i denotes GNP of country i, Y_j denotes GNP of country j and D_{ij} the distance between the two countries. He provides estimates that α_1, α_2 are close to 1 and α_3 is close to -1. Estimates of these parameters have been observed for aggregate trade flows as well as a wide range of goods, and they have been remarkably persistent over time (Disdier and Head 2008). While the reason for the persistence of the coefficients indicating economic size (α_1 and α_2) is predicted by many of the current standard models of international trade, the persistent role of distance (α_3) continues to puzzle economists (Chaney 2013).[1]

The main contribution of this paper is to emphasize two analogies to gravity equations in international trade. The first is with gravity in physics. I describe the geometric reason that leads to the inverse square relationship for Newtonian gravity in three dimensional space and argue that this same logic can explain the estimates of $\alpha_1 = 1$, $\alpha_2 = 1$, and $\alpha_3 = -1$ described above, given that a large class of processes lead to these relationships, and many of these processes have an interpretation as trade. Trade takes place in two dimensional space, and thus the answer to some of the empirical regularities is found in elementary geometry (see also Rauch 2014 which shows how Zipf's Law for cities can be explained as a feature of clustering in 2-dimensional space). Head and Mayer (2013) write that gravity equations "emerge from mainstream modeling frameworks in economics and should no longer be thought of as deriving from some murky analogy with Newtonian physics". Yet the analogy with physics can explain the persistence of estimates of -1 on the log of distance, which is typically not explained by trade

[1] Disdier and Head (2008) is titled "The puzzling persistence of the distance effect on bilateral trade". Chaney (2013) writes "to this day no explanation for the role of distance [...] has been found."

models. I believe this is the first paper to show point this out. The second analogy concerns foraging behavior of animals, which also conforms with gravity equations in trade. This might be of interest to natural scientists, as it provides a large range of testable hypotheses from the literature on international trade to them, but it might also be of interest to economists as it raises the possibility of a new field of experiments in international economics.

This simple geometric argument, explained in greater detail further below, is based on the geometry of two-dimensional space. A ray sent out randomly from the center of a circle is twice as likely to hit a given pixel on a circle with radius r than on a circle with radius $2r$. We can build on this elementary observation to see that random shapes intersected by rays sent from a point outside them will lead to intersections that approximately obey Tinbergen's equations. We can think of the ray as a transport route along which a producer sells goods. This relationship emerges quite generally from intersections of one dimensional lines with two dimensional areas, and a large class of processes in two dimensional space will thus mechanically conform to gravity with the coefficients estimated.

This explanation is simple, plausible, allows to incorporate a large number of assumptions on behavior of trading partners and is analogous in its intuition to Newton's theory of gravity. A robust empirical regularity such as the -1 estimates for α_3 would invite an explanation with these properties (see discussion in Rauch 2014). I do not claim to be the first to see this natural theory of gravity in trade. Given the similarity of the intuition to gravity in physics I find it very likely that Tinbergen, who named gravity equations in economics after having studied physics, was aware of it. I do however find it worthwhile to emphasize this relationship given that currently it seems not to be known widely in economics, and that I was unable to find a written source for it in the existing literature.

This paper highlights a simple mathematical property, and does not present a model of trade. At the very least its contribution is to provides an argument that could be the foundation of a model of contacts and information in the style of Chaney (2013) that is simpler and requires fewer structural assumptions. Its mechanics could also be incorporated into more complex models of trade. I provide a few examples of trade that should be expected to lead to gravity equations with these parameters. The most noteworthy is that I bring results from biology to economics that study the foraging behavior of various animals moving on land (deer), in water (sharks) and in the air (albatross and bees). I show that these animals trade[2] consistently with the stylized facts on international human trade uncovered by Tinbergen (1962), including distance coefficients of $\alpha_3 \approx -1$.

This paper proceeds as follows: Section 2 describes the geometric process that leads to this result, Section 3 gives examples of trade in humans and animals that follows this process, Section 4 concludes.

2 Geometry

The reason for the persistence of the parameters estimated in gravity equation in trade may be found in the geometry of two dimensional space, yet it is instructive to start with the three dimensional case commonly used in physics. Consider Figure 1. Four rays labeled a, b, c and d are sent out from a point O and cross three areas at distances r, $2r$, and $3r$. The areas that the intersections of the rays and the areas span increase quadratically in the distance of the areas to O. If the rays represent some physical force in three dimensional space this force will decrease in the distance squared, a common finding called the 'inverse square law' in physics. It applies to gravitation as well as electric, magnetic, light, sound or radiation phenomena. All these forces weaken linearly in the distance squared.

An analogous observation can be made on phenomena in 2-dimensional space. Consider the area created by the rays c and d. The lines they intersect at distances r, $2r$ and $3r$ now decrease linearly in distance, not in squared distance, as evident from looking at the lines they intersect. The point O in the above example can represent a very small origin country. The ray can represent trade. For example, we may assume that the ray represents a road along which a car

[2]See the discussion of foraging further down for a comment of the use of the term trade in the context of animal behavior.

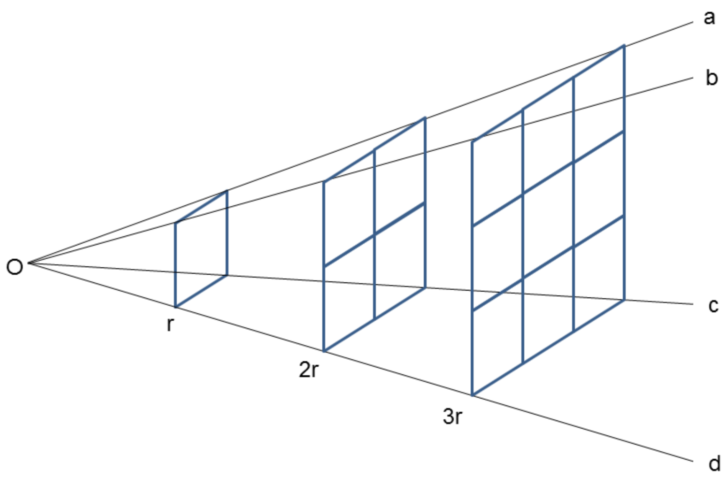

Figure 1: Rays a, b, c, and d emanating from O intersect areas at distances r, $2r$ and $3r$.

coming from O sells. Further assume that the angle at which the road leads out of O is random. Then we can use the above geometric example for a thought experiment in which the size of the origin country and the size of the destination country do not change (leaving both very small) but double the distance between the two countries. Following the notation in equation 1, if we normalize units such that at distance $D = r$ value $E = e$, where e denotes the probability that a pixel at that distance is intersected by the ray, then we would observe at $D = 2r$ a measure of $E = e/2$. This is consistent with equation 1 if and only if $\alpha_3 = -1$.[3]

We may think of a destination country as a number of small squares, to which I refer as pixels, grouped together. If all of these pixels are at the same distance from the origin, doubling the number of pixels doubles the probability that the ray intersects these pixels. This is consistent with equation 1 if and only if $\alpha_2 = 1$. Equally assuming that there are twice the number of origins of rays at constant distance to one pixel, the probability that it is hit by at least one ray will be double. This is consistent with equation 1 if and only if $\alpha_1 = 1$. This interpretation of a country as a set of pixels introduces some noise, as a pixel might intersect a ray with longer

[3]This is clear from solving the system of equations $e = cr^{\alpha_3}$ and $e/2 = c(2r)^{\alpha_3}$, where c is a constant in this thought experiment.

or shorter length of intersection. However, these distortions can be made small by reducing the size of pixels.

This concludes the basic explanation of the geometric intuition for gravity equations, yet two remarks are necessary, first on transport routes that deviate from the assumption of linearity, and second on the measurement of distance in practice.

First, it is easy to see that the geometric intuition extends to rays that move out from an origin in another way than linear. For example, a ray that moves outwards along a sine curve would fit the theory. In fact any stationary process (a process that follows the same movement generating algorithm at any distance from the origin) will lead to gravity equations in this setting in expected value. There is one important condition, this will only hold up to a distance of d from O provided the force sent out reaches a distance of d. This is immediately apparent from Figure 1, as all such stationary processes would conform with the argument about probabilities made there.

Second, if the origin and the destination country in this example are thought of as a number of grouped pixels that are at varying distances from one another there is some ambiguity how to measure the distance between the two countries, as there is in applying gravity equations in practice. Denote by $r_1, r_2, ...r_n$ all pairwise distances between the full set of pixels in the origin and the full set of pixels in the destination country, ordered by increasing distance. Then the undistorted measure of distance between the two countries relative to equation 1 would be the harmonic mean $D_{ij}^{-1} = cn/(1/r_1 + 1/r_2 ... + 1/r_n)$, where c is a constant capturing the strength of the gravitational pull.[4] Note that this measure only requires pairwise distances and is independent of the shape of both countries. This insight has implications on how distance should be measured in the application of gravity equations.

[4]In this way the distance measure is proportional to the total gravitational pull, which is the sum of all pairwise forces $k(1/r_1 + 1/r_2 ... + 1/r_n)$, where k is a constant.

3 Examples

This paper does not develop a trade model, it merely highlights a mathematical property which is consistent with a few instances that could be called trade, and that could be incorporated into a fuller model of trade. This section introduces a few simple processes that have an interpretation as trade that are consistent with the geometrical model discussed in the previous section. The main exception to these examples is that trade volumes that increase (for example through higher prices at greater distance) or decrease (for example trade prohibited by too high trade costs) in distance violate the stationarity condition and thus are not consistent with this explanation. Another limitation of the approach is that it abstracts in its simple form from multilateral resistance.

Information

The most straight forward incorporation of this argument into a model of trade would be to see trade frictions as largely informational in the spirit of Chaney (2013). We could think of trade emerging from random encounters between a seller and a buyer. Any functions describing the movement of the seller and the buyer that is stationary up to a distance of d would lead to gravity equations for information up to that distance d. If information is correlated to the movement of banknotes then this explanation would be supported by findings from Brockman et al (2006) who show that banknotes travel according to Lévy flight discussed in the foraging section below. The encounter of a seller and a buyer is a sufficient condition to trade, not a necessary one. This explanation requires that these informational encounters are important quantitatively relative to other functions of distance such as the increase of trade costs or the divergence of preferences in distance.

Balloon flight, message in a bottle drifting on the ocean

These two examples are admittedly quantitatively unimpressive, yet they clearly fit the geometric examples discussed above. A bottle drifting around in the ocean to be picked up by a ship, or a hot air balloon could be thought to follow processes that are consistent with this interpretation of gravity. Even if they are released on a point where a wind or stream carries them into some directions with higher probability than others, they will lead to gravity equations as long as the probability that they are picked up at a certain distance is the same for every distance, and as long as gravity is measured only within an observed maximum travel distance d.

Transport networks

It is plausible to think of trade to take part along rays going out of a center, for example trucks going out in various directions along a road network from a factory, ship going down a river or river delta, water or gas being pumped using a pipeline. All these will lead to gravity equations empirically up to a distance d if the goods that are traded are shipped on a road that stretches out at a distance larger than d, if sales are equally likely or frequent at any point along that road, at any distance to the origin, and if economic activity is uniformly distributed in space.

This last condition is likely invalid in the example of a navigable river that attracts economic activity (for quantifications see Michaels and Rauch 2014). Yet even if this condition is invalidated the gravity equations might apply. Consider the extreme example where all the population lives along a big river uniformly distributed along its shore, and all other areas are uninhabited (we may think of the river Nile). Then there will nevertheless be twice as much land at distance $2r$ from the origin than at r, albeit uninhabited. There may be the same level of trade at distances r and $2r$, yet relative to the land that exists at these distances trade is half at $2r$ to what it is at r. Even the world in this extreme example would conform to Tinbergen's gravity.

Foraging[5]

Many animal species including albatrosses, bony fishes, bumblebees, deer, microbes, penguins, sea turtles and sharks seem to conform to the same search behavior when searching for food,

[5] Is it reasonable to speak of trade in the case of animals? Merriam Webster's Dictionary (2014) defines trade among other definitions as "the activity or process of buying, selling, or exchanging goods and services" and "the act of exchanging one thing for another". When a bee visits a flower it provides the flower the service of pollination, while receiving the good nectar in return. By the dictionary's definition the bee trades with the flower. (Adrian Wood suggested to me that the definition in the dictionary should include the word 'voluntary'. I agree, but feel that it is still justified to speak of trade in the case of a bee and a flower.)

a model labeled the Lévy flight foraging hypothesis, even humans conform to these movement patterns in some circumstances (Brockmann et al 2006, Edwards et al 2007, Schuster et al 1996, Sims et al 2008). By Lévy flight search behavior animals searching for food would chose its step length l from the probability density function $P(l)\ l^{-\mu}$, with $1 < \mu \leq 3$. This fat tailed distribution leads to occasional long steps, which imply that a given spot is revisited with much lower probability than under Brownian motion. Under some conditions this type of search behavior is thought optimal (Viswanathan et al 1999), with the optimal search parameter close to $\mu = 2$. This process fulfills the condition of stationarity required for the geometrical argument above to be valid. Thus foraging behavior described by this theory is consistent with the geometrical reasoning in the previous section and thus lead to gravity equations with $\alpha_1 = 1$, $\alpha_2 = 1$, and $\alpha_3 = -1$. Lévy flight search predicts movement with an infinite expected length. In the context of the geometric discussion this is helpful given the requirement that the ray exceeds a certain maximum distance.

The analogy with animals allows to test the model empirically. Zurbuchen et al (2010) develop a methodology to study flight distances of bees that only forage a single plant genus. Females were forced to collect pollen from plotted host plants that were successively placed in increasing distances from the fixed nesting stands. They provide numbers of bees by distance collected in different years. For none of the sets of numbers provided in their paper did a regression of the log number of bees by distance on log distance allow to reject a coefficient of -1 on log distance at 5 percent level of significance.

4 Conclusion

Any process of human exchange of goods, information, or services by which traded items or information originate at one point and randomly spread out from there by a stationary process in two dimensional space up to a distance d leads to gravity equations up to d where trade levels are exactly proportional to the size of both countries and exactly inversely proportional to distance. This fact is equivalent to Newtonian gravity in three dimensional space. It can explain why coefficients of distance in estimates of gravity equations have been so persistent over time. The mathematics that this paper is based on is simple and well known in other subjects. Yet I believe this result is important as it provides the basis for a simple, plausible, and very general explanation for coefficients found in empirical gravity equations in trade that seems not to be widely known.

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