A dynamic econometric analysis of the dollar-pound exchange rate in an era of structural breaks and policy regime shifts

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Abstract

We employ a newly-developed partial cointegration system allowing for level shifts to examine whether economic fundamentals form the long-run determinants of the dollar-pound exchange rate over a recent period characterised by structural breaks and policy regime shifts. The paper models both long-run and short-run dynamic features of the exchange rate using a set of economic variables that explicitly reflect quantitative monetary policy and the influence of a forward exchange market. Out-of-sample forecasts comparing the model with economic fundamentals to benchmarks including the random walk indicate that fundamentals can help at short horizons but less so at longer horizons.

Keywords: Exchange rates, Unconventional monetary policies, Structural breaks, Economic fundamentals, General-to-specific approach, Partial cointegrated vector autoregressive models

JEL classification codes: C22, C32, C52, F31.

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1 Introduction

Relationships between foreign exchange rates and economic fundamentals have been thoroughly investigated in the international economics and finance literature since Meese and Rogoff (1983, 1988) but there is still no consensus with regard to deviations of observed exchange rates from their fundamental-based values, see e.g., Engel and West (2005), Sarno and Sojli (2009), Bacchetta and Van Wincoop (2013) and Balke, Ma, and Wohar (2013). Applied research has found weak support for the view that macroeconomic variables are critical factors in exchange rate fluctuations; see Baxter (1994), Isaac and De Mel (2001), Cheung, Chinn, and Pascual (2005), and Sarno (2005), *inter alia*.

This paper addresses the role of economic fundamentals in exchange rate determination by adopting a class of novel econometric techniques that can explicitly model structural breaks and by selecting those variables that explicitly reflect unconventional monetary policy and influences of a forward market. We focus on modeling the dollar-pound exchange rate but the methodology is general. The paper develops a cointegrated vector autoregressive (CVAR) system for processes integrated of order 1 (denoted $\mathsf{I}(1)$), pioneered by Johansen (1988, 1996), which provides a basis for a feasible general-to-specific modeling scheme applicable to multivariate non-stationary time series data. See Hunter, Burke, and Canepa (2017) for various recent developments in CVAR-based methods.

There have been various CVAR analyses conducted in the exchange rate literature, such as Johansen and Juselius (1992), Juselius and MacDonald (2004), Kurita (2007), Juselius and Assenmacher (2017) and Juselius and Stillwagon (2018). In this paper we make four novel contributions. First, a classical theory in international economics is adapted in such a manner that it directly reflects quantitative monetary policy (see Joyce, Miles, Scott, and Vayanos, 2012) adopted by the central banks in two countries (UK and US) and some other aspects of international financial markets. More specifically, both countries' monetary bases are incorporated into a theoretical model proposed by Frankel (1979), along with an expectation formation based on a foreign-exchange forward premium. Second, the empirical investigation commences with a partial CVAR (PCVAR) system with level shifts, a member of a new class of econometric models introduced by Kurita and Nielsen (2019). This avoids an intractable large-scale econometric system subject to a number of problems such as vanishing degrees of freedom, and includes a structural break caused by the financial crisis and subsequent global recession starting in September 2008. The PCVAR system reveals the underlying long-run relationships between the two countries. It is further reduced to a trivariate vector equilibrium correction model (VECM) by exploiting revealed evidence for the existence of additional weakly exogenous variables.

Having developed the system for the dollar-pound exchange rate, we then focus on the exchange rate equation in the system and apply impulse-indicator saturation (IIS: Hendry, Johansen, and Santos, 2008, and Johansen and Nielsen, 2009) to test for breaks and location shifts. This is particularly beneficial towards the end of sample when the Brexit referendum led to two large falls in the spot exchange rate in close succession. Finally, the data sample is extended and we evaluate the model based on forecast performance for different horizons over 2018-2020. The out-of-sample forecasting exercise shows that economic fundamentals do help to forecast at short horizons but the random walk is hard to beat at longer horizons, in line with Meese and Rogoff (1983a). Our results also align with Engel and West (2005) who use an asset pricing model to argue that the exchange rate can exhibit close to random walk behaviour if a fundamental variable has a unit root and the discount factor is near unity.

The modeling approach we use embeds testable theory relationships within an empirical model. If the model satisfies all assumptions needed for valid inference (i.e. is congruent) then the tests of theory relationships are valid. Therefore, we start by outlining the underlying theory relationships in the empirical model. §2 re-visits a classical exchange-rate theory to obtain a set of candidate variables for long-run economic relationships underlying the UK and US, with §3 outlining the econometric methods used in the empirical study. §4 provides an overview of the data. The empirical analysis is given in §5, where a level-shift PCVAR system is employed to reveal the long-run economic relationships, and the system is reduced using general-to-specific principles. The system-based analysis paves the way for a single-equation analysis in §6, where we focus on achieving a congruent model for the dollar-pound rate using saturation methods. §7 conducts the forecasting exercise for the spot exchange rate, and finally §8 concludes.[1]

[1]The software *OxMetrics* version 8 and *PcGive* (Doornik and Hendry, 2013a,b) was used in all the

2 Resuscitating a classical exchange-rate theory

We reformulate a theoretical model by Frankel (1979) in such a way that the model can be empirically relevant to modern financial and macroeconomic environments in which monetary policy and a forward exchange market play more critical roles than before. In view of its application to the dollar-pound exchange rate data, the model presented here is composed of UK and US macroeconomic variables, the time series data of which are all realisations of processes that are wide-sense non-stationary, both $\mathsf{I}(1)$ and subject to distributional shifts.

First, consider the uncovered interest parity (UIP) condition based on bond yields:

$$s^e_{t+1} - s_t = i_t - i^*_t + \rho_t, \tag{1}$$

where s_t is the log of the dollar-pound spot exchange rate, s^e_{t+1} is its one-period ahead expectation, i_t is the US bond yield and i^*_t is the UK counterpart (the superscript $*$ denotes a UK variable hereafter), and ρ_t represents a risk premium term. Introduce the following Frankel (1979)-type expectation formation in the foreign exchange market of the spot rate dynamics:

$$\Delta s^e_{t+1} = s^e_{t+1} - s_t = -\theta(s_t - \bar{s}_t) + \pi^e_{t+1} - \pi^{*,e}_{t+1} \qquad \text{for} \quad 0 < \theta < 1, \tag{2}$$

where $\bar{s}_t$ represents a fundamental-based value of the exchange rate and π^e_{t+1} denotes the market expectation of the US inflation rate. Equation (2) implies that relative purchasing power parity (PPP) holds in the formation of the expected rate of change Δs^e_{t+1} when s_t equals $\bar{s}_t$. The level $\bar{s}_t$ was frequently specified using absolute PPP in the exchange rate literature (see MacDonald and Nagayasu, 1998, *inter alia*):

$$\bar{s}_t = p_t - p^*_t, \tag{3}$$

where p_t is the log of the US price level. The specification (3) leads to the definition of the real exchange rate $q_t = s_t - p_t + p^*_t$, which, by noting $\pi^e_{t+1} = p^e_{t+1} - p_t$, enables us to restate

empirical analyses performed in this paper.

(2) in terms of q_t as $q^e_{t+1} = (1-\theta)\, q_t$, (where $0 < \theta < 1$) for $q^e_{t+1} = s^e_{t+1} - p^e_{t+1} + p^{*,e}_{t+1}$. Hence, if θ is small, this is consistent with a vast number of econometric studies on near unit root properties of real exchange rates. As reviewed by Sarno and Taylor (2002, Ch.3), scalar unit root tests are liable to suffer power deficiency, with the result that we have to rely on long-span studies, such as century-long data analysis, to estimate θ precisely in an AR regression. See Lothian and Taylor (1996) for a study employing two centuries of data for real exchange rates and Hendry (2001) who showed that very long-run PPP held for the UK versus the world; see also Johansen (2006) for inferential problems with modeling near unit root data as stationary in a small-sample context.

Furthermore, in a class of standard monetary models in the literature (see Sarno and Taylor, 2002, Ch.4, *inter alia*), each of the price levels in the PPP formulation (3) was replaced by a broad money stock measure (M2 and M3, for example), along with other factors such as real income. In this paper we explore a new approach, albeit in the spirit of monetary models, to modeling s_t such that it explicitly reflects the two countries' quantitative monetary easing (see Joyce, Miles, Scott, and Vayanos, 2012, for further details):

$$\bar{s}_t = \; g\{\underset{+}{MB_t}, \underset{-}{MB^*_t}\} + \; \eta_t, \tag{4}$$

where $g\{\cdot\}$ denotes a monotonically increasing function of MB_t (the US monetary base) and a monotonically decreasing function of MB^*_t, and η_t denotes an intercept determining the level of $\bar{s}_t$ and it is changeable over time according to regime shifts in monetary policy. The formulation (4) can still be viewed as a PPP-based specification of $\bar{s}_t$ as its basis is given by (3), but it reflects the era of unconventional monetary policies in response to economic shocks and crises. Quantitative easing is usually accompanied by the expansion of the central bank's balance sheet pursued by open market operations, and therefore entails an increase in the monetary base. In addition, there is evidence indicating a close linkage between the Japan-US monetary base ratio and the yen-dollar exchange rate in recent years, as discussed by Kano and Morita (2015). Given the fact that both the UK and US have adopted quantitative monetary easing as one of the policy responses to the global recession starting in 2008, formulation (4) is considered to be plausible. With the log of the US monetary base denoted by mb_t, we assume further the function $g\{\cdot\}$ is restricted in such a way that it is in a testable linear form:

$$g\left\{MB_t, MB_t^*\right\} = \delta mb_t - \xi mb_t^* \qquad \text{for} \quad \delta, \xi \geq 0, \tag{5}$$

in which the weak inequality is allowed with respect to the parameters δ and ξ.

Combining (1), (2), (4) and (5) then leads to:

$$s_t = \delta mb_t - \xi mb_t^* - \frac{1}{\theta}\left(i_t - i_t^* + \rho_t\right) + \frac{1}{\theta}\left(\pi_{t+1}^e - \pi_{t+1}^{*,e}\right) + \eta_t,$$

which indicates a close connection between the exchange rate and a class of macroeconomic variables. By assuming a mean-zero stationary risk premium, this equation allows us to conceive the following linear combination as a candidate of the underlying long-run economic relationships:

$$s_t - \delta mb_t + \xi mb_t^* + \frac{1}{\theta}\left(i_t - i_t^*\right) - \frac{1}{\theta}\left(\pi_{t+1}^e - \pi_{t+1}^{*,e}\right) - \eta_t \sim \mathsf{I}\left(0\right), \tag{6}$$

which means the linear combination is a mean-zero stationary or $\mathsf{I}(0)$ series. This combination, (6), lays a theoretical foundation for empirical cointegration analysis, in which π_{t+1}^e and $\pi_{t+1}^{*,e}$ need to be represented by some observable variables.

Suppose that π_{t+1}^e and $\pi_{t+1}^{*,e}$ are approximated as π_t and π_t^*, respectively, as a result of a random-walk expectation formation; we are then justified in suggesting:

$$s_t - \delta mb_t + \xi mb_t^* + \frac{1}{\theta}\left\{\left(i_t - \pi_t\right) - \left(i_t^* - \pi_t^*\right)\right\} - \eta_t \sim \mathsf{I}\left(0\right), \tag{7}$$

as a candidate long-run combination incorporating an observable real interest differential. Alternatively, $\pi_{t+1}^e - \pi_{t+1}^{*,e}$ may be approximated as a linear function of the forward premium fp_t defined as $fp_t = f_{t+1}^t - s_t$, where f_{t+1}^t is the dollar-pound forward exchange rate applicable at $t+1$ (but contracted at t). The justification of this approximation stems from the combination of covered interest rate parity (CIP) and a forward guidance strategy in inflation targeting policy, which utilises short-term interbank interest rates as policy tools to influence inflation expectations. The CIP tells us $fp_t = i_t^s - i_t^{s,*} + \sigma_t$, where i_t^s is the US interbank rate and σ_t denotes transaction costs, which could be marginal in the current liberalised global financial market. The conjunction of CIP and monetary policy with forward guidance thus leads to:

$$\pi_{t+1}^{e} - \pi_{t+1}^{*,e} = \psi\left(i_t^s - i_t^{s,*}\right) \approx \psi f p_t \qquad \text{for} \quad \psi > 0. \tag{8}$$

Using (8), we then arrive at the other observable long-run relationship:

$$s_t - \delta m b_t + \xi m b_t^* + \frac{1}{\theta}\left(i_t - i_t^*\right) - \frac{\psi}{\theta} f p_t - \eta_t \sim \mathsf{I}(0)\,. \tag{9}$$

We can employ $i_t^s - i_t^{s,*}$ instead of fp_t to derive (9), but the use of fp_t is justified in that it may have the advantage of capturing the underlying complex interdependency between the spot and forward foreign exchange markets.

The set of conceivable long-run combinations, (7) and (9), allows us to formulate an empirical system for X_t defined as:

$$X_t = \left(s_t, i_t, i_t^*, \pi_t, \pi_t^*, m b_t, m b_t^*, f p_t\right)', \tag{10}$$

which gives a basis for the starting point of a general unrestricted model, thereby allowing us to proceed to a nested specific model centering on the dollar-pound exchange rate.

3 A PCVAR system with structural breaks

We build on the new class of CVAR models introduced by Kurita and Nielsen (2019) which consist of PCVAR systems allowing for structural breaks in deterministic components, such as a linear trend or intercept.

Consider a p-variate $\mathsf{I}(1)$ vector sequence X_t for $t = 1, ..., T$, which is further decomposed as $X_t = (Y_t', Z_t')'$, in which the dimensions of Y_t and Z_t are m and $p - m$ respectively under $m < p$. A system for X_t is a stochastic autoregressive system driven by the innovation sequence ε_t, which is assumed to be a martingale difference sequence with time-varying conditional variances and the sample average of the variances is assumed to converge to a positive-definite matrix $\Omega \in \mathbf{R}^{p \times p}$; see Kurita and Nielsen (2019) for further details. Corresponding to the decomposition $X_t = (Y_t', Z_t')'$, the innovation sequence and variance matrix are also broken down as follows, along with the introduction of two combinational expressions:

$$\varepsilon_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{pmatrix}, \ \Omega = \begin{pmatrix} \Omega_{yy} & \Omega_{yz} \\ \Omega_{zy} & \Omega_{zz} \end{pmatrix}, \ \varepsilon_{y.z,t} = \varepsilon_{y,t} - \omega \varepsilon_{z,t} \text{ for } \omega = \Omega_{yz}\Omega_{zz}^{-1}.$$

The PCVAR system is based on the assumption that Z_t is weakly exogenous (see Engle, Hendry, and Richard, 1983) with respect to parameters of interest, cointegrating vectors $(\beta', \gamma) \in \mathbf{R}^{r\times(p+q)}$ and adjustment vectors $\alpha_y \in \mathbf{R}^{m\times r}$. In order to permit deterministic shifts in the level of X_t, we pre-determine the number of sub-sample periods, q, corresponding to the length of each sub-sample period: $0 = T_0 < T_1 < \cdots < T_q = T$. It is also assumed that the underlying VAR system for X_t satisfies all conditions stated in Theorem 2.1 in Johansen, Mosconi, and Nielsen (2000), so that X_t has an $\mathsf{I}(1)$ moving-average representation presented in that theorem.

Given a period j under $1 \leq j \leq q$ and a time point t under $T_{j-1} + k < t \leq T_j$, we present the following PCVAR(k) system for Y_t given Z_t, allowing for the presence of $q-1$ level shifts and k lagged dynamics:

$$\begin{aligned} \Delta Y_t = \omega \Delta Z_t + \alpha_y (\beta', \gamma) \begin{pmatrix} X_{t-1} \\ E_t \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_{y.z,i} \Delta X_{t-i} \\ + \sum_{i=1}^{k} \sum_{j=2}^{q} \Psi_{j,i} D_{j,t-i} + \Phi_{y.z} d_t + \varepsilon_{y.z,t} \quad \text{for} \quad t = k+1, ..., T, \quad (11) \end{aligned}$$

where both $\alpha_y \in \mathbf{R}^{m\times r}$ and $\beta \in \mathbf{R}^{p\times r}$ are of full column rank r, while the other parameters are subject to: $\gamma \in \mathbf{R}^{r\times q}$, $\Phi_{y.z} \in \mathbf{R}^{m\times s}$, $\Gamma_{y.z,i} \in \mathbf{R}^{m\times p}$ for $i = 1, \ldots, k-1$, and $\Psi_{j,i} \in \mathbf{R}^m$ for $i = 1, \ldots, k$ and $j = 2, \ldots, q$. With regard to the other variables appearing in (11), $D_{j,t}$ is defined as:

$$D_{j,t} = \begin{cases} 1 & \text{for } t = T_{j-1}, \\ 0 & \text{otherwise}, \end{cases} \quad \text{for } j = 1, ..., q \quad \text{and} \quad t = 1, ..., T,$$

which results in $D_{j,t-i} = 1$ if $t = T_{j-1} + i$, allowing us to construct a conditional likelihood function for each sub-sample period, and this variable is also employed to define:

$$E_{j,t} = \sum_{i=1}^{T_j - T_{j-1}} D_{j,t-i} = \begin{cases} 1 & \text{for } T_{j-1} < t \leq T_j, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad E_t = (E_{1,t}, \ldots, E_{q,t})',$$

so that E_t enables us to include level shifts in the system with their corresponding parameters γ, or more specifically, $\gamma = (\gamma_1, \ldots, \gamma_q)$. Finally, d_t represents an unrestricted s-variate vector of dummy variables and second-order difference variables, which are included in the system for the purpose of capturing various irregularities in the process.

In addition to (11), Kurita and Nielsen (2019) develop a PCVAR system allowing for structural breaks in a linear trend term, a broken-trend system useful for the purpose of modeling a macroeconomic system that incorporates the logs of GDP, consumption or production indices. The analysis of this paper in §5.2, however, will show that the level-shift specification (11) turns out to be sufficient to represent the data being studied.

In system (11), the underlying long-run economic relationships are interpreted as being represented by the combinations $\beta' X_{t-1} + \gamma E_t$, which are of testable linear form in view of the theory in §2. The combinations also allow for the possibility of level shifts in the process. In order to reveal the underlying long-run linkages, we follow a multi-step modeling strategy: we begin by estimating a well-formulated general unrestricted partial VAR (PVAR) model assuming known location shifts and then proceed to test the cointegrating rank r by using a quasi-likelihood testing procedure in Kurita and Nielsen (2019); after determining r, we seek those structures of the quasi-maximum likelihood estimates $(\hat{\beta}', \hat{\gamma})$ which are consistent with the theoretical formulations given in §2. In addition, the adjustment vectors α_y represent how each variable in the system reacts to disequilibrium errors expressed by $\beta' X_{t-1} + \gamma E_t$. Hence, checking the structure of $\hat{\alpha}_y$ is also important in terms of detailed economic interpretations of $(\hat{\beta}', \hat{\gamma})$.

The PCVAR system is a tractable model for exploring long-run dynamics subject to regime shifts. The partial model focusing on Y_t given information on Z_t, allows us to avoid a large-dimensional system which poses difficulties in correctly choosing the cointegrating rank in finite samples. Furthermore, the partial system can be reduced to a smaller system by testing for evidence indicating the presence of additional weakly exogenous variables in the set of Y_t; let $Y_t = (Y_{1t}', Y_{2t}')'$ and let $\alpha_y = (\alpha_{y1}', \alpha_{y2}')'$ accordingly, and if

$\alpha_{y2} = 0$, then Y_{2t} is judged to be weakly exogenous for α_{y1} and (β', γ) in the same sense as the assumption about Z_t.

The system acts as a bridge connecting the system analysis with a single-equation analysis. The reduction from the system to single equation analysis requires that there is no loss of information by focusing on the conditional model for one variable, marginalizing with respect to the other variables. A conditional-marginal factorization can be applied to any joint density, but the parameter space from the joint model must be the cross product of the individual parameter spaces, ensuring the parameter spaces of the marginal models are not linked to the parameter spaces of the conditional model. If these conditions are satisfied we can proceed to a single equation analysis for the spot exchange rate. Although endogeneity is no longer modeled, the single equation approach allows for a more general analysis of structural breaks and outliers by applying Impulse Indicator Saturation (see Hendry, Johansen, and Santos, 2008, Johansen and Nielsen, 2009 and Hendry and Doornik, 2014).[2] We take the PCVAR as given from the multivariate modeling approach outlined above and check for robustness of the model specification to outliers and location shifts using IIS.

4 An overview of the data

Figure 1 records the monthly log dollar-pound spot exchange rate (s_t), along with its time series properties and distribution.[3] The estimation period for the subsequent analysis is October 2003 to April 2018, denoted as 2003.10 - 2018.4. The spot exchange rate is non-stationary, with a stochastic trend and location shifts, highlighted by panel b which records a time-varying mean obtained using Step Indicator Saturation (SIS: Castle, Doornik, Hendry, and Pretis, 2015) at 0.1% with a fixed intercept. There is a large location shift in 2008.9 corresponding to the financial crisis and subsequent great recession, with a bimodal distribution for the data, but there are additional location shifts, notably a decline in s_t over 2015-2017 which also need to be modeled.

Figure 2 records other market-based variables (bond and goods markets), with the US

[2]Saturation techniques can also be applied to the system but the impulse indicators are detected based on tests for joint restrictions across all equations in the system.

[3]See the Data Appendix for further details including data sources.

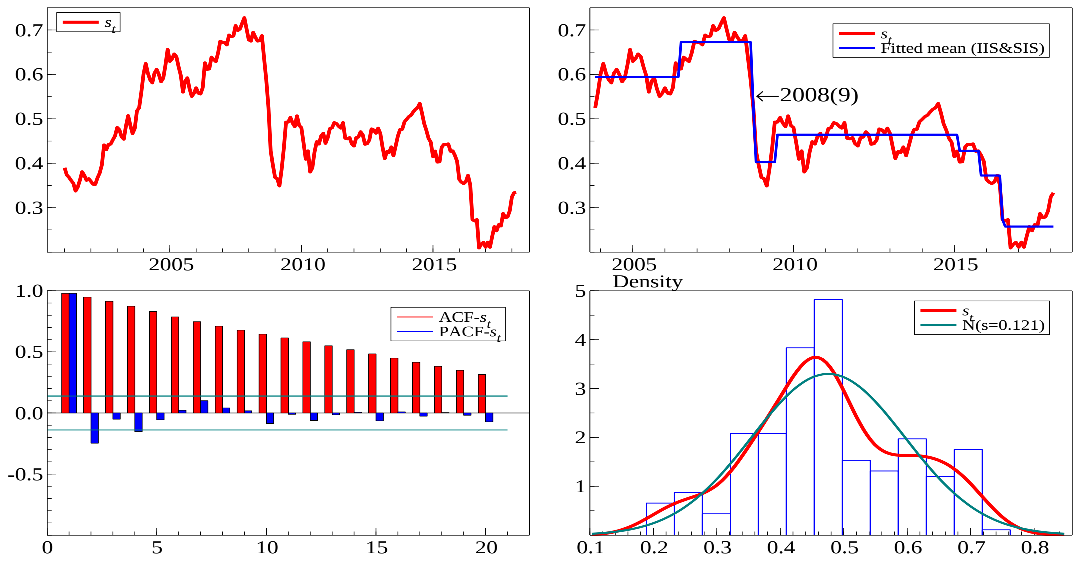

Figure 1: The log dollar-pound spot exchange rate (s_t), with a fitted mean using SIS (panel b), The autocorrelation and partial autocorrelation function for s_t and the histogram for s_t, recorded against a normal distribution.

(domestic) bond yield and inflation rate in solid red and the UK (foreign) bond yield and inflation rate in dashed blue. The nominal interest rates exhibit similar declines over the last decade, although the differences in annual inflation over the period are more stark.

Figure 3 records a class of policy-oriented variables, with the first panel plotting the US (domestic) and UK (foreign) monetary base, scaled to match means to view on the same figure (with original units in billions of US dollars and millions of pounds sterling before transformations). The periods of expansion of the monetary base due to active policy are evident, but are not aligned in the two countries. The US quantitative easing policy started around the end of 2008, while the quantitative easing policy in the UK commenced in March 2009. Despite this difference, there are similarities in the overall upward trending behaviour of the two series, which may indicate the presence of co-trending (see Hatanaka and Yamada, 2003), so that the inclusion of linear trend in the model may not be required in empirical analysis. In addition, although the monetary base is a stock variable and may thus be viewed as $\mathsf{I}(2)$ in general, checking Δmb_t and Δmb_t^* leads us to the judgement that both of the differenced series are non-integrated, or $\mathsf{I}(0)$, over the sample period of interest. Treating mb_t and mb_t^* as $\mathsf{I}(1)$ is therefore justified. The second panel of Figure 3 records the forward premium which is also subject to the two countries' monetary policies under the CIP condition.

All the data series exhibit unit roots and are treated as $\mathsf{I}(1)$, and they are subject to

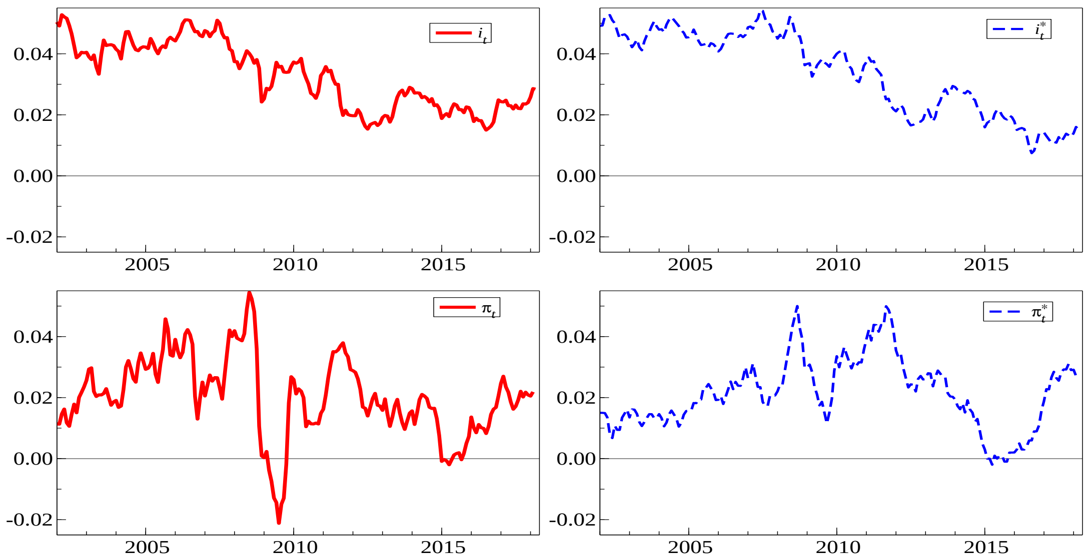

Figure 2: The US 10-year bond yield (i_t), the UK 10-year bond yield (i^*_t), the US annual CPI inflation rate (π_t) and the UK annual CPI inflation rate (π^*_t)

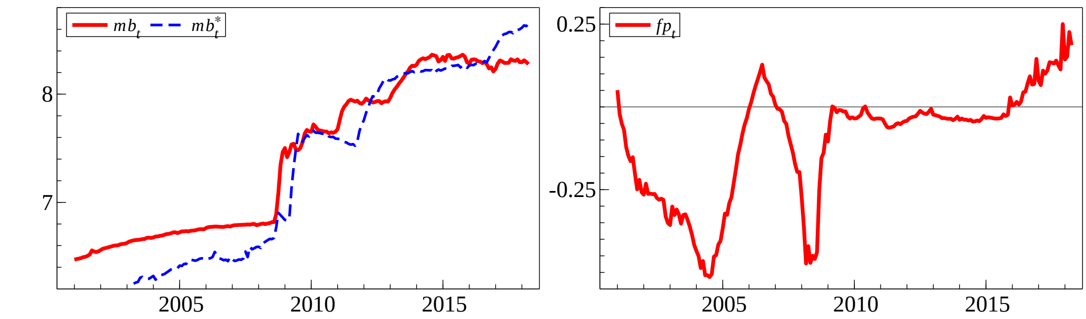

Figure 3: The US and UK monetary base in logs (mb_t, mb^*_t) and the forward premium (fp_t)

distributional shifts, with abrupt changes in mean and variance. Hence, the econometric methodology employed must handle both forms of non-stationarity to obtain a congruent model.

Referring to the PCVAR model with structural breaks in (11), we are justified, on several grounds, in classifying X_t into endogenous or modeled variables Y_t and conditioning variables Z_t which are assumed to be weakly exogenous for $(\alpha'_y, \beta', \gamma)$ in the following manner:

$$Y_t = (s_t, i_t, i^*_t, \pi_t, \pi^*_t)' \quad \text{and} \quad Z_t = (mb_t, mb^*_t, fp_t)' \,. \tag{12}$$

The first ground for this classification is that all the variables in Z_t in (12) are policy-driven, so that they exhibit too erratic behaviour to be modeled in the linear VAR framework; see Figure 3. This type of argument is found in Section 4 of Harbo, Johansen, Nielsen, and Rahbek (1998), and it provides a strong basis for a modeling strategy based on a partial model, instead of a large-dimensional full system. The second argument is that, given our research interest, we can regard Y_t in (12) as a class of variables of interest, which contains s_t and several other market-based variables, while modeling Z_t in (12) is considered to be a secondary interest. Lastly, we may be encouraged by the finding that a short-term interest rate, a representative policy-oriented variable, was judged to be approximately weakly exogenous in an empirical UK money demand model in Ericsson, Hendry, and Mizon (1998). Our US-UK empirical system may also be of some parallel structure such that (12) could be justifiable. The above classification of variables in view of the PCVAR system (11) sets the stage for general-to-specific econometric modeling.

5 A partial-system analysis with a shift in the level

A system-based analysis is conducted in this section. §5.1 determines the cointegrating rank of a PVAR model, and §5.2 then specifies cointegrating vectors consistent with the theoretical model presented in §2 and seeks evidence for the validity of further model reductions. §5.3 then arrives at a trivariate equilibrium correction model.

Given the variable classification in (12) we build a level-shifting partial system for Y_t conditional on the three policy variables Z_t, with the aim of revealing the underlying long-run structure. This framework also allows us to explore the possibility that Z_t is not only weakly exogenous but also super exogenous with respect to the parameters of interest. We need to ensure the invariance condition (Engle and Hendry, 1993) is satisfied for the validity of super exogeneity, also see Ericsson (1994). We note that there are three potentially coherent normalizations, see Boswijk (1996), and we motivate the preferred orientation in §5.3.

5.1 Testing for cointegrating rank

First, we introduce an unrestricted level-shift PVAR system for Y_t given Z_t, which is defined in (12), so that, with $p = 8$ and $m = 5$, the variable set is $X_t = (Y_t', Z_t')'$. The level-shift PCVAR system (11) in §3 is nested in this 5-dimensional PVAR system as a result of the cointegrating-rank restriction, which is determined by a sequence of partial log-likelihood ratio (PLR) tests, see Kurita and Nielsen (2019). As described in §3, Z_t is assumed to be weakly exogenous for the underlying parameters of interest, and, given the wide-sense non-stationarity of the system subject to distributional shifts we can also test for super exogeneity which combines weak exogeneity with the invariance of conditional parameters to interventions changing marginal parameters, see Hendry and Santos (2010).

We first determine the number of breaks q, the lag length k, and a class of unrestricted variables d_t; see equation (11). The data indicates a structural break around the end of 2008, which corresponds to the global economic recession triggered by the US financial crisis starting in 2008.9. Thus, selecting $q = 2$ with a break point 2008.9 is justifiable based on both the data and the historical record. This selection results in a classification of two regimes for the whole sample period, with the first regime's relative length $T_1/T = 0.352$, for $T_1 = 63$ and $T = 179$, when the lag length is 4 ($k = 4$). This lag order is chosen on the basis of $F-$test statistics for lag-order selection in preliminary regression analysis.

The impact of the Brexit referendum (23 June 2016) was also noticeable (see Powdthavee, Plagnol, Frijters, and Clark (2019) for further details of the referendum). Its magnitude was much smaller than the global recession, reflecting the effect of shocks on the dominant economy versus a shock on the UK alone which did not feed back to the US economy. Therefore, we use unrestricted dummy variables for possible outliers due to the Brexit referendum. We find outliers in 2016.7 and 2016.10 in the residuals of the s_t equation, which are attributable to the Brexit referendum and its aftermath. These outliers are handled by an unrestricted dummy variable being 1 in both 2016.7 and 2016.10, and 0 otherwise.[4] There is also an outlier in 2006.9 in the residuals of the π_t equation, caused by a decrease in petrol prices in the US. This outlier is modeled by an unrestricted dummy being 1 in 2006.9 and 0 otherwise. In addition, there is some evidence indicating seasonality-related autocorrelations in the residuals of the equation for π_t. Hence, an unrestricted 12-lagged second-order differenced series, $\Delta^2\pi_{t-12}$, is added to the PVAR(4) system, as proposed by Kurita and Nielsen (2009); this adjustment addresses the autocorrelation without affecting the underlying asymptotic theory in cointegration analysis.

We check diagnostic tests for the estimated level-shift PVAR model, in order to verify the specification adopted here is judged to be a valid statistical representation of the

[4]Unrestricted means that the dummy variable is not constrained to lie within the cointegrating space.

Table 1: Diagnostic tests for the PVAR(4) system with a level shift

Single-equation tests	s_t	i_t	i_t^*	π_t	π_t^*
Autocorr.[F_{AR}(7,124)]	1.50[0.17]	1.29[0.26]	1.33[0.24]	1.00[0.43]	1.22[0.30]
ARCH [F_{ARCH}(7,161)]	0.43[0.89]	0.99[0.44]	1.10[0.37]	0.34[0.94]	0.36[0.93]
Hetero.[F_{HET}(74,95)]	0.90[0.69]	1.21[0.19]	1.01[0.48]	0.64[0.98]	1.24[0.17]
Normality [χ^2_{ND}(2)]	1.44[0.49]	2.49[0.29]	4.19[0.12]	4.71[0.10]	2.27[0.32]

Vector tests		
	Autocorr.[F_{AR}(175,461)]	1.08[0.25]
	Hetero.[F_{HET}(370,459)]	1.16[0.07]
	Normality [χ^2_{ND}(10)]	11.43[0.33]

Note. Figures in square brackets are p-values.

Table 2: Testing for cointegrating rank in the PVAR(4) system with a level shift

	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	$r \leq 4$
PLR	141.02[0.00]**	94.17[0.04]*	58.33[0.15]	25.93[0.61]	6.03[0.93]

Note. Figures in square brackets are p-values according to Kurita and Nielsen (2018). $**$ and $*$ significance at the 1% and 5% level, respectively.

data. Table 1 displays a battery of residual diagnostic test outcomes for the PVAR(4) model.[5] We interpret the PVAR system as a well-specified general model and confirm the validity of the level-shift specification characterised by $q = 2$ and the break point 2008.9. In contrast to the clean diagnostics for the level-shift model in Table 1, the use of the broken-trend model of Kurita and Nielsen (2019) gives rise to some autocorrelation problems, possibly a reflection of collateral effects of the included piecewise linear trend. We thus proceed to the analysis of cointegrating rank in the level-shift PVAR model.

A sequence of PLR test statistics is presented in Table 2, in which the p-values of the statistics are calculated by using a response-surface table reported in Kurita and Nielsen (2019). We conclude that $r = 2$ is selected at the 5% significance level, which is also supportive of the theoretical exchange-rate model proposed; either of the estimated cointegrating combinations may correspond to one of the conceivable long-run relationships derived in §2. We next explore the underlying structure of the two cointegrating vectors.

[5] Test statistics are rounded to two decimal places, and most of them are presented in the form $\mathrm{F}_j(\cdot,\cdot)$, which indicates an approximate F test against the alternative hypothesis j. A class of the alternative hypotheses is given as follows: 7th-order serial correlation (F_{AR}: see Godfrey, 1978; Nielsen, 2006), 7th-order autoregressive conditional heteroscedasticity (F_{ARCH}: see Engle, 1982), heteroscedasticity (F_{HET}: see White, 1980), and a chi-squared test for normality (χ^2_{ND}: see Doornik and Hansen, 2008).

Table 3: Preliminary tests for a class of sub-hypotheses

	H1	H2	H3	H4	H5
PLR	$11.05[0.00]^{**}_{(df=2)}$	$2.3[0.13]_{(df=1)}$	$0.32[0.57]_{(df=1)}$	$4.73[0.09]_{(df=2)}$	$19.42[0.00]^{**}_{(df=1)}$

Note. Figures in square brackets are p-values according to $\chi^2(df)$, in which df denotes degree of freedom.

5.2 Theory-consistent long-run economic relationships

Having selected $r = 2$ in the level-shift model (11), we can examine various restrictions on $(\hat{\beta}', \hat{\gamma})$, along with those on $\hat{\alpha}$, to see if we can reveal empirical cointegrating relationships consistent with the theoretical formulations, such as (9). This approach towards theory-consistent long-run linkages was also adopted by Kurita (2007); see also Almaas and Kurita (2019). We have a class of four sub-hypotheses represented as a set of restrictions on $\hat{\beta}$ under $r = 2$, which are specified as follows:

H1 : Exclusion of s_t from both of the cointegrating relations

H2 : Exclusion of π_t and π_t^* from the first cointegrating relation

H3 : H2 $\cap$ The interest rate spread $i_t - i_t^*$ in the first cointegrating relation

H4 : H3 $\cap$ Exclusion of s_t, i_t and i_t^* from the second cointegrating relation

The test results are recorded in Table 3, which shows that the above hypotheses, except H1, are not rejected at the 5% level. The rejection of H1 implies either of the cointegrating relationships can work as a long-run exchange rate equation. We now normalise the first cointegrating vector with respect to s_t. The non-rejection of H2 against no restrictions is consistent with the theoretical formulation excluding the inflation rates, equation (9). The non-rejection of H3 against H2 also supports the view that the first cointegrating combination is interpretable as a long-run exchange rate equation given by (9).[6] Lastly, the non-rejection of H4 against H3 conveys useful structural information on the second long-run relationship concerning π_t and π_t^*.

[6] We have also checked the possibility of excluding each of i_t and i_t^* from the cointegrating relations; the corresponding test statistics are $PLR = 7.63[0.02]^*$ and $PLR = 10.46[0.005]^{**}$, respectively, according to $\chi^2(2)$. Thus, normalising the cointegrating vectors on these variables is also conceivable, i_t in particular, judging from its non-weak exogeneity status found in Table 4. However, the evidence for strong and highly-significant feedback effects with respect to s_t and π_t^*, as reported in Table 4, allows us to assign importance to the roles of s_t and π_t^* in the empirical long-run relationships.

The results obtained so far allow us to interpret the first cointegrating linkage as a long-run equation for s_t and the second linkage as a long-run equation for either π_t or π_t^*. Since additional analysis indicates π_t is weakly exogenous for the cointegrating vectors (as discussed below), it is natural to normalise the second vector for π_t^*. This normalisation can be justified by the examination of $\mathsf{H5}$:

$$\mathsf{H5}: \ \mathsf{H4} \cap \ \text{Exclusion of } \pi_t^* \text{ from the second cointegrating relation}$$

Table 3 shows $\mathsf{H5}$ against $\mathsf{H4}$ is rejected; in other words, the addition of the zero restriction to $\mathsf{H4}$ is not acceptable. The overall evidence in Table 3 is interpretable in terms of Boswijk (1996), thus allowing us to normalise the two vectors for s_t and π_t^*, respectively.

Reordering the variables in the level-shift model according to this normalisation scheme for s_t and π_t^*, we arrive at a set of restricted cointegrating relationships, along with the corresponding adjustment structure, reported in Table 4. The rounded estimates, their standard errors and the corresponding PLR statistic are provided in the table. The hypothesis of the overall restrictions is not rejected at the 5% level. The first 2×2 block of $\widehat{\beta}'$ is an identity matrix corresponding to an identification rule in Boswijk (1996), while the first 2×2 block of $\hat{\alpha}'$ is a diagonal matrix, so the structure is partially interpretable in the context of separate cointegration (see Konishi, 1993 and Granger and Haldrup, 1997), and can be related to a quasidiagonal form for the alpha matrix which has been considered in the context of exchange rate models by Hunter (1992). In addition, $\hat{\alpha}$ for π_t and i_t^* are zero, implying that these two variables are weakly exogenous for the parameters of interest β.

As a comparison, the use of the broken-trend model also permits the same restrictions on $\hat{\alpha}$ and $\hat{\beta}$ as those adopted in Table 4; moreover, the set of broken-trend terms are all excludable from the cointegrating combinations of the model, with the test statistic for the additional zero restrictions being $PLR = 5.44[0.25]$ according to $\chi^2(4)$. That is, no broken-trend terms are judged to be required. This comparative study ensures the validity of the level-shift specification, (11).

The panel for $(\widehat{\beta}', \widehat{\gamma})$ in Table 4 shows that the first cointegrating relationship leads to

Table 4: Restricted adjustment and cointegrating vectors

$\widehat{\alpha}'$	s_t	π_t^*	i_t	π_t	i_t^*
	-0.126 (0.032)	0 $(-)$	-0.005 (0.0018)	0 $(-)$	0 $(-)$
	0 $(-)$	-0.157 (0.022)	0 $(-)$	0 $(-)$	0 $(-)$

$(\widehat{\beta}', \widehat{\gamma})$	s_t	π_t^*	i_t	π_t	i_t^*	mb_t	mb_t^*	fp_t	$E_{1,t}$	$E_{2,t}$
	1 $(-)$	0 $(-)$	16.456 (2.437)	0 $(-)$	-16.456 $(-)$	-0.056 (0.0015)	0 $(-)$	-0.224 (0.081)	-0.257 (0.02)	0 $(-)$
	0 $(-)$	1 $(-)$	0 $(-)$	-1 $(-)$	0 $(-)$	0.044 (0.012)	-0.014 (0.009)	0 $(-)$	-0.132 (0.042)	-0.184 (0.048)

Note. The PLR test statistic for the joint restrictions is 19.45[0.194], where the figure in square brackets denotes a p-value according to $\chi^2(15)$.

an equilibrium correction term, $ecm_{1,t}$, defined as:

$$ecm_{1,t} = s_t - 0.055mb_t + 16.456\,(i_t - i_t^*) - 0.224fp_t - 0.257E_{1,t},$$

for $E_{1,t} = 1$ for $0 < t \leq T_1$ and 0 otherwise, with the break point 2008.9. The relationship $ecm_{1,t}$ matches (9), so that all the coefficients in $ecm_{1,t}$ are interpretable from the theory in §2. The adjustment coefficient for s_t in Table 4 is -0.126 and is highly significant, so the dollar-pound exchange rate reacts to $ecm_{1,t-1}$ by steadily adjusting disequilibrium errors. Hence, Frankel (1979)'s theoretical model is judged to be empirically relevant as a long-run equation for s_t.

Note the US monetary base is highly significant in $ecm_{1,t}$, while that of the UK is insignificant and thus removed from the combination. This asymmetry indicates that US monetary policy has a more dominant role in the determination of the dollar-pound exchange rate. Since i_t and i_t^* in $ecm_{1,t}$ are both annual rates, it is necessary to interpret their coefficients in terms of monthly rates $16.456\,(i_t - i_t^*) \approx 197.47\,(i_t - i_t^*)\,/12$, resulting in $\hat{\theta} = (197.47)^{-1} \approx 0.0051$. This estimate is seen as evidence for a near unit value of $1 - \theta$, consistent with numerous econometric studies on persistent deviations from PPP-based values in small samples, as reviewed by Sarno and Taylor (2002, Ch.3). The level parameter for $ecm_{1,t}$ is significant only in the first sub-sample period. This level shift is interpreted as a structural shift in the policy-oriented value of the exchange rate $\overline{s}_t$, as defined in equation (5); this could reflect the impact of quantitative easing adopted by the US monetary authority, a policy regime change in response to the global financial crisis.

Table 5: Testing for no level shift against the level shift alternative

	No level shift in $ecm_{1,t-1}$	No level shift in $ecm_{2,t-1}$
PLR	14.437[0.000]**	18.117[0.000]**

Note. Figures in square brackets are p-values according to $\chi^2(1)$.
$**$ denotes significance at the 1% level.

Similarly, the second cointegrating combination in Table 4 leads to:

$$ecm_{2,t} = \pi_t^* - \pi_t - 0.014mb_t^* + 0.044mb_t - 0.132E_{1,t} - 0.184E_{2,t},$$

in which $E_{1,t} = 1$ for $0 < t \leq T_1$, $E_{2,t} = 1$ for $T_1 < t \leq T$ and both 0 otherwise, with the break point 2008.9. This combination is interpreted as a long-run representation connecting the two countries' monetary base measures with the rates of inflation. Monetary expansion in the US given UK monetary policy tends to increase inflation in the US relative to that in the UK, although the UK monetary base plays a marginal role in the relationship. Since one of the primary objectives of monetary policy is to control inflation, this empirical evidence is informative in terms of macroeconomic policy coordination between the two countries. In addition, the adjustment coefficients in the table show that there is a sole significant feedback mechanism for the UK inflation rate π_t^*, so $ecm_{2,t-1}$ is critical for the stability of UK inflation.

The level shifts in the two cointegrating linkages highlight the importance of employing a PCVAR model allowing for structural breaks in the data. We test how significant the level shift is in each of the cointegrating combinations and check the possibility of co-breaking (see Hendry and Massmann, 2007) simultaneously. We remove the zero restriction on $E_{2,t}$ (see Table 4) and calculate the likelihood for a level shift specification. We then restrict the coefficients for $E_{1,t}$ and $E_{2,t}$ to be of the same value in each cointegrating combination, so that we can check the PLR test statistic for this restriction against the level shift alternative. Table 5 records the test statistics and their p-values, which indicate that the null hypothesis of no level shift is rejected at the 5% level for both of the relationships; the possibility of co-breaking is thereby excluded.

Figure 4 (a) and (b) record plots of $ecm_{1,t}$ and $ecm_{2,t}$, which are based on Table 4 and are mean-reverting stationary series except for some noticeable fluctuations around the

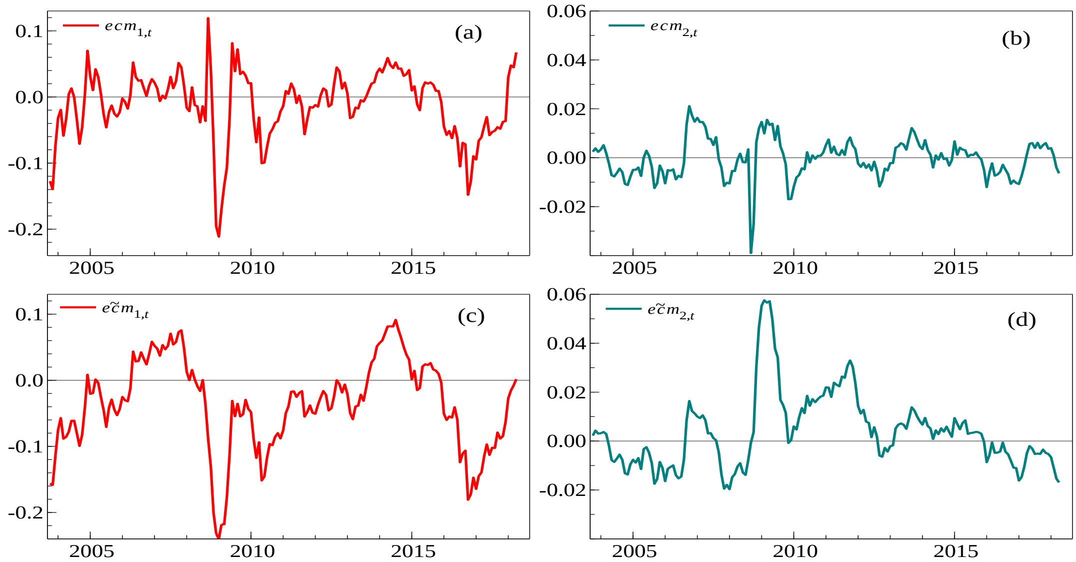

Figure 4: Plots of cointegrating combinations with level shift (a) and (b), and without level shift (c) and (d).

global recession in 2008 - 2009. These cointegrating combinations are clearly free from upward or downward trending features, so the sufficiency of the level-shift model (instead of the broken-trend one) has been graphically affirmed. As a comparison, Figure 4 (c) and (d) display the no level shift versions of $ecm_{1,t}$ and $ecm_{2,t}$ (denoted as $\widetilde{ecm}_{1,t}$ and $\widetilde{ecm}_{2,t}$ in the figure), which are based on Table 5; recall that this restriction was rejected for both of them as reported in the table. The series displayed in panels (c) and (d) suffer influences of the uncaptured level shift around 2008 - 2009. Both $\widetilde{ecm}_{1,t}$ and $\widetilde{ecm}_{2,t}$ appear to be non-stationary indicating a significant problem with the restriction of no level shift. The overall evidence supports the level shift specification in Table 4; modeling the structural break is essential in enabling the derivation of theory-consistent cointegrating linkages.

Lastly, we employ a set of *ex post* tests suggested in Harbo, Johansen, Nielsen, and Rahbek (1998), *inter alia*, to check if each of $Z_t = (mb_t, mb_t^*, fp_t)$ can be viewed as weakly exogenous for the parameters of interest fixed at their estimated values. First, we set up an $\mathsf{I}(0)$ marginal VAR system for ΔZ_t, which contains the two restricted cointegrating relationships ($ecm_{1,t-1}$ and $ecm_{2,t-1}$) as lagged regressors. Second, the marginal VAR model is estimated to check if the cointegrating relationships can be excluded from each equation in the model. The estimated marginal model is nothing but a preliminary data-representation, inevitably suffering from a number of diagnostic problems due to policy-driven irregularities in the data (see Figure 3), so any likelihood-based tests for this model

Table 6: Restricted adjustment and cointegrating vectors in a reformulated level-shift PCVAR system

$$\widehat{\alpha}' \begin{bmatrix} s_t & \pi_t^* & i_t & mb_t^* \\ \underset{(0.031)}{-0.141} & \underset{(-)}{0} & \underset{(0.0017)}{-0.004} & \underset{(0.057)}{-0.215} \\ \underset{(-)}{0} & \underset{(0.031)}{-0.156} & \underset{(-)}{0} & \underset{(-)}{0} \end{bmatrix}$$

$$(\widehat{\beta}', \widehat{\gamma}) \begin{bmatrix} s_t & \pi_t^* & i_t & mb_t^* & \pi_t & i_t^* & mb_t & fp_t & E_{1,t} & E_{2,t} \\ \underset{(-)}{1} & \underset{(-)}{0} & \underset{(2.104)}{18.811} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{-18.811} & \underset{(0.0013)}{-0.056} & \underset{(0.071)}{-0.276} & \underset{(0.018)}{-0.262} & \underset{(-)}{0} \\ \underset{(-)}{0} & \underset{(-)}{1} & \underset{(-)}{0} & \underset{(0.009)}{-0.013} & \underset{(-)}{-1} & \underset{(-)}{0} & \underset{(0.012)}{0.043} & \underset{(-)}{0} & \underset{(0.041)}{-0.137} & \underset{(0.047)}{-0.190} \end{bmatrix}$$

Note. The quasi PLR test statistic for the joint restrictions is 18.886 [0.091] , where the figure in square brackets denotes a p-value according to $\chi^2(12)$.

should be seen as quasi-type tests. The observed test statistics for exclusion restrictions on the cointegrating combinations are: 4.3[0.117] for the Δmb_t equation, 20.544[0.000]** for the Δmb_t^* equation and 2.015[0.365] for the Δfp_t equation.[7] We may have to doubt the weak-exogeneity property of mb_t^*, due to the quasi likelihood-based evidence for some feedback effects from the cointegrating combinations on Δmb_t^*.

As a further check of robustness, we have re-estimated the above restricted PCVAR model but treated mb_t^* as an additional modeled (left-hand side) variable and rendered π_t and i_t^* non-modeled (right-hand side) variables, so the model is now a 4-dimensional sub-system. Table 6 reports a set of updated estimates, which are almost equal to those in Table 4, a confirmation of the validity of the identifying restrictions employed in Table 4. Table 6 also provides the additional evidence that mb_t^* reacts solely to $ecm_{1,t-1}$, so that $ecm_{2,t-1}$ can naturally be interpreted as a long-run equation for π_t^*.

5.3 An I(0) vector equilibrium correction system

All the variables are now transformed to I(0) by using $ecm_{1,t-1}$, $ecm_{2,t-1}$ and first-order differences. Commencing with the general trivariate model for $\Delta Y_{1t} = (\Delta s_t, \Delta i_t, \Delta\pi_t^*)'$ conditional on $\Delta Y_{2t} = (\Delta i_t^*, \Delta\pi_t)'$ and ΔZ_t for $Y_t = (Y_{1t}', Y_{2t}')'$, we test reductions of the model to arrive at the following parsimonious vector equilibrium correction (VEC)

[7] Figures in square brackets are p-values according to $\chi^2(2)$.

system:

$$\Delta s_t = \underset{(0.061)}{0.179}\ \Delta s_{t-1} + \underset{(0.061)}{0.163}\ \Delta s_{t-3} - \underset{(0.449)}{1.308}\ \Delta \pi^*_{t-2} - \underset{(0.026)}{0.114}\ ecm_{1,t-1} + \underset{(0.785)}{1.883}\ \Delta i^*_t$$
$$+ \underset{(0.273)}{0.722}\ \Delta \pi_t - \underset{(0.023)}{0.114}\ (\Delta fp_t - \Delta mb^*_t + \Delta mb_t) - \underset{(0.012)}{0.075}\ d_{t,Brexit}, \tag{13}$$

$$\Delta i_t = -\underset{(0.003)}{0.009}\ \Delta s_{t-1} + \underset{(0.015)}{0.03}\ \Delta \pi_{t-1} + \underset{(0.003)}{0.007}\ \Delta s_{t-3} - \underset{(0.015)}{0.052}\ \Delta \pi_{t-3}$$
$$- \underset{(0.0014)}{0.005}\ ecm_{1,t-1} + \underset{(0.042)}{0.931}\ \Delta i^*_t + \underset{(0.0014)}{0.0068}\ \left(\Delta mb^*_{t-1} - \Delta mb^*_{t-3}\right) \tag{14}$$
$$+ \underset{(0.0024)}{0.0085}\ \Delta fp_t + \underset{(0.0025)}{0.0057}\ \Delta fp_{t-3} - \underset{(0.001)}{0.0026}\ d_{t,2008.9} - \underset{(0.0011)}{0.006}\ d_{t,2008.12},$$

$$\Delta \pi^*_t = \underset{(0.042)}{0.132}\ \Delta \pi_{t-1} - \underset{(0.024)}{0.076}\ ecm_{2,t-1} + \underset{(0.043)}{0.266}\ \Delta \pi_t - \underset{(0.006)}{0.015}\ \Delta mb_t$$
$$+ \underset{(0.0022)}{0.0059}\ d_{t,2008.9} + \underset{(0.0028)}{0.0065}\ d_{t,2008.11}, \tag{15}$$

Vector tests : $F_{AR}(63{,}430) = 1.13[0.24]$, $F_{HET}(222{,}791) = 1.10[0.19]$, $\chi^2_{ND}(6) = 4.15[0.66]$,

where standard errors are given in parentheses. A number of contemporaneous regressors play significant roles in accounting for the dynamics of the three endogenous variables. If we focus on influences of $\Delta Y_{2t} = (\Delta i^*_t, \Delta \pi_t)'$, we find Δi^*_t is significant in the equations for Δs_t and Δi_t, while $\Delta \pi_t$ is in the equations for Δs_t and $\Delta \pi^*_t$; their coefficients are all consistent with economic theory. Various influences from $\Delta Z_t = (\Delta mb_t, \Delta mb^*_t, \Delta fp_t)'$ are also noted; in particular, they play important roles in the Δs_t and Δi_t equations. Recall the CIP condition justifies $fp_t \approx i^s_t - i^{s,*}_t$. We can, therefore, interpret Δfp_t as a difference between the dynamics of the two countries' short-term policy interest rates, implying that a relative increase in the US rate to the UK rate results in an appreciation in the US dollar against the UK pound as well as a rise in the US long-term interest rate. Positive and negative coefficients for Δmb^*_t and Δmb_t in the Δs_t equation indicate the presence of an overshooting phenomenon in the foreign exchange market. A monetary expansion in the UK, for example, brings about a rapid overshooting depreciation in pound sterling, so that it then moves back afterwards to cancel out the overshooting part. This cancellingout effect is captured by Δmb_t and Δmb_t^* in the above system. It should be noted that various dummy variables are significant in this conditional system, indicating the failure of an invariance condition required for super exogeneity of ΔY_{2t} and ΔZ_t with respect to the parameters of interest; see Engle and Hendry (1993).

Figure 5 (a), (c) and (e) compare the actual values of ΔY_{1t} with their fitted values derived from the VEC system, while Figure 5 (b), (d) and (f) present a set of scaled residuals of the equations in the system. Consistent with the vector diagnostic tests reported above, there are no significant mis-specification problems in the figure. Overall, the VEC system is judged to be a data-congruent representation subject to economic interpretations.

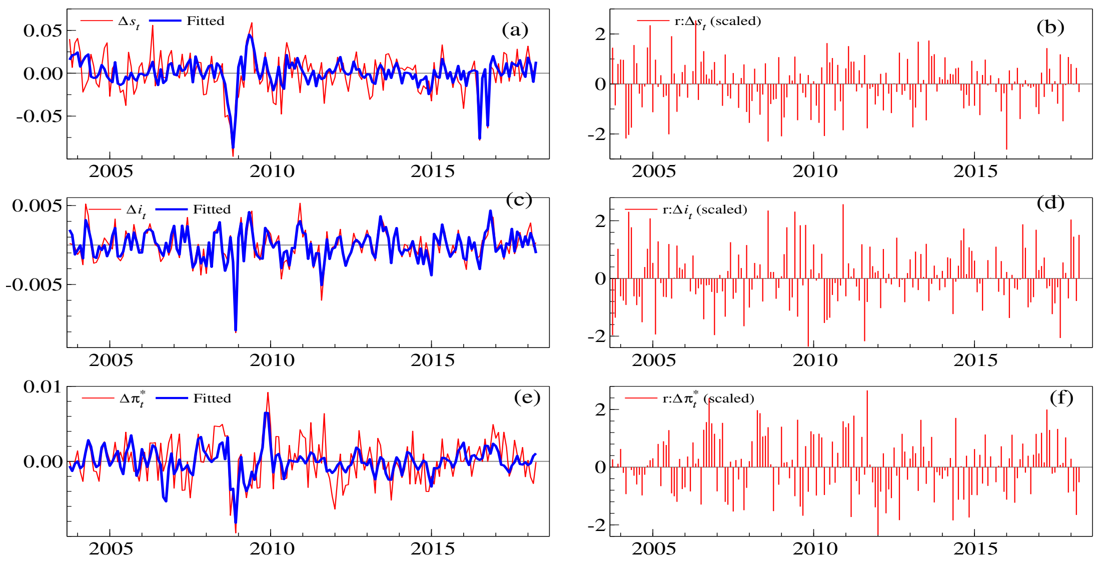

Figure 5: Actual values, fitted values and scaled residuals

The equation for Δs_t in the system, equation (13), contains lagged regressors along with a class of contemporaneous explanatory variables. This structure indicates that the dynamics of the dollar-pound exchange rate are predictable to some extent by means of lagged information. We have checked that the lagged variables are all highly significant in equation (13) even after we removed the contemporaneous variables (that is, Δi_t^*, $\Delta \pi_t$ and $\Delta fp_t - \Delta mb_t^* + \Delta mb_t$) from it. The finding of forecastability is remarkable but in line with existing studies such as MacDonald and Marsh (1997), and it is encouraging for a comparative forecasting analysis pursued in §7. We will focus on the refinement of equation (13) in §6 to assess the impact of the Brexit referendum on the dollar-pound exchange rate dynamics in a single-equation framework. Since neither i_t nor

π_t^* was judged to be weakly exogenous for the parameters of interest, focusing on the single-equation model for the exchange rate dynamics may give rise to some loss of information for statistical inference, but this is of second order importance when forecasting in §7.

6 Single-equation analysis of the spot exchange rate

We apply IIS to the Δs_t equation to check for unmodeled outliers. The retained variables from (13) are not selected over, but the Brexit dummy is omitted to see if saturation picks it up. The significance level for selection of impulse indicators is $\alpha = 0.0057\,(\approx 1/T)$. The resulting model picks up 6 impulse indicators. The two Brexit dummies for 2016.7 and 2016.10 are selected and are highly significant ($\mathsf{t}_{(2016.7)} = -4.92$ and $\mathsf{t}_{(2016.10)} = -4.55$) and could be combined as in the multivariate model. We also detect four further outliers, namely 2004.3, 2004.12, 2006.5 and 2016.1. They are recorded in Figure 6, with $\pm 2\sigma$ error bars around the impulse indicators. The additional indicators do not change the model significantly. The equation standard error is reduced from $\widehat{\sigma} = 1.67\%$ to $\widehat{\sigma}_{IIS} = 1.57\%$ and the short run effect of i_t^* weakens to become marginally insignificant at the 5% level. Interestingly, there are no indicators picked up over the financial crisis period, so the level shift included in the equilibrium correction mechanism is sufficient to model this period.

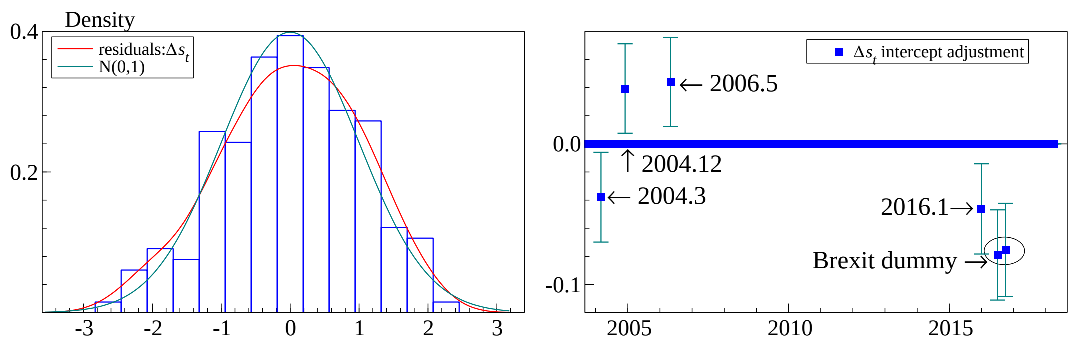

Figure 6: Single equation model of Δs_t with IIS residual density and intercept adjustment with error bars plotting $\pm 2\sigma$ for retained impulse indicators.

7 Forecasting the spot exchange rate

To evaluate the model out-of-sample we extend the dataset to 2020.10, recorded in figure 7.[8] The forecasting exercise was undertaken after the model was established so there is no knowledge of the out-of-sample data embodied within the econometric model. The gains seen in the exchange rate over 2017, following the sharp decline due to the Brexit referendum, have since been eroded over the forecast horizon. The forecast period covers many events including the Covid-19 pandemic, the US election and the UK leaving the European Union. These structural breaks, along with many other global events, likely impact the exchange rate, making the period particularly interesting to forecast. Clements and Hendry (2011) and Castle, Clements, and Hendry (2015), *inter alia*, discuss the main culprit of forecast failure, namely unanticipated location shifts. As such, we also consider a robust device based on the VECM to forecast both the spot exchange rate and the change in the spot exchange rate at horizons of 1-month, 6-months and 12-months ahead. Section §7.1 outlines the models considered and §7.2 presents the results.

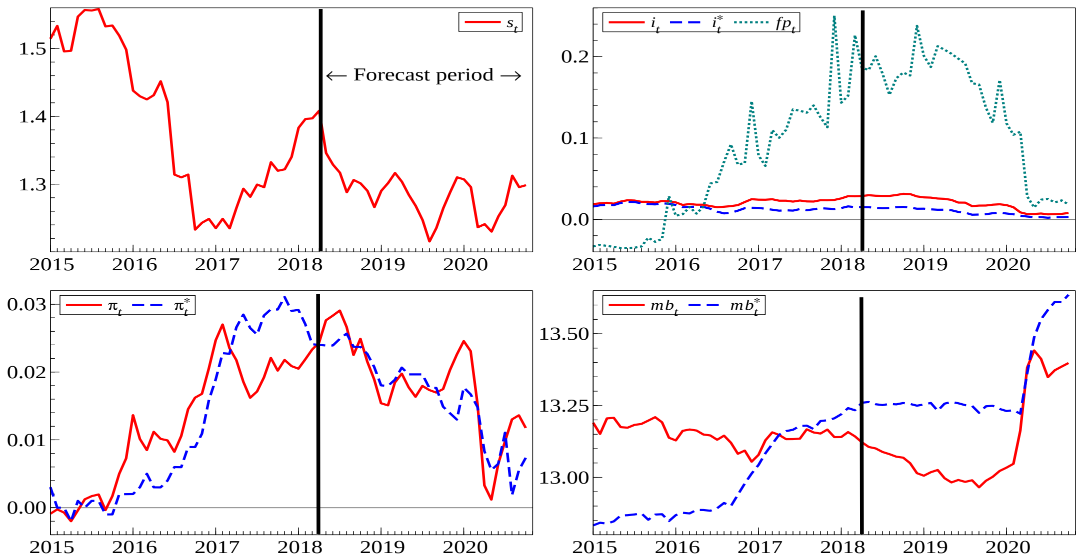

Figure 7: Extended sample for s_t; i_t and i_t^* along with fp_t; π_t and π_t^*; and mb_t and mb_t^*, with the forecast period covering 2018.5-2020.10.

[8] The Data Appendix details the updated data.

7.1 Forecasting Models

Six forecasting models are considered, including multivariate and single equation equilibrium correction models, a robust variant, and some naïve benchmarks.[9] Forecasts are produced from the models for $\widehat{\Delta s}_{T+h+j|T+j}$, where T is the last in-sample observation, h defines the forecast horizon ($h = 1, 6, 12$) and j defines the forecast period ($j = 2018.5 - 2020.10$). Median level forecasts for the spot exchange rate are obtained using $\widehat{S}_{T+h+j|T+j} = \exp\left(\sum_{i=1}^{h} \widehat{\Delta s}_{T+h+j|T+j} + s_{T+j}\right)$ for $h > j$, with dynamic level forecasts for the first h forecasts.[10]

The equilibrium correction models belong to the class of open models where the exogenous regressors must also be forecast to produce *ex ante* spot exchange rate forecasts. The forecasting models include:[11]

1. VECM (§5.3).

2. Robust model given by the differenced VECM (Castle, Clements, and Hendry, 2015) obtained by differencing (11), and using adaptive estimates of the equilibrium mean and growth rate. Two levels of smoothing are compared, including [2a] using the last in-sample observation to compute the equilibrium mean and growth rate and [2b] using the last 12 in-sample observations to introduce smoothing, but will reduce adaptability to shifts.

3. ECM (§6).

   Two alternative specifications are considered depending on how the exogenous regressors are treated. Offline variables can be [3a] forecast or [3b] ignored, where direct projections are made including just the lagged dependent variable and ecm_1 term.

4. VAR(1) including all 8 variables defined in X with one lag.

[9]Non-linear forecasting models such as SETAR (Clements and Smith, 2001) and STAR (Sarantis, 1999) are not included in our forecasting exercise as Boero and Marrocu (2002) argue that linear models demonstrate better forecast performance when evaluating point forecasts across regimes, as our set-up does.

[10]We compute median level forecasts rather than mean level forecasts which require an additional variance correction. Assuming $s_{T+h|T} \sim \mathsf{N}\left[\widehat{s}_{T+h|T}, \widehat{\sigma}^2_{s,T+h|T}\right]$, then $\mathsf{E}\left(S_{T+h|T}\right) = \exp\left(\widehat{s}_{T+h|T} + \frac{1}{2}\widehat{\sigma}^2_{s,T+h|T}\right)$, see Doornik and Hendry (2013a, ch.18).

[11]Details of the forecasting models are given in the supplementary material.

5. AR(1) for Δs_t. $h > 1$-step ahead forecasts are computed by iterating forward by $h-$steps.

6. Random Walk for either the level or change in the exchange rate, i.e. $\widehat{\Delta s}_{T+h+j|T+j} = \Delta s_{T+j}$ and $\widehat{S}_{T+h+j|T+j} = S_{T+j}$.

7.2 Forecasting Results

Table 7 reports summary statistics including the Root Mean Square Forecast Error (RMSFE) and the log determinant of the general matrix of the forecast-error second moment (GFESM), see Clements and Hendry (1993) and Hendry and Martinez (2017). Define the forecast errors as $\widehat{v}_{T+h+j|T+j} = \Delta s_{T+h+j} - \widehat{\Delta s}_{T+h+j|T+j}$ or $\widehat{v}_{T+h+j|T+j} = S_{T+h+j} - \widehat{S}_{T+h+j|T+j}$, where $\widehat{\cdot}$ denotes the forecast. Stack the forecast errors over j such that $\underset{(1\times J)}{\widehat{\mathbf{v}}_h} = \left(\widehat{v}_{T+1|T}, \ldots, \widehat{v}_{T+h+1|T+1}, \ldots, \widehat{v}_{T+J|T+J-h}\right)$, where the first h forecast errors are dynamic, and $h-$step ahead from thereon, and then stack the different horizon forecast errors to obtain $\underset{(3\times J)}{\widehat{\mathbf{W}}} = (\widehat{\mathbf{v}}_1, \widehat{\mathbf{v}}_6, \widehat{\mathbf{v}}_{12})'$. The forecast statistics are given by $\mathsf{RMSFE}_h = \sqrt{(\widehat{\mathbf{v}}_h \widehat{\mathbf{v}}_h')}$, and $\mathsf{GFESM} = \log\left|\frac{1}{J}\left(\widehat{\mathbf{W}}\widehat{\mathbf{W}}'\right)\right|$. GFESM has advantages over RMSFE because its rankings for forecasts can differ, both depending on whether the levels or differences are evaluated, and over the horizon. Christoffersen and Diebold (1998) highlight that standard forecast accuracy measures (the trace MSE, of which the reported RMSFE is the univariate equivalent) fail to value the maintenance of cointegrating relations at longer horizons, so the more general GFESM may be preferable.

Accurately forecasting the exchange rate is difficult, as even for 1-month ahead the smallest RMSFE for Δs_t is 1.88%, which compares to the VECM equation standard error over the full sample of $\widehat{\sigma} = 1.67\%$.

The VECM, our theory consistent and data admissible model, produces the best forecasts at the 1-month horizon for both the level and differences, implying it contains useful information over and above the benchmark devices. At longer horizons it performs worse, with the largest RMSFE at the 1-year horizon for S_t. Despite this the GFESM, which accounts for correlations across horizons, ranks it as producing the most accurate forecasts for the level of the exchange rate.

The robust device with 1 month of smoothing frequently ranks as one of the poorest

Table 7: RMSFE $\times 100$ and GFESM for forecast period 2018.5–2020.10.

	Δs_t				S_t			
	RMSFE ($\times 100$)			GFESM	RMSFE ($\times 100$)			GFESM
	1m	6m	12m	$\forall h$	1m	6m	12m	$\forall h$
1. VECM	**1.88**	2.88	2.81	-26.89	**2.43**	11.63	20.79	**-25.33**
2a. Robust (1)	2.26	3.33	3.05	-22.24	2.92	14.13	18.30	-15.02
2b. Robust (12)	*1.88*	2.55	2.35	-25.65	*2.43*	11.70	16.58	-16.27
3a. ECM	2.05	2.73	2.70	-29.35	2.66	12.89	19.99	-16.13
3b. ECM direct	2.24	2.07	*1.93*	-26.89	2.91	6.01	9.62	-17.53
4. VAR	2.04	*2.00*	2.00	*-34.61*	2.64	6.60	9.77	-17.50
5. AR	1.89	**1.93**	**1.93**	**-41.45**	2.44	*5.97*	*9.04*	-17.97
6. RW	2.45	2.80	2.72	-22.36	2.48	**5.47**	**5.84**	*-19.13*

Note. Bold indicates smallest RMSFE/GFESM and italic indicates second smallest RMSFE/GFESM, with underline indicating largest RMSFE/GFESM. The 1−step ahead forecasts in levels and differences are not identical because the levels are given by the spot exchange rate (S_t) and not the log spot exchange rate (s_t).

forecasting models. This device is designed to adapt to sudden location shifts but the trade-off for such adaptability is a larger forecast variance. Over this period the costs of increasing the forecast error variance outweigh any gains from rapidly updating a shifting mean. At shorter horizons the robust device using a year of smoothing does produce accurate forecasts (ranked second after the VECM). The robust devices are not designed for long range forecasts where the available information set does not allow for rapid updating.

For the single equation models, at longer horizons it clearly pays to ignore conditioning variables rather than attempting to forecast them. Forecasting the exogenous regressors adds further forecast uncertainly which is costly in forecasting S_t, and the gains from excluding the exogenous regressors more than halve the RMSFE at the 6-month and 12-month horizons. This is in keeping with the results from Hendry and Mizon (2012) who show that many more mistakes can be made when forecasting exogenous regressors offline in addition to the variable of interest.

For the change in the exchange rate, the AR(1) benchmark is very competitive at all horizons. At longer horizons the autoregressive model converges to its unconditional mean, which is approximately zero in differences. This uninformative forecast mitigates risks and performs well over the given forecast period where there are no systematic swings in the exchange rate. The random walk, although not competitive for the difference forecasts, is hard to beat at longer horizons for the level of the exchange rate. This is in keeping with Meese and Rogoff (1983b), where the random walk has dominated as the benchmark forecasting device, with models evaluated on their ability to produce superior forecasts to the random walk.

In many cases, forecast accuracy of Δs_t can improve with forecast horizon. Such a result refutes standard forecasting theory that forecasts worsen as the forecast horizon increases and is due to the non-stationarity of the data.

The results highlight the distinction between empirical modeling to test theories or understand economic phenomena and forecasting. The VECM model forecasts are the best at 1-month ahead but are worse than the random walk at 6-months and 12-months ahead. Ever since Meese and Rogoff (1983a, 1988), attempts to outperform a random walk in out-of-sample forecasting have struggled at short horizons, although the relative performance of structural models tends to improve at longer horizons. Our results contrast this, suggesting that the short-run dynamics embodied in the VECM are useful in forecasting at short horizons but less so further out. The implications of the forecasting exercise are that congruent models should not be discarded but should be seen as complimentary to benchmark devices.

8 Conclusions

The paper develops a fundamental-based model of the dollar-pound exchange rate in which the monetary stances of the central banks are incorporated into a sticky-price monetary model, with the foreign exchange forward premium capturing expectations. The model hinges on the explicit modeling of a structural break for the financial crisis and subsequent global recession, triggered in September 2008. Modeling the structural break is essential to finding stable, mean-reverting long-run equilibrium relationships that capture the cointegrating relations in the model. The partial cointegrated VAR with level shifts is a new class of econometric models and reveals theory-based relationships that would otherwise have been obscured by the non-stationary data subject to distributional shifts. The resulting parsimonious vector equilibrium system is a data-congruent representation with theoretically justified economic interpretations.

The financial crisis and global recession is not the only structural break that needs modeling. The Brexit referendum took place near the end of the sample period, and had a large effect on the dollar-pound exchange rate, detected using IIS. IIS also checks the robustness of the VECM specification, demonstrating that the modeled level shift in the cointegrating relations was sufficient to capture the structural break.

A forecasting exercise is undertaken to assess how well the model performs out-of-sample. The model forecasts well 1-month ahead, but the random walk benchmark for the exchange rate is hard to beat at longer horizons. We show that (i) forecast accuracy need not decline with forecast horizon; (ii) forecast performance depends on the transformation of the dependent variable so it matters if forecasting the exchange rate or exchange rate appreciation; and (iii) in open models it pays to omit contemporaneous regressors when forecasting, rather than forecasting the exogenous variables separately. Our results highlight that a model should not be judged on its forecast performance as this could be influenced by unanticipated events occurring in the forecast period.

Our research highlights the importance of taking the data seriously when modeling the exchange rate. Non-stationarity, both in the form of unit roots and distributional shifts, are handled with care to develop a congruent, theory-consistent and data-admissible empirical model of the dollar-pound exchange rate. Anticipating the distributional shifts would resolve the apparent disconnect between modeling and forecasting, but that is left for future research.

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Data Appendix

(Data definitions, sources and notes)

Data definitions (figures in angle brackets represent source numbers):

s_t	The log of the spot dollar-pound exchange rate; Series code: XUMAUSS in <1>.
i_t	US 10-year treasury constant maturity rate in decimal form; Series code: GS10 in <3>.
i_t^*	UK 10-year government securities yield in decimal form; IUMAMNPY in <1>.
p_t	The log of the US consumer price index for all commodities; Series code: CPIAUCNS in <4>.
p_t^*	The log of the UK consumer price index for all commodities; Series code: D7BT in <2>.
π_t	12th-order difference of p_t, *i.e.* $\Delta^{12}p_t$
π_t^*	12th-order difference of p_t^*, *i.e.* $\Delta^{12}p_t^*$.
mb_t	The log of the US monetary base (seasonally adjusted); Series code: AMBSL in <5>.
mb_t^*	The log of the UK monetary base (constructed series; see the notes below); <1>.
fp_t	Forward premium (discount) rate, 1 month; Series code: XUMADF1 in <1>.

Sources (all the sources below were accessed on 7-9/07/18 and then again on 25/11/20):

<1>	Bank of England (BoE) Statistical Interactive Database (http://www.bankofengland.co.uk/boeapps/iadb/).
<2>	Office for National Statistics (https://www.ons.gov.uk/economy/inflationandpriceindices/timeseries/d7bt/mm23#).
<3>	Board of Governors of the Federal Reserve System (US), 10-Year Treasury Constant Maturity Rate, retrieved from FRED, Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/GS10).
<4>	U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items, retrieved from FRED, Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/CPIAUCNS).
<5>	Federal Reserve Bank of St. Louis, St. Louis Adjusted Monetary Base, retrieved from FRED, Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/AMBSL).

Notes:

i_t, i_t^* The original series were divided by 100 to obtain interest rates in decimal form.

mb_t Updates of AMBSL ceased in 2019.12. Data for 2020 on total monetary base obtained from H.6 statistical release from Board of Governors (https://www.federalreserve.gov/releases/H6/default.htm). As Board of Governors data is not seasonally adjusted, the difference between the SA and NSA data for each month in 2019 is used to adjust the NSA data in 2020 to give a SA series.

mb_t^* Since UK data equivalent to those for the US monetary base are unavailable, we constructed data for the UK monetary base by adding data for notes and coins in circulation (Series code: LPMB8H4; seasonally adjusted) to those for bank reserves (Series codes: RPWAEFI and LPMBL22). That is, we constructed the following series, depending on the availability of the data:

M_t^* = (LPMB8H4 + RPWAEFI for 2003.6 - 2006.4)
and (LPMB8H4 + LPMBL22 from 2006.5 onwards),

so that $\log M_t^* = m_t^*$. Note that the data for RPWAEFI are available on a weekly basis, so we averaged the weekly data to obtain the corresponding monthly data. Extending LPMB8H4 to 2020.10, the data are revised historically back to 2001, although not substantially. We use the original data to 2018.4 and then splice the revised data to extend the dataset.