

Supporting Information

Coarse-graining the fluid flow around a human sperm

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1. Supplementary Data

RegStokesletParameters.txt

This file provides the regularized Stokeslet parameters for each PCA mode of the velocity field, labelled by s in the Main Text. We consider the decomposition $\tilde{\mathbf{u}}^s(\mathbf{x}) = \sum_{k=1}^K \mathbf{f}^{(s,k)} \cdot \mathbf{G}_{\epsilon^{(s,k)}}(\mathbf{x}, \mathbf{x}_0^{(s,k)})$ as in the main text, where \mathbf{G}_{ϵ} is the regularized Stokeslet. The rows of the .txt file correspond to the Stokeslets data for the lowest five PCA modes, in this case $K = 3, 3, 4, 6, 5$ for each mode respectively. The set of parameters are: $x_0^{(s,k)}, y_0^{(s,k)}, f_x^{(s,k)}, f_y^{(s,k)}, \epsilon^{(s,k)}$, where $\mathbf{x}_0^{(s,k)} = (x_0^{(s,k)}, y_0^{(s,k)}, 0)$ shows positions of the regularized Stokeslet k of the mode s . Analogously $\mathbf{f}^{(s,k)} = (f_x^{(s,k)}, f_y^{(s,k)}, 0)$ indicates the strength of the regularized Stokeslet, and $\epsilon^{(s,k)}$ is the regularized parameter. These values are shown in this order. All the values are non-dimensionalized using the flagellar length $L = 50\mu\text{m}$, the flagellar beat period $T \approx 0.424\text{sec}$ and a mass scale such that viscosity of water non-dimensionalizes to unity. The configurations of the regularized Stokeslet approximation are illustrated in Supplementary Figure 1.

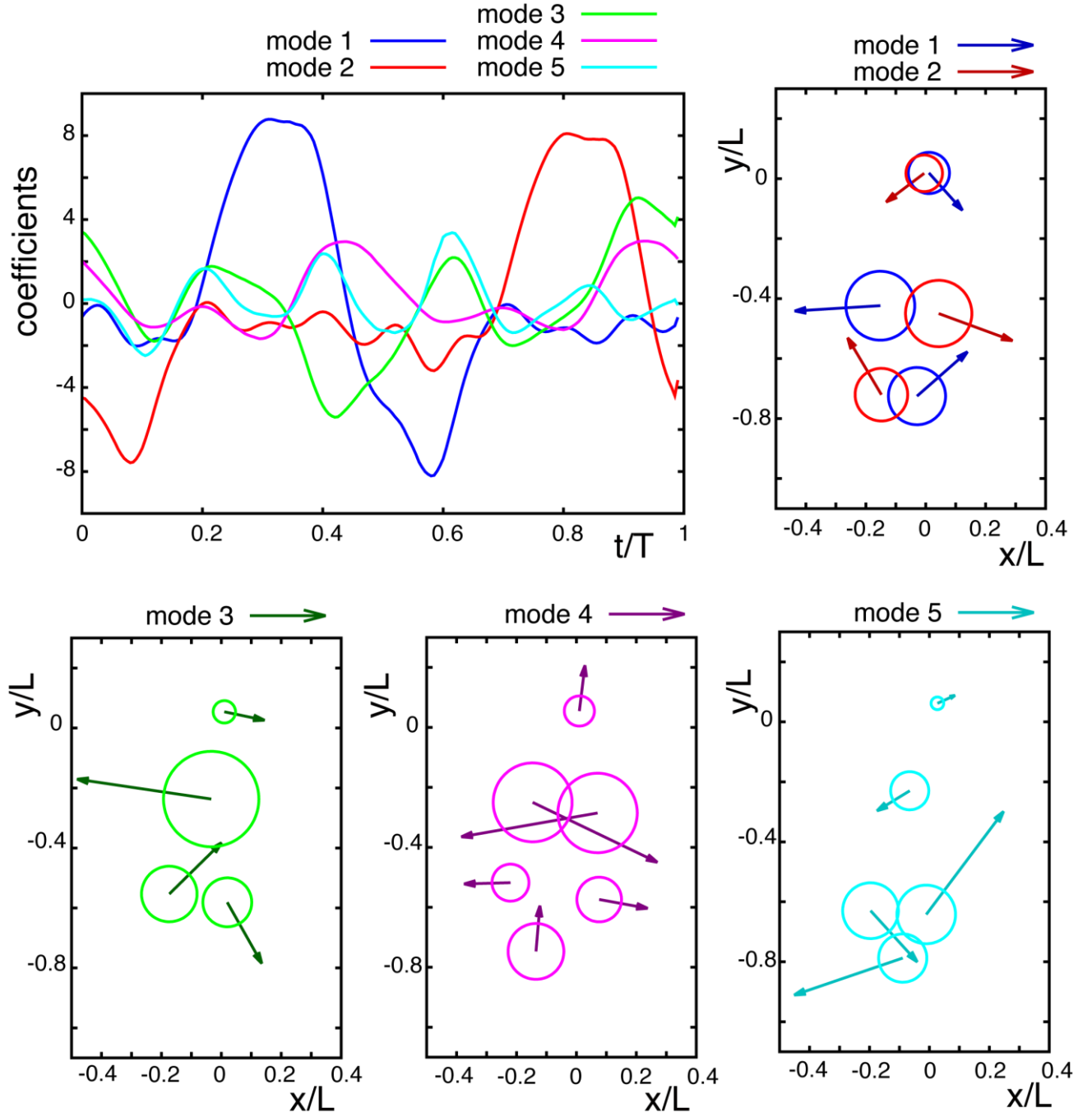
RegStokesletCoef_time.txt

This file has the time-evolution of the time dependent coefficients for each Regularized Stokeslet basis extracted from the associated Principal Component Analysis (PCA) mode decomposition of the velocity field. The columns show the time evolution of the coefficients for the lowest five regularized multiplet bases. The instantaneous velocity field at time t is reconstructed via $\mathbf{u}(\mathbf{x}, t) = \sum_{s=1}^S w_s(t) \tilde{\mathbf{u}}^s(\mathbf{x})$, where $\tilde{\mathbf{u}}^s(\mathbf{x})$ is the regularized Stokeslet approximation for the velocity field of PCA mode s , and S denotes the number of the bases for the reconstruction. In this text file, the time-dependent weight $w_s(t)$ is listed. The first column shows the non-dimensional time t/T and the following five columns show the weight $w_s(t)$ for the PCA modes $s = 1, 2, \dots, 5$. All the values are non-dimensionalized using the flagellar length $L = 50\mu\text{m}$ and flagellar beat period $T \approx 0.424\text{sec}$. The time evolution of the weights is plotted in Supplementary Figure 1.

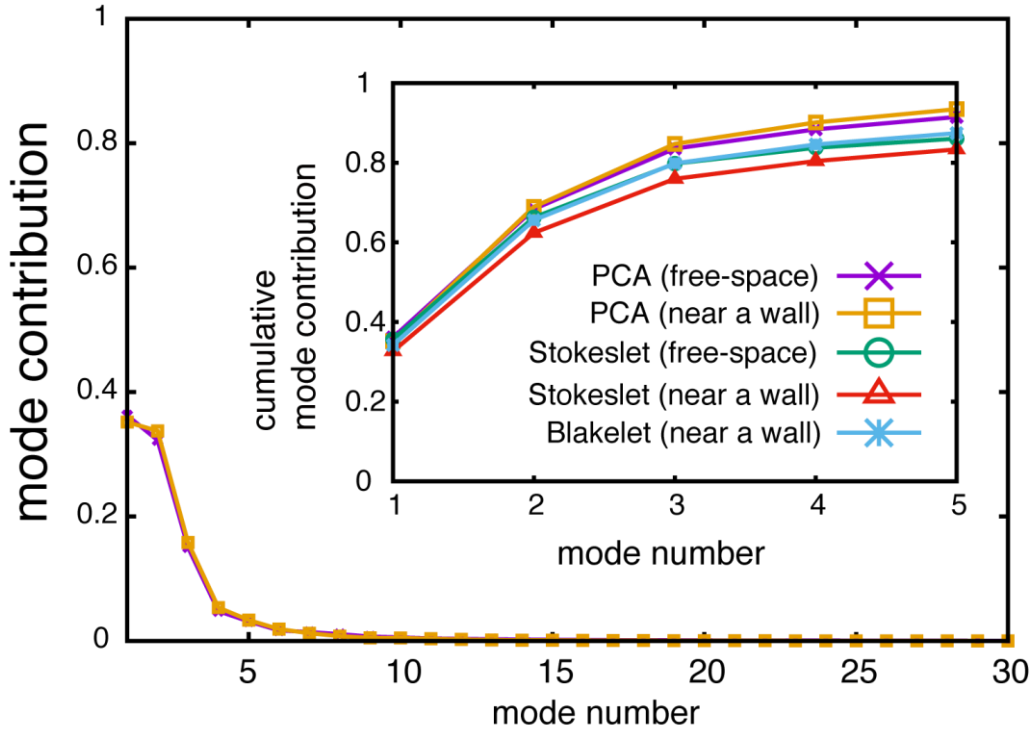
LabFrameTrajectory.txt

The reconstructed velocity field $\mathbf{u}(\mathbf{x}, t)$ has its origin at the head-tail junction, thus this junction's trajectory in the laboratory frame is required in order to obtain the velocity field in the laboratory fixed frame of reference. This .txt file contains the time evolution of the head-tail junction location for the sperm computed by the boundary element simulation, with the flagellar waveform given by the limit cycle associated with three PCA flagellar shape modes, as Fig. 2 of the main text. The three columns show the time evolution of sperm trajectory with parameters: $t/T, X/L, Y/L$, where $\mathbf{X} = (X, Y, 0)$ are the position for the head-tail junction, the flagellar length $L = 50\mu\text{m}$ and flagellar beat period $T \approx 0.424\text{sec}$.

2. Supplementary Figures



Supplementary Figure 1. Upper left plot. Time-evolution of the basis coefficients for each PCA mode, and its associated regularized singularity expansion, for the velocity field, with T denoting the beat period, as in Fig.4d and Fig.4e of the main text. These coefficients are available in the file `RegStokesletCoef_time.txt`. Upper right and lower plots. The force multiplet approximation of PCA mode 1 to 5 for the fluid velocity field. The origin, magnitude and direction of the regularized force singularities, with the circle radius corresponding to the magnitude of the regularization parameter, as in Fig. 4c of the Main Text. The regularized Stokeslet parameters are available in the file `RegStokesletParameters.txt`.



Supplementary Figure 2. The contributions for the velocity field PCA modes associated with a swimming spermatozoa in free space, labelled PCA (free-space), and for a sperm swimming parallel to a wall with a height of $z = 0.3L$, denoted PCA (near a wall). The inset depicts the cumulative mode contributions of the PCA modes, and the regularised Stokeslet approximations for the flow field in the free space and near the wall cases, both for free-space approximations and for a flow with no slip at $z=0$, labeled via “near a wall”. In addition cumulative contributions for the singularity representation using regularized Blakelets [1,2], which exactly enforce no slip on the wall, are presented.

- [1] J. Ainley, S. Durkin, R. Embid, P. Boindala and R. Cortez, 'The method of images for regularized Stokeslets', J. Comput. Phys., 227 (2008) 4600-4616.
- [2] D. J. Smith 'A boundary element regularized Stokeslet method applied to cilia- and flagella-driven flow, Proc. R. Soc. A, 465 (2009) 3605-3626.

3. Supplementary movies

Supplementary Movie 1

Digital video microscopy of a swimming human sperm cell, analysed in the Main Text. For more empirical details see Main Text and Smith *et al.*, *Cell. Motil. Cytoskel.* **2009**, 66, 220-236.

Supplementary Movie 2

Boundary element simulation of a swimming sperm near a no-slip boundary. The flagellar shape was extracted from empirical data via the three-dimensional limit cycle of the PCA flagellar shape phase space, as detailed in the Main Text.

Supplementary Movie 3

Instantaneous flow field and streamlines around a freely swimming sperm from BEM simulations. The flagellar shape was reconstructed from the three-dimensional limit cycle of the PCA flagellar shape phase space, presented analogously to Fig. 3 and Fig. 4a of the main text.