Review of Multidisciplinary Homogenization Techniques applied to Electric Machines

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***Abstract*—Cost reduction of any design process is always of interest for industries. Simulation work packages tackle this problem since they can quickly provide reliable results that permit detection of critical design issues prior to the prototype phase. A trade-off is then often made between model accuracy and computation speed. In the particular case of electric machines, homogenization techniques are used in order to keep high accuracy while running fast calculations. They are involved in multiple disciplines in which the machine performances are verified such as electromagnetic, mechanical, thermal and acoustic domains. This paper aims at defining whether these homogenization methods can be extended from one discipline to another by reviewing them independently of the physical domain.**

***Keywords*—Homogenization, multidisciplinary, electric machine, modelling**

I. INTRODUCTION

Driven by the increasing need for more sustainable and ecological vehicles, the automotive industry invests time and effort in electric powertrain technologies. Global industrial projects prove this interest, especially to lower the costs of the electrical machine by using novel motor technologies that try to get rid of the expensive permanent magnets [1][2]. On the other hand, total cost is not only dependent on the materials used in the machine but also on its design phase cost. For instance the prototype manufacturing phases are substantially expensive time-wise and hence money-wise. Fortunately nowadays computers are powerful enough such that numerical and/or analytical simulations can be performed to collect the machine performances prior this prototype phase; and eventually reveal critical design issues to be tackled. Yet this modelling process often involves a trade-off between model accuracy and computation speed. Homogenization methods are one solution to reduce the model complexity, without affecting the final solution but improving the computational efficiency. Indeed, appearing more than 100 years ago in the electromagnetism domain [3], the homogenization approach is literally trying to replace an heterogeneous material with an "equivalent" homogeneous one; i.e. it is an approach to study the macro-behaviour of a medium by its micro-properties. This technique has been widely applied for the complex laminated stator/rotor and the windings of the machine.

It is also interesting to note that the homogenization principle is used in a broad range of disciplines such as electromagnetics, thermal analysis and mechanics. Most of the time these different domains are studied independently from one to another, although they are inter-related. However lately research institutions try to address that e.g. ADEPT [1] pushes for the necessity of the interaction within multi-disciplinary networks.

In this paper, a literature review is done on multi-disciplinary homogenization techniques applied for cost-effective simulations of electric machines. The homogenization methods used for electromagnetic simulations, thermal modelling and structural dynamics are respectively reviewed in Section II, III and IV. Then Section V compares the methodologies depending on the domain of competences.

II. MOTOR DESIGN AND ELECTROMAGNETICS

An electric machine is firstly designed to achieve the required mechanical torque and speed with the highest efficiency, lowest cost, smallest volume, etc... In short, the feeding current input flowing through the windings of the machine produces magnetic flux that itself induces mechanical rotation speed and torque. Electromagnetic numerical and/or analytical analyses allow for obtaining these simulated machine output prior any prototype phase, which significantly decreases design costs.

However the machine complexity does not facilitate the analyses to be performed, such that homogenization techniques become relevant for computational speed and accuracy.

The windings distribution for instance varies from one machine to another. In order to avoid modelling each winding configuration, homogenization is performed, substituting the windings compound with a solid conductor, where the current density J is assumed to be uniform. This simplification is effective for stranded conductors [4], where eddy currents can be neglected. Windings homogenization techniques that account for AC losses are given in [5][6]. Homogenization of form-wound windings in frequency and time domain was presented in [7]. Skin and proximity effects in one stator conductor are taken into account by complex frequency-dependent coefficients derived from a simple low cost FE analysis. These complex coefficients are next translated into real-valued constant coefficient for time-domain homogenization [8].

Another geometry complexity is situated at the stator core of the machine. Composed of hundreds of thin steel laminations to reduce eddy current losses, its electromagnetic behaviour is still non-trivial. A complete 3D FE simulation of such compound by modelling every single lamina leads to millions of nodes models and thus non-relevant analysis. However, 2D FE simulation are often used in electric machine simulations such that homogenization is not necessary for transversally laminated cores since the magnetic flux is assumed to be axially independent. On the other hand, one cannot assume a uniform magnetic flux distribution along the laminations in the case of Synchronous Reluctance Machines with axially laminated rotor. Hence, when the magnetic field in each lamination of the rotor does not have to be known, the laminated rotor structure can be replaced by an homogenized rotor which significantly reduces simulation time. A model of such a homogenized laminated structure is shown in Fig. 1 [9].

The obtained reluctance ν from the homogenized medium then depends on the direction i.e. anisotropic, and also on the filling ratio of the steel laminations α. ν is then used to solve the magnetic vector potential A of equation (1).

$$\bigtriangledown \times \nu \bigtriangledown \times (A \ \mathbf{e_z}) = J \ \mathbf{e_z} \tag{1}$$

It is important to notice that this technique can be applied to any equipment that has laminated cores. For instance, laminated transformers are extensively studied especially for eddy current losses determination. Ziske et al. [10] replace the laminated structure with single solid anisotropic material which has a similar macroscopic behaviour. The electrical conductivity is adapted such that the Ohmic resistance in the homogenized core is equal to that of the current path in the laminated structure. The permeability is further homogenized by the rules of mixture technique using the filling ratio α. Yet a fine and time-consuming model of the laminated structure is needed to evaluate the homogenized parameters. The homogenized permeability assumption is also used as input data in structure optimization process, for instance to obtain the optimal shape of H-magnets structure [11]. This homogenization design method is further extended in [12] and is applied to a three-dimensional case which can take the saturation of the material into account.

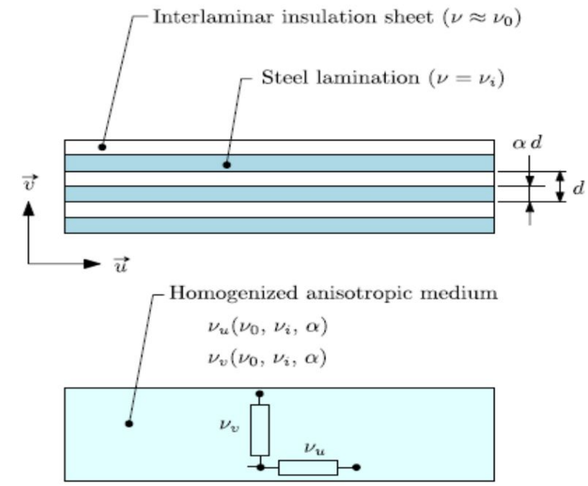


Fig. 1. Homogenization of a stack of steel sheets with interlaminar insulation sheets - Picture from [9]

Another approach, commonly called Multi-Scale approach, is also used to improve the solvability of electromagnetic problems of magnetic cores. The microscopic behaviour of a unit cell composing the periodic structure is extended to the macroscopic level by means of asymptotic expansion, which allows determining the anisotropic properties of the homogenized material. De Rochebrune et al. [13] defines an MS method in which the laminated core is homogenized through a macroscopic permeability tensor. The same approach may be extended to any stratified and/or periodic material [13][14].

Soft magnetic composites materials are rather studied with homogenization due to their structure. MS theory can be applied again to study these periodic materials [15]. The method leads to the identification of equivalent electric (resistivity) and magnetic (hysteresis curve) features. Niyonzima et al. [16] deal with two-dimensional magnetostatic of composite material problems as well and focuses on heterogeneous MS method. In this last case, the time saved is proved to be quite significant and justifies the added value of such technique.

Optimization of structures was also performed with homogenization techniques. In [11] the homogenization technique is applied to obtain the optimal shape of H-magnet structure in magnetic fields. As input data in optimization process, the authors used homogenized permeability of a microstructure. Homogenization design method is next extended in [12] where it was used to obtain the optimal shape of magnetic devices. This method was applied to a three-dimensional case, taking into account the saturation of the material.

III. Thermal management

Thermal analysis is fundamental for a good machine design. It is indeed very important to ensure that the critical components, such as the windings or the permanent magnets, do not overheat in any allowed working condition, which might lead to the failure of the machine. To reduce cost and time-to-market this analysis is performed with models that simulate the heat transfer within and between the machine's components. Thermal homogenisation allows a model order reduction of the thermal problem that in turn reduces the computational burden and the solution time. This is crucial for the simulation of long driving cycles or optimisation routines. Similarly to other scientific fields, thermal homogenisation means translating a complex and multi-phase domain into a homogeneous mixture with equivalent thermal features, i.e. equivalent thermal conductivity k_{eq} $[\mathrm{W/mK}]$, equivalent heat capacity c_{eq} $[\mathrm{J/kgK}]$ and equivalent density ρ_{eq} $[\mathrm{kg/m^3}]$. This means that the thermal conduction problem within a multi-phase domain reduces to a simplified "heat equation":

$$\rho_{eq} c_{eq} \frac{\partial T}{\partial t} = k_{eq} \nabla^2 T + \dot{q}_{eq} \tag{2}$$

where k_{eq} is assumed to be in each direction a constant and $\dot{q}_{eq}$ $[\mathrm{W/m^3}]$ is the equivalent internal heat generation. For what electrical machines are concerned, the domains where thermal homogenisation is applicable are mainly electrical windings and laminations. Homogenising each of the mentioned domains implies losing local temperature peaks or thermal gradients which may rise in correspondence to material discontinuities. However, a proper homogenisation gives a good estimation of the macroscopic thermal behaviour of the analysed domain. It can be shown [17] that to estimate the equivalent heat capacity we should just volume average the contributions of each of the phase part of the mixture. If we define $C_{eq} = c_{eq}\rho_{eq}$, then:

$$C_{eq} = \frac{1}{\|\Omega\|} \sum_p c_p \rho_p \|\Omega\|_p \tag{3}$$

where $\|\Omega\|$ is volume and the subscript p refers to the pth phase. The complexity comes with the estimation of k_{eq} as it is influenced by many factors, such as geometry, material properties, dispersion and filling ratio $\mu = \frac{n_w \|\Omega_c\|}{\|\Omega\|}$, where n_w is the number of conductors, $\|\Omega_c\|$ the volume of a single conductor and $\|\Omega\|$ the volume of the windings domain. In the next sections we will present the methodologies to estimate this parameter depending on the domain.

A. Electrical windings

The electrical windings are a very complex domain to be modelled from a thermal point of view as they are composed of a mixture of insulated conductors and impregnation, as described in Fig. 2. There are many methodologies to estimate k_{eq} for this extremely complex domain. The first one is based on experimental measurements [18][19]. This consists in building a specimen reproducing the windings arrangement that will be employed in the machine. The k_{eq} is then measured applying a ΔT to two of the boundaries and evaluating the heat flowing through it. However, any experimental-based technique is not widely applied as it is not cost-effective, since it implies building multiple specimens and acquiring the testing equipment. The common approach is based on analytical formulae or numerical methods. The simplest formula is the lower of the bounds given by Wiener [20], as applied by [21]; this consists of calculating the series of the varnish and the conductor contributions

$$k^{-}_{eq,\mathrm{Wiener}} = \left(\frac{\mu}{k_c} + \frac{1-\mu}{k_f} \right)^{-1} \tag{4}$$

where k_c and k_f refer to the conductors and varnish thermal conductivity respectively. The limitations of this formula is that it cannot account for the insulation phase, wire disposition and wire shape. Idoughi [22] proposed the use of the formulae representing the lower bounds given by Hashin and Shtrikman (HS) [23] or Milton [24]. These formulae still neglect the contribution of the insulation but considered the inclusion shape:

$$k^{-}_{eq,\mathrm{HS}} = k_f \frac{(1+\mu)k_c + (1-\mu)k_f}{(1-\mu)k_c + (1+\mu)k_f} \tag{5}$$

$$k^{-}_{eq,\mathrm{Milton}} = k_f \frac{(\mu k_c + (1-\mu)k_f + k_c)(k_c + k_f) - \mu\xi(k_c - k_f)^2}{((1-\mu)k_c + \mu k_f + k_f)(k_c + k_f) - \mu\xi(k_c - k_f)^2} \tag{6}$$

where ξ is a factor that changes with inclusion shape and filling ratio (the values for the case of circular inclusions are given in [22]), whereas (5) refers to circular discontinuities only. On the other hand, Kanzaki [25] derived two formulae that account for insulated circular wires on a square or hexagonal lattice solved numerically.

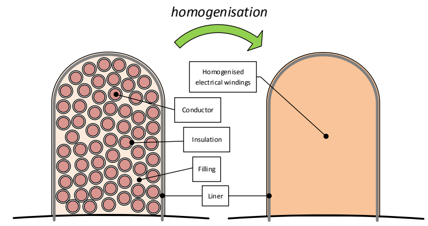
Another way of simplifying the thermal problem in the electrical windings domain is the "multi-layer" method developed by Nategh [26][27]. It consists of modelling the windings domain with a series of concentric elliptic layers of conductor, insulation and varnish material. The number of layers and their thickness are defined iteratively [26]. The limitation of the method is that it estimates the hot-spot location always in the middle of the windings domain, which is not the case in particular when AC losses are considered [28].

To overcome the limitations related to the analytical approaches, numerical methods are employed [29][25]. The equivalent thermal conductivity is estimated modelling each single wire of the full windings compound, calculating the heat flowing through it when a ΔT is applied. We will refer to the solution with $k_{eq,\mathrm{FE}}$, where the subscript FE refers to the Finite Element method employed to solve this problem. However, due to different length scales involved, e.g. insulation thickness or slot dimension, the mesh size grows very quickly. The downside of this approach is the increased time to set up and run this complex model. A possible way to reduce the complexity of the model might be the application of the multiple-scales method [30].

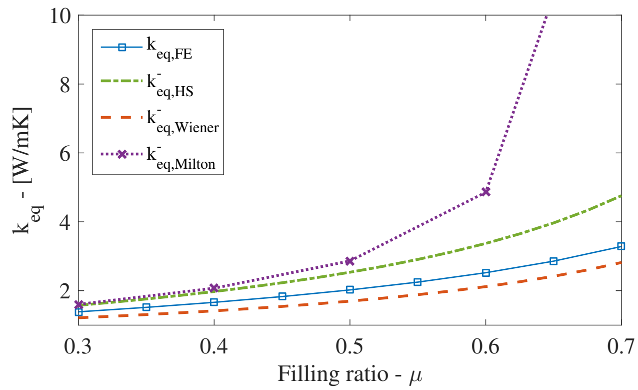
In Fig. 2(b) we collected the estimations given by (4), (5) and (6) along with the results estimated numerically ($k_{eq,\mathrm{FE}}$), where the wires were distributed on a square lattice. The material data was taken from [29]. As we can see from the figure, each of these methods gives a different estimation of the same parameter. The FE based method gives the highest accuracy [29], whereas Wiener formula is the most conservative. In general, the bigger the difference of the material properties of the various phases, the higher the mismatch between the estimations given by the analytical formulae.

B. Laminations

The stator and rotor active part of an electrical machine are usually composed of a set of very thin electrical sheets glued together with epoxy or varnish, or sometimes simply pressed together. This structure, usually referred to as "laminations", is needed to reduce the available path for the induced eddy currents and accordingly reduce the losses. Thermally speaking, the effect of the lamination structure can be seen only along axial direction. On the other directions the thermal conductivity is almost the same as the one of electrical steel. When a machine is cooled via a water-jacket, which is typically the case for an electric powertrain, the main heat path goes from the windings to the coolant, through the laminations. The coolant usually runs in the housing and, accordingly, the impact of the laminations thermal conductivity along direction 3 is not very significant. For this reason not many works covering this topic are available in the literature.



(a)



(b)

Fig. 2. (a) Homogenisation of electrical windings, (b) variation with μ of k_{eq} estimated with various techniques

Since the structure of the laminations consists of parallel electrical sheet, the equivalent thermal conductivity in the axial direction can be simply estimated using (4) [31]. For this problem (4) we should be filled with the suitable thermal conductivities of electrical steel and varnish or air. However, measurements carried out by the NREL laboratory [32] showed how the equivalent thermal conductivity of the laminations is affected by the pressure that constrains the laminations together. We think it would be useful to correlate the effect of pressure in this domain to improve the accuracy of thermal models.

IV. Structural Dynamics

Structural dynamics and especially acoustics is a topic that is gaining interest in the electric machine industry. Acoustic comfort plays a strong role when driving the vehicle in front of a prospect, and novel electric machines of increasing interest [1], e.g. Synchronous Reluctance Machine, Switched Reluctance Machine or DC Excited Flux-Switching Machine might produce critical vibration levels and/or acoustic noise. Hence it is necessary to predict the structural behaviour of the machine. As mentioned in section I, the choice of the structural dynamics model to use depends on the required accuracy and speed.

The homogenization technique is an interesting strategy in which the computational speed is increased and the accuracy is kept to a more acceptable level. The corresponding equivalent material properties then need to describe at best the original complexity introduced by the components. In the literature, mostly the stator core lamination stack is of interest in structural homogenization since it is generally the main contributor to the global stiffness and mass of the complete machine; this does not necessarily apply for small machines. Therefore only the homogenization techniques applied to lamination stacks for structural dynamics prediction are further reviewed.

In order to perform structural dynamics simulations, it is important to know the constitutive parameters of the material. Any component is in fact characterized by its mass density ρ and its elasticity matrix $[C]$ in Pascal. This fourth-order matrix is defined by Hooke's law $\{\sigma\} = [C]\{\epsilon\}$ in its general (i.e. anisotropic) formulation; where $\{\sigma\}$ is the stress tensor, $\{\epsilon\}$ is the strain tensor and $[C]$ is a 6×6 matrix with 36 independent coefficients.

Hence the simplification of the complex steel stack to an homogeneous component referring to an equivalent elasticity matrix $[C]_{eq}$ is an homogenization by definition. However the difficulty arises when trying to identify the coefficients C_{ijkl} of $[C]_{eq}$ which best emulate the system complexity. Indeed 36 independent coefficients need to be determined, which reduces to 21 if the material is conservative leading the elasticity matrix to be symmetric.

Several authors firstly assume that the steel stack behaves identically in every direction i.e. isotropic behaviour [33][34][35], since it reduces the number of unknowns to 3: the Young's modulus E, the poisson ratio ν and the mass density ρ. Manufacturer sheets usually provide such data so that its corresponding elasticity matrix can be calculated. Chauvicourt et al. [36] show promising results from models using isotropic properties and raw data obtained from manufacturers.

Improvements may be achieved using experimental-based model updating e.g. M. van der Griet explains in [37] an empirical method to identify the 3 constitutive unknowns. The isotropic modelling approach also limits the amount of novel modelling possibilities, it makes strong assumptions and the results accuracy is directly impacted. Experimental work [38] also shows that the Young's modulus in the stack direction is lower than in the other directions.

To better emulate the stacking influence on the structural behaviour, researchers mostly consider transverse isotropy of the stacks i.e. the structure has the same properties in one plane (parallel to the cross section) and different properties in the direction normal to this plane. In order to compute the new compliance matrix, five independent variables have to be determined but are not available in common manufacturer data sheets.

Hence researchers come up with a variety of identification methods. It is important to note that the expensive construction of a prototype limits the experimental model updating use such that it becomes interesting to calculate the constitutive parameters as accurately as possible prior any prototype phase of the design. Thus no model updating is further presented in this paper.

Analytical models shall be utilized to determine these parameters. The steel stack is often a composition of insulating resin (matrix m) and steel (fiber f) layers which form a composite material. Hence the rule of mixtures presented in [39] can be exploited. The basic idea is to take the volume ratio ϕ_n of each of the component into account to calculate the weighted averages of each constitutive parameters. Details about this method and equations to retrieve the unknown parameters are available in the work of M. van der Griet [38] and Millithaler [40][41].

A more elaborated analytical method is developed by Begis et al. [42][43] to obtain the Y-periodic composite structure's equivalent elasticity matrix $[C_{eq}]$ from an asymptotic homogenisation approach. The global idea is to solve equation (7) to get Y-periodic vectors $\{W^{pq}(y)\}$ and finally obtain the homogenised coefficients $C_{ijkl,eq}$ using (8).

$$\frac{\partial}{\partial y_j}\big(C_{ijkl}(y)\epsilon_{kl}(\{W^{pq}(y)\})\big) = \frac{-\partial}{\partial y_j}C_{ijkl}(y) \tag{7}$$

$$C_{ijkl,eq} = \frac{1}{vol_Y}\int_Y \big[C_{ijkl}(y) - C_{ijpq}(y)\epsilon_{kl}(\{W^{pq}(y)\})\big]\,dy \tag{8}$$

Although the computational time is significantly reduced and the steel stacks can be assumed to be a composite material, mathematical difficulties may emerge to resolve equation (7). Nevertheless Begis et al. [42] apply

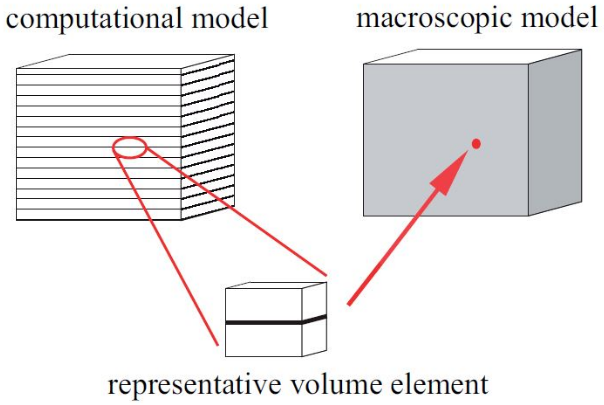


Fig. 3. Principle of multi-scale homogenization for a laminated stack - Picture from [45]

it for laminated structures. By assuming isotropy for each constitutive layer one obtains the equivalent elasticity matrix $[C]_{eq}$, equation available in [42] and [41]. It is worth noting that the matrix verifies transverse isotropy property $C_{1111,eq} = C_{1122,eq} + 2C_{1212,eq}$.

The homogenized structural behaviour can also be obtained using a two-step approach and is often referred as a multi-scale technique. Hirschberger et al. describe in [44] the process of modelling heterogeneous material layers. Firstly the behaviour of a Representative Volume Element (RVE) is studied. It represents a microscopic portion of the macroscopic structure where the heterogeneous property takes its origin. Secondly one can relate the macro level deformations to the averaged stress and strain tensors at micro level by applying correct RVE volumes and boundary conditions, using an iterative nested solution procedure. One of the advantages of the multi-scale method is the possibility to emulate nonlinear behaviour without increasing greatly the model complexity. Hence, it is applied by Luchscheider et al. in [45] in the case of laminated stack. Indeed in order to simulate the nonlinear contact between the laminations, the authors make use of a repeated RVE which integrates the nonlinearity by assuming the contact surfaces' roughness to be stack pressure dependent. This periodic RVE corresponds to a portion of the stacks as thick as a single sheet which contains the plane contact in the middle as shown in Fig. 3. The stack's material properties, i.e. nonlinear stiffness and damping matrices, are then identified by applying different static load conditions to the RVE. Being more accurate with the nonlinear implementation in the model, this multi-scale homogenization technique might still require high computational time because of the nonlinearitites themselves.

The latest referenced approach belongs to Millithaler et al. [41][40][46]. His work introduces a novel approach in which transverse anisotropic and orthotropic [40] or anisotropic [41][46] equivalent material properties are identified for laminated steel stacks. The technique is also a two-step approach similar to multi-scale method. At first a base unit cell, composed of three isotropic layers (here steel, epoxy and steel) perfectly connected to each other (see Fig. 4), is put in different pre-stress conditions to collect the equivalent constitutive parameters. In short, static displacements are enforced at particular nodes of the unit cell such that the FEM computed nodal displacements and reaction forces permit to calculate the constitutive parameters (equations available in [41]. Several combinations of enforced displacements are then necessary to provide every constitutive component $C_{ijkl,eq}$ since they activate different independent motions e.g. pure tension, pure shear, sliding shear, etc. Secondly this calculated data fills the elasticity matrix in order to perform linear and fast FEM simulations of a complete laminated stack. Therefore in this method, the complete laminated stack is linearly homogenized which reduces the computational time. On the other hand, the user is first obliged to perform several identification simulations. Moreover the approach does not take into account the contact modelling complexity and assumes perfect nodal contacts (i.e. equal nodal motion) between the different isotropic component layers.

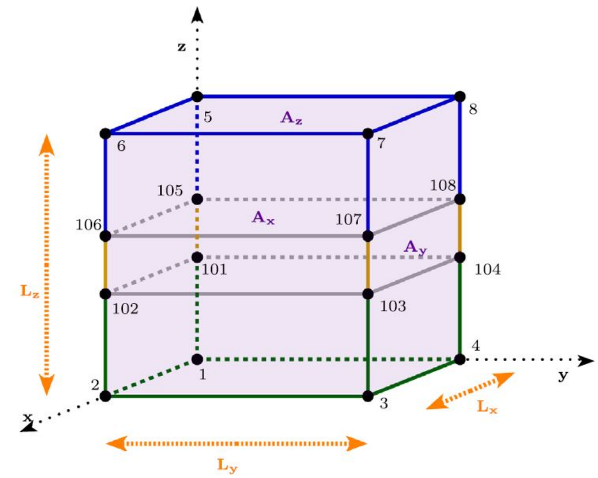


Fig. 4. Base unit cell used for coefficient identification - Picture from [40]

V. Discussions and Conclusions

Homogenization techniques are extensively referenced and progressively used thanks to their added value in computational speed. It is interesting to note that in structural dynamics simulations, homogenization techniques are also considered as state-of-the-art accuracy providers. The available literature demonstrates an increasing interest in this type of method not only in the initially mentioned electromagnetic topic but also in the thermal and vibration studies of electrical machines. Similarities between the domains can even be extracted.

Indeed, resemblances can be noticed between every domain of expertise under review. It all starts with a common problem to tackle; here, either the non-continuous windings or the complex lamination stack forming the stator/rotor. In any of these cases and disciplines, one wants to find an homogeneous domain in which equivalent parameters will best emulate the complex behaviour. Electromagnetic simulations for instance look for the equivalent magnetic permeability μ_l and conductivity σ. Thermal simulation issues occur as well within the windings where equivalent heat capacity c_{eq}, the equivalent conductivity k_{eq} and the equivalent density ρ_{eq} need to be estimated. Vibration studies then focus on the mechanical properties of the component and therefore its equivalent elasticity matrix $[C]_{eq}$ needs to be defined. Analytical-based methodologies are easily applicable for the equivalent parameters identification of every discipline since they most commonly refer to the mathematical rules of mixture. Afterwards, two step approaches such as FEM-based methods can be used in the identification process. These types of techniques allow for a better understanding of the component intrinsic behaviour but significantly increase both the simulation preparation and computation times.

Multi-Scale methods can homogenize all the analysed periodic structures. Still on a two-step approach basis, they are used in the electromagnetic, thermal and vibration domains. The idea is to simulate the behaviour of a micro-component which contains the behaviour particularities of the corresponding macro-component and to inject the obtained homogenized parameters to the homogeneous macro-component. Therefore the principle can be compared to a model order reduction technique and permits to reduce the computational burden while keeping the accuracy to an acceptable level. To the authors' knowledge, this method has not been applied yet for electromagnetic simulations for electric machines but may be feasible, if more accuracy is necessary.

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