C.F.I. OCCASIONAL PAPERS

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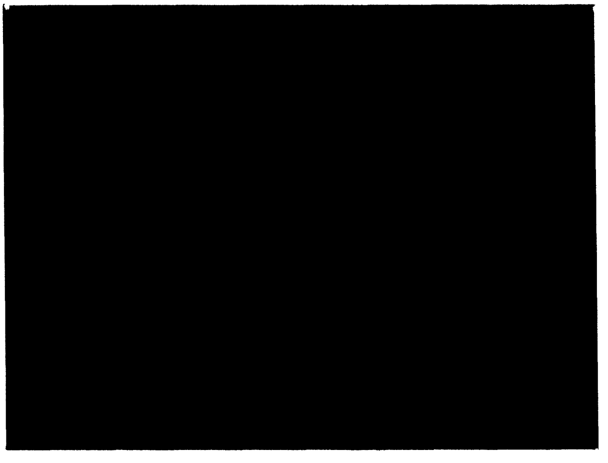
No. 3

A regional volume table
for *Gmelina arborea* Roxb.

A. Greaves
Commonwealth Forestry Institute
Oxford University
1978

DEPARTMENT OF FORESTRY
COMMONWEALTH FORESTRY INSTITUTE
UNIVERSITY OF OXFORD



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Summary

Four non-weighted and six weighted volume equations were fitted by the method of least squares to volume data compiled from 400 felled trees of *Gmelina arborea*. Using Furnival's Index as the criterion for judging the goodness of fit, the Australian formula weighted by $1/D^2$ was selected as being the most efficient. A volume table is presented, based on the equation

$$V = -10.8495 + 0.01259D^2 + 0.8583H + 0.02543\,D^2H$$

Where V = volume (cubic decimetres)
D = diameter breast height over bark (cm)
H = total height (m)

Acknowledgements

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The data on which the paper is based were compiled during field work undertaken in Nigeria during the period 1971-72. The author is indebted to Mr. Obaseki, then Chief Conservator of Forests, Mid-Western State, for his permission to carry out the field work.

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Abbreviations

The following abbreviations are used in the text:

b_o = regression constant

$b_1 b_2 \ldots b_n$ = regression coefficients

V_n = tree volume

D = diameter breast height

H = total tree height

d.o.b. = diameter over bark

d.b.h.o.b. = diameter breast height over bark

b.h. = breast height

1.0 Methods of volume table construction

In the past the harmonized curve method of volume table construction (Chapman and Meyer, 1949) was widely used. The method has the disadvantage of the accuracy of the curve being dependent on the skill of the investigator; neither can confidence limits be placed on the estimate of volume.

An approach which is intermediate between the graphical and mathematical methods is that used in the construction of the British volume tables (Hummel, 1955). Linear regressions of volume over basal area are determined for each height class. Curves of the regression constants and coefficients over height are then constructed. The final volumes are calculated using the smoothed values of these parameters.

The method has the advantage of fitting curves to the data rather than assuming that the data follows a preconceived pattern, which is implied when an equation is fitted. This permits a curvilinear relationship between height and the coefficients and constants of the regressions of volume over basal area if the data require it. However, the method does require a large number of volume sample tree data.

The fitting of a regression equation to volume sample tree data by the method of least squares is now the generally accepted procedure for volume table construction. Numerous tree volume equations have been developed, some of which are discussed by Spurr (1952). The method has the advantage of objectivity, the provision for tests of significance and the defining of confidence limits.

If the volume table is to be applied on a very restricted local scale it is sometimes possible to estimate volume efficiently from measurements of diameter breast height squared, or basal area, alone. Thus Sandrasegaran (1966) produced a local volume table for *Gmelina arborea* based on basal area.

For use on a regional or general scale the tree volume equations which have most frequently fitted the sample data satisfactorily are:

(i) The combined variable formula

$$V = b_o + b_1 D^2 H$$

(ii) The Australian formula

$$V = b_o + b_1 D^2 + b_2 H + b_3 D^2 H$$

Unlike the constant form factor equation

$$V = b_o D^2 H$$

the combined variable equation permits the form factor of the trees to vary according to the magnitudes of D^2 and H.

The Australian formula is derived from the local volume equation

$$V = b_o + b_1 D^2$$

in which the constants b_o and b_1 are directly proportional to height. If, in the procedure adopted for the preparation of the British volume tables, the values of b_o and b_1 plot at straight lines over H, then the Australian formula is applicable.

An intermediate equation in which only the regression coefficient (b_1) varies with H whilst the regression constant (b_o) is fixed is

$$V = b_o + b_1 D^2 + b_2 D^2 H$$

All these equations assume a linear trend between volume and the stem dimensions. This is not always the case. Curvilinearity may be accounted for by introducing additional terms of diameter and height raised to higher powers. More frequently logarithmic equations are used.

The equation for the local volume table may be expressed as:

$$\log V = \log b_o + b_1 \log D$$

When height is taken into consideration an equation frequently used is the Schumacher formula:

$$\log V = \log b_o + b_1 \log D + b_2 \log H$$

Solutions of the logarithmic models tend to be more difficult to interpret than solutions of the arithmetic. The arithmetic mean is replaced by the geometric mean and confidence limits are in logarithmic values.

Spurr (1952), following a review of the general suitability of the different volume estimation methods, concluded that the harmonized curve method was not to be recommended in view of its dependency on the skill of the investigator for its accuracy. Of the least squares methods he was of the opinion that the logarithmic equations were apt to be less precise than the arithmetic. Finally he recommended that for samples of 500 trees or more the method adopted in the construction of the British volume tables should be used. For 100 to 500 trees the Australian formula was recommended, and for 50 to 100 trees the combined variable formula.

As the number of volume sample trees available was to be 400 it was decided that the fitting of a regression equation by the least squares method would be the most practical approach.

2.0 Field procedure

2.1 Selection of sample trees

The locations of 400 volume sample trees were purposely allocated to the plantation compartments so as to achieve an approximately uniform distribution by age and site. The sample trees were then randomly located within each compartment. Dead, dying, and broken trees were excluded from the selection.

2.2 Measurement of sample trees

Before felling, the d.b.h.o.b. of the tree was measured and recorded to the completed 0.1 cm. The position of breast height was marked.

After felling the total height of the tree was measured and recorded to the completed centimetre.

The stem from the butt to a point 5.0 cm d.o.b. was divided into sections of two metres, commencing from the base of the tree, and the mid-points of the sections marked. In order to allow for a 0.25 m stump the centre of the first section was located 5 cm below the b.h. position. If the length of the final section was less than 1.0 m it was added to the previous section and the position of the mid-point recalculated.

The lengths of the sections were recorded to the completed centimetre and the mid-point diameters over bark to the completed 0.1 cm.

At the mid-point of every section, and at breast height, the bark thickness at two positions at right angles to each other was measured to the nearest 0.1 cm.

3.0 Computational procedure

3.1 Volume calculation

The mid-point diameters under bark of the stem sections were calculated by the equation:

$$D_2 = D_1 - (t_1 + t_2)$$

where D_2 = diameter under bark
D_1 = diameter over bark
t_1 and t_2 = the measurements of bark thickness

The volume under bark of each section was calculated by Huber's formula:

$$V = L \times A$$

where V = volume under bark
L = length of the section
A = section mid-point cross-sectional area under bark

The volumes of the individual sections were then summed to give the tree volume.

3.2 The relationship between diameter breast height and bark thickness

Using the data from the 400 sample trees bark thickness at breast height was plotted over d.b.h.o.b. The trend of the scatter of points was linear. A simple linear regression equation was fitted to the data and found to have the parameters:

$$T = 0.7639 + 0.009323D$$

where T = bark thickness in cm
D = d.b.h.o.b. in cm
coefficient of determination $(r^2) = 0.1$

According to this equation the mean bark thickness at 5.0 cm d.b.h.o.b. is 0.8 cm and at 30.0 cm d.b.h.o.b. it is 1.0 cm.

Compensating for a difference in bark thickness of 0.2 cm over a d.b.h.o.b. range of 25.0 cm was not considered to be of any practical significance since the experimental error involved in the measuring of the volume of the sample trees would invalidate such a fine correction. Moreover, although the coefficient of determination is significant at the 95% level, such a small value cannot be regarded as being of practical use for purposes of prediction. Bark thickness was therefore regarded as being constant.

Since D^2 varies directly with $(D\text{-constant})^2$ a volume equation could be fitted to the data, D being measured over bark and V under bark.

3.3 Potential volume equations

With regard to the observations made in Section 1.0, it was decided that the following equations should be fitted and examined for their efficiencies:

(1) $V = b_o + b_1 D^2$

(2) $V = b_o + b_1 D^2 H$ (combined variable equation)

(3) $V = b_o + b_1 D^2 + b_2 D^2 H$

(4) $V = b_o + b_1 D^2 + b_2 H + b_3 D^2 H$ (Australian formula)

3.4 Weighting the equations

Wright (1964) and Cunia (1964) discuss the need to weight volume equations in most circumstances in order to equalise the variance in volume along the regression line or surface, this being necessary before valid tests of significance can be applied to the regression equation. Wright concluded that the variance of volume is usually proportional to the square of tree size as expressed by $(D^2)^2$, V^2 or $(D^2 H)^2$. The general rule for weighting equations to induce homogeneity of variance is that if the variance is directly proportional to a function then the

Table 1

Volume equations

1	2	3	4	5	6
Volume equation	Standard deviation	R^2	Geometric of	mean =	Furnival's index
1. $V = -50.9076 + 0.6998D^2$	33.90	0.982	1.0	1.0	33.90
2. $V = 8.2063 + 0.02512D^2H$	24.86	0.990	1.0	1.0	24.86
3. $V = 1.2877 + 0.07786D^2 + 0.02238D^2H$	24.69	0.990	1.0	1.0	24.69
4. $V = -40.8132 + 0.09317D^2 + 3.0974H + 0.01998D^2H$	22.8	0.992	1.0	1.0	22.80
5. $V/D^2 = 0.6083 - 29.6996/D^2$	0.1154	0.704	D^2	240.25	27.72
6. $V/D^2H = 0.02754 - 3.04/D^2H$	0.0044	0.320	D^2H	3,907.38	17.19
7. $V/D^2 = 0.001302 - 2.07697/D^2 + 0.02714\ H$	0.0655	0.915	D^2	240.25	15.74
8. $V/D^2H = 0.02838 - 2.2465/D^2H - 0.01789/H$	0.004359	0.324	D^2H	3,907.38	17.03
9. $V/D^2 = 0.01259 - 10.8495/D^2 + 0.8583H/D^2 + 0.02543H$	0.064	0.92	D^2	240.25	15.38
10. $V/D^2H = 0.02707 - 5.5673/D^2H - 0.00712/H + 0.377/D^2$	0.0042	0.342	D^2H	3,907.38	16.41

V = volume under bark down to 5.0 cm d.o.b. in cubic decimetres. D = d.b.h.o.b. in cm. H = total height in m.

equation should be weighted by the reciprocal of that function. In this case the weighting functions for volume equations become $1/D^4$, $1/V^2$ or $1/(D^2 H)^2$.

In view of the difficulties of determining the manner in which the variance varies with tree size Wright suggests that it should be assumed to be directly proportional to the square of tree size as outlined above. A number of weighted and unweighted models can then be fitted to the data and the one of best fit selected for the construction of the volume table.

An equation of variable weight 'w' may be reduced to an equation of unit weight by multiplying both sides of the equation by $\sqrt{w}$. It follows from above that the functions to be used in this case are $1/D^2$, $1/D^2 H$ or $1/V$. The particular function chosen must be such that the weighted equation contains a constant term.

Referring to the equations cited in the previous section, the functions $1/D^2$ and $1/D^2 H$ are suitable for models (1) and (2) respectively. Either term may be used for models (3) and (4). Thus it was possible to fit six weighted equations as follows:

(5) $V/D^2 = b_o + b_1 D^2$

(6) $V/D^2 H = b_o + b_1/D^2 H$

(7) $V/D^2 H = b_o + b_1/D^2 + b_2 /H$

(8) $V/D^2 H = b_o + b_1/H + b_2/D^2 H$

(9) $V/D^2 = b_o + b_1/D^2 + b_2 H/D^2 + b_3 H$

(10) $V/D^2 H = b_o + b_1/D^2 H + b_2/H + b_3/D^2$

The parameters of both the weighted and non-weighted equations are given in Table 1.

3.5 Testing the goodness of fit

The precision of a regression equation is usually measured by the standard deviation from regression or the coefficient of determination (R^2). However, these statistics do not take into account other factors such as heterogeneous variance. A more suitable index for comparing regression equations has been devised by Furnival (1961).

Furnival's index is based on the concept of maximum likelihood. Its value is increased by large residuals, departures from linearity, non-normality, and heterogeneous variance. Thus a decrease in its value indicates an improved fit to the data.

The expression for the calculation of Furnival's index is:

$$I = \left[f^1 (V)\right]^{-1} S$$

where I = Furnival's index
$f^1(V)$ = the derivative of the dependent variable with respect to volume
S = the standard deviation from regression
The square brackets indicate the geometric mean.

The derivative of the dependent variable V of the unweighted equations is 1.0. The derivatives of the dependent variables V/D^2 and $V/D^2 H$ of the weighted equations are D^2 and D^2H respectively. The geometric means are obtained by:

$$\text{geometric mean } D^2 = \text{antilog } \frac{\Sigma \log D^2}{n}$$

$$\text{geometric mean } D^2H = \text{antilog } \frac{\Sigma \log D^2H}{n}$$

The values of Furnival's index are entered in Table 1. The superior efficiency of the weighted equations is clearly demonstrated. The equation with the smallest index value is the Australian formula weighted by $1/D^2$ (equation 9 in table 1). This equation was therefore adopted as having the best fit to the data. Multiplying both sides by D^2 produces the final tree volume equation:

$$V = -10.8495 + 0.01259D^2 + 0.8583H + 0.02543D^2 H$$

A volume table based on this model is presented in Appendix ii.

4.0 Discussion

4.1 Applicability of the results

The volume equation should only be applied in circumstances which lie within the conditions of the original sample data.

The volume sample trees were selected from unthinned plantations of Gmelina arborea having a planting espacement of 2.44 m x 2.44 m (8ft x 8ft) on the square, and aged between 5 and 15 years. Rates of growth, as indicated by site index, ranged from 9 to 29, site index being the top height achieved at 10 years. Top height is defined as the arithmetic mean height of the 100 largest basal area trees per hectare (40 per acre).

Mean annual rainfall over the study area ranges from 1,300 mm to 1,840 mm with a distinct dry season from November to March.

Before the equation can be used in other conditions both its accuracy in the new environment, and applicability to tree dimensions outside those of the original sample (see Appendix 1), must be verified.

4.2 Accuracy of the volume equation

The statistics of the volume equation indicate a very satisfactory degree of fit to the data, thus implying an equally satisfactory degree of predictive accuracy. However, reference to Appendix 1 shows that within each height class the extremes of d.b.h.o.b. are poorly sampled. Consequently the predictive accuracy of the equation will be least satisfactory when applied to trees of these dimensions. This should not have an unacceptable influence on the results of a plantation enumeration since most of the trees encountered will fall in the mid-range d.b.h.o.b. for each height class, provided that the conditions set down in section 4.1 above are maintained.

Improved accuracy could be gained by including in the original sample more trees from the dimensional extremes. Ten trees per height/diameter cell would be a suitable objective. This would necessitate not only a larger sample but also a more tedious sampling design involving prior stratification by diameter in addition to age and site. If this could be achieved the method of data analysis as used in the construction of the British volume tables (see section 1.0) would be more appropriate.

4.3 Variation in bark thickness

The argument used in Section 3.2 for regarding bark thickness as being constant is based on observations taken from trees growing on distinctly different sites with a wide range in rate of growth. A sample of this nature was necessary if the volume equation was to have regional applicability.

It is highly probable that a distinct correlation between diameter and bark thickness would be found for trees growing in areas of similar growth rates.

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Appendix 1

Distribution of volume sample tree data by height and diameter

d.b.h.o.b. class (cm)	Height class (m)												
	5	7	9	11	13	15	17	19	21	23	25	27	29
6		2		1									
8	1	5	6	5	3	2							
10		3	11	9	7	6	1						
12	1		6	12	14	15	19	2	1				
14			1	13	8	18	16	11	2				
16			1	5	4	9	10	12	5	1			
18				1	1	6	1	10	12	4			
20						2	4	5	8	10	2		
22					1		3	4	6	11	2		
24							1		4	6	8		
26								1	1	7	3	1	
28										1	2	3	
30										1	3	1	1
32											2	1	1
34											2	3	
36												3	1
38											1		
40													1

Appendix 2

Volume table

Under bark volumes in cubic decimetres

d.b.h.o.b. (cm)	Total height (m)												
	5	7	9	11	13	15	17	19	21	23	25	27	29
8	2	7	12	17	22	27	32						
10	7	14	21	28	35	41	48	55					
12	14	23	32	41	50	59	68	77	86				
14	21	32	44	56	68	79	91	103	114	126			
16	29	44	59	73	88	103	118	132	147	162	176		
18		57	75	93	111	130	148	166	184	202	221	239	
20			93	116	138	160	182	204	226	248	270	292	314
22				140	166	193	219	245	272	298	324	351	377
24					198	229	260	291	322	353	384	415	446
26						268	304	340	377	413	449	485	521
28							352	394	436	477	519	560	602
30								452	499	547	594	642	689
32									567	621	674	728	782
34										700	760	820	880
36											851	918	986
38												1022	1097
40													1214