Letter

Symmetry restoration and vacuum decay from accretion around black holes

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Vacuum decay and symmetry breaking play an important role in the fundamental structure of the matter and the evolution of the Universe. In this work we study how the purely classical effect of accretion of fundamental fields onto black holes can lead to shells of symmetry restoration in the midst of a symmetry broken phase. We also show how it can catalyze vacuum decay, forming a bubble that expands asymptotically at the speed of light. These effects offer an alternative, purely classical mechanism to quantum tunneling for seeding phase transitions in the Universe.

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Introduction. Throughout its history, the Universe has undergone a cascade of phase transitions [1]. These transitions have been associated with symmetry breaking, in which the vacuum structure for fundamental fields has morphed from one of higher symmetry to one of lower symmetry [2]. It has also been proposed that our Universe could, now or in the past, have lived in a false vacuum of some field, and that phase transitions may then be triggered that take it to a true minimum [3–6].

The case of quantum transitions between vacuum states has been extensively studied, including the impact of black holes in early works by Hawking and Moss [7,8], and their potential to enhance the transition rate in more recent work by Gregory *et al.* [9–12], see also [13–19]. In this work we will study alternative, purely classical transitions that arise around black holes when they are immersed in coherent scalar fields (such as those that model wavelike dark matter with small mass and high number density, see [20] for a review). Such effects are the extreme end of the study of self-interactions in the field, where attractive and repulsive forces arising from the shape of the potential play a role in the classical evolution. In the repulsive case (which we study in more detail in [21]) the self-interactions saturate the growth of density around the black hole (BH), and so the field is confined to one region of the potential. In the attractive case, the growth does not saturate (at least initially), and as a result the field explores greater and greater regions of the potential. It is then affected by the wider shape and in particular the presence of other minima.

The effects that attractive corrections to a quadratic potential have on scalar (and vector) clouds near BHs have previously been considered for the case of self-interacting dark matter like axions [22–29], including their potential to create so-called "bosenova" explosions in the case of superradiant clouds, where the BH must have a relatively high spin and the scalar mass must satisfy the superradiant condition [30–38] (see [39] for a review). There, however, self-interactions can cause the saturation and dissipation of the cloud, which radiates and transitions to other bound states (although additional accretion may help [40].) The accretion case we study is therefore arguably more generic and inescapable, provided a (stable, sufficiently abundant) scalar with some multiminima potential exists.

Other studies of classical transitions have focused on more exotic overdensity scenarios, such as soliton stars [41–44], spontaneous scalarization [45] or extra couplings to matter [46,47], or been related to attempts to model the quantum process as a classical but stochastic process [48–56], with simulations that indicate that the transition (or decay rate) matches the quantum, path integral estimate up to factors of $O(1)$. The process we describe here is inherently classical and deterministic, and simply a consequence of the scalar field evolution on a curved background, rather than an approximation to a quantum system (although it may correspond to some semiclassical limit of a quantum process, it does not rely on fluctuations to occur).

In this work, we characterize how the BH accretion process can lead the scalar field to gradually explore

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multiple vacua. Depending on the form of the potential, this may result in symmetry restoration or trigger a phase transition through vacuum decay. Such processes could be ubiquitous throughout the history of the Universe; BHs exist over a range of mass scales and over a range of times, and may serve as catalysts for such transitions. We consider an isolated Schwarzschild BH, characterizing it through numerical simulations and extracting the key features and timescales that impact on observables.

The model. In a cosmological setting, phase transitions arise from radiative corrections in field theory, which modify the effective potential seen by the field.[1] In classical field theory we consider the scalar field potential to be fixed and study the dynamics of the field within it. We study the dynamics of a real scalar field ϕ, solving the Klein-Gordon equation,

$$\nabla^{\mu}\nabla_{\mu}\phi - V'(\phi) = 0, \tag{1}$$

where ∇_{μ} is the covariant derivative operator associated to the metric $g_{\alpha\beta}$. We solve the Klein-Gordon equation on a fixed Schwarzschild background, in horizon penetrating Cartesian Kerr-Schild coordinates [59]

$$\mathrm{d}s^2 = \left(\eta_{\mu\nu} + \frac{2M}{r} l_{\mu} l_{\nu}\right)\mathrm{d}x^{\mu}\mathrm{d}x^{\nu}, \tag{2}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ and M is the mass of the black hole. Here $l_{\mu} = (1, x/r, y/r, z/r)$ is the ingoing null vector with respect to both $g_{\mu\nu}$ and $\eta_{\mu\nu}$. We use t to represent the Kerr-Schild time coordinate, which is related to the usual Schwarzschild $\bar{t}$ as $\bar{t} = t - 2M\log(r - 2M)$. The radial coordinate r is the same in both systems. These coordinates allow us to evolve the fields smoothly across the horizon without the freezing that would occur in Schwarzschild-like coordinates, but we still need to excise the diverging region around the singularity. We focus in the plots on the behavior in the observable region outside the horizon.

To illustrate both the mechanisms of symmetry restoration and vacuum decay, we choose a scalar potential with multiple minima,

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \eta^2)^2 + \frac{g}{3}\eta(\phi + \eta)^3, \tag{3}$$

where λ and g are dimensionless coupling constants and η parametrizes the symmetry breaking scale. The cubic term controls the difference in the energy of each vacuum $\Delta V \equiv V(\phi_+) - V(\phi_-)$, which is positive if $g > 0$, negative if $g < 0$, and in the $g = 0$ limit, the potential simplifies to the usual double-well potential with degenerate vacua at $\phi_{\pm} = \pm\eta$.

To lowest order around these minima, the potential is $V(\phi) \approx \mu^2\phi^2/2$, with mass $\mu \approx \sqrt{2\lambda}\eta$. In a flat spacetime, a spatially homogeneous value of the field ϕ close to a minimum would oscillate around it with frequency given by the mass μ. Thus, asymptotically far from the black hole the solutions of the scalar field have the form $\phi \propto \cos(\mu\bar{t})$ (which differs only by a slowly varying phase for Kerr-Schild time t at large r). In the purely massive case, closer to the black hole, the BH gradually accretes a cloud of scalar field [60–65], described by the Heun functions as in [66–68], with a power law envelope and characteristic oscillations of the spatial profile on length scales set by the scalar wavelength. Whilst the scalar in principle extends to infinity, in a physical scenario the accretion will only occur over a finite radius. In practice, we can treat it as infalling from the boundary [66]. The presence of repulsive self-interactions ($V'' > 0$), can stabilize the accretion process and prevent the field amplitude from growing [21]. Attractive self-interactions ($V'' < 0$), on the other hand, result (at least initially, near the minimum of the potential, and as long as there is an asymptotic reservoir of scalar field to feed it) in a continuous growth of the profile, such that closer to the horizon, the amplitude of oscillations increases [21]. This means that the scalar field can explore other features of the potential, such as adjacent vacuum states.

It will be useful to define a mean, time averaged, value of the scalar field at some coordinate radius r,

$$\langle\phi(r,t)\rangle \approx \frac{1}{T}\int_t^{t+T}\phi(r,t')dt', \tag{4}$$

where $T \simeq 2\pi/\mu$ is the local period of the oscillation. If the field oscillates around the negative minimum of the potential $\langle\phi(r,t)\rangle = -\eta$.

In this paper we study the dynamics of scalar fields in black hole spacetimes for potentials with multiple minima. We fix $\eta = 1$, $\lambda = 0.1\ \mathrm{M}^{-2}$ so that $\mu M \sim 0.5$ and vary g to study symmetry restoration and vacuum decay (we use geometric units $G = c = 1$, so that $[\phi] = 1$, $[\rho] = M^{-2}$). We choose the initial homogeneous value of the scalar field to be $\phi_0 = -0.8\eta$, so that it resides near the negative vacuum of a symmetry broken phase. These choices fix the asymptotic energy density to $\rho_0 \approx 0.03\lambda\eta^4$. If the local density of the scalar environment is much smaller than the curvature scale of the BH, $\rho M^2 \ll 1$, we can neglect the backreaction of the scalar field on the metric. This allows us to rescale the results to different physical densities, as the equation of motion is invariant under the transformation $\phi_0 \to a\phi_0$ (or equivalently $\rho_0 \to a^2\rho_0$), as long as we keep μ constant via $\eta \to a\eta$ and $\lambda \to \lambda/a^2$. We perform the

[1]In the case of a scalar field, Coleman and Weinberg [2] explored how corrections to the potential could modify the vacuum structure, restoring symmetry at high energies. A chemical potential [57] or finite temperature corrections [58] can have a similar effect.

numerical simulations of the growth of the field using the $3+1$ open-source code GRDZHADZHA [69,70], with octant symmetry to simulate a physical box size of $L = 576M$, a coarsest grid resolution $N^3 = 256^3$ grid points, and six levels of refinement, which gives the finest resolution $dx_{\rm finest} = 0.035M$ at the BH horizon, located at $r = 2M$.

Results. We consider in turn the case of symmetry restoration, for degenerate minima, and then vacuum decay between false and true vacua. Movies of the two processes can be found in [71].

Symmetry restoration: We study the case with $g = 0$, where $\Delta V = 0$ and, thus, the potential has two degenerate vacua at $\phi_\pm = \pm\eta$. In the left panel of Fig. 1 we plot the potential, and in the right panel the evolution of the scalar field at the black hole horizon $r = 2M$ (note the use of the Kerr-Schild coordinates, corresponding to infalling observers). While asymptotically far from the black hole the field oscillates around the negative minimum ϕ_-, at smaller radii the amplitude of the oscillations grows due to the increased density resulting from the accretion. The scalar field starts to probe the quartic part of the potential, and when the cloud has become dense enough, it transcends the barrier and reaches the other vacuum at $\phi_+ = +\eta$. The region of restored symmetry phase extends beyond the horizon and gradually expands as more matter is accreted—its extent will ultimately depend on the asymptotic reservoir available.

We can derive conditions on the timescale for the transition using the following argument. The field ϕ is at its maximum, when its time derivative is zero. The energy density is then approximately given by $\rho \approx V(\phi) + k^2(\phi+\eta)^2/2$, where $k \approx \mu$ is the typical inverse length scale of the gradient. When the field reaches the positive vacuum, at $\phi_+ = \eta$, the gradient term is the only contribution to the energy density, so that $\rho \approx 2k^2\eta^2 = 4\lambda\eta^4$. We choose the asymptotic, background, value of the density to be $\rho_0 \approx 0.03\lambda\eta^4$, which means the density near the black hole has to grow by $\approx$100 times to reach $\rho \approx 4\lambda\eta^4$. The timescale for the transition Δt is related to the scalar accretion rate $\dot\rho$. Using that the accretion by the black hole is approximately given by $\dot M \sim \rho_0 v R^2$, with $v \approx 0.5$ the radial velocity,

$$\Delta t \approx \frac{4\lambda\eta^4 - \rho_0}{\dot\rho} \approx 4M\left(\frac{4\lambda\eta^4}{\rho_0} - 1\right). \tag{5}$$

See the Supplemental Material [72] for more details. Hence, we find that symmetry should be restored at $t/M \approx 400M$, which is consistent with what we observe in Fig. 1. After this state is reached the mean of the field near the black hole $\langle\phi\rangle_{\rm bh}$ appears to remain roughly constant, it now effectively evolves in a symmetric potential.

In Fig. 2 we plot the radial profiles of the mean value $\langle\phi(r,t)\rangle$ at different times. Initially the scalar field oscillates around the negative minimum everywhere, so that $\langle\phi(r,t)\rangle = -\eta$ for all radii. The scalar amplitude grows near the black hole due to the accretion, so that the field transcends the barrier near the horizon and starts to oscillate periodically over the full double-well potential. Hence, $\langle\phi(t)\rangle_{\rm bh} = 0$ and symmetry is restored at that radius. However, the effects of gradients and asymptotic boundary conditions play an important role keeping the field oscillating around $\langle\phi(t)\rangle_\infty = -\eta$ at large distances. Since the symmetry restored phase requires the field to reach a

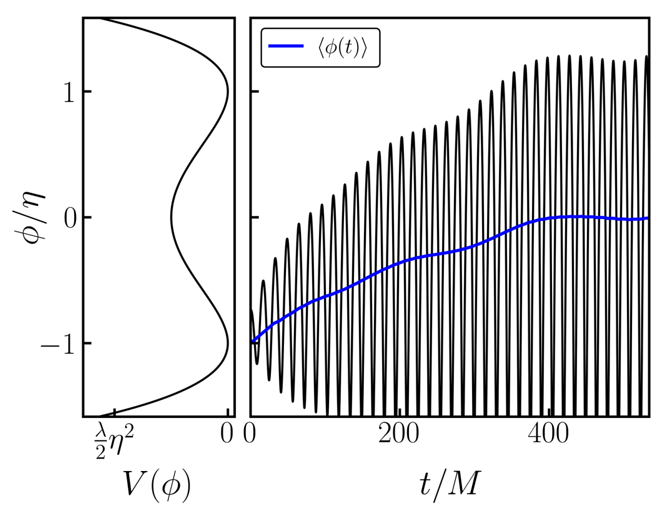

FIG. 1. Symmetry restoration case, where the value of the scalar field at $r = 2M$ transitions the barrier and explores the other vacuum (black line). In blue we plot the mean value of the scalar field, which transitions from a symmetry broken phase at $\langle\phi(t)\rangle_{\rm bh} = -\eta$ to a restored phase at $\langle\phi(t)\rangle_{\rm bh} = 0$.

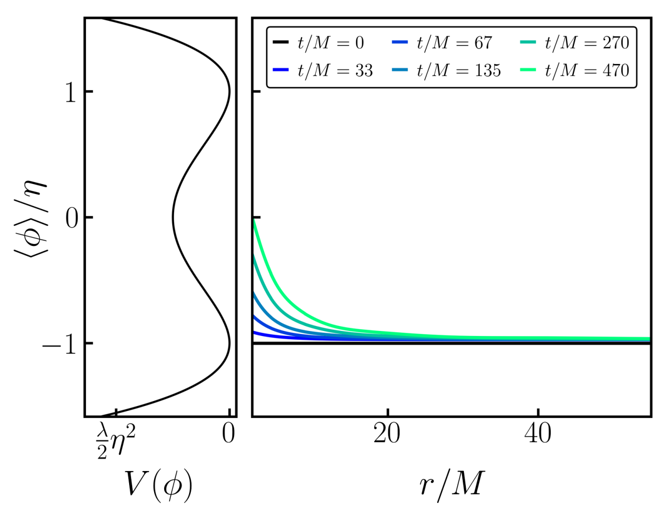

FIG. 2. Evolution of the mean scalar field radial profile over time. Symmetry is restored near the black hole horizon as $\langle\phi(t)\rangle_{\rm bh} = 0$, and the field remains pinned down at $\langle\phi(t)\rangle_\infty = -\eta$ asymptotically. The restoration phase slowly grows outwards as the black hole accretes more scalar field.

certain density, it will only extend out as far as this density is reached. Hence, the timescale for symmetry restoration will strongly depend on the accretion rate. From Fig. 2 we can infer that the symmetry restored phase propagates outwards, into the symmetry broken phase. This is a slow process, driven by the buildup of the scalar matter via accretion, and so is different from the well-known expanding bubble of a new phase that is nucleated when there is a vacuum decay, which we will see in the following section.

What is the end state of this process? The density will only continue to increase whilst there is an asymptotic reservoir of the scalar feeding the growth. So only in the infinite accretion limit would the field fully restore symmetry. At some point the reservoir may be depleted and the cloud will then gradually decay into the black hole (a process that can be extremely long on astrophysical timescales for light scalars [63]).

Vacuum decay: We now study how black holes can catalyze vacuum decay. To do so, we vary g in Eq. (3), breaking the degeneracy between the minima by introducing a difference in vacuum potential energy densities ΔV. If $g < 0$, the energy difference between the minima is negative $\Delta V < 0$, and ϕ_- and ϕ_+ correspond to a false (metastable) and a true vacuum, respectively. This means that, as opposed to the degenerate case we have discussed before, there is now a preference for the field to decay to the lower energy state at ϕ_+.

The process for $g = -0.04\ \mathrm{M}^{-2}$ is illustrated in Fig. 3, which shows how the profile of the time averaged scalar field $\langle\phi(r,t)\rangle$ changes over time. Close to the black hole, accretion drives the growth of the amplitude of the scalar field until it transcends the barrier and starts exploring the true vacuum ϕ_+ by forming a true vacuum bubble. When

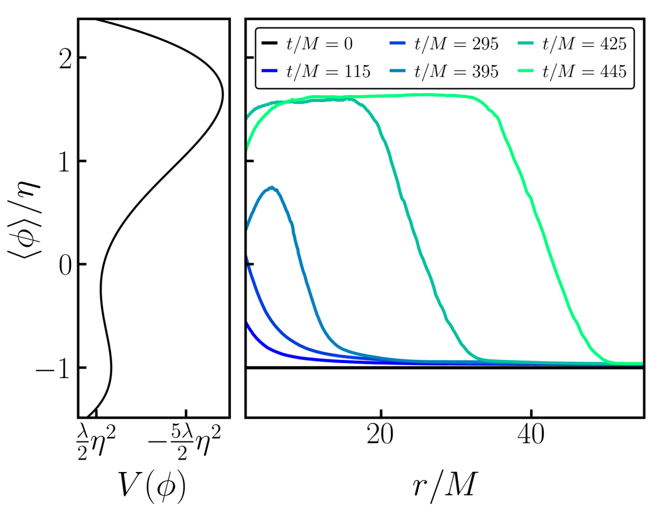

FIG. 3. Evolution of the mean $\langle\phi(r,t)\rangle$ over time. The dynamics near the black hole triggers the nucleation of a true vacuum bubble that expands at ultrarelativistic speeds.

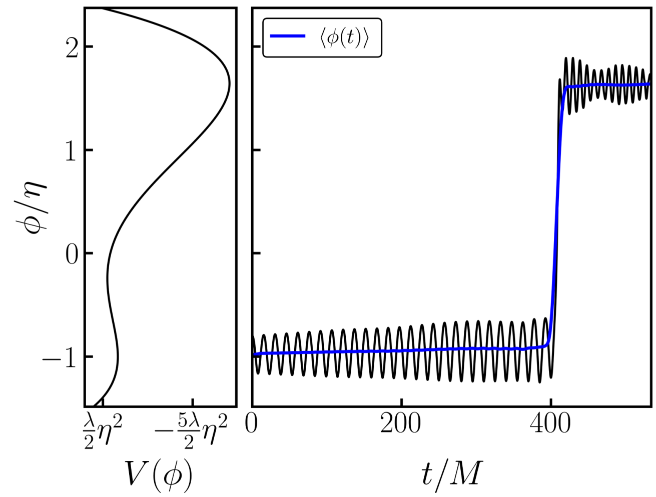

FIG. 4. Vacuum decay case, showing the value of the scalar field at $r = 20M$ (black line). In blue we plot the mean value of the scalar field $\langle\phi(t)\rangle$, which transitions from ϕ_- to ϕ_+.

this bubble is large enough so that the energy difference between vacua can overcome the wall tension, it is more favorable for the bubble to expand radially outwards. The energy difference ΔV is transferred to the bubble wall velocity, so that it quickly accelerates and expands at ultrarelativistic speeds. Gradients in the field once again play a crucial role. In this case, as opposed to keeping the field around the false minimum, they are responsible for dragging the rest of the field down to the true minimum, see Fig. 4. Whilst this is similar to how vacuum bubbles that formed via quantum tunneling propagate [3–5], the timescale for the decay here is completely deterministic. In our classical scenario, the associated timescale is given by the initial conditions around the black hole, the details of the potential, and the accretion rate (which itself depends on the ratio μM).

So what is the end state of this process? As the true vacuum bubble expands, it drags down the field in the false vacuum, which remains oscillating around the true minimum ϕ_+, as shown in Fig. 4. More interestingly, very close to the black hole, the symmetry restoration mechanism remains at play: the scalar field may sweep the whole potential, revisiting the false vacuum if the cloud density is large enough, see Fig. 3. It would be interesting to study the possibility that this process could trigger the nucleation of a separate de Sitter universe when including backreaction, as observed in upward transitions of flyover vacuum decay [51]. If, on the other hand, the field is decaying to an AdS vacuum $V(\phi_+) < 0$, the universe would transition to a collapsing phase that should eventually result in a big crunch, (although see also [73] where scalar dynamics play a role). So although in this work we have ignored the effects of backreaction, this new vacuum decay mechanism triggered by black holes could have important gravitational consequences.

Discussion. In this work we have proposed a novel mechanism by which black holes can alter the vacuum structure of the Universe. The presence of compact objects leads to the formation of overdensities in scalar fields in their vicinity via accretion, which translate to large-amplitude oscillations in the potential. In the case of a scalar field embedded in a symmetry broken phase, these oscillations can transcend the potential barrier separating different vacua. As a result, shells of symmetry restored phase may be created around black holes. In the case in which the field is in a false vacuum, this mechanism can also trigger vacuum decay near the black hole, forming a true vacuum bubble that propagates outwards at ultra-relativistic speeds and changes the vacuum structure even far away from the black hole.

Whilst here we focus on the case of $\mu M \sim 0.5$ (with $\mu^2 = 2\lambda\eta^2$), this process should be relevant for scalar fields and black holes whenever $\mu M \gtrsim 1$—the accretion clouds that we study are suppressed in the case of very light scalars [66]. That is, for $(\mu/10^{-11}\ \mathrm{eV})(M/M_\odot) \gtrsim 1$. The decay rate of this classical mechanism is completely deterministic and depends on the accretion rate of the black hole. The timescale derived in Eq. (5), which is valid for $\mu M \sim 1$, can be written in terms of physical quantities as

$$\Delta t \approx 1\ \ \mathrm{sec}\left(\frac{\eta}{\mathrm{GeV}}\right)^2\left(\frac{\mathrm{GeV/cm^3}}{\rho_0}\right)\left(\frac{M}{M_\odot}\right)^{-1}, \tag{6}$$

showing that this mechanism could take place on short astrophysical timescales. The assumption that backreaction can be neglected is valid if the maximum density remains small relative to the curvature scale of the BH, which imposes the condition

$$\left(\frac{\rho_{\max}}{10^{42}\ \mathrm{GeV/cm^3}}\right)\left(\frac{M}{M_\odot}\right)^2 \ll 1. \tag{7}$$

This should be the case for most physically relevant scenarios involving stellar-mass black holes (the present average dark matter density is of order $\mathrm{GeV/cm^3}$).

Here we have studied the simplest case of a minimally coupled scalar field around an isolated black hole, but similar transitions could be catalyzed by accretion onto binary black holes [74,75]. We conclude that the presence of scalar fields around black holes could have implications for a range of dark matter and early Universe scenarios, the study of which we leave to future work.

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