Distinguishability and many-particle interference

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Quantum interference of two independent particles in pure quantum states is fully described by the particles' distinguishability: the closer the particles are to being identical, the higher the degree of quantum interference. When more than two particles are involved, the situation becomes more complex and interference capability extends beyond pairwise distinguishability, taking on a surprisingly rich character. Here, we study many-particle interference using three photons. We show that the distinguishability between pairs of photons is not sufficient to fully describe the photons' behaviour in a scattering process, but that a collective phase, the *triad* phase, plays a role. We are able to explore the full parameter space of three-photon interference by generating heralded single photons and interfering them in a fibre tritter. Using multiple degrees of freedom—temporal delays and polarisation—we isolate three-photon interference from two-photon interference. Our experiment disproves the view that pairwise two-photon distinguishability uniquely determines the degree of non-classical many-particle interference.

The famous Hong-Ou-Mandel (HOM) experiment in 1987 provided the first important example of non-classical two-photon interference [1]. Two independent photons impinging on a beam splitter exhibit bunching behaviour at the output ports that cannot be explained by a classical field model. The degree of bunching depends on how similar the two photons are in all degrees of freedom, for example time, frequency, polarisation, and spatial mode. Extending the study of interference to many particles is of interest from a fundamental as well as from a technological viewpoint [2–7]. The scattering of multiple photons in linear networks is related to solving problems in quantum information processing, metrology, and quantum state engineering [8–16]. Thus, understanding multiphoton interference is also of great relevance for practical applications.

Here, we demonstrate how many-particle interference is fundamentally richer than two-particle interference [17]. Two situations with the *same* pairwise distinguishability can lead to a *different* output distribution. This is due to a phase, the *triad* phase, that occurs only when more than two photons interfere.

We use independent photons and a tritter, a three-port symmetric beam splitter to investigate many-particle interference. We isolate the triad phase for the first time by interfering three photons in a tritter and exploiting multiple degrees of freedom, here time and polarisation. We show that interfering three identical photons and varying time delays between them, as demonstrated in previous work [5, 18, 19], is not sufficient to study three-photon interference in full generality [20, 21]. Our experiment allows us to isolate and tune the three-photon interference term as distinct from two-photon interference. In particular manipulation of the triad phase goes beyond what is possible using temporal delays alone [5, 6, 19].

Theory

The inner scalar product of two pure states $|\phi_i\rangle$ and $|\phi_j\rangle$ is:

$$\langle\phi_i|\phi_j\rangle = r_{ij} e^{i\varphi_{ij}}, \tag{1}$$

where $r_{ij} \in (0, 1)$ is the real modulus and $\varphi_{ij} \in (0, 2\pi)$ is the argument. The modulus r_{ij} can be interpreted as a measure of the distinguishability of two photons in states $|\phi_i\rangle$ and $|\phi_j\rangle$, and equals zero (one) for two orthogonal (identical) states [22]. The argument φ_{ij} has, so far, received little attention due to its irrelevance in two-photon interference.

We consider two examples of devices that can be used to probe interference: a beam splitter and a tritter. The simplest device to probe interference is a balanced two-port beam splitter (see Fig. 1a). When two photons $|\phi_1\rangle$ and $|\phi_2\rangle$ are injected into the beam splitter, the output statistics depend on the pairwise distinguishability of the incident photons:

$$P_{11} = \frac{1}{2}\left(1 - r_{12}^2\right), \tag{2}$$

where P_{11} is the probability for detecting one photon in each of the output ports. If the photons are completely indistinguishable they always exit the same output port, in contrast to the classical behaviour.

A tritter maps three spatial input modes onto three spatial output modes (see Fig. 1b); the linear transformation for a balanced tritter is given by the unitary matrix:

$$U_{tritter} = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \zeta^2 & \zeta\\ 1 & \zeta & \zeta^2\end{pmatrix}, \tag{3}$$

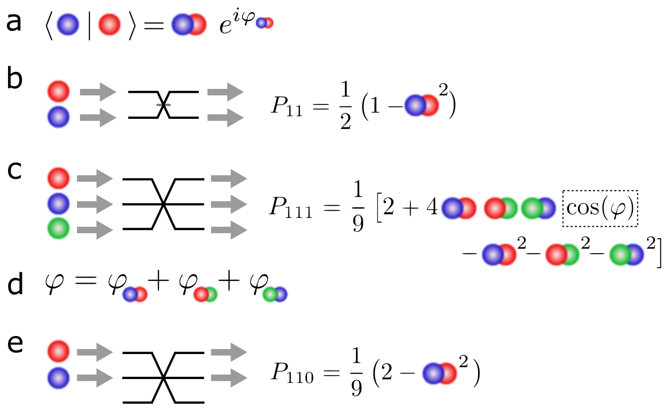

Figure 1. Interference of photons in balanced beam splitters and tritters. **a.**, **b.** The output statistics of two photon interfering in a beam splitter can be described via the pairwise distinguishability of the photons. **c.** In the case of a tritter, the output statistics depend on an additional phase φ. **d.** This *triad* phase φ is defined by the arguments of the pairwise complex scalar products. **e.** φ only occurs in the interference of more than two photons.

where each output is equally likely and $\zeta = e^{i2\pi/3}$.

If we inject three photons into the tritter—a single photon in state $|\phi_i\rangle$ into each mode i for each $i = 1, 2, 3$—the probability P_{111} of having one photon in each of the output modes of the tritter is (see Appendix) [23, 24]:

$$P_{111} = \frac{1}{9}\left[2 + 4\, r_{12} r_{23} r_{31} \cos(\varphi) - r_{12}^2 - r_{23}^2 - r_{31}^2\right] \quad (4)$$

where we define the collective *triad* phase $\varphi = \varphi_{12} + \varphi_{23} + \varphi_{31}$ as the sum of the three arguments. The dependence on φ appears only if the photons are partially distinguishable. If the states are orthogonal, the three moduli are zero; if they are identical, their scalar product will be equal to one and φ vanishes. Similar expressions can also be derived for the probabilities of having two or three photons in one of the output modes of the tritter (see Appendix).

Note that a global phase applied onto one of the input states does not lead to any change in the triad phase φ. Each phase φ_{ij} is only defined up to a global arbitrary phase. The sum of the phases, the triad phase, has physical meaning and is a measurable quantity. It remains unaffected by any global phase transformation and is crucial for the description of *partially distinguishable* photons [25, 26].

However, dependence on the triad phase φ only occurs in measurements with more than two photons. The two-photon output coincidence probabilities P_{011} (one photon in outputs 2 and 3), P_{101}, P_{110} when sending two photons into different input ports of the tritter (as in Fig. 1e) are:

$$P_{011} = P_{101} = P_{110} = \frac{1}{9}\left(2 - r_{ij}^2\right),\ i, j = 1, 2, 3;\ i \neq j \quad (5)$$

and depend only on the mutual distinguishability of the incident photons $|\phi_i\rangle$ and $|\phi_j\rangle$.

Probing the triad phase and genuine three-photon interference

We introduce a convenient implementation that allows us to control the moduli r_{ij} and the arguments φ_{ij} independently. We use two degrees of freedom for each spatial mode—time and polarisation—to show that the addition of non-identical polarisation states can be used to create a non-zero φ. We consider the following input states to the tritter (see Fig. 2):

$$|\phi_i\rangle = |t_i\rangle \otimes \left(\cos\alpha_i |H\rangle + e^{i\eta_i} \sin\alpha_i |V\rangle\right) \quad (6)$$

where $|t_i\rangle$ is a temporal mode delayed by time t_i, $|H\rangle$ and $|V\rangle$ denote horizontal and vertical polarisation, respectively, and $i = 1, 2, 3$ denotes the spatial mode. Using only temporal modes, $|t_1\rangle, |t_2\rangle, |t_3\rangle$, and otherwise identical photons with symmetric spectral intensities, the triad phase would always vanish, since $\langle t_1|t_2\rangle\langle t_2|t_3\rangle\langle t_3|t_1\rangle$ is real and non-negative (see Appendix).

In a first experiment, we aim to probe the triad phase directly. As a first step, we prepare the photons with the same polarisation $|H\rangle$ in states

$$|\phi_i\rangle = |t_i\rangle \otimes |H\rangle \quad (7)$$

for $i = 1, 2, 3$, which sets $\varphi = 0$.

In the next step, we prepare photons in states (as depicted in the inset in Fig. 3b):

$$\begin{aligned} |\phi_1'\rangle &= |t_1\rangle \otimes |H\rangle \\ |\phi_2'\rangle &= |t_2\rangle \otimes \frac{1}{2}(|H\rangle + \sqrt{3}|V\rangle) \\ |\phi_3'\rangle &= |t_3\rangle \otimes \frac{1}{2}(|H\rangle - \sqrt{3}|V\rangle). \end{aligned} \quad (8)$$

Here the scalar products $\langle\phi_1'|\phi_2'\rangle = 1/2\langle t_1|t_2\rangle$ and $\langle\phi_3'|\phi_1'\rangle = 1/2\langle t_3|t_1\rangle$, but $\langle\phi_2'|\phi_3'\rangle = -1/2\langle t_3|t_1\rangle$, setting $\varphi = \pi$. These two configurations demonstrate that using polarisation as an additional degree of freedom allows us to vary the triad phase φ (see Appendix for more details).

In a second experiment, we *isolate* three-photon interference from two-photon interference. We explicitly show that control of φ allows manipulation of the three-photon term whilst leaving the two-photon interference terms constant. To do so, we prepare the following as input states to the tritter:

$$\begin{aligned} |\phi_1''\rangle &= |t_1\rangle \otimes [\cos(2\theta)|H\rangle + i\sin(2\theta)|V\rangle] \\ |\phi_2''\rangle &= |t_2\rangle \otimes \left[\frac{1}{2}(\sqrt{3}|H\rangle + |V\rangle)\right] \\ |\phi_3''\rangle &= |t_3\rangle \otimes \left[\frac{1}{2}(\sqrt{3}|H\rangle - |V\rangle)\right], \end{aligned} \quad (9)$$

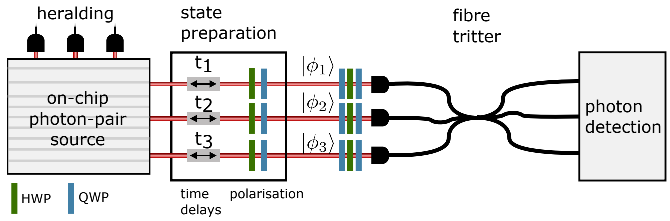

Figure 2. Scheme of the experimental setup. We generate three photon pairs using spontaneous four-wave mixing (SFWM) in silica-on-silicon waveguides [19]. The relative temporal delays of the three photons are adjusted using delay stages. We use sets of quarter-wave plates (QWPs) and half-wave plates (HWPs) to prepare the polarisation state of each photon and to compensate for polarisation rotations in the fibres. The outputs of the fibre tritter are monitored using commercial avalanche photodiodes and additional fibre beam splitters and a tritter for pseudo-number resolution. The average rate of sixfold coincidences is about 0.5 Hz (see Appendix).

where the state $|\phi_1''\rangle$ depends on a polarisation rotation with angle θ and the polarisations of $|\phi_2''\rangle$ and $|\phi_3''\rangle$ are kept constant. With these states, we obtain the following moduli

$$r_{12} = |\langle t_1|t_2\rangle| \times \frac{1}{2}\sqrt{2+\cos(4\theta)} \qquad (10)$$

$$r_{31} = |\langle t_3|t_1\rangle| \times \frac{1}{2}\sqrt{2+\cos(4\theta)} \qquad (11)$$

$$r_{23} = |\langle t_2|t_3\rangle| \times \frac{1}{2} \qquad (12)$$

and the triad phase

$$\varphi = 2\,\mathrm{Arg}\left(\sqrt{3}\cos(2\theta) + i\sin(2\theta)\right). \qquad (13)$$

The angle θ affects both the triad phase φ and the moduli r_{12}, r_{31}; the temporal state $|t_1\rangle$ only affects r_{12} and r_{31}, but not φ. Combining control of both θ and $|t_1\rangle$ allows us to manipulate φ whilst r_{12} and r_{31} remain unchanged. For example, to keep $r_{12} = r_{23} = r_{31} = 1/2$, $|t_1\rangle$ must be chosen such that

$$|t_1 - t_2| = |t_1 - t_3| = \sigma\sqrt{2\ln[2+\cos(4\theta)]} \qquad (14)$$

with $t_2 = t_3$ and σ^2 being the variance in time of the Gaussian wave packet (see Appendix).

Experiment and Results

To study the triad phase experimentally, we generate three heralded photons using spontaneous four-wave mixing (SFWM) in silica-on-silicon waveguides [19] (see Fig. 2 and Appendix).

We first probe the triad phase φ directly by choosing the input polarisations of the photons as given in Eqns. (7) and (8). By setting $t_1 = t_2 - \tau/2$, and $t_3 = t_2 + \tau/2$, and varying τ smoothly over the range shown in Fig. 3c and d, we tune the degree of two- and three-photon interference. The results are shown in Fig. 3; we see a clear qualitative difference in behaviour for the two cases of $\varphi = 0$ and $\varphi = \pi$.

We then demonstrate genuine three-photon interference by choosing the input states as given in Eqn. (9), but now setting the time delay differences as in Eqn. 14. We determined σ from a set of two-photon HOM dips with polarisations chosen as in Fig. 4a (first and third panel). The results are shown in Fig. 4; we observe good agreement of the measured curves with the theoretical prediction. The three-photon data follow a cosine shape as predicted by Eqn. 4. The two-photon contributions P_{110}, P_{101}, P_{011} (see Eqn. 5) are nearly constant and show fluctuations of only on average 6% and the single photon detections at the tritter outputs vary only by a maximum of 3% due to polarisation dependence. This verifies that these two-photon contributions are independent of the arguments. Detailed analysis suggests that polarization dependence of the tritter contributes to these fluctuations (see Appendix).

Our experimental data, both in Fig. 3 and Fig. 4, show the expected behaviour, but there are some deviations from the probabilities given by Eqns. (7), (8), and (9). To understand the influence of experimental imperfection, we performed a simulation of our experiment. Based on our model, we calculated theory curves including realistic experimental parameters (see dashed lines in Fig. 3 and Fig. 4 and Appendix for more details) [27]).

Conclusion

In this work, we identify and describe a new phase that arises at the level of three photons: the triad phase. This new phase manifests itself in quantum interference and therefore has implications for the scattering of many particles. In particular, the outcome of scattering events of more than two particles is determined not only by pairwise distinguishabilities of the particles' wavefunctions, but also on the collective properties of the particles. In this context, the triad phase initially emerges as a formal artifact [17, 18, 24, 28–34]; we show here that it is of physical relevance.

There is a formal similarity between the triad phase and the geometric phases that can be acquired by single photons, for instance in the Pancharatnam-Berry phase [22, 35–37]. Scaling up our study to more than three photons is ongoing work, but for example four interfering photons can be described by six two-photon measurements and three three-photon measurements.

Our work has implications for both linear-optical quantum computing and general multiparticle scattering. It shows having truly indistinguishable particles is a crucial ingredient for all types of scattering experiments. How-

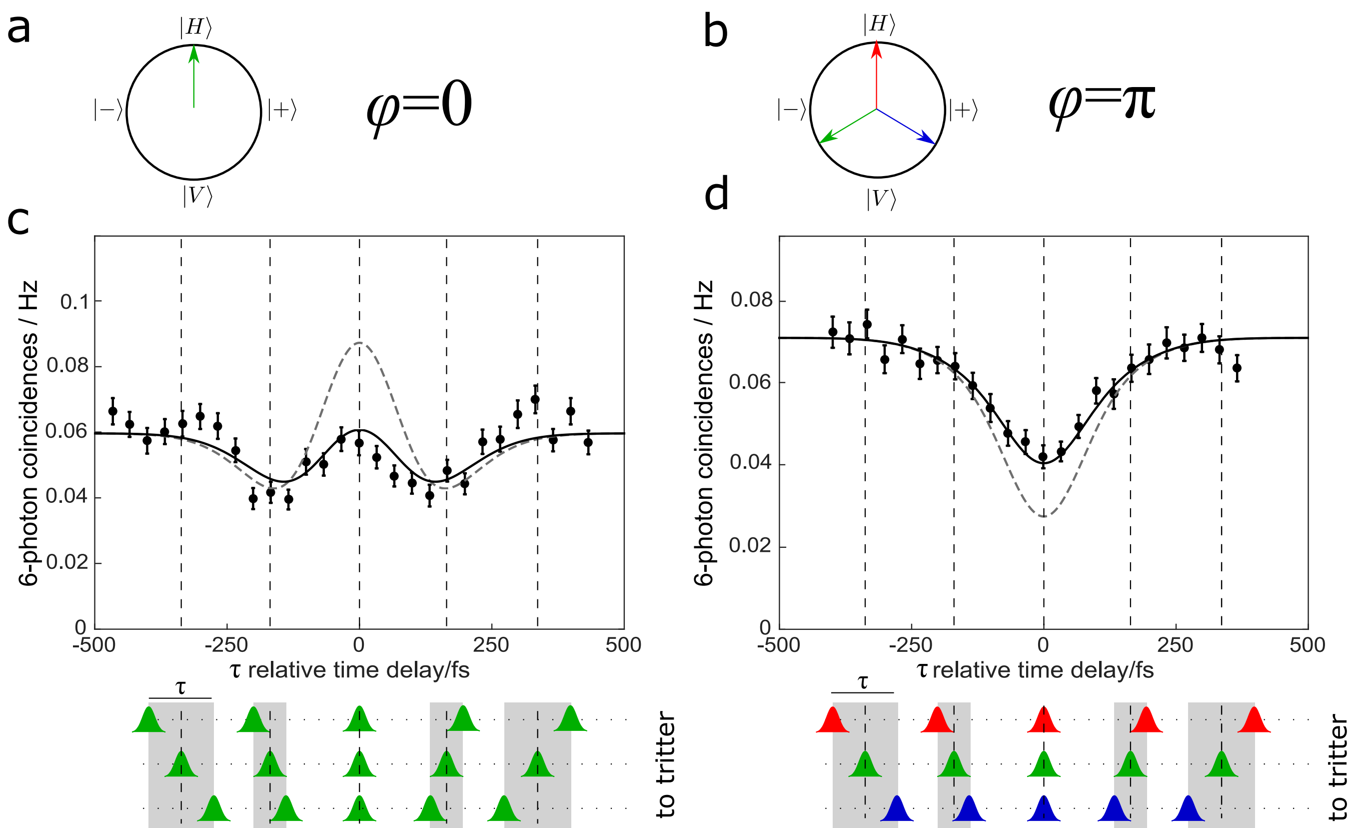

Figure 3. Experimental heralded three-photon coincidences at the output of a fibre tritter for two values of the triad phase φ. **a**, **b**, We choose two polarisation configurations so that $\varphi = 0$ (**a**) and $\varphi = \pi$ (**b**), see Eqns. 7 and 8. **c**, **d**, We measure heralded three-fold coincidences ($\propto P_{111}$) between the different output ports of the tritter whilst varying the temporal delays of the photons. As shown pictorially beneath the plots, we start in a configuration where the photons are completely distinguishable in time; two of the photons are then scanned symmetrically across the third photon ($t_1 = t_2 - \tau/2$, $t_3 = t_2 + \tau/2$). The grey boxes show the region of temporal overlap of the photons. The non-monotonic behaviour in **c** arises because $\varphi = 0$ causes the three-photon interference term in Eqn. 4 to have a contribution of opposite sign to those of the two-photon terms described by r_{ij}^2. In **d** $\varphi = \pi$ and so the contribution is of the same sign, resulting in monotonic behaviour of the statistic. The grey curve shows the theoretical prediction and the dashed black curve is calculated using a model which includes experimental imperfections (see main text for details). The absolute number of counts per point were between 200 and 350 (250 and 450) for **a** (**b**). Error bars are calculated from repeated measurements.

ever, our work also opens up new opportunities as the triad phase can be seen as a tool to engineer the output state of a scattering process. Furthermore, extending applications such as boson sampling to partial distinguishabilities and using multiple degrees of freedom will be an interesting avenue to explore.

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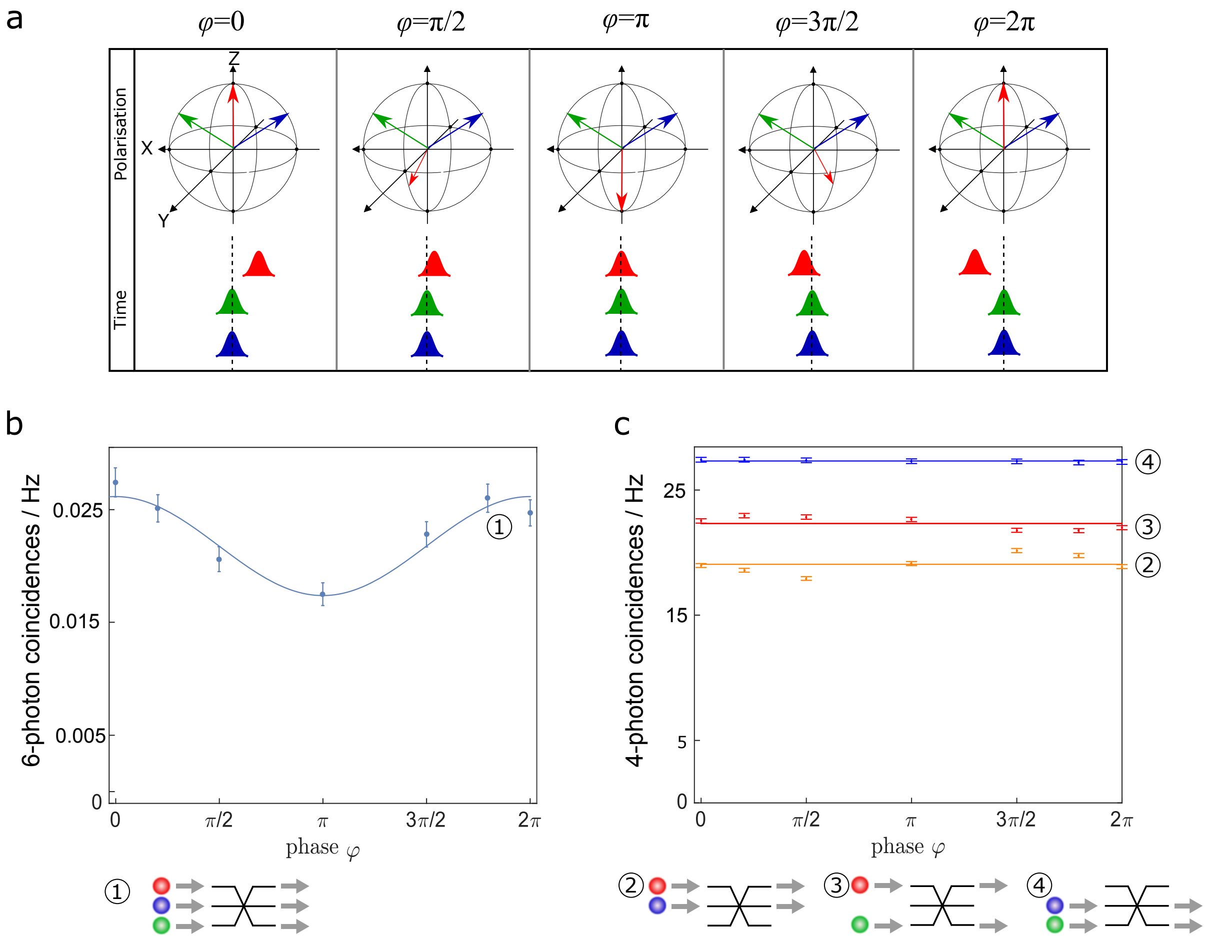

Figure 4. Isolating two-photon from three-photon interference. **a**, We vary the triad phase by rotating the polarisation of photon $|\phi_1''\rangle$ as given in Eqn. 9, leaving the polarisation states of the two other photons fixed. To keep the moduli r_{12} and r_{31} constant, we adapt the temporal overlaps of the photons by tuning $|t_1\rangle$. **b**, The three-photon signal P_{111} varies with the triad phase (absolute number of counts per data point is between 330 and 515). The plotted curve is a theory curve calculated based on our model of the experiment. **c**, We plot a subset of two-photon distinguishability terms to demonstrate that these are kept constant.

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AUTHOR CONTRIBUTIONS

A.J.M. and A.E.J. performed the experiments, theoretical modeling, and data analysis. B.J.M., S.B., and W.S.K assisted with data-taking and data analysis. All authors discussed the results. M.C.T. developed the theory. B.J.M., S.B., W.S.K. and I.A.W conceived the project. S.B., W.S.K. and I.A.W. supervised the project. All authors wrote the manuscript.