Supporting Information for Magneto-optical Kerr switching properties of $(CrI_3)_2$ and $(CrBr_3/CrI_3)$ bilayers

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SI. Selection Rules

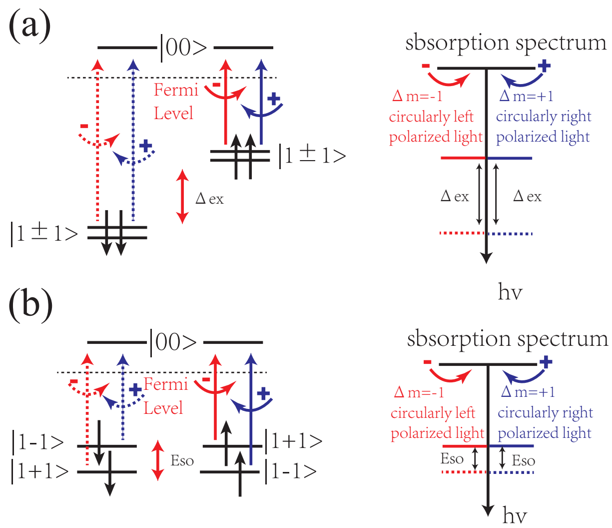

Figure S1: The simplified electronic structure and absorption spectrum. (a) the p⟶s transition without spin-orbit coupling. (b) the p⟶s transition without exchange splitting.

The real part σ'_ω and imaginary part σ''_ω of the conductivity tensor are, respectively, even and odd with respect to ω and are linked by the well-known Kramers-Kronig transformation.[1]

$$\sigma'_{ij}(\mathbf{k},\omega) = \frac{2}{\pi} P \int_0^\infty \frac{\omega' \sigma''_{ij}(\mathbf{k},\omega')}{(\omega'^2 - \omega^2)} d\omega'$$

$$\sigma''_{ij}(\mathbf{k},\omega) = -\frac{2}{\pi} \omega P \int_0^\infty \frac{\sigma'_{ij}(\mathbf{k},\omega')}{(\omega'^2 - \omega^2)} d\omega'$$

In such case, the absorptive parts of optical conductivity tensor (real diagonal and imaginary off-diagonal elements) due to interband transitions can be obtained using Kubo's formula within linear response theory:[2–4]

$$\sigma'_{xx}(\omega) = \frac{\lambda}{\omega} \sum_{k,jj'} \left[\left|\Pi^+_{jj'}\right|^2 + \left|\Pi^-_{jj'}\right|^2 \right] \delta\left(\omega - \omega_{jj'}\right)$$

$$\sigma''_{xy}(\omega) = \frac{\lambda}{\omega} \sum_{k,jj'} \left[\left|\Pi^+_{jj'}\right|^2 - \left|\Pi^-_{jj'}\right|^2 \right] \delta\left(\omega - \omega_{jj'}\right)$$

where $\lambda = \frac{\pi e^2}{2\hbar m^2 V}$ is a material specific constant, $\hbar\omega$ is the photon energy, $\hbar\omega_{jj'}$ is the energy difference between the occupied and unoccupied bands at the same k point, and $\Pi^{\pm}_{jj'} = \left\langle \mathbf{k}j \left| \frac{1}{\sqrt{2}} \left(\hat{p}_x \pm i\hat{p}_y \right) \right| \mathbf{k}j' \right\rangle$ are the dipole matrix elements for circularly polarized light with $\hat{p}_+ = \frac{1}{\sqrt{2}}(\hat{p}_x + i\hat{p}_y)$ (left) and $\hat{p}_- = \frac{1}{\sqrt{2}}(\hat{p}_x - i\hat{p}_y)$ (right), respectively. Besides, according to Kubo's formula, the selection rules for electric dipolar transitions must be satisfied, i.e., $\Delta l = \pm 1$, and $\Delta m_l = \pm 1$. The first selection rule implies that only transitions between s and p levels or between p and d levels are allowed. For the second selection rule, the transitions with $\Delta m_l = +1$ and $\Delta m_l = -1$ correspond to left and right circularly polarized light, respectively.[5,6] More generally, absence of the Kerr effect is due to the fact that the value of the numerator of Eq. (1) off diagonal element of the optical conductivity tensor σ_{xy} is zero, *i.e.*,

$$|\langle \mathbf{k}j \uparrow | \hat{p}_+ | \mathbf{k}j' \uparrow \rangle| = |\langle \mathbf{k}j \downarrow | \hat{p}_- | \mathbf{k}j' \downarrow \rangle|$$

$$|\langle \mathbf{k}j \uparrow | \hat{p}_- | \mathbf{k}j' \uparrow \rangle| = |\langle \mathbf{k}j \downarrow | \hat{p}_+ | \mathbf{k}j' \downarrow \rangle|$$

(corresponding to the case without exchange splitting see FIG.S1(b))

$$|\langle \mathbf{k}j \downarrow | \hat{p}_+ | \mathbf{k}j' \downarrow \rangle| = |\langle \mathbf{k}j \downarrow | \hat{p}_- | \mathbf{k}j' \downarrow \rangle|$$

$$|\langle \mathbf{k}j \uparrow | \hat{p}_+ | \mathbf{k}j' \uparrow \rangle| = |\langle \mathbf{k}j \uparrow | \hat{p}_- | \mathbf{k}j' \uparrow \rangle|$$

(corresponding to the case without spin-orbit coupling see FIG.S1(a))

In other words, both the spin-orbit coupling and exchange splitting have an essential role in the occurrence of MOKE.

SII. Symmetry-adapted form of the optical conductivity tensor σ_{ij} for all the magnetic point groups

It is well known that the magneto-optical Kerr effect is related to off-diagonal components of the optical conductivity tensor. In this section we consider the optical conductivity tensor rather than the corresponding dielectric tensor. The quantities are related by the identity.[7]

$$\varepsilon_{ij}(\omega) = \delta_{ij} + i\frac{4\pi}{\omega}\sigma_{ij}(\omega)$$

The general form of the Kerr angle in arbitrary symmetry only involves the antisymmetric part of the optical conductivity tensor.[8]

$$\begin{array}{l} \tilde{\sigma}_{ij} = (\sigma_{ij} - \sigma_{ji})/2, i \neq j \\ \tilde{\sigma}_{ji} = -(\sigma_{ij} - \sigma_{ji})/2, i \neq j \end{array} \quad (\text{ i, j} = 1,\ 2,\ 3)$$

Now consider the two kinds of forms of the conductivity tensor:

$\sigma_{ij} = \sigma_{ji}$ $(i \neq j)$, there is no antisymmetric part and therefore no MOKE;

$\sigma_{ij} \neq \sigma_{ji}$ $(i \neq j)$, there is an antisymmetric part, consequently, MOKE is allowed.

Based on this fact, we list below the conductivity tensor σ_{ij} of total 122 magnetic point groups.[9] In order for MOKE to be allowed, the magnetic point group should be one of the 31 pyromagnetic point groups which have an antisymmetric part: (1, –1, 2, 2′, m, m′, 2/m, 2′/m′, 2′2′2, m′m2′, m′m′2, m′m′m, 4, –4, 4/m, 42′2′, 4m′m′, –42′m′, 4/mm′m′, 3, –3, 32′, 3m′, –3m′, 6, –6, 6/m, 62′2′, 6m′m′, –6m′2′,6/mm′m′). More detail can be seen below, particularly in the tensors marked with red labels.

$$\begin{pmatrix} 1 & & \mathbf{1} & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & \sigma_{13} \\ 2 & \sigma_{21} & \sigma_{22} & \sigma_{23} \\ 3 & \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \begin{pmatrix} 2 & & 11' & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & \sigma_{13} \\ 2 & \sigma_{12} & \sigma_{22} & \sigma_{23} \\ 3 & \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix} \begin{pmatrix} 3 & & \mathbf{-1} & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & \sigma_{13} \\ 2 & \sigma_{21} & \sigma_{22} & \sigma_{23} \\ 3 & \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \begin{pmatrix} 4 & & -11' & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & \sigma_{13} \\ 2 & \sigma_{12} & \sigma_{22} & \sigma_{23} \\ 3 & \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$$

5	$-1'$		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	σ_{13}
2	σ_{12}	σ_{22}	σ_{23}
3	σ_{13}	σ_{23}	σ_{33}

6	**2**		
σ_{ij}	1	2	3
1	σ_{11}	0	σ_{13}
2	0	σ_{22}	0
3	σ_{31}	0	σ_{33}

7	$21'$		
σ_{ij}	1	2	3
1	σ_{11}	0	σ_{13}
2	0	σ_{22}	0
3	σ_{13}	0	σ_{33}

8	**2'**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	σ_{13}
2	$-\sigma_{12}$	σ_{22}	σ_{23}
3	σ_{13}	$-\sigma_{23}$	σ_{33}

9	**m**		
σ_{ij}	1	2	3
1	σ_{11}	0	σ_{13}
2	0	σ_{22}	0
3	σ_{31}	0	σ_{33}

10	$m1'$		
σ_{ij}	1	2	3
1	σ_{11}	0	σ_{13}
2	0	σ_{22}	0
3	σ_{13}	0	σ_{33}

11	**m'**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	σ_{13}
2	$-\sigma_{12}$	σ_{22}	σ_{23}
3	σ_{13}	$-\sigma_{23}$	σ_{33}

12	**2/m**		
σ_{ij}	1	2	3
1	σ_{11}	0	σ_{13}
2	0	σ_{22}	0
3	σ_{31}	0	σ_{33}

13	$2/m1'$		
σ_{ij}	1	2	3
1	σ_{11}	0	σ_{13}
2	0	σ_{22}	0
3	σ_{13}		σ_{33}

14	$2'/m$		
σ_{ij}	1	2	3
1	σ_{11}	0	σ_{13}
2	0	σ_{22}	0
3	σ_{13}	0	σ_{33}

15	$2/m'$		
σ_{ij}	1	2	3
1	σ_{11}	0	σ_{13}
2	0	σ_{22}	0
3	σ_{13}	0	σ_{33}

16	**2'/m'**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	σ_{13}
2	$-\sigma_{12}$	σ_{22}	σ_{23}
3	σ_{13}	$-\sigma_{23}$	σ_{33}

17	222		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{22}	0
3	0	0	σ_{33}

18	$2221'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{22}	0
3	0	0	σ_{33}

19	**2'2'2**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	0
2	$-\sigma_{12}$	σ_{22}	0
3	0	0	σ_{33}

20	$mm2$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{22}	0
3	0	0	σ_{33}

21	$mm21'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{22}	0
3	0	0	σ_{33}

22	**m'm2'**		
σ_{ij}	1	2	3
1	σ_{11}	0	σ_{13}
2	0	σ_{22}	0
3	$-\sigma_{13}$	0	σ_{33}

23	**m'm'2**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	0
2	$-\sigma_{12}$	σ_{22}	0
3	0	0	σ_{33}

24	mmm		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{22}	0
3	0	0	σ_{33}

25	$mmm1'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{22}	0
3	0	0	σ_{33}

26	$m'mm$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{22}	0
3	0	0	σ_{33}

27	**m'm'm**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	0
2	$-\sigma_{12}$	σ_{22}	0
3	0	0	σ_{33}

28	$m'm'm'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{22}	0
3	0	0	σ_{33}

29	**4**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	0
2	$-\sigma_{12}$	σ_{11}	0
3	0	0	σ_{33}

30	$41'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

31	$4'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

32	**-4**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	0
2	$-\sigma_{12}$	σ_{11}	0
3	0	0	σ_{33}

33	$-41'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

34	$-4'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

35	**4/m**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	0
2	$-\sigma_{12}$	σ_{11}	0
3	0	0	σ_{33}

36	$4/m1'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

$$\begin{pmatrix} 37 & 4'/m & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 38 & 4/m' & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 39 & 4'/m' & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 40 & 422 & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 41 & 4221' & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 42 & 4'22' & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 43 & \textbf{42'2'} & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 44 & 4mm & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 45 & 4mm1' & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 46 & 4'm'm & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 47 & \textbf{4m'm'} & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 48 & -42m & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 49 & -42m1' & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 50 & -4'2'm & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 51 & -4'2'm' & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 52 & \textbf{-42'm'} & & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

53	$4/mmm$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

54	$4/mmm1'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

55	$4/m'mm$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

56	$4'/mm'm$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

57	$4'/m'm'm$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

58	**4/mm'm'**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	0
2	$-\sigma_{12}$	σ_{11}	0
3	0	0	σ_{33}

59	$4/m'm'm'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

60	**3**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	0
2	$-\sigma_{12}$	σ_{11}	0
3	0	0	σ_{33}

61	$31'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

62	**-3**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	0
2	$-\sigma_{12}$	σ_{11}	0
3	0	0	σ_{33}

63	$-31'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

64	$-3'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

65	32		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

66	$321'$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

67	**32'**		
σ_{ij}	1	2	3
1	σ_{11}	σ_{12}	0
2	$-\sigma_{12}$	σ_{11}	0
3	0	0	σ_{33}

68	$3m$		
σ_{ij}	1	2	3
1	σ_{11}	0	0
2	0	σ_{11}	0
3	0	0	σ_{33}

$$\begin{pmatrix} 69 & & 3m1' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 70 & & \mathbf{3m'} & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 71 & & -3m & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 72 & & -3m1' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 73 & & -3'm & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 74 & & -3'm' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 75 & & \mathbf{\text{-}3m'} & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 76 & & \mathbf{6} & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 77 & & 61' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 78 & & 6' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 79 & & \mathbf{\text{-}6} & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 80 & & -61' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 81 & & -6' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 82 & & \mathbf{6/m} & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 83 & & 6/m1' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 84 & & 6'/m & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 85 & & 6/m' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 86 & & 6'/m' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 87 & & 622 & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 88 & & 6221' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 89 & & 6'22' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 90 & & \textbf{62'2'} & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 91 & & 6mm & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 92 & & 6mm1' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 93 & & 6'mm' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 94 & & \textbf{6m'm'} & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 95 & & -6m2 & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 96 & & -6m21' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 97 & & -6'm'2 & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 98 & & -6'm2' & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 99 & & \textbf{-6m'2'} & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 100 & & 6/mmm & \\ \sigma_{\mathrm{ij}} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 101 & 6/mmm1' & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 102 & 6/m'mm & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 103 & 6'/mmm' & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 104 & 6'/m'mm' & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} 105 & \mathbf{6/mm'm'} & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & \sigma_{12} & 0 \\ 2 & -\sigma_{12} & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 106 & 6/m'm'm' & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} 107 & 23 & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix} \begin{pmatrix} 108 & 231' & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix}$$

$$\begin{pmatrix} 109 & m-3 & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix} \begin{pmatrix} 110 & m-31' & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix} \begin{pmatrix} 111 & m'-3' & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix} \begin{pmatrix} 112 & 432 & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix}$$

$$\begin{pmatrix} 113 & 4321' & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix} \begin{pmatrix} 114 & 4'32' & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix} \begin{pmatrix} 115 & -43m & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix} \begin{pmatrix} 116 & -43m1' & & \\ \sigma_{\rm ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix}$$

$$\begin{pmatrix} 117 & -4'3m' & & \\ \sigma_{ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix} \begin{pmatrix} 118 & m-3m & & \\ \sigma_{ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix} \begin{pmatrix} 119 & m-3m1' & & \\ \sigma_{ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix} \begin{pmatrix} 120 & m'-3'm & & \\ \sigma_{ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix}$$

$$\begin{pmatrix} 121 & m-3m' & & \\ \sigma_{ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix} \begin{pmatrix} 122 & m'-3'm' & & \\ \sigma_{ij} & 1 & 2 & 3 \\ 1 & \sigma_{11} & 0 & 0 \\ 2 & 0 & \sigma_{11} & 0 \\ 3 & 0 & 0 & \sigma_{11} \end{pmatrix}$$

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