Essays in Financial Econometrics

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University College

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In memory of Alexander Temple McCune

20.01.1995 - 13.08.2021

Statement of Authorship

I, Tales Padilha, declare that this thesis represents my own work, and that none of it has already been accepted, or is currently being submitted, for any degree, diploma, certificate, or other qualification in this University or elsewhere. The first chapter of this thesis is entirely my own work. The second chapter is a result of joint work with Susana Campos-Martins and the third one is joint with Serhan Cevik.

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Abstract

This dissertation consists of three chapters discussing issues in the field of financial econometrics. All three chapters are largely empirical, with some theoretical developments in second moments modelling in the second chapter.

The first chapter of this thesis analyses the market neutrality of Pairs Trading, a statistical arbitrage trading technique, from a second moments perspective. In this study, I analyse how market and idiosyncratic news affect the profitability of this trading strategy. I propose a conditional covariance framework based on Kroner and Ng (1998) extension of the BEKK Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model to analyse the dependence of second moments between different portfolios of pairs and the returns of the market. In contradiction to what is generally assumed about the market neutrality of this strategy, the results indicate the existence of significant spillovers from market news to different portfolios of pairs. In a second step of the study, I analyse the contribution of both idiosyncratic and market components to pairs volatility over time in an asynchronous panel of pairs. This analysis shows that the volatility of the pairs strategy has become more dependent on idiosyncratic rather than market shocks. In this sense, although Pairs Trading cannot be said to be market neutral from a second moments perspective if we look at the full sample from 1962 to 2018, the strategy has certainly become more market neutral as markets have evolved over the last two decades.

In the second chapter of this thesis, Susana Campos-Martins (Oxford) and I propose an econometric framework that explains the Purchasing Power Parity (PPP) Puzzle as common volatility shocks. Most of the discussion about the PPP Puzzle of Rogoff (1996) has pertained to the reversion speed of deviations from PPP. Much less attention, however, has been given to the other component of the puzzle: the high volatilities of real exchange rates. In this paper, we provide a framework that is capable of explaining the econometric sources of these volatilities. First, we study the drivers of real exchange rate volatilities using a Cross-Sectionally Augmented Autoregressive Distributed Lag (CS-ARDL) panel framework and the conditional covariance matrices of the system with nominal exchange rates and price differentials. This analysis indicates that, for both emerging and developed markets, common factors are the main drivers of volatility. With this result in hand, we propose a novel econometric framework that explains the sources of these volatilities as common second moment shocks. This model allows us to gives structure to the origins of these high volatilities and propose an extension to study their macrofinancial drivers.

The third and final chapter of this thesis is an adapted version of a current IMF working paper which introduces the IMF Soft Power Index. In this chapter, Serhan Cevik (IMF) and I introduce a new composite Global Soft Power Index (GSPI) composed of six dimensions for a broad sample of 72 countries across the world over the period 2007-2019. The proposed framework allows for comparisons not only at the "headline" level of the GSPI, but also at the level of the sub-indices, which allows us to identify and study how countries differ at a granular level of soft power. In a final step of the analysis, we present a possible macro-financial application to analyse the relationship between soft power and the volatility of Real Effective Exchange Rates (REER) across countries and over time. The results indicate that some dimensions of the GSPI play an important role in explaining real exchange rate volatility at almost all significance levels. Overall, our framework presents a systematic approach to measure soft power and its dimensions. By capturing the matrix of soft power characteristics, the GSPI offers significant advantages in comparative analysis of soft power across countries and over time.

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1 | Accounting for the Variance: An analysis of Pairs Trading market neutrality

Abstract

Pairs Trading is commonly referred to as the most straightforward example of a market neutral trading strategy. In this study, we analyse how market news and idiosyncratic news affect the profitability of this statistical arbitrage trading technique. We propose a conditional covariance framework based on Kroner and Ng (1998) extension of the BEKK Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model to analyse the dependence in second moments between different portfolios of pairs and the returns of the market. In contradiction to what is generally assumed about the market neutrality of this strategy, our results indicate the existence of significant spillovers from market news to different portfolios of pairs. In fact, market shocks seem to play a significant role in explaining conditional second moments of the returns of these portfolios, particularly in more turbulent periods. Not only do market shocks have a significant impact on the volatility of different portfolios, but they also appear to influence the correlation of this strategy with the market. In a second step of the study, we analyse the contribution of both idiosyncratic and market components to pairs volatility over time in an asynchronous panel of pairs. This analysis shows that the volatility of the pairs strategy has become more dependent on idiosyncratic rather than market shocks. In this sense, although Pairs Trading cannot be said to be market neutral from a second moments perspective if we look at the full sample from 1962 to 2018, the strategy has certainly become more market neutral as markets have evolved over the last two decades.

1.1 Introduction

Over the last quarter of a century, financial markets across the world have experienced the emergence of statistical arbitrageurs. Together with this increasing interest of practitioners, the mere existence of these arbitrage opportunities triggered financial economists into studying if these strategies are, indeed, profitable and how can we explain their anomalous returns.

Pairs Trading was pioneered by Nunzio Tartaglia's quant group at Morgan Stanley in the 1980's and has since then been used by different players of financial markets. It is a statistical arbitrage technique with a disarmingly simple concept. First, one needs to find two stocks whose prices move together. Then, when spread between them widens, short the winner and buy the loser. If there is a equilibrium relationship between the stocks, prices will converge and the arbitrageur will profit. If equity markets were efficient at all times, risk-adjusted returns from Pairs Trading should not be positive. Nevertheless, evidence from the literature suggests otherwise.

Even though the study of pairs strategies has not had the same academic popularity of other statistical arbitrage techniques, such as the momentum from Jegadeesh and Titman (1993), after Gatev et al. (2006) the literature on this topic has been expanding. Nevertheless, although many authors have developed studies trying to answer questions related to whether this strategy is profitable, much less attention has been given to trying to understand the properties of its returns. In particular, very little research has been done in studying the market neutrality of this strategy.

Inspired by the empirical findings of Engelberg et al. (2008) and Jacobs and Weber (2015) on the heterogeneous performance of Pairs Trading depending on different factors, this study focuses on analysing the different effects of market and idiosyncratic movements on the volatility of returns of this strategy. The objective is to develop a parsimonious and tractable framework to study how much of the dynamics in the second moments of

Pairs Trading is due to movements in the market and to idiosyncratic variations in the stocks that are being used to form the pairs. Understanding these sources of volatility is particularly important in the case of Pairs Trading, once that much of attraction of this strategy comes from its assumed market neutrality.

We begin by using a conditional covariance framework to study the market neutrality hypothesis of this strategy and how market shocks are transmitted into the volatility of different portfolios of pairs. In a second step, we incorporate information about idiosyncratic movements in each of the stocks to identify the effects of market and idiosyncratic terms on the second moment dynamics of this strategy.

Our results indicate that not only the assumption of market neutrality of Pairs Trading fails to hold, but market shocks seem to explain a significant part of the variation in the portfolios of pairs, particularly during periods of great market instability. As one would expect, as the number of pairs of stocks in the portfolio increases, the volatility of this portfolio in normal times is reduced. Nevertheless, results from the conditional covariance framework indicate that a strong transmission in volatility between the market and the portfolio of pairs persists in periods with high market volatility, even for portfolios with a large number of pairs.

The significant effect of the market on the volatility of pairs persists even when idiosyncratic information about the stocks that form the pairs are included in the analysis. The estimation results also suggest, however, that this relationship, albeit significant, is much less important than the relationship between the volatility of pair returns and the components of system with idiosyncratic returns. A time-varying analysis of the drivers of pairs volatility indicates that this market impact has been decreasing since the mid-to-late nineties and has become almost irrelevant after the 2010s.

Taken all together, these results show that, from a second moments perspective, Pairs Trading has not been historically market neutral. Evidence does seem to suggest, however, that this strategy has become increasingly more market neutral as markets have evolved over the last two decades.

In order to have results which are comparable to the literature, we focused on the Pairs Trading strategy of Gatev et al. (2006), the paper that outlined the anomalous returns of this strategy and still the most influential one in the study of Pairs Trading. Moreover, the dataset is also selected with the intention to generate results which are commensurable with previous studies. Therefore we consider daily returns for all stocks traded on NYSE, NASDAQ and AMEX (prior to NYSE acquisition of AMEX) from 1962 to 2018, expanding the traditional timeline on studies of Pairs Trading to the end of 2018.

This research relates to several other puzzles in financial markets. In a broader sense, an analysis of Pairs Trading market neutrality helps to better understand how the interactions of news affects the efficiency of fundamentally linked asset prices. Within the next pages, Section 1.2 presents a brief literature review on Pairs Trading and some relevant studies on statistical arbitrage. Section 1.3 describes the methodology of the Pairs Trading strategy in more details. The dataset used for this research, the transformations required, and some preliminary results are presented in Section 1.4. In Section 1.5, we present an econometric framework to model conditional second moments and study the relationship between the returns of Pairs Trading portfolios and the returns of the market, as well as its results. Section 1.6 expands the analysis by introducing the idiosyncratic components of the stocks that form the pairs. The linear independence between the market and idiosyncratic terms is used to develop an econometric framework that allows for the decomposition of the volatility of Pairs Trading into each of these components. Section 1.7 concludes and presents suggestions for further research in the topic.

1.2 Literature Review

The first comprehensive study on Pairs Trading was done by Gatev et al. (2006). Using a simple algorithm for choosing pairs, the authors test the profitability of several straightforward self-financing trading rules. They find an average annualized excess return of about

11% by interpreting the practitioners description of pairs trading as simply as possible. That is, their rule follows a general outline of first "finding stocks that move together" and then taking a long-short position when they diverge.

In their method, Gatev et al. (2006) calculate the spread between the normalized prices of all possible combinations of stock pairs during the formation period. They then select the combinations that have the least sum of squared spread. When the spread diverges by two or more historical standard deviations (as calculated in the formation period), a position is opened. The position is then closed once the spread reverses to zero. As the opening threshold is always set to two standard deviations from the formation period, the divergence required to open a position will be different for each of the pairs.

The out of sample results from Gatev et al. (2006) suggest that these economically and statistically significant profits are not simply an artifact of the training sample, over which Pairs Trading was known to be popular, but it also suggests that the public dissemination of the results has apparently not affected the general risk and return characteristics of the strategy. Remarkably, despite the simplicity of the trading rule and the attention that their paper attracted to this strategy, excess returns of Pairs Trading have persisted. Very few trading rules have stood the test of time. Price momentum of Jegadeesh and Titman (1993) is probably the most well known exception.

Do and Faff (2010) follow the same framework for building the pairs as Gatev et al. (2006). They study the performance of the strategy between July 1962 and June 2009. To do so, the authors look at two measures of excess return. The return on committed capital, which is given by the total market-to-market payoff for all pairs divided by the number of pairs in the portfolio, and the return on employed capital, which is the total payoff divided by the number of pairs that are actually traded.

Results from Do and Faff (2010) conclude that there has been a general decline in the performance of Gatev et al. (2006) type of Pairs Trading strategy. Nevertheless, their results also show that this strategy had a solid performance over two turbulent periods:

from January 2000 to December 2002 ("Dotcom" Bubble) and from July 2007 to June 2009 (2008 Crisis).

The Pairs Trading strategy from Gatev et al. (2006) is based exclusively in the historical closeness in pricing between the securities. As a result, this methodology became known as distance method Pairs Trading. Besides applying the Gatev et al. (2006) method, Do and Faff (2010) also examine ways to improve the distance method by including more information. Their result indicates that the number of zero crosses (number of times the normalized spread crosses the zero line) in the formation period seems to have some information in predicting future convergence. Intuitively, good pairs are those that frequently deviate from each other and reverse back to set the spread to zero. Moreover, restricting the pairs to belong to the same industry results in a significant improvement in profits.

Nevertheless, the results of the distance method for Pairs Trading from Gatev et al. (2006) are not unanimous. Do and Faff (2012) show that this method is largely unprofitable after 2002, once the trading costs are considered. For this reason, Rad et al. (2016) consider other tools, such as cointegration and copulas, to implement Pairs Trading. The objective of Rad et al. (2016) is to study how increasing the sophistication in the methods by which pairs are selected and traded can affect the quality and precision of the captured relationship between the pairs and improve the performance of the Pairs Trading strategy. In their results, the distance method is the approach with highest monthly returns, but the cointegration method exhibits a slightly higher Sharpe ratio. The performance of the copula approach is significantly worse than the first two.

Overall, the most cited paper and the most established methodology to form pairs is the distance method proposed by Gatev et al. (2006). A simple yet compelling algorithm for trading in the U.S. equity market. More importantly, its profitability cannot be explained by previously documented reversal profits. Despite these findings, academic research into pairs trading is still small when compared to other techniques, such as momentum trading or more exotic strategies with options.

1.2.1 On the properties of pairs trading returns

One of the areas of interest of financial economists is understanding the profitability of various forms of statistical arbitrage trading strategies and properties of their returns. One obvious reason for this is with regards to market efficiency, once that the profitability of such strategies violates even the weakest form of market efficiency. Despite the existence of plenty of studies analysing whether the Pairs Trading strategy yield positive returns, very little is known about the properties of these returns.

A few studies have been done on trying to further understand the profitability of Pairs Trading by the use of market fundamentals. Papadakis and Wysocki (2007) analyse how this strategy responds to earnings events. The authors found that about 15% of the pairs openings are triggered by accounting events, either official (earnings announcements) or unofficial (e.g. third party forecasts). The authors also found that trades triggered by accounting events perform worst than non-event trades.

Engelberg et al. (2008) follow the simple approach of Gatev et al. (2006). By analysing the behaviour of Pairs Trading strategies, they find that the profitability of this strategy is higher on days close to the divergence between the two stocks of the pairs. This indicates that the such divergence date is not some random date in which pairs spread reaches an arbitrary threshold, but an informative component to profitability.

While Engelberg et al. (2008) find profits from a Pairs Trading strategy to be larger near the divergence date and then decline monotonically, their profitability remains economically and statistically significant for months. These observations suggests that profits from Pairs Trading could come from different sources. That is, while some factors may contribute to profits at the shorter horizons, some others may contribute to profits at a longer horizons.

Jacobs and Weber (2015) also try to contribute to the understanding of what is driving the profitability of Gatev et al. (2006) type of Pairs Trading strategies. The authors argue that abnormal returns from this type of strategies might come from two general sources. First, investors can overreact to firm specific shocks. Or, in the other case, shocks that affect both stocks of the pair to a similar degree might have different response speeds in each due to investors under-reaction. Although no study before Jacobs and Weber (2015) had explicitly focused on these different possible responses, the empirical literature on Pairs Trading[1] tends to support the latter idea of heterogeneous responses to shocks in factors that affect the two stocks of the pair in a similar way.

Jacobs and Weber (2015) start by analysing the link between different types of information shocks on the day of divergence and subsequent abnormal returns. As proxies for firm specific news, the authors consider earnings announcements and dividend announcements from CRSP. For common news, they rely on the same macroeconomic news as Savor and Wilson (2013). For firm specific news as well as combined macroeconomic news, the authors construct a dummy variable that is one (zero) if a news event does (does not) take place. They then analyse one-month event-time returns of pairs depending on the type of news identifiable on the day of pairs divergence.

The analysis of Jacobs and Weber (2015) finds that pairs that diverge on days with firm-specific news yield considerably lower returns. All firm-specific news are negatively related to pairs trading profitability in a statistically significant and economically meaningful matter. On the other hand, pairs that diverge on days with macroeconomic news tend to generate slightly higher returns.

Overall, Jacobs and Weber (2015) do look at the effect of stock specific shocks and macro shocks but they do so from an event-study perspective. They analyse the link between different types of information shocks on the day when the pairs diverged and the subsequent returns. Without analysing information in days other than when the pair divergence occurred, they bring the idea that idiosyncratic and macroeconomic shocks are important drivers of the profitability of Pairs Trading.

The objective of this piece is therefore to study the dependence in second moments between Pairs Trading returns and both market and idiosyncratic returns via a tractable and

[1] Engelberg et al. (2008), Chen et al. (2017), Deaves et al. (2013), and others.

parsimonious econometric framework. By first analysing the conditional covariance between market returns and the returns of different portfolios of pairs, we are able to track the effects of shocks in the markets to both the volatility of the returns of these portfolios and the covariance term. Moreover, by modelling the series of idiosyncratic volatility for each stock rather than just considering firm specific events, we increase the possible scope of the analysis and build a bridge between the Pairs Trading literature and the literature regarding the study of idiosyncratic volatility.

1.3 Pairs Trading Methodology

Previous studies analysing the effects of different factors on the returns of Pairs Trading have focused on the distance method of Gatev et al. (2006) when forming their pairs. Although most of the literature regarding Pairs Trading focuses on studying properties and the profitability of this distance method, we believe there is no reason for such a constraint. Following the results from Rad et al. (2016) that their cointegration method has a similar performance and an even better Sharpe ratio than the distance method, we initially include pairs formed using the cointegration method in our analysis.

The choice of also considering a cointegration approach is a logical extension. Engle and Granger (1987) have formalized, using an Error Correction Model, how to model short run deviations between variables that cointegrate; that is, that have some long-run stationary relationship. Since the underlying idea of Pairs Trading is exactly to explore these deviations, the inclusion of a cointegration framework to conduce pairs trading in our analysis is straightforward.

In order to generate results which are comparable to previous studies, the methodology used for the formation of pairs follows a similar pattern of the literature. Since Gatev et al. (2006), studies have considered a twelve months pairs formation period followed by a six months trading period. This means that, at any given month m, pairs are formed based on data from the previous twelve months and then traded during the next six months. This is done at the beginning of every month in the trading period. This setting of one year pair formation period followed by a six months trading period is used in the two strategies of Pairs Trading being considered. That is, the distance method and the cointegration approach are implemented using one year of daily data to form the pairs and six months of trading period.

1.3.1 Distance method

Our distance method is implemented following the original procedure of Gatev et al. (2006). In their seminal work, the authors form their pairs by first constructing a cumulative returns index for each stock, scaled to $1 at the first day of the formation period. This "normalized price" is calculated for each of the stocks that will be considered to form pairs. They then choose the matching pair for each stock by finding the security that minimizes the sum of squared deviations between the two normalized price series. In other words, pairs are formed by matching in the normalized daily price space. There are no industry or sector restrictions when performing this match, meaning that all stocks in the sample are pair candidates to each other.

After finding which stocks will belong to each pair, we need to decide when to trade them. At the beginning of each trading period, prices are again normalized to $1. The spread of each of the pairs is then monitored. Following Gatev et al. (2006), we base the trading rules on a standard deviation metric. A position is opened when this spread between the two normalized prices diverges more than two standard deviations (as estimated during the pairs formation period). The position is then closed under two circumstances. We stop trading the pair if either the spread goes back to zero (i.e. when prices converge) or at the end of the trading period. If there is no convergence before the end of the trading period, gains and losses are calculated based on the last trading day. In the case one of the stocks in the pair stops being traded, we close the position of the pair using the information on the last day of trade. Figure 1.1 presents an example of a pair being traded during the first semester of 2009 (between the 1^{st} of January 2009 to the 30^{th} of June 2009).

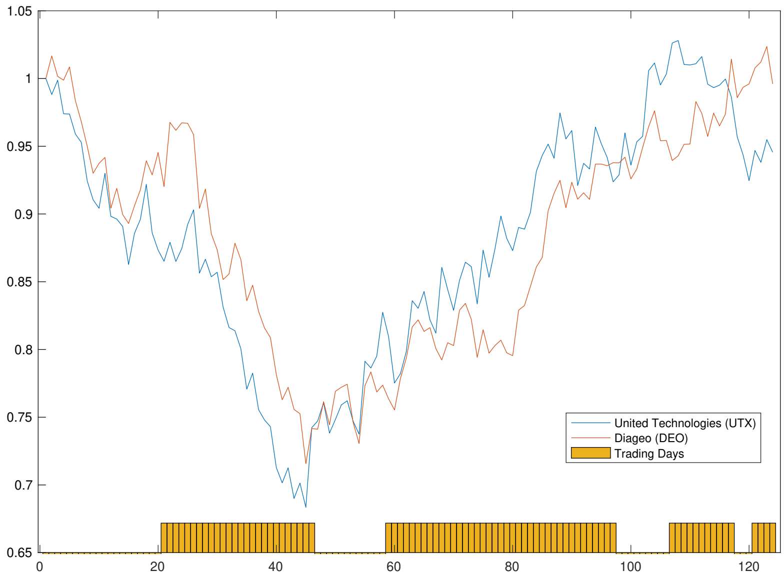


Figure 1.1 Trading the pairs United Technologies (UTX) and Diageo (DEO) during the first semester of 2009 according to the distance method. The X axis is the number of days in the trading period and the Y axis stands for the normalized prices. The yellow bars indicate the days when the pair is being traded.

1.3.2 Cointegration

From Engle and Granger (1987) we know that if $X_{1,t}$ and $X_{2,t}$ are two $I(1)$ time series and there exist a linear combination of these two series that is stationary, then $X_{1,t}$ and $X_{2,t}$ are said to cointegrate. That is, if $X_{1,t}$ and $X_{2,t}$ cointegrate, there exists a nonzero β such that we can write:

$$X_{2,t} - \beta X_{1,t} = \mu_t \tag{1.1}$$

where β is the cointegration coefficient and μ_t is the resulting spread, which is a stationary time series.

Following the error correction framework from Engle and Granger (1987), Vidyamurthy (2004) provides a basic framework on how to apply a cointegration approach to Pairs Trading. The cointegration relationship from Equation (1.1) shows the deviation of the process from its long-term equilibrium. Vidyamurthy (2004) then defines the spread as being the scaled difference in the price of the two stocks:

$$spread_t = X_{2,t} - \beta X_{1,t} \tag{1.2}$$

Assuming one buys one share of X_2 and sells β shares of X_1 at time $t-1$, the profit at time t is given by:

$$(X_{2,t} - X_{2,t-1}) - \beta(X_{1,t} - X_{1,t-1}) \tag{1.3}$$

Which can be re-arranged, in order to compare with the spread term, as:

$$spread_t - spread_{t-1} = (X_{2,t} - \beta X_{1,t}) - (X_{2,t-1} - \beta X_{1,t-1}) \tag{1.4}$$

Therefore, the spread gives exactly the payoff of getting a position in the portfolio above. But recall that this spread is stationary, and hence it has mean reverting properties. We can then use this to build a trading strategy that arbitrates deviations from the long-run equilibrium.

Vidyamurthy (2004) sets two criteria to select the stocks that will be on each pair. First, as in Gatev et al. (2006), we calculate the square distance between normalized prices for all possible pairs and rank the pairs according to this distance method. We then test each of these pairs with lowest sum of squared difference for cointegration[2], until this algorithm has selected the desired number of cointegrated pairs.

Rad et al. (2016) further formalize the cointegration framework from Vidyamurthy (2004). In their procedure, the cointegration regression is estimated using OLS to then estimate the Error Correction Model. For each of the pairs being analysed, we construct the series of spreads, as described by Equation (1.2). Then, using data from the pairs formation period, both the spread mean μ_e and spread standard deviation σ_e are calculated. Following

[2]Using the Engle-Granger test (Engle and Granger, 1987).

Rad et al. (2016), we then normalize the cointegration spread by:

$$spread_{t,norm} = \frac{spread_t - \mu_e}{\sigma_e} \tag{1.5}$$

The trading strategy is finally followed in the same way as in the distance method. That is, positions are opened once the normalized spread, as described in Equation (1.5), goes either above 2 or bellow -2. The position is then closed in case of convergence, which means the spread going back to zero, or at the end of the trading period. As in the distance method, if one of the stocks of the pair stops being traded, the position is closed using the information from the last day of trade.

1.4 Data and Preliminary Results

1.4.1 Constructing the series of Pairs Trading returns

In their seminal paper Gatev et al. (2006) analyse the profitability of portfolios of different sizes (i.e. considering a different number of pairs) between 1962 and 2002. In order to make our results comparable not only to Gatev et al. (2006), but to other papers that followed and focused on similar datasets, we consider a dataset with daily returns on all stocks traded on AMEX (prior to NYSE acquisition of AMEX), NYSE and NASDAQ from the 1^{st} of January 1962 to the 31^{st} of December 2018. The raw data with daily prices has been imported from CRSP data available in the WRDS database for all listed stocks, just adding the restriction that the stock is traded in one of the exchanges mentioned above. The initial full dataset has information on $30,006$ stocks over $14,349$ days.

Given the high dimensionality of the raw data matrix, this includes many stocks that cannot be easily traded. As a result, applying a strategy that requires a reasonable level of liquidity on this plethora of stocks is not realistic. Therefore, when analysing each of the twelve months period to match the pairs, we follow the methodology of Asness et al. (2013) and considered our universe of stocks to be the assets that add up to 90% of the volume in that year. Asness et al. (2013) also argue that stocks with price of less than one Dollar in any given day of the training set should not be consider in a backtesting strategy. As a result, we exclude these "penny stocks" from the pool of possible candidates to from pairs[3].

Since at the beginning of each month pairs are formed based on data from the previous twelve months and then traded during the next semester, at each month we have the subset of possible stocks to be considered to form pairs. The clean data has 667 of these subsets; one for every month in the test set (Jan 1963 to Dec 2018).

Once that the subsets of stocks to be considered to form pairs at the beginning of each month is known, the methodologies described in Section 1.3 are used to match these pairs.

1.4.2 Preliminary results

1.4.2.1 Distance and cointegration methods

We begin by discussing the results of portfolios of pairs based on both the distance method and the cointegration method. Following the results from Gatev et al. (2006), where best performance is achieved in portfolios with 5 and 20 pairs, we construct portfolios with 5 and 20 pairs for each of the strategies. This means that, in the beginning of every month, 5 or 20 pairs are added to the portfolio. These pairs are then traded according to the methodology for the next six months.

Figure 1.2 presents the daily returns for each of the mentioned portfolios. From analysing the returns plotted in Figure 1.2 we get a few initial insights. First, the volatility of the series of returns seems to be significantly lower towards the end of the sample (after 2010). More importantly, one can notice that the profits of the two methodologies are

[3] All code used for data transformation and analysis can be found in the remote repository for this paper at: https://github.com/talespadilha/pairs_trading.

very similar, given the same number of pairs. This makes sense. From Section 1.3, recall that the only difference between the distance method and the cointegration method when selecting pairs is that in the latter the spread is tested for stationarity. But since in the distance method we choose the pairs that have minimum distance between normalized prices in the twelve months before the selection, these tend to be pairs where the difference is stationary. The statistical summary presented in Table 1.1 shows that the series of returns for the distance method and the cointegration method are indeed very alike. Given the same number of pairs, they present almost identical first and second moments, and a correlation coefficient higher than 0.9. The similarity between other properties of returns of the two methodologies can be further observed in Figures A.1 and A.2 in Appendix A.3.

Table 1.1 Means, standard deviations, and correlations of daily portfolio returns.

	Portfolios	Mean	Std	Correlations			
				1.	2.	3.	4.
1.	Distance Method - 5 Pairs	0.0757%	0.4379%				
2.	Distance Method - 20 Pairs	0.0646%	0.2798%	0.55			
3.	Cointegration Method - 5 Pairs	0.0701%	0.4177%	0.92	0.55		
4.	Cointegration Method - 20 Pairs	0.0629%	0.2798%	0.51	0.94	0.55	

1.4.2.2 Including different portfolio sizes

Because returns of the portfolio of pairs vary more according to the number of pairs considered in the strategy than depending on which strategy is being considered, we decided to also include portfolios with 10 and 40 pairs in our analysis[4]. By doing so, we can more clearly identify the impact that the number of pairs in the portfolio has on the properties of returns. Moreover, as can be seen in Figure 1.2 and Figures A.1 and A.2 in Appendix A.3, the returns of portfolios using the distance method or the cointegration methodology are very similar. We have hence decided to focus our analysis on the distance method, once that this is the methodology used by the great majority of the literature on Pairs Trading.

[4]We tested 50 different portfolio sizes, from 5 to 100 pairs. The properties of the returns did not change much for portfolios with more than 40 pairs.

From now on, we will consider portfolios built according to the distance method of Gatev et al. (2006) with 5, 10, 20 and 40 pairs. Figure 1.3 presents the daily returns for each of these portfolios. One can notice that the volatility of daily returns decreases as we increase the number of pairs in the portfolio. The evidence from Figure 1.3 and the results presented in Figures A.3 and A.4 in Appendix A.3 also indicate that this decrease is barely noticeable when we increase the number of pairs from 20 to 40. Moreover, even though volatility in normal times decreases as we increase the number of pairs in the portfolio, some days with very large volatility shocks seem to persist.

The persistence of days with high volatility, even in portfolios with as many as 40 pairs, presents an interesting property of Pairs Trading. By analysing Figure 1.3 and the information in Table 1.2 and Table 1.3, one can notice that not only periods with more market turbulence are reflected in periods with higher volatility in the Pairs Trading portfolios, but days with extreme performance (both positive and negative) are also during major crises in stock markets. Almost all days presented in Table 1.2 and Table 1.3 are during a period of high market instability. These include the 1973–75 recession, the Black Monday in 1987 and the following days, the "Dotcom" bubble, and the 2008 financial crisis. Even if we extend this analysis to the Top 10 or Top 20 days with most extreme returns, this pattern persists.

Table 1.2 Top 5 days with best performance.

5 Pairs	**10 Pairs**	**20 Pairs**	**40 Pairs**
02/05/2001 (5.83%)	20/10/1987 (4%)	20/10/1987 (4.03%)	21/10/1987 (2.83%)
11/04/2001 (5.67%)	02/01/1975 (3.53%)	21/10/1987 (3.25%)	13/10/2008 (2.75%)
20/10/1987 (4.5%)	02/05/2001 (3.24%)	02/05/2001 (3.07%)	20/10/1987 (2.69%)
16/05/1974 (4.48%)	16/05/1974 (2.96%)	02/01/1975 (2.79%)	16/03/2000 (2.14%)
02/01/1975 (4.35%)	05/04/2001 (2.81%)	04/01/1963 (2.46%)	02/01/1975 (1.99%)

Finally, Figure 1.4 displays the performance of the four portfolios being considered against

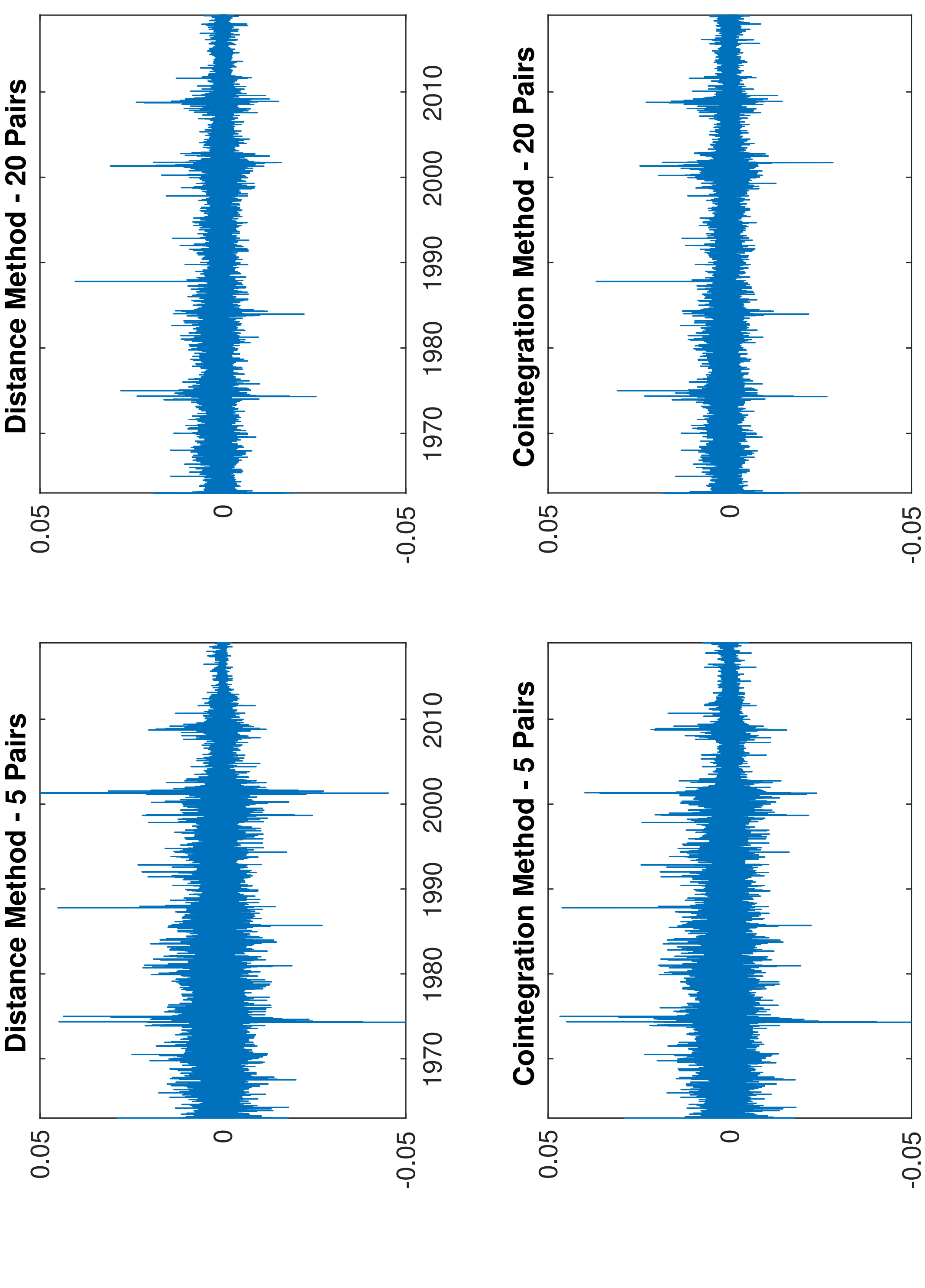


Figure 1.2 Daily Returns - Distance and Cointegration Methods

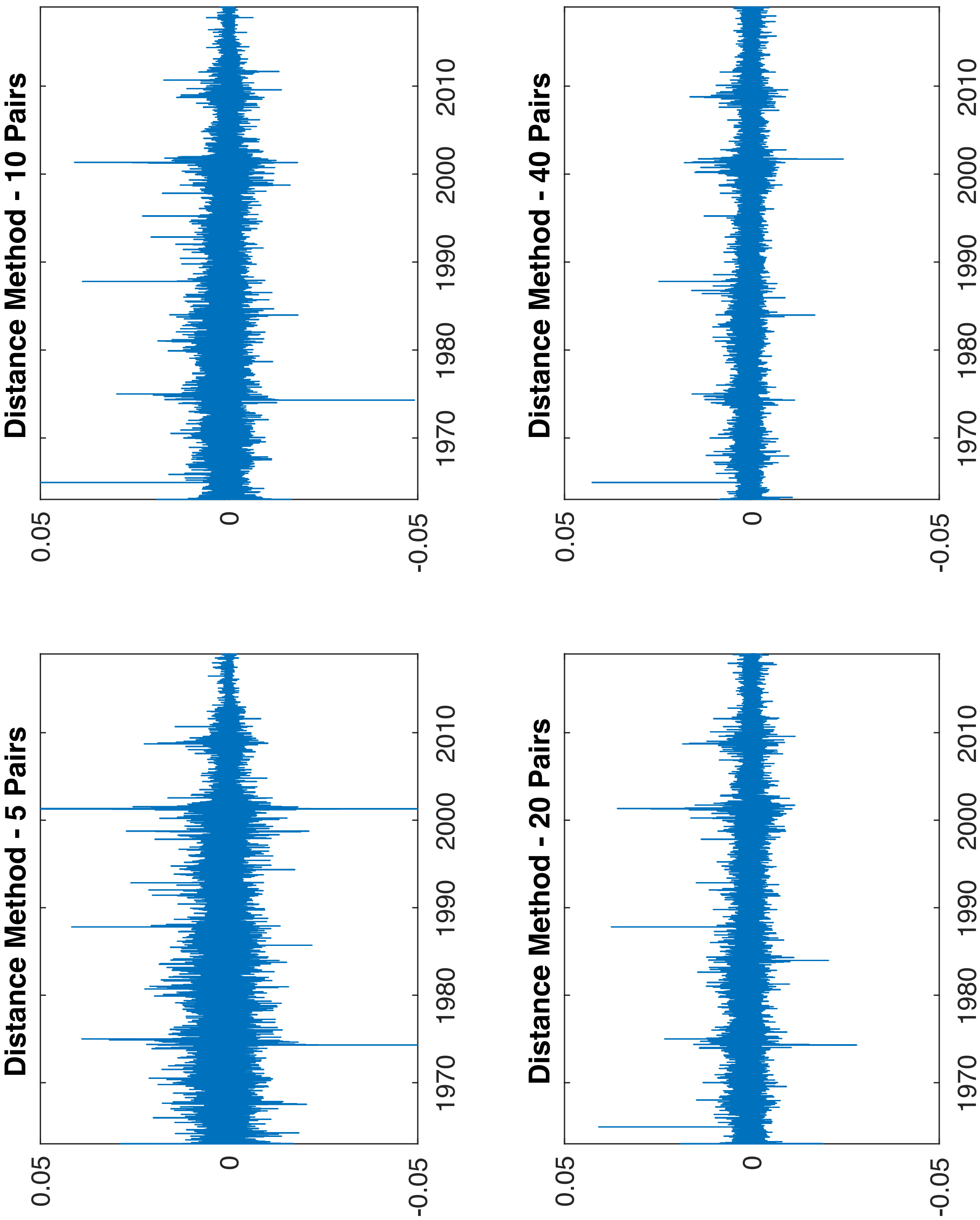


Figure 1.3 Daily Returns - Distance Method

the S&P 500 Index. It is clear that, considering the full time window from 1963 to 2018, all Pairs Trading portfolios outperform the market by a significant amount (notice that the y-axis is in log scale). Nevertheless, the performance of the pairs strategy seems to be decreasing since the mid nineties; particularly after 2010, when the performance of all Pairs Trading strategies is worst than that of the market (see Figure A.5 in Appendix A.3). Figure 1.4 strengthens the view outlined in the previous paragraph. By looking at the lines representing the portfolios of pairs, one can see that the significant jumps occur exactly during the periods of great market turbulence discussed above.

These initial evidences suggest the existence of a relationship between the level of risk, or volatility, in the market and the behaviour of the different portfolios of Pairs Trading. In the next section, we present a conditional covariance framework to analyse the relationship between second moments of market returns and those of Pairs Trading in order to formally assess the level of market neutrality of this strategy from a second moments perspective.

Table 1.3 Top 5 days with worst performance.

5 Pairs	10 Pairs	20 Pairs	40 Pairs
23/04/1974	23/04/1974	23/04/1974	17/09/2001
(-7.92%)	(-4.45%)	(-2.54%)	(-2.35%)
19/04/2001	29/04/1974	22/12/1983	22/12/1983
(-4.52%)	(-1.99%)	(-2.22%)	(-1.83%)
10/05/1974	22/12/1983	14/01/1963	20/09/2001
(-3.81%)	(-1.84%)	(-1.97%)	(-1.77%)
29/04/1974	10/05/1974	14/05/1974	20/11/2008
(-3.7%)	(-1.79%)	(-1.81%)	(-1.57%)
10/07/2001	28/05/1974	10/01/1963	06/11/2008
(-2.75%)	(-1.67%)	(-1.63%)	(-1.23%)

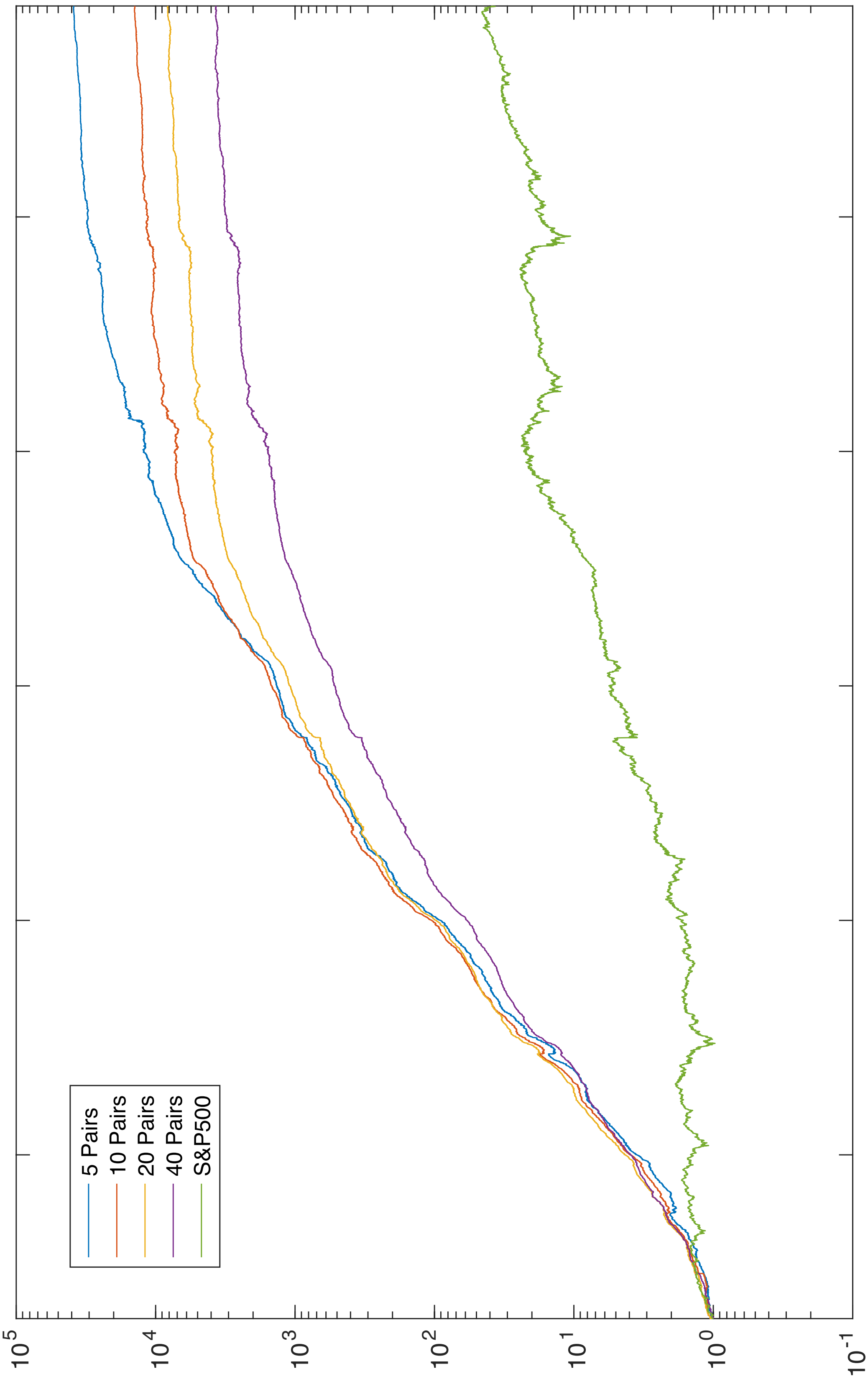


Figure 1.4 Performance of Different Portfolios of Pairs and S&P 500 Index

1.5 Market Volatility and Portfolios of Pairs

Pairs Trading is constantly described as a "market neutral" strategy. That is, an investment strategy that is hedged against market risk. Both academic researchers[5] and practitioners generally refer to Pairs Trading as one of the most traditional examples of market neutral strategies. Of course, there are different possible definitions of market neutrality, regarding the avoidance of specific types of market risk. Nevertheless, the preliminary results from Section 1.4.2 indicate that Pairs Trading might not be as hedged against market risks as its commonly understood.

In this section we propose a conditional covariance framework to analyse the dependence of second moments between the different portfolios of pairs and the returns of the market. The idea is that, by modelling the conditional variance of the returns of these portfolios and the conditional covariance between these returns and those of the market in a framework that allows for cross dependence, we can understand if and how shocks in the overall market are transmitted to the Pairs Trading portfolios.

1.5.1 Econometric framework

The Autoregressive Conditional Heteroskedasticiy (ARCH) model of Engle (1982) is arguably the most influential tool to model conditional second moments. Engle (1982) suggested that unobservable second moments can be modelled by specifying a functional form for the conditional variance and modelling first and second moment jointly. His proposition that these conditional variances depend on elements in the information set in an autoregressive manner has become widely accepted and was even responsible for yielding a Nobel Prize to Engle in 2003. This model was later expanded by Bollerslev (1986) by allowing lags of the conditional variance to affect the current conditional variance. This generalization was called the Generalized ARCH or GARCH.

[5] Gatev et al. (2006), Elliott et al. (2005), Rad et al. (2016), and many other studies describe Pairs Trading as being a market neutral strategy.

The works of Engle (1982) and Bollerslev (1986) have inspired an entire field of research in econometrics, particularly in financial econometrics, with the objective of modelling conditional second moments. Many extensions to the GARCH framework were proposed and received considerable attention in the following years. These include the Exponential GARCH (or EGARCH) of Nelson (1991), which models the logarithm of the variance rather than the variance directly, the GJR-GARCH of Glosten et al. (1993), which allows for asymmetric effects of news impact on conditional volatility, and many other improvements.

The extension of the GARCH toolbox from the univariate to a multivariate universe has also been the focus of many studies since the late nineties. This extenssion, however, implies thinking of a conditional covariance matrix. Let ϵ_t be a n by 1 zero mean random vector. Moreover, let $\mathcal{F}_t$ denote the sigma field generated by the past values of ϵ_t. If we assume that the conditional covariance matrix $\mathbf{\Sigma}_t$ is measurable with respect to $\mathcal{F}_{t-1}$, the conditional distribution of ϵ_t in a multivariate GARCH model can be written as:

$$\epsilon_t|\mathcal{F}_{t-1} \sim N(0, \mathbf{\Sigma}_t) \tag{1.6}$$

Different multivariate GARCH models have hence been proposed in order to model Σ_t as a function of $\mathcal{F}_{t-1}$. The Vector GARCH of Bollerslev et al. (1988) was the first multivariate GARCH specification. In this model the elements in the vec[6] of the conditional variance are allowed to depend on all elements of ϵ_{t-1}, their interactions, and on its lags. That is, in a baseline Vector GARCH the conditional covariance evolves according to:

$$vec(\mathbf{\Sigma}_t) = vec(\mathbf{C}) + \mathbf{A}vec(\epsilon_{t-1}\epsilon'_{t-1}) + \mathbf{B}vec(\mathbf{\Sigma}_{t-1}) \tag{1.7}$$

where $\mathbf{C}$ is a k by k positive definite matrix and $\mathbf{A}$ and $\mathbf{B}$ are k^2 by k^2 parameter matrices.

[6] $vec(\cdot)$ is the operator that stacks the columns of the matrix in a single vector.

The Vector GARCH is the most flexible multivariate GARCH model. In practice, however, its estimation is difficult given the large number of parameters. A more significant drawback of this model regards the positive definiteness of Σ_t. Even when the dimensionality of ϵ_t is small, it is hard to impose restrictions on $\mathbf{A}$ and $\mathbf{B}$ that will guarantee Σ_t to be positive definite.

In order to deal with the positive definiteness problem of the Vector GARCH, Engle and Kroner (1995) proposed a multivariate GARCH model that became known as the BEKK GARCH. Their specification economizes the number of parameters (in comparison to the Vector GARCH), but still preserves great flexibility once that allows every possible element on the right hand side of Equation (1.7) to affect each element of Σ_t. The original BEKK GARCH of Engle and Kroner (1995) specifies the evolution of the conditional covariance as follows:

$$\Sigma_t = \mathbf{C}\mathbf{C}' + \mathbf{A}\epsilon_{t-1}\epsilon_{t-1}'\mathbf{A}' + \mathbf{B}\Sigma_{t-1}\mathbf{B}' \tag{1.8}$$

where $\mathbf{C}$ is a k by k lower triangular parameter matrix and $\mathbf{A}$ and $\mathbf{B}$ are k by k matrices of coefficients.

The BEKK GARCH specification models the conditional covariance as the outer product matrices of the vector of past return shocks and past conditional covariance. Since all terms on the right hand side of Equation (1.8) are expressed in quadratic forms, the positive definiteness of Σ_t is guaranteed.

Other multivariate GARCH models have been proposed by the literature. The Matrix GARCH of Ding and Engle (2001), the Constant Conditional Correlation (CCC) GARCH of Bollerslev et al. (1990) and the Dynamic Conditional Correlation (DCC) GARCH of Engle and Sheppard (2001) are among the most popular ones. Nevertheless, these models generally impose a more restrictive structure either to the covariance or the variance components of Σ_t, when compared to the BEKK GARCH. Therefore, since we are interested in allowing news impacts on the market, the pairs and their crossed terms to affect both the conditional variance of the portfolios of pairs and the covariance term, we have chosen a BEKK GARCH framework to study this dependence.

1.5.1.1 A model for Pairs Trading and market returns

In order to allow for the widely reported asymmetric effects of positive and negative returns in financial markets[7], we will follow the Kroner and Ng (1998) extension to the BEKK GARCH of Engle and Kroner (1995) that allows for asymmetric effects from positive and negative news impacts on different elements of the conditional covariance.

To begin with our analysis, we can define $r_{m,t}$ to be the daily return of the market and $r_{p,t}$ to be the daily return of a portfolio of pairs (as defined by the methodology in Section 1.3). Then, to step into the multivariate framework, let:

$$\mathbf{r}_t \equiv \begin{bmatrix} r_{m,t} & r_{p,t} \end{bmatrix}' \tag{1.9}$$

Where the multivariate process $\{\mathbf{r}_t\}$ can be decomposed, given $\mathcal{F}_{t-1}$, in terms of conditional mean and conditional innovation as:

$$\mathbf{r}_t = \mu_t + \epsilon_t \tag{1.10}$$

Since we are dealing with a vector of returns $\mathbf{r}_t$, we can also refer to the term ϵ_t above as being the vector of demeaned returns. Moreover, given that we are interested in studying the properties of the second moments of this vector of demeaned returns and not the time-varying mean, we won't focus on developing a model for the conditional mean μ_t. Following Kroner and Ng (1998), we simply model the mean of the return vector as a 10^{th} order Vector Autogressive (VAR) process. In order to allow for cross interaction, constant

[7] See Campbell and Hentschel (1992), Bekaert and Wu (2000), and Cappiello et al. (2006) for evidences on the asymmetric behaviour of financial markets.

term and trends in the model for the first conditional moments, we set the following VAR specification:

$$\mathbf{r}_t = \phi_0 + \sum_{s=1}^{10} \mathbf{\Phi}_s \mathbf{r}_{t-s} + \delta t + \epsilon_t \tag{1.11}$$

The estimation is hence done is two steps. We first estimate the mean equation (Equation (1.11)) to obtain the series of vectors of demeaned returns $\{\epsilon_t\}$ and then we proceed to the estimation of the Asymmetric BEKK GARCH for the conditional covariance matrix.

Once we have obtained this vector of demeaned returns, we can focus on the analysis of the conditional second moments. We begin by decomposing:

$$\epsilon_t = \mathbf{\Sigma}_t^{1/2} \mathbf{e}_t \tag{1.12}$$

where $\mathbf{\Sigma}_t \equiv \mathbb{E}_{t-1}[\epsilon_t \epsilon_t']$ is matrix of conditional covariance (assumed to be positive definite) and $\mathbf{e}_t$ is a vector of multivariate standard normally distributed innovations. As a result from this decomposition, we have $\mathbb{E}_{t-1}[\mathbf{e}_t \mathbf{e}_t'] = \mathbf{I}_2$. The assumption regarding the distribution of the residuals $\mathbf{e}_t$ is what characterizes the likelihood function used for estimation in the second step.

From this point onward, we will focus our analysis on the conditional covariance matrix, $\mathbf{\Sigma}_t$. As discussed in the previous section, the BEKK GARCH (Engle and Kroner, 1995) provides a flexible yet tractable structure, in which all news shocks and lags of the conditional covariance are allowed to affect every element of $\mathbf{\Sigma}_t$. Following the asymmetric extension of Kroner and Ng (1998), we set the following process for the conditional covariance:

$$\mathbf{\Sigma}_t = \mathbf{C}\mathbf{C}' + \mathbf{A}\epsilon_{t-1}\epsilon_{t-1}'\mathbf{A}' + \mathbf{G}\eta_{t-1}\eta_{t-1}'\mathbf{G}' + \mathbf{B}\mathbf{\Sigma}_{t-1}\mathbf{B}' \tag{1.13}$$

where $\mathbf{C}$ is a 2 by 2 lower triangular parameter matrix and $\mathbf{A}$, $\mathbf{G}$ and $\mathbf{B}$ are 2 by 2 matrices of coefficients. Moreover, the asymmetric component η_{t-1} is defined as:

$$\eta_t \equiv 1[\epsilon_t < 0] \circ \epsilon_t \quad (1.14)$$

where $\circ$ indicates the Hadamard Product.

Other than estimating the conditional variances and conditional covariances, the structure presented in Equation (1.13) also allows for the construction of news impact surfaces. Kroner and Ng (1998) introduced news impact surfaces as a multivatiate extension of the news impact curves of Engle and Ng (1993). In the news impact curves of Engle and Ng (1993), the conditional variance is plotted against last period's shock by using the coefficients estimated in the univariate GARCH. In the multivariate generalization of Kroner and Ng (1998), we use the coefficients estimated in multivariate GARCH model to study the effects of "news" from different components in the previous period ($\mathbf{e}_{t-1}$) in whichever element of $\mathbf{\Sigma}_t$ we are interested in. In both cases, past conditional variances and covariances are hold at their unconditional sample mean levels in order to build the news impact curves or surfaces.

In our case, we are interested in analysing the effect of these news, particularly market news, on the variance of the portfolios of pairs (σ_p^2) and the covariance of these portfolios with the market (σ_{mp}).

Let h_{ijt} denote the element in the i^{th} row and j^{th} column of $\mathbf{\Sigma}_t$. Then let $\mathbf{z}_{t-1}$ denote the vector of inputs known at time $t-1$ for the determination of h_{ijt}, excluding the components in $\mathbf{e}_{t-1}$. We can hence denote the news impact surface as the three-dimensional graph of the function:

$$h_{ijt} = h_{ijt}(\mathbf{e}_{t-1} | \mathbf{z}_{t-1} = \overline{\mathbf{z}}) \quad (1.15)$$

where $\overline{\mathbf{z}}$ stands for the unconditional sample means of the elements in $\mathbf{z}_{t-1}$.

1.5.2 Estimation results

Before proceeding to the analysis of the conditional second moments, we estimate the model as described in Equation (1.11) for the conditional mean of $\mathbf{r}_t$ for the different portfolios of Pairs Trading. Recall that $\mathbf{r}_t \equiv [r_{m,t} \quad r_{p,t}]'$. In Section 1.3 we have already described the methodology followed to construct the series of returns of the portfolios of pairs r_t^p. For the return of the market r_m^p, we use as proxy the CRSP Value-Weighted Market Return Index as our sample contains stocks being traded in many exchanges.

Once that we have estimated the model for the conditional mean and, as a result, obtained the vector of demeaned returns ϵ_t, we begin our analysis of conditional second moments. The model outlined in Equation (1.13) is estimated for each of the portfolios being considered. Figure 1.5 presents the Asymetric BEKK GARCH estimates for the conditional standard deviations of the different portfolios, the conditional standard deviations of the market and the conditional correlations.

By analysing the “Sigmas” graphs in Figure 1.5, we can see that, for all portfolios being considered, periods of high market volatility are associated with periods of high volatility in the portfolios of pairs. Even though the unconditional volatility is reduced as we increase the number of pairs (see Table 1.4), periods of very high volatility seem to persist even for the portfolio with 40 pairs.

Table 1.4 Unconditional standard deviations and correlations (with the market) for demeaned returns.

Portfolio	σ_p	ρ_{mp}
5 Pairs	0.4265%	0.0512
10 Pairs	0.3306%	0.0480
20 Pairs	0.2681%	0.0413
40 Pairs	0.2255%	0.0364

The graphs “Rho and Market Sigma” in Figure 1.5 present the conditional correlation between the portfolios of pairs and the market and the conditional standard deviations of the market. The results suggest that portfolios with fewer pairs (5 and 10 pairs) present a

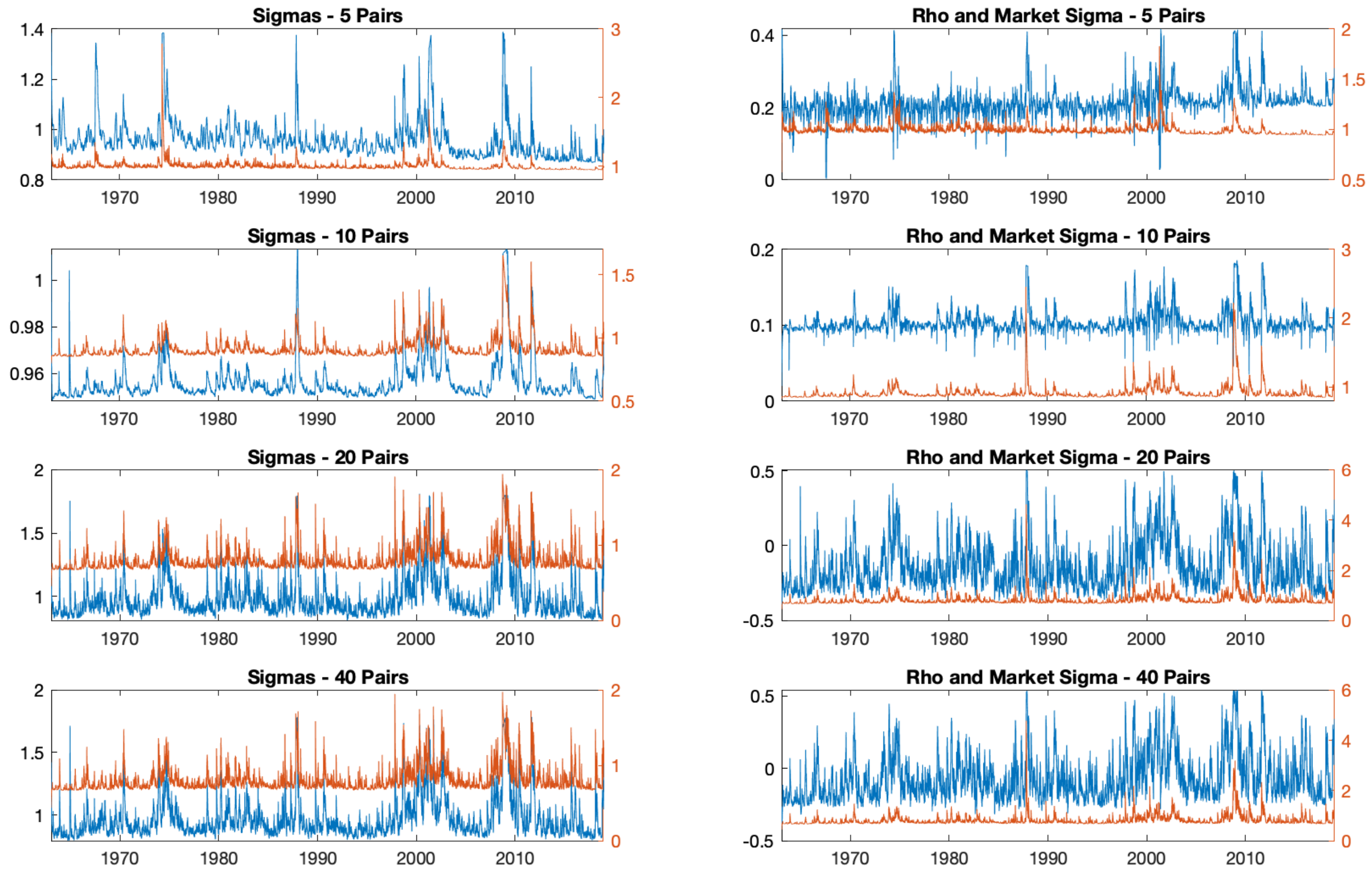


Figure 1.5 The "Sigmas" graphs present the Asymmetric BEKK GARCH estimated conditional standard deviations of the portfolios (blue line) against the conditional standard deviations of the market (orange line). The "Rho and Market Sigma" graphs present the Asymmetric BEKK GARCH estimated conditional correlations between the portfolios of pairs and the market (blue line) against the conditional standard deviations of the market (orange line).

conditional correlation which varies around zero in normal times. However, these conditional correlations increase significantly during periods of greater market instability. Interestingly, the conditional correlations of portfolios with more pairs (20 and 40 pairs) are more stable over time. Nevertheless, these are significantly higher than those of smaller portfolios. This makes sense as once we start trading a larger number of pairs, these are more likely to be not very good pairs hence resulting in higher exposure to the market.

Table A.1 in Appendix A.3 presents the coefficients for this Asymmetric BEKK GARCH model. Using these coefficients, we build the news impact surfaces for the conditional variance of the portfolios of pairs and the conditional covariance between them and the market. These can be found in Figures A.6, A.7, A.8 and A.9 in Appendix A.3. The results from these news impact surfaces are straightforward: market news are the main drivers of changes in both the volatility of the portfolios of pairs and their covariance with the market. The degree by which news of the portfolios of pairs also affect these components varies according to the size of the portfolios. The strong effect of market news, however, is constant across all the portfolios.

Overall, the empirical results from the analysis of these conditional second moments demonstrate significant spillovers from market news to the volatility of Pairs Trading. Not only do market shocks have a significant impact on the volatility of different portfolios, but they also appear to influence the correlation of this strategy with the market. By bringing information about second moment dependencies, these results strongly reject the common understanding regarding the market neutrality of Pairs Trading.

1.6 Incorporating Idiosyncratic Terms

In the multivariate GARCH framework presented in Section 1.5, we have seen that there are significant spillovers from market news to the different portfolios of pairs being considered in this study. This is a meaningful result that goes against the general understanding about the market neutrality of the Pairs Trading strategy. Nevertheless, by itself, this result does little in helping to explain the drivers of the volatility of the returns of this strategy.

In this section, we propose a framework for the decomposition of the volatility of the returns of the Pairs Trading strategy into idiosyncratic and market components. Using this framework, we then turn to the analysis of the driving components of this volatility. By analysing the dependency of the realised volatility of individual pairs on the conditional idiosyncratic components and the market, we can better understand the drivers of pairs volatility and even think of improvements to the strategy from a hedging perspective.

1.6.1 Defining idiosyncratic returns

Always when referring to any form of idiosyncratic return or volatility, it is of extreme importance to be clear about the definition of what we mean by this component. Idiosyncratic returns are model dependent; that is, the series we get for the idiosyncratic returns depends on the model that we use to identify them.

Since Fama and French (1993), the standard of the literature for identifying idiosyncratic returns has been to consider a three factor Fama and French model[8], also incorporating industry factors whenever relevant. Nevertheless, the main objective of this study is to analyse the dependence in second moments between the returns of Pairs Trading and that of the market. In this section, we are particularly interested in analysing this dependence in the light of the decomposition between market and idiosyncratic returns. As a result, the most important property that we look for when building our series of idiosyncratic returns is that they are orthogonal to market returns.

Let $r_{i,t}$ be the return of a given stock at time t. We consider the following market factor model of Fama and French (1993):

$$r_{i,t} = \beta_i r_{m,t} + \tilde{r}_{i,t} \tag{1.16}$$

[8]Considering a model that adds size risk and value risk factors to the market risk factor in the CAPM.

That is, we define as idiosyncratic returns every variation in $r_{i,t}$ that is orthogonal to market returns. This definition simplifies the decomposition of the variance of the returns of pairs and doesn't affect the relationship between the market and the pairs, our main object of interest. However, as all approaches that rely on estimate of factor loadings to find the idiosyncratic orthogonal component, our framework suffers from the fact that these loading can change with time, hence compromising the identification of the idiosyncratic terms. We will deal with this problem when we study the time varying properties of our estimates.

1.6.2 Decomposing pairs volatility

Once that we have defined the idiosyncratic component of returns, we can turn to the decomposition of the volatility of the returns of pairs into market and idiosyncratic components. From Section 1.3, recall that each pair is defined by long and short positions in each of the stocks that forms the pair. Let $r_{l,t}$ be the return of the long stock (the stock we take a long position) at time t and $r_{s,t}$ be the return of the short stock (the stock we take a short position) at time t. We then have that, at any given t, the return of the pair is given by:

$$r_{p,t} = w_{l,t} r_{l,t} - w_{s,t} r_{s,t} \tag{1.17}$$

where $w_{l,t}$ and $w_{s,t}$ are the weights of the portfolios in the long and short stocks, respectively, at time t. As it was described many times above, these weights start as $w_{l,t_0} = 1$ and $w_{s,t_0} = -1$ when the pair starts to trade. But as the price of the stocks themselves evolve, the relatives weights in the pair will also change.

We can then use the definition of the market and idiosyncratic components of each of the stocks given by Equation (1.16) to replace in Equation (1.17). By doing this, we get that the expression for the conditional variance of the return of the pair is given by[9]:

$$VAR_{t-1}[r_{p,t}] = (w_{l,t}\beta_l - w_{s,t}\beta_s)^2 VAR_{t-1}[r_{m,t}] + w_{l,t}^2 VAR_{t-1}[\tilde{r}_{l,t}] \\ + w_{s,t}^2 VAR_{t-1}[\tilde{r}_{s,t}] - 2w_{l,t}w_{s,t}COV_{t-1}[\tilde{r}_{l,t}, \tilde{r}_{s,t}] \tag{1.18}$$

which, for a clearer interpretation, we can re-write as:

$$VAR_{t-1}[r_{p,t}] = \gamma_m VAR_{t-1}[r_{m,t}] + \gamma_l VAR_{t-1}[\tilde{r}_{l,t}] \\ + \gamma_s VAR_{t-1}[\tilde{r}_{s,t}] + \gamma_{cov} COV_{t-1}[\tilde{r}_{l,t}, \tilde{r}_{s,t}] \tag{1.19}$$

Our objective is hence to study the significance of each of the components in Equation (1.19) in driving the volatility of the pairs. In order to do so, we will need estimates of $VAR_{t-1}[r_{p,t}]$, $VAR_{t-1}[r_{m,t}]$, $VAR_{t-1}[\tilde{r}_{l,t}]$, $VAR_{t-1}[\tilde{r}_{s,t}]$, and $COV_{t-1}[\tilde{r}_{l,t}, \tilde{r}_{s,t}]$. As it has already been noted above, in this stage we are analysing the volatility of each individual pair rather than a portfolio of pairs. This will imply in some restrictions when estimating the listed second moments of the data thus requiring a different methodology.

From Section 1.3 and Section 1.4 one can recall that we used daily returns from twelve months to match pairs and traded the pairs during the following six months. We will hence use this 18-month interval to estimate the components described in Equation (1.19) for each pair. The same interval is used to estimate the market loading and obtain the idiosyncratic returns for each stock as described by Equation (1.16).

We start by describing the methodology used to estimate the regressors of Equation (1.19). For the conditional variance of market returns ($VAR_{t-1}[r_{m,t}]$), we estimate this term in each of the mentioned 18-month windows with daily returns using a GJR-GARCH(1,1,1) of Glosten et al. (1993)[10]. We decided to use a GJR-GARCH(1,1,1) to keep a parsimo-

[9] See Appendix A.1 for derivation of the expression for the conditional variance of pairs returns.

[10] For more information about the model specification of the GJR-GARCH(1,1,1) model, check Appendix A.2.1

nious model for the variance of market returns but also allow for asymmetric effect of shocks during different periods.

We then turn to the analysis of the other elements on the right-hand-side in Equation (1.19). Since we need not only an estimate for the conditional variance of the idiosyncratic returns of each of the stocks that for the pair ($VAR_{t-1}[\tilde{r}_{l,t}]$ and $VAR_{t-1}[\tilde{r}_{s,t}]$), but also an estimate of the conditional covariance of these two terms($COV_{t-1}[\tilde{r}_{l,t}, \tilde{r}_{s,t}]$), these three elements are jointly estimated in a multivariate GARCH framework. To keep the same logic from the model used to estimate the conditional variance of market returns, we use a multivariate equivalent model and estimate the conditional covariance matrix of the system of the idiosyncratic returns using the AG-DCC-GARCH(1,1,1) from Cappiello et al. (2006)[11]. This model allows for both the asymmetric effect of negative and positive shocks as well as an evolution of the conditional correlation in an autoregressive manner.

The final term we need to estimate from Equation (1.19) is the dependent variable. That is, the conditional variance of the returns of the pair itself ($VAR_{t-1}[r_{p,t}]$). However, this is where the aforementioned restrictions come into play. As we need the pairs to be trading to have information about their returns, the series of pairs returns present some inconsistencies. Since pairs do not trade every day of the 18-month interval, we only have returns for some of the days in this interval. Moreover, the days for which we have information on returns are at inconsistent intervals within and across pairs. This generates series which are discontinuous and highly heterogeneous for each of the pairs. As a result, we are not able to estimate a structured parametric model for the variance of the returns of pairs as we are for the other components. Because we are interested in decomposing the variance of the returns of the pairs into the elements in the right-hand-side of Equation (1.19) and not estimating a model for it, we use the simple realised variance ($r^2_{p,t}$) as a proxy in our analysis.

By using the methodology described above, for every day (t) a given pair (p) has been

[11]For more information about the model specification of the AG-DCC-GARCH(1,1,1) model, check Appendix A.2.2

traded we now have the following series:

- Realised pair variance: $r_{p,t}^2$
- Estimated market variance: $\hat{\sigma}_{m,t}^2 \equiv V\hat{A}R_{t-1}[r_{m,t}]$
- Estimated variance of idiosyncratic component of stock l: $\hat{\sigma}_{l,t}^2 \equiv V\hat{A}R_{t-1}[\tilde{r}_{l,t}]$
- Estimated variance of idiosyncratic component of stock s: $\hat{\sigma}_{s,t}^2 \equiv V\hat{A}R_{t-1}[\tilde{r}_{s,t}]$
- Estimated covariance: $\hat{\sigma}_{ls,t} \equiv C\hat{O}V_{t-1}[\tilde{r}_{l,t}, \tilde{r}_{s,t}]$

And we can finally use these terms to perform the decomposition form Equation (1.19). Nevertheless, the inconsistency of the $r_{p,t}^2$ data provides one last challenge. Because t is inconsistent within and across pairs with some pairs being traded for just a few days and others being traded all over the sample, we cannot use a traditional panel technique to perform the decomposition considering each pair as an entity. Moreover, there are not enough observations in many pairs to conduce the estimation individually. We will present an alternative way of preserving the time information later on, but, initially, we choose to pool the time dimension across all pairs as a solution to this problem. This means that we will have as many i observations as the sum of the number of days that each of the pairs was traded.

Putting more formally, by pooling across the time dimension we estimate the following model:

$$(r_{p,i})^2 = \gamma_m \hat{\sigma}_{m,i}^2 + \gamma_l \hat{\sigma}_{l,i}^2 + \gamma_s \hat{\sigma}_{s,i}^2 + \gamma_{cov} \hat{\sigma}_{ls,i} + \mu_i \tag{1.20}$$

where $i \in 1, \ldots, I$ with $I = \sum_1^N T_{pr}$. In which N is the total number of pairs and T_{pr} is the number of days in which the pair $\{pr\}$ is traded.

By estimating the model above, we pool the time dimension and consider all information about days when pairs were traded (with respective pair information $r_{p,i}$, $\hat{\sigma}_{m,i}^2$, $\hat{\sigma}_{l,i}^2$, $\hat{\sigma}_{s,i}^2$, and $\hat{\sigma}_{ij,i}$) in a very large – from 216,302 observations when trading 5 pairs to 1,930,709 observations when trading 40 pairs – cross-section. A Newey-West estimator is used to account for the serial correlation in μ_i that occurs from pooling the time dimension and having, in some cases, multiple is from the same day.

Table 1.5 presents the results for the estimated model from Equation (1.20). We decided to include the t-statistics rather than p-values for the coefficients because all estimates are significant at all significance levels. As a result, t-statistics are more informative than p-values. There are a few key insights we can get from the results shown in Table 1.5. First, as it was noted in Section 1.5, there seems to be significant spillovers from market volatility to the volatility of pair returns. Nevertheless, when we include information about the idiosyncratic volatilities of each stock and their covariance, these components are shown to be more significant than the volatility of the market. That is, the results from Table 1.5 suggest that the Pairs Trading strategy is indeed not market neutral from a second moments perspective, as there is a significant relationship between the volatility of pair returns and the volatility of the market. The estimation results also suggest, however, that this relationship, albeit significant, is much less important than the relationship between the volatility of pair returns and the components of the system with idiosyncratic returns. A point worth noting is the significance of covariance term between the two idiosyncratic returns that form the pair. This seems to be the most significant term in the analysis. This makes sense as the key idea behind Pairs Trading is precisely the relationship between the two stocks that form the pair. The more these stocks move together, the more stable the pair being traded will be. A final point that can be noted in Table 1.5 is that the effect of the market on the volatility of pairs increases with the number of pairs being traded. As noted in Section 1.5, this makes sense as once we start trading a larger number of pairs, these are more likely to be not very good pairs hence resulting in higher exposure to the market.

1.6.3 Allowing for time varying dynamics

The results presented in Table 1.5 are very interesting and provide important insights regarding the sources of the volatility of Pairs Trading returns. The sample that we perform the analysis is, however, very long from a time perspective (from 1962 to 2018). It would

Table 1.5 Pooled Newey-West regression estimates and t-stats.

	γ_m	γ_l	γ_s	γ_{cov}
5 Pairs	0.1329	0.7282	0.9018	-1.6584
	(8.82)***	(18.13)***	(36.77)***	(-48.010)***
10 Pairs	0.1233	0.82504	1.0133	-1.894
	(6.03)***	(24.05)***	(36.30)***	(-55.16)***
20 Pairs	0.2998	0.7585	0.8467	-1.7009
	(9.72)***	(18.74)***	(15.69)***	(-36.7)***
40 Pairs	0.3250	0.8714	0.8127	-1.8038
	(15.08)***	(36.97)***	(29.86)***	(-61.98)***

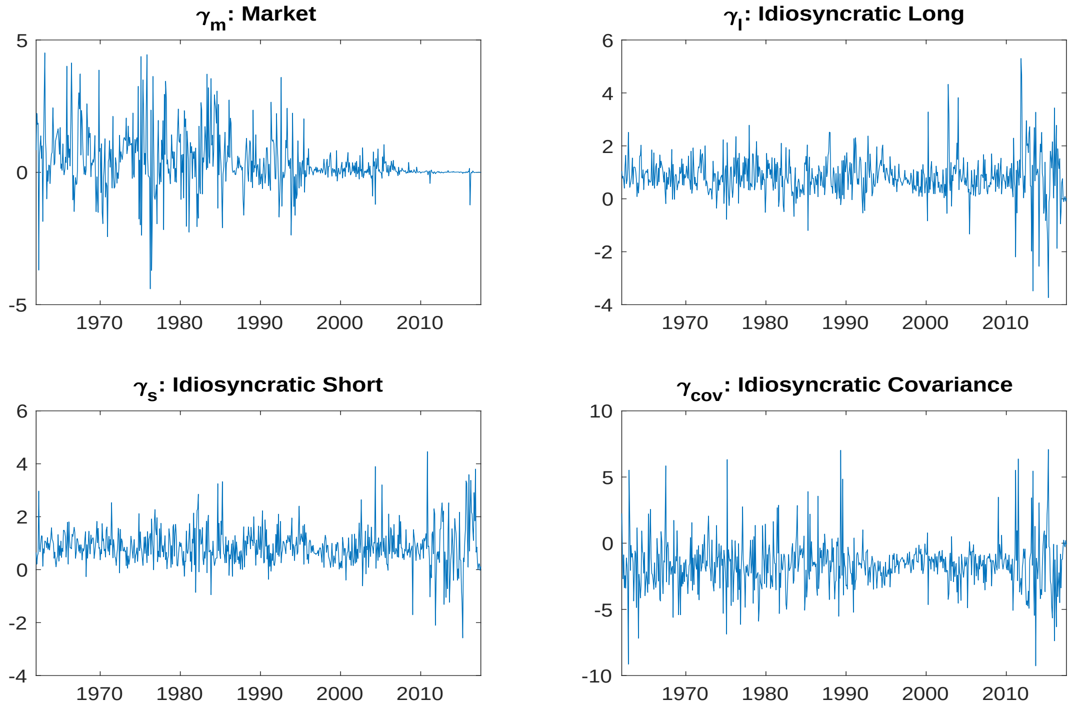
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

therefore be relevant to include information about the time dimension in our estimation framework. In this subsection we propose an extension that does precisely that.

The reason why we pooled the time dimension in the model presented in Equation (1.20) is because such dimension is inconsistent within and across pairs. In the previous subsection, we pooled the time dimension all over our sample. One can, however, pool it within smaller sub-samples. Recall that during the entire trading sample, at each month we start trading 5, 10, 20, or 40 pairs for the next six months. A very granular manner to pool the time dimension is to use these sets of portfolios which are opened at each month to estimate monthly time varying coefficients.

Each of these monthly sets has enough observations to perform the pooled regression described in Equation (1.20) within the set. And because we have one of these sets for each month in the trading period, we can use these estimates to build a monthly series of each of the coefficients $\{\gamma_{m,m}, \gamma_{l,m}, \gamma_{s,m}, \gamma_{cov,m}\}_{m=1}^{M}$. We proceed in this manner and estimate the pooled model described in Equation (1.20) using each of the monthly sets mentioned above and build a series of monthly coefficients $\{\gamma_{m,m}, \gamma_{l,m}, \gamma_{s,m}, \gamma_{cov,m}\}_{m=1}^{M}$. We do this for the sets considering 5, 10, 20 and 40 pairs. The series of coefficients for sets considering 5 pairs can be seen in Figure 1.6. The series for the other numbers of pairs can be found in Appendix A.3 as they don't vary significantly from the results plotted in Figure 1.6.

Figure 1.6 Time varying coefficients when trading 5 pairs.



The results from Figure 1.6 are extremely interesting. They show that up to the 2000s, the coefficient estimates of the impact of market volatility on the volatility of pairs were indeed very relevant. In a lot of the times, being as relevant as or even more relevant than the idiosyncratic components. Nevertheless, after the late 90s, this effect of the market on the volatility of pairs has diminished significantly, becoming pretty much irrelevant after the 2010s. The idiosyncratic components, on the other hand, seem to have become more relevant as the impact of the market has vanished.

In order to evaluate this idea more formally, we re-estimate the model from Equation (1.20) but now pooling over observations available in the sample after 2010. The results from such estimation are displayed in Table 1.6.

The overall results from Table 1.6 are very similar to those from Table 1.5. They present, however, one key difference. The impact the market volatility on the volatility of pairs is significantly reduced. Although being statistically significant (a result of having many observations in our sample), the impact of the market on the volatility of pairs is almost

Table 1.6 Pooled Newey-West regression estimates and t-stats after 2010.

	γ_{vm}	γ_{vi}	γ_{vj}	γ_{cov}
5 Pairs	0.0139	0.6131	0.5687	-1.1491
	(5.37)***	(9.89)***	(9.11)***	(-10.90)***
10 Pairs	0.0418	0.6221	0.5887	-1.1794
	(10.00)***	(14.02)***	(12.71)***	(-16.71)***
20 Pairs	0.0842	0.6184	0.7814	-1.3784
	(13.85)***	(18.72)***	(29.79)***	(-32.97)***
40 Pairs	0.1463	0.6812	0.7042	-1.3733
	(23.05)***	(34.57)***	(39.35)***	(-56.91)***

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

irrelevant in comparison to other components.

The results from Table 1.6 reinforce the result from Figure 1.6. That is, taken together, all the results from this section indicate that, over the entire sample from 1962 to 2018, the market has a meaningful impact on the volatilities of the Pair Trading strategy. Nevertheless, this impact has been decreasing since the mid-to-late nineties and has become almost irrelevant, depending on the measure used to evaluate, after the 2010s. In a sense, these results show that, from a second moments perspective, Pairs Trading has not been historically market neutral. Evidence does seem to suggest, however, that this strategy has become increasingly more market neutral over the last two decades.

1.7 Concluding Remarks

The Pairs Trading strategy is commonly referred to as the most straightforward example of a market neutral trading strategy by both academic researchers and practitioners. Inspired by the empirical findings of Engelberg et al. (2008) and Jacobs and Weber (2015) – on the heterogeneous performance of pairs depending on different factors – and the results presented in Section 1.4 regarding the behaviour of the volatility of the Pairs Trading strategy, the objective this paper was to use modern techniques from second moments literature to analyse the market neutrality of this strategy.

We perform this analysis in two steps. In Section 1.5 we propose a conditional covariance framework to analyse the dependence in second moments between different portfolios of pairs and the market. Overall, the empirical results from Section 1.5 demonstrate significant spillovers from market news to the volatilities of these portfolios. Not only market shocks have a significant impact on the volatility of different portfolios, but they also appear to influence the correlation of this strategy with the market.

The results from Section 1.5 go against the general understanding regarding the market neutrality of Pairs Trading. They do not say, however, how the influence of the market on the volatility of pairs compares to the influence of other, idiosyncratic, components. Inspired by these results, in Section 1.6 we propose a framework for the decomposition of this volatility and the analysis of its driving components. Taken together, the results from the analysis performed in Section 1.6 show that, as presented in Section 1.5, the market has a meaningful impact on the volatilities of Pairs Trading over the entire sample even when idiosyncratic components are included in the analysis. This impact, however, has been decreasing since the mid-to-late nineties and has become almost irrelevant after the 2010s. In one sentence, the results from this study suggest that although Pairs Trading cannot be said to be market neutral from a second moments perspective if we look at the full sample from 1962 to 2018, the strategy has certainly become more market neutral as markets have evolved over the last two decades.

This research expects to contribute to a better understating of the properties of the Pairs Trading strategy and, more broadly, to the field of study of market neutral statistical trading techniques. Further research could address the changes in market behaviour and market micro-structure that, according to our results, seem to be related to a structural break in the link between the volatility of pairs and that of the overall market. Future studies could also try to understand the impact of such regime changes in overall performance and hedging properties of Pairs Trading and other statistically based trading strategies.

2 | Global Volatility Shocks and the PPP Puzzle

This chapter is based on joint work with Susana Campos-Martins[†].

Abstract

Most of the discussion about the Purchasing Power Parity (PPP) Puzzle of Rogoff (1996) has pertained to the reversion speed of deviations from PPP. Much less attention, however, has been given to the other component of the puzzle: the high volatilities of real exchange rates. In this paper, we provide a framework that is capable of explaining the econometric sources of these volatilities. First, we study the drivers of real exchange rate volatilities using a Cross-Sectionally Augmented Autoregressive Distributed Lag (CS-ARDL) panel framework and the conditional covariance matrices of the system with nominal exchange rates and price differentials. This analysis indicates that, for both emerging and developed markets, common factors are the main drivers of volatility. With this result in hand, we propose a novel econometric framework – based on the endogenous common volatility shocks model of Engle and Campos-Martins (2020) – that explains the sources of these volatilities as common second moment shocks. This framework allows us to give structure to the origins of these high volatilities and propose an extension to study their macro-financial drivers.

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2.1 Introduction

Since the breakdown of the Bretton Woods system, major shifts in the global economy and financial markets have exacerbated the magnitude of exchange rate fluctuations. While Friedman (1953) notoriously argued that exchange rate volatility is a manifestation of macroeconomic volatility, empirical studies have uncovered a range of anomalies and puzzles that contradict theoretical models of exchange rates. Among the many unanswered questions raised by the empirical international finance literature, one of the most persistent ones has been the Purchasing Power Parity (PPP) Puzzle of Rogoff (1996).

In his words, the puzzle presented by Kenneth Rogoff in his seminal 1996 paper is: "How can we reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out?". The very slow[1] speed of adjustment of shocks to real exchange rates has been the source of considerable theoretical and empirical research, with relative success (Taylor, 2002). From the empirical side, Cheung and Lai (2000) argue that the persistence of these deviations from PPP is mainly due to the non-linearity of the adjustment process. From the theory side, Carvalho and Nechio (2011) propose a model that is capable of generating the persistence observed in the data by introducing heterogeneity in the frequency of price changes across sectors.

Most of the discussion about the PPP Puzzle has, however, only pertained to the reversion speed of deviations from PPP. As Taylor (2002) argues, much less attention has been given to the other component of the puzzle: the high short-term volatilities of real exchange rates. Even if we consider that the recent literature has established that exchange rates do revert to the PPP equilibrium rate over the medium term at a speed that is consistent with theory, the volatilities present in the data in the short-run, at least under floating regimes, still remain a puzzle. According to Ganguly and Breuer (2010), this piece of the puzzle is arguably more important to understand than the first because of its implications for trade, investment, and economic growth.

[1]The literature half-life estimates for real exchange rates are generally between 3 and 6 years.

Mussa (1986) was the first piece of research to analyse second moments of real exchange rates with a focus on short-run fluctuations. The author calculates unconditional variances and covariances for real exchange rates, nominal exchange rates, and price differentials for fixed and flexible exchange rate periods. This exercise shows that not only these variances and covariances changed from one regime to the other, but real exchange rates present significantly higher variances under flexible regimes. This finding surprised the field, as theories going back to Friedman (1953) maintain that a flexible exchange rate should be useful as an alternative adjustment mechanism of relative prices when nominal prices are not free to adjust. The critique of Mussa (1986) was, and still is, extremely influential, changing the course of exchange rate models. More recently, Taylor (2002) updated the analysis of Mussa (1986) by allowing slowly evolving deterministic trends and studying deviations from these trends. The author finds important quantitative differences in the residual variances with floating regimes exhibiting much larger shocks to the real exchange rate process, accounting for the significantly larger deviations from PPP in these eras. Ganguly and Breuer (2010) also explore the short-run volatility of real exchange rates. The authors conduct a simple unconditional variance decomposition of real exchange rate into nominal exchange rate volatility and relative price volatility, after controlling for real and nominal factors. Finally, Bergin et al. (2014) study changes in variances and covariances between different periods using simulations from a Vector Error Correction framework of Cheung et al. (2004).

Although the literature presented above has achieved important conclusions regarding the short-run volatilities of real exchange rates during different regimes, it has not been able to answer more meaningful questions about these short-run dynamics. As, for instance, what drives these high and persistence variations. Moreover, in all studies, the analyses focus on calculating realised unconditional volatilities and covariances for different periods and drawing inference from their differences. Even though this might be useful to superficially understand the differences in unconditional second moments between different currency regimes, it is certainly not the most recommended econometric framework to study the dynamics of short-run volatilities.

In this paper, we use latest develops from the financial econometrics literature in modelling second moments dynamics to study this remaining part of the PPP Puzzle. In a first step of the analysis, we present an econometric framework based on the Cross-Sectionally Augmented Autoregressive Distributed Lag (CS-ARDL) model of Pesaran (2006) for the decomposition of real exchange rate volatility into its building blocks. The results from our proposed framework indicate that the most important driver of real exchange rate volatilities are the common dynamics (considered as group means in our framework). This suggests that a key part of understanding the remaining part of the PPP Puzzle – that is understanding the high short-to-medium-term volatilities of real exchange rates – is modelling the cross-sectional correlations in real exchange rate volatilities.

Inspired by the importance of cross-sectional correlations in explaining real exchange rate volatilities, we propose an econometric model based on the endogenous common volatility shocks framework of Engle and Campos-Martins (2020) to model the dynamics of real exchange rate volatilities in a second step of this study. This framework presents encouraging results when modelling the aforementioned cross-sectional correlations. It successfully gives structure to these common volatility dependencies in real exchange rates and further allows us to propose an extension to the framework in order to study the impact of other macro-financial variables on this common volatility movements. In fact, this extension is a general framework and can be considered as an extension of the model of Engle and Campos-Martins (2020) to allow for exogenous drivers of common volatility shocks. The results from our proposed extension show common shocks to interest rate differentials as a meaningful driver of common volatility shocks in exchange rates, hence building a bridge between our analysis of the PPP Puzzle and the Interest Rate Parity literature.

This research relates to the empirical international finance literature and, specifically, to the study of volatility of real exchange rates. By using latest developments in second moments modelling to study the dynamics and sources of real exchange rate volatility, we expect to shed some light on the the remaining part of the PPP Puzzle. In a brother sense, the results from this study can also be seen as a motivation for applications of the endogenous common volatility shocks framework of Engle and Campos-Martins (2020) and our extension to exogenous drivers to other asset classes. Within the next pages, Section 2.2 presents a brief literature review on the PPP Puzzle and the analysis of real exchange rate volatility. Section 2.3 describes the dataset used for this research, the transformations required and some preliminary results. In Section 2.4, we present the econometric framework used for estimating second moments and decomposing real exchange rate volatility into its building components. Based on the results from Section 2.4, Section 2.5 presents a model that gives structure to the common volatility shocks to exchange rates and expands this model to allow for exogenous drivers of these common shocks. Section 2.6 concludes by linking our findings regarding the PPP Puzzle to other topics in the empirical international finance literature.

2.2 Literature Review

2.2.1 The PPP Puzzle

Purchasing Power Parity (PPP) is the disarmingly simple empirical proposition that, once converted to a common currency, national price levels should be equal. It was was articulated by scholars of the Salamnca school in the sixteenth century in Spain but first proposed by Swedish economist Gustav Cassel (Cassel, 1921; Cassel, 1922) as mean for setting relative gold parities in exchange rates after World War I. Though PPP had been discussed previously by classical economists such as John Stuart Mill, Alfred Marshall, and Ludwig von Mises, Cassel was really the first to treat PPP as a practical empirical theory.

The basic idea is that if the goods market arbitrage enforces broad parity in prices across a sufficient range of goods via law of one price, then, by construction, there should also be a high correlation in aggregate price levels. Some might say that, given the observed volatilities in exchange rates and differences in prices of the same good across the world, the PPP is only a theoretical construct that does not apply in practice. Nevertheless, "while a few empirically literate economists take PPP seriously as a short-term proposition, most instinctively believe in some variant of PPP as an anchor for long-run real exchange rates" Rogoff (1996).

Empirical support for PPP has changed over the years. From a historical standpoint, there have been numerous studies of PPP with various datasets[2]. McCloskey and Zecher (1984) argue that PPP worked very well under the gold standard before 1914. Diebold et al. (1991) explore a very long run panel of nineteenth-century data for six countries and find support for PPP based on the low-frequency information lacking in short-sample studies. Abuaf and Jorion (1990) study a century of Dollar-Franc-Sterling exchange rate data and verified PPP. Lothian and Taylor (1996) further confirm the results from Abuaf and Jorion (1990) using two centuries of Dollar-Franc-Sterling. Lothian (1990) also finds evidence that real exchange rates were stationary in Japan, the US, the UK and France for the period 1975-1986. More recently, Engel et al. (2015) and Ca'Zorzi et al. (2020) find that PPP based forecasts for exchange rates have the best out-of-sample performance from all models considered.

By the late 90s, the empirical international finance literature had arrived at a surprising degree of consensus over some basic facts regarding exchange rates. First, a number of studies had presented evidence that points towards a PPP equilibrium of exchange rates in the long-run. Second, that short-run deviations from PPP are large and volatile. Puzzled by this empirical dichotomy, Rogoff (1996) proposed the following PPP Puzzle: How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp? The most obvious explanation for the short-run volatility of real exchange rates would be price stickness. This is the essence of the Dornbusch (1976) overshooting model of nominal and real exchange rate volatility.

[2] For a full review of the literature up to the 90s, one can refer to Froot and Rogoff (1995).

Consensus estimates for the rate at which PPP deviations damp, however, suggest a half-life of three to six years, seemingly far too long to be explained by nominal rigidities.

The puzzle proposed by Rogoff (1996) created a new sub-field within the international finance literature, and inspired countless papers in both the theory and empirics of PPP. In one of the first attempts to solve the puzzle, Clarida and Gali (1994) and Rogers (1999) identify the relevance of multiple shocks in explaining the variability of real exchange rates, but their results still do not resolve the PPP Puzzle. The first meaningful progress in "solving" the PPP Puzzle is the work of Cheung and Lai (2000). Using impulse response analysis, Cheung and Lai (2000) analyse the adjustment dynamics of real exchange rates by evaluating both the sample and half-life measure and its estimation accuracy. The impulse response analysis shows that the shocks impact tends to amplify first before it dissipates. The full impact of the shock is not felt immediately but until a few periods after the initial shock. Hence, following the shock, the real exchange rate does not revert to its long-run value monotonically, but in hump-shaped manner. Cheung and Lai (2000) find that this non-monotonic adjustment contributes considerably to generate persistency in real exchange rates.

In a following paper, Cheung et al. (2004) present additional evidence on the convergence speeds of nominal exchange rates and prices. Using Vector Error Correction (VEC) analysis, the authors estimate the speeds at which the individual variables revert to their long-run values. The VEC analysis provides an alternative, easier way to measure those convergences speeds than the previous state-space studies (as Engel and Morley, 2001). While taking a different approach, the results from Cheung et al. (2004) corroborate those of Engel and Morley (2001) that nominal exchange rates do converge to at a much slower rate than prices. Half-lives of nominal exchange rates are estimated to be from 3 to 6 years, whereas half-lives of prices are found to be substantially shorter (mostly about 1 to 2 years). Cheung et al. (2004) also show that about 60% to 90% of PPP disequilibrium adjustment takes place through nominal exchange rate adjustment. Hence, it is mostly nominal exchange rate adjustment – not price adjustment – that drives real exchange rates towards parity. As such, the observed rate of PPP reversion reflects primarily the speed of nominal exchange rate convergence. Should nominal exchange rates converge much more slowly than prices, the PPP reversion speed can be slower than the price convergence speed, as described by the PPP puzzle.

Trying to further address the PPP Puzzle, Taylor (2002) recreates the analysis of Mussa (1986) with empirical innovations by controlling for long-run deviations from PPP – Balassa (1964) and Samuelson (1964) like effects – and using longer span of historical data. When investigating four different currency regimes, Taylor (2002) finds important differences in the residual variance, with the floating regimes exhibiting much larger shocks to the real exchange rate process accounting for the much larger deviations from PPP during these eras. According to the author, these results show that there was relatively little change in the ability of international market integration to smooth out real exchange rate shocks. Instead, Taylor (2002) argues, the changes in the variance of the shocks reinforce the conclusion of Mussa (1986) of seeing exchange rate regimes as a major determinant of real exchange rate behaviour. More importantly, the author concludes that changes in the persistence of the process play little role in explaining why the behaviour of real exchange rates changes so much from one regime to the other. “Changes in the volatility of the shocks explain virtually all changes in the volatility in the real exchange rate across space and time” (Taylor, 2002). Therefore, understanding the dynamics and sources of these shocks is crucial for a better understanding of the PPP Puzzle. In fact, Taylor (2002) defends that “further study will be needed to incorporate these dynamics into an econometric PPP model and measure them in historical and contemporary samples”.

As suggested by Cheung and Lai (2000), one approach to resolving part the PPP Puzzle of Rogoff (1996) lies in allowing for nonlinear dynamics in real exchange rate adjustment. More recently, however, a contribution from the theory side has also been able to solve the persistency part of the puzzle. Carvalho and Nechio (2011) study the PPP Puzzle in a multisector, two-country, sticky-price model. In their model, sectors differ in the extend of price stickiness, leading to heterogeneous sectoral real exchange rate dynamics.

By introducing heterogeneity in the frequency of price changes across sectors, Carvalho and Nechio (2011) are capable of generating the persistence in deviations from PPP as observed in the data.

2.2.2 The remaining PPP Puzzle: High short-term volatilities

Empirical work that focuses on understanding the dynamics of real exchange rates and the PPP Puzzle must grapple with the two key properties: the reversion speed of deviations from PPP and the high short term volatility of the disturbance term. Most of the discussion of the literature up to this point has pertained to the reversion speed, which is a medium-to-long-term phenomenon. Nevertheless, the literature also acknowledges that more attention should be given to the other part of the puzzle. As Taylor and Taylor (2004) put it "even if the current work can establish that exchange rates do revert to the PPP rate over the medium term at a more reasonable speed, the volatilities present in the data in the short-run, at least under floating regimes, still cause considerable mystification".

While much work has been done directed at the first piece of Rogoff (1996) PPP Puzzle – studying the speed of convergence of deviations from PPP equilibrium – Ganguly and Breuer (2010) also defend that the high short-to-medium-term volatility piece is arguably more important to understand because of its implications for trade, investment, and economic growth. Yet, "real exchange rate volatility has received sporadic attention[3], at best" Ganguly and Breuer (2010).

Ganguly and Breuer (2010) build on the work of Hausmann et al. (2006), who find that the volatility of real exchange rates in developing markets is 2.5 times higher than for industrialized countries, even when controlling for real shocks. Like the model of Hausmann et al. (2006), Ganguly and Breuer (2010) include real factors but also includes domestic and external monetary and financial factors and trade balances. With the aim of better

[3] Contributions include Edwards (1987), Côté (1994), Hausmann and Gavin (1996), McKenzie (1999), Hau (2000), Hau (2002), Clark et al. (2004), Hausmann et al. (2006), Morales-Zumaquero and Sosvilla-Rivero (2010), and Cevik et al. (2017).

understanding the reasons for the high volatilities of real exchange rates, Ganguly and Breuer (2010) also conduct a simple variance decomposition of the real exchange rate, after controlling for real and nominal factors. This decomposition of the residual variance allow the authors to calculate the contributions of unexplained nominal exchange rate volatility, unexplained relative price volatility and their covariance to the residual proportion of real exchange rate volatility.

The analysis of Ganguly and Breuer (2010) produces three main findings. With the inclusion of nominal factors, their model substantially reduces the real exchange rate volatility spread between developing and developed economies, hence helping to explain Hausmann et al. (2006) finding. The authors also find evidence that nominal factors matter in both the short and long-run. Nominal factors can have long-lived effect on the volatility of real exchange rate. Ganguly and Breuer (2010) also find that for developing countries, a much larger share of real exchange rate volatility stems from relative price than for industrial countries. This finding persists in both the short and the long-run.

Bergin et al. (2014) develop an updated version of the Mussa (1986) critique. The authors ask whether recent findings regarding dynamics of real exchange rate studying the standard post-Bretton Woods dataset apply also to the Bretton Woods period of generally fixed exchange rates. Specifically, the method of Pesaran (2006) is adapted to estimate an autoregression of the real exchange rate over the Bretton Woods and post-Bretton Woods periods for a panel of 20 industrialized countries. In addition, the authores estimate a two-equation Vector Error Correction Model (VECM) to decompose the real exchange rate into its nominal exchange rate and relative price components.

The key finding of Bergin et al. (2014) is that the dynamic properties of the real exchange rate differ between these two periods, in accordance with the original results from Mussa (1986). The methodology of Bergin et al. (2014) for decomposing real exchange rate changes into their underlying components is closely related to Cheung et al. (2004), but the latter are interested only in the flexible exchange rate period and do not implement panel techniques.

Overall, the empirical international finance literature has achieved a fair consensus that some sort of PPP equilibrium holds in the long-run. Moreover, both advances from the empirical side and the theory side have addressed why deviations from this PPP equilibrium might be so persistent. As indicated by Taylor (2002), Taylor and Taylor (2004), and Bergin et al. (2014), a more important and interesting question regarding the PPP Puzzle that still remains unanswered is why real exchange rates are so volatile in the short-run. In this paper, we apply late developments from the financial econometrics literature to study the drivers of real exchange rate volatility and propose a novel econometric framework that is capable of explaining the sources of these volatilities as common second moment shocks.

We divide this analysis into two steps. In the first one, we propose a panel model for the decomposition of real exchange rate volatility into its building components. This decomposition allows us to analyse the importance of each of the components and serves as a guideline to the model proposed in the following section. Inspired by the results from step one, we propose an econometric framework based on the work of Engle and Campos-Martins (2020) that is able to model the origins of the short-term volatility in real exchange rates. This framework further allows us to give structures to these volatilities and study their macro-financial drivers.

2.3 Data and Transformations

The main object of our study is the real exchange rate series for a multiplicity of countries. In order to obtain a set of countries which is representative for both emerging and developed markets but, at the same time, only selects relevant currencies with enough liquidity, we follow the methodology of BIS (2019) and select the thirty most traded currencies in the world. A list with the full set of currencies and their market classification according to the MSCI (2020) Emerging-Developed Market classification can be found in Appendix B.1.

Because we need both nominal exchange rates and price series in order to construct the series for the real exchange rates, we consider the data at a monthly frequency. This is the highest frequency for the price series and usually what is referred to when analysing the short-run behaviour of real exchange rates. The series for CPI and nominal exchange rates (period mean) were extracted from the IMF International Financial Statistics for all countries considered in the study from Jan 1990 to December 2020. The CPI series are standardized to be unity at the most recent observation. This makes price levels comparable and allows for easy interpretation of real exchange rates. The nominal exchange rate series are considered as the home currency unit of one US Dollar. We decided to use a currency as base – rather than using trade weighted measures of real exchange rates – to directly evaluate the impact of changes in the nominal exchange rates and price differentials on the real exchange rates. We decided to use the US Dollar as base because most currencies are usually denominated in this base and this is the exchange rate in which most of the trading takes place[4]. In a later step of the analysis, we also consider trade weighted Real Effective Exchange Rates for all countries in our sample. These were also extracted from the the IMF International Financial Statistics.

With this data in hand, for each of the countries, we define the following series:

- Nominal exchange rates (USD base): $E_{i,t}$
- Price ratios (to USD): $\tilde{P}_{i,t} = \frac{P_{USD,t}}{P_{i,t}}$
- Real exchange rate (USD base): $R_{i,t} = E_{i,t}\tilde{P}_{i,t}$

where i represents the country indicator for each of the countries considered in our sample and t stands for the month indicator from January 1990 to December 2020.

By constructing the real exchange rate in the above manner, we keep the standard procedure from the literature. Moreover, the base one in the latest observation of the price ratio allows us to calculate the level of real exchange that is comparable to the latest value of

[4]As a robustness check, we have also estimated our models with other currencies as base in order to guarantee results were not being driven by changes in the base currency. These results are available on request.

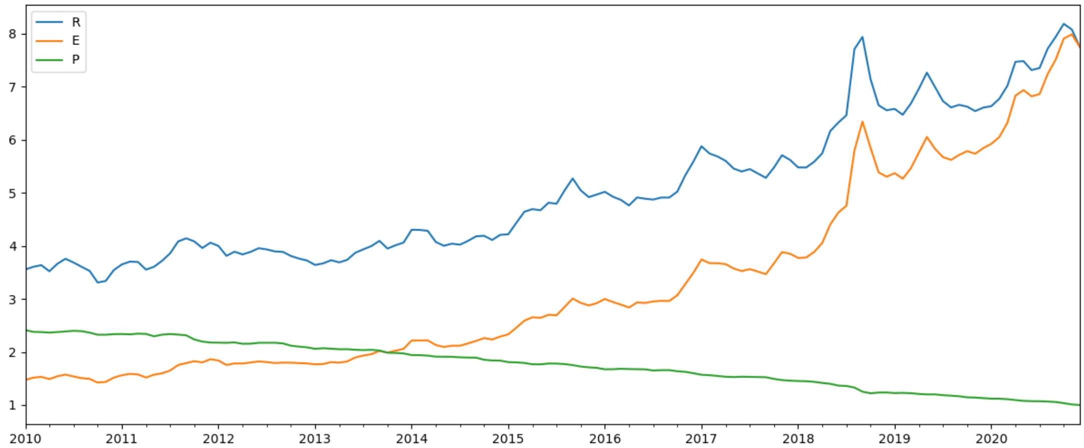

Figure 2.1 Real exchange rate ($R_{i,t}$), nominal exchange rate ($E_{i,t}$) and price ratio ($\tilde{P}_{i,t}$) for the Turkish Lira (TRY) since 2010.

the nominal exchange rate. Figure 2.1 plots these three series for TRY, the Turkish Lira. By looking at Figure 2.1, one can see what would be the equivalent real exchange rate at today's prices and compare it with the nominal exchange rate at the time.

As it was noted by Mussa (1986) and we can also see in Figure 2.1, both exchange rate series are generally significantly more volatile than the price series in the short-to-medium-run, even for countries with considerable inflation. As a result, in order to control for this significant difference in the behaviour of prices and exchange rates volatilities, we follow Andersen et al. (2000) and focus on the standardized returns[5] of each of the series to study the second moments dynamics of real exchange rates. By focusing on unconditionally standardized returns rather than the returns themselves, we control for the differences in unconditional volatilities between prices and exchange rates when conducing the analysis presented in Section 2.4. From now on, we will refer to $r^R_{i,t}$ as the standardized return of $R_{i,t}$, $r^E_{i,t}$ as the standardized return of $E_{i,t}$, and $r^P_{i,t}$ as the standardized return of $\tilde{P}_{i,t}$.

[5]At this point, we simply standardize returns by dividing by their unconditional standard deviations.

2.4 Decomposing Real Exchange Rate Volatility: A Panel Approach

Once we have the standardized series of returns for each of the variables we are interested in, we can proceed to the first step of our formal analysis. The objective of this first step is to develop an econometric framework to decompose the dynamics of real exchange rate volatility into its building blocks. We start with a description of the econometric methodology used to estimate second moments. We then introduce an econometric framework for the decomposition of real exchange rate volatility and present the estimation results. These open the door for the model proposed in the second step of the analysis in Section 2.5.

2.4.1 Estimating conditional second moments

A crucial part of studying second moments of any given series is the methodology used to compute or estimate them. There are many ways of studying second moments. One can simply calculate the sample variances and covariances as realised second moments over some arbitrary time period, as introduced in modern econometrics by Andersen and Bollerslev (1998). Another approach when studying volatility is to use the implied volatility given a model for asset prices; as, for example, the VIX measure of volatility from CBOE (2009). Since we need a dynamic measure of second moments that is as agnostic as possible, we will use estimates of conditional second moments as introduced in the literature by Engle (1982) and Bollerslev (1986).

In this subsection we will present the methodologies used to estimate the conditional volatilities of real exchange rates and the conditional covariance matrices of the system with price differentials and nominal exchange rates. We base our methodology in the work of Cappiello et al. (2006) to select the best univariate and multivariate second moments models for global equities and bonds. As it was discussed in the previous section, the object of study here will be the standardized returns of each of these series.

2.4.1.1 Modelling real exchange rate volatilities

The first step is to build the series of conditional variances for the real exchange rate standardized returns. For a given country i, let $r_{i,t}^{R}$ be the standardized return of real exchange rates. Moreover, let $\mathcal{F}_t^R$ denote the sigma field generated by the past values of $r_{i,t}^{R}$. Following the approach of Engle (1982) and Bollerslev (1986), we can then write the conditional variance of $r_{i,t}^{R}$ as:

$$VAR[r_{i,t}^{R}|\mathcal{F}_{t-1}^{R}] = VAR_{t-1}[r_{i,t}^{R}] \equiv \sigma_{i,t}^{2,R} \tag{2.1}$$

which we define as $\sigma_{i,t}^{2,R}$ for easiness of notation.

As we want to keep our analysis as agnostic as possible regarding the model for $\sigma_{i,t}^{2,R}$, we follow the approach of Cappiello et al. (2006) and do an specification search on the following models:

- The TARCH of Zakoian (1994);
- The GJR-GARCH of Glosten et al. (1993);
- The EGARCH of Nelson (1991).

Allowing for up to two lags of each possible element in each of the specifications above. The best model is the chosen according to the Bayesian Information Criterion (BIC) of Schwarz et al. (1978). The model specification of each of these models used in the specification search can be found in Appendix B.2, as well as the model chosen for each of the real exchange rate series.

2.4.1.2 Second moments of prices and nominal exchange rates

The next step is to estimate the conditional second moments of the system with price differentials and nominal exchange rates. For a given country i, we begin by defining the following standardized return vector:

$$\mathbf{r}_t \equiv \begin{bmatrix} r^E_{i,t} \\ r^P_{i,t} \end{bmatrix} \tag{2.2}$$

where $r^E_{i,t}$ represents the standardized returns of nominal exchange rate and $r^P_{i,t}$ the standardized returns of price differentials.

As the components of $\mathbf{r}_t$ are standardized, we assume it to be a mean zero random vector. Moreover, let $\mathcal{F}^{EP}_{t-1}$ denote the sigma field generated by the past values of $\mathbf{r}_t$. If we assume that the conditional covariance matrix $\mathbf{\Sigma}_t$ is measurable with respect to $\mathcal{F}^{EP}_{t-1}$ and that $\mathbf{r}_t$ is conditionally normal[6], then the conditional distribution of $\mathbf{r}_t$ can be written as:

$$\mathbf{r}_t|\mathcal{F}^{EP}_{t-1} \sim N(0, \mathbf{\Sigma}_t) \tag{2.3}$$

where:

$$\mathbf{\Sigma}_t = \begin{bmatrix} VAR[r^E_{i,t}|\mathcal{F}^{EP}_{t-1}] & COV[r^E_{i,t}, r^P_{i,t}|\mathcal{F}^{EP}_{t-1}] \\ COV[r^E_{i,t}, r^P_{i,t}|\mathcal{F}^{EP}_{t-1}] & VAR[r^P_{i,t}|\mathcal{F}^{EP}_{t-1}] \end{bmatrix} \equiv \begin{bmatrix} \sigma^{2,E}_{i,t} & \sigma^{EP}_{i,t} \\ \sigma^{EP}_{i,t} & \sigma^{2,P}_{i,t} \end{bmatrix} \tag{2.4}$$

In this study, we estimate the components of $\mathbf{\Sigma}_t$ according to the Asymmetric Generalized Dynamic Conditional Correlation (AG-DCC) GARCH of Cappiello et al. (2006). As we did for the univariate volatility models for real exchange rates, we allow for up to two lags of each component of the model and choose the model for each country according to the

[6] Standard assumptions of multivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models.

BIC of Schwarz et al. (1978). The full details about the AG-DCC GARCH of Cappiello et al. (2006) can be found in Appendix B.3, as well as the structure of the AG-DCC GARCH model chosen for each country.

2.4.2 Decomposing real exchange rate volatility

We now turn to the decomposition of real exchange rate volatility into its building components. After estimating the models described in Section 2.4.1, we end up with the following series for each of the i countries in our sample:

- $\hat{\sigma}_{i,t}^{2,R}$: Fitted real exchange conditional variance;
- $\hat{\sigma}_{i,t}^{2,E}$: Fitted nominal exchange rate conditional variance;
- $\hat{\sigma}_{i,t}^{2,P}$: Fitted price differentials conditional variance;
- $\hat{\sigma}_{i,t}^{EP}$: Fitted conditional covariance between nominal exchange rates and price differentials.

Recall that we have monthly series for 29 exchange rates and price differentials against the United States from January 1990 to December 2020. As a result, for each of the estimated series described above, we have a panel dataset with a small N (29) and a fairly large T (372). We can therefore use a panel econometric technique to perform the decomposition of real exchange rate volatility into its building components.

The most straightforward way to perform this decomposition is to follow from our definition of real exchange rate in Section 2.3 and consider the real exchange rate volatilities as the dependent variables and the variables from the system with price differentials and nominal exchange rates as explanatory variables. Because, by construction, these volatilities are correlated over time, we also need to account for their time dependencies in the model.

Auto-Regressive Distributed Lag (ARDL) models are standard least squares regressions that include lags of both the dependent variable and explanatory variables as regressors

(Greene, 2003). In our setting, a standard ARDL model takes the form:

$$\sigma^{2,R}_{i,t} = \eta_i + \alpha_i \sigma^{2,R}_{i,t-1} + \beta_i \mathbf{x}_{i,t} + v_{i,t} \quad (2.5)$$

for i in $\{1, \ldots, N\}$ and t in $\{1, \ldots, T\}$ and where $\mathbf{x}_{i,t} = [\sigma^{2,E}_{i,t}, \sigma^{2,P}_{i,t}, \sigma^{EP}_{i,t}]'$ and β_i is a vector of coefficients.

Nevertheless, the panel of real exchange rate volatilities has one very important property. As it will be extensively studied in Section 2.5, real exchange rate volatilities are significantly and positively correlated. This means that the residuals from Equation (2.5) will be cross-sectionally correlated, hence violating the key assumption of cross-sectional independence from the ARDL model.

In order to address the cross-sectional correlation in real exchange rate volatilities, we propose using the Cross-Sectionally Augmented Autoregressive Distributed Lag (CS-ARDL) model with the Common Correlated Effects Mean Groups (CCE-MG) estimator of Pesaran (2006) to study the decomposition of real exchange rate volatilities. Pesaran (2006) allows for a form of cross-sectional dependence by introducing an error component with a factor structure. The author shows that one can allow for the presence of this unobserved common factor with a heterogeneous loading parameter. We can control for the presence of this error component by augmenting the ARDL model above by including time-specific means as additional explanatory variables. That is, by estimating the following:

$$\sigma^{2R}_{i,t} = \eta_i + \alpha_i \sigma^{2R}_{i,t-1} + \beta_i \mathbf{x}_{i,t} + \gamma_i \bar{\sigma}^{2R}_{t-1} + \delta_i \bar{\mathbf{x}}_{i,t} + \omega_i \bar{\sigma}^{2R}_{t} + v_{i,t} \quad (2.6)$$

for i in $\{1, \ldots, N\}$ and t in $\{1, \ldots, T\}$ and where $\mathbf{x}_{i,t} = [\sigma^{2,E}_{i,t}, \sigma^{2,P}_{i,t}, \sigma^{EP}_{i,t}]'$, the $^{-}$stands for the group mean values and β_i and δ_i are vector of coefficients.

The parameters from the model described in Equation (2.6) are then estimated using the mean groups estimator of Pesaran and Smith (1995), generating the Common Correlated

Effects Mean Groups (CCE-MG) estimator of Pesaran (2006). We perform the mean groups estimation considering all countries as one group as well as clustering the countries in groups according to the MSCI (2020) market classification into emerging and developed markets that we have been using throughout this paper[7].

The estimation results for the model from Equation (2.6) can be found in Table 2.1. By looking at Table 2.1, one can notice a few results. First, real exchange rate volatilities seem to be more persistent in developed markets than in emerging markets. Moreover, as one would expect, nominal exchange rate volatilities are the most significant driver of real exchange rate volatilities from the covariance matrix of the system with price differentials and nominal exchange rates.

Although nominal exchange rate volatilities are shown to play a significant role in all specifications, the main results from Table 2.1 are regarding the (ω) coefficients for the simultaneous real exchange rate variance group mean ($\bar{\sigma}_t^{2,R}$). These estimation results indicate that not only is this term significant for all group mean specifications considered, but that the most important driver of real exchange rate volatilities are the group means. This result holds for all groups considered in our group means estimator at most significance levels.

To check for parameter stability, we estimate the model described in Equation (2.6) recursively in a monthly manner from the half point of the sample (from January 2005 to December 2020). Figure 2.2 presents these recursive estimates for the impact of the simultaneous real exchange rate variance group mean on real exchange rate volatilities (ω), the main parameter of interest. By analysing Figure 2.2, we see that the impact of this group mean on the volatilities is, overall, fairly stable. According to the series plotted in Figure 2.2, the only point at which we might have a structural break in this relationship is after the Financial Crisis at the end of 2008.

In order to test the hypothesis that we might have a structural break in the relationship

[7]For each of these groups, the group means used as factors for estimating the model from Equation (2.6) are the means of the countries considered in the respective group means estimator.

Table 2.1 Estimated coefficients and p-values from the Common Correlated Effects Mean Groups (CCE-MG) estimator for Cross-Sectionally Augmented Autoregressive Distributed Lag (CS-ARDL) model for $\sigma_{i,t}^{2,R}$.

	All	**DMs**	**EMs**
η	0.1582	0.2129	0.1316
(intercept)	(0.21)	(0.09)*	(0.12)
α	0.3865	0.5149	0.2857
$(\sigma_{i,t-1}^{2,R})$	(0.04)**	(0.04)**	(0.17)
β_1	0.3415	0.1517	0.4158
$(\sigma_{i,t}^{EP})$	(0.20)	(0.10)	(0.20)
β_2	0.1761	0.1556	0.1511
$(\sigma_{i,t}^{2,E})$	(0.05)*	(0.04)**	(0.06)*
β_3	0.0396	0.0095	0.0726
$(\sigma_{i,t}^{2,P})$	(0.13)	(0.23)	(0.09)*
γ	-0.4915	-0.2860	-0.3143
$(\bar{\sigma}_{t-1}^{2,R})$	(0.17)	(0.04)**	(0.28)
δ_1	-0.0850	0.0875	0.4133
$(\bar{\sigma}_{t}^{EP})$	(0.32)	(0.18)	(0.37)
δ_2	-0.0905	-0.0414	-0.1002
$(\bar{\sigma}_{t}^{2,E})$	(0.32)	(0.16)	(0.25)
δ_3	-0.0100	-0.0003	0.2788
$(\bar{\sigma}_{t}^{2,P})$	(0.22)	(0.23)	(0.20)
ω	0.8593	0.3933	0.9415
$(\bar{\sigma}_{t}^{2,R})$	(0.04)**	(0.07)*	(0.00)***

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 2.2 Estimated coefficients and p-values from the Common Correlated Effects Mean Groups (CCE-MG) estimator for Cross-Sectionally Augmented Autoregressive Distributed Lag (CS-ARDL) model for $\sigma_{i,t}^{2,R}$ including DXY volatility.

	All	**DMs**	**EMs**
η	0.1814	0.1957	0.1600
(intercept)	(0.21)	(0.06)*	(0.16)
α	0.3601	0.4580	0.2777
$(\sigma_{i,t-1}^{2,R})$	(0.04)**	(0.02)**	(0.19)
β_1	0.3384	0.1358	0.4244
$(\sigma_{i,t}^{EP})$	(0.24)	(0.17)	(0.19)
β_2	0.1788	0.1595	0.1690
$(\sigma_{i,t}^{2,E})$	(0.06)*	(0.09)*	(0.07)*
β_3	0.0344	0.0065	0.0625
$(\sigma_{i,t}^{2,P})$	(0.15)	(0.26)	(0.19)
γ	-0.4780	-0.2601	-0.2864
$(\bar{\sigma}_{t-1}^{2,R})$	(0.19)	(0.03)**	(0.30)
δ_1	-0.0612	0.0469	0.2185
$(\bar{\sigma}_{t}^{EP})$	(0.31)	(0.24)	(0.34)
δ_2	-0.1135	-0.0636	-0.1078
$(\bar{\sigma}_{t}^{2,E})$	(0.28)	(0.09)*	(0.25)
δ_3	0.0125	-0.0049	0.1911
$(\bar{\sigma}_{t}^{2,P})$	(0.23)	(0.26)	(0.17)
ω	0.9637	0.4498	1.0339
$(\bar{\sigma}_{t}^{2,R})$	(0.06)*	(0.04)**	(0.00)***
θ_i	-0.1138	-0.0092	-0.1711
(σ_{t}^{DXY})	(0.31)	(0.27)	(0.23)

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

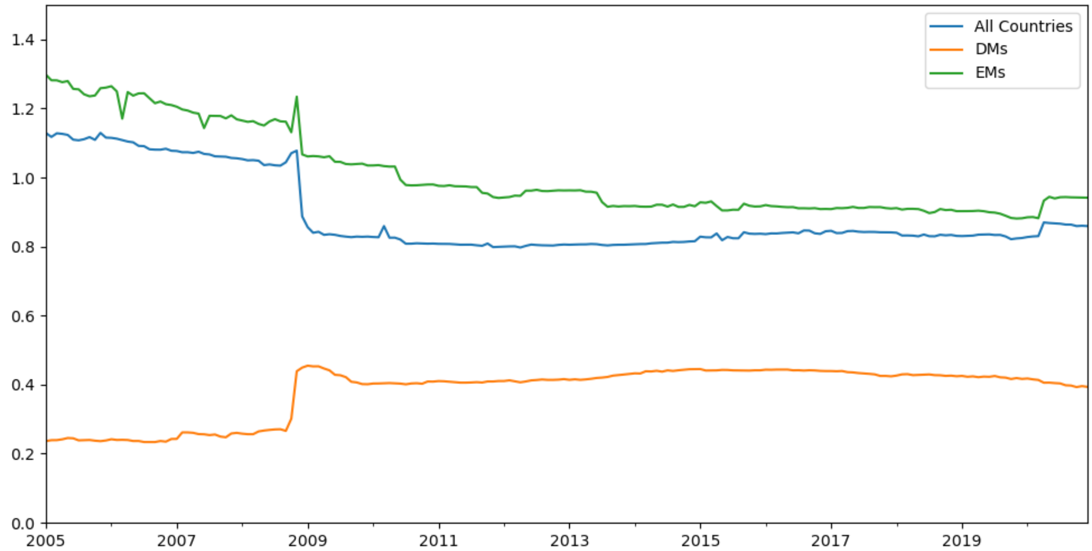

Figure 2.2 Recursive estimates for the mean group parameter from January 2005 (half-point of the sample) to December 2021

between the real exchange rate mean factor and real exchange rate volatilities after the Financial Crisis of 2008, we perform a Chow (1960) Test to compare the full sample model with the pre and post Financial Crisis models. That is, we set January 2009 as a break point and use a Chow (1960) Test to compare the full sample (January 1990 to December 2020) model with the ones estimated separately using data from January 1990 to December 2008 and from January 2009 to December 2020. This Chow (1960) Test applied to the model described in Equation (2.6) yields a test statistic of 0.29. This means that, at any reasonable significance level, we have no evidence to believe that the 2008 Financial Crises resulted in a structural break in the relationships described by the model in Equation (2.6).

Sceptical eyes can look at the results from Table 2.1 and suppose that the significance of the group mean in explaining real exchange rate volatilities is due to the fact that the currencies in our sample are in US Dollar terms. This makes sense as the volatility of the US Dollar itself could be behind the importance of the mean factor as the driver of real exchange rate volatilities.

In order to control for the volatility of the base currency (the US Dollar), we incorporate the volatility of the Dollar itself into our framework. The ICE's US Dollar index (DXY) is a benchmark measure of the value of the United States dollar relative to a basket of foreign currencies, often referred to as a basket of U.S. trade partners' currencies. We therefore import the DXY from Yahoo Finance for the period of our sample. We then apply the framework described in Section 2.4.1.1 to find the best volatility model for real exchange rates to find the volatility model that best describes the returns of the DXY[8].

With the estimates for the volatility of DXY returns in hand, Equation (2.6) is expanded to account for the volatility of the the US Dollar itself. In order to account for the possibility that our results might be driven by the volatility of the US Dollar, we estimate the following model:

$$\sigma_{i,t}^{2R} = \eta_i + \alpha_i \sigma_{i,t-1}^{2R} + \beta_i \mathbf{x}_{i,t} + \gamma_i \bar{\sigma}_{t-1}^{2R} + \delta_i \bar{\mathbf{x}}_{i,t} + \omega_i \bar{\sigma}_t^{2R} + \theta_i \sigma_t^{2,DXY} + v_{i,t} \tag{2.7}$$

for i in $\{1, \ldots, N\}$ and t in $\{1, \ldots, T\}$, where $\mathbf{x}_{i,t} = [\sigma_{i,t}^{2,E}, \sigma_{i,t}^{2,P}, \sigma_{i,t}^{EP}]'$, the $^{-}$ stands for the group mean values, β_i and δ_i are vector of coefficients, and where σ_t^{DXY} is the volatility of the US Dollar index (DXY) estimated using an EGARCH(1,1,1).

The estimation results for the model that includes the US Dollar volatility from Equation (2.7) can be found in Table 2.2. The results for this table are marginally different from the results from Table 2.1. However, by looking at Table 2.2, one can see that the volatility of the US Dollar (represented by σ_t^{DXY}) does not seem to affect the main results from Table 2.1. More importantly, even when the volatility of the US Dollar (i.e. the base currency) is taken into account, the the most significant drivers of real exchange rate volatilities are still the group means.

As proposed by Taylor (2002), Taylor and Taylor (2004), Ganguly and Breuer (2010) and many others, understanding the drivers of real exchange rate volatilities in the short-to-medium-run is the key remaining part of the PPP Puzzle. Our estimation results from the decompositions proposed by Equation (2.6) and Equation (2.7) show that this task can be

[8]This framework selects a EGARCH(1,1,1) for the volatility of the DXY.

re-framed. According to the results presented in Table 2.1 and Table 2.2, if one seeks to understand the sources of high real exchange rate volatility – and hence understand the remaining part of the PPP Puzzle – one must address the cross-sectional correlation in volatilities across currencies. In the next section, we propose an econometric framework that is capable of explaining this cross-sectional correlation in volatilities as common volatility shocks. This framework allows us to give structure to the origins of these high volatilities and study their macro-financial drivers.

2.5 A Model of Volatility Co-movements

The results from the previous section show the key for understanding the remaining part of the PPP Puzzle – that is understanding the high short-to-medium-term volatilities of real exchange rates – is understanding the cross-sectional correlations in real exchange rate volatilities. We begin this section by presenting the endogenous model of volatility co-movements introduced by Engle and Campos-Martins (2020). We then apply a version of this model to address the cross-sectional correlation in real exchange rate volatilities. Finally, we conclude by expanding the so-called GEOVOL model to allow for exogenous variables when estimating common volatility shocks. This allows us to study the influence of other macro-financial variables on the volatility co-movements of real exchange rates.

2.5.1 The GEOVOL model

As exemplified in Section 2.4, volatilities of many financial returns tend to rise and fall together. In the endogenous model of common volatility shocks of Engle and Campos-Martins (2020), a volatility factor-like structure is introduced to explain the cross-sectional correlations in second moments as joint shocks to volatilities.

The standard asset pricing model can be formulated for $NX1$ vector of returns $\mathbf{r}_t \equiv$

$(r_{1,t}, \ldots, r_{N,t})$ as:

$$\begin{aligned} \mathbf{f}_t &= \mathbf{w}'_{t-1}\mathbf{r}_t \\ \mathbf{r}_t &= r^f + \beta\mathbf{f}_t + diag\{\sqrt{\mathbf{h_t}}\}\mathbf{e}_t \end{aligned} \tag{2.8}$$

where β is a (NXK) matrix of risk exposures, $\mathbf{f}_t$ is a $(KX1)$ vector of factors and $\mathbf{e}_t \equiv (e_{1,t}, \ldots, e_{N,t})'$ is the vector of residuals from factors, and $\mathbf{h}_t \equiv (h_{1,t}, \ldots, h_{N,t})'$ contains the conditional variances. We define $diag\{\alpha\}$ as a matrix with the vector α on the diagonal and 0 elsewhere.

If Model (2.8) is correctly specified and factors fully explain the cross-sectional correlation, then $\mathbf{e}_t$ contains idiosyncratic returns and $\mathbf{h}_t$ idiosyncratic conditional variances. The standard assumptions on $\mathbf{e}_t$ state that the standardized residuals are uncorrelated in both time series and cross-section with unit variance. Hence, if factors are sufficient to reduce the contemporaneous correlations to zero we have that:

$$\mathbb{E}_{t-1}(\boldsymbol{e}_t\boldsymbol{e}'_t) = \mathbb{I} \tag{2.9}$$

Satisfying this assumption does not imply that the elements of $\mathbf{e}_t$ are independent, only that they are uncorrelated. If they were independent, then all functions of the elements of $\mathbf{e}_t$ would also be independent and there would be no co-movements of any kind. As they are not independent, the square of $\mathbf{e}_t$ may be correlated in the cross-section.

The assumption above means that for each $i \in N$ we have $\mathbb{E}_{t-1}[e^2_{i,t}] = 1$. One can then evaluate deviations from this expectation and define $\psi_{i,t}$ as a volatility shock in the univariate case as follows:

$$\psi_{i,t} \equiv e^2_{i,t} - 1 = \frac{(r_{i,t} - r^f - \beta'_i\mathbf{f}_t)^2 - h_{i,t}}{h_{i,t}} \tag{2.10}$$

The volatility shock $\psi_{i,t}$ represents the proportional difference between the squared i^{th} idiosyncrasy and its expectation. In univariate settings, the realised $e_{i,t}^2$ are on some dates larger than one and on some dates smaller than one. If many assets have $e_{i,t}^2$ larger than one at the same time, this can be interpreted as a common volatility shock.

Engle and Campos-Martins (2020) state that there is very strong evidence that the squared standardized residuals of returns net of factors are positively correlated for many assets. This observation over the time series dimension was the motivation for the original ARCH model of Engle (1982). The same observation over the cross-sectional dimension is the motivation for the endogenous common volatility shocks model of Engle and Campos-Martins (2020). The authors define a univariate factor, called *GEOVOL*, that dictates these volatility shocks that are common to all observations. This *GEOVOL* is a measure of the magnitude of shocks to volatility that are common to a collection of assets, meaning that it will be high when squared standardized residuals are high for a wide range of assets.

To estimate this *GEOVOL* factor and the loadings of each of the cross-sectional entities, Engle and Campos-Martins (2020) introduce parametric assumptions on the form of this relationship. Let *GEOVOL* be represented by $\sqrt{x}$ where x is a $(TX1)$ vector of latent variables and let s be an $(NX1)$ vector of parameters interpreted as factor loadings satisfying the assumptions:

$$\mathbb{E}_{t-1}(x_t) = 1 \tag{2.11}$$

$$\mathbb{E}_{t-1}(x_t - 1)^2 = v_t \tag{2.12}$$

$$\epsilon_t \sim IIN(0, 1) \tag{2.13}$$

$$s_i \in [0, 1] \tag{2.14}$$

where $t = \{1, \ldots, T\}$ and $i = \{1, \ldots, N\}$.

The authors then specify a function $g(s_i, x_t)$ that is a data generating process for the random variables e_{it} from x_t, s_i and ϵ_{it}. They propose the following structure:

$$e_{it} = \sqrt{g(s_i, x_t)}\epsilon_{i,t} \tag{2.15}$$
$$g(s_i, x_t) \equiv s_i x_t + 1 - s_i$$

Specification (2.15) implies that g is non-negative with expected value 1 and therefore satisfies the condition from (2.9). From the specification described by the structure in (2.15) it follows that:

$$\Psi_{ij,t} = \mathbb{E}_{t-1}[\epsilon_{i,t}^2\epsilon_{j,t}^2(s_i s_j(x_t - 1)^2 + (s_i + s_j)(x_t - 1) + 1) - 1] = s_i s_j v_t \tag{2.16}$$
$$\Psi_{ii,t} = \mathbb{E}_{t-1}[\epsilon_{i,t}^4(s_i^2(x_t - 1)^2 + 1) - 1] = 3s_i^2 v_t + 2 \tag{2.17}$$

Using these, the sample covarianve matrix can be constructed by averaging over t:

$$\Psi = \frac{1}{T}\mathbb{E}\left[\sum_{t=1}^{T} \mathbf{e}_t^2 \mathbf{e}_t^{2\prime}\right] = \frac{1}{T}\Psi_t = \mathbf{s}\mathbf{s}'\bar{v} + D \tag{2.18}$$
$$D = diag\{3s_i^2\bar{v} + 2\}$$

where $\bar{v} = (1/T)\sum_{t=1}^{T} v_t$. One can hence state that Ψ is a factor matrix with x as the factor and s as the vector of factor loadings.

2.5.1.1 Estimating the model for volatility co-movements

This covariance matrix Ψ is observable and has information on the parameters of the model. However, it is not identifiable unless additional assumptions are made on the unknown parameter s.

If all the elements of s are multiplied by a scalar and v is divided by the square of this scalar, the covariances will be unchanged. Engle and Campos-Martins (2020) therefore propose normalizing the factor loadings by requiring that:

$$\mathbf{s}\mathbf{s}' = 1 \tag{2.19}$$

This normalization is consistent with the requirement that the loadings are in the unit interval.

To gain efficiency, Engle and Campos-Martins (2020) suggest using not only the unconditional covariances but also the observed heteroscedasticity relationships as in Equation (2.15). These equations can be used to estimate s conditional on $\hat{x}$ by time series analysis or to estimate x conditional on $\hat{s}$ from cross-sectional analysis. The likelihood function can be written for this model as follows:

$$L(\mathbf{s}, \mathbf{x}; \mathbf{e}) = -\frac{1}{2} \sum_{i=1,t=1}^{N,T} \left[log(g(s_i, x_t)) + \frac{e_{i,t}^2}{g(s_i, x_t)} \right] \tag{2.20}$$

This is not a classical likelihood function since x is considered a latent variable rather than a parameter. However, Engle and Campos-Martins (2020) propose using the likelihood as if x was observable. Hastie and Tibshirani (2009) calls this procedure data augmentation. The iteration solves the first order conditions:

$$\frac{\partial L(\mathbf{s}, \mathbf{x}; \mathbf{e}))}{\partial s_i} = 0 \tag{2.21}$$

$$\frac{\partial L(\mathbf{s}, \mathbf{x}; \mathbf{e}))}{\partial x_t} = 0 \tag{2.22}$$

sequentially until parameters are found that solve both jointly.

This algorithm can be interpreted as an Expectation-Maximization (EM) algorithm where the cross-sectional regression estimates the unobserved value of x in the expectation step and then the time series regression maximizes the likelihood function conditional on the estimated latent variable. Since the expectation step is also a maximization, Hastie and

Tibshirani (2009) call this a Maximization-Maximization procedure. Each step therefore increases the likelihood function. The algorithm stops when the parameters become constant and hence the likelihood function has reached an extremum which can be verified to be a maximum.

2.5.1.2 Testing for common volatility shocks

An observable implication of the GEOVOL model is that, even though the elements of $\mathbf{e}_t^2$ are orthogonal in both time series and cross-section, they may not be independent. This means that their squares and can be correlated in the cross-section. These co-movements in $\mathbf{e}_t^2$ are induced by the common volatility factor x_t. It hence follows that detecting this common endogenous volatility factor structure involves testing whether the squared standardized innovations are correlated.

Empirical evidence for these common volatility shocks can be found using the sample covariance matrix. The null hypothesis of no correlation in $\mathbf{e}_t^2$ with $\bar{v} = 0$ is:

$$\mathbb{H}_0 : \Psi = 2\mathbb{I} \tag{2.23}$$

The two in this equation is a result of assuming normality. Otherwise it would be the kurtosis of each return minus one. When the factor loading model is the alternative, all assets are affected by the same shock. The "no common volatility shocks" null holds when x_t is constant, that is when $v = 0$. In this setting, $\mathbf{e}_t$ are independent and no co-movements of any kind can be observed across the standardized residuals. The null hypothesis $\mathbb{H}_0 : v = 0 \Rightarrow \bar{\rho}_{e^2} = 0$, where $\bar{\rho}_{e^2}$ denotes the average empirical correlation of $\mathbf{e}^2$.

2.5.2 Volatility co-movements of real exchange rates

Going back to the object of our study, consider the vector representing the standardized residuals of real exchange rates $\boldsymbol{e}_t^R \equiv (e_{1,t}^R, \ldots, e_{N,t}^R)'$ and assume factors are sufficient to reduce the contemporaneous correlations to zero:

$$\mathbb{E}_{t-1}(\boldsymbol{e}_t^R \boldsymbol{e}_t^{R\prime}) = \mathbb{I} \tag{2.24}$$

To obtain the series of standardized residuals, we assume and estimate for each series of real exchange rate returns a single factor model with a first-order auto-regressive term (conditional upon rejecting the null of time independence in the first moment), where the cross-sectional average of returns is used as the single factor, with GARCH$(1, 1)$ errors (conditional upon rejecting the null of time independence in the second moment) as described in Section 2.5.1.

The average of returns seems to capture most of the correlation between real exchange rates. At it can be seen in Table 2.3, the average correlation of the raw real exchange returns is 0.298 whereas of standardized residuals of real exchange rates is -0.019.

Table 2.3 Average cross-sectional correlations for real exchange rates.

	$\bar{\rho}_{[r_R]}$	$\bar{\rho}_{[\hat{e}_R]}$	$\bar{\rho}_{[\hat{e}_R^2]}$	$\bar{\rho}_{[\hat{e}_R^2/\hat{g}_R]}$
Correlation	0.298	−0.019	0.049	0.004
GEOVOL test statistic			21.58	11.70

Assumption (2.24) implies that the standardized residuals are orthogonal with unit variance. It does not mean however that they are independent. In fact, the squared standardized residuals of real exchange rates are correlated. Referring back to Table 2.3, we can see that their average correlation is 0.049, which is positive and statistically significant according to the statistical test proposed by Engle and Campos-Martins (2020).

Because the squares of the standardized residuals are correlated, the co-movements of volatilities are most likely caused by the correlation between shocks to volatility. This is confirmed by the results showed in Table 2.3. Once we consider the information about the endogenous common volatility shocks model of Engle and Campos-Martins (2020) described in Section 2.5.1 via the $g(s_i, x_t)$ function from Equation (2.15), we are able to capture most of the cross-sectional correlations. That is, even the standard one-factor common volatility shocks model of Engle and Campos-Martins (2020) described in Section 2.5.1 is capable of successfully give structure to a large portion of the common volatility dependencies in real exchange rates.

Assuming the volatility factor structure described by $g(s_i, x_t)$ in (2.15), the estimated most extreme common shocks to the volatilities of real exchange rates and their factor loadings are summarized, respectively, in Table B.4 and in Table B.5 in Appendix B.4. Many common shocks can be easily identified such as those during the COVID-19 pandemic in 2020, the Asian financial crisis in 1997, the global financial crisis in 2008, among others. Different currencies have different volatility factor loadings. This means that currencies with larger loadings have larger fractions of the volatility factor affecting their volatilities and so are more exposed to common volatility shocks than others. This gives room for hedging against common shocks, which traditional diversification strategies do not allow. We refer to Engle and Campos-Martins (2020) for the portfolio optimization criterion when in the presence of this risk structure.

These results indicate that the volatility factor structure of Engle and Campos-Martins (2020) performs well in modelling the cross-sectional correlation in real exchange rates volatilities. This finding gives econometric structure to the results from Section 2.4 – regarding the importance of cross-sectional correlations in explaining real exchange rate volatilities – indicating the importance of common shocks in driving the volatilities of real exchange rates. The results are even more encouraging when we consider Real Effective Exchange Rates (REER) from the International Monetary Fund as our measure of real exchange rates to control for base currency effects. Details about this analysis can be found in Appendix B.5.

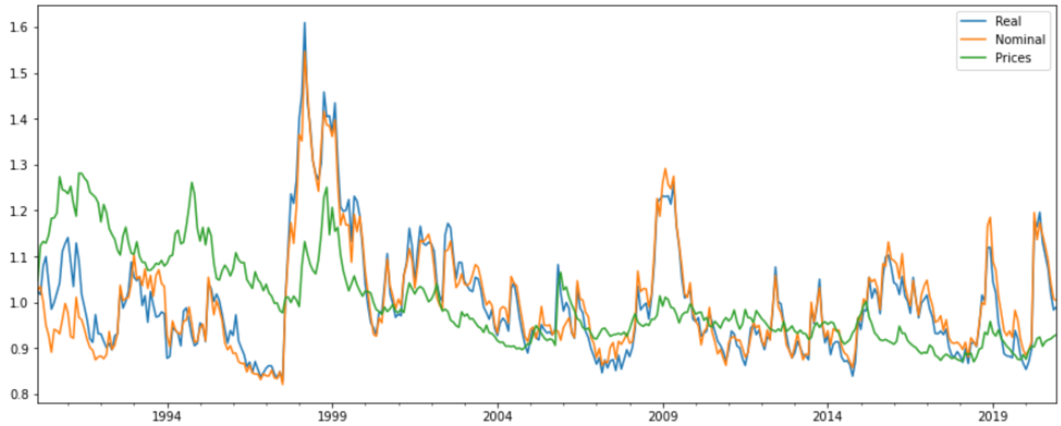

Figure 2.3 The cross-sectional average of the estimated GARCH-type volatilities of the real (blue) and nominal (orange) exchange rates, and of the relative prices (green).

2.5.2.1 Incorporating nominal exchange rates and prices

In order to get a better understanding of the drivers of common volatility shocks to real exchange rates, we now introduce information about nominal exchange rates and relative prices. Figure 2.3 shows the cross-sectional average of the estimated volatilities for these variables. Volatilities seem to not only co-move within the same set of variables but also between sets. This co-movement is particularly noticeable between the volatilities of real and nominal exchange rates.

In Equation (2.10) from Section 2.5.1 we define $\psi_{i,t}$ as a general volatility shock. Focusing on real exchange rates, define a shock to the i^{th} standardized residual at time t as follows:

$$\psi_{i,t}^R \equiv \left(e_{i,t}^R\right)^2 - 1 = \frac{(r_{i,t}^R - \alpha_i^R r_{i,t-1}^R - \beta_i^R \bar{r}_t^R)^2 - h_{i,t}^R}{h_{i,t}^R}.$$

The volatility shock $\psi_{i,t}^R$ represents the proportional difference between the squared real exchange rate idiosyncrasy and its expectation. To study the determinants of volatility shocks to the real exchange rates, we regress volatility shocks to the real exchange rates ($\psi_{i,t}^R$) on the volatility shocks to the nominal exchange rates ($\psi_{i,t}^E$) and to the relative prices ($\psi_{i,t}^P$). For each currency, we run the regression (by taking the volatility shocks as if they were observed):

$$\psi_{i,t}^R = \delta_i^E \psi_{i,t}^E + \delta_i^P \psi_{i,t}^P + \delta_i^{EP} \psi_{i,t}^{EP} + v_{i,t},$$

where $\psi_{i,t}^{EP} = \psi_{i,t}^E \times \psi_{i,t}^P$ is an interaction term.

For most currencies, volatility shocks to both nominal exchange rates and relative prices seem to explain the volatility shocks to real exchange rates. The average R^2 for the regressions with all three variables is 0.653. With only $\psi_{i,t}^E$ as a regressor, the average among all regressions is 0.606, which supports the view that the nominal exchange rates component has much higher explanatory power compared to relative prices component of real exchange rates. In terms of the magnitude of the effects (among the statistically significant coefficients), on average, $\bar{\bar{\delta}}^E = 0.905$, $\bar{\bar{\delta}}^P = 0.017$ and $\bar{\bar{\delta}}^{EP} = 0.110$.

These results suggest that co-movements of real exchange rates seem to mostly arise from volatility shocks to nominal exchange rates. In other words, shocks affecting nominal exchange rates are the main drivers of simultaneous changes in the volatilities of real exchange rates. Nominal shocks appear to have real effects at the global scale.

We can now start bringing information from the volatility factor model of Engle and Campos-Martins (2020) into this analysis. Section 2.5.1 introduces the framework for estimating the one factor common endogenous volatility shocks model. We follow the procedure described in Section 2.5.1 and estimate this common volatility factor for real exchange rates (x_t^R), nominal exchange rates (x_t^E), and price differentials (x_t^P).

The 12-month rolling-window average of the estimated volatility factors $\hat{x}_t^R$, $\hat{x}_t^E$ and $\hat{x}_t^P$ are plotted in Figure 2.4. The volatility factor shows high variability for both real and nominal exchange rates. The common shocks to real exchange rates seem to be almost entirely driven by the common shocks to nominal exchange rates. Even though at the beginning of the sample there appears to be some similarity between the common shocks to both real exchange rates and the relative prices, during the last two decades there seemed to be little co-movement of relative price volatilities.

As in Table 2.3, denote the average empirical correlation across the pairwise correlations

Figure 2.4 The 12-month rolling window average of the endogenous volatility factor of the real $\hat{x}^R$ (blue) and nominal $\hat{x}^E$ (orange) exchange rates, and of the relative prices $\hat{x}^P$ (green).

of the squared standardized residuals of real exchange rates, $\hat{\boldsymbol{e}}_R^2$, as $\bar{\rho}_{\hat{\boldsymbol{e}}_R^2}$. In Table 2.4, we summarize $\bar{\rho}_{\hat{\boldsymbol{e}}_R^2}$ for the raw and standardized $\hat{\boldsymbol{e}}_R^2$, namely standardized by the estimated volatility factor of real exchange rates $\hat{\boldsymbol{g}}^R$ (whose elements are defined in $g(s_i, x_t)$ function from Equation (2.15)), and similarly of nominal exchange rates $\hat{\boldsymbol{g}}^E$, and relative prices $\hat{\boldsymbol{g}}^P$.

Table 2.4 Average correlation of $\hat{\boldsymbol{e}}_R^2$ for different standardization procedures.

	$\bar{\rho}_{\hat{\boldsymbol{e}}_R^2}$	$\bar{\rho}_{(\hat{\boldsymbol{e}}_R^2/\hat{\boldsymbol{g}}^R)}$	$\bar{\rho}_{(\hat{\boldsymbol{e}}_R^2/\hat{\boldsymbol{g}}^E)}$	$\bar{\rho}_{(\hat{\boldsymbol{e}}_R^2/\hat{\boldsymbol{g}}^P)}$
Correlation	0.049	0.004	0.011	0.053
GEOVOL test statistic	21.58	11.70	12.95	26.35

Comparing the last two columns, we conclude that $\bar{\rho}_{\hat{\boldsymbol{e}}_R^2}$ can be significantly reduced when $\hat{\boldsymbol{e}}_R^2$ are standardized by the estimated volatility factor of nominal exchange rates, $\hat{\boldsymbol{g}}^E$ (rather than relative prices i.e., $\hat{\boldsymbol{g}}^P$). In fact, this nominal exchange rates factor is able to gauge almost as much of the cross-sectional correlation as the standardization performed using real exchange rates. This further supports the idea that common shocks to nominal exchange rates are the main drivers of volatility co-movements of real exchange rates.

2.5.3 The GEOVOL model with exogenous information

Given that the common volatility of real exchange rates is mainly driven by common volatility shocks to the nominal exchange rates, we will focus on the volatility co-movements of the nominal exchange rates when expanding the model of Engle and Campos-Martins (2020) to allow for exogenous volatility factors.

In order to also include exogenous information in the volatility factor model, we use two standard monetary drivers of exchange rates in the short-to-medium run: interest rate differentials[9] (with respect to the U.S. interest rate) and inflation differentials (with respect to the U.S. inflation rate). We take first-differences to compute both interest and inflation volatility shocks.

In order to incorporate information about these other variables in the volatility factor model, the standardized residuals are obtained by regressing the nominal exchange rate returns on not only the return cross-sectional average (proxy for market factor) but also the interest rate and inflation differentials (after computing their first-differences).

We can specify a multiplicative function $g_{i,t}^E$ that is a data generating process for the squared standardized residuals of the nominal exchange rates, $e_{i,t}^E$, from the endogenous volatility factor x_t^G (and s_i^G, where the superscript G is introduced to distinguish it from the other factors), the exogenous volatility factors x_t^i, x_t^π (and s_i^i, s_i^π), which are assumed as observed, and $\epsilon_{i,t}$ as follows:

$$e_{i,t}^E = \sqrt{g_{i,t}^E}\epsilon_{i,t}^E,$$

[9] We use the shortest maturity money market rates as measures of interest rates.

Figure 2.5 The 12-month rolling window average of the volatility factors $\hat{x}^G_{(2.25)}$ (blue), $\hat{x}^i_t$ (orange), and $\hat{x}^\pi_t$ (green).

where $g^E_{i,t}$ can be specified as either:

$$g^E_{i,t,(2.25)} \equiv \left[s^G_{i,(2.25)}\left(x^G_{t,(2.25)} - 1\right) + 1\right] \tag{2.25}$$

$$g^E_{i,t,(2.26)} \equiv \left[s^G_{i,(2.26)}\left(x^G_{t,(2.26)} - 1\right) + 1\right] \times \left[s^i_{i,(2.26)}\left(x^i_t - 1\right) + 1\right] \tag{2.26}$$

$$g^E_{i,t,(2.27)} \equiv \left[s^G_{i,(2.27)}\left(x^G_{t,(2.27)} - 1\right) + 1\right] \times \left[s^i_{i,(2.27)}\left(x^i_t - 1\right) + 1\right] \times \left[s^\pi_{i,(2.27)}\left(x^\pi_t - 1\right) + 1\right] \tag{2.27}$$

Other specifications are certainly possible as long as $g^E_{i,t}, i = 1, \ldots, n$, is non-negative with expected value 1 such that (2.24) is satisfied. For comparison, the exogenous volatility factors $\hat{x}^i_t$ and $\hat{x}^\pi_t$ are depicted in Figure 2.5, alongside $x^G_{t,(2.25)}$.

We proceed to the estimation of the volatility factor models (2.25)-(2.27). The 12-month rolling average of the endogenous volatility factor for all models are depicted in Figure 2.6. Results point out a similar trajectory across all models, either including or excluding exogenous factors. Nevertheless, some differences are noticeable around crisis periods such as the global financial crisis.

The correlations between the estimated endogenous volatility factors $\hat{x}^G_{(\cdot)}$ for the different volatility factor model specifications from (2.25)-(2.27) depicted in Figure 2.6 are summarised below in Table 2.5.

Figure 2.6 The 12-month rolling window average of the endogenous volatility global factor from different specifications: no exogenous factors, $\hat{x}^G_{(2.25)}$ (blue), with exogenous factor $x^i_{(2.26)}$, $\hat{x}^G_{(2.26)}$ (orange), and with exogenous factors x^i and x^π, $\hat{x}^G_{(2.27)}$ (green).

Table 2.5 Correlations between the endogenous volatility factor $\hat{x}^G_{(.)}$ obtained from different specifications.

	$\hat{x}^G_{(2.25)}$	$\hat{x}^G_{(2.26)}$	$\hat{x}^G_{(2.27)}$
$\hat{x}^G_{(2.25)}$	1	0.800	0.693
$\hat{x}^G_{(2.26)}$	0.800	1	0.890
$\hat{x}^G_{(2.27)}$	0.693	0.890	1

Denote the average correlation of the squared standardized residuals of nominal exchange rates $\hat{e}_E^2$ as $\bar{\rho}_{\hat{e}_E^2}$. In Table 2.6, we summarize $\bar{\rho}_{\hat{e}_E^2}$ for the squared residuals of nominal exchange rates standardized by the volatility factors from the baseline GEOVOL model (2.25) and the models with exogenous information (2.26) and (2.27).

Table 2.6 Average correlation of $\hat{e}_E^2$ for different standardization, where $\bar{\rho}_{\hat{e}_E^2} = 0.043$.

	$\bar{\rho}[\hat{e}_E^2/\hat{g}^E_{(2.25)}]$	$\bar{\rho}[\hat{e}_E^2/\hat{g}^E_{(2.26)}]$	$\bar{\rho}[\hat{e}_E^2/\hat{g}^E_{(2.27)}]$
Correlation	0.003	0.002	0.008
GEOVOL test statistic	5.583	6.552	10.17

A comparison of the performance in capturing common volatility shocks between the different models reveals that $\bar{\rho}_{\hat{e}_E^2}$ is minimised for $\hat{e}_E^2$ standardized by the estimated composite volatility factor where an endogenous factor and an exogenous factor measuring common volatility shocks to the interest rate differentials are both included[10] (i.e., by $\hat{g}^E_{(2.26)}$). Overall, volatility co-movements in nominal exchange rates seem to be best explained by the volatility factor model that addresses common shocks to nominal exchange rates themselves but also incorporates common shocks to interest rate differentials.

2.5.4 Discussion of results

The results presented in this section show that the endogenous common volatility shocks model of Engle and Campos-Martins (2020) performs well in modelling the cross-sectional correlation in real exchange rates volatilities (Tables 2.3 and 2.4). Moreover, a significant amount of evidence suggests that this cross-sectional correlation in real exchange rates is, as expected, a result of common volatility shocks to nominal exchange rates rather than price differentials. As the return of real exchange rates are simply a linear combination of the returns of nominal exchange rates and price differentials, we decided to focus the analysis on the exogenous drivers of the common volatility shocks to nominal exchange rates.

[10] Although there is only a marginal difference between this average correlation and that of the simple univariate volatility factor model with only exchange rates.

In Section 2.5.3, we present a framework that allows us to study how exogenous factors may drive this common volatility shocks to nominal exchange rates. In fact, this is a general framework and can be considered as an extension of the model of Engle and Campos-Martins (2020) to allow for exogenous drivers of common volatility shocks. In order to introduce our framework, we consider two major drivers of nominal exchange rate fluctuations in the short-to-medium run as exogenous variables: interest rate differentials and inflation differentials. The results from Section 2.5.3 show common shocks to interest rate differentials as a relevant driver of common volatility shocks in nominal exchange rates. The model with both the endogenous term x_t^G and the interest rate differentials term x_t^i seems to perform better than the other models considered in purging the cross-sectional correlation left in the volatilities of nominal exchange rates. These results provide an interesting link between the sources of the remaining part of the PPP Puzzle and another major topic of study in empirical international finance: the Interest Rate Parity.

2.6 Concluding Remarks

Most of the discussion about the Purchasing Power Parity (PPP) Puzzle of Rogoff (1996) has pertained to the reversion speed of deviations from PPP. Much less attention, however, has been given to the other component of the puzzle: the high volatilities of real exchange rates. In this paper, we use latest developments from the financial econometrics literature in second moments dynamics to provide a framework that is capable of explaining the econometric sources of these volatilities and further provides a framework to link these to their possible macro-financial drivers.

In Section 2.4, we present an econometric framework based on the Cross-Sectionally Augmented Autoregressive Distributed Lag (CS-ARDL) model of Pesaran (2006) for the decomposition of real exchange rate volatility into its building blocks. As discussed by Taylor (2002), Taylor and Taylor (2004), Ganguly and Breuer (2010) and many others, understanding the drivers of real exchange rate volatilities in the short-to-medium-run is the key remaining part of the PPP Puzzle. Our estimation results from the decomposition show that this task can be re-framed. If one seeks to understand the sources of high real exchange rate volatility – and hence understand the remaining part of the PPP Puzzle – one must address the cross-sectional correlation in volatilities across currencies.

Inspired by the results from Section 2.4 regarding the importance of cross-sectional correlations in explaining real exchange rate volatilities, in Section 2.5 we propose an econometric model based on the endogenous common volatility shocks framework of Engle and Campos-Martins (2020) to model the dynamics of real exchange rate volatilities. The proposed framework presents encouraging results when modelling these cross-sectional correlations. It successfully gives structure to these common volatility dependencies in real exchange rates and further allows us to propose an extension to the framework in order to study the impact of other macro-financial variables on this common volatility movements. In fact, this is a general framework and can be considered as an extension of the model of Engle and Campos-Martins (2020) to allow for exogenous drivers of common volatility shocks. The results from our proposed extension show common shocks to interest rate differentials as a significant driver of common volatility shocks in exchange rates, hence building a bridge between our analysis of the PPP Puzzle and the Interest Rate Parity literature.

There is a vast body of research regarding the links between interest rate differentials and exchange rate dynamics. However, a topic that is particularly interested in studying the relationship between these two variables is the study of the Interest Rate Parity[11]. The failure of the Interest Rate Parity in providing useful guidance to exchange rate behaviour has, in fact, been a topic of intense study[12] and the source of another puzzle in the international finance literature, known as the Forward Premium Puzzle. Although the Forward Premium Puzzle of Fama (1984) refers to futures of exchange rates, we find it noteworthy

[11] For an intro and review of Interest Rate Parity see Stein (1962), Glahe (1967), Aliber (1973), Wu and Chen (1998), among others.

[12] See Fama (1984) for original Forward Premium Puzzle and Bansal (1997), Bansal and Dahlquist (2000), Burnside et al. (2009), and others for more recent interpretations.

the link between these two variables presented in our results as being, at least partially, the source of the PPP Puzzle. An interesting way forward in both empirical and theoretical international finance would be to study how common volatility shocks, shown to be the main source of the PPP Puzzle, also affect Interest Rate Parity and hence the findings that originated the Forward Premium Puzzle.

This research expects to contribute to a better understanding of the PPP Puzzle. More specifically, to the question of why real exchange rates are so volatile in the sort-to-medium run. Our application of the exogenous drivers extension to the endogenous common volatility shocks framework of Engle and Campos-Martins (2020) was limited to the two series which are more meaningfully related to exchange rate dynamics according to the literature. Further research could study other drivers of common shocks to exchange rate volatilities in a more holistic approach, diving into even higher frequencies by considering daily financial series. Future studies could, more broadly, seek to understand exogenous drivers of common volatility shocks in other asset classes by re-framing the framework presented in Section 2.5. Engle and Campos-Martins (2020) show that international equity markets present a similar behavior regarding common volatility shocks. This feature in multiple asset classes suggests an avenue to be explored by linking these common volatility shocks to possible exogenous drivers.

3 | Soft Power: Introducing a Global Index

This chapter is based on joint work with Serhan Cevik[‡].

Abstract

Soft power is difficult to measure directly and alternative measures mostly rely on subjective data that are not always transparent. This paper introduces a new composite Global Soft Power Index (GSPI) composed of six dimensions for a broad sample of 72 countries across the world over the period 2007-2019. The proposed framework allows for comparisons not only at the "headline" level of the GSPI, but also at the level of the sub-indices, which allows us to identify and study how countries differ at a granular level of soft power. In a final step of the analysis, we present a possible macro-financial application to analyse the relationship between soft power and the volatility of Real Effective Exchange Rates (REER) across countries and over time. The results indicate that some dimensions of the GSPI play an important role in explaining real exchange rate volatility at almost all significance levels. Overall, our framework presents a systematic approach to measure soft power and its dimensions. By capturing the matrix of soft power characteristics, the GSPI offers significant advantages in comparative analysis of soft power across countries and over time.

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3.1 Introduction

Power is the ability to affect others to get the outcomes one prefers, and that can be accomplished by coercion, payment, or attraction and persuasion. Soft power, on the other hand, is the ability to obtain preferred outcomes by attraction rather than coercion or payment. As an analytic concept in international relations, soft power is popularized by Nye (1990) to capture intangible resources beyond material considerations. With digitization accelerating the diffusion of power within and across countries, soft power has increasingly become more and more important to the shaping of global outcomes in an interconnected world – whether they be driven by state or non-state actors.

Composite indicators are popular tools for monitoring and assessing the performance of countries on a wide spectrum of issues ranging from human development, environmental sustainability, corruption, innovation, competitiveness, or other complex phenomena that are not directly measurable and not uniquely defined[1]. There have been several attempts to measure soft power and analyse its influence on political and macroeconomic developments. Treverton and Jones (2005) discuss measuring soft power in terms of how effective non-state actors like corporations and humanitarian organizations are in international affairs. Operationalizing Nye's conceptual framework, McClory (2015) and McClory and Harvey (2016) build the Soft Power 30 index that compares the relative strength of countries' soft power resources by combining objective and subjective data. While these efforts provide insight into soft power, their sources of data are not always transparent, or readily replicate over time beyond a small set of countries.

Accordingly, in this paper, we construct a new composite Global Soft Power Index (GSPI) based on 29 indicators covering a wide spectrum of variables for a broad sample of 72 countries across the world over the period 2007-2019. The GSPI is constructed using a three-step approach to reduce multidimensional data into a single composite index: (i)

[1]Examples include the Human Development Index (UNDP, 1990), the Sustainable Society Index (Van de Kerk and Manuel, 2008), and the Environmental Performance Index (Hsu and Zomer, 2014).

normalization of variables; (ii) aggregation of normalized variables into the sub-indices representing a particular functional dimension; and (iii) aggregation of the sub-indices into the final index. Being a systematic framework, the approach used in calculating the GSPI captures the matrix of soft power characteristics and offers significant advantages as it allows us to compare the level of soft power for different countries in both the cross-section and time series dimensions.

A key characteristic of the GSPI indicator is that it is an aggregation of different sub-indices, each representing a particular functional dimension of soft power. The composite GSPI is composed of six of these dimensions (or sub-indices): Commercial, Culture, Digital, Education, Global Reach, and Institutions. This allows for comparisons not only at the "headline" level of the GSPI, but also at the level of the sub-indices.

Our GSPI presents results in accordance with what one would expect from an index measuring soft power. By using information about the sub-indices, we are able to identify four group of countries. The first one is the group of low soft power nations with mostly developing countries that are considerably behind others in the Education and Institutions dimensions. The second group is the group of medium soft power countries. These are mostly developed nations that present high levels of Education and Institutions, but do not have much of an impact on other countries via the dimensions measured by the Culture and Global Reach sub-indices. The third group is the group of high soft power countries. Japan and South Korea are their own group. They are similar to other high soft power countries but have significantly higher Commercial Prowess and perform meaningfully worse in the Culture dimension.

In a final step of the analysis, we present a possible macro-financial application of our proposed index by using the composite GSPI and its sub-indices to study the volatility of Real Effective Exchange Rates (REER). Previous studies (see Cevik et al. 2017) have already documented a meaningful relationship between soft power like variables and real exchange rate volatility. Overall, our results are in line with the findings from Cevik et al. (2017). Although the composite GSPI doesn't seem to be significant at conventional levels, the Culture and Global Reach dimensions of soft power are shown to be relevant in explaining real exchange rate volatility at almost all significance levels. This is an interesting result because these are precisely the dimensions that are responsible for most of the differentiation between medium and high soft power countries. Moreover, the results also suggest that Global Reach, the dimension that one would think to be the "purest" measure international soft power reach, is the the sub-index of soft power with the most impact on real exchange rate volatility.

This research relates to other empirical works that aim to measure and evaluate soft power. The objective is to therefore contribute to the growing literature on the study of this important dimension of power and its influence on political and macroeconomic developments. The remainder of this study is organized as follows. Section 3.2 provides a data overview, including the dimensions of the index, the data sources and data normalization and transformations performed. Section 3.3 describes the salient features of the weighting and aggregation methodology used in constructing the GSPI. Section 3.4 introduces our soft power index and its sub-components with greater granularity as well as some properties of these indices. Section 3.5 presents an application of the GSPI as a tool to model real exchange rate volatility. Finally, Section 3.6 offers concluding remarks with respect to our proposed soft power index.

3.2 Data Overview

In determining the soft power characteristics of countries, rather than relying on an arbitrary choice of a small set of variables, we take an agnostic view and initially consider a wide range of demographic, institutional, political, and social indicators. We start with the broadest possible sample, including as many countries as possible. But, due to data availability regarding this plethora of indicators for some countries, we were able to construct the final GSPI for a set of 72 countries. The list of countries included in our sample is presented in Table C.1 in Appendix C.1.

Our main proposed objective is to develop the GSPI as a comprehensive composite index of soft power across the world. Nevertheless, soft power is a broad concept that encompasses many dimensions. In order to incorporate these dimensions, we follow the approach of the Technology Achievement Index (UNDP, 2001; Sen et al., 2003), the Global Innovation Index (Dutta and Lanvin, 2012), and many other composite indices in the literature and construct the GSPI based on 6 individual sub-indices. This gives clarity to what we are defining as the main components of our index and allows for an intermediate link between measurable variables and the fairly broad concept of soft power. Moreover, this framework also allows us to measure each of the dimensions of soft power individually and to study them separately as well as jointly.

We measure soft power in six dimensions: Commercial, Culture, Digital, Education, Global Reach, and Institutions. Table C.3 in Appendix C.2 reports the dimensions and indicators as well as detailed information on data sources. In total, we consider 29 variables to construct our index. For most variables, the series are already available in annual frequency. For series that are released less often than on a yearly basis, as, for example, the number of Olympic medals, we use the latest information available as the yearly realisation.

Table C.2 in Appendix C.2 presents a summary of statistics for each of the variables included in our study. As can be clearly observed in Table C.2, the set of variables that we consider are not easily comparable. Therefore, before analysing these series jointly, we need to apply a normalization procedure. This procedure needs to take into account the properties of the data with respect to the measurement units in which the indicators are expressed and their robustness against possible outliers in the data (Ebert and Welsch, 2004).

Since the 29 variables presented in Table C.2 in Appendix C.2 have not only very different means and extreme values, but also differ significantly with respect to their standard deviations, we opt for following the z-score standardization approach recommended by the OECD Handbook on Constructing Composite Indicators of the Joint Research Centre of the European Commission (OECD, 2008). For each individual indicator $x_{i,t}$ representing the value of the indicator for country i at time t, the average across countries $x_{i=\bar{i},t}$ and the standard deviation across countries $\sigma_{i=\bar{i},t}$ are calculated. The normalization is then applied for each observation according to the following z-score transformation:

$$z_{i,t} = \frac{x_{i,t} - x_{i=\bar{i},t}}{\sigma_{i=\bar{i},t}} \tag{3.1}$$

This approach converts indicators into a common scale with an average of 0 and standard deviation of 1. The average of 0 avoids introducing aggregation distortions stemming from differences in the indicator means. The formula to calculate the z-score is the value of an indicator minus the average of the indicator across countries, divided by the standard deviation. In this manner, all $z_{i,t}$ have similar dispersion across countries. With this standardized set of variables in hand, we can proceed to the construction of the soft power indices.

There is a trade-off between creating a comprehensive measure of soft power and data availability. More data is available for a larger sample of countries in the most recent fifteen years rather than earlier in the sample. The extent of missing data varies considerably across indicators. For example, data coverage is strong for Commercial and Education variables, but weak for Culture variables. In some cases, such as Cultural Exports, data were not being collected before 2007 on a comprehensive basis. Where data are not yet available for the latest year (e.g., 2019), the values are set equal to the latest available observations (e.g. 2018). Regarding data availability in the early sample, we decided to only start constructing the indices from the date that information was available for all variables of interest. For more information about the availability of each of the series used, please refer to Table C.3 in Appendix C.2.

3.3 Index Methodology

The GSPI is constructed using a standard three-step approach found in the literature on reducing multidimensional data into one summary index: (i) normalization of variables; (ii) aggregation of normalized variables into the sub-indices representing a particular functional dimension; and (iii) aggregation of the sub-indices into the final index. This procedure follows the OECD Handbook on Constructing Composite Indicators (OECD, 2008), which is a broadly used reference for methodological suggestions.

Our objective is to construct not only a “headline” soft power index, but also to have measures for each of the 6 dimension which we use to construct the GSPI: Commercial, Culture, Digital, Education, Global Reach, and Institutions. We hence begin by constructing a sub-index for each of these dimensions of soft power. The variables included in the construction of each of the sub-indices can be found in the table with the information regarding the data sources (Table C.3 in Appendix C.2).

3.3.1 Weighting

A crucial part of any index construction is the methodology used for the weighting of the variables considered to form the index. Since there is inevitably a high degree of collinearity among some of the variables we take into account, we consider a variable weighting and aggregation technique that systematically eliminates those variables in the original set that are best explained by the remaining variables. When used in a benchmarking framework, weights can have a significant effect on the overall composite indicator and country rankings. Some weighting techniques are derived from statistical models, such as factor analysis, others from participatory methods, like analytical hierarchy process. Regardless of which method is used, weights are essentially value judgments. While some analysts might choose weights based only on statistical methods, others might reward components that are deemed more influential, depending on expert opinion, to better reflect policy priorities or theoretical factors. In constructing the GSPI, we follow the weighting methodology based on principal component analysis recommended by OECD (2008).

First proposed by Pearson (1901), principal components analysis, or simply PCA, is one of the most successful approaches to the problem of creating low dimensional data representation. The literature on principal components and classical factor models is large and well known[2]. PCA (or factor analysis) groups together individual indicators which are collinear to form a composite indicator that captures as much as possible of the information common to individual indicators. Each factor, estimated using principal component analysis, reveals the set of highest possible variation.

More formally, these factors are constructed using the eigenvectors with the largest eigenvalues of the empirical covariance matrix of the data for which we want to extract the factors (Murphy, 2012)[3]. The factor (eigenvector) associated with the highest eigenvalue will be the one explaining most of the variation in the data, the factor (eigenvector) associated with the second highest eigenvalue will be the one explaining most of the variation in the data that is orthogonal to the first factor, and so on. The eigenvalues of the empirical covariance matrix hence tell us about the variation explained by each factor while the associated eigenvectors tell us about the weights of each variable in this factor.

The idea of using PCA as a weighting mechanism is to account for the highest possible variation in the indicator set using the smallest possible number of factors. The choice of the number of factors is a crucial one when conducing PCA. Regarding this choice, we follow the approach of the Handbook on Constructing Composite Indicators (OECD, 2008) for selecting weights and choose all factors that contribute individually to at least ten percent of the overall variance.

For clarification, let's consider the PCA methodological approach for selecting index weights from OECD (2008) for a specific sub-index constructed in our study. Consider the example of the sub-index for the Education dimension. To measure this dimension

[2]For a detailed review of PCA and the literature, see Kim and Mueller (1978).
[3]For proof of and estimation procedure please refer to Murphy (2012).

of soft power, we use the following variables: Education Expenditure, Journal Articles, PISA: Maths, PISA: Reading, PISA: Science, Primary Completion, Tertiary Education, and Years of Schooling (for more details on each of these variables please refer to Table C.3 in Appendix C.2).

We begin by estimating all possible factors of these series based on the principal component analysis methodology described above. The information about all possible factors that can be extracted from the Education variables in our dataset is displayed below in Table 3.1.

Following the methodology from the Joint Research Centre of the European Commission (OECD, 2008) described above, we use all factors that have "Variance Explained" of at least 0.1 (i.e., explain at least 10 percent of the variation in the data) to construct the weights. In the case of the Education sub-index described in Table 3.1, this means using Factor 1 and Factor 2 to calculate the weights, as these are the only factors that explain more than ten percent of the total variance.

After selecting which factors will be taken to account, the next step is to use the weights from these factors to calculate the weights for each variable in the sub-index. In accordance to the guidance of OECD (2008) and following Nicoletti et al. (1999), we begin by setting all weights which are less than 0.10 in the selected factors to 0.00. After eliminating these small weights, the final weightings for the sub-index are calculated as a weighted sum of the factor weights where the explained variances are used as weights.

Using the Education sub-index as an example for the approach of Nicoletti et al. (1999), Table 3.2 provides an illustration of the steps to obtain the aforementioned sub-index weights. We begin by calculating the "Weighted Total". This is just the sum of all weights for a factor times the variance explained by that factor. The sub-index weights for each variable can then be calculated as the weighted sum of the factor weights for that variable – using the variance explained by each factor as weights – divided by the sum of the weighted totals. To clarify, consider the case of the PISA: Maths variable. The sub-index

Table 3.1 Variance Explained by Education factors and respective variable weights.

	Variance Explained	Education Expenditure	Journal Articles	PISA: Maths	PISA: Reading	PISA: Science	Primary Completion	Tertiary Education	Years of Schooling
Factor 1	0.71	0.01	0.29	0.20	0.20	0.20	0.00	0.06	0.04
Factor 2	0.11	0.12	0.28	0.13	0.10	0.14	0.00	0.18	0.05
Factor 3	0.07	0.01	0.31	0.01	0.01	0.01	0.00	0.62	0.04
Factor 4	0.05	0.50	0.02	0.00	0.02	0.01	0.01	0.01	0.43
Factor 5	0.03	0.35	0.09	0.00	0.00	0.01	0.09	0.11	0.35
Factor 6	0.02	0.02	0.01	0.00	0.00	0.00	0.89	0.00	0.08
Factor 7	0.01	0.00	0.00	0.32	0.62	0.04	0.00	0.01	0.01
Factor 8	0.00	0.00	0.00	0.34	0.05	0.61	0.00	0.00	0.00

weight can be calculated as:

$$w_{PISA:\ Maths} = \frac{(0.20 * 0.71) + (0.13 * 0.11)}{0.63 + 0.09} = 0.22 \tag{3.2}$$

From Table 3.2, we get that the final weights for the Education sub-index are: 0.02 for Education Expenditure, 0.33 for Journal Articles, 0.22 for PISA: Maths, 0.19 for PISA: Reading, 0.21 for PISA: Science, and 0.03 for Tertiary Education. The variables Primary Completion and Years of Schooling are not included in the index as their factor weights in the most relevant factors are lower than 0.10.

We follow the same methodology described above for each of the sub-indices. The complete set of weights for the Commercial, Culture, Digital, Education, Global Reach, and Institutions sub-indices can be found in Table C.5 in Appendix C.4.

3.3.2 Aggregation

Once we have obtained the weights for each of the sub-indices using the methodology described above, we use them to calculate these sub-indices and aggregate them into the overall Global Soft Power Index (GSPI). To perform this aggregation, we follow the additive aggregation method recommended by the Handbook on Constructing Composite Indicators of the Joint Research Centre of the European Commission (OECD, 2008). That is, we follow a similar approach to Fagerberg (2002) and calculate the overall index as the average values of our sub-indices. For each country i at year t, we aggregate:

$$Index_{i,t} = \frac{\sum_s SubIndex_{i,t,s}}{n_s} \tag{3.3}$$

where $s = \{\text{Commercial, Culture, Digital, Education, Global Reach, Institutions}\}$ represents all sub-indices averaged to construct the final composite Soft Power Index.

Because all variables were normalized to become standard z-scores and we only apply

Table 3.2 Variance Explained by selected Education factors with respective variable weights and final weights for the Education sub-index according to the methodology of Nicoletti et al. (1999).

	Variance Explained	Education Expenditure	Journal Articles	PISA: Maths	PISA: Reading	PISA: Science	Primary Completion	Tertiary Education	Years of Schooling
Factor 1	0.71	0.01	0.29	0.20	0.20	0.20	0.00	0.06	0.04
Factor 2	0.11	0.12	0.28	0.13	0.10	0.14	0.00	0.18	0.05
Weights		**0.02**	**0.33**	**0.22**	**0.19**	**0.21**	**0.00**	**0.03**	**0.00**

linear combinations of these variables when constructing the sub-indices, the sub-indices themselves are standardized (Greene, 2003). The fact that these sub-indices are standardized is what allows us to take a simple average to aggregate them, as they are all represented as cross-sectional z-scores. Moreover, because in this final aggregation step we are also only preforming a linear transformation of the sub-indices into the aggregated index, this final index does not require any further normalization.

3.4 Index Results

With the sub-indices and the aggregated GSPI in hand, we can finally turn to the analysis of their results. In this section we discuss how countries differ with respect to their latest sub-indices and overall GSPI reading. We use information from all sub-indices to classify countries according to their level of soft power and discuss possible applications of our proposed Global Soft Power Index (GSPI).

We are able to construct sub-indices and the GSPI for a broad sample of 72 countries on a yearly basis from 2007 to 2019. As data is released by the sources that we use to collect the series described in Table C.3 in Appendix C.2, we will update our index and release updated versions. At the time that we write this, however, 2019 is the latest year for which we have observations for most variables used to construct the GSPI. The values for the sub-indices as well as the GSPI for each country can be found in Table C.6 in appendix C.5.

To give an idea of how the values of the GSPI changed over our time frame, let's take a look into the GSPI values for China (CHN) and Italy (ITA) from 2007 to 2019. The values for both of these countries are plotted in Figure 3.1 below. We can observe that back in 2007 Italy had a significantly higher level of soft power (as measured by the GSPI) than China. However, between the years of 2007 and 2019, Chinese soft power has increased significantly while Italy's soft power has decreased. As a result, in 2019 the GSPI shows China as having a higher level of soft power than Italy.

Figure 3.1 GSPI for China and Italy from 2007 to 2019.

Given the fairly short length of the time series dimension in our panel of countries, a more interesting analysis is to focus on the cross-sectional differences at a given year. We will now focus on the overall GSPI values and sub-indices for our latest observation available; that is, the values for different countries in 2019. Figure 3.2 displays the 2019 GSPI values for all countries in our sample. By checking Figure 3.2, we can analyse the ranking of the different countries with respect to the GSPI. As expected, countries like Germany, Japan, South Korea, the United Kingdom, and the United States are among the countries with highest levels of soft power. What is perhaps a bit surprising the extremely high level of soft power for Japan and South Korea, the two countries with the highest GSPI values. In order to understand not only this but also what drives the overall differences in the GSPI between different countries we now turn to the analysis of the sub-indices used to construct the GSPI.

A better way to understand the results from our GSPI and its sub-indices is visualizing the values of the different sub-indices in a scatter-plot. However, before plotting the different sub-indices for the 72 countries, we use the information from the sub-indices themselves to group the countries in an agnostic manner.

We follow the approach of Likas et al. (2003) and group the countries in our sample according to the 2019 values for the sub-indices using K-Means Clustering. The K-Means

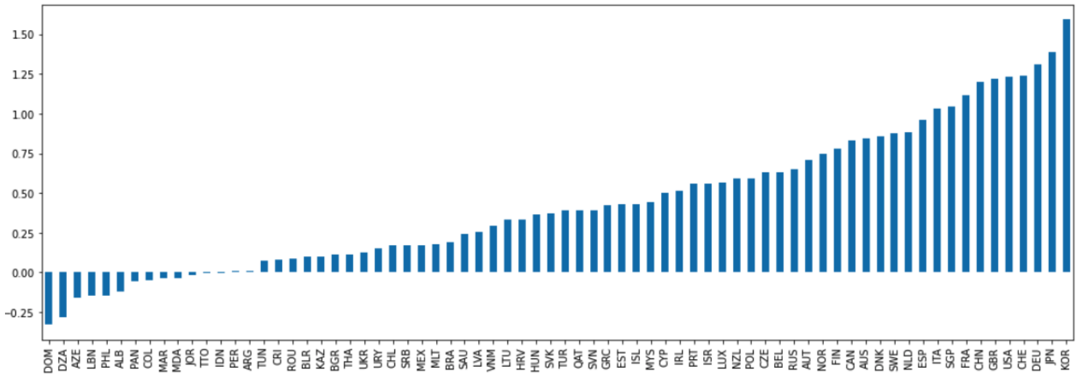

Figure 3.2 GSPI 2019 values for all countries.

algorithm is a popular data-clustering methodology. The term was first used by MacQueen et al. (1967), though the algorithm was first proposed by Stuart Lloyd in 1957 as a technique for pulse code modulation[4]. The idea is to group the observations into clusters based on the observation feature values. Given the desired number of clusters, the algorithm will start with a random allocation of each observation into these clusters and iterate until it achieves the classification that results in the least squared Euclidian distance between the observations classified in each cluster and the cluster centroids. A detailed description of the algorithm can be found in Likas et al. (2003) and Na et al. (2010).

In our framework, the objective is to group the 72 countries into clusters according to the sub-index values for each country. We follow the approach of Pham et al. (2005) to select the number of clusters according to the improvement in the sum of square distances between the cluster centroids and the feature observations and group the 72 countries into 4 groups. The groups chosen by the K-Means Clustering algorithm are the following:

- Group 1: Albania, Algeria, Argentina, Azerbaijan, Belarus, Brazil, Bulgaria, Chile, Colombia, Costa Rica, Dominican Republic, Indonesia, Jordan, Kazakhstan, Lebanon, Malaysia, Malta, Mexico, Moldova, Morocco, Panama, Peru, Philippines, Qatar, Romania, Saudi Arabia, Serbia, Thailand, Trinidad and Tobago, Tunisia, Turkey, Ukraine, Uruguay, and Vietnam.

- Group 2: Australia, Austria, Belgium, Canada, Croatia, Cyprus, Czech Repub-

[4]Only published in 1982 as Lloyd (1982).

lic, Denmark, Estonia, Finland, Greece, Hungary, Iceland, Ireland, Israel, Latvia, Lithuania, Luxembourg, Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, Slovakia, Slovenia, and Sweden.

- Group 3: China, France, Germany, Italy, Russian Federation, Spain, Switzerland, United Kingdom, and United States.
- Group 4: Japan and Republic of Korea (South Korea).

From the list of countries in each of the groups above we can already start to guess which characteristics are responsible for driving the classification. In order to see how the sub-indices differ for these groups, we plot each of the sub-indices against the GSPI. These plots can be observed in Figure 3.3 below.

Figure 3.3 helps us to understand where the differences between the four groups of countries identified by the K-Means Clustering algorithm come from. Group 1 is the set of countries with low soft power. These countries have, on average, lower levels for each of the sub-indices but are considerably behind others in the Education and Institutions dimensions. Group 2 represents the countries with medium level of soft power. These are mostly developed nations that present high levels of Education and Institutions, but do not have much of an impact on other countries via the dimensions measured by the Culture and Global Reach sub indices. Finally, Groups 3 and 4 are the countries with high levels of soft power. By looking at Figure 3.3, we can see that the only reason why these are considered as different groups is because Japan and South Korea (the only countries in Group 4) have significantly higher Commercial Prowess and perform meaningfully worse in the Culture dimension in comparison to other high soft power countries.

Taking into account both the set of countries in each of the groups and the information displayed in Figure 3.3, a superficial description of the identified groups would be that Group 1 represents the set of developing countries with overall low soft power, Group 2 represents the developed countries with overall medium soft power, and finally, Groups 3 and 4 represent the countries with high soft power (mostly developed nations but also

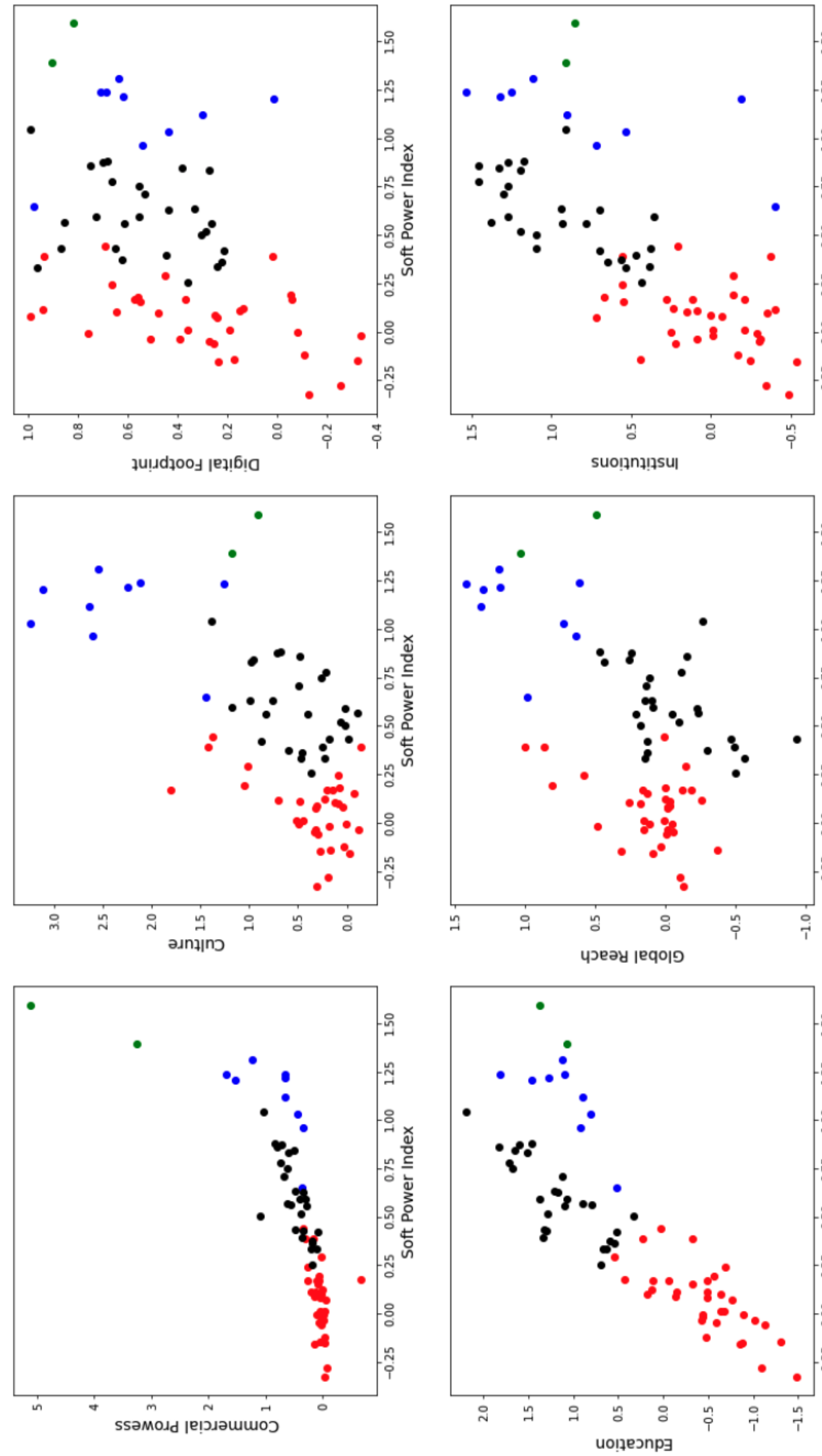

Figure 3.3 Sub-indices plotted against the Global Soft Power Index (GSPI) with countries split according to K-Means Clustering classification: Group 1 (red), Group 2 (black), Group 3 (blue), and Group 4 (green).

with "soft-powerful" developing nations such as China and Russia).

The information presented here regarding both the values of the sub-indices and the GSPI confirm what one would expect regarding the level of soft power for these 72 nations. However, the merit of the the proposed GSPI is that we have arrived at the information presented in this section in a purely systematic manner. In doing so, we propose a measure of soft power and its dimensions that can be used to formally evaluate different countries and study the relationship between soft power and its dimensions and many variables of interest. In the next section, we consider one of such possible applications of the GSPI and its composing sub-indices to study the relationship between soft power and real exchange rate volatility.

3.5 Soft Power and Real Exchange Rate Volatility

Since the breakdown of the Bretton Woods system, major shifts in the global economy and financial markets have exacerbated the magnitude of exchange rate fluctuations. While Friedman (1953) famously argued that exchange rate volatility is a manifestation of macroeconomic volatility, empirical studies have uncovered a range of anomalies and puzzles that contradict the theoretical models of exchange rates. Meese and Rogoff (1983), for example, showed that there is no stable relationship between exchange rate fluctuations and fundamental factors, conflicting with the theoretical models predicting that exchange rate volatility can only increase when the variability of the underlying fundamentals increases. Exchange rate volatility is still of great interest to academics, policymakers, and market practitioners because of the potential linkages between the behaviour of exchange rates and other economic and financial variables. The general consensus in the literature[5] is that exchange rate volatility reflects a variety of global and country-specific factors, such as income growth, inflation, fiscal and current account balances,

[5]Contributions include Edwards (1987), Côté (1994), Hausmann and Gavin (1996), McKenzie (1999), Hau (2000), Hau (2002), Clark et al. (2004), Hausmann et al. (2006), and Morales-Zumaquero and Sosvilla-Rivero (2010).

foreign exchange reserves, financial and trade openness, and the size and type of capital flows.

In recent research, Cevik et al. (2017) find evidence of a possible link between soft power variables – that encapsulate a country's demographic, institutional, political, and social underpinnings that are generally ignored in the literature – and the volatility of REER when considering a panel of developed and emerging market economies. The objective of this section is hence to use our proposed GSPI and its sub-indices to formally evaluate this relationship and map which dimensions from soft power are more useful to explain real exchange rate volatility.

Using the REER series from the IMF's International Financial Statistics database and following the methodology used in Cevik et al. (2017), we calculate the annual REER volatility as the realised volatility[6] of the log returns of the REER sampled at monthly frequencies. That is, we estimate each country's annual REER volatility as:

$$VOL_{i,y} = \sum_{m=1}^{12} r_{i,m-y}^2 \tag{3.4}$$

where $r_{i,m-y} = log(REER_{i,m-y}) - log(REER_{i,(m-1)-y})$ represents the monthly log returns of the real effective exchange rate for country i on month m of year y.

Focusing only on the countries that allow their exchange rates to fluctuate – that is, countries that have floating exchange rate regimes according to the IMF (2014) Monetary Policy Framework – we proceed to plot the GSPI against REER volatility (as defined in Equation (3.4)). We consider all years for these countries since 2007, the first observation that we have for our soft power index. This plot is presented in Figure 3.4 below. For this plot, we consider Groups 3 and 4 as a single high soft power group as Group 4 only contains Japan and South Korea.

[6]Realised volatility estimates volatility as the sum of sample variances over some period. For full details please refer to Andersen and Bollerslev (1998).

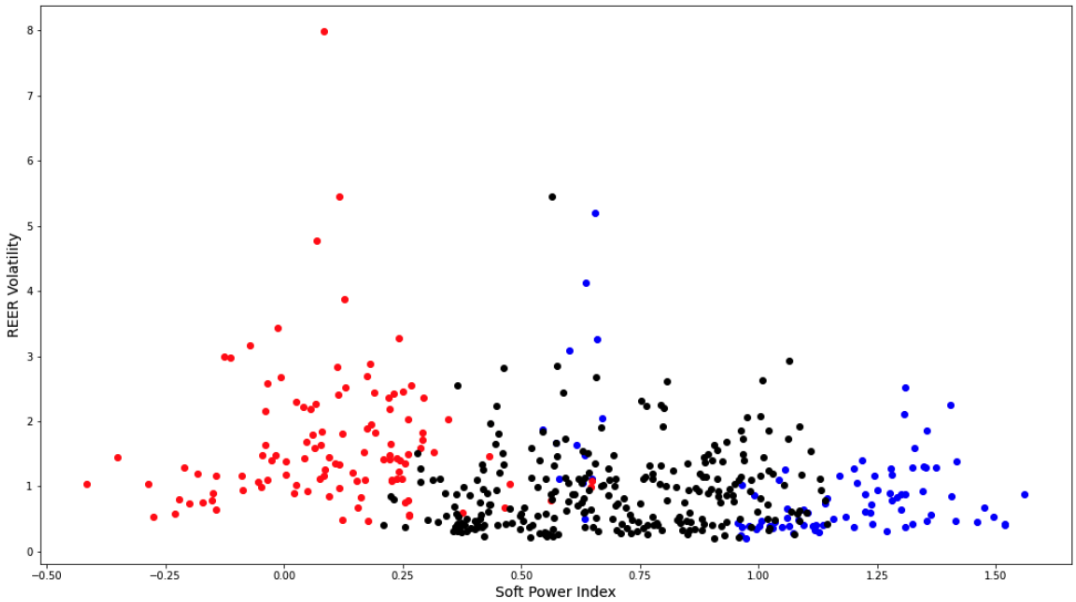

Figure 3.4 Real Effective Exchange Rate (REER) volatility and Global Soft Power Index (GSPI) for low soft power countries (red), medium soft power countries (black), and high soft power countries (blue).

Although this is just simple a scatter-plot representation of REER volatility against our GSPI, one can observe the negative relationship between the two variables. This point is further emphasized by checking the means and medians for each variable in each group presented below in Table 3.3. Taken together, the results from Figure 3.4 and Table 3.3 seem to suggest that low soft power countries present a dynamic between these two variables that is different from that for medium and high soft power countries. This may just be due to the fact that low soft power countries are mostly developing economies whereas medium and high soft power countries are mostly developed economies.

Table 3.3 Means and medians REER volatility and Global Soft Power Index (GSPI) for different country groups.

Variable	Group	Mean	Median
REER Volatility	Low Soft Power	1.66	1.42
REER Volatility	Medium Soft Power	0.97	0.80
REER Volatility	High Soft Power	0.93	0.72
Soft Power Index	Low Soft Power	0.10	0.12
Soft Power Index	Medium Soft Power	0.69	0.69
Soft Power Index	High Soft Power	1.12	1.16

3.5.1 A model for soft power and real exchange rate volatility

In order to control for other possible covariates that might influence real exchange rate volatilities, we follow Cevik et al. (2017) and study the relationship between our proposed indices and real exchange rate volatility in a panel framework. The authors argue that soft power variables are more likely to have an impact on exchange rate volatility in the cross-section rather than in the time series and thus a panel regression should be used in an attempt to uncover this relationship.

In order to capture the fundamental macroeconomic drivers of exchange rate volatility, we include the same nine control variables from Cevik et al. (2017) drawn from the literature on exchange rate modelling. The control variables are, in alphabetical order, credit, current account, export concentration, inflation, stock market capitalization, trade openness, volatility of government consumption, volatility of labour productivity growth, and volatility of terms of trade. More information about the sources used and any transformations to these series can be found in Table C.4 in Appendix C.3.

Moreover, exchange rate volatilities present an auto-regressive behavior (Rapach and Strauss, 2008). In order to control for this, we continue to follow Cevik et al. (2017) and also include the lags of $VOL_{i,t}$ in our panel framework.

In their study, Cevik et al. (2017) consider a standard entity fixed effects panel model in order to allow for country specific heterogeneity. However, because the time horizon of our analysis (from 2007 to 2019 on a yearly basis) is significantly shorter than the one considered in Cevik et al. (2017), we also allow for time specific heterogeneity by including yearly time dummies. That is, we consider a model with both time and entity fixed effects. Common shocks are a key driver of the volatilities of real exchange rates (Campos-Martins and Padilha, 2021) and such a framework allows to account for this feature.

We begin by analysing the relationship between real exchange rate volatility and the overall GSPI. Following the framework described in the previous paragraphs, we consider the following specification:

$$VOL_{i,t} = \mu_i + \gamma_t + \sum_{k=1}^{K} \alpha_k VOL_{i,t-k} + \delta' \mathbf{Z}_{i,t} + \beta GSPI_{i,t} + \epsilon_{i,t} \tag{3.5}$$

where $VOL_{i,t}$ is the volatility of REER as described above, $\mathbf{Z}_{i,t}$ is the $9X1$ vector with the control variables, and $GSPI_{i,t}$ is the Global Soft Power Index (GSPI) for country i at year t. The lag length K is selected via Bayesian Information Criterion (BIC).

We also consider the expanded specification where, rather than using just the composite GSPI, we include the sub-indices themselves. This expanded model takes the following form:

$$VOL_{i,t} = \mu_i + \gamma_t + \sum_{k=1}^{K} \alpha_k VOL_{i,t-k} + \delta' \mathbf{Z}_{i,t} + \beta' \mathbf{X}_{i,t} + \epsilon_{i,t} \tag{3.6}$$

where we have replaced the $GSPI_{i,t}$ scalar from Model (3.5) by the $6X1$ vector $\mathbf{X}_{i,t}$ representing each of the sub-indices that compose the $GSPI_{i,t}$ and the scalar parameter β by the $6X1$ vector of coefficients β.

Both Model (3.5) and Model (3.6) are then estimated via the fixed effects estimator. Moreover, BIC selects lag length (K) equal to 2 for the two models. The results from these estimations can be found in Table 3.4 below. In this table, we have omitted both entity and time fixed effects for presentation.

Many insights can be derived by analysing the results from Table 3.4. First, focusing on the results from Model (3.5), we can see that the most meaningful variables in explaining real exchange rate volatility are inflation, the volatility of labour productivity growth, and the volatility of the terms of trade index. Trade openness is also relevant to a lesser extent. However, the main result for our analysis is that the overall GSPI does not seem to be significant in explaining real exchange rate volatility. Although the coefficient is in the the direction one would expect, with higher index levels leading to lower real exchange rate volatility, its p-value is fairly high (0.22), indicating that GSPI is not significant at any relevant significance level.

Table 3.4 Estimated coefficients and p-values from the fixed effects estimator for Model (3.5) and Model (3.6).

	Model (3.5)	**Model** (3.6)
α_1	0.1120	0.0989
$(VOL_{i,t-1})$	(0.01)**	(0.02)*
α_2	-0.1989	-0.2009
$(VOL_{i,t-2})$	(0.00)***	(0.00)***
δ_1	0.0110	0.0076
(Current Account)	(0.17)	(0.36)
δ_2	-0.5394	-0.6603
(Export Concentration)	(0.40)	(0.32)
δ_3	-0.0017	-0.0015
(Credit)	(0.24)	(0.32)
δ_4	0.0598	0.0497
(Gov. Cons. Vol.)	(0.46)	(0.54)
δ_5	0.1109	0.1098
(Inflation)	(0.00)***	(0.00)***
δ_6	-0.0964	-0.1043
(Labour Prod. Growth Vol.)	(0.00)***	(0.00)***
δ_7	-0.0016	-0.0010
(Stock Market Cap.)	(0.18)	(0.45)
δ_8	-0.1496	-0.1433
(Terms of Trade Vol.)	(0.00)***	(0.00)***
δ_9	0.0042	0.0057
(Trade Openness)	(0.05)*	(0.01)**
β	-0.3918	
(GSPI)	(0.22)	
β_1		0.0153
(Commercial)		(0.86)
β_2		-0.5116
(Culture)		(0.04)**
β_3		0.0276
(Digital)		(0.82)
β_4		-0.0884
(Education)		(0.72)
β_5		-0.9937
(Global Reach)		(0.01)**
β_6		-0.1093
(Institutions)		(0.33)

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

In Model (3.6) – where we incorporate information about the sub-indices rather than using the composite GSPI index – the parameters and significance levels for the control variables remain very similar to those of Model (3.5). The main difference is that now, by considering each dimension of soft power as a separate variable, some of them are shown to be significant. The results presented in Table 3.4 suggest that both the Culture and the Global Reach dimensions of soft power are significant in explaining real exchange rate volatility even when we control for fundamental macroeconomic drivers. As expected, both coefficients are negative indicating that the higher the level of these sub-indices the lower the level of real exchange rate volatility. Furthermore, Global Reach, the "purest" measure international soft power reach[7], seems to be the most significant of the two sub-indices according to the results from Table 3.4.

On the whole, the results presented in Table 3.4 are in line with the findings from Cevik et al. (2017) where some soft power variables are shown to meaningfully explain real exchange rate volatility. We consider both a model with our headline GSPI and a model with each individual soft power sub-index. Although the composite GSPI doesn't seem to be significant in explaining real exchange rate volatility, the Culture and Global Reach dimensions of soft power are shown to be relevant at almost all significance levels. Moreover, the results also suggest that Global Reach is the dimension of soft power with the most impact on real exchange rate volatilities.

3.6 Concluding Remarks

Soft power – defined as the ability to obtain preferred outcomes by attraction rather than coercion or payment – has grown in importance in an increasingly interconnected world. In this paper, we develop a new composite index of soft power based on 29 indicators covering a wide spectrum of variables for a broad sample of 72 countries across the world over the period 2007-2019. The Global Soft Power Index (GSPI) is constructed using a

[7] See Table C.3 in Appendix C.2 for information about variables in each sub-index.

three-step approach to reduce multidimensional data into a single composite index: (i) normalization of variables; (ii) aggregation of normalized variables into the sub-indices representing a particular functional dimension; and (iii) aggregation of the sub-indices into the final index.

Additionally to presenting the index methodology, we discuss how countries differ with respect to their latest sub-indices and overall GSPI reading. By using the latest information about the sub-indices, we are able to identify four group of countries. The first one is the group of low soft power countries with mostly developing nations that are considerably behind others in the Education and Institutions dimensions. The second group is the group of medium soft power countries. These are mostly developed nations that present high levels of Education and Institutions, but do not have much of an impact on other countries via the dimensions measured by the Culture and Global Reach sub-indices. The third group is the group of high soft power countries. Japan and South Korea are their own group. They are similar to other high soft power countries but have significantly higher Commercial Prowess and perform meaningfully worse in the Culture dimension.

To assess the GSPI's macro-financial relevance, we look at the effect of soft power on real exchange rate volatility as discussed in Cevik et al. (2017). According to our analysis, low soft power countries present a dynamic between these two variables that is significantly different from that for medium and high soft power countries. In order to control for other possible covariates that might influence real exchange rate volatilities, we study the relationship between our proposed indices and real exchange rate volatility in a panel framework that also includes fundamental macroeconomic drivers drawn from the literature on exchange rate modelling. Our results are in line with the findings from Cevik et al. (2017). Although the composite GSPI doesn't seem to be significant in explaining real exchange rate volatility, the Culture and Global Reach dimensions of soft power are shown to be relevant at almost all significance levels, with Global Reach being the sub-index of soft power with the most impact. This is an interesting result because these are precisely the dimensions that are responsible for most of the differentiation between medium and high soft power countries.

Overall, our proposed composite Global Soft Power Index (GSPI) and its sub-indices present a systematic approach to measure soft power and its dimensions. They capture the matrix of soft power characteristics and offer significant advantages as they allow one to compare the level of soft power for different countries in both the cross-section and time series dimensions. The application presented in Section 3.5 is only one of the many possible use cases of the GSPI. We hope that our proposed framework for measuring and evaluating soft power contributes to the growing literature on the study of this important dimension of power and its influence on political and macroeconomic developments.

Bibliography

Abuaf, N., Jorion, P., 1990. Purchasing power parity in the long run. The Journal of Finance 45, 157–174.

Aliber, R.Z., 1973. The interest rate parity theorem: A reinterpretation. Journal of political economy 81, 1451–1459.

Andersen, T.G., Bollerslev, T., 1998. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. International economic review , 885–905.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P., 2000. Exchange rate returns standardized by realized volatility are (nearly) gaussian. NBER Working Paper .

Asness, C.S., Moskowitz, T.J., Pedersen, L.H., 2013. Value and momentum everywhere. The Journal of Finance 68, 929–985.

Balassa, B., 1964. The purchasing-power parity doctrine: a reappraisal. Journal of political Economy 72, 584–596.

Bansal, R., 1997. An exploration of the forward premium puzzle in currency markets. The Review of Financial Studies 10, 369–403.

Bansal, R., Dahlquist, M., 2000. The forward premium puzzle: different tales from developed and emerging economies. Journal of international Economics 51, 115–144.

Bekaert, G., Wu, G., 2000. Asymmetric volatility and risk in equity markets. The review of financial studies 13, 1–42.

Bergin, P.R., Glick, R., Wu, J.L., 2014. Mussa redux and conditional ppp. Journal of Monetary Economics 68, 101–114.

BIS, 2019. Triennial Central Bank Survey. Technical Report. Bank for International Settlements.

Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. Journal of econometrics 31, 307–327.

Bollerslev, T., Engle, R.F., Wooldridge, J.M., 1988. A capital asset pricing model with time-varying covariances. Journal of political Economy 96, 116–131.

Bollerslev, T., et al., 1990. Modelling the coherence in short-run nominal exchange rates: a multivariate generalized arch model. Review of Economics and statistics 72, 498–505.

Burnside, C., Eichenbaum, M., Rebelo, S., 2009. Understanding the forward premium puzzle: A microstructure approach. American Economic Journal: Macroeconomics 1, 127–54.

Campbell, J.Y., Hentschel, L., 1992. No news is good news: An asymmetric model of changing volatility in stock returns. Journal of financial Economics 31, 281–318.

Campos-Martins, S., Padilha, T., 2021. Global volatility shocks and the ppp puzzle .

Cappiello, L., Engle, R.F., Sheppard, K., 2006. Asymmetric dynamics in the correlations of global equity and bond returns. Journal of Financial econometrics 4, 537–572.

Carvalho, C., Nechio, F., 2011. Aggregation and the ppp puzzle in a sticky-price model. American Economic Review 101, 2391–2424.

Cassel, G., 1921. Worlds monetary problems: Two memoranda. Constable and Company Limited, London.

Cassel, G., 1922. Money and foreign exchange after 1914. Constable and Company Limited, London.

Ca'Zorzi, M., Cap, A., Mijakovic, A., Rubaszek, M., 2020. The predictive power of equilibrium exchange rate models .

CBOE, C.B.O.E., 2009. The cboe volatility index–vix. White paper .

Cevik, S., Harris, R.D., Yilmaz, F., 2017. Soft power and exchange rate volatility. International Finance 20, 271–288.

Chen, H., Chen, S., Chen, Z., Li, F., 2017. Empirical investigation of an equity pairs trading strategy. Management Science .

Cheung, Y.W., Lai, K.S., 2000. On the purchasing power parity puzzle. Journal of international Economics 52, 321–330.

Cheung, Y.W., Lai, K.S., Bergman, M., 2004. Dissecting the ppp puzzle: the unconventional roles of nominal exchange rate and price adjustments. Journal of International Economics 64, 135–150.

Chow, G.C., 1960. Tests of equality between sets of coefficients in two linear regressions. Econometrica: Journal of the Econometric Society , 591–605.

Clarida, R., Gali, J., 1994. Sources of real exchange-rate fluctuations: How important are nominal shocks?, in: Carnegie-Rochester conference series on public policy, Elsevier. pp. 1–56.

Clark, P.B., Tamirisa, N., Wei, S.J., Sadikov, A., Zeng, L., 2004. A new look at exchange rate volatility and trade flows. volume 235. International Monetary Fund Washington, DC.

Côté, A., 1994. Exchange rate volatility and trade: A survey. Technical Report. Bank of Canada.

Deaves, R., Liu, J., Miu, P., 2013. Pairs trading in canadian markets: Pay attention to inattention. Can. Invest. Rev .

Diebold, F.X., Husted, S., Rush, M., 1991. Real exchange rates under the gold standard. Journal of Political Economy 99, 1252–1271.

Ding, Z., Engle, R.F., 2001. Large scale conditional covariance matrix modeling, estimation and testing .

Do, B., Faff, R., 2010. Does simple pairs trading still work? Financial Analysts Journal 66, 83–95.

Do, B., Faff, R., 2012. Are pairs trading profits robust to trading costs? Journal of Financial Research 35, 261–287.

Dornbusch, R., 1976. Expectations and exchange rate dynamics. Journal of political Economy 84, 1161–1176.

Dutta, S., Lanvin, B., 2012. The global innovation index 2012. Stronger innovation linkages for global .

Ebert, U., Welsch, H., 2004. Meaningful environmental indices: a social choice approach. Journal of Environmental Economics and Management 47, 270–283.

Edwards, S., 1987. Real exchange rate variability: An empirical analysis of the developing countries case. International Economic Journal 1, 91–106.

Elliott, R.J., Van Der Hoek*, J., Malcolm, W.P., 2005. Pairs trading. Quantitative Finance 5, 271–276.

Engel, C., Mark, N.C., West, K.D., 2015. Factor model forecasts of exchange rates. Econometric Reviews 34, 32–55.

Engel, C., Morley, J.C., 2001. The adjustment of prices and the adjustment of the exchange rate. Technical Report. National Bureau of Economic Research.

Engelberg, J., Gao, P., Jagannathan, R., 2008. An anatomy of pairs trading: the role of idiosyncratic news, common information and liquidity .

Engle, R.F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. Econometrica: Journal of the Econometric Society , 987–1007.

Engle, R.F., Campos-Martins, S., 2020. Measuring and hedging geopolitical risk .

Engle, R.F., Granger, C.W., 1987. Co-integration and error correction: representation, estimation, and testing. Econometrica: journal of the Econometric Society , 251–276.

Engle, R.F., Kroner, K.F., 1995. Multivariate simultaneous generalized arch. Econometric theory 11, 122–150.

Engle, R.F., Ng, V.K., 1993. Measuring and testing the impact of news on volatility. The journal of finance 48, 1749–1778.

Engle, R.F., Sheppard, K., 2001. Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH. Technical Report. National Bureau of Economic Research.

Fagerberg, J., 2002. Europe at the crossroads: the challenge from innovation-based growth. The globalizing learning economy , 45–60.

Fama, E.F., 1984. Forward and spot exchange rates. Journal of monetary economics 14, 319–338.

Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. Journal of financial economics 33, 3–56.

Friedman, M., 1953. The case for flexible exchange rates. Essays in positive economics 157, 203.

Froot, K.A., Rogoff, K., 1995. Perspectives on ppp and long-run real exchange rates. Handbook of international economics 3, 1647–1688.

Ganguly, S., Breuer, J.B., 2010. Nominal exchange rate volatility, relative price volatility, and the real exchange rate. Journal of International Money and Finance 29, 840–856.

Gatev, E., Goetzmann, W.N., Rouwenhorst, K.G., 2006. Pairs trading: Performance of a relative-value arbitrage rule. The Review of Financial Studies 19, 797–827.

Glahe, F.R., 1967. An empirical study of the foreign-exchange market: test of a theory. 20, International Finance Section, Department of Economics, Princeton University.

Glosten, L.R., Jagannathan, R., Runkle, D.E., 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. The journal of finance 48, 1779–1801.

Greene, W.H., 2003. Econometric analysis. Pearson Education India.

Hastie, T., Tibshirani, R., 2009. & friedman, j.(2008). the elements of statistical learning; data mining, inference and prediction.

Hau, H., 2000. Exchange rate determination: The role of factor price rigidities and non-tradeables. Journal of International Economics 50, 421–447.

Hau, H., 2002. Real exchange rate volatility and economic openness: theory and evidence. Journal of money, Credit and Banking , 611–630.

Hausmann, R., Gavin, M., 1996. Securing stability and growth in a shock prone region: the policy challenge for latin america .

Hausmann, R., Panizza, U., Rigobon, R., 2006. The long-run volatility puzzle of the real exchange rate. Journal of International Money and Finance 25, 93–124.

Hsu, A., Zomer, A., 2014. Environmental performance index. Wiley StatsRef: Statistics Reference Online , 1–5.

IMF, 2014. Monetary policy framework. Annual report on exchange arrangements and exchange restrictions .

Jacobs, H., Weber, M., 2015. On the determinants of pairs trading profitability. Journal of financial markets 23, 75–97.

Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. The Journal of finance 48, 65–91.

Van de Kerk, G., Manuel, A.R., 2008. A comprehensive index for a sustainable society: The ssi—the sustainable society index. Ecological Economics 66, 228–242.

Kim, J.O., Mueller, C.W., 1978. Factor analysis: Statistical methods and practical issues. volume 14. sage.

Kroner, K.F., Ng, V.K., 1998. Modeling asymmetric comovements of asset returns. The review of financial studies 11, 817–844.

Likas, A., Vlassis, N., Verbeek, J.J., 2003. The global k-means clustering algorithm. Pattern recognition 36, 451–461.

Lloyd, S., 1982. Least squares quantization in pcm. IEEE transactions on information theory 28, 129–137.

Lothian, J.R., 1990. A century plus of yen exchange rate behavior. Japan and the world economy 2, 47–70.

Lothian, J.R., Taylor, M.P., 1996. Real exchange rate behavior: the recent float from the perspective of the past two centuries. Journal of political economy 104, 488–509.

MacQueen, J., et al., 1967. Some methods for classification and analysis of multivariate observations, in: Proceedings of the fifth Berkeley symposium on mathematical statistics and probability, Oakland, CA, USA. pp. 281–297.

McClory, J., 2015. The Soft Power 30: A global ranking of soft power. Portland.

McClory, J., Harvey, O., 2016. The soft power 30: getting to grips with the measurement challenge. Global Affairs 2, 309–319.

McCloskey, D.N., Zecher, J.R., 1984. The success of purchasing-power parity: historical evidence and its implications for macroeconomics, in: A Retrospective on the Classical Gold Standard, 1821-1931. University of Chicago Press, pp. 121–172.

McKenzie, M.D., 1999. The impact of exchange rate volatility on international trade flows. Journal of economic Surveys 13, 71–106.

Meese, R.A., Rogoff, K., 1983. Empirical exchange rate models of the seventies: Do they fit out of sample? Journal of international economics 14, 3–24.

Morales-Zumaquero, A., Sosvilla-Rivero, S., 2010. Structural breaks in volatility: Evidence for the oecd and non-oecd real exchange rates. Journal of International Money and Finance 29, 139–168.

MSCI, 2020. MSCI 2020 Market Classification Review. Technical Report. MSCI.

Murphy, K.P., 2012. Machine learning: a probabilistic perspective. MIT press.

Mussa, M., 1986. Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications, in: Carnegie-Rochester Conference series on public policy, Elsevier. pp. 117–214.

Na, S., Xumin, L., Yong, G., 2010. Research on k-means clustering algorithm: An improved k-means clustering algorithm, in: 2010 Third International Symposium on intelligent information technology and security informatics, Ieee. pp. 63–67.

Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: A new approach. Econometrica: Journal of the Econometric Society , 347–370.

Nicoletti, G., Scarpetta, S., Boylaud, O., 1999. Summary indicators of product market regulation with an extension to employment protection legislation .

Nye, J.S., 1990. Soft power. Foreign policy , 153–171.

OECD, 2008. Handbook on constructing composite indicators: methodology and user guide. OECD publishing.

Papadakis, G., Wysocki, P., 2007. Pairs trading and accounting information. Boston university and mit working paper .

Pearson, K., 1901. On lines and planes of closest fit to systems of points in space. The London, Edinburgh, and Dublin philosophical magazine and journal of science 2, 559–572.

Pesaran, M.H., 2006. Estimation and inference in large heterogeneous panels with a multifactor error structure. Econometrica 74, 967–1012.

Pesaran, M.H., Smith, R., 1995. Estimating long-run relationships from dynamic heterogeneous panels. Journal of econometrics 68, 79–113.

Pham, D.T., Dimov, S.S., Nguyen, C.D., 2005. Selection of k in k-means clustering. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science 219, 103–119.

Rad, H., Low, R.K.Y., Faff, R., 2016. The profitability of pairs trading strategies: distance, cointegration and copula methods. Quantitative Finance 16, 1541–1558.

Rapach, D.E., Strauss, J.K., 2008. Structural breaks and garch models of exchange rate volatility. Journal of Applied Econometrics 23, 65–90.

Rogers, J.H., 1999. Monetary shocks and real exchange rates. Journal of International Economics 49, 269–288.

Rogoff, K., 1996. The purchasing power parity puzzle. Journal of Economic literature 34, 647–668.

Samuelson, P.A., 1964. Theoretical notes on trade problems. The review of economics and statistics , 145–154.

Savor, P., Wilson, M., 2013. How much do investors care about macroeconomic risk? evidence from scheduled economic announcements. Journal of Financial and Quantitative Analysis 48, 343–375.

Schwarz, G., et al., 1978. Estimating the dimension of a model. The annals of statistics 6, 461–464.

Sen, A., Fukuda-Parr, S., et al., 2003. Development as capability expansion. Readings in human development .

Stein, J.L., 1962. The nature and efficiency of the foreign exchange market. 40, Princeton, NJ: International Finance Section, Department of Economics ....

Taylor, A.M., 2002. A century of purchasing-power parity. Review of economics and statistics 84, 139–150.

Taylor, A.M., Taylor, M.P., 2004. The purchasing power parity debate. Journal of economic perspectives 18, 135–158.

Treverton, G.F., Jones, S.G., 2005. Measuring national power. Technical Report. Rand corp Arlington VA national security research div.

UNDP, 1990. Human Development Report 1990: Concept and Measurement of Human Development. New York: United National Development Program.

UNDP, 2001. Human Development Report 2001: Making New Technologies Work for Human Development. New York: United National Development Program.

Vidyamurthy, G., 2004. Pairs Trading: quantitative methods and analysis. volume 217. John Wiley & Sons.

Wu, J.L., Chen, S.L., 1998. A re-examination of real interest rate parity. Canadian Journal of Economics , 837–851.

Zakoian, J.M., 1994. Threshold heteroskedastic models. Journal of Economic Dynamics and control 18, 931–955.

A | Appendix to Chapter 1

A.1 Derivation of Pairs Variance

From Equation (1.17) we know that the return of a pair is given by:

$$r_t^p = w_{l,t} r_{l,t} - w_{s,t} r_{s,t}$$

We can then use the decomposition of stock returns into market and idiosyncratic components given by Equation (1.16) to replace $r_{l,t}$ and $r_{s,t}$ above. This yields:

$$\begin{aligned} r_{p,t} &= w_{l,t}(\beta_l r_{m,t} + \tilde{r}_{l,t}) - w_{s,t}(\beta_s r_{m,t} + \tilde{r}_{l,t}) \\ &= (w_{l,t}\beta_l - w_{s,t}\beta_s) r_{m,t} + (w_{l,t}\tilde{r}_{l,t} - w_{s,t}\tilde{r}_{l,t}) \end{aligned} \tag{A.1}$$

Which decomposes the returns of the pair into its market ($(w_{l,t}\beta_l - w_{s,t}\beta_s) r_{m,t}$) and idiosyncratic ($w_{l,t}\tilde{r}_{l,t} + w_{s,t}\tilde{r}_{l,t}$) terms.

We can now turn to the analysis of the conditional variance. From Equation (A.1), we have that the expression for the variance of $r_{p,t}$ is given by:

$$
\begin{aligned}
VAR_{t-1}[r_{p,t}] &= VAR_{t-1}[(w_{l,t}\beta_l - w_{s,t}\beta_s)r_{m,t} + (w_{l,t}\tilde{r}_{l,t} - w_{s,t}\tilde{r}_{s,t})] \\
&= VAR_{t-1}[(w_{l,t}\beta_l - w_{s,t}\beta_s)r_{m,t}] + VAR_{t-1}[(w_{l,t}\tilde{r}_{l,t} - w_{s,t}\tilde{r}_{s,t})] \\
&+ 2COV_{t-1}[(w_{l,t}\beta_l - w_{s,t}\beta_s)r_{m,t}, (w_{l,t}\tilde{r}_{l,t} - w_{s,t}\tilde{r}_{s,t})] \\
&= VAR_{t-1}[(w_{l,t}\beta_l - w_{s,t}\beta_s)r_{m,t}] + VAR_{t-1}[(w_{l,t}\tilde{r}_{l,t} - w_{s,t}\tilde{r}_{s,t})] \\
&+ 2(w_{l,t}\beta_l - w_{s,t}\beta_s)COV_{t-1}[r_{m,t}, (w_{l,t}\tilde{r}_{l,t} - w_{s,t}\tilde{r}_{s,t})] \\
&= VAR_{t-1}[(w_{l,t}\beta_l - w_{s,t}\beta_s)r_{m,t}] + VAR_{t-1}[(w_{l,t}\tilde{r}_{l,t} - w_{s,t}\tilde{r}_{s,t})] \\
&+ 2(w_{l,t}\beta_l - w_{s,t}\beta_s)(0) \\
&= VAR_{t-1}[(w_{l,t}\beta_l - w_{s,t}\beta_s)r_{m,t}] + VAR_{t-1}[(w_{l,t}\tilde{r}_{l,t} - w_{s,t}\tilde{r}_{s,t})] \\
&= (w_{l,t}\beta_l - w_{s,t}\beta_s)^2 VAR_{t-1}[r_{m,t}] + w_{l,t}^2 VAR_{t-1}[\tilde{r}_{l,t}] \\
&+ w_{s,t}^2 VAR_{t-1}[\tilde{r}_{s,t}] - 2w_{l,t}w_{s,t}COV_{t-1}[\tilde{r}_{l,t}, \tilde{r}_{s,t}]
\end{aligned}
\tag{A.2}
$$

where we have used the fact that $w_{l,t}$ and $w_{s,t}$ are conditionally non-stochastic at $t-1$ and that, by definition, the idiosyncratic returns are orthogonal to market returns and therefore $COV[r_{m,t}, (\tilde{r}_{l,t} + \tilde{r}_{l,t})] = 0$.

The last line of Equation (A.2) gives the expression that will be used for the decomposition of the variance of each of the pairs being traded in our analysis.

A.2 Other Conditional Second Moments Models

A.2.1 GJR-GARCH Model

We estimate a GJR-GARCH(P,O,Q) model of Glosten et al. (1993) with the following model specification:

$$
\begin{aligned}
r_t &= \epsilon_t \\
\epsilon_t &= \sigma_t e_t \\
\sigma_t^2 &= \omega + \sum_{p=1}^{P} \alpha_p \epsilon_{t-p}^2 + \sum_{o=1}^{O} \gamma_o \epsilon_{t-o}^2 1_{[\epsilon_{t-o}<0]} + \sum_{q=1}^{Q} \beta_q \sigma_{t-q}^2 \\
e_t &\overset{i.i.d.}{\sim} N(0,1)
\end{aligned}
$$

A.2.2 The AG-DCC Multivariate GARCH Model

We begin by defining:

$$\begin{aligned}
\boldsymbol{r}_t &= \boldsymbol{\epsilon}_t \\
\boldsymbol{\epsilon}_t &= \boldsymbol{\Sigma}_t^{1/2} \boldsymbol{e}_t \\
\boldsymbol{e}_t &\overset{i.i.d.}{\sim} N(\boldsymbol{0}, \boldsymbol{I}_2)
\end{aligned}$$

And then we model $\boldsymbol{\Sigma}_t$ according to the AG-DCC GARCH(M,L,N) specification of Cappiello et al. (2006):

$$\begin{aligned}
\boldsymbol{\Sigma}_t &= \boldsymbol{D}_t \boldsymbol{P}_t \boldsymbol{D}_t \\
\boldsymbol{P}_t &= \boldsymbol{Q}_t^* \boldsymbol{Q}_t \boldsymbol{Q}_t^* \\
\boldsymbol{Q}_t &= (\bar{\boldsymbol{P}} - \sum_{m=1}^{M} \boldsymbol{A}'_m \bar{\boldsymbol{P}} \boldsymbol{A}_m - \sum_{l=1}^{L} \boldsymbol{G}'_l \bar{\boldsymbol{N}} \boldsymbol{G}_l - \sum_{n=1}^{N} \boldsymbol{B}'_n \bar{\boldsymbol{P}} \boldsymbol{B}_n) + \sum_{m=1}^{M} \boldsymbol{A}'_m \boldsymbol{e}_{t-m} \boldsymbol{e}'_{t-m} \boldsymbol{A}_m \\
&\quad + \sum_{l=1}^{L} \boldsymbol{G}'_l \boldsymbol{n}_{t-l} \boldsymbol{n}'_{t-l} \boldsymbol{G}_l + \sum_{n=1}^{N} \boldsymbol{B}'_n \boldsymbol{Q}_{t-n} \boldsymbol{B}_n \\
\boldsymbol{Q}_t^* &= (\boldsymbol{Q}_t \odot \boldsymbol{I}_2)^{\frac{1}{2}}
\end{aligned}$$

Where $\boldsymbol{D}_t$ is a diagonal matrix of conditional standard deviations, $\boldsymbol{P}_t$ is the correlation matrix with diagonal one.

A.3 Additional Graphs and Tables

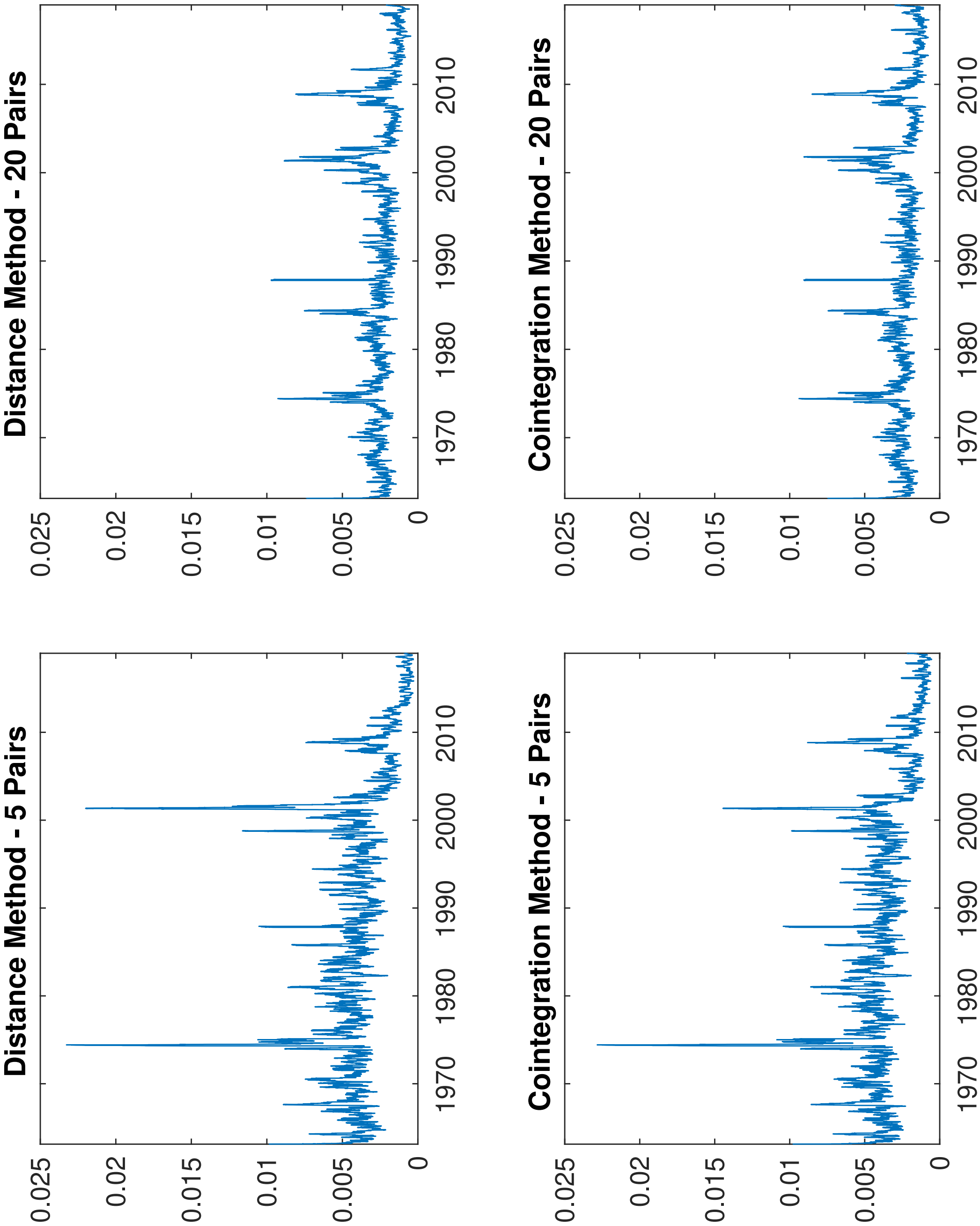

Figure A.1 Rolling Sample 30 Days Standard Deviations

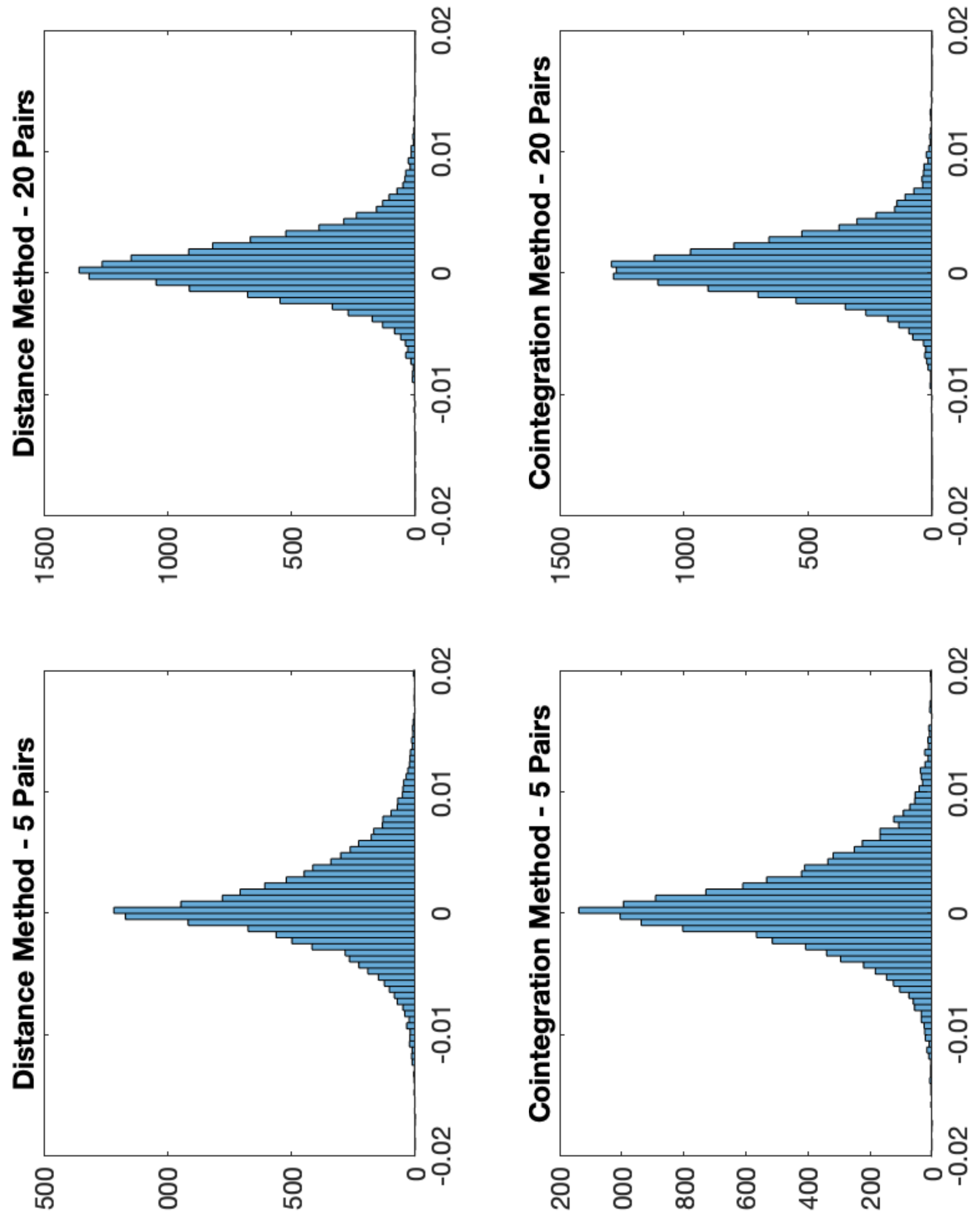

Figure A.2 Histogram of Daily Returns

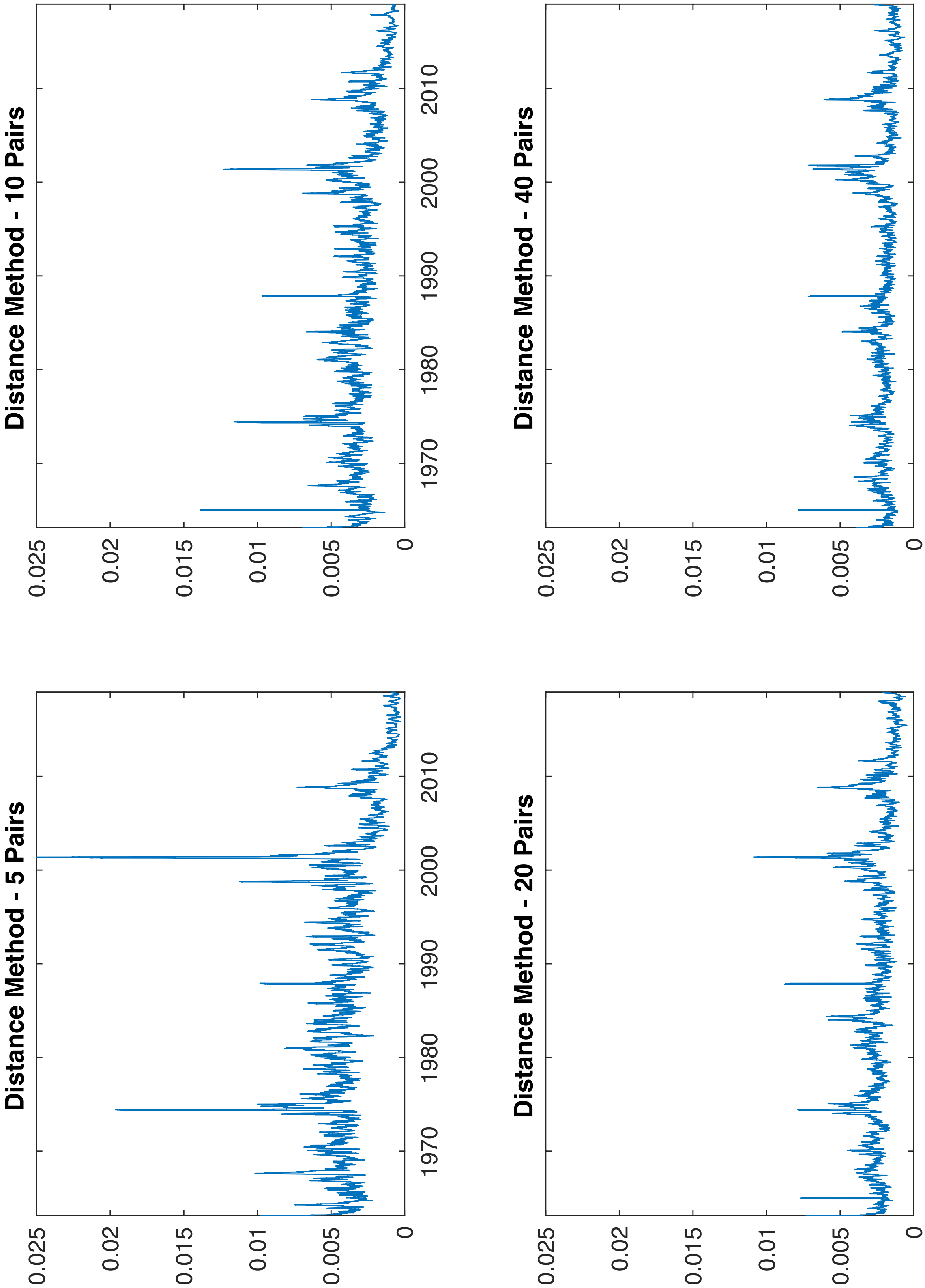

Figure A.3 Rolling Sample 30 Days Standard Deviations

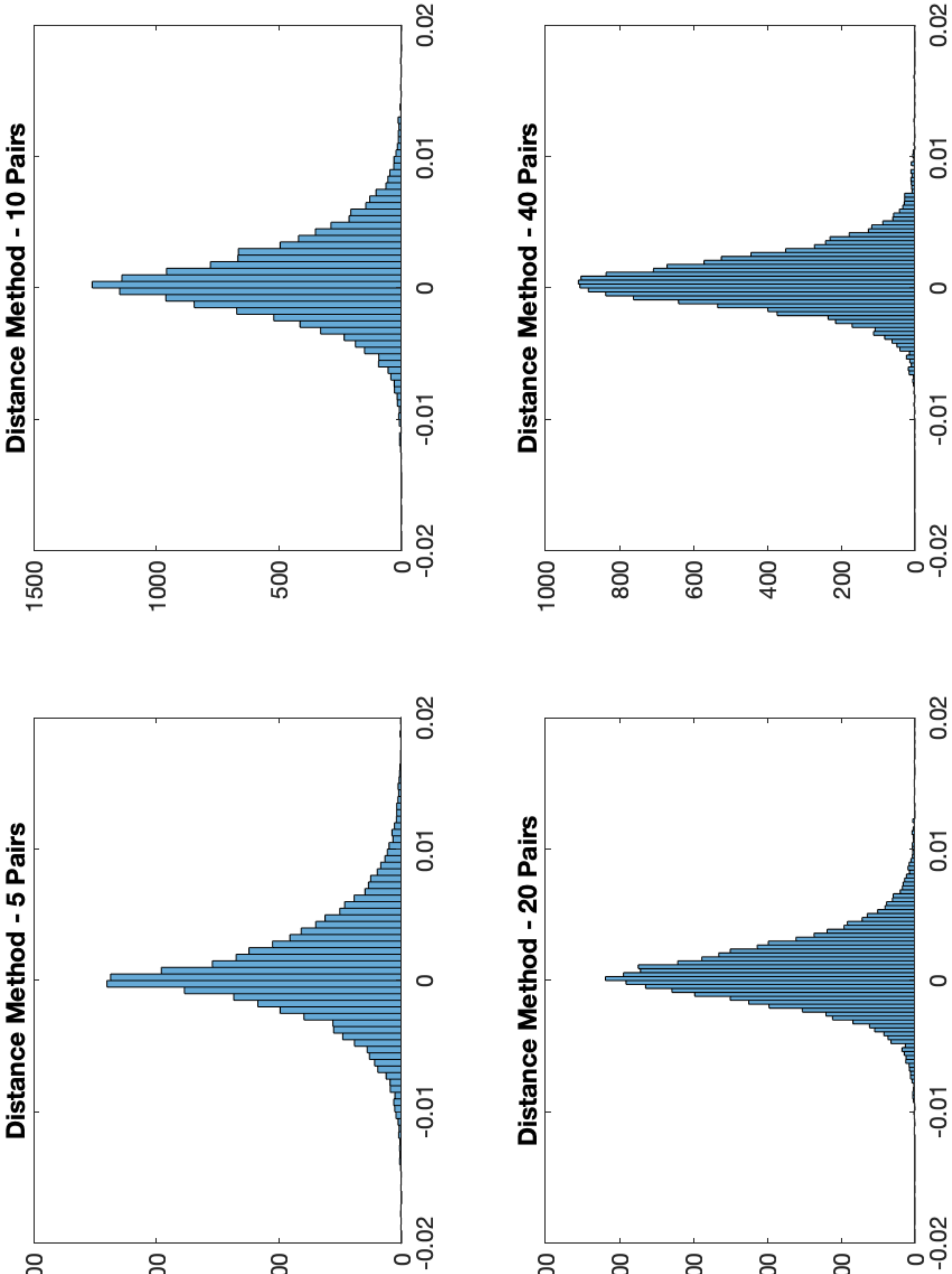

Figure A.4 Histograms of Daily Returns

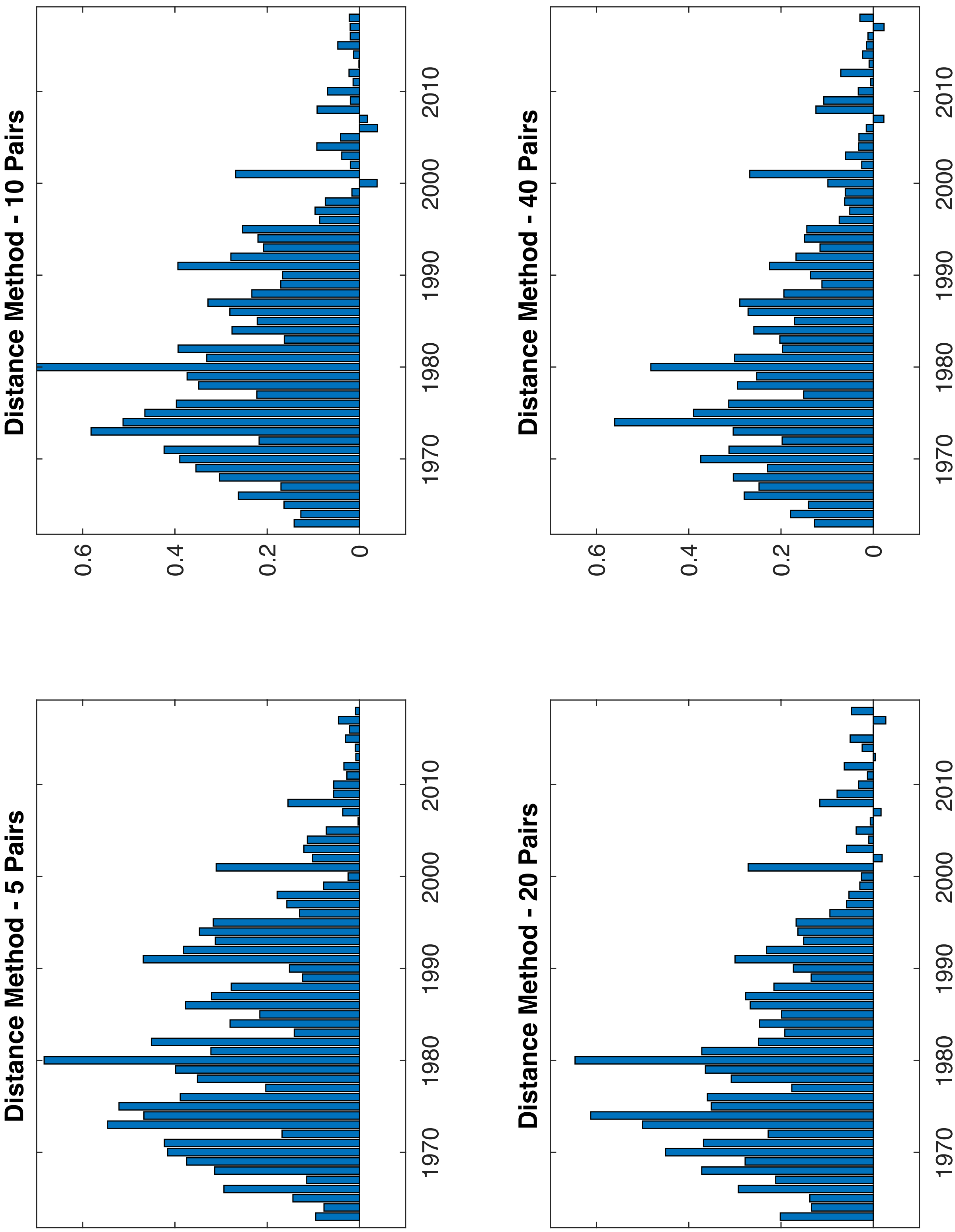

Figure A.5 Graphs of Yearly Returns

Table A.1 Asymmetric BEKK GARCH estimates.

	5 Pairs	**10 Pairs**	**20 Pairs**	**40 Pairs**
C(1,1)	0.2877***	0.1383***	0.1010***	0.1798
	(0.0197)	(0.0049)	(0.0050)	(0.1105)
C(2,1)	0.0043**	-0.0022	-0.0250***	-0.0271*
	(0.0017)	(0.0018)	(0.0032)	(0.0153)
C(2,2)	0.0780***	0.0092***	0.0099	0.0205***
	(0.0062)	(0.0006)	(0.0085)	(0.0025)
A(1,1)	-0.0282***	-0.0084***	0.0576***	0.1402***
	(0.0091)	(0.0028)	(0.0028)	(0.0298)
A(1,2)	0.0704*	0.1095***	0.0088	-0.0533
	(0.0422)	(0.0113)	(0.0110)	(0.1432)
A(2,1)	-0.0415***	-0.0674***	-0.1223***	0.0046
	(0.0064)	(0.0085)	(0.0073)	(0.1071)
A(2,2)	0.0219***	0.0451***	-0.0299	-0.0205
	(0.0075)	(0.0157)	(0.0235)	(0.0844)
G(1,1)	0.1712***	-0.0070*	0.1260***	0.2710***
	(0.0065)	(0.0042)	(0.0161)	(0.0771)
G(1,2)	-0.0994***	0.1007***	0.0050	-0.0285
	(0.0273)	(0.0069)	(0.0138)	(0.0800)
G(2,1)	0.0891***	0.0841***	0.0002	0.0373
	(0.0066)	(0.0192)	(0.0004)	(0.2089)
G(2,2)	-0.0604***	0.0695*	0.0028	0.0353
	(0.0054)	(0.0387)	(0.0073)	(0.1071)
B(1,1)	0.9491***	0.9410***	0.9594***	0.9465***
	(0.0045)	(0.0009)	(0.0013)	(0.0016)
B(1,2)	0.0168**	0.0567***	-0.0201***	-0.0360***
	(0.0071)	(0.0056)	(0.0013)	(0.0047)
B(2,1)	-0.0106***	-0.0472***	0.0779***	0.0358
	(0.0010)	(0.0109)	(0.0009)	(0.0412)
B(2,2)	0.9725***	0.9734***	0.9544***	0.9490***
	(0.0032)	(0.9734)	(0.0021)	(0.0212)

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

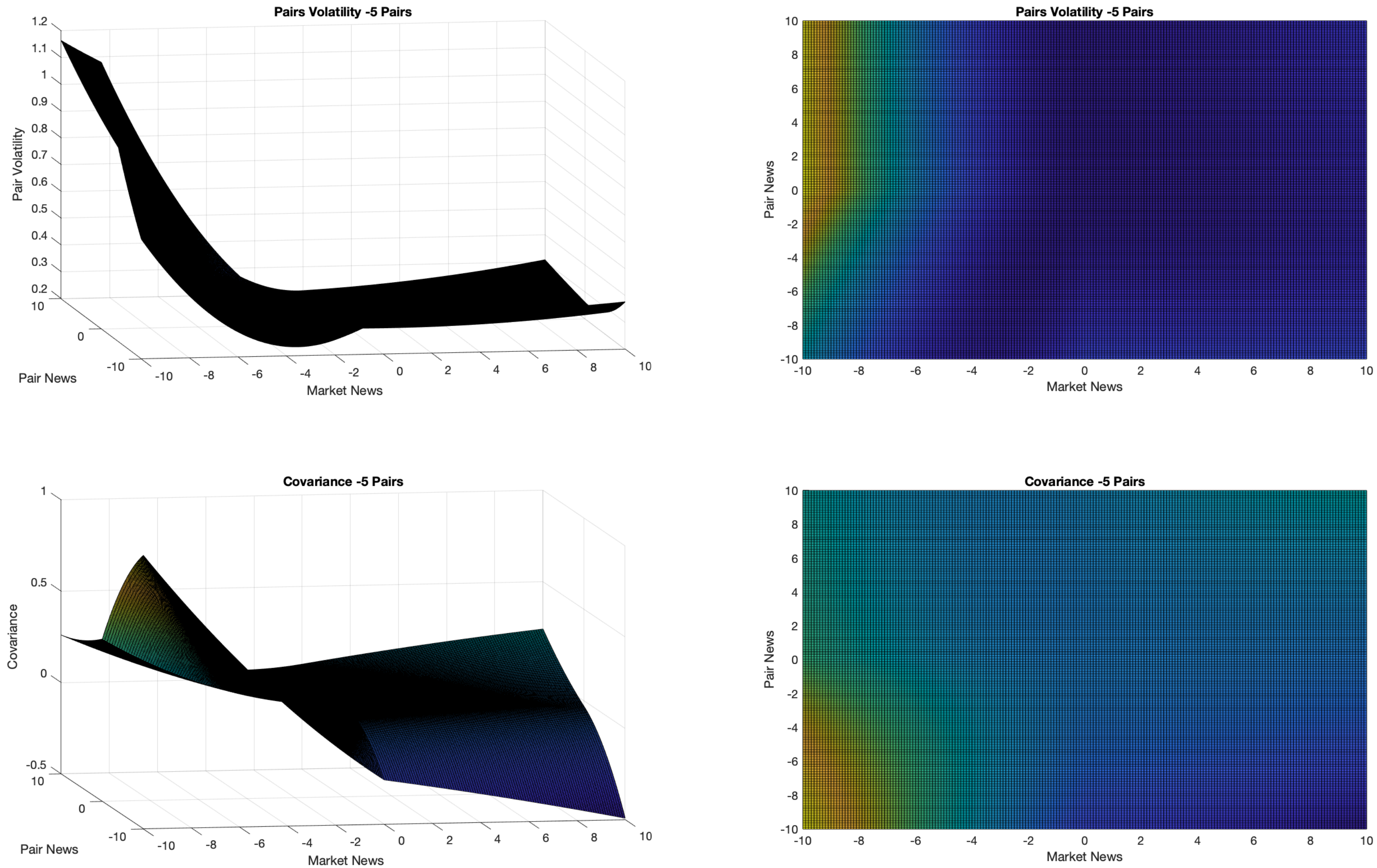

Figure A.6 News impact surfaces for the volatility of the portfolio of 5 pairs and for the covariance between this portfolio and the market.

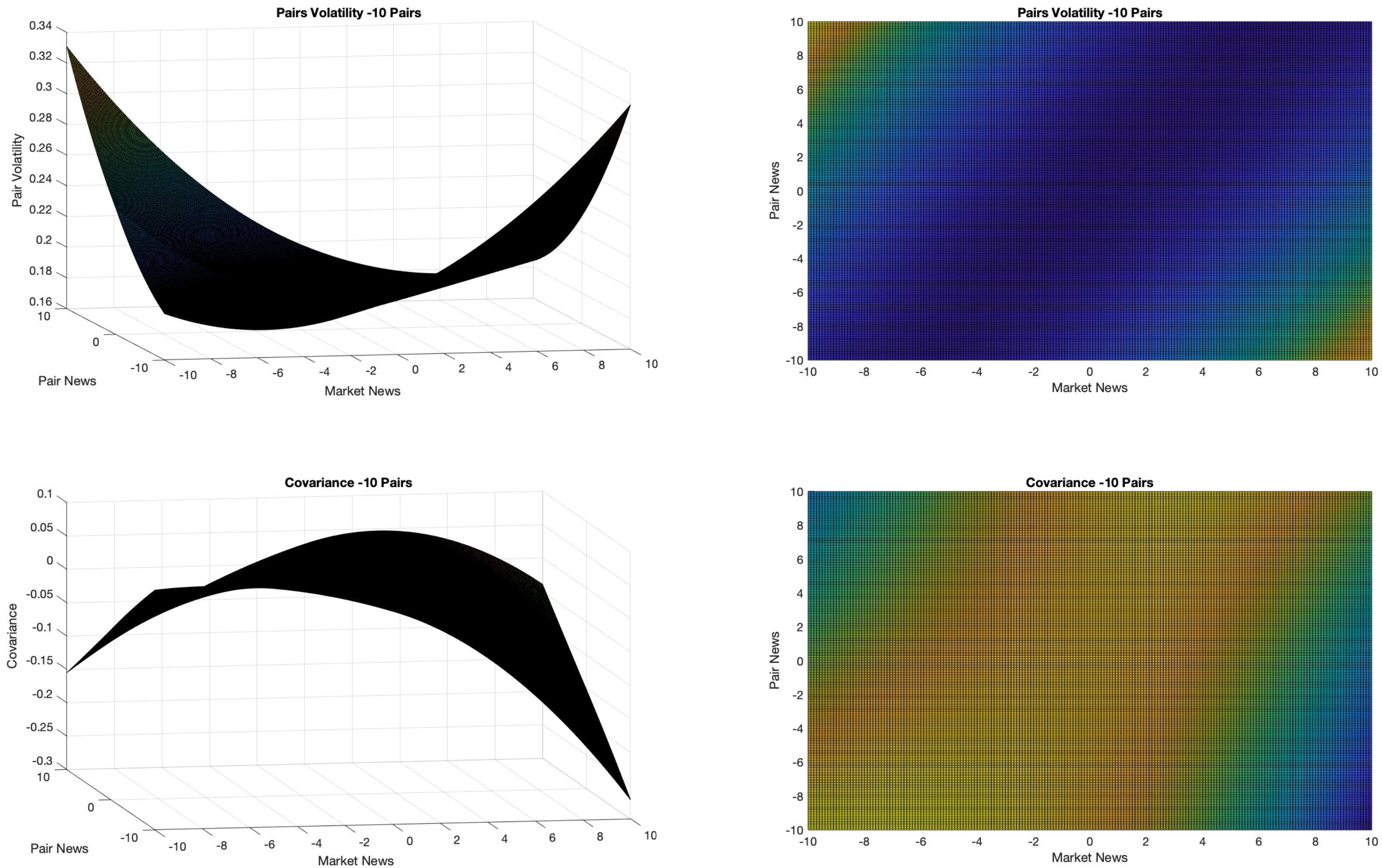

Figure A.7 News impact surfaces for the volatility of the portfolio of 10 pairs and for the covariance between this portfolio and the market.

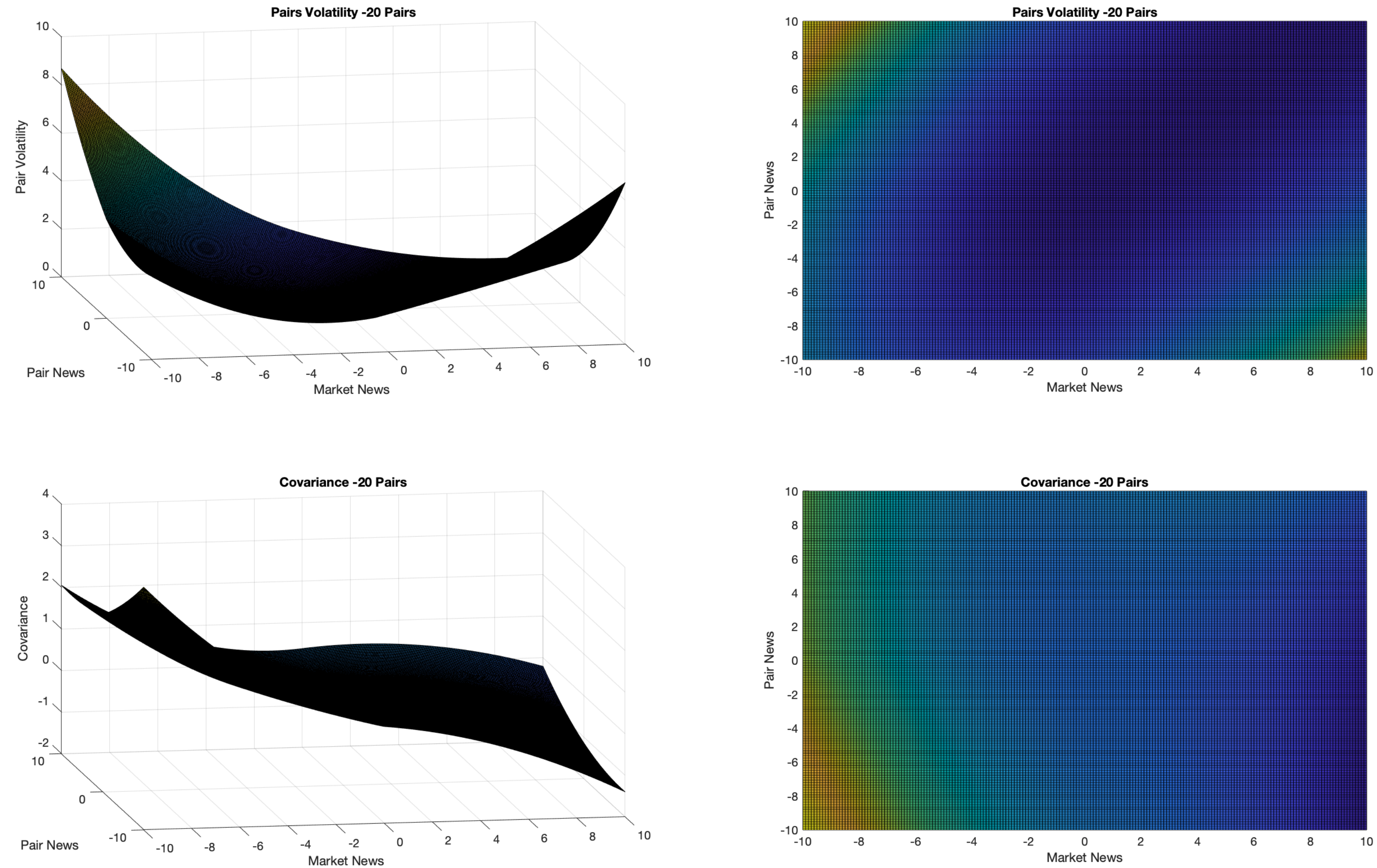

Figure A.8 News impact surfaces for the volatility of the portfolio of 20 pairs and for the covariance between this portfolio and the market.

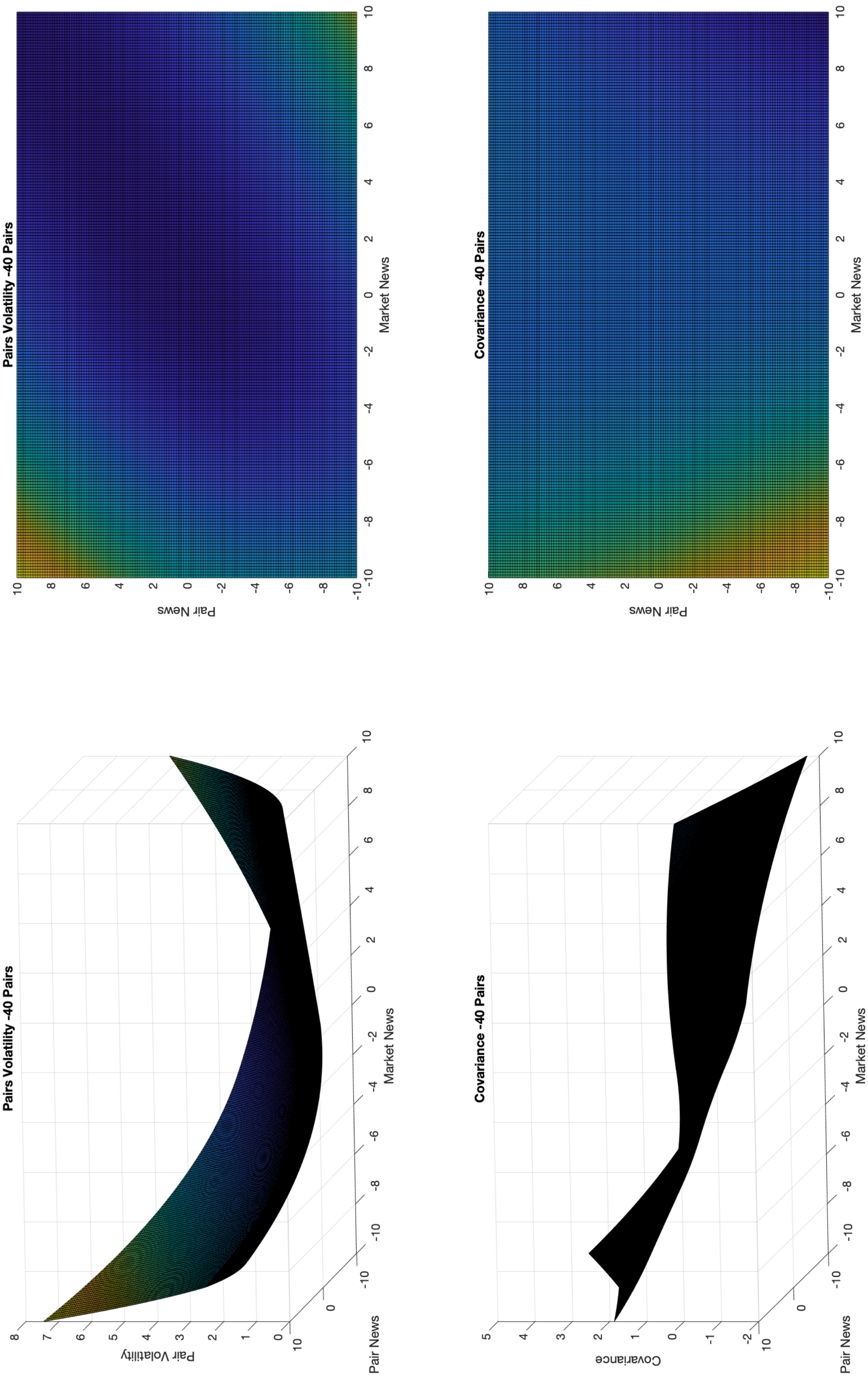

Figure A.9 News impact surfaces for the volatility of the portfolio of 40 pairs and for the covariance between this portfolio and the market.

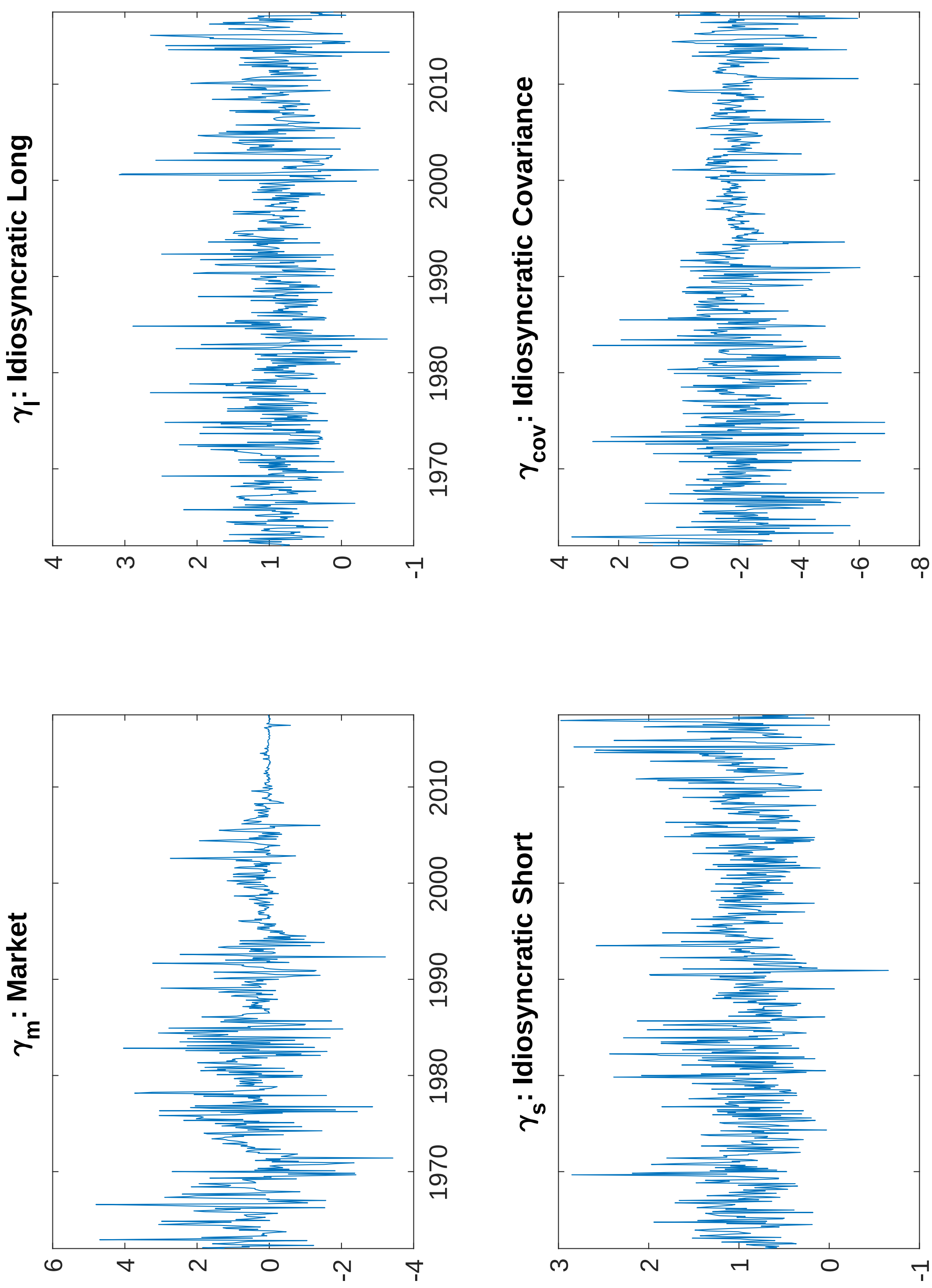

Figure A.10 Time varying coefficients when trading 10 pairs.

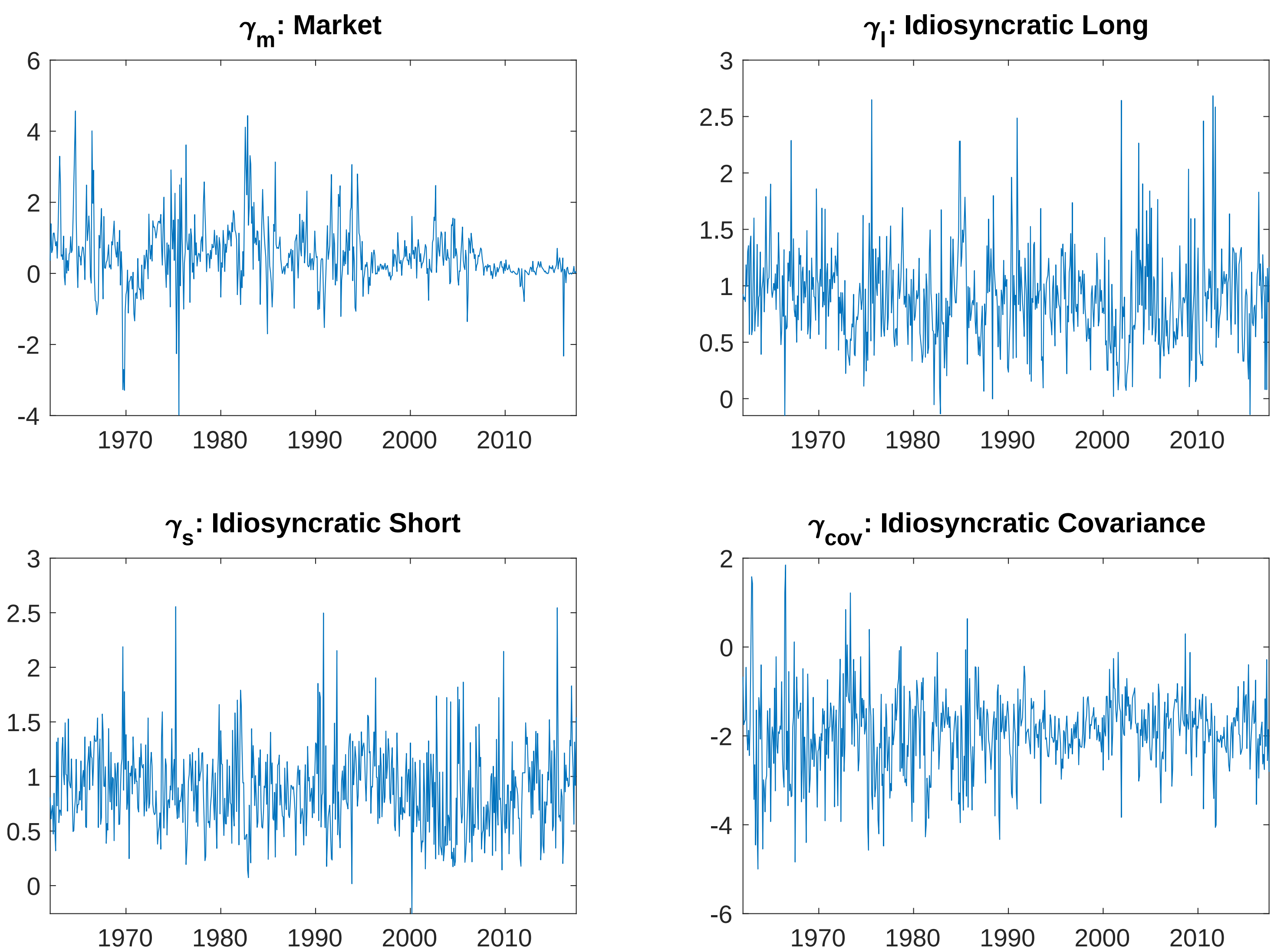

Figure A.11 Time varying coefficients when trading 20 pairs.

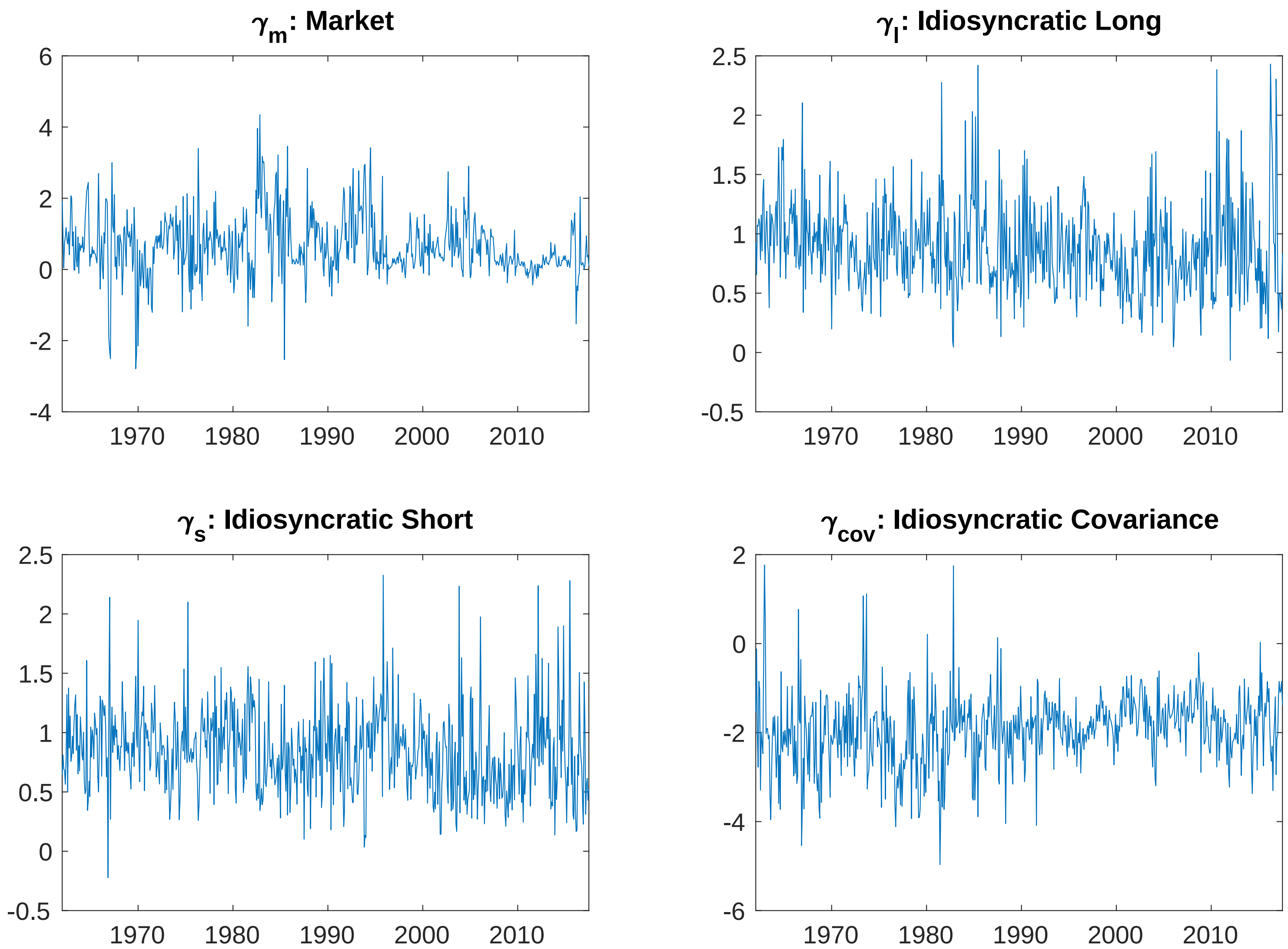

Figure A.12 Time varying coefficients when trading 40 pairs.

B | Appendix to Chapter 2

B.1 List of Countries and Market Classification

Table B.1 List of countries and respective MSCI (2020) market classification.

Currency	Country	MSCI Market Classification
AUD	Australia	Developed Market
BRL	Brazil	Emerging Market
CAD	Canada	Developed Market
CHF	Switzerland	Developed Market
CLP	Chile	Emerging Market
CNY	China	Emerging Market
CZK	Czech Republic	Emerging Market
DKK	Denmark	Developed Market
EUR	Euro Area	Developed Market
GBP	United Kingdom	Developed Market
HKD	Hong Kong	Developed Market
HUF	Hungary	Emerging Market
IDR	Indonesia	Emerging Market
ILS	Israel	Developed Market
INR	India	Emerging Market
JPY	Japan	Developed Market
KRW	South Korea	Emerging Market
MXN	Mexico	Emerging Market
NOK	Norway	Developed Market
NZD	New Zealand	Developed Market
PHP	Philippines	Emerging Market
PLN	Poland	Emerging Market
RUB	Russia	Emerging Market
SEK	Sweden	Developed Market
SGD	Singapore	Developed Market
THB	Thailand	Emerging Market
TRY	Turkey	Emerging Market
TWD	Taiwan	Emerging Market
ZAR	South Africa	Emerging Market

B.2 Volatility Models for Real Exchange Rates

B.2.1 TARCH Model

We estimate a TARCH(P,O,Q) model of Zakoian (1994) with the following model specification:

$$
\begin{aligned}
r_t &= \epsilon_t \\
\epsilon_t &= \sigma_t e_t \\
\sigma_t &= \omega + \sum_{p=1}^{P} \alpha_p |\epsilon_{t-p}| + \sum_{o=1}^{O} \gamma_o |\epsilon_{t-o}| 1_{[\epsilon_{t-o}<0]} + \sum_{q=1}^{Q} \beta_q \sigma_{t-q} \\
e_t &\overset{i.i.d.}{\sim} N(0,1)
\end{aligned}
$$

B.2.2 GJR-GARCH Model

We estimate a GJR-GARCH(P,O,Q) model of Glosten et al. (1993) with the following model specification:

$$
\begin{aligned}
r_t &= \epsilon_t \\
\epsilon_t &= \sigma_t e_t \\
\sigma_t^2 &= \omega + \sum_{p=1}^{P} \alpha_p \epsilon_{t-p}^2 + \sum_{o=1}^{O} \gamma_o \epsilon_{t-o}^2 1_{[\epsilon_{t-o}<0]} + \sum_{q=1}^{Q} \beta_q \sigma_{t-q}^2 \\
e_t &\overset{i.i.d.}{\sim} N(0,1)
\end{aligned}
$$

B.2.3 EGARCH Model

We estimate a EGARCH(P,O,Q) model of of Nelson (1991) with the following model specification:

$$
\begin{aligned}
r_t &= \epsilon_t \\
\epsilon_t &= \sigma_t e_t \\
ln(\sigma_t^2) &= \omega + \sum_{p=1}^{P} \alpha_p \left(\left| \frac{e_{t-p}}{\sigma_{t-p}} \right| - \sqrt{\frac{2}{\pi}} \right) + \sum_{o=1}^{O} \gamma_o \frac{e_{t-o}}{\sigma_{t-o}} + \sum_{q=1}^{Q} \beta_q ln(\sigma_{t-q}^2) \\
e_t &\overset{i.i.d.}{\sim} N(0,1)
\end{aligned}
$$

B.2.4 Selected models

Table B.2 Real exchange rate volatility model selected for each currency.

Currency	Country	Model Selected
AUD	Australia	GJR-GARCH(1,0,1)
BRL	Brazil	EGARCH(2,1,1)
CAD	Canada	TARCH(1,0,0)
CHF	Switzerland	EGARCH(0,1,1)
CLP	Chile	EGARCH(1,1,1)
CNY	China	EGARCH(2,0,1)
CZK	Czech Republic	TARCH(1,0,1)
DKK	Denmark	EGARCH(1,0,1)
EUR	Euro Area	GJR-GARCH(0,1,2)
GBP	United Kingdom	GJR-GARCH(1,0,1)
HKD	Hong Kong	GJR-GARCH(1,1,1)
HUF	Hungary	TARCH(1,0,1)
IDR	Indonesia	EGARCH(1,0,1)
ILS	Israel	GJR-GARCH(1,0,1)
INR	India	TARCH(1,0,1)
JPY	Japan	GJR-GARCH(1,0,0)
KRW	South Korea	EGARCH(2,1,1)
MXN	Mexico	EGARCH(2,1,1)
NOK	Norway	GJR-GARCH(1,0,0)
NZD	New Zealand	GJR-GARCH(1,0,1)
PHP	Philippines	GJR-GARCH(1,1,1)
PLN	Poland	GJR-GARCH(1,0,1)
RUB	Russia	EGARCH(1,1,2)
SEK	Sweden	GJR-GARCH(1,0,0)
SGD	Singapore	GJR-GARCH(1,0,1)
THB	Thailand	GJR-GARCH(1,0,1)
TRY	Turkey	TARCH(1,1,0)
TWD	Taiwan	EGARCH(1,0,1)
ZAR	South Africa	EGARCH(2,2,1)

B.3 Covariance Model for Nominal Exchange Rates and Price Differentials

B.3.1 The AG-DCC Multivariate GARCH Model

We begin by defining:

$$
\begin{aligned}
\boldsymbol{r}_t &= \boldsymbol{\epsilon}_t \\
\boldsymbol{\epsilon}_t &= \boldsymbol{\Sigma}_t^{1/2} \boldsymbol{e}_t \\
\boldsymbol{e}_t &\overset{i.i.d.}{\sim} N(\mathbf{0}, \boldsymbol{I}_2)
\end{aligned}
$$

And then we model $\boldsymbol{\Sigma}_t$ according to the AG-DCC GARCH(M,L,N) specification of Cappiello et al. (2006):

$$
\begin{aligned}
\boldsymbol{\Sigma}_t &= \boldsymbol{D}_t \boldsymbol{P}_t \boldsymbol{D}_t \\
\boldsymbol{P}_t &= \boldsymbol{Q}_t^* \boldsymbol{Q}_t \boldsymbol{Q}_t^* \\
\boldsymbol{Q}_t &= (\bar{\boldsymbol{P}} - \sum_{m=1}^{M} \boldsymbol{A}_m' \bar{\boldsymbol{P}} \boldsymbol{A}_m - \sum_{l=1}^{L} \boldsymbol{G}_l' \bar{\boldsymbol{N}} \boldsymbol{G}_l - \sum_{n=1}^{N} \boldsymbol{B}_n' \bar{\boldsymbol{P}} \boldsymbol{B}_n) + \sum_{m=1}^{M} \boldsymbol{A}_m' \boldsymbol{e}_{t-m} \boldsymbol{e}_{t-m}' \boldsymbol{A}_m \\
&\quad + \sum_{l=1}^{L} \boldsymbol{G}_l' \boldsymbol{n}_{t-l} \boldsymbol{n}_{t-l}' \boldsymbol{G}_l + \sum_{n=1}^{N} \boldsymbol{B}_n' \boldsymbol{Q}_{t-n} \boldsymbol{B}_n \\
\boldsymbol{Q}_t^* &= (\boldsymbol{Q}_t \odot \boldsymbol{I}_2)^{\frac{1}{2}}
\end{aligned}
$$

Where $\boldsymbol{D}_t$ is a diagonal matrix of conditional standard deviations and $\boldsymbol{P}_t$ is the correlation matrix with diagonal one.

B.3.2 Selected models

Table B.3 AG-DCC GARCH(M,L,N) model selected for each currency.

Currency	Country	Model Selected
AUD	Australia	AG-DCC(1,0,1)
BRL	Brazil	AG-DCC(1,0,1)
CAD	Canada	AG-DCC(1,0,1)
CHF	Switzerland	AG-DCC(1,0,1)
CLP	Chile	AG-DCC(1,0,1)
CNY	China	AG-DCC(1,0,1)
CZK	Czech Republic	AG-DCC(1,0,1)
DKK	Denmark	AG-DCC(1,0,0)
EUR	Euro Area	AG-DCC(1,1,0)
GBP	United Kingdom	AG-DCC(1,0,1)
HKD	Hong Kong	AG-DCC(1,0,1)
HUF	Hungary	AG-DCC(1,0,1)
IDR	Indonesia	AG-DCC(1,0,1)
ILS	Israel	AG-DCC(1,0,0)
INR	India	AG-DCC(1,0,1)
JPY	Japan	AG-DCC(1,0,0)
KRW	South Korea	AG-DCC(1,0,1)
MXN	Mexico	AG-DCC(2,0,0)
NOK	Norway	AG-DCC(1,0,1)
NZD	New Zealand	AG-DCC(1,0,1)
PHP	Philippines	AG-DCC(1,0,2)
PLN	Poland	AG-DCC(1,0,1)
RUB	Russia	AG-DCC(1,1,1)
SEK	Sweden	AG-DCC(1,0,0)
SGD	Singapore	AG-DCC(1,0,1)
THB	Thailand	AG-DCC(1,0,1)
TRY	Turkey	AG-DCC(1,0,0)
TWD	Taiwan	AG-DCC(1,0,1)
ZAR	South Africa	AG-DCC(1,0,0)

B.4 Factor Loading and Extreme Days

Table B.4 The most extreme common volatility shocks to real exchange rates using the baseline GEOVOL model.

Date	$\hat{x}$	$\bar{r}^R$
2020-03	8.176	3.666
1993-12	5.862	−0.817
1991-02	5.351	−1.566
1997-07	4.573	1.592
2009-01	4.561	1.322
2020-02	4.262	1.808
2009-03	4.122	0.219
2015-01	4.113	2.006
1993-03	4.108	−0.115
2008-09	4.079	3.622
1998-09	4.047	−1.269
1991-07	4.043	0.203
2008-08	3.933	2.431
1997-08	3.766	1.986
2008-10	3.658	5.809

Table B.5 The GEOVOL factor loadings for real exchange rates.

THBr	0.262	JPYr	0.176
PLNr	0.259	BRLr	0.173
TRYr	0.241	ILSr	0.167
RUBr	0.230	CZKr	0.166
PHPr	0.224	ZARr	0.161
HKDr	0.212	SEKr	0.157
SGDr	0.204	NOKr	0.154
DKKr	0.202	TWDr	0.150
EURr	0.201	AVGreal	0.134
INRr	0.198	CNYr	0.131
IDRr	0.198	CLPr	0.124
HUFr	0.193	AUDr	0.123
MXNr	0.188	NZDr	0.122
CHFr	0.187	GBPr	0.115
KRWr	0.179	CADr	0.103

Table B.6 The most extreme common volatility shocks to nominal exchange rates (volatility factor model (2.25), i.e., with no exogenous information).

Date	$\hat{x}^G_{(2.25)}$	$\bar{r}^E$
2020-03	8.961	3.755
1998-09	8.393	−0.655
1997-07	7.342	1.983
1993-02	6.586	1.493
2020-02	6.163	1.773
1999-01	5.734	0.919
1991-06	5.543	2.445
1998-08	5.436	1.391
1991-02	5.335	0.126
2002-06	5.289	−1.120
2013-05	4.740	0.888
1991-03	4.710	3.183
2009-02	4.692	3.419
2008-10	4.558	6.555
2008-09	4.369	3.987

Table B.7 The volatility factor loadings on $x^G_{(2.25)}$.

CNYn	0.379	IDRn	0.165
EURn	0.273	CZKn	0.164
THBn	0.271	MXNn	0.159
HKDn	0.239	ZARn	0.159
TWDn	0.233	JPYn	0.125
HUFn	0.214	BRLn	0.121
DKKn	0.211	ILSn	0.114
PHPn	0.208	NOKn	0.111
PLNn	0.204	AVGnominal	0.111
RUBn	0.194	CLPn	0.105
INRn	0.193	GBPn	0.101
SGDn	0.186	SEKn	0.089
KRWn	0.181	AUDn	0.086
TRYn	0.181	CADn	0.067
CHFn	0.175	NZDn	0.043

B.5 The Study of REER Volatilities

Consider the vector representing the standardized residuals of real effective exchange rates $\boldsymbol{e}_t^{REER} \equiv (e_{1,t}^{REER}, \ldots, e_{N,t}^{REER})'$ and that factors are sufficient to reduce the contemporaneous correlations to zero. This is achieved by estimating an AR(1) model with GARCH(1,1) errors whenever necessary.

The average of returns seems to capture most of the correlation between real effective exchange rates. The average correlation of the raw real effective exchange rate returns is 0.036 whereas of standardized residuals of the real effective exchange rates is -0.037 as shown in Table B.8.

Table B.8 Average cross-sectional correlations for real effective exchange rates.

	$\bar{\rho}_{[r_{REER}]}$	$\bar{\rho}_{[\hat{e}_{REER}]}$	$\bar{\rho}_{[\hat{e}^2_{REER}]}$	$\bar{\rho}_{[\hat{e}^2_{REER}/\hat{g}_t]}$
Correlation	0.036	−0.037	0.061	−0.005
GEOVOL test statistic	–	–	17.69	−1.709

Table B.9 The most extreme GEOVOL shocks to real effective exchange rates.

t	$\hat{x}_t^{GEOVOL}$	$\bar{r}_t^{REER}$
2008-10	13.461	−2.638
2015-01	13.277	−0.283
2020-03	12.033	−2.624
1990-02	9.471	−0.284
1998-09	7.716	−1.368
2008-12	7.557	−1.051
1998-10	7.307	−0.664
2020-04	6.790	−0.911
1993-09	6.380	−0.767
1998-11	6.273	1.278
2006-05	6.226	−0.015
1990-11	6.196	−1.239
1994-01	5.991	0.717
1990-03	5.967	1.039
1991-03	5.914	1.901

Assumption (2.24) implies that the standardized residuals are orthogonal with unit variance. It does not mean however that they are independent. In fact, the squared standardized residuals of real effective exchange rates are correlated. Referring back to Table B.8, we can see that their average correlation is 0.061, which is positive and statistically significant. For the sample of squared standardized residuals of real effective exchange rates, the test statistic is 17.69.

However, according to Table B.8, once we consider the information about the endogenous common volatility shocks model of Engle and Campos-Martins (2020) described in Section 2.5.1 via the $g(s_i, x_t)$ function from Equation (2.15), we are able to get rid of these cross-sectional correlations. That is, even the standard one factor common volatility shocks model of Engle and Campos-Martins (2020) described in Section 2.5.1 is capable of successfully giving structure to common volatility dependencies in real effective exchange rates.

Table B.10 The GEOVOL loadings, $\hat{s}_i^{REER}$.

Average	0.530	Norway	0.169
Denmark	0.281	Brazil	0.168
United States	0.258	Russia	0.168
China	0.258	Australia	0.158
Euro Area	0.246	Israel	0.155
Switzerland	0.222	United Kingdom	0.133
Hungary	0.198	Canada	0.111
Poland	0.198	Japan	0.091
Philippines	0.194	Sweden	0.082
Czech Republic	0.188	Chile	0.060
Mexico	0.178	Singapore	0.038
South Africa	0.169	New Zealand	0.037

$\text{Average}_t = 1/N \sum e_{i,t}^{REER}$.

C | Appendix to Chapter 3

C.1 List of Countries

Table C.1 This table reports the sample of 72 countries included in our final index.

Albania	Denmark	Lebanon	Russian Federation
Algeria	Dominican Republic	Lithuania	Saudi Arabia
Argentina	Estonia	Luxembourg	Serbia
Australia	Finland	Malaysia	Singapore
Austria	France	Malta	Slovak Republic
Azerbaijan	Germany	Mexico	Slovenia
Belarus	Greece	Moldova	Spain
Belgium	Hungary	Morocco	Sweden
Brazil	Iceland	Netherlands	Switzerland
Bulgaria	Indonesia	New Zealand	Thailand
Canada	Ireland	Norway	Trinidad and Tobago
Chile	Israel	Panama	Tunisia
China	Italy	Peru	Turkey
Colombia	Japan	Philippines	Ukraine
Costa Rica	Jordan	Poland	United Kingdom
Croatia	Kazakhstan	Portugal	United States
Cyprus	Korea, Rep.	Qatar	Uruguay
Czech Republic	Latvia	Romania	Vietnam

C.2 Soft Power Variables

Table C.2 Summary statistics calculated pooling across the time and country dimensions.

Variable Name	Start Date	Mean	Standard Deviation	5^{th} Percentile	95^{th} Percentile
Competitiveness	01/01/2007	15.97	23.58	3.23	70.20
Outward Foreign Investment	01/01/1970	2.28	25.87	-0.25	5.27
Patents	01/01/1980	0.01	0.03	0.00	0.04
Trademarks	01/01/1980	0.29	1.40	0.00	0.53
Cultural Exports	01/01/2007	0.17	0.38	0.00	0.63
International Tourists	01/01/1995	123.60	337.74	0.53	644.25
Olympic Medals	01/01/1960	9.45	22.60	0.19	33.24
World Heritage Sites	01/01/1978	3.78	6.64	0.00	16.00
Internet Access	01/01/1990	28.03	29.87	0.03	86.53
Mobile Phones Access	01/01/1980	57.29	54.10	0.03	146.92
Education Expenditure	01/01/1970	4.34	2.68	1.54	7.38
Journal Articles	01/01/2000	0.03	0.05	0.00	0.15
PISA: Maths	01/01/2000	461.39	59.83	361.53	543.80
PISA: Reading	01/01/2000	458.29	55.26	358.31	526.89
PISA: Science	01/01/2000	464.82	54.74	375.37	539.47
Primary Completion	01/01/1970	78.41	24.78	28.31	105.67
Tertiary Education	01/01/1970	22.16	23.21	0.68	72.45
Years of Schooling	01/01/1970	6.18	3.29	1.09	11.53
Aid and Assistance	01/01/1960	6.35	10.96	0.01	24.69
Diplomatic Events	01/01/1979	9.51	7.10	3.97	18.18
Embassies	01/01/2006	78.57	40.54	24.00	152.00
Migrants	01/01/1960	11.13	15.87	0.26	50.33
Refugees	01/01/1960	1.24	5.16	0.00	4.51
Bureaucratic Effectiveness	01/01/1984	2.17	1.15	0.00	4.00
Corruption	01/01/1984	2.92	1.32	1.00	5.50
Democratic Accountability	01/01/1984	3.81	1.65	1.00	6.00
Government Stability	01/01/1984	7.45	2.11	4.00	11.00
Rule of Law	01/01/1984	3.65	1.42	1.00	6.00

Table C.3 List of variables, definitions and data sources.

Variable Name	Dimension	Source	Definition
Competitiveness	Commercial Prowess	World Economic Forum	World Economic Forum Global Competitiveness Index
Outward Foreign Investment	Commercial Prowess	UNCTAD	Outward Foreign Investment (% of GDP)
Patents	Commercial Prowess	World Bank	Number of international patents (per population)
Trademarks	Commercial Prowess	World Bank	Number of trademarks (per population)
Cultural Exports	Culture	UNCTAD	Exports of cultural goods (% of GDP)
International Tourists	Culture	World Bank	Number of international tourists (per population)
Olympic Medals	Culture	Olympic Committee	Total number of medals in latest Olympic games (per population)
World Heritage Sites	Culture	UNESCO	Number of UNESCO World Heritage sites
Internet Access	Digital Footprint	World Bank	Number of internet users (per population)
Mobile Phones Access	Digital Footprint	World Bank	Number of mobile phones (per population)
Education Expenditure	Education	World Bank	Government expenditure on education (% of GDP)
Journal Articles	Education	World Bank	Number of journal articles (per population)
PISA: Maths	Education	World Bank	Mean performance on the PISA mathematics exam scale
PISA: Reading	Education	World Bank	Mean performance on the PISA reading exam scale
PISA: Science	Education	World Bank	Mean performance on the PISA science exam scale
Primary Completion	Education	World Bank	Primary completion rate both sexes (% of population)
Tertiary Education	Education	World Bank	Gross tertiary educational enrolment rate
Years of Schooling	Education	World Bank	Barro-Lee: Average years of total schooling age 25+
Aid and Assistance	Global Reach	World Bank	Official development assistance and aid (% of GDP)
Diplomatic Events	Global Reach	GDELT	Share of diplomatic cooperation events in a year (% total events)
Embassies	Global Reach	Lowy Institute	Lowy Institute number of embassies abroad
Migrants	Global Reach	World Bank	Number of migrants (per population)
Refugees	Global Reach	World Bank	Number of refugees (per population)
Bureaucratic Effectiveness	Institutions	ICRG	The PRS Group International Country Risk Guide (ICRG) Bureaucratic Effectiveness Index
Corruption	Institutions	ICRG	The PRS Group International Country Risk Guide (ICRG) Corruption Index
Democratic Accountability	Institutions	ICRG	The PRS Group International Country Risk Guide (ICRG) Democratic Accountability Index
Government Stability	Institutions	ICRG	The PRS Group International Country Risk Guide (ICRG) Government Stability Index
Rule of Law	Institutions	ICRG	The PRS Group International Country Risk Guide (ICRG) Rule of Law Index

C.3 Control Variables

Table C.4 List of variables, definitions and data sources.

Variable Name	Definition	Source
Credit	Domestic credit to private sector (%GDP)	World Bank
Current Account	Annual current account balance (%GDP)	World Bank
Export Concentration	Export Concentration Index	UNCTAD
Inflation	Inflation, consumer prices (annual %)	World Bank
Stock Market Cap	Market capitalization of listed domestic companies (% of GDP)	World Bank
Trade Openness	Trade (% of GDP)	World Bank
Volatility of Government Consumption	5y rolling standard deviation of government consumption (% of GDP)	
Volatility of Labour Productivity Growth	5y rolling standard deviation of annual output per worker growth	ILO
Volatility of Terms of Trade	5y rolling standard deviation of commodity terms of trade index growth	IMF

C.4 Sub-index Weights

Table C.5 Weights for each variable in their respective sub-index.

Variable Name	Sub-index	Weight
Competitiveness	Commercial Prowess	33.82%
Outward Foreign Investment	Commercial Prowess	19.99%
Patents	Commercial Prowess	46.19%
Trademarks	Commercial Prowess	0.00%
Cultural Exports	Culture	44.38%
International Tourists	Culture	0.00%
Olympic Medals	Culture	0.00%
World Heritage Sites	Culture	55.62%
Internet Access	Digital Footprint	49.91%
Mobile Phones Access	Digital Footprint	50.09%
Education Expenditure	Education	1.87%
Journal Articles	Education	32.67%
PISA: Maths	Education	22.03%
PISA: Reading	Education	19.14%
PISA: Science	Education	21.47%
Primary Completion	Education	0.00%
Tertiary Education	Education	2.82%
Years of Schooling	Education	0.00%
Aid and Assistance	Global Reach	0.00%
Diplomatic Events	Global Reach	14.81%
Embassies	Global Reach	63.68%
Migrants	Global Reach	21.51%
Refugees	Global Reach	0.00%
Bureaucratic Effectiveness	Institutions	17.70%
Corruption	Institutions	18.04%
Democratic Accountability	Institutions	22.74%
Government Stability	Institutions	21.95%
Rule of Law	Institutions	19.57%

C.5 Index Values

Table C.6 Latest (2019) values for the Global Soft Power Index (GSPI) and soft power sub-indices for each country in our sample.

Country	Commercial Prowess	Culture	Digital Footprint	Education	Global Reach	Institutions	GSPI
Albania	-0.03	0.03	-0.11	-0.48	0.03	-0.17	-0.12
Algeria	-0.07	0.19	-0.26	-1.09	-0.10	-0.35	-0.28
Argentina	-0.04	0.44	0.36	-0.64	0.15	-0.21	0.01
Australia	0.50	0.96	0.38	1.65	0.25	1.33	0.84
Austria	0.68	0.49	0.53	1.12	0.14	1.30	0.71
Azerbaijan	0.15	-0.03	0.24	-0.85	0.09	-0.54	-0.16
Belarus	0.03	0.09	0.48	0.17	0.18	-0.35	0.10
Belgium	0.47	0.76	0.33	1.21	0.10	0.94	0.63
Brazil	0.06	1.05	-0.05	-0.57	0.81	-0.14	0.19
Bulgaria	0.13	0.49	0.15	-0.15	-0.03	0.09	0.11
Canada	0.60	0.98	0.27	1.51	0.44	1.20	0.83
Chile	0.27	0.14	0.57	-0.06	-0.19	0.28	0.17
China	1.53	3.11	0.01	1.46	1.30	-0.19	1.20
Colombia	0.07	0.33	0.27	-0.59	-0.06	-0.30	-0.05
Costa Rica	0.04	0.04	0.99	-0.49	-0.02	-0.07	0.08
Croatia	0.10	0.47	0.24	0.67	0.15	0.39	0.34
Cyprus	1.09	0.02	0.30	0.33	0.18	1.10	0.50
Czech Republic	0.33	0.99	0.44	1.17	0.15	0.70	0.63
Denmark	0.80	0.48	0.75	1.83	-0.15	1.46	0.86
Dominican Republic	-0.04	0.31	-0.13	-1.49	-0.13	-0.49	-0.33
Estonia	0.34	0.18	0.87	1.29	-0.47	0.38	0.43
Finland	0.74	0.22	0.66	1.71	-0.12	1.46	0.78
France	0.67	2.64	0.30	0.90	1.31	0.90	1.12
Germany	1.24	2.55	0.63	1.12	1.19	1.12	1.31
Greece	0.09	0.88	0.21	0.52	0.13	0.70	0.42
Hungary	0.18	0.46	0.22	0.54	0.13	0.65	0.36
Iceland	0.49	-0.02	0.65	1.32	-0.94	1.09	0.43
Indonesia	0.10	0.50	-0.08	-0.90	0.11	0.25	0.00
Ireland	0.37	0.06	0.29	1.28	-0.09	1.20	0.52
Israel	0.56	0.41	0.61	0.80	0.21	0.78	0.56
Italy	0.45	3.24	0.43	0.80	0.72	0.54	1.03
Japan	3.25	1.17	0.91	1.07	1.03	0.91	1.39
Jordan	0.02	0.18	-0.34	-0.44	0.48	-0.01	-0.02
Kazakhstan	0.09	0.12	0.64	-0.64	0.26	0.15	0.10
Latvia	0.18	0.37	0.36	0.70	-0.50	0.43	0.25
Lebanon	-0.04	0.28	-0.32	-0.88	0.31	-0.24	-0.15

Table C.6 Latest (2019) values for the Global Soft Power Index (GSPI) and soft power sub-indices for each country in our sample.

Country	Commercial Prowess	Culture	Digital Footprint	Education	Global Reach	Institutions	GSPI
Lithuania	0.20	0.23	0.96	0.63	-0.57	0.54	0.33
Luxembourg	0.62	-0.10	0.85	0.90	-0.23	1.38	0.57
Malaysia	0.34	1.37	0.69	0.03	0.01	0.21	0.44
Malta	-0.66	0.07	0.56	0.43	0.00	0.67	0.18
Mexico	0.11	1.80	-0.06	-0.49	-0.12	-0.21	0.17
Moldova	-0.02	-0.12	0.51	-0.42	0.15	-0.31	-0.03
Morocco	0.02	0.32	0.39	-1.01	-0.01	0.09	-0.03
Netherlands	0.83	0.68	0.68	1.46	0.46	1.17	0.88
New Zealand	0.40	0.01	0.72	1.37	-0.23	1.27	0.59
Norway	0.62	0.27	0.55	1.67	0.11	1.27	0.75
Panama	0.02	0.29	0.25	-1.13	-0.01	0.22	-0.06
Peru	0.04	0.52	0.19	-0.68	0.01	-0.01	0.01
Philippines	0.04	0.17	0.17	-1.31	-0.37	0.44	-0.14
Poland	0.31	1.18	0.55	1.07	0.09	0.36	0.59
Portugal	0.28	0.83	0.26	1.10	-0.05	0.93	0.56
Qatar	0.30	-0.14	0.94	-0.32	1.00	0.56	0.39
Republic of Korea	5.12	0.91	0.82	1.37	0.49	0.85	1.59
Romania	0.14	0.31	0.25	-0.14	-0.03	0.00	0.09
Russian Federation	0.37	1.44	0.97	0.52	0.98	-0.40	0.65
Saudi Arabia	0.26	0.09	0.66	-0.69	0.58	0.56	0.24
Serbia	0.06	0.21	0.37	0.11	0.16	0.12	0.17
Singapore	1.04	1.39	0.99	2.19	-0.27	0.91	1.04
Slovakia	0.18	0.60	0.62	0.59	-0.30	0.57	0.37
Slovenia	0.36	0.25	0.45	1.33	-0.50	0.47	0.39
Spain	0.35	2.60	0.54	0.92	0.63	0.72	0.96
Sweden	0.71	0.72	0.70	1.60	0.24	1.27	0.87
Switzerland	0.66	2.11	0.69	1.81	0.61	1.53	1.24
Thailand	0.20	0.70	0.94	-0.49	-0.26	-0.40	0.12
Trinidad and Tobago	-0.01	0.00	0.76	-0.45	-0.05	-0.29	-0.01
Tunisia	-0.05	0.32	0.24	-0.77	-0.02	0.72	0.07
Turkey	0.17	1.42	0.02	0.23	0.87	-0.38	0.39
Ukraine	0.01	0.22	0.14	0.13	0.00	0.24	0.12
United Kingdom	0.67	2.24	0.62	1.27	1.18	1.33	1.22
United States	1.68	1.25	0.71	1.10	1.42	1.25	1.24
Uruguay	0.09	-0.08	0.55	-0.32	0.13	0.55	0.15
Vietnam	0.04	1.01	0.45	0.54	-0.15	-0.14	0.29