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Depletion and Development:

Natural resource supply with endogenous field opening

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Depletion and development: natural resource supply with endogenous field opening

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Abstract

Supply of a non-renewable resource adjusts through two margins: the rate at which new fields are opened, and the rate of depletion of open fields. The paper combines these margins in a model in which there is a continuum of fields with varying capital costs. Opening a new field involves sinking a capital cost, and the date of opening is chosen to maximize the present value of the field. Depletion of each open field follows a Hotelling rule, modified by the fact that faster depletion reduces the amount that can ultimately be extracted. The paper studies the equilibrium paths of output and price. Under specific but reasonable assumptions on demand and the cost distribution of deposits it is found that the rate of growth of price is constant and independent of the rate of interest, depending instead on characteristics of demand and geology.

Keywords: non-renewable resource, depletion, exhaustible, Hotelling, fossil fuel, carbon tax.
JEL classification: D9, Q3, Q4, Q5

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1. Introduction:

The benchmark theory of non-renewable resource extraction is that of Hotelling (1931). Its insight is that, because of intertemporal arbitrage, the rent element of the resource price should be expected to increase at the rate of interest. This is often interpreted as a theory of the resource price and hence also of the flow of resources extracted and sold at each period of time.

There are several problems with this interpretation. The first is that there are two elements in the Hotelling condition. One is the resource price, and the other is the level of extraction costs. The condition links changes in the difference between these two variables to the rate of interest. Evidently (and as widely recognized), this provides no predictive power for the movement of the resource price without a theory of extraction costs; varying extraction costs could, in principle, make any price path compatible with Hotelling relationship. The second is that simple models that generate the Hotelling result typically make no distinction between the intensive and extensive margins at which supply decisions are made. By the intensive margin, we mean the rate of depletion of existing oil fields or mineral deposits. The extensive margin is the development and opening of new fields, an action that typically involves large sunk costs. These sunk costs are now the greater part of the costs of extracting a resource, and should be explicitly taken into account in a model of resource extraction. The third problem is empirical: as is well known, the change in prices of non-renewable resources has little or no relationship with the rate of interest (for discussion of the issues and survey of evidence see Gaudet 2007 and Livernois 2009).

This paper addresses the first two of these issues. First, it develops a model of field (intensive margin) depletion in which a faster rate of depletion may reduce the overall yield of a field, building on the work of Nystad (1985, 1987). Extraction costs increase with the rate of depletion and are 'iceberg', using up the resource itself. These assumptions are supported in the technical literature on oil extraction which suggests that faster depletion means that less of the resource may be ultimately recoverable. The approach also provides a transparent way of highlighting the crucial role of technical flexibility in extraction. The extent to which price changes are linked to the rate of interest varies with a technical parameter that captures this flexibility. The pure Hotelling case holds if the rate of depletion is completely flexible, while the other extreme is complete inflexibility in which case output from a particular field declines at a constant exponential rate, regardless of price and the interest rate.[1]

[1] If fields are flexible, as in the Hotelling-like cases, short run supply elasticities might be expected to be large (although dependent on price expectations). Industry experts use extremely low supply elasticities (the US Energy Information Administration uses short-run supply elasticity of 0.02 and long run 0.1, see Smith 2009), on the basis of limited ability to adjust depletion rates.

Second, we develop a model of the extensive margin, i.e. the development of new fields. The key element here is that capital has to be sunk before a new field is opened, a feature that accords with reality and is a quantitatively important feature of major mining developments and oil investments in offshore and deep fields.[2] Fields differ in capital cost per unit reserve and producers decide when to sink capital in order to open a field.[3] It is this that produces, in equilibrium, a sequence of field openings through time.

Combining the two models yields a central result that there exists a combination of demand, technological and geological characteristics such that price grows at a constant rate that is independent of the rate of interest. The rate of price increase depends on demand parameters (price elasticity and growth) and features of the geology and technology of supply (intensive margin yield curve and extensive margin distribution of costs of developing new fields). This outcome is consistent with intertemporal arbitrage. At the intensive margin depletion rates on individual fields adjust according to price growth, the rate of interest, and the flexibility of technology. At the extensive margin, owners of fields choose the date at which to open the field and start production, the choice depending on (amongst other things) the rate of interest and the rate of price increase. Yet long-run price growth is, as one would expect for other commodities, determined by fundamentals of demand and supply.

This central result holds in the long-run, and the paper also looks at the medium and short run effects of demand or supply shocks. Because of the fixed costs of field development the level, as well as the rate of change, of the price is important in shaping response to shocks. Thus, a permanent proportional price reduction postpones field opening, reducing the quantity produced in the short run, raising it in the long run, and reducing the cumulative quantity produced at all future dates. A permanent reduction in the rate of growth of price increases production in the short run (bringing forward depletion of existing fields and, temporarily, field opening) but, in the long run, the lower price reduces both the flow of output and the cumulative quantity extracted. It follows that, while demand reduction policies motivated by climate change may bring forward depletion of existing fields (the 'green paradox' noted by Sinn 2008), they will also cause postponement of the development of new fields so that overall supply and emissions are reduced.

The literature on resource depletion is extensive. Our focus on the opening of new fields is in contrast with much of the literature where additions to stock are modelled as the outcome of a continuous variable (exploration) that adds to the capacity and reduces

[2] In the Middle East and onshore North America 'lifting costs' and 'finding and development costs' are approximately equal. Offshore and in other regions of the world, finding and development costs per barrel are 3-4 times larger than lifting costs (Energy Information Administration 2011).

[3] Other sources of field heterogeneity include mineral quality (Cairns and Laserre 1986) and pollution intensity (Chakravorty et al. 2008).

extraction costs of the existing field (as in Pindyck 1978, Dasgupta and Heal 1979).[4] The sequence in which new fields is opened is studied in a variety of contexts (for a recent example see Chakravorty et al. 2008 where resources differ by pollution intensity). Existing literature in which there are field set-up costs includes Hartwick et al. (1986), Holland (2003), and Livernois and Uhler (1987). Hartwick et al. assume zero extraction costs, in which case only one field is operated at any time, and Holland (2003) looks at cases where marginal extraction costs are either constant or infinite. Livernois and Uhler (1987) look at the rate of discovery of new fields with field-specific extraction costs, characterising first order conditions for the problem but doing little subsequent analysis of the equilibrium. We go beyond these models, fully integrating intensive and extensive margin choices.

The next two sections of the paper concentrate on depletion of open fields, taking the dates at which new fields are opened as exogenous. This gives a simple demonstration of how the relationship between the resource price and the rate of interest depends on the ease with which the rate of depletion can be varied. Sections 4 and 5 endogenise field opening decisions and derive aggregate supply. Sections 6 and 7 look at the market equilibrium of the full model and derive the central results of the paper, the long run independence of the rate of increase of price from the rate of interest. They also conduct a series of comparative dynamic experiments.

2. Field depletion:

We look first at choice of the rate at which to deplete a single field (or deposit). Our central assumption is that varying the rate of depletion may be costly, as increasing the rate reduces the yield of the field. For example, too rapid pumping or extraction may reduce the overall recoverable capacity of the field. This approach has foundations in the energy literature, as we discuss later in the section. It also has the advantage of allowing easy aggregation of fields, as is necessary when we move to the multi-field model. Using this approach, the current costs of extraction are recoverable output foregone. The capital costs associated with opening a new field are described in section 4.

Formally, output of a particular field at date t is $xq(z)$, where x is the stock remaining and z is the rate of depletion, defined as the proportionate rate of decline of remaining stock, so $\dot{x} = -xz$. While z is the rate of depletion of the field and xz is the reduction in the stock, $xq(z)$ is the recovered output. The expression $q(z)/z \leq 1$ is the yield curve, giving the fraction of the reduction in stock that is marketable output. All current extraction costs are

[4] See Krautkraemer (1998) for a survey. Swierzbinski and Mendelsohn (1989) aggregate separate fields, but assuming no fixed costs and constant returns to scale in exploration and extraction.

subsumed in this yield curve.[5] The rate of depletion is non-negative, and we also allow for the possibility that there is a minimum value of *z*, denoted $z_m \geq 0$ below which *q*(*z*) = 0. For $z > z_m$, *q*(*z*) is increasing and concave in *z*, with strict concavity implying that increases in the rate of depletion yield less than proportionate increases in output. An example is given below and illustrated in figure 1.

The rate of depletion is determined by profit maximization. The present value at date *t* of a field with stock *x* is $PV \equiv \int_t^\infty pxq(z)e^{-\rho(\tau-t)}d\tau$, where *p* is the price, time varying but exogenous to the firm, and *ρ* is the interest rate. The firm's problem is to choose *z* to maximize this subject to depletion, i.e. maximize

$$PV \equiv \int_t^\infty pxq(z)e^{-\rho(\tau-t)}d\tau \quad \text{subject to } \dot{x}/x = -z, \quad z \geq 0,\ x \geq 0, \text{ and given } x(t). \tag{1}$$

We assume that as $t \to \infty$ the rate of growth of price tends to some constant exponential growth rate, denoted $\hat{p}_\infty$, less than or equal to *ρ*; this ensures that the objective is bounded.[6] The maximization problem can be solved by calculus of variations (see appendix 1) or by the following more direct argument. Consider a perturbation *dz* in the rate of depletion at date *t*. The value of this to profits at date *t* is *pxq'*(*z*)*dz*. At all future dates $\tau \geq t$, the stock of reserves is lower by amount $dx(\tau) = -x(\tau)dz$.[7] Differentiating the objective in (1) with respect to this change in the path of *x* gives the marginal cost of the perturbation as $dz\int_t^\infty pxq(z)e^{-\rho(\tau-t)}d\tau$. The change in *PV* is therefore

$$\frac{dPV}{dz} = pxq'(z) - \int_t^\infty pxq(z)e^{-\rho(\tau-t)}d\tau. \tag{2}$$

This must be equal to zero at an interior maximum, a condition known as the Hotelling valuation condition.[8] The condition holds for perturbations at all dates and, differentiating with respect to *t*, (appendix 1) gives the Euler equation

$$\frac{\dot{p}}{p} - z + \frac{q''(z)\dot{z}}{q'(z)} = \rho - \frac{q(z)}{q'(z)}, \quad \text{or} \quad \dot{z} = \left[\rho - \frac{\dot{p}}{p} + z - \frac{q(z)}{q'(z)}\right]\frac{q'(z)}{q''(z)}. \tag{3}$$

[5] The value of resource foregone in extraction costs is, with resource price *p*, $(z - q(z))px$. This is more restrictive than some of the literature, in which costs are modelled as a function of extraction and the stock of resource remaining. For example, Dasgupta and Heal (1979) assumes that costs are increasing in extraction and decreasing in remaining stock. The rate of extraction is the ratio of these variables. Nystad (1985,1987) assumes that recoverable output is a concave function of a (constant) peak depletion rate.

[6] We use ^ to denote a proportionate rate of change, so $\hat{p} \equiv \dot{p}/p$.

[7] For $\tau > t$, $x(\tau) = x(t)\exp(-\int_t^\tau z(s)ds)$. Differentiating with respect to *z*(*t*) gives $dx(\tau) = -x(\tau)dz(t)$

[8] See Miller and Upton 1985. It says that the market value of reserves (the second term) is equal to the current net price (*pq'*) times the amount of reserves.

This is a differential equation for *z*, depending on the difference between the rate of interest and rate of price increase, and also on the curvature of *q*(*z*), indicating the yield loss from varying the rate of depletion. Equation (3) has stationary value *z** at

$$\rho - \hat{p}_\infty = q(z^*)/q'(z^*) - z^*, \quad \text{or} \qquad z^* = Z(\rho - \hat{p}_\infty), \qquad Z' > 0. \tag{4}$$

The function $Z(\rho - \hat{p}_\infty)$ summarizes the long-run relationship. If the rate of growth of price is constant for all *t* then *z* simply jumps to the stationary value and remains constant.[9] For more general price paths which converge to $\hat{p}_\infty$, concavity of *q*(*z*) ensures that *z* converges to the stationary value *z** given by (4).

The slope of Z is $Z' = -qq''/(q')^2$ which is positive if *q*(*z*) is strictly concave (for $z > z_m$). Thus, a higher value of $(\rho - \hat{p}_\infty)$ raises the rate of extraction, and the slope of this relationship depends on the concavity of *q*(*z*) . Since $z \geq 0$ (depletion cannot be negative) there is a minimum value of $(\rho - \hat{p}_\infty)$ consistent with an interior solution. In the limiting case in which *q*(*z*) is linear the relationship $Z(\rho - \hat{p}_\infty)$ becomes vertical at $\rho - \hat{p}_\infty = 0$. This is the pure Hotelling case; if price increases faster than *ρ* then depletion is delayed, *z* = 0, and if it increases more slowly depletion is instantaneous.

This modeling of extraction costs and depletion is grounded in the technical literature on resource depletion, particularly in the oil sector. In this literature the benchmark assumption is that output from a field follows an exponential rate of decline (Adelman 1990, 1993); in our framework this would mean constant *z*.[10] Varying the rate of depletion has a cost primarily by its impact on total recoverable reserves. This variation is typically achieved by altering the rate of water or gas injection which pressurizes the well, and its effects are geology dependent; Nystad (1985, 1987) categorises fields as ‘Hotelling’, ‘intermediate’, and ‘geosensitive’, in increasing order according to loss of recoverable reserves from faster depletion. This is captured in the relationship *q*(*z*).

Understanding these relationships is facilitated by a particular functional form that will be used in simulations later in the paper. Suppose that *q*(*z*) takes the form:

$$z > m\lambda, \quad q(z) = a(z - m\lambda)^{1-\lambda}, \text{ with } 1 \geq a > 0,\ m \geq 0, \text{ and } \lambda \leq 1: \tag{5}$$

$$z < m\lambda, \quad q(z) = 0,$$

[9] Derivation of the optimal depletion rate is simple in this case as the integral in (1) is $\int_0^\infty p_0 x_0 q(z) e^{(\hat{p}-z-r)\tau} d\tau = p_0 x_0 q(z) / [r + z - \hat{p}]$ and condition (4) comes from setting the derivative of this with respect to *z* equal to zero.

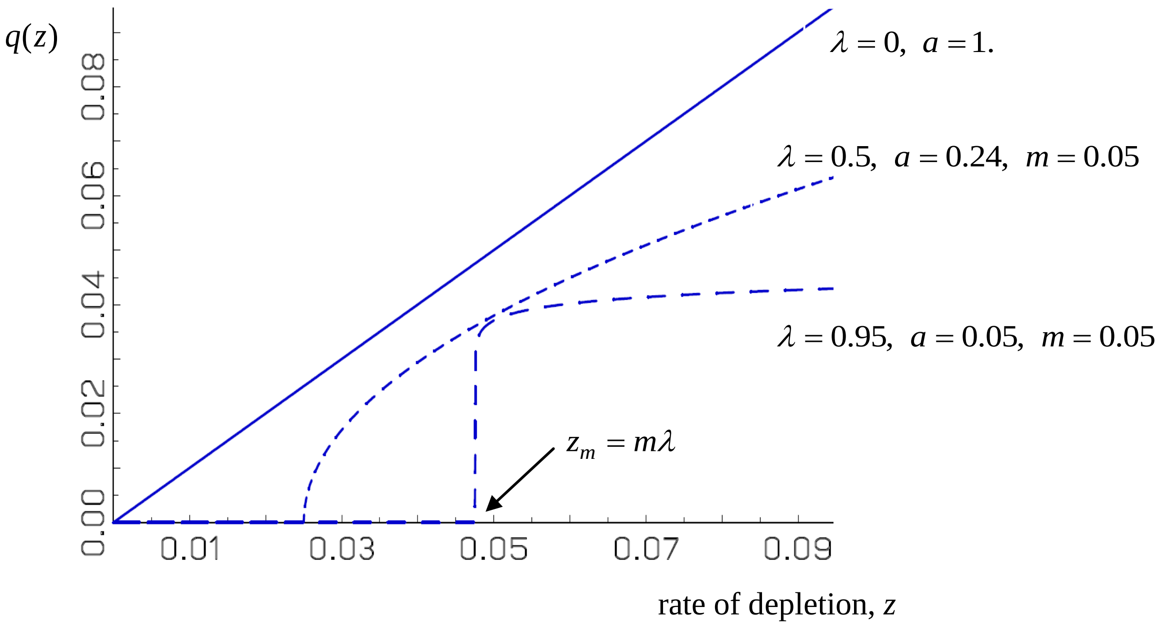
[10] A constant *z* means exponential decline in remaining stock *x*, and hence in output *q*(*z*)*x*.

(so $z_m = m\lambda$). With this specification the Euler condition (3) and long-run value of the rate of depletion (4) are,

$$\dot{z} = \left(\frac{z - m\lambda}{\lambda}\right)\left[\frac{\dot{p}}{p} - \rho + \frac{\lambda(z-m)}{1-\lambda}\right], \quad z^* = m + \frac{(1-\lambda)(\rho - \hat{p}_\infty)}{\lambda}. \tag{6}$$

Examples are given in figure 1. The key parameter is λ which captures the 'geosensitivity' of the field, and hence also the extent to which optimal depletion is sensitive to price. The slope of $Z(\rho - \hat{p}_\infty)$ is increasing in λ, $Z' = -qq''/(q')^2 = \lambda/(1-\lambda)$. If $\lambda = 0$ (solid line in figure 1) the rate of depletion is infinitely sensitive to the gap between $\hat{p}$ and ρ so continuing extraction over an interval of time is possible only if these are equal. At the other extreme, as $\lambda \to 1$ the rate of depletion is equal to $m \to z_m$, and completely independent of the rate of price increase or rate of interest (the long-dashed line has $\lambda = 0.95$). This is consistent with Adelman's (1990) view that the rate of depletion from a particular reservoir is quite insensitive to price, and well approximated by a constant exponential rate of decline (at rate *m* in this specification). For cases with intermediate degrees of 'geosensitivity' depletion is faster (the extraction path is more tilted towards the present) the larger is $\rho - \hat{p}$.

Figure 1: Examples of extraction costs, $q(z) = a(z - m\lambda)^{1-\lambda}$



3. Market equilibrium with exogenous field opening

The equilibrium price path is determined by depletion of open fields, supply from new fields that are opened, and the level and change in demand. We now put together supply (from all fields) and demand in order to derive the equilibrium price path, conditional on a given rate of field opening; (field opening is endogenised in the next section). At each date a number of fields are open, and the stock remaining in all open fields is denoted *R*. The volume of new reserves opened at each date, expressed relative to the stock *R*, is denoted *r*; in this section *r is* taken to be exogenous. We assume that open fields differ only in the stock of reserves remaining, and all have the same function *q*(*z*); the rate of depletion at any date is therefore the same in all fields The total stock of reserves in open fields changes according to

$$\hat{R} = r - z \ . \tag{7}$$

This is simply the rate at which reserves are opened minus the rate at which they are depleted.

Total supply is $Q_S = q(z)R$, so changes according to

$$\hat{Q}_S = \frac{q'(z)}{q(z)}\dot{z} + r - z \ . \tag{8}$$

The path of *z* comes from the Euler equation (3). Using this with our particular functional form, i.e. using (5) and (6) in (8), the rate of growth of supply is, $\hat{Q}_S = r - m + (\hat{p} - \rho)(1-\lambda)/\lambda$. The growth of demand is $\hat{Q}_D = g - \eta\hat{p}$, where *g* measures an exogenous shift in the demand curve and η is the price elasticity of demand (both *g* and η being exogenous but potentially time varying). The equilibrium rate of change of price comes from equating the growth rates of supply and demand, giving Proposition 1.

Proposition 1:

If $q(z) = a(z - m\lambda)^{1-\lambda}$, new reserves open at rate *r*, and $r > g + m - \eta\rho$, then the resource price increases at rate:

$$\hat{p} = \frac{\lambda(g - r + m) + (1-\lambda)\rho}{\lambda\eta + 1 - \lambda} \ . \tag{9}$$

The proposition demonstrates how the role of the interest rate depends on the cost of varying the rate of depletion, λ. If λ = 0 then faster depletion does not decrease yield, so supply is infinitely sensitive to $\hat{p} - \rho$ and the equilibrium price must satisfy $\hat{p} = \rho$. λ = 1 gives the opposite case in which there is no flexibility; the rate of depletion is simply the constant *m*. The equilibrium $\hat{p}$ is then the difference between the rate of growth of demand, *g*, and supply, $r – m$ (the rate of opening of new fields minus depletion of open fields), divided by the price elasticity of demand.

Notice that proposition 1 and eqn. (9) are, given the technology, properties that hold at all t and do not require constant proportionate rates of change. If the instantaneous rate of growth of demand or of field opening change, then so too do the rate of price increase and the rate of depletion of existing fields, as given by (9) and (6) respectively. The restriction $r > g + m - \eta\rho$ ensures that $\hat{p} - \rho > 0$, this ensuring that the optimization problem has interior solution (as assumed by use of the Euler condition to derive equation (8)). Failing this, there is a corner solution to the optimization problem, with $z = 0$. For example, if the rate of growth of demand, g, is extremely high and field opening r is low, then price growth is rapid; producers cut depletion to zero, a corner solution of problem (1). We do not analyse the equilibrium in this case, restricting analysis throughout the paper to interior solutions.

4. Field opening

To complete the supply side of the model we now endogenise field opening. We start by looking at the supply responses of firms, so in this and the next section the path of price is taken to be exogenous, returning in section 6 to market equilibrium determination of the price. We suppose that there is a continuum of fields all of which are known at date 0, and are owned by price-taking profit maximizing agents. Each field contains one unit of the resource but cannot produce until a field specific fixed cost $Ke^{-\theta T}$, $\theta \geq 0$, has been paid, where $e^{-\theta T}$ captures technical progress in field development that has taken place by date $t = T$ when the field is opened.[11] K varies across fields, and we will use K as the index of field types, with $K > 0$ and running to plus infinity. The date at which a particular field is opened is endogenous and the number (measure) of fields of type K is $S(K)$.[12]

Profits are as before, but we now look at the choice of date T at which to open a field with fixed cost K. The present value at date 0 of profits on field type K opened at T is,

$$PV(K,T) \equiv \int_T^{\infty} px(t,T)q(z)e^{-\rho t}dt - Ke^{-(\theta+\rho)T}, \tag{10}$$

$$\text{with } x(t,T) = \exp\left[-\int_T^t z(\tau)d\tau\right] \text{ for } t \geq T.$$

The firm is a price-taker, and in this expression p and z are functions of time (z given by

[11] All capital spending is incurred at the date the field is opened. Campbell (1980) develops a model in which this timing of investment is optimal.

[12] Assuming each field contains one unit of resource is without loss of generality as K can be interpreted as capital cost per unit capacity. The total stock of resource in fields of type K is $S(K)$.

equation (3)).[13] The stock remaining in the field at each date, $x(t, T)$, depends on cumulative depletion from the date at which the field opened. The profit maximizing date T, at which to spend $Ke^{-\theta T}$ and open the field is therefore given by

$$\frac{\partial PV(K,T)}{\partial T} = -pq(z)e^{-\rho T} + z\int_T^{\infty} px(t,T)q(z)e^{-\rho\tau}dt + (\theta+\rho)Ke^{-\theta T} = 0\,. \qquad (11)$$

Intuition comes from thinking about the value of delaying the date of opening by dT. Revenue in this instant is foregone, giving the first term on the right hand side (derivative with respect to the lower limit of integration). Not depleting in this instant raises the remaining stock at all future dates by $\partial x(t,T)/\partial T = z(T)x(t,T)$, the value of which is the second term. The third term is the benefit of pushing the capital cost, K, further into the future.[14]

The implications of this are most readily seen by looking at the case in which price is growing at constant rate $\hat{p}$ (with price level p_0 at $t = 0$), so z is at its stationary value z^* and $x(t,T) = e^{-z^*(t-T)}$, $t \geq T$, The present value of profits on field K, equation (10) is then

$$PV(K,T) = p_0 q(z^*)e^{z^*T}\int_T^{\infty} e^{(\hat{p}-\rho-z^*)t}dt - Ke^{-(\theta+\rho)T}$$
$$= \frac{p_0 q(z^*)e^{(\hat{p}-\rho)T}}{z^*+\rho-\hat{p}} - Ke^{-(\theta+\rho)T} = p_0 e^{(\hat{p}-\rho)T}q'(z^*) - Ke^{-(\theta+\rho)T} \qquad (12)$$

where the second line comes from integrating and then simplifying using equation (4). The first and second order conditions for choice of T are

$$\frac{\partial PV(K,T)}{\partial T} = (\hat{p}-\rho)p_0 e^{(\hat{p}-\rho)T}q'(z^*) + (\theta+\rho)Ke^{-(\theta+\rho)T} = 0\,, \qquad (13)$$

$$\frac{\partial^2 PV(K,T)}{\partial T^2} = -(\hat{p}+\theta)(\theta+\rho)Ke^{-(\theta+\rho)T} < 0\,, \qquad (14)$$

(where the second order condition is evaluated at $\partial PV/\partial T = 0$). The first order condition, equation (13), gives the profit maximizing date of opening fields of each type K if two conditions are satisfied, $\hat{p}-\rho < 0$ and $\hat{p}+\theta > 0$. The first of these is necessary since, as $\theta + \rho > 0$, equation (13) can have an interior solution only if $\hat{p}-\rho < 0$. The other is the second

[13] The field owner's objective, eqn. (10), is written in terms of a field of size one ($x(0) = 1$) developed at cost K. Setting the size of each field at unity is a normalization, and the key measure is size per unit capital cost. Furthermore, K can be thought of as the expected sunk cost, rather than the actual one. The role of K is to induce a sequence of dates of field openings, and the realization of K plays no role in the model. This important source of uncertainty is therefore consistent with our framework.

[14] The first order condition is sometimes known as the r (or ρ) percent stopping rule. For an application to resource extraction see Cairns and Davis (2007).

order condition, requiring that $\hat{p}+\theta>0$. These conditions are intuitive. If $\hat{p}-\rho>0$ then field opening is postponed indefinitely. And if $\hat{p}+\theta<0$ then a declining price means that all fields should be opened instantaneously.

Our primary interest is the rate of field opening through time, so it is helpful to rearrange to give the field type, *K*, which opens at date *T*. The general form comes from equation (11) which, with constant price growth, becomes (13) and hence

$$K(T)=\frac{p_0 e^{(\theta+\hat{p})T} q'(z^*)(\rho-\hat{p})}{(\rho+\theta)}, \quad \text{and } \hat{K}=\hat{p}+\theta \geq 0. \qquad (15)$$

While this expression is derived with the assumption of constant proportional price growth, some important general points come from it. First, fields open in increasing order of *K*, so that at any date *T* all fields with $K \leq K(T)$ are open.[15] Second, the rate of change of *K* is $\hat{K}=\hat{p}+\theta$, and does not depend on the rate of interest. This is because, for a particular field, both revenue and costs in (10) are discounted by ρ, while revenue is increasing at $\hat{p}$ and costs decreasing at rate θ. It is this rate of change of *K* that drives the rate of field opening, and hence *r*, additions to the stock of reserves.

5. Aggregate supply

We now aggregate over fields to derive the stock of reserves in open fields, *R*, and hence aggregate supply, $q(z)R$, for a given price path. Fields of type $K(T)$ open at date *T*, and the measure of fields of type *K* is $S(K)$. The total number of fields that are open at date *t* is therefore $\int_{-\infty}^{t} \dot{K}(T) S(K(T)) dT$, i.e. the integral over all previous dates of the set of field types that opened at each date, $\dot{K}(T)$, times the number of fields of type *K*, $S(K(T))$. Open reserves *R* are given by

$$R=\int_{-\infty}^{t} \dot{K}(T) S(K(T)) x(t,T) dT. \qquad (16)$$

R moves according to differential equation

$$\dot{R}=\dot{K}S(K)-zR, \qquad (17)$$

derived by differentiating (16) with respect to *t* and using $x(t,t)=1$ and $\dot{x}=-zx$. This is the same as equation (7) of section 3, with *r* now taking the form $r=\dot{K}S(K)/R$.

The number of fields of each type, *S*(*K*), is clearly crucial for long run supply, as it

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[15] This is general, since , $dT/dK=-\left(\partial^2 PV/\partial T\partial K\right)/\left(\partial^2 PV/\partial T^2\right)>0$ with $\partial^2 PV/\partial T^2<0$ by the second order condition and $\partial^2 PV/\partial T\partial K>0$ by inspection of (11).

captures the geology of available reserves. We assume that this relationship is iso-elastic, with $S(K) = sK^{-(\sigma+1)}$. Parameter *s* is a constant scale factor, and σ measures the rate at which the number of fields associated with each value of *K* declines.[16] We assume that σ > 0, as is necessary for the remaining resource stock to be finite. If $\bar{K}$ fields are open, the stock remaining unopened is then $\int_{\bar{K}}^{\infty} S(K)dK = s\int_{\bar{K}}^{\infty} K^{-\sigma-1}dK = s\bar{K}^{-\sigma}/\sigma$. (The integral is unbounded if σ < 0). Since opening a field (of size unity) costs *K*, this relationship implies that 1/ σ measures the rate at which the costs of opening a field rises as remaining (unopened) reserves decline.

This iso-elastic form of *S*(*K*) is a crucial assumption, implying that if the rate of growth of price is constant, then so too is the long-run rate of growth of output. In the following section (looking at the full market equilibrium), it is this assumption, together with a constant rate of growth of demand and an iso-elastic demand curve, that give a long run equilibrium path in which all variables are changing at constant exponential rates.

With $S(K) = sK^{-(\sigma+1)}$ the dynamics of *K* and *R* (and hence of output, $Q_S = q(z)R$) can be derived analytically for the case of constant price growth. Constant price growth means that *z** and $\hat{K}$ are both constant (equations (3) and (15) respectively. The differential equation for open reserves, (17), then has explicit solution,

$$R = K^{-\sigma}\left[\frac{s\hat{K}}{z^* - \sigma\hat{K}}\right] + e^{-z^*t}\left[R_0 - \frac{s\hat{K}K_0^{-\sigma}}{z^* - \sigma\hat{K}}\right] \qquad (18)$$

where K_0 and R_0 are the values of *K* and *R* at date zero. The effect of these initial values goes to zero with e^{-z^*t}, so *R* converges asymptotically to path given by $R = K^{-\sigma}\left[s\hat{K}/\left(z^* - \sigma\hat{K}\right)\right]$. The denominator of this is *r*, newly opened reserves relative to the existing stock, $r = z^* - \sigma\hat{K}$ (see (17)), and must be positive: we give parameter restrictions sufficient to ensure this in the next section. The terms in square brackets in (18) are constant, so the long run rate of change of open reserves is $\hat{R} = -\sigma\hat{K} = -\sigma(\hat{p} + \theta)$, (the second of these equalities coming from equation (15) and subsequent discussion). Thus, with σ > 0, open reserves decline exponentially.[17] Furthermore, since $Q_S = q(z^*)R$, output also declines at rate $\hat{Q} = -\sigma(\hat{p} + \theta)$. We summarize these properties as follows.

[16]This relationship can be given a micro-foundation. The size distribution of oil fields is well approximated by a power law (Laherrere 2000). If the elasticity of capital costs with respect to field size is less than unity and greater than the absolute value of the exponent in the power law, then the relationship $S(K) = sK^{-\sigma-1}$ with σ > 0 follows (see appendix 2).

[17] σ > 0 ensures both that the stock is finite and that, as $K \to \infty$, open reserves tend to zero.

Proposition 2:

If price grows at constant rate $\hat{p}$ at all dates and $\hat{p} \in [-\theta, \rho]$, then:

i) z, the rate of depletion of each field is constant, and is faster the larger is $\rho - \hat{p}$ (equation (4)).

ii) Fields open in increasing order of their sunk cost per unit reserve, K, with type of field opening at date T given by $K(T) = \frac{p_0 e^{(\theta+\hat{p})T} q'(z^*)(\rho - \hat{p})}{(\rho + \theta)}$, and $\hat{K} = \hat{p} + \theta \geq 0$ (equation (15)).

If, additionally, the number of fields of type K is $S(K) = sK^{-\sigma-1}$, with σ > 0 then:

iii) The rate of growth of open reserves and of supply converge asymptotically to

$$\hat{Q}_S = \hat{R} = -\sigma(\hat{p} + \theta) < 0.$$

iv) On the long run (asymptotic) growth path R and Q are given by

$$R = K^{-\sigma}\left[\frac{s(\hat{p}+\theta)}{z^* - \sigma(\hat{p}+\theta)}\right], \qquad Q_S = q(z^*)R. \tag{19}$$

Section 7.1 explores the comparative dynamics of these supply paths, but two points about comparisons of long run (asymptotic) paths can be made immediately. First, comparing two such paths, the one with higher $\hat{p}$ has more rapid field opening and more rapidly declining reserves and output. Second, comparing two paths, that with higher initial price level, p_0, has more fields opened at each date (higher K, (15)) and lower open reserves and less supply at each date (19). From (15) and (19) the long run elasticity of supply with respect to the price level is therefore $-\sigma$ (the exponent on K in (19)). The intuition behind this negative supply elasticity is that a higher level of prices means that more fields have been opened and (partially) depleted so current output is lower. We emphasize that this is a comparison across asymptotic growth paths. The effects of shocks to these paths are discussed in section 7, after establishing equilibrium prices.

6. Market equilibrium:

To characterize the market equilibrium we add demand and endogenise price. The demand curve is now assumed to have constant price elasticity $\eta > 0$, constant exogenous rate of growth g, and level parameter D,

$$Q_D = Dp^{-\eta}e^{gt}, \text{ so } \hat{Q}_D = g - \eta\hat{p}. \tag{20}$$

The equilibrium price path comes from equating Q_D to Q_S. Section 5 established that if price is growing at a constant rate in the interval $[-\theta, \rho]$ and $S(K) = sK^{-(\sigma+1)}$ then the long run rate of growth of supply is constant at $\hat{Q}_S = -\sigma(\hat{p} + \theta)$ (proposition 2). Equating this with the rate of growth of demand, the asymptotic quilibrium rate of growth of price is

$$\hat{p} = \frac{g + \sigma\theta}{\eta - \sigma}. \tag{21}$$

Recalling that -σ is the (asymptotic) price elasticity of supply, this expression links a demand shift (demand growth g) to price change via elasticities of supply and demand in the usual way.[18] A number of points are noteworthy.

First, the long run equilibrium rate of price increase is independent of the rate of interest. A higher interest rate means faster depletion of existing fields, as we saw in section 2. However, once the extensive margin is included in the supply response the long run rate of growth of price depends on parameters of demand and supply (geology), and not at all on the interest rate.

Second, we assume that $\eta - \sigma > 0$. In a static supply and demand context a demand shift changes price according to the shift divided by the elasticity of demand plus the elasticity of supply, and equilibrium is stable if the sum of these elasticities is positive. Equation (21) is analogous, linking changes in the growth of demand to the growth of price; (although since this is comparing growth paths the stability analogy is not exact).

Third, the value of $\hat{p}$ given by equation (21) does not necessarily lie in the interval $[-\theta, \rho]$ as required in proposition 2. The parameter restrictions necessary for this to hold are, providing $\eta - \sigma > 0$, that $\rho(\eta - \sigma) - \sigma\theta > g > -\eta\theta$. If the second of these inequalities fails then $\hat{p} + \theta < 0$ and the second order condition (14) fails; firms will seek to open all field instantaneously. If the first fails then $\hat{p} - \rho > 0$ and firms will postpone opening indefinitely (equation (13)). It is beyond the scope of this paper to investigate all these regimes.[19] We simply note that the equilibrium described here applies only in the subset of parameter space indicated by the above inequality. A sufficient condition for these inequalities to be satisfied is that the price elasticity of demand, η, is large enough. As discussed in the previous section we also require that $r = sK^{-\sigma}\hat{K}/R = z^* - \sigma\hat{K} = z^* - \sigma(\hat{p} + \theta) > 0$. The rate of depletion, z^*, is as given in section 2, and appendix 3 gives sufficient conditions for this to be satisfied.

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[18] Notice that $r = sK^{-\sigma}\hat{K}/R = z - \sigma\hat{K} = z - \sigma(\hat{p} + \theta)$. In the example of section 2 with $q(z) = a(z - m\lambda)^{1-\lambda}$, eqn. (6) gives the long run value of z. Substituting this gives $r = m - \sigma\theta + \{\rho(1-\lambda) - \hat{p}(\lambda\sigma + (1-\lambda))\}/\lambda$. Using this in eqn. (9) for equilibrium $\hat{p}$ gives (21).

[19] Modifications of the model would then be desirable. For example, capital costs of drilling fields could become endogenous to spikes in drilling activity (for evidence, see Toews and Naumov 2014).

With these conditions, long run equilibrium values of other variables in the system follow directly from the price growth given by (21) together with proposition 1. The long run rates of growth of open fields, open reserves, and output are

$$\hat{K} = \frac{g + \eta\theta}{\eta - \sigma} \geq 0, \qquad \hat{Q} = \hat{R} = \frac{-\sigma(g + \eta\theta)}{\eta - \sigma} < 0. \tag{22}$$

The initial price equates supply and demand so, using (14) and (18) in (19) at $T = 0$, p_0 satisfies

$$p_0^{-(\eta-\sigma)} = \left(\frac{s}{D}\right) \cdot \frac{(\hat{p} + \theta) q(z^*)}{z^* - \sigma(\hat{p} + \theta)} \left[\frac{q'(z^*)(\rho - \hat{p})}{(\rho + \theta)}\right]^{\sigma}. \tag{23}$$

These properties of the long run equilibrium are summarized as:

Proposition 3:

If $S(K) = sK^{-(\sigma+1)}$ and parameters $\rho, \eta, g, \theta, \sigma$ are constant and satisfy $\rho(\eta - \sigma) - \sigma\theta > g > -\eta\theta$ then, on the long run (asymptotic) path:

i) The rate of growth of price is independent of the rate of interest, and given by $\hat{p} = (g + \sigma\theta)/(\eta - \sigma)$.

ii) The rate of depletion is constant and output is declining at rate $\sigma(g + \eta\theta)/(\eta - \sigma)$.

iii) The elasticity of the equilibrium price with respect to the level of demand is $1/(\eta - \sigma)$.

Comparing across long run equilibrium paths, parameters *s*, *D* and *ρ* determine the levels of variables, while other parameters also influence rates of change. For example, routine calculation indicates that a higher demand parameter, *D*, lower *s*, or higher *ρ* is associated with higher price and lower supply at all dates on the long-run path. The higher price is intuitive, and is associated with higher *K* at each date; this means that more has been depleted on the transition to the long run path, giving the lower level of output. Faster demand growth, *g*, or technical progress, *θ*, is associated with a lower value of *K* at each date. Field opening is postponed in anticipation of future demand or technical improvement. Correspondingly, current output is higher and price lower (as low *K* is associated with large *S*(*K*)); the rate of growth of price and rate of fall of output are larger.

These comparative dynamics are hard to interpret as they are the outcome of a long-run process which generally involves a transitional dynamic. We now turn, therefore, to the effect of shocks and the short and medium run price and quantity responses that they create.

7. Responses to shocks

Shocks create a new long run path to which the model converges, but adjustment is slow because open reserves are determined by the past history of field opening. We focus on demand shocks, looking at changes in both the level of demand, *D*, and the rate of growth, *g*. These shocks may be due to policy changes, such as climate policy. We proceed in two stages, looking first at the supply response, i.e. taking a price shock as given, and then turning to the full equilibrium response to shifts in the demand curve.

7. 1 Price shocks and supply.

Suppose that an unanticipated upwards jump in *p* occurs at date 0 and lifts the price path equi-proportionately at all future dates. Since this is a price level (not growth) effect it has no effect on the rate of depletion (intensive margin, equations (3), (4)), in which price enters only as future price growth. However, an increase in p_0 affects the extensive margin through the timing of field openings, causing an equi-proportionate increase in *K* as given by equation (15). An upwards jump in *K* means that a discrete number of new fields are opened as the shock occurs but, since $\sigma > 0$, fewer fields are opened at every date thereafter. This initial jump and subsequently lower rate of field opening works through into the stock of open reserves and hence output through equation (18). *R* jumps and then converges asymptotically to $R / K^{-\sigma} = \hat{K} / \left(z^* - \sigma\hat{K}\right)$; the right hand side of this expression is unchanged, but since $K^{-\sigma}$ is lower at each date, so too is *R*. Output is proportional to open reserves, so a permanent proportional price increase elicits a positive short to medium run supply response which turns negative as fewer new fields are opened. While the short run price elasticity of supply is positive, the long run supply elasticity is negative, as discussed above. Since field openings are brought forward, cumulative supply (cumulated from the date of the shock) and hence total resource extracted is increased by a positive price level shock.

We illustrate these effects, for a downwards price jump, on figure 2a in which the horizontal axis is time, solid lines give variables on the initial path, and dashed lines give variables with a 20% lower price at all dates.[20] The price fall causes a pause in field opening (the shift in *K*, top left panel). During this pause open reserves fall, as does output. Once field openings resume *K* is lower and *S*(*K*) is higher, so more capacity is opened at each date. Open reserves recover and overtake what they otherwise would have been. Combining the short run reduction in output and long run increase, the effect is to decrease cumulative output (and hence total stock depleted) at all dates. We summarize these effects in Proposition 4.

[20] See appendix 4 for details of simulation.

Proposition 4:

A permanent proportionate change in the price ($\hat{p}$ constant and unchanged) has no effect on the rate of depletion or the long run rate of growth of supply. A price increase brings forward the opening of fields. Supply increases before eventually falling below what it otherwise would have been (with long run price elasticity of supply of -σ). Cumulative supply is increased at all dates. A price decrease has reverse effects, leading to a reduction in cumulative supply at all dates.

A change in the rate of growth of price affects both the intensive and the extensive margin. At the intensive margin, a permanent increase in price growth causes an immediate and permanent fall in the rate of depletion, *z* (equation (4)). Slower depletion means less supply from a given quantity of open reserves but more open reserves at all future dates, so a short run reduction in supply is followed by higher supply in future, the Hotelling-like response that would be expected.

The extensive margin now operates in a similar manner to the intensive as higher future prices create an incentive to postpone field opening. Field opening is reduced (or ceases altogether) for a period, and then resumes at a faster rate, since $\hat{K} = \hat{p} + \theta$. The tension between these forces can be seen by using equation (3), $z^* + \rho - \hat{p} = q(z^*)/q'(z^*)$, in equation (15) to give

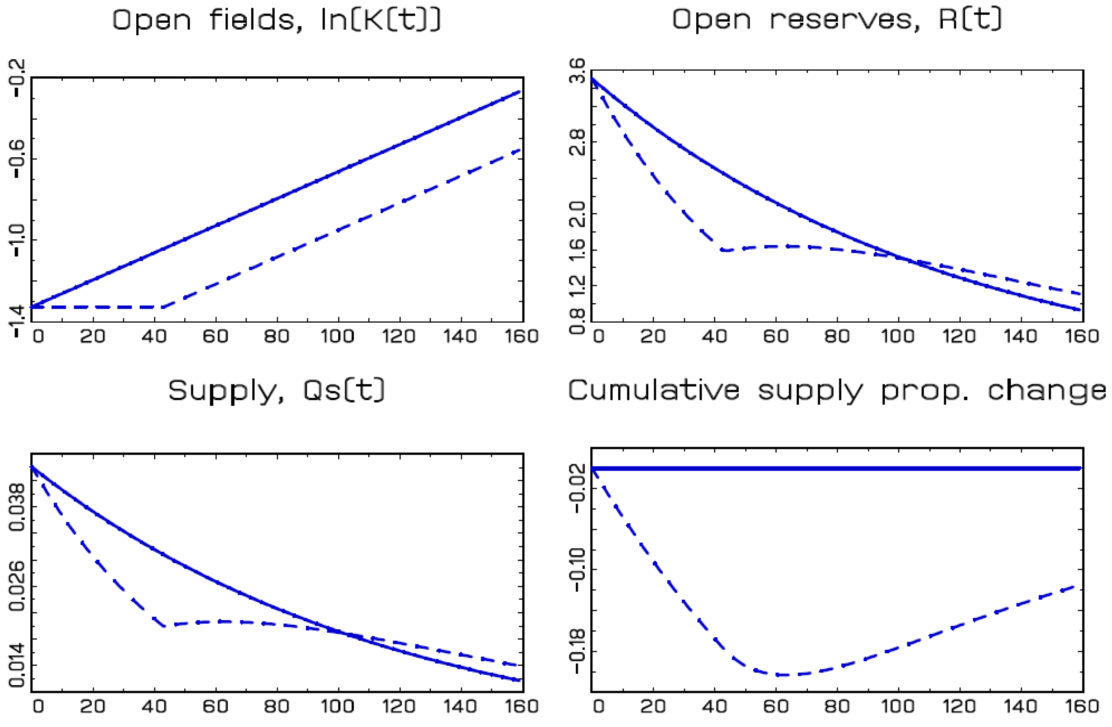
$$K(T) = \frac{p_0 e^{(\theta+\hat{p})T} q'(z^*)(\rho - \hat{p})}{(\rho+\theta)} = \frac{p_0 e^{(\theta+\hat{p})T}\left[q(z^*) - z^* q'(z^*)\right]}{(\rho+\theta)} \tag{24}$$

and differentiating with respect to *T* giving

$$\frac{dK(T)}{d\hat{p}} = \frac{p_0 e^{(\theta+\hat{p})T}}{(\rho+\theta)}\left[\{q(z^*) - z^* q'(z^*)\}T - z^* q''(z^*).\frac{dz^*}{d\hat{p}}\right]. \tag{25}$$

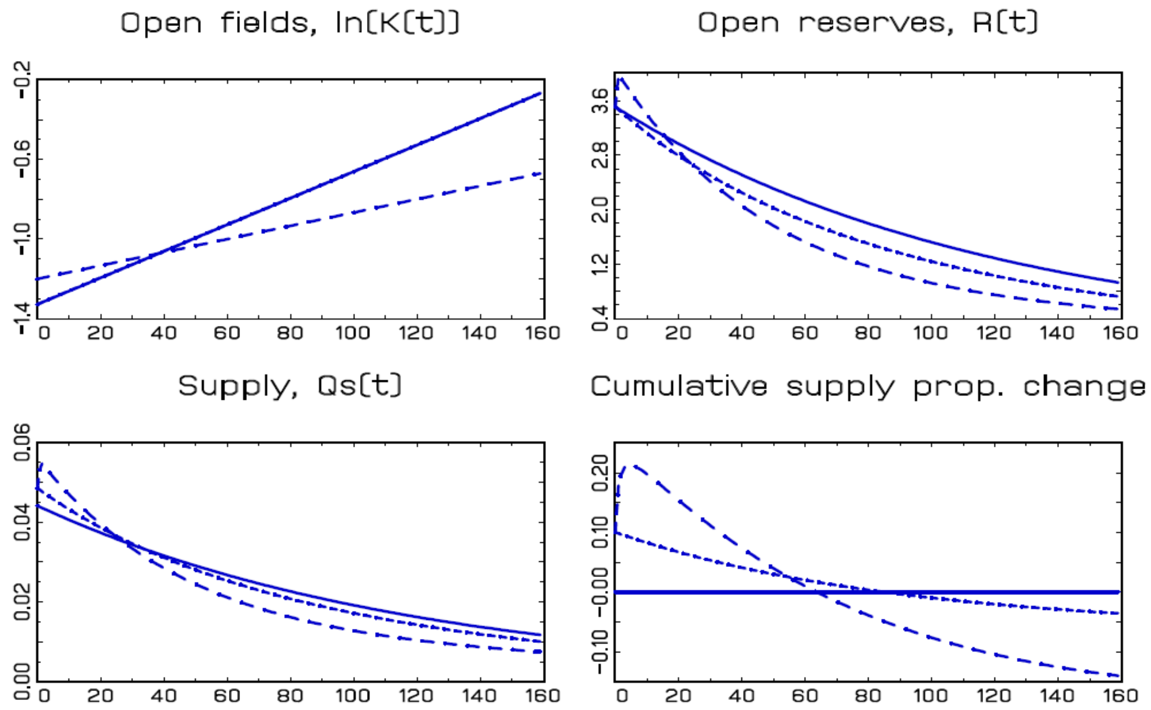
This expression is negative for small *T* (since $q'' < 0$ and $dz^*/d\hat{p} < 0$) and positive for large *T*, when the first term in the square brackets comes to dominate. There is therefore a period in which field openings are reduced (or cease altogether), following which more fields are opened at each date and the new path overtakes the old.

Figure 2a: Price decrease



Solid line: original path. Dashed line: new path.

Figure 2b. Slower price growth



Short dash: Intensive margin, *K* constant, *z* adjusts.

Long dash: Intensive and extensive margin, *K* and *z* adjust.

Figure 2b illustrates for a permanent reduction in $\hat{p}$. This increases the rate of depletion and brings forward field opening, giving the K crossing that we noted in equation (25), (top left hand panel). The top right and bottom left panels give the paths of R and Q_S, giving initial path (solid), intensive margin only (K constant, short dash) and full adjustment (long dash). Faster depletion alone (short dash) gives a fall in open reserves at all dates, associated with higher output in the short run and lower output in the long run. Combining this with the change in field openings (long dash), the effect is magnified with a larger output increase in the short run, but a sharper fall in the long run. Cumulative output is raised for a short period, but then permanently reduced as lower prices have a major impact in reducing field openings (bottom left panel). We summarize in proposition 5:

Proposition 5:

A permanent increase in the rate of growth of price tilts production to the future. Depletion of existing fields is slowed down, and opening of new fields postponed. Supply is reduced for a period, after which it overtakes its previous level. The converse holds for a permanent decrease in the rate of price growth.

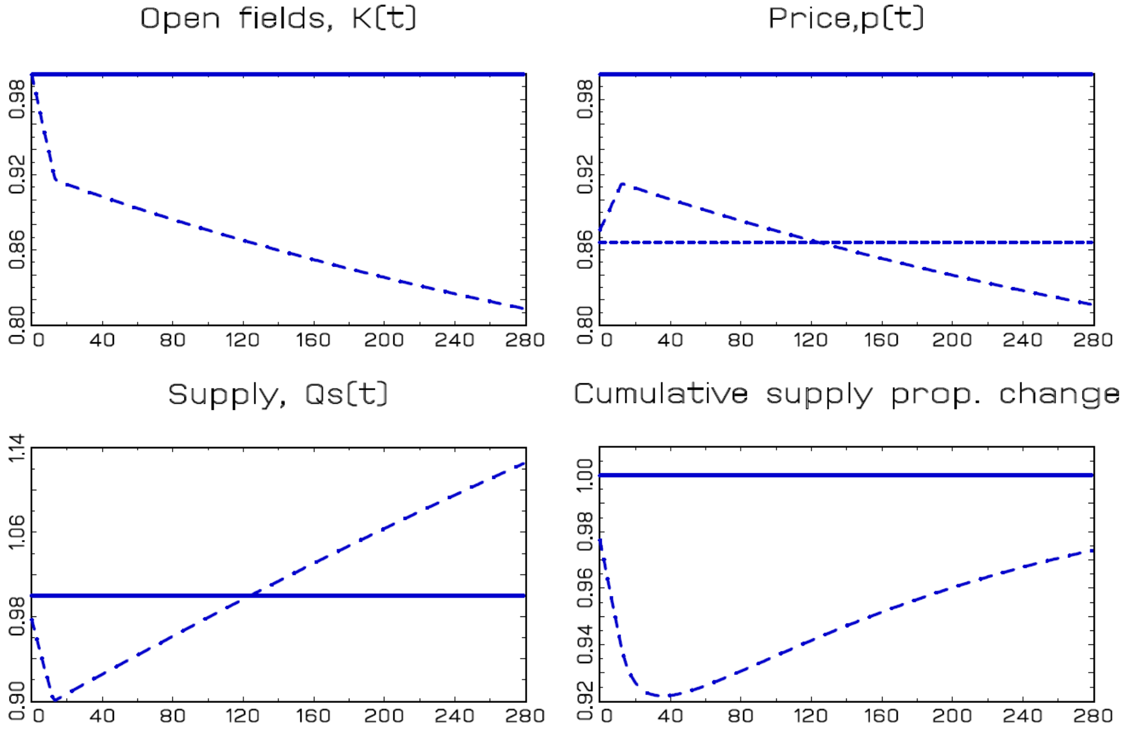
7.2 Demand shocks and equilibrium responses:

Now consider a change in the level of demand at all dates, i.e. a shift in D. We know from section 6 that there is no effect on long rate rates of growth of p, Q_S, or R, or on the level of z, although there is a change in the price level. If there were no extensive margin effects (the path of K held constant) then there would be no short-run effects either; all quantities would be unaffected and the demand change would be shifted wholly to the price level. However, as seen in the previous sub-section, the extensive margin depends on the level of prices as well as their rate of change; a change in the price level changes the timing of field opening, this changing supply and inducing a transitional dynamic response.

Figure 3a illustrates the effect of a permanent decrease in demand (D falling to 75% of its previous value), with all variables now expressed relative to the initial path. The top right hand panel gives the price path. The short dashed line gives the price path in the absence of extensive margin effects: a one-off drop to 0.866 = $0.75^{1/\eta}$ of its previous value. Including extensive margin effects, the long dashed line indicates a larger ultimate price fall, asymptoting to 0.68= $0.75^{1/(\eta-\sigma)}$ of its previous value. As we saw in the preceding section, a price fall leads to postponement of field opening; a pause (top left), and resumption with K smaller and $S(K)$ larger. This means that supply falls and then overtakes its previous path (bottom left). This now has a feedback effect on price; price drops abruptly as demand falls, increases when supply is falling, and then falls to its asymptotic path (top right). The main message concerns the equilibrium path of supply, particularly cumulative supply (bottom right). Without the extensive margin, a demand change would have no effect whatsoever on output. With the extensive margin operating, a reduction in demand cuts supply in the short run, raises it in the long run, and has a negative impact on the cumulative quantity extracted and supplied to all dates.

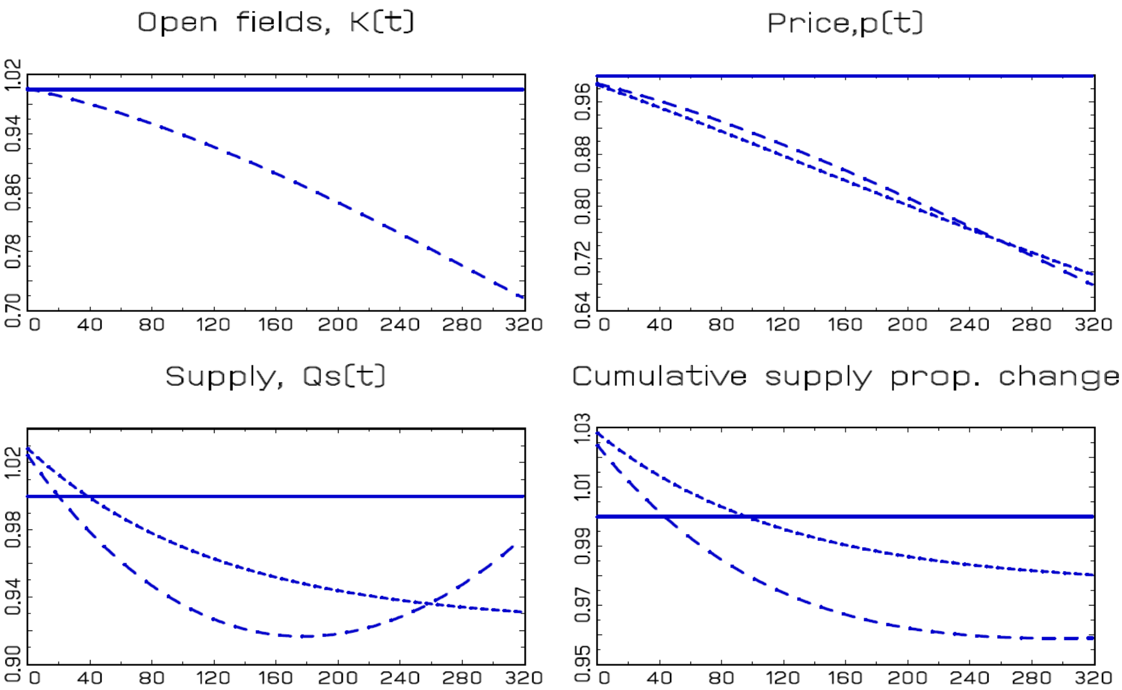
A permanent change in demand growth affects the long run growth of variables as well as transitional dynamics. Long run growth rates of variables can be found explicitly (appendix table 1); a reduction in the rate of growth of demand gives a lower long run rate of price increase and a less rapid decline of output. The full dynamic story is illustrated in figure 3b. Following the reduction in demand growth inter-temporal substitution creates an incentive to shift both depletion and field opening from the future to the present, but this is combined with a price level effect that deters field opening. If adjustment were to take place only at the intensive margin, then the path of supply would be unambiguously tilted towards the present (short dashes); price growth is slower, the rate of depletion faster, and the increase in present supply leads to an immediate fall in price. The extensive margin of field opening responds both to this fall in the price level, and to the slow future growth of prices. The combined effect is to slow the rate of field opening and push opening new capacity into the future giving the U-shaped path of output (bottom left). In the short run, the faster extraction of open fields dominates and supply increases. In the medium run supply is lower because open fields have been depleted faster and because fewer new fields have been opened. In the long run supply turns up, because the high $S(K)$ field types, opening of which was postponed, are coming on stream. Looking at cumulative supply, we see that adding the extensive margin effect mitigates the shift in supply towards the present; cumulative supply is raised for a shorter period, beyond which it is associated with larger reductions in cumulative output and cumulative stock of resource extracted.

Figure 3a: Decrease in demand: relative to constant growth path



Solid line: original path. Dashed line: new path.

Figure 3b: Slower growth of demand: relative to constant growth path



Short dash: Intensive margin, *K* constant, *z* adjusts:
Long dash: Intensive and extensive margin, *K* and *z* adjust.

8. Concluding comments

The paper has developed a model of the supply of a non-renewable resource in which the empirically compelling fact that large sunk costs are associated with the development of new deposits or fields is put centre stage. The model encompasses both depletion of existing fields and the development of new fields, thereby providing a modest step towards greater reality. New insights come from the approach. The most fundamental is that while the rate of interest may matter for depletion rates and short run transitional dynamics, it has no impact on the long run behaviour of resource prices; long run price growth depends on demand and underlying supply considerations (the geology of available fields). The approach also provides perspective on some ‘paradoxes’ that have gained recent attention. For example, emissions taxes may tend to bring forward depletion of existing resources, but they also discourage the development of new fields, so are likely to have to the desired effect of pushing production into the future, reducing cumulative output and any associated stock of emissions.

The approach suggests a number of extensions and applications. For example, we have assumed throughout that (following a shock) future price paths are known with certainty and that owners of fields will postpone opening until the date at which the present value of the field is maximized. Allowing price uncertainty and placing the field opening decision in a stochastic context is clearly important. Lags in opening fields will introduce a more complex dynamic response to shocks. The development of substitutes provides a further supply margin. The main results in the paper are derived with iso-elastic functional forms; whether there exist other combinations of demand, technology and geology capable of producing them is an open question. On the applied side, the model provides a relatively tractable framework for thinking about a number of practical and policy issues such as climate policy. The model also provides a framework for analysis of taxes (royalties, production sharing arrangements and corporate income taxes) which have to balance the need to capture rent with incentives for field development.

Appendix 1:

Substituting the constraint into the objective, (1), gives $PV \equiv \int_0^\infty pxq(-\dot{x}/x)e^{-\rho\tau}d\tau$.

The Euler-Lagrange equation is $\frac{d\left[-pq'(-\dot{x}/x)e^{-\rho\tau}\right]}{d\tau} = p\left[q + q'\dot{x}/x\right]e^{-\rho\tau}$, giving eqn. (3).

The derivative of the first term on the right hand side of eqn. (2) is: $pxq'\left[\frac{\dot{p}}{p} + \frac{\dot{x}}{x} + \frac{q''(z)\dot{z}}{q'(z)}\right]$.

The derivative of the second term is: $-pxq + \rho\int_t^\infty pxq(z)e^{-\rho(\tau-t)}d\tau = pxq'\left[-\frac{q(z)}{q'(z)} + \rho\right]$,

(using eqn. (2) set equal to zero), this giving eqn. (3) of the text.

Appendix 2:

Fields vary in capital cost K, with the number of fields of type K denoted $S(K)$. This can be derived from the following set up. Suppose that fields are ordered by size, s, with $n(s)$ fields of size s, $n' < 0$. $n(s)$ follows a power law, so $n(s) = s^\alpha$, $\alpha < 0$. The total capacity of fields of size s is $sn(s) = s^{1+\alpha}$. The capital cost of a field of size s is $k(s)$, and we suppose $k(s) = s^\kappa$, $0 < \kappa < 1$, so costs are increasing and strictly concave in field size; the capital cost of one unit of capacity on a field of size s is $s^{\kappa-1}$, i.e. $K = s^{\kappa-1}$. Since the capacity associated with fields of size s is $S = s^{1+\alpha}$, we have, eliminating s, $S(K) = K^{(1+\alpha)/(\kappa-1)}$. Thus, $\sigma + 1 = (1+\alpha)/(1-\kappa)$ and hence $\sigma = (\kappa+\alpha)/(1-\kappa)$, which is positive if $1 > \kappa$ and $\kappa+\alpha > 0$.

Appendix 3:

Using equations (9) and (21) gives

$$r = z^* - \sigma\hat{K} = m + (\rho - \hat{p})\frac{(1-\lambda)}{\lambda} - \sigma(\hat{p} + \theta) = m - \sigma\left(\frac{g + \eta\theta}{\eta - \sigma}\right) + \frac{(1-\lambda)}{\lambda}\left(\frac{\rho(\eta-\sigma) - \sigma\theta - g}{\eta - \sigma}\right)$$

The condition in proposition 1, is $r - g - m + \eta\rho = \left(\eta + \frac{(1-\lambda)}{\lambda}\right)\left(\frac{\rho(\eta-\sigma) - \sigma\theta - g}{\eta - \sigma}\right)$ which is satisfied by the parameter restriction in proposition 3. A sufficient condition for $r > 0$ is that $m - \sigma\left(\frac{g + \sigma\theta}{\eta - \sigma}\right) > 0$. This is necessary only if λ is close to unity, and ensures that depletion is fast enough (m large enough) for open reserves not to become unbounded.

Appendix 4:

Simulation undertaken with yield curve equation (5) with $a = 0.1$; $m = 0.005$; $\lambda = 0.5$.
Other parameter values, $\rho = 0.02$; $g = 0.005$; $\eta = 2$; $\sigma = 1.25$.
figures 2 and 3: Long run equilibrium $\hat{p} = 0.067$ (exogenous in figure 2).
Figure 2a: initial price p_0 reduced by 20%. Figure 2b: $\hat{p}$ halved to 0.0025
Figure 3a: demand, D, cut by 25%. Figure 3b: growth rate g halved to 0.0025

Table 1: Asymptotic growth rates for a reduction in the rate of growth of demand:
g_I = initial growth of demand; g_N = new growth of demand. $g_I < g_N$.

	Initial, g_I		New, g_N Intensive margin only		New, g_N Intensive & extensive margin		
$\hat{K}$	$\frac{g_I+\eta\theta}{(\eta-\sigma)}$	=	$\frac{g_I+\eta\theta}{(\eta-\sigma)}$	>	$\frac{g_N+\eta\theta}{(\eta-\sigma)}$	>	0
$\hat{Q}_S$	$\frac{-\sigma(g_I+\eta\theta)}{(\eta-\sigma)}$	=	$\frac{-\sigma(g_I+\eta\theta)}{(\eta-\sigma)}$	<	$\frac{-\sigma(g_N+\eta\theta)}{(\eta-\sigma)}$	<	0
$\hat{Q}_D$	$g_I-\eta\hat{p}$	=	$g_N-\eta\hat{p}$	<	$g_N-\eta\hat{p}$	<	0
$\hat{p}$	$\frac{g_I+\sigma\theta}{\eta-\sigma}$	>	$\frac{g_N+\sigma\theta-\sigma(g_N-g_I)/\eta}{\eta-\sigma}$	>	$\frac{g_N+\sigma\theta}{\eta-\sigma}$	>	0

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