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Multi-Product Firms and Flexible Manufacturing in the Global Economy*

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Abstract

We present a new model of multi-product firms (MPFs) and flexible manufacturing and explore its implications in partial and general equilibrium. International trade integration affects the scale and scope of MPFs through a competition effect and a demand effect. We demonstrate how MPFs adjust in the presence of single-product firms and in heterogeneous industries. Our results are in line with recent empirical evidence and suggest that MPFs in conjunction with flexible manufacturing play an important role in the impact of international trade on product diversity.

Keywords: Multi-Product Firms, Flexible Manufacturing, General Oligoplistic Equilibrium (GOLE), International Trade, Product Diversity

JEL Classification: F12, L13

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1 Introduction

Multi-product firms are omnipresent in the modern world economy, especially in technologically advanced countries. Their importance is documented in a recent study of U.S. firms by Bernard, Redding and Schott (2006).[1] This shows that multi-product firms are present in all industries; they typically coexist with single-product firms, accounting for less than half (41%) of the total number of firms but a much greater fraction (91%) of total output; and they are very active in varying their product mix: 89% of multi-product firms do so on average every five years. Yet, despite this empirical importance, and despite the interest in trade as a source of increased product diversity, multi-product firms have received relatively little attention in the theory of international trade.

General equilibrium models of international trade typically rely on single-product firms only. In such a framework, intra-firm adjustments are limited to changes in the scale of production. Changes in diversity are linked exclusively to changes in the number of firms. In contrast to the theory of international trade, multi-product firms have received more attention in the field of industrial organization (Brander and Eaton (1984), Ottaviano and Thisse (1999), Hallak (2000), Baldwin and Ottaviano (2001), Grossmann (2003), Johnson and Myatt (2003a, 2003b), Ju (2003), Baldwin and Gu (2005), Allanson and Montagna (2005)). These studies have emphasized that, because of supply and demand linkages, intra-firm adjustments within multi-product firms are significantly different from adjustments via exit and entry. However, studies in industrial organization are commonly conducted in partial equilibrium, so that they cannot capture feedback effects through factor markets.[2] But given the omnipresence and empirical importance of multi-product firms across industries, these general equilibrium effects can be significant and should be included in an analysis of multi-product firms in the global economy. In this paper, we develop a new model of multi-product firms that incorporates both supply and demand linkages and explore its implications in partial and general equilibrium. Our findings show that intra-firm adjustments imply quite different predictions regarding the impact of international trade on factor prices and product diversity than traditional models of international trade.

The supply and demand linkages in our framework capture important differences between multi-product and single-product firms, which have been highlighted in the theory of industrial organization but largely neglected in the literature on international trade. First, in contrast to single-product firms, multi-product firms internalize demand linkages between the varieties they produce. This feature is called the "cannibal-

[1] This uses a longitudinal database derived from the U.S. Census of Manufactures with observations at five-yearly intervals between 1972 and 1997. Over 140,000 surviving firms are present in each census year. In this study a "product" is defined at the five-digit Standard Industry Classification (SIC) level.

[2] Ottaviano and Thisse (1999) allow for labour market equilibrium in their framework, but since they use quasi-linear preferences, they cannot address income effects. The same point applies to Hallak (2000) and Baldwin and Gu (2005), who use the Ottaviano and Thisse approach.

ization effect" and it is generally considered as a defining feature of multi-product firms. The existence of a cannibalization effect requires that firms are large in their markets and behave like oligopolists. It gives rise to strategic interactions that are of particular importance for a firm's reaction to changes in competition. Second, the varieties within a firm's product line are linked on the cost side through a flexible manufacturing technology (Milgrom and Roberts (1990), Eaton and Schmitt (1994), Norman and Thisse (1999), Eckel (2005)). Flexible manufacturing emphasizes the fact that firms typically possess a "core competence" in the production of a particular variety and that they are less efficient in the production of varieties outside their core competence. In our framework, this inefficiency translates into higher marginal labor requirements. Hence, flexible manufacturing allows firms to expand their product lines, but this expansion is subject to diseconomies of scope and creates cost heterogeneities within these product lines. These cost heterogeneities are important for the general equilibrium effects of changes in product ranges. The two types of linkages, cannibalization and flexible manufacturing, are the driving forces behind the intra-firm adjustments in our framework.

The type of cost linkages and the existence of demand linkages and cannibalization distinguish our work from recent papers by Allanson and Montagna (2005), Bernard, Redding and Schott (2005) and Nocke and Yeaple (2005). Allanson and Montagna assume both firm- and variety-specific fixed costs; Bernard, Redding and Schott develop a model where the fixed costs of production vary with the product range of multi-product firms; and Nocke and Yeaple assume that unit costs of all products are positively related to the range of products produced. Even more significantly, all three papers analyze multi-product firms in models of "large-group" monopolistic competition. In such a framework, demand linkages and strategic behaviour are excluded, making it impossible to address the issue of cannibalization.

This paper addresses the role of adjustment processes within multi-product firms and linkages with factor and goods markets in a global economy. In particular, we analyze how multi-product firms react to different globalization shocks (both higher foreign productivity and greater international market integration), how these intra-firm adjustments affect the demand for labour, and how induced changes in wages affect the optimal product range and the distribution of outputs within a firm's product range. Furthermore, we extend our framework to allow for heterogeneous industries and illustrate how global shocks can have asymmetric effects on multi-product firms in different industries. In order to isolate adjustments within firms from adjustment via exit and entry, we focus on oligopolistic markets where barriers to entry are prohibitively high and the number of firms is exogenously given. Our analysis provides plausible explanations for observable facts about multi-product firms and presents testable propositions with respect to the impact of economy-wide shocks on the scale and scope of multi-product firms.

2 Scale and Scope of Multi-Product Firms

We begin by considering the behaviour of consumers and multi-product firms in a single industry. In Section 4 we will look at the consumers' optimization problem in detail. For now we assume that preferences exhibit symmetric horizontal product differentiation, and give rise to a linear inverse demand function for each good or variety:

$$p_j(i) = a' - b'\left[(1-e)\,x_j(i) + eY\right]. \tag{1}$$

Here, $p_j(i)$ and $x_j(i)$ denote the price of good i and its quantity produced by firm j, and $Y = \int_0^N x(i)\,di$ denotes the output of the entire industry. The total mass of differentiated goods is given by N. The parameters a', b' and e denote the consumers' maximum willingness to pay, the inverse market size and the inverse degree of product differentiation respectively. The primes attached to a' and b' are a reminder that these parameters, taken as given by firms, are endogenous in general equilibrium, as will be explained in Section 4. If $e = 1$, the goods are homogeneous (perfect substitutes) so that demand depends on aggregate output only. On the other hand, $e = 0$ describes the monopoly case where the demand for each good is completely independent of other goods.

Each multi-product firm produces a mass of products which is denoted by δ_j. Profits for a multi-product firm j are then given by

$$\pi_j = \int_0^{\delta_j} \left[p_j(i) - c_j(i)\right] x_j(i)\,di, \tag{2}$$

where $c_j(i)$ denotes the marginal cost of producing good i. This is constant with respect to the quantity produced, but varies between varieties.

As explained in the introduction, the technology of multi-product firms can be characterized by a core competence and flexible manufacturing. We assume that each firm has a core competence in producing a particular variety, which describes the production process at which the firm is most efficient, i.e. where it exhibits the lowest marginal production costs. We set a firm's core competence at $i = 0$ with $c_j(0) = c_j^0$ and $c_j^0 < c_j(i)\ \forall\ i > 0$. In addition to producing its core competence variety, the firm can add new products to its product line via flexible manufacturing. This describes a firm's ability to produce additional varieties with only a minimum of adaptation. However, some adaptation is necessary, so each addition to the product line incurs a higher marginal production cost but leaves the marginal production costs of existing products unchanged. Marginal production costs for variety i are therefore an increasing function of the mass of products produced: $\frac{\partial c_j(i)}{\partial i} > 0$. Furthermore, we assume that the increase in marginal production costs is increasing in the length of the product line: $\frac{\partial^2 c_j(i)}{\partial i^2} > 0$.

Firms simultaneously choose the quantity produced of each good and the mass of products produced.

The first-order condition with respect to the scale of production of a particular good i is given by

$$\frac{\partial \pi_j}{\partial x_j(i)} = p_j(i) - c_j(i) - b'[(1-e)x_j(i) + eX_j] = 0, \tag{3}$$

where $X_j = \int_0^{\delta_j} x_j(i)\,di$ denotes the firm's aggregate output. The second-order condition is easily verified: $\frac{\partial^2 \pi_j}{\partial x_j(i)^2} = \frac{\partial p_j(i)}{\partial x_j(i)} - b'(1-e) - b'e\frac{\partial X_j}{\partial x_j(i)} < 0$. Eliminating the price from equations (1) and (3) gives the output of a single variety:[3]

$$2b'(1-e)x_j(i) = a' - c_j(i) - b'e(X_j + Y). \tag{4}$$

Equation (4) reflects the cannibalization effect discussed in the introduction. Because a larger output of one variety tends to lower the demand for all other varieties, a multi-product firm has an additional incentive to restrict its output of each variety beyond the familiar own-price effect. This is shown in equation (4) by the fact that the output of a single variety is decreasing in the aggregate size of the firm: $\frac{\partial x_j(i)}{\partial X_j} = -e/2(1-e) < 0$. The effect is also illustrated in Figure 1. Because of the cannibalization effect, the marginal revenue curve is lower than it would be for a single-product firm, so other things equal a multi-product firm produces less of each good.

Consider next the firm's choice of product line. Multi-product firms add new products as long as marginal profits are positive. The first-order condition with respect to the scope of production is then:

$$\frac{\partial \pi_j}{\partial \delta_j} = [p_j(\delta_j) - c_j(\delta_j)]\, x_j(\delta_j) = 0. \tag{5}$$

As $\frac{\partial c_j(\delta_j)}{\partial \delta_j} > 0$ and, thus, $\frac{\partial x_j(\delta_j)}{\partial \delta_j} = -\frac{1}{2b'(1-e)}\frac{\partial c_j(\delta_j)}{\partial \delta_j} < 0$, the second-order condition is easily verified: $\frac{\partial^2 \pi_j}{\partial \delta_j^2} = [p_j(\delta_j) - c_j(\delta_j)]\frac{\partial x_j(\delta_j)}{\partial \delta_j} < 0$. From (3), $p_j(\delta_j) - c_j(\delta_j)$ cannot be zero. Equation (5) therefore implies that profit-maximizing multi-product firms choose their product range so that the output of the marginal variety is zero: $x_j(\delta_j) = 0$. Combining this with equation (4), the first-order condition with respect to scope can also be expressed as

$$c_j(\delta_j) = a' - b'e(X_j + Y). \tag{6}$$

The determination of the profit-maximizing product range is illustrated in Figure 2. The firm's marginal cost of production is lowest for its core competence and rises at an increasing rate as it expands its product line. The firm will add new varieties up to the point where the marginal cost of producing the marginal

[3] Alternatively we can solve for the price of each variety: $2p_j(i) = a' + c_j(i) + b'e(X_j - Y)$. The firm charges higher prices for products that are further from its core competence, by contrast with models where economies of scope arise from fixed costs, or where producing more varieties raises marginal costs for all varieties, as in Nocke and Yeaple (2005).

variety equals the marginal revenue at zero output.[4]

The cannibalization effect not only affects the scale of production, it also influences the scope of production. Total differentiation of (6) shows that $\frac{\partial \delta_j}{\partial X_j} = -\frac{b'e}{\partial c_j(\delta_j)/\partial \delta_j} < 0$. Because firms internalize the impact of one variety's output on the demand for all of their varieties, they not only produce less of each product, they also produce fewer products.

Taken together, the two first-order conditions provide a nice expression for the output of a single variety. Substitute (6) into (4) to obtain:

$$2b'(1-e)\,x_j(i) = c_j(\delta_j) - c_j(i). \tag{7}$$

Equation (7) expresses the output of a single variety in terms of the difference in marginal costs between this variety and the marginal variety. It also shows that if preferences (b' and e) and technology ($\{c_j(i)\}$) do not change, then the output of each variety is positively related to the firm's product range: $\frac{\partial x_j(i)}{\partial \delta_j} = \frac{1}{2b'(1-e)}\frac{\partial c_j(\delta_j)}{\partial \delta_j} > 0$.

Integrating (7) over the entire mass of products produced yields

$$2b'(1-e)\,X_j = A_j(\delta_j), \tag{8}$$

where $A_j(\delta_j) = \delta_j c_j(\delta_j) - \int_0^{\delta_j} c_j(i)\,di$ and $\frac{\partial A_j(\delta_j)}{\partial \delta_j} = \delta_j \frac{\partial c_j(\delta_j)}{\partial \delta_j} > 0$. $A_j(\delta_j)$ measures the total cost savings from flexible manufacturing and is represented by the shaded region in Figure 2. This summarizes the impact of the firm's technology on its total output. Equation (8) provides an expression for the output of firm j as a function of its product range δ_j.

The first-order condition for scope implies, from (6), that higher firm output encourages a fall in product range because of the cannibalization effect. The first-order conditions for scale and scope combined imply, from (8), that an increase in product range encourages an increase in firm output. Taken together, these two equations jointly determine scale and scope, X_j and δ_j, for given industry output Y. They can be combined to yield a single equation that describes the product range setting behavior by multi-product firms:

$$c_j(\delta_j) + \frac{e}{2(1-e)}A_j(\delta_j) = a' - b'eY \tag{9}$$

This implies that $\delta_j = \delta_j[a', b', \{c_j(i)\}, e, Y]$, and, since the left-hand side is increasing in δ_j, it is clear that $\frac{\partial \delta_j}{\partial a'} > 0$, $\frac{\partial \delta_j}{\partial b'} < 0$, $\frac{\partial \delta_j}{\partial e} < 0$, and $\frac{\partial \delta_j}{\partial Y} < 0$. Profit-maximizing multi-product firms broaden their product range

[4] Combining (6) with footnote 3, we see that the price charged for the marginal variety is $p_j(\delta_j) = a' - b'eY$, which from (1) is just sufficient to induce zero demand.

if demand for their products increases (a' rises or b' falls) and reduce it if competition intensifies (e or Y rises). In addition, the product range also depends on the exact location and shape of the marginal cost curve. It is clear from Figure 2 that the product range contracts if the core competence marginal production cost $c_j(0)$ rises (for a given shape of the $c_j(i)$ curve) or if the $c_j(i)$ curve becomes more convex (for a given $c_j(0)$). More complex shifts in the cost schedule (for example, if more flexible manufacturing requires an increase in the core competence cost) have ambiguous effects on δ_j. Lemma 1 summarizes the determinants of the profit maximizing product range:

Lemma 1 *The profit maximizing product range is given by the following:*

$$\delta_j = \delta_j[\underset{+}{a'}, \underset{-}{b'}, \underset{+/-}{\{c_j(i)\}}, \underset{-}{e}, \underset{-}{Y}]. \tag{10}$$

While all of these determinants are exogenous to an individual firm, they are affected by changes in the industry or in the economy. In partial equilibrium, industry output is endogenous, and in general equilibrium, a', b' and $\{c_j(i)\}$ are also endogenous. In the next section we show how industry output is determined and in the following sections we show how demand and cost parameters are determined in general equilibrium.

3 Partial Equilibrium

The market structure in a typical industry is characterized by a heterogeneous Cournot oligopoly where multi-product firms and single-product firms compete side by side. Since we wish to focus on intra-firm adjustments as opposed to adjustments via exit and entry, we assume that both the number of multi-product firms m and the number of single-product firms n are exogenously given. Assuming for simplicity that the single-product firms are symmetric, industry output is then given by

$$Y = \sum_{j=1}^{m} X_j + nx^s, \tag{11}$$

where x^s is the output of a single-product firm.[5] Single-product firms face the same demand function (1) and are subject to constant marginal production costs c^s. Hence, their output is given by

$$b'(2-e)x^s = a' - c^s - b'eY. \tag{12}$$

[5] It may seem strange to add the output of a finite number of single-product firms to that of the multi-product firms, each of which produces a continuum of products. However, this poses no problems since the total output of each multi-product firm, X, is itself finite. It may be helpful to think of the single-product firms as producing a continuum of identical products along the unit interval. Because their output is homogeneous, equation (12) is identical to the output of a single variety of a multi-product firm, equation (4), in the special case where $x_j(i) = X_j$.

Naturally, there is no cannibalization effect for single-product firms, so equation (12) is independent of X.

By substituting (8) and (12) in (11) we derive a single expression for industry output:

$$\zeta b' Y = \frac{\sum_{j=1}^{m} A_j(\delta_j)}{2(1-e)} + \frac{n(a' - c^s)}{2-e}, \tag{13}$$

where $\zeta \equiv 1 + \frac{e}{2-e}n$. Equation (13) expresses the industry's output for a given product range δ. Naturally, when the product range of any multi-product firm rises, industry output also rises: $\frac{\partial Y}{\partial \delta_j} = \frac{m\delta_j}{2\zeta b'(1-e)} c_\delta^j(\delta_j) > 0$, where $c_\delta^j(\delta_j) = \frac{\partial c^j(\delta_j)}{\partial \delta_j} > 0$.

Equation (9), which gives the product range of each multi-product firm for a given industry output, and equation (13), which gives industry output for given product ranges, yield $m+1$ equations in δ_j and Y that allow us to solve the partial equilibrium. To get some intuition for the workings of the model, we begin with the case where all multi-product firms are identical, so there are just two equations in δ and Y. The equilibrium in that case can be illustrated in (δ, Y) space as in Figure 3. From equation (9), an increase in industry output Y implies an increase in the competition facing each multi-product firm, so product range δ contracts and the curve labeled $Scope|_{MPF}$ is downward-sloping. By contrast, from equation (13), an increase in the product range of every multi-product firm implies an increase in industry output Y, so the curve labeled $IE|_{PE}$ is upward-sloping.

Figure 3 provides some quick comparative static results. Changes in the number of firms (m and n) and changes in the marginal production costs of single-product firms (c^s) shift the $IE|_{PE}$ curve but leave the $Scope|_{MPF}$ curve unaffected. Hence, $\frac{\partial Y}{\partial m}, \frac{\partial Y}{\partial n}, \frac{\partial Y}{-\partial c^s} > 0$ and $\frac{\partial \delta}{\partial m}, \frac{\partial \delta}{\partial n}, \frac{\partial \delta}{-\partial c^s} < 0$. These shocks are pure supply shocks that either increase competition directly via an increase in the number of competitors (m, n rises) or indirectly via an increase in the competitiveness of the competitors (c^s falls). On the other hand, a change in the market size parameter b' shifts both curves rightwards or leftwards to an identical extent, so that $\frac{\partial Y}{\partial b'} = -\frac{Y}{b'} < 0$ and $\frac{\partial \delta}{\partial b'} = 0$. Hence, an increase in the size of the market (a fall in b) has no impact on the product range of multi-product firms, with the full adjustment borne by equiproportionate increases in the outputs of all firms ($\frac{dx(i)}{x(i)} = \frac{dX}{X} = \frac{dx^s}{x^s} = -\frac{db'}{b'}$). Finally, the impact of changes in a' and e on the product range δ are the same as the impacts laid out in lemma 1: $\frac{\partial \delta}{\partial a'} > 0$ and $\frac{\partial \delta}{\partial e} < 0$.

When multi-product firms are heterogeneous, these results continue to hold qualitatively for the effects of exogenous shocks on industry output and on the product ranges of all multi-product firms.[6] In addition, we can compare the responses of different multi-product firms. The relative responses of the product ranges of any two multi-product firms j and h to changes in n, c^s or a' are given by:

[6] Strictly speaking, we cannot use calculus to determine the effects of entry by a new multi-product firm on the equilibrium. However, inspection of equations (9) and (13) confirms that it has the same effects as in the homogeneous firms case.

$$\frac{d\delta_j}{d\delta_h} = \frac{\varphi_j \phi_j}{\varphi_h \phi_h} \tag{14}$$

where:

$$\varphi_j \equiv \frac{(2-e)\,\delta_j}{2\,(1-e) + e\delta_j} \qquad \text{and} \qquad \phi_j \equiv \left[\delta_j c_\delta^j\,(\delta_j)\right]^{-1} \tag{15}$$

Here φ_j is an increasing concave function of the product range δ_j, while ϕ_j is the inverse semi-elasticity of marginal cost, evaluated at the marginal variety, and so can be interpreted as a measure of firm j's flexibility in manufacturing. Equation (14) shows that firms with longer product lines (for a given flexibility) and with more flexible technology at the margin (for a given length of product line) tend to respond more to shocks. The former result is consistent with the empirical finding of Bernard, Redding and Schott (2006) that larger firms are more active in changing their product mix.

These results can be summarised as follows:

Proposition 1 *In partial equilibrium, an increase in competition reduces the product range δ_j of all multi-product firms and raises industry output Y. An increase in the size of the market also leads to an increase in industry output Y but leaves the product ranges δ_j unaffected. Multi-product firms with longer product lines and with more flexible technology tend to respond more to shocks.*

From a welfare perspective, the impact on the product range of individual firms is not as important as the impact on the overall diversity of products offered. The total number of varieties in the market is given by $N = \sum_{j=1}^{m} \delta_j + n$. If m and n stay constant, the changes in product ranges determine the change in diversity: $dN = \sum_{j=1}^{m} d\delta_j$. However, if the number of firms changes, the impact on diversity consists of two effects: a direct effect through the change in the number of firms and an indirect effect through induced adjustments of the product range. As product range is decreasing in n, and also in m when firms are homogeneous ($\frac{\partial \delta}{\partial m}, \frac{\partial \delta}{\partial n} < 0$), these two effects work in opposite directions so that the overall impact on diversity is ambiguous. This is an important observation because it highlights a major difference between our framework and models of international trade with only single-product firms. In the latter case, an increase in the number of firms always increases diversity because, by definition, these models cannot take account of adjustments in the product range. In our framework we see that changes in the product range are an important margin of adjustment that has a non-trivial impact on diversity.

Given (9) and (13), the impact of a change in the number of single-product firms n on industry diversity N is given by:[7]

[7] The determinant of the system is denoted by Δ' and is always positive: $\Delta' \equiv \frac{e}{2(1-e)} + \frac{\zeta}{\varepsilon} > 0$.

$$\frac{\partial N}{\partial n} = 1 - \frac{b' e x^s}{\Delta'} \phi, \qquad \text{where:} \qquad \phi \equiv \frac{\Sigma_{j=1}^m \varphi_j \phi_j}{\Sigma_{j=1}^m \varphi_j}. \tag{16}$$

Here ϕ is a weighted average of the flexibilities of all m multi-product firms, where the weights $\varphi_j / \Sigma_{k=1}^m \varphi_k$ are larger for firms with longer product lines. When multi-product firms are homogeneous they all have flexibility ϕ. In that case the impact on diversity of a change in the number of multi-product firms m is:

$$\frac{\partial N}{\partial m} = \delta - \frac{b' e X}{\Delta'} \phi. \tag{17}$$

Clearly both derivatives can become negative if ϕ is sufficiently large: $\frac{\partial N}{\partial n} < 0$ if $\phi > \frac{\Delta'}{b' e x^s}$ and $\frac{\partial N}{\partial m} < 0$ if $\phi > \frac{\delta \Delta'}{b' e X}$. Hence, ϕ is an important determinant of the change in diversity. If flexibility as measured by ϕ is low, changes in the product range lead to large cost effects. Hence adjustments take place primarily via adjustments of output levels and less via changes in the product range. Traditional trade models correspond to the extreme case where ϕ is zero. On the other hand, if ϕ is high, changes in the product range lead to only small cost effects. In this case, adjustments take place primarily via changes in the product range, and the entry of either type of firm can reduce diversity. Summarizing:

Proposition 2 *In partial equilibrium, the impact of changes in the number of firms on diversity depends on the degree of flexibility in manufacturing. If flexibility is low, diversity rises when the number of firms increases, otherwise diversity falls.*

4 General Equilibrium

We now turn to the level of the economy as a whole, extending the model of general oligopolistic equilibrium (GOLE) set out in Neary (2002) to allow for multi-product firms. We assume that the world economy consists of a continuum of industries, each of which has an oligopolistic market structure, and a finite number of countries, all with fully integrated goods markets but no international factor mobility.

Each consumer maximizes a two-tier utility function that depends on their consumption levels $q(i, z)$ of all $N(z)$ goods produced in each industry z, where z varies over the interval $[0, 1]$. The upper tier is an additive function of a continuum of sub-utility functions, each corresponding to one industry:

$$U \langle u [q(0, z), ..., q\{N(z), z\}] \rangle = \int_0^1 u [q(0, z), ..., q\{N(z), z\}] \, dz. \tag{18}$$

Each sub-utility function in turn is quadratic:

$$u\left[q\left(0,z\right),...,q\left(N\left(z\right),z\right)\right] = a\int_0^{N(z)} q\left(i,z\right)di - \frac{1}{2}b\left(1-e\right)\int_0^{N(z)} q\left(i,z\right)^2 di - \frac{1}{2}be\left[\int_0^{N(z)} q\left(i,z\right)di\right]^2. \tag{19}$$

The utility parameters a, b and e are assumed to be identical for all consumers. Consumers maximize utility subject to the budget constraint

$$\int_0^1 \int_0^{N(z)} p\left(i,z\right)q\left(i,z\right)didz \leq I, \tag{20}$$

where I denotes individual income. This leads to the following individual inverse demand functions:

$$\lambda p\left(i,z\right) = a - b\left(1-e\right)q\left(i,z\right) - be\int_0^{N(z)} q_j\left(i,z\right)di. \tag{21}$$

The parameter λ is the Lagrange multiplier, which denotes the consumer's marginal utility of income.

To move from individual to aggregate demands, we assume that there are L consumers located in the home country, and L^* consumers in each of k identical foreign countries.[8] In spite of the differences in nationalities, all consumers (domestic and foreign) have identical preferences. However, as incomes may differ between countries, they may have different consumption levels and, thus, different marginal utilities of income. Because the goods markets of all countries are completely integrated in a single world market and free trade prevails, the price of a given variety is the same everywhere. Therefore, the market demand for a particular variety i in industry z, $x\left(i,z\right)$, facing a firm in any country consists of demand from domestic consumers, $Lq\left(i,z\right)$, plus demand from foreign consumers, $kL^*q^*\left(i,z\right)$. The inverse world market demand function for good i in industry z can then be written exactly as in (1):

$$p\left(i,z\right) = a' - b'\left[\left(1-e\right)x\left(i,z\right) + eY\left(z\right)\right]. \tag{22}$$

where

$$a' \equiv \frac{a}{\bar{\lambda}}, \qquad b' \equiv \frac{b}{\bar{\lambda}\left(L+kL^*\right)} \tag{23}$$

and

$$\bar{\lambda} \equiv \frac{L}{L+kL^*}\lambda + \frac{kL^*}{L+kL^*}\lambda^*, \tag{24}$$

The parameter $\bar{\lambda}$ is a population-weighted average of the home and foreign marginal utilities of income and so can be interpreted as the average world marginal utility of income. Because they depend on $\bar{\lambda}$, the parameters

[8] Foreign variables are denoted by an asterisk throughout.

a' and b' are endogenously determined in general equilibrium. However, with a continuum of industries they are perceived as exogenous by individual firms. Hence firms are "large" in their own market but "small" in the economy as a whole, which permits a consistent analysis of oligopoly in general equilibrium. (See Neary (2002) for details.)

Turning to production, we assume a Ricardian technology with labour as the only factor of production. For tractability, we also assume from now on that firms are homogeneous within each sector, so we can suppress the j subscript. The marginal production costs $c(i,z)$ of each multi-product firm can therefore be decomposed into marginal labor requirements $\gamma(i,z)$ and the economy-wide wage rate w:

$$c(i,z) = w\gamma(i,z). \tag{25}$$

The flexible manufacturing features of the cost function, a core competence and increasing convex marginal costs of new varieties, are now imposed on the marginal labor requirements, i.e. $\gamma(0,z) = \gamma^0(z)$ and $\frac{\partial\gamma(i,z)}{\partial i}, \frac{\partial^2\gamma(i,z)}{\partial i^2} > 0$. The marginal production costs of single-product firms at home and abroad are simply $c^s(z) = w\gamma^s(z)$ and $c^*(z) = w^*\gamma^*(z)$. It is convenient to define real wages at home and abroad W and W^* not in units of a particular good or a basket of some kind, but in terms of utils at the margin. Thus, the nominal wage is weighted by the average marginal utility $\bar{\lambda}$:

$$W = w\bar{\lambda} \qquad W^* = w^*\bar{\lambda}. \tag{26}$$

Labor markets are perfectly competitive and fully integrated within each country, so the wage rate is the same for all firms and all industries within each country. However, there is no international labor mobility so national labor markets are segmented. The labor demand of a multi-product firm in industry z consists of the total labor requirements for each variety over the entire product range:

$$l_{MPF}^D(z) = \int_0^{\delta(z)} \gamma(i,z)\, x(i,z)\, di. \tag{27}$$

The labor demand of a single-product firm in industry z is simply $l_{SPF}^D(z) = \gamma^s(z)\, x^s(z)$. Labor market equilibrium requires that the entire labor demand over all industries equals the endowment of labor, L:

$$\int_0^1 \left[m(z)\, l_{MPF}^D(z) + n(z)\, l_{SPF}^D(z)\right] dz = L. \tag{28}$$

In principle, the same holds for the foreign labor market. However, we assume that multi-product firms are located in the home country only. First and foremost, this assumption is a simplification that allows us to concentrate on the home country for adjustments within multi-product firms. Once these adjustments are understood, extending multi-product firms to all countries is just a technicality. Secondly, this assumption introduces an asymmetry between countries that allows us to interpret the home country as a fully industrialized country and the foreign countries as developing countries or emerging market economies that are not yet advanced enough to implement flexible manufacturing technologies. Hence, foreign labor market equilibrium is given by

$$\int_0^1 n^* (z) \gamma^* (z) x^* (z) \, dz = L^*. \tag{29}$$

We can now set out the full description of an equilibrium in the world economy. Given equations (23), (25) and (26), the first-order condition for scale, equation (6), can be rewritten as

$$be [X (z) + Y (z)] = [a - W\gamma (\delta, z)] (L + kL^*) \tag{30}$$

and that for scale and scope combined, equation (8), can be rewritten as

$$2b (1 - e) X (z) = W\alpha (\delta, z) (L + kL^*) , \tag{31}$$

where $\alpha (\delta, z) \equiv \delta (z) \gamma (\delta, z) - \int_0^{\delta(z)} \gamma (i, z) \, di$, the real component of the total cost savings from flexible manufacturing $A (\delta, z)$. The output of domestic and foreign single-product firms can now be expressed as

$$b (2 - e) x^s (z) = [a - W\gamma^s (z)] (L + kL^*) - beY (z) \tag{32}$$

and

$$b (2 - e) x^* (z) = [a - W^*\gamma^* (z)] (L + kL^*) - beY (z) . \tag{33}$$

The expression for industry output takes into account that there are domestic and foreign single-product firms:

$$Y (z) = m (z) X (z) + n (z) x^s (z) + kn^* (z) x^* (z) . \tag{34}$$

Equations (30) to (34) can be solved for $\delta (z)$, $X (z)$, $x^s (z)$, $x^* (z)$ and $Y (z)$ for each industry z for given values of the two economy-wide real wage rates W and W^*. The two labor market clearing conditions (28) and (29) then provide the final two equations.

5 Globalization with Symmetric Industries

Our general setup allows for two different types of heterogeneities: heterogenous firms (multi-product and single-product firms) and heterogenous industries. To simplify the analysis we look at one heterogeneity at a time. In this section, we assume that all industries are identical, while in the next section we consider the case where industries are heterogeneous but have only one kind of firm. We first illustrate the determination of equilibrium, and then show how it is affected by two globalization shocks: an increase in the productivity of foreign firms located in emerging market economies (a reduction in γ^*) and an increase in the number of countries participating in the world market (k).

5.1 Equilibrium

When all industries are symmetric, the index z can be omitted. In this case, the full general equilibrium can be described by only four equations. First, equations (30) and (31) can be combined and the output of multi-product firms X eliminated to give:

$$\gamma(\delta) + \frac{e}{2(1-e)}\alpha(\delta) = \frac{1}{W}\left(a - be\frac{Y}{L+kL^*}\right) \tag{35}$$

This equation is the general equilibrium equivalent of (9). It determines δ, the product range of a typical multi-product firm, for given Y and W. Next, we can use equations (29), (31) and (32) to eliminate firm outputs x^*, X and x^s from the expression for industry output (34):

$$\zeta Y = \frac{1}{b}\left[\frac{mW\alpha(\delta)}{2(1-e)} + \frac{n(a-W\gamma^s)}{2-e}\right](L+kL^*) + \frac{kL^*}{\gamma^*}, \tag{36}$$

where $\zeta \equiv 1 + \frac{e}{2-e}n$ as before. Equation (36) is the general equilibrium equivalent of (13). It determines industry output Y for given δ and W.

The remaining two equations give the conditions for labor-market equilibrium at home and abroad. Using equations (7), (27) and (32), the domestic labor market equilibrium (28) can be expressed as

$$\frac{1}{b}\left[\frac{mW\beta(\delta)}{2(1-e)} + \frac{n\gamma^s(a-W\gamma^s)}{2-e}\right](L+kL^*) = \frac{e}{2-e}n\gamma^s Y + L, \tag{37}$$

where $\beta(\delta) \equiv \int_0^\delta \gamma(i)\left[\gamma(\delta)-\gamma(i)\right]di$ measures the average labor requirement of a multi-product firm, corrected for the cost savings from flexible manufacturing: $\beta(\delta) = \alpha(\delta)\frac{l^D_{MPF}}{X}$. Naturally, the domestic labor market clearing condition determines W for given δ and Y. Finally, the foreign labor market equilibrium condition comes from equations (29) and (33):

$$W^* = \frac{1}{\gamma^*}\left[a - b\left(2-e\right)\frac{1}{n^*\gamma^*}\frac{L^*}{L+kL^*} - be\frac{Y}{L+kL^*}\right] \tag{38}$$

This determines the foreign real wage W^* as a function of Y only. Hence, we can concentrate on equations (35) to (37) which uniquely determine the equilibrium values of the three key variables, industry output Y, the product range of multi-product firms δ, and the domestic real wage W, for a given number of firms (m, n, and n^*) and countries (k).

To illustrate the equilibrium diagrammatically, we can reduce the number of equations to two. Figure 4 provides explicit solutions for the two domestic variables W and δ, with implicit solutions for Y and W^*. The IE contour describes the industry equilibrium in (W, δ) space. It is derived by solving (35) for Y and substituting into (36):

$$\frac{e}{2\left(1-e\right)}\left(m+\zeta\right)\alpha\left(\delta\right) + \zeta\gamma\left(\delta\right) - \frac{e}{2-e}n\gamma^s = \frac{1}{W}\left(a - \frac{be}{\gamma^*}\frac{kL^*}{L+kL^*}\right), \tag{39}$$

The left-hand side is increasing in δ and the right-hand side is decreasing in W, so the IE curve has a negative slope.[9] If W rises for a given δ, equation (35) implies that competition (Y) falls. This tends to boost outputs (both X and x^s rise)[10]. In this case, restoring industry equilibrium requires that δ falls, thus the negative slope of the IE curve.

The LL contour describes the labor market equilibrium in (W, δ) space. It is derived by substituting Y from (35) into (37):

$$m\beta\left(\delta\right) + \frac{2\left(1-e\right)}{2-e}n\gamma^s\left[\frac{e}{2\left(1-e\right)}\alpha\left(\delta\right) + \gamma\left(\delta\right) - \gamma^s\right] = \frac{2b\left(1-e\right)L}{W\left(L+kL^*\right)}. \tag{40}$$

The slope of the LL curve is also negative. Again, equation (35) implies that if W rises, competition (Y) falls for a given δ. The implicit increase in outputs creates an excess demand for labor. Hence, labor market clearing also requires that δ falls. We show in the appendix that the LL curve must be steeper than the IE curve. Hence the intersection of the two curves as illustrated in Figure 4 determines the domestic real wage W and the product range of multi-product firms δ in a global general equilibrium.

[9] See the Appendix for a formal proof.

[10] Equations (30) to (32) imply $X = W\alpha\left(\delta\right)\frac{L+kL^*}{2b(1-e)}$ and $x^s = W\left[\gamma\left(\delta\right) - \gamma^s + \frac{e}{2(1-e)}\alpha\left(\delta\right)\right]\frac{L+kL^*}{b(2-e)}$, so that *for a given* δ, $\frac{\partial X}{\partial W} > 0$ and $\frac{\partial x^s}{\partial W} > 0$.

5.2 An Increase in Foreign Productivity

Having established the general equilibrium we can now turn to the comparative statics of globalization. We begin with an increase in the productivity of foreign firms (a fall in γ^*). This is a pure competition shock which from equations (39) and (40) shifts the IE curve inwards, but leaves the LL curve unaffected. As Figure 5 shows, this leads to a fall in the domestic wage and a rise in the product range of multi-product firms. Explicit calculations (given in the appendix) confirm these results and show also that industry output Y rises:

$$\frac{dY}{d\gamma^*} = -\frac{1}{\Delta}\frac{kL^*}{(\gamma^*)^2}\left[2(1-e)\left\{\varphi m\mu'_\gamma(\delta)^2 + n(\gamma^s)^2\right\} + (2-e)\, m\delta\sigma^2_\gamma(\delta)\right] < 0, \tag{41}$$

$$\frac{d\delta}{d\gamma^*} = -\frac{1}{\Delta}\frac{ekLL^*}{(\gamma^*)^2}\left[\frac{2b(1-e)}{W(L+kL^*)}\right]^2 \varphi\phi < 0, \tag{42}$$

$$\frac{dW}{d\gamma^*} = \frac{1}{\Delta}\frac{kL^*}{L+kL^*}\frac{2be(1-e)}{(\gamma^*)^2}\left[\varphi m\mu'_\gamma(\delta) + n\gamma^s\right] > 0, \tag{43}$$

Here the determinant of the equation system Δ is positive,[11] and, as in Section 3, $\varphi \equiv \frac{(2-e)\delta}{2(1-e)+e\delta}$ and $\phi \equiv [\delta\gamma_\delta(\delta)]^{-1}$, with the latter measuring the flexibility of production by multi-product firms. We have expressed $\alpha(\delta)$ and $\beta(\delta)$ in terms of the first and second moments of the distribution of $\gamma(i)$. Define the first moment about zero (the mean) as $\mu'_\gamma(\delta) \equiv \frac{1}{\delta}\int_0^\delta \gamma(i)\,di$ and the second moment about zero as $\mu''_\gamma(\delta) \equiv \frac{1}{\delta}\int_0^\delta \gamma(i)^2\,di$. Then, $\alpha(\delta) = \delta\left[\gamma(\delta) - \mu'_\gamma(\delta)\right]$ and $\beta(\delta) = \delta\left[\gamma(\delta)\mu'_\gamma(\delta) - \mu''_\gamma(\delta)\right]$. The variance of $\gamma(i)$ is then given by $\sigma^2_\gamma(\delta) = \mu''_\gamma(\delta) - \mu'_\gamma(\delta)^2$. Summarizing:

Proposition 3 *With symmetric industries, an increase in the productivity of foreign firms (a fall in γ^*) raises industry output, increases the product range of multi-product firms and lowers the domestic real wage.*

The increase in foreign productivity raises the output of foreign firms $[dx^*/d\gamma^* = -L^*/n^*(\gamma^*)^2 < 0]$, which encourages an increase in industry output. The outputs of domestic firms tend to contract as a consequence of the increase in competition from abroad, but the direct effect on industry output dominates. However, the incipient decrease in domestic output lowers demand for labor at home, so that the wage rate falls [equation (43)]. Moreover this fall in the wage is sufficient to offset the contractionary effect of the increase in competition on the product range of multi-product firms. (Recall equation (35) which shows that changes in Y and W have opposite effects on δ.) Figure 5 and equation (42) show that the effect of the decrease in the wage rate dominates so that the product range of multi-product firms rises.

[11] $\Delta = \{(2-e) + e(\varphi m + n)\}\, m\delta\sigma^2_\gamma(\delta) + 2(1-e)\left[\left\{\varphi m\mu'_\gamma(\delta)^2 + n(\gamma^s)^2\right\} + \frac{en}{2-e}\varphi m\left\{\gamma^s - \mu'_\gamma(\delta)\right\}^2\right] > 0.$

The increase in foreign competition can be compared to an increase in the competitiveness of single-product firms (a fall in c^s) in the partial equilibrium analysis of Section 3. The shocks are similar but the general equilibrium result presented here differs substantially from the partial equilibrium result. While industry output rises in both cases, the product range of multi-product firms falls in partial equilibrium but rises in general equilibrium. The difference between these two findings is due to the missing wage effect in the partial equilibrium analysis. In partial equilibrium, the wage rate remains constant, so from (35) changes in the product range are determined entirely by changes in competition (Y). Hence, the product range falls. But in general equilibrium, the decrease in domestic labor demand brought about by an increase in foreign competition lowers the wage rate. The associated cut in costs allows multi-product firms to expand their product range. This difference in adjustments between partial and general equilibrium is illustrated in Figure 5. The partial equilibrium effect is shown by the fall in the product range to δ^{PE} for a *given* W. By contrast, in general equilibrium the fall in the wage rate encourages a rise in the product range to δ^{GE}.

Although the product range of multi-product firms expands, it does not follow that the output of each variety must rise. Since the unit labor requirements of the various products produced are different, the absolute cost reductions induced by a fall in the wage rate also differ across products. These cost reductions are highest for the marginal product (the one with the highest unit labor requirement) and smallest for the core competence product. In general, the change in each infra-marginal variety is:

$$\frac{dx(i)}{d\gamma^*} = \frac{1}{\Delta} \frac{kL^* e\left[\varphi m \mu_\gamma'(\delta) + n\gamma^s\right]}{(\gamma^*)^2} \left[\bar{\gamma}(\delta) - \gamma(i)\right] \gtreqless 0, \tag{44}$$

where $\bar{\gamma}(\delta) = \frac{\varphi m \mu_\gamma''(\delta)}{\varphi m \mu_\gamma'(\delta) + n\gamma^s} + \frac{n\gamma^s}{\varphi m \mu_\gamma'(\delta) + n\gamma^s} \frac{e\delta\mu_\gamma'(\delta) + 2(1-e)\gamma^s}{e\delta + 2(1-e)}$ is a weighted average of $\mu_\gamma''(\delta)/\mu_\gamma'(\delta)$, $\mu_\gamma'(\delta)$ and γ^s. Equation (44) shows that whether the output of an individual variety rises or falls depends on its unit labor requirement $\gamma(i)$. The output of the marginal product, with unit labour requirement $\gamma(\delta)$, must rise since the product range expands. (Output $x(\delta)$ is initially zero and increases as δ rises.) As for the core competence variety, if γ^0 is sufficiently low (so that $\gamma^0 < \bar{\gamma}(\delta)$), its output $x(0)$ falls. In this case, the outputs of different varieties produced by a single multi-product firm move in different directions, with the outputs of products close to the "core" of a firm's product range falling while varieties closer to the margin expand. Even if the outputs of all varieties increase, differentiating (44) with respect to i shows that a fall in γ^* flattens the size distribution of varieties in a firm's product range:

$$\frac{d^2x(i)}{d\gamma^* di} = -\frac{1}{\Delta} \frac{kL^* e\left[\varphi m \mu_\gamma'(\delta) + n\gamma^s\right]}{(\gamma^*)^2} \frac{\partial \gamma(i)}{\partial i} < 0. \tag{45}$$

Hence, because of the flexible manufacturing technology (which implies that marginal costs $\gamma(i)$ increase in i, $\frac{\partial \gamma(i)}{\partial i} > 0$), the impact of a fall in γ^* on individual outputs is more positive (or less negative), the further away a variety is located from the firm's core competence. This asymmetry is driven by the asymmetry in unit labor requirements between products. All products are subject to the same two forces: an increase in competition (Y rises) and a decrease in costs (W falls). The increase in competition tends to lower the outputs of all products and the cut in costs tends to raise outputs. The former affects all products in the same way, while the latter varies in magnitude with a product's unit labor requirements, being smallest for the core competence variety and largest for the marginal variety. It is this asymmetric sensitivity to costs which leads to asymmetric adjustments within a firm's product range.

Changes in the aggregate sizes of both types of firms depend on their mean labor requirements γ^s and $\mu'_\gamma(\delta)$, and on the variance in labor requirements within the product ranges of multi-product firms $\sigma^2_\gamma(\delta)$:

$$\frac{dX}{d\gamma^*} = \int_0^\delta \frac{dx(i)}{d\gamma^*} di = \frac{1}{\Delta} \frac{kL^* e\varphi}{(\gamma^*)^2} \left[\frac{2(1-e)}{2-e} n\gamma^s \left\{ \gamma^s - \mu'_\gamma(\delta) \right\} + m\delta\sigma^2_\gamma(\delta) \right] \gtreqless 0. \tag{46}$$

$$\frac{dx^s}{d\gamma^*} = \frac{1}{\Delta} \frac{kL^* em}{(\gamma^*)^2} \left[\frac{2(1-e)}{2-e} \varphi\mu'_\gamma(\delta) \left\{ \mu'_\gamma(\delta) - \gamma^s \right\} + \delta\sigma^2_\gamma(\delta) \right] \gtreqless 0. \tag{47}$$

Inspecting these equations shows that both types of firms contract if they are equally efficient on average ($\mu'_\gamma(\delta) = \gamma^s$), but that the *less* efficient type may expand. For example, if single-product firms are much less efficient ($\mu'_\gamma(\delta) << \gamma^s$), then they gain more from the fall in wages and their output may rise following a fall in γ^*. Only one kind of firm can have a higher total output, however, and total domestic output of all firms must fall:[12]

$$m\frac{dX}{d\gamma^*} + n\frac{dx^s}{d\gamma^*} = \frac{1}{\Delta} \frac{kL^* em\delta}{(\gamma^*)^2} \left[\frac{2(1-e)}{2-e} \varphi n \left\{ \mu'_\gamma(\delta) - \gamma^s \right\}^2 + \left\{ \varphi m\delta + n \right\} \sigma^2_\gamma(\delta) \right] > 0 \tag{48}$$

Proposition 4 *An increase in foreign productivity flattens the distribution of outputs within a multi-product firm's product range. Products at the margin of the product range always expand while those near the core may contract. Aggregate firm outputs can also rise or fall depending on the first and second moments of the distribution of* $\gamma(i)$*, though total output of all home firms must fall.*

Note finally that the expansion in the product range means that an increase in foreign productivity leads to an increase in diversity (measured by $N = m\delta + n + kn^*$) even without firm entry. However, this result need not hold when we turn to consider the effects of international market integration in the next sub-section.

[12] Note that the reduction in total output by multi-product firms is fully consistent with the expansion in their product range. This is most easily seen in the special case with no single-product firms ($n = 0$), when equation (46) reduces to (48).

5.3 International Market Integration

A different type of globalization shock is an increase in the number of countries participating in the world market, k. Inspecting (39) and (40) we can see that this shifts both curves inwards. Explicit solutions for the changes in the endogenous variables (derived in the appendix) are as follows:

$$\frac{dY}{dk} = \frac{L^*}{\Delta}\left\{\frac{1}{\gamma^*}\left[2(1-e)\left\{\varphi m \mu'_\gamma(\delta)^2 + n(\gamma^s)^2\right\} + (2-e)m\delta\sigma^2_\gamma(\delta)\right] + \frac{a}{b}m\left[\frac{2(1-e)}{2-e}\varphi n\left\{\mu'_\gamma(\delta)-\gamma^s\right\}^2 + (\varphi m + n)\delta\sigma^2_\gamma(\delta)\right]\right\} > 0, \tag{49}$$

$$\frac{d\delta}{dk} = -\frac{LL^*}{\Delta}\left(\frac{a}{b}-\frac{e}{\gamma^*}\right)\left[\frac{2b(1-e)}{W(L+kL^*)}\right]^2\varphi\phi < 0, \tag{50}$$

$$\frac{dW}{dk} = \frac{L^*}{\Delta}\frac{2b(1-e)L}{(L+kL^*)^2}\frac{2-e+e(\varphi m+n)}{\gamma^*}\left[\gamma^* - \tilde{\gamma}(\delta)\right] \gtreqless 0. \tag{51}$$

Here $\tilde{\gamma}(\delta) = \frac{e\left[\varphi m \mu'_\gamma(\delta) + n\gamma^s\right]}{2-e+e(\varphi m+n)}$ is a weighted sum of $\mu'_\gamma(\delta)$ and γ^s, where the weights add to less than one. Note that a positive value for the foreign wage in autarky requires that $a\gamma^* > be$.[13] This inequality is used in deriving the sign of equation (50).

The results in equations (49) to (51) can be summarized as follows:

Proposition 5 *With symmetric industries, international market integration raises industry output and lowers the product range of multi-product firms. The impact on the wage is ambiguous: it rises if and only if* $\gamma^* > \tilde{\gamma}$.

The ambiguity in the wage change arises because the increase in k affects the domestic economy through two channels, a competition effect and a demand effect. An increase in k increases competition on the product market because the integration of new countries into the world trading system also brings in new firms. The primary effect (before firm adjustments take place) can be derived from equation (34): $\left.\frac{\partial Y}{\partial k}\right|_{\text{Primary}} = n^* x^* > 0$. This channel which we call the *competition effect* is qualitatively identical to the effect of a fall in γ^* considered in the previous sub-section. In addition, an increase in k increases demand for all products because the number of consumers rises: $\left.\frac{\partial(L+kL^*)}{\partial k}\right|_{\text{Primary}} = L^* > 0$. We call this channel the *demand effect.* Both the competition effect and the demand effect tend to increase industry output, but they have opposing effects on the domestic real wage W and the product range δ. Equation (50) shows that with respect to δ, the demand effect dominates. But the impact on the real wage is ambiguous.

[13] This comes from solving the autarky equilibrium conditions in the foreign country: $b(2-e)x^* = (a - W^*\gamma^*)L^* - beY$ (where $W^* = \lambda^* w^*$), $Y = n^* x^*$, and $n^*\gamma^* x^* = L^*$. Hence, the foreign autarky wage is $W^* = \frac{1}{(\gamma^*)^2}\left[a\gamma^* - be - \frac{1}{n^*}b(2-e)\right]$.

An increase in competition reduces the market shares of domestic firms and demand for domestic labor falls. Hence, the competition effect tends to lower the domestic wage. But an increase in demand from the newly integrated economies raises demand for labor at home, so that the demand effect tends to raise the real wage. In fact, equation (51) can be expressed explicitly in terms of the two primary effects, $\frac{\partial Y}{\partial k}\Big|_{\text{Primary}} = n^* x^*$ and $\frac{\partial (L+kL^*)}{\partial k}\Big|_{\text{Primary}} = L^*$:

$$\frac{dW}{dk} = \frac{1}{\Delta} \frac{2b(1-e)L[2-e+e(\varphi m+n)]}{(L+kL^*)^2} \left[\underbrace{L^*}_{\text{Demand Effect}} - \underbrace{\tilde{\gamma}(\delta) n^* x^*}_{\text{Competition Effect}} \right]. \tag{52}$$

The net change in the wage depends on the relative competitiveness of domestic firms vis-à-vis foreign firms. If all firms have identical unit labor requirements (on average), so that $\mu'_\gamma(\delta) = \gamma^s = \gamma^*$, then the demand effect dominates: $\frac{dW}{dk} = \frac{2b(1-e)(2-e)}{\Delta} \frac{LL^*}{(L+kL^*)^2} > 0$. But if foreign firms have relatively low unit labor requirements, $\gamma^* < \tilde{\gamma}(\delta)$, the competition effect dominates and the domestic real wage falls.

The competition effect and the demand effect can be illustrated separately in Figure 6. The competition effect leads to an inward shift of the IE curve, very similar to the effects of a fall in γ^* in the previous subsection (a move from point A to B). It is derived by letting $\frac{kL^*}{\gamma^*} (= kn^* x^*)$ on the right-hand side of equation (39) rise while keeping $L + kL^*$ constant. The demand effect is derived by letting $L + kL^*$ rise while keeping $\frac{kL^*}{\gamma^*}$ constant. Hence, the demand effect corresponds to a partial backward shift of the IE curve and a shift of the LL curve to the left (a move from B to C). On aggregate, both curves are shifted to the left and the new equilibrium is at a lower δ, but the impact on the wage rate is ambiguous.

The diagrammatic analysis confirms that the impact on δ and W of the demand effect is exactly opposite to that of the competition effect. Consequently, the demand effect has a different impact on infra-marginal outputs as well. The increase in both Y and W tends to reduce all outputs because competition and production costs both rise. As the increase in production costs is largest for products with high unit labor requirements, the increase in W reinforces a steeper output distribution within the product range. In addition, an increase in the size of the world market (as implied by an increase in $L + kL^*$) also shifts the marginal revenue curve outwards. This demand side expansion is a proportional shock, so it is largest for products close to the core (with low marginal costs) and smallest for products close to the margin. In sum, the demand effect steepens the size distribution.[14]

Mathematically, the impacts of an increase in k on the output of any variety and on the distribution of

[14] The impact of a shift in the marginal revenue curve on the distribution of $x(i)$ is also present in partial equilibrium. Equation (7) shows that $\frac{d^2 x_j(i)}{di db'} = \frac{1}{2(b')^2(1-e)} \frac{\partial c_j(i)}{\partial i} > 0$, so that the distribution becomes steeper when b' falls. However, asymmetric adjustments are not possible in partial equilibrium.

outputs are given by

$$\frac{dx\,(i)}{dk} = \frac{L^*}{\Delta}\left(\frac{a}{b} - \frac{e}{\gamma^*}\right)\left[\varphi m \mu'_\gamma(\delta) + n\gamma^s\right]\left[\bar{\gamma}(\delta) - \gamma(i)\right] \gtreqless 0 \tag{53}$$

and

$$\frac{d^2x\,(i)}{dkdi} = -\frac{L^*}{\Delta}\left(\frac{a}{b} - \frac{e}{\gamma^*}\right)\left[\varphi m \mu'_\gamma(\delta) + n\gamma^s\right]\frac{\partial\gamma(i)}{\partial i} < 0. \tag{54}$$

Again, since $a\gamma^* > be$, the demand effect clearly dominates and the size distribution steepens when k rises. The dominance of the demand effect on individual outputs is most obvious when $\frac{dW}{dk} = 0$. In this case, the impact on outputs is determined solely by the interplay between the increase in competition and the increase in marginal revenue. But even if $\frac{dW}{dk} < 0$, the effect of a lower wage is dominated by the increase in marginal revenue in its impact on the size distribution. By analogy with our findings in the previous section, asymmetric adjustments are possible here, too. If $\gamma^0 < \bar{\gamma}(\delta)$, $\frac{dx(0)}{dk} > 0$ and $\frac{dx(\delta)}{dk} < 0$. The only difference is that in the case of international market integration, products close to the core expand while products further away from the core contract. Finally, the effect on total firm output is given by

$$\frac{dX}{dk} = \frac{L^*\varphi}{\Delta}\left(\frac{a}{b} - \frac{e}{\gamma^*}\right)\left[\frac{2(1-e)}{2-e}n\gamma^s\left\{\gamma^s - \mu'_\gamma(\delta)\right\} + m\delta\sigma^2_\gamma(\delta)\right], \tag{55}$$

so that $\frac{dX}{dk} > 0$ if $\gamma^s > \mu'_\gamma(\delta)$.

The impact on diversity also consists of two effects. Since $N = m\delta + n + kn^*$, we obtain

$$\frac{dN}{dk} = m\frac{d\delta}{dk} + n^* \gtreqless 0. \tag{56}$$

The addition of new firms from new countries to the world market raises the choices available to consumers ($n^* > 0$), while the dropping of products from the product ranges of existing multi-product firms lowers diversity ($d\delta/dk < 0$). The overall impact on diversity depends on the flexibility in multi-product manufacturing. Diversity actually falls if

$$\phi > \frac{LL^*}{\Delta}\left(\frac{a}{b} - \frac{e}{\gamma^*}\right)\left[\frac{2b(1-e)}{W(L+kL^*)}\right]^2\frac{m\varphi}{n^*}, \tag{57}$$

where $\phi = [\delta\gamma_\delta(\delta)]^{-1}$ is our measure of flexibility. Note that there is a striking correspondence with the corresponding partial equilibrium result in proposition 2. Again, the degree of flexibility ϕ is a key determinant of whether overall diversity rises or falls.

Proposition 6 *If flexibility as measured by ϕ is low overall diversity rises, whereas if flexibility is high overall diversity falls.*

Proposition 6 presents a result that differs fundamentally from the predictions of standard trade theory. Because conventional workhorse models in international trade theory do not allow for multi-product firms, they cannot account for the effects of globalization on the degree of diversity within firms. With single-product firms only, there is a direct correspondence between the number of firms and diversity. Hence, an increase in the number of firms in the world market raises diversity by assumption. Here, however, we show that an increase in the number of producers can lead to counteracting adjustment processes within firms that can lower overall product diversity.

6 High-Tech and Low-Tech Industries

In this section we relax our previous assumption regarding the perfect symmetry of industries. Instead, we assume that the mass of industries can be divided into two groups: high-tech and low-tech industries. The difference between these is that low-tech industries are subject to competition from developing countries whereas high-tech industries are located entirely in the industrialized world. In our two-country framework this translates into assuming that the home country possesses both types of industries whereas the foreign country only has access to the low-tech technology and thus hosts only single-product firms in this group of industries. For simplicity, we assume that all firms in the home country are multi-product firms. The interaction between single-product firms and multi-product firms within an industry has been described in great detail in the previous section, so we can focus on inter-industry adjustments in this section.

Let low-tech industries be in the interval $z \in (0, \theta)$ and high-tech industries in the interval $z \in (\theta, 1)$, so that θ denotes the share of low-tech industries. Otherwise, firms and consumers in all industries continue to be symmetric. With two groups of industries in the home country there must be a set of equations for firm behavior and industry equilibrium for each group. Only the labor market equilibrium is common to both groups. In addition, we need to adjust the labor market equilibrium for the fact that the demand for labor can differ between firms in high-tech and low-tech industries.

In both low-tech (L) and high-tech (H) industries, the equilibrium product ranges of multi-product firms δ_L and δ_H and the industry outputs Y_L and Y_H are determined by an equation similar to (35):

$$\gamma(\delta_L) + \frac{e}{2(1-e)}\alpha(\delta_L) = \frac{1}{W}\left(a - be\frac{Y_L}{L + kL^*}\right) \tag{58}$$

$$\gamma(\delta_H) + \frac{e}{2(1-e)}\alpha(\delta_H) = \frac{1}{W}\left(a - be\frac{Y_H}{L + kL^*}\right), \tag{59}$$

The industry outputs in turn are determined by equations similar to (36):

$$Y_L = \frac{m_L W \alpha(\delta_L)}{2b(1-e)}(L + kL^*) + kn^* x^*, \tag{60}$$

$$Y_H = \frac{m_H W \alpha(\delta_H)}{2b(1-e)}(L + kL^*). \tag{61}$$

where the parameters m_L and m_H denote the number of multi-product firms in each group of industries. Finally the model is closed by the domestic and foreign labor market equilibrium conditions:

$$\theta m_L \beta(\delta_L) + (1-\theta) m_H \beta(\delta_H) = \frac{2b(1-e) L}{W(L + kL^*)} \tag{62}$$

$$\theta n^* \gamma^* x^* = L^*. \tag{63}$$

Labor demand at home comes from both types of industries, weighted by their shares θ and $1-\theta$, respectively, but labor demand abroad comes from low-tech firms only, since there are no high-tech firms there.

In this setup the high-tech industries are shielded from direct foreign competition, so there is no direct competition effect. Firms in the high-tech industries are only affected indirectly through changes in the economy-wide wage rate W. The product range of multi-product firms in these high-tech industries can be determined by eliminating Y from equations (59) and (61):

$$\frac{e}{2(1-e)}(m_H + 1)\alpha(\delta_H) + \gamma(\delta_H) = \frac{a}{W} \tag{64}$$

Equation (64) provides a unique relation between the real wage W and the product range δ_H in high-tech industries with $\frac{d\delta_H}{dW} < 0$. If the wage rate rises, production costs in the high-tech industries increase and firms react to the cost increase by pruning their product range. Note that this relation is independent of any foreign influences, and in particular of γ^* and k. It is represented by the IE_{HT} locus in the left-hand quadrant of Figure 7, which is negatively sloped in (W, δ_H) space.

In the low-tech industries, the corresponding relationship is not independent of foreign parameters because the low-tech industry is subject to foreign competition. Combining the two equations for those industries, (58) and (60), shows that equilibrium depends on γ^* and k as well as on δ_L and W:

$$\frac{e}{2(1-e)}(m_L + 1)\alpha(\delta_L) + \gamma(\delta_L) = \frac{1}{W}\left(a - \frac{be}{\theta\gamma^*}\frac{kL^*}{L + kL^*}\right). \tag{65}$$

This condition is illustrated by the IE_{LT} curve in the right-hand panel of Figure 7, and it exhibits very similar features to the industry equilibrium curve (39) in the previous section. It is also negatively sloped and it is shifted to the left if γ^* falls or if k rises, with a negative competition effect outweighing a positive demand effect in the latter case.

Finally, equation (62) describes the domestic labor market equilibrium as a function of the wage rate W and the two product ranges δ_L and δ_H. It is most convenient to illustrate this in (W, δ_L) space (again referred to as the LL locus), since this allows us to focus on how the competition and demand effects influence the equilibrium product range in the low-tech industries and how general equilibrium feedback effects influence the product range in the high-tech industries. Hence, using (64) to eliminate δ_H from (62), the slope of the LL locus is given by

$$\frac{dW}{d\delta_L} = \frac{\theta e m_L \beta_\delta (\delta_L) W^2}{2(1-e)\left[(1-\theta)a\tilde{\gamma}(\delta_H) - \frac{beL}{L+kL^*}\right]}, \tag{66}$$

where $\tilde{\gamma}(\delta_H) \equiv \frac{e\varphi_H m_H \mu'_\gamma(\delta_H)}{2(1-e)}$ and $\varphi_H \equiv \frac{2(1-e)\delta_H}{2(1-e)+e(m_H+1)\delta_H}$. Note that this slope depends on the share of low-tech industries, θ. If $\theta = 1$, then equation (65) reduces to (39) with $n = 0$. In this case, $\frac{dW}{d\delta_L}$ is negative and the LL locus is downward-sloping as in Section 5 (and, as there, it must be more negatively sloped than the IE_{LT} locus). But if θ is less than $1 - \frac{beL}{a\tilde{\gamma}(\delta_H)(L+kL^*)}$, then the LL locus is upward-sloping. The slope of the LL locus in (W, δ_L) space varies with θ because changes in δ_L have a smaller impact on labor demand when θ is small than when it is large. This is most obvious in the extreme case of $\theta = 0$, when changes in δ_L have no impact on labor demand, so the LL curve is vertical.

We are now ready to consider the effects of an increase in foreign productivity (a fall in γ^*). Inspecting equations (62), (64) and (65), only the latter is affected: the IE_{LT} curve is shifted to the left and so the wage unambiguously falls. As a result the product range in high-tech sectors *increases*, since their costs have fallen and they face no foreign competition. By contrast the change in the product range in low-tech sectors depends on the slope of the LL locus. These changes in δ_L, δ_H and W are shown formally as follows:

$$\frac{d\delta_L}{d\gamma^*} = \frac{2b(1-e)k}{\Delta_T \theta (W\gamma^*)^2}\left[(1-\theta)a\tilde{\gamma}(\delta_H) - \frac{beL}{L+kL^*}\right]\varphi_L \phi_L \gtreqless 0, \tag{67}$$

$$\frac{d\delta_H}{d\gamma^*} = -\frac{2ab(1-e)k}{\Delta_T (W\gamma^*)^2}\tilde{\gamma}(\delta_L)\varphi_H \phi_H < 0, \tag{68}$$

$$\frac{dW}{d\gamma^*} = \frac{2b(1-e)k}{\Delta_T (\gamma^*)^2}\tilde{\gamma}(\delta_L) > 0, \tag{69}$$

where $\phi_L \equiv [\gamma_\delta(\delta_L)]^{-1}$ and $\phi_H \equiv [\gamma_\delta(\delta_H)]^{-1}$ measure the flexibility of production by multi-product firms in the two sectors, $\varphi_L \equiv \frac{2(1-e)\delta_L}{2(1-e)+e(m_L+1)\delta_L}$ and $\tilde{\gamma}(\delta_L) = \frac{e\varphi_L m_L \mu'_\gamma(\delta_L)}{2(1-e)}$.[15]

The difference in responses between the two sectors reflects the asymmetry in their exposure to foreign

[15] $\Delta_T \equiv \left[\theta m_L \left\{\varphi_L \mu'_\gamma(\delta_L)^2 + \delta_L \sigma^2_\gamma(\delta_L)\right\} + (1-\theta)m_H\left\{\varphi_H \mu'_\gamma(\delta_H)^2 + \delta_H \sigma^2_\gamma(\delta_H)\right\}\right]\frac{L+kL^*}{L^*} > 0.$

competition. The increase in foreign competition has a direct effect on low-tech industries only. At the initial wage rate, this tends to lower the product range in low-tech industries and to reduce demand for labor. The induced fall in the wage rate is an economy-wide shock, which tends to raise the product range in all industries. On balance, the product range in high-tech industries clearly rises, but the impact on the product range in low-tech industries is ambiguous. When θ is high, so the share of low-tech sectors which are subject to foreign competition is large, the lower wage raises the product range in both sectors. This case is qualitatively identical to that in the last section: the LL curve is downward sloping, W falls and δ_L rises.[16] By contrast, if θ is less than $1 - \frac{beL}{a(L+kL^*)\tilde{\gamma}(\delta_H)}$, the LL curve is upward sloping as illustrated in Figure 7. Now the increased competition faced by low-tech sectors has a relatively small impact on aggregate labor demand at home and so the wage change is small. As a consequence, the product range falls in low-tech sectors.

Having laid out the mechanisms driving inter-industry adjustments, the case of international market integration (an increase in k) is straightforward. The explicit solutions are:

$$\frac{d\delta_L}{dk} = -\frac{2b(1-e)L}{\Delta_T(L+kL^*)W^2\gamma^*}\frac{a}{\gamma^*}\left[\gamma^* - \frac{be}{\theta a} + \frac{1-\theta}{\theta}\tilde{\gamma}(\delta_H)\right]\varphi_L\phi_L < 0, \tag{70}$$

$$\frac{d\delta_H}{dk} = -\frac{2ab(1-e)L}{\Delta_T(L+kL^*)W^2\gamma^*}\left[\gamma^* - \tilde{\gamma}(\delta_L)\right]\varphi_H\phi_H \gtreqless 0, \tag{71}$$

$$\frac{dW}{dk} = \frac{2b(1-e)L}{\Delta_T(L+kL^*)\gamma^*}\left[\gamma^* - \tilde{\gamma}(\delta_L)\right] \gtreqless 0. \tag{72}$$

As in the previous section, the additional demand effect pushes wages up sufficiently that the product range in the low-tech industries unambiguously falls.[17] However, the impact on W (and hence on δ_H) is ambiguous, and depends on the average labor requirements in low-tech industries at home compared to the unit labor requirements of foreign competitors, $\tilde{\gamma}(\delta_L) \gtreqless \gamma^*$. (Note that it does not depend on the share of low-tech industries θ.) Hence, asymmetric adjustments are still possible though they arise from a different mechanism than in the case of a fall in γ^*. If foreign firms are sufficiently competitive that $\gamma^* < \tilde{\gamma}(\delta_L)$, an increase in k leads to a fall in the wage rate, so that the product range in the high-tech industries expands. Since the product range in the low-tech industries always contracts, multi-product firms adjust differently to the same economy-wide shock depending on the type of industry they belong to.

We can summarize the results in this section as follows:

Proposition 7 *With heterogeneous industries, shocks exclusive to one industry are transmitted to other*

[16] When θ equals one, the results here are quantitatively identical to those in the previous section with $n = 0$. Under these assumptions the two industry equilibrium conditions, (39) and (65), and the two labour-market equilibrium conditions, (40) and (62), are identical to one another.

[17] As in the corresponding condition in Section 5 (where $\theta = 1$), positive foreign outputs in autarky require that $\theta a\gamma^* > be$.

industries via wage adjustments. Hence globalization can lead to asymmetric product range adjustments between high-tech and low-tech industries. An increase in the competitiveness of foreign firms always lowers domestic wages, so the product range in low-tech industries is subject to conflicting influences and expands if and only if $\theta > 1 - \frac{beL}{a\tilde{\gamma}(\delta_H)(L+kL^*)}$. *By contrast, the demand effect of greater international market integration leads to less pressure on wages, with the result that low-tech industries always prune their product range. High-tech industries shielded from foreign competition expand their product range whenever domestic wages fall, which always happens following an increase in foreign competitiveness but occurs after greater international market integration if and only if foreign firms are sufficiently competitive that* $\gamma^* < \tilde{\gamma}(\delta_L)$.

7 Conclusion

In this paper we have developed a new model of multi-product firms which highlights the role of flexible manufacturing but which is sufficiently tractable that it can be embedded in a model of general oligopolistic equilibrium. Our analysis shows that the GOLE model provides a coherent framework within which the implications of multi-product firms and the associated supply and demand linkages can be addressed. Our focus is on the intra-firm adjustments within multi-product firms and we find that economy-wide shocks can have a considerable impact on both the scale and scope of multi-product firms. In addition, our analysis shows that the general equilibrium feedback effects, through changes in wages and income, are an important determinant of changes in product ranges.

Our results suggest that adjustment processes within multi-product firms are significantly different from adjustments within industries through exit and entry. Standard trade theory based on single-product firms in monopolistic competition predicts that international market integration raises the real wages of all participating countries and unambiguously increases the choices available to consumers. While this outcome is still possible in our framework, our results show that other outcomes are also possible depending on the competitiveness of foreign firms, on consumer preferences and on the degree of flexibility in manufacturing. First, the change in the real wage depends on whether the impact of an increase in competition from abroad is accompanied by an increase in foreign demand, because the competition effect tends to lower the real wage while the demand effect tends to raise it. Second, the overall change in diversity depends on the degree of flexibility in manufacturing. If manufacturing technologies are highly flexible, multi-product firms respond to shocks more by altering their product range than their total output, which as we have shown implies that overall product diversity can fall when new countries enter the world market.[18] These results are substan-

[18] This is quite consistent with the findings of Broda and Weinstein (2006) that the diversity of *imports* has increased as a result of trade liberalization. Moreover, their study assumes CES preferences, which place a higher premium on diversity than quadratic preferences.

tially different from the predictions of standard trade theory even though both sets of results are driven by the same forces, an increase in the number of firms and an increase in the size of the market. This difference in predictions underlines the importance of intra-firm adjustments.

Furthermore, our look inside a firm's product range reveals new and testable insights into how infra-marginal products adjust. Because flexible manufacturing creates cost heterogeneities within firms, asymmetric adjustment processes are possible that differ significantly from adjustments via exit and entry. We show that these processes are driven to a large degree by changes in factor prices, underlining the importance of a general equilibrium approach.

Our framework can be extended in various directions. We present an extension that analyzes the general equilibrium feedback effects between asymmetric industries. This provides insights into how adjustments within multi-product firms can differ between industries and shows that industries which are not subject to direct foreign competition in their own markets are still affected by a competition effect through the labor market. We also allow for heterogeneous firms in our partial equilibrium analysis. Further extensions, to allow for heterogeneous firms in general equilibrium, and to consider how firms choose their degree of flexibility, seem well worth exploring in our framework.

Empirical evidence suggests that multi-product firms are an important feature of modern industries. Our results show that adjustment processes within multi-product firms differ substantially from adjustments via exit and entry and that globalization can be a driving force of these adjustment processes.

8 Appendix

8.1 Ranking of Elasticities

The elasticity of the IE curve is given by

$$\left.\frac{\partial W}{\partial \delta}\frac{\delta}{W}\right|_{IE} = -\frac{\left[\left(m+1+\frac{e}{2-e}n\right)\frac{e\delta}{2(1-e)}+1+\frac{e}{2-e}n\right]\delta\gamma_{\delta}(\delta)}{\left(m+1+\frac{e}{2-e}n\right)\frac{e}{2(1-e)}\alpha(\delta)+\left(1+\frac{e}{2-e}n\right)\gamma(\delta)-\frac{e}{2-e}n\gamma^{s}} < 0. \tag{73}$$

Note that the denominator is clearly positive because $\gamma(\delta)-\gamma^s+\frac{e}{2(1-e)}\alpha(\delta)=\frac{2b(1-e)x^s}{W(L+kL^*)}>0$.

The elasticity of the LL curve is given by

$$\left.\frac{\partial W}{\partial \delta}\frac{\delta}{W}\right|_{LL} = -\frac{\left[m\delta\mu'_{\gamma}(\delta)+\frac{2(1-e)}{2-e}n\gamma^{s}\left\{\frac{e\delta}{2(1-e)}+1\right\}\right]\delta\gamma_{\delta}(\delta)}{m\beta(\delta)+\frac{2(1-e)}{2-e}n\gamma^{s}\left\{\frac{e}{2(1-e)}\alpha(\delta)+\gamma(\delta)-\gamma^{s}\right\}} < 0. \tag{74}$$

Recall that $\alpha(\delta)=\delta\left[\gamma(\delta)-\mu'_{\gamma}(\delta)\right]$ and $\beta(\delta)=\delta\left[\gamma(\delta)\mu'_{\gamma}(\delta)-\mu'_{\gamma}(\delta)^2-\sigma^2_{\gamma}(\delta)\right]$. Subtracting (74) from (73), the LL curve is more steeply sloped (in absolute value) than the IE curve provided that:

$$\frac{\varphi m\frac{e}{2-e}n\left[\mu'_{\gamma}(\delta)-\gamma^{s}\right]^2+\varphi m\mu'_{\gamma}(\delta)^2+n(\gamma^{s})^2}{\left(m+1+\frac{e}{2-e}n\right)\frac{e\delta}{2(1-e)}+1+\frac{e}{2-e}n} > -\varphi m\sigma^2_{\gamma}(\delta), \tag{75}$$

which always holds.

8.2 Comparative Statics with Homogeneous Industries

The full equilibrium is described by the following set of equations:

$$be(X+Y) = [a-W\gamma(\delta)](L+kL^*) \tag{76}$$

$$2b(1-e)X = W\alpha(\delta)(L+kL^*) \tag{77}$$

$$2b(1-e)x(i) = [\gamma(\delta)-\gamma(i)]W(L+kL^*) \tag{78}$$

$$b(2-e)x^s = (a-W\gamma^s)(L+kL^*)-beY \tag{79}$$

$$Y = mX+nx^s+k\frac{L^*}{\gamma^*} \tag{80}$$

$$2b(1-e)L = m(L+kL^*)W\beta(\delta)+2b(1-e)n\gamma^s x^s \tag{81}$$

Taking derivatives and rewriting the equations in a matrix format we obtain

$$\underline{\Delta}\overrightarrow{\upsilon} = \overrightarrow{\omega}_k L^* dk + \overrightarrow{\omega}_{\gamma^*} \frac{kL^*}{(\gamma^*)^2} d\gamma^*, \tag{82}$$

where

$$\underline{\Delta} = \begin{pmatrix} be & 1 & 0 & 0 & be & \gamma(\delta) \\ 2b(1-e) & -\delta & 0 & 0 & 0 & -\alpha(\delta) \\ 0 & -1 & 1 & 0 & 0 & -(\gamma(\delta)-\gamma(i)) \\ 0 & 0 & 0 & b(2-e) & be & \gamma^s \\ -m & 0 & 0 & -n & 1 & 0 \\ 0 & m\delta\mu'_\gamma(\delta) & 0 & 2b(1-e)n\gamma^s & 0 & m\beta(\delta) \end{pmatrix}$$

$$\overrightarrow{\upsilon} = \begin{pmatrix} dX \\ W(L+kL^*)\gamma_\delta(\delta)d\delta \\ 2b(1-e)dx(i) \\ dx^s \\ dY \\ (L+kL^*)dW \end{pmatrix}, \overrightarrow{\omega}_k = \begin{pmatrix} (a - W\gamma(\delta)) \\ W\alpha(\delta) \\ (\gamma(\delta)-\gamma(i))W \\ (a - W\gamma^s) \\ \frac{1}{\gamma^*} \\ -mW\beta(\delta) \end{pmatrix} \text{ and } \overrightarrow{\omega}_{\gamma^*} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

The determinant of coefficients $|\underline{\Delta}| = \frac{\Delta}{b^2[2(1-e)+e\delta]}$ is clearly positive: see the explicit expression for Δ in footnote 11. Cramer's rule then provides the results presented in Sections 5.2 and 5.3.

8.3 Comparative Statics with Heterogeneous Industries

By taking derivatives of equations (62), (64) and (65) we obtain the following set of equations:

$$\begin{gathered} \theta m_L \delta_L \mu'_\gamma(\delta_L) W\gamma_\delta(\delta_L) d\delta_L + (1-\theta) m_H \delta_H \mu'_\gamma(\delta_H) W\gamma_\delta(\delta_H) d\delta_H \\ + \frac{2b(1-e)L}{W(L+kL^*)} dW = -2b(1-e)\frac{L^*L}{(L+kL^*)^2} dk, \end{gathered} \tag{83}$$

$$\begin{gathered} W\delta\gamma_\delta(\delta_L) d\delta_L + \frac{\varphi_L}{W}\left(a - \frac{be}{\theta\gamma^*}\frac{kL^*}{L+kL^*}\right) dW \\ = \frac{\varphi_L be}{\theta(\gamma^*)^2}\frac{kL^*}{L+kL^*} d\gamma^* - \frac{\varphi_L be}{\theta\gamma^*}\frac{L^*L}{(L+kL^*)^2} dk, \end{gathered} \tag{84}$$

$$W\delta\gamma_\delta(\delta_H) d\delta_H + \frac{a\varphi_H}{W} dW = 0, \tag{85}$$

where φ_H and φ_L are defined in the text.

In matrix format, this can be written as:

$$\underline{\Delta}\overrightarrow{v} = \overrightarrow{\omega}_{\gamma^*}\frac{\varphi_L be}{\theta\delta\left(\gamma^*\right)^2}\frac{kL^*}{L+kL^*}d\gamma^* + \overrightarrow{\omega}_k\frac{L^*L}{\left(L+kL^*\right)^2}dk, \tag{86}$$

where:

$$\underline{\Delta} = \begin{pmatrix} 1 & 0 & \frac{\varphi_L}{W}\left(a - \frac{be}{\theta\gamma^*}\frac{kL^*}{L+kL^*}\right) \\ 0 & 1 & \frac{a\varphi_H}{W} \\ \theta m_L\delta_L\mu'_\gamma\left(\delta_L\right) & \left(1-\theta\right)m_H\delta_H\mu'_\gamma\left(\delta_H\right) & \frac{2b(1-e)L}{W(L+kL^*)} \end{pmatrix}, \quad \overrightarrow{v} = \begin{pmatrix} W\gamma_\delta\left(\delta_L\right)d\delta_L \\ W\gamma_\delta\left(\delta_H\right)d\delta_H \\ dW \end{pmatrix} \tag{87}$$

and

$$\overrightarrow{\omega}_{\gamma^*} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \overrightarrow{\omega}_{\gamma^*} = \begin{pmatrix} -\frac{\varphi_L be}{\theta\gamma^*} \\ 0 \\ -2b\left(1-e\right) \end{pmatrix}. \tag{88}$$

The determinant of coefficients $|\underline{\Delta}| = -\frac{L^*}{L+kL^*}\Delta_T$ is clearly signed:

$$|\underline{\Delta}| = -\left[\theta m_L\left\{\varphi_L\mu'_\gamma\left(\delta_L\right)^2 + \delta_L\sigma^2_\gamma\left(\delta_L\right)\right\} + \left(1-\theta\right)m_H\left\{\varphi_H\mu'_\gamma\left(\delta_H\right)^2 + \delta_H\sigma^2_\gamma\left(\delta_H\right)\right\}\right] < 0 \tag{89}$$

Cramer's rule provides the results presented in Section 6 with $\Delta_T = -\frac{L+kL^*}{L^*}|\underline{\Delta}|$.

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Figure 1: The Scale of Production and the Cannibalization Effect

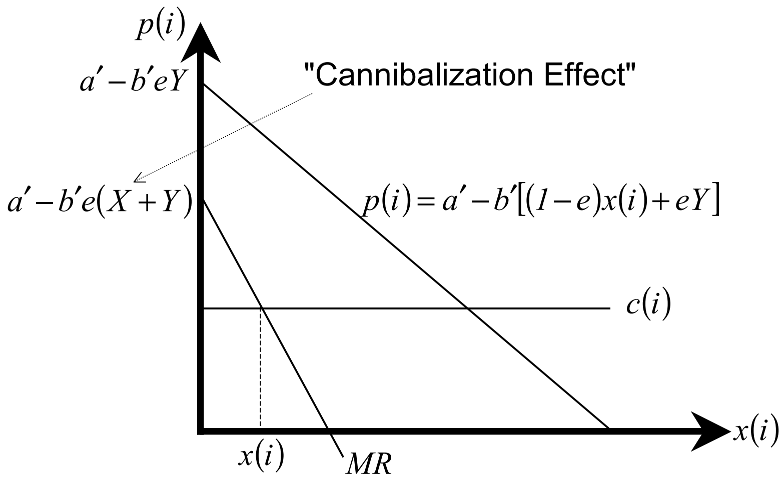

Figure 2: Core Competence and Flexible Manufacturing: The Profit-Maximizing Product Range

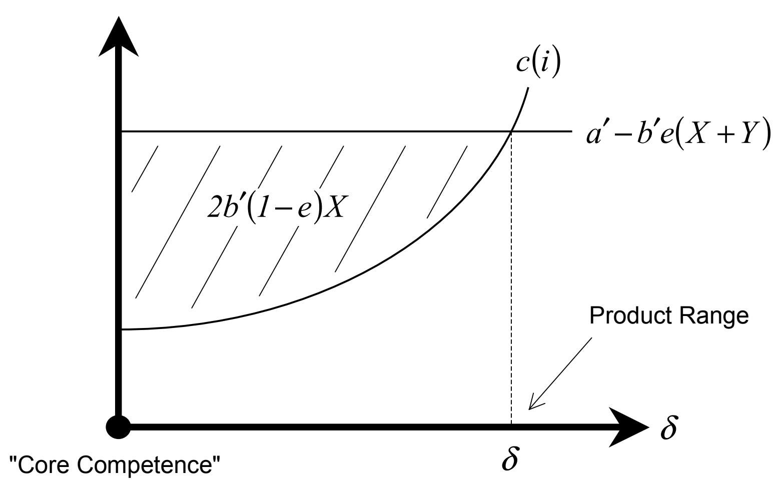

Figure 3: Partial Equilibrium

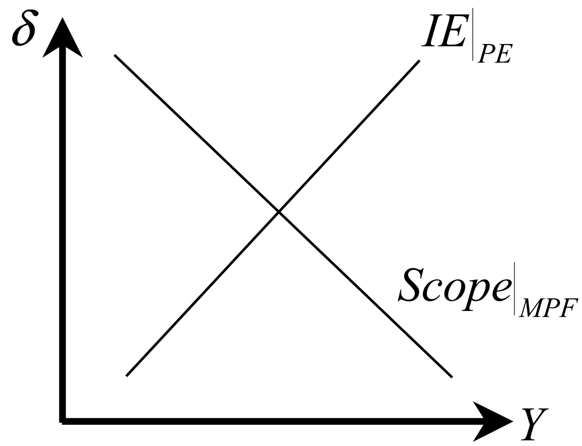

Figure 4: General Equilibrium

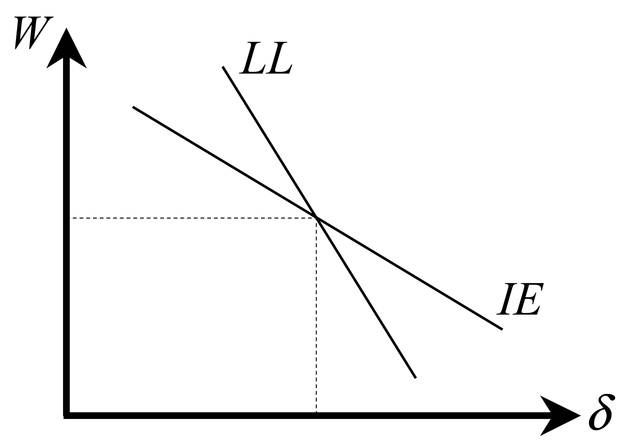

Figure 5: An Increase in Foreign Productivity

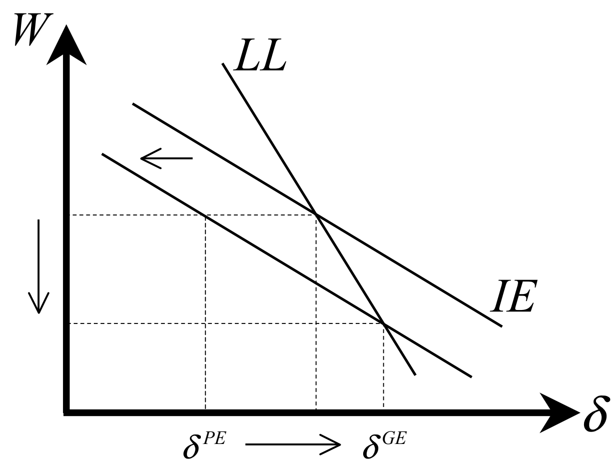

Figure 6: International Market Integration

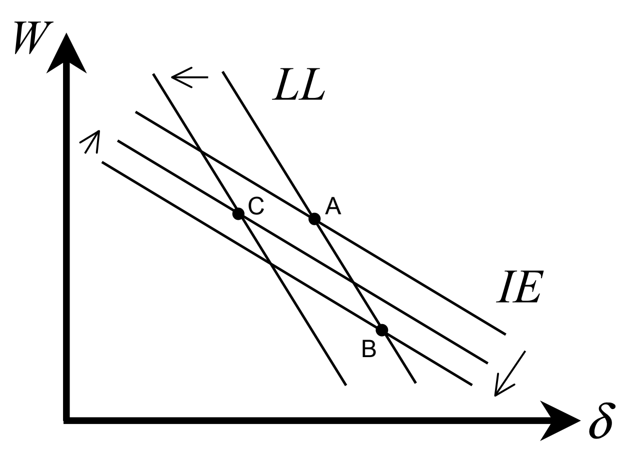

Figure 7: Asymmetric Adjustments in High-Tech and Low-Tech Industries