

Figure 1. Application of composites in the areas of renewables, civil infrastructure, aerospace, and automotive. Reprinted with permission from ref. [1], Creative Commons Attribution CC BY license.

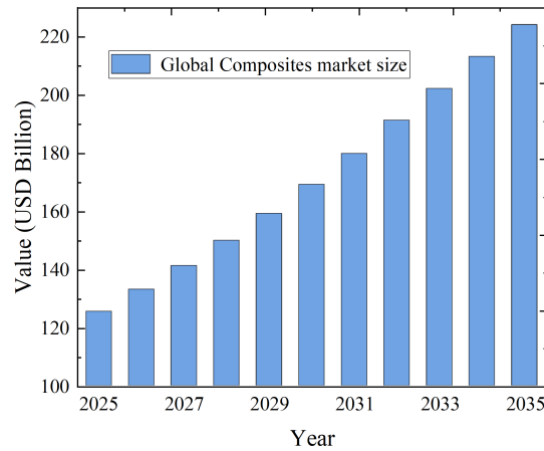


Figure 2. Predicted composites market size. Data adapted with permission from ref. [2], Precedence Research.

This growth is driven by the demand for lightweight, high-strength materials across industries. For example, in aerospace, composites extend flight range and reduce fuel consumption; in automotive manufacturing, they enhance fuel efficiency and lower emissions; and in renewable energy, they are essential for lightweight wind turbine blades. Research activity mirrors this trend, with studies on "lightweight composites" increasing nearly fivefold in the last decade in Fig. 3. The broader advanced lightweight material market is also expanding rapidly, projected to grow from USD 42.71 billion in 2024 to USD 69.4 billion by 2035 [3]. The concurrent upward trends of the two curves indicate that as lightweight composites gain wider adoption, there is a corresponding and vital need to investigate their complex dynamic characteristics.

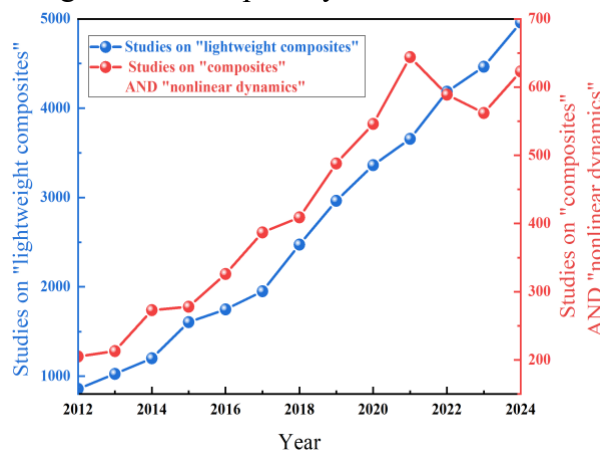
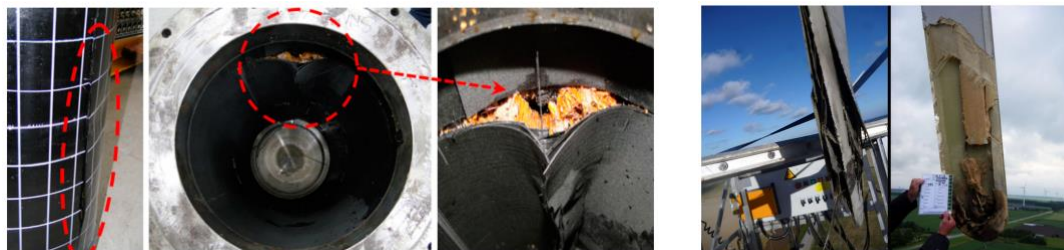


Figure 3. Trend for the number of studies addressing “lightweight composites”, “composites” AND “nonlinear dynamics” based on Web of Science.

Lightweight composite structures, as one of the composite structures, are mainly used to reduce weight while maintaining structural strength and efficiency. They can offer a high strength-to-weight ratio, strong corrosion resistance, and greater design flexibility, and reduced cost. These structures also have consistent potential advantages in terms of noise and vibration reduction, impact resistance and energy absorption

capacity. They also offer advantages in the manufacturing process, such as the reduced cost of producing small batch parts and the possibility of integration, i.e., structures can be manufactured with fewer sub-parts. Lightweight composite structures also have a unique function: with the right choice of the orientation of the constituent materials and reinforcing fibers, they can be customized to meet design requirements that cannot be matched by traditional materials. This is essential for performance optimization, where the goal is to minimize mass and/or stress concentration, thereby guaranteeing the desired performance.

With the wide application of advanced lightweight composite structures, the applicable environment has become more diverse and complex, such as blades and wings will be subject to aeroelastic coupling effects; rocket, satellite and other aerospace vehicles are subject to external high temperature; ships and warships are subject to fluid-structure coupling, high pressure and so on. In addition, the lightweight composite structures are mostly thin-walled structures, so they will vibrate greatly under complex physical fields, and the mechanical behavior of structures composed of advanced composite materials is much more complex than that of isotropic structures. Problems such as matrix cracking, interfacial debonding and interlayer stress component hopping may occur in these structures [4–6] as shown in Fig. 4. They can affect the normal operation of the equipment and even pose a danger to the operator. Therefore, we focus on the nonlinear dynamics of lightweight composite structures, trying to reveal the physical phenomena behind the numerical observation, and provide theoretical guidance and structural design optimization for practical engineering applications.



(a) Composite shells for underwater vehicle [4] (b) Wind turbine composite blade [5]

Figure 4. Structural damage or destruction of composite structures subjected to complex external loads. (a) Reprinted with permission from ref. [4], Elsevier; (b) Reprinted with permission from ref. [5], Creative Commons Attribution CC BY license.

Research on the nonlinear dynamics and vibration of novel lightweight composite structures is a highly active and burgeoning area of study. However, the knowledge remains fragmented across the literature. This review therefore seeks to consolidate the latest research results, provide a structured overview of the current state-of-the-art, and refine the most critical unsolved problems and challenges to guide future investigations. In this case, the structure of this paper is arranged as follows: In section 2, the classification of lightweight composite structures is given. The mechanical modeling of these structures is explained in section 3. In section 4, the corresponding solution methods and nonlinear dynamic behavior analysis are shown. In section 5, the future challenges and research prospects of nonlinear dynamics and vibration are presented. Finally, section 6 gives the main conclusions.

2. Material Constituents and Reinforcement Architectures

Lightweight composite structures are increasingly adopted across multiple industries due to their high strength-to-weight ratios and design flexibility. A clear understanding of their material constituents is essential for interpreting their mechanical behavior and dynamic performance [7]. This section classifies lightweight composite structures according to matrix materials and reinforcement architectures

2.1 Matrix Materials: Functions and Key Types

The matrix materials are the continuous phase in a composite material that supports and surrounds the reinforced materials. They can provide shape and volume to the composite structure, and can also help to transfer loads between the reinforcement materials and protects them from environmental factors, such as moisture and chemical exposure [8]. The three main matrix categories of lightweight composites are metal matrix composites (MMCs), polymer matrix composites (PMCs), and ceramic matrix composites (CMCs) as presented in Tab. 1.

Table 1. The micromechanical models of effective elastic properties.

Category	Metal Matrix Composites (MMCs)	Polymer Matrix Composites (PMCs)	Ceramic Matrix Composites (CMCs)
Matrix	Metal	Polymer	Ceramic
Reinforcement	Fiber/Particle	Fiber/Particle	Fiber/Whisker
Advantages	High Strength, Temp, Conduct.	Lightweight, Low Cost, Moldable	High Temp, Oxidation Resist.
Disadvantages	High Cost, Hard to Process	Low Temp, Low Strength	Brittle, Hard to Process
Density	Medium/High	Low	Medium
Temperature Range	High	Low/Medium	Very High
Cost	High	Low/Medium	High
Key Features	Strength, Temperature, Conductivity	Lightweight, Moldability, Cheap	Temperature, Oxidation, Corrosion

MMCs offer superior thermal and electrical conductivity compared to PMCs and many monolithic metals, making them attractive for applications requiring efficient thermal management or electromagnetic shielding [8] Aluminum-based MMCs are particularly prevalent due to their favorable balance of low density, strength, conductivity, manufacturability, and cost [9–11]. Overall, MMCs exhibit higher strength, toughness, and environmental durability than PMCs, and greater fracture resistance than CMCs.

PMCs are widely employed due to their low density, corrosion resistance, cost-effectiveness, and ease of processing. However, the inherent insulating nature of most polymer matrices limits their applicability in areas such as medical devices and flexible electronics [12,13]. Current research focuses on incorporating conductive fillers like carbon nanotubes, graphene, and metal nanowires/particles to enhance the electrical

properties of PMCs [14–16]. Despite these advancements, PMCs still lag behind MMCs in terms of electrical conductivity and electromagnetic shielding effectiveness.

CMCs combine the high-temperature capability, hardness, and chemical stability of ceramics with improved toughness introduced by reinforcing phases [17–19]. Common ceramic matrices include silicon carbide and alumina. Despite their brittleness and processing challenges, CMCs are indispensable in extreme environments, such as aerospace propulsion systems and thermal protection applications, due to their exceptional thermal and oxidation resistance.

Each matrix type presents a unique set of advantages and trade-offs that must be carefully considered during material selection for a specific application. Given that this review primarily focuses on the nonlinear dynamic behavior of lightweight composite structures, detailed discussions of specific fabrication approaches and application-specific examples for each composite type will be limited.

2.2 Reinforcement Architectures: Tailoring Dynamic Performance

Reinforcements dictate anisotropy, stiffness, and damage propagation mechanisms, all critical to vibration and nonlinear dynamics. Three key reinforcement types are distinguished by their morphology and load-transfer efficiency: continuous fibers (CFs), discontinuous fibers (DFs) and particulate reinforcements (PRs) [20]. The examples of these composites are shown in Fig. 5.

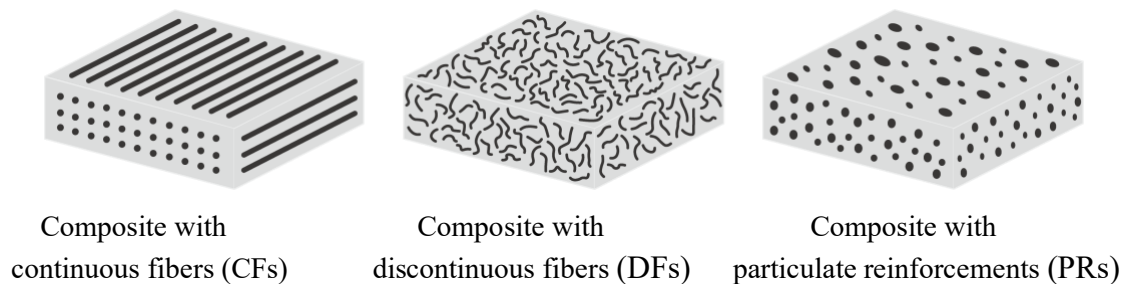


Figure 5. Schematics of CFs, DFs and PRs common reinforcement materials.

CF-reinforced composites offer the highest specific stiffness and strength, with excellent tunability through fiber orientation and stacking sequences, making them dominant in high-performance structural applications [21,22]. DFs, including chopped fibers, provide improved manufacturability and reduced cost at the expense of directional control, and are therefore widely adopted in high-volume industrial applications [23].

PRs, such as ceramic particles and graphene nanoplatelets, are increasingly employed to enhance multifunctional properties, including wear resistance, thermal conductivity, and electrical performance [24–26]. Although their contribution to load-bearing capacity is generally lower than that of fiber reinforcements, PRs are particularly attractive for non-structural or hybrid composite systems.

Overall, each matrix system presents distinct advantages and trade-offs, and material selection is inherently application-driven. Given the focus of this review, detailed fabrication techniques and application-specific examples are not discussed.

3. Mechanical Modeling of Typical Lightweight Composite Structures

This chapter focuses on the mechanical modeling of lightweight composite

structures, which achieve performance through two main strategies: geometric optimization at the meso/macro scale and tailoring material property distribution. We first review structures defined by geometry: corrugated, honeycomb, porous foam, and lattice is provided in Tab. 2 (Sections 3.1-3.4). Due to complex internal topologies, modeling emphasizes effective macroscopic properties rather than microstructural details. We then discuss Functionally Graded Materials (FGMs), where performance is optimized through continuous material gradients (Section 3.5).

Table 2. Comparison of Typical Lightweight Composite Structures based on geometric architecture.

Structure Type	Geometry	Fabrication	Mechanical Properties	Mechanical Behavior	Applications
Corrugated Structures	2D periodic waves	Stamping, folding	High in-plane stiffness, directional strength	Anisotropic	Packaging, sandwich panels
Honeycomb Structures	Hexagonal cells (2D/3D)	Expansion, 3D printing	High out-of-plane compression resistance	Orthotropic	Aerospace, armor
Porous Foam Structures	Random open/closed pores	Foaming, sintering	Low stiffness, high energy absorption	Isotropic	Cushioning, thermal insulation
Lattice Structures	3D periodic truss/beam networks	Additive manufacturing	High strength-to-weight, tunable anisotropy	Tunable (Anisotropic, Orthotropic & Isotropic)	Aerospace, biomedical implants

3.1 Corrugated Structures

A corrugated structure is characterized by parallel ridges and furrows, exhibiting pronounced anisotropy: high flexibility along the corrugation direction and increased transverse stiffness. The use of composite materials enhances the potential for performance optimization by tailoring design parameters.

Research includes novel composite corrugations to prevent core buckling [27,28] and the integration of multistable properties [29,30]. Multistability, arising from the interaction between prestressed laminates and nonlinear geometric deformation, enables large, stable shape changes without locking mechanisms while maintaining a high strength-to-weight ratio. As shown in Fig. 6, parametric studies allow for customized stiffness to meet specific application needs, driving increased use in both research and industry [31] [32].

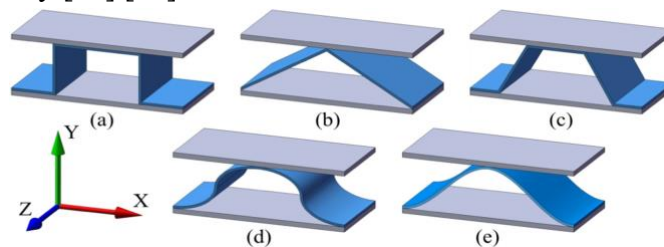


Figure 6. Structure diagram of variable corrugated unit cells. Reprinted with permission from ref. [32], Creative Commons Attribution CC BY license.

Mechanical modeling relies on equivalent models. Early analyses employed mesh-free methods [33–39], followed by homogenization techniques that account for local deformation [40]. Recent approaches favor numerical and hybrid methods, using techniques like numerical homogenization within finite element models to evaluate tensile, bending, and shear stiffness [41–47]. Current trends prioritize refined analytical methods; for instance, building on an equivalent anisotropic plate model [40], a novel corrugated shell model uses a stiffness matrix and the equivalent force method to determine effective properties [48,49].

3.2 Honeycomb Structures

Honeycomb structures are lightweight, two-dimensional periodic lattices, and their hexagonal tiling efficiently minimizes material use while maximizing specific stiffness, strength, and energy absorption [50–53]. Through rational microstructural design, these structures can exhibit remarkable properties such as a negative Poisson’s ratio (auxetic behavior) [54,55], negative stiffness [56], compression-torsion coupling [57], and negative thermal expansion [58]. Due to their superior performance in vibration damping, impact absorption, and thermal management, they are widely used in aerospace, automotive, and construction [59–62].

Research on mechanical modeling typically focuses on unit cell analysis. Based on deformation mechanisms, honeycombs are classified here into three main categories: traditional, auxetic, and combined honeycomb structures.

3.2.1 Traditional Honeycomb Structures

The mechanical modeling of traditional hexagonal honeycombs is well-established. Gibson’s constitutive framework [63,64] remains fundamental. Subsequent studies have refined this model by addressing its initial simplifications [65].

3.2.2 Auxetic Honeycomb Structures

In contrast to traditional designs, auxetic honeycombs exhibit a zero-Poisson’s ratio or negative Poisson’s ratio, leading to superior impact absorption and shear stiffness. This behavior is achieved by modifying joint geometry, with three primary unit cell types: re-entrant, chiral, and rotating structures.

Re-entrant structures

This category includes concave hexagonal, double arrowhead, and star-shaped configurations as shown in Fig. 7. Their auxetic behavior is highly sensitive to rib angles. Early beam-bending models by Gibson and Ashby [63] fail at extreme angles, a limitation addressed by Masters and Evans [66] through the inclusion of hinging and stretching deformations. Further refinements by Grima et al. [67] and Liu et al. [54,55] who derived a modified Gibson function by accounting for geometric nonlinearity and tensile/compressive deformation.

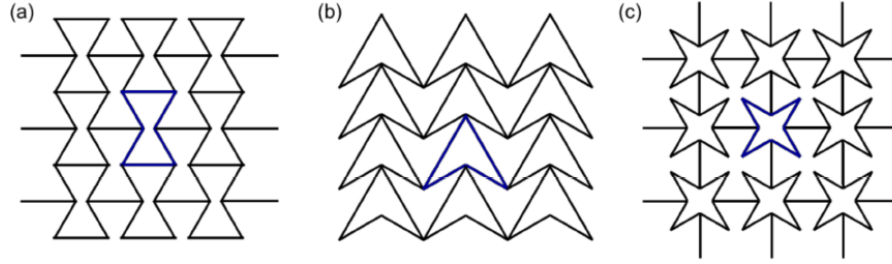


Figure 7. Re-entrant structures. (a) concave hexagonal honeycomb, (b) double arrowhead, (c) star-shaped honeycomb. Reprinted with permission from ref. [68], Creative Commons Attribution CC BY license.

Double arrowhead structures, first identified by Larsen et al. [69], are less studied; Berinskii [70] provided a comprehensive analytical model for their in-plane response. Star-shaped honeycombs, analyzed by Grima et al. [71] and Mizzi et al. [72], generally show less auxetic behavior.

Chiral structures

Chiral honeycombs achieve auxetic behavior through ligaments that wrap around nodes as presented in Fig. 8. Specifically, when ligaments are secured on the same side of both nodes, causing them to rotate in opposite directions during deformation, anti-chiral configurations are formed [73]. Spadoni and Ruzzene [74] used a micropolar method to model them, quantifying the previously indeterminate shear modulus and showing its dependence on ligament thickness. This approach was extended by Jin et al. [75], who derived buckling strengths and continuum approximations.

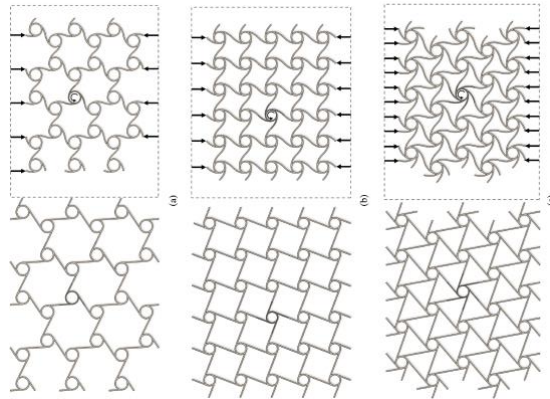


Figure 8. Typical types of chiral structures: (a) tri-chiral; (b) tetra-achiral, and (c) hex-achiral structures. Reprinted with permission from ref. [73], Creative Commons Attribution CC BY license.

3.2.3 Combined Honeycomb Structures

Combined structures merge different unit cells to enhance performance or achieve new properties in Fig. 9. Studies have established models for their effective in-plane mechanical [76] and thermo-elastic properties [77], as well as for structures with tunable Poisson's ratios [78]. Analytical models for specific designs like the AuxHex structure have been validated experimentally [79,80]. Recently, Some new combined honeycomb structures are expanded and studied [81–90].

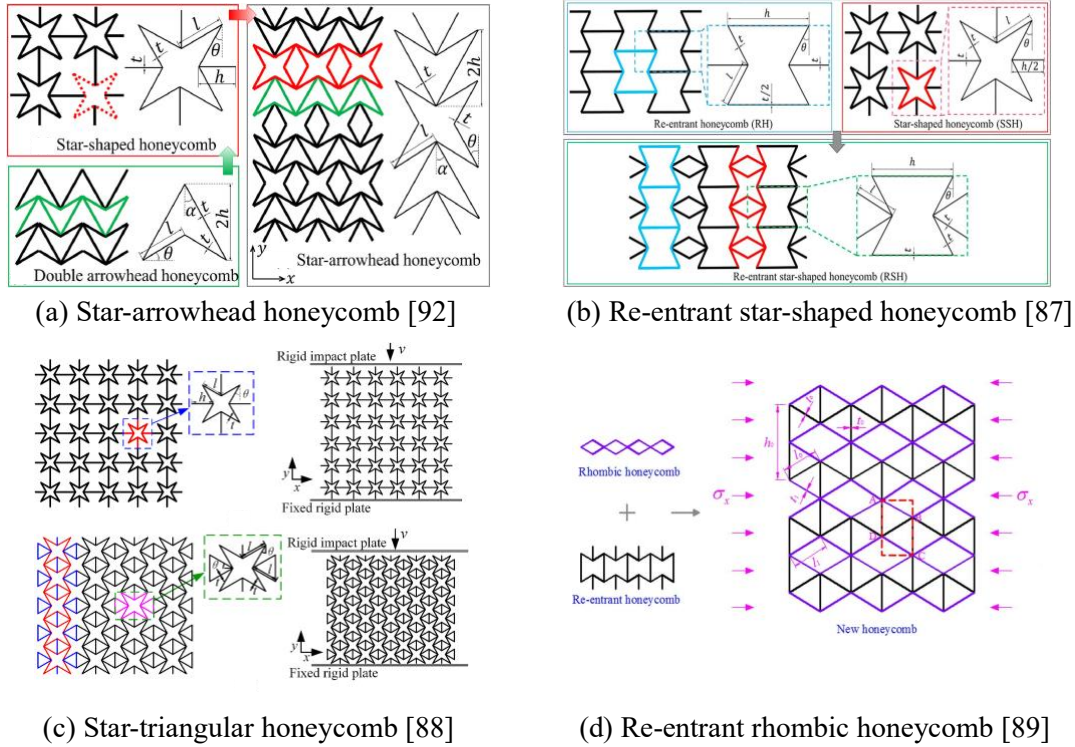
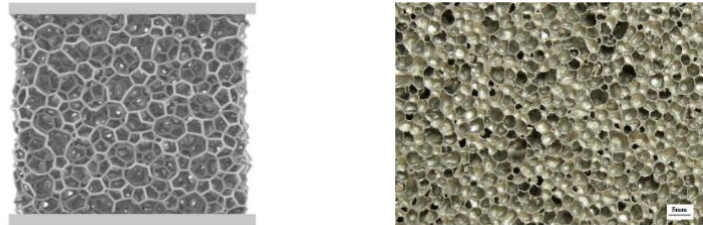


Figure 9. Various combined honeycombs. (a)-(d) Reprinted with permission from refs. [87–90], Elsevier.

3.3 Porous Foam Structures

Unlike regular 2D honeycombs, three-dimensional porous materials, or foam structures, feature a complex, non-uniform pore distribution as presented in Fig. 10. These intentionally introduced pores reduce weight and can improve vibration damping and impact resistance with minimal loss of load-bearing capacity [91,92].



(a) Open-cell porous foam [93]

(b) Closed-cell porous foam [94]

Figure 10. Typical types of porous foam structures. (a)-(b) Reprinted with permission from refs. [93,94], Elsevier.

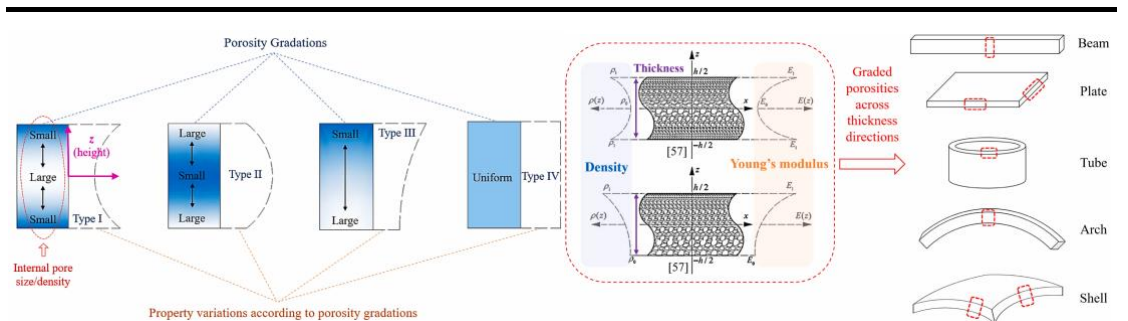
Foams are primarily classified as open-cell or closed-cell in Fig. 12. Their elastic properties are commonly evaluated using micromechanical models where relative density is the key predictive parameter as shown in Tab. 3. These semi-empirical or numerical models, applicable to both foam types with adjusted parameters, include:

The Menges-Knipschild model [95], which relates modulus to relative density but is limited to low-density foams and omits Poisson's ratio; The widely used Gibson-Ashby model [64,96,97], a semi-empirical framework where mechanics are primarily governed by relative density. This method has also seen extensive application and further development in recent research [98–100]; The Christensen model [101], based on membrane deformation and applicable only to very low-density structures; The

Roberts-Garboczi model [102], a statistical FEM-based approach for closed-cell foams within a specific density range; Modified mixing rules [103–105], used for micro-porosity in FGMs but requiring further experimental validation.

Table 3. The micromechanical models of effective elastic properties. $E_s, \rho_s, \nu_s,$ and G_s are Young’s modulus, Mass density, Poisson’s ratio and shear modulus of the solid material, respectively; E_o, ρ_o, ν_o and G_o are the elastic properties of the open-cell porous foam structures, respectively; E_c, ρ_c, ν_c and G_c are the elastic properties of the close-cell porous foam structures, respectively; P and V are elastic properties and volume fraction, with the subscripts “1” and “2” indicating the different constituents, respectively; α is the porosity; and ξ_1 and ξ_2 are constants to be determined by experiment or simulation.

Mechanical models	Effective elastic properties
Menges–Knipschild model [95]	$\frac{E_{o,c}}{E_s} = \frac{\xi_1 (\rho_{o,c} / \rho_s)^2}{\rho_{o,c} / \rho_s + \xi_2}$
Gibson–Ashby model [96]	$\begin{cases} E_o / E_s = \xi_1 (\rho_o / \rho_s)^2, \\ G_o = 3 E_o / 8, \nu_o = E_o / (2 G_o) - 1, \text{ for open-cell foams} \\ E_c / E_s = \xi_2 \left[\frac{1}{2} (\rho_c / \rho_s)^2 + \frac{3}{10} (\rho_c / \rho_s) \right], \\ G_c = 3 E_c / 8, \nu_c = E_c / (2 G_c) - 1, \text{ for closed-cell foams} \end{cases}$
Christensen model [101]	$\begin{cases} \frac{E_o}{E_s} = \frac{1}{6} \frac{\rho_o}{\rho_s}, \nu_o = \frac{1}{4}, \text{ for open-cell foams} \\ \frac{E_c}{E_s} = \frac{2(7-5\nu_s)}{3(1-\nu_s)(9+5\nu_s)} \frac{\rho_c}{\rho_s}, \nu_c = \frac{1+5\nu_s}{9+5\nu_s}, \text{ for closed-cell foams} \end{cases}$
Roberts–Garboczi model [102]	$\frac{E_c}{E_s} = \left(\frac{\rho_c / \rho_s + 0.14}{1 + 0.14} \right)^{2.09} \quad (0.15 < \rho_c / \rho_s < 1), \text{ for closed-cell foams}$
Modified rule of mixture [103–105]	$P = P_1 \left(V_1 - \frac{\alpha}{2} \right) + P_2 \left(V_2 - \frac{\alpha}{2} \right), \quad (p \square 1)$



(a) Typical porous foam types (b) Non-uniform porous foam in different structures
Figure 11. Non-uniform porous foam structures. Reprinted with permission from ref. [106], Creative Commons Attribution CC BY license

Recent applications increasingly utilize non-uniform porous foams with graded

porosity across the thickness, leading to novel functionally graded properties in beams, plates, and shells as presented in Fig. 11.

3.4 Lattice Structures

Lattice structures are engineered, three-dimensional periodic materials composed of interconnected unit cells formed by struts, plates, or shells. Their design is highly controllable, allowing programmable tuning of mechanical, thermal, and fluidic properties through parameters like topology, strut thickness, and relative density [107]. This multifunctional nature combines high specific strength, energy absorption, and surface-to-volume ratios, making them suitable for aerospace components, biomedical implants, and heat exchangers [108,109]. Mechanical modeling depends on the unit cell type, broadly categorized into strut-based and surface-based designs, as illustrated in Fig. 12 [107].

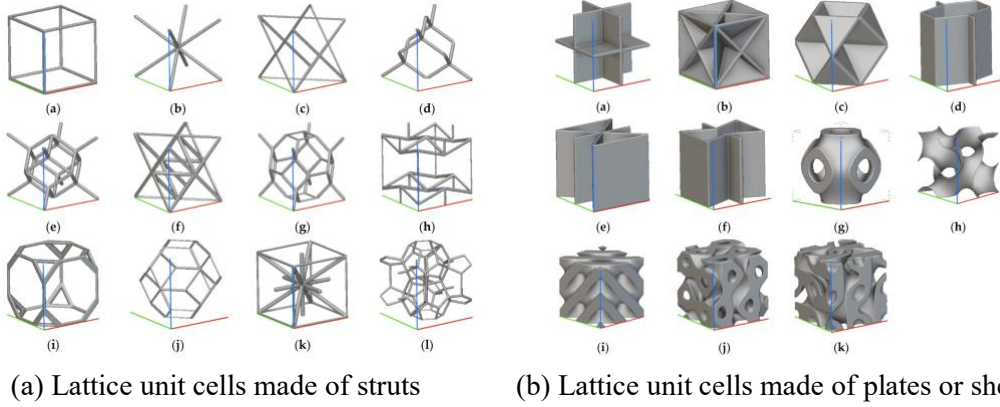


Figure 12. Representative unit cells of lattice structures. Reprinted with permission from ref. [107], Creative Commons Attribution CC BY license.

Strut-dominated unit cells, such as simple cubic (SC) and body-centered cubic (BCC), share cubic symmetry with varying beam complexity [110–112]. Octet-truss units evolve from face-centered cubic (FCC) arrangements [113], while modified Gibson-Ashby and Wallach-Gibson cells achieve enhanced tunability through beam reinforcement [111,114,115]. In contrast, surface-based systems use continuous plates or shells, resulting in distinct deformation mechanics.

The macroscale mechanical response depends on hierarchical interactions between global topology and local cell behavior. Lattices are typically classified into two deformation modes [116]: Stretch-dominated systems (e.g., Octet, Kelvin, FCC), where axial deformation leads to an elastic modulus with a near-linear relationship to relative density; Bend-dominated systems (e.g., BCC, stochastic foams), where transverse deflection causes the modulus to scale with the square of the relative density. These relationships are expressed as [117,118]:

$$E^* / E_s = C_E \bar{\rho}^n \quad \text{Stretch - dominated} \quad (1)$$

$$E^* / E_s = C_E (\bar{\rho})^2 \quad \text{Bending - dominated} \quad (2)$$

where, E^* and E_s are the equivalent and matrix elastic moduli, $\bar{\rho}$ is the relative density, ρ^* and ρ_s are the equivalent and matrix densities, and C_E is a topology-dependent constant. For stretch-dominated structures, the exponent $n \approx 1$ (ignoring

nodes), though manufacturing defects can increase it to 1.5–2. In some hierarchical structures, n can also be negative [119].

The relative density is principally determined through geometric volume fraction analysis, and it can be respectively obtained as follows:

$$\bar{\rho} = V_{struts} / V_{element} \quad \text{Stretch - dominated} \quad (3)$$

$$\bar{\rho} \propto (t_b / l_b)^2 \quad \text{Bending - dominated} \quad (4)$$

For more complex lattice unit cell configurations, the elastic properties are mainly determined through experiments or simulations [118,120–122].

3.5 Functionally Graded Materials Composite Structures

Beyond geometric optimization, lightweighting can be achieved through strategic material design, notably via Functionally Graded Materials (FGMs). FGMs feature a continuous spatial gradient in microstructure and properties. By eliminating the sharp interfaces of traditional laminates, they reduce stress concentrations and delamination, creating more efficient and reliable structures under dynamic loads [123–125].

The inherent material gradient in FGM structures results in unique nonlinear dynamic behaviors and presents distinct challenges for mechanical modeling. To accurately predict these behaviors, a variety of methods have been developed to model the equivalent material properties of FGM structures. The main modeling methods are summarized in Tab. 4.

Table 4. Comparison of main modeling methods for the effective material properties of FGM structures.

Method / Model	References	Principle
Rule of mixtures (Voigt model)	[126–129]	A simple volume-weighted average of constituent properties based on an iso-strain assumption
Power-Law	[130–137]	Volume fraction varies as $V(z) = (z/h)^k$
Sigmoid law	[138–145]	Provides a smooth, S-shaped transition of volume fraction between the two constituent phases
Exponential law	[146–148]	Volume fraction varies exponentially across the thickness, modeling phenomena like diffusion processes
Mori-Tanaka model	[149–154]	A micromechanics model treating one phase as the matrix and the other as inclusions, accounting for stress fields

The classical Rule of Mixtures applies a simple volume-weighted average under an iso-strain assumption, with some studies proposing refined versions for greater accuracy [129]. The Power-Law method is the most common, where properties vary as a function of thickness; it has been shown to fundamentally influence the vibration characteristics of plates and shells [130–137].

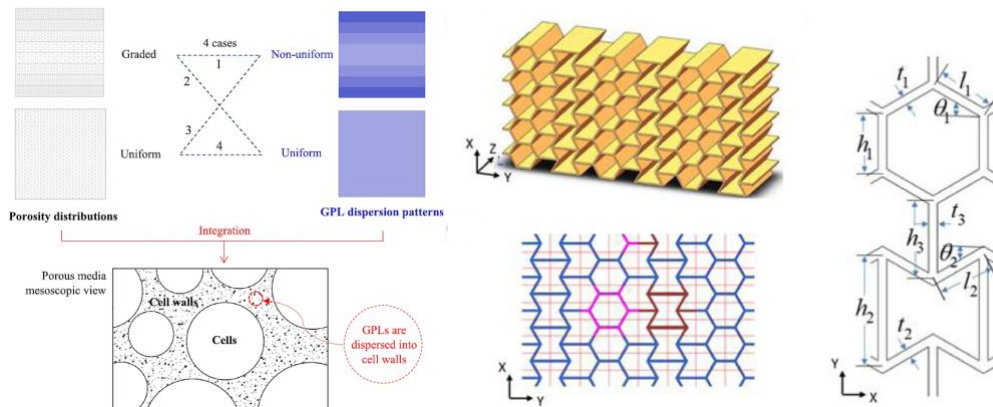
Alternative gradient descriptions include the Sigmoid law, which provides a smooth S-shaped transition that minimizes stress at interfaces [138–145], and the Exponential law, suitable for modeling processes like diffusion or infiltration [146–148]. For greater micromechanical accuracy, the Mori-Tanaka model treats one phase as inclusions within a matrix, accounting for interaction stresses, and is often combined

with a distribution law [149–154].

Recent research extends this concept by using discrete layers with varying concentrations of nanofillers like graphene platelets (GPLs) to achieve FGM-like performance [155]. A detailed discussion of nanocomposite modeling is beyond this review's scope, as comprehensive surveys exist [156–158].

3.6 Hybrid Lightweight Composite Structures

The growing industrial demand for composite structures drives the development of hybrid designs, which combine multiple materials or structural types for tailored performance as illustrated in Fig. 13. These hybrids offer significant advantages, including weight reduction, enhanced mechanical properties, improved durability, and greater design flexibility.



(a) GPL-reinforced FG porous structures [159] (b) Hybrid honeycomb unit cell structures [79]

Figure 13. Hybrid lightweight composite structures from different materials or structural types.

(a)-(b) Reprinted with permission from refs. [79,159], Elsevier.

A forward-looking extension of this approach involves embedding active functionality into the composite architecture itself. A prime example is the development of self-healing composites [160–162], which autonomously repair damage, fundamentally altering a structure's lifecycle response. From a modeling perspective, self-healing introduces a central challenge: time-varying material properties. The continuous evolution of mechanical response during damage-healing cycles requires new theoretical frameworks that integrate structural mechanics with material science, an area still in its early stages [163–165].

Beyond such actively triggered variations, the intrinsic viscoelastic nature of polymer matrices in lightweight composites introduces time-dependent mechanical behavior (e.g., creep and stress relaxation), which is essential for predicting long-term dynamic performance [166]. Future modeling efforts should integrate viscoelasticity to enhance the accuracy and reliability of nonlinear dynamic analyses across various lightweight configurations.

4. Solution Methods and Nonlinear Dynamic Analysis

The inherent anisotropy and heterogeneity of lightweight composite structures introduce complex mechanical couplings (e.g., bend-twist, extension-shear). These couplings facilitate energy transfer between vibration modes, leading to dense internal resonances and intricate modal interactions that linear models cannot capture. Accurate

design requires methods that can resolve the full nonlinear response, including bifurcations and instability thresholds.

Solution approaches are examined in two categories: transient analysis for impact/shock response, and steady-state analysis for periodic loading and stability.

4.1 Transient Dynamics: Impact-Induced Nonlinearity

In terms of investigating transient responses, the impact dynamics and energy absorption capacities of lightweight composite structures are of paramount importance. It is worth noting that high-velocity impacts are frequently adopted to examine the extreme energy absorption of honeycomb, porous foam, and lattice composites for applications like aerospace shielding and structural protection. Under such ballistic conditions, energy is rapidly dissipated through localized mechanisms such as cell-wall folding, core crushing, and structural perforation [167–169]. However, in routine service, these structures are predominantly subjected to low-velocity impact loads. Unlike high-velocity impact, low-velocity impact induces complex global bending and hidden internal damage, which are the primary drivers of significant structural nonlinearity. Therefore, this review will concentrate primarily on low-velocity impact scenarios. Common transient solution methods for these problems are summarized as presented in Tab. 5.

Table 5. Common transient solution methods of lightweight composite structures.

Solution methods	References
Duhamel integration method	[49,170–175]
Direct integration method	[54,176–179]
Newmark method	[159,180,181]
FEM	[182–188]
AI-based method	[189–192]

Impact contact problems are strongly nonlinear; however, low-velocity impact responses are often analyzed using mass–spring models with the Duhamel integral as presented in Fig. 14 [49,170–175], which is effective for approximately linear systems but limited for complex nonlinear behaviors and geometries. Studies based on this approach show that corrugated, porous, auxetic, and re-entrant core designs significantly improve impact resistance and energy absorption compared with conventional configurations. Material enhancements, such as graphene platelet nanofillers and chopped fiber rod–reinforced foam cores, further increase stiffness and reduce displacement, contact duration, and vibro-impact responses.

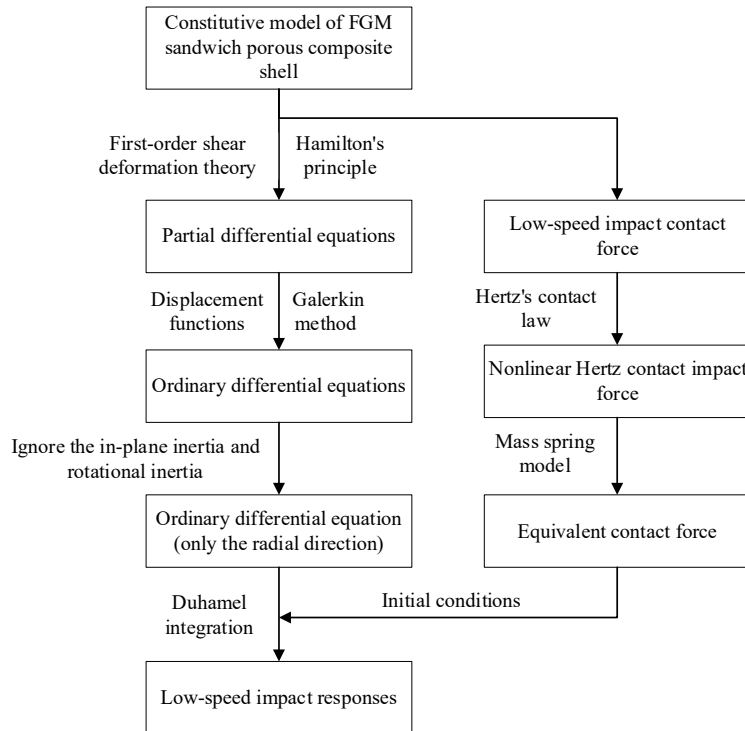


Figure 14. A summary of the Duhamel integral flowchart of FGM sandwich porous composite shells [175].

Direct integration methods provide high accuracy and flexibility for dynamic impact analysis but at increased computational cost. Studies using this method consistently report improved vibration damping and impact resistance from porous foams [176,177], auxetic honeycombs [54,179], and CNT-reinforced FGMs [193,194], though specific effects vary with structure and loading.

The Newmark method is widely employed for low-velocity impact analysis of lightweight composites due to its stability and ability to handle both linear and nonlinear responses [159,180,181,195–197]. Applications of this method reveal strong sensitivity of structural responses to load pulse characteristics, with rectangular pulses producing the most severe dynamic effects [159]. For highly complex geometries and material behaviors, the finite element method (FEM) remains the more comprehensive tool, enabling detailed modeling of nonlinear impact responses at the expense of higher computational cost [182–188], as shown in Figs. 15-16.

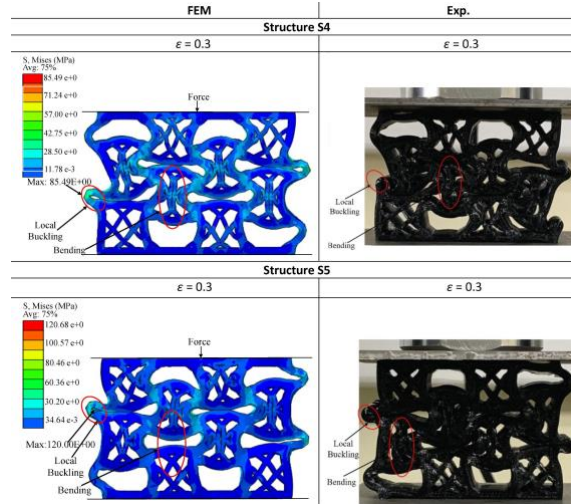


Figure 15. Comparison between FEM and experiment of fiber-reinforced composite plates under low-velocity oblique impact. Reprinted with permission from ref. [198], Elsevier.

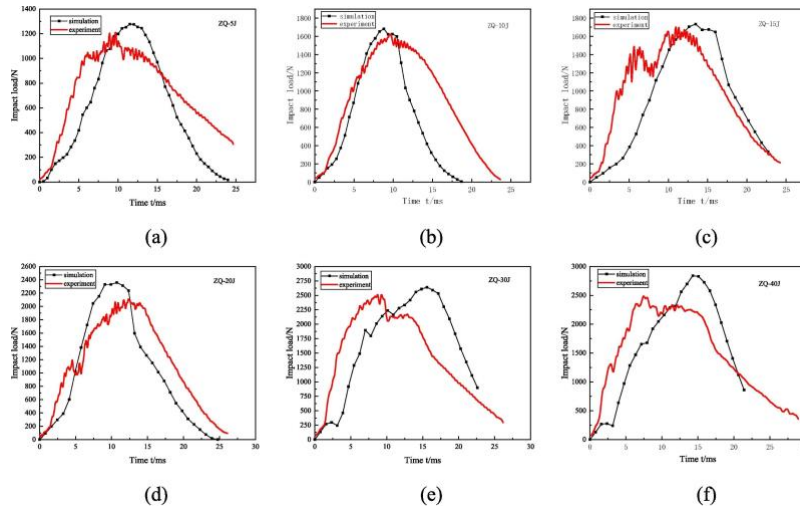


Figure 16. Comparison of impact load-time curves with different impact energies: (a) 5 J, (b) 10 J, (c) 15 J, (d) 20 J, (e) 30 J, and (f) 40 J. Reprinted with permission from ref. [199], Elsevier.

Recently, Artificial Intelligence (AI) has been increasingly utilized to address the computational challenges in composite dynamics. Common approaches include surrogate models for rapid prediction of impact and vibration responses [189,190,200–202], deep learning for damage evaluation [192], and Physics-Informed Neural Networks (PINNs) that embed physical laws for data-efficient, physics-consistent simulations [191,203]. As composite structures become increasingly complex, these AI-based methods provide an indispensable framework for both forward dynamic prediction and inverse damage identification.

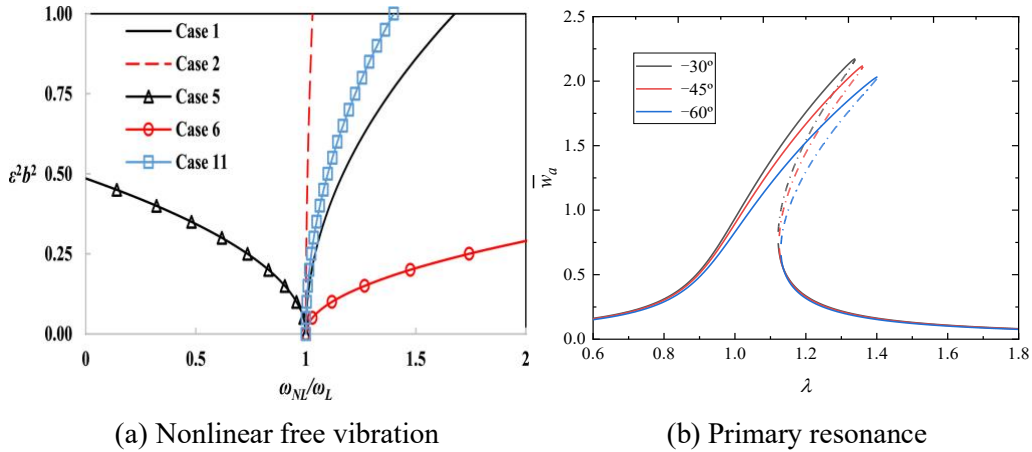
4.2 Steady-State Dynamics: Bifurcations and Chaos

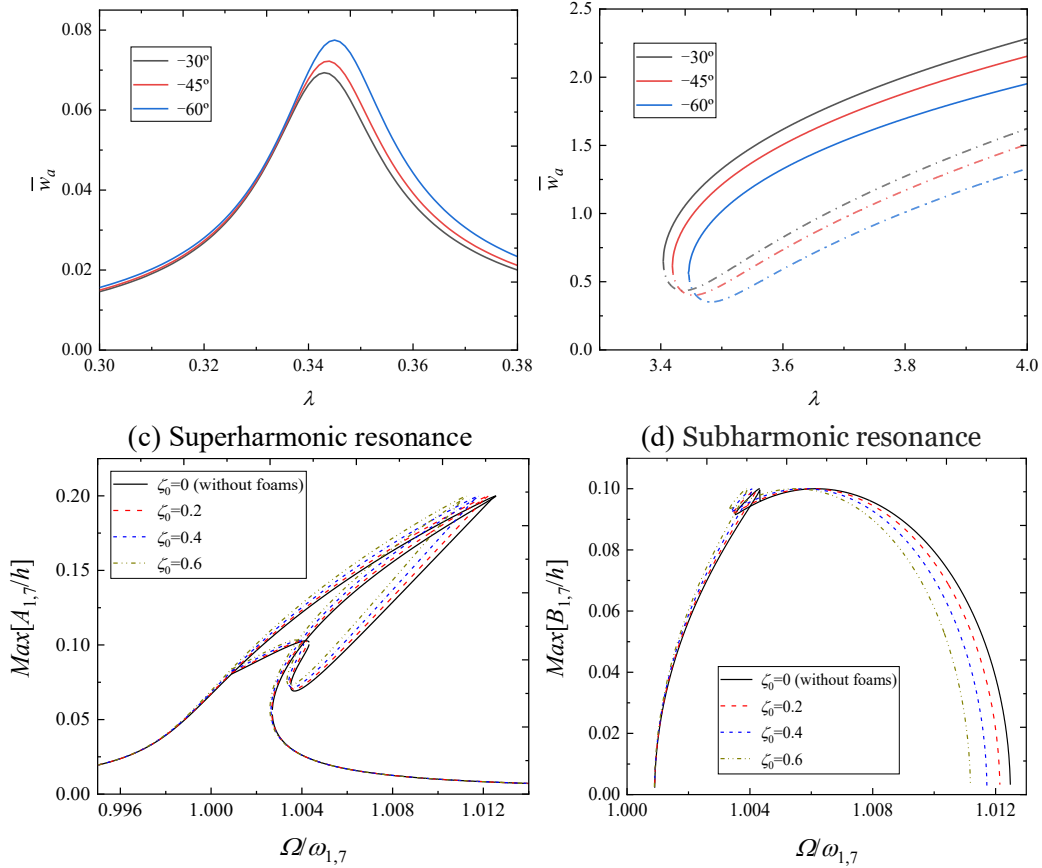
Analyzing the steady-state, nonlinear dynamics of lightweight composite structures requires specialized solution methods, broadly categorized as approximate analytical or numerical as shown in Tab. 6.

Table 6. Common steady-state solution methods of lightweight composite structures.

Solution methods	References
MSM	[55,100,204–212]
HBM	[213–218]
Perturbation method	[219–226]
FEM	[65,227–229]
Direct integration method	[230–237]
Quadrature method	[238–244]
Pseudo-arclength continuation method	[137,245–250]
AI-based method	[200–203]

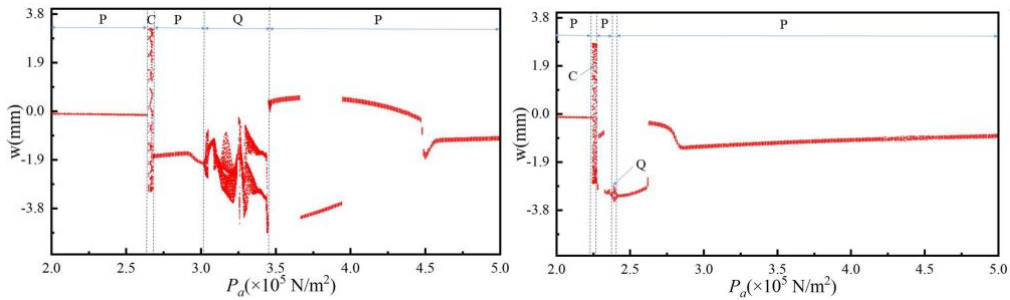
Analytical methods, including the Multi-Scale Method (MSM), Harmonic Balance Method (HBM), and Perturbation Method, are effective for simpler systems. Extensive investigations have addressed nonlinear free vibration, primary and secondary resonances, internal resonances, and super-/sub-harmonic responses in lightweight composite structures as presented in Figs. 17-18. Results consistently demonstrate strong designability of nonlinear responses, enabling tuning between hardening and softening behaviors through structural parameters such as core thickness, porosity distribution, and material gradation. For instance, honeycomb core thickness [209,251–253], foam distribution [207] and core-to-face ratio [100], and environmental factors such as temperature and humidity [204] were shown to significantly influence resonance characteristics and vibration amplitudes.





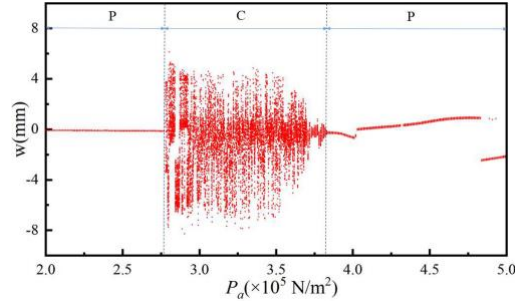
(e) 1:1 internal resonance (driven mode) (f) 1:1 internal resonance (companion mode)

Figure 17. Closed-form solutions of lightweight composite structures: (a) Nonlinear free vibration of fiber-reinforced hexagon honeycomb shells (where different cases denote different honeycomb core layer thicknesses) [209], (b)-(d) Primary resonance, super-/sub- harmonic resonance of auxetic honeycomb plates [55], and (e)-(f) 1:1 internal resonance of FGM porous shells [100] (λ : dimensionless frequency; dotted line: unstable solution; solid line: stable solution). (a)-(d) Reprinted with permission from refs. [100,209], Elsevier; (e)-(f) Reprinted with permission from ref. [100], Creative Commons Attribution CC BY license.



(a) Type-1 porosity distribution

(b) Type-2 porosity distribution



(c) Type-3 porosity distribution

Figure 18. The bifurcation diagram of porous metal conical shells with different porosity distribution. Reprinted with permission from ref. [239], Elsevier.

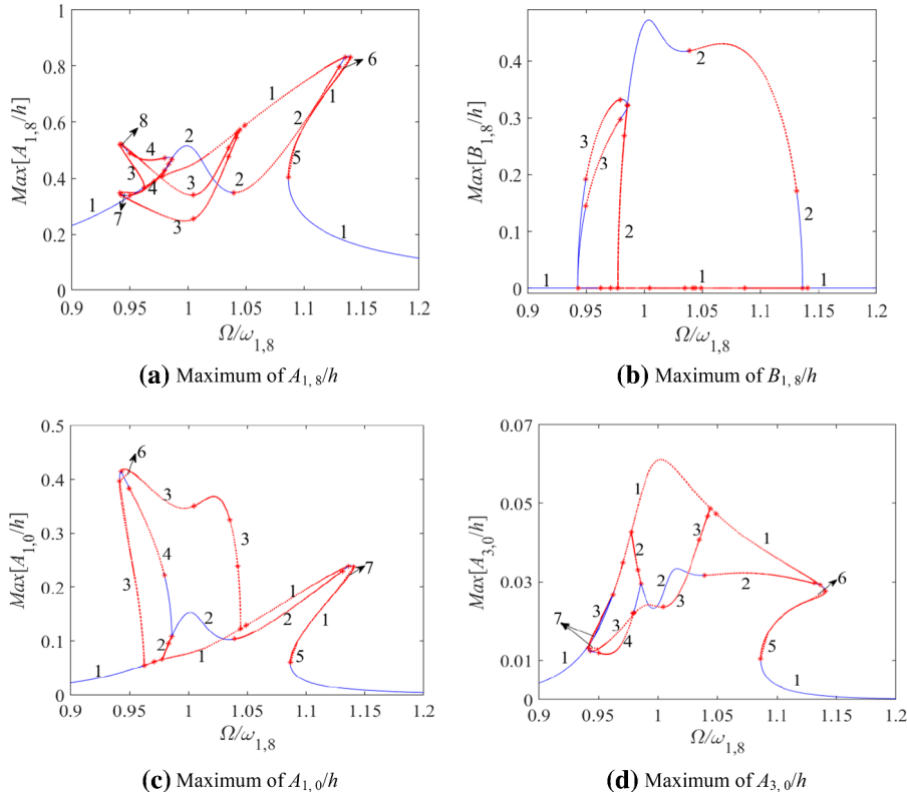


Figure 19. Nonlinear frequency–response curves of FGM composite shells with porosities (red dotted line: unstable solution; blue solid line: stable solution). Reprinted with permission from ref. [246], Springer Nature.

For complex excitation conditions or strong nonlinearities, analytical methods become inadequate, and numerical approaches are required. These include FEM, direct integration, quadrature-based methods, and pseudo-arclength continuation techniques. Quadrature methods [238–244], such as the differential quadrature method (DQM), offer high accuracy with relatively few discretization points but may have difficulty capturing unstable or chaotic responses. In contrast, pseudo-arclength continuation method [254,255] is particularly effective for tracing stable and unstable solution branches, bifurcations, and internal resonances in high-degree-of-freedom systems. For example, applications of these methods have revealed chaotic responses, mode energy transfer, and large-amplitude vibrations driven by nonlinear internal resonances in porous and functionally graded composite shells as shown in Fig. 19 [246].

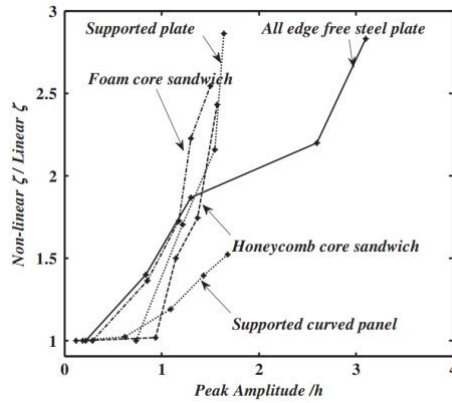


Figure 20. Non-linear variation of damping (normalized with respect to the linear damping for small vibration amplitude) versus the peak vibration amplitude (normalized with respect to the thickness). Reprinted with permission from ref. [256], Elsevier.

Damping plays a critical role in nonlinear vibration behavior. Experimental and numerical studies show that damping in lightweight composite structures is strongly amplitude-dependent, with large-amplitude vibrations exhibiting substantially higher energy dissipation than predicted by linear models as presented in Fig. 20 [256–258]. Amabili and co-workers further demonstrated that nonlinear damping can be rigorously derived from viscoelastic constitutive laws, rather than phenomenological assumptions [259–262]. Since key dissipation mechanisms—such as matrix viscoelasticity, interfacial slip, and interlaminar friction—are inherently nonlinear, this physics-based framework provides a solid foundation for future nonlinear dynamic analyses of lightweight composite structures.

5. Challenges and research prospects

The preceding sections provided a summary of advancements in nonlinear dynamics and vibration issues pertaining to lightweight composite structures. While significant progress has been made in this field, numerous challenges and unresolved problems remain, and to further enhance the practical application of theoretical insights in engineering contexts, the outline future research directions are as follows:

5.1 Challenges

(1) A major challenge is linking microscopic features to macroscopic nonlinear dynamic responses. This includes understanding how unit-cell geometries give rise to global geometric nonlinearities, as well as how microscale damage and material nonlinearities propagate to affect overall structural behavior under dynamic loading;

(2) Inherent manufacturing and material variability complicates the design of controlled experiments, especially for validating predicted complex phenomena like bifurcations and chaos;

(3) The various design space offered by composites (e.g., topology, material gradation, ply orientation) combined with the computational cost of high-dimensional nonlinear analysis creates a major bottleneck. Developing efficient computational methods that can rapidly explore this design space to find robust, optimized solutions for structures operating in complex, multi-physical environments remain a key hurdle;

(4) Accurate modeling of energy dissipation is a critical challenge. Linear damping

assumptions are generally invalid for large-amplitude nonlinear vibrations, and reliable experimental characterization and theoretical formulation of nonlinear, amplitude-dependent damping remain limited. Without high-fidelity damping models, nonlinear analyses may capture qualitative trends but fail to deliver the quantitative accuracy required for engineering applications.

5.2 Research Prospects

(1) Develop multi-scale models that integrate microscopic and macroscopic nonlinear behaviors, allowing for a more comprehensive understanding of how microstructural changes influence overall material properties based on the hierarchical modeling approaches or advanced computational techniques;

(2) Current dynamic analyses often rely on low DOF models, which lose accuracy under complex excitations. Future work must establish novel algorithms capable of handling high-dimensional, strongly nonlinear systems to accurately capture extreme dynamic phenomena, such as chaotic vibrations, bifurcation, and nonlinear damping prior to material failure;

(3) To facilitate the computationally intensive tasks highlighted above, incorporating AI is a critical future direction. Specifically, developing AI-based methods, such as PINNs, offer a technically sound and highly efficient approach to solving multi-degree-of-freedom nonlinear systems, thereby establishing more robust design frameworks.

6. Conclusion

This paper summarized the recent developments in nonlinear dynamics and vibrations for typical lightweight composite structures. It provided a selective yet comprehensive review of the general concepts, classifications, mechanical models, structural dynamics modeling, solution methods, and nonlinear dynamic behavior analysis. The conclusions can be summarized as follows:

(1) A variety of mechanical models have been established to correlate micro-scale unit cell behavior with macro-scale mechanical properties in lightweight composite structures. These approaches account for the inherent anisotropy and multi-scale interactions spanning from fiber-matrix interfaces to global structural response, enabling more accurate prediction of performance under diverse loading conditions;

(2) The review highlights the use of various solution methods, including analytical approaches (e.g., Galerkin procedures and perturbation techniques) to numerical simulations (e.g., FEM, Pseudo-arclength continuation method), each offering unique advantages for analyzing the nonlinear dynamic behavior of lightweight composite structures;

(3) The lightweight composite material structures, particularly honeycombs, porous foams, and lattice metamaterials, exhibit varying nonlinear vibration characteristics, bifurcation points, and instability thresholds and strengthening effects based on unit cell type or distribution;

(4) Continued investigation into strongly nonlinear, multi-degree-of-freedom composite systems remains critical. To address the computational bottlenecks associated with these high-dimensional dynamic analyses, it is highly recommended to

integrate emerging AI-based methods to facilitate data-efficient, physics-consistent structural design and prediction.

Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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