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THE TIME INCONSISTENCY OF MONETARY POLICY WITH INFLATION PERSISTENCE

Richard Mash

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Manor Road Building, Oxford OX1 3UQ

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Richard Mash

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Department of Economics, University of Oxford, OX1 3UQ, UK

Tel. +44-1865-274553, Fax. +44-1865-274558

email address: richard.mash@economics.ox.ac.uk

Abstract:

In a monetary policy model incorporating partial persistence in inflation it is shown that inflation bias is reduced and the response to shocks improved if the policy maker has a discount rate lower than its true social value. Thus a *patient* central banker is shown to be a third mechanism for offsetting time inconsistency problems in addition to Rogoff's conservative central banker and the principal-agent approach of Walsh. The paper also analyses outcomes under the latter regimes and the optimal rule, finding important differences from the results of earlier literature that excludes inflation persistence.

Keywords: Monetary policy, Time inconsistency, Inflation persistence

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"Maintaining a long time horizon is perhaps the principal

raison d'etre for central bank independence"

Alan Blinder, Distinguished Lecture on Economics in Government: What Central Bankers Could Learn from Academia - and Vice Versa, Journal of Economic Perspectives 1997.

Introduction

Following the seminal contributions of Kydland and Prescott (1977) and Barro and Gordon (1983) a substantial literature has developed on the time inconsistency of monetary policy. When policy makers have a target level of output in excess of its natural rate and the private sector forms rational expectations an average inflation rate in excess of its socially optimal level arises while average output is no higher as a result. Prominent suggestions as to how to reduce this problem include Rogoff's (1985) conservative central banker who has higher aversion to inflation away from its target value, in which case the inflation bias is reduced but at the cost of sub-optimal responses to shocks, and Walsh's (1995) demonstration that inflation bias may be removed completely while retaining optimal responses to shocks if the central bank faces an appropriately specified penalty that is linear in the rate of inflation.[1]

While tremendously influential, the underlying structure of these models has been criticised for being unrealistically static and a growing literature has addressed the implications of allowing for persistence in the key variables of firstly output or unemployment (Lockwood, 1997; Svensson, 1997; Lockwood, Miller and Zhang, 1998; Goodhart and Huang, 1998), and secondly inflation (Clark, Goodhart and Huang, 1999). The latter paper explores the responses to supply shocks in a model with inflation persistence, contrasting optimal behaviour under rules and discretion, while assuming that the policy maker targets the natural rate of output so there is no inflation bias. It is shown that even in the absence of a higher output target, the usual source of time inconsistency, outcomes under the optimal rule and discretion differ when inflation is persistent.

The contribution of the current paper is to extend the work of Clark et. al. (1999) on inflation persistence, generalising their model to include a source of inflation bias while also moving beyond the rules vs. discretion dichotomy to consider outcomes under a Walsh inflation penalty, a Rogoffian conservative central banker and, a new case, a patient central banker whose discount rate is lower than the true social value. The main results are as follows.

Firstly, building on the result that the outcome in response to shocks is no longer optimal under discretion when inflation is persistent, we analyse the effect of the conservative central banker and Walsh regimes on the loss from shocks. In contrast to the standard case without persistence it is found that mild conservatism reduces the loss arising from shocks while the Walsh case retains the sub-optimal properties of discretion when inflation persists.

Secondly, we restore the standard assumption of an output target in excess of its natural rate and analyse the resulting bias to average inflation. Under discretion the usual inflation bias result from the static literature is repeated though the size of the bias is decreasing in the degree of inflation persistence. A conservative central banker still reduces average inflation, as does a Walsh inflation penalty except that it no longer achieves the first best inflation rate unless initial inflation at the start of the regime coincides with its desired long run value.

Thirdly, the most innovative extension, we introduce the concept of a patient central banker whose time discount rate is lower than the true social discount rate. Under inflation persistence an increase in output and inflation in a given period will result in inflation costs in subsequent periods and thus the policy maker's discount rate becomes an important determinant of outcomes. It is shown that a patient central banker reduces both inflation bias and the loss from shocks. Hence a patient central banker offers a third potential mechanism for reducing time inconsistency problems in addition to the Rogoff and Walsh approaches.

The importance of the discount rate under inflation persistence also adds a further explanatory variable to the positive analysis of inflation under discretionary policy and the interpretation of empirical evidence on central bank independence.[2] In the formal analysis we follow the standard convention of assuming that discretionary policy optimises according to true social preference parameters (the discount rate and the relative importance of output and inflation in the loss function). In practice, however, political economy factors may raise a government's discount rate above its social value in which case average inflation and the loss from shocks will both be higher. Thus worse outcomes under an impatient government are the mirror image of better outcomes under a patient central banker. A further possibility is that a government's discount rate may vary over the electoral cycle.

It should be noted that we do not consider reputational effects as a possible offset to the time inconsistency problem.[3] The existence and stability of reputational equilibria are sensitive to private sector beliefs and the observability of the policy making process, which is problematic in a shock prone environment, but if these constraints are overcome reputational effects may of themselves remove the time inconsistency problem and remove the need for the Rogoff, Walsh or patient central banker approaches. The latter is, however, more open to this critique since reputational equilibria are more sustainable with a low discount rate.

A further perspective on the inflation bias issue, from McCallum (1997), Blinder (1997) and Bean (1998) amongst others, is that the act of delegation to an independent central bank may of itself solve the inflation bias problem because an independent institution free of (most) political pressure will tend to target the natural rate of output rather than a higher value. As with successful reputation building, the need for alternative solutions to the inflation bias problem disappears if this is correct. If some gap between the "true" output target and its natural rate remains, however, these arguments are harder to sustain despite their plausibility. The results of this paper provide some general support in this context since if inflation is persistent, and it is natural for independent central banks to have a lower discount rate than governments, they will give rise to lower inflation and behave as if they were targeting the natural rate of output. It should be noted, however, that even if inflation bias disappears after delegation to a central bank that targets the natural rate of output, the issue of sub-optimal responses to shocks identified by Clark et. al. (1999) and extended below remains in place.

The remainder of the paper is structured as follows. The following section presents the model while Sections 2 and 3 derive the outcomes under rules and discretion/delegation respectively. The formal analysis uses standard methods, discussed in depth in Lockwood et. al. (1998) and Clark et. al. (1999), and hence we give a concise treatment. Section 4 presents results and compares the outcomes under different policy regimes while Section 5 concludes.

1. The Model

We make use of a simple model whose core components comprise a standard social loss function (1), defined over outcomes for inflation and output from period t=0 onwards, and a Phillips curve describing the short run relationship between them given by (3) below. In (1), δ is the true social discount factor (the inverse of one plus the discount rate). K is the true social relative aversion to deviations of inflation, π_t, away from its socially optimal value which we take to be zero for simplicity. The level of log output is given by y_t with its natural rate equal to y^*. If k=1 the policy maker targets y^* but we adopt the standard assumption of k>1 that gives rise to inflation bias in the static case (assumed away in Clark et. al., 1999).

$$(1) \qquad L_0 = \Sigma_{t=0}^{\infty} \delta^t \ [\ K\pi_t^2 + (y_t - ky^*)^2\ ]$$

When we consider delegation we allow for the central banker to have different values of δ and K, denoted δ_b and K_b, as well as a possible Walsh inflation penalty ($\varphi\pi_t$), while otherwise retaining the structure of (1) for their loss function which is shown by (2).

$$(2) \qquad L_0 = \Sigma_{t=0}^{\infty} \delta_b^t \ [K_b\pi_t^2 + (y_t - ky^*)^2 + \varphi\pi_t]$$

Under delegation, policy choices are modelled as minimising (2) so δ_b and K_b are relevant for behaviour while the outcomes generated by those choices are evaluated using the true δ and K according to (1). We assume that k>1 is a fundamental feature of the social loss function, rather than reflecting political economy factors and keep k constant across (1) and (2) so inflation bias is not directly eliminated by delegation to an independent central bank. In practice some fraction of k-1 under discretion is likely to arise from the political process but for analytic reasons we keep k-1 constant.[4] If in fact k equals unity under delegation the analysis of discretion as well as the response to shocks (which does not depend on k) across all the regimes remains unchanged. The last term in (2) is present if policy making is delegated to an independent central bank with a Walsh linear inflation penalty in place.

The policy regimes considered are as follows. We follow the literature in assuming that under discretion the policy maker minimises (1), the true social loss function, while noting that in practice governments may not necessarily behave according to true social preferences. Under delegation monetary policy is determined by an agent (central bank) who minimises (2). There are three variants to delegation which are not necessarily mutually exclusive but for clarity we treat them independently. Under a Rogoff conservative central banker $K_b>K$ while $\delta_b=\delta$ and $\varphi=0$. For the Walsh case $K_b=K$ and $\delta_b=\delta$ but φ is set in advance by government in order to minimise (1) subject to the subsequent choices made by the Walsh central banker who minimises their version of (2). The third and new case is that of a patient central banker where $\delta_b>\delta$ while $K_b=K$ and $\varphi=0$. We constrain δ_b to be no greater than unity since it is unlikely that an independent central bank would value the future more than the present. Unless λ is very close to one this constraint binds since the unconstrained optimal choice of central banker would have $\delta_b>1$. In all these cases inflation expectations are a constraint on the policy maker's ex post choices whereas under commitment we assume that the policy maker may credibly commit to a rule, designed in advance to minimise the expected value of (1), in which case the determination of expectations is incorporated within the optimisation. It should also be noted that the paper is focused on the implications of given values for δ_b and K_b rather than their origin though we follow the convention of referring to policy decisions by an agent whose preferences differ from social ones as delegation to a central banker.[5]

We use the short run Phillips equation (3) as the key reduced form relationship between inflation, expected inflation and output that acts as the constraint on the policy maker's optimisation. The parameter λ in (3) allows for inflation persistence.

$$\pi_t = \lambda\pi_{t-1} + (1-\lambda)E_{t-1}[\pi_t] + \gamma(y_t - y^*) + \varepsilon_t \tag{3}$$

If $\lambda=0$ the model reduces to the standard static case. Clark et. al. (1999) also use (3), and discuss it more fully, as does Bean (1998) apart from a one year lag between the output gap and inflation. Both papers argue strongly for the empirical realism of inflation persistence. Fuhrer (1997) finds it difficult to reject the hypothesis that $\lambda=1$ for US data while arguing that in practice λ is likely to be less than unity, though Gali and Gertler (1999) find that the backwards looking element is statistically significant but question its quantitative importance. In simulations for the UK, Blake and Westaway (1996) use $\lambda=0.75$ (α_1 in their notation) and Batini and Haldane (1999) $\lambda=0.8$ as a central value. This paper is concerned with the consequences of inflation persistence rather than its causes but the presence of λ in (3) may be motivated by either a direct appeal to empirical evidence or the presence of rational behaviour combined with a Buiter and Jewitt (1981) or Fuhrer and Moore (1995) contracting structure. A further alternative is less than fully rational expectations or rational expectations with significant information constraints (Ball, 1991). We refer to $\lambda=1$ as complete inflation persistence and note that in this limit case inflation expectations no longer matter and as a result discretionary policy coincides with that under a rule. Hence with $\lambda=1$ there is no time inconsistency problem, while the existing literature applies if $\lambda=0$, so the analysis of this paper is relevant to intermediate values of λ. This range is empirically plausible and generates results that differ from those at the extremes of $\lambda=0$ or 1.

We also assume that expectations are formed rationally and the shock term, ε, is iid with zero expected value and $E[\varepsilon^2]=\sigma^2$. The time structure of the model is standard. At time t-1 the first two terms in (3) for period t are determined. In period t the inflation shock, ε_t, is realised and observed by the policy maker who determines the outcome for inflation and output according to their loss function, (1) or (2), subject to the constraint (3) under discretion/delegation or following a pre-determined rule under commitment. By using the reduced form expression (3) we abstract from the policy instrument used to control inflation and output, as if the policy maker directly sets either of these with the other determined by (3). We also do not consider lags between policy choices and outcomes which are very important in practice but less so analytically in this context. For example a realistic one year lag between changes in output and changes in inflation would make little difference since output and inflation are separable in the loss function.[6] A further implication is that demand shocks do not matter (assuming no control errors) since the contemporaneous nature of the policy choices and outcomes means that policy can immediately offset demand shocks. This property is shared by the Rogoff (1985) model, for example, but natural extensions would allow for demand shocks, control errors and lags.

We conclude this section by mentioning possible alternative modelling approaches that are potentially important but beyond the scope of the current paper. Firstly we follow much of the literature by ignoring the possibility that the Phillips curve may be non-linear. Secondly, we do not allow for parameter uncertainty, either symmetric (across both the private sector and the policy maker) or asymmetric including uncertainty about the policy maker's preferences and possible reputation effects.[7] Thirdly, a further possibility is to replace $E_{t-1}[\pi_t]$ in (3) with $E_t[\pi_{t+1}]$, as in Clarida, Gali and Gertler (1999) for example, but we retain the formulation used by Clark et. al. (1999) for simplicity and to make the results directly comparable. Fourthly we assume that wage and price setting is decentralised so that there is no strategic behaviour on the part of the private sector.[8] Lastly we do not consider political economy factors that may affect outcomes under discretion or independent central banks (Persson and Tabellini, 1999; Keefer and Stasavage, 1999) though the possibility of governments having a sub-optimally high discount rate was noted above.

2. Optimal Rule With Commitment

We derive the optimal policy rule, assuming access to a credible commitment technology, and use it to derive the minimised value of the loss function given the ability to commit to a rule. From the linear-quadratic structure of the model, the optimal policy rule will be of the form $\pi_t=a+b\pi_{t-1}+c\varepsilon_t$ where a, b and c are pre-committed at values to be determined below. It is convenient to define the expected value of π_t by $E_{t-1}[\pi_t]=\bar{\pi}_t=a+b\pi_{t-1}$ and use the notation $\pi_t=\bar{\pi}_t+c\varepsilon_t$. Given $E[\varepsilon_t]=0$ and the linearity of the policy rule in ε, the optimal choice of c may be made independently from the optimal choice of the coefficients a and b that determine $\bar{\pi}_t$ for given π_{t-1} so we may think of the policy maker optimising separately with respect to both $\bar{\pi}_t$ and the response to shocks. As a first step towards deriving the optimal rule we may substitute $\pi_t=\bar{\pi}_t+c\varepsilon_t$ into (3) to yield (4). We assume that the choice of rule is made in period t = -1 and hence analyse the effect of that choice from period 0 onwards.

$$(4) \qquad y_0-ky^* = -y^*(k-1) + \frac{\lambda}{\gamma}(\bar{\pi}_0-\pi_{-1}) - \varepsilon_0\frac{(1-c)}{\gamma}$$

We define the value function under commitment by $V^R(\pi_t)=\beta_0^R+\beta_1^R\pi_t+\beta_2^R\pi_t^2$ which is the expected value of the loss function from period t+1 onwards, given adherence to the rule, discounted to period t+1, as a function of π_t (the value of the state variable inherited at the start of period t+1). Using this value function and the short run Phillips curve under commitment from (4), the loss function (1) may be expressed in terms of π_0 by (5), in which the last term represents the effect of the period 0 choice on the loss in future periods. The appendix shows that minimisation of (5) implies (6) where $\alpha^R = \gamma^2(K + \delta\beta_2^R)$.

$$L_0 = K\pi_0^2 + (y_0 - ky^*)^2 + \delta[\beta_0^R + \beta_1^R\pi_0 + \beta_2^R\pi_o^2] \tag{5}$$

$$\pi_0 = \frac{2\gamma\lambda y^*(k-1) - \gamma^2\delta\beta_1^R}{2(\lambda^2 + \alpha^R)} + \frac{\lambda^2}{\lambda^2 + \alpha^R}\pi_{-1} + \frac{1}{1 + \alpha^R}\varepsilon_0 \tag{6}$$

To complete the derivation of the optimal rule the appendix solves for β_0^R, β_1^R and β_2^R in (7)-(10) from the recursion equation $V^R(\pi_{-1}) = E[K\pi_0^2 + (y_0 - ky^*)^2 + \delta V^R(\pi_0)]$.

$$\gamma^2\beta_2^R(\lambda^2 + \alpha^R) - \alpha^R\lambda^2 = 0 \tag{7}$$

$$\beta_2^R = \frac{-[\gamma^2K + \lambda^2(1-\delta)] + \sqrt{[\gamma^2K + \lambda^2(1-\delta)]^2 + 4\gamma^2\delta K\lambda^2}}{2\gamma^2\delta} \tag{8}$$

$$\beta_1^R = \frac{2y^*(k-1)\lambda\alpha^R}{\gamma[\lambda^2(1-\delta) + \alpha^R]} \tag{9}$$

$$\beta_0^R = \frac{1}{1-\delta}\Big[\ [y^*(k-1)]^2\Big[1 - \frac{\lambda^2(1-\delta)^2(\lambda^2 + \alpha^R)}{[\alpha^R + \lambda^2(1-\delta)]^2}\Big] + \frac{\sigma^2\alpha^R}{\gamma^2(1 + \alpha^R)}\ \Big] \tag{10}$$

Substituting (9) into (6) allows us to derive the long run average inflation rate under the rule, π_s^R given by a/(1-b) and shown in (11) where (7) is used to simplify the left hand part. This is greater than its target level of zero because, unlike the static case, the backwards looking element in (3) gives rise to a (small) tradeoff between inflation and output, even under a rule, and the quadratic structure of preferences means that it is optimal to exploit it to some extent.

$$\pi_s^R = \frac{\gamma y^*(k-1)(1-\delta)\lambda(\lambda^2 + \alpha^R)}{\alpha^R[\lambda^2(1-\delta) + \alpha^R]} = \frac{y^*(k-1)(1-\delta)\lambda}{\gamma K} \tag{11}$$

Having derived the optimal policy rule we conclude this section by expressing the expected value of the social loss function (1) under the optimal rule. For ease of comparison we present the annuity value of the expected loss from t = 0 onwards, evaluated at t = -1. From the definition of the value function this is straightforwardly given by (12).

$$(1-\delta)E_{-1}[L_0] = (1-\delta)[\beta_0^R + \beta_1^R \pi_{-1} + \beta_2^R \pi_{-1}^2] \tag{12}$$

The intuition behind the results may be seen more readily if we consider the special case where π_{-1} equals π_s^R so the initial inflation rate happens to be equal to its average level under the optimal policy rule. For this case (12) simplifies to (13) in which the first term is the loss arising from the deviation of desired output (ky^*) from its average level (y^*), the second term the deviation of actual average inflation, π_s^R, from its (ceteris paribus) ideal level of zero, weighted by the relative aversion to inflation, K, and the third term is the loss arising from shocks that take the economy away from its average output and inflation levels. Thus (13) is the total loss when the system happens to start in its ergodic stationary state and the difference between (13) and (12), where π_{-1} is kept general, reflects the costs (benefits) of bringing inflation down (up) from an initial value above (below) its optimal long run average.

$$(1-\delta)E_{-1}[L_0] = [y^*(k-1)]^2 + (\pi_s^R)^2 K + \frac{\sigma^2 \alpha^R}{\gamma^2(1+\alpha^R)} \tag{13}$$

Our main purpose in considering the outcome under an optimal commitment rule is to provide a benchmark with which to compare the outcomes under discretion and delegation. It may be noted, however, that its implementation would be difficult unless ε is fully observable since the policy maker would have an ex post incentive to deviate from the optimal commitment values of a and b and could do so by misrepresenting ε_t if only the outcome, π_t, is observed (King, 1997). A further point is the general inflexibility of rules in the face of parameter changes over time unless they are specified in a fully contingent form. Flood and Isard (1989), Fischer (1990), and Svensson (1999) provide much broader analyses of rules.

3. Discretion and Delegation Without Commitment

We analyse the discretion, Rogoff, Walsh and patient central banker cases initially together by solving the model in the absence of a commitment technology and with the policy maker's loss function of the general form (2). Separate results for the different cases are subsequently derived by imposing the relevant values of δ_b, K_b and φ.

It is convenient to define $\pi^c_t=\lambda\pi_{t-1}+(1-\lambda)E_{t-1}[\pi_t]$, the first two terms in (3) which, in the absence of commitment and unlike the case of a credible rule, the policy maker must treat as given. Once the (constrained) optimal choice of π_t has been made, its expected value solves for π^c_t given π_{t-1} and thus π_t in full. Using this notation (3) may be expressed by (14).

$$(14) \qquad y_t - ky^* = -y^*(k-1) + \frac{1}{\gamma}(\pi_t-\pi^c_t-\varepsilon_t)$$

We make use of the value function $V(\pi_t)=\beta_0+\beta_1\pi_t+\beta_2\pi^2_t$ which is defined analogously to that under the rule as the expected value of the policy maker's loss function from period t+1 onwards given their optimal policy choices in the future (without the ability to commit), discounted to period t+1, as a function of π_t. Using this value function and (14) we may express the loss function of the policy maker from period 0 onwards by (15) in which the term $\delta_b V(\pi_0)$ captures the effect of the period 0 decision on the loss from period 1 onwards.

$$(15) \qquad L_0 = K_b\pi_0^2 + [-y^*(k-1)+\frac{1}{\gamma}(\pi_0-\pi^c_0-\varepsilon_0)]^2 + \varphi\pi_0 + \delta_b V(\pi_0)$$

The first order condition for the minimisation of (15) implies (16) in which $\alpha=\gamma^2(K_b+\delta_b\beta_2)$.[9]

$$\pi_0 = \frac{2\gamma y^*(k-1) + 2(\pi_0^c+\varepsilon_0)-\gamma^2(\varphi+\delta_b\beta_1)}{2(1+\alpha)} \qquad (16)$$

Taking expectations of (16), noting that $E_{-1}[\varepsilon_0]=0$, allows us to solve for π_0^c in (17) which may be substituted back into (16) to give (18).

$$\pi_0^c = \frac{2\lambda(1+\alpha)\pi_{-1} + 2(1-\lambda)\gamma y^*(k-1) - (1-\lambda)\gamma^2(\varphi+\delta_b\beta_1)}{2(\lambda+\alpha)} \qquad (17)$$

$$\pi_0 = \frac{2\gamma y^*(k-1) - \gamma^2(\varphi+\delta_b\beta_1)}{2(\lambda+\alpha)} + \frac{\lambda}{\lambda+\alpha}\pi_{-1} + \frac{1}{1+\alpha}\varepsilon_0 \qquad (18)$$

Rearranging (15), making use of (17), into terms in π_0 and its square allows us to express the recursion equation for the policy maker's choice problem by (19).

$$\beta_0+\beta_1\pi_{-1}+\beta_2\pi_{-1}^2 = \begin{array}{l} [y^*(k-1)]^2 + \delta_b\beta_0 + [\frac{\gamma}{2}(\varphi+\delta_b\beta_1)]^2 \\ +E_{-1}[\pi_0^2\frac{\alpha}{\gamma^2}(1+\alpha)+\pi_0(\varphi+\delta_b\beta_1)(1+\alpha)] \end{array} \qquad (19)$$

Substituting (18) into (19) and equating coefficients on both sides on π_{-1} and its square gives us two Riccati equations from which β_2 and β_1 may be determined. The first yields (20).

$$\beta_2\gamma^2(\lambda+\alpha)^2 - \lambda^2\alpha(1+\alpha) = 0 \qquad (20)$$

This is a cubic equation in β_2 (since α is linear in β_2) which may be shown to have a single (economically meaningful) positive root.[10] The analytic solution for this root (from Maple) is too complex to use but comparative statics results may be derived without an explicit solution for β_2. The Riccati equation in π_{-1} gives (21), the right hand part of which uses (20) to simplify the left hand part.

(21) $$\beta_1 = \frac{2\alpha\lambda(1+\alpha)y^*(k-1) + \varphi\gamma\lambda^2(1+\alpha)}{\gamma[(\lambda+\alpha)^2 - \delta_b\lambda^2(1+\alpha)]} = \frac{\beta_2[2\alpha y^*(k-1)+\gamma\lambda\varphi]}{\gamma\lambda K_b}$$

Substituting (21) into (18) allows us to express the optimal choice of π_t in the absence of commitment by (22) where π_s is the long run average inflation rate given by (23).

(22) $$\pi_t = \frac{\alpha}{(\lambda+\alpha)}\pi_s + \frac{\lambda}{\lambda+\alpha}\pi_{t-1} + \frac{1}{1+\alpha}\varepsilon_t$$

(23) $$\pi_s = \frac{y^*(k-1)[\lambda(1-\delta_b)+\alpha(1-\delta_b\lambda)]}{\gamma K_b(\lambda+\alpha)} - \frac{\varphi}{2K_b}$$

Before turning to the overall welfare outcome under discretion/delegation we briefly comment on the reasons why the optimal policy choices summarised by (22) and (23) differ from those under the optimal rule in (6) and (11), stressing points not emphasised in the discussion of Clark et. al. (1999). Firstly it may be noted that if $\lambda=1$ the two regimes coincide since expectations (and thus commitment) no longer matter. Also if $\lambda=0$ the standard static model applies and α (with $K_b=K$) will equal α^R and thus the coefficients on lagged inflation and the shock term in (6) and (22) are also equal while the first term differs reflecting the usual inflation bias result. These coefficients also coincide (at zero) if the policy maker does not care about output (infinite K) in which case inflation is always returned to target each period and inflation persistence becomes irrelevant. For the general case of $0<\lambda<1$ and finite K, Clark et. al. (1999) show that the coefficient on lagged inflation is smaller under the rule while that on the shock variable is larger. These coefficients do not depend on $k>1$, the source of inflation bias, and hence are unchanged from Clark et. al., the new feature being the effect of changes in the behavioural parameters shown below.

Figure 1 illustrates the discrepancy between discretion and the optimal rule identified by Clark et. al. (1999), particularly with respect to the lagged inflation rate, and provides some intuition for why the conservative and patient central bankers may improve on the discretionary outcome in response to shocks. It is assumed for simplicity that there are no new shocks and that the previous period's inflation rate corresponds to point A. The natural rate of output is given by y^*. Under discretion the outcome is at point D on the ex post short run Phillips curve YY which has gradient γ from (3). The position of YY is determined, through expectations, together with the point chosen on it such that point B corresponds to the first two terms in (3). In the absence of shocks, expectations are realised and hence from (3) point D is also on line XX which has gradient γ/λ. If policy can pre-commit the policy maker optimises on this line and chooses point R. For a given ex post Phillips curve under the optimal rule, ZZ, the policy maker would like to deviate from the rule and move towards point C but of course if this was anticipated ZZ would have coincided with YY. From Clark et. al. (1999), inflation at R is lower than at D and R is preferred to D since D is within the feasible set shown by XX under commitment.[11] Thus the coefficient on lagged inflation is smaller under the rule (observed inflation will be less persistent) since the policy maker faces smaller output costs of reducing inflation given that the gradient of XX (γ/λ) is steeper than YY (γ) unless $\lambda=1$. In turn this means that the loss from shocks to inflation away from its long run average, both from the initial effect of shocks and the costs of returning to the steady state, will be higher under discretion.

Figure 1 is also suggestive of why the loss from shocks is reduced by mild conservatism or patience on the part of the policy maker (Propositions 2 and 3 below). These changes in preferences imply less tolerance of inflation which is known to the private sector and thus influences expectations such that line YY moves down in anticipation of lower inflation. This gives rise to a first order benefit whereas the resulting distortion to the output/inflation mix chosen for a given ex post Phillips curve is only second order. Lastly it may be noted that the policy choices made under both the optimal rule and discretion imply that inflation is returned to its target gradually and at a speed that depends on policy preferences and subject to the occurrence of further shocks. This means that observed inflation will be serially correlated but with an autoregressive coefficient not (in general) equal to λ. Output will also be serially correlated despite the absence of any structural persistence in output in the model. These predictions are empirically supported but do not arise in the standard static model where the outcome in one period has no bearing on the outcome in future periods.

Having derived the policy maker's optimal behaviour in the absence of commitment it remains to evaluate the expected social loss arising from that behaviour. The loss as perceived by the policy maker may be obtained from the value function but, given that we wish to vary the policy maker's preference parameters away from their social values, the loss perceived by the policy maker may differ from the true social loss. Hence instead we substitute the policy maker's choices derived above, which are contingent on their preferences, directly into the social loss function (1) and solve for its expected value. For this we require expected values at t = -1 for inflation and its square at all t which are derived by iterating (22) to give (24).

$$
\begin{aligned}
E_{-1}[\pi_t] &= \pi_s + (\pi_{-1}-\pi_s)\left(\frac{\lambda}{\lambda+\alpha}\right)^{t+1} \\
E_{-1}[\pi_t^2] &= \left[\pi_s + (\pi_{-1}-\pi_s)\left(\frac{\lambda}{\lambda+\alpha}\right)^{t+1}\right]^2 + \frac{\sigma^2}{(1+\alpha)^2}\left[\frac{1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{2(t+1)}}{1-\left(\frac{\lambda}{\lambda+\alpha}\right)^2}\right]
\end{aligned}
\tag{24}
$$

We also substitute (22) and (23) into the first term in (1), and (14) with (16) and (17) into the second term. Using (24) and these substitutions we solve for the expected infinite horizon sum in (1) and present the annuity value of the expected social loss in (25) where π_s is given by (23) and S, the coefficient on the variance of shocks, σ^2, by (26). The true social parameters, K and δ appear in (25) since we evaluate outcomes using the social loss function (1) while their behavioural values, K_b and δ_b, determine α, β_2 and π_s. φ affects π_s only.

$$(25) \qquad (1-\delta)\, E_{-1}[L_0] = \begin{aligned} &[y^*(k-1)]^2 + K\pi_s^2 + \frac{\sigma^2}{\gamma^2}S \\ &+2(\pi_{-1}-\pi_s)[K\pi_s+\frac{\alpha}{\gamma}y^*(k-1)][\frac{\lambda(1-\delta)}{\lambda(1-\delta)+\alpha}] \\ &+(\pi_{-1}-\pi_s)^2(1-\delta)\beta_2 \end{aligned}$$

$$(26) \qquad S = \frac{(\gamma^2 K+\alpha^2)(\lambda+\alpha)^2}{(1+\alpha)^2[(\lambda+\alpha)^2-\delta\lambda^2]}$$

The top line in (25) shows the loss that would arise if the initial inflation rate happened to be equal to its long run average value and may be interpreted in the same way as (13) above, while the remaining terms show the costs or benefits of adjusting from a different initial inflation rate. In relation to S in (26), if $\delta_b=\delta$ and $K_b=K$ (as under the discretion and Walsh cases), we may use (20) to simplify (26) to $\alpha/(1+\alpha)$ which is of the same form as the equivalent term in (13). The value of (26) could also be split between the loss arising from inflation and output variability separately though this is not necessary for the results below.

4. Results

We present key results on the properties of the different monetary policy regimes based on the analysis above. The most striking results are stated as Propositions 1-3 and concern the patient central banker, the case entirely new to this paper, and the response to shocks under a conservative central banker. Further findings are given by Propositions 4-9. These are also new to the literature but mostly comprise either properties of the static case which are shown to still hold under inflation persistence or are implied by the analysis of Clark et. al. (1999) though not stated by them. Collectively these analytic results provide a comprehensive overview of each of the policy regimes but for further illustration, and to facilitate comparisons between the regimes, we also present the outcomes of numerical simulations. These use the parameter values K=1, $\delta=1/(1+0.03)$, $\sigma=0.01$, k-1=0.03, $\gamma=0.2$ and $\pi_{-1}=0$ together with an assumed value of λ of 0.75 unless shown otherwise. Since the key results are derived analytically we do not present separate simulations that vary these parameters. Before turning to the different regimes it may be noted that the comparative statics depend on how the variables α and α^R respond to changes in key parameters and we show these first in (27) and (28).[12]

$$(27)\qquad \frac{d\alpha^R}{d\lambda}=\frac{2\delta\lambda(\alpha^R)^2}{(\lambda^2+\alpha^R)^2-\delta\lambda^4} > 0 \ ; \quad \frac{d\alpha}{d\lambda}=\frac{2\delta_b\alpha^2\lambda(1+\alpha)}{(\lambda+\alpha)^3-\delta_b\lambda^2(\lambda+2\alpha\lambda-\alpha)} > 0$$

$$(28)\qquad \frac{d\alpha}{d\delta_b}=\frac{\alpha\lambda^2(1+\alpha)(\lambda+\alpha)}{(\lambda+\alpha)^3-\delta_b\lambda^2(\lambda+2\alpha\lambda-\alpha)} > 0 \ ; \quad \frac{d\alpha}{dK_b}=\frac{\gamma^2(\lambda+\alpha)^3}{(\lambda+\alpha)^3-\delta_b\lambda^2(\lambda+2\alpha\lambda-\alpha)} > 0$$

Patient Central Banker ($\delta_b>\delta$, $K_b=K$, $\varphi=0$)

Proposition 1: The average inflation rate is decreasing in the patience of the policy maker ($d\pi_s/d\delta_b<0$).

Differentiating (23) with respect to δ_b yields (29) which may be shown to be negative if we substitute for $d\alpha/d\delta_b>0$ from (28).

$$(29)\qquad \frac{d\pi_s}{d\delta_b} = \frac{y^*(k-1)\lambda[\frac{d\alpha}{d\delta_b}\delta_b(1-\lambda)-(1+\alpha)(\lambda+\alpha)]}{\gamma K_b(\lambda+\alpha)^2} < 0$$

Inflation persistence implies that the present value of inflation costs from a given expansion of output perceived by the policy maker are increasing in their discount factor, δ_b. As a result the temptation to expand output by raising inflation is smaller, the policy maker is more tolerant of the output costs of lowering inflation and the long run average inflation rate is reduced. Figure 2(i)a shows the inflation rate falling as the behavioural discount rate falls to zero (δ_b tends to unity), together with the optimal rate under the rule in (11) for comparison.

Proposition 2: Increased patience of the policy maker (up to a threshold value of the discount factor, δ^**) improves the response to shocks under incomplete inflation persistence (*$dS/d\delta_b<0$ *if* $0<\lambda<1$ *and* $\delta_b<\delta^*$ *where* $\delta^*>\delta$*;* $dS/d\delta_b>0$ *if* $\delta_b>\delta^*$*).*

Differentiating (26) with respect to δ_b and simplifying yields (30).

$$\frac{dS}{d\delta_b} = \frac{2\frac{d\alpha}{d\delta_b}(\lambda+\alpha)[(\delta_b-\delta)\alpha(1+\alpha)(\lambda+\alpha)-\delta(1-\lambda)(\gamma^2K+\alpha^2)]}{(1+\alpha)^3[(\lambda+\alpha)^2-\delta\lambda^2]^2} \tag{30}$$

From (28) $d\alpha/d\delta_b>0$ and hence (30) is negative from the second term in the square bracket of the numerator if it is evaluated at $\delta_b=\delta$. As δ_b rises above δ the first term in the square bracket becomes positive and there will be a threshold value of δ, δ^*, at which the numerator of (31) becomes zero, after which it will be positive and the loss from shocks will be increasing in δ_b. Figure 2(i)b shows the loss from shocks falling as the discount rate falls (with a minimum, not shown, at $\delta_b>1$). If $\lambda=1$ the minimum is at $\delta_b=\delta$. In Figure 1 an increase in δ_b, known to the private sector, lowers the short run Phillips curve YY while a small movement of δ_b away from δ has only a second order effect on welfare from the choice of position on the short run tradeoff.

Taken together, Propositions 1 and 2 show that welfare is increasing (over a range) in the degree of patience of the policy maker since a higher discount factor both lowers average inflation, reducing the inflation bias present with discretion (see Proposition 7 below) and also improves the response to shocks. Thus the patient central banker offers a third approach to the problem of time inconsistency in monetary policy in addition to the Rogoff and Walsh mechanisms. Figure 2(i)c shows that for the parameter assumptions made the total social loss (from average inflation and the response to shocks) is continuously decreasing as the behavioural discount factor falls towards zero (reaching a minimum beyond this point). The total loss under discretion is given by the height of the line in Figure 2(i)c when the behavioural discount rate is at its true social value (assumed to be 3%) and from the chart more than half this loss is removed by a 3 percentage points fall in the behavioural discount rate to zero. The maximum available gain is larger if the true discount rate is greater than 3% since δ_b may rise further above δ before reaching the imposed ceiling of unity.

<u>Conservative Central Banker</u> ($\delta_b=\delta$, $K_b>K$, $\varphi=0$)

Proposition 3: Mild conservatism of the policy maker reduces the loss from shocks under incomplete inflation persistence ($dS/dK_b<0$ *if* $0<\lambda<1$ *and* $K_b<K^*$ $(K^*>K)$*;* $dS/dK_b>0$ *if* $K_b>K^*$*).*

Differentiating (26) with respect to K_b and simplifying gives (31).

$$\text{(31)} \qquad \frac{dS}{dK_b} = \frac{2\dfrac{d\alpha}{dK_b}(\lambda+\alpha)[\gamma^2(K_b-K)(\lambda+\alpha)^3-\delta(1-\lambda)(\gamma^2K+\alpha^2)]}{(1+\alpha)^3[(\lambda+\alpha)^2-\delta\lambda^2]^2}$$

In the static case ($\lambda=0$), varying K_b away from K immediately worsens the loss from shocks, a result that reflects the optimality of the discretionary response to shocks in this case. With inflation persistence the discretionary response to shocks is no longer optimal (implicit in Clark et. al. 1999, and stated formally in Proposition 8 below) and an increase in K_b above K over a limited range reduces the loss from shocks. Evaluated at $K_b=K$, dS/dK_b in (31) is unambiguously negative so the conservative central banker results in a lower loss from shocks than discretion. If K_b is increased above K there will come a point where the positive first term in the square bracket of the numerator outweighs the second and the sign of the derivative reverses. Figure 2(ii)b shows this relationship with a minimum where $K_b>K$ given $\lambda<1$. The minimum is at $K_b=K$ if $\lambda=1$.

Proposition 4: The average inflation rate is decreasing in the conservatism of the policy maker ($d\pi_s/dK_b<0$).

This result reproduces that from the static case (Rogoff, 1985) where a lower average inflation rate is the raison d'etre of the conservative central banker. It may be derived by differentiating (23) with respect to K_b using $d\alpha/dK_b$ from (28) and is shown in Figure 2(ii)a.

Thus under inflation persistence the conservative central banker reduces average inflation, as in the static case, but over a range it also improves the response to shocks whereas in the absence of persistence, conservatism immediately worsens the response to shocks. Figure 2(ii)c illustrates these gains with the total loss falling from its value under discretion (shown by the height of the line when $K_b=K=1$) to a minimum when K_b is between 1.5 and 2. With reference to Figures 2(ii)a and 2(ii)b this minimum represents a balance between an ideal K_b (given the other parameter values) of 5.5-6.00 for average inflation and 1-1.5 for shocks.

Optimal Policy Rule

Proposition 5: The long run average inflation rate under the optimal policy rule is strictly positive under inflation persistence if the discount rate is strictly positive ($\pi_s^R>0$ if $\lambda>0$, $\delta<1$, $k>1$ and K finite) and is increasing in the degree of inflation persistence ($d\pi_s^R/d\lambda>0$).

This result contrasts with the static case ($\pi_s^R=0$ if $\lambda=0$) and follows directly from (11). It is not present in Clark et. al. (1999) who assume k=1 which results in $\pi_s^R=0$ (or more generally its target level) for any λ. With $\lambda>0$ there is some tradeoff between output and inflation even with rational expectations and the structure of the loss function (1) implies that social welfare is higher if the tradeoff is exploited when k>1. Figure 3a, however, shows the positive desired average inflation rate under the rule to be low. The figure also shows the optimal inflation rate increasing with λ which follows from (11).

Proposition 6: The loss from shocks under the optimal policy rule is increasing in the degree of inflation persistence.

From (10) the loss arising from shocks under the optimal rule is proportional to $\alpha^R/(1+\alpha^R)$. This is increasing in α^R which in turn is increasing in λ from (27). This is shown in Figure 3b and is to be expected since a rise in the degree of inflation persistence worsens the set of intertemporal inflation/output combinations available to the policy maker following shocks.

Discretion ($\delta_b=\delta$, $K_b=K$, $\varphi=0$)

Proposition 7: The average inflation rate is higher under discretion compared with the optimal rule under incomplete inflation persistence ($\pi_s>\pi_s^R$ if $\lambda<1$, $k>1$ and K finite) and is decreasing in the degree of inflation persistence.

From (23) and (11) we may derive (32) which equals zero if $\lambda=1$, or if k=1 in which case there is no source of inflation bias and both π_s and π_s^R are zero. For the standard case of k>1, (32) is positive if $\lambda<1$, including the standard static case result when $\lambda=0$. A further special case is if $\delta=1$ (so the future is not discounted) in which case $\pi_s^R=0$ from (11) but $\pi_s>0$. Thus the standard inflation bias result from the static model is repeated under inflation persistence unless $\lambda=1$ and that persistence is complete (see Figure 3a).

$$\pi_s-\pi_s^R = \frac{y^*(k-1)(1-\lambda)[\alpha+\lambda(1-\delta)]}{\gamma K(\lambda+\alpha)} \tag{32}$$

In relation to the size of the inflation bias, differentiating (23) yields (33) which may be shown to be negative using (27). The opposite signs of (33) and $d\pi_s^R/d\lambda$ from (11) show that the size of inflation bias falls with inflation persistence, disappearing once $\lambda=1$ (Figure 3a).

$$\frac{d\pi_s}{d\lambda} = \frac{y^*(k-1)\delta[\lambda(1-\lambda)\frac{d\alpha}{d\lambda}-\alpha(1+\alpha)]}{\gamma K(\lambda+\alpha)^2} < 0 \tag{33}$$

Proposition 8: The loss from shocks is higher under discretion than the optimal rule if $0<\lambda<1$.

This result is implicit in Clark et. al. (1999) who find that behaviour under discretion differs from that under the optimal rule. Given that behaviour under the rule is first best, the loss from shocks under discretion is larger. The analysis above confirms the difference in behaviour though the cubic equation (20) in β_2 makes direct verification of a higher loss from shocks under discretion difficult. From (13) the loss from shocks under the rule is proportional to $\alpha^R/(1+\alpha^R)$. Under discretion, from (25) and (26), simplified using (20) given that $K_b=K$ and $\delta_b=\delta$ under discretion, it is proportional to $\alpha/(1+\alpha)$ and from (7) and (20), α differs from α^R unless $\lambda=1$ or 0. Clark et. al. show that $\alpha>\alpha^R$ ($-\gamma_2^d>-\gamma_2^c$ in their notation) which confirms Proposition 8. If $\lambda=0$ the static model applies and the discretionary response to shocks (around higher average inflation) is first best. Outcomes coincide if $\lambda=1$ but for $0<\lambda<1$ the loss under discretion is higher (Figure 3b).

Propositions 7 and 8 imply that for incomplete inflation persistence discretion is inferior to the optimal rule, as in the static case, but stronger persistence may narrow this gap (by reducing inflation bias) or worsen it (by raising the cost from shocks). Figure 3c shows that for the assumed parameter values the reduction in inflation bias dominates.

Walsh Linear Inflation Penalty ($\delta_b=\delta$, $K_b=K$, φ chosen to minimise social loss)

Corollary 1: The loss from shocks under a Walsh contract is higher than that under the optimal rule.

In the static case the Walsh inflation penalty (φ) does not affect the response to shocks which as a result is the same as under discretion (which is itself first best). Expressions (25) and (26) confirm that φ does not affect the response to shocks under inflation persistence either so the Walsh case also corresponds to the discretionary outcome. From Proposition 8, however, that outcome is no longer first best if $0<\lambda<1$.

Proposition 9: Under a Walsh contract any long run average inflation rate may be achieved without affecting the response to shocks by appropriate choice of the linear inflation penalty, φ, but its optimal value depends on the initial inflation rate unless δ=1.

From (25) and (26), the Walsh inflation penalty, φ, affects the total loss only through π_s which is given by (23). Hence any desired π_s may be achieved through the appropriate choice of φ as in the static case. Unlike the static case, however, differentiating (25) to derive the optimal π_s, given by $\pi_s(\varphi^*)$ in (34), where φ^* is the value of φ that equates (23) with (34), shows that the optimal value of φ depends on π_{-1}, the inflation rate inherited from the past at the start of the Walsh regime. For example, if π_{-1} is high, a larger value of φ (known to the private sector and incorporated into expectations) will reduce the output losses associated with reducing inflation towards its ergodic stationary state. A larger φ, however, means that the long run average inflation rate will be sub-optimally low and hence the optimal φ balances these two factors.[13] In the special case where π_{-1} equals π_s^R this tension disappears and the optimal φ achieves the first best inflation rate in (11).

$$\pi_s(\varphi^*) = \frac{(1-\delta)\lambda[\gamma y^*(k-1)(\lambda+\alpha)-\pi_{-1}\alpha(1-\lambda)]}{\gamma^2 K(\lambda+\alpha)-(1-\delta)\lambda\alpha(1-\lambda)} \tag{34}$$

Thus the Walsh linear inflation penalty retains the power to influence long run average inflation as in the static case but if φ is fixed once and for all, this regime cannot attain the first best outcome which it achieves in a static setting both because of the trade-off in the setting of the optimal φ referred to and because of the sub-optimal response to shocks when inflation is persistent. Both these difficulties would, in principle, be resolved if φ could be made state dependent so φ_t is a function of π_{t-1} (Lockwood, 1997, demonstrates the equivalent result with real persistence).

Comparisons Between Regimes

We briefly compare the total loss across the regimes (for the assumed parameter values) shown in Figure 4, focusing on the range $0.5<\lambda<1$, while also showing the effect of a combined conservative and patient central banker. The comparisons should be regarded as indicative since for reasons of space we do not present a comprehensive set of simulations. The parameters φ, K_b and δ_b are chosen to minimise the total loss at each value of λ shown. From Figure 3c the total loss under the optimal rule is increasing in λ and that from discretion tends to decrease. Figure 4 shows that these regimes straddle the conservative and patient central banker cases whose relative ranking depends on the degree of inflation persistence. If persistence is low the discount rate has a weak influence on behaviour (especially given that the optimal δ_b would be greater than the imposed ceiling of unity) but for λ above approximately 0.7 the patient central banker outperforms the conservative one. The outcome under the jointly optimal δ_b and K_b is superior to both so the ideal central banker combines these characteristics. The Walsh regime results in a total loss close to that achieved by the optimal rule so the key issue with this regime remains the potential difficulty of implementing a feasible Walsh penalty rather its desirability.

5. Conclusion

The paper has analysed the time inconsistency of monetary policy when inflation is persistent and the target rate of output exceeds its natural rate. In common with models that exclude persistence there is an inflation bias under discretion and this may be reduced by a conservative central banker regime or a Walsh inflation penalty. Unlike the static case, however, the optimal inflation rate under a rule is higher than its target in the social loss function, the discretionary response to shocks (and that under a Walsh penalty) is no longer first best and a mildly conservative central banker reduces the loss from shocks. We also introduce the concept of a patient central banker, whose discount rate is lower than the social discount rate, and show that this regime outperforms discretion in relation to both inflation bias and the response to shocks.

In future work it would be useful to combine the insights of the current paper with those that consider output or unemployment persistence. In these models a low discount rate may worsen the inflation bias since the output benefits of expansionary policy last longer (assuming that output persistence is symmetric above and below its natural rate) and hence analysis of the role of the policy maker's discount rate when both forms of persistence are present would be informative. Clearly the determinants of that discount rate (and its relation to the terms of office of decision makers and their democratic accountability) is also of interest when persistence is strong. The paper's results are also likely to be relevant to the interpretation of empirical evidence on central bank independence and the political economy of monetary policy under discretion if central bank discount rates tend to be lower (and potentially more stable) than those of elected governments.

Figure 1: Policy Choices Under Discretion and the Optimal Rule

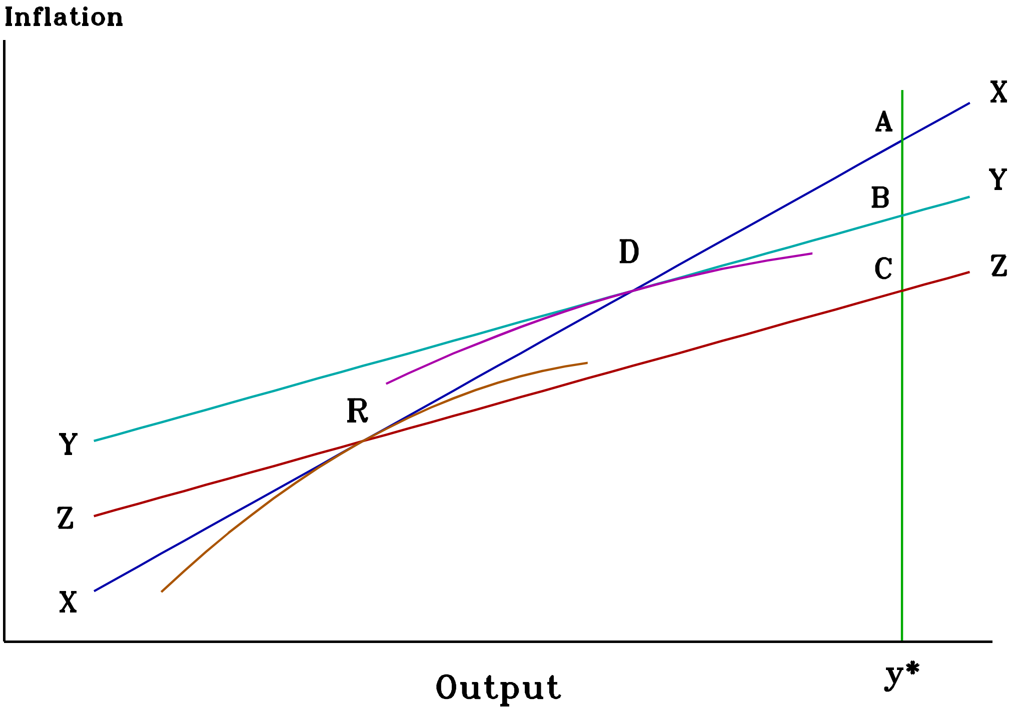
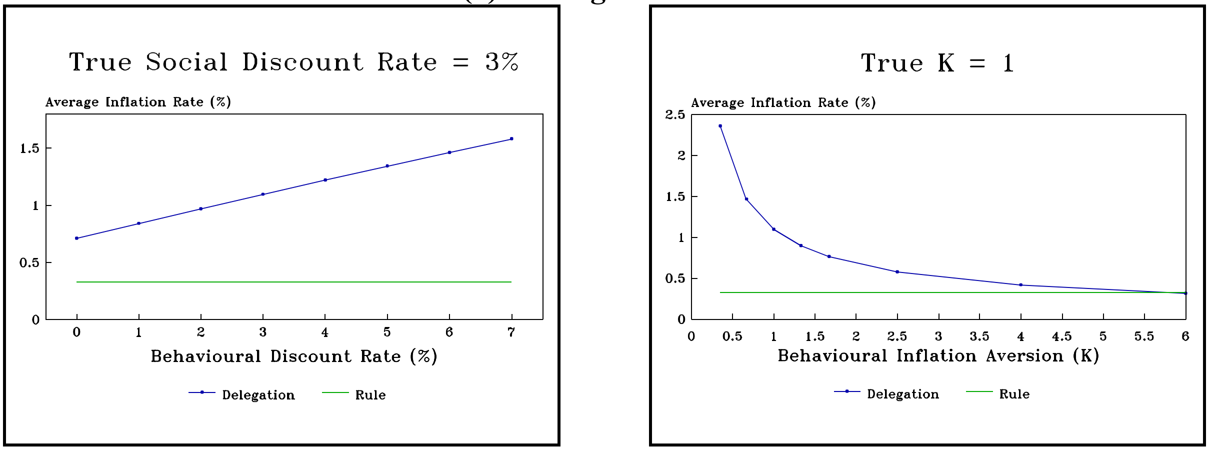


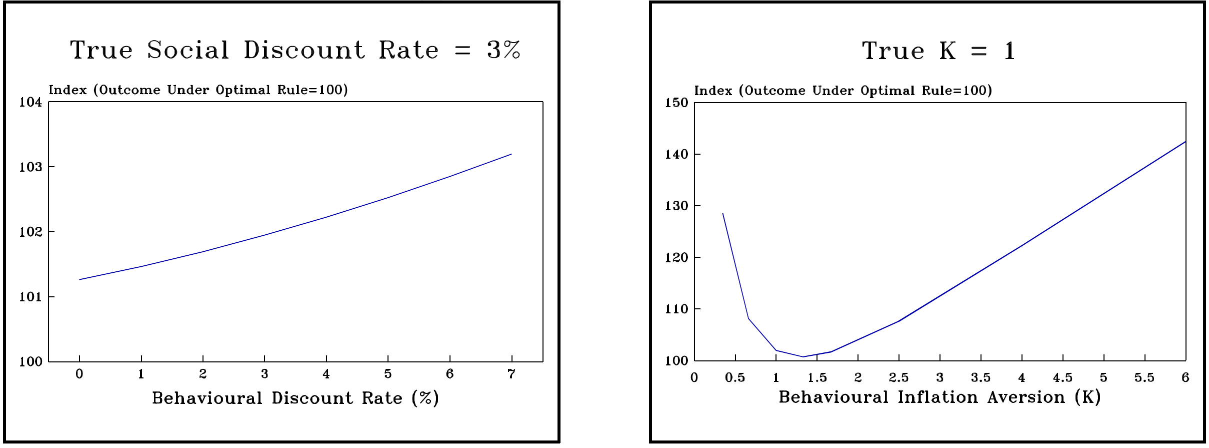
Figure 2: Outcomes Under Patient and Conservative Central Bankers

i) Patient Central Banker **ii) Conservative Central Banker**

(a) Average Inflation



(b) Loss From Shocks Alone



(c) Total Loss

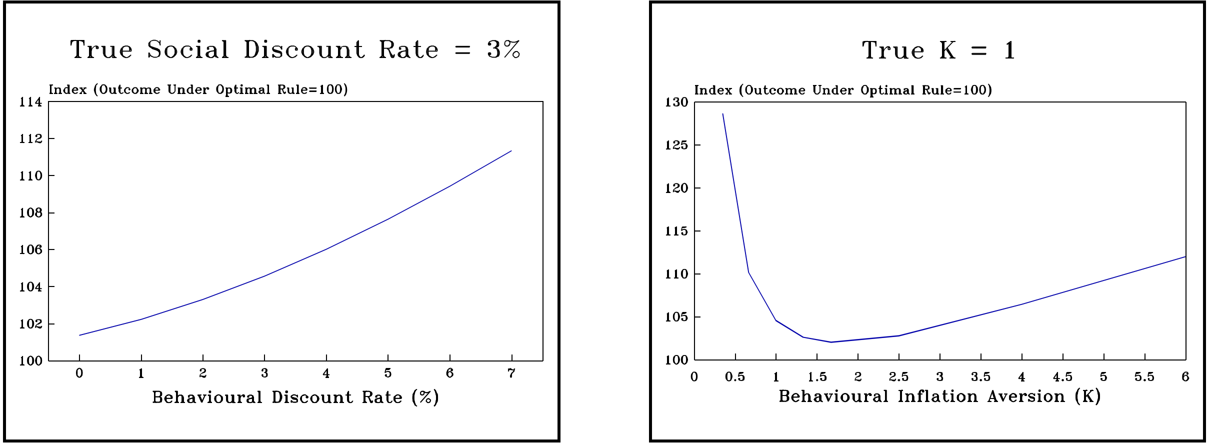
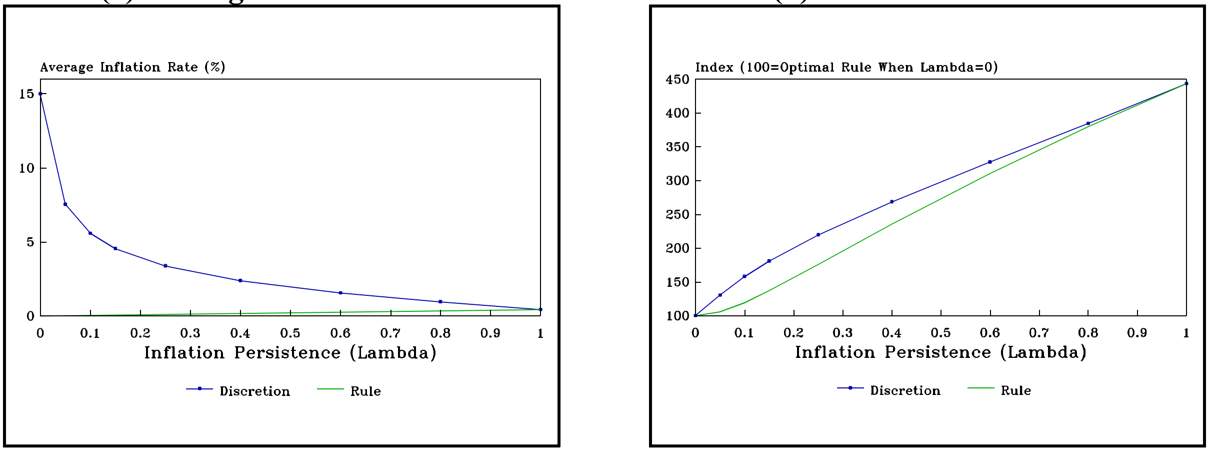


Figure 3: Outcomes Under Discretion and the Optimal Rule



(a) Average Inflation

(b) Loss From Shocks Alone

(c) Total Loss

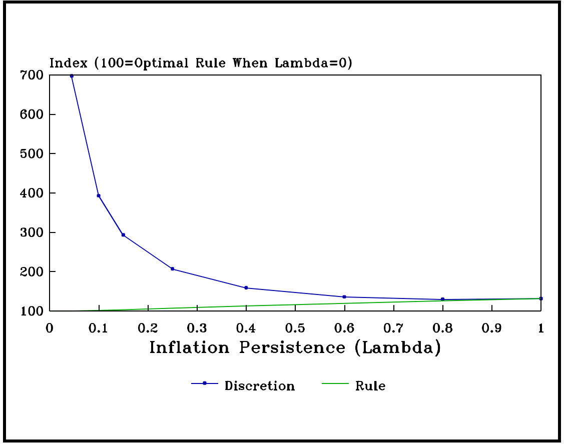
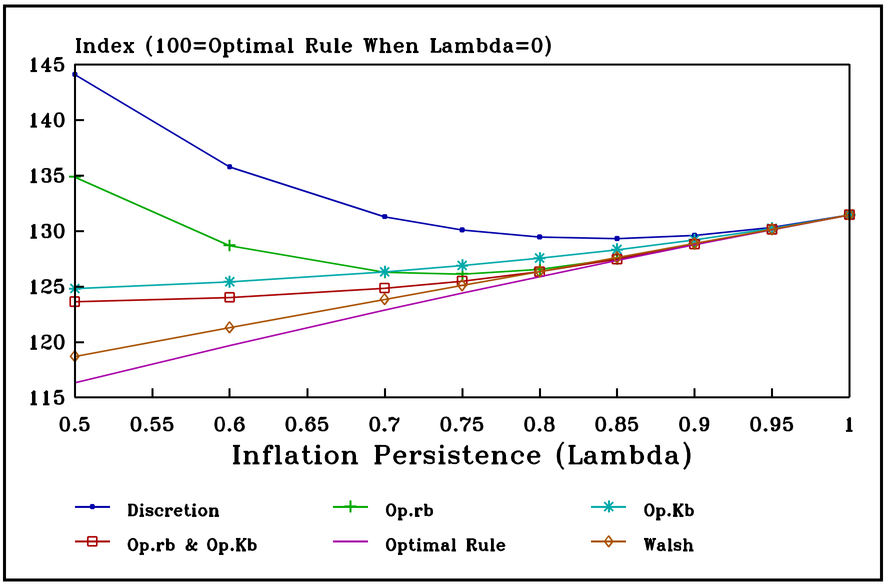


Figure 4: Comparison Between Regimes (Total Loss)



Appendix to Section 2: Derivation of the Optimal Rule

Expanding (5) using (4) and taking its expected value gives (A1) where $\alpha^R=\gamma^2(K+\delta\beta_2^R)$. Since the rule coefficients a, b and c must be chosen in advance (at time -1) they are optimally set to minimise the expected loss from period 0 onwards shown by (A1).

$$\text{(A1)} \qquad \gamma^2 E_{-1}[L_0] = \begin{array}{c} \bar{\pi}_0^2(\alpha^R+\lambda^2) + \sigma^2[c^2\alpha^R+(1-c)^2] \\ +\bar{\pi}_0[\gamma^2\delta\beta_1^R-2\lambda^2\pi_{-1}-2\lambda\gamma y^*(k-1)] \\ +\gamma^2\delta\beta_0^R+[\gamma y^*(k-1)]^2+\lambda^2\pi_{-1}^2+2\lambda\gamma y^*(k-1)\pi_{-1} \end{array}$$

From (A1) the first order conditions for $\bar{\pi}_0$ and c are given by (A2) and (A3) which in turn yield (6) in the text.[14]

$$\text{(A2)} \qquad \gamma^2\frac{\partial E[L_0]}{\partial\bar{\pi}_0} = 2(\alpha^R+\lambda^2)\bar{\pi}_0 + \gamma^2\delta\beta_1^R - 2\lambda^2\pi_{-1} - 2\gamma\lambda y^*(k-1) = 0$$

$$\text{(A3)} \qquad \gamma^2\frac{\partial E[L_0]}{\partial c} = \sigma^2[2c\alpha^R-2(1-c)] = 0$$

To solve for the β coefficients in the value function we set up the recursion equation (A4) using (4).

$$\text{(A4)} \qquad \gamma^2(\beta_0^R+\beta_1^R\pi_{-1}+\beta_2^R\pi_{-1}^2) = \begin{array}{c} E_{-1}[[-\gamma y^*(k-1)+\lambda(\bar{\pi}_0-\pi_{-1})-(1-c)\varepsilon_0]^2] \\ +\gamma^2 K E_{-1}[\pi_0^2]+E_{-1}[\gamma^2\delta(\beta_0^R+\beta_1^R\pi_0+\beta_2^R\pi_0^2)] \end{array}$$

Substituting (6) into (A4), with $E[\varepsilon_0]=0$, and simplifying gives (A5).

$$\text{(A5)} \qquad \gamma^2(\beta_0^R+\beta_1^R\pi_{-1}+\beta_2^R\pi_{-1}^2) = \begin{array}{c} \gamma^2\delta\beta_0^R + [\gamma y^*(k-1)]^2 - a^2(\lambda^2+\alpha^R) + \dfrac{\sigma^2\alpha^R}{1+\alpha^R} \\ +\pi_{-1}\left(\dfrac{2\lambda\gamma y^*(k-1)\alpha^R+\gamma^2\delta\beta_1^R\lambda^2}{\lambda^2+\alpha^R}\right) + \pi_{-1}^2\left(\dfrac{\alpha^R\lambda^2}{\lambda^2+\alpha^R}\right) \end{array}$$

Equating the coefficients on either side of (A5) in π_{-1} and its square, and the terms that do not involve π_{-1}, gives us Riccati equations for β_1^R, β_2^R, and β_0^R respectively. These yield the quadratic in β_2^R given by (7), of which we take the (economically meaningful) positive root given by (8), and the values of β_1^R by (9) and β_0^R by (10). For the values of the rule coefficients, a, b and c we may substitute (8)-(10) into (6), recalling that $\alpha^R=\gamma^2(K+\delta\beta_2^R)$ and $\pi_t=a+b\pi_{t-1}+c\varepsilon_t$, to give:

$$a = \frac{2\gamma y^{*}(k-1)(1-\delta)\lambda}{\gamma^2K+\lambda^2(1-\delta)+\sqrt{[\gamma^2K+\lambda^2(1-\delta)]^2+4\gamma^2\delta K\lambda^2}} \tag{A6}$$

$$b = \frac{2\lambda^2}{\gamma^2K+\lambda^2(1+\delta)+\sqrt{[\gamma^2K+\lambda^2(1-\delta)]^2+4\gamma^2\delta K\lambda^2}} \tag{A7}$$

$$c = \frac{2}{2+\gamma^2K-\lambda^2(1-\delta)+\sqrt{[\gamma^2K+\lambda^2(1-\delta)]^2+4\gamma^2\delta K\lambda^2}} \tag{A8}$$

The expressions for the optimal b and c in (A7) and (A8) may be shown to be equal to A.4 and A.7 in Clark et. al. (1999) whereas that for the constant term in the policy rule, a given by (A6), differs from their A.6. In Clark et. al., the optimal a is proportional to a positive target inflation rate. This is set to zero in the model above and hence the comparable expression here would be a=0. However, Clark et. al. assume that the policy maker targets the natural rate of output which in the notation above implies that k=1 and hence (A6) would generate a=0. With the target rate of output greater than its natural rate (k>1) a positive value for the constant, a, is restored if $\lambda>0$, $\delta<1$ and K is finite. In turn this means that long run average inflation, which is given by a/(1-b), is also positive.

Notes

1. See also Persson and Tabellini (1993, 1999). Fischer (1990, 1995) and Clarida, Gali and Gertler (1999) review the literature. Lohmann (1992) extends the Rogoff model and McCallum (1997) questions the applicability and credibility of the Walsh approach.

2. See Alesina and Gatti (1995) and Berger, de Haan and Eijffinger (2000) for example. A full discussion of the analytic results in relation to the empirical evidence is beyond the scope of the paper. Nordhaus (1975) presents an early model where the discount rate is important.

3. See Barro and Gordon (1983), Fischer (1990), and Persson and Tabellini (1999).

4. Bean (1998) suggests that both the social loss function and that guiding government behaviour should have $k=1$, inflation bias under government discretion arising from a politically driven extra term that is negative and linear in the level of output.

5. See Forder (1998) for a sceptical view of the benefits of central bank independence.

6. In common with virtually the entire literature in this area, the model is vulnerable to the criticism that the policy lags involved may be long enough for policy choices to be observed before nominal contracts are set for the period in which those choices will take effect. In these circumstances dynamic inconsistency will not arise. See Goodhart and Huang (1998).

7. See Backus and Driffill (1985a, 1985b), Vickers (1986), Schaling, Hoeberichts and Eijffinger (1998), Haldane and Quah (1999), Martin (1999), Sibert (1999), and Soderstrom (2000).

8. See Cubitt (1992), Cukierman and Lippi (1999), Guzzo and Velasco (1999), Lippi (1999).

9. The definition of α here is similar to that for α^R under the rule but the two differ in value (unless $\lambda=0$ or 1), even if δ_b and K_b are equal to δ and K, due to the absence of commitment under discretion.

10. This follows from the left hand side of (20) being negative at $\beta_2=0$, positive for large β_2 and it has at most one turning point at $\beta_2>0$.

11. This result is robust but otherwise Figure 1 should be interpreted with care since the contours of the loss functions differ between rules and discretion. The within-period loss functions coincide but the difference in the output/inflation tradeoff implies that after allowing for intertemporal effects the marginal rates of substitution between π and y will differ.

12. These are derived from differentiating (7) and (20) having substituted $(1/\delta)(\alpha^R-\gamma^2K)$ and $(1/\delta_b)(\alpha-\gamma^2K_b)$ for $\gamma^2\beta_2^R$ and $\gamma^2\beta_2$ respectively. Sufficient conditions for the signs of the derivatives are $K>0$, $0<\delta<1$, $\delta_b<1$ and $\lambda<1$.

13. The optimal values of K_b and δ_b will in general also depend on π_{-1} but we emphasise this point in relation to the Walsh contract because it achieves the optimal inflation rate in the static case and this is no longer possible with inflation persistence.

14. The second order conditions here (and in the rest of the paper) may readily be shown to be satisfied.

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