Fiscal Regime Uncertainty, Risk Aversion, and Exhaustible Resource Depletion

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ABSTRACT

How does uncertainty about future rent tax liability affect the competitive supply pattern for an exhaustible resource? Historically, changes in tax and regulatory clauses have been a frequent occurrence in the Petroleum industry, and appear to have contributed to the climate of uncertainty about future rent appropriation. This paper develops a generally applicable framework to tackle this question.

The analysis modifies the classic Hotelling problem of exhaustible resource management to embody producer risk-aversion in terms of the underlying portfolio allocation behaviour of firms' owners, using a simple mean-variance approach. The construct is then used to derive equilibrium price profiles for the resource under a number of different methods of characterizing the risk, including a "continuous" variety under which mean-variance analysis gives general results.

By and large, the results suggest that this type of uncertainty promotes excessively rapid depletion. Important exceptions arise where rent variability (i) does not increase the more distant the horizon; and (ii) exhibits a negative correlation with dominant sources of risk in investors' portfolios.

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1. INTRODUCTION

Over the past decade or so, a good deal of the conceptual work on exhaustible resources has directed some attention to the problem of uncertainty. As Hotelling (1931) illustrated early on, even the simplest problems of exhaustible resource management are inherently dynamic. It is thus hardly surprising that an uncertain future should be a pressing issue, and that its treatment in the literature should yield a number of interesting results. Future additions to resource stocks, technological improvements (including those that make a substitute for the resource in question available, or available more cheaply), demand shifts, and changes in environmental and fiscal legislation are all relevant data about which resource managers often have little prior knowledge. For concreteness, this paper examines the effects on resource depletion of uncertainty in the industry about future tax liability. The method developed is, however, equally applicable to other types of uncertainty. The treatment is distinguished by its allowance, in a rigorous manner, for risk aversion on the part of resource-extracting firms.

In fact the problem posed here is a pervasive feature in the petroleum industry. For example, Devereux and Morris (1983, Chs. 1-3) give an account of the evolution of North Sea Oil taxation in the United Kingdom for the period 1974-83, and note that "... in that time the North Sea tax system ... changed significantly no less than 13 times" creating, by the end of the period "... a deep-seated scepticism about future actions of governments on the part of oil companies, who are now probably less, rather than more, likely to take long-term marginal risks." Similarly, Seymour (1980, Chs. 3-5) documents the movement, following the formation of OPEC, by member countries (particularly in the Middle East) to improve unit revenues from their oil concessions. Though change initially was somewhat sluggish, it appears that by the late 1960's, as OPEC established its credibility, oil companies began to exhibit serious concern about the hazard of changes in their concession agreements. Even in the United States, widely viewed as one of the most "stable" production venues, a record of significant fiscal and regulatory revision can be traced.[1]

The sections that follow build a framework to analyze the effect of uncertainty of this kind on production decisions for an exhaustible resource. The construct used is a very simplified one: typically several distinct fiscal implements, each with scores of attached provisions and clauses, and all subject to imperfectly anticipated changes, are applied to extractive industries. In the case of petroleum, more direct regulatory measures usually supplement the different taxes. Collectively, these affect not only the intertemporal allocation of extraction from a stock of given magnitude (the issue addressed below) but also exploratory effort, investment in drilling and extractive equipment of various kinds, and total cumulative recovery of the resource. Although these latter issues are neglected here, it is important to bear them in mind because they will in general alter predictions about the effect of uncertainty. If, for example, there is a risk that a given set of oilfields will be nationalized without full compensation, there is on the one hand an incentive for current owners to deplete the fields quickly, but on the other a disincentive to undertake exploratory activity and invest in extractive capacity, an effect that retards extraction. The net bias caused by the presence of risk, as compared with the risk-free case, is in general indeterminate.[2]

Another important point concerns the large number of different taxes in use. The focus here is on the distortions caused by uncertainty about possible tax changes, but the majority of taxes in place entail distortions (relative to the tax-free outcome) even if they do not change at all or all changes are perfectly anticipated. A general discussion of the issues involved in assessing the impact of taxation on resource production can be found in Church (1981, Ch. 3). Dasgupta and Heal (1979, Ch.12), Dasgupta, Heal and Stiglitz (1980) and Conrad and Hool (1980, Ch.3) show how different tax instruments alter the path of marginal profit implied by any given production plan, and consequently the allocation of output over time.[3] Heaps (1985) extends the model to incorporate non-constant costs of extraction and depletion effects, and Lewis and Slade (1985) develop a framework wherein the extracted resource ("crude oil" or "ore") combines with other inputs in the production of a refined product ("gasoline" or "metal"). These extensions are important, because on occasion they reverse the conclusions of the simpler models. For example, a time-invariant severance tax (a tax per unit of the resource extracted) is unambiguously conservation-inducing where demand does not choke off at high prices and depletion effects are neglected. But when both cumulative resource recovery and the date at which production ceases are endogenous (in addition to the time-profile of production), the result is no longer clear-cut. If the effect of the tax is to shorten production life, then cumulative extraction is reduced. If however production life is lengthened, it is possible for total recovery either to exceed or fall short of the tax-free outcome, though in all cases initial rates of production are lower with the severance tax in place. Results about the distortionary effects of taxes and other fiscal controls are further complicated if a refining process is taken into account.

To keep the discussion manageable and its point clearly in focus, the model developed in this paper has a number of simplifying features. Firstly, as far as the <u>effects</u> of fiscal implements are concerned, attention is restricted to the cake-eating problem: how the intertemporal allocation of a given total stock of a resource that is costless to extract and is never optimally exhausted in finite time changes with the introduction of uncertainty. Depletion effects, demand chokeoffs at high prices, and the impact of uncertainty on exploratory and investment activity are all neglected. Secondly, to fix ideas, it helps to concentrate on a fiscal instrument that is not, but for the uncertainty about its future movement, distortionary.[4] Accordingly, the analysis is confined to uncertainty about the future evolution of an otherwise neutral resource rent tax.

The rest of this paper is structured as follows. The remainder of this section consists of a brief review of the relevant work on "demand" uncertainty in exhaustible resource production (uncertainty about the future payoff to resource sales that does not stem from supply conditions). Section 2 then lays the foundations for a rigorous incorporation of risk aversion into firms' objective functions. This forms the basis of the analysis in subsequent sections. Sections 3 and 4 analyze two different methods of characterizing expectations about tax changes, and what each implies for the intertemporal production profile. In the former, a rent tax (change) of uncertain magnitude occurs at a known date; in the latter, the magnitude is known but the event date uncertain. Section 5 presents another characterization of beliefs about future taxes where the risk of fiscal change recurs continually and examines the properties of the accompanying extraction profile. Section 6 introduces a "continuous" representation of future rent uncertainty under which mean-variance results attain generality. Finally, Section 7 retains the "continuous" characterization of risk, but qualifies the general drift of the results by introducing other risky assets (besides shareholdings in firms that have rights to the resource deposits). Section 8 contains some concluding remarks.

In his review of the literature, Long (1984) outlines the sources of uncertainty that arise in exhaustible resource management and surveys the theoretical contributions on the subject. A large number of these focus on responses to uncertainty at the level of the individual extractive firm (as opposed to, say, the state planner level). Most papers within this class take firms' objectives to be the maximization of expected present value (PV) rents. The underlying assumption there is that firms are risk-neutral: their owners hold sufficiently widely diversified portfolios for the risk associated with their performance to be of little consequence. The formal argument is given in, among others, Nickell (1978, pp.84-5).

Long (1975) uses the criterion of PV rent maximization to investigate the effect of an anticipated risk of nationalization. The resource-owning firm there possesses a belief (in the form of a subjective probability distribution defined over an occurrence date) about the danger of being nationalized without adequate compensation. The result is higher rates of extraction initially, and, in the case of a finite chokeoff price, a nearer exhaustion date as compared with the case where the firm is certain that nationalization will never occur. The result is intuitively appealing - when there is a lingering risk that any reserves not extracted and sold today will be subject to a discontinuous fall in value, it pays to extract faster than otherwise, to the point where marginal profit today is less than discounted marginal profit (conditional on nationalization not occurring) tomorrow.

Interestingly, Hartwick and Yeung (1985) actually establish a preference for price uncertainty on the part of the competitive resource-owning firm. This is so in the following sense: if the firm knows that at some future (known) date the resource price will change, (expected) PV of rents is higher in the case where the future price is currently random than in the case where the future price is known with certainty today. Furthermore, the firm depletes less of its available stock prior to the price change in the random price case. The result does, however, require a time-invariant resource price (except of course at the date the new price is introduced) and therefore implicitly the assumption that the source of risk is firm-specific, so that the change in behaviour which the randomness causes leaves the resource price unaffected. Where this is not so, the result is invalid (this is implicit in the results of section 3 below).

In Pindyck (1980) firms face an uncertain rate of growth of market demand modelled as a Wiener diffusion process (this is the approach used in sections 6 and 7 below). In the competitive case, the intertemporal equilibrium allocation of the resource is shown to be determined so that, at every date, the expected instantaneous rate of price growth equals the rate of interest. However, the accompanying rate of expected output decline, even in the case of constant or zero unit extraction costs, does not in general coincide with the rate of decline under certainty. Briefly, the reason is that although fluctuations in the rate of demand growth cancel out on average, the adjustments in output required to accompany them do not, in general: the net tendency depends on the characteristics of the market demand function.

In a subsequent paper (Pindyck, 1981), competitive firms likewise face a stochastically evolving resource price. Pindyck points out that, once the possibility of a binding non-negativity constraint on resource output is taken into account, an additional incentive to delay extraction emerges. Bohi and Toman (1984, pp. 74-77) demonstrate this clearly in a two-period linear-quadratic model where the second period resource price is random. They show that if there is a positive probability that second-period price, once revealed, will induce a corner solution, a certainty-equivalent calculation on the basis of the expected price will overstate optimal first-period extraction. In other words, there may be an incentive to delay production which a certainty-equivalence approach does not capture.

Other authors, in an attempt to rectify the shortcomings of assuming risk-neutrality on the part of resource-producing firms, have experimented with objective functions that capture an aversion to risk. In Weinstein and Zeckhauser (1975), firms choose extraction profiles to maximize the expectation of a concave (utility) function of the PV of rents. The source of uncertainty lies in future demand; the problem is essentially a discrete-time version of the one addressed by Pindyck. The solution has the intertemporal allocation of the resource determined so that the resource price grows at a rate exceeding the safe rate of interest (less is conserved relative to the risk-neutral case). The result is heuristically attractive (the average rate of return on a risky asset should exceed the safe rate of return to equilibriate the market if participants are risk-averse), but the weak point of the argument is the rather arbitrary choice of objective.[5]

A similar criticism applies to the models of Lewis (1977) and Burness (1978). In each of these, the resource-owning firm selects its production profile to maximize a discounted stream where the component at each point in time is a (concave, in the risk-averse case) function of current rents. The resource price is assumed in both papers to have a random component that is independently and identically distributed in every period, but it is significant that the evolution of the price has no chain property (see also section 5 below). Under these conditions, the risk-averse firm overconserves in comparison with the PV rents-maximizing firm. Given the structure of the problem, it is clear why this is so: period-to-period fluctuations in profits are undesirable, so it pays to push them into the future where their impact - in PV terms - is softer. Since profits are an increasing function of output, and fluctuations in profits proportional to profits, this is achieved by delaying extraction. The result does however depend strongly on both the specification of demand uncertainty and the firm's objective function, highlighting the need for these to be chosen carefully and consistently.

Another class of papers has covered uncertainty and its effect on resource pricing from a social management viewpoint. The criterion function there is typically a discounted stream of consumption felicities, which the resource contributes to the production of, or, more directly, a discounted stream of "net social benefit" (consumer surplus plus rent) accruing from the use of the resource. Dasgupta and Heal (1974) introduce a backstop technology with a random invention date into an optimal growth model where the resource is necessary for production. They show that, compared with the case where there is no possibility that a backstop technology will ever become available, the required modification to the solution is equivalent to raising the felicity discount rate. (The upper bound on the required increase at each date is the probabilistic rate of occurrence of the invention, though this will generally overestimate the required increase unless the invention makes the available stocks of capital and the resource totally worthless). In general, a positive probability associated with the invention of a backstop means that relatively more favourable treatment is given to current generations, on the anticipation that future generations are likely to be able to benefit from the availability of the backstop technology.

Dasgupta and Stiglitz (1981) pose the question whether uncertainty in the date at which a backstop is to become available should hasten or slow depletion compared with the case when the availability date is known with certainty. Somewhat counter-intuitively, it transpires that under an optimal plan this type of uncertainty should encourage conservation if reserves are large, but discourage it (provided the elasticity of demand at high prices is not too low) if reserves are small. This is so because, when the resource stock is large relative to the invention date, there is no gain to having the backstop earlier, but a delay hurts; an increase in variability increases the marginal value (MV) of postponed resource use, but does not change the MV of current use. So there is an incentive to conserve the resource. In the case of small reserves, the effects of greater variability are more or less symmetric, but the MV of postponed resource utilization is relatively unaffected (while the MV of current use is increased) provided the elasticity of demand at high prices is not too low (there is less of an incentive to conserve it if, should it chance that invention is delayed so that the resource must be rationed, demand for the resource falls quickly). Finally, in Deshmukh and Pliska (1985), the rate of research and development effort influences the probability distribution associated with the date of invention of a backstop technology. Deshmukh and Pliska show that along the optimal solution, the present value of the shadow price of the resource _in situ_ is a martingale (that is, its expected change at each date is zero) provided that the current resource stock does not affect the probability distribution of the invention date. Furthermore, the optimal rate of resource utilization is a monotonically increasing, and the rate of expenditure on R&D a monotonically decreasing, function of the remaining resource stock.

Though the above survey is by no means an exhaustive one, it does scan much of the relevant work on "demand" uncertainty in exhaustible resource management.[6] The next section turns to the problem of deriving an objective function for the individual resource-owning firm that takes risk-averse attitudes into account.

2. INVESTOR BEHAVIOUR AND THE OBJECTIVES OF RESOURCE - OWNING FIRMS

The progression from individual to firm objectives is adapted from Stevens (1974). The setting is a resource-rich intertemporal exchange economy with two assets: one a safe (numeraire) asset yielding a known rate of return $r>0$ (for simplicity time-invariant) and the other a risky asset (in a sense to be specified in subsequent sections), namely ownership of the entities ("firms") with property rights over the resource.[7] There is in addition a central authority that, depending upon the context, currently appropriates, or may in the future appropriate, a share of the rents from the resource. All production (other than resource extraction) is abstracted from entirely.

Let individual i, $i=1,..,I(t)$ of "generation" $t \geq 0$ possess an endowment of $w_i(t)$ at date t, expressed in terms of units of the numeraire asset. There is assumed to be a complete set of markets for contingent claims at the initial date $t=0$. Each member i of generation t is an expected "lifetime" utility maximizer with criterion function

$$(1) \quad E_t \; W^i \; (C_i(t), \; \tilde{C}_i(t+\theta))$$

where $C_i(t)$ denotes consumption at date t and $\tilde{C}_i(t+\theta)$ consumption at date $t+\theta$ $(\theta>0)$. The tilde indicates a random outcome, and the probability distribution of outcomes is conditioned on the information set at t. Because this is assumed to be identical across individuals, E_t is the mathematical expectation taken at t. θ is the length of investors' horizons. Note that the assumption of a complete set of markets ensures an ex ante Pareto efficient allocation (assuming that nothing can be done about the source of the uncertainty) and can thus, by an appropriate set of lump-sum transfers, sustain an ex ante welfare optimum. In general, however, it cannot be expected to sustain an ex post welfare optimum (Hammond, 1981).[8]

Now $C_i(t)$ and $\tilde{C}_i(t+\theta)$ are constrained by

(2) $w_i(t) - C_i(t) + B_i(t) \equiv \alpha_i(t)V(t)$, and

(3) $\tilde{C}_i(t+\theta) = \alpha_i(t)\tilde{x}(t,t+\theta) - (1+r\theta)B_i(t)$

where $\tilde{x}$ is the total return (income plus principal) to an investment of \$V(t) in the risky asset at t. x is subject to a given probability distribution of outcomes $\Omega(x)$ (a joint probability distribution if x and α_i are n-vectors). Equation (2) states that the difference between the endowment and what is consumed immediately plus net borrowing (B_i) equals i's share (α_i) of the total expenditure on ownership shares of the firms in the resource-producing industry. Equation (3) states simply that i's share of the total return on investment, less debts to be repaid, is all spent on "second-period" consumption.

Let $(\alpha_i{}^*, B_i{}^*)$ be the solution to the problem of maximizing (1) subject to (2) and (3). For the analysis that follows, the starting point is to enquire about the conditions under which it is valid to write (1) as

(1') $U^i(C_i(t),\bar{C}_i(t+\theta),s_i(t,t+\theta))$

where $s_i(t,t+\theta)=\text{var}_t\,\tilde{C}_i(t+\theta)$, and the bar denotes the expectation (formed at t). (1') will be used below in the characterization of the investor's portfolio allocation problem. The well-known result (Feldstein, 1969; Tobin, 1969) is that to write (1) in the form (1') is legitimate, in the case of a single risky asset, provided either (i) the utility function is quadratic (so that, if one expands it by Taylor's theorem around mean "second-period" consumption, higher than second-order terms vanish); or (ii) the possible investment outcomes are assumed to follow a two-parameter probability distribution (so that third- and higher-order moments can be expressed in terms of the first two).

In the case of several risky assets, the conditions under which (1) can be written in the form (1') are even more stringent. If condition (i) is not met, any linear combination of assets must follow a distribution that belongs to the two-parameter family. If individual asset returns are (jointly) normally distributed, the requirement is met, but this is the only "common" probability distribution with this property. Very restrictive assumptions therefore underpin the analysis of investor behaviour in terms only of the mean and variance of the portfolio return, and the corresponding results about firms' behaviour lack generality.

There is, however, one important result that assures generality in an important class of cases considered in this chapter. The fundamental approximation theorem of portfolio analysis (Samuelson, 1971) states the following: provided the (joint) distribution $\Omega(.)$ belongs to a family of "compact" distributions (in the sense that all the distributions converge to sure outcomes as a given parameter approaches zero), then the solution (α_i, B_i) that is obtained from maximizing the quadratic approximation to (1) subject to (2) and (3) tends to the true solution (α_i^*, B_i^*) as the parameter in question goes to zero.[9] As is made clear below (see section 6 and Appendix B), the result can be put to use in the present context if, as the length of investors' horizon goes to zero, the variability in asset returns disappears.

Return now to the characterization of investor behaviour. Individual i of generation t is assumed to maximize (1') subject to (2) and (3). The idea is to derive an expression, in the form of a difference equation, for V(t), the market value of the risky asset at t. First note that

$$(4) \quad s_i(t,t+\theta) = \mathrm{var}_t \tilde{C}_i(t+\theta) = \alpha_i^2 \ \sigma(t,t+\theta),$$

where $\sigma(t,t+\theta)$ is the variance of the total return to the risky asset, x, computed from the standpoint of date t. Now let R(t) denote the total (that is, industry) rate of extraction and sales of the resource over the interval $(t,t+\theta)$, and p(t) the resource price prevailing then. Under the assumption - preserved for the rest of this paper - that firms incur no costs in extracting the resource, the total return is written

$$(5) \quad \tilde{x}(t,t+\theta) = \theta p(t)R(t) + V(t+\theta) .$$

(5) will be used presently. Substituting now equations (2)-(4) in (1') and maximizing with respect to α_i and B_i, necessary

conditions for an interior solution are

(6.1) $U^i{}_1 - U^i{}_2(1+r\theta) = 0$, and

(6.2) $-U^i{}_1 V(t) + U^i{}_2 \bar{x}(t,t+\theta) + 2U^i{}_3 \alpha_i \sigma(t,t+\theta) = 0$,

where $U^i{}_j$ denotes the partial derivative with respect to the jth argument of U(.,.,.), evaluated at the optimal choice. Eliminating $U^i{}_1$ between (6.1) and (6.2) yields that

(7) $U^i{}_2\{\bar{x}(t,t+\theta) - (1+r\theta)V(t)\} + 2U^i{}_3\ \alpha_i \sigma(t,t+\theta) = 0$.

Equation (7) has the usual interpretation that investor i determines her portfolio mix to equate her subjective marginal rate of indifferent "substitution" of variance for mean consumption $(-U^i{}_3/U^i{}_2)$ to the market rate of "substitution" (that is, the increment in variance that an extra unit addition to mean consumption entails). On rearrangement, (7) gives the expression for the equilibrium market value of firms in the resource-producing industry:

(8) $$V(t) = \frac{\bar{x}(t,t+\theta) - \gamma(t,t+\theta)\ \sigma(t,t+\theta)}{(1+r\theta)}$$

where $\gamma(t,t+\theta) \equiv -\ 2\alpha_i U^i{}_3/U^i{}_2$ is identical in equilibrium across individuals i=1,..,I(t). The total value placed upon resource-owning firms by the market therefore emerges as the total mean return to their ownership over the interval less the term $\gamma\sigma$ (discounted over the interval). The latter term is of course the "allowance" for risk, and may be interpreted as the unit cost (or "price") of risk (variance) in the market multiplied by total variance. Alternatively, using (5) in (8) yields

$$(8') \quad r\theta V(t) = V(t+\theta) - V(t) + \theta p(t)R(t) - \gamma(t,t+\theta)\sigma(t,t+\theta).$$

That is, in equilibrium the return on shares equals capital gain plus profit, less the allowance for risk.

Now given the way x is defined (see equation (5)), it is evident that (8') is a difference (or, in the limiting case where θ approaches 0, differential) equation in V. From it, we shall be able to deduce the characteristics of the production profile $\{R(t)\}_{t=0}^{\infty}$ in specific instances, once the nature of the risk is elucidated. Subsequent sections will be concerned with doing this.

To conclude the list of components and assumptions of the construct used in the sections that follow, denote by D(p) demand for the resource (say from abroad), and by p(R) its inverse function. For expositional convenience, it is assumed that $\lim_{R\to 0} p(R)=+\infty$. This ensures that output of the resource remains positive throughout. The "industry", which has (partial) property rights to the resource, is assumed always to consist of a large number of price-taking (and unlevered) firms. "Firms" are defined as entities that, at any given date, formulate production policies to maximize their market value then. The behavioural criterion is therefore <u>market value maximization</u>.

Finally, there is of course one important condition that production policies must satisfy: the sum over time of total production and sales of the resource cannot exceed the reserves available at the start of the planning period. S(t) will denote the total remaining resource stock at the start of period t (the interval $(t,t+\theta)$), $t \geq 0$. Finally, the remaining stock is assumed homogeneous, non-augmentable, and unconcentrated in ownership.

3. A TAX OF RANDOM SIZE IMPOSED AT A KNOWN FUTURE DATE

Suppose there is a common "belief" among investors that a profits (rents) tax at rate $1-\phi$, where $\phi \in [0,1]$ is a random variable with a two-parameter distribution, will materialize at some known date $T>0$. At date T the true value of ϕ is revealed, but no new information about the possible value of ϕ is acquired until then. For simplicity the interval $[0,T)$ is assumed tax-free, and the interval (T,∞) free of further tax changes.

Consider first the situation for $t \geq T$, once the tax has been imposed (the "post-event regime"). Let $\hat{\phi}$ denote the value that is revealed. Since ownership of the resource-producing firms is then no longer risky, equation (8') has, for $t \geq T$, the straightforward solution

$$(9) \quad \hat{V}(t) = \sum_{t=T}^{\infty} \frac{\theta \hat{\phi} \hat{p}(t) \hat{R}(t)}{(1+r\theta)^{(t-T+\theta)/\theta}}$$

(where again the generic index of time, t, assumes values θ units apart). $\hat{V}(t)$ denotes the post-event (maximized) market value of the firms in the industry, $\hat{R}(t)$ the optimal extraction policy of $t \geq T$, and $\hat{p}(t) = p(\hat{R}(t))$ the associated resource price.

What about the situation before T? For generations before the $(T-\theta)$th, there is likewise no risk associated with holding a unit of shares in the resource-producing industry, in the sense that no new information will be acquired about the true value of ϕ before the shares are resold (so that, under the assumption of forward transactions, the true value of the total return x that accrues at $t+\theta$ will be known at t). Thus (again from (8')) for $0 \leq t \leq T-2\theta$,

$$(10) \quad (1+r\theta)V(t) = \theta p(t)R(t) + V(t+\theta)$$

However, for $t=T-\theta$, there is no escape from the loss in share value associated with the appearance of the tax, and so

$$(11) \quad (1+r\theta)V(T-\theta) = x(T-\theta,T) - \gamma(T-\theta,T)\ \sigma(T-\theta,T)$$

$$= \theta p(T-\theta)R(T-\theta) + EV(T) - \gamma(T-\theta,T)\sigma(T-\theta,T),$$

where E denotes the mathematical expectation. The assumption is that dividends to the ownership shares held over $(T-\theta,T)$ are paid prior to the imposition of the tax. The revealed value of ϕ therefore only determines the magnitude of the capital loss on the asset, and not income from it. Using (10) and (11), one now obtains, by repeated substitution,

$$(12)\ V(0) = \sum_{t=0}^{T-\theta} \frac{\theta\ p(t)R(t)}{(1+r\theta)^{(t+\theta)/\theta}} + \frac{EV(T) - \gamma(T-\theta,T)\ \sigma(T-\theta,T)}{(1+r\theta)^{T/\theta}},$$

Note that it is immaterial whether the expectation is taken at date 0 or just before T, since the information set remains the same over [0,T).

It is now straightforward to derive the properties of the optimal (privately, of course) production programme. The standard approach is to work backwards. Return to equation (9), and for convenience pass to continuous time by evaluating the limit as $\theta \to 0$. At date T,

$$(13)\quad V(T) = \int_T^\infty e^{-r(t-T)}\, \hat{\phi}\, \hat{p}(t)\hat{R}(t)\, dt$$

$$= \max_{\{R(t)\}^\infty_{t=T}} \int_T^\infty e^{-r(t-T)}\, \hat{\phi}\, p(t)R(t)\, dt$$

subject to the condition that

$$(14)\quad \int_T^\infty R(t)\, dt = S_T$$

in other words, that the resource should be exhausted asymptotically. In addition, $p(t) = p(R(t))$ for each date, although for the purposes of the optimization problem $p(t)$ is treated as parametic at date t (underlying the aggregate optimization problem is a large number of firms who are price-takers). The solution $\hat{R}_t$ must satisfy

$$(15)\quad e^{-r(t-T)}\, \hat{\phi}\, p(t) = \lambda$$

for $t \geq T$, where λ is a positive constant, and where of course in equilibrium $\hat{p}(t) = p(\hat{R}(t))$. From (15) it is clear that if $\hat{\phi}$ is, as assumed, time-invariant, the extraction profile is chosen so that $\hat{p}$ simply appreciates at the rate r. So for $t > T$

$$(16)\quad \hat{p}(t) = \hat{p}(T)\, e^{r(t-T)}.$$

Write $\hat{p}(T) = \hat{p}(S_T)$ to indicate that the resource price (rate of extraction) at T is chosen so that (14) is satisfied (i.e., reserves are just exhausted asymptotically). (13) may thus be re-written as

$$(13') \quad V(S_T, \hat{\phi}) = \hat{\phi}\ \hat{p}(S_T)\ S_T$$

Note that $\hat{p}$ is completely independent of $\hat{\phi}$, so long as $\hat{\phi}$ is constant for $t > T$.

The next step is to return to (12), the expression for market value prior to the arrival of the tax. Passing to continuous time and using (13') to substitute for V(T) and σ:

$$(17) \quad V(0) = \int_0^T e^{-rt}\ p(t)R(t)\ dt$$

$$+ \quad e^{-rT}\ \hat{p}(S_T)\ S_T\ \{\bar{\phi} - \gamma(T)\ \hat{p}(S_T)\ S_T\ \sigma_\phi\},$$

where $\bar{\phi}$ denotes the expectation of ϕ and σ_ϕ its variance. The production programme that maximizes V(0) is intertemporally consistent - that is, it will also maximize V(t) for 0<t<T, given the resource stock that remains then. The maximum of (17) subject to the constraint

$$(18) \quad S_0 - S_T = \int_0^T R(t)\ dt$$

must satisfy the conditions

$$(19) \quad e^{-rt}\ p(t) = \mu \qquad 0 \leq t < T$$

where μ is a positive constant, and

$$(20) \quad \mu = e^{-rT}\ \bar{\phi}\ \hat{p}(S_T) - 2\ \gamma(T)\ e^{-rT}\ \hat{p}(S_T)^2\ S_T\ \sigma_\phi.$$

(19) therefore indicates that on the interval [0,T) extraction is determined so that the resource price grows at the rate r (it has

already been noted that the Hotelling r per cent rule applies equally on $[T,\infty)$). Equation (20) states that the amount of stock carried over into the post-event regime should be so as to equate the shadow price of the resource then with that in the pre-event regime. Equations (19) and (20) together imply a discontinuity in the resource price at date T, since

$$(20')\quad \{\hat{p}(S_T) - \lim_{t \to T^-} p_t\}/\hat{p}(S_T) = \{1 - \bar{\phi} + 2\gamma(T)\hat{p}(S_T)S_T\sigma_\phi\}.$$

In words, the equilibrium extraction profile for the industry has a (positive) price jump at T. Figure 1 illustrates a price path for given values of the parameters. The gross price of the resource is identical to the net price in the pre-event, but not post-event, regime. It follows the solid trajectory. The broken trajectory indicates the <u>expectation</u> (made at some date $0 \leq t < T$) of the evolution of the <u>net</u> price for $t \geq T$ (the actual such path will naturally differ from this in general). Note that the PV of this net price exceeds the PV price in the pre-event regime. The difference (the risk "premium" in Figure 1) represents the portion of the jump in the resource price that just compensates owners for bearing the risk: since they are risk-averse ($\gamma(T) > 0$), the expectation, made before T, of a random PV marginal return in the post-event regime must exceed the (known) PV marginal return in the pre-event regime to effect indifference between the two. Under risk-neutrality ($\gamma(T) = 0$), sales of the resource are planned to ensure that the <u>net</u> price of the resource (the price received by the owners) remains constant in PV terms over $[0,\infty)$. The solid and broken trajectories would coincide in this case.

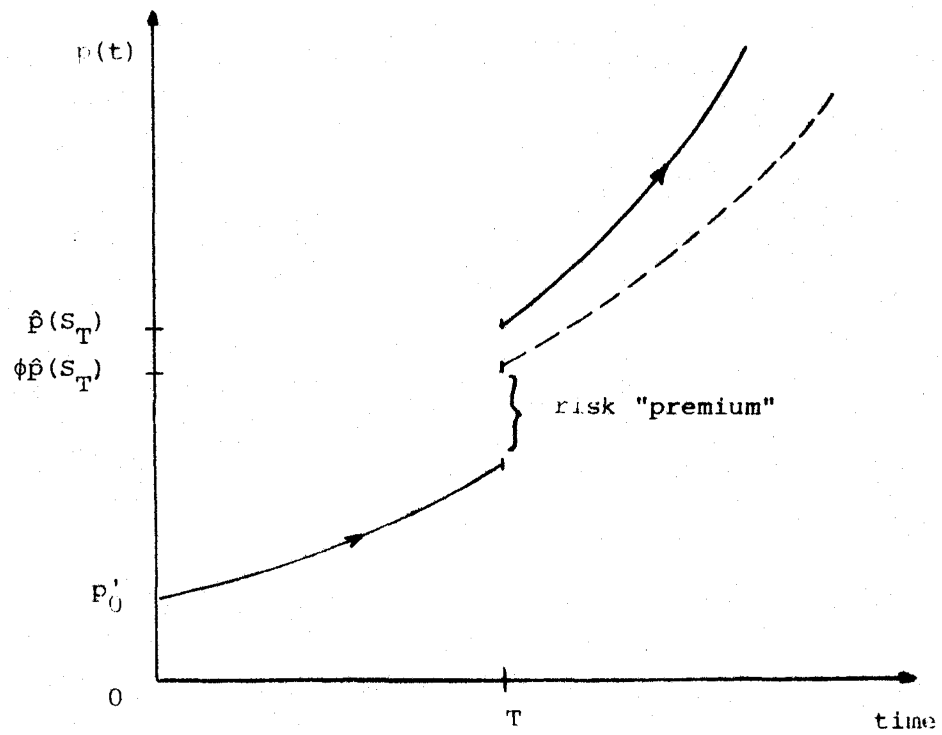

FIGURE 1

The magnitude of the price jump at T (as a proportion of the after-tax price) will be larger, *ceteris paribus*,

(i) the smaller is $\bar{\phi}$;

(ii) the larger is σ_{ϕ}[10]; and

(iii) the larger is (the exogenous component of) $\gamma(T)$.

Broadly, (i) - (iii) represent increases in the undesirability (to the private sector) of the post-event regime. It is therefore intuitive that firms should ensure that, prior to it, they dispose of relatively more of the available resource stock. From (20'), it can be seen that a smaller $\bar{\phi}$ and larger σ_{ϕ} and γ must imply, *ceteris paribus*, a smaller S_T in equilibrium, and therefore a larger price jump at T. Equivalently, they imply a lower initial price p_0 (and thus a greater rate of extraction and sales up to T), and therefore magnify the intertemporal misallocation of the resource (vis-a-vis a no-tax programme). Note that the only attitudes to risk that are relevant to the misallocation are those of investors at T. The signal to extract faster is transmitted to earlier generations via the (resale) value of their ownership shares on forward markets. The risk aversion or otherwise of investors at other dates is irrelevant. Finally, a *cet. par.* increase in T means that the prospective loss implied by the tax threat is - in PV terms - lessened. The magnitude of the allocative bias is in consequence reduced. Under risk-neutrality, it can be shown that p_0 is a monotonically increasing function of T. However, expression (20') reveals that the proportional jump in the market price at T is invariant, *ceteris paribus*, to the magnitude of T under risk neutrality.

It is noteworthy also that what matters for the allocation is how undesirable the post-event regime looks (to the firm) *ex ante*, and not how undesirable it turns out in effect to be. The magnitude of the jump in the resource price is totally insensitive to the actual value of ϕ revealed at T (as opposed to the expectation of ϕ and its variance prior to T). All that matters is that the tax threat subsides. This implies, for example, that if a profits tax were to be imposed at some date, or change at some date in a totally unanticipated fashion, the price profile sustained both before and after the event would coincide exactly with a perpetually tax-free profile. Also interesting is the point that, if at some date $t>0$ resource owners suddenly perceive - following an announcement, say - the risk of a tax (increase) at T>t where previously they did not, their response actually induces a price _fall_ at t (and a price rise later, at T).

Another interesting result pertains to a paradoxical property of the extraction profile that is pursued when it is known with certainty that a rents tax will materialize at date T>0. Suppose that the tax rate $1-\phi$ is known with certainty _ex ante_. The following example shows that, for the isoelastic class of demand functions, the reallocation of the extraction profile as a tax avoidance strategy may, in certain cases, make matters worse _ex post_ for resource owners.

Example Let $D(p)=p^{\eta}$ $(\eta<0)$.

Take first the case where it is known with certainty that a rents tax at known rate $(1-\phi)$ will be imposed at T. From (19) and (20), and using the boundary conditions $S(0)=S_0$ and $\lim_{t\to\infty} S(t)=0$ to evaluate the constants of integration, one obtains, for $t\leq T$

$$S(t) = S_0 \frac{\{e^{-rt} - (1-\phi)e^{-rT}\}}{1 - (1-\phi)e^{-rT}}$$

and the PV of rents accruing to the resource owners is given by

$$\text{(E1.1)}\quad V' = p_0'(S_0 - S'(T)) + e^{-rt}\phi p'(T)S'(T) = p_0'S_0$$

because $e^{-rT}p'(T)=p_0'/\phi$ (eliminating μ between (19) and (20)). p_0' denotes the equilibrium initial price on the taxed profile.

Now compare (E1.1) with the analogous expression under the profile that results if precisely the same event (rents tax at known rate $1-\phi$) occurs at T but is _completely_ unanticipated. In this case the equilibrium initial price, $p_0{*}$, is identical to what it would be if the profile were entirely tax-free. The PV of rents that go to resource owners here is

$$(E1.2)\quad V^* = p_0^*\{S_0 - (1-\phi)S^*(T)\}$$

It remains to compare (E1.1) and (E1.2). Now

$p^*(t) = p_0^* e^{rt}$, $t \geq 0$, while

$p'(t) = p_0' e^{rt}$, $0 \leq t < T$; $p'(t) = (p_0'/\phi)e^{rt}$, $t \geq T$.

Using this information and the requirement that reserves be asymptotically depleted along both profiles, p_0' and p_0^* can be computed as

$$(E1.3)\quad p_0' = [-r\eta S_0\{1-(1-\phi^{-\eta})e^{r\eta T}\}^{-1}]^{1/\eta}$$

$$(E1.4)\quad p_0^* = [-r\eta S_0]^{1/\eta}$$

Finally, using (E1.3) in (E1.1) and (E1.4) (along with the information that $S^*(T)=S_0 e^{r\eta T}$) in (E1.2), it transpires that

$V^* > (=) (<) V'$

if and only if

$$\{1 - (1-\phi)e^{r\eta T}\}^{-\eta} > (=) (<) 1 - (1-\phi^{-\eta})e^{r\eta T}$$

and it is a relatively straightforward matter to confirm that $V^*>V'$ if and only if the elasticity of demand is less than unity in absolute value, $|\eta|<1$.[11] □

The example shows that the equilibrium where resource owners have *no* prior information about tax changes actually implies that in some cases they are better off *ex post* than when they have *perfect* prior information (to which they respond fully). In other words, if resource owners could somehow conspire to ignore the information, they would - under inelastic demand conditions - be better off.[12] This point is a logical consequence of the observation by Dasgupta (1983) that, where the average elasticity of demand over the optimal production profile is less than unity, the value of a resource deposit (the PV of the stream of rents that it generates) is a declining function of the size of the stock.[13] The larger the resource stock, the smaller the benefit to the owners. The relevance of this to the current example is this: if resource owners anticipate the tax, and therefore ensure that *less* of the stock remains by the time it arrives, the total value of the remaining stock (and therefore the total tax burden) is actually *larger* than if nothing is done and larger reserves remain when the tax is imposed.

To sum up: this section has indicated that the greater the variability about the payoff to a future holding of an exhaustible resource, and the greater the degree of risk aversion, the more profligate the resource extraction profile for a given mean payoff. Note that even if, say, the rate of tax is positive prior to T and a change of mean zero (but positive variance) is expected to occur at date T, depletion is more rapid than in the tax-free case unless investors are risk-neutral. Note also that for given parameter values a certainty-equivalent $\phi^c < \bar{\phi}$ can be found such that, if ϕ^c is known with certainty to be the post-event value of ϕ, the outcome is identical - in terms of the extraction programme - to that where ϕ is random. But there is no increase in the discount rate to allow for risk. The opposite holds true for the reverse case of a known value of ϕ with a random arrival date, to which the analysis now turns.

4. A KNOWN RATE OF TAX WITH A RANDOM IMPOSITION DATE

Suppose now that there is a common belief that a known value of ϕ is to materialize at some date $T\in(0,\infty)$ that is not known with certainty. Denote by

$$F(t) = \mathrm{Prob}(T\geq t) = \int_t^{\infty} f(s)ds$$

unity less the cumulative probability distribution. If $f(t)=-F'(t)$ is the probability density at t, $\lambda_t=f(t)/F(t)$ is then the probabilistic rate of occurrence of the event (the imposition of the tax) at date t, given that it has not occurred up to t. In other words, λ_t is the "hazard rate" associated with the imposition date T materializing then.[14] These changes aside, the setting is precisely that used in the previous section.

Analogously with the previous section, the starting point is the remaining optimization exercise once the value of T has been revealed, whatever it turns out to be. Given T and S_T, the stock of the resource remaining then, the market value of the firms in the industry is again given by (13'). Next, consider the generation of investors at t (<T) that holds ownership shares in the industry over the small interval of time $(t,t+\theta)$. The market value of these shares will satisfy equation (8) with

$$(21)\qquad \bar{x}(t,t+\theta) = (1-\theta\lambda_t)\, V(t+\theta) + \theta\lambda_t\, \hat{V}(t+\theta) + \theta p(t)R(t)$$

assuming that dividends are paid out prior to the arrival of the tax (if the event does indeed occur during the interval $(t, t+\theta)$). (21) holds almost exactly for small θ. $\theta\lambda_t$ is approximately the conditional probability that the tax will be imposed during $(t, t+\theta)$ given that it has not been imposed up to t. $\hat{V}(t+\theta)$ and $V(t+\theta)$ denote the market value of the shares at $t+\theta$ if, respectively, the tax is and is not imposed during the interval. Also, recalling that σ denotes the variance of the total return,

$$(22) \quad \sigma(t,t+\theta) = \theta\lambda_t \ (1-\theta\lambda_t) \ \{V(t+\theta) - \hat{V}(t+\theta)\}^2$$

so that (8) can be written, after some rearrangement, as

$$(23) \quad (1+r\theta)V(S(t),t) =$$

$$(1-\theta\lambda_t) \ \{1 + 2\theta\lambda_t \gamma(t,t+\theta)\hat{V}(S(t+\theta))\} \ V(S(t+\theta),t+\theta)$$

$$- \ \theta\lambda_t \gamma(t,t+\theta)(1-\theta\lambda_t) \ V(S(t+\theta),t+\theta)^2$$

$$+ \ \theta\lambda_t \{1-\gamma(t,t+\theta)(1-\theta\lambda_t)\hat{V}(S(t+\theta))\}\hat{V}(S(t+\theta)) + \theta p(t)R(t)$$

where the dependence of the market value of the shares on the remaining stock as well as the date in question is now explicitly recognized.[15] (23) can now be employed directly to trace the evolution of the resource price along an optimal programme.

Fix t at some arbitrary t_0, and let S denote the stock of the resource remaining then. Suppose that firms are pursuing optimal extraction policies, so that $V(S,t_0)$ denotes the maximum attainable market value at t_0, and consider equation (23) over the small interval $(t_0, t_0+\theta)$. The principle of optimality dictates that any portion of the optimal programme must itself be optimal, so if the resource is extracted at the constant rate $\bar{R}$

over the interval,

$$(24)\quad (1+r\theta)V(S,t_0) = \max_R \{ (1-\theta\lambda)(1+2\theta\lambda\gamma\hat{V}(S-\theta R))\ V(S-\theta R,t_0+\theta) - \theta\lambda\gamma(1-\theta\lambda)V(S-\theta R,t_0+\theta)^2 + \theta\lambda[1-\gamma(1-\theta\lambda)\hat{V}(S-\theta R)]\ \hat{V}(S-\theta R) + \theta pR \}$$

where time-arguments have been omitted (variables are evaluated at t_0). Expanding $\hat{V}$ and V around S (and t_0 in the latter case), substituting in (24), then dividing through by θ and evaluating the limit as $\theta \to 0$ yields that

$$(25)\quad rV(S,t_0) = \max_R \{ -RV_s(S,t_0) + V_t(S,t_0) - \lambda V(S,t_0) + 2\lambda\gamma\hat{V}(S)V(S,t_0) - \gamma\lambda V(S,t_0)^2 + \lambda(1-\gamma\hat{V}(S))\hat{V}(S) + pR \}.$$

Denote by $R^*(S,t_0)$ the maximizing choice. Clearly for $R^*>0$ $V_s(S,t_0) = p$; that is, extraction proceeds up to the point where immediate gain from incremental extraction is matched by long run loss. Differentiating (25) with respect to S and using this information then yields

$$(26)\quad rV_s(S,t_0) = -\lambda V_s(S,t_0) + V_{st}(S,t_0) + 2\lambda\gamma\{\hat{V}_s(S)V(S,t_0) + V_s(S,t_0)\hat{V}(S)\} - R^*(S,t_0)V_{ss}(S,t_0) - 2\gamma\lambda V(S,t_0)\ V_s(S,t_0) + \lambda\hat{V}_s(S)\{1 - \gamma\hat{V}(S)\} - \lambda\gamma\hat{V}(S)\hat{V}_s(S).$$

Alternatively, noting that $V_S(S,t_0) = p(S,t_0)$ (the pre-event regime resource price), $\hat{V}_S(S) = \phi\hat{p}(S)$, where $\hat{p}(S)$ denotes the "fallback" price that becomes effective once the risk subsides and no tax-avoidance strategy is any longer possible, and noting also that

$$-V_{SS}(S,t_0)R^*(S,t_0) + V_{St}(S,t_0) = \frac{d}{dt_0}\, p(S,t_0)$$

one obtains, on rearrangement,

$$(27) \quad \frac{\dot{p}}{p} = r + \lambda\,(1 - \phi\hat{p}(S)/p)\{1 + 2\gamma(V(S,t_0) - \hat{V}(S))\}$$

where $p = p(S,t_0)$ and the dot denotes a time derivative. Under the type of uncertainty considered in this section, equation (27) replaces the r per cent price growth rule under certainty (it holds here if the hazard rate is zero over some interval of time). Since t_0 was arbitrary, equation (27) holds for any $t>0$, and the implied resource extraction programme is intertemporally consistent - so long, of course, as the tax has not materialized.

One would expect that, given the risk associated with holding ownership shares in resource producing firms, the resource price would appreciate at a rate exceeding the safe rate of return (the risk is one of a capital loss). This is readily confirmed by noting that $V(S,t) \geq \hat{V}(S)$, all $S>0$ (with strict inequality unless $\phi=1$). It is also clear intuitively that $\phi\hat{p}(S) < p$ at any date: a unit of the resource cannot be worth more to firms once the tax is actually imposed than before it is in place. In fact it can be shown that, for $\gamma\geq 0$,

$$\phi\hat{p}(S) < p < \hat{p}(S),$$

all S>0. Appendix A demonstrates this.

Equation (27) is now straightforward to interpret. Extraction is adjusted to the point where the rate of return on the risky asset (i.e., the rate of appreciation of the resource price) equals the rate of return on the safe asset plus an additional component to compensate for the risk of a capital loss on the shareholding. This term is the product of the hazard rate and the loss per unit of the resource that is incurred once the tax is imposed, weighted by a term exceeding unity that accounts for the risk aversion of the firms' owners. The weight exceeds unity by the equilibrium "price" of risk in the market times twice the capital loss on the asset (the total market value of the shares) should the tax be imposed.[16]

Figure 2 illustrates a typical realization of the resource price path. The contingent (i.e., pre-event) price trajectory emanates at p_0'' and follows the path marked by arrows, along which its evolution is described by equation (27). At date T the tax appears, and the market price jumps to $\hat{p}(S_T)$. Thereafter the tax in place is a <u>fait accompli</u> and the industry can do no better than to adjust extraction so that the market price follows the path $\hat{p}(S_T)e^{r(t-T)}$, which emanates from the path labelled $\hat{p}(S_t)$ at date t. Note that the latter path caters specifically to the depletion profile implied by the contingent price path. It is drawn as a continuous function of time, since along the contingent price path S(t) is a continuously declining function of time. In fact we have that

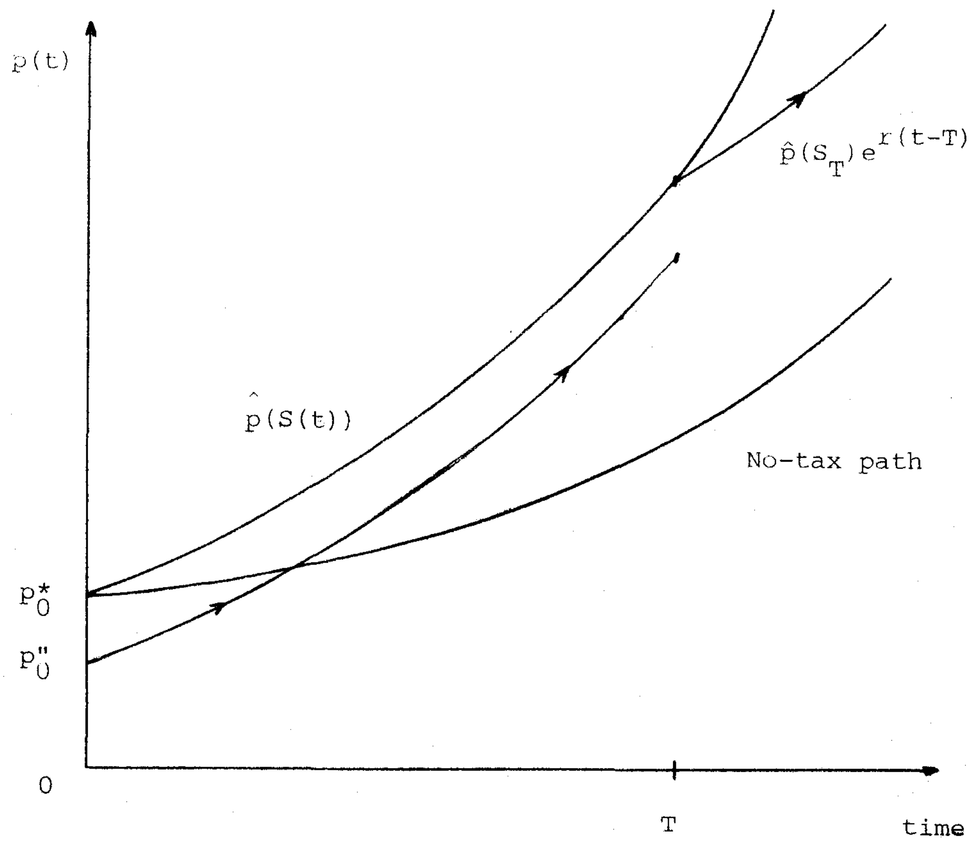

FIGURE 2

$$\frac{d}{dt}\{\log p(S(t))\} = -\{\eta(S(t))\}^{-1} R(t)/S(t)$$

where $\eta(S(t)) = \hat{p}(S(t)))/\hat{p}'(S(t)))S(t)$. Now $\hat{p}(S)$ is a monotonically declining function, so it can be inverted. Denote by $S(\hat{p})$ its inverse. But $\hat{p}$ satisfies (equations (14) - (16))

$$\int_t^{\infty} D(\hat{p}(t)e^{r(s-t)})ds = S(\hat{p}(t)),$$

whereupon differentiation with respect to $\hat{p}$ yields that

$$\eta(S(t)) = \int_t^{\infty} \{\hat{p}(S(t))e^{r(s-t)}D'(.)\}\ \{D(.)/S(t)\}\ ds,$$

where the argument of D(.) is understood to be $\hat{p}(S(t))e^{r(s-t)}$. The momentary (proportional) rate of increase of the "fallback" price $\hat{p}(S)$ at date t thus depends on the resource utilization ratio R/S then, as well as the average of the elasticity of demand along the (future) price path which would prevail were the tax to be imposed at t. Note that the weights in the computation of the average are the quantity extracted at each future date as a fraction of the stock remaining at date t.

A number of additional points should be made about the equilibrium depletion profile. The first point is that the rate of increase of the resource price is, in equilibrium, greater the greater the hazard rate (λ) and the greater the "price" of risk determined in the market (γ).[17] The second point is that the initial price, p_0'', is chosen so that, with equation (27) satisfied at all dates, the resource stock is just exhausted asymptotically. The reason is clear: if the initial price were set below p_0'', resource exhaustion in finite time would have a positive probability attached to it - no matter how distant a future date is contemplated, the tax may not yet have materialized then, so that the evolution of price continues to be given by (27). Since by assumption the demand function has no chokeoff price, firms will wish to avoid this when (hypothetically or otherwise) making contingent sales at the initial date. In so doing they will similarly wish to avoid a situation where a little more of the resource could be sold at each date without the stock ever being exhausted. So the initial price is not determined above p_0'' either.

The third point is that along the contingent price trajectory, less of a given total resource stock will at any date remain unextracted as compared with the benchmark (untaxed) programme (or, for example, one where any tax (change) is completely unanticipated).[18] The risk of a capital loss therefore - as is intuitive - discourages conservation, the more so the greater the hazard rate and the extent of risk aversion. Fourthly, it is worth noting that the magnitude of the jump is independent of whether the point-expectation of ϕ turns out to be correct or not, and that there is no certainty-equivalent event date. Once the event date is known with certainty, there is no longer any instantaneous risk of a capital loss, and the rate of growth of the resource price no longer embodies a risk premium. So it is not possible to find a non-random event date that reproduces the depletion profile under a random event date.

Finally, in both this and the previous section, the rate of resource depletion is hastened intrinsically because of investors' knowledge that *some* capital loss (whether known or not) at some future date (whether known or not) is inevitable. On the one hand, where the magnitude of the rents tax to be imposed is random (but the date of imposition known), profligacy is accentuated vis-a-vis the case of a known tax due to arrive at a known date, more so the more risk-averse the "average" investor. On the other hand, where the imposition date is random (but the magnitude known), depletion is more rapid under risk aversion than under risk neutrality.

One question of interest concerns the distortion attributable to the randomness in the imposition date *per se* for a tax of given magnitude. Does uncertainty in the imposition date (for a given mean imposition date) under all circumstances discourage conservation compared with a scenario where the date is known (and equal to the mean in the uncertain case)? This is what one might expect: after all, so long as the hazard rate is positive, there is a risk of being "caught out" at any time in the former case, and therefore a possible incentive to deplete faster. Under a sufficiently high degree of risk aversion, this may well be the case but, at least in the risk-neutral case, the opposite can be true. The following example provides an instance of this.

<u>Example</u> Suppose that $D(p)=p^{-1}$ and $f(t)=\lambda e^{-\lambda t}$, and let y^c and y^u denote a variable in the case of a certain and random T, respectively. Assume also that investors are risk-neutral ($\gamma=0$).

In the uncertain case, using equation (27), and noting that $p=R^{-1}=(-\dot{S}^{-1})$, one obtains the following second-order differential equation for $S^u(t)$:

$$\text{(E2.1)}\quad \ddot{S}/\dot{S} = (r+\lambda) + \phi(\lambda/r)\dot{S}/S$$

which, using the conditions $S(0)=S_0$ and $\lim_{t\to\infty} S(t)=0$ to evaluate the constants of integration, has solution

$$\text{(E2.2)}\quad S^u(t)= S_0 \exp\left\{ \frac{-(r+\lambda)t}{1 + \phi(\lambda/r)} \right\} .$$

Under certainty, the use of equations (19) and (20) with the same boundary conditions shows remaining reserves at date t to be given by

$$\text{(E2.3)}\quad S^c(t) = S_0 \,\frac{\{e^{-rt} - (1-\phi)e^{-rT}\}}{1 - (1-\phi)e^{-rT}},$$

where by assumption $T = \int_0^{\infty} tf(t)dt = 1/\lambda$

denotes the date at which the tax is imposed at rate $1-\phi$. A little manipulation then shows that $S^c(T) < S^u(T)$. Less is conserved up to the expected tax imposition date when it is known with certainty. □

The basic idea is of course that in the uncertainty case the benefits of quick depletion must be balanced at the margin against the benefits forgone if it turns out that reserves have been run down to very low levels but the tax has not actually materialized. It is the latter consideration that may provide an incentive for greater conservation in the uncertain case.

5. RECURRENT RISKS OF FISCAL CHANGE

It may be argued that the foregoing sections do not capture beliefs about tax changes very adequately. For one thing, such changes are likely to be expected to occur with greater regularity than previously assumed. This section considers a further approach to the problem that meets this criticism. It may be viewed as a limiting case of the characterization in section 2, in that there is a risk of change at every date. In general the tax rate changes in every period and, once it does, the process starts again. As will become clear below, the stochastic process considered in this section does not have a chain property.

At any date, let the proportion of rents retained by firms in the resource-extracting industry ($\phi \in [0,1]$) be a random variable subject to a distribution that is assumed to retain the same mean ($\bar{\phi}$) over time. Again, the starting point is the equilibrium condition for the market value of the firms in the resource-extracting industry. This is given by equation (8):

$$(8) \quad (1+r)\, V(t) = \bar{x}(t,t+1) - \gamma(t,t+1)\sigma(t,t+1)$$

where time is now measured in intervals of unit length. Recall that x denotes the total return, over the interval (t,t+1) (henceforth period t), to shares in the resource-producing industry that are worth V(t) at the beginning of the period. Also $\sigma(t,t+1)$ is the variance - evaluated at the beginning of the period - of this return (which accrues at the end of the period). It is assumed that $\sigma(s,t+1) = \sigma(t,t+1)$, $0 \leq s < t$.

The first step is to specify $x(t,t+1)$. Define x as dividends (rents accruing from the sale of the resource) paid over the period plus the total resale value of the shares. The dividends paid naturally depend on the rate of tax that prevails at the time: the applicable rate is assumed to be revealed at the end of the period just before dividend payments are made. Moreover, the total resale value of the shares is non-random at the start of period t. The reason is simply that nothing which occurs over (t,t+1) has any bearing on the pattern of returns to the lottery - the bundle of shares - that is put on the market at the end of period t. Therefore,

$$(28.1) \quad \bar{x}(t,t+1) = \bar{\phi}\, p(t)R(t) + V(t+1), \text{ and}$$

$$(28.2) \quad \sigma(t,t+1) = \{p(t)R(t)\}^2 \sigma_{\phi}(t,t+1)$$

where σ_{ϕ} denotes the variance of ϕ. Using (28) in (8), we obtain that

$$(1+r)V(t) = \bar{\phi} p(t)R(t) + V(t+1) - \gamma(t,t+1)(p(t)R(t))^2\sigma_{\phi}(t,t+1)$$

from which can be obtained the solution (assuming the market value of the resource firms approaches zero in the long run)

$$(29) \quad V(0) = \sum_{t=0}^{\infty} \left\{\frac{\bar{\phi} p(t)R(t) - \gamma(t,t+1)(p(t)R(t))^2\sigma_{\phi}(t,t+1)}{(1+r)^t}\right\}$$

An extraction profile $\{R(t)\}_{t=0}^{\infty}$ that maximizes V(0) subject to

$$(30) \quad S_0 = \sum_{t=0}^{\infty} R(t)$$

will be intertemporally consistent, in the sense that successive generations of owners will wish to pursue the original production plan. As indicated previously, the basic reason for this is that no new information about the distribution of future returns is acquired over time, so there is no need for an adaptive rule.

A maximizing solution must satisfy (30) and the difference equation

$$(31) \quad p(t) \frac{\{\bar{\phi} - 2\gamma(t,t+1)p(t)R(t)\sigma_{\phi}(t,t+1)\}}{\{\bar{\phi} - 2\gamma(t+1,t+2)p(t+1)R(t+1)\sigma_{\phi}(t+1,t+2\}} = p(t+1)/(1+r)$$

for $t \geqq 0$. Under risk neutrality, this reduces to the Hotelling r per cent rule. That is, the extraction programme is adjusted to keep the PV of the resource price constant over time. Under risk aversion, it is clear that the outcome hinges on the magnitude, at different periods of time, of the ratio in the curly brackets. Assume that the distribution of ϕ is stationary, so that $\sigma_{\phi}(t,t+1) = \sigma_{\phi}(t+1,t+2) \equiv \sigma_{\phi}$, all t. Then,

$p(t) > (=) (<) p(t+1)/(1+r)$

if and only if

$\gamma(t,t+1)p(t)R(t) > (=) (<) \gamma(t+1,t+2)p(t+1)R(t+1)$.

To get a clear indication of the forces at work, suppose that $\gamma(t,t+1) = \gamma(t+1,t+2)$, all t.[19] It then becomes evident that the characteristics of the demand function for the resource play a pivotal role in the outcome. Specifically, if the elasticity of demand (between the quantities R(t+1) and R(t)) exceeds unity, it is straightforward to deduce that $p(t) > p(t+1)/(1+r)$; that is, the PV of the resource price is lower in period t+1 than in period t. The converse holds when demand is inelastic, and if the elasticity of demand equals unity over the relevant range, the PV of the resource price is simply held constant over the two periods.

A rather sharp form of the result is obtained if it is assumed that the elasticity of demand stays on the same side of unity over the entire range of output envisaged under the optimal plan. In the case where demand is inelastic, the result is then that the proportional rate of growth of the resource price is larger than the rate of return on the safe asset, and it is not difficult to deduce that the extraction profile is more profligate than under risk neutrality. In the opposite case of elastic demand, the rate of growth of price is less than r per cent and the production plan overconserves relative to the case where investors at each date are risk-neutral.

The heuristic explanation for this apparently odd result is very simple. Firm owners at t have, when considering whether or not to produce an extra unit of the resource then, the following choice: to produce and sell now (a risky option, the return to which depends on the realization of the profits tax rate for the current period), or to leave the unit in the ground, an option that, <u>ceteris paribus</u>, enhances the resale value of the shares. The question then arises as to how much the incremental unit contributes not only to revenue, but also to the variability of revenue (a "bad"), in each of the two alternatives. If in the future, when resource prices are "high", revenue (hence its variability) is "low", it is better to leave the incremental unit in the ground for a future generation to buy (from the preceding generation), extract, and sell it. Thus, when demand is elastic, it pays to push extraction into the future because that is the best way of allocating risk at the margin. The opposite case applies when demand is inelastic. Using these guidelines, modifications to the analysis when σ_* is non-stationary, and when γ varies from period to period are straightforward, but in general they prevent clear-cut results from emerging.

However, the drawbacks to the approach of this section are twofold. Firstly, one cannot, strictly speaking, work in continuous time because the ex post stream of rents cannot be defined: the tax rate jumps discontinuously, in general, from period to period, and the limiting path of its evolution, as period length is shrunk to zero, does not exist. One is therefore left working with rather an awkward device. The second point is related, and concerns the adequacy of the representation of risk: to describe the (believed) evolution of a tax system as a random variable independent of the current state of the system hardly seems adequate. Otherwise put, one would expect knowledge of today's tax rate somehow to comprise part of the information set on the basis of which the expectation of tomorrow's tax rate is formed. It also seems sensible, by and large, to suppose that the variability (today) associated with the outcome (the state of the tax system) tomorrow is less than that of the outcome one year hence, and that in turn less than the variability of the outcome five years hence. The formulation in the next section captures these features by modelling the process that drives the evolution of the tax rate as a simple Markov chain. Perhaps more importantly, the fact that it is a "continuous" characterization of uncertainty implies that the validity of the results is not tied to the conditions under which mean-variance analysis is generally valid. A formal demonstration of this point will be given in Appendix B.

6. "CONTINUOUS" RISK OF FISCAL CHANGE: THE SINGLE ASSET CASE

The analyses of the preceding sections had the drawback that the private sector did not, as time passed, receive (or seek to acquire) further information about its likely tax treatment at a given future date. In section 3 the peculiarity was that as the transition date T approached, no further information was acquired on the basis of which beliefs about the likely outcome at T could be revised. Similarly in section 4 the only information which individuals received with the passage of time was whether or not a transition had occurred. Beliefs about when a tax change would occur did not, for example, become increasingly concentrated around a given date as time passed.

These lines of analysis may be approximately credible in some contexts (an example might be where the known transition date T of section 3 represents the date at which a host country is set to achieve independence). In general, however, one would expect signals to filter through with time that would permit individuals to reassess the tax treatment that resource-producing firms can be expected to confront at some future date. The desired property can be elucidated as follows: suppose today's date is date 0. The state of the fiscal system today (summarized in the proportion ϕ of rents that firms are currently permitted to retain) is known with certainty, but the state at $t>0$ is not. However, as time unfolds on the interval $(0,t)$, the variability associated with the outcome at t decreases, and disappears completely as date t is approached. This property will be captured in the analysis of this section.

It would clearly be preferable to allow the private sector actively to choose the rate at which it acquires signals (firms could have the option of incurring costs to gather information and on this basis compile assessments of likely future budgetary requirements, political motivation, etc.). The value of a programme of active information acquisition would lie in the fact that the signals received could be used to revise current beliefs about likely future outcomes, enabling current extraction decisions to be better tailored - in an expected sense - to future conditions. Such programmes have been modelled in the context of exploration to determine whether a resource deposit is commercially viable (see Howe, 1979, pp. 212-218, and especially Campbell and Lindner, 1985). Because in this context the decision rule involved in the choice of information structure is complex, studies which have characterized it have typically abstracted from the subsequent time-allocation of output from the deposit. Here it is precisely the latter aspect which is of interest. It will emerge below that decision rules in this regard are already quite complex, so it appears necessary, if regrettable, to abstract from studying active signal acquisition by the private sector. That is, individuals' information structure is assumed to be exogenously given at any moment in time.

To proceed with the analysis, the proportion of rents that firms are permitted to retain is assumed to be given by $\phi(z(t))$ at date t. It is supposed that ϕ is an increasing and twice-continuously differentiable transformation which maps the real line onto the unit interval. For the present, ϕ simply ensures that whatever the realization of z, the proportion of rents accruing to the private sector lies between 0 and 1. It is assumed only that $\phi'(z)>0$, $\phi(-\infty)\geq 0$, and $\phi(+\infty)\leq 1$. The interpretation which attaches to the curvature of ϕ is discussed below.

The random process $\{z(t):t\geq 0\}$ is assumed to be a diffusion process defined as the solution to a stochastic differential equation given by the limiting form of equation (32') below (see Merton, 1971, pp. 374-377). Note immediately that $\{z\}$ has the Markov property, which is that for $s\geq t$ the density function of $z(s)$ conditioned on the value of $z(t)$ is completely independent of the history of the process up to t. This is clearly a restrictive attribute: in general the historical pattern of fiscal revision can be expected to have a bearing on individuals' beliefs about the likelihood and extent of future revision. The present formulation is adopted to keep the analysis manageable, but this point is brought up again briefly at the end of this section.

To begin with, $\{z\}$ is assumed to be a Wiener process. This means that $\{z\}$ has independent increments and that $z(t+\theta)-z(t)$ is distributed $N(0,\theta\sigma_1)$ for $\theta>0$, all $t\geq 0$, where σ_1 is a positive constant. The motion of the process at date t can thus be written as

$$(32) \quad z(t+\theta) = z(t) + \epsilon\sqrt{\theta}$$

(neglecting an error of magnitude $o(\theta)$), where ϵ is a normally distributed random variable with mean 0 and variance σ_1. This formulation will provide a clear illustration of the mechanics of resource allocation where fiscal uncertainty increases with the time horizon, particularly in relation to the role of risk aversion. The results under weaker assumptions about {z} are similar and are considered briefly afterwards.

Following the discussion in section 4, consider the extraction decision on an arbitrary interval of time $(t, t+\theta)$. At date t the relevant "generation" of investors acquires the ownership shares. It is assumed to know the value of ϕ that will prevail over the interval, and be applied to the dividends paid over the interval, but (in general) the value of ϕ will have changed before the asset can be resold.

Denote by $V(S,\phi(z),t)$ the (maximized) market value of the resource-producing firms at date t, and by $V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta)$ the same at date $t+\theta$ if the resource is extracted at the constant rate R over the interval. Again invoking the principle that any portion of an optimal programme is itself optimal, equilibrium requires (see equation (8)) that

$$(33) \quad (1+r\theta)\, V(S,\phi(z),t) = \max_{R} \{\, \theta\phi(z)pR + E_t(V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta)) - \gamma\, var_t(V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta)) \,\}.$$

where time-arguments have been omitted. Now expanding $V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta)$ around $(S,\phi(z),t)$ (neglecting higher than second-order terms which are of order $o(\theta)$), and applying the operators E_t and var_t yields that

(34.1) $E_t\ V(S-\theta R, \phi(z+\varepsilon\sqrt{\theta}), t+\theta) = V - V_S\ \theta R$

$$+ 1/2\ V_\phi\ \phi''(z)\theta\sigma_1 + \theta V_t + 1/2\ V_{\phi\phi}\ \phi'(z)^2\theta\sigma_1$$

and

(34.2) $\mathrm{var}_t V(S-\theta R, \phi(z+\varepsilon\sqrt{\theta}), t+\theta) = \{V_\phi \phi'(z)\}^2\theta\sigma_1$

where the argument of V and its partial derivatives is understood to be $(S,\phi(z),t)$. Note that in (34) all subscripts, <u>including t</u>, denote partial derivatives. Using (34) in (33), dividing through by θ and taking the limit as $\theta\to 0$, one obtains

(35) $rV = \phi(z)\ pR^* - V_S R^* + 1/2\ V_\phi\ \phi''(z)\sigma_1 + V_t$

$$+ 1/2\ V_{\phi\phi}\ \phi'(z)^2\sigma_1 - \gamma\{V_\phi \phi'(z)\}^2\sigma_1$$

where $R^*=R^*(S,\phi(z),t)$ is the optimal output rate at date t. (35) has the interpretation that the (maximal) return on the shares equals the capital gain plus profit minus the allowance for risk. Since demand does not choke off, $R^*>0$, and the required stationarity of the right-hand side of (35) with respect to R implies

(36) $\phi(z)p(t) = V_S(S,\phi(z),t)$.

The task that remains now is to use (35) to find the <u>expected</u> rate of resource price growth that will be sustained when the resource is competitively supplied. First, differentiate both sides of (35) with respect to S, treating γ as parametric. This gives

$$(37)\quad rV_s = -R^* V_{ss} + 1/2\, V_{\phi s}\, \phi''(z)\, \sigma_1 + V_{st}$$

$$+ 1/2\, V_{s\phi\phi}\, \phi'(z)^2\sigma_1 - 2\gamma\phi'(z)^2\sigma_1 V_\phi\, V_{\phi s}$$

Next, expanding $V_s(S-\theta R, \phi(z+\epsilon\sqrt{\theta}), t+\theta)$ around $(S, \phi(z), t)$ and applying the operator E_t yields

$$(38)\quad E_t\ \{V_s(S-\theta R, \phi(z+\epsilon\sqrt{\theta}), t+\theta) - V_s\} = -\theta R V_{ss}$$

$$+ 1/2\, V_{s\phi}\, \phi''(z)\theta\sigma_1 + \theta V_{st}$$

$$+ 1/2\, V_{s\phi\phi}\, \phi'(z)^2\theta\sigma_1 .$$

But, in view of the fact that (36) holds at all dates,

$$(39)\quad E_t\ \{V_s(S-\theta R, \phi(z+\epsilon\sqrt{\theta}) - V_s\}$$

$$= E_t\ \{\phi(z+\epsilon\sqrt{\theta})p(t+\theta) - \phi(z)p(t)\}.$$

Now using (39) in (38) and the result, as well as equation (36), in equation (37) reveals that

$$(40)\quad r\phi(z)p(t) = \frac{1}{\theta}\, E_t\ \{(\phi(z) + \phi'(z)\epsilon\sqrt{\theta}$$

$$+ 1/2\phi''(z)\epsilon^2\theta)p(t+\theta)$$

$$- \phi(z)p(t)\} - 2\gamma(t, t+\theta)\phi'(z)^2\sigma_1 V_\phi\, V_{\phi s}$$

(where $\phi(z+\epsilon\sqrt{\theta})$ has been expanded around z and higher than second-order terms neglected). Rearranging (40) yields that

$$(41)\quad \frac{1}{\theta}\, E_t\ \{p(t+\theta) - p(t)\} = rp(t) - 1/2\, \frac{\phi''(z)}{\phi(z)}\, E_t\{\epsilon^2 p(t+\theta)\}$$

$$+ 2\gamma(t, t+\theta)\, \frac{\phi'(z)^2}{\phi(z)}\, \sigma_1\, V_\phi\, V_{\phi s} .$$

Finally, evaluating the limit of both sides of (41) as $\theta \to 0$ (and noting, from (36), that $V_{\phi s} = p$) gives

$$(42) \quad \frac{1}{dt} E_t d(p(t)) = p(t) \left\{ r - 1/2 \frac{\phi''(z(t))}{\phi(z(t))} \sigma_1 + 2\gamma(t) \frac{\phi'(z(t))^2}{\phi(z(t))} \sigma_1 \; V_\phi \right\}$$

where the operator $\frac{1}{dt} E_t \, d(.)$ is known as Itô's differential generator.[20] Equation (42) is the stochastic analogue of an equilibrium resource price growth rule under certainty.

Equation (42) has a straightforward interpretation. Note first that in the absence of any unanticipated risk of fiscal change ($\sigma_1 = 0$), the equilibrium condition reduces to the Hotelling rule as the required no-arbitrage condition in the asset market. Where $\sigma_1 > 0$, condition (42) states that the expected instantaneous growth rate of the resource price should equal the rate of return on the numeraire asset, adjusted by two different terms (the second and third terms in the curly brackets).

To focus attention on the role of the second term, suppose momentarily that $\gamma(t)=0$ (that is, the current generation of investors is risk-neutral). Then, whether expected price growth should in equilibrium be less or greater than r depends on the shape of ϕ. Recall that ϕ was chosen only to be an increasing function which took values on the unit interval. However, as well as (32), the shape of ϕ contains information about private sector beliefs regarding the risk of fiscal revision. According to (32), individuals at date t expect the increment $z(t+\theta)-z(t)$ to be zero over a short period of time. However, this does not mean that individuals expect ϕ to remain constant for that period of time, unless ϕ is locally linear in z. If, for example, $\phi''<0$ locally, the assertion that individuals believe that z will stay constant on average translates to the assertion that individuals expect ϕ to fall. This is because under concavity an upward fluctuation in z raises ϕ by less than a downward fluctuation of equal magnitude lowers it. It is therefore intuitive that if $\phi''<0$ locally, instantaneous expected price growth should exceed r to compensate for the expected net loss, and that the excess should increase with the variance σ_1. By analogous reasoning, $\phi''>(=)0$ locally implies that expected price growth should be less than (equal to) r. This is precisely what condition (42) requires.

Next, the third term on the right-hand side of equation (42) is positive if investors are risk-averse ($\gamma(t)>0$). To fix ideas, suppose that $\phi''(z)=0$ locally. This makes the second term zero. (42) then states that the (expected) rate of return on the resource should, in equilibrium, exceed the rate of return on the numeraire asset. In other words, the expected rate of price appreciation should be larger than r. Equivalently, the expected rate of output decline should be larger compared with the risk-neutral or risk-free case; more is extracted than otherwise at earlier dates (and less at later dates), and remaining reserves are at any date lower. In fact, the third term on the right-hand side of (42) is proportional to the increment in the variance of the resale value of the shares if an incremental unit of the resource is left in the ground rather than extracted and sold today, multiplied by the unit cost of risk. This makes it clear that it is a penalty component associated with leaving the marginal unit in the ground that is not there when there is no risk of fiscal revision or when investors are risk-neutral. It is this that provides the incentive for greater profligacy.

What of the case where individual beliefs can only be represented by a diffusion process {z} that has increments with a time- and state-dependent mean and variance? In this case, again neglecting an error of order $o(\theta)$, motion at date t is given by

$$(32') \quad z(t+\theta) = z(t) + m(t,z(t))\theta + k(t,z(t))\varepsilon\sqrt{\theta} ,$$

where ε is now assumed to have a standard normal distribution. $m(t,z)$ and $k(t,z)^2$ are, respectively, the instantaneous mean and variance of the increment in z(t).

By following exactly the same steps as those used to derive the equilibrium condition (42) from condition (33) when motion was given by (32), it can be checked that where {z} evolves according to (32') the equilibrium price growth rule is given by

$$(42') \quad \frac{1}{dt} E_t d(p(t)) = p(t) \left\{ r - 1/2 \frac{\phi''(z)}{\phi(z)} k(t,z)^2 - \frac{\phi'(z)}{\phi(z)} m(t,z) + 2\gamma(t) \frac{\phi'(z)^2}{\phi(z)} k(t,z)^2 \; V\phi \right\}$$

where z is shorthand for z(t).

Condition (42') looks much like condition (42) and has, <u>mutatis mutandis</u>, the same interpretation. The instantaneous variance of the increment in z(t) is of course no longer constant; other things equal, those periods for which the variance is large will be characterized by a larger expected rate of price growth to compensate investors for the increased riskiness. The principal difference between (42') and (42) is that the third term in the curly brackets on the right-hand side of (42') does not appear in (42). The effect of this term on the expected equilibrium growth rate of the resource price depends on the sign of $m(t,z)$ (recall that (42) was derived on the assumption that the expected increment in z was always zero). If, for example, $m(t,z)$ is positive, the assertion is that on average individuals expect the instantaneous increment in z to be positive. Because ϕ is an increasing function, this enhances the attractiveness of holding the resource as compared with the case where the expected increment is zero, *ceteris paribus*. It is then intuitive that the expected rate of resource price growth that is required to equilibrate the asset market should be slightly lower than otherwise if $m(t,z)>0$. Using a symmetric argument, it should be slightly larger than otherwise if $m(t,z)<0$. This is exactly what condition (42') reveals.

Before concluding this section, it is worth considering briefly how the results given in (42) and (42') might be affected if individuals based their beliefs about future fiscal revision on the magnitude and frequency of observed past changes. Suppose, for example, that individual beliefs about the movement of $z(t)$ are given by (32'), but where m and k are "estimates" based on original beliefs as well as trends and a measure of variability therein observed over some period up to t. It seems intuitive that, other things equal, shares in the resource-producing industry would become relatively more attractive over time if events turned out to be "more favourable" than originally expected. If, for example, individuals originally believed that ϕ would be consistently eroded and would exhibit great volatility over time but actual experience had shown ϕ to remain stable over time, then later generations of investors would become increasingly willing to hold the resource even though earlier generations had shunned it. In this case, the equilibrium expected rate of price growth would be much in excess of the safe rate of return early on in the extraction programme, but would decline progressively towards the safe rate with the passage of time (neglecting the effect of changes in attitudes to risk as the extraction programme unfolds). The opposite would occur if actual experience was consistently "unfavourable" vis-à-vis the beliefs of the initial generation of investors.

To conclude this section, it should be stressed that the results in (42) and (42') are <u>not</u> confined in validity to cases where investor behaviour can be analyzed in terms of the mean and variance of returns only. This is so even though the assumption that investors' utility functions can be written in the form (1') was the starting premise for the analysis. Appendix B demonstrates this for the case where z evolves according to (32) (that is, for the case corresponding to the result in (42)), but the assertion holds equally for the case where z evolves according to (32') (that is, for the case corresponding to the result in (42')). The basic intuition behind the result is this: if the source of randomness can be represented by a diffusion process which evolves according to (32') (of which a special case is (32)), then the instantaneous mean and variance characterize instantaneous motion completely. A complete neglect of higher-order moments thus entails no error in the limit as period length (θ) approaches zero.

7. "CONTINUOUS" RISK OF FISCAL CHANGE: THE MULTI-ASSET CASE

The preceding sections have indicated that, by and large, if the future is relatively "more uncertain" the more distant it is, or if there is a risk of a loss (but no risk of a gain) in the future, then firms will adopt a policy of quicker depletion. In other words, extraction is adjusted so that, in equilibrium, a (random) future PV return of given mean exceeds the current but known return. This section draws attention to what, in portfolio theory, is a well-known point: if the pattern of returns to share ownership in the resource-producing industry is inclined to vary inversely with the returns on a sufficient number of other risky assets in investors' portfolios, it will be used as an insurance vehicle, and investors will not wish to reduce its variability. In other words, investor attitudes - as far as the management of that particular asset is concerned - may be risk neutral, or even risk-seeking, in spite of an underlying aversion to risk.

The importance of this point in the context of extractive industries cannot, of course, be determined on a priori theoretical grounds. Certainly in the present case, where the source of risk in resource ownership is the risk of tax revision, the argument for a systematic (inverse) link between the evolution of share values in resource-producing industries and (say) those in manufacturing industries is somewhat tenuous. But if, to give another example, the risk in resource share values stems from the possibility of resource-saving technical progress, it is possible that, in states of nature where progress is poor, some resource-producing industries will do well while manufacturing industries do poorly, and vice-versa. It is then in theory possible that risk associated with resource holdings could induce greater conservation.

The point can be made relatively simply by retaining the Wiener process characterization of risk used in the preceding section. Suppose that there are N+1 risky assets in addition to the numeraire asset (that yields a safe rate of return r). Let asset N+1 be shares in the resource-owning firms. Over a typical interval $(t,t+\theta)$ investor i $(i=1,..,I(t))$ of generation t maximizes the criterion function (1'), subject now to

$$(43)\quad w_i + B_i = C_i(t) + \sum_{j=1}^{N} \alpha_{ij} V_j + \alpha_i V,$$

$$(44)\quad \bar{C}_i(t+\theta) = \sum_{j=1}^{N} \alpha_{ij} \bar{x}_j + \alpha_i \bar{x} - (1+r\theta)B_i$$

and

$$(45)\quad s_i(t,t+\theta) = \sum_{j=1}^{N} \sum_{k=1}^{N} \alpha_{ij} \alpha_{ik} \operatorname{cov}_t(\tilde{x}_j, \tilde{x}_k) + 2 \sum_{j=1}^{N} \alpha_{ij} \alpha_i \operatorname{cov}_t(\tilde{x}_j, \tilde{x}) + \alpha_i^2 \operatorname{var}_t(\tilde{x})$$

where the notation is as follows: V_j denotes the total market value of asset j $(j=1,...,N)$ at date t (say the value of shares in manufacturing sector j abroad), V the total market value of the resource-owning firms (asset N+1), α_{ij} the ith individual's share of asset j, and α_i his share of asset N+1. $\bar{x}_j$ denotes the total (expected) return to asset j (i.e., dividend paid over the interval plus resale value) evaluated at t, and $\bar{x}$ the total expected return to asset N+1.

To capture the randomness in the movement of share values, let $V_j = V_j(.,z_j,t)$ denote the market value of asset j at date t, and write

$$(46)\quad \bar{x}_j = \theta d_j + E_t V_j(.,z_j+\varepsilon_j\sqrt{\theta},t+\theta),\ j=1,...,N$$

where d_j is the rate of dividend payment over the interval, assumed non-random, and z_j is a Wiener process (ε_j is a normally distributed random variable with zero mean and constant variance). Similarly

$$(47)\quad \bar{x} = \theta pR + E_t V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta)$$

denotes (as in the preceding section) the expected total return to asset N+1.

Now using (43) - (45) to substitute out $C_i(t)$, $\bar{C}_i(t+\theta)$ and $s_i(t,t+\theta)$ in individual i's utility function (1'), and maximizing with respect to B_i, α_{ij}, j=1,...,N and α_i, one obtains in the same way as before an expression for the equilibrium market value of the ownership shares in the resource-producing industry at date t. This is derived in Appendix C, and is given by

$$(48)\quad (1+r\theta)\ V(S,\phi(z),t) = \max_R \{\theta\phi(z)pR + E_t\ V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta)$$

$$- \gamma(t,t+\theta)\ [\sum_{j=1}^{N} \mathrm{cov}_t(\tilde{x}_j,\tilde{x}) + \mathrm{Var}_t(\tilde{x})]\}$$

where again R is the constant rate of production over the interval $(t,t+\theta)$. (48) is the analogue of equation (8) in the case of several risky assets. Note that γ continues to measure the unit cost that the market ascribes to the risk (variance) in asset returns (see Appendix C). It is taken to be parametric by the individual resource-extracting firms.

Now interest focuses on the sign of the term in the square brackets in equation (48). Using (46) and (47),

$$(49)\quad \mathrm{cov}_t(\tilde{x}_j,\tilde{x}) = \mathrm{cov}_t(V_j(.,z_j+\varepsilon_j\sqrt{\theta},t+\theta),V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta))$$

$$= \theta V_{jz}(.,z_j,t)\ V_\phi(S,\phi(z),t)\phi'(z)\mathrm{cov}_t(\varepsilon_j,\varepsilon)$$

$$+\ o_j(t,t+\theta)$$

where $\lim_{\theta\to 0}\frac{o_j(t,t+\theta)}{\theta} = 0$. the second step in (49) has used the expansions of $V_j(.,z_j+\varepsilon_j\sqrt{\theta},t+\theta)$ and $V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta)$ around $(.,z_j,t)$ and $(S,\phi(z),t)$ respectively, and V_{jz} is the partial derivative of V_j with respect to z_j.

In view of (49), (48) can be written

$$(50)\quad (1+r\theta)\ V(S,\phi(z),t) = \underset{R}{\mathrm{Max}}\ \{\theta\phi(z)pR$$

$$+\ E_t\ V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta)$$

$$-\ \gamma(t,t+\theta)\ [\theta\sum_{j=1}^{N} V_{jz}(.,z_j,t)\ V_\phi(S,\phi(z),t)\phi'(z)\mathrm{cov}_t(\varepsilon_j,\varepsilon)$$

$$+\ o_j(t,t+\theta) + \theta\ \sigma_1\ V_\phi^2\ \phi'(z)^2]\}.$$

The instantaneous expected growth rate of the resource price in equilibrium can now be derived from equation (50) in a manner precisely analogous to that used in the previous section. It is given by

$$(51)\quad \frac{1}{dt} E_t d(p(t)) = p(t) \{ r - 1/2 \frac{\phi''(z(t))}{\phi(z(t))} \sigma_1$$

$$+ 2\gamma(t)\frac{\phi'(z(t))}{\phi(z(t))} [\phi'(z(t))\sigma_1 V_\phi + 1/2 \sum_{j=1}^{N} V_{jz} \ \mathrm{cov}_t(\varepsilon_j, \varepsilon)] \}.$$

The equilibrium condition (51) has the same interpretation as (42), and differs only as regards the third term in the curly brackets. If the term in the square bracket is negative, then, assuming also that ϕ is locally non-concave, this is sufficient for the expected growth rate of the resource price to be less than r. This condition could be satisfied if the evolution of share values in the resource-producing industry exhibits a strong negative correlation with the variability in the value of a sufficient number of other assets.

As in the preceding section, under the present formulation of uncertainty the validity of rule (51) does not depend on individuals having quadratic utility functions or the returns on linear combinations of asset holdings being normally distributed, even though that was the starting point for the analysis. As before, the reason is that as period length (θ) shrinks to zero, the motion of the random process {z} is completely described by the instantaneous mean and variance of its increments, so a neglect of the higher-order moments causes no error.

8. CONCLUDING COMMENTS

Without repeating any of the results of the preceding sections, a few general comments are in order. To begin with, it is clear that the mean-variance approach offers a good deal of flexibility in rationalizing firm behaviour in terms of underlying investor behaviour. In general, however, its results are unacceptably restrictive, although the approach arguably does offer some conceptual insights that a complete neglect of risk aversion does not. Its power in giving more general results becomes evident in the context of "small-risk" distribution representations of uncertainty, an important example of which arises when states within which firms operate are driven by the family of random processes considered in section 6.

What of the other assumptions used? Firstly, it is not clear how the results are affected if one relaxes the assumption of a two-period lifespan for individuals. Logically, individual investors should employ adaptive (closed-loop) portfolio choice rules, and it is these that should underlie the control paths chosen by firms. It seems apparent that for tractability one would have to assume that individuals' preferences are additively separable in time. Even then, however, the problem would seem to be a daunting one. Secondly, the progression from individual investor to firm objectives that is postulated here is excessively naive: there is no allowance for (say) owner/manager conflicts of interest or shareholder disputes. In other words, market value maximization may not be a sensible objective on which to base behavioural predictions. Thirdly, the analysis rests on a host of other assumptions, including the presence of a safe (and exogenously determined) rate of return, the full set of markets for contingent claims (that serves to eschew the question of how expectations are formed), and the tacit assumption that the revenue collected by the fiscal authority does not find its way back into the system.

It is by no means clear that dropping these simplifying devices would leave unchanged the basic thrust of the results about the pace of exhaustible resource depletion under uncertainty. In spite of its drawbacks, however, the approach formulated in this paper does appear to have a slight edge over more _ad hoc_ methods of dealing with producer risk aversion, or simply postulating risk neutrality.

APPENDIX A

This Appendix verifies the claim in section 4 that $\phi\hat{p}(S)<p<\hat{p}(S)$, all $S>0$. As in the text, let p_t denote the resource price at t if the rent tax has not been imposed (the contingent resource price), and $\hat{p}_t$ if it has (the fallback price). Also let p_t^* denote the price (the benchmark resource price) that would prevail along a perpetually tax-free profile (which implies that $p_t^* = p_0^* e^{rt}$).

(i) Suppose that, over an initial interval $[0,T_1)$, $p_t<\phi\hat{p}_t$ or equivalently, $\dot{p}_t/p_t<r$. In particular, at $t=0$, $\hat{p}_0-p_0/\phi=\beta$, for some $\beta>0$. Notice also that $p_0^*=\hat{p}_0$ (if a time-invariant rent tax is imposed at the initial date, the price profile simply coincides with the tax-free price profile). Now for $0\leq t<T_1$, $\dot{p}_t^*/p_t^*=r$, while $\dot{p}_t/p_t<r$. Since p is a continuous function of time, it follows that $p^*(T_1)-p(T_1)/\phi>\beta$. But cumulative extraction (up to T_1) along the contingent price trajectory will have exceeded that along the benchmark profile. Consequently $\hat{p}(T_1)>p^*(T_1)$, and <u>a fortiori</u> $\hat{p}(T_1)-p(T_1)/\phi>\beta$.

No finite T_1 therefore exists, and $\dot{p}_t/p_t<r$ for all $t\geq 0$. This then means that the implied extraction profile is infeasible, since the contingent price always remains below the benchmark price, and therefore implies greater cumulative extraction.

(ii) Suppose instead that the contingent price profile displays $\phi\hat{p}_t = p_t$, or $\dot{p}_t/p_t = r$, on the initial interval in question. Here $\phi\hat{p}_0 = p_0$ (and of course $\hat{p}_0 = p_0^*$). Now p^*, p and ϕp all appreciate at the proportional rate r. So $p_t/\phi = p_t^*$ on the entire interval. But this means that for any t>0, more will have been depleted along the contingent than along the benchmark price profile. Thus $\hat{p}_t > p_t^* = p_t/\phi$, implying that $\dot{p}_t/p_t < r$ and contradicting the initial supposition that $\dot{p}_t/p_t = r$ over the interval.

(iii) The remaining possibility is that the contingent price profile exhibits $\dot{p}_t/p_t > r$, at least on an initial interval of time. In particular, this means that $p_0 > \phi\hat{p}_0 = \phi p_0^*$. The case where $p_0 > p_0^* (= \hat{p}_0)$ can be ruled out immediately, because in this case it can be shown that $\dot{p}_t/p_t > r \; \forall t>0$ (using a parallel argument to that in (i) above), which means that the implied extraction profile does not exhaust the available stock. Thus $\hat{p}_0 > p_0 > \phi\hat{p}_0$.

Since the contingent price trajectory starts off below the benchmark profile, the paths must cross (otherwise one of the two is either infeasible or inefficient). This means that $\dot{p}_t/p_t > r$ until the paths cross (if $\dot{p}_t/p_t \leq r$ at any date prior to this, this remains the case subsequently, and the paths cannot intersect). Consider then the possibility of a switch from $\dot{p}_t/p_t > r$ to $\dot{p}_t/p_t \leq r$ (at some date T_2) after the paths have crossed. Using continuity, the switch requires $\phi\hat{p}(T_2) = p(T_2)$, so that $\hat{p}(T_2) > p(T_2)$.

By definiton, however, extraction along a profile starting at $\hat{p}(T_2)$ and growing at the rate r just exhausts the available stock asymptotically. It follows that, if the contingent price

$p(T_2)$ is below $\hat{p}(T_2)$, more is being depleted along the contingent profile than this hypothetical r per cent path allows for. So the fallback price $\hat{p}$ (and therefore $\phi\hat{p}$) must be growing at a bigger percentage rate than r. This means that p will not be able to cross $\phi\hat{p}$ again (because p, when it is below $\phi\hat{p}$, is rising at a rate less than r). But this in turn means that cumulative depletion must be larger along the contingent than along the benchmark price path, implying that the former is infeasible.

Thus $\dot{p}_t/p_t > r$, or $\phi\hat{p}_t < p_t$, all $t \geq 0$. Cumulative depletion up to any finite date is greater along the contingent than along the benchmark profile (see note 18), implying $\hat{p}_t > p_t$, all $t \geq 0$.

APPENDIX B

This Appendix verifies the claim in sections 6 and 7 that the mean-variance results are asymptotically valid in the general case as the parameter θ (period length) approaches zero. The utility function (1) can be expanded as a Taylor series around mean "second-period" consumption. Omitting the subscript i that identifies the investor, this yields that

$$(b.1)\quad E_t\ W(C(t),\tilde{C}(t+\theta)) = \sum_{j=0}^{\infty} W^{(j)}(C(t),\bar{C}(t+\theta))\ E_t\ \frac{(\tilde{C}(t+\theta)-\bar{C}(t+\theta))^j}{j!},$$

where $W^{(j)}$ denotes the jth order partial derivative of $W(.,.)$ with respect to its <u>second</u> argument. Of course the quadratic approximation truncates the series after j=2. Take for simplicity the case (as in section 6) of a single risky asset (shares in the resource-owning firms). The individual thus maximizes (b.1) subject to equations (2) and (3) in the text. Using (2) and(3) to eliminate C(t) and $\tilde{C}(t+\theta)$ from (b.1) and maximizing with respect to α and B yields, for an interior solution,

$$(b.2)\quad \frac{\partial}{\partial\alpha} E_t\ W(.,.) = \sum_{j=1}^{\infty} W^{(j)}(.,.)\ \alpha^{(j-1)}\ \frac{\mu(j)}{(j-1)!} + \sum_{j=0}^{\infty} \alpha^j \frac{\mu(j)}{j!}\{-V\ W^{(j)}_1(.,.) + \bar{x}\ W^{(j+1)}(.,.)\} = 0,\quad \text{and}$$

$$(b.3)\quad \frac{\partial}{\partial B} E_t\ W(.,.) = \sum_{j=0}^{\infty} \alpha^j\ \frac{\mu(j)}{j!}\{W^{(j)}_1(.,.) - (1+r\theta)W^{(j+1)}(.,.)\} = 0$$

where $W^{(j)}{}_1(.,.)$ is the partial derivative of $W^{(j)}(.,.)$ with respect to its <u>first</u> argument (i.e., C(t)), and

$$\mu(j) \equiv E_t \{\tilde{C}(t+\theta)-\bar{C}(t+\theta)\}^j / \alpha^j$$

$$= E_t \{\tilde{x}(t,t+\theta)-\bar{x}(t,t+\theta)\}^j$$

$$= E_t\{V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta) - E_t V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta)\}^j$$

where the equalities follow from (3) and (5) in the text (section 2). Now using equations (b.2) and (b.3), one derives the general analogue of expression (33) for the market value of the shares, that is

$$\text{(b.4)}\quad (1+r\theta)V(S,\phi(z),t) = \max \{\theta\phi(z)pR + E_t V(S-\theta R,\phi(z+\varepsilon\sqrt{\theta}),t+\theta) + \frac{\sum_{j=1}^{\infty} W^{(j)}(.,.)\,\alpha^{j-1}\,\frac{\mu(j)}{(j-1)!}}{\sum_{j=0}^{\infty} W^{(j+1)}(.,.)\,\alpha^{j}\,\frac{\mu(j)}{j!}}.$$

Recall now that, in order to derive the price growth rule, (b.4) is divided through by θ and the limit of the expression as θ goes to zero taken. Thus consider the limit approached by the ratio of the power series divided by θ, as $\theta \to 0$. By referring to (34), it can be seen that moments of order 1 are zero, moments of order zero unity, and the lowest power of $\sqrt{\theta}$ in moments of order j is j (note that this holds equally for the more general case where the random process {z} is described by (32')). Third- and higher-order moments are therefore $o(\theta)$. Consequently

$$\lim_{\theta\to 0} \frac{\sum_{j=1}^{\infty} W^{(j)}(.,.)\ \alpha^{j-1}\ \frac{\mu(j)}{(j-1)!}}{\theta\{\sum_{j=0}^{\infty} W^{(j+1)}(.,.)\ \alpha^{j}\ \frac{\mu(j)}{j!}\}} = \lim_{\theta\to 0} \frac{\alpha\ W_{22}(.,.)\mu(2)}{W_2(.,.)}.$$

But in fact the right-hand side is simply the expression for $\gamma(t)$ that is used in the text. To see this, define the quadratic approximation to $E_t\ W(.,.)$ as

$$U(C(t),\bar{C}(t+\theta),s(t,t+\theta)) = W(C(t),\bar{C}(t+\theta))$$

$$+ W_2(C(t),\bar{C}(t+\theta))\ E_t\{\tilde{C}(t+\theta)-\bar{C}(t+\theta)\}$$

$$+ 1/2\ W_{22}(C(t),\bar{C}(t+\theta))\ E_t\{\tilde{C}(t+\theta)-\bar{C}(t+\theta)\}^2$$

where $s(t,t+\theta) \equiv E_t\{\tilde{C}(t+\theta)-\bar{C}(t+\theta)\}^2$. Then it is clear that

$$\gamma(t,t+\theta) \equiv -2\alpha\ U_3(.,.,.)/U_2(.,.,.) = -\alpha\ W_{22}(.,.)/W_2(.,.)\ .$$

In other words, the analysis that uses a quadratic approximation to the utility function is, as θ goes to zero, asymptotically valid in the general case. It follows that, provided the characterization of risk is of the "continuous" variety, where the variance of returns grows linearly with the relevant horizon, a result derived from mean-variance underpinnings becomes a general result in continuous time. Thus where uncertainty is characterized by a random process described by (32), equation (42) is a general result and not restricted in validity to cases where (1) can be written as (1'). By similar reasoning, the same holds true where uncertainty is characterized by a random process satisfying (32'), as well as for the multi-(risky) asset case of section 7.

APPENDIX C

This Appendix derives condition (48), section 7. Given the criterion function

$$(1')\quad U^i(C^i(t),\bar{C}_i(t+\theta),s_i(t,t+\theta))$$

use (43)-(45) to substitute for $C_i(t)$, $\bar{C}_i(t+\theta)$ and $s_i(t,t+\theta)$, and choose α_{ij} $j=1,..,N$, α_i and B_i. The necessary conditions for an interior solution are

$$(c.1)\quad -U^i{}_1V_j + U^i{}_2\bar{x}_j + U^i{}_3\{\ 2\alpha_{ij}\mathrm{var}(x_j) + 2\sum_{k\neq j}\alpha_{ik}\mathrm{cov}(x_j,x_k) + 2\alpha_i\mathrm{cov}(x_j,x)\ \} = 0.$$

for $j=1,..,N$, as well as

$$(c.2)\quad -U^i{}_1V + U^i{}_2\bar{x} + U^i{}_3\{\ 2\alpha_i\mathrm{var}(x) + 2\sum_{j=1}^{N}\alpha_{ij}\mathrm{cov}(x_j,x)\ \} = 0,$$

and

$$(c.3)\quad U^i{}_1 - (1 + r\theta)U^i{}_2 = 0.$$

Using (c.3) to eliminate $U^i{}_1$ from (c.1) and (c.2) yields that

$$(c.4)\quad -U^i{}_2(\bar{x}_j - (1+r\theta)V_j) + U^i{}_3\{\ 2\alpha_{ij}\mathrm{var}(x_j) + 2\sum_{k\neq j}\alpha_{ik}\mathrm{cov}(x_j,x_k) + 2\alpha_i\mathrm{cov}(x_j,x)\ \} = 0,$$

for $j=1,..,N$, and

$$(c.5)\quad -U^i_2(\bar{x} - (1+r\theta)V) + U^i_3\{\ 2\alpha_i \operatorname{var}(x) + 2\sum_{j=1}^{N} \alpha_{ij}\operatorname{cov}(x_j, x)\ \} = 0.$$

The ratio U^i_2/U^i_3 can be eliminated between (c.4) and (c.5), so that

$$(c.6)\quad \frac{[\bar{x} - (1+r\theta)V]}{\operatorname{var}(x) + \sum_{j=1}^{N}\operatorname{cov}(x_j, x)} = \frac{[\bar{x}_j - (1+r\theta)V_j]}{\operatorname{var}(x_j) + \sum_{k\neq j}\operatorname{cov}(x_j, x_k) + \operatorname{cov}(x_j, x)} = \gamma \text{ say,}$$

Rearranging (c.6) and using (47) to substitute out $\bar{x}$ gives expression (48).

To interpret γ, rewrite equations (c.6) as

$$\gamma = a/b = a_j/b_j$$

$j=1,..,N$. Now note that

$$a + \sum_{j=1}^{N} a_j = a + \sum_{j=1}^{N}(a/b)b_j = (a/b)\{b + \sum_{j=1}^{N} b_j\},$$

so that

$$(c.7)\quad \gamma = a/b = (a + \sum_{j=1}^{N} a_j) / (b + \sum_{j=1}^{N} b_j).$$

Finally, using (c.7) and (c.6), one obtains that

$$\gamma(t,t+\theta) = \frac{\bar{x} + \sum_{j=1}^{N} \bar{x}_j - (1+r\theta)\,[V + \sum_{j=1}^{N} V_j]}{\sum_{j=1}^{N} \sum_{k=1}^{N} \mathrm{cov}_t(x_j, x_k) + 2 \sum_{j=1}^{N} \mathrm{cov}_t(x_j, x) + \mathrm{var}_t(x)}.$$

The numerator is the difference between the expected future return on the assets and their current market valuation. It therefore captures the value that the market places on the removal of the risk in asset returns. The denominator measures the total variance in all the assets taken together. Thus γ measures the unit cost that the market ascribes to the risk (variance) in asset returns (see Nickell, 1978, pp. 160-165).

NOTES

1. The recent history of federal measures in the United States is characterized by import restraints, price control, and, more recently, excise taxes. The evolution of these measures has exhibited a considerable amount of volatility. Kalt (1981, pp. 4-23) gives a brief overview of the developments.

2. In this example, the magnitude of the bias is determined, among other things, by the size of remaining reserves. This is because the effect of the risk of a capital loss on extraction behaviour can be proxied by an increase in resource owners' discount rates. A discount rate increase may lower or raise the pace of depletion, depending on whether or not the negative effect of investment in extractive capacity outweighs the "impatience" effect that makes for quicker depletion. If stocks are "large" (small), the former (latter) effect dominates. On this, see for example Farzin (1984) or Khadr (1987, section 2C).

3. Also, Conrad and Hool (1981) use a two-period, two-grade model to analyze the effect of different taxes on "cutoff grade selection" for a resource. The focus, in other words, is on how taxes determine how much of the resource is ultimately recovered.

4. Starting from a risk-free, distortion-free state also allows one to state that any distortion induced by the introduction of the (endogenous) tax risk is welfare-reducing vis-a-vis the risk-free state. However, to the extent that there are other (exogenous) sources of risk already in existence, optimality properties cannot be inferred for the initial state, and thus the welfare effects of the introduction of the tax uncertainty cannot in general be assessed; See section 2 and note 8.

5. As Weinstein and Zeckhauser point out, resource owners must take borrowing and lending decisions without waiting for the actual realization of resource rents. Under these circumstances, a recursive (closed-loop) solution technique should be used, which is incompatible with putting a (non-linear) cardinal utility function on the sum of increments to rent over time. The closed-loop approach is outlined in Spence and Zeckhauser (1972).

6. Other work that considers the issue of "demand" uncertainty in the context of exhaustible resource extraction includes Dasgupta, Eastwood and Heal (1978), Kamien and Schwartz (1978) and Hoel (1978). Examples of papers that deal with the question of uncertainty about available reserves of the resource are Loury (1978), Gilbert (1979) and Heal (1979).

7. It is perhaps easiest to think of a small open economy where the numeraire asset corresponds to "safe foreign assets", the rate of return on which is given exogenously.

8. An *ex ante* welfare optimum is an allocation that maximizes a social ordering on individuals' *ex ante* (expected) utilities computed according to individuals' subjective probabilities. An *ex post* welfare optimum is an allocation that maximizes a weighted sum of social orderings on individuals' *ex post* utilities, where the weights are the social probabilities. It is only under very stringent conditions that an *ex ante* welfare optimum (achieved under a complete set of contingent markets with appropriate lump-sum transfers) also realizes an *ex post* welfare optimum. Even a more powerful device whereby the social planner disallows private forward transactions and dictates instead a profile of contingent lump-sum transfers does not, in general, sustain an *ex post* welfare optimum unless individual preferences have a property of backward separability (see Hammond, 1981). In the present context, it seems rather pointless to enquire about whether an *ex post* welfare optimum is achievable, since it is the central authority - presumably the social planner himself - who introduces the source of the problem (the tax risk) in the first place. But, where the environment is beleaguered with other sources of uncertainty (see section 7), the argument in this note says that (*ex post*) optimality cannot in general be ascribed to the initial state.

9. More generally, Samuelson demonstrates that the solution obtained from maximizing the nth order approximation to the utility function coincides with the true solution (as the parameter goes to zero) up to its (n-2)th order derivative (with respect to the parameter).

10. Assuming, of course, that an increase in the variance does not actually decrease the price of risk in the market: risk, in other words, is more costly to bear at the margin the more of it there is.

11. Suppose that $0 > \eta > -1$. Clearly the left-hand side (lhs) is strictly greater than the right-hand side (rhs) for ϕ close to zero. Moreover the two sides approach one another as ϕ approaches unity. If the rhs is to become greater than or equal to the lhs, the derivative of the rhs with respect to ϕ must be greater than or equal to the derivative of the lhs given at a value of ϕ (say ϕ^*) where the two sides are equal. But the ratio of the derivative of the lhs to the derivative of the rhs at ϕ^* can be written as

$$\frac{1 - e^{r\eta T}(1-\phi^{-\eta})}{\{1 - e^{r\eta T}(1-\phi)\}\phi^{-\eta-1}}$$

which exceeds unity, and therefore contradicts the possibility that the rhs could be greater than or equal to the lhs at any value of $\phi < 1$ under inelastic demand conditions.

12. Of course, if resource owners *could* collude, they would supply negligible amounts of the resource under inelastic demand.

13. In fact Dasgupta is concerned with showing the rapidity of

convergence to zero of the value of resource deposits as their size is increased. Convergence is guaranteed provided the average of the elasticity of demand along the optimal profile is less than unity in absolute value.

14. An example of this characterization of risk in the context of consumption-portfolio choice over an uncertain lifespan appears in Yaari (1965) and Merton (1971, section 8). The "event" in this case is "death". More akin to the problem under consideration here is the analysis of Dasgupta and Stiglitz (1981) of resource depletion under the "hazard" that a backstop technology will become available.

15. $\hat{V}$ depends only on the remaining stock because once the tax materializes, a Hotelling r per cent path is pursued with the initial price chosen to ensure asymptotic exhaustion. The initial price depends only on the remaining stock.

16. The latter term in the product is in fact the instantaneous variance of the conditional distribution of the share value at date t.

17. For given V and $\hat{V}$, the instantaneous variance of returns is larger the larger the hazard rate; again the proviso of note 10 applies.

18. Suppose that one profile (say 1) exhibits a higher rate of price growth than another (profile 2) at all dates. Then clearly 1 must have a lower initial price associated with it, otherwise cumulative depletion along profile 2 exceeds that along 1. Also, the reverse holds unless the price paths cross. If τ is the date at which the paths cross, then clearly for $t \leq \tau$,

$$S_0 - S^1(t) = \int_0^t D(p^1(s))ds > \int_0^t D(p^2(s))ds = S_0 - S^2(t)$$

so that $S^1(t) < S^2(t)$. In addition, for all $t > \tau$, $D(p^1) < D(p^2)$, therefore

$$S^1(t) = \int_t^\infty D(p^1(s))ds < \int_t^\infty D(p^2(s))ds = S^2(t),$$

where the equalities follow from the stock exhaustion condition.

19. This supposition is just a heuristic device and in general could only happen by sheer coincidence.

20. Merton (1971, pp. 374-7) reviews this concept. That $\lim_{\theta \to 0} E_t\{\varepsilon^2 p(t+\theta)\} = \sigma_1 p(t)$ follows from the (joint) right-continuity of this term as θ approaches zero (see Merton, footnote 9).

REFERENCES

Bohi, D.R. and M.A. Toman (1984) Analyzing Nonrenewable Resource Supply, Resources for the Future, Washington, D.C.

Burness, H.S. (1978) "Price uncertainty and the exhaustive firm" Journal of Environmental Economics and Management 5, pp. 128-149.

Campbell, H.F. and R.K. Lindner (1985) "A model of mineral exploration and resource taxation" Economic Journal, 95, pp. 146-160.

Church, A.M. (1981) Taxation of Nonrenewable Resources, Lexington Books, D.C. Heath & Co., Lexington, Mass.

Conrad, R.F. and R.B. Hool (1980) Taxation of Mineral Resources, Lexington books, D.C. Heath & Co., Lexington, Mass.

Conrad, R.F. and R.B. Hool (1981) "Resource taxation with heterogeneous quality and endogenous reserves" Journal of Public Economics 16, pp. 17-33.

Dasgupta, P.S. (1983) "The value of large resource deposits or, how precious are the seabed nodules?" Economics Letters 12, pp. 369-375

Dasgupta, P.S. and G.M. Heal (1974) "The optimal depletion of exhaustible resources" Review of Economic Studies, Symposium, pp. 3-28.

Dasgupta, P.S. and G.M. Heal (1979) Economic Theory and Exhaustible Resources James Nisbet & Co., Welwyn, and Cambridge University Press, Cambridge.

Dasgupta, P.S. and J.E. Stiglitz (1981) "Resource depletion under technological uncertainty" Econometrica 49, pp. 85-104.

Dasgupta, P.S., R.K. Eastwood and G.M. Heal (1978) "Resource management in a trading economy" Quarterly Journal of Economics, May, pp. 297-305.

Dasgupta, P.S., G.M. Heal and J.E. Stiglitz (1980) The Taxation of Exhaustible Resources, National Bureau of Economic Research Reprint no. 227, Cambridge, Mass.

Deshmukh, S.D. and S.R. Pliska (1985) "A martingale characterization of the price of a non-renewable resource in decisions involving uncertainty" Journal of Economic Theory 35, pp. 322-342

Devereux, M.P. and C.N. Morris (1983) North Sea Oil Taxation Institute for Fiscal Studies report series No. 6, London.

Farzin, Y.H. (1984) "The effect of the discount rate on depletion of exhaustible resources" Journal of Political Economy 92, pp. 841-51.

Feldstein, M.S. (1969) "Mean-variance analysis in the theory of liquidity preference and portfolio selection" Review of Economic Studies 36, pp. 5-12.

Gilbert, R. (1979) "Optimal depletion of an uncertain stock" Review of Economic Studies 46, pp. 47-57.

Hammond, P.J. (1981) "Ex-ante and ex post welfare economics under uncertainty" Economica 48, pp. 235-50.

Hartwick, J.M. and D. Yeung (1985) "Preference for output price uncertainty by the non-renewable resource extracting firm" Economics Letters 19, pp. 85-89

Heal, G.M. (1978) "Uncertainty and the optimal supply policy for an exhaustible resource" in Pindyck, R.S. (ed.) Advances in the Economics of Energy and Resources, vol. 2, JAI Press, New York.

Heaps, T. (1985) "The taxation of nonreplenishable natural resources revisted" Journal of Environmental Economics and Management 12, pp. 14-27.

Hoel, M. (1978) "Resource extraction when a future substitute has an uncertain cost" Review of Economic Studies 45, pp.637-644.

Howe, C.W. (1979) Natural Resource Economics, John Wiley & Sons, New York.

Kalt, J.P. (1981) The Economics and Politics of Oil Price Regulation, MIT Press, Cambridge, Mass.

Kamien, M.I. and N.L. Schwartz (1978) "Optimal exhaustible resource depletion with endogenous technical change" Review of Economic Studies 45, pp. 179-96.

Khadr, A.M. (1987) "Capacity constraints and the production of nonrenewable resources", Working Paper no. EE5, Oxford Institute for Energy Studies.

Lewis, T.R. (1977) "Attitudes towards risk and the optimal exploitation of an exhaustible resource" Journal of Environmental Economics and Management 4, pp. 111-119.

Lewis, T.R. and M. Slade (1985) "The effects of price controls, taxes, and subsidies on exhaustible resource production" in A. Scott et al. (eds.) Progress in Natural Resource Economics, Clarendon Press, Oxford.

Long, N.V. (1975) "Resource extraction under the uncertainty about possible nationalization" Journal of Economic Theory 10, pp. 42-53.

Long, N.V. (1984) "Risk and resource economics: the state of the art" in D.W. Pearce et al. (eds.) Risk and the Political Economy of Resource Development, Macmillan Press, London.

Loury, G.C. (1978) "The optimum exploitation of an unknown reserve" Review of Economic Studies 45, pp. 621-636.

Merton, R.C. (1971) "Optimum consumption and portfolio rules in a continuous-time model" Journal of Economic Theory 3, pp. 373-413.

Nickell, S.J. (1978) The Investment Decisions of Firms, James Nisbet & Co., Welwyn, and Cambridge University Press, Cambridge.

Pindyck, R.S. (1980) "Uncertainty and exhaustible resource markets" Journal of Political Economy 88, pp. 1203-1225.

Pindyck, R.S. (1981) "The optimal production of an exhaustible resource when price is exogenous and stochastic" Scandanavian Journal of Economics 83, pp. 277-289.

Samuelson, P.A. (1971) "The fundamental approximation theorem of portfolio analysis in terms of means, variances, and higher moments" Review of Economic Studies 38, pp. 537-542.

Seymour, I. (1980) OPEC: Instrument of Change, Macmillan Press, London.

Spence, M. and R. Zeckhauser (1972) "The effect of the timing of consumption and the resolution of lotteries in the choice of lotteries" Econometrica 40, pp. 401-403.

Stevens, G.V.G. (1974) "On the impact of uncertainty on the value and investment of the neoclassical firm" American Economic Review 64, pp. 319-336.

Tobin, J. (1969) "Comment on Borch and Feldstein" Review of Economic Studies 36, pp. 13-14.

Weinstein, M.C. and R.J. Zeckhauser (1975) "The optimal consumption of depletable natural resources" Quarterly Journal of Economics 89, pp. 371-92.

Yaari, M.E. (1965) "Uncertain lifetime, life insurance and the theory of the consumer" Review of Economic Studies 32, pp. 137-150.

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