

Stirling Pulse Tube Cryocoolers for Space Science Missions



Hannah Rana

Magdalen College

University of Oxford

A thesis submitted for the degree of

Doctor of Philosophy

Michaelmas 2021

Abstract

Cryocoolers are compact and effective devices that provide cooling to cryogenic temperature. They serve to reduce thermal noise on focal plane assemblies in space telescopes, cool down superconducting devices, and present many other applications for a wide range of technologies. A Stirling cryocooler operates by running the Stirling cycle and reaches high efficiencies. They have an established spaceflight heritage, and in the case of the Oxford-style cryocooler, they employ flexure bearing springs that enhance their longevity and make them suitable for long lifetime space missions. In more recent times, pulse tube cryocoolers have garnered great interest specifically due to the benefit they pose of having no moving parts at the cold end. This is beneficial for astrophysics purposes where vibration requirements are especially stringent.

In this thesis, a hybrid Stirling pulse tube cryocooler (SPTC) was developed, which incorporates a warm end displacer as is used in Stirling systems, whilst hybridised with a pulse tube assembly at the cold end. This enabled the pulse tube to reach higher efficiencies and for the displacer to recover PV expansion power back into the system at the warm end. This was developed in a coaxial configuration using an active displacer with an additional motor. The subsequent aim of this research project was to reiterate this design for a coaxial SPTC that reaches comparable cooling capabilities, whilst running the displacer passively. The objective in doing so was to remove the second phase in the power electronics and reduce the complexity of the active displacer design. The active displacer SPTC was successful in experimentally cooling to a cold temperature of 80 K with an input power of 79 W, an optimum cooling power of 2.4 W and a relative Carnot efficiency of 8.3%. Similarly, the designed, built, and tested passive displacer SPTC achieved cooling to 80 K with an optimum input power of 97 W, a cooling power of 3 W, and a relative Carnot efficiency of 9.4%. Numerical simulations were completed using Sage software for both SPTCs and the models were validated with the experimental findings to show similar trends for cooling power and efficiency.

As part of the design and physical development of the passive displacer SPTC, new design flexure bearing springs underwent stiffness, fatigue, and resonance testing in order to characterise their behaviour and performance. This enabled a suitable selection for the passive displacer. Moreover, the suitability of a strain gauge circuit in a full Wheatstone bridge configuration was investigated as a novel technique for in-situ cryocooler stroke measurements. The technology was found to be successful in serving this purpose, and further, was not subject to drift or phase lag, which makes it an advantageous technology compared to commonly used LVDTs which are subject to the latter. A good frequency response was also demonstrated by conducting a range of impact tests on the spring-strain gauge setup and assessing the strain gauge bridge response.

Finally, theoretical and computational studies were undertaken to better understand the inner workings of the pulse tube cryocooler. Phasor analysis was conducted to derive mathematical expressions for mass flow, pressure pulse, and the phase angle between the two. Fluid flow studies were undertaken to gain insight into the hydrodynamics within the coaxial regenerator and thermal simulations were completed using ESATAN-TMS to quantify conductive and convective losses in the SPTC assembly, where strong agreement with an error of under 7% was achieved between the thermal model and the experimentally validated Sage model.

This thesis is dedicated to my beloved grandfathers:

Mr Rana Rafiq Parwaz

&

Mr Rana Abdul Haq.

Acknowledgements

This D.Phil. would not have been possible without the guidance and support of countless people. My first thanks go to Professor Richard Stone, my supervisor, whose technical excellence, professionalism, and humility I derive lessons from on a daily basis, and will do for years to come. Thank you for your guidance throughout this journey, and particularly, for your patience and understanding during the last year. I feel privileged and grateful to have had the opportunity to learn from a truly brilliant scholar. Sincere thanks to Mike Dadd for always being available to answer queries on pulse tubes and for bringing so much energy and enthusiasm to the lab. My further thanks to Paul Bailey for his help in answering queries and providing insightful technical advice. I must also thank him for the endless supply of biscuits in the Cryo lab. My particular thanks to Amin Abolghasemi for his assistance in a variety of tasks towards my research and general support throughout my D.Phil. I will miss working with this distinguished group of researchers, not only for their technical excellence, but also the tea breaks in which we discussed a range of topics; from Brexit politics to international development to how birds breathe (thanks, Mike). Thank you to each of you for making our research such an enjoyable and stimulating endeavour.

Thank you to the EPSRC and Honeywell Aerospace for funding this thesis work. Thanks also to Zonta International for the Amelia Earhart fellowship towards my research. I'd like to thank my peers Maruthi, Kharthik, and Priyav in the Thermal Propulsion Systems research group, for so many things - the daily support and encouragement as we all progressed in our research has been invaluable. Together we managed to keep each other's spirits high in the face of the pandemic, lab shutdowns, research re-directions, and countless other challenges in between. You are like brothers to me and this would not have been possible without your support. Further thanks to Sam S, Laurent, Leo, Chris, Ruixuan, Sam B, Sam W, Varun, Tejo, Qichi, and Niko from the TPSRG for creating a friendly working environment in the Thom office.

A thank you to Rabia, Alaa, Naiyerah, Selena, and all the girls that make up the 'Roti Lovers' - your support and friendship during this D.Phil. has been invaluable. This, of course, includes your collective companionship in efforts to systematically identify and rank the best eateries in Oxford. A huge thank you to my dear Magdalen friends for always being up for tea, chess, boardgames, political discussions, and terrible Hall food. My sincere thanks to the ISoc at Oxford, my indebted thanks to Dr Nadwi for his generous time over the years offering me guidance on all matters in life, my recitation teacher and dear friend Nabeela for the immense support throughout my academic journey, and everyone at ASI for making weekly classes so enriching and providing welcome perspective during my D.Phil.

I'd like to extend a warm thank you to Thierry Tirolen, who supervised me at the European Space Agency and who first sparked my interest in the field of space cryogenics. He taught me much of what I know today about space thermal control. My thanks also to Dr Olivier Pin at ESA for encouraging me to do this D.Phil. Thank you to Dr Ed Brooks from the Oxford Character Project and Prof. Carole Mundell, my GLI mentor, for our conversations on research, character, leadership, and pretty much everything else in life. I've learned so much from you. I'd also like to thank Dr Fabiola Gianotti, who

over the years has been both a mentor and a dear friend to me. I'm truly grateful for all the wisdom, advice, and encouragement.

Finally, but most significantly, my deepest and warmest thanks go to my family. Thank you to Mariam api and Omar bhai, Ayesha api, Zeynab, and little Abdullah and Emaan, for your good humour, example, love, and immeasurable support.

Above all, thank you to my beloved parents, Tasnim and Sohail, my Mama jaan and Abu jaan, for your endless love and unwavering support. You have both worked tirelessly to give us the very best of everything and are behind everything we do. Words will never be able to express my full gratitude.

وَالْحَمْدُ لِلّٰهِ رَبِّ الْعَالَمِينَ، الْوَلِيّ الْأَعْظَم.

Contents

Nomenclature

viii

I	Background and Theoretical Treatment	1
1	Introduction	2
1.1	Motivation	2
1.1.1	Background	2
1.1.2	Space Telescopes	3
1.2	Thesis Layout	5
1.3	List of Published Works	6
2	Stirling and Pulse Tube Cryocoolers: A Review	9
2.1	Stirling Cryocooler	9
2.1.1	The Stirling Refrigeration Cycle	10
2.1.2	Spaceflight Stirling Cryocoolers	11
2.2	Pulse Tube Cryocooler	13
2.2.1	The Pulse Tube	13
2.2.2	Stirling Pulse Tube Cryocoolers	16
2.2.3	Current Needs and Objectives	17
3	Theoretical Treatment	20
3.1	Background	20
3.2	Modelling Rationale	21
3.3	Energy Flows	22
3.3.1	Enthalpy and Entropy	22
3.3.2	Work and Exergy Flows	23
3.4	Phasor Analysis	25
3.4.1	Pressure and Mass Flow Phase	25
3.4.2	Mass Flow at the Pulse Tube Cold End	25
3.4.3	SPTC Component Phasors	28
3.4.4	Effect of Phase Angle	30
3.4.5	Comparative Phasor Analysis of PTC Configurations	31
3.5	Summary	33

II	Testing and Measurement Techniques	36
4	Flexure Spring Resonance and Fatigue	37
4.1	Introduction	38
4.1.1	The Oxford Cryocooler	38
4.1.2	Moving Magnet Compressor	39
4.1.3	Oxford Flexure Springs	41
4.2	Theory	42
4.2.1	Damped Harmonic Oscillator	42
4.2.2	Spring Resonance	43
4.2.3	Axial and Radial Spring Stiffness	43
4.2.4	Oscillatory Loading	44
4.3	Design and Modelling Analysis	46
4.3.1	Spring Designs	46
4.3.2	Finite Element Analysis	46
4.4	Experimental Testing and Analysis	47
4.4.1	Testing Rig	47
4.4.2	Electromagnetic Modelling of Testing Rig	50
4.4.3	Fatigue Testing	51
4.5	Summary	57
5	A Novel Measurement Technique for Cryocooler Stroke	59
5.1	Introduction	60
5.1.1	LVDTs	61
5.1.2	Strain Gauge Bridge Configuration	62
5.1.3	Finite Element Analysis	63
5.2	Experimental Design	65
5.2.1	Strain Gauge Installation	65
5.2.2	Testing Rig	66
5.2.3	Isolation Amplifier Design	67
5.2.4	Static Spring Offset	68
5.2.5	System Characterisation	69
5.2.6	Data Acquisition	71
5.3	Experimental Testing	72
5.3.1	Omron Calibration	72
5.3.2	Strain Gauge Bridge Calibration	72
5.3.3	FEA Validation	73
5.3.4	Dynamic Tests	74
5.3.5	Impact Tests	75
5.4	Summary	83

III	From Active to Passive Displacer	86
6	A Hybrid Coaxial Stirling Pulse Tube Cryocooler	87
6.1	Introduction	88
6.2	Thermodynamics and Harmonic Approximation	88
6.3	Design and Coaxial Configuration	91
6.4	Sage Numerical Simulation	93
6.4.1	Key Parameters	93
6.4.2	Modelling Theory	94
6.4.3	Modelling Approach	97
6.5	Hydrodynamic Phenomena in Coaxial Regenerator	102
6.5.1	Objectives	102
6.5.2	Simulated Coaxial Regenerator	103
6.5.3	Experimental Flow Testing	105
6.6	Thermal Simulation of Coaxial Regenerator	110
6.6.1	Introduction	110
6.6.2	Theory	112
6.6.3	Model Setup and Properties	114
6.6.4	Steady State Conduction	115
6.6.5	Radiative Losses	116
6.7	Simulated Cooling Performance at 80 K	118
6.8	Experimental Layout and Instrumentation	125
6.9	Experimental Performance	126
6.9.1	Cryocooler Performance	126
6.9.2	Pressure Pulses and Mass Flows	129
6.9.3	Optimum Performance	132
6.9.4	Work Flows	132
6.9.5	PV Powers	134
6.10	Summary of Research Findings	137
7	A Novel Passive Displacer Design	139
7.1	Objectives	140
7.2	Theoretical Analysis of Warm End Displacer	141
7.2.1	Displacer Design	141
7.2.2	Thermodynamic and Harmonic Analysis	142
7.3	SPTC Experimental Setup	143
7.3.1	Instrumentation	143
7.3.2	Pressure and Laser Displacement Transducers	144
7.3.3	Damping Plate	145
7.3.4	Displacer Assembly and Full SPTC	145

7.4	Passive Displacer Experimental Performance	147
7.4.1	Testing Campaign	147
7.4.2	Displacer Resonant Frequency	147
7.4.3	Clamped Displacer Tests	149
7.4.4	No Load T_c	150
7.4.5	Experimental Displacer Resonance	150
7.4.6	Displacer Dynamics Experimental Validation	151
7.5	SPTC Experimental Performance at 80 K	155
7.5.1	Cooling Power and Carnot Efficiency	155
7.5.2	Displacer Stroke and Phase	158
7.5.3	Pressure Oscillations and Pressure Driving Forces	159
7.5.4	Harmonic Forces and Effect of Damping	160
7.6	Energy Recovery in Pulse Tube Cryocoolers	162
7.6.1	Pulse Tube Design Rationale	162
7.6.2	Comparison of Cryocooler Performance at 80 K	164
7.7	Summary	166
IV	Summary of Research Findings	170
8	Conclusions and Future Developments	171
8.1	Summary of Research Findings	171
8.1.1	Testing and Measurement Techniques for Cryocoolers	172
8.1.2	Coaxial Stirling Pulse Tube Cryocoolers with Warm End Displacers	173
8.1.3	Theoretical Analysis and Thermohydrodynamic Computation	174
8.2	Future Work	175
8.2.1	Component Testing	175
8.2.2	Coaxial SPTCs	176
	References	177
	Appendices	
A	Appendices	181

Nomenclature

AC		Alternating Current
CCD		Charge Coupled Device
COP		Coefficient of Performance
COMP		Compressor
CHX		Cold End Heat Exchanger
CPU		Central Processing Unit
DAQ		Data Acquisition System
DC		Direct Current
DFT		Discrete Fourier Transform
DISP		Displacer
EMF		Electromotive Force
ECSS		European Cooperation for Space Standardization
ESA		European Space Agency
FEA		Finite Element Analysis
FEMM		Finite Element Method Magnetics
FFT		Fast Fourier Transform
FS		Flow Straightener
Ge:X		Germanium detector (Ge_x)
GF		Gauge Factor
HDAQ		High Speed Data Acquisition
HEMT		High Electron Mobility Transistor
HTSSE		High Temperature Superconductivity Space Experiment
ISAMS		Improved Stratospheric and Mesospheric Sounder
LDAQ		Low Speed Data Acquisition
LVDT		Linear Variable Displacement Transducer
MMF		Magneto-motive Force
NASA		National Aeronautics and Space Administration
PCB		Printed Circuit Board
PID		Proportional, Integral and Derivative control
Pk-Pk		Peak to Peak

PV diagram	Pressure-Volume thermodynamic graph
PT	Pulse Tube
QWIP	Quantum Well Infrared Photodetector
RAL	Rutherford Appleton Laboratory
REG	Regenerator
RMS	Root Mean Square
SG	Strain Gauge
STJ	Superconducting Tunnel Junction detector
TC	Thermocouple
TS diagram	Temperature-Entropy thermodynamic graph
UARS	Upper Atmosphere Research
UV	Ultraviolet
WHX	Warm End Heat Exchanger

A	Geometric area (m^2)
A_o	Amplitude of oscillation (m)
C	Ratio of cumulative damage
C_c	Capacitance (F)
C_i	Inertial coefficient (m^{-1})
C_j	Viscous coefficient (m^2)
C_n	Compression force (N)
C_p	Specific heat capacity (J/kgK)
c	Damping coefficient
d_h	Hydraulic diameter (m)
d_w	Mesh wire diameter (m)
E_t	Total energy (J)
F	Force (N)
f	Frequency (Hz)
f_f	Friction factor
f_s	Tortuosity
I	Current (A)
k	Thermal conductivity (W/m K)
k_r	Spring radial stiffness (N/m)
k_z	Spring axial stiffness (N/m)

L	Characteristic length (m)
L_i	Inductance (H)
m	Mass (kg)
N	Number of turns in coil
N_i	Total number of cycles before failure
N_k	Axial-conductivity enhancement ratio
n	Sampling number
P	Pressure (Pa)
Q	Heat flow (W)
R	Resistance (Ω)
r_g	Clearance seal radial thickness (μm)
S	Entropy (J/K)
S_e	Instrumentation sensitivity (unit/V)
S_i	Stress at cycle i (N/m^2)
S_t	Moment source term (N/m^3)
T_C	Temperature at cold end (K)
T_H	Temperature at hot end (K)
T_n	Tension force (N)
t	Time (s)
u	Axial velocity (m/s)
V	Volume (m^3)
V_{EX}	Excitation voltage (V)
V_{RMS}	Root mean square voltage (V)
W_{in}	Input power (W)
W_{sh}	Shaft power (W)
x	Piston or spring stroke (m)
β	Mesh porosity
ε	Emissivity
ϵ_s	Strain
ζ	Damping ratio
η_r	Relative Carnot efficiency
μ	Viscosity (Pa s)
μ_0	Magnetic permeability (H/m)

ρ		Density (kg/m ³)
σ_{sd}		Standard deviation
σ_v		Von Mises stress (N/m ²)
σ_y		Yield strength (N/m ²)
$\bar{\bar{\tau}}$		Stress tensor (N/m ²)
ϕ		Phase angle (deg)
ψ		Exergy, available work (W)
Ω		Resonant frequency (Hz)
ω		Angular frequency (rad/s)

Part I

Background and Theoretical Treatment

1

Introduction

Contents

1.1	Motivation	2
1.1.1	Background	2
1.1.2	Space Telescopes	3
1.2	Thesis Layout	5
1.3	List of Published Works	6

1.1 Motivation

1.1.1 Background

A cryogenic cooling device is a single or group of equipment that refrigerates to temperature below 120 K [1]. The science of cryogenics is exploited in a broad range of areas, including; industrial liquefaction and separation processes, quantum effects and superconductivity, cryotherapy and cryosurgery, minimising thermal noise for camera imaging, and preserving biological material. The first historical use of refrigeration dates back to 1877, was shipborne refrigeration employing a Bell-Coleman steam driven air-cycle refrigerator for meat transportation from South America and Australasia to Britain [2]. In 1877, Cailletet and Pictet were then successful in achieving liquefaction of oxygen [3], and, after a further few achievements in liquefaction, of hydrogen and helium, low temperature superconductivity was discovered by Kamerlingh Onnes in 1911 [4], for which he received the 1913 Nobel Prize in Physics. In the late 1950s and 60s, open loop (Joule-Thomson coolers) and closed loop cryocoolers for detector cooling were developed, and in the late 1980s, the long life Stirling Cycle ‘Oxford Cryocooler’ was developed for the cooling of the ISAMS instrument (Improved Stratospheric and Mesospheric Sounder) on board the Upper

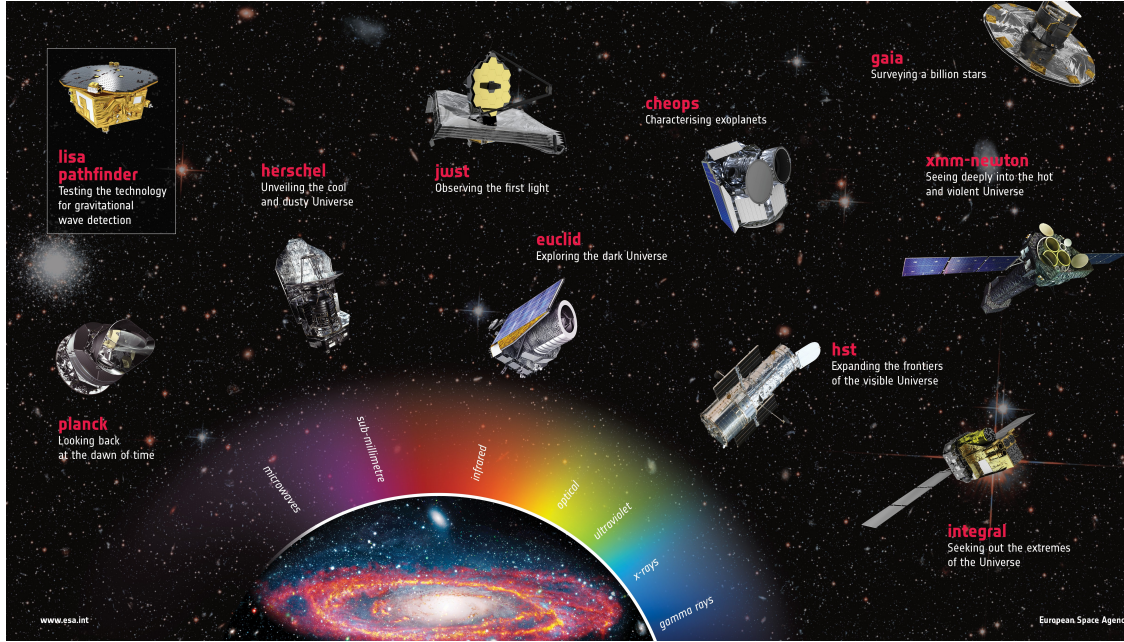


Figure 1.1: The ESA fleet of space telescopes that detect along the entire electromagnetic spectrum using cryogenic instruments [7]. From left to right, Planck, LISA Pathfinder, Herschel, the James Webb Space Telescope, Euclid, Cheops, the Hubble Space Telescope, Gaia (top far right), XMM-Newton (middle far right), and Integral (bottom far right).

Atmospheric Research (UARS) platform, which was then launched on the NASA Space Shuttle in 1991. This was achieved in a joint effort between the University of Oxford and the Rutherford Appleton Laboratory (RAL) [5].

1.1.2 Space Telescopes

Given that the space environment to which spacecraft are exposed is at a temperature of approximately 3 K^1 , it is noteworthy to highlight why active cooling through cryogenic technology may need to be utilised. Spacecraft coolers exist in two forms; passive and active. Passive cooling exploits the cold space environment by devising energy pathways within the spacecraft that enable heat to be rejected by radiators to the exterior of the spacecraft. Multi-Layer Insulation (MLI), a thermal blanket consisting of plastic films in honeycomb mesh, typically made of Kapton or Mylar, and an outer layer of reflective golden coloured aluminium [6], is further used to protect the spacecraft in its entirety from heat flow due to solar flux, infrared radiation or planetary albedo flux. In addition to these passive cooling methods, stored cryogenics are utilised to passively cool

¹Space is referred to as being at a temperature of 3 K . This is a rounded value of 2.73 K , which is the average measured temperature in outer space due to cosmic microwave background radiation that spreads throughout the universe.

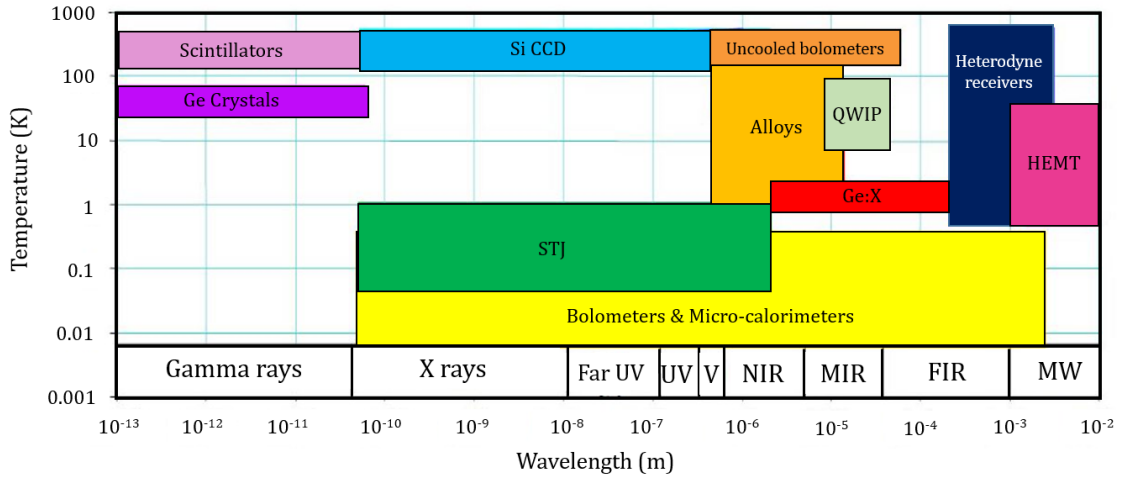


Figure 1.2: Overview of photon detectors and the temperature and wavelength at which they operate (adapted from [10]). V indicates visible light. NIR, MIR, FIR, and MW are near-infrared, mid-infrared, far-infrared and microwave, respectively. All other abbreviations are as previously stated.

further below the temperature achieved by radiators, through means of venting the effluent vapour produced by the liquid during heating. The issue with the latter is maintaining an insulated closed cycle, and undoubtedly this method has its lifetime limitations as well as posing as a significant added mass to the spacecraft [6].

In the space industry, operation of spacecraft instrumentation below 200 K is referred to as under cryogenic conditions [8]. This is difficult to achieve passively, and thus gives rise to the need for active cooling solutions. The need for cryogenics in a space environment is motivated by varying activities, such as focal plane cooling, boil-off mitigation of propellants, superconducting applications, and in the preservation of biological samples carried on spacecraft. Focal plane cooling is the predominant driving need for space cryogenic technology given its wide use in Earth Observation (EO) and scientific spacecraft missions. In EO, focal plane cooling is required for civil or military imaging, meteorology, land monitoring, and atmospheric chemistry. Focal plane cooling is necessary in detectors in order to reduce thermal noise and achieve a clearer detection signal [9]. Scientific missions employ focal plane cooling in endeavours focused on investigating black holes, exoplanet chemistry, gamma rays, and cosmic microwave background radiation. Spacecraft missions that have utilised the aforementioned include Rosetta, Planck, Herschel, and in more recent times, the Sentinel group of satellites dedicated to EO. The current ESA fleet for scientific missions is depicted in Figure 1.1. The specific detectors used at differing



wavelengths are depicted in Figure 1.2. It can be seen that operating temperature needed for detection covers a wide temperature range from 0.05 K to room temperature, with the majority of detectors requiring cryogenic temperature.

1.2 Thesis Layout

This thesis has been laid out in four parts. Part I is the background and theoretical study, consisting of this chapter, followed by chapter 2 and chapter 3. Chapter 2 gives an overview of the existing literature and fundamentals on both Stirling and pulse tube cryocooler systems. This starts with the various advantages and disadvantages of Stirling systems, after which the Stirling cycle is explained, and the various types of pulse tubes are then described, where different methods for generating the pressure pulse in the cryocooler are shown. Gaps in the field are identified and the objectives of this thesis are further elaborated. Chapter 3 presents the theoretical background for the operation of Stirling and pulse tube cryocoolers. Moreover, phasor analyses are conducted in which insights in the mass flow, pressure pulse, and phase angle between the two are mathematically explored. Further, mathematical expressions are derived for these relationships and a phasor diagram is constructed for the in-line SPTC with a warm end displacer.

Part II addresses the testing and measurement techniques explored in this thesis. Chapter 4 presents the work undertaken in testing flexure springs using the low cost flexure spring testing setup. The testing rig has been electromagnetically modelled and characterised, the springs have been modelled for stress using SolidWorks FEA, and the fatigue testing results are presented. The testing for a set of springs towards the ESA MREP programme is presented, where the resonant frequency is determined and the regions of highest stress are explored both by FEA and experimentally. Thereafter, a second series of spring testing is completed - in this case, the different springs tested are for use towards the passive displacer and vary in terms of batch number, spring arm design, and manufacturer. Chapter 5 follows and, keeping within the metrological genre, the work undertaken is in developing a novel measurement technique for in-situ measurement of cryocooler stroke using strain gauges. The strain gauges are used in a full Wheatstone bridge circuit and they installed onto the spring arms on the coordinates identified by the FEA as the region of highest stress. The strain gauge bridge circuit is then compared with an Omron laser



displacement transducer in order to compare the measurements and the frequency response. A series of dynamic and impact tests are conducted to assess the range of measuring ability and frequency response of the bridge configuration.

Part III delves into the research conducted on the hybrid Stirling pulse tube cryocooler in a coaxial configuration using a warm end displacer. Chapter 6 explores the coaxial SPTC using an active displacer, meaning, a motor is used at the warm end to run the displacer. In this chapter, the design of the SPTC is discussed, followed by details on the steps to developing and experimentally validating the numerical simulation using Sage. The numerical simulation is validated by conducting flow tests on the physical sub-component models that are the components that serve as the building blocks for the numerical simulation. The thermal modelling using ESATAN-TMS, in order to estimate the conductive and radiative losses, is exhibited. Fluid flow throughout the coaxial regenerator and other components is explored, where mass flow and pressure drop are analysed. The physical SPTC is then experimentally tested and the performance results are discussed. Insights into the work flows throughout the SPTC are exhibited and key findings are presented. The optimum case for the active displacer SPTC then served as inputs for the design of the passive SPTC. In chapter 7, the passive SPTC is discussed. This involves the harmonic design of the passive displacer, the experimental validation of the displacer dynamics at 250 K, and finally, the experimental testing at 80 K cooling. Lastly, a comparative analysis is conducted on the different designs and configurations for pulse tube cryocooler systems and their capacity for PV expansion power recovery. Finally, in part V, all research findings from this thesis work are summarised in the conclusions and future work is discussed.

1.3 List of Published Works

Below are the published works relating to the D.Phil. research undertaken, in reverse chronological order.

Peer Reviewed Publications

- **H. Rana**, M.A. Abolghasemi, M. Dadd, P. Bailey, R. Stone. "Energy recovery in pulse tube cryocoolers", (in preparation).



- **H. Rana**, M. Dadd, P. Bailey, R. Stone. "In-situ measurements of cryocooler stroke and frequency response using a strain gauge bridge circuit", (in preparation).
- **H. Rana**, M. Dadd, M.A. Abolghasemi, P. Bailey, R. Stone. "Stirling pulse tube cryocooler with a passive displacer", *Cryogenics*, (under review).
- **H. Rana**. "Pulse tube cryocoolers in European space missions", *Acta Astronautica*, 2022.
- **H. Rana**, M. Dadd, M. A. Abolghasemi, P. Bailey, R. Stone. "Experimental performance of a passively driven displacer", *Cryocoolers*, 2022.
- **H. Rana**, M. Dadd, P. Bailey, R. Stone. "Cryocooler compressor stroke measurements using strain gauges", *Cryocoolers*, 2022.
- **H. Rana**, P. Bailey, M. Dadd, R. Stone. "Experimental fatigue testing of cryocooler flexure springs", *IOP Conf. Ser. Material Sci. and Eng.*, 2022.
- **H. Rana**, M.A. Abolghasemi, R. Stone, M. Dadd, P. Bailey. "Thermal losses in a coaxial pulse tube cryocooler", *Cryocoolers*, 2021.
- **H. Rana**, M.A. Abolghasemi, R. Stone, M. Dadd, P. Bailey. "Design of a passive displacer for a Stirling pulse tube cryocooler", *Cryocoolers*, 2021.
- M.A. Abolghasemi, **H. Rana**, R. Stone, M. Dadd, P. Bailey. "Detailed analysis of a coaxial Stirling pulse tube cryocooler with an active displacer", *Cryocoolers*, 2021.
- **H. Rana**, M.A. Abolghasemi, R. Stone, M. Dadd, P. Bailey. "Numerical modelling of a coaxial Stirling pulse tube cryocooler with an active displacer for space applications", *Cryogenics*, 2020.
- M.A. Abolghasemi, **H. Rana**, R. Stone, M. Dadd, P. Bailey, K. Liang. "Coaxial Stirling pulse tube cryocooler using active displacer", *Cryogenics*, 2020.
- P. Bailey, **H. Rana**, G. Gilley. "Low cost flexure spring testing", *IOP Conf. Ser. Material Sci. and Eng.*, 2019.



Technical Reviews

- **H. Rana.** "From particle physics to astrophysics: a cryogenic solution for all", *Cold Facts*, Cryogenic Society of America, 2021.
- **H. Rana.** "Space cryocoolers can open doors in electric propulsion for applied-field magnetoplasma dynamic thrusters", *Cold Facts*, Cryogenic Society of America, 2022.

Books

- **H. Rana.** "Mathematical methods for cryocoolers", IOP Publishing, publish date 2022.

2

Stirling and Pulse Tube Cryocoolers: A Review

Contents

2.1 Stirling Cryocooler	9
2.1.1 The Stirling Refrigeration Cycle	10
2.1.2 Spaceflight Stirling Cryocoolers	11
2.2 Pulse Tube Cryocooler	13
2.2.1 The Pulse Tube	13
2.2.2 Stirling Pulse Tube Cryocoolers	16
2.2.3 Current Needs and Objectives	17

2.1 Stirling Cryocooler

There are many different types of coolers that achieve cooling to a cryogenic temperature. One of the most widely used and the first to be developed, the Stirling cycle cryocooler, is a reliable and efficient cryocooler. As of 2019, over 200,000 Stirling cryocoolers have been made [11]. Table 5.1 highlights some of the key advantages and disadvantages of Stirling cryocoolers.

Table 2.1: Advantages and disadvantages of Stirling cryocoolers.

Advantages	Disadvantages
High efficiency	No lubrication is permissible
Moderate cost	Intrinsic vibration from displacer
Small size & weight	Long lifetime so expensive

Achieving high efficiency operation in a Stirling cryocooler is the result of many carefully designed components, running a Stirling thermodynamic cycle. These are discussed in the sections below.



2.1.1 The Stirling Refrigeration Cycle

The ideal Carnot cycle cannot be employed in practice for cryogenic cooling. An unfeasibly high volume and pressure would be needed to generate the necessary amount of refrigeration for the significant temperature range of the gaseous working fluid in a Carnot cycle system [12]. The Stirling cycle is largely similar to the Carnot cycle, with the key difference lying in the replacement of the isentropic processes with regenerative heat transfer processes at constant volume. The regenerator is essentially a form of thermodynamic ‘sponge’ that retains and rejects absorbed heat, which can typically be made of metal wire meshing. The working gas used is typically helium [11].

The four steps of the Stirling cooling cycle are shown in Figure 5.1. The system consists of two pistons working on gaseous chambers in conjunction with one another, and are connected by a regenerator.

- *Step (a) to step (b)*: The right piston remains fixed whilst the left piston moves to the right in an isothermal compression process. Heat, Q , is given off to the surroundings.
- *Step (b) to step (c)*: Both pistons shift to the right in an isochoric process causing the hot working fluid to pass through the regenerator, transferring heat energy.
- *Step (c) to step (d)*: The expansion piston moves to the right in an isothermal expansion process. Heat is taken up from the surroundings during this stage as a result of the isothermal expansion; this is the useful cooling power achieved in the cycle.
- *Step (d) to (a)*: The right piston now moves back inwards and the left piston moves outwards forcing the working gas through the regenerator. The heat stored in the regenerator is recovered by the gas, causing it to return to the starting state at temperature T_H .

Figure 5.1 (right) shows the P - V and T - S diagram. It is important to note that the temperature gradient in the isochoric processes between (b) to (c) and from (d) to (a) are the same. The heat that flows in and out of the regenerator are equal in quantity due to the fact that a fixed working fluid

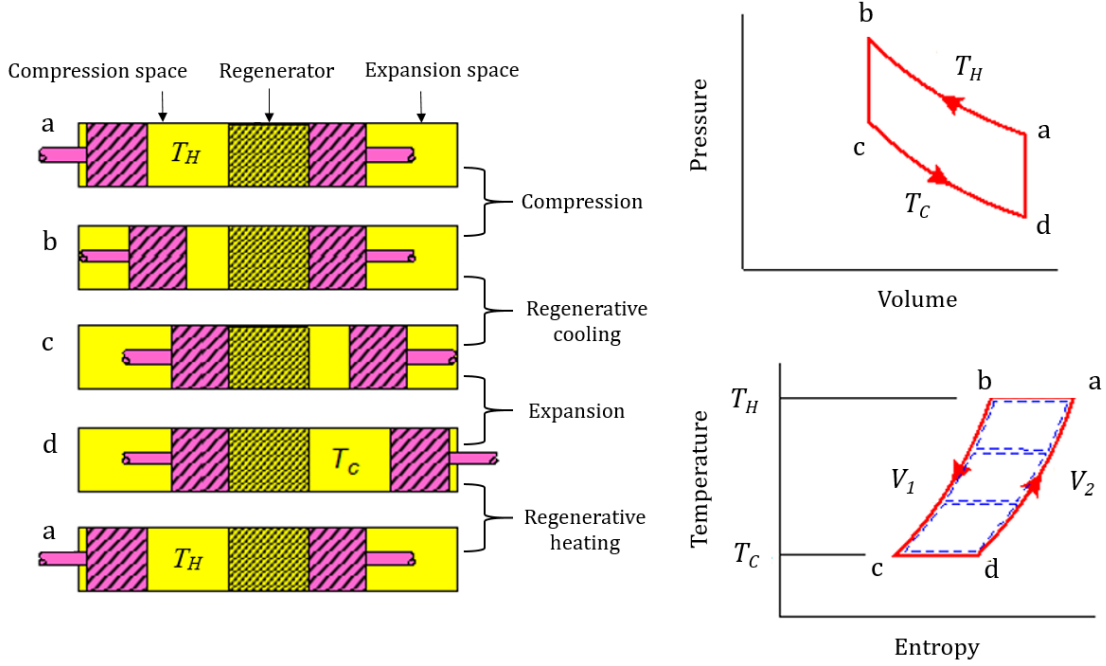


Figure 2.1: A Stirling cycle cooler system (left) indicating the piston movement in the compression and expansion spaces, and the regenerator in between the two. The P - V and T - S diagrams are shown (right) (adapted from [11]).

mass is being used in the system. Furthermore, the replacement of the two isentropic processes featured in the Carnot cycle with regenerative isochoric processes in the Stirling cycle result in a higher amount of work completed for the same thermal efficiency. As the processes in the Stirling cycle are all reversible, and the only external heat exchange is isothermal (at T_H and T_C), the performance is the same as the Carnot cycle. The additional amount of heat transferred in a Stirling cycle increases with the same proportion as the increase of work done [12].

$$COP = \frac{\text{heat lift}}{\text{work done}} = \frac{T_C}{T_H - T_C} \quad (2.1.1.1)$$

The coefficient of performance for a refrigerator, COP, is the ratio of heat extracted to work done and is given by Equation 2.1.1.1. This is the same for a Carnot cycle and is the maximum possible COP that a refrigeration cycle operating between certain temperature limits can achieve [1].

2.1.2 Spaceflight Stirling Cryocoolers

After the ISAMS cryocooler development in the 1980s, successful attempts were made to transfer the Oxford University technology to industry. Over the last 20 years, Stirling cycle cryocoolers



have evolved greatly, with roughly 4 identifiable categorical developments.

1. 80 K coolers: these coolers were the first widely used in space missions, and 4 batches were utilised. The cold tip temperature stability achieved was ± 50 mK, proven to be maintained for 1.5 years on the High Temperature Superconductivity Space Experiment (HTSSE-II) [13].
2. 50-80 K coolers: the achievable range of cooling was lowered, where modifications to achieve this lay in altering the displacer cold finger size, notable optimisation design to the compressor drive motor, and a higher volume covered by the compressor [14]. These coolers remain in use, with the latest batch having been used on the Sentinel-3 project. The first 50-80 K cooler pair, made by Matra Marconi (now Airbus DS), was flown on NASA's Measurement of Pollution in the Troposphere (MOPITT) instrument on board the Terra spacecraft in 1999. The displacer piston in Cooler B failed, a conclusion that was eventually reached with the use of indirect telemetry data [15]. Figure 2.2 displays the telemetry data for temperature of one of the detectors, where sharp temperature variation can be seen prior to the 2001 interruption. This highlighted the importance of stroke measurement technology for cryocoolers.
3. 10-20 K coolers: these are now widely developed throughout the international cryogenic market [16].
4. Small scale cryocoolers: these form part of the latest initiatives, and are being developed by RAL Space UK and Honeywell Hymatic, weighing less than 1 kg cooling to 77 K with under 25 W input of electrical power [17].

Amongst all the Stirling coolers, strong temperature stability has been achieved, where both stroke-governed and temperature driven approaches have been demonstrated. The key parameters assessed during flight design of Stirling coolers include achievable temperature range for cooling, cold tip stability, cold tip heat lift removal, compressor/displacer stroke, reduced cryocooler vibration, and ultimately, efficiency. It is apparent that compressor/displacer piston movement

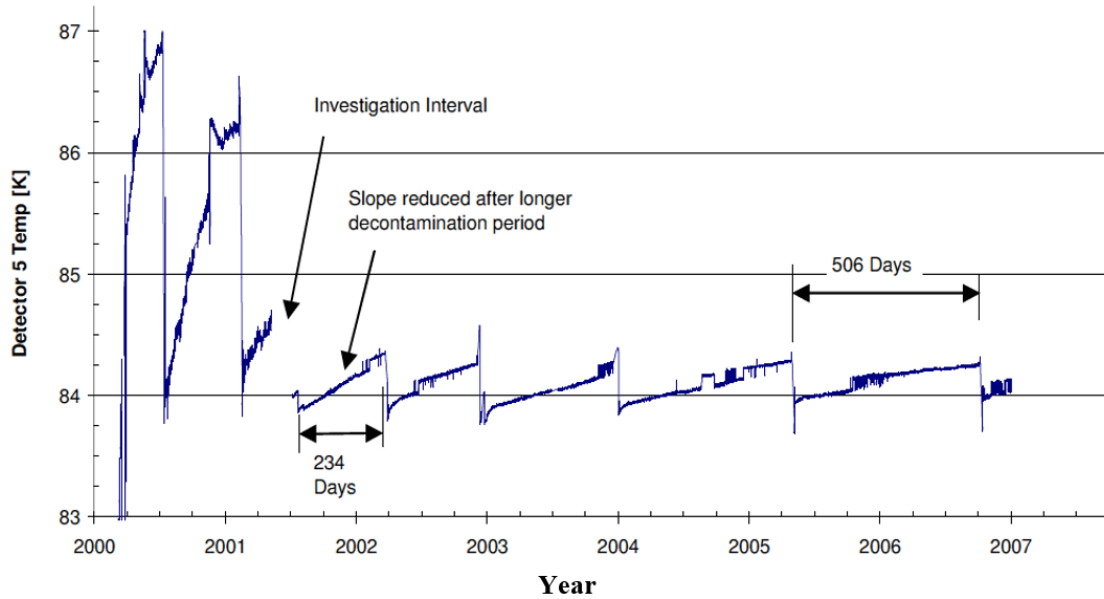


Figure 2.2: Telemetry data for detector 5 temperature for MOPITT on board the TERRA spacecraft. The interruption for cooler investigation can be seen in 2001 (adapted from [18]).

becomes compromised in most instances of failure. It is therefore of the utmost importance that techniques for reliably assessing and quantifying stroke are enhanced.

2.2 Pulse Tube Cryocooler

2.2.1 The Pulse Tube

The progress achieved in the development of Stirling cryocoolers has permitted high efficiency cooling of infrared sensors for space applications and otherwise. The reduction of thermal noise has been achieved and Stirling cryocoolers have been widely used. A further issue, however, of vibration at the cold end arises due to the motor required for the displacer in the Stirling design. The introduced vibrations interfere with detector signals and thus require elimination. This issue gave rise in the 1960s to the design of the pulse tube cryocooler, and by the 1980s, the pulse tube cryocooler became a practical alternative to Stirling cryocoolers [11]. A pulse tube cryocooler operates with no moving parts at the cold end, unlike the Stirling cryocooler which utilises a displacer, and is driven by a variety of pulse-generating systems. These are depicted in Figure 2.3.

In Figure 2.3, type (a) is a valveless continuous pressure oscillator, which operates in Stirling systems as described in section 4.1.2, and image (b) depicts an orifice pulse tube cryocooler driven by such. The pressure oscillator in (c) is a valved compressor running a recuperative cryocooling

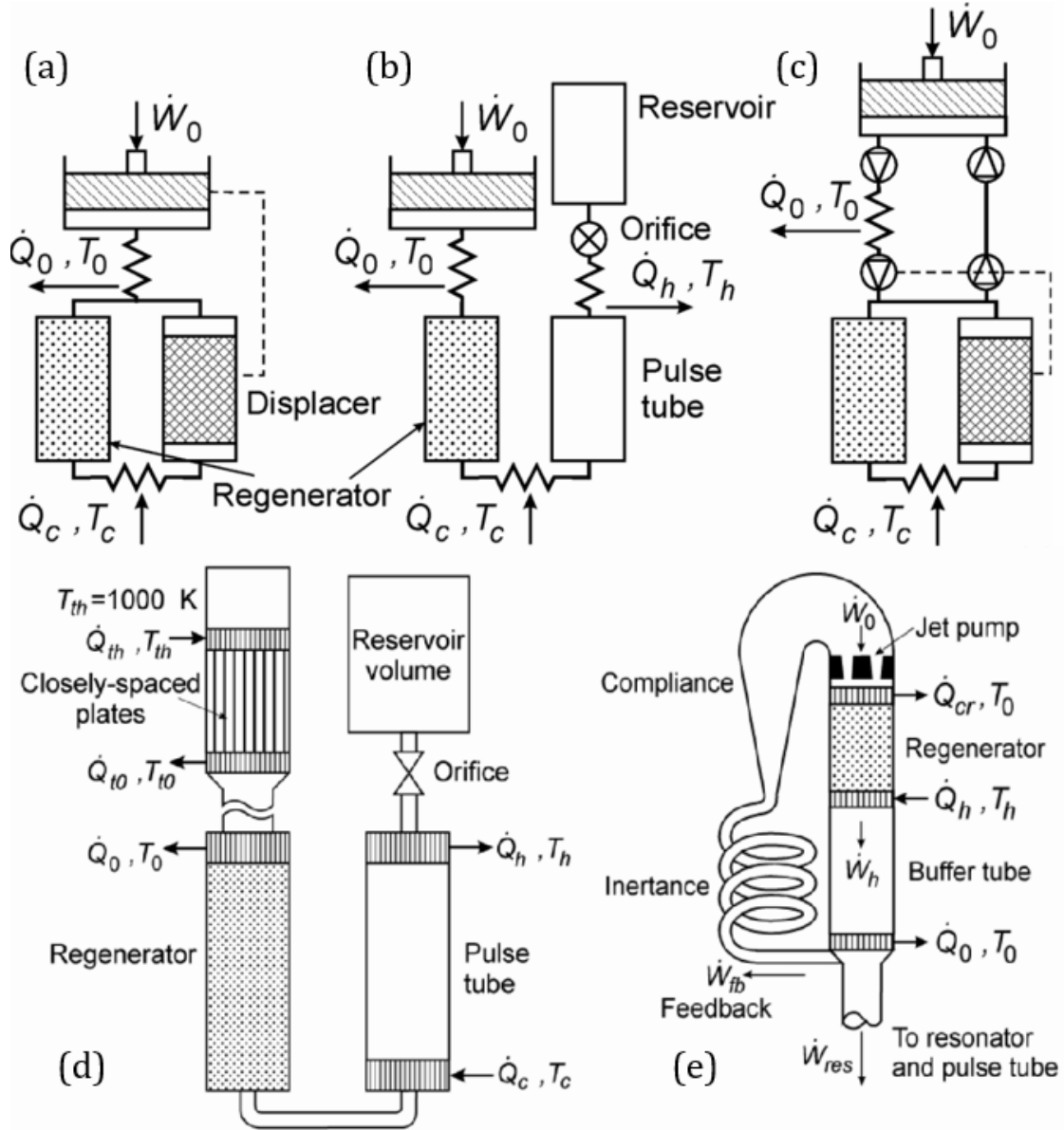


Figure 2.3: The different types of pressure oscillators that can be used to drive a pulse tube cryocooler. (a) depicts a valveless compressor (Stirling-type) driving a Stirling cryocooler design, (b) is a orifice pulse tube being run by a valveless compressor as shown (a), (c) is a valved compressor running a Gifford-McMahon cryocooler, (d) is a thermoacoustic driver (TAD) attached to an orifice, and (e) is a thermoacoustic Stirling heat engine (TASHE) [11].

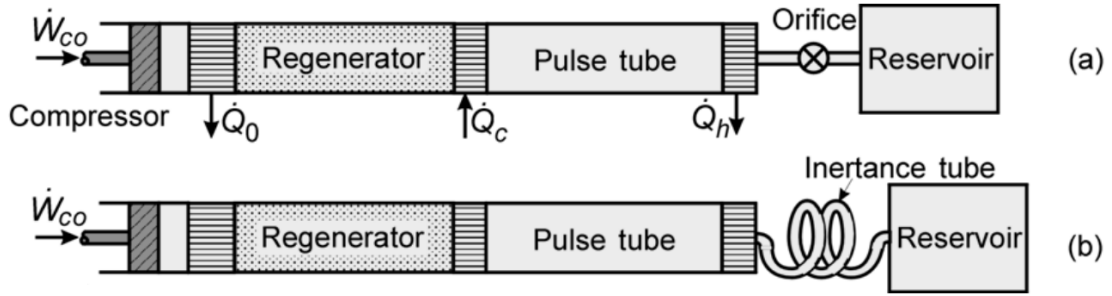


Figure 2.4: (a) The orifice pulse tube cryocooler and (b) inertance tube pulse tube cryocooler designs (adapted from [19]).

system, and cryocoolers that employ such a pressure oscillator are known as a Gifford McMahon cryocoolers. The pressure wave is generated by use of a rotary valve that alternates between low and high pressure sources, and the pressure pulse is generated in a DC fashion. The thermoacoustic driver (TAD) orifice pulse tube shown in Figure 2.3 (d) generates pressure oscillations created as a result of a large temperature gradient over closely-spaced plates. One plate is heated to a very high temperature, usually around 1000 K, whilst the other is kept at ambient temperature. This generates spontaneous acoustic oscillations which are carried forward into the pulse tube to drive it. This was the first cryogenic refrigerator with no moving parts [11], and reaches a no-load temperature of 90 K. Similarly, the thermoacoustic Stirling heat engine (TASHE) pressure oscillator presented in (e) operates by creating a similar magnitude temperature gradient, with a hot temperature of 900 K, and a regenerator used in place of closely-space plates. The acoustic power (or PV power) is amplified due to the increased specific volume of gas at the higher temperature. A TASHE can produce 3 kW of input power at the hot end, a portion of which is then fed back into the ambient end of the TASHE regenerator [11] while the remainder drives the pulse tube. Whilst no moving parts are desirable in spacecraft missions, the vastly high temperatures at the TAD and TASHE oscillator end is difficult to achieve in a confined manner, and the purpose of the TAD and TASHE is to be used for natural gas liquefying on an industrial scale. Hence, further research is required to develop suitably-sized cryocoolers with no moving parts at the cold end operable for long-life space missions.

The two key designs for the pulse tube cryocooler are shown in Figure 2.4. In the orifice pulse tube, the orifice has a similar function to the displacer in the Stirling cryocooler. As the



pressure difference across the orifice fluctuates continuously as a result of the compressor (or pressure oscillator), gas flow within gas parcel local to the orifice enters in and out of the pulse tube warm end/reservoir. The inertance tube shown in (b) utilises the inertia of the oscillating flow in order to create a phase difference between the mass flow and pressure pulse throughout the system [11]. An optimal phase difference improves cooling performance and efficiency of the cryocooler. Originally design (b) featured an orifice fitted in the same location as in design (a), however, this was eventually removed in design (b) to avoid any DC flow. In both designs, the pulse tube must be large enough to ensure that the warm end and cold end gaseous parcels are essentially insulated from one another, but constricted enough to ensure stratified flow. This is addressed in greater detail in chapter 7.

2.2.2 Stirling Pulse Tube Cryocoolers

Stirling pulse tube cryocoolers (SPTCs) unite the established technologies of Stirling cycle refrigerators with pulse tubes in a hybrid fashion. As discussed, in traditional pulse tube cryocoolers, orifices and inertance tubes are used at the warm end to achieve the required phase difference between mass flow and pressure pulse in order to drive the thermodynamic cycle within the cryocooler [11] [20] [21]. These methods do not allow for the phase difference to be readily adjusted amid operation, and as such, an active displacer at the warm end of the cryocooler can be utilised to permit fine-tuning of this phase interplay [22] [23]. Thus, in a hybrid SPTC, the use of a warm end displacer can be exploited to create this phase difference and achieve a higher performance efficiency [24]. Moreover, expansion power that would otherwise be lost at the warm end of a traditional pulse tube system is recovered in an SPTC through the use of the warm end displacer, further enhancing the performance and efficiency [25]. At present, the SPTCs developed are large systems that cater to large cooling power requirements through means of very high power inputs in the range of 200-300 W [26]. These are undoubtedly unsuitable for spacecraft missions with limited power supply and system integration space on-board. A smaller system requiring a power input of 69 W total shaft power SPTC has previously been developed and tested in the Oxford Cryogenic research group with an in-line design. This cryocooler was able to cool to 80 K while achieving a cooling power of 3.8 W with a 28 bar fill pressure, and a displacer phase, ϕ_d , of 40° [24].



2.2.3 Current Needs and Objectives

Coaxial Configuration

A key element that requires development is reconfiguring the hybrid SPTC technology into a coaxial arrangement. Being able to achieve this would pose many benefits. A coaxial configuration entails fitting the pulse tube inside the surrounding regenerator, with the cold head serving as a region for flow reversal within the local gas parcels into either component. Designing the SPTC system in a coaxial manner allows for better access and integration of the cold head to the focal plane assemblies of detectors and instruments [27] [28]. Furthermore, this configuration presents the added advantage of reducing dead volume of the cryocooler; a feature which is particularly of benefit for spaceflight design given that mass, power, and configuration budgets are limited. Hence, coaxial configurations are considered a more compact design choice that novel technologies must aspire towards in terms of development [29].

Undoubtedly, with this configuration change comes a range of physical challenges. One key challenge that requires careful consideration is the impact that the coaxial integration of the regenerator, pulse tube, and cold head has on the fluid flow throughout the system. This in turn impacts the heat transfer and thus overall cooling performance of the SPTC. A particularly noteworthy issue is the difficulty to predict the extent to which fluid mixing occurs at the coaxial cold head and the degree to which this influences enthalpy transfer in the regenerator and entropy transfer in the pulse tube [30].

Passive Displacer

Whilst the literature indicates that SPTCs running an active displacer demonstrate good performance and higher efficiencies than pulse tube cryocoolers that utilize orifices, using an active displacer introduces a second phase from the power electronics into the system and involves a more complicated design in order to incorporate the second motor or generator. Moreover, an active displacer introduces additional vibrations at the cold end which are highly undesirable, considering the increasingly stringent vibration tolerances for the devices being cooled. It is therefore desirable to aim to reduce the mass and the number of additional moving parts, as well as simplify the cryocooler design. This improves reliability and longevity of the cryocooler. Inertance tubes can



be utilized to achieve this, however, they do not recycle power back into the cryocooling system. A passively driven displacer is therefore a suitable solution to address each of these issues.

Thesis Objectives

This thesis sets out to address the following pressing needs in SPTC development to propose suitable cryocooling technology for space science missions:

- Developing a coaxial configuration design for a SPTC with an active displacer at the warm end. As discussed, this would serve as a more compact design that integrates into spacecraft more effectively and allows for better access to the cold head for detector integration. This includes conducting a 1D numerical simulation using Sage software to predict and optimise the design performance of the sub-components as well as the overall active displacer SPTC. These objectives are addressed in chapter 6.
- Moreover, the physical build and experimental testing of the coaxial pulse tube assembly is required following the design phase. The experimental performance in terms of key parameters such as cooling power and efficiency need to be ascertained. This further allows for validation of the numerical simulation. These efforts are presented in chapter 6.
- The flow and thermal performance of the coaxially-configured pulse tube assembly requires investigation to better understand the underlying physics influencing the thermodynamic performance of the experimentally built and tested coaxial SPTC with an active displacer. Additional sensitivity studies, both numerical and experimental, permit identifying the parameters of influence on components of the SPTC and shedding light on insightful and exploitable performance factors. The latter allows for the passive displacer to be designed in order to address the need for passively-run warm end displacers. These objectives are addressed in chapter 6.
- The need for eliminating additional vibrations introduced into the cold end as well as the second phase in the power electronics can be tackled through the use of a passive displacer. The design and simulation of the passive displacer is explored in chapter 7, using necessary



inputs from the conclusions from the studies conducted on the active displacer SPTC of the same dimension and broader assembly.

- The building and testing of a passive displacer SPTC in a coaxial configuration is required to ascertain its experimental performance. The ability of the warm end displacer to effectively recover PV power at the warm end back into the system needs to be investigated. These objectives are explored in chapter 7.

Furthermore, the need for further in-situ measurement methods and advancement of instrumentation for space SPTCs is necessary. The following additional objectives in this domain are explored in this thesis as below:

- Advancing the understanding of the inner workings of an SPTC using a warm end displacer through phasor analyses allows for mathematical relationships on the phase between the pressure pulse and mass flow, allowing for the cooling power and efficiency of the SPTC to be readily obtained. This phasor study is conducted in chapter 3.
- The designing and testing of the harmonics and fatigue life of various flexure springs for the passive displacer must be completed for a suitable design and operation. The spring activity affects the damping, and hence, motion of the displacer which in turn impacts the thermodynamic performance of the overall SPTC. Thus, the springs must be carefully designed and a suitable fatigue testing method must be developed and experimentally assessed. Moreover, determining the fatigue limit of the spring sets the stroke limit for displacer activity and recommends the SPTC for long lifetime missions. These objectives are addressed in chapter 4.
- Lastly, a novel method for in-situ cryocooler measurement stroke needs to be developed to address the setbacks presented by the use of Linear Variable Displacement Transducers (LVDTs). The design, testing and suitability of a method using strain gauges is explored in chapter 5.

3

Theoretical Treatment

Contents

3.1	Background	20
3.2	Modelling Rationale	21
3.3	Energy Flows	22
3.3.1	Enthalpy and Entropy	22
3.3.2	Work and Exergy Flows	23
3.4	Phasor Analysis	25
3.4.1	Pressure and Mass Flow Phase	25
3.4.2	Mass Flow at the Pulse Tube Cold End	25
3.4.3	SPTC Component Phasors	28
3.4.4	Effect of Phase Angle	30
3.4.5	Comparative Phasor Analysis of PTC Configurations	31
3.5	Summary	33

3.1 Background

The pulse tube cryocooler is a complex system that exploits a variety of physical phenomena in order to achieve cryogenic cooling. As shown in the previous chapters, the parameters affecting the thermodynamic performance of a PTC are multifactoral and often interplay closely with one another. Devising suitable strategies to simulate the cryocooler is vital to ensuring a reliable design of the physical build and acquiring a means for validating the experimental results. Furthermore, it has already been highlighted how crucial it is to achieve the correct phase difference between the compressor and displacer; moreover, it is paramount that the mass flow and pressure pulse also be maintained within the correct phase for an efficient cooler performance. Developing a framework to mathematically express these relationships enables light to be shed on how exactly



each parameter affects the other and how better designs for PTCs can be achieved as a result. In this chapter, the modelling rationale for a coaxial SPTC with a warm end displacer for energy recovery is outlined. Thereafter, the relevant mathematical and theoretical frameworks for pulse tube cryocooler phenomena are presented, and ultimately, how they can be tailored to represent the Stirling pulse tube cryocooler system is discussed. The factors affecting an SPTC performance such as mass flow, pressure pulse, phase angle, instantaneous thermodynamic behaviour in the regenerator and pulse tube, energy flows, and loss terms that degrade the thermodynamic efficiency of an SPTC are explored.

3.2 Modelling Rationale

In literature, efforts have been made to find suitable methods to mathematically describe and represent the thermodynamic, hydrodynamic, harmonic and mechanical activity within the pulse tube [11] [20] [31] [32] [25] [33]. Much of this is achieved by completing phasor analyses which allows for the sinusoidal behaviour of oscillatory pressure and mass flows to be manipulated with ease [31] [34]. In this way, this allows for expressions for the cooling power and efficiency of a specific system to be readily obtained. Thereafter, 1D modelling using Sage can be exploited to determine the heat transfer between components within the SPTC modelled as nodes. This allows for the design and simulation of the thermodynamic performance of the cryocooler, as well as an understanding of the energy flows within the system. Temperatures, pressures and mass flows are required at warm and cold ends of the system as boundary conditions in order to solve for the SPTC performance, hence these simulations are heavily dependent on empirical correlations and experimental inputs. A further limitation to the 1D Sage modelling is the lack of fluid flow effects modelling, such as mixing at the coaxial cold end, and the inaccurate estimation of thermal losses due to the difficulty in simulating complex SPTC geometry in a 1D nodal system. Hence, ESATAN-TMS can be used to explore the thermal losses, where 2D shells are modelled to assess conductive links and radiative view factors between surfaces. To address the fluid mixing effects at the coaxial cold end, 3D CFD simulations would be required. In this thesis, the mathematical analysis was conducted through phasor analysis, the thermodynamic performance was investigated through 1D Sage modelling, and the thermal losses were verified using ESATAN-TMS 2D modelling. Further simulations were completed using SolidWorks to conduct FEA studies of stress and stiffness on



the flexure springs and strain gauge setups, and FEMM software was used to electromagnetically model the fatigue testing setup up in order to ascertain the force-motor constant of the moving magnet motor.

3.3 Energy Flows

3.3.1 Enthalpy and Entropy

The range of complex components working within the SPTC demonstrate differing thermodynamic behaviour. Namely, the regenerator can be assumed to behave as a perfectly isothermal element, meaning that the temperature does not change with time at any given location [31]. The pulse tube is considered a perfectly adiabatic element, in that there is no heat transfer between the gas and the pulse tube wall. Given the following equation for time-averaged enthalpy flow, $\langle \dot{H} \rangle$:

$$\langle \dot{H} \rangle = \frac{1}{\tau} \int_0^\tau \dot{m} h \, dt = \frac{1}{\tau} \int_0^\tau \dot{m} c_p T \, dt \quad (3.3.1.1)$$

where τ is the time constant and all other variables are as previously defined, it can be seen that in the isothermal regenerator for an ideal gas, the enthalpy flow will equal zero whenever $\langle \dot{m} \rangle$ equates to zero, given that c_p and h are independent of pressure [31]. As entropy, s , is a function of pressure at a constant temperature, this means $\langle \dot{S} \rangle$ is finite in the isothermal regenerator, however, since there is no time-averaged heat flow to the system in an ideal regenerator, $\langle \dot{S} \rangle$ is constant throughout the regenerator.

Inversely, in the ideal adiabatic pulse tube, s is independent of time due to there being no heat transfer resulting in any changes. This is evident from the following equation for time-averaged entropy as following:

$$\langle \dot{S} \rangle = \frac{1}{\tau} \int_0^\tau \dot{m} s \, dt \quad (3.3.1.2)$$

Hence, $\langle \dot{S} \rangle$ is zero in the adiabatic pulse tube. $\langle \dot{H} \rangle$ will be constant throughout the pulse tube provided there is no work done along it, as $\langle \dot{Q} \rangle$ is zero. These conditions for energy flow in the regenerator and the pulse tube can be summarised as given in Table 3.1.

**Table 3.1:** Enthalpy and entropy behaviour of ideal isothermal and adiabatic elements.

Thermodynamic Behaviour	Enthalpy	Entropy
Ideal isothermal element	$\langle \dot{H} \rangle = 0$	$\langle \dot{S} \rangle = \text{constant}$
Ideal adiabatic element	$\langle \dot{H} \rangle = \text{constant}$	$\langle \dot{S} \rangle = 0$

3.3.2 Work and Exergy Flows

Figure 3.1 shows how the energy flows through a pulse tube cryocooler with an orifice, where the top graph is for an ideal system and the lower graph indicates the energy flows for a real system with losses [31]. $\langle P\dot{V} \rangle$ is the PV power flow and is given by the following:

$$\langle P\dot{V} \rangle = \langle \dot{H} \rangle - T_a \langle \dot{S} \rangle \quad (3.3.2.1)$$

In Figure 3.1, at the compressor a introduction of PV power flow is brought into the system as the piston moves to the right, which results in an oscillating mass flow and pressure within the helium working fluid. This results in enthalpy, entropy and exergy, $\langle \dot{E}_x \rangle$ flows. Hence, the graph shows how $\langle P\dot{V} \rangle$ power travels along the regenerator as a result, where $\langle \dot{H} \rangle$ is zero and $\langle \dot{S} \rangle$ remains constant throughout in the ideal system, as has been previously shown. The entropy flow travels 'down' the regenerator to the left, and is negative based on the sign convention for this analysis, where energy flow to the right is positive. This is also consistent with Eqn. 3.3.2.1, whereby $\langle P\dot{V} \rangle = -T_a \langle \dot{S} \rangle$ given that the enthalpy is zero. Similarly, as shown the inverse conditions are true for the ideal pulse tube, and hence, the enthalpy in the pulse tube is $\langle P\dot{V} \rangle = \langle \dot{H} \rangle$ and is constant throughout the pulse tube, where there is no heat or work input. This is what enables the pulse tube to serve its unique purpose in the cryocooler, whereby PV power can be transferred across a large temperature gradient without changing value, hence serving as a 'PV work transfer tube'. This also means that the heat absorbed at the cold end heat exchanger and the heat rejected at the warm end heat rejector are equal to $\langle \dot{H} \rangle$ in the pulse tube, as expressed by Eqn. 3.3.2.2 below based on the second law of thermodynamics [31].

$$\frac{\langle \dot{Q}_h \rangle}{T_h} = \frac{\langle \dot{Q}_c \rangle}{T_c} \quad (3.3.2.2)$$

In the lower graph, entropy generation is seen due to the losses in a real system. The losses therefore require a larger input power from the compressor, hence the higher $\dot{W}_{co}$ input compared to the ideal case and increased magnitudes of other energy flow terms. This also means that there

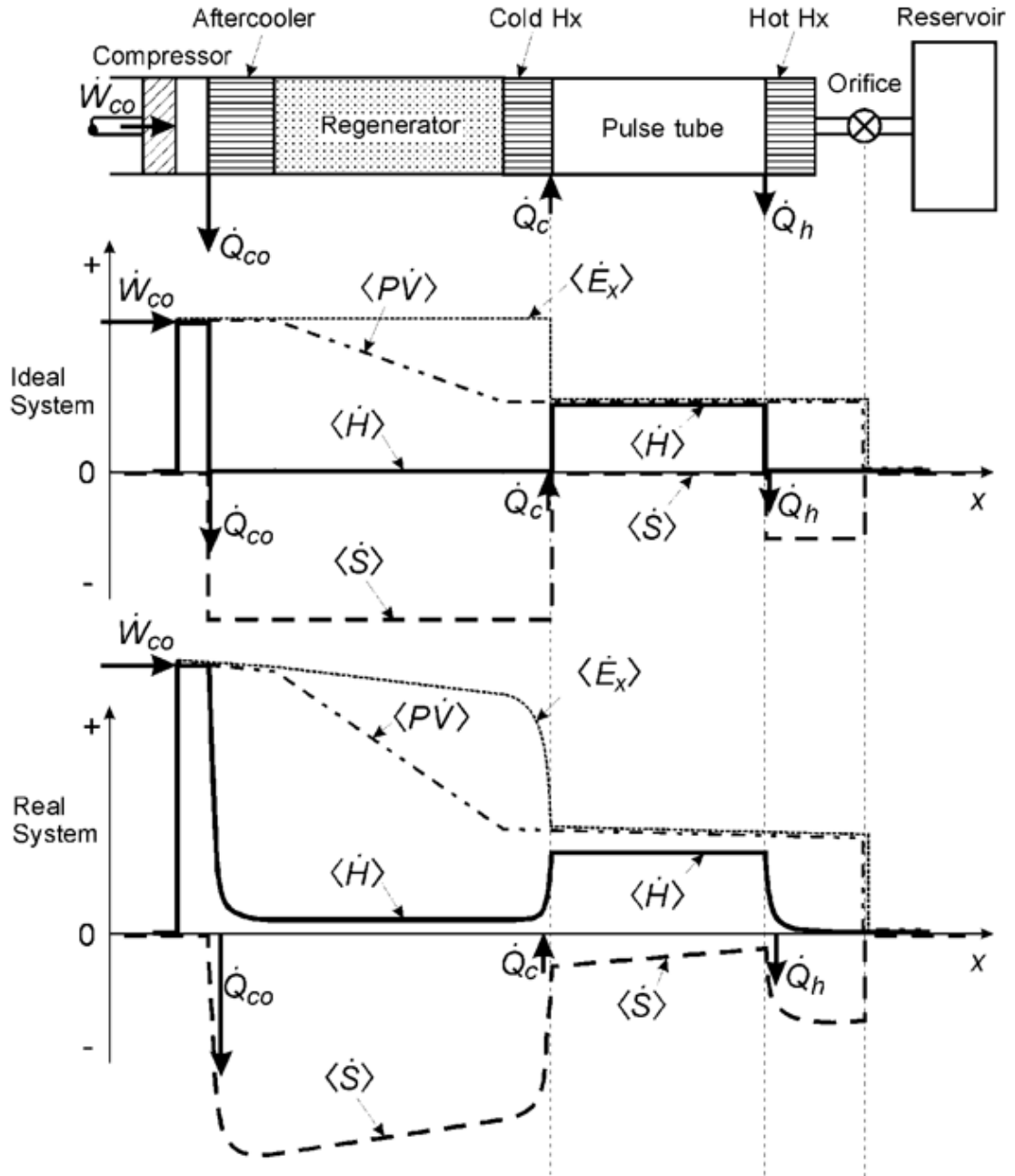


Figure 3.1: The enthalpy, entropy, heat, work, and PV power flows in an ideal and real orifice pulse tube cryocooler, where positive flow is to the right (adapted from [31]).

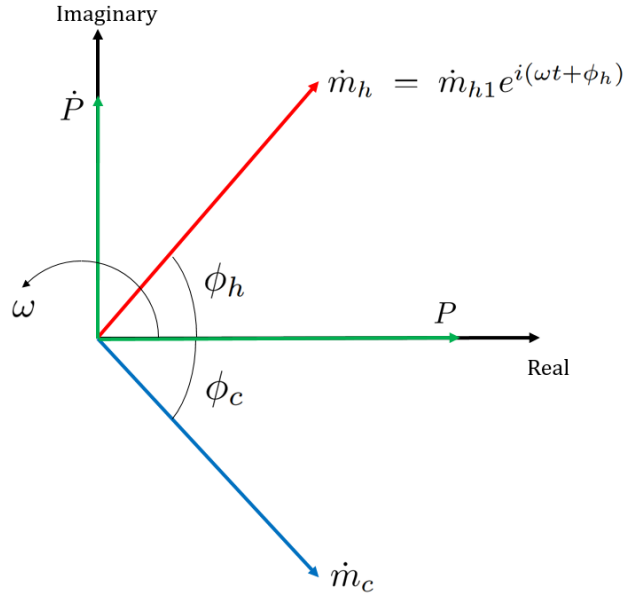


Figure 3.2: Mass flow at the warm end and cold end expressed as phasors against pressure in the real and imaginary axes.

is irreversible entropy generation at the pulse tube hot end heat exchanger. This demonstrates the benefit once again of a warm end displacer that is able to recover PV power and feed it back into the system as available work.

3.4 Phasor Analysis

3.4.1 Pressure and Mass Flow Phase

The key characterising physical components in the pulse tube, pressure and mass flow, can be expressed as a complex number consisting of a complex and real part due to their oscillatory operation in the cryocooler. These can be plotted as phasors on a phasor diagram, as shown in Figure 3.2. Given that the pressure in the cryocooler experiences a smaller phase change ($5-10^\circ$) compared to the mass flow (anywhere between $50-90^\circ$), pressure is used as the real axis. The imaginary axis thus becomes the first derivative of pressure, and so on. The phasor diagram expressing mass flow at the cold and and warm end is shown in Fig. 3.2.

3.4.2 Mass Flow at the Pulse Tube Cold End

Taking first the case of an in-line pulse tube cryocooler using an orifice at the warm end where the displacer would be situated, where helium is the working fluid, the pressure pulse $P(t)$ can



be written as given in Eqn. 3.4.2.1, where P_0 is the average pressure, A_p is the amplitude of the pressure wave, and all other terms are as previously defined.

$$P(t) = P_0 + A_p \cos(2\pi ft) \quad (3.4.2.1)$$

Similarly, the change in mass in the pulse tube can be written as follows, where dm_h and dm_c are the change in mass at hot end and cold end of the pulse tube, respectively [34]:

$$dm_{PT} = dm_h - dm_c \quad (3.4.2.2)$$

The mass flow at the hot end where the orifice is situated is given by:

$$\dot{m}_h(t) = C_{or} \cdot \Delta P(t) \quad (3.4.2.3)$$

where C_{or} and $\Delta P(t)$ are the flow conductance of the orifice, which is determined as the ratio of mass flow to the pressure drop, and the pressure difference across the orifice, respectively. For an adiabatic process using helium as a monatomic ideal gas, the following thermodynamic relationships hold true and can be utilised [35]:

$$P \cdot dV = RT \cdot dm \quad (3.4.2.4)$$

$$P \cdot V_{PT}^\gamma = \text{constant} \quad (3.4.2.5)$$

$$\left(\frac{T}{T_0}\right) = \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}} \quad (3.4.2.6)$$

$$c_p = \left(\frac{R\gamma}{\gamma-1}\right) \quad (3.4.2.7)$$

where R , γ , V_{PT} , and c_p are the ideal gas constant, the ratio of specific heat for helium, the pulse tube volume, and the specific heat capacity of helium for constant pressure. T_0 and P_0 are the average temperature and pressure states. Differentiating Eqn. 3.4.2.5 gives:

$$V_{PT} \cdot dP + \gamma P \cdot dV_{PT} = 0 \quad (3.4.2.8)$$

Substituting this and Eqn. 3.4.2.4 into Eqn. 3.4.2.2 now allows for an expression for the change in mass flow at the cold end to be obtained:

$$\frac{dm_c(t)}{dt} = \frac{V_{PT}}{\gamma R T_c} \frac{dP(t)}{dt} + \frac{T_h}{T_c} \frac{dm_h(t)}{dt} \quad (3.4.2.9)$$

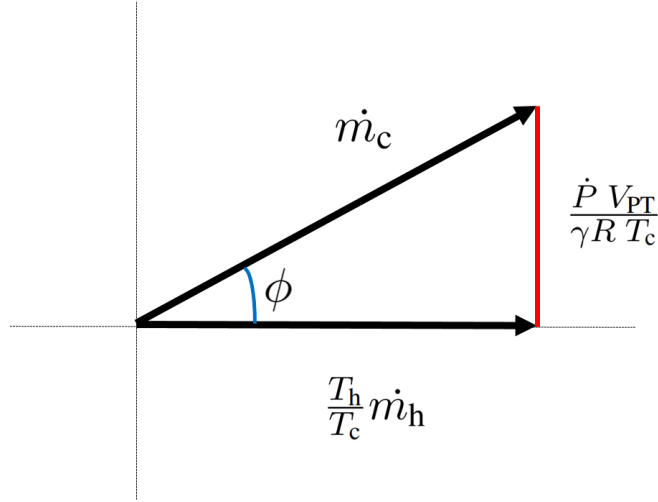


Figure 3.3: Phasor diagram showing the mass flow rates at the cold end and hot end of the pulse tube. The mass flow rate at the cold end is the sum of the two vectors shown and as given by Eqn. 3.4.2.10.

In order to obtain a full relation for the gas mass flow rate Eqn. 3.4.2.3, which provides the mass flow at the hot end as a function of time, can be substituted into Eqn. 3.4.2.9. Moreover, a differential form of the pressure pulse can be obtained by differentiating Eqn. 3.4.2.1, in order to relate the pressure change terms. This gives the following equation for mass flow rate at the cold end:

$$\dot{m}_c(t) = \frac{\omega P V_{PT}}{\gamma R T_c} \cos\left(2\pi ft + \frac{\pi}{2}\right) + \frac{T_h}{T_c} \underbrace{C_{or} P \cos(2\pi ft)}_{\dot{m}_h} \quad (3.4.2.10)$$

From Eqn. 3.4.2.10 the sinusoidal nature of the mass flow waveform at the cold head can be seen. It is now further possible to understand how the different parameters hold influence over the mass flow. Notably, the pressure amplitude P and frequency f are variables upon which $\dot{m}_c$ depends, as well as the pulse tube volume V_{PT} and the cold end temperature T_c . Furthermore, it can be seen that the expression formulates as two vectors, one of which incorporates the expression for $\dot{m}_h$ as shown in Eqn. 3.4.2.10. Eqn. 3.4.2.11 expresses this in simplified form, where $\dot{P}$ approximates the angular pressure wave term.

$$\dot{m}_c = \frac{\dot{P} V_{PT}}{\gamma R T_c} + \dot{m}_h \frac{T_h}{T_c} \quad (3.4.2.11)$$

Figure 3.3 represents these vectors and their summing to give the mass flow rate at the cold end in the form of a phasor diagram. Hence, the phase angle between the mass flow and the pressure pulse can be formulated accordingly. Eqn. 3.4.2.12 formulates this based on the phasor diagram



shown in Figure 3.3, where the T_c terms cancel. Interestingly, this now indicates that the mass flow to pressure pulse phase is not influenced by the cold end temperature.

$$\phi = \tan^{-1} \left(\frac{\dot{P} V_{PT}}{T_h \dot{m}_h \gamma R} \right) \quad (3.4.2.12)$$

3.4.3 SPTC Component Phasors

Now, for a pulse tube cryocooler using a warm end displacer (hence, an SPTC), the cold end mass flow can be expressed following a similar logic with a few modifications. Radebaugh highlights that, given there is no mass flow crossing the moving system boundary but work cross the boundary at that location, an additional expander term is required that accounts for the expander gas volume due to displacer activity [31]. Integrating Eqn. 3.4.3.1, the equation for mass conservation, over the length of the pulse tube, and approximating the pressure pulse to $\dot{P}$ allows Eqn. 3.4.3.2 to be expressed [31], where A_g is the gas cross-sectional area and T_a is the average temperature in the component.

$$\frac{\partial}{\partial x} \left(\frac{\dot{m}}{A_g} \right) = -\frac{\partial \rho}{\partial t} \quad (3.4.3.1)$$

$$\dot{m}_h = \dot{m}_c + \frac{\dot{P} V}{RT_a} \quad (3.4.3.2)$$

Hence, Eqn. 3.4.3.3 gives the cold end mass flow rate for an SPTC using a warm end displacer, assuming isothermal expansion, in terms of T_c . Given that for SPTCs, comparatively smaller pressure amplitudes are achieved, the average volume during a cycle is taken as half that of the volume for the total swept volume of the displacer [31], hence the latter term being halved. As the displacer is situated at the warm, the approximation $T_{disp} \approx T_h$ is made. V_d and V_D denote the instantaneous displacer gas volume and the total displacer swept volume, respectively.

$$\dot{m}_h = \frac{P \dot{V}_d}{RT_h} + \frac{\dot{P} V_D}{2RT_h} \quad (3.4.3.3)$$

Similarly, an equivalent relationship can be formulated for an isothermal compressor, where V_{co} and V_{co} are the total swept compressor volume and instantaneous compressor gas volume, respectively.

$$\frac{-P \dot{V}_{co}}{RT_{co}} = \dot{m}_{co} + \frac{\dot{P} V_{co}}{2RT_{co}} \quad (3.4.3.4)$$



Similarly, the mass flow can be expressed for the regenerator in the same manner:

$$\dot{m}_h = \dot{m}_c + \frac{\dot{P}V_{\text{reg}}}{RT_{\text{reg}}} \quad (3.4.3.5)$$

The regenerator must serve as a very good heat exchanger between the working gas and the regenerator matrix. Across any cross-sectional area at any location within the regenerator, the temperature is assumed as constant, hence $\Delta T_{\text{reg, cs}}=0$. Conversely, a large temperature gradient of $\Delta T=300 \text{ K}-80 \text{ K}$ occurs across its length. Generally, T_a is taken as the average temperature in the case of the regenerator [36].

It can be seen in Eqn. 3.4.3.5 that with an increase in frequency or regenerator volume there is a proportional increase in the second term, which forms the vertical phasor vector. As explored in Eqn. 3.4.5.3, the mass flow magnitude in the regenerator is directly proportional to regenerator losses. This is due to the fact that with an increase magnitude of $\dot{m}_h$, are larger heat exchanger is required for effective heat transfer, which results in an increased pressure drop and a degraded overall efficiency as a consequence. Secondly, for a fixed average pressure, an increased $\dot{m}_h$ results in a proportionally increased volumetric flow. This therefore would require a higher swept volume within a larger compressor. This in turn would also decrease the overall efficiency of the cryocooler. Hence, for a given operating frequency, a smaller gas volume is desirable, concurrently with a high effective heat transfer and low pressure drop, for best performance efficiency.

The impact of the phase difference between the mass flow and pressure pulse is most significant in the regenerator [37]. The optimum phase requires that the mass flow be in phase with the pressure at the midpoint of the regenerator. This minimises the magnitude of the mass flow at either end of the regenerator and reduces the losses, for a given PV power. The phasor diagram for this condition is illustrated in Figure 3.4, where a Stirling system is shown as an example of a system with a regenerator. Eqn. 3.4.3.5, the mass flow for the regenerator, is the phasor represented. The phasors for the compressor and displacer have been omitted.

Once again, the phasor diagram can be used to derive an expression for the phase angle between the mass flow and pressure pulse throughout the regenerator, and thus, the wider Stirling

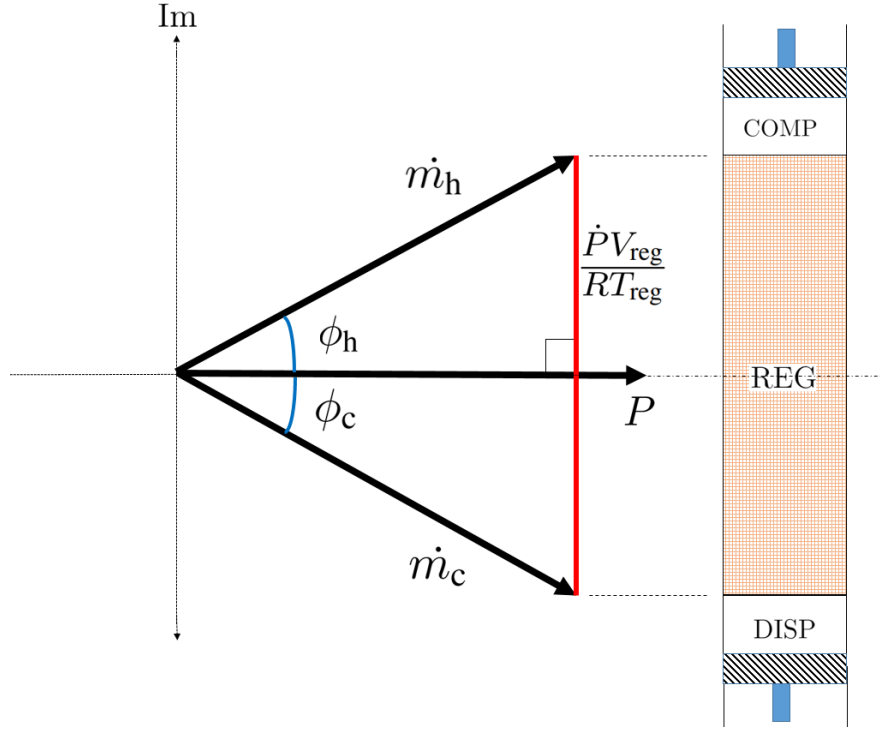


Figure 3.4: Phasor diagram showing the mass flow in phase with the pressure at the midpoint of the regenerator, for minimal regenerator losses. The pressure is arbitrarily chosen as the reference for the real axis, against which the mass flow phase is determined. The mass flow vertical vectors are represented in the imaginary axis, denoted 'Im'. A Stirling system is shown, as an example of a regenerative system, however the phasors for the compressor and displacer are omitted.

system. Given that the warm end mass flow leads the pressure pulse and that the pressure aligns with the midpoint of the regenerator, in the case of Figure 3.4, the following is true:

$$\phi_h = \phi_c = \phi \quad (3.4.3.6)$$

Hence,

$$\phi = \sin^{-1} \left(\frac{\dot{P} V_{\text{reg}}}{2 \dot{m}_h R T_{\text{reg}}} \right) \quad (3.4.3.7)$$

3.4.4 Effect of Phase Angle

The expressions established in the previous section can be utilised to relate the effect of the phase angle on the cooling power [34]. Firstly, considering the enthalpy travelling through the regenerator by applying the first law of thermodynamics, the cooling power $\dot{Q}_c$ can be expressed, where $\dot{H}_{\text{PT}}$ and $\dot{H}_{\text{reg}}$ are the enthalpy flow rates in the pulse tube and the regenerator, respectively.

$$\dot{Q}_c = \dot{H}_{\text{PT}} - \dot{H}_{\text{reg}} \quad (3.4.4.1)$$



Now, assuming that the regenerator is ideal and does not store any energy for an averaged cycle, the latter term in Eqn. 3.4.4.1 can be reduced to zero. Hence,

$$\dot{Q}_c = \dot{H}_{PT} = \frac{c_p}{\tau} \int_0^\tau \dot{m}_c(t) T \cdot dt \quad (3.4.4.2)$$

where τ is the period of the pressure cycle. Taking the ratio of specific heats for helium as $\gamma \sim 1.67$, a linear regression was conducted by Almtireen et al. [34] on Eqn. 3.4.2.6 as the following:

$$\left(\frac{T}{T_0} \right) = 0.4 \left(\frac{P}{P_0} \right) \quad (3.4.4.3)$$

By integrating Eqn. 3.4.4.2, substituting Eqn. 3.4.4.3 and Eqn. 3.4.2.10, and cancelling sinusoidal terms, the following approximation for cooling power is achieved [34]:

$$\dot{Q}_c \approx \frac{R T_c P}{2 P_0} |\dot{m}_c| \cos(\phi) \quad (3.4.4.4)$$

The above confirms the dependence that the cooling power has on the phase angle between the mass flow and pressure pulse.

3.4.5 Comparative Phasor Analysis of PTC Configurations

Figure 3.5, adapted from Radebaugh et al. [31], shows the phasor diagram for a pulse tube cryocooler using an orifice at the warm end, where Eqn. 3.4.2.11, Eqn. 3.4.3.4 and Eqn. 3.4.3.5 are plotted. Activity in the compressor is assumed to be isothermal and the pulse tube is assumed to be adiabatic. Conversely, a system such as this, in which the mass flow is not in phase with the pressure pulse, will achieve a higher cooling power. This, however, results in a reduced efficiency as the mass flow magnitude at the warm end (or cold end, depending on the direction of shift of the phase angle) is increased and regenerator losses are amplified. There will be an increase in mass flow at the warm end in the case of a positive phase angle, without much increase in PV power at the cold end, resulting in a decreased efficiency. This relates to the notion explored that expansion volume is required at the warm end in order to recover PV power and recycle it back into the system, with an optimised phase angle ϕ .

The acoustic power, or PV power, for the pulse tube system is given by equation 3.4.5.1.

$$\langle \dot{W} \rangle = \langle P \dot{V} \rangle = \frac{1}{2} |P| |\dot{V}| \cos\theta = \frac{1}{2} \left| \frac{P}{P_0} \right| |\dot{m}| RT \cos\theta \quad (3.4.5.1)$$



And hence,

$$\langle \dot{W} \rangle \propto |\dot{m}| \cos \theta \quad (3.4.5.2)$$

Since acoustic power is measured in the real axis (zeroth derivative), an increase in the amplitude of $\dot{m}$ does not increase PV in the real axis, as per 3.5. This therefore implies that a higher amplitude of $\dot{m}$ results in losses (namely in the regenerator) with no increase in refrigeration power, leading to the relationship of proportionality shown in 3.4.5.3.

$$\text{regenerator losses} \propto |\dot{m}| \cos \theta \quad (3.4.5.3)$$

It is thus favourable to ensure that the amplitude of vertical vector pertaining to the mass flow is minimised. In the phasor diagram for an orifice pulse tube cryocooler shown in Fig. 3.5, the vertical component of the mass flow vector at the cold end, $\dot{m}_c$, is in the positive imaginary axis, as no warm end heat exchanger or inertance tube features at the base of the pulse tube to enable $\dot{m}_c$ to lag behind the pressure where the orifice is located. This results in a higher amplitude of the vertical component of the mass flow vector at the warm end, $\dot{m}_h$, in the imaginary axis, leading to higher regenerator losses with no increase in refrigeration power. It is therefore desirable to ensure that the mass flow at the cold end is out of phase such that it lags the pressure pulse, in order to reduce losses in the system. This is why the use of an inertance tube or a displacer to create a phase difference at the other end of the pulse tube is beneficial for achieving higher efficiency in the overall cryocooler system.

Furthermore, this sheds light on the results of the comparative study conducted in chapter 7 section 7.6, where the in-line pulse tube cryocooler using an inertance tube has been found to achieve a higher cooling power than the in-line SPTC with a warm end displacer, however, the SPTC yielded a much higher efficiency due to PV recovery at the warm end. This comparative outcome is reflected in the phasor theory that has been discussed in this chapter, where better alignment of the mass flow to pressure phase towards the midpoint of the regenerator results in reduced losses due to minimised mass flow magnitudes and expansion power recovered at the warm end. The phase input into the inertance tube cryocooler numerical model was the optimised phase angle that was experimentally determined for the in-line SPTC system; it is unlikely this phase matched with the optimum ϕ required for the inertance tube pulse tube, in order for the

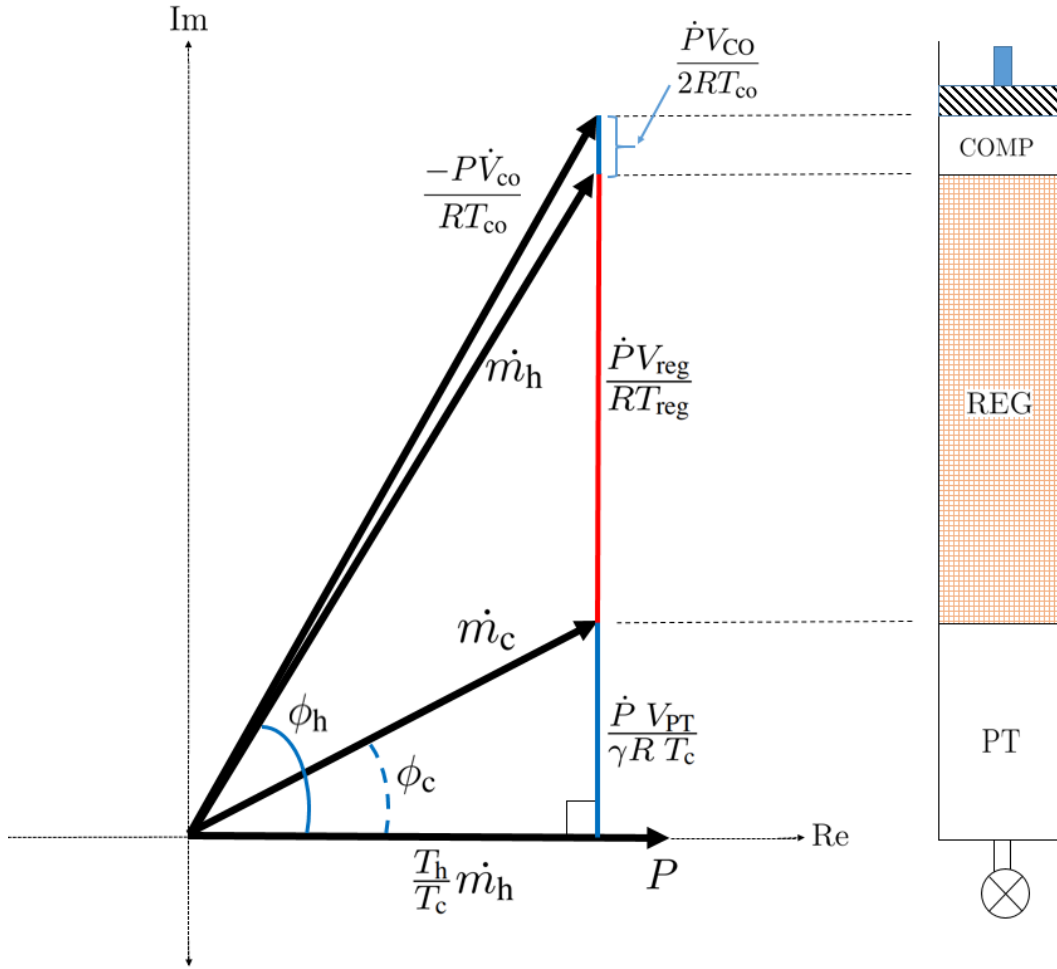


Figure 3.5: Phasor diagram for an orifice pulse tube cryocooler, where Eqn. 3.4.2.11, Eqn. 3.4.3.4 and Eqn. 3.4.3.5 are plotted. 'Re' denotes the real axis and 'Im' denotes the imaginary axis.

pressure pulse and mass flow to be in phase at the regenerator midpoint. The high value for Q_c and lower value for η_r in the inertance tube simulation suggests the ϕ input was shifted in the positive direction, as depicted for the system shown in Figure 3.5. In addition to the under-estimation of conductive and convective losses within the cryocooler, this is a likely explanation for the inertance tube simulated results. Finally, for an in-line SPTC, Figure 3.6 depicts the different phasor vectors for the compressor, regenerator, pulse tube, and displacer at the warm end.

3.5 Summary

In this chapter, Stirling and pulse tube cryocooler systems were theoretically studied and mathematically explored. The rationale for the varying modelling processes throughout this thesis work

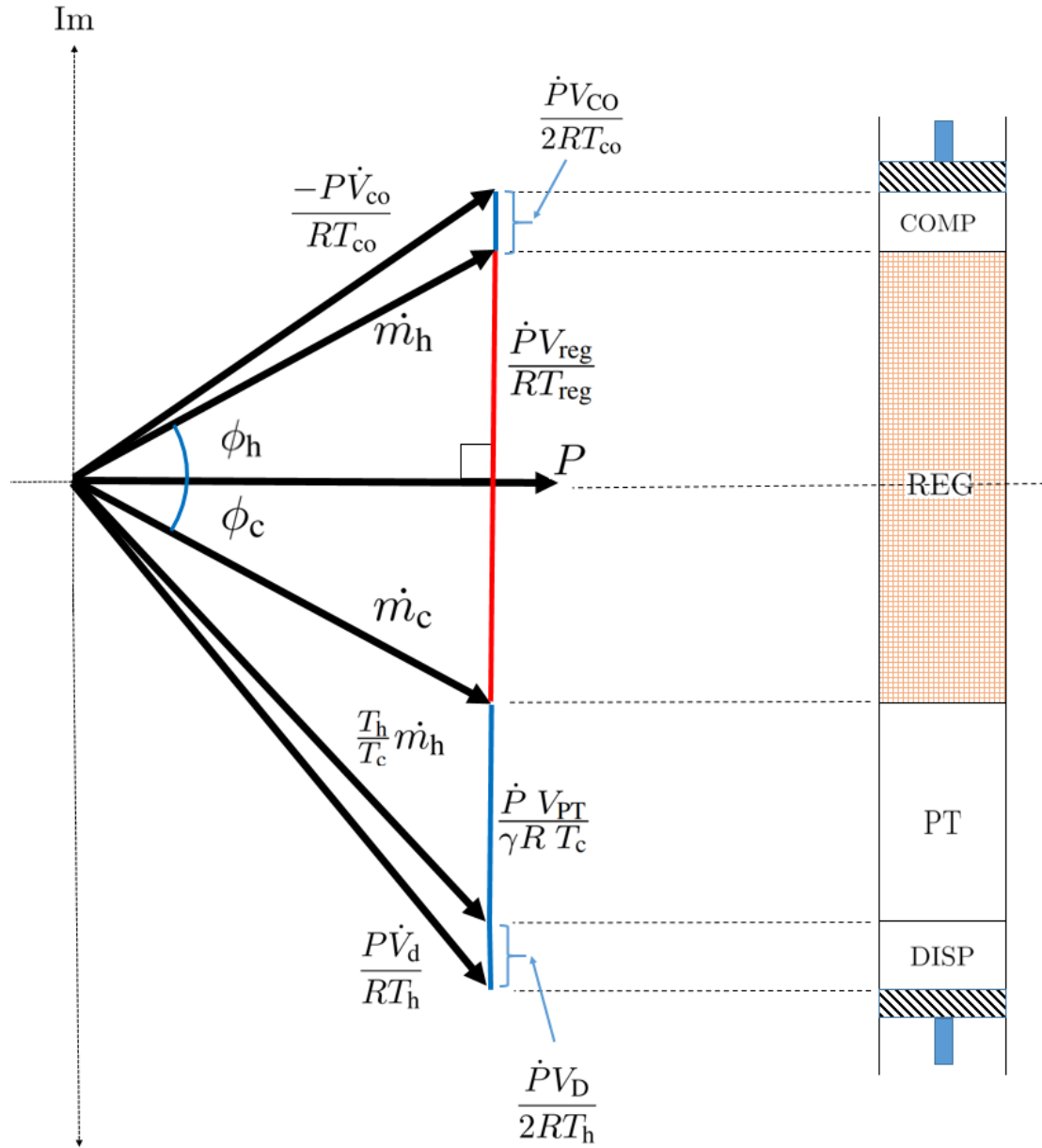


Figure 3.6: Phasor diagram for an in-line SPTC with a warm end displacer. The cold and warm end heat exchangers have been omitted in order to convey the main component phasors based on the formulated expressions Eqn.s 3.4.2.11-3.4.3.5.



were discussed and their advantages and limitations were briefly outlined. Thereafter, the energy flows throughout an ideal and real pulse tube cryocooler system, in terms of enthalpy, entropy, exergy, heat flows and loss terms, were explored. This shed light on the fundamental and unique operation of the pulse tube as well as the thermodynamic behaviour of all other key elements within a pulse tube cryocooler, putting the phasor analysis and further SPTC analysis to follow in this thesis into beneficial perspective.

The relationship between mass flow and pressure pulse was presented and the impact it has on cooling power and regenerator losses was outlined from first principles using phasor analysis. Furthermore, this permitted the formulation of mathematical expressions for phase angle. This is beneficial as it enables theoretical study and interpretation of the impact of the mass flow and pressure pulse phase angle on cooling power and the degradation of efficiency, but also, it provides insight for design purposes in inspiring suitable operating conditions. Phasor relationships and a phasor diagram for an in-line Stirling pulse tube cryocooler has also been developed. Lastly, the advantage of using a warm end displacer was once again demonstrated, putting into context the motivators for the key objectives of the studies presented in chapters 6 and 7.

Part II

Testing and Measurement Techniques

4

Flexure Spring Resonance and Fatigue

Contents

4.1	Introduction	38
4.1.1	The Oxford Cryocooler	38
4.1.2	Moving Magnet Compressor	39
4.1.3	Oxford Flexure Springs	41
4.2	Theory	42
4.2.1	Damped Harmonic Oscillator	42
4.2.2	Spring Resonance	43
4.2.3	Axial and Radial Spring Stiffness	43
4.2.4	Oscillatory Loading	44
4.3	Design and Modelling Analysis	46
4.3.1	Spring Designs	46
4.3.2	Finite Element Analysis	46
4.4	Experimental Testing and Analysis	47
4.4.1	Testing Rig	47
4.4.2	Electromagnetic Modelling of Testing Rig	50
4.4.3	Fatigue Testing	51
4.5	Summary	57

This chapter presents the theory, design, and testing of flexure springs used in SPTC linear motor systems. The work presented in this chapter is solely of the author's undertaking, except in instances where otherwise stated, and expands upon the research presented in the following publications:

- H. Rana et al. "Experimental fatigue testing of cryocooler flexure springs", *IOP Conf. Ser. Material Sci. and Eng.*, (2022).
- P. Bailey, H. Rana et al., "Low cost flexure spring testing", *IOP Conf. Ser. Material Sci. and Eng.*, (2019).



4.1 Introduction

Flexure springs are beneficial as they have a high radial stiffness and a low axial stiffness, preventing off-axis forces acting on the linear motor piston. The springs used must prove to have a longevity suitable for the length of an entire space mission; for example, a 10-year mission requires up to 10^{10} cycles of the flexure spring within the cryocooler. A method for fatigue testing of the springs must therefore be developed. Moreover, a study of different spring designs and thicknesses is required to better understand the effect these parameters have on fatigue limit, spring stiffness, and resonance. These outputs will ultimately inspire the harmonic displacer design for use in the SPTC with a passive displacer (detailed in chapter 7). This chapter details the background, design, theoretical analyses and experimental testing of stress, stiffness, resonance, and fatigue for different spring designs; firstly an ESA-RAL spring design for the Mars Robotic Exploration Programme (MREP), and secondly, a set of springs produced by 2 different industrial manufacturers, named 'ET' and 'QT' in this thesis, for use in the Stirling pulse tube cryocooler investigated in chapters 6 and 7 [38] [39].

4.1.1 The Oxford Cryocooler

The 'Oxford' Cryocooler was developed in the 1980s for the ISAMS instrument, employing the Stirling cycle, and was launched into space in 1991. It served to demonstrate that Stirling cycle cryocoolers have operational longevity fit for space applications. This is due to several modifications to the mechanical equipment that ensured a reduction of friction and wear. Figure 4.1 shows the first generation Oxford cryocooler.

The design of the cryocooler is beneficial due to several engineered adaptations [40]:

1. *Split design*: the cryocooler is divided into two segments consisting of a compressor part and a displacer part connected by a pipe. This enabled a smoother testing campaign and simplified the assembly and integration of the structure into its entirety.
2. *Flexure springs*: the spiral flexure springs, a signature design of the Oxford cryocooler, demonstrate high radial stiffness that ensures purely axial movement of the driving coil and

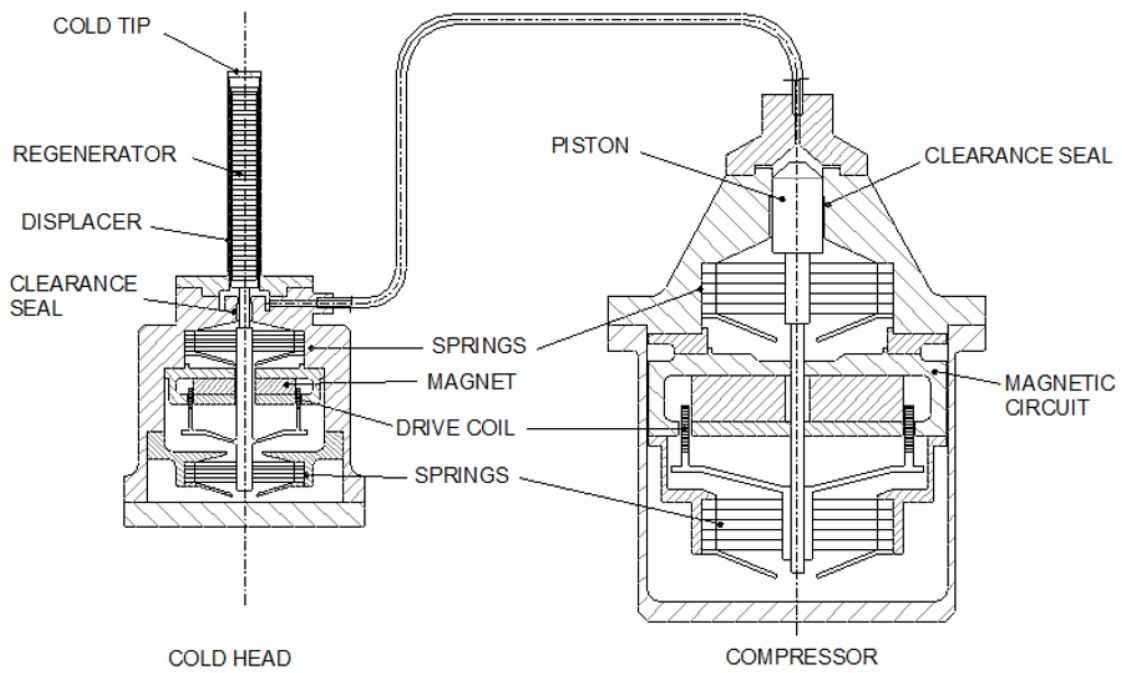


Figure 4.1: Schematic of the first generation Oxford split-design cryocooler for space applications [40].

piston so as to permit the tight tolerances necessary for piston/cylinder clearance seals. A spiral flexure spring is shown in Figure 4.2.

3. *Clearance seals*: these seals are utilised to sustain the pressure within the compressor and the displacer portions of the cooler. Fluid bearings cannot be used in space-qualified seals as they become subject to contamination from wear.
4. *Linear motor*: a moving coil motor similar to that in a loudspeaker, the novel linear motor design avoids side loads and thus addresses the wear issues faced in reciprocating compressors.

In recent years, two-stage Stirling cycle coolers have been developed that permit the previous temperature range of approximately 60-80 K to reach 15-30 K.

4.1.2 Moving Magnet Compressor

Two key components in the compressor system within the cryocooler are the motor (either moving coil or moving magnet) and the flexure springs [40]. This pairing forms part of the bespoke



Figure 4.2: The Oxford design stiff spiral flexure bearing spring used in linear motors [40].

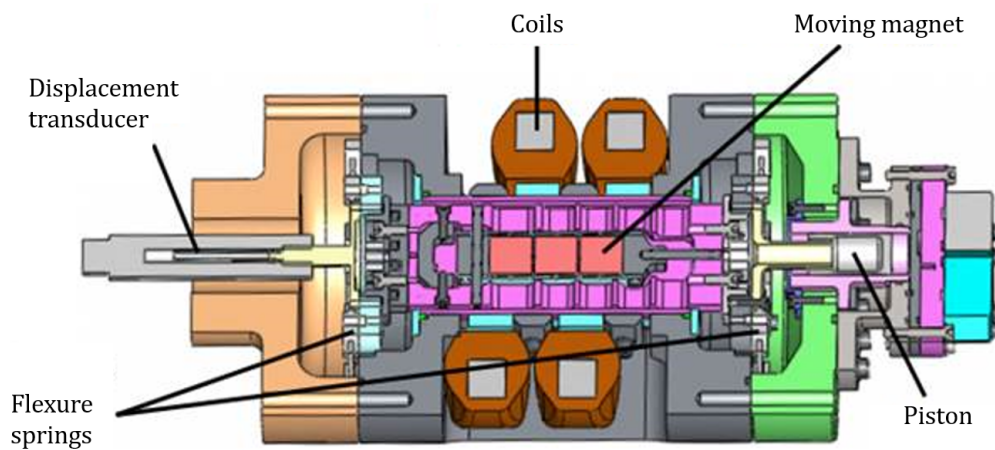


Figure 4.3: The coils drive the moving magnet, which is attached to the piston [42].

compressors/expanders developed at Oxford. The compressor serves to create the pressure wave that drives the thermodynamic cycle in the cooler. Research and development has focused on reducing their size given that they account for a significant part of the cooler's mass and volume. In every design optimisation in spacecraft engineering, reducing mass and the space occupied by equipment within the spacecraft are of utmost importance. Depending on the launcher, it costs space agencies approximately \$10,000-\$20,000 per kg to send a payload into orbit [41]. This section will briefly address the two optimised components that make up the linear compressor technology.

A novel magnetic assembly has been devised as a simple, low cost and efficient alternative to the more established moving coil motor systems [40], as is depicted in 4.3. An alternating current in the coils results in the movement of magnet motor. The coils possess opposing polarity, and thus applying an alternating current will result in an alternating force.



The key disadvantage associated with moving magnet motors compared to moving coil motors is that reluctance forces are generated orthogonally to the linear motion, even when the coil is not energised. Given the symmetry of the setup, if the central magnets are maintained centrally and not allowed to deviate off-axis, the off-axis reluctance forces can be minimised to zero. This is achieved with the use of radially stiff flexure springs.

Completing a force balance over the moving magnet motor operating between suspensions-controlled compression chambers, as depicted in Figure 4.4, Equation 4.1.2.1 is obtained [43].

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + k_m x + F_s + A(P - P_b) = 0 \quad (4.1.2.1)$$

In Equation 4.1.2.1, the following force terms are balanced:

- Inertial force: $m \, d^2x/dt^2$, where m is the moving mass and d^2x/dt^2 is the acceleration of the mass while moving.
- Damping: $c \, dx/dt$, where c is the damping coefficient and dx/dt is the velocity of the motor.
- Spring stiffness: $k_m x$, where k_m is the spring constant and x is the spring displacement, and $k_m x$ is the restoring force as given by Hooke's Law.
- Shaft force: F_s is the force produced by the shaft. It is generated electromagnetically and is a function of the motor constants that depend on current and displacement.
- Compression force: $A(P - P_b)$, where A is the area of the motor, P is the pressure in the compression area, and P_b is the motor body pressure.

4.1.3 Oxford Flexure Springs

The 'Oxford' spiral flexure spring was shown in Figure 4.2. The springs are designed such that they can readily move in the linear direction, however, exhibit high radial stiffness. Hence, the flexure springs permit extremely fine clearances (approximately 10-20 μm) to ensure no contact between moving parts during high frequency operation without the use of oil. Moreover, the

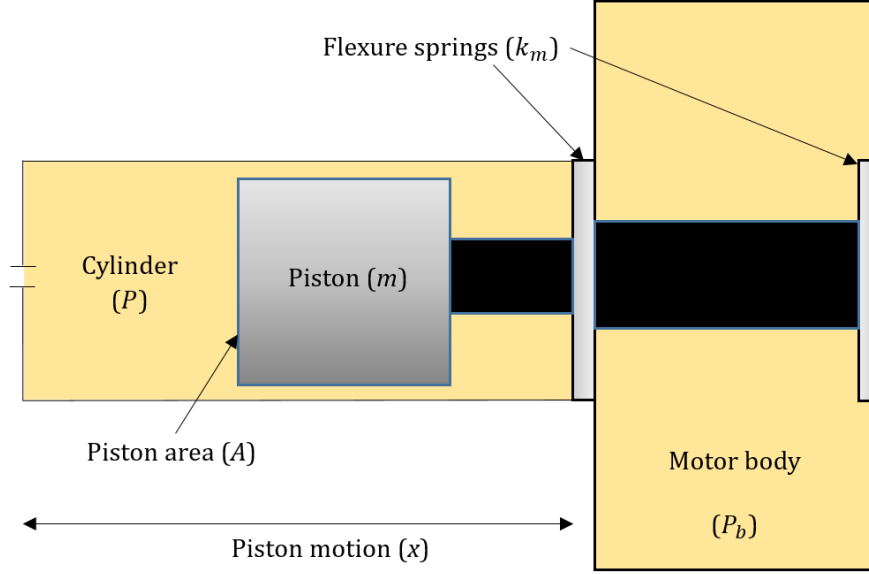


Figure 4.4: Moving mass motor operating between two cylindrical chambers with suspension springs.

springs are therefore a solution to the undesirable off-axis reluctance forces in moving magnet motors, as discussed in section 4.1.2. Furthermore, given that they remain sealed they present the ability to be used for continuous operation for more than 15 years without requiring maintenance [14]. For spacecraft missions, this is of particular significance. One key disadvantage associated with the springs is that the material often used, martensitic stainless steel, is subject to a fatigue limit. Fatigue limits and fatigue testing of flexure springs are discussed at greater length in section 4.

4.2 Theory

4.2.1 Damped Harmonic Oscillator

For a moving motor and oscillating spring system, such as that in Figure 4.4, the forces to be considered are reduced to the inertial force, damping and spring stiffness [43]. Eqn. 4.1.2.1 can be re-formulated as a sinusoidal function for running the oscillator, as below:

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + k_m x = P A \cos(\omega t + \phi) \quad (4.2.1.1)$$

where $\omega^2 = k_m/m$ and is the angular frequency of oscillation, P is the amplitude of the pressure oscillation, ϕ is the phase angle, and all other variables are as defined in equation 4.1.2.1. Incorporating $\omega_0 = \sqrt{k/m}$, the undamped natural frequency of the system; $F_P(t) = P A \cos(\omega t + \phi)$,



the sinusoidal forcing function; and $\zeta = c/c_c$, the damping ratio of actual damping (c) to critical damping (c_c), the below equation for the normalised amplitude ratio X/X_0 (also known as the magnification factor) is obtained:

$$\frac{X}{X_0} = \frac{1}{\sqrt{(1 - (\frac{\omega}{\omega_0})^2)^2 + (2\zeta\frac{\omega}{\omega_0})^2}} \quad (4.2.1.2)$$

The damping ratio ζ is a system parameter that indicates how damped a system is. For the flexure spring systems being analysed using the experimental testing rig in this study, the system is underdamped.

4.2.2 Spring Resonance

Resonance is an occurrence in which an oscillating system can reach significantly higher amplitudes if a periodically applied force is equal to the natural frequency or a Fourier component of the natural frequency of the system [43]. The resonant frequency of a system, denoted Ω , is therefore the frequency at which this relative maximum occurs. Operating the flexure springs at resonance is hence of particular benefit and efficiency. This phenomenon occurs due to the transfer of stored energy in the system to kinetic energy. The natural frequency of a system is the frequency at which a system oscillates when unforced vibrations occur in the absence of damping. If damping is low in a system, the resonant frequency will roughly equal the undamped natural frequency, and the magnification factor will be significantly large, tending to infinity for a completely undamped system.

4.2.3 Axial and Radial Spring Stiffness

The Oxford-style flexure bearing springs are made of a thin material with a high fatigue stress limit to Young's modulus ratio. Suitable materials that fit this criterion are stainless steel and beryllium copper, which also permit a design with low axial stiffness, k_z , and high radial stiffness, k_r . The clearance seal is able to keep its close alignment whilst a small increase in moving mass occurs due to the axial spring mass [44]. The radial stiffness required for the spring design in normal gravity is given in Eqn. 4.2.3.1, where g is gravitational acceleration, m is the moving mass, r_g is the clearance seal radial thickness, and C_r is the allowable ratio of radial displacement to r_g .

$$k_r = \frac{mg}{C_r r_g} \quad (4.2.3.1)$$

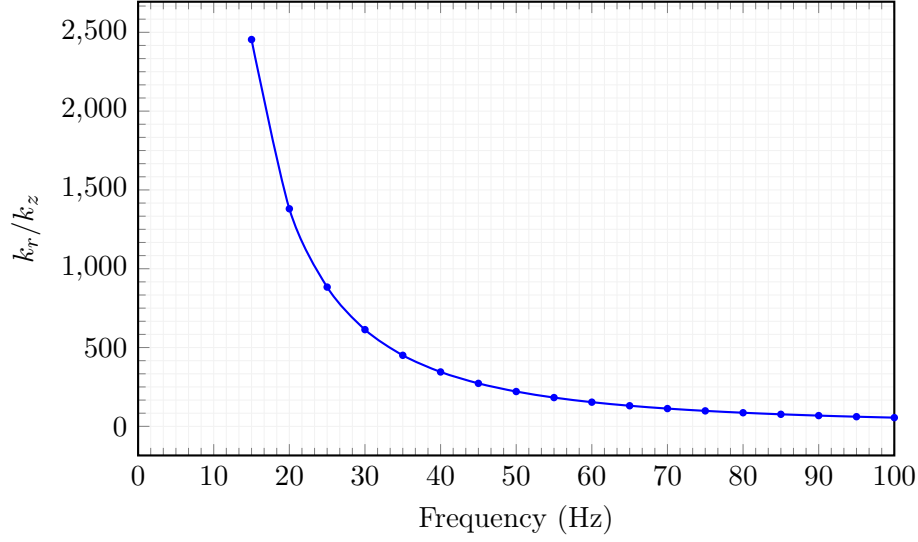


Figure 4.5: Sensitivity of the ratio of axial to radial stiffness with frequency for a typical Oxford-style flexure bearing spring, where $C_r=0.15$, $C_z=0.20$, and $r_g=15 \mu\text{m}$.

Moreover, the desired k_z is given as below, where C_z is the ratio of axial spring force to the inertial spring force.

$$k_z = 4C_z\pi^2 m f^2 \quad (4.2.3.2)$$

The desired ratio of radial to axial spring stiffness is hence given by Eqn. 4.2.3.3.

$$\frac{k_r}{k_z} = \frac{g}{4\pi^2 C_r C_z r_g f^2} \quad (4.2.3.3)$$

The value of C_z should be maintained above 15% in order to prevent instabilities being introduced into the system when utilising the springs in a dual-opposed set of compressors and values that are larger than 25% have been found to significantly increase the effective moving mass of the system. Similarly, C_r should be kept at a value less than 0.20 in order to ensure no contact during operation that could arise due to the large gas side forces associated with a slight axial misalignment [44]. Figure 4.5 demonstrates how the ratio of radial to axial spring stiffness changes with operating frequency. The decreased sensitivity to increased frequency can be seen for a flexure bearing spring with typical values of $C_r=0.15$, $C_z=0.20$ and $r_g=15 \mu\text{m}$.

4.2.4 Oscillatory Loading

The springs are subjected to oscillatory loading during operation and their operating limit, known as the fatigue limit, is the operating stroke at which the spring will fail. The fatigue lifetime of a spring can be measured by subjecting it to fatigue testing. The total amount of stress to be exerted

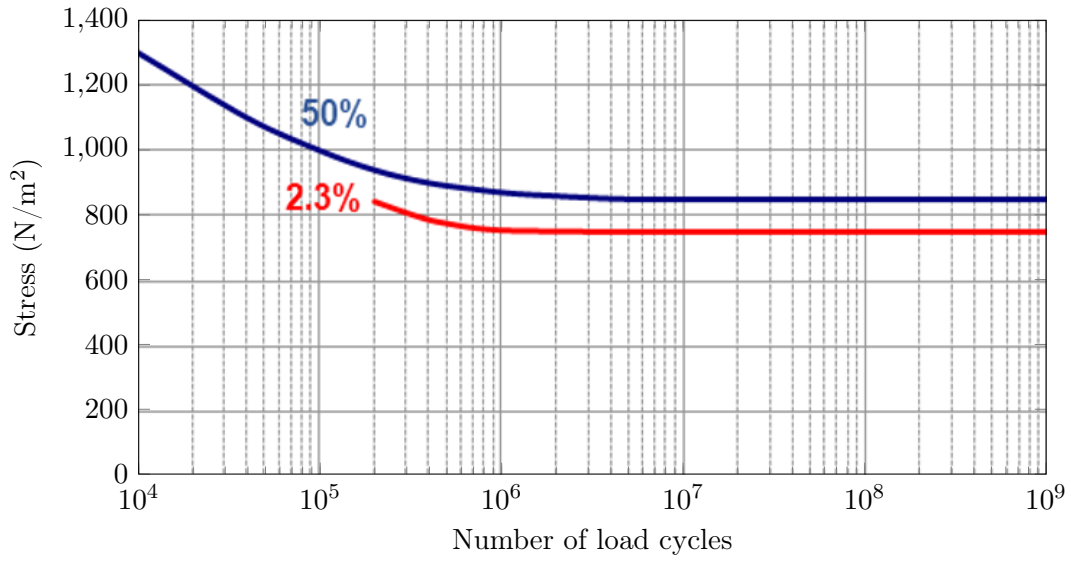


Figure 4.6: S-N curves indicating probability of failure for martensitic stainless steel springs (Sandvik 7C27Mo2).

on the spring in order to achieve failure is reached through a cumulative number of cycles, as given by Miner's rule below [45].

$$\sum \frac{n_i}{N_i} = C \quad (4.2.4.1)$$

In equation 4.2.4.1, n_i is the number of cycles at stress S_i , N_i is the total number of cycles before failure at stress S_i , and C is the ratio for cumulative damage, where $C = 1$ refers to failure. This rule can be used to determine the acceptable stress loads, and by extension, amplitude that the spring can operate at for lengths of time before failure occurs.

Figure 4.6 presents a Stress-Number of Load Cycles (S-N) curve (also known as a Wöhler curve) for martensitic stainless steel springs (Sandvik 7C27Mo2) [46]. The plots indicate the probability of failure for a certain number of load cycles at a certain stress loading, where each point on the plot represents a set of conditions that would result in failure. The springs are designed such that the maximum stress applied does not exceed that which corresponds to the fatigue limit. Spring arm reliability is usually designed as greater than 0.9998 and the number of cycles completed during fatigue testing is approximately 10 million at a 25% over-stroke.

Further to the S-N curve, von Mises yield criterion can be quantified to predict failure due to complex loading, based on models for uniaxial loading and stress. Von Mises stress, σ_v , is a scalar

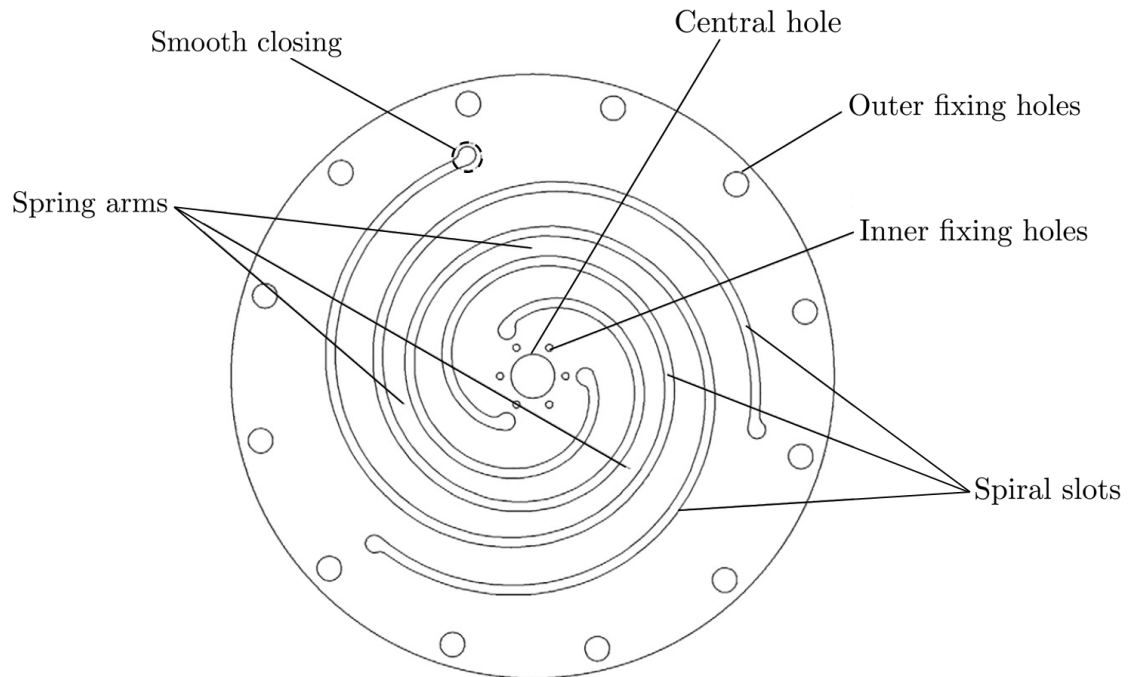


Figure 4.7: A schematic of the typical Oxford flexure bearing spring (adapted from [47]).

value that expresses the tensile stress a material is subjected to. Yielding is defined as occurring when the von Mises stress equates to the yield strength, σ_y , and a von Mises 2D plot can map the projected region in which a material's tensile stress must remain in order to prevent yielding.

4.3 Design and Modelling Analysis

4.3.1 Spring Designs

Figure 4.7 shows the Oxford flexure spring design. The smooth closing indicates where the 'teardrop' region exists at the start of the spiral slots for the spring arms. The springs are photo-etched to improve their surface finishing, where three different series of springs were tested from different manufacturers each. This introduced variation in the resultant photo-etching quality, which can affect the performance and fatigue limit of the springs. This was experimentally investigated and is presented in section 4.4.

4.3.2 Finite Element Analysis

Finite Element Analysis (FEA) was performed on the first design of flexure spring, which is representative of the springs tested in this chapter. The springs tested were made from a hardened 302-grade stainless steel. The displacement stroke was set as 8 mm stroke as a boundary condition.

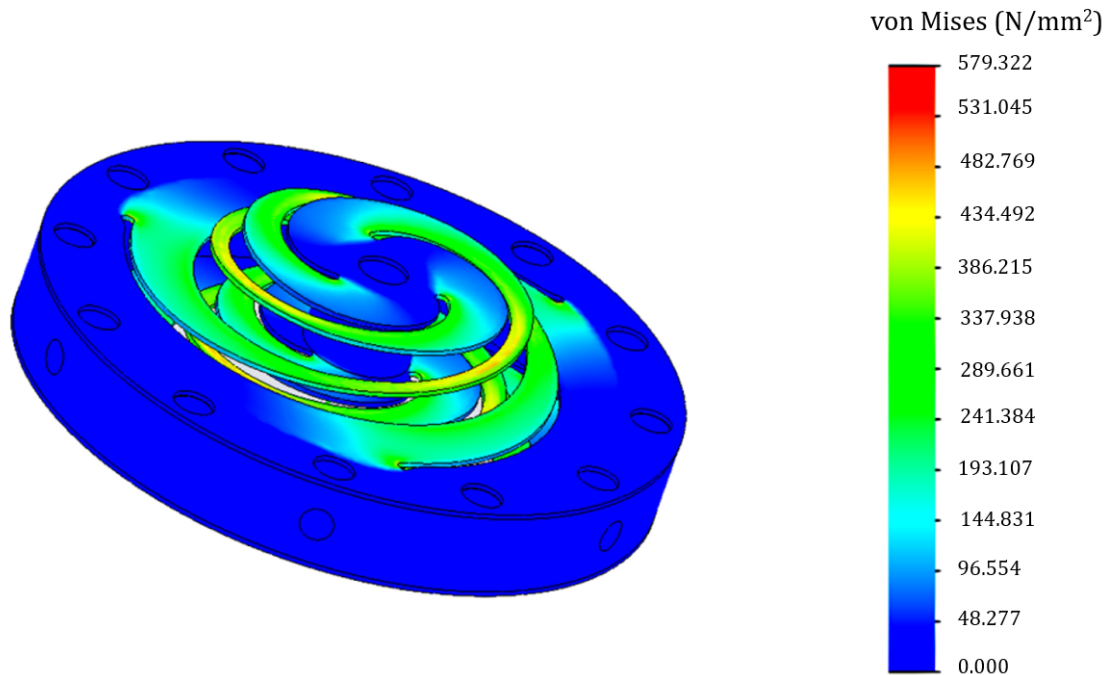


Figure 4.8: The Solidworks simulation plot for von Mises stress on the spring displaced by 4 mm amplitude.

The material tensile strength was set as 1800 N/mm². The analysis assessed the von Mises stress, and the regions of highest von Mises stress were found to be at the edges of the curves of the spirals. A maximum von Mises stress of approximately 450 MPa was computed.

4.4 Experimental Testing and Analysis

4.4.1 Testing Rig

A setup for testing the fatigue limits of flexure springs was assembled using an existing testing rig in the research group, as shown in Figure 4.9. It was then upgraded to include an Omron laser, as shown in Figure 4.10. The fatigue testing setup is driven using a moving magnet linear motor. The motor consists of an axially magnetised Neodymium-Boron-Iron magnet in the centre of a pair of coils. Each coil has 600 turns and the coils are connected such that the magnetic flux from each opposes the other. The magnet is bonded to a small cup, which is hung from a shaft attached to the centre of a pair of springs. As the AC current passes through the coil, an electromagnetic field is generated that displaces the magnet and thus generates the oscillating motion of the spring. This enables the springs to be subjected to oscillatory loading at varying strokes in order to determine the failing stroke due to fatigue. There is no iron circuit, so the AC flux density produced by each coil is very low and the motor efficiency is also very low. This does not matter, however, if

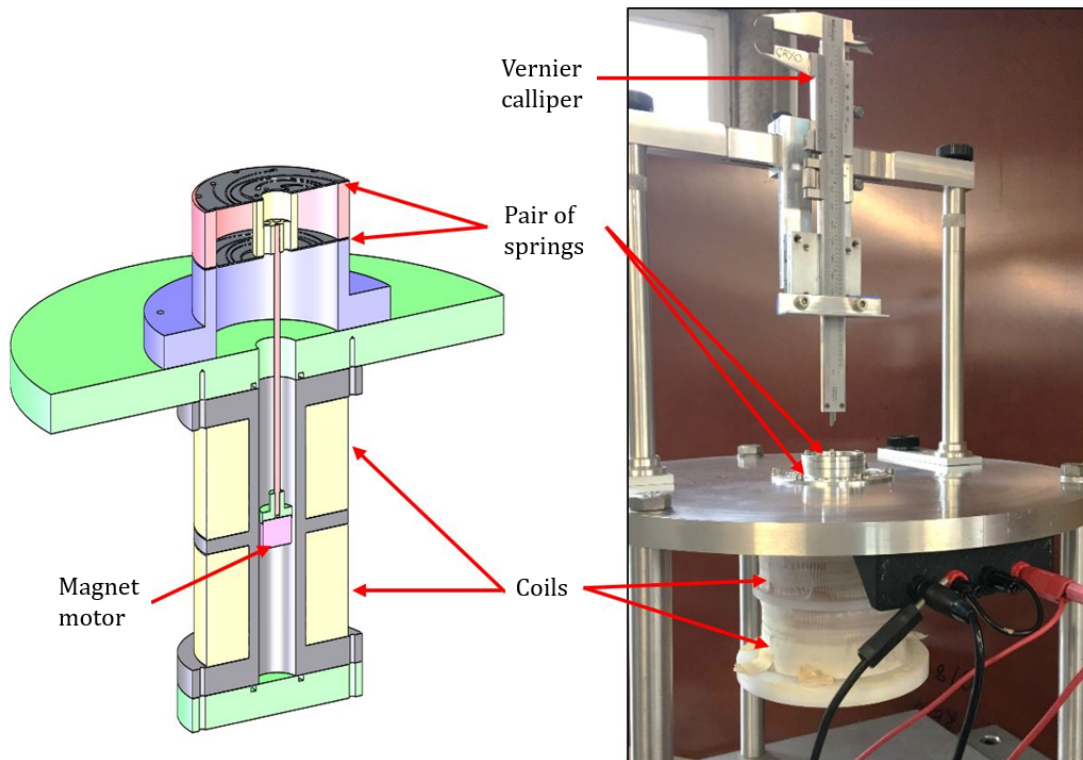


Figure 4.9: A cross-sectional view of the spring attached to the magnet that extends into the magnetic coil (left) and the full fatigue testing test setup (right).

the motor is driven at the resonant frequency of the mass-spring system, as the force required is relatively small. The typical input power for the motor was within the range of a few Watts. The motor is powered by an AC amplifier, driven by an oscillator, and the displacement of the spring can be measured by the digital calliper and the Omron laser displacement transducer.

The springs were operated at resonant frequency, which was ascertained by finding the specific range in which the current decreases with a fixed applied voltage, giving a minimum current associated with resonance. The benefit of operating at resonance is that much higher testing amplitudes can be reached for a lower input, and the setup ‘fails safe’ due to failure inducing off-resonance operation at a much lower frequency. The setup with the Omron laser is shown in Figure 4.10, which was calibrated against the digital calliper to convert the peak-to-peak voltage values to displacement stroke. The stroke was incrementally increased by increasing the input signal frequency, within a pre-established range close to the spring resonant frequency. This was continued until the springs ultimately failed. An automated continuous logger recorded the voltage and the recording was saved at hourly intervals, so that the failure time could be identified and the

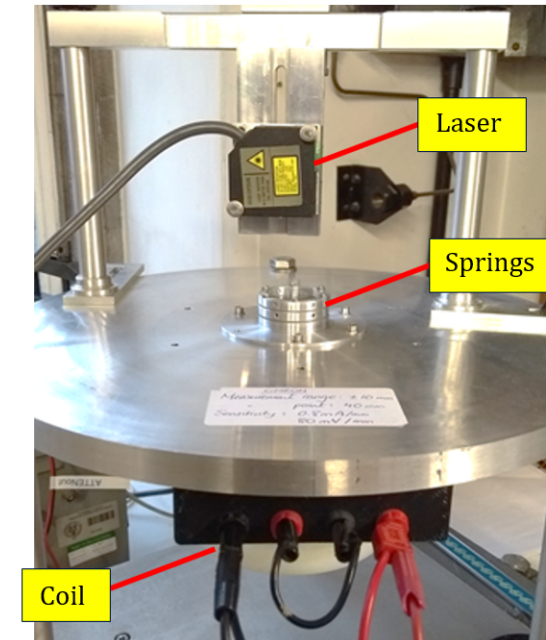


Figure 4.10: The fatigue testing setup with an Omron laser installed to measure spring stroke.

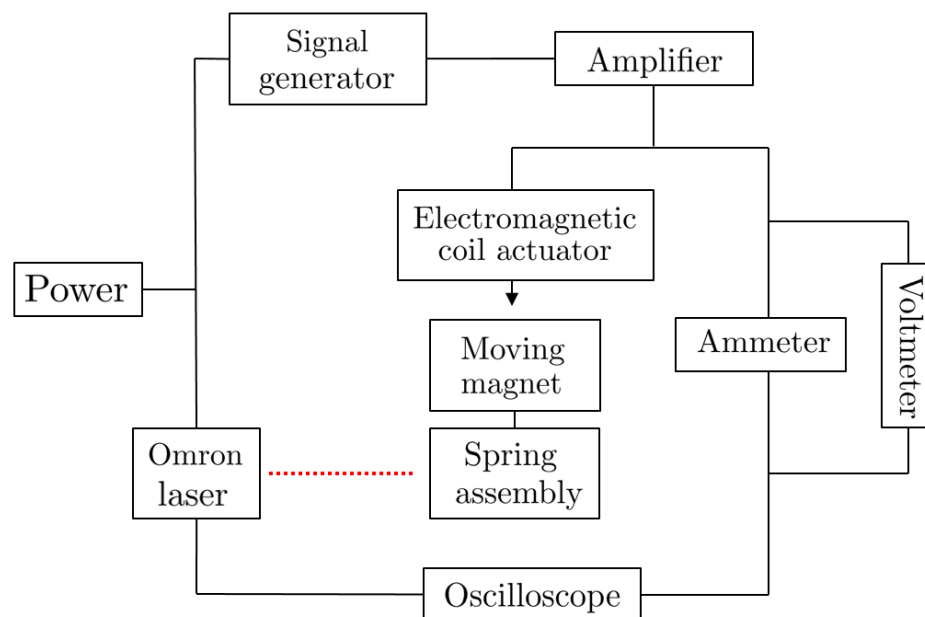


Figure 4.11: The block diagram for the spring testing setup and instrumentation.



number of cycles completed could be computed.

4.4.2 Electromagnetic Modelling of Testing Rig

Methodology

A moving magnet motor was used in the flexure spring displacement assembly for the fatigue and strain gauge bridge measurements. The electromagnetic coil generates a magnetic field when supplied with a current, which in turn results in the motion of the moving magnet motor. In order to achieve this, a driving voltage is thus required to drive the current through the coil in order to run the motor. The excitation voltage, V_{in} , into the coil can be broken down into constituent components, as shown in Eqn. 4.4.2.1. V_R is the resistance of the coil, V_L is the inductance voltage, and V_{EMF} is the voltage corresponding to the magneto-motive force (analogous to an electromotive force). There are two components in turn to the voltage associated with the magneto-motive force; one as a result of the current into the coil, and the other due to the magnet moving within the coil. Modelling the setup on FEMM permits assessing electromagnetic properties such as these which are difficult to measure. The only voltage that can directly be measured is the excitation voltage.

$$V_{EX} = V_{EMF} + L \frac{dI}{dt} + IR + \frac{1}{C} \int I \cdot dt \quad (4.4.2.1)$$

The electromagnetic setup was modelled using the Finite Element Method Magnetics (FEMM) software in order to study certain key properties such as magnetic flux distribution, inductance, and magneto-motive forces (MMF). The model setup is shown in Figure 4.12. The input electrical conductivity of the coils was 597 MS/m and the current density was 1.21 MA/m².

Simulation Results

Figure 4.13 shows the magnetic field lines and flux density of the 2D axisymmetric model of the coil and magnet motor. The highest flux density is expectedly around the magnet motor. The field lines can be seen to tend towards an upwards angle. This is due to the fact that the top coil is reinforcing the magnetic field and the bottom coil is opposing it as a result of opposite polarity. Using the simulation, a value of 3.73 N/A was calculated as the force constant of the motor. This value is corroborated by that found in the force-current motor constant tests completed, averaging at 3.71 N/A as presented in chapter 4 Figure 5.14. The inductance was computed to be 91 mH,

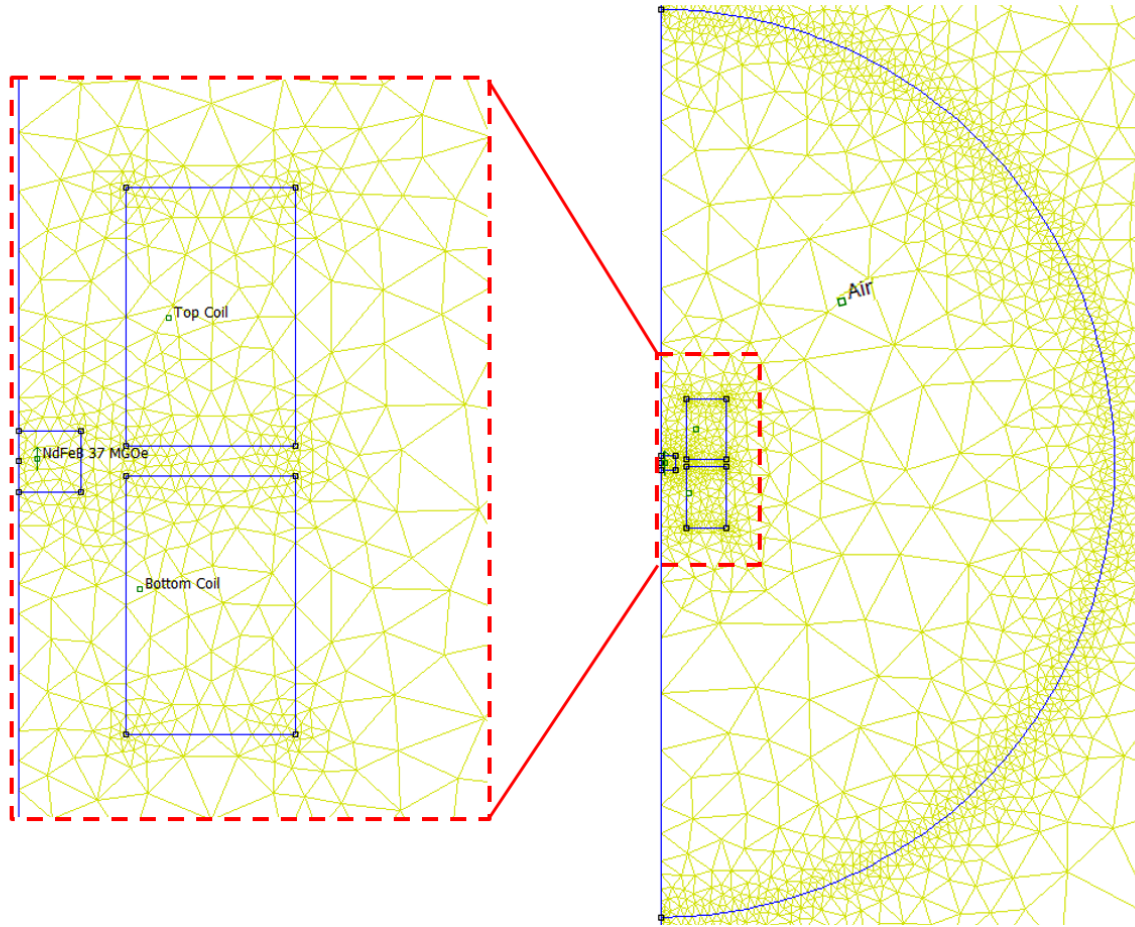


Figure 4.12: 2D axisymmetric model mesh in FEMM with the NdFeB 37 MGOe magnet, top and bottom coils, and surrounding air reservoir components modelled as part of the moving magnet testing rig.

closely matching that obtained using the LCR bridge in chapter 4. The slightly higher bridge value is likely due to the presence of the magnet within the coil setup.

4.4.3 Fatigue Testing

ESA MREP Springs

Table 4.1 indicates the settings that induced spring failure. The springs were determined as failed upon observing that they had ceased to oscillate, and further, that the current had increased significantly, e.g. ~ 30 mA to ~ 102 mA for Set A in Table 4.1. For the voltage strokes in Table 4.1, a corresponding amplitude percentage was calculated as a proportion of the settings for the springs at rest (100%). Thus it can be seen that the springs begin to approach failure once oscillating at roughly 140% amplitude.

In Figure 4.14 the set A disc can be seen to have failed on the thinnest part of the spring

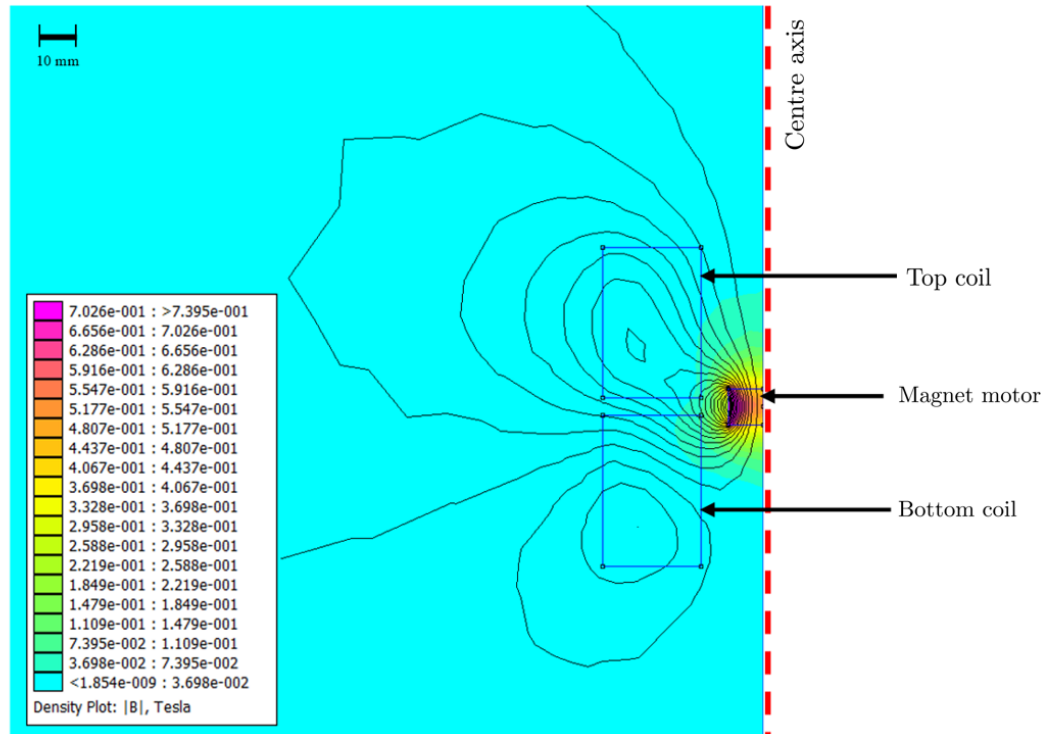


Figure 4.13: 2D axisymmetric electromagnetic model of the moving magnet motor indicating flux density distribution, B , with a current of 1 A as generated using FEMM software.



Figure 4.14: Comparison of the failed flexure springs of set A (left) and set B (right), where failure points are indicated by red circling. These are the three key stress points where failure is expected to occur, as per FEA predictions in section 4.3.2 [blurred due to confidentiality].

**Table 4.1:** Measured voltage and current of spring setup around time of failure for spring Set A and Set B.

Spring Set A			
Voltage (V)	Amplitude (%)	Time (h:m)	Current (mA)
7.10	140.8	21:00	29.99
		22:00	30.02
		23:00	102.89
		00:00	102.91
Spring Set B			
Voltage (V)	Amplitude (%)	Time (h:m)	Current(mA)
7.13	142.4	23:00	35.47
		00:00	49.02
		01:00	102.32
		02:00	102.33

arm, further out from the centre. The set B disc, however, failed on two locations; it is suspected that it firstly experienced the crack seen on the outer circle, which then may have accelerated the development and ultimate failure crack that can be seen in the centre of the disc at the base of one of the spring arms. This likely correlates with the recordings of change in current seen in Table 4.1, where for spring set B the current increases firstly from 35.47 mA to 49.02 mA, suggesting that the initial outer crack had formed. The current then increases to 102.32 mA after what is likely the formation of the detrimental central spring arm rupture.

The FEA presented in Figure 4.8 indicates that the region of highest principal stress is towards the outer edge of the spring arm on a flexure spring, closer to where spring set B experienced its initial rupture. Whilst one might expect set A to have failed in a similar area, there are multiple other factors that affect the spring arm failing. A possible influencing factor on where the failure occurs are likely the variations in surface finishing. In terms of repeatability of the results, it is evident that performing fatigue tests on two sets of springs is insufficient data to allow reliable prediction of the location of future spring failure. It is clear, however, that a corresponding amplitude of 140% can be established as the failing limit for these springs.

Figure 4.16 shows the S-N plot for typical austenitic stainless steel [48] [49] and indicates the location of the two spring sets tested along this curve. The loading stress amplitude, S_a , was determined by re-computing the FEA model using the failure displacement amplitude for the

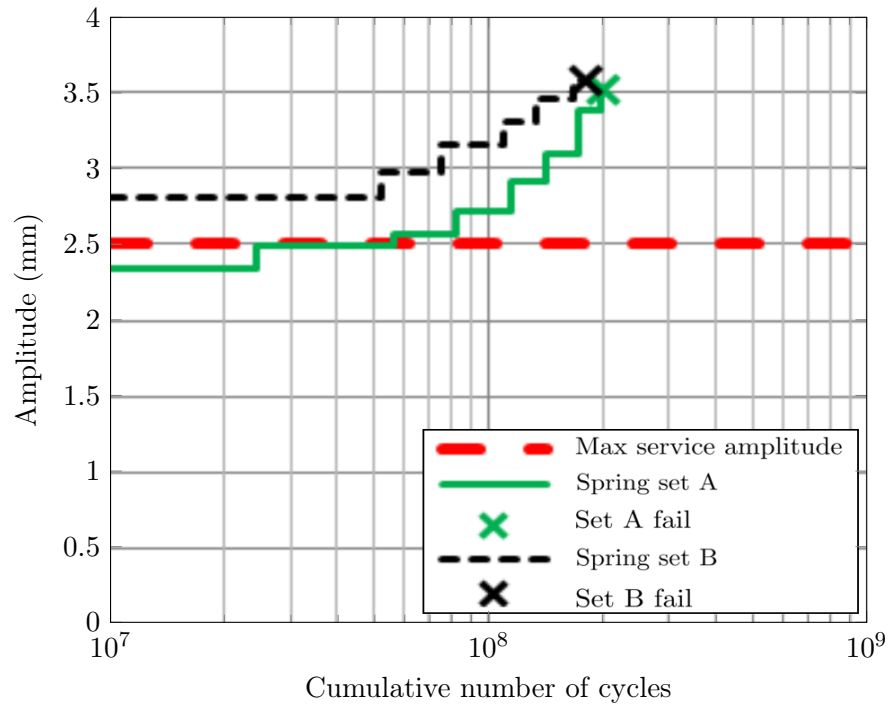


Figure 4.15: Fatigue test results for the ESA MREP springs, showing the maximum service amplitude at 2.5 mm amplitude.

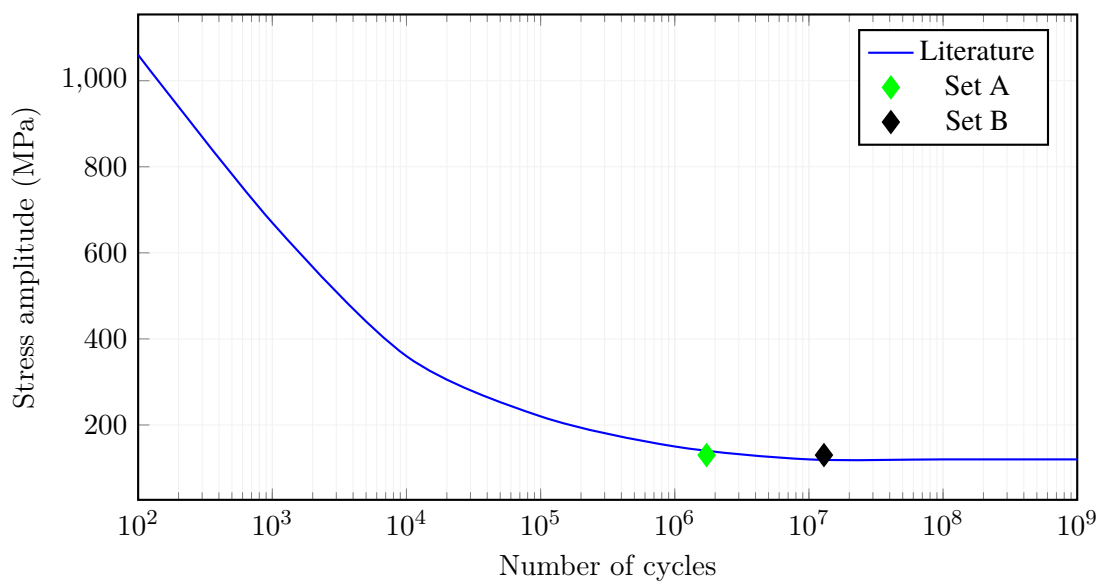


Figure 4.16: S-N plot for austenitic stainless steel data in literature [48], and Set A and Set B failure points.



springs as an input. It can be seen that the springs failed at failure conditions approximate to those expected for a 300 series stainless steel spring, with 2.8% error of Set A from the S-N curve and 8.4% error of Set B. Set B deviates further from the S-N curve given that failure occurred at an unexpectedly region on the spring.

Displacer Springs ET and QT

Thereafter, two sets of springs of thickness 0.3 mm were designed in the group and were produced by two manufacturers, who will be referred to as 'ET' and 'QT' in this thesis. Two spring arm designs were investigated; one which follows the standard pattern, similar to that seen in the ESA MREP springs, and the second which included a pronounced 'teardrop' at the region on the outer circumference where the assembly screws are inserted. The two manufacturers utilised differing etching processes for the surface finishing of the springs, which resulted in relatively noticeable differences in how the springs looked, by the naked eye. Namely, this resulted in visible burrs which were sanded down to be removed. Table 4.2 provides an overview of the spring fatigue results.

Table 4.2: Overview of the fatigue testing results for the displacer springs manufactured by ET and QT for designs A and B, all of spring thickness of 0.3 mm.

Manufacturer	Batch	Design	No. of cycles $\times 10^7$	Failing stroke (mm)	Cum. no. cycles $\times 10^7$
ET	1	A	0.45	10.72	10.12
ET	2	A	1.15	10.50	9.36
QT	1	A	0.18	12.00	10.73
QT	2	A	0.52	11.00	4.48
ET	1	B	0.05	7.00	5.83
QT	1	B	0.15	7.50	7.17

Figure 4.17 presents the fatigue plot for the spring design A, where both manufacturers' springs were tested and twice over in order to assess repeatability of the fatigue results. The fatigue tests can be seen to start at different strokes; the later tests (QT springs) were started at higher strokes once the range for expected failure was ascertained, in order to save testing time. As previously, the prior number of oscillatory cycles completed at a non-failing stroke do not impact the fatigue test results at higher strokes. Given that many factors are at play in the spring assembly and the flexure spring testing setup that may contribute towards failure, ensuring some level of

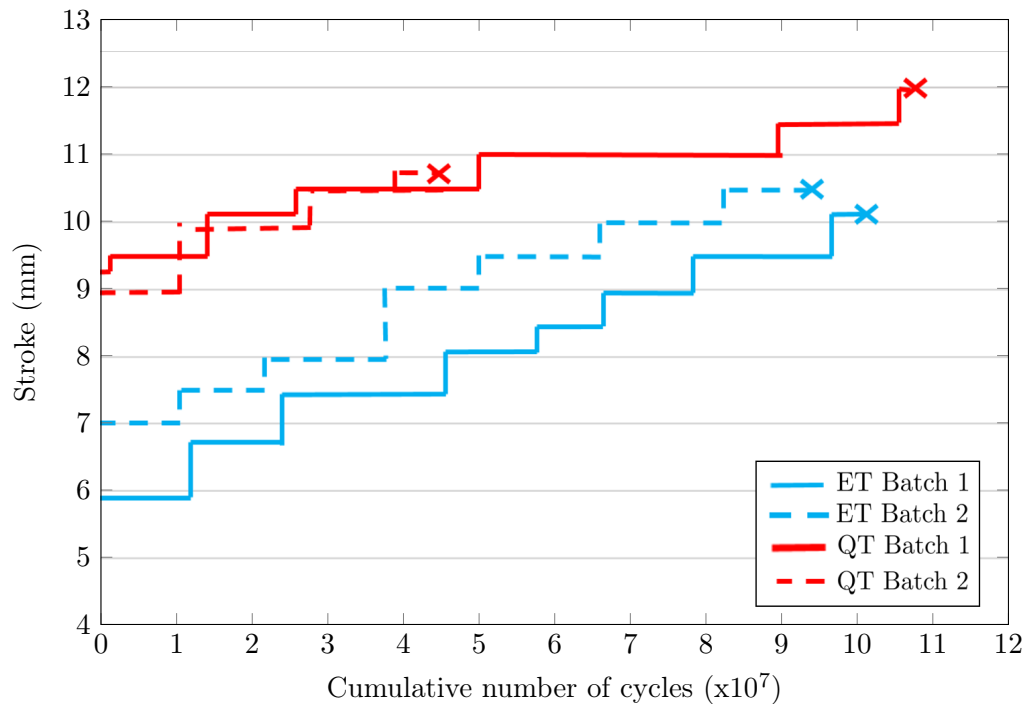


Figure 4.17: Spring design A fatigue testing results for two batches of each manufacturer, where the 'x' indicates failure.

repeatability of the springs testing was important as there is potential for variation in the fatigue limits reached. In Figure 4.17, it is clear that the QT springs are able to reach a higher fatigue life with a 10-20% higher failing stroke observed by the two batches compared to the ET springs. It is worth noting that, although only two data points to draw from in this instance, the ET springs failing strokes of 10 mm and 10.5 mm respectively, are in closer range of one another compared to the larger variation between the batches tested for the QT springs, for a fixed design and a fixed thickness of 0.3 mm.

Figure 4.18 depicts the fatigue testing results for the two spring manufacturers for design B. It is immediately clear that between designs A and B, the springs with design A reached higher failing strokes by between 25% to 37.5% higher. Moving forward, the springs selected for use in the displacer were the QT manufactured 0.3 mm thick design A springs, which demonstrated the highest failing stroke (between 10-12 mm). Given that the fatigue tests take several days to conduct, it is difficult to repeat the results in order to acquire a larger database of fatigue data. Nonetheless, in the case of the ESA MREP springs, it was shown that the failing points when computed to fit be formulated for an S-N, fit the data in literature very well. As the materials being used for the

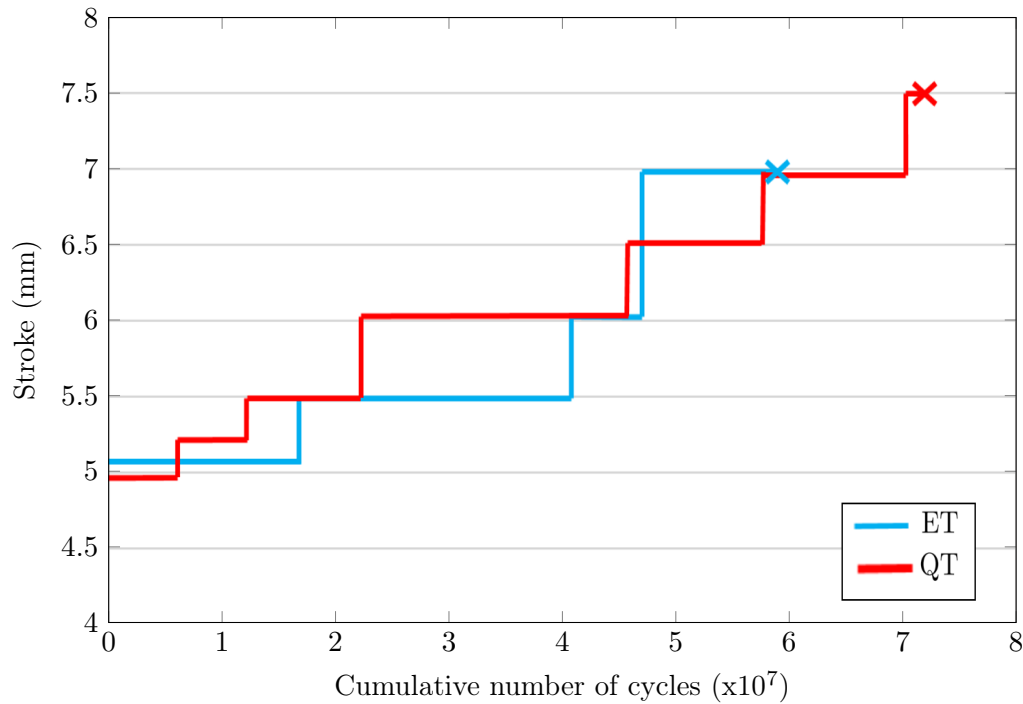


Figure 4.18: Spring design B fatigue testing results for each manufacturer, where the 'x' indicates failure.

springs are well documented, fatigue data is readily available and accessible for stainless steel fatigue limits. The issue is effectively predicting the influence of the very specific flexure bearing design, the associated stiffness, and establishing some level of predictability about where on the spring failure is likely to occur, such that there is good agreement with the predictions made by the FEA models. This undoubtedly requires further experimental fatigue testing data to be catalogued for the flexure springs.

4.5 Summary

In this chapter, the theory surrounding the operation and fatigue life of flexure bearing springs is presented. The method for modelling oscillatory loading is discussed and FEA models were developed to model the flexure springs to understand the magnitudes of failing stress, where failure on the spring is likely to occur in terms of highest stress points, as well as other aspects. The low cost flexure spring testing setup consisting of an electromagnetic motor and coil testing rig was electromagnetically modelled in order to understand the force-motor constant and the operation of the testing rig. The low cost flexure spring testing setup presented demonstrates a 'fail safe' feature and can be logged 24-hour, enabling hands-off fatigue testing for longer running times.



For the ESA MREP springs, resonance of the springs occurred at 90-100 Hz and approximately 10^8 cumulative cycles were carried out for each spring set, with under 2 million cycles at 130 MPa leading to failure. The first set of springs failed along the thinnest part of the spring arm, as corroborated by the FEA results. The second set failed at the central clamping region, which is likely due to a discrepancy in the spring surface finishing or defects, or fretting at the clamping location. An $S-N$ curve was produced from literature for the spring material and the data points obtained through testing for the commercial spring sets were compared with respect to the curve. The springs tested failed within a 3-8% error range of the expected failure conditions for a 300 series stainless steel.

For use in the SPTC passive displacer, two designs of springs, A and B, were developed in the group. Two different manufacturers were engaged for the production of identical springs with the same thickness and same designs. The different manufactured springs were fatigue testing and variation was noted between the manufacturers. 'QT' springs demonstrate a higher fatigue limit, and hence, design A of the QT manufactured springs were selected for use in the passive displacer.

5

A Novel Measurement Technique for Cryocooler Stroke

Contents

5.1	Introduction	60
5.1.1	LVDTs	61
5.1.2	Strain Gauge Bridge Configuration	62
5.1.3	Finite Element Analysis	63
5.2	Experimental Design	65
5.2.1	Strain Gauge Installation	65
5.2.2	Testing Rig	66
5.2.3	Isolation Amplifier Design	67
5.2.4	Static Spring Offset	68
5.2.5	System Characterisation	69
5.2.6	Data Acquisition	71
5.3	Experimental Testing	72
5.3.1	Omron Calibration	72
5.3.2	Strain Gauge Bridge Calibration	72
5.3.3	FEA Validation	73
5.3.4	Dynamic Tests	74
5.3.5	Impact Tests	75
5.4	Summary	83

This chapter presents the design and experimental research undertaken in testing a novel method for measuring cryocooler stroke using strain gauges. The work presented in this chapter is solely of the author's undertaking, except in instances where otherwise referenced, and expands upon the research presented in the following publications:

- H. Rana et al., "Compressor stroke and frequency response measurements using strain gauges", *Cryocoolers*, (2022).



5.1 Introduction

In chapter 2, the importance of long lifetime oscillatory operation of flexure bearing springs was detailed. The lifetime operation of the springs is paramount in order to enable the continuous operation of cryocoolers for long-duration space missions. In chapter 4, the design and experimental analysis of the flexure bearing springs and the characterisation of their operation up to failure was explored. At failure, the off-axis forces on the moving motor shaft will increase and lead to a compressor failure that will halt the operation of the cryocooler (see Figure 2.2 in chapter 2). It is therefore important that a suitable method for detecting cryocooler stroke is in place during operation in order to detect early signs of off-axis movement and potential failure. Further, the measurement technique for compressor stroke would need to be suitably accurate to detect small impacts at high sensitivity in order to enable altering operating conditions of the cryocooler compressor in-situ.

Existing established metrology for these purposes includes LDVTs, which are the most commonly used displacement transducer devices integrated into cryocooler compressors. They present the particular drawback of phase lag in the measurement signal. In this chapter, the suitability of a novel method using strain gauges in a full Wheatstone bridge configuration, integrated onto a spring-moving magnet motor setup, is investigated. The testing rig is the same as the one utilised in chapter 4 experiments, however, the spring used in this chapter has a higher diameter, reduced spring arm quantity, and different spring arm design. These differences result in an enhanced spring stiffness, which is useful for assessing the sensitivity of a displacement measurement device.

This chapter presents the strain gauge bridge configuration design and the finite element analysis conducted to explore regions of highest strain in the spring for a given displacement. The latter permits appropriate strain gauge installation. Thereafter, the testing equipment and data acquisition is discussed, and a variety of tests are conducted with the system to understand its harmonic characteristics. The strain gauge bridge configuration is then calibrated with both a digital caliper and an Omron laser displacement transducer and the results from the integrated arrangement tests are used to validate the FEA model. Finally, a series of dynamic and impact tests

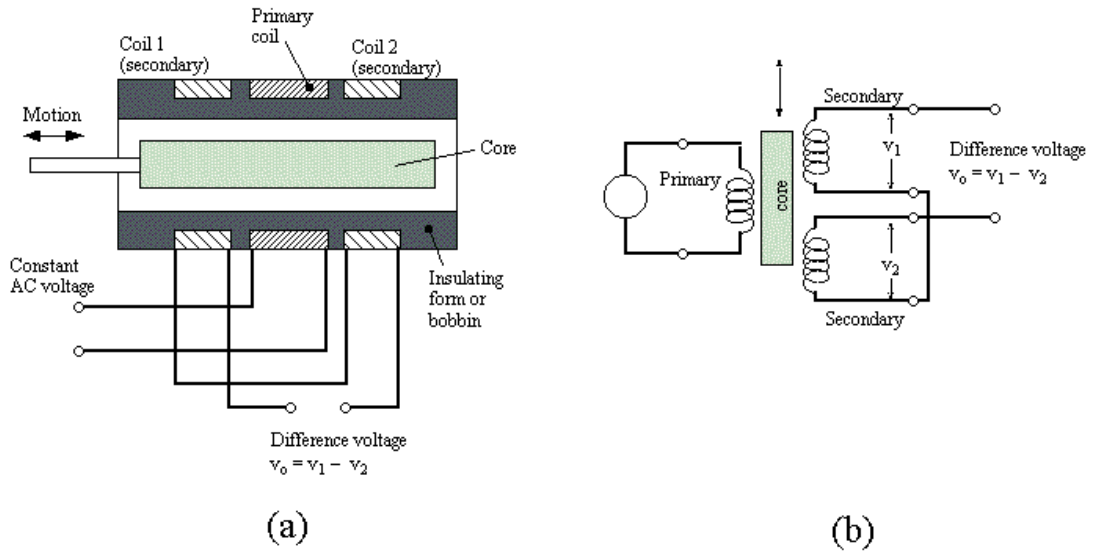


Figure 5.1: (a) the LVDT design is shown, where motion of the moving part results in movement of the inner core, and (b) shows the electrical circuit [43].

are conducted, where measurement during continuous operation and during impact collisions of increasing force is assessment. An analysis of the signal processing, strain gauge bridge sensitivity, and phasor characteristics of the integrated setup is discussed [50].

5.1.1 LVDTs

Linear variable differential transducers (LVDTs) are commonly utilised to measure displacement on spacecraft cryocoolers, as they can withstand high vibrations and shock levels, can operate at cryogenic temperature, and are a well-established and widely used technology. They are able to measure displacement or amplitude by converting a change in linear position into an electrical signal by means of electromagnetic coupling [51]. The latter therefore means that no electrical circuit setup is required, making it a technology involving limited contact between moving parts. Figure 5.1 shows the design of the LVDT. Motion of the moving core being measured changes the magnetic flux within the assembly and the coupling between the primary (excitation) coil and the secondary (output) coils to give a voltage change proportional to the displacement. The disadvantages of LVDTs are firstly, shielding is required from surrounding equipment due to the magnetic region being utilised for operation. Furthermore, LVDTs demonstrate distortions in output due to phase lag experienced as a result of the filtering that is needed to remove the carrier wave, and can be difficult to install. Alternatives to LVDTs are the use of laser transducers,

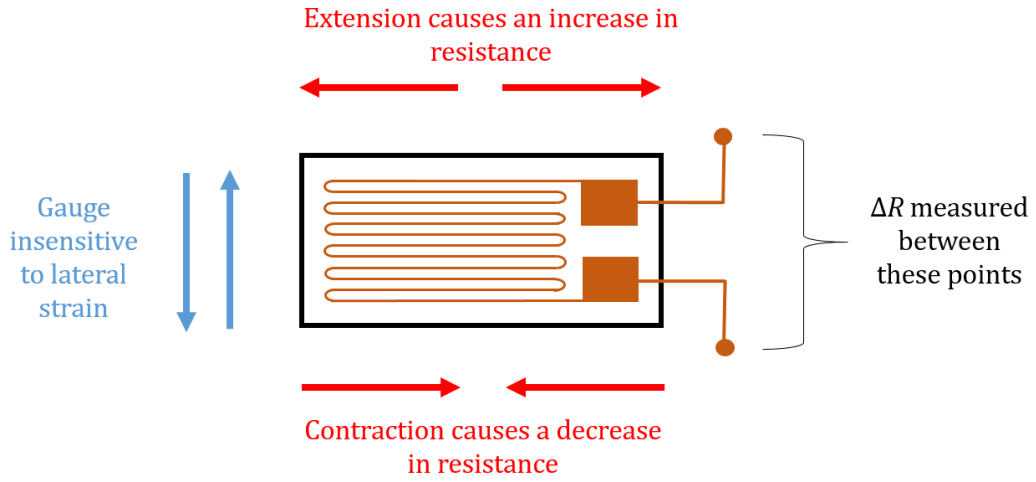


Figure 5.2: A single strain gauge bonded to a material surface is able to register changes in tension and compression to assess change in strain.

although undoubtedly, these are near impossible to integrate for in-situ use during spaceflight. Capacitive displacement sensors are further used in some applications, however, they present similar issues to LVDTs in terms of phase lag. The possibility of using strain gauges on flexure bearing springs, particularly for the application of cryocooler compressors, remains unaddressed.

5.1.2 Strain Gauge Bridge Configuration

Strain gauges are devices that can be used to measure changes in strain through a change in resistance, ΔR . Figure 5.2 depicts a single strain gauge that would be bonded to a material surface. It is able to measure a change in tension and compression as a voltage due to resistance change. The unstressed strain gauge typically has a resistance, R_G , within the range of 30-3000 Ω [52] and a 0.1% strain leads to a change in resistance of approximately 0.2%. In order to measure small changes in resistance with higher accuracy, the strain gauges need to be in a bridge configuration. Furthermore, the resistance of the strain gauge may not solely change because of a change in strain, but also because of a temperature change. The use of four active strain gauges in a full Wheatstone bridge configuration, as shown in Figure 5.3, mitigates for any changes in resistance due to temperature (R_T) being intrinsically compensated for, as it cancels in Eqn. 5.1.2.1. The strain gauges are used in pairs, where one will respond to compression and the other will register tension, by being mounted in parallel on either side of a surface. Any effect of temperature on the gauge resistance will therefore be cancelled. The sensitivity of the strain gauges is maximised

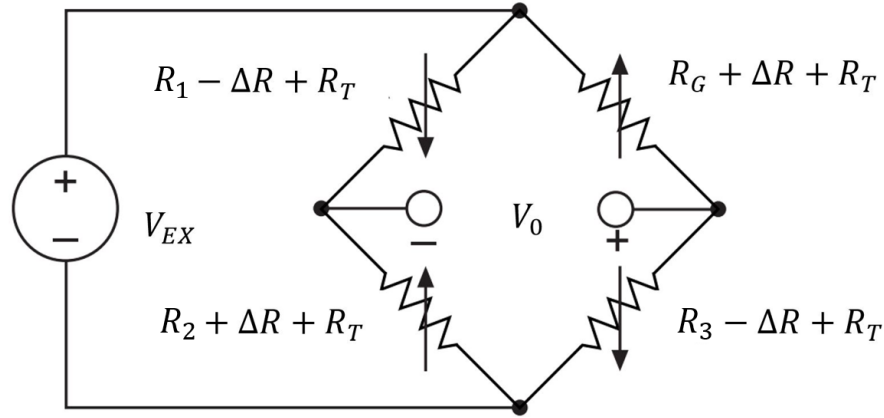


Figure 5.3: Full Wheatstone bridge configuration with 4 active strain gauges. V_{EX} is the excitation voltage.

when two gauges are mounted in tension and two in compression. The circuit shown in Fig. 5.3 allows the output voltage to be related to the resistance in the strain gauge as below.

$$V_0 = V_{EX} \left[\frac{R_3}{R_3 + R_G} - \frac{R_2}{R_1 + R_2} \right] \quad (5.1.2.1)$$

The gauge factor, GF, is the ratio by which the electrical resistance changes due to a change in strain, and is given by Eqn. 5.1.2.2. This is formulated with the assumption that temperature compensation is accounted for by the bridge circuit.

$$GF = \frac{\Delta R}{\epsilon_s R_G} \quad (5.1.2.2)$$

In Eqn. 5.1.2.2, R_G is the unstrained resistance of the strain gauge, ΔR is the change in resistance, and ϵ_s is the strain. If the current supplied to the circuit remains constant, the ratio of change in voltage to supplied voltage can be used to obtain values of strain measured by the strain gauge.

5.1.3 Finite Element Analysis

An existing FEA model of the spring arm was manipulated in order to simulate the direction and magnitude of principal stresses acting on the spring arm during displacement in the designed setup. A strain gauge was then installed onto each spring arm in the location and along the direction vector that the highest principal stress was observed in the simulation. Furthermore, the strain at the location of the installed strain gauge was computed for a parametric sweep of spring arm displacement. Figure 5.4 shows the contour plot of strain along the spring arm along the x -axis, which is indicated by the coordinate system depicted. This coordinate system was defined with its

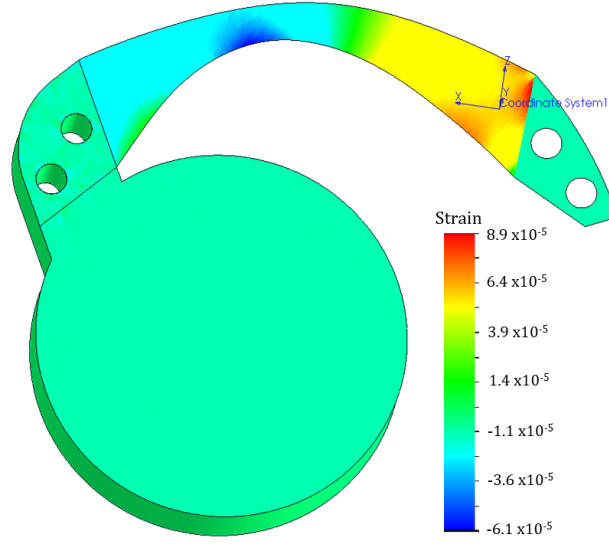


Figure 5.4: The strain along the spring arm in the x -axis direction while the spring is displaced by 8 mm; the origin point of the coordinate system indicates the location of the strain gauge.

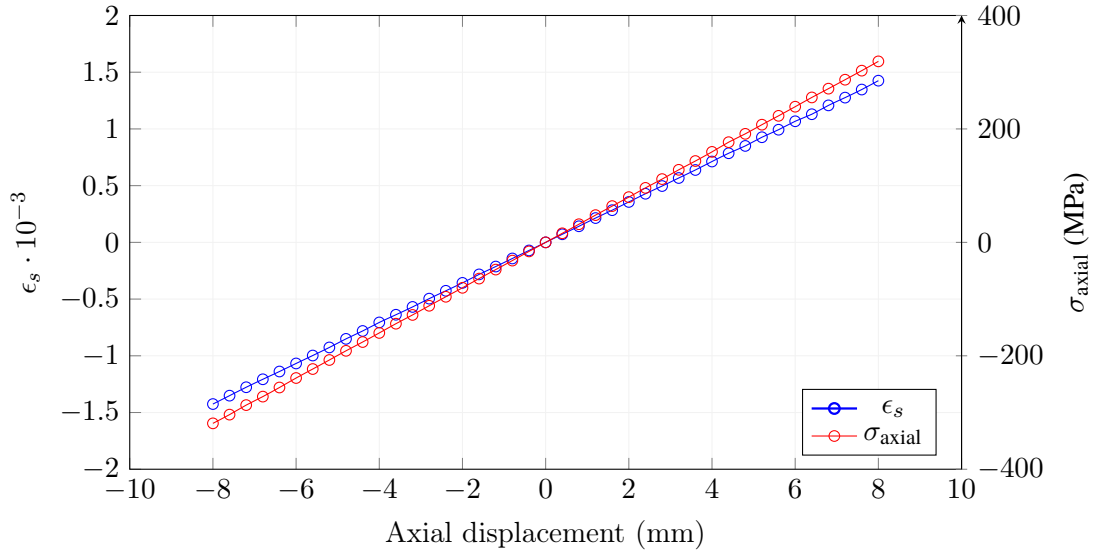


Figure 5.5: Strain in the axial direction at the location of the strain gauge on the spring arm during axial displacement, as computed by the SolidWorks simulation.

origin at the location of the highest principal stress, and the x -axis was established along the vector direction of the highest principal stresses. The origin therefore also represents where the strain gauge is to be installed. Expectedly, the strain can be seen to be the highest in the region where the principal stress is the highest. Using the results obtained from the Figure 5.4 simulation, the strain and the axial stress, σ_{axial} , over a ± 8 mm range of spring displacement is presented, as shown in Figure 5.5. This is expectedly linear due to being in the linear elastic region of the spring.

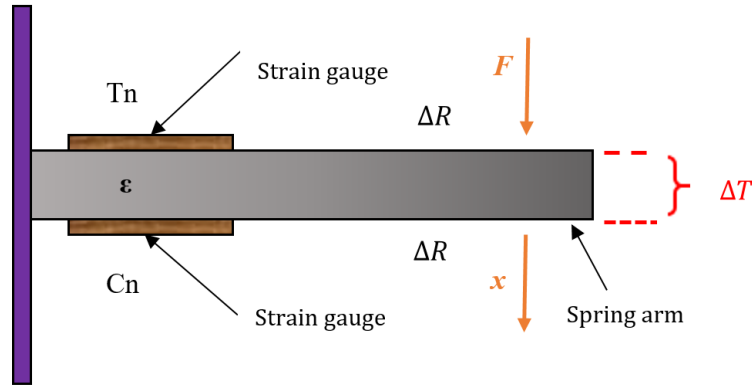


Figure 5.6: A strain gauge installed onto either side of the spring arm. T_n refers to tension, C_n is compression, ΔT , all other symbols are as previously defined.

5.2 Experimental Design

5.2.1 Strain Gauge Installation

The $120\ \Omega$ foil strain gauges of size 2.5 mm by 6 mm obtained from RS Components were installed in full bridge configuration. Two strain gauges were installed per spring arm, with one being placed on the top and one directly in parallel on the underside of the spring arm, as shown in Figure 5.6. In this way, the strain gauges on either side of the spring arm are able to respond to tension and compression due to a force applied, F . For a change in temperature, ΔT , the full bridge configuration provides intrinsic temperature compensation, as outlined in section 5.1.2. The strain gauges were installed through a standard procedure; undertaking surface finishing on the spring arms prior to installation, and applying acid and alkali solutions to ensure the surface was properly decontaminated. M-Bond 200 Adhesive from Micro Measurements, a strain gauge specific superglue, was utilised to bond the strain gauge to the spring arms. By addressing the FEA completed for the spring arms in question, the region where the highest principal stress is experienced was identified. The strain gauge was placed onto the spring arm in this region, and was aligned with the direction of the principal stress vectors, as shown in Figure 5.7. The GF for the strain gauges was provided by the manufacturers as 2.00.

Figure 5.8 shows the strain gauges connected into a full bridge configuration on the compressor spring. This is then connected to the instrumentation amplifier and data acquisition.

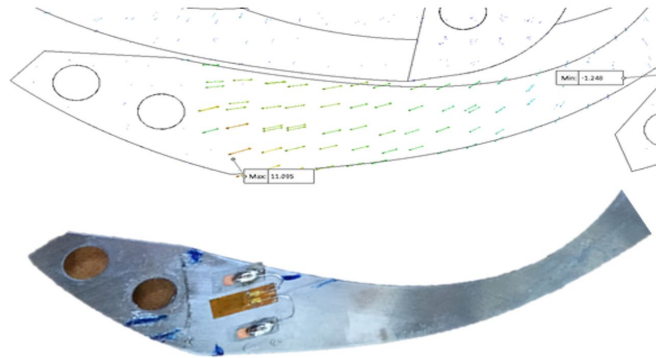


Figure 5.7: Placing the strain gauge onto the region with the highest principal stress and aligning it with the direction of the stress vector.



Figure 5.8: The strain gauges can be seen as installed in a full bridge configuration on two of the 3 spring arm pairs [blurred due to confidentiality].

5.2.2 Testing Rig

The strain gauge bridge testing setup is similar to the fatigue testing rig, in that the spring is mounted on the same stand and largely powered by the same equipment. An isolation amplifier was built to attenuate the signal reaching the Data Acquisition System (DAQ) from the electromagnetic motor coil. An Omron laser displacement transducer was mounted directly above one of the SG-installed spring arms, permitting calibration measurements of displacement in order to validate the output signals received from the strain gauge bridge, and recorded displacement with an error of ± 0.05 mm.

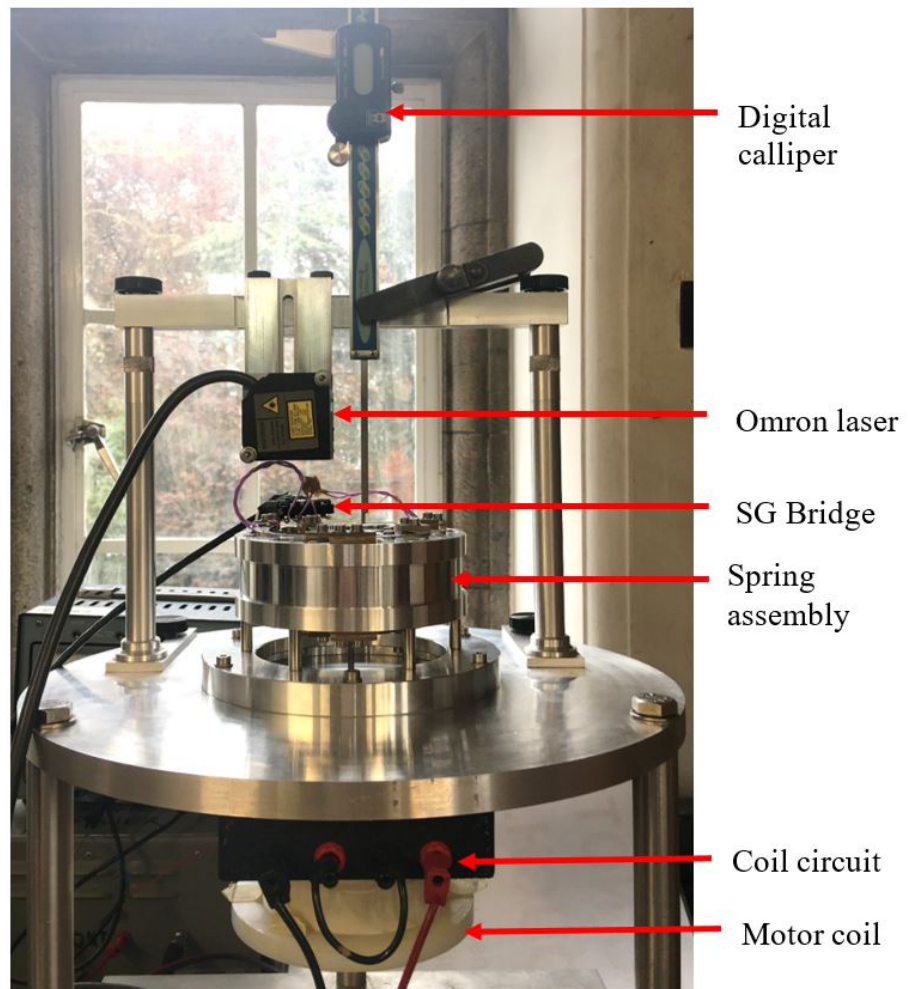


Figure 5.9: Strain gauge-installed spring testing setup. A digital calliper and Omron laser displacement transducer are utilised for SG bridge output calibration.

5.2.3 Isolation Amplifier Design

For a setup in which signals of high voltage need to be recorded, an isolation amplifier is required to attenuate and isolate the signal; the DAQ input is limited to ± 10 V. An isolation amplifier circuit was built as shown in Figure 5.10, which included a Hall effect current transducer LA 25-NP manufactured by LEM Electronics and an AD620 amplifier from Analog Devices. The total input resistance is $100\text{ k}\Omega$ and resistors R1 ($47\text{ k}\Omega$), R2 ($10\text{ k}\Omega$) and R3 ($43\text{ k}\Omega$) act as 10:1 potential dividers. The isolation amplifier was then assessed for gain and phase lag, as shown in Figure 5.11, in order to ensure that the correct voltage outputs were being acquired and constructed by the DAQ system.

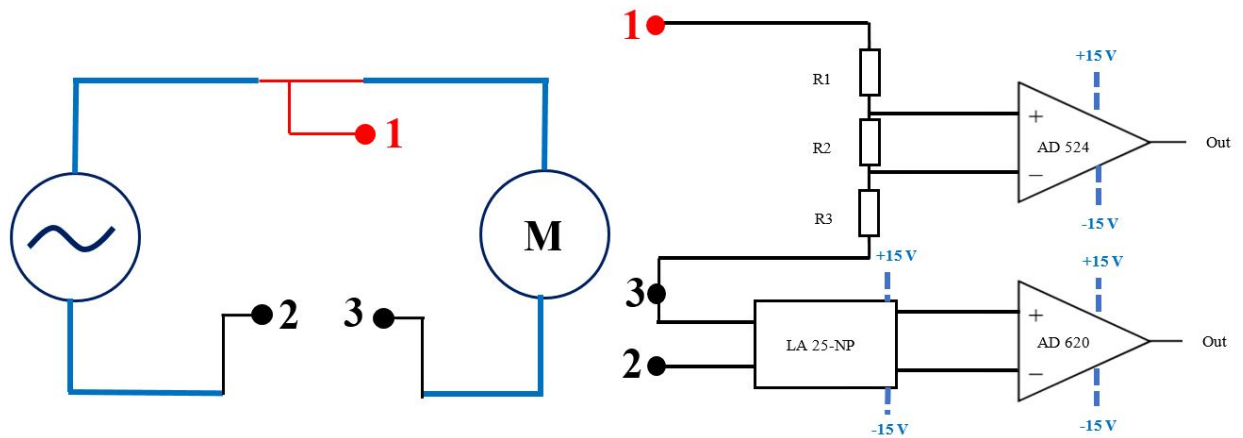


Figure 5.10: The isolation amplifier simplified circuit diagram. An attenuation by a factor of 10 is achieved.

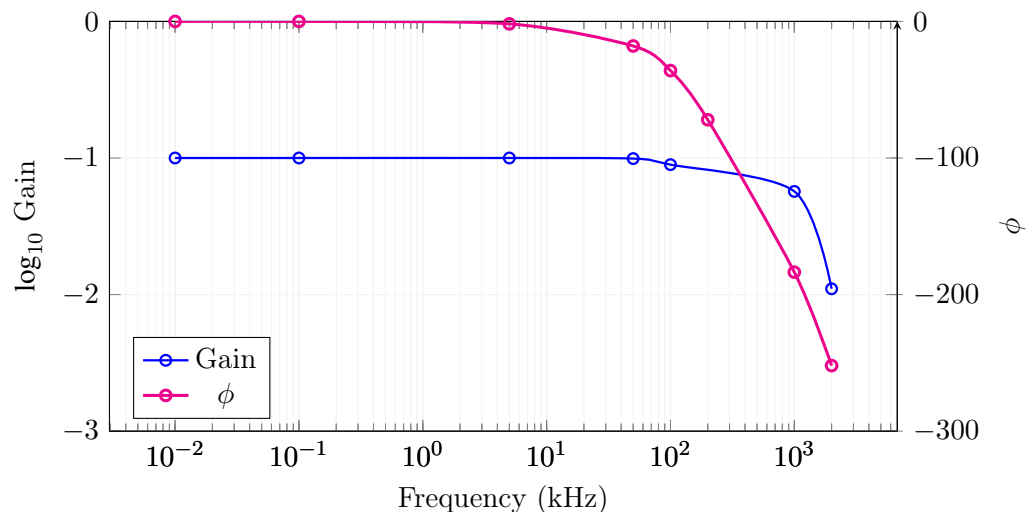


Figure 5.11: Logarithmic gain and phase lag of the isolation amplifier.

5.2.4 Static Spring Offset

Both the digital caliper and the Omron laser displacement transducer have arbitrary datums, therefore their outputs need to be determined when the springs have zero displacement. Given that the spring assembly is loaded by the weight of the magnet and central spring mounting when set up vertically, the true zero displacement position of the spring is offset due to gravity. This offset was determined by recording the voltage measured by the Omron laser position sensor whilst the setup is in three configurations; upright, horizontal at 90° , and upside-down at 180° . Thereafter the Omron calibration was utilised to determine the corresponding displacement for this voltage, as given by Eqn. 5.2.4.1. The values recorded are shown in Table 5.1 and the midpoint voltage is 1.058 V.

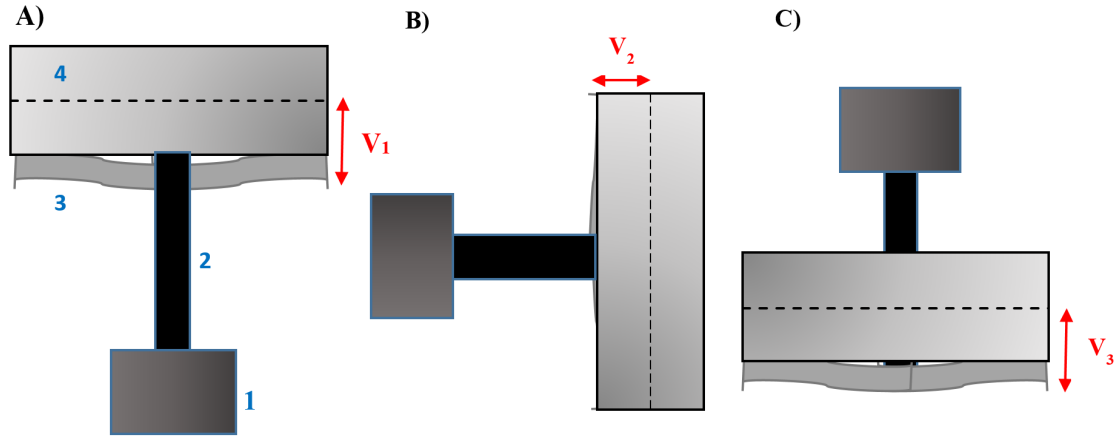


Figure 5.12: Omron displacement measurements recorded for three orientations of the setup to determine the true zero displacement position of the spring for zero gravity. (1) indicates the magnetic actuator attached to the spring corpus (4) with a metal rod (2) which causes displacement of the spring arms (3) in configurations A and C.

Table 5.1: Recorded Omron voltage values for the static spring configurations A, B and C, and the determined spring static offset (mm).

V_1 (V)	V_2 (V)	V_3 (V)	$\frac{V_1+V_3}{2}$ (V)	Static Offset (mm)
1.18	0.937	1.056	1.058	-1.53

$$\delta_{s,Omron} = -12.594V_{Omron} + 13.659 \quad (5.2.4.1)$$

Where $\delta_{spring, Omron}$ (mm) is the true spring displacement determined from the Omron calibration. The offset due to gravity of the static spring at rest was therefore found as -1.53 mm.

5.2.5 System Characterisation

System Resonance

The system resonance was assessed as shown in Figure 5.13a. A peak of much greater current over amplitude can be observed at a log frequency of approximately 1.155, indicating resonance at this point, where the signal tends to infinity. Figure 5.13b indicates the spring setup pk-pk response to input drivers of voltage and current into the electromagnetic coil, both are expressed as voltages, by the strain gauge bridge and the Omron laser. It can be seen the moving magnet motor is very sensitive to smaller changes of current and voltage input into the electromagnetic coil, resulting in rapid increases in the resultant pk-pk spring stroke.

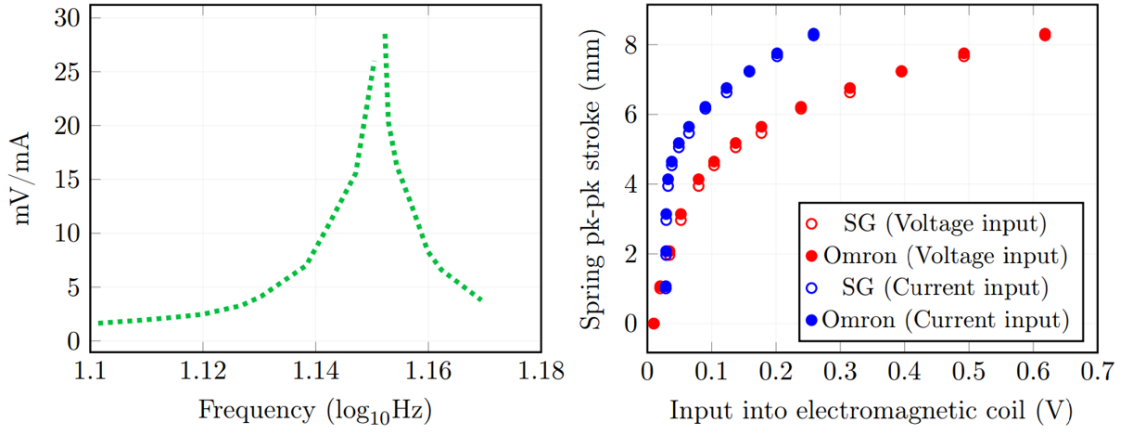


Figure 5.13: (a) Characterising the system resonance and (b) the spring setup pk-pk response to input drivers of voltage and current into the electromagnetic coil, expressed as voltages, by the strain gauge bridge and the Omron laser.

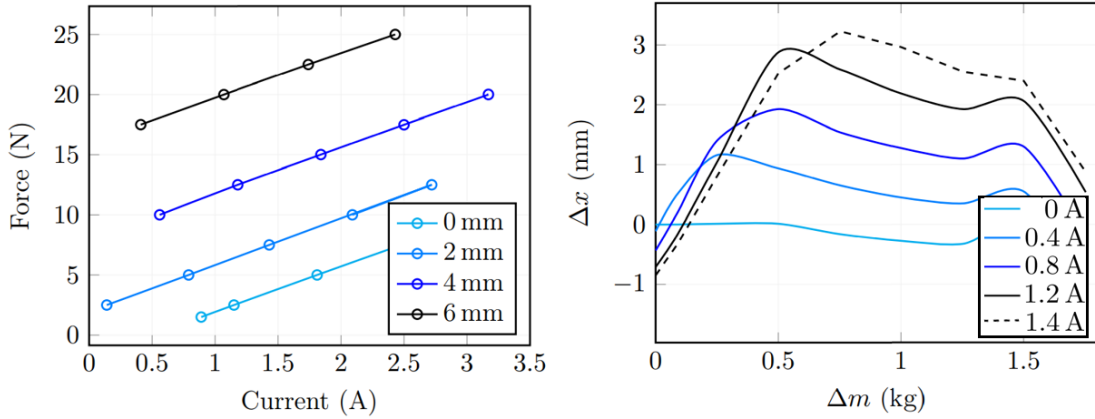


Figure 5.14: (a) Characterising the linearity and motor-force constant for the electromagnetic drive for the integrated spring testing setup at varying spring displacements and (b) system characterisation for stiffness with added mass at fixed current inputs to the electromagnetic coil drive.

Force-Motor Constant and Added Mass

The variation of force as a result of current applied to the magnetic coil is a key parameter that characterises the moving magnet's use as a motor in the spring setup. This variation must therefore be assessed in order to quantify the factor by which the motor is additionally displaced per unit current delivered to the motor coil. This constant, achieved by establishing the force-current relationship, was determined by conducting a series of static tests that recorded the displacement of the motor with varying weights (force) applied to the motor. The displacement of the motor was measured with an Omron laser displacement transducer. The first series of tests were completed with no current passing through the system, the second with 1 A of current in one direction, and

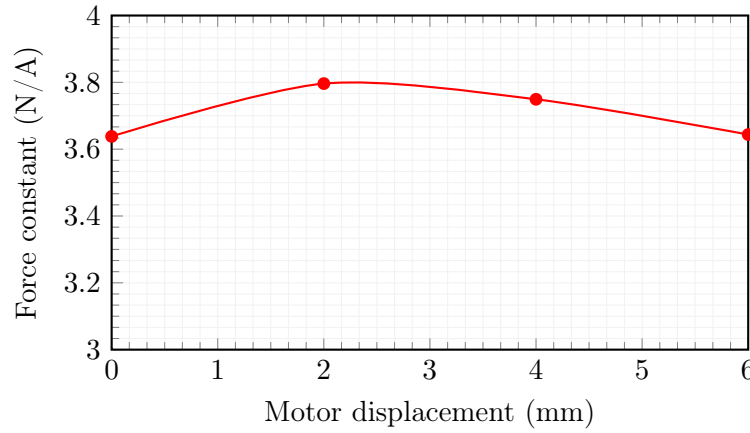


Figure 5.15: Little variation between the force-current constants measured at varying displacements with different forces applied.

the third with 1 A in the other direction. This therefore allowed the resultant force with and without current to be assessed and hence, quantifying the current-force constant; the current that allows the motor to be brought back to a specified position with different weights added.

Figure 5.14a demonstrates the linearity of the relationship between the current through the motor system and the force applied to the spring. The gradient of these relationships are expectedly similar in value as shown in Figure 5.15, giving the force-current constant for the spring-motor assembly, averaging as a constant of 3.71 N/A. This matches the force-current constant simulated using FEMM software in chapter 4.

5.2.6 Data Acquisition

In order to register and record signals in the test rigs used for analysing the flexure springs, a LabVIEW program with a Data Acquisition (DAQ) card was programmed. Four channels were set up to log current and voltage driving the moving magnet motor, as well as the voltage output from the strain gauge bridge configuration and the laser transducer for displacement measurement.

A sample rate of 100 ksamples/s was used in acquisition operations. As per the Nyquist-Shannon theorem, for a maximum frequency f of a component within a signal, the minimum sampling rate must be set as $2f$, in order to ensure a sample period suitable for data collection of the sine functions expected. This prevents aliasing, where sampling of the data does not occur frequently enough for a higher frequency signal to be constructed from the data.

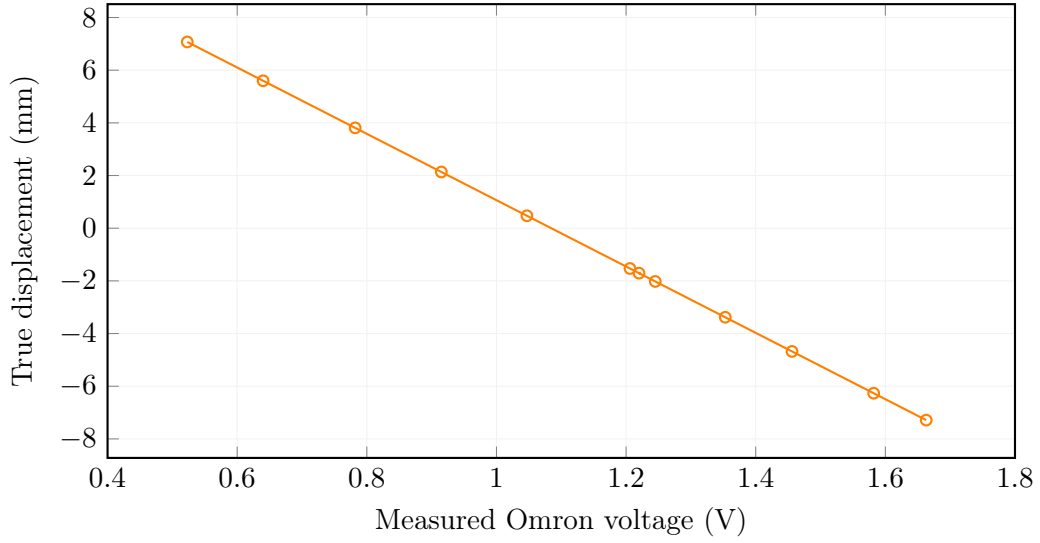


Figure 5.16: Static calibration for measured Omron voltage against true spring displacement.

5.3 Experimental Testing

5.3.1 Omron Calibration

The reliability of the Omron displacement transducer was verified and calibrated with the use of a digital caliper. The caliper was used to create and record incremental spring displacement in a downward direction. A mounted plastic rod was used to displace the spring in the upwards direction, making use of packing pieces to control the displacement. With incremental displacement of the spring in either direction, the Omron voltage was recorded both using a digital voltmeter and LabVIEW. The latter two tools recorded comparable values for Omron voltage for a given displacement applied, confirming the validity of the LabVIEW program devised. The spring displacement values were then corrected to account for the static displacement offset of the spring, to yield values for true displacement. The relationship for Omron voltage and true spring displacement was hence obtained, and is represented in Figure 5.16. The Omron measured voltage corresponding to the spring at rest whilst subjected to gravity, $V_{s,Omron}$, was found to be 1.206 V, while the Omron voltage reading for zero-gravity displacement of 0 mm, $V_{0,Omron}$, was established as 1.08 V; where Eqn. 5.2.4.1 crosses the x -axis.

5.3.2 Strain Gauge Bridge Calibration

The strain gauge bridge configuration was calibrated in a similar manner to that of the Omron displacement transducer. The digital caliper was used for calibration and the corresponding true

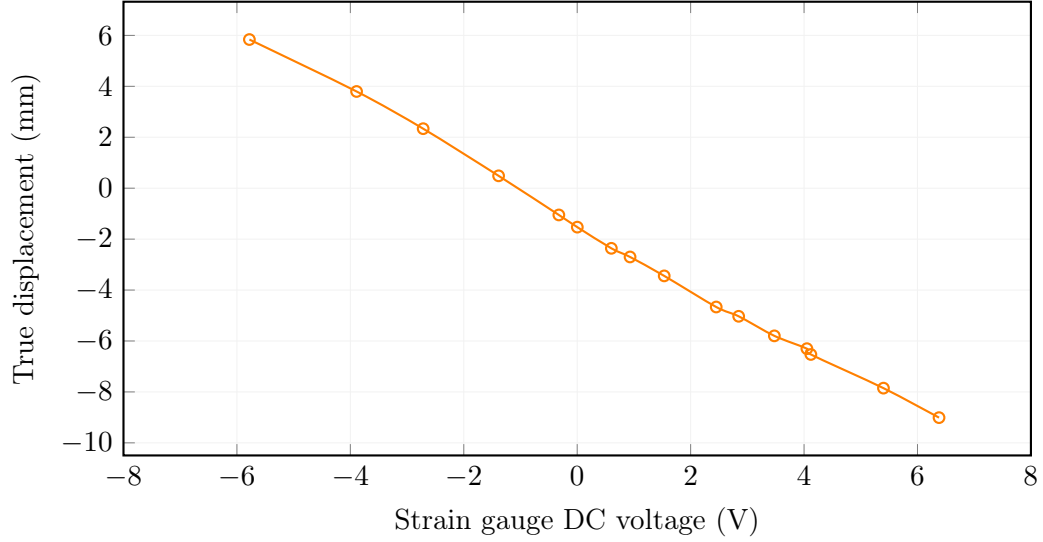


Figure 5.17: Measured Strain Gauge DC voltage against true displacement, as obtained with the Omron calibration.

displacement values were determined by using the Omron calibration. A Fylde bridge amplifier (model FE-537-SGA) was used with a bridge voltage of 2 V and a gain of 1000. The Fylde amplifier has an automatic zeroing facility that compensates for any initial imbalance in the bridge voltage, so 0 V corresponds to the assembly having an offset due to gravity of 1.53 mm. Recording the static voltage of the strain gauge, $V_{s,SG}$, and plotting against the corresponding true displacement using the Omron calibration, the calibration is achieved as shown in Figure 5.17.

The calibration equation for the strain gauge bridge configuration can now be used to calculate the corresponding spring displacement, as given in Eqn. 5.3.2.1.

$$\delta_{s,SG} = 0.0027V_{SG}^3 + 0.0085V_{SG}^2 - 1.3238V_{SG} - 1.4545 \quad (5.3.2.1)$$

A cubic has been fitted for the calibration because the spring as a whole demonstrates non-linear behaviour as force is applied to it (as shown by Figure 5.14b).

5.3.3 FEA Validation

The performance of the strain gauges in the experimental rig can be compared to that simulated in the FEA in section 5.1.3. The voltage output from the strain gauge full bridge can be converted to strain using Eqn. 5.1.2.2. The constant voltage supplied to the bridge is 2 V and the amplifier gain is 1,000. Figure 5.18 shows the change in strain over a spring displacement range of ± 8 mm as

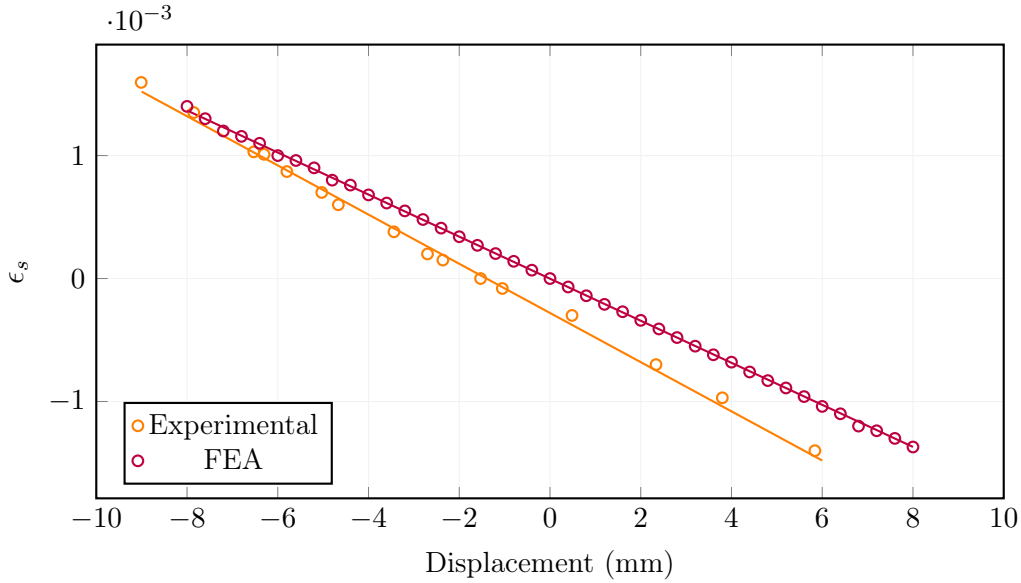


Figure 5.18: Change in strain as the spring over a ± 8 mm range, as measured experimentally and as simulated in the SolidWorks FEA.

simulated by the SolidWorks FEA compared with that processed from experimental data. Whilst there is some discrepancy between the achieved values from the strain gauge bridge testing and the FEA, both relationships are linear.

The differences between the computed strain and measured strain could be due to potential misalignment of the strain gauge along the axis of principal stress and location of the strain gauges. Nonetheless, the linearity of both the computed and strain gauge measured relationships implies that the strain gauge bridge can suitably measure a linear change in displacement for a proportional change in strain. It is therefore suitable for the purposes of measuring compressor stroke for isolated displacement. Hereafter, dynamic tests were completed to determine the suitability of the strain gauge bridge for measuring continuous operation compressor stroke.

5.3.4 Dynamic Tests

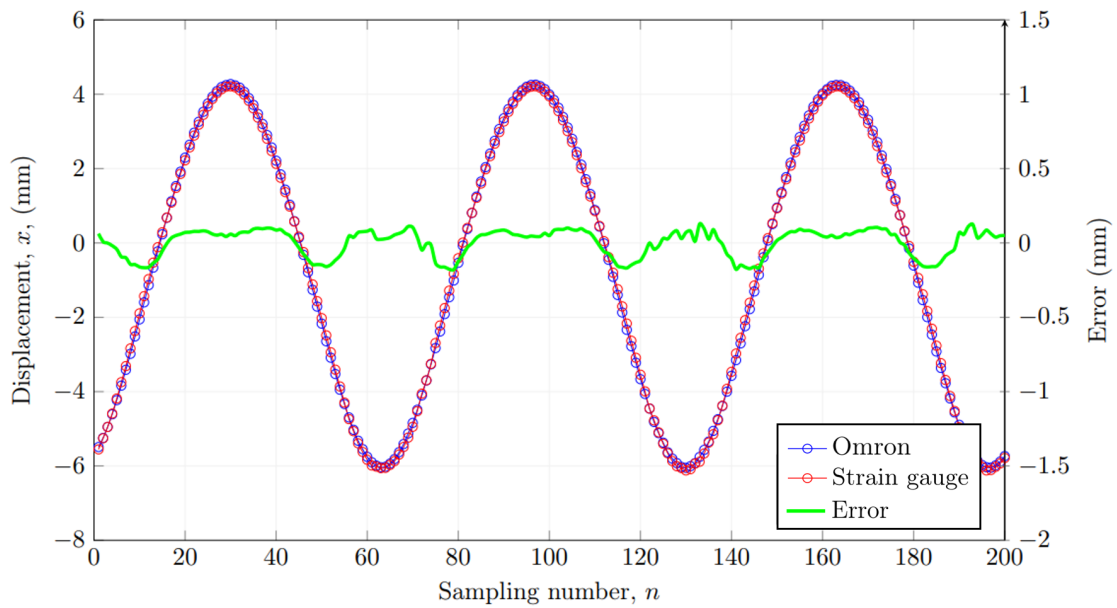
The spring was oscillated at 15 Hz, its resonant frequency, while the RMS voltage was incrementally increased by altering the signal amplitude, to achieve the corresponding displacement values as given in Table 5.2.

$$\delta_{pk-pk} = \frac{2\sqrt{2}}{0.0794} V_{RMS} \quad (5.3.4.1)$$

The theoretical pk-pk values were obtained by using Eqn. 5.3.4.1, where 0.0794 V/mm is the

**Table 5.2:** RMS voltages and corresponding pk-pk displacements (mm) applied to the oscillating spring.

RMS Voltage (V)	Theoretical pk-pk Displacement (mm)
0.07	2.5
0.14	5
0.21	7.5
0.28	10

**Figure 5.19:** The output signals from the Omron transducer and the SG bridge processed as compressor stroke for 10 mm pk-pk amplitude. Error, the difference between the two, is displayed on the secondary axis.

gradient obtained from the Omron transducer calibration. The calibration equation was then utilised to process the output signals and convert into displacement.

It can be seen from Figure 5.19 that the two displacements calculated from the strain gauge and Omron laser displacement transducer measurements agree, with an average error of 3.13% between the two methods and an average standard deviation of 1.44%, as indicated in Figure 5.20. It can be seen that the absolute error increases as the amplitude of the spring oscillation increases, but decreases as a percentage of the stroke. The present calibration is valid within certain stroke limits; beyond these limits, a new calibration would need to be established.

5.3.5 Impact Tests

The strain gauge bridge configuration has so far proven successful in measuring static displacement and continuous operation dynamic spring motion. One key thing that requires assessing is the bridge

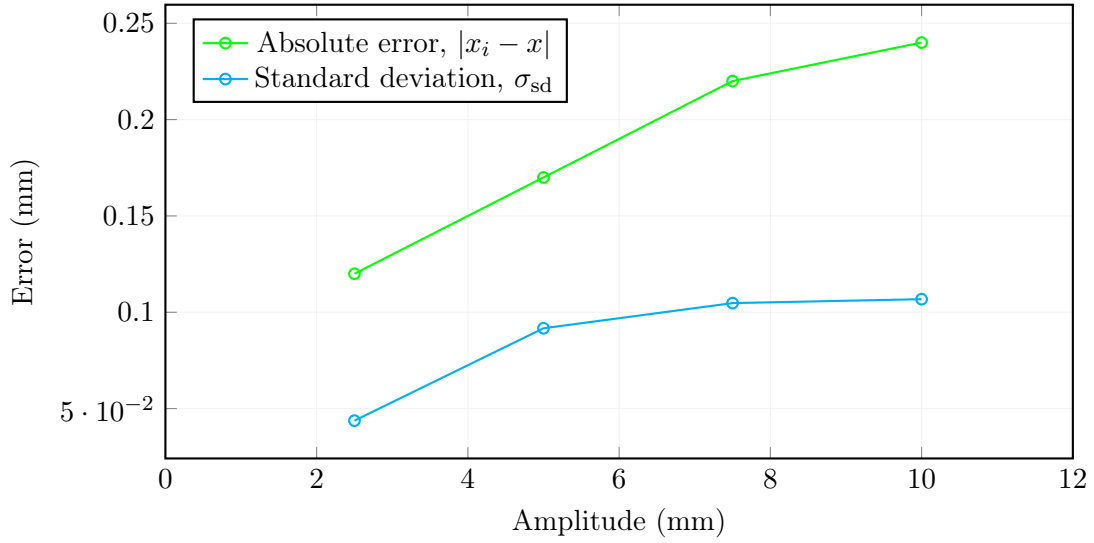


Figure 5.20: Average error and standard deviation between Omron and SG bridge for pk-pk amplitude.

configuration's suitability for measuring impacts, given the importance this plays in cryocooler failure, as discussed in section 5.1. To assess the different natural frequencies that the spring will exhibit during impact, and by extension, ascertain the ability of the strain gauge bridge to detect additional harmonics, a series of impact tests were conducted at 14 Hz (resonant frequency). Either a low pass or high pass filter can accordingly be utilised depending on the purpose; it is therefore useful to further explore the strain gauge circuit's sensitivity in this way. Impact was created in two manners; one by generating a pluck force on the spring arm and recording the strain gauge bridge response, and the other by installing a fixed boundary that the spring will repeatedly hit at 4 mm amplitude on one end of the axial displacement direction during continued forced oscillation. The data acquired showed how the frequency of the spring changes as the impact is made and as the input into the motor coil is increased. The harmonics in the system were then assessed using MATLAB Fast Fourier Transform (FFT) functions, utilising Hanning or Tukey windows where indicated. These analyses are detailed in the discussion below.

No Impact and Pluck Test

When no impact occurs on the spring, the output signal from the strain gauge bridge can be assessed for usual harmonic frequencies present in the spring oscillation. Figure 5.21 shows the FFT analysis. The principal frequency features at 14 Hz (f), a second harmonic can be seen at 28 Hz ($2f$), and a third at 42 Hz ($3f$). Next, a pluck force was applied to the spring arm, generating

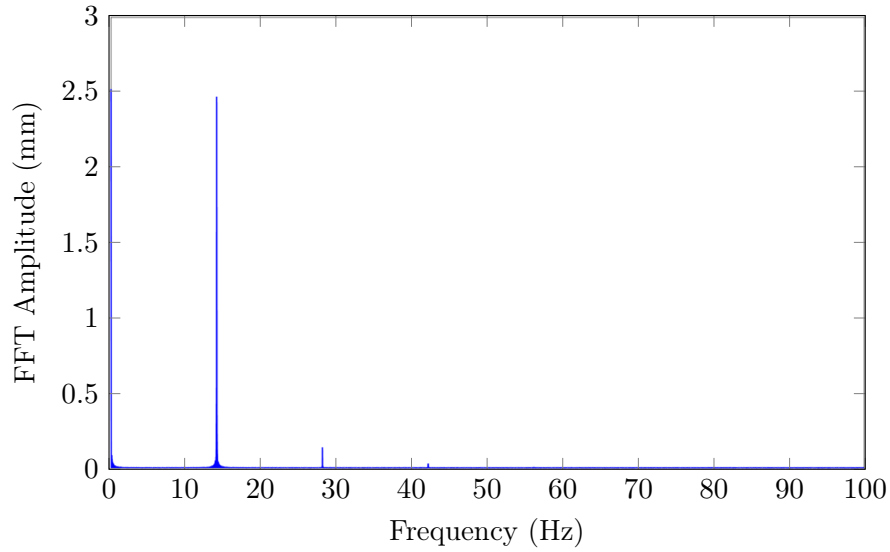


Figure 5.21: FFT of 5 mm pk-pk amplitude signal from the strain gauge bridge with no additional impact.

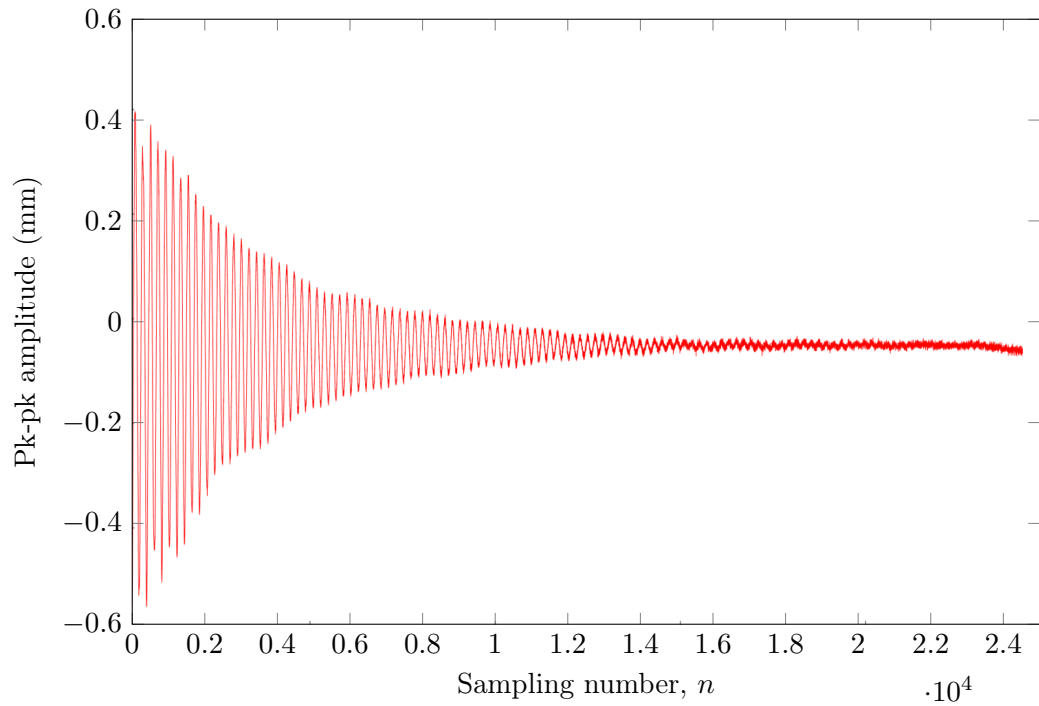


Figure 5.22: Output signal of the pluck test from the SG bridge.

the signal output shown in Figure 5.22, and the FFT plot to indicate the harmonics of the plucking is shown in Figure 5.23. A Tukey window was applied to the signal to 90%, in order to avoid artefacts being introduced when sampling part of a continuous signal and produce clearer harmonic peaks during the FFT. A harmonic can be seen at approximately 4.3 kHz.

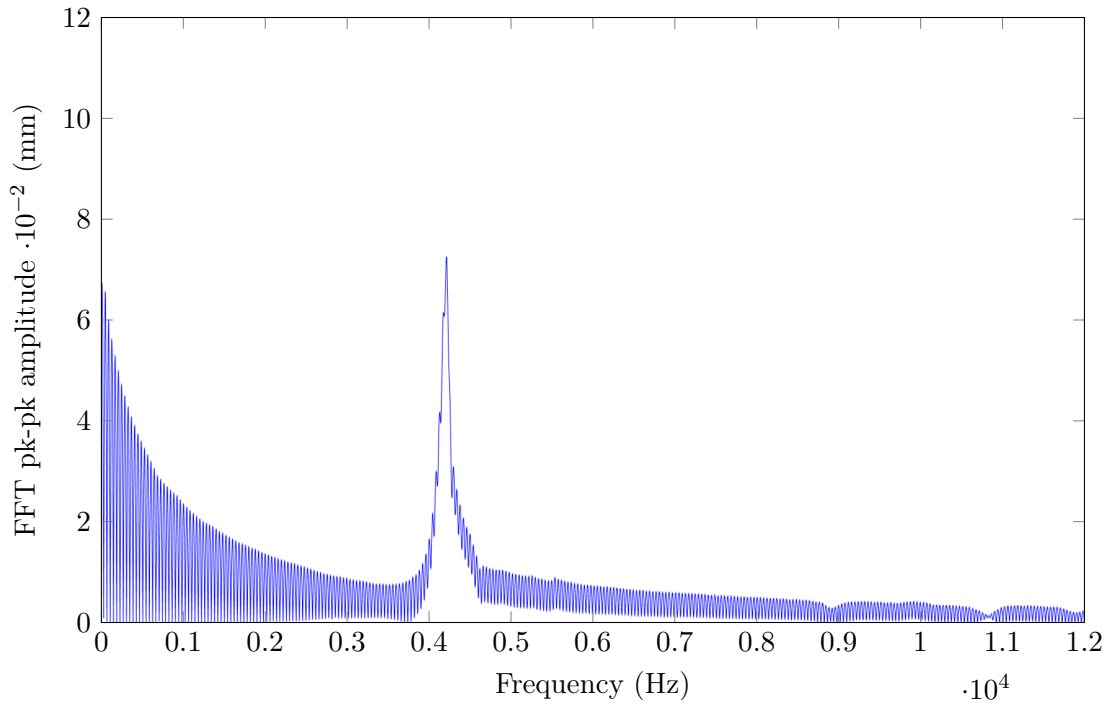


Figure 5.23: FFT plot of the signal indicating the harmonic peaks in the pluck test signal shown in Figure 5.22.

Fixed Impact Tests

The fixed impact tests were completed by installing a fixed boundary at 4 mm distance above the spring which the spring arm would then repeatedly hit whilst made to oscillate at varying amplitudes. Signal power was supplied to equate to strokes of 7 mm up until 11.5 mm at 0.5 mm increments. Tests were not completed beyond 11.5 mm as the noise generated during impact was significantly loud at this stage and further impact force posed the threat of irreparably destroying the setup. Nonetheless, a suitable range and significant impact force was achieved at the upper end of the testing range for the purposes of the tests.

The output signal from the strain gauge bridge, shown in Figure 5.24, corresponds to operation at an input signal for 8.5 mm stroke with an impact occurring at 4 mm. In the signal, sharpened motion can be seen in response to the impact, producing oscillations in the spring that the strain gauge bridge is able to detect. A Hann window was applied to the FFT signal and the harmonic peak can be seen at 14 Hz in Figure 5.25. The impact has led to the broadening of the resonant frequency, as more than one mode is now present. The zoomed-in view of the impact in Figure

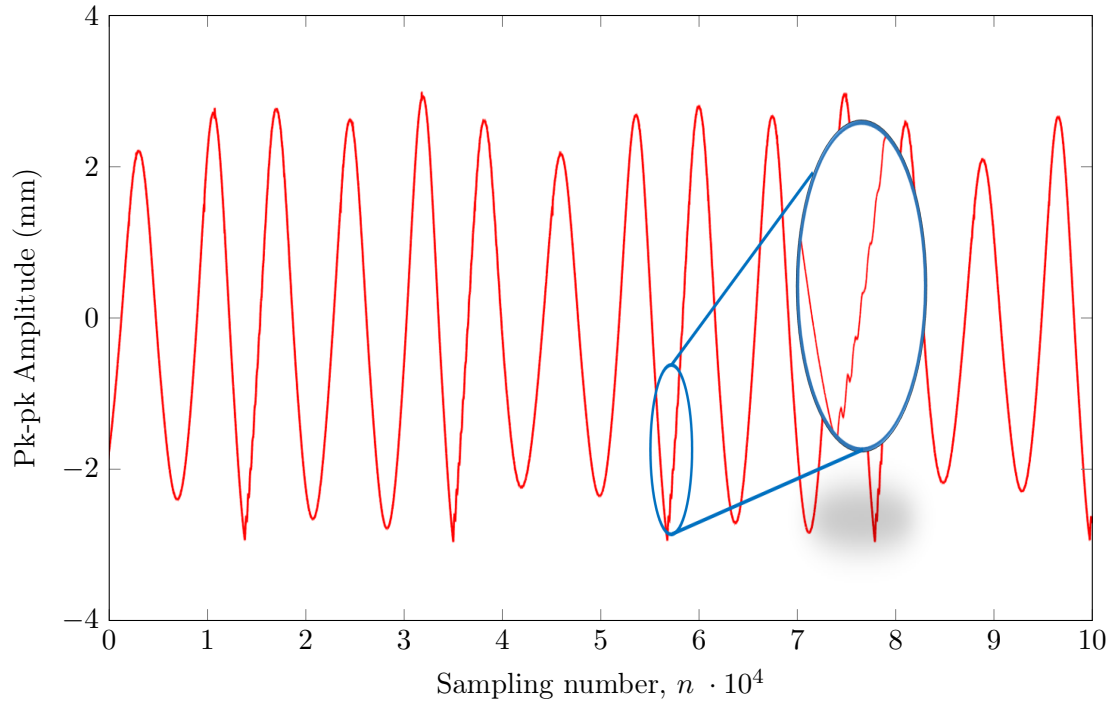


Figure 5.24: The signal at 8.5 mm stroke and a resonant frequency of 14 Hz whilst subjected to a repeated fixed impact is shown, with a zoom-in on the signal post-impact.

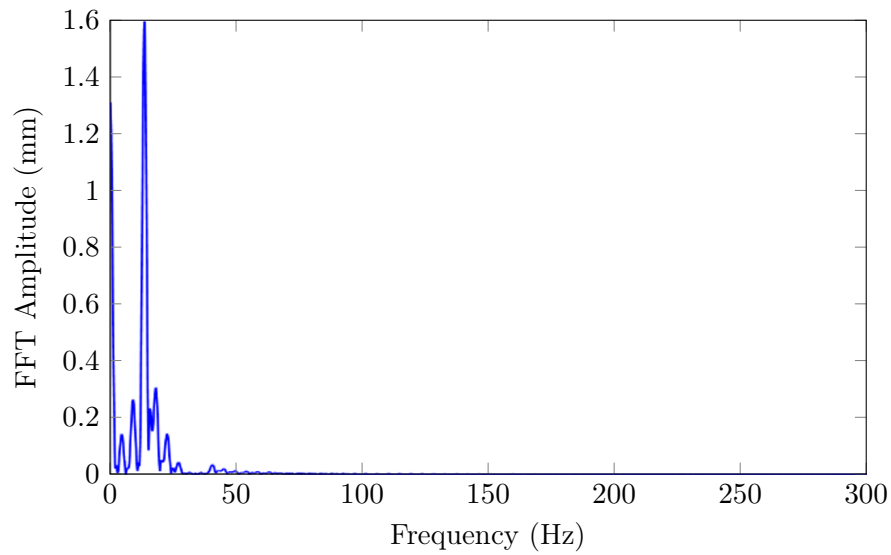


Figure 5.25: The FFT plot of the signal in Figure 5.24 using a Hann window is shown.

5.24 shows that the higher frequency components are present.

Figure 5.26 shows a selection of impact output signals from the strain gauge bridge for varying stroke inputs between 7 mm to 11.5 mm and a frequency of 14 Hz. The increased distortion of the output signal can be seen with increasing impact force due to increased operating stroke.

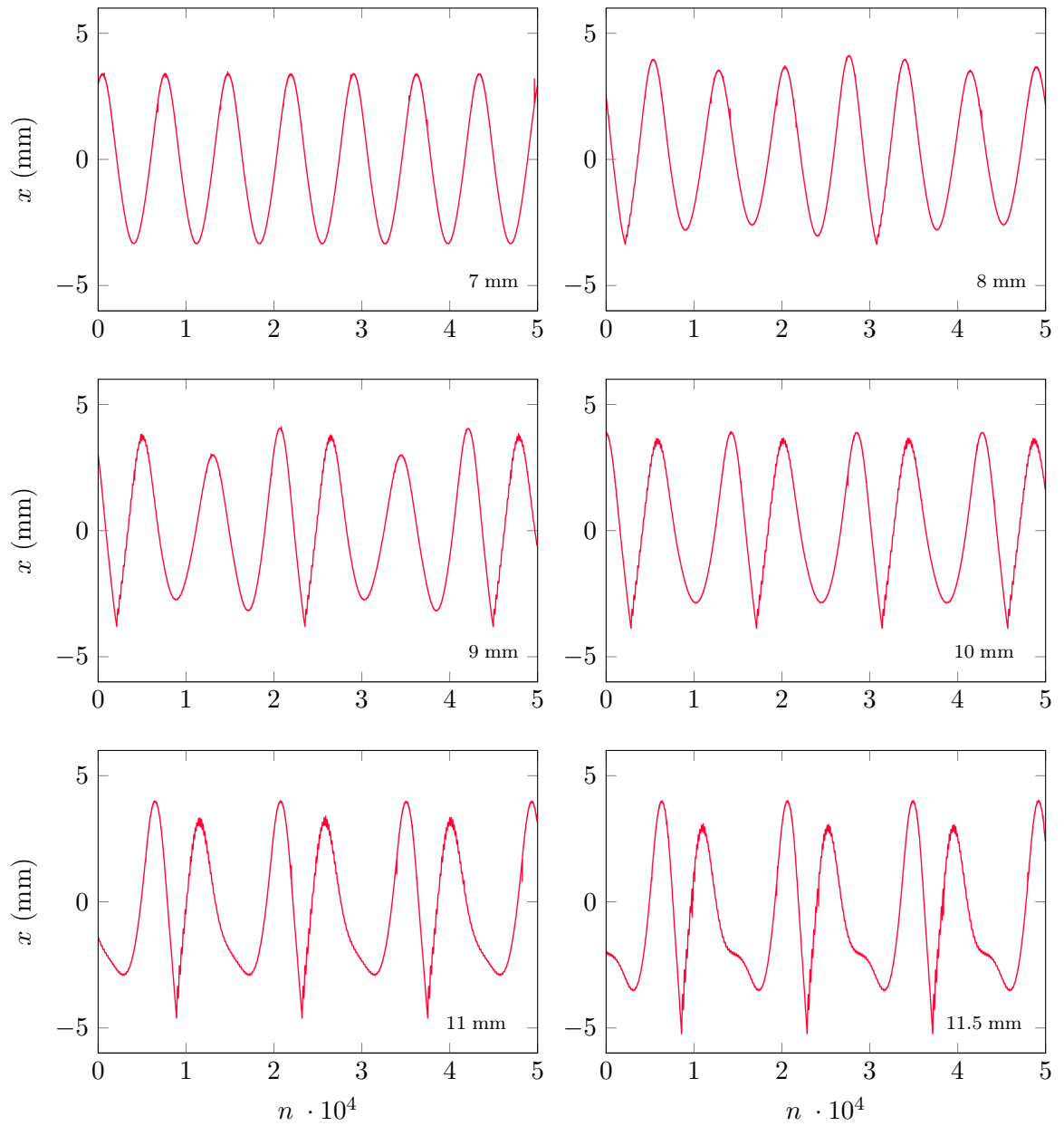


Figure 5.26: Displacement, x , as measured by the SG-spring setup, where the effect of the fixed impact on the SG-spring setup driven at signal inputs for strokes of 7 mm, 8 mm, 9 mm, 10 mm, 11 mm, and 11.5 mm at a frequency of 14 Hz can be seen. The fixed impact is situated at a 4 mm height from the rest position.

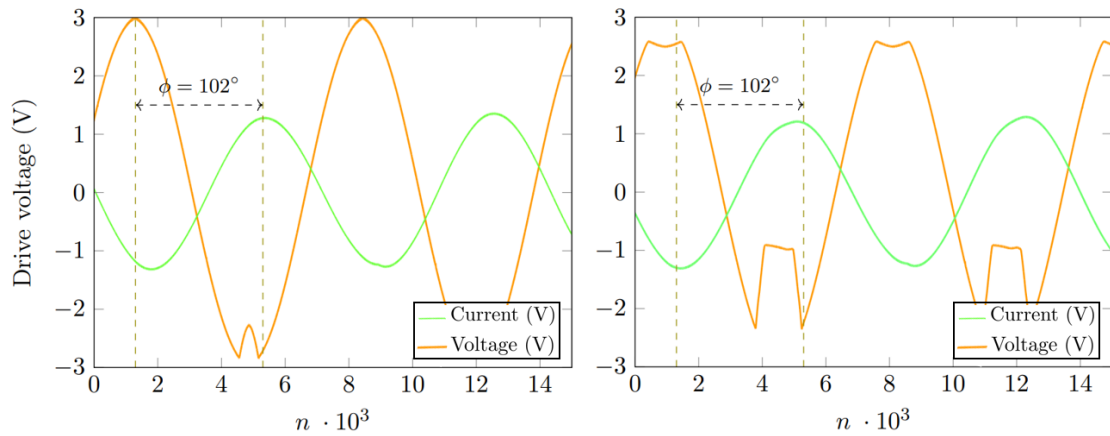


Figure 5.27: The drive voltage and current into the coil, expressed as a voltage, during the impact tests for a signal input for 11 mm (left) and 11.5 mm (right) stroke.

Impact Forces and Back-EMF

In Figure 5.27, the input drive voltage and current, expressed as a voltage, can be seen for a signal input equivalent to 11 mm (left) and 11.5 mm stroke (right) where impact occurs at 4 mm height from the rest position at one end of the axial direction of displacement. It can be seen that the voltage input signal experiences a distortion as a result of the input. This is likely due to the back electromotive force (emf) in the magnetic coil following the reactive force generated at impact. A phase difference between the voltage and current signal can be observed, where the voltage leads the current by approximately 100° , as is expected in an inductive circuit due to impedance.

The increasing distortion in the signal waveform that was observed with increasing impact force exerted on the oscillating SG-spring setup in Figure 5.26 can be investigated in further detail. The 'ridges' observed in the waveform, as shown in Figure 5.24, demonstrate the motor force driving the moving magnet being perturbed by the distortion in force due to impact. A further distortion that occurs following impact is depicted in Figure 5.28, where the signal demonstrates inertia towards motion as characterised by the waveform.

Frequency Response Compared to Omron sensor

In Figure 5.29, the Omron laser transducer output signal and the strain gauge bridge output can be seen, at 11.5 mm input stroke conditions and an impact at 4 mm. Greater frequency response to smaller distortions in signal can be seen in the strain gauge signal output, where higher frequency

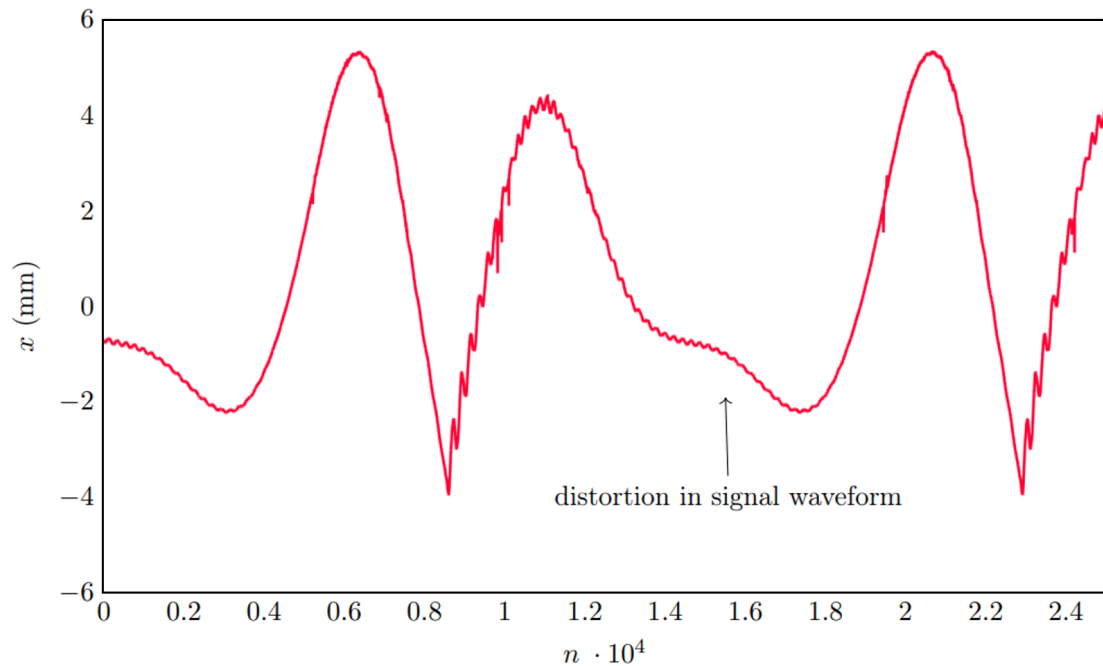


Figure 5.28: The strain gauge bridge output signal following fixed impact for a signal input of 14 Hz and stroke corresponding to 11.5 mm. The distortion occurring in the waveform following impact is indicated.

response following impact is observed between $800 \leq n \leq 1400$. This is reflected in the difference between the two signals. It is clear the strain gauge bridge configuration demonstrates a higher frequency response compared to the Omron laser transducer. The difference in signals is due to vibrations of the spring arm, presumably at the natural frequency which was identified during the pluck small impact test. The Omron laser is likely correctly measuring the displacement of the centre of the setup but superimposed on this motion are the spring vibrations.

Phasor Analysis

Figure 5.30 presents the input voltage, input current, strain gauge bridge voltage output, and the Omron laser transducer voltage output for stroke inputs relating to 7 mm, 9 mm and 11.5 mm. The phasor diagram for each is plotted to indicate how the difference in phase between the signals varies with increased impact force. The difference in phase between the voltage and current signals differs by approximately 100° throughout the varying impact tests, as can be seen in the phasor plots. The phase difference between output voltages for the strain gauge bridge and Omron laser can be seen to increase with increased impact.

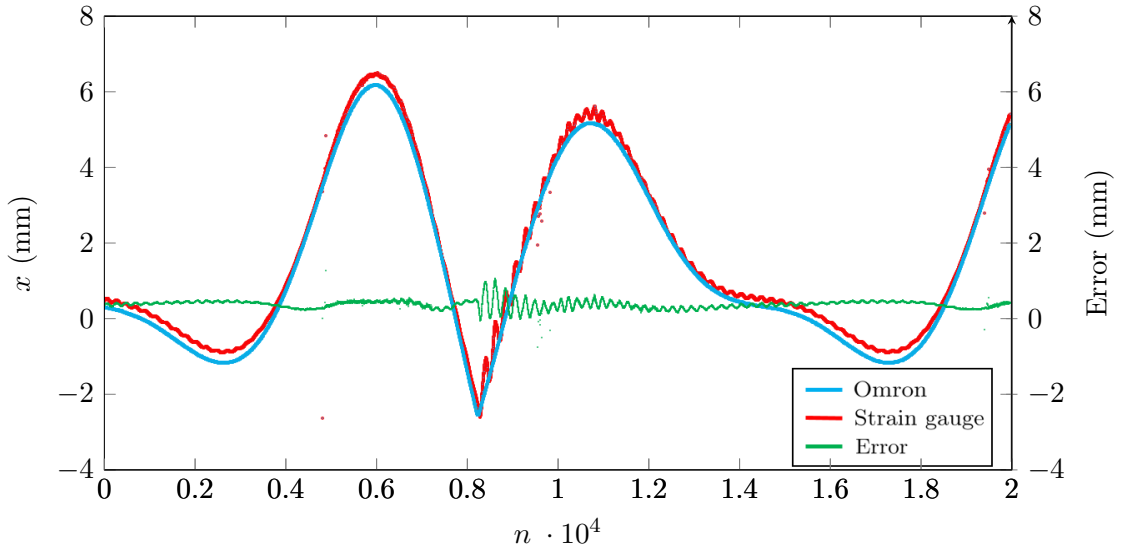


Figure 5.29: The computed output for displacement as measured by the strain gauge bridge and Omron laser transducer during a fixed 4 mm impact and a stroke input of 11.5 mm and 14 Hz. The difference in sensitivity between the computed signals can be seen, which is further demonstrated by the error difference shown.

5.4 Summary

In this chapter, the use of strain gauges in a full Wheatstone bridge configuration for measuring spring displacement, and by extension, compressor stroke during spring operation within the electromagnetic coil-moving magnet motor setup, was explored. A LabVIEW data acquisition process was programmed to monitor and control the outputs of the spring operation. The strain gauges were installed onto the spring arms in a full bridge configuration to provide intrinsic temperature compensation. An isolation amplifier was built to isolate the signal entering LabVIEW and the coil drive. An FEA model was simulated using SolidWorks to determine the vector location of highest principal stress on the spring arm, indicating the most suitable region for the strain gauges to be installed. The system was harmonically characterised to better understand the kinetic and resonant properties, and the motor force-current constant was determined experimentally as 3.71 N/A and matched that found by the FEMM simulation presented in chapter 4. The strain gauge bridge circuit was compared with a laser displacement transducer during dynamic spring tests at a frequency of 14 Hz, with an error of 3.13% between the two measurement methods. The FEA model further validated the suitability of the strain gauge measurements for compressor stroke.

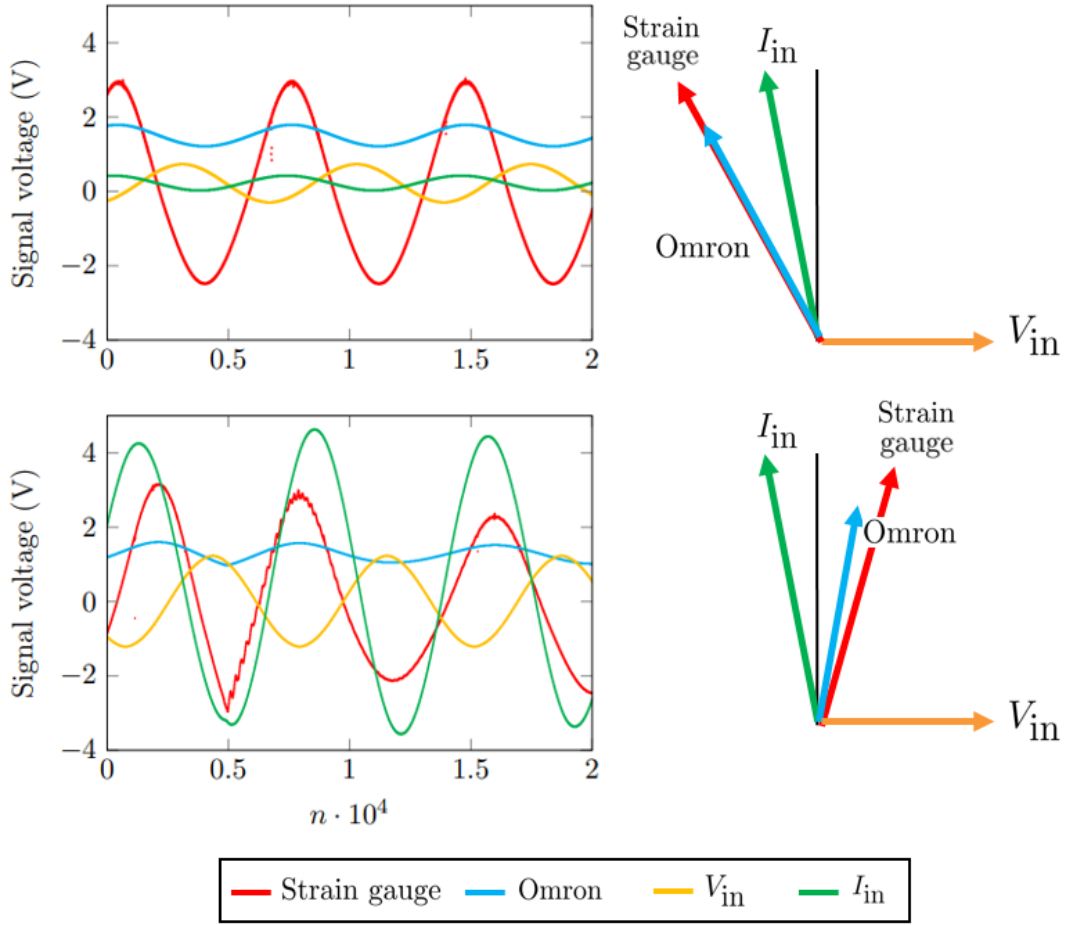


Figure 5.30: (Left) Voltages for current input, voltage input, SG-bridge voltage output, Omron laser voltage output, for stroke inputs of 7 mm and 9 mm, and (right) the phasor diagrams indicating the differences in phase between the signals and the variation with increased stroke input.

Additionally, the circuit's ability to detect impact, both large and subtle, was investigated and FFT analyses were completed to study the harmonic frequencies produced by the spring. A sampling rate of 100 ksamples/s with LabVIEW permitted very fine recording of the output signal post-impact. Insight into the operation of the spring and moving magnet motor within the electromagnetic coil was achieved as a result of testing response signals to static displacement, pluck excitation and fixed impact forces.

Overall, the strain gauge bridge circuit has been shown to be suitable for detecting impacts that occur on the spring, as well as being sensitive to more subtle contacts such as a pluck or small signal distortions. This is a useful finding given that friction rubs and impacts are a common cause of cryocooler stroke inefficiency or even failure, and presents itself as a promising measurement



technology for cryocooler stroke measurement. For integration into other systems of varying designs, the calibration methods and instrumentation would have to be carefully designed, and the phase lag would have to be suitably characterised and accounted for for varying potential impact scenarios. Moreover, a more compact electronic design and wiring would need to be considered for integration into a spaceflight cryocooler, where configuration and mass budget are important factors to take into account.

Part III

From Active to Passive Displacer

6

A Hybrid Coaxial Stirling Pulse Tube Cryocooler

Contents

6.1	Introduction	88
6.2	Thermodynamics and Harmonic Approximation	88
6.3	Design and Coaxial Configuration	91
6.4	Sage Numerical Simulation	93
6.4.1	Key Parameters	93
6.4.2	Modelling Theory	94
6.4.3	Modelling Approach	97
6.5	Hydrodynamic Phenomena in Coaxial Regenerator	102
6.5.1	Objectives	102
6.5.2	Simulated Coaxial Regenerator	103
6.5.3	Experimental Flow Testing	105
6.6	Thermal Simulation of Coaxial Regenerator	110
6.6.1	Introduction	110
6.6.2	Theory	112
6.6.3	Model Setup and Properties	114
6.6.4	Steady State Conduction	115
6.6.5	Radiative Losses	116
6.7	Simulated Cooling Performance at 80 K	118
6.8	Experimental Layout and Instrumentation	125
6.9	Experimental Performance	126
6.9.1	Cryocooler Performance	126
6.9.2	Pressure Pulses and Mass Flows	129
6.9.3	Optimum Performance	132
6.9.4	Work Flows	132
6.9.5	PV Powers	134
6.10	Summary of Research Findings	137

This chapter presents the design and experimental resesarch undertaken in testing a hybrid Stirling pulse tube cryocooler (SPTC) in a coaxial configuration using an active displacer. The work presented in this chapter is solely of the author's undertaking, except in instances where otherwise

stated, and expands upon the research presented in the below publications.

- H. Rana et al., "Numerical modelling of a coaxial Stirling pulse tube cryocooler with an active displacer for space applications", *Cryogenics*, (2020).
- M.A. Abolghasemi, H. Rana et al., "Coaxial Stirling pulse tube cryocooler using active displacer", *Cryogenics*, (2020).
- H. Rana et al. "Thermal losses in a coaxial pulse tube cryocooler", *Cryocoolers*, 2021.

6.1 Introduction

In this chapter, the design, building and testing of a hybrid SPTC into a coaxial configuration is presented. The design of the pulse tube and regenerator into a coaxial design is completed using the existing in-line design for the compressor and displacer units as starting points. The design theory, governed by thermodynamic and harmonic approximations, is further presented. The development of a numerical model for the full cryocooler is explored, which was completed using the commonly employed cryocooler software Sage [53]. The numerical model permits the prediction of the heat transfer throughout the various cryocooler components and the overall thermodynamic performance of the cryocooler. Experimental flow testing was completed in order to produce data that could be input as boundary conditions for the numerical simulations, in terms of pressure and mass flow. The flow testing further allowed various hydrodynamic parameters to be determined and compared with theoretically calculated values. The equipment assembly and data acquisition integration is then discussed and the various tests and sensitivity analyses on the cryocooler are described. Lastly, the performance of the coaxial SPTC is compared with the previously designed and tested in-line SPTC [24] in order to acquire noteworthy insights about the impact of the coaxial configuration on performance [54] [55] [56] [57].

6.2 Thermodynamics and Harmonic Approximation

The fundamental relations that thermodynamically define the cryocooler system are presented here. Fig. 6.1 shows the energy flow throughout the system. The function of the refrigerator is to absorb heat flow Q_c from the cold head where the instrument is integrated, thus achieving cryogenic refrigeration. T_c is the cold head temperature that must be maintained for instrument

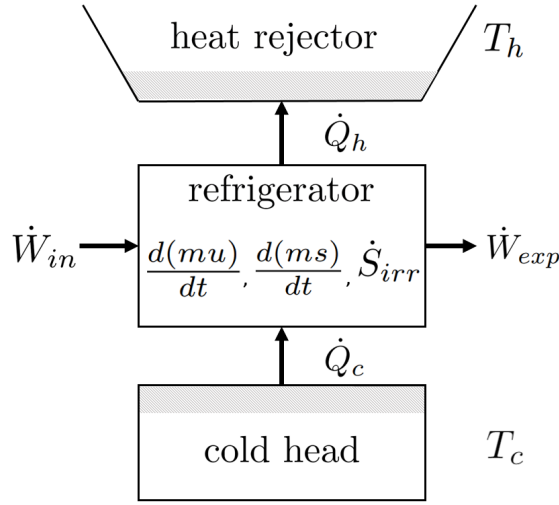


Figure 6.1: Schematic of a closed cycle refrigerator and the thermodynamic parameters that characterise the system (adapted from [31]).

or detector operation, Q_h is the heat flow rejected by the system which is dissipated by the heat rejector, referred to in this thesis as warm end heat exchanger (WHX), and T_h is the warm end temperature, which is ambient temperature. The actual coefficient of performance (COP) is given by Eq. 6.2.0.1, where Q_c (W) is the net refrigeration power (or cooling power) and W_{in} is the net input power from the compressors. An expander provides external power flow, W_{exp} , which represents the expansion work crossing the system boundary. The rate of change of internal energy, u , for a refrigerator mass m , is reduced to zero for a steady state balance. The steady state first law balance is hence given by Eq. 6.2.0.2. Similarly, the second law entropy balance is given by Eq. 6.2.0.3, where the rate of change of entropy, s , is reduced to zero for a steady state balance, and S_{irr} denotes the rate of entropy generation due to irreversible processes, which is always positive [31]. The entropy balance relates the warm and cold end cryocooler heat flows to the temperatures.

$$COP = \frac{Q_c}{W_{in}} \quad (6.2.0.1)$$

$$W_{in} - W_{exp} = Q_h - Q_c + \frac{d(mu)}{dt} \quad \therefore W_{in} - W_{exp} = Q_h - Q_c \quad (6.2.0.2)$$

$$\frac{Q_c}{T_c} - \frac{Q_h}{T_h} + S_{irr} = \frac{d(ms)}{dt} \quad \therefore \frac{Q_c}{T_c} - \frac{Q_h}{T_h} + S_{irr} = 0 \quad (6.2.0.3)$$

Solving Eq. 6.2.0.2 for Q_h and substituting into Eq. 6.2.0.3 and Eq. 6.2.0.1 achieves the expression for COP under steady state conditions, as shown in Eq. 6.2.0.4.

$$COP \equiv \frac{Q_c}{W_{in}} = \frac{T_c}{T_h - T_c} \left[1 - \frac{(T_h S_{irr} + W_{exp})}{W_{in}} \right] \quad (6.2.0.4)$$

Eq. 6.2.0.4 highlights that if the expansion work, W_{exp} , leaves the system as opposed to being recycled internally, the overall COP of the refrigerator is degraded. This highlights the significance of utilising a warm end displacer to recycle expansion power back into the SPTC in order to improve cryocooler performance. Moreover, the irreversible entropy produced multiplied by the warm end temperature, $T_h S_{irr}$, reflects an additional loss term and for an ideal refrigerator this is considered to be zero. Thus, assuming an ideal refrigerator ($S_{irr} = 0$) with internally recovered expansion power by a warm end displacer ($W_{exp} = 0$), the Carnot COP can be expressed by Eq. 6.2.0.5. The performance efficiency of cryocooling refrigerators can hence be evaluated in relation to the Carnot COP, where η_r represents the relative Carnot efficiency, and is given by Eq. 6.2.0.6.

$$\text{COP}_{\text{Carnot}} = \frac{T_c}{T_h - T_c} \quad (6.2.0.5)$$

$$\eta = \frac{\text{COP}}{\text{COP}_{\text{Carnot}}} \quad (6.2.0.6)$$

These equations all deal with flow in one direction. Instantaneous flow can be accounted for by factoring the change in entropy and internal energy terms in the balances presented. In the case of regenerative systems, such as the SPTC in question which utilises a regenerator, the mass flow and pressure pulse are oscillating. Hence, time averaged values for mass flow and pressure pulse are approximated at a component level within the regenerative cryocooler. This enables employing the open cycle thermodynamic balances within the component. For an open system with time averaged values, the first and second law are given in Eqn.s 6.2.0.7 and 6.2.0.8, where $\langle U \rangle$ and $\langle S \rangle$ are the internal energy and entropy averaged over one cycle, respectively.

$$\langle \dot{Q} \rangle = \langle \dot{W} \rangle + \langle \dot{H} \rangle_e - \langle \dot{H} \rangle_i + \frac{d\langle U \rangle}{dt} \quad (6.2.0.7)$$

$$\frac{\langle \dot{Q} \rangle}{T_j} = \langle \dot{S} \rangle_e - \langle \dot{S} \rangle_i - \langle \dot{S}_{irr} \rangle + \frac{d\langle S \rangle}{dt} \quad (6.2.0.8)$$

For a pulse tube cryocooler the cooling power can be expressed as the difference between the enthalpy flows between the regenerator and the pulse tube, given that there is no work power extracted at the cold end, as given by Eqn. 6.2.0.7. Once again, the first and second law can be combined for a steady-state operation balance over the component, as given by Eqn. 6.2.0.9, and the maximum reversible power that can be extracted from the system is given by Eqn. 6.2.0.10, where ψ is the exergy; the total available work in the system at any given location.

$$\langle \dot{W} \rangle_{rev} = \langle \dot{W} \rangle + T_j \langle \dot{S} \rangle_{irr} = \left[\langle \dot{H} \rangle - T_j \langle \dot{S} \rangle \right]_i - \left[\langle \dot{H} \rangle - T_j \langle \dot{S} \rangle \right]_e \quad (6.2.0.9)$$

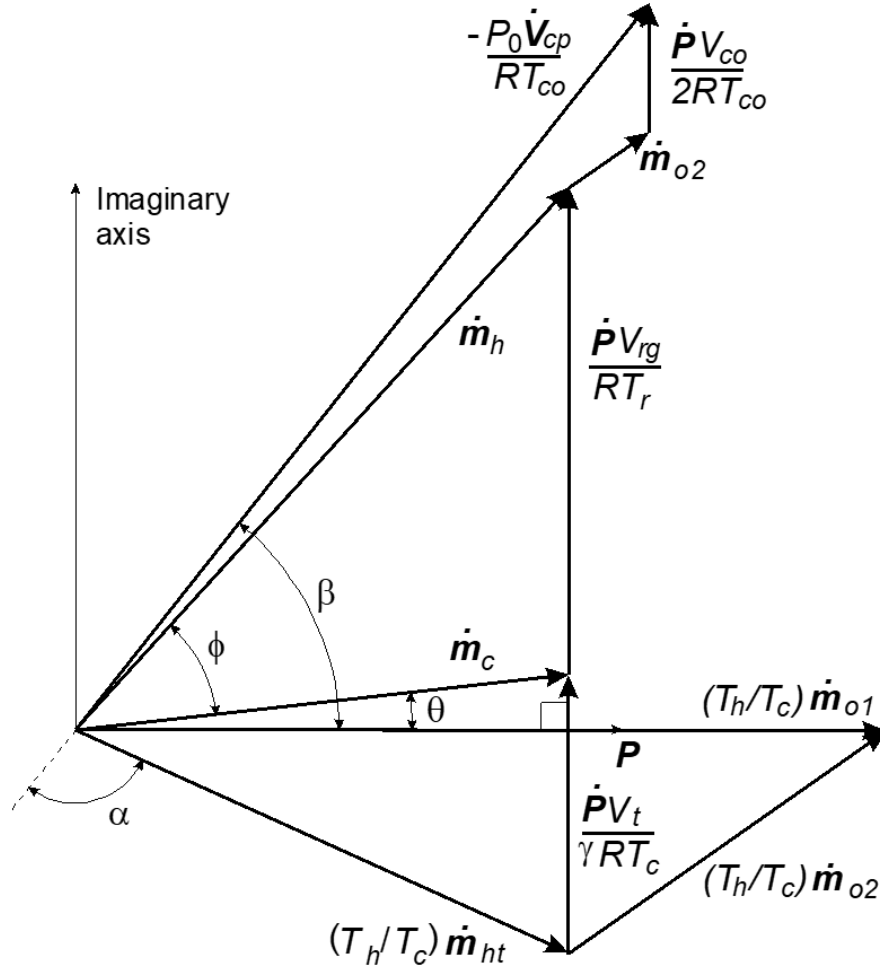


Figure 6.2: The phasor diagram for a double inlet pulse tube cryocooler (adapted from [11]).

$$\langle \psi \rangle = \langle \dot{W} \rangle_{max} = \langle \dot{H} \rangle - T_0 \langle \dot{S} \rangle \quad (6.2.0.10)$$

Fig. 6.2 represents the phasor diagram for a pulse tube cryocooler with a double inlet orificed pulse tube. The second inlet is represented by the phasor $T_h/T_c \dot{m}_{o2}$, where 'o2' indicates the second orifice entry back into the system. This is hence a representative phasor diagram for the double-inlet SPTC in this study, whereby one inlet occurs into the regenerator and the second into the pulse tube from the back end of the displacer. All other phasors are as discussed in chapter 3. The design for the coaxial SPTC is laid out in section 6.3.

6.3 Design and Coaxial Configuration

The pulse tube configurations for an in-line is shown in Figure 6.3 and both the in-line and coaxial configuration design are shown side by side in Figure 6.4. The in-line design shown on the left

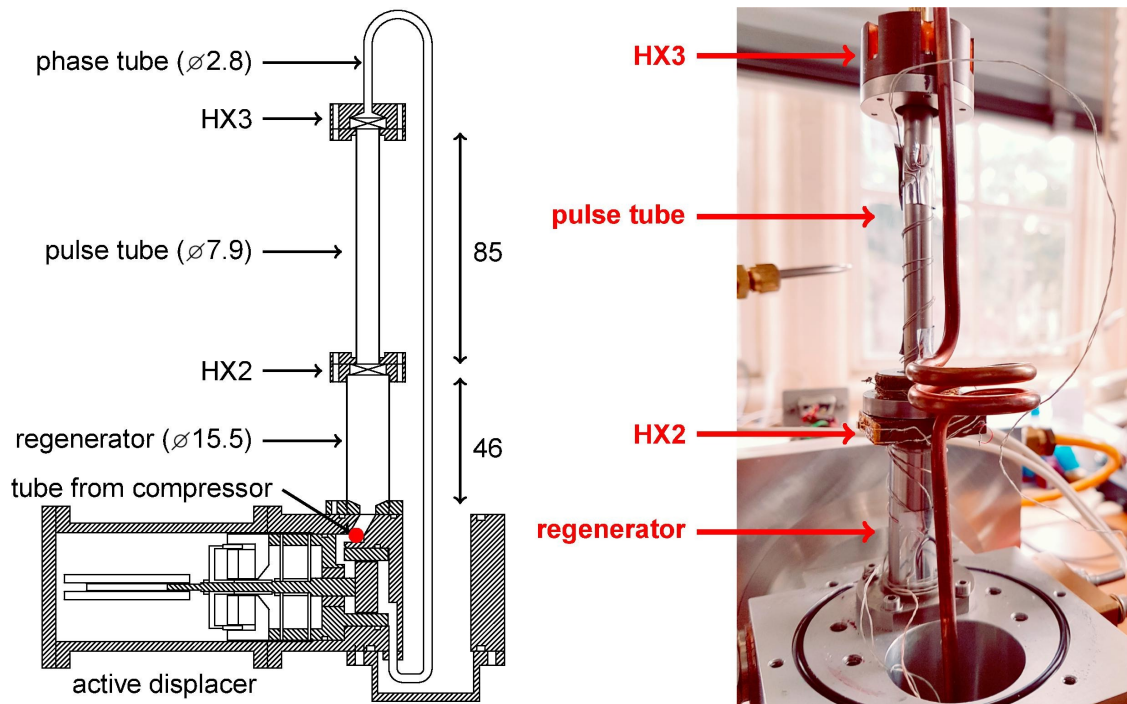


Figure 6.3: The cold head assembly with the vacuum chamber removed and without radiation shields. HX2 and HX3 are the cold end and warm end heat exchangers respectively. The drawing on the left indicates the connection location of the regenerator into the back end of the displacer, where all dimensions are in millimeters.

has previously been developed and tested in the Cryogenic Engineering Group. The new coaxial design, shown on the right, indicates a significant reduction in envelope and a better exposed cold head, permitting improved focal plane cooling integration. The compression unit, consisting of two dual-opposed compressors (COMP), connects to the displacer as well as the heat rejector (WHX) at the bottom of the annular regenerator (REG). The heat rejector cools the gas prior to its entry into the annular regenerator.

Table 6.1: Dimensions of the pulse tube and regenerator in the in-line and coaxial configurations of the cryocooler design.

	Pulse Tube		Regenerator		
Configuration	Length (mm)	ID (mm)	Length (mm)	Diameter (mm)	Area (mm ²)
In-Line	85	7.7	46.3	15.5	189
Coaxial	91	7.9	60	11.1(ID) / 18.5(OD)	173

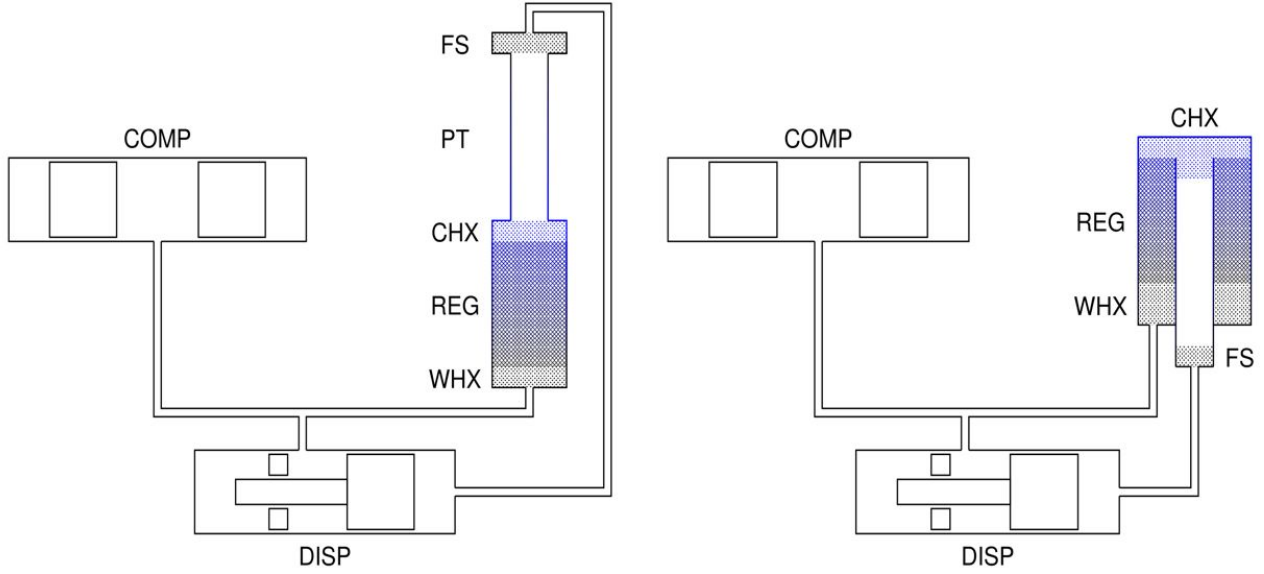


Figure 6.4: Schematic of (left) the in-line cryocooler with an active displacer and (right) a coaxial cryocooler with an active displacer. COMP is the compression unit, DISP is the displacer, WHX is the warm end heat exchanger (i.e. heat rejector), PT is the pulse tube, CHX represents the cold head heat exchanger, REG is the regenerator, and FS is the flow straightener.

6.4 Sage Numerical Simulation

6.4.1 Key Parameters

There are several key parameters that affect the numerical modelling of the cryocooler design, and ultimately, the performance. For a given length and flow area, the wire mesh controls the pressure drop within the regenerator and pulse tube unit. Therefore, its properties must be carefully designed to achieve the correct desired cooling. Whilst developing a numerical sub-model for the heat exchanger mesh, the mesh block porosity, wire diameter, and block diameter were investigated to select optimal values. Porosity of the mesh block is difficult to estimate, where a rough approximation is obtained by calculating the void fraction using the ratio between the wire volume and the block volume. This can alter significantly once the mesh block is tightly squeezed and packed, hence a numerical optimal porosity value must be calculated for which a minimum error between the numerical model output and the flow tests occurs. Secondly, a similar error-minimised optimum for mesh block diameter must be determined, as the outer diameter of the block may fluctuate in value due to external solder seeping through in the heat exchanger, as this could limit the pathway of the gas through the mesh. The dimensions of the pulse tube required preliminary sizing, which was largely inspired by the performance calculations completed for the axially-sized

pulse tube [24]. The gaseous fill pressure was set at 30 bar and a sensitivity analysis was later completed, as discussed in section 1.6. A compressor developed within the Cryogenic Engineering group at Oxford [40] was implemented into the cryocooler design, with similar equipment being utilised for the displacer setup.

6.4.2 Modelling Theory

Sage numerical simulation

The Sage numerical program allows for the cryocooler system design, thermodynamic performance, axial distribution of key parameters, as well as parametric optimisation to be simulated. This is achieved by solving for the steady state operation of a network of elements within the cryocooler system, arranged in a user-defined configuration with user inputs. The program consists of elements that are fundamental for the operation of all regenerative cryocoolers, such as regenerators, pistons, phase shifters, compliance tubes (which can be codified to physically resemble a pulse tube), woven screen matrices, piping, amongst other key components. The governing equations for continuity, momentum, and energy in the gas domain and the ideal gas equation of state are solved, as given below [53].

$$\frac{\partial \rho A}{\partial t} + \frac{\partial \rho u A}{\partial x} = 0 \quad (6.4.2.1)$$

$$\frac{\partial \rho u A}{\partial t} + \frac{\partial u \rho u A}{\partial x} + \frac{\partial P}{\partial x} A - F_{vp} A = 0 \quad (6.4.2.2)$$

$$\frac{\partial \rho e A}{\partial t} + P \frac{\partial A}{\partial t} + \frac{\partial}{\partial x} (u \rho e A + u P A + q) - Q_w = 0 \quad (6.4.2.3)$$

$$P = \rho R T \quad (6.4.2.4)$$

where

$$e = \kappa + \frac{u^2}{2} \quad (6.4.2.5)$$

where P , ρ , u and A are the pressure, gas density, axial flow velocity, and cross-sectional area, respectively. The mass-specific total gas energy, e , is given by Eqn. 6.4.2.5, where κ is the mass-specific internal gas energy. The heat flow per unit length is denoted Q_w , the factor F_{vp} is the viscous pressure gradient, and q is the axial heat flow due to conduction. These are determined in Sage by the following equations [53].

$$F_{vp} = - \left(\frac{f_D}{d_h} + \frac{K}{l} \right) \rho u \frac{|u|}{2} \quad (6.4.2.6)$$

$$Q_w = \text{Nu} \left(\frac{k_g}{d_h} \right) S_x (T_w - T) \quad (6.4.2.7)$$

In the above, f_D is the Darcy friction factor, d_h is the hydraulic diameter, K is the total local loss coefficient, Nu is the Nusselt number, giving the ratio of heat transfer by convection to conduction at a boundary in the fluid, S_x is the wetted perimeter, k_g is the gas conductivity, and $(T_w - T)$ is the temperature difference between the section-average and surface.

In order to solve for the governing equations, the gas dynamics equations are discretised over a series of uniformly spaced grid of points (x_j, t_j) , throughout the domain and over a cycle period. Using this grid, the variables ρ , $\rho u A$, and ρe are either interpolated explicitly or solved implicitly based on a staggered-grid formulation. Solution instability is avoided by solving ρ and ρe on alternate nodes, which are solved at the midpoints, to $\rho u A$, which is solved at the boundary, hence conforming to overall mass, momentum, and energy conservation. An integral number of control volumes constitutes the domain across which this is performed. First-order central differencing is used when solving for ρ and ρe in the continuity and energy equations in order to ensure that quantities exiting one control volume enter into the adjacent one, ensuring global conservation of mass and energy.

Axial conduction and conductivity

The heat flux in the axial direction, q , is given below where all terms are as previously stated and x the length covered.

$$q = -N_k k A \frac{\partial T}{\partial x} \quad (6.4.2.8)$$

As the values for k programmed within Sage inherently account for turbulent flow enhancement, an additional factor of N_k , an empirically correlated axial-conductivity enhancement ratio, is employed to ensure a more representative molecular conductivity of the gas being modelled [53]. The values of k are interpolated in Sage solely as a function of temperature. In instances where pressure dependence would need to be accounted for when interpolating for conductivity and viscosity, tabular inputs for helium gas can be mapped within Sage at different pressures.

Woven screen matrix flow and tortuosity

The Reynolds number provides the ratio of the inertial to viscous forces in the fluids, as given by Eqn. 6.4.2.9, where D_h is the characteristic length, which in this case is the hydraulic diameter, μ is the dynamic viscosity of the fluid. The friction factor f for a packed column is given by the Ergun equation as given by Eqn.s 6.4.2.10 and 6.4.2.11, using a modified Reynolds number, Re_m , as given by Eqn. 6.4.2.12 [58].

$$Re = \frac{\rho u D_h}{\mu} \quad (6.4.2.9)$$

$$f_f = \frac{150}{Re_m} + 1.75 \quad (6.4.2.10)$$

where

$$f_f = \frac{\Delta p}{L} \frac{d}{\rho v_s^2} \left(\frac{\beta^3}{1 - \beta} \right) \quad (6.4.2.11)$$

$$Re_m = \frac{Re}{1 - \beta} \quad (6.4.2.12)$$

Similarly, the pressure drop is given by Eqn. 6.4.2.13 [58].

$$\Delta p = \frac{150\mu L (1 - \beta)^2}{d^2 \beta^3} v_s + \frac{1.75L\rho}{d} \frac{1 - \beta}{\beta^3} v_s |v_s| \quad (6.4.2.13)$$

These can be applied to model a woven screen matrix in order to model the mesh and have been formulated empirically, as given by Eqn 6.4.2.14 [59], where Δp is the pressure drop, L is the regenerator length, d is the regenerator diameter, v_s is the superficial velocity, and β is the mesh porosity. The critical Reynolds number is 2300 [53].

$$f_{f, \text{empirical}} = \frac{129}{Re} + 2.91 Re^{-0.103} \quad (6.4.2.14)$$

The average conductivity through a tubular canister containing wired mesh is calculated within Sage using Maxwell's calibrated formulation for tortuosity, f_s , of a matrix-type heat exchanger, as given below [53] [60].

$$f_s = \left(\frac{k_s}{k_g} \right)^{m-1} \left[\frac{3 \left(\frac{k_s}{k_g} - \beta \right) + \left(2 + \frac{k_s}{k_g} \beta \right)}{3 (1 - \beta) + \left(2 + \frac{k_s}{k_g} \beta \right)} \right] \quad (6.4.2.15)$$

The variables k_s and k_g are the solid and gas conductivity respectively. The calibration factor $\left(\frac{k_s}{k_g} \right)^{m-1}$ uses m to account for different mesh materials, as empirically correlated [61]. This corrected tortuosity factor is then implemented into standard equations for conduction to model heat exchange from the gas to the mesh throughout the cylinder.

The empirical expressions for Nu and f_D as well as the temperature-dependent inputs for helium gas conductivity can be found in [53], where all standard correlations for the default Stirling pulse tube model type were employed. It is important to note that Sage modelling occurs in 1D and hence the conduction and pressure drop only vary axially. This therefore means that radial flow is not considered in the tortuosity determined for flow through the mesh. Moreover, porosity will be assumed as isotropic in Sage, however, flow is likely to be easier in the radial direction rather than axially given the packing of the mesh, particularly at the cold head. A suitable method to determine a representative porosity, and therefore, pressure drop has been discussed in section 6.4.3.

Pulse tube compliance gas model

A compliance-duct gas model [53] adds to the axial conduction model as presented in Eqn. 6.4.2.8 and is used to model the gas in the pulse tube. This is through the inclusion of terms that account for the convective losses that occur in the wall boundary layer of a pulse tube cylinder with a large temperature gradient.

$$\frac{q}{q_m} = 1 + \frac{q_t}{q_m} + \frac{q_f}{q_m} + \frac{q_b}{q_m} + \frac{q_s}{q_m} \quad (6.4.2.16)$$

This can be expressed as a dimensionless term q/q_m which then replaces the N_k term in the original conduction equation, and is comprised of molecular conduction q_m , turbulent conduction q_t , free convection q_f , boundary convection q_b , and streamline convection q_s . This inclusion is critical given that these additional heat loss terms thermally travel from the warm end to the cold head and reduce the cold end heat lift. Further clarification on the default inputs and computing methods are beyond the scope of this discussion and can be found in the Sage manual [53].

6.4.3 Modelling Approach

The key sub-assemblies of interest for numerical modelling were the heat rejector that meets the gas after compression, serving as a cooler for the gas, and the radial flow cold head in the coaxial pulse tube unit. These numerical models are outlined in the following two sections, as well as discussion on the flow tests carried out on these sub-assemblies to allow for model validation. The incorporation of these sub-assemblies into a full cryocooler numerical model is then explored.

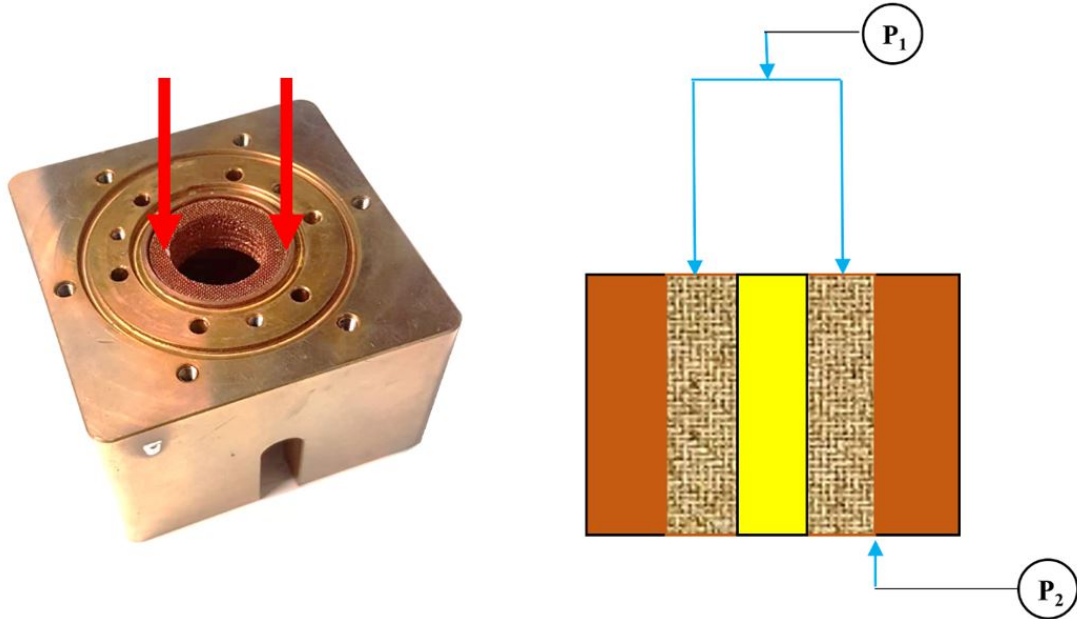


Figure 6.5: The copper block heat rejector with arrows indicating the annular flow through the mesh (left) and the flow test setup through the block measuring the pressure drop across the system (right). The yellow central cylinder indicates the aluminium used uniquely in the flow test setup, in the place of the pulse tube in the cryocooler.

Pulse tube heat rejector

The rejector is a copper block of 11 mm height, with an annular flow passage with an inner diameter of 18 mm and outer diameter of 26 mm, as shown in Figure 6.5. The heat rejector serves to remove heat from the gas as it enters the regenerator (as shown in Figure 6.4. The mesh in the annular channel consists of 40 discs of copper 60 (60 wires/inch) with a wire diameter of 0.16 mm. The block was flow tested to measure the change in pressure with varying mass flow, with an error of ± 0.1 bar, as shown in the flow testing setup in Figure 6.5. A mass flow range of 0 – 0.02 g/s was used under ambient temperature.

An ‘annular canister’ was utilized to represent the rejector in the numerical sub-model within Sage and the mesh was modelled as a ‘woven screen matrix’; the sub-model was then validated with the flow test data. The sub-model was further used to compare varying error between the test data and the model with change in modelled porosity of the mesh, in order to evaluate the most accurate value for porosity matching the physical build. Similarly, an optimal value for the outer diameter of the mesh block which yields the lowest error between the test data and the model was determined. The porosity and outer diameter were addressed in this sequence given that porosity

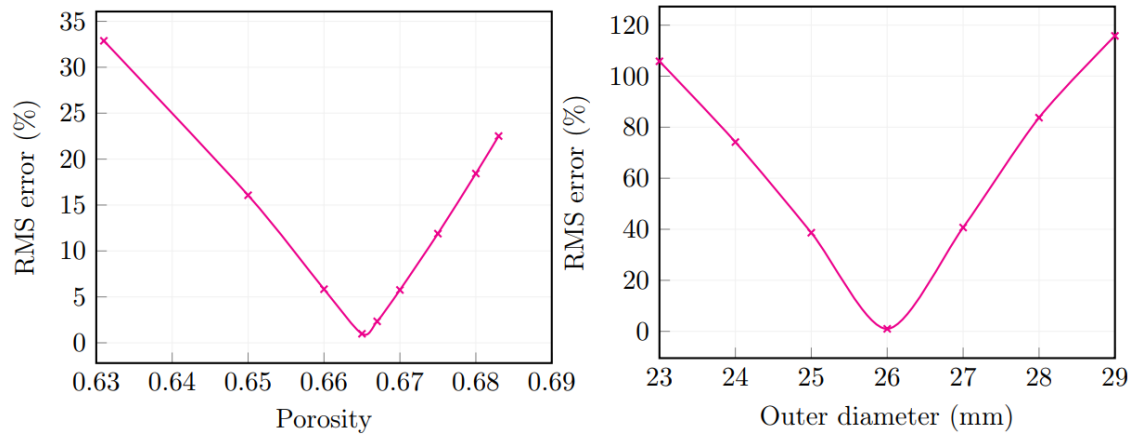


Figure 6.6: Change in RMS error (%) with (a) varying porosity inputs for the mesh block within the heat rejector and (b) varying outer diameter inputs for the mesh block within the heat rejector in the numerical sub-model in Sage.

is most likely to vary more significantly from the physically estimated value compared to other parameters, introducing the most error. This is due to the difficulty in estimating porosity, as discussed in section 6.4.2.

Figure 6.6a indicates that an input porosity value of 0.665 yields the lowest percentage error between the flow test data and the model. This is lower compared to the estimated void fraction initially calculated as 0.683. Figure 6.6b indicates that the physical length of the outer diameter of 26 mm also gives the lowest percentage error between the test data and the model, giving confidence to the fact that no external solder has seeped into the mesh internally. Both simulations provide an indication of the baseline error inherent in the numerical model.

Cold head

The cold head modelling approach was to develop a numerical sub-model that represents, irrespective of geometry, the pressure drop measured across the coaxial cold during flow tested. The use of Sage as the numerical modelling tool meant that flow occurring in a radial direction could not specifically be modelled, however, a ‘tubular canister’ cylindrical input could be used to represent the activity at the cold head, which was developed and optimized for the specific cold head dimensions and design in question. This representative sub-model was then implemented into the full numerical model for the entire cryocooler.

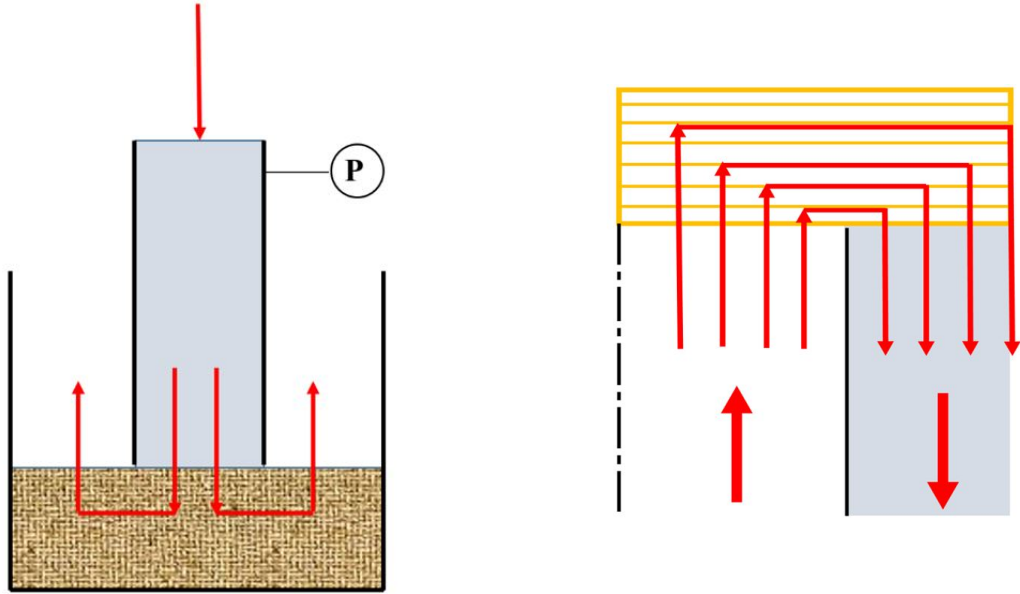


Figure 6.7: (Left) Flow test setup for the cold head coaxial flow. The pressure drop occurs between the inlet flow pressure and the outlet which is open to atmosphere. The meshed cold head is the region of interest and depicted by the brown mesh-shading. (Right) Axisymmetric cross-section of the coaxial flow through the cold head mesh (yellow lines). The SPTC coaxial flow setup is rotated 180° compared to the flow testing setup.

Figure 6.7 depicts the flow test setup up for the coaxial cold head, devised to deduce the pressure drop from the inlet end of the mesh to atmospheric pressure. The length of the cold head (meshed region only) is 6 mm, the width is 15.5 mm, the mesh is copper 60, the wire diameter of the mesh is 0.16 mm, and the porosity is 0.665. A mass flow range of 0 – 0.02 g/s was used.

A simplified yet representative model in terms of pressure drop was selected for the modelling approach due to the fact that using the full dimensions of the cold head within an annular canister flow would generate pressure drops far larger than those experimentally observed. This is due to the fact that as the gas flow changes course and direction from one axis to another, it is difficult to identify the effective gas flow length and diameter, as depicted in Figure 6.7. The model is representative of the cold head - the geometry is consistent with the cold head geometry but not the full pulse tube assembly. Hence, the pressure drop model is suitable in terms of verifying the pressure drop and effective flow length at the coaxial cold head. Furthermore, given the fact that radial flow for the gas may be easier than axial flow in the block of layers of woven screen mesh, the one dimensional isotropic numerical model that Sage permits is limited, which would yield further higher pressure drops than those observed in reality if physical dimensions of the

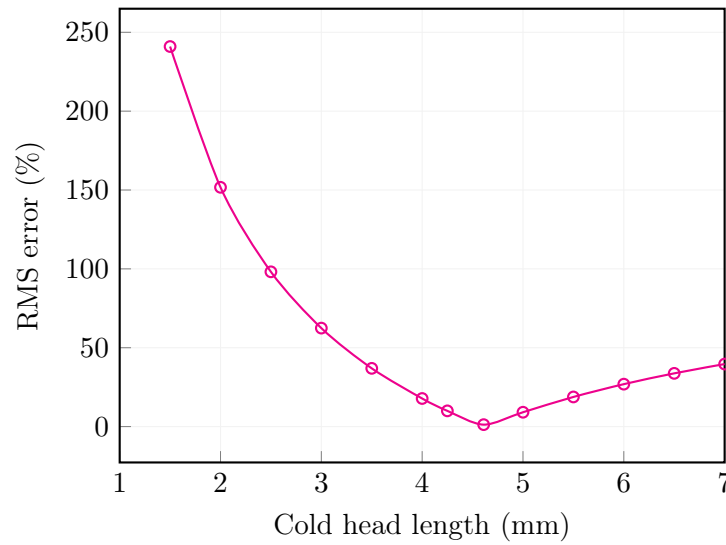


Figure 6.8: Effective cold head length of 4.6 mm determined where percentage RMS error between model and flow test is at a minimum. The physical length of the cold head is 6.7 mm.

cold head are employed. It is important to note, however, that the sub-model is validated purely based on pressure drop, and is specific to the particular geometry. The effective parameters derived for this particular heat exchanger cannot be readily used for a different component unless key dimensionless parameters are equivalent. Furthermore, the heat transfer due to radial mixing of the gas in this region is not accounted for, and this would require more detailed fluid simulation for further accuracy, using CFD.

Figure 6.8 shows the effective tubular canister length that represents the pressure drop at the cold head with the lowest percentage RMS error between the flow tests and numerical model. The effective length is 4.6 mm, compared to the physical length of the cold head of 6.7 mm; hence approximately 70% of the physical length is taken as the input length for the canister representing the cold head. This percentage is utilized in the full cryocooler model when later small-scale modifications are made to the physical cold head length.

Full cryocooler model

The validated sub-models discussed were implemented into a full coaxial cryocooler numerical model in Sage, with components modelled to numerically simulate mass flow and heat transfer throughout each unit indicated in Figure 6.4 and all connecting components. A high-level overview of the numerical model through the Sage user interface is shown in Figure 6.9. Figure 6.10

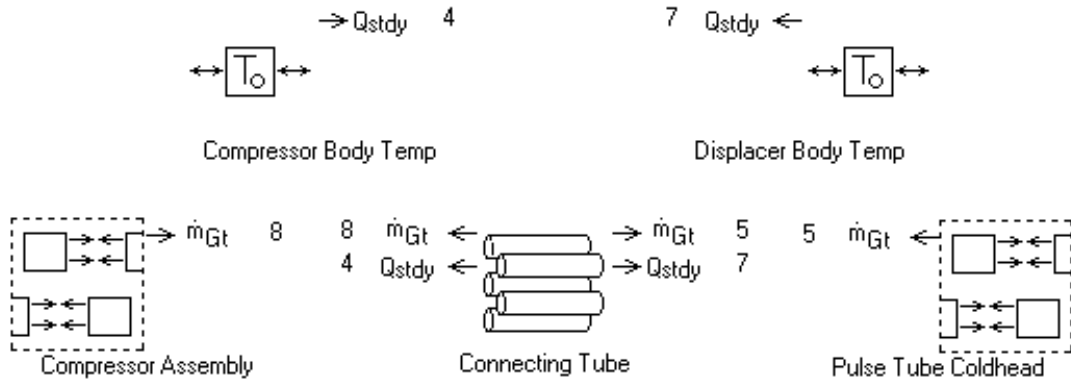


Figure 6.9: Overview of the numerical model for the full coaxial cryocooler at the system level. For the full model at component level, please see Appendix A.

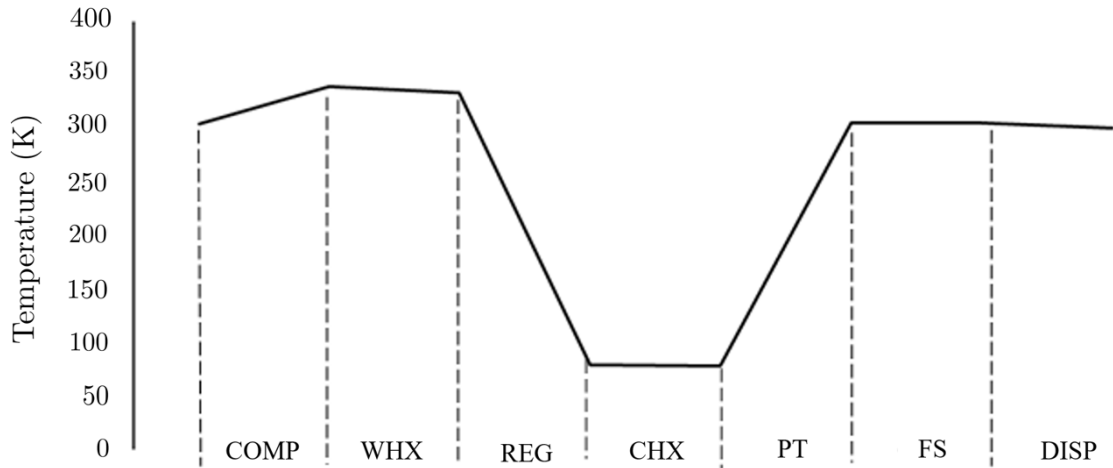


Figure 6.10: Temperature map of simulated gas through key components in the coaxial cryocooler model. COMP is the compressor unit, WHX is the heat rejector, REG is the regenerator, CHX is the cold head heat exchanger, PT is the pulse tube, FS is the warm end flow straightener, and DISP is the displacer.

indicates the final temperature map of the full numerical model with the key components marked. The detailed breakdown of the full Sage model for the displacer unit and the pulse tube unit can be found in Appendix A.

6.5 Hydrodynamic Phenomena in Coaxial Regenerator

6.5.1 Objectives

The regenerator, as discussed, is a key component in governing the cryocooler operation. In order to better understand how the heat transfer matrix (or mesh) within the regenerator affects the fluid flow throughout its length, as well as the pressure drop, it is important to be able to numerically

simulate this. This is particularly useful for determining input values of mass flow at the warm end and pressure pulse at the cold end, or vice versa, when aiming to design for the optimum phase angle for a regenerator-pulse tube system being designed. Moreover, the impact of the coaxial configuration needs to be assessed. In this section, the previously validated numerical model using Sage is exploited to better understand the flow through the coaxial regenerator. Moreover, the flow results from the numerical model were used to inspire starting inputs for flow testing of the regenerator, warm end flow straightener, cold head, and warm end heat rejector components. These flow tests ultimately result in a better understanding of which physical components play the greatest influence on the pressure drop throughout the cryocooler and allow for parameters such as the porosity of the warm end flow straightener and the regenerator to be empirically determined.

The governing equation set for solving for the gas domain in the numerical model is given by the equations for continuity, momentum and gas energy, as presented in section 6.4.2. The dimensions and other key input parameters for the coaxial regenerator and other components can also be found there.

6.5.2 Simulated Coaxial Regenerator

The numerical model was used to gain insight into how the regenerator length and porosity affect flow parameters such as mean Reynolds number, the peak-to-peak mass flow, and the peak-to-peak pressure at the warm end and cold end of the regenerator, across a 300 K to 80 K difference in temperature. Figure 6.11 shows how the peak-to-peak mass flow varies with regenerator length. It can be seen that the difference in mass flow is greatest at smaller regenerator lengths. However, the difference in peak-to-peak pressure increases with an increase in regenerator length. Figure 6.12 shows how the average Reynolds number in the regenerator varies with the mesh porosity of the regenerator matrix. Expectedly, for a higher void fraction, there is an increase in the average Reynolds number. This shows that at higher porosities the mesh enhances inertial forces; although the Reynolds number throughout the sweep for the coaxial regenerator is still very low and very much laminar despite variation in porosity. Conversely, for a coaxial regenerator mesh porosity of 0.68 and all other parameters as previously stated, the mean Reynolds number in the pulse tube is found to be much higher, at 8,755. This is undoubtedly expected given the lack of mesh within the pulse tube.

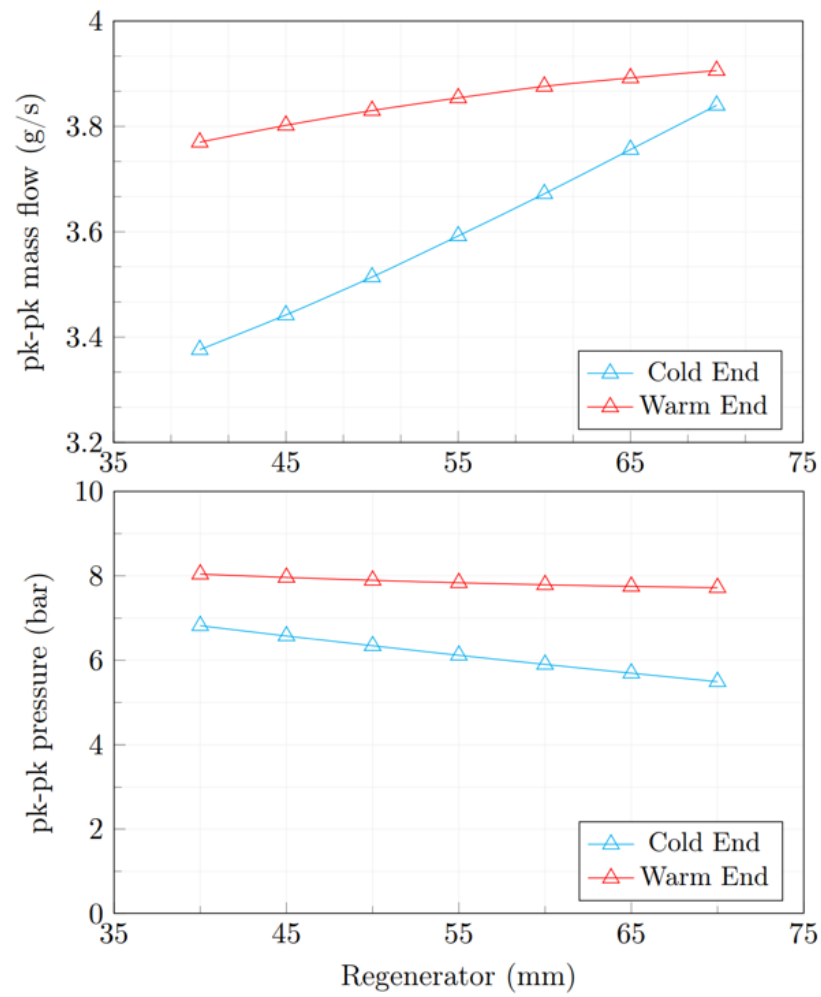


Figure 6.11: The pk-pk mass flow and pressure with varying regenerator for the coaxial regenerator, at both the warm end and the cold end.

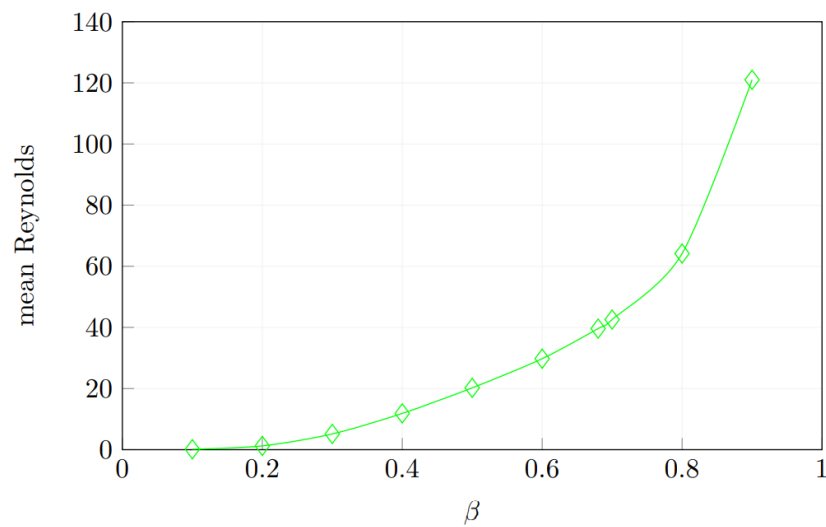


Figure 6.12: The variation of the mean Reynolds number with changing mesh porosity, β , of the coaxial regenerator is shown.

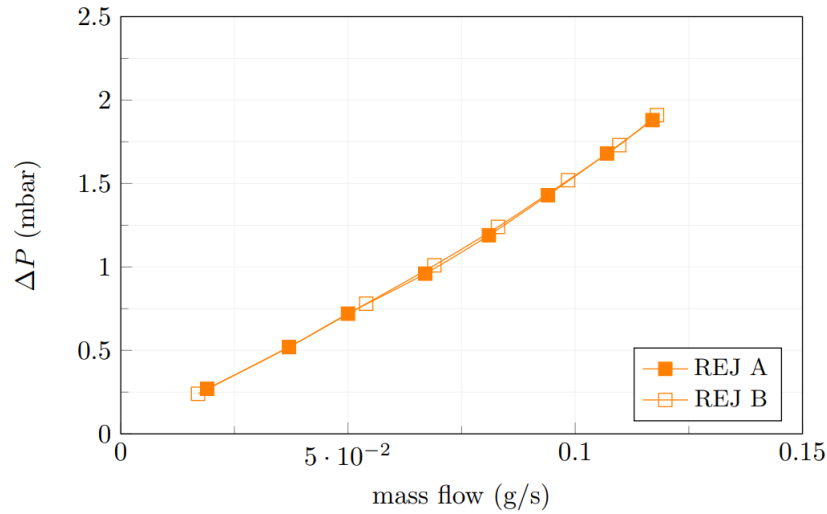


Figure 6.13: The effect of varying mass flow on the overall pressure drop across the warm end heat rejector.

6.5.3 Experimental Flow Testing

The pressure drop across different key components was experimentally assessed in a sequential manner through flow testing. More on the general methodology and setup of flowing of sub-components of the coaxial cryocooler is discussed in section 6.4.3. In the case of the sequential flow testing, this was achieved as follows: flow testing was conducted on one component, and thereafter, at each subsequent stage the next component was installed one by one. Hence, by deduction, the pressure drop and flow could be determined across each component. The experiments were conducted in the following stages: firstly, the coaxial copper block heat rejector was flow tested. Thereafter, the rejector was installed with the cold head, and the same sweep of mass flows were conducted where the pressure at either end of the component was recorded. Following on from this, the warm end flow straightener and a 4 mm tube were added. Lastly, the regenerator was installed in order to complete the assembly that would give an representative indication of the total pressure drop across the coaxial pulse tube assembly.

Figure 6.13 indicates the effect of varying the mass flow on the overall pressure drop across the warm end heat rejector. The flow test was conducted in both directions, A and B, as shown. It is clear there is no effect on the pressure drop results when changing direction of the flow test. A relationship for the two variables has been formulated as a power function, with good agreement.

Figures 6.14 and 6.15 further show the pressure drop with the next two stages of the sequence

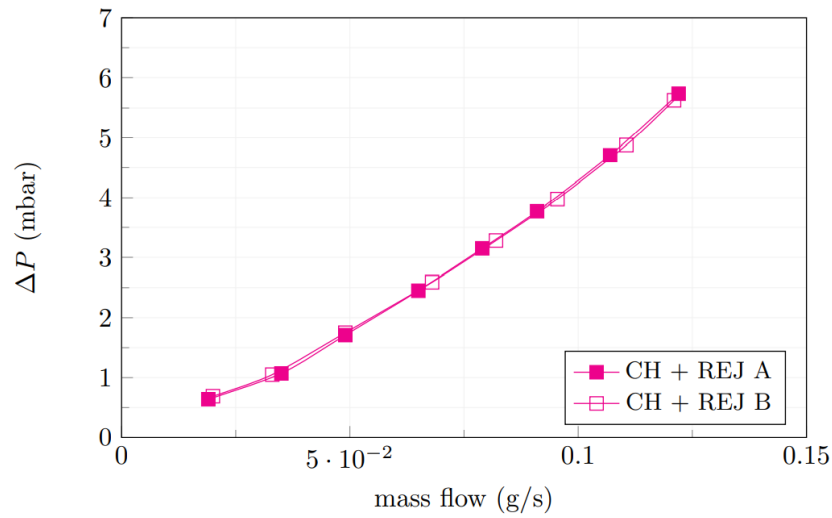


Figure 6.14: Flow test of the rejector installed with the cold head.

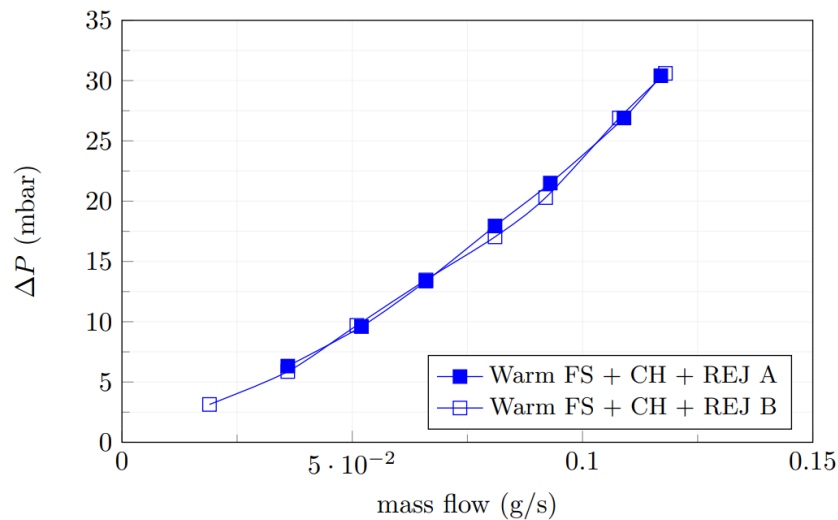


Figure 6.15: The overall pressure drop with varying mass flow for the rejector, cold head, warm end flow straightener, and tube.

being flow tested; firstly the cold head now installed into the warm end heat rejector, and then the warm end flow straightener and tube added. In Figure 6.14, a good agreement is still seen between test run A and B, and the pressure drops remain within the same order of magnitude as with the results from the heat rejector flow test alone. The addition of the warm end flow straightener and tube, however, there is a sharp increase in recorded pressure drop.

The pressure drop across the warm end flow straightener is very high, as can be seen in Figure 6.15, where the overall pressure drop is 5 times higher than that observed in the rejector and cold head assembly in Figure 6.14. It was deemed most likely that the warm end flow straightener,

which is filled with mesh, was the responsible component for the greater pressure drop (as opposed to the added 4 mm tube). This led to an investigation into the appropriate design for the mesh in the flow straightener. A range of alternative mesh options made of different materials with different wire diameters were explored. Table 6.2 provides an overview of the different mesh options explored for the warm end flow straightener. These different mesh designs were each flow tested in order to select an alternative to the existing mesh of copper 60 (60 wires/inch) with a soldered exterior to the stacking, which was responsible for producing a higher than desirable pressure drop in the flow straightener.

Table 6.2: Different mesh stacks tested for the warm end flow straightener mass flow and pressure testing experiments. The first row indicates the original flow straightener tested, as shown in the results in Figure 6.15. Duplicates of each stack were tested.

FS Tested	Material	Wire Diameter	No. of Discs	Stacking
1	Copper	60 wires/inch	18	Soldered
2	Copper	40 wires/inch	12	Loose
3	Copper	60 wires/inch	10	Loose
4	Brass	80 wires/inch	10	Loose
5	Copper	40 wires/inch	10	Loose

In terms of stacking type, 'soldered' refers to whether the stack of discs were lightly soldered on the outside in order to keep the discs compact. After the first experiment, which produced a very high pressure drop as shown in Figure 6.15, the discs were stacked without external soldering, hence 'loose', in an attempt to reduce the very high pressure drop. Duplicates of each stack were tested to assess repeatability. Figure 6.16 shows the results of the experiments. Aside from the tightly packed original flow straightener with soldered stacking, the brass flow straightener demonstrated the highest pressure drop. Experiment 2, in which the flow straightener consisted of copper 40 (40 wires/inch), the lowest pressure drop was observed across the assembly. Hence, the flow straightener tested in experiment 5 was a repeat of the stack parameters tested in experiment 2 for further repeatability, as this was the strongest candidate for being used in the warm end flow straightener to achieve a lower pressure drop. The flow test for the full component assembly was conducted until 0.06 g/s to match the expected inlet mass flow rate as determined by the simulation. Figure 6.17 shows the pressure drop variation for the complete assembly, where at 0.06 g/s, a total overall pressure difference of 124 mbar was recorded.

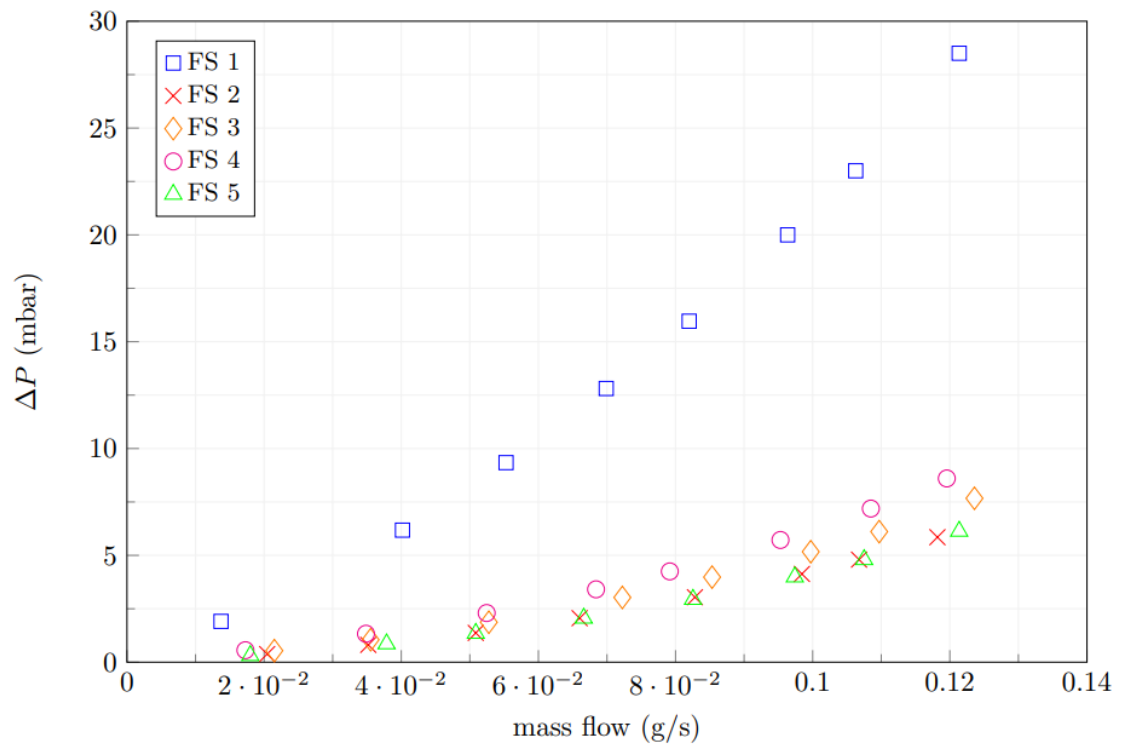


Figure 6.16: Pressure drop across different warm end flow straightener assemblies with a variety of different mesh with different parameters installed, as per Table 6.2.

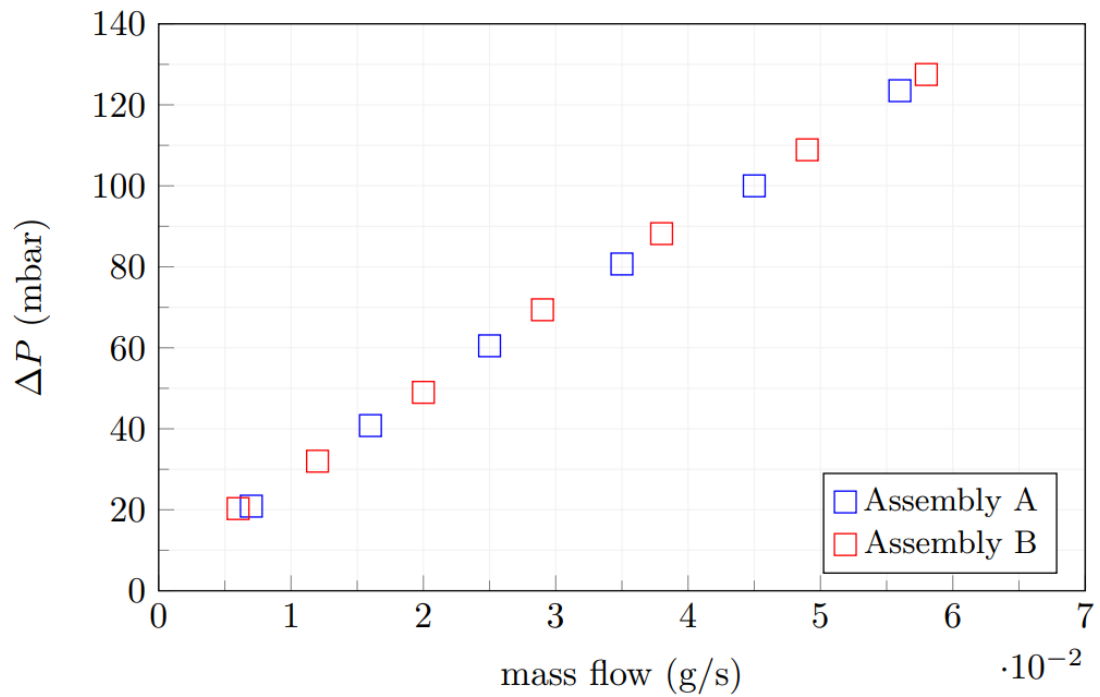


Figure 6.17: Pressure drop with varying mass flow across the full assembly consisted of the rejector, cold head, warm end flow straightener, 4 mm tube and the regenerator. The original and new warm end flow straightener parameters are as given by Table 6.2 'FS Tested' 1 and 2, respectively.

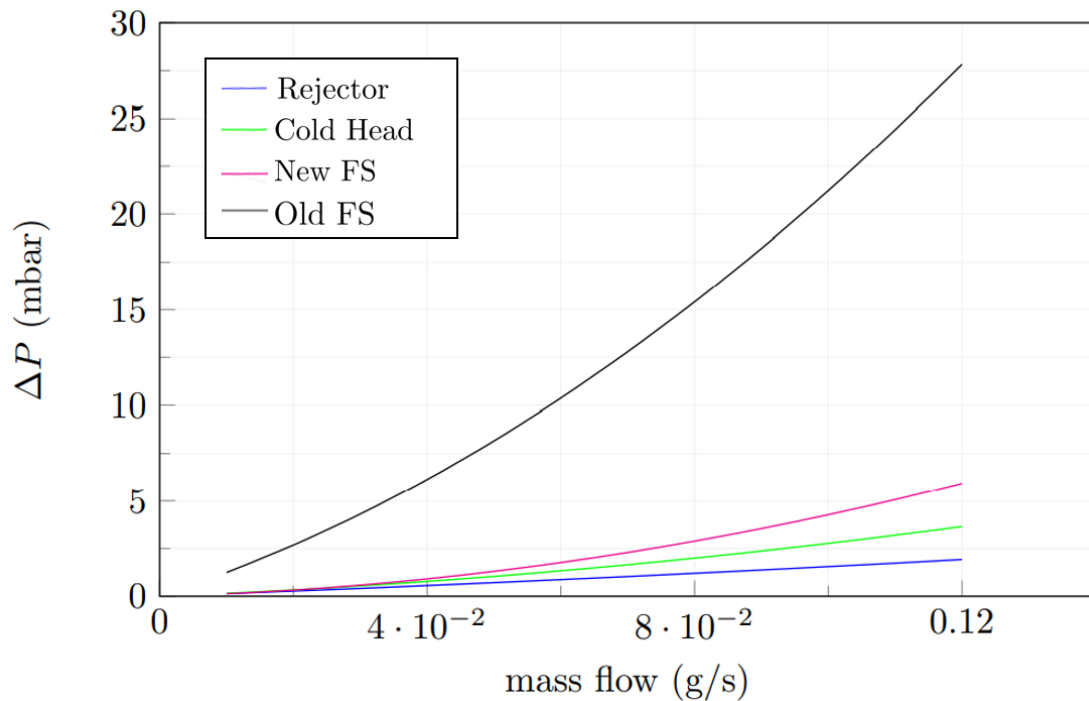


Figure 6.18: The pressure drops across the different components. The regenerator is not shown given that, being responsible for the largest proportion of the total pressure drop, the scale is very different. Figure 6.19 represents the regenerator and total pressure drop.

Figures 6.18 and 6.19 present the pressure drop for each individual component, achieved by sequential deduction. The regenerator pressure difference accounts for the majority of the pressure drop, even in the case of the component pressure drops that factor the original warm end flow straightener design with the higher pressure drop. Figure 6.20 indicates the proportions for the two design cases in the form of pie charts. The total overall pressure drop for the full assembly was found to be 133 mbar and 125 mbar at 0.06 g/s inlet mass flow rate, respectively.

Lastly, the porosity of the warm end flow straightener and the regenerator was determined, by calculating the ratio of the wire volume and mesh block volume of each stack, using the wire diameter, number of discs, mesh number per inch and block dimensions. The porosity of the regenerator and the new warm end flow straightener was calculated as 0.68 and 0.79, respectively.

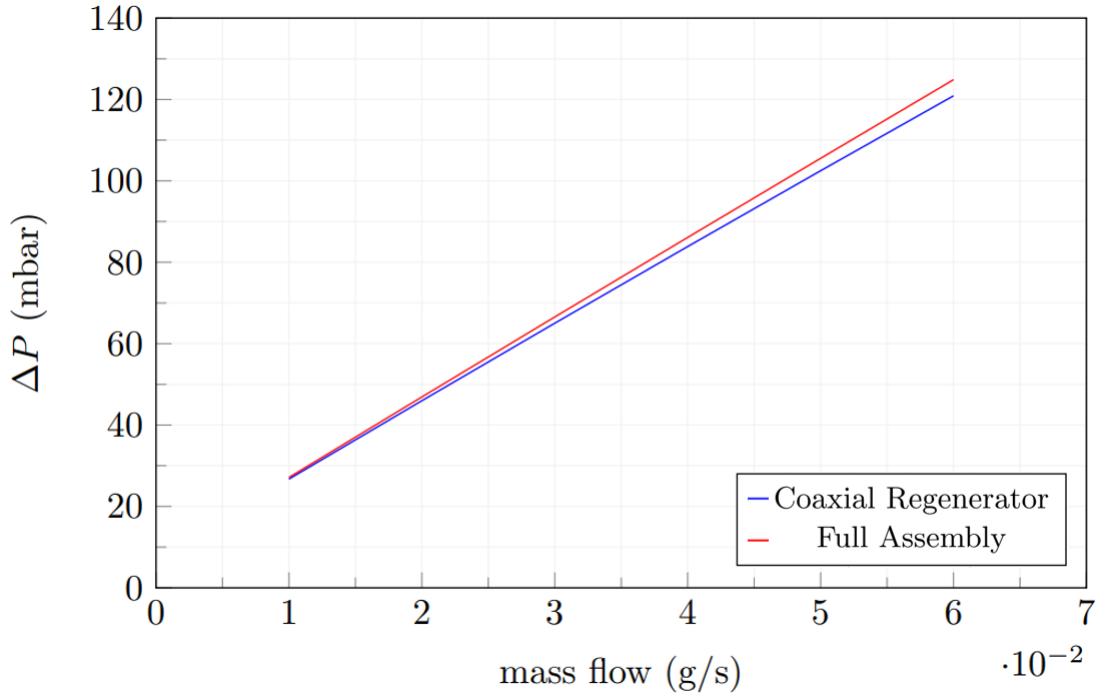


Figure 6.19: The pressure drop across the coaxial regenerator, as determined experimentally through a sequential component testing analysis, and the total pressure drop of all components assembled. It can be seen that the coaxial regenerator is mostly responsible for the total pressure drop across the assembly.

6.6 Thermal Simulation of Coaxial Regenerator

6.6.1 Introduction

As numerical models are widely used to predict the performance of the pulse tube cryocooler, it is important to be able to achieve a simulated performance that is as representative of the experimental findings as possible. This will permit better future designs of SPTC components and the cryocooler overall, as well as allow for simulation to be trusted to provide useful insights into the cryocooler operation. Moreover, variables that cannot be physically measured due to instrumentation limits or difficult in accessing regions of interest. In this instance, simulations are of particular importance.

The thermal losses in the pulse tube cryocooler can be difficult to predict numerically in Sage, one of the most widely used and most comprehensive cryocooler modelling software available. This is especially true when simulating a cryocooler in a coaxial configuration, and thus, these thermal losses should be quantified accurately as they will undoubtedly affect the overall cooling performance and efficiency of the cryocooler. In this section, efforts are made to study and model the conductive, radiative, and convective flow throughout the coaxial pulse tube assembly of the

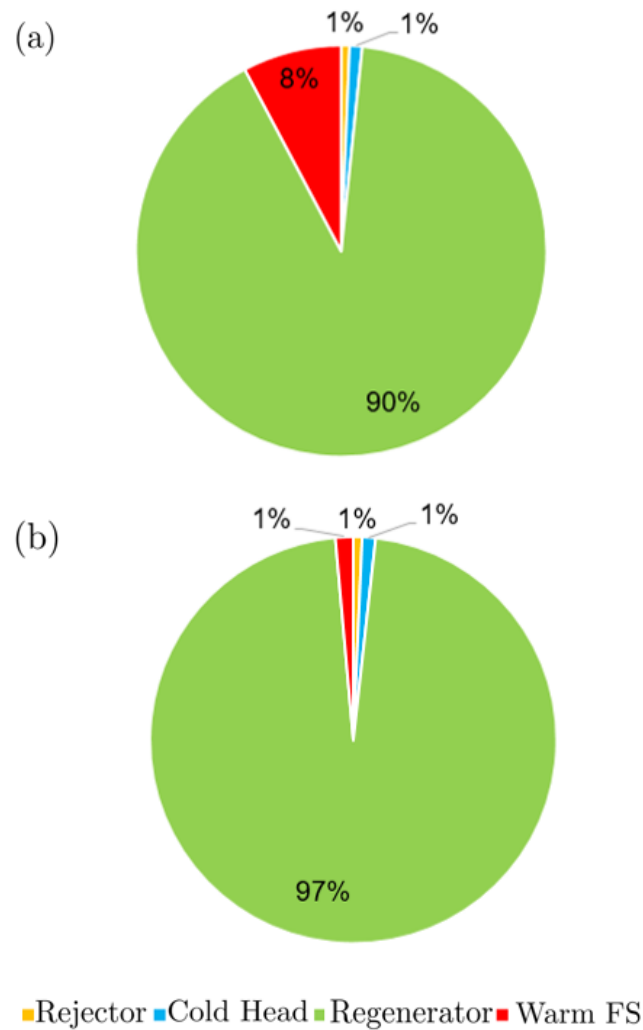


Figure 6.20: Pie charts indicating the components responsible for the proportions of the total pressure drop, where (a) is with the original warm end flow straightener giving a total pressure drop of 133 mbar at 0.06 g/s, and (b) is with the newly selected warm end flow straightener giving a total pressure drop of 125 mbar at 0.06 g/s.

cryocooler.

ESATAN-TMS was selected as a suitable software for modelling the thermal losses [62]. This permitted the creation of a lumped parameter model of the coaxial pulse tube in order to predict the thermal losses. The output from these simulations were then fed back into the existing numerical model to re-validate the experimental findings of cryocooler performance with the numerical simulation. The modelling methodology and findings of thermal losses are outlined, focusing on conductive and radiative thermal losses, and addresses the velocity profiles and pressure drop within the coaxial regenerator filled with mesh.

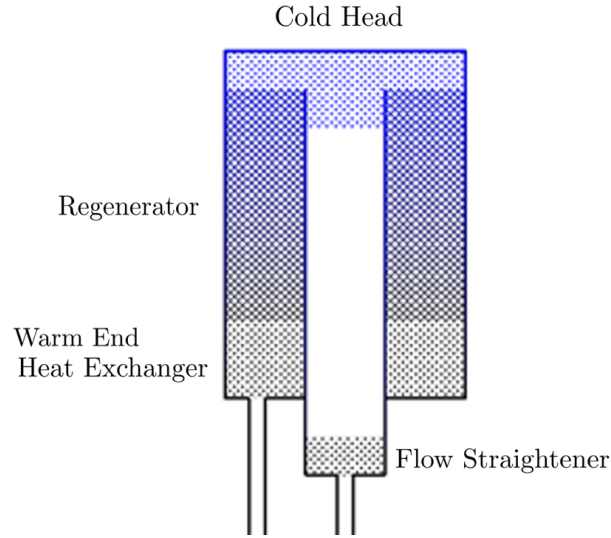


Figure 6.21: A schematic of the pulse tube and coaxial regenerator.

The performance of the SPTC with an active displacer is shown in Figure 6.22, indicating how the cooling power and relative Carnot efficiency changes with displacer phase (left) and stroke (right). In blue, the cooling power for both experimental data and the simulated performance (using Sage software) can be seen. The Sage model was improved by tying certain boundaries to temperatures measured during experiments. The discrepancy in the model validation is approximately 0.5 W when averaged across the sweeps in terms of cooling power, and an average of 4% difference in relative Carnot efficiency. This discrepancy could potentially be attributed to the underestimation in the numerical model of the conductive losses in the system. These can be difficult to model due to the complex coaxial geometry of the pulse tube and regenerator. These losses are critical as they transfer thermal energy from the warm end to the cold end, degrading the cooling performance. The Sage numerical model computes 0.65 W of conductive losses in the coaxial SPTC.

6.6.2 Theory

Fourier's law for heat conduction for local heat flux density, $\mathbf{q}$ (W/m²) in the differential form below:

$$\mathbf{q} = -k\nabla^2 T = -k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (6.6.2.1)$$

where k is the material's thermal conductivity (W/mK). Similarly, the Stefan-Boltzmann equation gives the power radiated from a black body as a function of its temperature, and can be

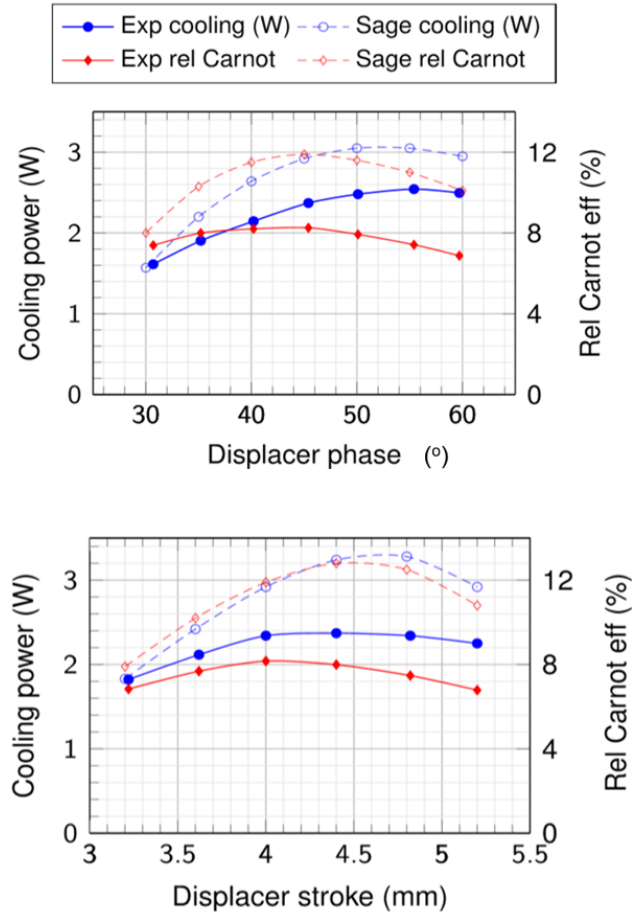


Figure 6.22: Change in cooling power and relative Carnot efficiency as displacer phase is varied (left) and displacer stroke is varied (right). The discrepancy between the Sage model output compared to the experimentally determined performance can be observed.

expressed as the following:

$$\mathbf{q} = \varepsilon \sigma T^4 \quad (6.6.2.2)$$

where ε is the emissivity of the surface and ω is the Stefan-Boltzmann constant. Furthermore, ESATAN-TMS is able to conduct ray-tracing between various surfaces in order to map the radiative fluxes, and hence, impingent radiative heat flux on two surfaces. It achieves this by using Monte-Carlo ray tracing algorithms. Figure 6.23 depicts how two surfaces are in view of one another and the variables that can permit mathematically expressing their view factor with respect to each other. Hence, the expression for view factor can be given as below [63].

$$VF_{1 \rightarrow 2} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos \beta_1 \cos \beta_2}{\pi r_{12}^2} dA_2 dA_1 \quad (6.6.2.3)$$

where $VF_{1 \rightarrow 2}$ is the view factor between surface 1 and 2, A_1 and A_2 are the two surface areas

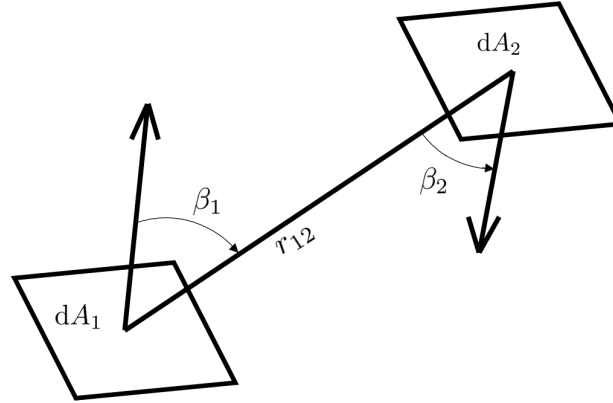


Figure 6.23: Two surfaces in view of each other, where the view factor is $VF_{1 \rightarrow 2}$ (adapted from [64]).

in view of each other, β is the angle of tilt between their respective centre lines, and r_{12} is the distance of separation between the two surfaces.

6.6.3 Model Setup and Properties

ESATAN-TMS

ESATAN-TMS is written in standard ANSI Fortran 77 and allows lumped parameter modelling. The prediction of temperature distributions can be achieved across modelled nodes, in which each are modelled to represent a key region in the pulse tube where the temperature is affected by significant heat transfer. The software does not permit convective modelling, however, and thus CFD and coupled multi-physics simulations will be required for this. Typical inputs that are applied to the nodes include thermal conductivity, specific heat capacity, and thermo-optical surface properties, such as emissivity and absorptivity. Further details on the default ESATAN-TMS software methods for conduction and radiation can be found in the ESATAN manual [65] [66].

Material Properties

The material properties of the surfaces used for the conductive and radiative model are shown in the Table 6.3 and Figure 6.24. The regenerator is made from stainless steel (SS316) and the pulse tube is fiberglass epoxy (G10). As a key input affecting the output of the model, the material properties need to be as accurate as possible. Furthermore, the variation due to temperature, particularly over a wide range of 80-300 K, has to be accounted for. The following curve fitting equation for material properties was utilised to obtain values across the 80 K to 300 K temperature

range in the pulse tube. The coefficients a - f , representing the material properties, are obtained from the NIST Cryogenic Material properties database [67]. The obtained distributions, shown in Figure 6.24 for thermal conductivity and specific heat capacity, are then fed into the model as temperature-dependent matrices. The curve fit equations have been generated by NIST for a 4-300 K temperature range.

$$\log_{10} y = a + b(\log_{10} T) + c(\log_{10} T)^2 + d(\log_{10} T)^3 + e(\log_{10} T)^4 + f(\log_{10} T)^5 \quad (6.6.3.1)$$

Table 6.3: The density values and thermo-optical emissivity coefficients used to model the regenerator (SS316) and the pulse tube (G10).

Material	Density, ρ (kg/m ³)	Emissivity, ε
Regenerator (SS316)	7880	0.910
Pulse tube (G10)	1320	0.350

6.6.4 Steady State Conduction

The conduction between nodes is calculated as shown in Eqn.6.6.4.1, for simplified conduction between two nodes, where l is the length. The linear conductance used in ESATAN-TMS that arises from thermal conductance is represented by GL , as given in Eqn. 6.6.4.2, which is the form of the output from the model.

$$Q = \frac{kA\Delta T}{l} \quad (6.6.4.1)$$

$$GL = \frac{kA}{l} \quad (6.6.4.2)$$

The contact conductance is best determined experimentally for materials, as they are affected by factors such as surface finish, temperature, as well as the pressure applied at the contact. Figure 6.25 shows the user interface representation of the axi-symmetrical model of the regenerator tubing and pulse tube. The coaxial regenerator outer tubing is shown in green and the pulse tube in blue, and the sections indicate the different nodes modelled. The heat flow was computed both radially and axially. The simulated total conductive losses were found to be approximately 0.7 W. Table 6.4 shows the conduction output from the simulation.

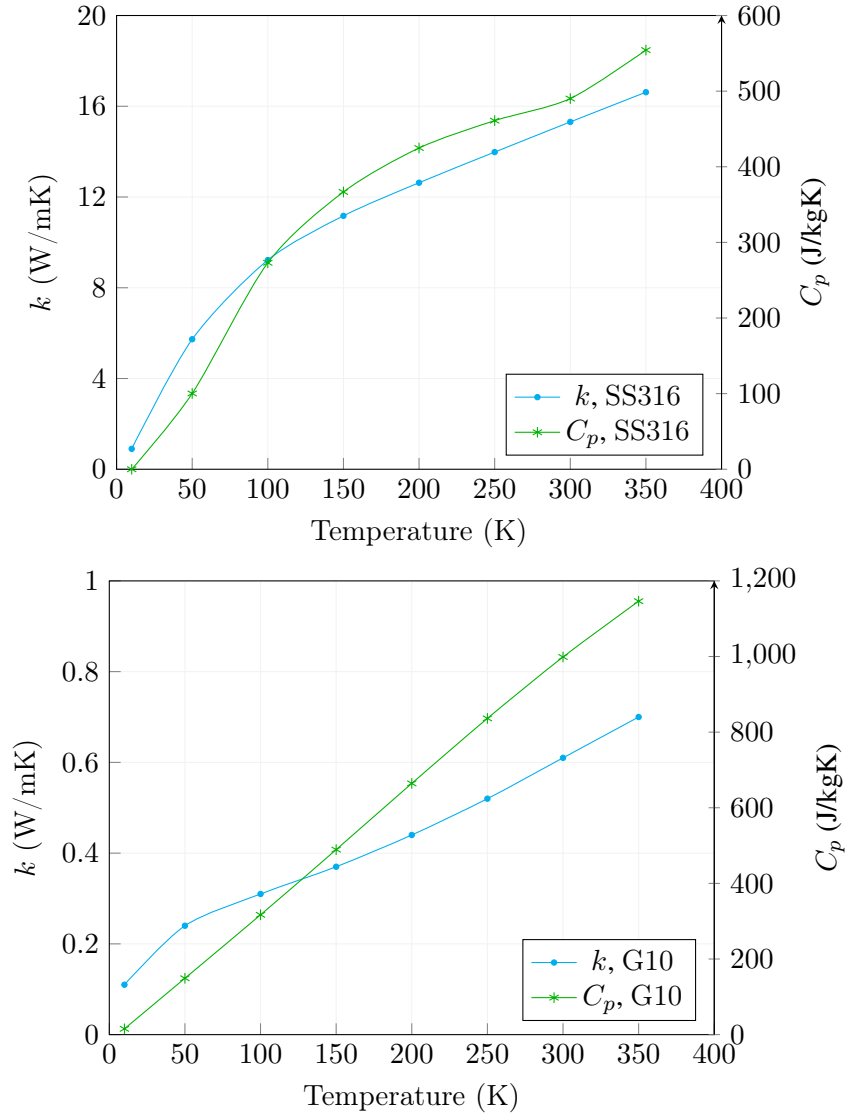


Figure 6.24: Thermal conductivity and specific heat capacity as a function of temperature for the SS316 regenerator (above) and the G10 pulse tube (below). These distributions were the material property inputs for both components, in addition to the parameters in Table 6.3.

6.6.5 Radiative Losses

The radiative losses from the pulse tube system required quantifying to ascertain their significance. ESATAN-TMS computes GR s to enable rapid analysis of radiative heat exchange between surfaces. The GR is the radiative coupling between two surfaces 1 and 2, is given by Equation 6.6.5.1, and is the form the model output takes.

$$GR_{12} = \varepsilon \sigma A V F_{12} (T_1^4 - T_2^4) \quad (6.6.5.1)$$

Table 6.4: The GL output and the corresponding conduction calculated for the regenerator tubing and the pulse tube in the coaxial arrangement shown in Figure 6.25.

Collective Nodes	GL Output (W/K)	Computed Conduction (W)
Regenerator tubing	0.141	0.64
Pulse tube	0.00149	0.034

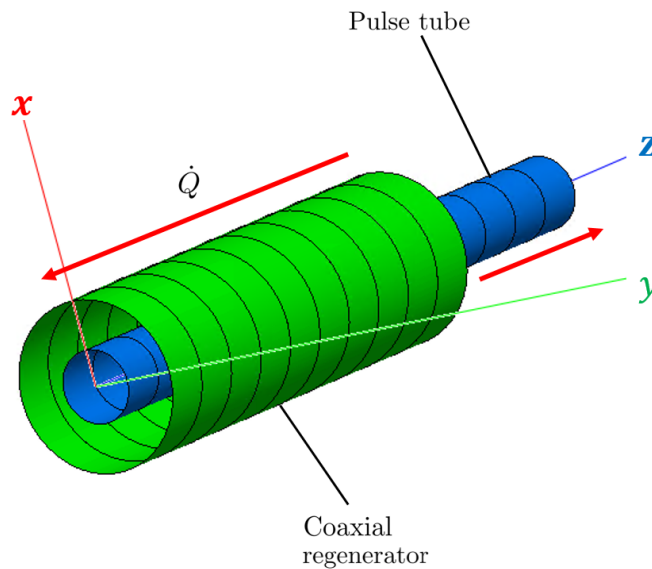
**Figure 6.25:** The coaxial regenerator tubing (green) and the pulse tube (blue) modelled in ESATAN-TMS. The sections indicate the nodes modelled. The x, y, and z model axes are also shown.

Table 6.5 shows the calculated total radiative heat flux from the coaxial regenerator-pulse tube arrangement. The computed values are expectedly low, and radiative heat flux from the G10 pulse tube was negligible.

Table 6.5: The calculated output for the radiative ray tracing model, as a result of computed GR values, between all node surfaces firstly from the regenerator to the pulse tube, and secondly, in the opposite direction.

Direction	Radiative Heat Flux (W)
Regenerator-PT	0.038
PT-Regenerator	Negligible

The conductive and radiative losses of the regenerator tubing and the pulse in the SPTC were computed using ESATAN-TMS. The combined total thermal losses were found to be approximately 0.7 W, closely matching the value of 0.65 W computed by the Sage numerical model. The radiative losses from the coaxial regenerator tubing and pulse tube arrangement were considerably lower.

The discrepancy in the model validation is due to many factors and does not seem attributable to errors in the conduction loss calculations alone. Figure 6.22 indicates that the difference in performance is not consistent; rather, it changes with varying stroke and phase angle. This matter requires further investigation, where unaccounted dead volumes in the experimental setup may need to be matched up with the numerical model. In addition to this, further analysis would require a thermo-fluid model to assess any convective losses, particularly those due to flow mixing, to complete a holistic assessment of all thermal losses within the system.

6.7 Simulated Cooling Performance at 80 K

The full model presented in this section was used to predict the performance of the cryocooler at 80 K and compared to numerically modelled performance results of the in-line cryocooler. The parameters that affect the performance of the cryocoolers that were addressed were the effect of the operating frequency on overall cooling power and pressure drop, the effect of shaft power input on cooling power and efficiency, and the effect of varying the phase angle between the linear compressor and the displacer. The inputs for each parameter were as shown in Table 6.6.

Table 6.6: Inputs for the coaxial and in-line cryocooler numerical models. One parameter was varied in step intervals to address the effect it had on the overall performance of the cryocooler, whilst all others were kept constant at the values found in this table.

Input	Value
Compressor stroke	9 mm
Displacer stroke	4.8 mm
Operating frequency	60 Hz
Phase angle	41°
Fill pressure	28 bar
T_c	80 K
T_h	300 K

Effect of operating frequency on cooling power and pressure drop

The operating frequency was varied to determine its effect on the cooling power of both cryocoolers at 80 K. Figure 6.26 indicates that at higher frequencies, higher cooling powers are obtained. Furthermore, the coaxial cryocooler is able to achieve a higher cooling power for a given frequency. A longer regenerator can permit a larger temperature difference to be generated at a given frequency, resulting in lower temperatures, but there are trade-offs between axial conduction, dead volume

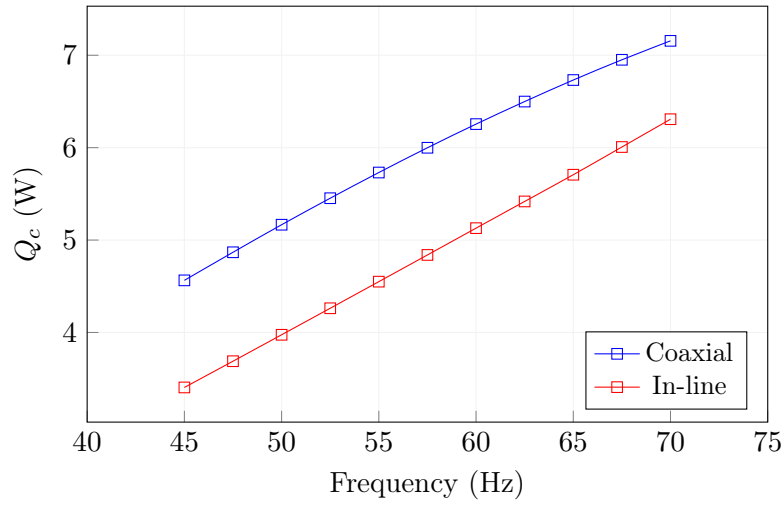


Figure 6.26: Cooling power (Q_c) of the coaxial and in-line cryocoolers as a function of varying the frequency with Table 2 inputs.

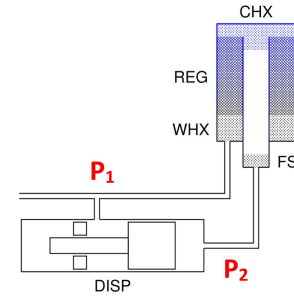
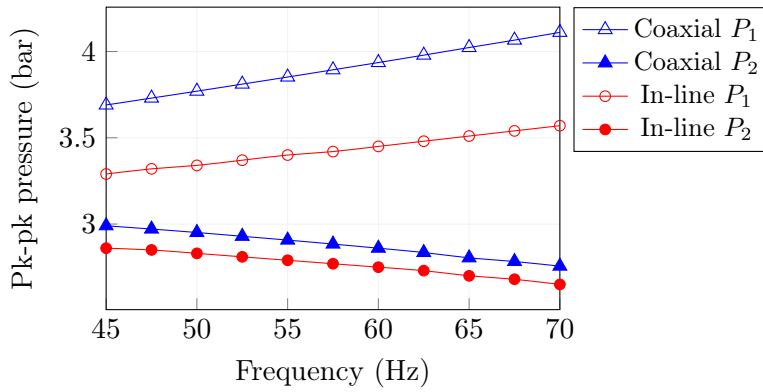


Figure 6.27: (Left) Effect of varying frequency on the pressure on either end of the coaxial and in-line cryocoolers and (right) the location of P_1 and P_2 in the coaxial cryocooler design, with Table 2 inputs. For both cryocoolers, P_1 is the pressure of the gas after the compression unit prior to entering the regenerator, and P_2 is the pressure of the gas after exiting the warm end heat exchanger. In this way, the difference between P_1 and P_2 indicates the pressure drop over the cryocooler.

and the pressure drop. It is worth noting that the conductive losses estimated by the numerical models for both the coaxial and the in-line simulate very small conductive losses in the cryocooler, as Sage does not account for the geometric complexity of all the conductive surfaces. This no doubt results in an overestimation of the cooling power.

The variation in pressure drop across the cryocooler as a result of varying operating frequency was also addressed as shown in Figure 6.27. The pressure drop across the cooler increases with an increase in operating frequency and the pressure drop across the coaxial pulse tube is larger than the in-line. Moreover, the pressure P_2 can be seen to decrease with an increase in operating

frequency. As the pressure amplitude increases in the cooler pulse tube unit due to an increased operating frequency, this leads to a decrease in pressure at the back end of the displacer.

Effect of shaft power on cooling power and efficiency

The effect of the shaft power on the cooling power and efficiency of the cryocoolers was determined by varying the compression volume by changing the piston stroke. Figure 6.28 demonstrates the interplay between the two parameters for both the coaxial and in-line cryocoolers. Initially, the in-line cryocooler is able to achieve a higher cooling power for a given shaft power. After a crossover point at approximately 68 W of shaft power, the coaxial cryocooler outperforms the in-line in terms of cooling power. It is evident therefore that the in-line requires less volume flow rate to achieve 80 K cooling at the cold end of the cryocooler, whereas the coaxial cryocooler must be operated at higher volume flow rate to achieve the same target cooling. Figure 6.28 demonstrates that whilst the cooling power of the coaxial cryocooler is higher at high shaft powers, the efficiency is not necessarily greater than the in-line cryocooler. The relative Carnot efficiency is determined by the equation below, where η_r is the relative Carnot efficiency, W_{sh} is the total shaft power, T_h is the warm end temperature (300 K) and T_c is the cold end temperature (80 K).

$$\eta_r = \frac{Q_c}{W_{sh}} \left(\frac{T_h}{T_c} - 1 \right) \quad (6.7.0.1)$$

Phase angle between compressor and displacer

Figure 6.29 shows the effect varying the phase angle between the compressor and displacer has on the cooling power and the relative Carnot efficiency, respectively. It can be seen that higher cooling power is consistently achieved by the coaxial cryocooler, however, again, the efficiency achieved is not significantly better than the in-line cryocooler. The peak cooling power for both the coaxial and the in-line occurs at an optimum phase angle of 55°, whereas the relative Carnot efficiency reaches a maximum at a phase angle of 40°. Hence a trade-off between the two would need to be considered.

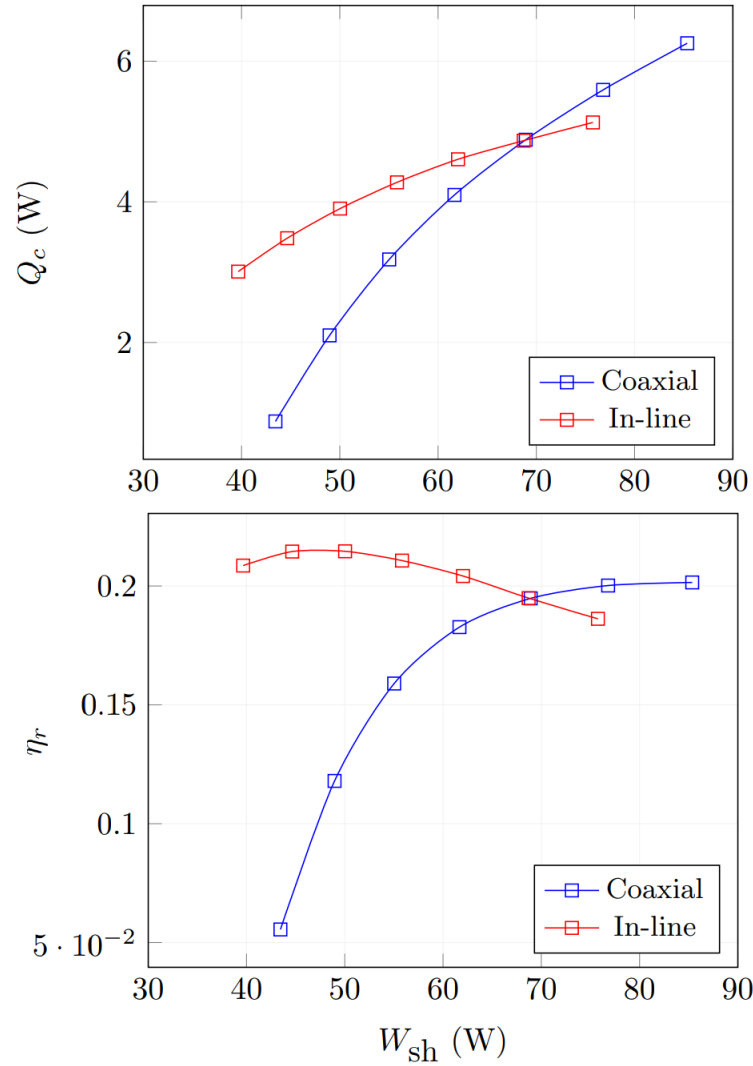


Figure 6.28: Effect of shaft power on cooling power Q_c (top) and relative Carnot efficiency η_r (bottom) as a function of compression volume, with Table 2 inputs.

Performance sensitivity to pulse tube and regenerator length

The numerical model can be used to determine the optimal length of the pulse tube and regenerator for highest net cooling power. Figure 6.30 indicates that the optimal regenerator length of 50 mm should be selected, and that the variation in the pulse tube length within this range only marginally affects the cooling performance.

The effect of the change in pulse tube and regenerator length on the variation in $\phi_{p,PT}$, the phase between the pressure pulse along the pulse tube length, and $\Delta P_{pk-pk,PT}$, the difference in peak-to-peak pressures on either end of the pulse tube, was investigated. The same was investigated

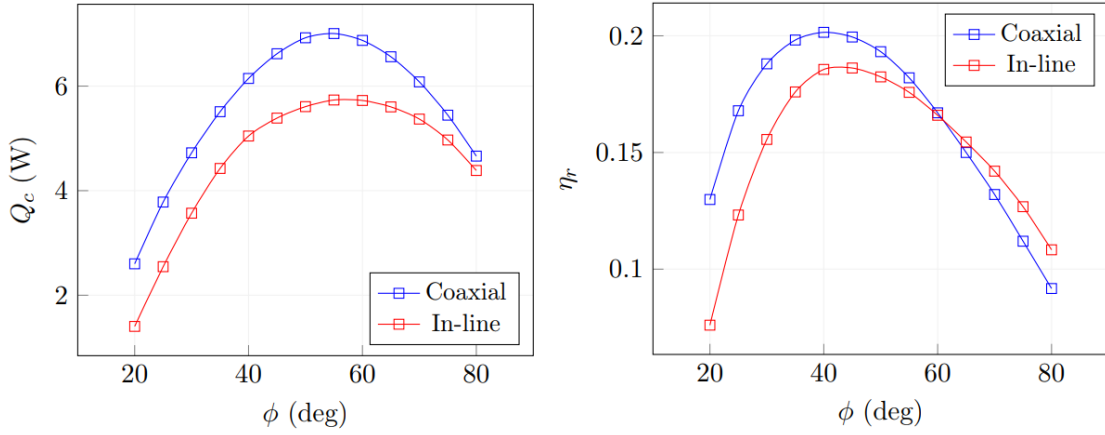


Figure 6.29: The effect of altering the phase angle between the linear compressor and displacer on (left) the cooling power Q_c and (right) the relative Carnot efficiency η_r of the coaxial and in-line cryocoolers, with Table 2 inputs.

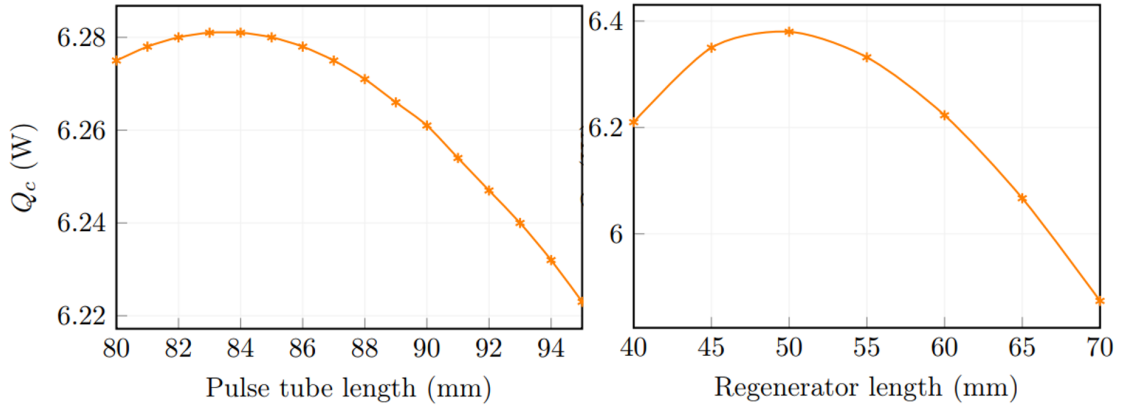


Figure 6.30: Change in cooling power Q_c (W) as pulse tube length (left) and regenerator length (right) varies in the coaxial cryocooler, with inputs as per Table 6.6.

for the regenerator. It was found that altering the pulse tube length does not greatly impact the $\phi_{p,PT}$ and $\Delta P_{pk-pk,PT}$, however, a noteworthy increase is observed in $\phi_{p,REG}$ and $\Delta P_{pk-pk,REG}$ with an increase in regenerator length, as shown in Figure 6.31. A variation in $\Delta \dot{m}_{pk-pk,PT}$, the difference in peak-to-peak mass flow, and $\phi \dot{m}_{PT}$, the mass flow phase along the pulse tube, was observed by varying the pulse tube length, as shown in Figure 6.32. This sheds light on the physical impact of length variation on the mass flow in the pulse tube. In the regenerator, in addition to the increased difference in peak-to-peak pressures with a longer length, the mass flow phase increases also, as shown in Figure 6.32. This highlights these as the factors responsible for the influence on the variation in cooling power with component length, as seen in Figure 6.30.

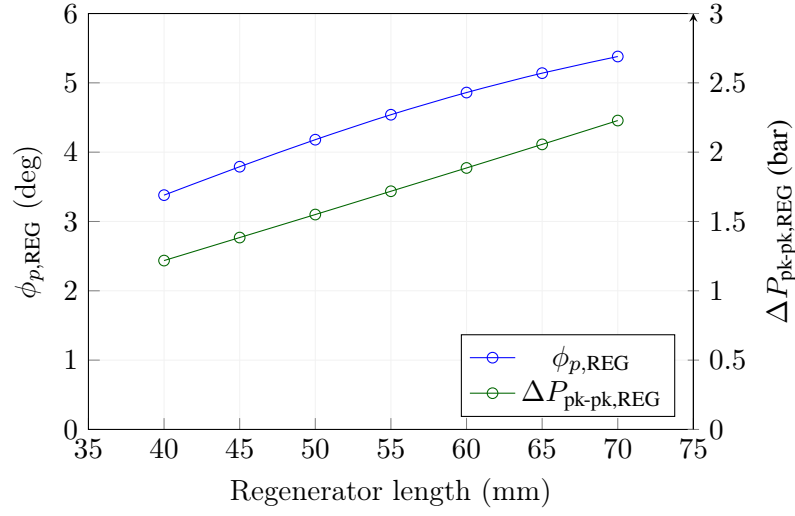


Figure 6.31: Variation of $\phi_{p,REG}$ and $\Delta P_{pk-pk,REG}$ with an increase in regenerator length, with all other inputs as per Table 6.6.

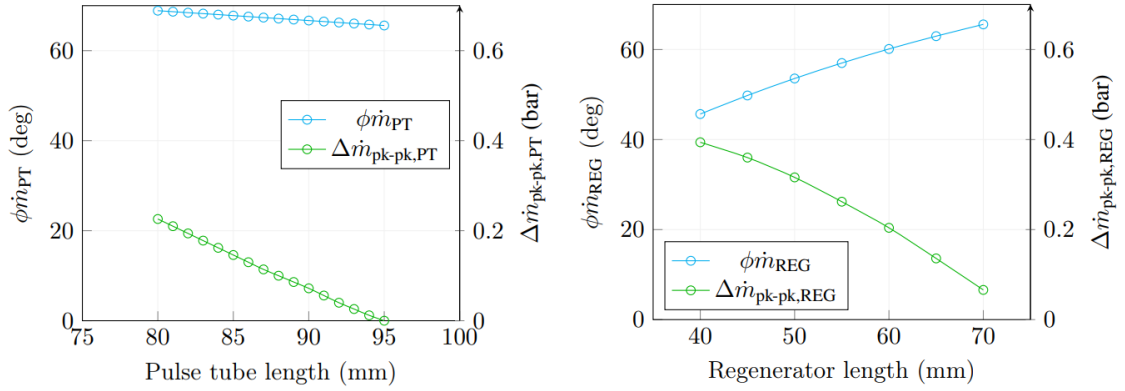


Figure 6.32: Variation of $\phi \dot{m}$ and $\Delta \dot{m}_{pk-pk}$ for the pulse tube (left) and regenerator (right).

Displacer insights from simulations

The desire to reduce additional moving parts and reduce the complexity of the cryocooler design would improve the reliability and longevity of a cryocooler selected for a spacecraft mission. Whilst inertance tubes succeed in tackling this problem, its use does not allow for power to be recycled back into the cryocooling system in the manner that a displacer does. A potential solution is to employ a passive displacer. The further challenge, however, would be to carefully design the correct dynamics of the displacer to achieve a high efficiency similar to the active displacer, as has been numerically simulated in this chapter and previously experimentally demonstrated [24] [23] [68] [69] [26]. A damper will likely be required to tune the phase angle, which will undoubtedly add an additional loss term due to its energy dissipation. Work by Wang et al. [70] have achieved

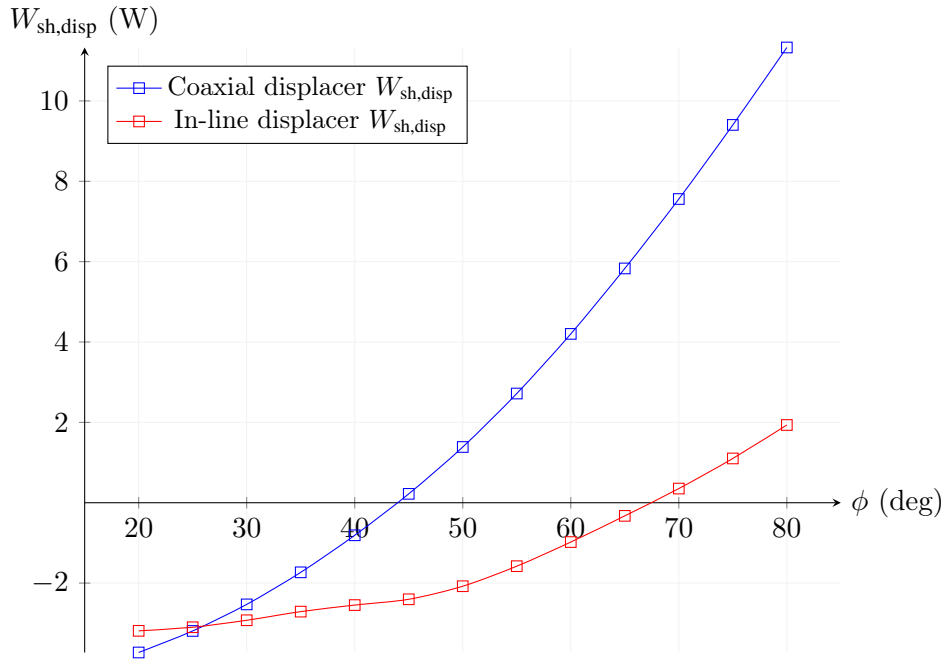


Figure 6.33: Displacer shaft power, $W_{sh,disp}$ (W), with varying compressor-displacer phase angle, ϕ (deg), with all other inputs as per Table 6.6

a relatively high Carnot efficiency by using a rod-less displacer.

Further analysis, shown in Figure 6.33, of the numerical performance data obtained for the coaxial and the in-line cryocoolers with active displacers demonstrated that the displacer shaft power computes to give negative values at lower phase angles. These results are useful in shedding light on the conditions in which the displacer can be driven passively. This will provide useful starting inputs for designing the damping parameters and sizing of a displacer to achieve the correct dynamics for passive operation. This will require the compressor to operate at resonance and the displacer to operate near resonance, in order to achieve the correct phase angle.

In summary, a numerical model has been devised for a complete 80 K SPTC with an active displacer in a coaxial configuration. Flow tests were completed on selected sub-assemblies such as the coaxial cold head to provide starting inputs for the simulation. The coaxial cryocooler numerically simulates 6 W of cooling at 80 K with an input power of 85 W, at a fill pressure of 28 bar, an operating frequency of 60 Hz and a compressor-displacer phase angle of 41° . The numerical performance indicates that the coaxial cryocooler may outperform the in-line cryocooler

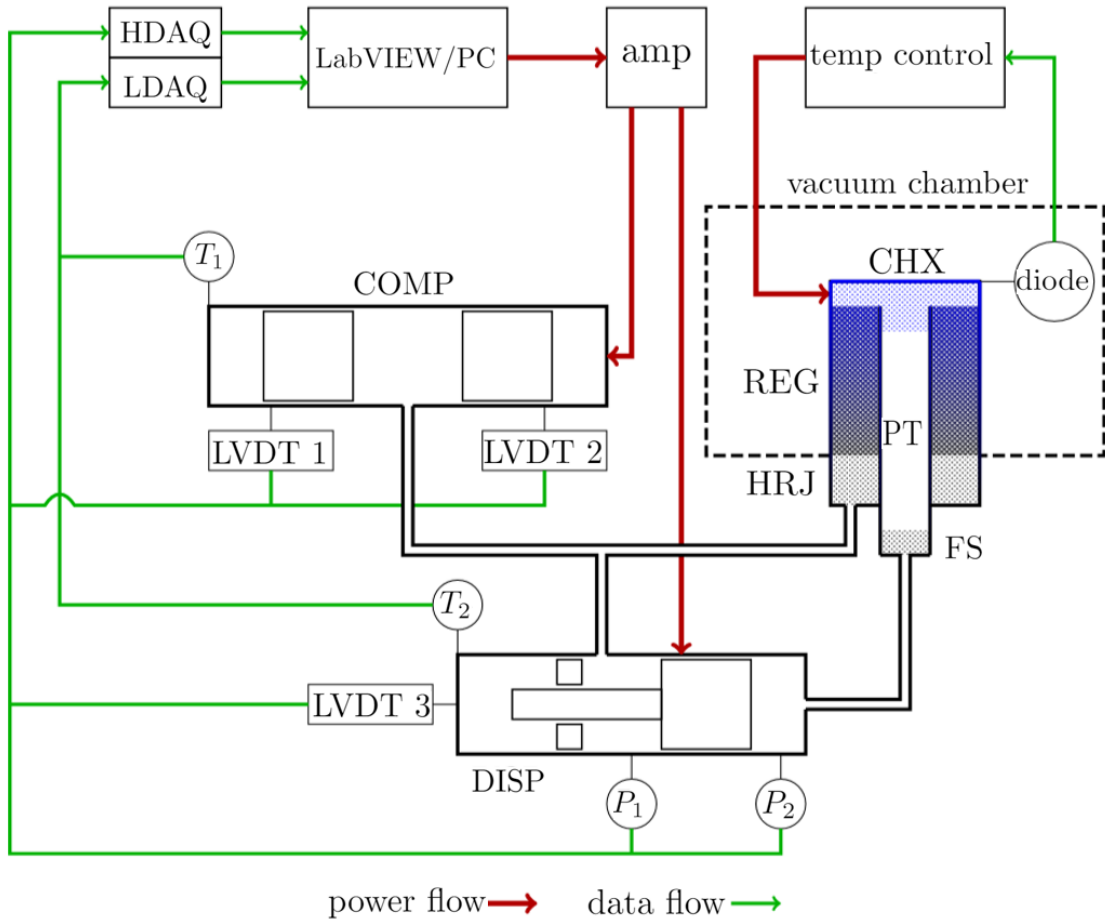


Figure 6.34: The experimental layout and instrumentation utilised to test the laboratory performance of the coaxial SPTC.

in terms of cooling power, but present no particularly significant improvement in terms of efficiency. These findings can be used to inspire the development of a passive displacer SPTC.

6.8 Experimental Layout and Instrumentation

Fig. 6.34 shows a schematic of the coaxial SPTC along with the instrumentation used [56]. The compression unit, marked 'COMP' in Figure 6.34, consists of a dual piston oil-free linear compressor which generates the pressure wave that runs the cryocooler. The operating frequency is typically within the range of 50-70 Hz. A single oil-free piston, serving as an active displacer, is situated at the warm end of the cryocooler and is operated at an input frequency which is the same as the drive frequency for the compressor. The active tuning of the motor inputs for the displacer allows for the phase difference between the compressor and displacer to be established, which ultimately allows for the control of the pressure pulse and mass flow throughout the system. An

annular heat rejector is positioned at the warm end, which is filled with copper mesh of 60 wires per inch, and the regenerator consists of stainless steel mesh of 400 wires per inch. A cold end heat exchanger is situated at the cold head region and serves to stratify the flow by operating as a flow straightener. Similarly, a flow straightener is inserted at the warm end, which consists of copper mesh of 60 wires per inch.

The temperature at the cold end was measured using a DT-670 LakeShore silicon diode cryogenic temperature sensor with an error of ± 0.5 K. A small heater was integrated at the cold head and a PID temperature controller was utilised to regulate the setpoint cold head temperature. The coaxial pulse tube, regenerator, and cold head setup were housed inside a vacuum chamber which maintained a vacuum at 1×10^{-5} mbar, in order to minimise convective losses. The latter setup was shrouded in radiation shields of aluminised Mylar to minimise radiative losses, as shown in Figure 6.35. The compressor and displacer drive inputs were controlled and monitored by a dedicated LabVIEW programme with an accuracy of 0.2% and LVDTs were used to record their displacement with an error of ± 0.05 mm. Furthermore, pressure transducers with an error of ± 0.1 bar were utilised to measure the pressures at P_1 and P_2 , the pressures at the front and back end of the active displacer, respectively. A phase meter was employed to monitor the phase angle between the compressor and displacer motion, which enabled the in-operation fine-tuning of the displacer motion relative to the compressor input stroke. Lastly, a sampling rate of 5 kHz was selected for the LabVIEW HDAQ for all sensors discussed and a LDAQ was set up to record the temperatures from the thermocouples T_1 and T_2 , as indicated in Figure 6.34, at a sampling rate of 10 Hz.

6.9 Experimental Performance

6.9.1 Cryocooler Performance

To determine the effect of displacer operation on the overall SPTC performance at 80 K cold head temperature, a range of sensitivity tests were conducted at a fill pressure of 28 bar, a constant compressor stroke of 8 mm, and an operating frequency of 60 Hz. To begin, a constant displacer stroke of 4 mm was maintained whilst the compressor-displacer phase was altered in increments of

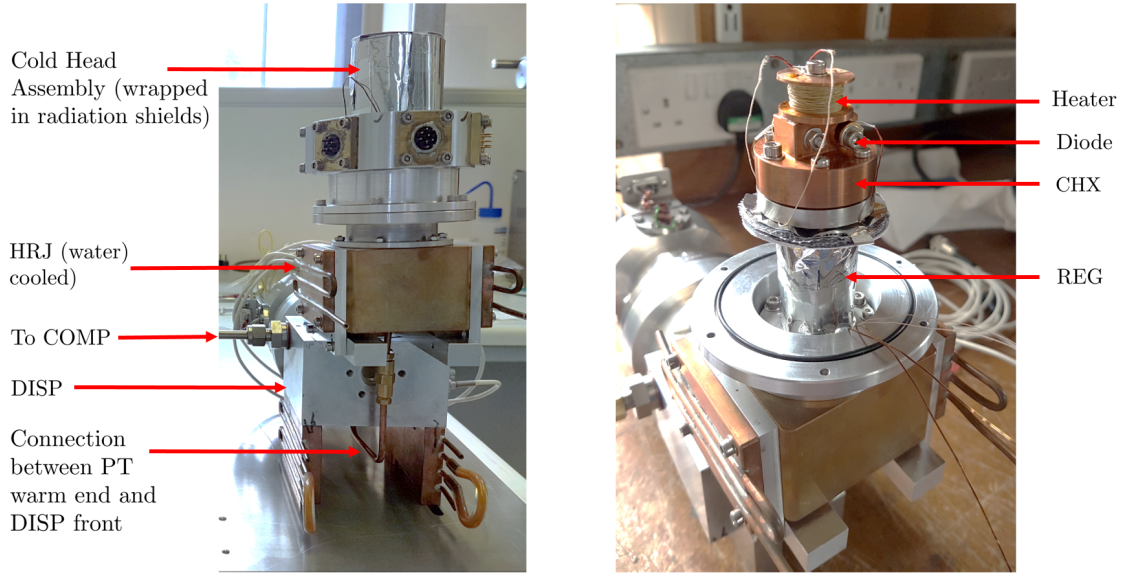


Figure 6.35: The coaxial regenerator and pulse tube assembly with the warm end heat rejector (HRJ) and displacer (left). The cold head assembly is pictured wrapped in radiation shields. The cold head assembly (right) is shown without the radiation shields, where the cold end heat exchanger (CHX), diode and heater can be seen installed onto the coaxial regenerator-pulse tube unit. All abbreviations are as indicated in Fig. 6.4.

5° over a range of 30° to 60°. Thereafter, the displacer stroke was varied over a range of 3.2 mm to 5.2 mm with a constant displacer phase of 45°. All testing inputs are summarised in Table 6.7.

The resulting cooling power and relative Carnot efficiency, η_r , which is given in Eq. 6.9.1.1, are shown in Figure 6.36. The predicted values simulated in the Sage numerical model are also shown.

$$\eta_r = \frac{Q_c}{\dot{W}_{in}} \left(\frac{T_h}{T_c} - 1 \right) \quad (6.9.1.1)$$

with

$$\dot{W}_{in} = \dot{W}_{in,c} + \dot{W}_{in,d} \quad (6.9.1.2)$$

where $\dot{W}_{in}$ is the total input power, $\dot{W}_{in,c}$ is the input power to the compressor and $\dot{W}_{in,d}$ is the input to the displacer. The compressor shaft power and motor efficiency are given by Eqn. 6.9.1.3 and Eqn. 6.9.1.4, where

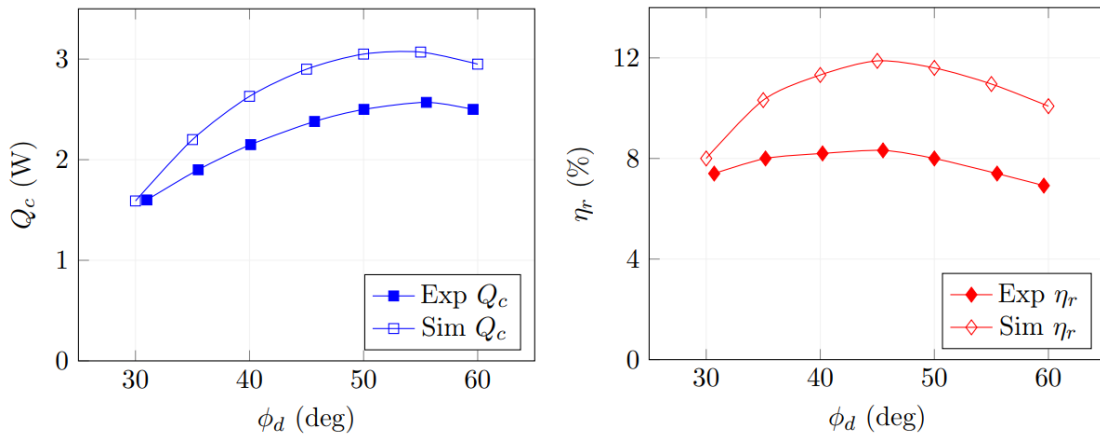
$$\dot{W}_{sh,c} = \dot{W}_{in,c} - I_c^2 R_c \quad (6.9.1.3)$$

$$\eta_m = \frac{\dot{W}_{sh,c}}{\dot{W}_{in,c}} \quad (6.9.1.4)$$

The optimum values for the displacer stroke and phase were determined through the experimental

Table 6.7: Inputs for the coaxial experimental conditions for the various performance sensitivities.

Quantity	Value
Fill pressure	28 bar
Frequency	60 Hz
Compressor stroke	8 mm
Displacer stroke, d_d	3.2 mm to 5.2 mm
Displacer phase, ϕ_d	30° to 60°
Cold end temperature, T_c	80 K
Heat rejection temperature, T_h	300 K

**Figure 6.36:** SPTC performance sensitivity to displacer phase with a constant displacer stroke of 4 mm and all other inputs as per Table 6.7.

testing. An optimum phase value of approximately $\phi_d = 50^\circ$ was obtained to achieve maximum Q_c . Optimum relative Carnot efficiency, however, was determined at a slightly lower value of $\phi_d = 45^\circ$. The optimum displacer stroke values were found at $d_d = 4.5$ mm and $d_d = 4$ mm for maximum Q_c and maximum η_r , respectively. It is worth noting that the variation in the values over the parametric sweep ranges is minimal and hence any slight deviation from the optimum will not have a significant adverse affect on overall SPTC performance.

All in all, the same trends observed in the numerical simulations presented section 6.7 can be seen in the experimental findings shown in Figures 6.36 and 6.37, where the peaks occur at approximately the same optimum operating conditions. It is important to note that the Sage model predicts higher values for cooling power and efficiency. This is likely due to the fact that conductive losses and heat transfer rates are difficult to model accurately in the Sage numerical model and hence are underestimated. It is difficult to numerically represent the complex geometry of the

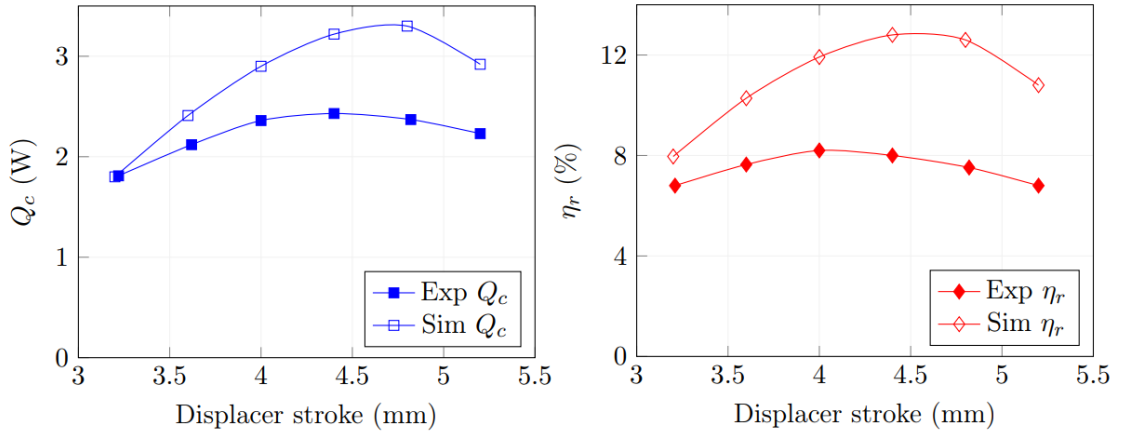


Figure 6.37: SPTC performance sensitivity to displacer stroke with a constant displacer phase of 45° and all other inputs as per Table 6.7.

intricate fully integrated system and all associated piping and bends. Moreover, the efficiencies achieved by the numerical simulation are particularly higher than the experimental values, given that the W_{in} input in the simulation accounts for all the power input, whereas in reality, there is a power loss between what is input into the power electronics and what transfers to shaft power entering the compressor and displacer pistons. The shaft power, W_{sh} , into the compressor is around 80% of the input power (at 60 Hz and 28 bar), whereas the displacer has a motor efficiency which is less than 50% for the same operating conditions. The significantly lower displacer motor efficiency is due to the fact that the displacer is being run further away from its resonant frequency. Nonetheless, this does not result in a notable loss given that the proportion of the input power entering the displacer is a small ($\sim 18\%$) compared to that entering the compressor ($\sim 82\%$). This is because the displacer is operating away from its optimum operating frequency; however, this loss does not result in a significant decline in efficiency as the displacer input power is a small fraction of the total input power as shown in Figure 6.38. At higher displacer stroke and phase operating conditions, this proportions changes and the displacer input accounts for a larger proportion of total W_{in} .

6.9.2 Pressure Pulses and Mass Flows

Active Displacer

The pressure transducers are situated on either end of the displacer piston to monitor the displacement during operation and to gain a better understanding of the how the motion impacts the

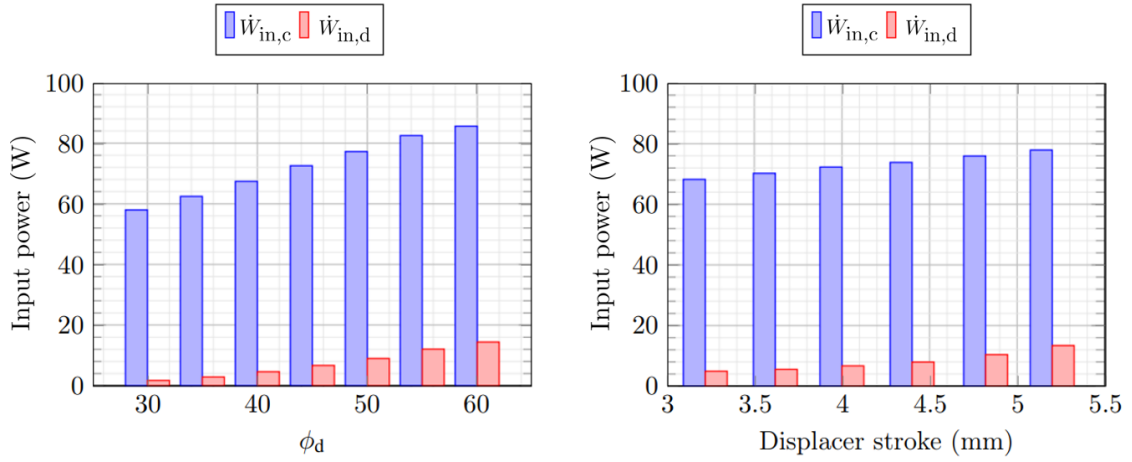


Figure 6.38: Compressor ($\dot{W}_{in,c}$) and active displacer ($\dot{W}_{in,d}$) contributions to total input power during the displacer phase and stroke sensitivity experiments, where the displacer stroke is 4 mm for the ϕ_d sensitivity (left), and the phase is 45° for the stroke sensitivity (right).

pressure pulse throughout the SPTC. Figure 6.39a presents how the peak-to-peak values for the displacer pressure varied across displacer phase, for both the numerically simulated values (as presented in section 6.7) and the experimentally measured results. The phase change across the displacer is further shown on the secondary axis. It can be seen that with a variation in displacer phase, P_1 remains unchanged however P_2 decreases by approximately 25% following a change in displacer phase of 30° . This results in an overall increase in pressure drop across the displacer of 0.9 bar for the 30° displacer phase change. The phase angle between the pressure pulses, however, remains relatively constant throughout the sensitivity sweep, at approximately a 6° phase angle. Likewise, Figure 6.39b demonstrates the variation in peak-to-peak pressure across the displacer and phase change between the pressure pulses at either end of the displacer with respect to the displacer stroke. In contrast to the sensitivity to displacer phase, doubling the displacer stroke only results in a 0.3 bar increase. A more drastic change in pressure phase can be seen over the range of displacer stroke variation, resulting in a phase change of 13° for an overall stroke increase of 2 mm. Hence, the peak-to-peak pressures are sensitive to the change in displacer phase and the phase between the pressure pulses are sensitive to displacer stroke. The peak-to-peak values are found to be slightly higher in the Sage numerical model, however, the numerical results display the same trends as observed in the experiments, as before. This provides confidence in the validity of the Sage predictions and enables the Sage model to be utilised for furthering understanding of what is taking place at the cold end of the SPTC.

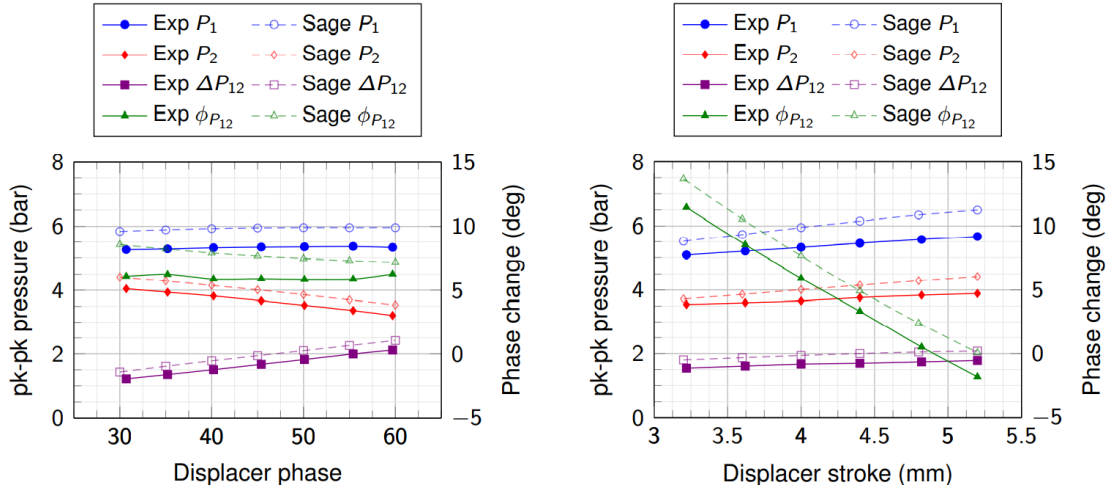


Figure 6.39: Pressure pulse peak-to-peak and phase change across the displacer as a result of varying the displacer phase and stroke, where (a) shows the sensitivity to displacer phase at a constant displacer stroke of 4 mm and (b) shows the sensitivity to displacer stroke at a constant displacer phase of 45°.

Cold Head

As previously discussed, the SPTC performance is governed by the interplay between the pressure pulse and the mass flow at the cold end. Figure 6.40 demonstrates how the peak-to-peak mass flow and phase angle with respect to the pressure pulse are predicted by the numerical model at the cold end. It can be seen that an increase in displacer phase results in an increase in mass flow from 2.5 g/s to 4.3 g/s. Moreover, an increase in displacer stroke yields a 1.4 g/s increase. Conversely, the mass flow phase relative to pressure behaves very differently in the two tests. A 30° increase in displacer phase results in a 9° reduction in ϕ_{pm} whereas a 2 mm increase in displacer stroke leads to a 33° increase.

This very phase relationship between mass flow and pressure pulse at the cold end is hence influential in the resultant cooling power that the SPTC can achieve. Figure 6.40 indicates the phase difference between pressure and mass flow at optimum operating conditions of $\phi_d = 45^\circ$ and $d_d = 4$ mm is around 20°. Changing the displacer phase and stroke away from these optimums of $\phi_d = 45^\circ$ and $d_d = 4$ mm results in a change in ϕ_{pm} away from its optimum value. Given that the greatest pressure drop in the SPTC occurs across the regenerator, it can be assumed that the pressure in the cold head is similar in magnitude to that determined at the front end of the displacer, P_2 . Figure 6.39 demonstrated that the peak-to-peak pressure decreases with an increase in displacer phase, however increases with displacer stroke. Conversely, the mass flow amplitude increases

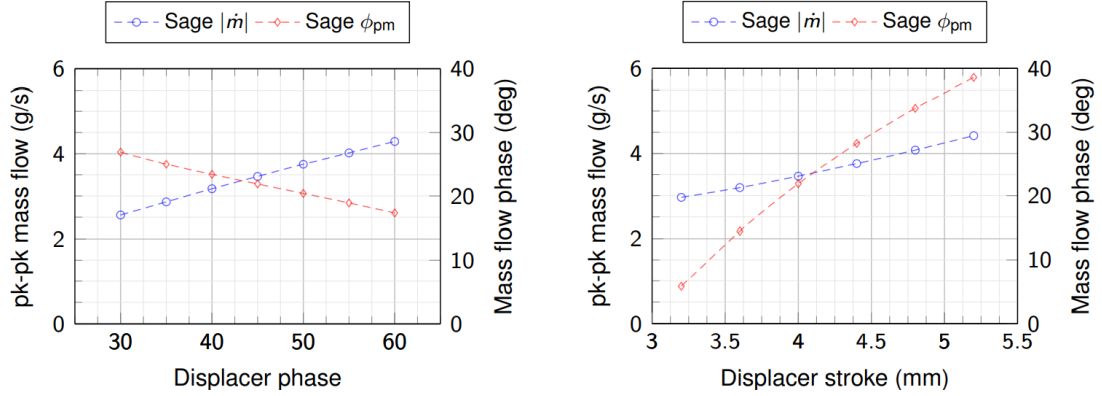


Figure 6.40: Variation in mass flow peak-to-peak and phase (relative to pressure) for different displacer phase and stroke values according to the Sage model, where (left) indicates the sensitivity to ϕ_d for a displacer stroke of 4 mm and (right) depicts the sensitivity to displacer stroke for a ϕ_d of 45° .

with increasing displacer stroke and displacer phase. In combination, these different variations all factor together to produce the cooling power trends in the SPTC that were presented in Figures 6.36 and 6.37.

6.9.3 Optimum Performance

Once the optimum displacer phase and stroke operating conditions were determined, further experimental tests were conducted to map out the SPTC performance for differing cold end temperatures between 70 K up to 120 K. The operating inputs for these tests were a displacer stroke of 4 mm and phase of 45° . These were the optimum values in terms of highest efficiency at 80 K (see Fig. 6.36 and Fig. 6.37). All other inputs were as given in Table 6.7. The cooling powers achieved results are shown in Fig. 6.41.

6.9.4 Work Flows

The validated Sage model permits an in-depth investigation into the work flows throughout the SPTC. The cyclic averaged PV powers, $\langle \dot{P}V \rangle$, enthalpy flows, $\langle \dot{H} \rangle$, and the heat transferred, $\langle \dot{Q} \rangle$, are shown in Figure 6.42. The PV powers indicate the power entering and leaving the compression space on either end of the displacer, as well as the power into the compressor. The values presented have been simulated for the optimum operating conditions where $\phi_d = 45^\circ$ and $d_d = 4$ mm.

The flow map demonstrates how the displacer is able to recover PV expansion work and recycle it back into the SPTC. Based on the optimum case presented, it can be seen that out of

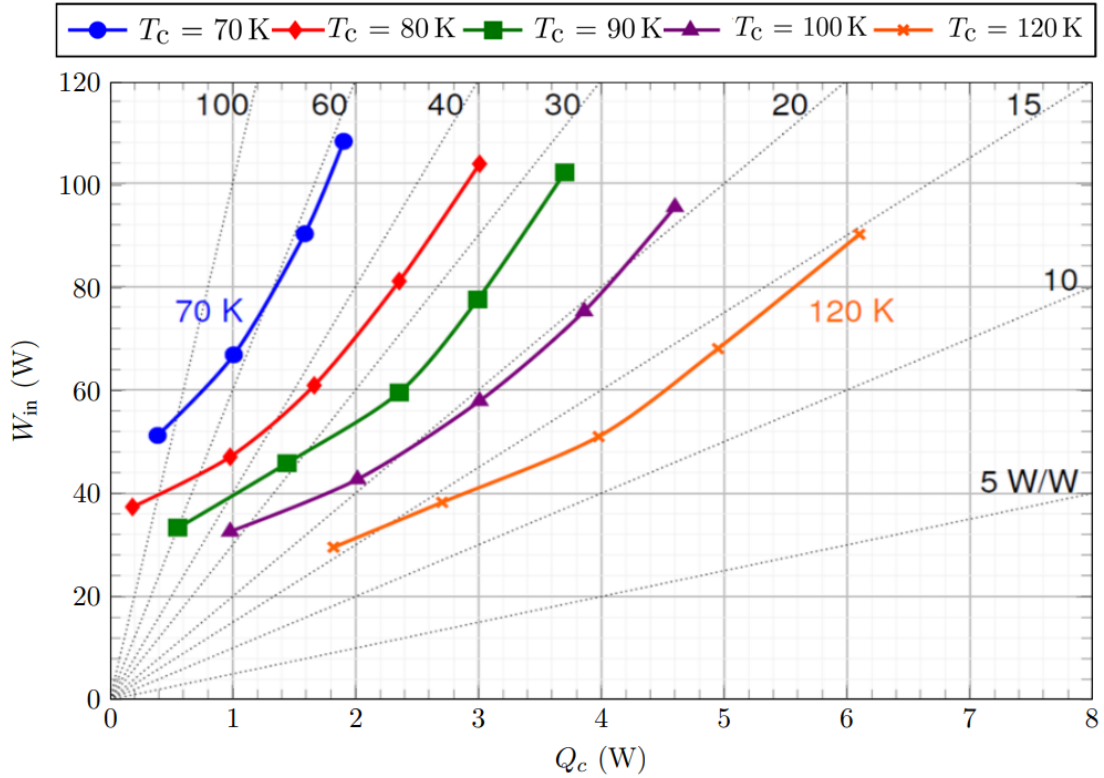


Figure 6.41: Coaxial SPTC performance at optimum displacer phase and stroke with a fill pressure of 28 bar and an operating frequency of 60 Hz. Lines of constant specific power (in W/W) are shown.

69 W of enthalpy flowing into the heat rejector, which drives the thermodynamic operation of the cryocooler, 4.5 W enters from the displacer. In the ideal case, there would be no enthalpy flow down the regenerator; this would be entirely delivered by the pulse tube from the cold head to the warm end displacer. It can be seen, however, that the regenerator transfers 3.5 W of enthalpy and that only 80% of the 7 W of enthalpy from the cold end reaches the displacer. This simulated value of enthalpy transferred down the pulse tube is likely to be even less in real life given that the heat transfer losses accounted for the Sage model are most likely underestimated, given that it is difficult to entirely replicate all intricacies of the geometry and configuration of the SPTC, and therefore, the associated heat transfer losses.

Additionally, the warm end flow straightener inadvertently acts in part as a heat exchanger that results in further expansion power being rejected as heat, given that it is difficult to isolate it from the heat rejector. Figure 6.43 depicts the heat lost through the warm end flow straightener, $\langle \dot{Q}_{FS} \rangle$, as well as the enthalpy recycled back into the system, $\langle \dot{H}_{FS} \rangle$, as a function of displacer stroke and phase. This demonstrates the benefit that a warm end displacer presents when compared

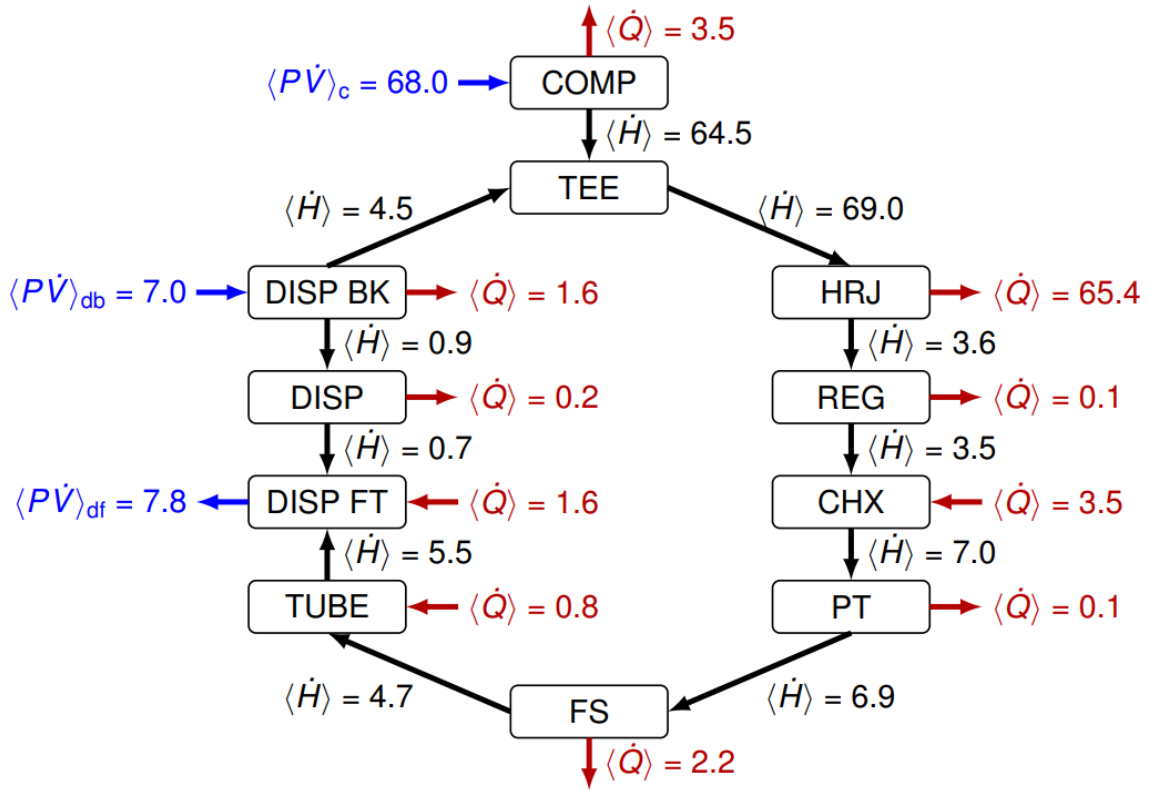


Figure 6.42: Cyclic averaged enthalpy flows (black), PV powers (blue) and heat flows (red) according to Sage in units of W. All abbreviations are as described in Fig. 6.4 and HRJ is the heat rejector and TEE refers to the 'T' junction that connects the back end of the displacer back into the warm end cycle. DISP BK and DISP FT stand for displacer back and displacer front, respectively. Results shown are for the optimum operating conditions with $\phi_d = 45^\circ$ and $d_d = 4$ mm.

to an inertance tube which does not allow for expansion power to be recovered back into the SPTC but is rather lost as heat. An increase in displacer phase shows an increased amount of enthalpy transferred into the displacer whilst a constant amount of heat is lost by the system. This indicates that the flow straightener at the warm is able to operate more effectively at higher displacer phase angles. The opposite is found with displacer stroke, in that an increase in stroke results in an increase in heat lost and a relatively constant amount of recycled enthalpy into the displacer.

6.9.5 PV Powers

The PV powers for the experimental SPTC performance can be determined using the pressure and LVDT displacement data obtained. Figure 6.44 shows the P - V plots for the compression space in the compressor and the displacer front and back spaces. These plots have been generated for the optimum operating conditions for the SPTC at 80 K, where $\phi_d = 45^\circ$ and $d_d = 4$ mm. The pressure in the compression and displacer back space were assumed to be the same and equal to P_1 . This

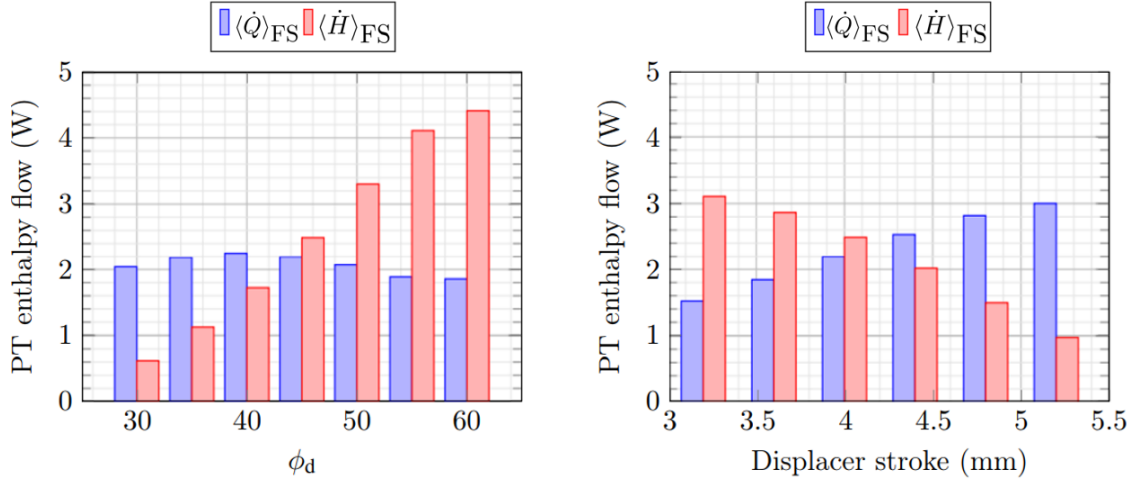


Figure 6.43: Proportions of pulse tube enthalpy flow which is lost as heat $\langle \dot{Q} \rangle_{FS}$ alongside that which is delivered to the displacer $\langle \dot{H} \rangle_{FS}$ according to the Sage model, where (a) is for a constant d_d of 4 mm and (b) is for constant $\phi_d=45^\circ$.

assumption is reasonable given the fact that the pressure drop across the connecting tube that joins these two gas compression spaces together is negligible compared to the pressure drop across the cold head assembly, where the pressure drop is largely dictated by the pressure difference across the regenerator. The PV powers were found to be 51.8 W and 3.7 W in the compression space and displacer front and back spaces, respectively. Despite these values being lower than those determined by Sage, as shown in the work flow schematic in Figure 6.42, there is good agreement with the Sage model in terms of the ratio of recycled $\langle P\dot{V} \rangle_{db}$ to $\langle P\dot{V} \rangle_c$, where the experimental data demonstrates 7% of recycled PV work and, similarly, Sage predicts 10%.

Figure 6.45 shows the change in PV powers of the compressor and displacer as displacer phase and stroke were increased. Once again, the Sage model and the experimental tests indicate the same trends. The PV powers into both the compressor, $\langle P\dot{V} \rangle_c$, and the displacer back space, $\langle P\dot{V} \rangle_{db}$, can be seen to increase with an increase of both phase and stroke. For the PV power for the displacer front space, $\langle P\dot{V} \rangle_{df}$, the gradient of increase is not as significant with an increase in displacer phase and it remains relatively constant with an increase in displacer stroke.

The SPTC using a passive displacer was designed and tested, as presented in chapter 7, following the design, simulation, testing and analysis of the active displacer, as presented in this chapter. In order for the displacer to be driven passively, the $\langle P\dot{V} \rangle_{df}$ extracted must equate the

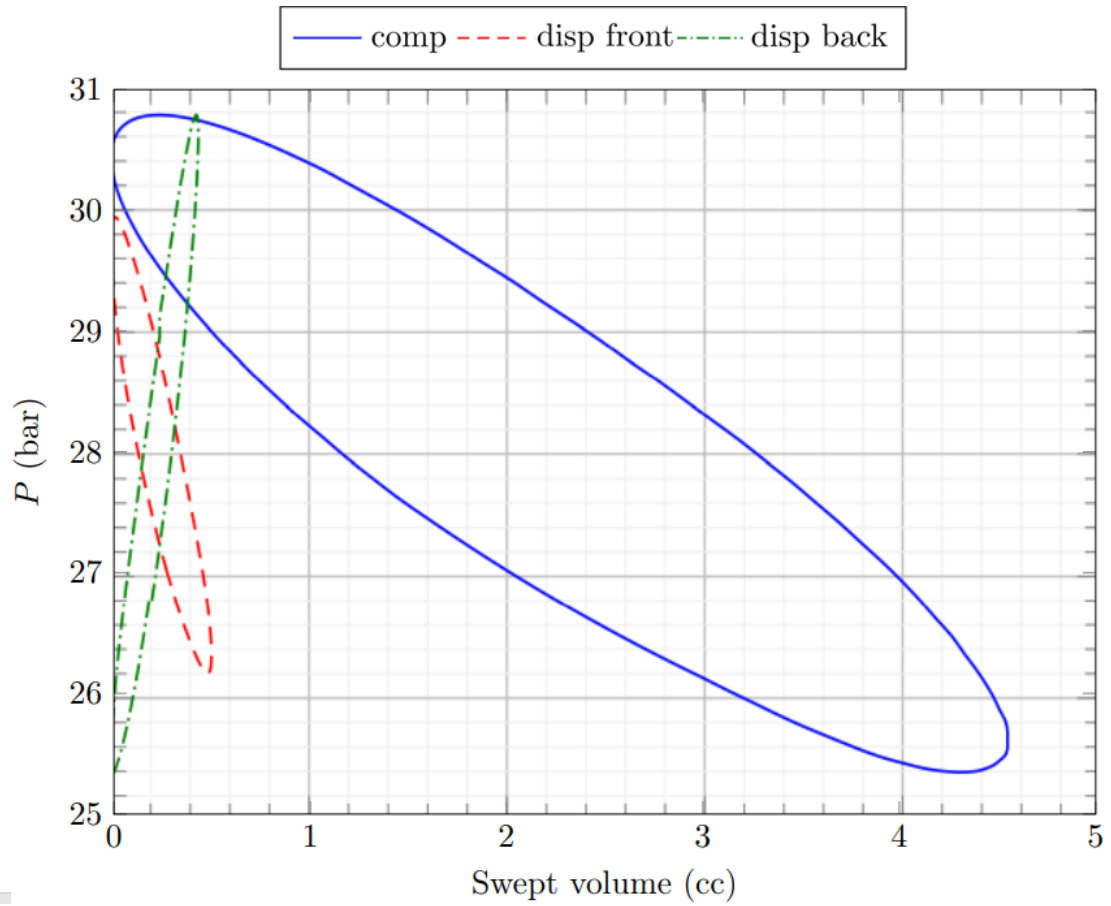


Figure 6.44: Experimental PV plot at the optimum operating conditions with $\phi_d = 45^\circ$ and $d_d = 4$ mm where $\langle P\dot{V} \rangle_c = 51.8$ W, $\langle P\dot{V} \rangle_{df} = 3.7$ W and $\langle P\dot{V} \rangle_{db} = 3.7$ W.

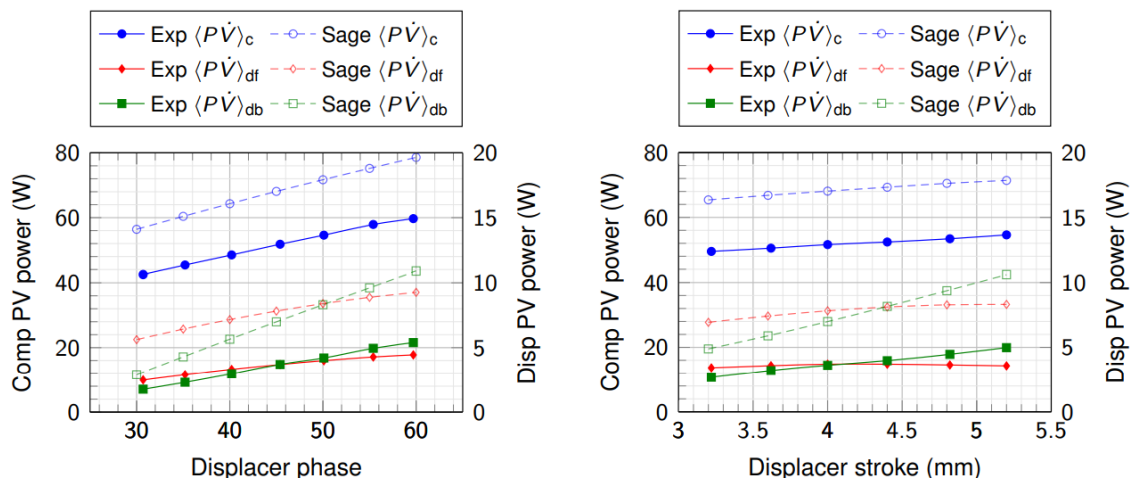


Figure 6.45: Compressor and displacer PV power sensitivity to active displacer motion, where the displacer stroke is constant at 4 mm (left) and ϕ_d is constant at 45° (right).

$\langle P\dot{V} \rangle_{db}$ entering the system. In actuality, the $\langle P\dot{V} \rangle_{df}$ would need to be slightly higher to overcome frictional forces, although these are estimated to be low given the clearance that is maintained between the displacer piston and the surrounding cylinder. Table 6.8 summarises the different scenarios in which the $\langle P\dot{V} \rangle_{db}$ and $\langle P\dot{V} \rangle_{df}$ dictate the operation of the active displacer.

Table 6.8: Different scenarios for displacer operation, as dictated by the PV power at the displacer front and back compression spaces.

Condition	Displacer operation
$\langle P\dot{V} \rangle_{df} < \langle P\dot{V} \rangle_{db}$	Active displacer motor provides additional power
$\langle P\dot{V} \rangle_{df} = \langle P\dot{V} \rangle_{db}$	Displacer is running passively
$\langle P\dot{V} \rangle_{df} > \langle P\dot{V} \rangle_{db}$	Active displacer acts as damper

6.10 Summary of Research Findings

In conclusion, a coaxial SPTC with an active displacer has been designed, simulated, built, and experimentally tested. A summary of the key findings are as follows:

- A coaxial Stirling pulse tube cryocooler with an active displacer was numerically simulated to reach a T_c of 80 K with a W_{in} of 85 W and achieve a Q_c of 6 W and a η_r of 18%. This was at a fill pressure of 28 bar, an operating frequency of 60 Hz, a ϕ_d of 41°, a compressor stroke of 9 mm and a displacer stroke of 4.8 mm.
- The Q_c and η_r values were predicted to be significantly higher than what would be expected in practice given that Sage is not able to fully account for heat transfer losses nor the complexity of the geometry and configuration of all components of the full cryocooler. Moreover, 3D flow effects are not modelled and further inhibit a physically representative model being achieved. A better understanding of the 3D flow within the SPTC can help further improve the Sage model.
- The numerical model was validated with sub-component flow testing and pressure measurements. Noteworthy insights into displacer phase and stroke sensitivity were gained and the overall trends were understood for the overall system at a component level. This further included understanding the crossover point at which the displacer can be run passively.

- The experimental coaxial SPTC was built and tested, achieving a T_c of 80 K with a W_{in} of 79 W and an optimum Q_c of 2.4 W, corresponding to a η_r of 8.3%.
- The active displacer permits the performance of the SPTC to be easily fine-tuned and optimised amid operation. This is achieved by varying the displacer motor stroke inputs, which alters the displacer stroke but also the phase between the compressor and displacer, ϕ_d .
- The displacer is able to recover expansion work at the warm end and recycle this back into the cryocooler system. This design hence improves efficiency of the cryocooler. The displacer provides 6% of the power needed to drive the cryocooler, and produces a more efficient SPTC compared to SPTCs that use inertance tubes.
- Noteworthy insights have been gained about the SPTC operation, including how the heat transfer, enthalpy flows, and PV work flows throughout the system, shedding light on its fundamental operation and the role key components play. Moreover, inputs for the design of a passively driven displacer have been acquired.

7

A Novel Passive Displacer Design

Contents

7.1	Objectives	140
7.2	Theoretical Analysis of Warm End Displacer	141
7.2.1	Displacer Design	141
7.2.2	Thermodynamic and Harmonic Analysis	142
7.3	SPTC Experimental Setup	143
7.3.1	Instrumentation	143
7.3.2	Pressure and Laser Displacement Transducers	144
7.3.3	Damping Plate	145
7.3.4	Displacer Assembly and Full SPTC	145
7.4	Passive Displacer Experimental Performance	147
7.4.1	Testing Campaign	147
7.4.2	Displacer Resonant Frequency	147
7.4.3	Clamped Displacer Tests	149
7.4.4	No Load T_c	150
7.4.5	Experimental Displacer Resonance	150
7.4.6	Displacer Dynamics Experimental Validation	151
7.5	SPTC Experimental Performance at 80 K	155
7.5.1	Cooling Power and Carnot Efficiency	155
7.5.2	Displacer Stroke and Phase	158
7.5.3	Pressure Oscillations and Pressure Driving Forces	159
7.5.4	Harmonic Forces and Effect of Damping	160
7.6	Energy Recovery in Pulse Tube Cryocoolers	162
7.6.1	Pulse Tube Design Rationale	162
7.6.2	Comparison of Cryocooler Performance at 80 K	164
7.7	Summary	166

This chapter presents the design and experimental research undertaken in developing a coaxial SPTC with a passively-driven displacer. The work presented in this chapter is solely of the author's undertaking, except in instances where otherwise referenced, and expands upon the research presented in the following publications:



- H. Rana et al., "Experimental performance of a passively driven displacer", *Cryocoolers*, (2022).
- H. Rana et al., "Design of a passive displacer for a Stirling pulse tube cryocooler", *Cryocoolers*, (2021).
- H. Rana et al., "Numerical modelling of a coaxial Stirling pulse tube cryocooler using an active displacer for space applications", *Cryogenics*, (2020).

7.1 Objectives

This chapter presents the work completed to study a coaxial SPTC with a warm end passive displacer and theoretically design its key components, using inputs from the coaxial SPTC with an active displacer performance. A design layout and harmonic analysis of the displacer is presented. Furthermore, the spring design analyses and experimental testing presented in chapter 4 are exploited for the passive displacer design. The outputs of these analyses provide design values for the displacer shaft diameter, spring stiffness, spring fatigue limit and stress regions, and displacer moving mass, enabling the manufacture and testing of a passive displacer for a high efficiency experimental coaxial SPTC. Following this, the experimental assembly, validation, and testing was undertaken. The passive displacer was then incorporated into the full SPTC and the overall cryocooler performance was investigated and analysed. The displacer dynamics were then experimentally validated and the overall cryocooler performance was compared with that of a numerically simulated in-line pulse tube cryocooler using an inertance tube in place of the displacer, as well as the in-line and coaxial SPTCs using active displacers which were previously experimentally tested, as presented in chapter 6 [71] [72].

The passive displacer is driven by the pressure difference across the displacer piston and displacer shaft which is created by the compressor unit. The passive displacer requires designing such that it operates at the optimal conditions in conjunction with the compressor to reach a similar efficiency and operation as the active coaxial SPTC. Inputs from the coaxial SPTC using an active displacer, which has the same design dimensions and materials of all other key components, are therefore required to perform design analysis of the passive displacer. Figure 7.1 indicates the

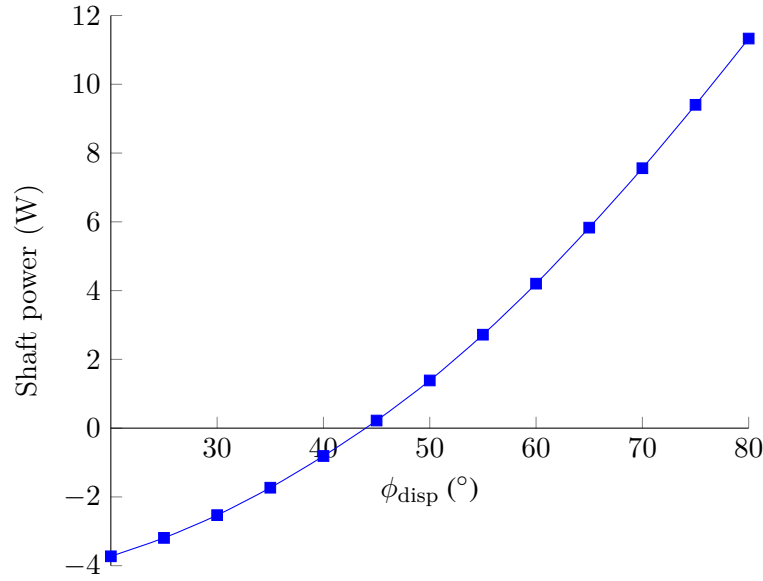


Figure 7.1: Simulated displacer shaft power with varying compressor-displacer phase angle (ϕ_{disp}) for a comparable coaxial SPTC with an active displacer. The compressor stroke is 9 mm, operating frequency is 60 Hz, and fill pressure is 28 bar.

simulated output of the coaxial SPTC with an active displacer, showing how the displacer shaft power varies with phase angle of the displacer relative to the compressor. Positive shaft power indicates the displacer is receiving work and negative indicates that it is putting work into the system. Negative values for shaft power of the displacer are obtained at lower phase angles, and the point of intersection at the axis indicates the conditions for phase angle in which the displacer would be running passively.

7.2 Theoretical Analysis of Warm End Displacer

7.2.1 Displacer Design

A schematic of the passive displacer is shown in Figure 7.2 that includes a damping piston and orifice. It shows a displacer piston of diameter d_d oscillating with displacement x due to the difference in pressure between the front space at p_f and the back space at p_b and the pressures either side of the damping piston. The displacer shaft with diameter d_s is connected to a damper piston of diameter d_p on the other side. The space in front of the damper piston is sealed from the rest of the back volume via an orifice plate of diameter d_o . Hence, the level of damping can be controlled by varying the size of the orifice. The dashed lines represent flexure bearings which are adjusted to achieve the necessary stiffness. Altogether, by varying the spring stiffness, moving

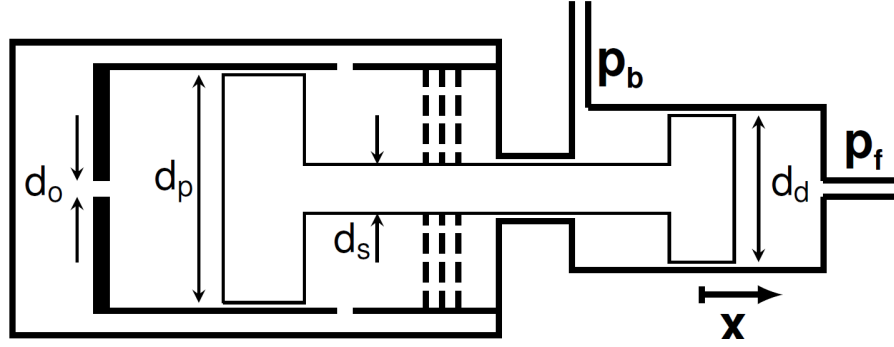


Figure 7.2: Schematic of the passive displacer that includes a damping piston and orifice is shown.

mass, damping and shaft diameter we can ensure that the passive displacer operates at the optimal phase and stroke in order to maximise cooling and efficiency.

7.2.2 Thermodynamic and Harmonic Analysis

A phasor analysis for the displacer was completed in which all forces acting on the displacer were balanced. The forces that make up the total force, F_t , are shown in Eqn. 7.2.2.1, where F_m is the motor force, F_p is the gas spring force, F_s is the spring force, F_d is the damping force, α is the motor force constant, I is the current, ΔP is the pressure difference across the displacer piston, A is the surface area of the piston, k is the spring constant, and c is the damping coefficient. As the different forces act sinusoidally, they can be converted to phasor form, where the steady state solution for the displacement, x , is given by Eqn. 7.2.2.2, and ω is the angular frequency. Eqn. 7.2.2.3 indicates the full force balance in phasor form, where the $e^{i\omega t}$ components cancel.

$$F_t = F_m - F_p - F_s - F_d = m\ddot{x} = \alpha I - \Delta P A - kx - c\dot{x} \quad (7.2.2.1)$$

$$x = x_1 e^{i\omega t} \quad (7.2.2.2)$$

$$\alpha I = kx + i c \omega x + \Delta P A - m \omega^2 x \quad (7.2.2.3)$$

A typical phasor diagram, as shown in Figure 7.3, can be produced to diagrammatically represent the magnitude of all forces acting on the displacer in the real and imaginary axes [31]. This allows for the resultant force F_m to be determined. The necessary value for the shaft diameter of the displacer can be determined to achieve a motor force of zero. At $F_m=0$, the displacer is being run passively and does not require a motor.

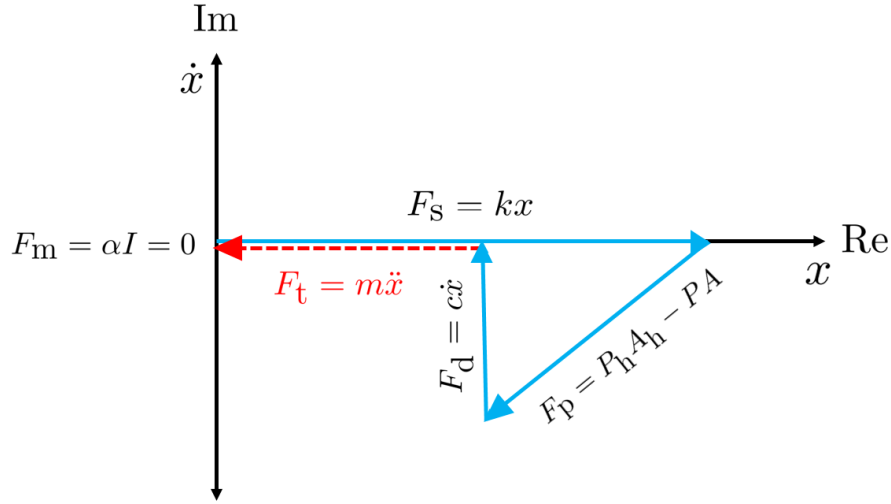


Figure 7.3: A typical phasor diagram for the forces acting on a displacer. When the forces balances to give a motor force F_m of zero the displacer is running passively.

Spring Stiffness

The springs utilized in the passive displacer require stiffness and fatigue testing in order to characterize their dynamics. The spring stiffness will allow for a decision to be made on the quantity of springs required in order to generate a given force. Measuring this experimentally allows for the simulated value through FE analysis to be validated. This was done by incrementally adding masses to the spring and observing how the natural frequency of the spring, ω_n , at which the spring oscillates changes with each mass.

$$\omega_n^2 = \frac{k}{m} \quad (7.2.2.4)$$

Given Eqn. 7.2.2.4, plotting the reciprocal of frequency squared against the added mass, as shown in Figure 7.4, results in a gradient equating to $\frac{1}{k}$ and a y-intercept of $\frac{k}{m_0}$. Hence, the spring stiffness and the required moving mass of the displacer, m_0 , can be determined. The values of k and m_0 per spring were found to be approximately 1000 N/m and 34 g respectively.

7.3 SPTC Experimental Setup

7.3.1 Instrumentation

The instrumentation utilised in the passive displacer SPTC testing matches that used in the active displacer SPTC testing presented in chapter 6, with a few modifications. The use of a passive displacer required that additional metrological equipment was installed onto the displacer.

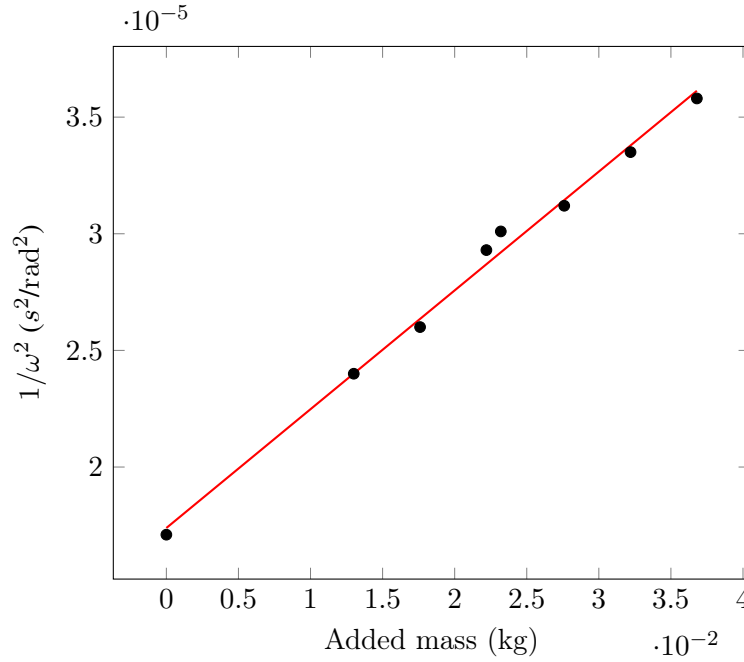


Figure 7.4: The reciprocal of the square of spring natural frequency against added mass to the spring, as experimentally measured.

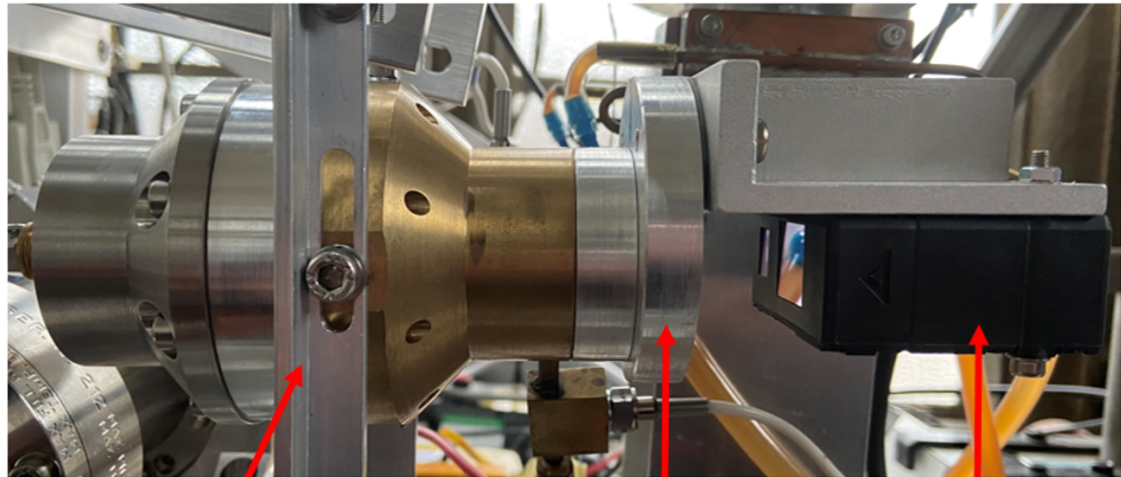
7.3.2 Pressure and Laser Displacement Transducers

In order to measure the pressure at P_1 and P_2 on either end of the passively driven displacer, pressure transducers were utilised. As the transducers are subject to drift, which may be the result of the diaphragm or amplifier drift, the calibration equation given by Eqn.7.3.2.1 is utilised. In this equation, P_o and V_o are the system pressure and voltage reading as recorded by LabVIEW when the system is static. During operation, the voltage reading V_i from the DAQ system corresponds to the pressure P_i , where S_e is the transducer sensitivity in bar/volt.

$$P_i = P_o + S_e(V_i - V_o) \quad (7.3.2.1)$$

The above does not account for drift during the experiment, hence, the mean signal level is determined at intervals throughout testing runs.

Figure 7.5 indicates the Omron laser displacement transducer that was installed onto the passive displacer assembly at a distance from the displacer backend window. A capacitance sensor was originally installed to measure the displacer motion, however, there was significant phase lag in the output signal and it proved difficult to sensibly calibrate. The Omron laser calibration achieved in chapter 5 was utilised for this assembly.



Passive displacer assembly

Displacer backend window

Omron laser

Figure 7.5: Omron laser displacement transducer setup adjacent to an installed window at the backend of the passive displacer assembly in order to measure motion amid operation.

7.3.3 Damping Plate

The damping plate is shown in Figure 7.6. On the left, the cross-sectional view of the plate can be seen with the rotating dial that allows for varying the setting of the diameter of the gas passage through the use of varying hole sizes. This enables the damping to be fine-tuned amid operation and permits the study of the effect of damping on the displacer performance and the overall SPTC performance. An additional piece was then attached to the damping plate outer rings in order to rotate the plate with the use of an external shaft.

7.3.4 Displacer Assembly and Full SPTC

The full SPTC with the passive displacer integrated is shown in Figure 7.7. A similar LabVIEW program was utilised that had previously been employed for logging the SPTC with an active displacer, as outlined in chapter 6. The program was modified to include logging channels for the pressure and Omron laser transducers, as well removal of channels logging aspects to the electromagnetic motor for the active displacer. The pressure transducer channels were validated through tests to ensure the channels were correctly assigned and that the calibration was suitably applicable.

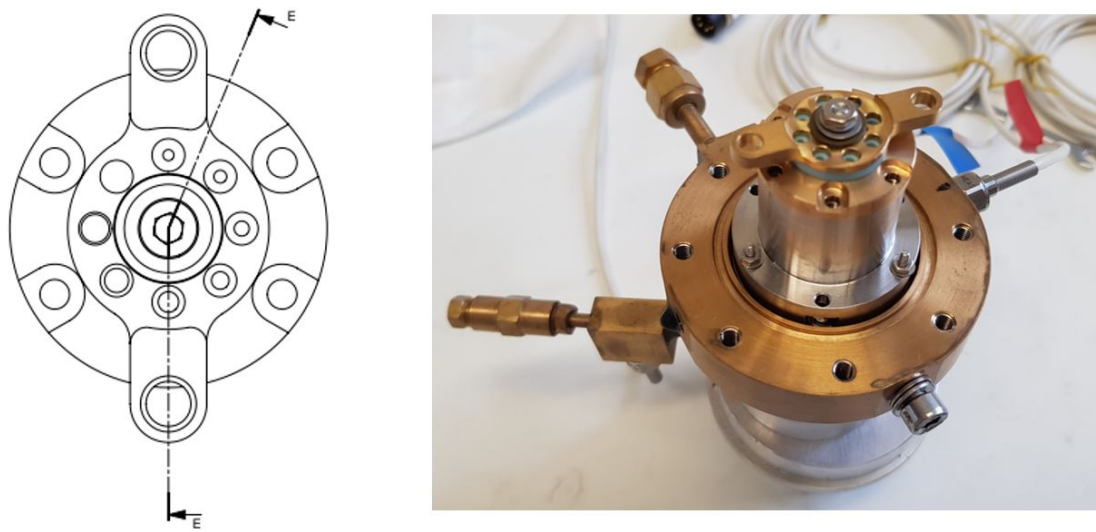


Figure 7.6: The damping plate cross-sectional view (left) and the physical assembly with the damping plate integrated (right).

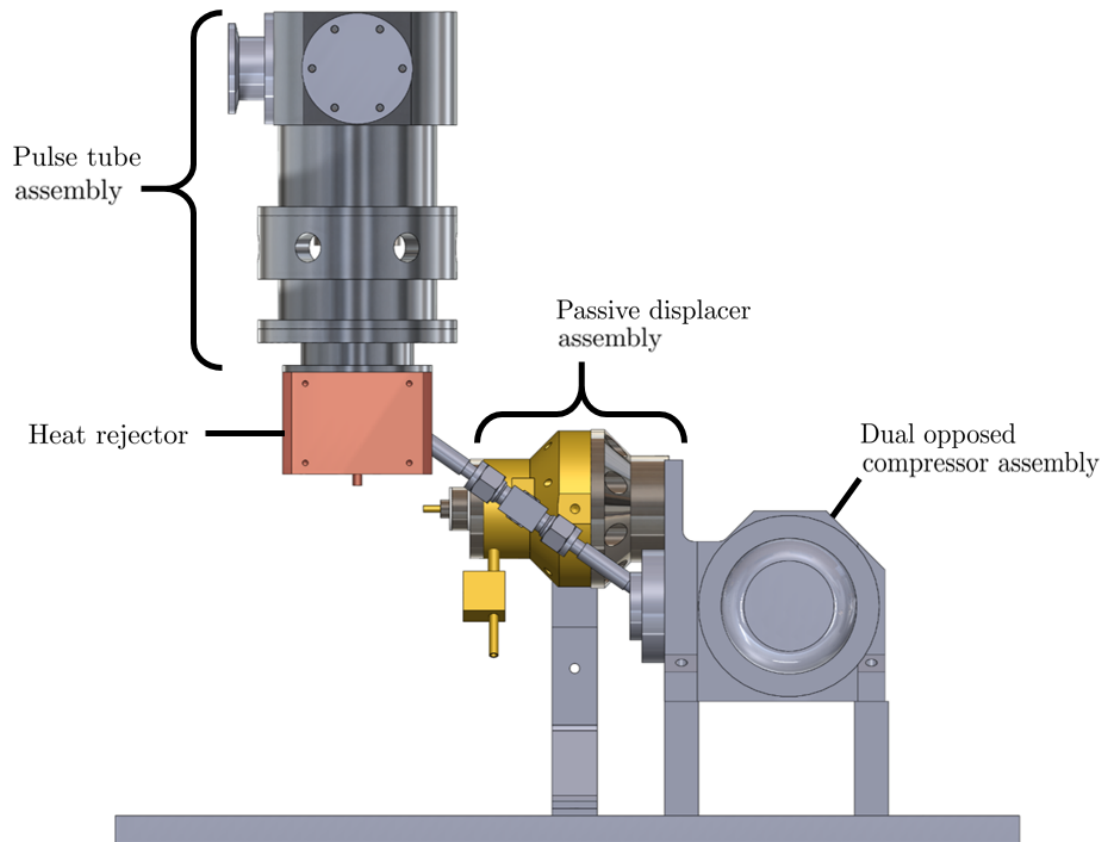


Figure 7.7: Full passive displacer SPTC 3D CAD assembly model with a cross-section of the dual-opposed compressors linked to the displacer subsystem and the pulse tube assembly shown. The support stands for the pulse tube assembly and water cooling circuit are not shown for clarity.



7.4 Passive Displacer Experimental Performance

7.4.1 Testing Campaign

Figure 7.8 provides a schematic to show the testing sequence mapped out prior to commencing the testing campaign of the passive displacer, both separately and whilst integrated with the full SPTC. The sequence of testing was followed through for cold end temperatures of 250 K, 90 K, 80 K and 70 K. Optimum operating conditions were iteratively achieved by alternating between keeping key parameters constant and varying others; namely the operating frequency, compressor stroke, displacer phase, and damping.

7.4.2 Displacer Resonant Frequency

Impact Test

Once assembled, the natural frequency of the displacer was determined by applying an instantaneous force to the assembly and measuring the frequency at which it oscillated. This process mimics that described in section 4.3.5. The natural resonating frequency of the displacer was found to be 103 Hz. Understanding this frequency gives a starting insight into the frequency ranges and limits for the passive displacer SPTC operation. Given the active displacer SPTC performance, it is expected that the frequency range for optimal operation will not result in the displacer assembly oscillating at its resonant frequency.

Theoretical Resonance

The displacer resonance was theoretically determined in order to corroborate the frequency found by the prior mentioned impact test. This was done by determining the mass of each moving component in the displacer, using the material density and volume. A summary of each moving component within the displacer and the calculated effective moving mass is presented in Table 7.1. The flexure springs utilised are those tested and discussed in chapter 4 by manufacturer B, with a thickness of 0.31 mm and all other properties as previously detailed in section 3.4.3. The total effective moving mass of the springs was verified using Eqn 7.4.2.1, where Eqn. 7.4.2.2 applies.

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2 \quad (7.4.2.1)$$

$$v = v_{\max} = 2\pi Af \quad (7.4.2.2)$$

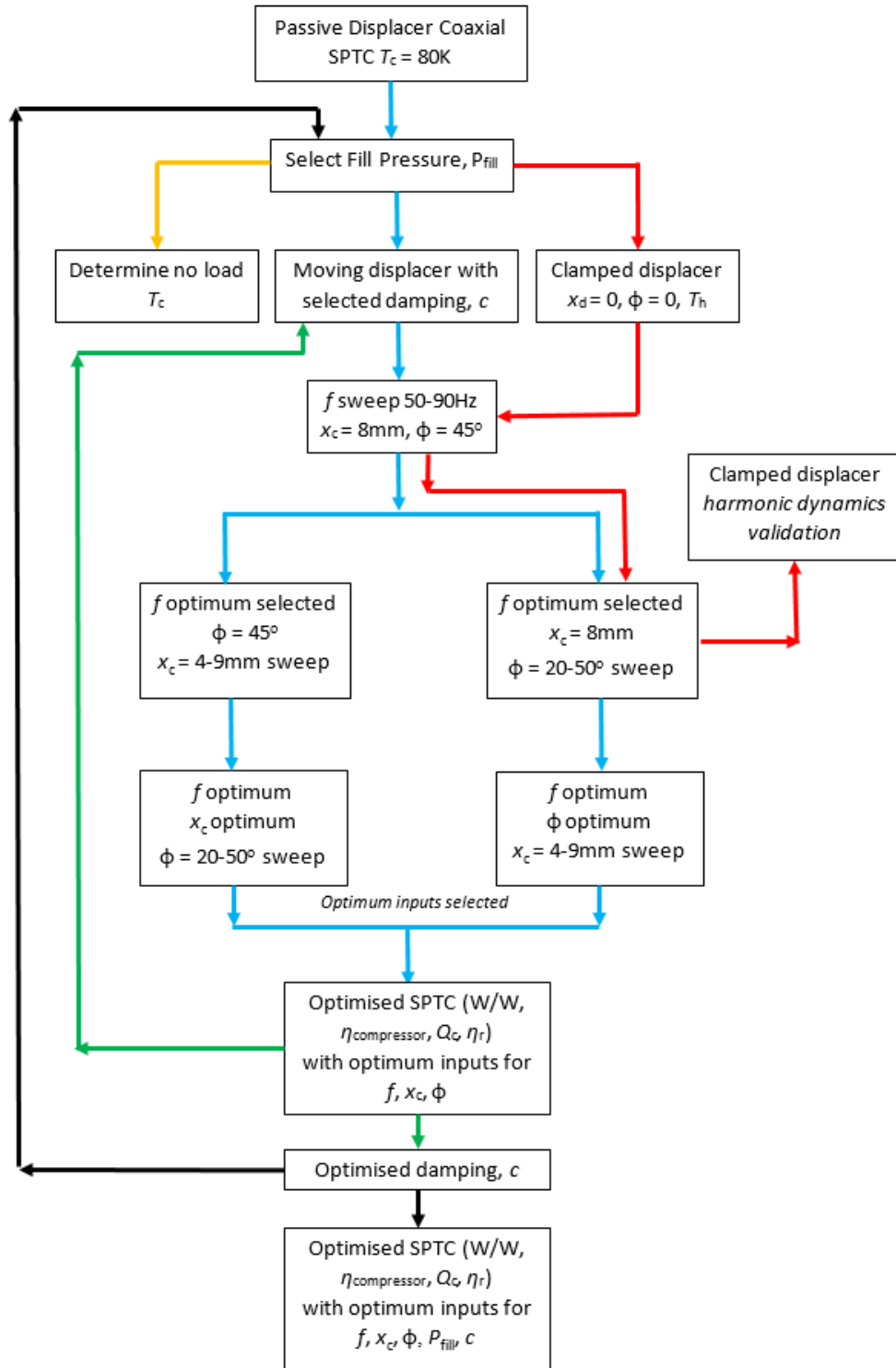
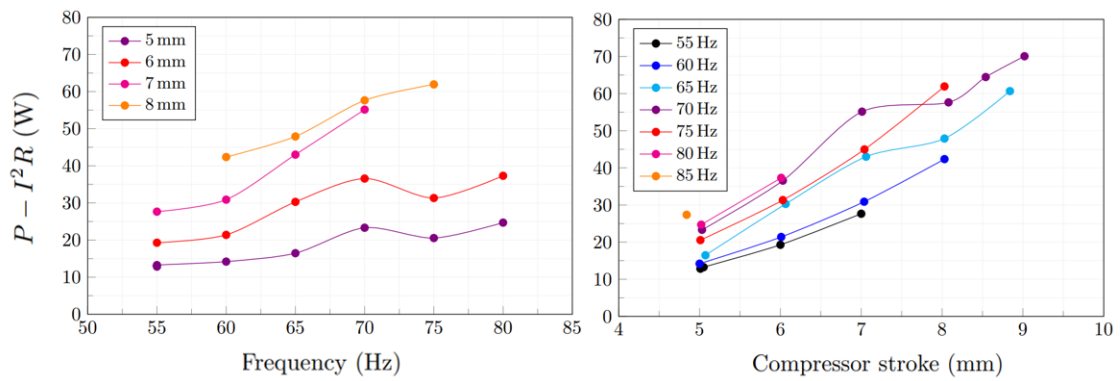


Figure 7.8: Testing logic for the passive displacer SPTC at 80 K cold temperature. Each step was repeated to assess variation and averaged values were utilised. The entire process was also instigated at $T_c=70\text{ K}$, $T_c=90\text{ K}$, and $T_c=250\text{ K}$, however, as the SPTC is designed for 80 K performance, a fully operational cryocooler could not be experimentally optimised at a lower temperature, i.e. $T_c=70\text{ K}$.

**Table 7.1:** Displacer component densities and their effective moving masses within the displacer assembly.

Component	Material	Density (kg/m ³)	Effective Moving Mass (g)
Shaft	SS316	7800	13.3
Piston	Fluorosint	2320	4.27
Short inner space	Aluminium 6082	2710	0.34
Long inner space	Aluminium 6082	2710	0.55
Damper piston	Fluorosint	2320	4.94
Moving mass block	Fluorosint	2320	2.54
16 flexure springs	SS304	7800	8.47

**Figure 7.9:** Compressor performance at ambient temperature whilst the displacer is clamped for varying frequency with constant compressor strokes (left) and stroke with constant frequencies (right).

The total effective moving mass of the displacer was calculated and verified to be approximately 27 g. For a total of 16 flexure springs, a k value of 14400 N/m was determined. The displacer resonance was calculated as 116 Hz using Eqn. 7.2.2.4, closely matching that found experimentally.

7.4.3 Clamped Displacer Tests

The displacer was clamped such that motion would not be permitted, in order to run the SPTC without the displacer and assess compressor activity. This was achieved by fixing a solid aluminium plate onto the assembly next to the springs, preventing the shaft from moving. The displacer assembly was then integrated into the crossing junction of the SPTC, where the back end is connected to the compressor-regenerator tube and the front end is connected to the warm end heat rejector at the base of the pulse tube. The SPTC was then run and the frequency, compressor stroke, and the pressure at either end of the displacer assembly were then recorded. A parametric sweep of the frequency and the compressor stroke were completed. The heater at the pulse tube cold end was kept off and the test was run at ambient temperature.

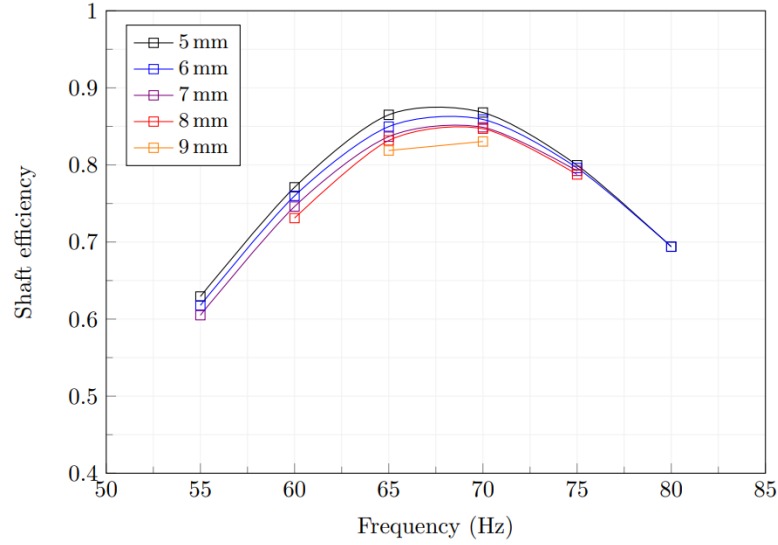


Figure 7.10: Compressor shaft efficiency at ambient temperature whilst the displacer is clamped for varying frequency at constant compressor strokes of 5 mm, 6 mm, 7 mm, 8 mm and 9 mm.

Figures 7.9 and 7.10 show that the highest efficiency of the compressor is reached at 65-70 Hz. The efficiency does not vary much over a range of varying stroke, however, the operating frequency has an expectedly significant influence on the compressor performance and efficiency, as a higher operating frequency increases the compressor drive.

7.4.4 No Load T_c

The passive displacer was unclamped and the SPTC was run with no heat load, in order to determine the lowest temperature achievable by the cryocooler that has been designed. The no load T_c was found to be 50 K, which is close in value to the active displacer SPTC.

7.4.5 Experimental Displacer Resonance

The passive displacer frequency response was determined by varying the operating frequency for the compressor drive whilst maintaining the compressor stroke constantly at 4.3 mm. Figure 7.11 presents the frequency response as a voltage, as recorded by LabVIEW with varying operating frequency. Peak resonance can be seen at 88 Hz for the passive displacer. This sheds light on the expected optimal operating frequency for the SPTC performance, as the ratio between the displacer resonance and optimal phase frequency from the active displacer SPTC (see section 6.9) can be applied for the passive displacer SPTC being investigated in this chapter. Hence, using the ratio for $f_\phi:f_d$ of 4:5, for a displacer resonance found as 88 Hz, the expected optimal SPTC operating

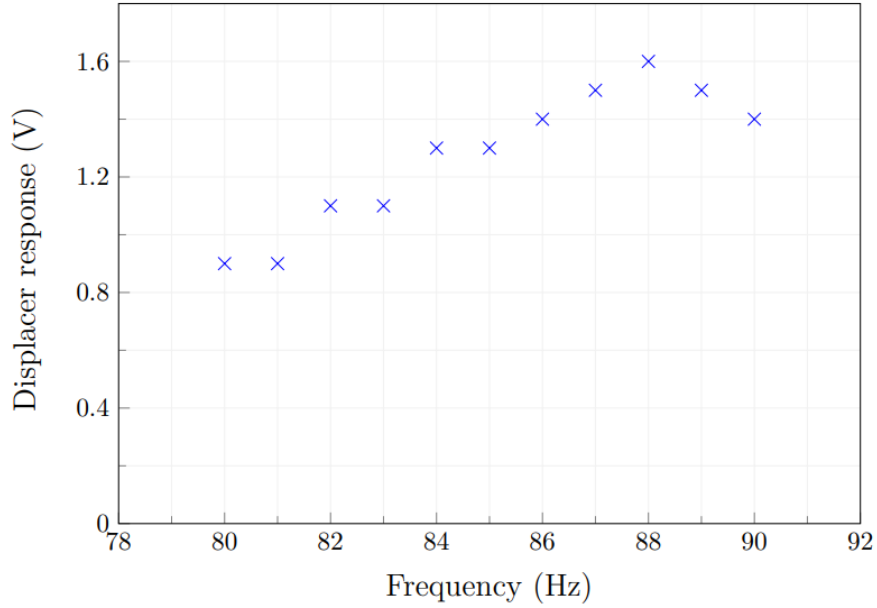


Figure 7.11: Displacer frequency response with varying operating frequency at a constant compressor stroke of 4.3 mm. The operating resonant frequency of the physical displacer once integrated into the full SPTC was hence ascertained.

frequency is 70 Hz. This will inspire the frequency range to be experimentally investigated, as presented in section 7.5.

7.4.6 Displacer Dynamics Experimental Validation

A balance was conducted across the displacer to characterise the forces resulting in its motion. Figure 7.12 indicates each physical area and its notation for the areas over which a force is exerted. Eqn. 7.4.6.1 gives the force balance with the notation as indicated in Figure 7.12.

$$P_{\text{body}} A_{\text{shaft}} + (A_{\text{PT}} - A_{\text{shaft}}) P_{\text{shaft}} - P_{\text{PT}} A_{\text{PT}} + m\ddot{x} + c\dot{x} + kx = 0 \quad (7.4.6.1)$$

As the passive displacer motion is driven by the pressure difference in the working gas on either end of the displacer, the assembly required experimental validation. The SPTC was tested with the operating conditions listed in Table 7.2, and the pressure was recorded on either end of the displacer in real-time during oscillatory operation. The calibration for the Omron laser displacement transducer voltage output was applied and the displacer motion was computed as shown in Figure 7.13. The computed displacement was further validated with Eqn. 7.4.6.2, where the values for amplitude, x_0 , angular frequency, ω , and displacer phase difference relative to the compressor, ϕ_d were calculated and utilised to theoretically reconstruct the displacement motion

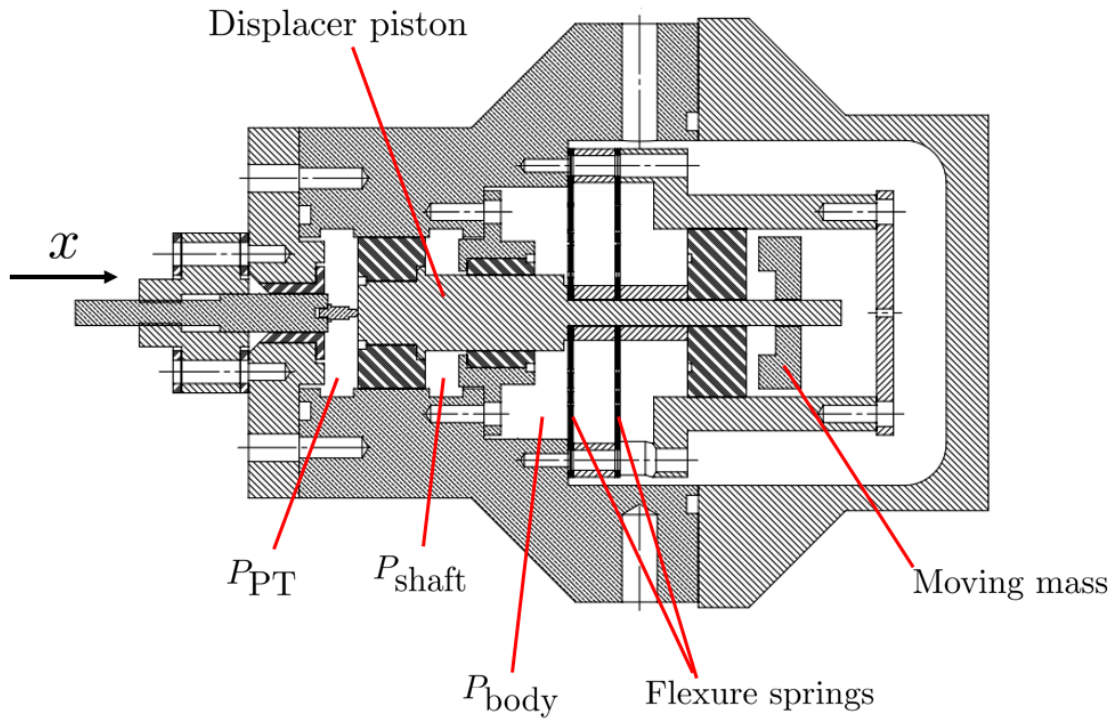


Figure 7.12: The passive displacer areas over which pressure forces are exerted and balanced in Eqn. 7.4.6.1.

Table 7.2: Testing input operating conditions and the performance results for displacer dynamics validation.

Test conditions	Input value	Result	Output value
f	55 Hz	x_d	3 mm
P_{fill}	30 bar	ϕ_d	72°
T_c	250 K	Q_c	1.6 W
x_c	7 mm		
Sampling rate	5000 s ⁻¹		

and validate the experimental findings.

$$x_d = x_o \sin(\omega t + \phi_d) \quad (7.4.6.2)$$

Similarly,

$$P = P_{b,f} \cos(\omega t + \phi_{b,f}) \quad (7.4.6.3)$$

where the subscripts 'b' and 'f' denote the back-end and the front-end of the displacer respectively. As previously discussed, the pressure wave follows the mass flow by 90°, hence the cosine expression of the pressure waveform. Incorporating this with Eqn. 7.4.6.1 and differentiating to the first and second order for the velocity, $\dot{x}$, and acceleration, $\ddot{x}$, the damping and the inertial

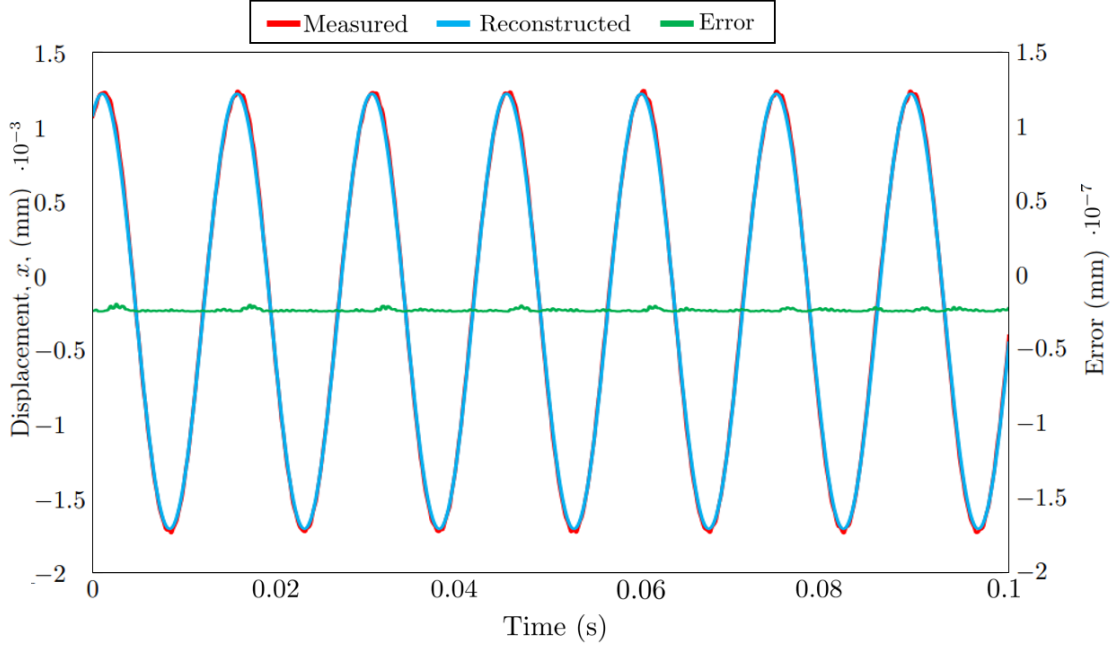


Figure 7.13: Displacer motion, both measured and reconstructed.

forces were formulated, and hence all forces acting on the displacer were balanced as given in Eqn. 7.4.6.4.

$$\underbrace{P_{\text{body}} A_{\text{shaft}}}_{F_{\text{inlet}}} + \underbrace{(A_{\text{PT}} - A_{\text{shaft}}) P_{\text{shaft}}}_{F_{\text{body}}} - \underbrace{P_{\text{PT}} A_{\text{PT}}}_{F_{\text{PT}}} = \underbrace{-c x_0 \omega \cos(\omega t + \phi_d)}_{F_{\text{damping}}} - \underbrace{(k - m\omega^2) (x_o \sin(\omega t + \phi_d))}_{F_{\text{spring}} F_{\text{inertial}}} \quad (7.4.6.4)$$

The forces computed from the experimental test results, with operating inputs and performance outputs as given in Table 7.2, are shown in Figure 7.14. The sum of the forces, F_{sum} , oscillates about zero and for a time-averaged cycle, the forces are balanced. This hence serves as an experimental validation of the passive displacer design operating as a harmonically balanced system. It is further noteworthy to observe that the shaft force, indicated as F_{inlet} in Figure 7.14, surpasses the front-end displacer force connecting to the pulse tube warm end, F_{PT} , in magnitude. This demonstrates the influence of the spring force, F_{spring} , and damping force, F_{damping} , in restoring the displacer whilst being passively driven. Figure 7.15 demonstrates the sum of the pressure forces and the sum of the displacer forces acting in balance, with a 180° phase difference, as expected.

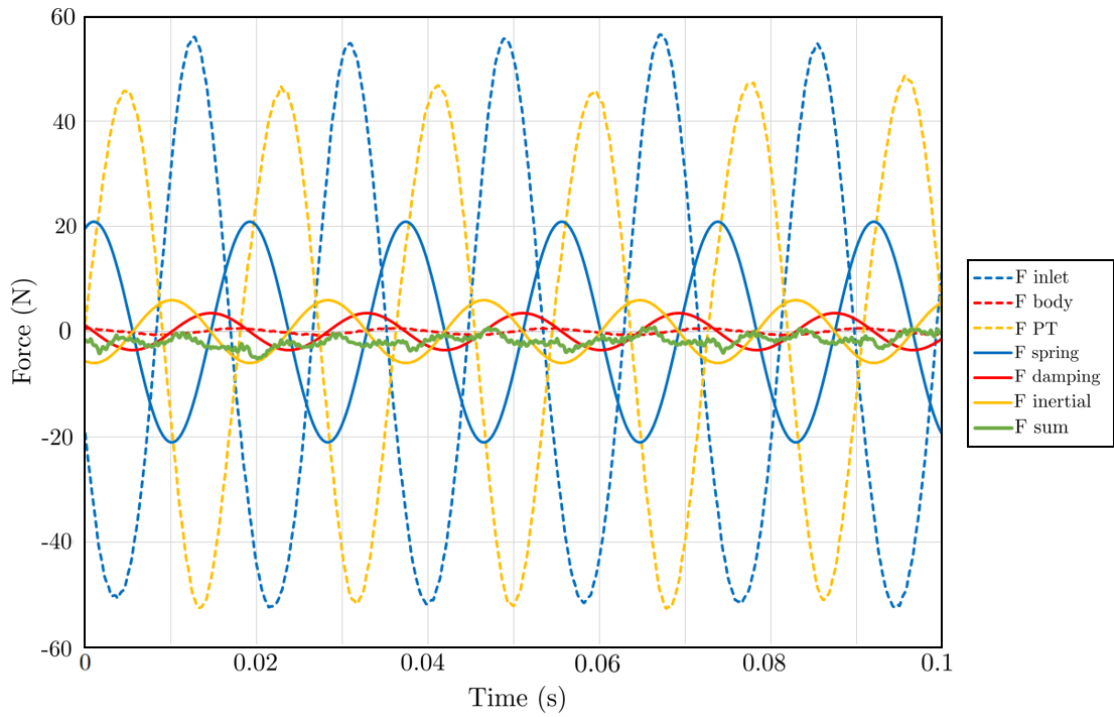


Figure 7.14: All forces acting on the displacer and the sum of all forces are shown. The F_{sum} can be seen to oscillate about zero, indicating that over a time-averaged cycle, the forces of the displacer running passively are balanced. The influence of the harmonic design parameters operating to counter the pressure forces driving the displacer can be seen.

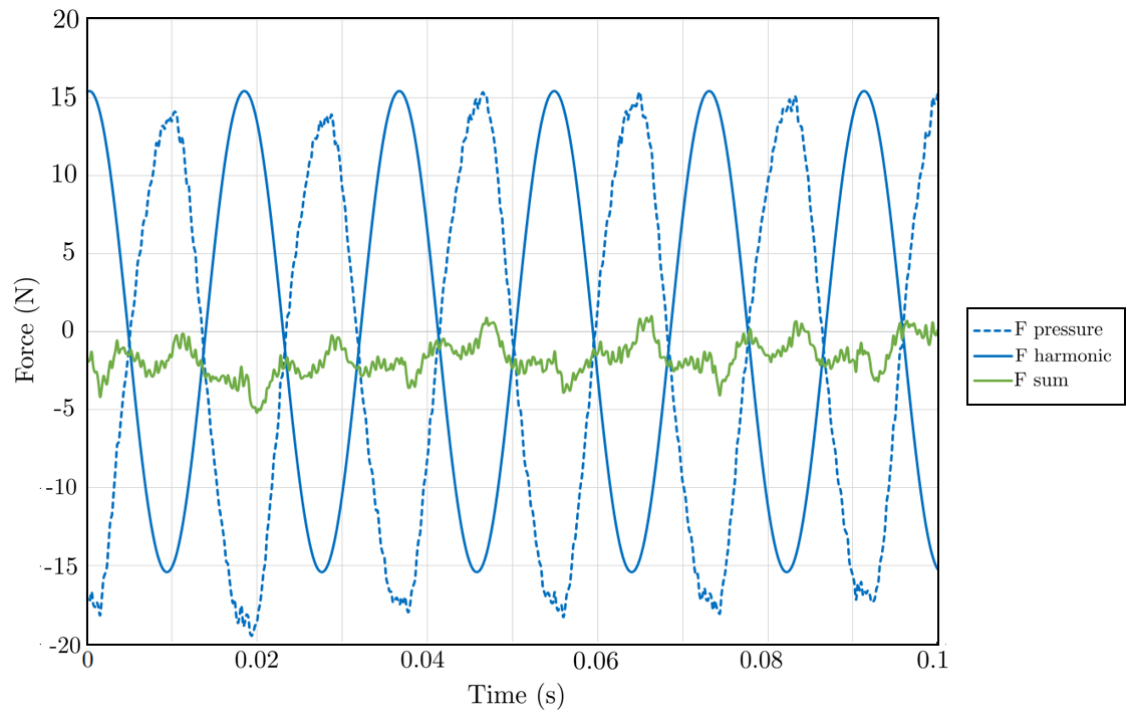


Figure 7.15: The pressure forces and the displacer motion forces are shown along with the sum of all forces on the displacer. The expected phase difference between the two opposing series of forces can be seen.



7.5 SPTC Experimental Performance at 80 K

7.5.1 Cooling Power and Carnot Efficiency

With an experimentally validated passive displacer integrated, the SPTC was run at 80 K to assess its performance. Firstly, the compressor performance was investigated to ascertain the compressor operation. Figure 7.16 demonstrates the W/W efficiency (defined as the ratio of cooling power to total input power) of the compressor whilst run with the inputs listed in Table 7.3 for fill pressures of 31 bar and 28 bar, where the lower value of x_c in Table 7.3 was utilised as a starting input for the frequency sweep, and lower value for f for the operating frequency during the compressor sweep. It can be seen that the compressor performance follows the same trends at both fill pressures and indicates the same optimal compressor stroke and operating frequency for both sweeps. A higher fill pressure yields a lower W/W compressor performance, indicating higher compressor efficiency at higher fill pressures. The optimal x_c can be seen between 8 mm and 8.5 mm, which was the range within which the optimal operating frequency was then determined. The sweeps were then repeated with input conditions varying within these ranges of x_c and f until a final optimal condition was found, as per the experimental process mapped out in Figure 7.8. It is further important to note that other damping settings were attempted in this iterative testing process, however, the displacer motion responding to maintain a ϕ_d that could permit achieving cooling to 80 K. Hence, the highest damping setting was maintained throughout the rest of the testing campaign.

Table 7.3: Testing input operating conditions kept as constant for frequency and compressor sensitivity sweeps at the highest damping setting.

Operating condition	Input value
P_{fill}	26-31 bar
x_c	8.5 mm
f	68 Hz - 72 Hz
T_c	80 K

Figures 7.17 and 7.18 present the cooling power and Carnot efficiencies achieved with varying operating frequency for the full SPTC with a passive displacer at 80 K cold end cooling. The optimum frequency can be seen to occur at 68 Hz, which is close to the empirically predicted

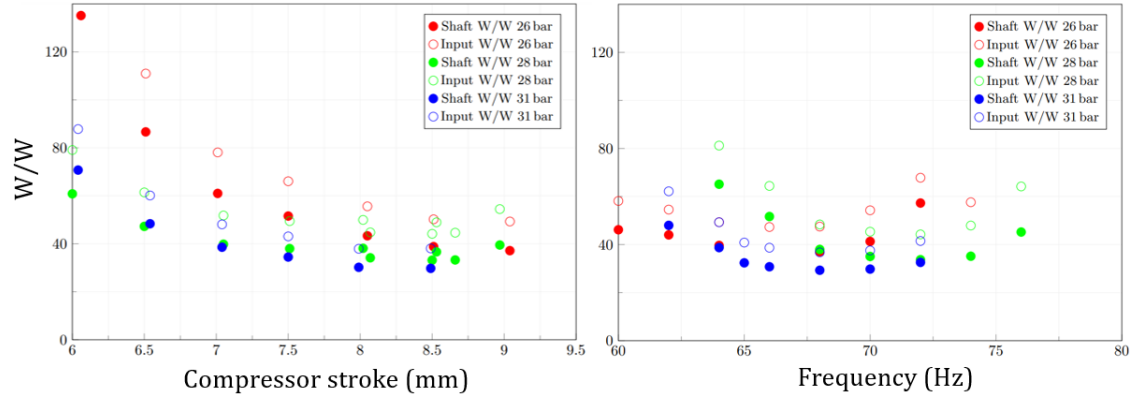


Figure 7.16: The compressor W/W efficiency is shown for varying compressor stroke (left) and for varying frequency (right). All other inputs for the experimental sweep are as indicated in Table 7.3.

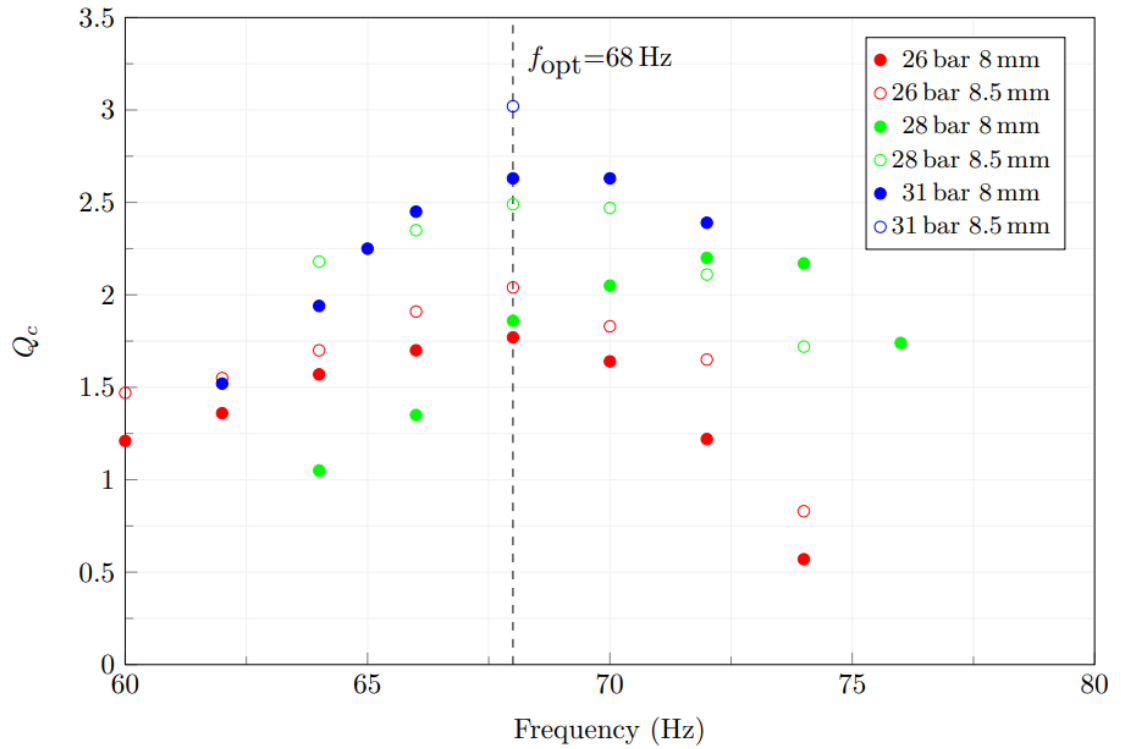


Figure 7.17: The SPTC performance in terms of Q_c at $T_c=80$ K for varying operating frequency at fill pressures of 31 bar, 28 bar and 26 bar at a fixed compressor strokes of 8 mm and 8.5 mm.

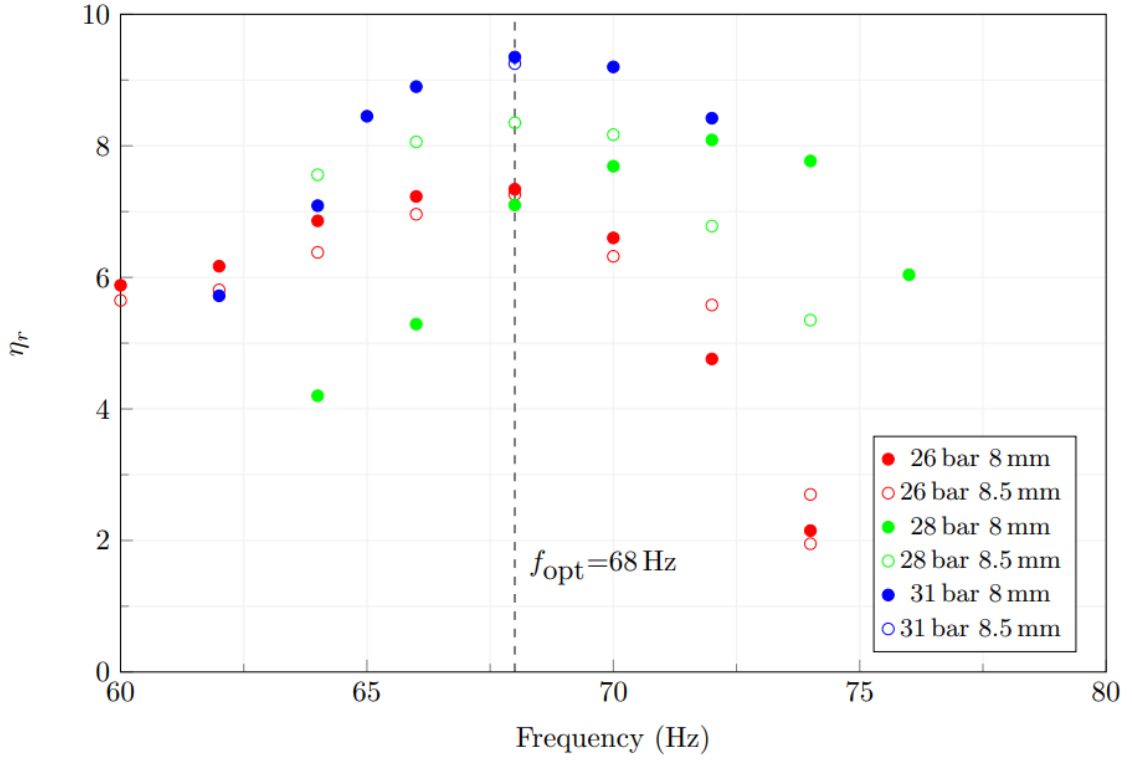


Figure 7.18: The SPTC performance in terms of η_r at $T_c=80$ K for varying operating frequency at fill pressures of 31 bar, 28 bar and 26 bar at a fixed compressor strokes of 8 mm and 8.5 mm.

value of 70 Hz as determined in section 7.4.5 based on the ratio of the resonant frequency of the compressor to displacer.

Moreover, it can be seen that a decreased fill pressure results in a decreased Q_c and η_r , as expected and as observed with the coaxial SPTC with an active displacer investigated in chapter 6. An increased fill pressure results in an increased peak-peak pressure pulse throughout the SPTC system. It may also be possible that it results in an increased peak-peak driving pressure force across the passive displacer, however, this would require investigating the displacer pressure forces for varying fill pressures. This was thus investigated in section 7.5.3.

Figure 7.19 indicates the effect of varying the compressor stroke on Q_c and η_r . It is clear that no optimum is achieved within the sensitivity sweep; the experimental equipment is subject to the fuse limit for the current going into the compressor. This therefore limits the upper values of the parametric sweeps experimentally conducted. It is clear that an increased compressor stroke results in an increased cooling rate per averaged cycle within the pulse tube. A plateau and eventual decline in efficiency would be expected to occur, whereby an increase in compressor stroke can

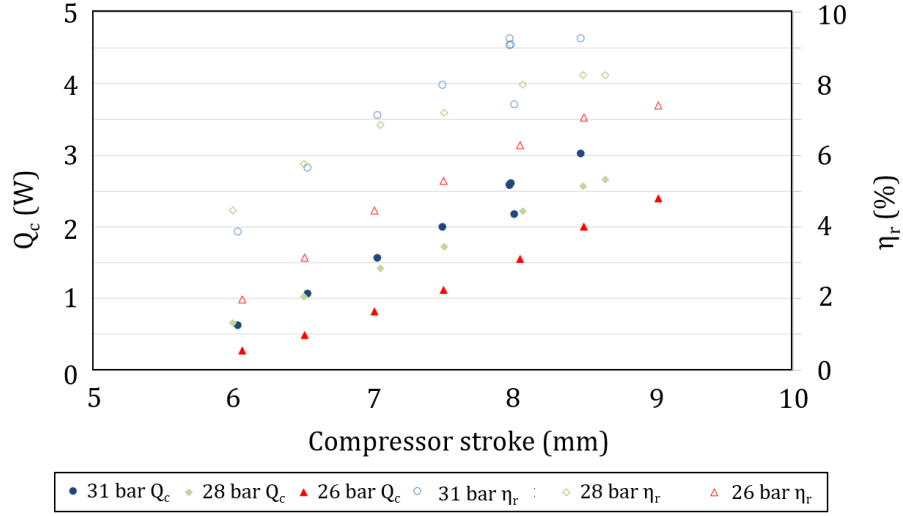


Figure 7.19: SPTC performance in terms of Q_c and η_r with varying compressor stroke at a constant operating frequency of 68 Hz.

no longer increase cooling as the limit for entropy transfer across the regenerator and enthalpy transfer down the pulse tube is reached. At 28 bar, a slight decline in cooling power can be seen at 8.7 mm compressor stroke compared to 8.5 mm, indicating a slight peak has occurred, which is further reflected in the Carnot efficiency at this fill pressure. Hence, an optimum compressor stroke was taken to be 8.5 mm.

7.5.2 Displacer Stroke and Phase

As explored, the compressor activity governs the passive displacer operation. Now that the trends for compressor stroke and their influence on the SPTC performance has been investigated and confirmed, the displacer activity was studied. The displacer stroke and phase sensitivity to operating frequency and compressor stroke are presented in Figure 7.20. Once again optimums are observed at approximately 68 Hz and with increased compressor stroke. Notably, the influence that operating frequency has on displacer phase is more significant than compressor stroke. This finding indicates that the compressor stroke activity is able to maintain a relatively constant phase between the compressor and displacer operation for a given frequency, where both systems are operating slightly off-resonance, however, a higher operating frequency perpetuates a greater phase lag between the two systems. Moreover, this increased phase difference is more notable at lower fill pressures, conversely to the influence of compressor stroke on the phase. This is consistent given that a lower fill pressure would result in a reduced peak-to-peak pressure pulse and hence

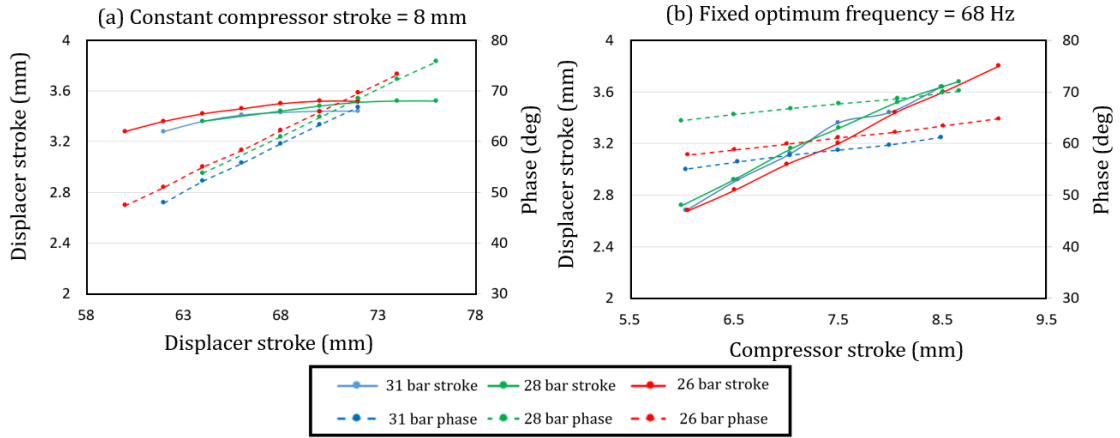


Figure 7.20: Displacer stroke and phase sensitivity to operating frequency and compressor stroke at varying fill pressures.

more variation between points P_1 at the front-end of the displacer close to the compressor, and P_2 , at the back-end of the displacer connecting to the outlet of the pulse tube. This further explains why higher displacer stroke values are passively achieved across the frequency sweep for the lower fill pressures, given that the displacer motion is governed by the pressure difference across P_1 and P_2 . It should be noted that the results for 28 bar fill pressure were completed at an operating frequency of 72 Hz, as opposed to $f=68$ Hz for $P_{\text{fill}}=26$ bar and $P_{\text{fill}}=31$ bar, as this was found to be the optimum frequency.

7.5.3 Pressure Oscillations and Pressure Driving Forces

The findings from sections 7.5.1 and 7.5.2 enticed an investigation into the pressure oscillations and driving forces across the displacer in order to better understand its passive activity within the SPTC. Figure 7.21 demonstrates the pk-pk pressures at P_1 , the front-end of the displacer close to the compressor, and P_2 , the back-end of the displacer connecting to the pulse tube outlet, as well as ΔP , the difference in the pk-pk waveforms across the displacer. The pressure waveforms were computed using the voltage outputs from the pressure transducers and applying the instrument calibration to achieve the pressure values. The displacer phase, ϕ_d , is further shown against varying compressor stroke. The increased compressor stroke results in increased pk-pk pressures across the displacer as well as at the individual points P_1 and P_2 . Moreover, ϕ_d increases with increased stroke, as is consistent with the findings shown in Figure 7.20. The higher phase values for $P_{\text{fill}}=28$ bar can once again be attributed to the higher operating frequency of 72 Hz, whereby

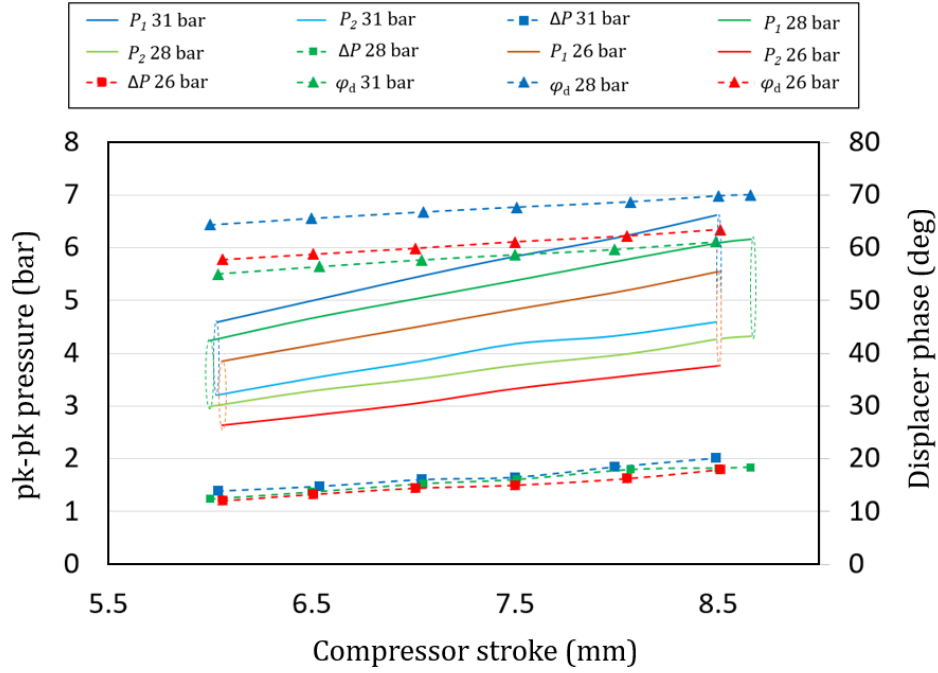


Figure 7.21: Peak-to-peak pressures at P_1 and P_2 on the displacer as well as ΔP across the displacer, for fill pressures of 31 bar, 28 bar and 26 bar. The couplings of P_1 and P_2 for each fill pressure are indicated by the dashed circle groupings in the vertical direction. The phase difference between the compressor and displacer, ϕ_d , is further shown with varying compressor stroke.

the a higher operating frequency results in a higher phase difference between the compressor and displacer motion.

Figure 7.22 indicates the simulated pk-pk pressure forces acting across the displacer, where the forces were determined by multiplying the computed pressure values with the respective area on the displacer upon which the force acts. An approximate phase difference of 180° can be seen between the forces acting at P_1 compared to those at P_2 , indicating how the pressure difference drives the oscillatory motion of the displacer piston. Figure 7.23 shows how the difference in acting force across the displacer varies with compressor stroke, indicating an overall increase in force difference with an increase in stroke. The fill pressure does not appear to have a great effect on this variation.

7.5.4 Harmonic Forces and Effect of Damping

The harmonic forces of the displacer motion were simulated as per the methodology outlined in section 7.4.6 and Eqn. 7.4.6.4, where the displacer motion output voltages from the Omron displacement laser transducer are converted to displacement using the calibration, and the waveform

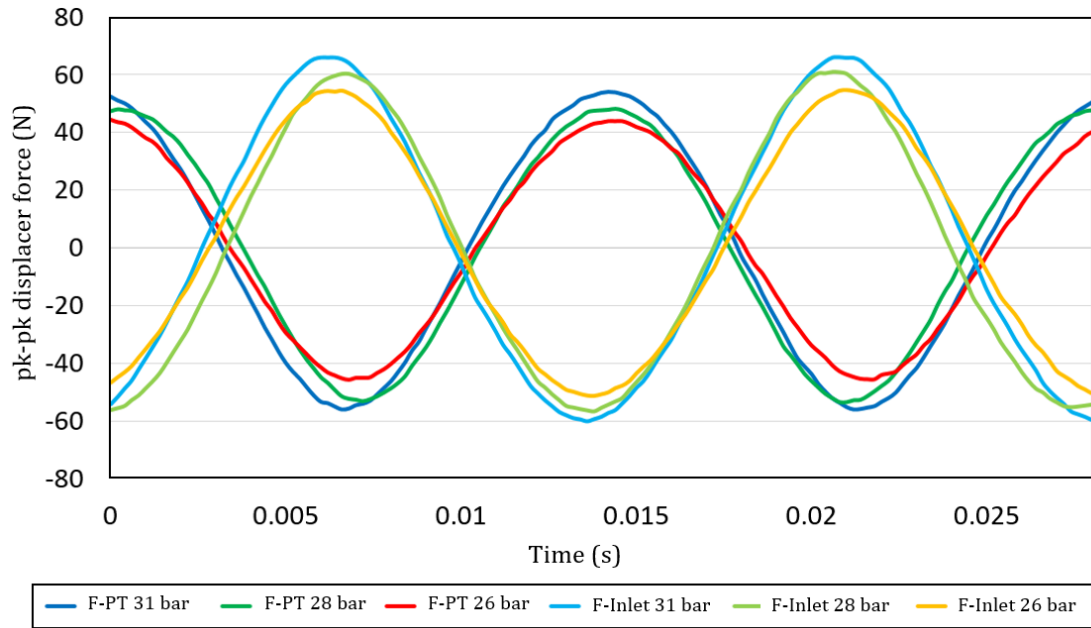


Figure 7.22: Peak-to-peak pressure driving forces acting across the displacer, where P_1 is the 'F-Inlet' force and P_2 is the 'F-PT' force indicated. The above test case is for varying fill pressures and an operating frequency of 68 Hz and 8 mm compressor stroke.

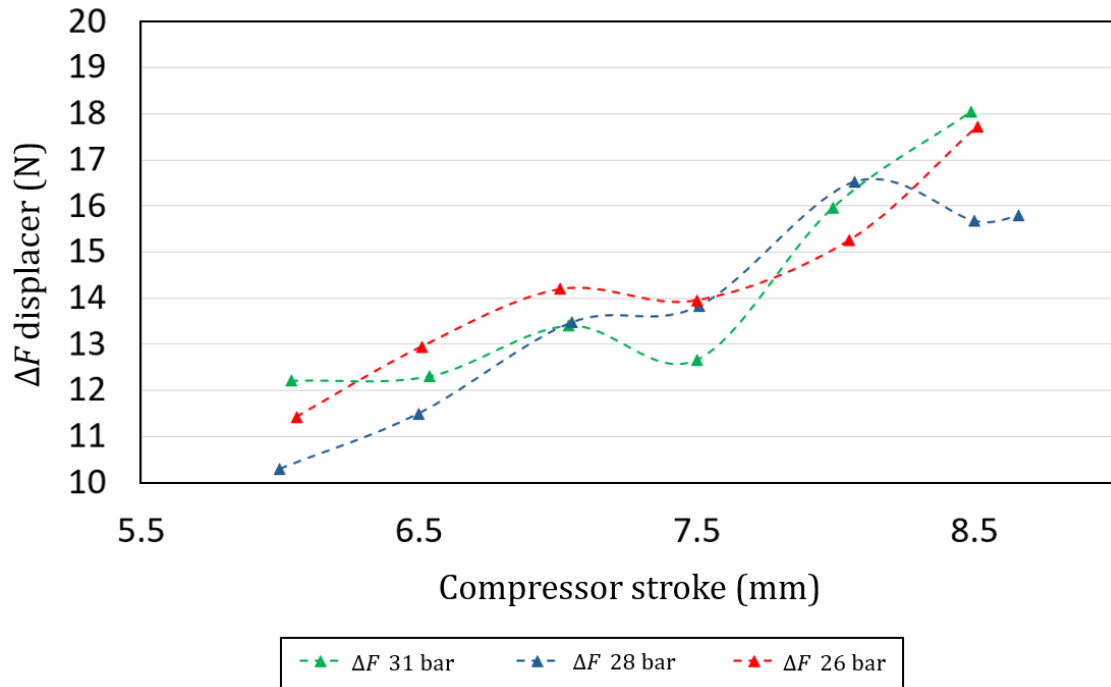


Figure 7.23: Difference in pressure forces acting across the displacer with varying compressor stroke for the different fill pressures.



is theoretically reconstructed to develop a mathematical expression for the motion as a waveform. The velocity and acceleration of the displacer are then achieved through differentiating the displacement waveform and the harmonic terms for the damping force, $c\dot{x}$, and inertial force, $m\ddot{x}$, are obtained. Figure 7.24 shows the varying phase at which the different harmonic forces act. The P_1 and P_2 pressure forces have been summed and are represented by the dashed line. The fill pressures of 31 bar and 26 bar are shown, where $x_c=8$ mm and $f_{\text{optimum}}=68$ Hz. The harmonic forces are summed as shown in yellow. The higher fill pressure of 31 bar indicates higher total pk-pk harmonic force achieved by the displacer, as well as a higher pk-pk damping force. The variation in the pk-pk force magnitude for the different fill pressures is overall quite small, suggesting that fill pressure does not greatly impact the displacer driving forces. Figure 7.25 indicates the influence of compressor stroke on specifically the pk-pk damping force for the same two fill pressures, where the difference between 31 bar and 26 bar can be clearly seen; a higher fill pressure results in a higher displacer damping force. For 31 bar the variation in pk-pk damping force with increased stroke is minimal up until $x_c=8$ mm, whereas the pk-pk damping force at 26 bar is fairly constant for changing compressor stroke. This is likely due to the fact that the resonant optimums for the displacer varies with different fill pressures. The phase angle between the harmonic and the pressure forces acting on the displacer, $\phi_{h,p}$, is further shown for varying compressor stroke. Similarly, $\phi_{h,p}$ is relatively constant across the stroke sweep at $P_{\text{fill}}=26$ bar, however, a decrease in $\phi_{h,p}$ can be observed at $x_c=8$ mm for $P_{\text{fill}}=31$ bar. Looking more closely at the effect of x_c on pk-pk damping force at $P_{\text{fill}}=31$ bar, Figure 7.26 shows a greater increase in pk-pk damping force magnitude at $x_c=8$ mm compared to 6 mm and 7 mm strokes. This again suggests that the optimum compressor stroke of 8 mm being reached results in the passive displacer acting more as a damper.

7.6 Energy Recovery in Pulse Tube Cryocoolers

7.6.1 Pulse Tube Design Rationale

In chapter 2, an overview of the state of the art pulse tube cryocoolers was presented where PTCs with inertance tubes and orifices, which are widely used, were discussed and the advantages of a warm end displacer was assessed. The rationale for developing the coaxial Stirling pulse tube cryocooler has been discussed in chapter 6. In this chapter, the benefits of a passive displacer were discussed, and subsequently, designed and tested. The aims within these studies have been

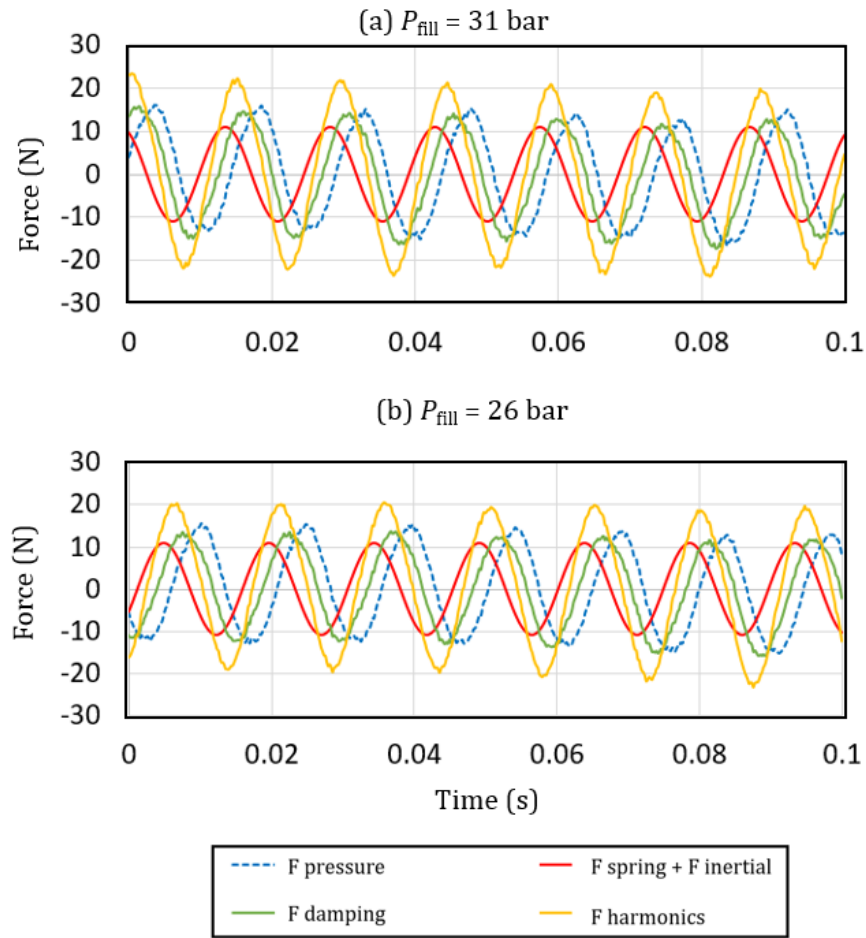


Figure 7.24: The summed pk-pk pressure force, the summed pk-pk harmonic, the component pk-pk damping force and the summed component spring and inertial forces are shown for fill pressures of 31 bar and 26 bar, where $f=68$ Hz and $x_c=8$ mm.

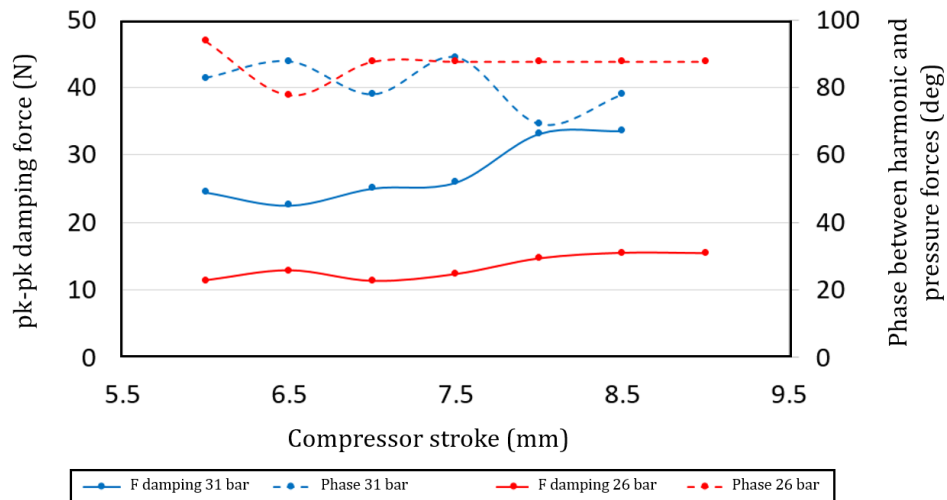


Figure 7.25: Pk-pk damping force and $\phi_{h,p}$ for 31 bar and 26 bar with varying compressor stroke.

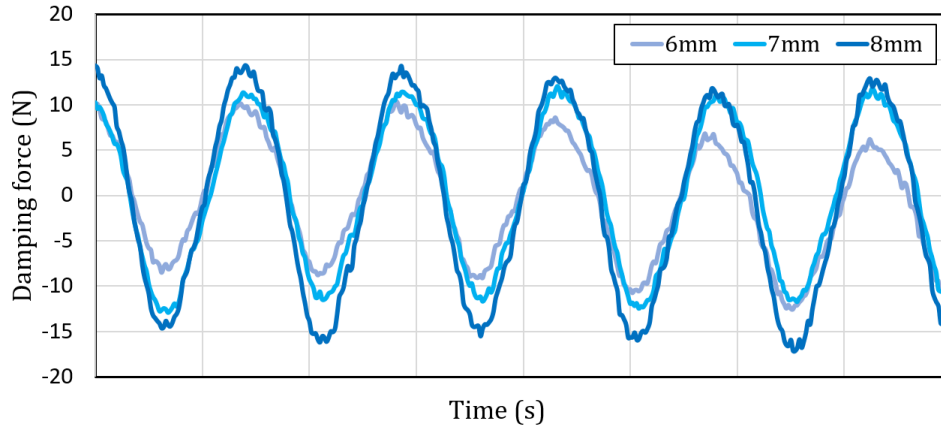


Figure 7.26: Pk-pk damping force at $P_{\text{fill}}=31$ bar for $x_c=6$ mm, $x_c=7$ mm, and $x_c=8$ mm, at $f=68$ Hz.

to make a series of fundamental modifications to the design, configuration, and thermodynamic performance of the traditional pulse tube cryocooler to achieve a more compact cryocooler with comparable cooling power capabilities and Carnot efficiencies. Figure 7.27 provides a visual overview of the series of designs and modifications to the pulse tube cryocooler, which ultimately results in the SPTC with a passive displacer. Figure 7.27(a) represents the in-line pulse tube cryocooler which utilises an inertance tube as a phase-shifter to create a phase lag between the pressure pulse and mass flow travelling within the gas parcels in the regenerator and pulse tube [11]. The inertance tube and designed reservoir volume do not allow for fine-tuning of the phase amid operation. No energy recovery occurs and the expansion power is dissipated as heat. This leads to the modification involving the warm end displacer as shown in (b), (c), and (d). The in-line active displacer SPTC is further modified into a coaxial design (c) in order to be compact and give access to the cold head. As explored, the passive displacer design shown in (d) is the result of desiring to remove the second phase from the power electronics and simplify the electro-motor design and complexity of the system.

7.6.2 Comparison of Cryocooler Performance at 80 K

The cooling performances of the different pulse tube cryocooler configurations and engineering designs were compared. The operating conditions for each of these inputs is as given in Table 7.4. Figure 7.28 shows the differing cooling powers achieved by each of these cryocoolers, where the in-line pulse tube cryocooler using an inertance tube has been simulated in Sage [24] and the SPTCs with active and passive displacers were experimentally built and tested. Table 7.4 presents

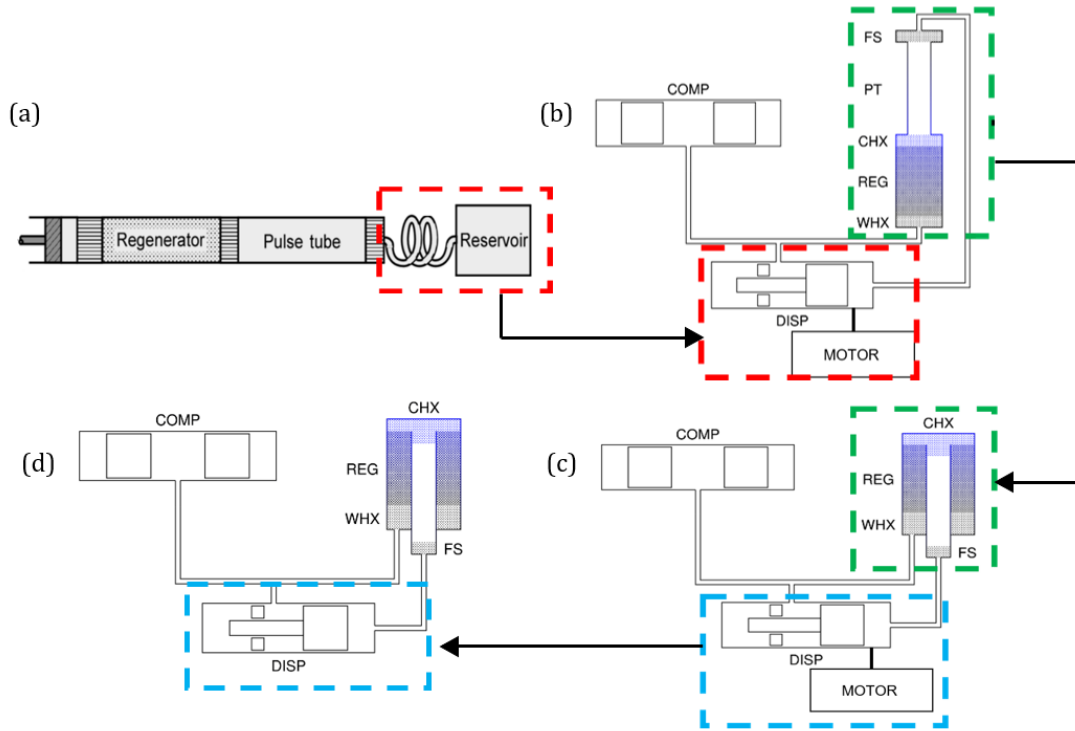


Figure 7.27: The different pulse tube cryocooler designs investigated, showing (a) an in-line pulse tube cryocooler with an in-line pulse tube as a phase-shifter [11], (b) an in-line Stirling pulse tube cryocooler with an active displacer, (c) a coaxial Stirling pulse tube cryocooler with an active displacer, and (d) a coaxial Stirling pulse tube cryocooler with a passive displacer.

the operating conditions and some of the simulation/experimental recordings for key parameters that affect the performance of the pulse tube cryocooler. The case presented for each cryocooler design is the optimum case for a fill pressure of 28 bar. It can be seen that the cryocooler with using the inductance tube requires a much higher shaft power for the optimum case, however, this also corresponds to a higher cooling power achieved. This can be seen in Figure 7.28, where the optimum case at 28 bar for the inductance tube cryocooler is over 5 W. Nonetheless, the in-line active displacer SPTC achieves a comparable cooling power of 4.9 W, using 45% less shaft power than the inductance tube. This could be attributed to the fact that the active displacer is able to recover PV expansion power back into the cryocooler. This is reflected in the very high Carnot efficiency value for the in-line active displacer SPTC, where a η_r of 21.5% was experimentally obtained. It is also worth noting that the Sage numerical software typically underestimates losses and hence overestimates cooling performance. This may further explain the higher cooling power achieved by the inductance tube cryocooler.



Table 7.4: Operating conditions for the optimal cases for different pulse tube cryocooler designs using different phase-shifters at a fill pressure of 28 bar for cooling at 80 K.

Design	Study	Phase- Shifter	f	$W_{\text{sh,c}}$	x_{c}	x_{d}	ϕ_{d}
In-line	Numerical	Inertance tube	60 Hz	115 W	8 mm		40°
In-line	Experimental	Active displacer	63 Hz	63 W	9 mm	5 mm	41°
Coaxial	Experimental	Active displacer	60 Hz	65 W	8 mm	4 mm	56°
Coaxial	Experimental	Passive displacer	72 Hz	82 W	8.5 mm	3.6 mm	62°

The coaxial active displacer SPTC and passive displacer SPTC demonstrate lower performances in terms of both cooling power and efficiency. It is noteworthy that the passive displacer SPTC compares well with the active displacer SPTC, indicating that the displacer harmonic design for the passive displacer was successful in replicating the dynamics of the active displacer SPTC in the optimum case. Hence, the passive displacer design was successful in achieving the primary objective set out in the investigation in this chapter. However, the lower performance of coaxial configuration SPTCs requires understanding. The key reasons why the coaxial configuration poses problems, as has been highlighted in both chapter 6 and in this chapter, is that it is difficult to simulate, and hence, predict and design the conductive and convective losses in the coaxial design. In chapter 6, both these losses have been addressed, however, there continues to be a need to enhance and improve the sophistication of the simulated hydrodynamic properties of the gas springs in the coaxial configuration, particularly at the cold head where flow reversal and flow mixing occurs. It is difficult to estimate the impact this has on cooling at the cold head, and further challenging to experimentally validate, but this may be partly responsible for losses and hence decline of the thermodynamic performance achievable.

7.7 Summary

In conclusion, a coaxial SPTC with a passive displacer has been designed, simulated, built, and experimentally tested. A summary of the key findings in this chapter are as follows:

- A coaxial Stirling pulse tube cryocooler with a passive displacer was harmonically designed

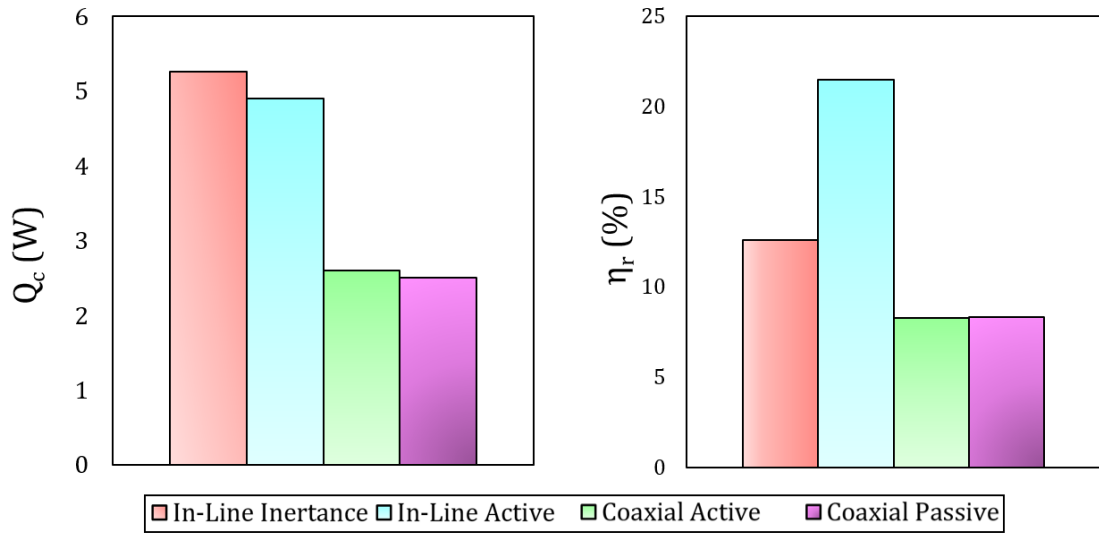


Figure 7.28: A comparison of the cooling power and Carnot efficiencies for the inertance tube PTC, the in-line active displacer SPTC, the coaxial active displacer SPTC, and the passive displacer SPTC.

using inputs from the SPTC with an active displacer, in order to achieve a comparable cryocooler performance with a displacer that is driven passively by the pressure difference across it, generated by the compressor motion.

- A series of springs with varying thicknesses and manufacturers were tested (as presented in chapter 4), whereby in this chapter, the spring design and thickness were utilised to complete the harmonic design of the other key parameters of the displacer, such as shaft diameter, spring number, required stiffness, moving mass, and damping.
- A numerical model was developed in Sage and the design parameters for the passive displacer were optimised using Sage. This then inspired the physical design and build of the passive displacer SPTC.
- The passive displacer components were physically built and assembled and the existing SPTC testing rig was customised with instrumentation access improved to the passive displacer assembly, in order to be able to measure key parameters amid operation. This includes installing an Omron laser displacement transducer onto the displacer assembly to measure displacer motion, as well as a damping plate to vary the damping in the displacer without requiring disassembly of the setup. The LabVIEW data acquisition was modified to



reflect new instrumentation calibrations and links, as well as re-routing channels that had previously been set up for the active design.

- The displacer assembly was theoretically determined and experimentally validated for its resonant frequency. The resonant frequency for the full displacer assembly was found to be approximately 103 Hz.
- The full SPTC with the passive displacer assembly installed was experimentally tested at 250 K in order to validate the harmonic dynamics of the displacer. This was achieved by determining the pressure forces acting on the displacer and computing the harmonic terms using the displacer motion. The displacer dynamics were thus validated.
- The passive displacer SPTC experimentally achieved a cooling power, Q_c , of 3 W and η_r of 9.35% for a T_c of 80 K at 31 bar fill pressure, 97 W input power, 8 mm compressor stroke, and a ϕ_d of 60° . This corresponded to a passively driven displacer stroke of 3.4 mm.
- Noteworthy insights into the displacer operation, the pressure driving forces and pressure phase, the harmonic driving forces and harmonic phase between the different forces driving the displacer, and the effect of displacer damping on the SPTC performance were made. This shed light on the role each fundamental parameter plays on running the displacer passively, as well as determining which components of the pulse tube assembly may be affecting the cryocooler performance beyond what was measured in this investigation. This provides helpful insight into what to investigate next with the SPTC.
- The results from the passive displacer SPTC were compared with the simulated inertance tube pulse tube cryocooler, the in-line active displacer SPTC, and the coaxial active displacer SPTC in order to understand how the design compares in terms of performance, as well as determine the aspects of the pulse tube cryocooler that affect performance the most. It was found that the passive displacer SPTC was successful in achieving a comparable performance to the coaxial active displacer designed and tested in chapter 6, which was the performance it was designed for.



- Whilst comparing energy recovery SPTCs, it was further found that the coaxial configuration SPTCs perform less well in terms of both cooling power and efficiency compared to the in-line design. This suggests that more studies need to be conducted on the loss terms associated with a coaxial configuration; most notably, the impact of the convective losses through the coaxial regenerator and at the cold head where flow reversal and flow mixing occur. It is further worthwhile to seek to quantify the conductive and radiative losses associated with such a configuration.

Part IV

Summary of Research Findings

8

Conclusions and Future Developments

Contents

8.1	Summary of Research Findings	171
8.1.1	Testing and Measurement Techniques for Cryocoolers	172
8.1.2	Coaxial Stirling Pulse Tube Cryocoolers with Warm End Displacers	173
8.1.3	Theoretical Analysis and Thermohydrodynamic Computation	174
8.2	Future Work	175
8.2.1	Component Testing	175
8.2.2	Coaxial SPTCs	176

8.1 Summary of Research Findings

In this research project, multiple aspects to the theory, design, and experimental testing of Stirling and pulse tube cryocooler systems have been explored. This thesis has presented bodies of work that fall under three key research approaches:

1. The development of measurement and testing techniques for components that are fundamental to the physical cryocooler operation.
2. The design, numerical simulation, building, and experimental testing of coaxial Stirling pulse tube cryocoolers, firstly whilst employing a warm end displacer with an active motor, and building on this, the development of a comparable performance coaxial SPTC using a passively driven warm end displacer.
3. The theoretical study to develop and enhance understanding of the SPTC system and



computationally simulate important aspects pertaining to the thermal and fluid behaviour of SPTC components that influence the cryocooler performance.

8.1.1 Testing and Measurement Techniques for Cryocoolers

Flexure Bearing Springs

The design, modelling, and testing of flexure bearing springs was conducted. FEA models were developed to model the flexure springs to understand the key locations and magnitudes of the highest stress points on the springs when undergoing oscillatory loading. A low cost and fail-safe fatigue testing rig was employed to run long cycle tests on a variety of springs. The springs tested fell into two categories - firstly, those to be exploited for the ESA MREP programme, where the springs are run at much higher frequencies of 90-100 Hz; and secondly, the springs to be used in the passive displacer. The testing data for the ESA MREP was correlated with the respective *S-N* curve from literature and good agreement was observed. The springs tested failed within a 3-8% error range of the expected failure conditions for a 300 series stainless steel. For the passive displacer springs, the use of two different spring manufacturers for an identical design shed light on the influence that manufacturing procedures, such as surface finishing, plays on the overall performance. An approximate 25% difference was observed between the manufacturers ET and QT. Ultimately, an experimentally tested spring design with a suitable manufacturer and a fatigue limit higher than the design stroke of 8 mm was selected for the passive displacer design.

Strain Gauges for Cryocooler Stroke Measurements

Based on the need for reliable methods of measuring cryocooler stroke that demonstrate high frequency response, the use of strain gauges in a full Wheatstone bridge configuration for measuring spring displacement, and by extension, compressor stroke during spring operation within the electromagnetic coil-moving magnet motor setup, was explored. This included developing data acquisition methods and electronics components to develop suitable instrumentation for the metrological testing and strain gauge bridge calibration, as well as an FEA model to understand where the best location of highest stress for the strain gauge would be on the spring arm. The strain gauge bridge circuit was calibrated with a laser displacement transducer during dynamic spring tests at a frequency of 14 Hz, with an error of 3.13% between the two measurement methods. The FEA model further validated the suitability of the strain gauge measurements for compressor

stroke. Additionally, the circuit's ability to detect impact, both large and subtle, was investigated and FFT analyses were completed to study the harmonic frequencies produced by the spring.

Overall, the strain gauge bridge circuit has been shown to be suitable for detecting impacts that occur on the spring and demonstrated good frequency response. This is a useful finding given that friction rubs and impacts are a common cause of cryocooler stroke inefficiency or even failure, and presents itself as a promising measurement technology for cryocooler stroke measurement.

8.1.2 Coaxial Stirling Pulse Tube Cryocoolers with Warm End Displacers

Coaxial SPTC with Active Displacer

A coaxial Stirling pulse tube cryocooler with an active displacer was numerically simulated to reach a T_c of 80 K with a W_{in} of 85 W and achieve a Q_c of 6 W and a η_r of 18%. This was at a fill pressure of 28 bar, an operating frequency of 60 Hz, a ϕ_d of 41°, a compressor stroke of 9 mm and a displacer stroke of 4.8 mm. The numerical model was experimentally validated, which provided noteworthy insights for the passive displacer design.

The experimental coaxial SPTC was built and tested, achieving a T_c of 80 K with a W_{in} of 79 W and an optimum Q_c of 2.4 W, corresponding to a η_r of 8.3%. It was found that the displacer provides 6% of the power needed to drive the cryocooler, and produces a more efficient SPTC compared to SPTCs that use inertance tubes. Lastly, noteworthy insights have been gained about the SPTC operation, including how the heat transfer, enthalpy flows, and PV work flows throughout the system, shedding light on its fundamental operation and the role key components play.

Coaxial SPTC with Passive Displacer

A coaxial Stirling pulse tube cryocooler with a passive displacer was harmonically designed using inputs from the SPTC with an active displacer, in order to achieve a comparable cryocooler performance with a displacer that is driven passively by the pressure difference across it, generated by the compressor motion. The passive displacer assembly was physically built and experimentally tested; firstly, its resonant frequency was determined as 103 Hz and it was then tested at 250 K once installed into the SPTC. This allowed for the harmonics validation of the displacer.

The passive displacer SPTC experimentally achieved a cooling power, Q_c , of 3 W and η_r of 9.35% for a T_c of 80 K at 31 bar fill pressure, 97 W input power, 8 mm compressor stroke, and a ϕ_d of 60° . This corresponded to a passively driven displacer stroke of 3.4 mm. Noteworthy insights into the displacer operation, the pressure driving forces and pressure phase, the harmonic driving forces and harmonic phase between the different forces driving the displacer, and the effect of displacer damping on the SPTC performance were made. This shed light on the role each fundamental parameter plays on running the displacer passively.

Pulse Tube Cryocooler Energy Recovery

The passive displacer SPTC experimental results were compared with the simulated inertance tube pulse tube cryocooler, the in-line active displacer SPTC, and the coaxial active displacer SPTC in order to understand how the design compares in terms of performance, as well as determine the aspects of the pulse tube cryocooler that affect performance the most. It was found that the passive displacer SPTC was successful in achieving a comparable performance to the coaxial active displacer SPTC. Whilst comparing energy recovery SPTCs, it was further found that the coaxial configuration SPTCs perform less well in terms of both cooling power and efficiency compared to the in-line design.

8.1.3 Theoretical Analysis and Thermodynamic Computation

Phasor and Thermodynamic Analyses

Through the use of phasor analyses, the relationship between mass flow and pressure pulse was explored and the impact it has on cooling power and regenerator losses was outlined. Furthermore, phasor analyses permitted the formulation of mathematical expressions for phase angle. This is beneficial as it enables theoretical study and interpretation of the impact of the mass flow and pressure pulse phase angle on cooling power and the degradation of efficiency, but also, it provides insight for design purposes in inspiring suitable operating conditions. Phasor relationships and a phasor diagram for an in-line Stirling pulse tube cryocooler has also been developed.

Thereafter, the energy flows throughout an ideal and real pulse tube cryocooler system, in terms of enthalpy, entropy, exergy, heat flows and loss terms, were explored. This shed light on the fundamental and unique operation of the pulse tube as well as the thermodynamic behaviour of all

other key elements within a pulse tube cryocooler, putting the analysis in the former part of this chapter and other chapters into beneficial perspective. Lastly, the advantage of using a warm end displacer was once again demonstrated, which adds further weighting to the underlying initiative in this D.Phil. project of incorporating warm end displacer technology into SPTCs for enhanced efficiency due to expansion power recovery.

Thermal Losses and Fluid Flow

A thermal model using ESATAN-TMS was developed for the regenerator-pulse tube assembly which investigated the conductive and radiative losses along the lengths of the coaxial regenerator tubing and the pulse tube. The combined total losses were found to be approximately 0.7 W, closely matching the 0.65 W conductive loss value computed by the numerical simulation in Sage for the active displacer SPTC. Hence, this study served as a useful validation of the conductive computation for the coaxial configuration within Sage. The radiative losses were found to be negligible, indicating the existing radiation shielding in the experimental SPTC setup is most definitely sufficient in thermally isolated the regenerator-pulse tube assembly.

Moreover, the flow within the coaxial regenerator was assessed experimentally using inputs from the validated Sage numerical model. The coaxial regenerator was found to exhibit the highest pressure drop across its length, where approximately 97% of the total overall pressure drop for a given inlet warm end mass flow rate was attributable to the regenerator. Different mesh for the warm end flow straightener were flow tested, allowing for the selection of the most suitable mesh design and parameters.

8.2 Future Work

8.2.1 Component Testing

To further explore the uses of the strain gauge bridge configuration as a suitable method for in-situ measurement of cryocooler stroke, the next most impactful step in this line of work is to install the bridge circuit onto the SPTC compressor or displacer and assess its performance amid SPTC operation. This would require calibration with the Omron laser displacement transducer that has been set up at the displacer backend.



There remains scope for further testing of different flexure bearing spring designs, with a need to delve further into understanding how the springs resonance response differs for varying spring thickness. This is currently lacking in literature and would provide useful insight into how to best design future flexure bearings for linear compressor applications that require being operated at higher frequencies. It would also be necessary to design experiments that shed light on how the flexure springs scale for larger systems, considering there is a growing need for Stirling systems in the energy sector.

8.2.2 Coaxial SPTCs

In terms of the physical build and experimental design, future work for the passive displacer SPTC would entail developing a more robust and readily fine-tuned damping system on the passive displacer. This would allow for changes to the damping inputs to be made amid operation, which would assist in achieving the right phase lag between the compressor and displacer stroke for optimum cryocooler performance.

Moreover, in order to gain a better understanding about the impact of the coaxial configuration on the SPTC operation and performance, flow mixing at the coaxial cold head where flow reversal occurs would be imperative to investigate. This would shed light on the magnitude of the convective loss terms and their impact on the SPTC cooling performance and efficiency. Once such a model is developed and experimentally validated, the cold head design can be optimised to improve the SPTC performance and another iteration of comparative study with the in-line SPTC design can be conducted. Lastly, a holistic thermo-fluid model that couples the transport phenomena physics that governs the pressure pulse and mass flow throughout the SPTC system should be developed, so as to better understand the relationship between the flow and the heat transfer in components that drive the cooling process, such as the regenerator.

References

- [1] Graham Walker. *Stirling-Cycle Machines*. Oxford, UK: Clarendon Press, 1973.
- [2] V. P. Dawson. “Engines and Innovation: Lewis Laboratory and American Propulsion Technology”. In: *NASA History Series* (1991).
- [3] N. Kurti. “From Cailletet and Pictet to Microkelvin”. In: *Cryogenics* (1978).
- [4] K. Onnes. “Further experiments with liquid helium: On the change of electric resistance of pure metals at very low temperature”. In: *Comm. Phys. Lab Univ. Leiden* (1911).
- [5] G. Davey. “Review of the Oxford cryocooler”. In: *Advances in Cryogenic Engineering* (1990).
- [6] J. Meseguer, I. Perez-Grande, and A. Sanz-Andres. *Spacecraft Thermal Control*. Woodhead Publishing, 2012.
- [7] European Space Agency. *ESA’s fleet across the electromagnetic spectrum*. 2017. URL: <http://sci.esa.int/education/59465-esa-fleet-across-the-spectrum-poster/> (visited on 11/01/2021).
- [8] European Space Agency. *Space Engineering: Thermal Design Handbook - Part 14: Cryogenic Cooling*. ESA, 2011.
- [9] M. Bruckmayer. *Imaging cryogenic detectors for astro and particle physics*. University of Oxford D.Phil. Thesis, 1999.
- [10] M. Linder et al. “Cryogenics in Space - A review of the missions and technologies”. In: *ESA Bulletin* 107 (2001).
- [11] Ray Radebaugh. “Development of the Pulse Tube Refrigerator as an Efficient and Reliable Cryocooler”. In: *Institute of Refrigeration* (2000), p. 1999.
- [12] Graham Walker. *Cryocoolers I*. New York: Springer US, 1983.
- [13] Thomas Kawecki. “High-Temperature Superconducting Space Experiment II (HTSSE II) Overview and Preliminary Cryocooler Integration Experience”. In: *Cryocoolers* 8 (1995), pp. 893–900.
- [14] Ronald G. Ross, Jr. “Aerospace Coolers: A 50-Year Quest for Long-Life Cryogenic Cooling in Space”. In: *Cryogenic Engineering: Fifty Years of Progress*. New York: Springer Publications, 2007, pp. 225–284.
- [15] G.S. Mand et al. “MOPITT On-Orbit Stirling Cycle Cooler Performance”. In: *Cryocoolers* 11 (1996), pp. 759–768.
- [16] Lionel Duband. “Space Cryocooler Developments”. In: *International Cryogenic Engineering Conference and the International Cryogenic Materials Conference*. Enschede, NL, 2014.
- [17] Martin Crook. “Small Scale Cooler”. In: *ESA Coordinated Final Presentation Days*. ESTEC, Noordwijk, 2016.
- [18] James R Drummond et al. “A Review of 9-year Performance and Operation of the MOPITT Instrument”. In: *Advances in Space Research* 45 (2010), pp. 760–774.
- [19] Michael Lewis, Peter Bradley, and Ray Radebaugh. “Experiments with linear compressors for phase shifting in pulse tube cryocoolers”. In: *AIP Conference Proceedings* 1434.1600 (2012).
- [20] P. Kittel. “Ideal orifice pulse tube refrigerator performance”. In: *Cryogenics* 32 (1992).
- [21] P. De Boer. “Performance of an inertance pulse tube”. In: *Cryogenics* 42 (2002).
- [22] M. Brito and G. Peskett. “Experimental analysis of free warm expander pulse tube”. In: *Cryogenics* 41 (2001).
- [23] Y. Shi and S. Zhu. “Experimental investigation of pulse tube refrigerator with displacer”. In: *International Journal of Refrigeration* 76 (2017).



- [24] M. A. Abolghasemi et al. "Stirling pulse tube cryocooler using an active displacer". In: *Cryogenics* 96 (2018).
- [25] S. Zhu and M. Nogawa. "Pulse tube Stirling machine with warm gas-driven displacer". In: *Cryogenics* 50 (2010).
- [26] Y. Zhang et al. "Advances in a high efficiency commercial pulse tube cooler". In: *IOP Conf. Ser.: Mater. Sci. Eng.* 278 (2017).
- [27] K.B. Wilson et al. "Sunpower's CPT60 pulse tube cryocooler". In: *Cryocoolers* 14 (2007).
- [28] T. Trolhier et al. "Status of Air Liquide space pulse tube cryocoolers". In: *Cryocoolers* 15 (2009).
- [29] D. Haizheng. "High-capacity 60 K single-stage coaxial pulse tube cryocoolers". In: *Cryogenics* 52 (2012).
- [30] "High-capacity 60 K single-stage coaxial pulse tube cryocoolers". In: *Cryogenics* 52 (2012).
- [31] Ray Radebaugh. "Thermodynamics of regenerative refrigerators". In: *Generation of Low Temperature and Its Applications* 3 (2003).
- [32] Y. Zhao and H. Dang. "CFD simulation of a miniature coaxial Stirling-type pulse tube cryocooler operating at 128 Hz". In: *Cryogenics* 73 (2016).
- [33] A. T. M. De Waele. "Pulse tube refrigerator with a warm displacer: Theoretical treatment in the harmonic approximation". In: *Cryogenics* 100 (2019).
- [34] N. Almtireen, J. J. Brandner, and J. G. Korvink. "Pulse tube cryocooler: Phasor analysis and one-dimensional numerical simulation". In: *Journal of Low Temperature Physics* 199 (2020).
- [35] P. Kittel et al. "General pulse tube theory". In: *Cryogenics* 10.36 (1996).
- [36] S. Vanapalli et al. "120 Hz pulse tube cryocooler for fast cooldown to 50 K". In: *Cryogenics* 90.36 (2007).
- [37] Ray Radebaugh and Peter Shirron. *Foundations of Cryocoolers*. Tech. rep. Burlington, Vermont: Cryogenic Society of America, 2018.
- [38] P.B. Bailey, H. Rana, and G. Gilley. "Low cost flexure spring testing". In: *IOP Conf. Ser.: Mater. Sci. Eng.* 502.012020 (2019).
- [39] H. Rana et al. "Experimental fatigue analysis of cryocooler flexure springs". In: *IOP Conf. Ser.: Mater. Sci. Eng.* (2022).
- [40] Kun Liang, Mike Dadd, and Paul Bailey. "Clearance seal compressors with linear motor drives. Part 1: Background and system analysis". In: *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy* 227.3 (2013), pp. 242–251.
- [41] Peter Fortescue, Graham Swinerd, and John Stark. *Spacecraft Systems Engineering*. Chichester, UK: Wiley, 2011.
- [42] Kun Liang et al. "Modelling and measurement of a moving magnet linear compressor performance". In: *Energy* 66 (2014), pp. 487–495.
- [43] J.M. Prentis. *Dynamics of Mechanical Systems*. Michigan, US: Horwood, 1980.
- [44] E. Marquardt, R. Radebaugh, and P. Kittel. "Design equations and scaling laws for linear compressors with flexure springs". In: *Cryocoolers* 7 (1992).
- [45] MA Miner. "Cumulative damage in fatigue". In: *Journal of Applied Mechanics* 12 (1945).
- [46] R Johansson and G Persson. "Influence of testing and material factors on the fatigue strength of valve steel". In: *International Compressor Engineering Conference* (1976).
- [47] H. Dang. "Development of high performance moving-coil linear compressors for space Stirling-type pulse tube cryocoolers". In: *Cryogenics* 68 (2015).
- [48] J. Keisler, O. K. Chopra, and W. J. Shack. "Fatigue stain-life behaviour of carbon and low-alloy steels, austenitic stainless steels, and alloy 600 in LWR environments". In: *US Nuclear Regulatory Commission* (1995).

- [49] Lampman S. R. and N. D. DiMatteo. "ASM Handbook Volume 19: Fatigue and fracture". In: *ASM International* (1996).
- [50] H. Rana et al. "Compressor stroke and frequency response using strain gauges". In: *Cryocoolers* (2022).
- [51] S. Maitra. "Two wire pressure transmitter using Bourdon tube pressure sensor and LVDT - an advanced pressure transmitter". In: *Communications on Applied Electronics* 3.6 (2015), pp. 2394–4714.
- [52] Agilent Technologies. *Application notes 290-1: practical strain gauge measurements*. Tech. rep. Manchester, UK: RS Components UK, 1999.
- [53] David Gedeon. *Sage User's Guide: For Stirling, Pulse Tube and Low-T Cooler Model Classes*. Ohio, US: Gedeon Associates, 2009.
- [54] H. Rana et al. "Numerical modelling of a coaxial Stirling pulse tube cryocooler with an active displacer for space applications". In: *Cryogenics* 106 (2020).
- [55] H. Rana et al. "Thermal losses in a coaxial pulse tube cryocooler". In: *Cryocoolers* (2021).
- [56] M. A. Abolghasemi et al. "Coaxial Stirling pulse tube cryocooler with active displacer". In: *Cryogenics* 111 (2020).
- [57] M. A. Abolghasemi et al. "Detailed analysis of a coaxial Stirling pulse tube cryocooler with an active displacer". In: *Cryocoolers* (2021).
- [58] S. Ergun. "Fluid flow through packed columns". In: *Chemical Engineering Process* 48.2 (1952), pp. 89–94.
- [59] F. Macdonald et al. "Flow through porous media - the Ergun equation revisited". In: *Ind. Eng. Chem. Fundamen.* 18.3 (1979), pp. 199–208.
- [60] H. S. Carslaw and J. C. Jaeger. *Conduction of heat in solids*. Oxford: Oxford: Clarendon Press, 1959.
- [61] M. A. Lewis and R. Radebaugh. "Measurement of heat conduction through metal spheres". In: *Cryocoolers* 11 11 (2001), pp. 419–425.
- [62] J. Fereday et al. "Cryocooler modelling methodology". In: *Cryogenics* 46.2 (2006).
- [63] R. Siegel and J. R. Howell. *Thermal radiation heat transfer*. Taylor and Francis, 2002.
- [64] F. Sabourin et al. "Application of 3D view factor method for heat fluxes deposition on ITER cryostat thermal shield". In: *Fusion Engineering and Design* 138 (2019).
- [65] Kan Yang. *The anatomy of ESATAN and ESARAD thermal model files*. Tech. rep. NASA Goddard Space Flight Center: NASA, 2012.
- [66] ITP Engines. *ESATAN-TMS 2017 Manual*. Tech. rep. ITP Engines, 2020.
- [67] NIST. *Cryogenic material properties methodology*. 2020. URL: <https://trc.nist.gov/cryogenics/materials/%20materialproperties.htm> (visited on 12/04/2020).
- [68] H. Dang. "40 K single-stage coaxial pulse tube cryocoolers". In: *Cryogenics* 52 (2012).
- [69] N. Wang et al. "A high efficiency coaxial pulse tube cryocooler operating at 60 K". In: *Cryogenics* 93 (2018).
- [70] X. Wang et al. "A high efficiency hybrid Stirling-pulse tube cryocooler". In: *AIP Advances* 5 (2015).
- [71] H. Rana et al. "A passive displacer for a Stirling pulse tube cryocooler". In: *Cryocoolers* (2021).
- [72] H. Rana et al. "Experimental performance of a passively driven displacer". In: *Cryocoolers* (2022).

Appendices



Appendices

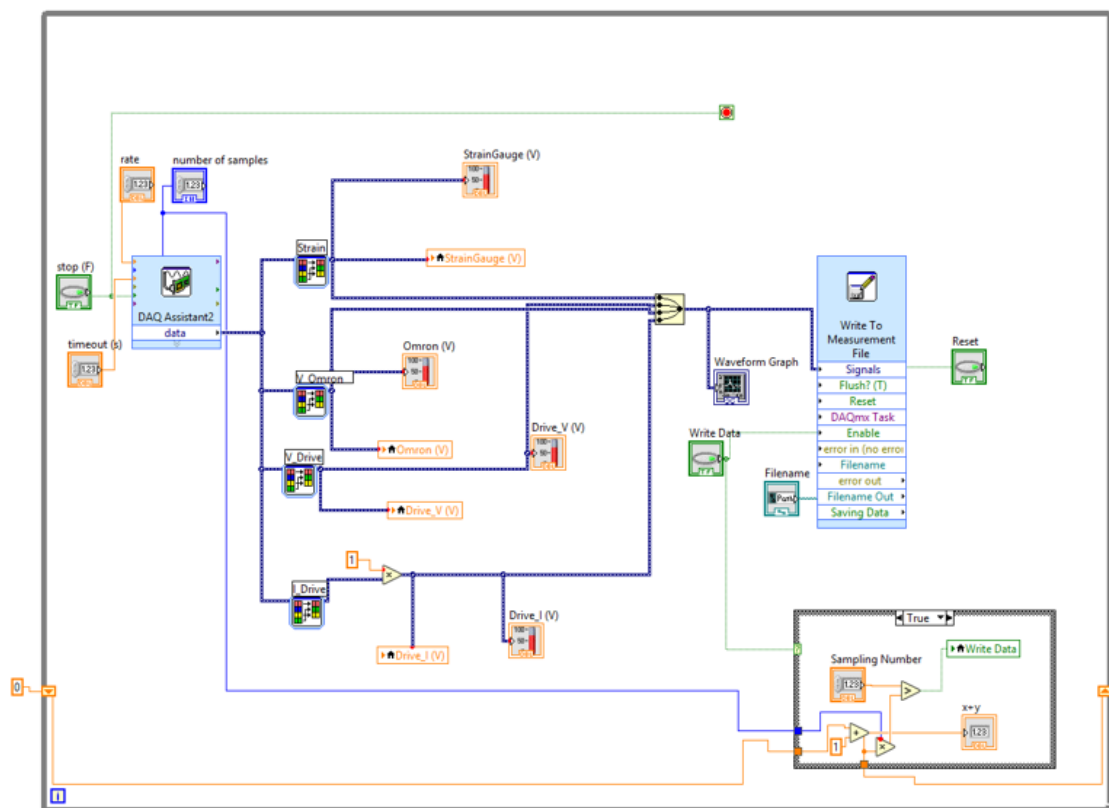


Figure A.1: LabVIEW data acquisition program for the strain gauge bridge configuration installed on the spring testing setup.

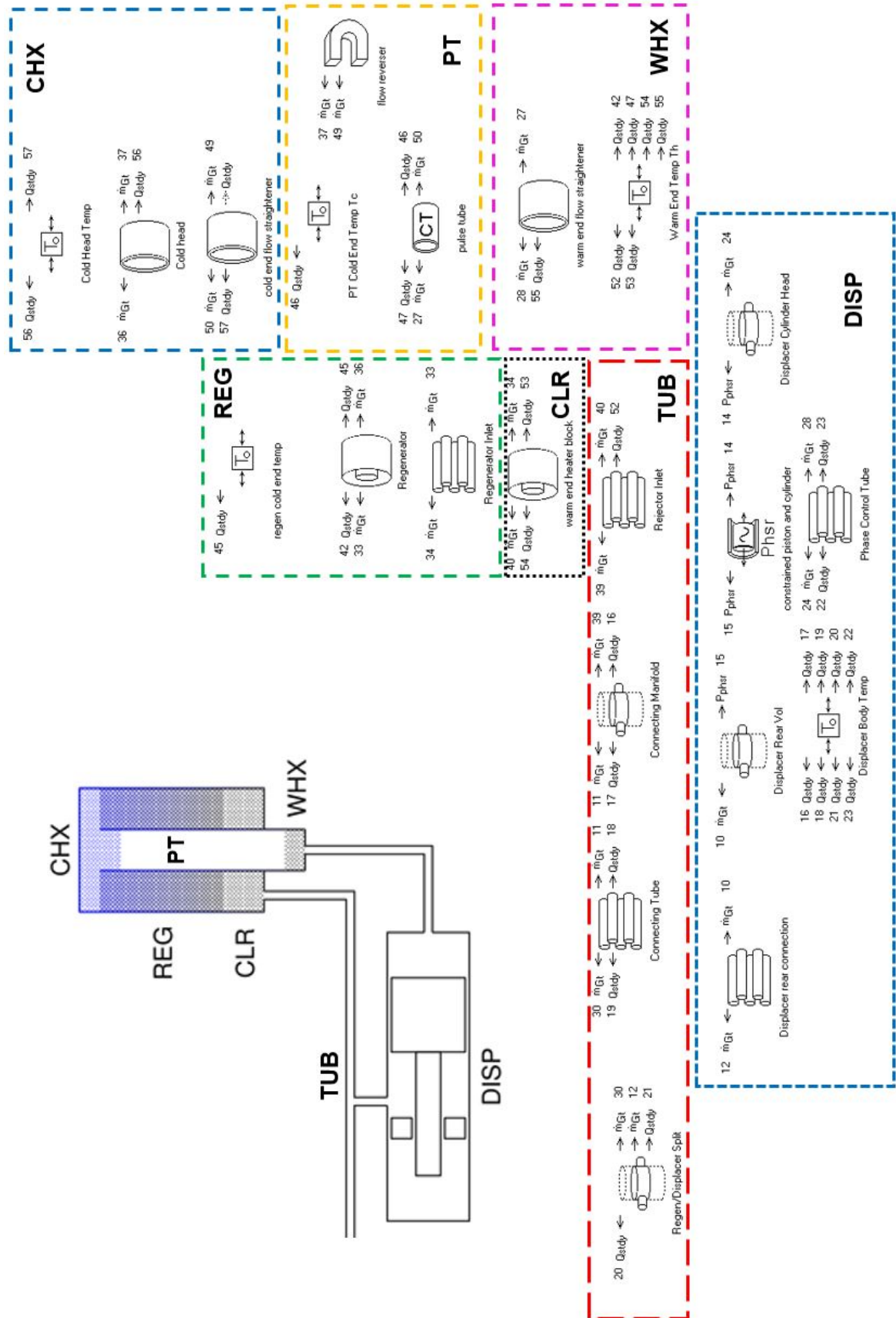


Figure A.2: Sage numerical model for an SPTC active displacer and the connected coaxial pulse tube unit.