

Energy Detection based Spectrum Sensing over Two-wave and Diffuse Power Fading Channels

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Abstract

One of the most important factors that affects the performance of energy detection based spectrum sensing is the fading channel between the wireless nodes. This paper investigates the performance of energy detection based spectrum sensing over two-wave and diffuse power fading channels. The two-wave and diffuse power fading model can characterize a variety of fading channels, including Rayleigh and Rician as special cases. It can also efficiently represent channels with more severe fading than Rayleigh, which is regarded as the worst case fading scenario for wireless communications. In this article novel analytic expressions for the average probability of detection over two-wave and diffuse power fading channels are derived that account for single-user, cooperative spectrum sensing and square law selection diversity reception. These expressions are used to describe the behavior of energy detection based spectrum sensing over severe fading conditions and to investigate cooperation and diversity as a means of mitigating the fading effects. Our results indicate that two-wave and diffuse power fading can significantly degrade the sensing performance; however, it is shown that detection performance can improve when cooperation and diversity are employed. The presented outcomes enable us to identify the limits of energy detection based spectrum sensing and quantify the trade-offs between detection performance and energy efficiency for cognitive radio systems that operate in confined environments such as in-vehicular wireless networks.

Index Terms

Cognitive radio, confined environments, cooperative spectrum sensing, diversity reception, energy detection, fading channels, spectrum sensing, TWDP fading

I. INTRODUCTION

ENERGY DETECTION (ED), also known as the radiometer, is a non-coherent detection method that measures the energy level of an unknown received signal and compares it with a predefined threshold in order to determine its presence or absence within a given bandwidth. The problem of detecting unknown deterministic signals was first studied by Urkowitz in [1], where a flat band-limited Gaussian

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channel was considered. The signal detection problem was revisited by Kostylev in [2] where signals of random amplitude, including Rayleigh, Rician and Nakagami distributions were investigated. ED has been widely used in a variety of wireless applications such as RADAR systems [3] and ultra-wideband communications [4]. Owing to its simplicity and no need for prior knowledge on the received signal, ED is considered as the most viable spectrum sensing method for cognitive radio (CR) networks [5]. CR networks are expected to improve spectrum utilization by allowing unlicensed secondary users (SUs) to dynamically access the licensed radio spectrum without interfering with the primary users (PUs) [6]. This can be achieved by deploying CR-enabled radios that are able to sense their spectral environment and adapt the transmission parameters accordingly. In this context, ED is used as a spectrum sensing mechanism in order for SUs to determine whether a PU is present or absent within a given band of interest.

The detection performance of ED-based spectrum sensing is significantly influenced by the fading channel between the SU and PU nodes. Therefore, ED-based spectrum sensing has been studied for a variety of fading channels such as Rayleigh, Rician and Nakagami [7]. In [8], the ED performance in the case of Nakagami- m fading has been evaluated. Likewise, the detection of unknown signals over K -distributed and generalized K -distributed channels has been analyzed in [9]. Furthermore, the performance of ED-based spectrum sensing over κ - μ and κ - μ extreme fading channels has been recently investigated in [10].

All of the aforementioned studies focus on the performance of ED-based spectrum sensing over traditional fading channels for outdoor and indoor propagation scenarios. Yet, as technology evolves, CR is expected to be applied in support of emerging applications such as Machine-to-Machine (M2M) communications systems and wireless sensor networks (WSN) as a practical realization of the Internet-of-Things concept [11], [12]. However, the operational environment of such communication systems has been found to exhibit fading conditions that are not adequately characterized by the existing fading models. Indicatively, WSNs deployed in cavity environments (e.g. aircraft, public transportation vehicles [13]), vehicle-to-vehicle communications [14], and enclosed structures (e.g. inside computer cabinets) [15] have been found to exhibit frequency-dependent and spatially-dependent fading whose severity exceeds those predicted by the Rayleigh fading model. Such small-scale fading conditions, named as “hyper-Rayleigh”, were experimentally verified to be adequately characterized by the two-wave and diffuse power (TWDP) model [16]. In this context, the problem of detection threshold optimization for ED-based spectrum sensing over “hyper-Rayleigh” was recently studied in [17]. However, this method considers only single user spectrum sensing and is limited for the case of a time-bandwidth product equal to 1.

To this end, in this article an analytic expression for the average probability of detection for ED-based

spectrum sensing over TWDP fading channels is derived which can be validated for any positive time-bandwidth product value. This expression is then extended to account for cooperative spectrum sensing and Square Law Selection (SLS) diversity reception. The performance of these schemes is evaluated as a means of mitigating the effects of fading and improving the sensing performance over TWDP fading channels. The obtained results provide a new insight into the performance of ED-based spectrum sensing over worse than Rayleigh fading conditions that can describe the behavior of ED-based spectrum sensing over fading conditions that occur within non-traditional environments such as metallic enclosed structures. The derived expressions can form the basis of analyzing the performance and designing future CR systems for emerging applications such as cognitive M2M systems and in-vehicular sensor networks.

The remainder of this article is organized as follows. Section II presents the fundamentals of ED and describes the theoretical underpinnings of the TWDP fading model. In Section III, analytic expressions for the average probability of detection for single user and cooperative spectrum sensing as well as for an SLS diversity reception scheme are derived. Numerical results and discussion for different fading scenarios are provided in Section IV, while Section V draws the conclusion of this work.

II. SYSTEM AND CHANNEL MODEL

A. Energy Detection Fundamentals

In the context of a communications link, the received signal, $y(t)$, can be mathematically described as,

$$x(t) = hs(t) + n(t), \quad (1)$$

where $s(t)$ is the transmitted signal, h denotes the channel gain, $n(t)$ is additive white Gaussian noise (AWGN) and t is the time index.

For ED, the received signal is filtered within a bandwidth W , squared, and integrated over an observation interval T . The output of the integrator is the received signal's energy which is then used as a test statistic [1]. By comparing the test statistic, y , with a predefined detection threshold, λ , the detector has to distinguish between the following hypotheses,

$$x(t) = \begin{cases} n(t) & ,H_0 \\ hs(t) + n(t) & ,H_1, \end{cases} \quad (2)$$

where H_0 and H_1 denote the hypothesis of the signal to be absent or present, respectively. Given the time-bandwidth product, $u = TW$, the test statistic follows a central chi-square distribution with $2u$ degrees of freedom under hypothesis H_0 and a non central chi-square distribution with $2u$ degrees of freedom under hypothesis H_1 [1]. As a result, the corresponding probability density function (PDF) of the test statistic

y is given by [7],

$$f_y(y) = \begin{cases} \frac{1}{2^u \Gamma(u)} y^{u-1} e^{-\frac{y}{2}} & , H_0 \\ \frac{1}{2} \frac{y}{2\gamma} \frac{u-1}{2} e^{-\frac{y+2\gamma}{2}} I_{u-1}(\sqrt{2y\gamma}) & , H_1, \end{cases} \quad (3)$$

where γ is the instantaneous SNR, $\Gamma(a) = \int_0^\infty t^{a-1} e^{-t} dt$ is the Gamma function and $I_n(x) = (1/\pi) \int_0^\pi \cos(n\theta) e^{x \cos(\theta)} d\theta$ is the modified Bessel function of the first kind [18].

The sensing performance of ED is evaluated in terms of the probability of false alarm P_{fa} , (i.e. false positive rate of signal detection), probability of detection, P_d , (i.e. true positive rate of signal detection), and probability of missed detection, $P_{md} = 1 - P_d$, (i.e. false negative rate of signal signal detection)

By integrating (3) over the limits of zero to infinity, the probabilities of false alarm and detection are obtained as [7],

$$P_{fa} = \frac{\Gamma(u, \lambda/2)}{\Gamma(u)} \quad (4)$$

$$P_d = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}), \quad (5)$$

where $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$ and $Q_m(a, b) = (1/a^{(m-1)}) \int_b^\infty x^m e^{-(x^2+a^2)/2} I_{(m-1)}(ax) dx$ denote the incomplete gamma function and the generalized Marcum Q-function, respectively [18], [19].

B. TWDP fading Model

Durkin et. al derived a new parametric family of PDFs that approximates the TWDP PDF, which describes small scale fading in the presence of two multipath components [20]. This family of PDFs characterizes a variety of fading scenarios including those experienced by narrow-band receivers, the use of directional antennas, and wide-band signals. In addition, the TWDP fading model has been found to be particularly versatile since it also reduces to the well-known Rician and Rayleigh distributions.

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$$f_r(r) = \frac{r}{\sigma^2} e^{(-\frac{r^2}{2\sigma^2} - K)} \sum_{i=1}^M a_i D\left(\frac{r}{\sigma}; K; a_i\right), \quad (6)$$

where,

$$D(x; K; a) = \frac{1}{2}e^{aK}I_0\left(x\sqrt{2K(1-a)}\right) + \frac{1}{2}e^{-aK}I_0\left(x\sqrt{2K(1+a)}\right), \quad (7)$$

with $I_0(\cdot)$ denoting the zero-order Bessel function of the first kind. The approximation coefficient, a_i , is given as $a_i = \Delta \cos(\pi(i-1)/2M-1)$, with M representing the order of approximation of the TWDP PDF. The minimum required approximation order, M , is determined by the product of the fading parameters K and Δ as $M_{min} \geq \lceil 1/2K\Delta \rceil$, with $\lceil \cdot \rceil$ denoting the ceiling function.

In two special cases, the TWDP PDF degenerates to the Rician PDF when $K \neq 0$ and $\Delta = 0$, and to the Rayleigh PDF when $K = 0$. Furthermore, for $K \rightarrow \infty$ and $\Delta = 1$ the TWDP PDF reduces to the two-ray PDF, which represents a fading channel that consists of two summed multipath components of equal weights. Under such fading conditions the envelope statistics do not follow a Rayleigh PDF and thus, it has been experimentally validated as a worse than Rayleigh fading scenario [16].

Given the PDF of the envelope over a TWDP fading channel in (6) and that $\gamma = \bar{\gamma}/2\sigma^2(K+1)$, the corresponding PDF of the SNR can be obtained by performing a squared transformation of the random variable r , in (6) [21]. Formally,

$$f_\gamma(\gamma) = \frac{K+1}{2\bar{\gamma}}e^{-K} \sum_{i=1}^M a_i \left[e^{a_i K} e^{-\frac{(K+1)\gamma}{\bar{\gamma}}} A + e^{-a_i K} e^{-\frac{(K+1)\gamma}{\bar{\gamma}}} B \right], \quad (8)$$

where,

$$A = I_0\left(2\sqrt{\frac{K(K+1)(1-a_i)\gamma}{\bar{\gamma}}}\right) \quad (9)$$

$$B = I_0\left(2\sqrt{\frac{K(K+1)(1+a_i)\gamma}{\bar{\gamma}}}\right), \quad (10)$$

with $\bar{\gamma}$ denoting the average SNR.

III. ENERGY DETECTION OVER TWDP FADING CHANNELS

A. Single User Spectrum Sensing

In a fading channel, where the channel gain, h , varies, the average probability of detection is obtained by averaging the probability of detection for a non-fading AWGN channel over the corresponding SNR fading statistics [22]. Formally,

$$\bar{P}_d = \int_0^\infty Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) f_\gamma(\gamma) d\gamma. \quad (11)$$

Hence, by substituting (8) into (11) yields,

$$\bar{P}_{d_{TWDP}} = \int_0^\infty Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) \frac{K+1}{2\bar{\gamma}} e^{-K} \sum_{i=1}^M \left[e^{a_i K} e^{-\frac{(K+1)\gamma}{\bar{\gamma}}} A + e^{-a_i K} e^{-\frac{(K+1)\gamma}{\bar{\gamma}}} B \right] d\gamma, \quad (12)$$

An analytic expression for (12) can be derived by using an infinite series representation of Marcum Q-function [23, eq. (27)]. Therefore, P_d can be rewritten as a sum of n terms as,

$$P_d = e^{-\frac{\lambda}{2}} \sum_{l=0}^{u-1} \frac{\left(\frac{\lambda}{2}\right)^l}{l!} + e^{-\frac{\lambda}{2}} \sum_{n=u}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^n}{n!} \left(1 - e^{-\gamma} \sum_{k=0}^{n-u} \frac{\gamma^k}{k!}\right). \quad (13)$$

By substituting (13) into (12) $\bar{P}_{d_{TWDP}}$ is obtained as,

$$\begin{aligned} \bar{P}_{d_{TWDP}} = & e^{-\frac{\lambda}{2}} \sum_{l=0}^{u-1} \frac{\left(\frac{\lambda}{2}\right)^l}{l!} + e^{-\frac{\lambda}{2}} \sum_{n=u}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^n}{n!} \left(1 - \frac{(K+1)e^{-K}}{2\bar{\gamma}} \right. \\ & \times \left. \sum_{k=0}^{n-u} \sum_{i=1}^M \frac{\int_0^{\infty} \gamma^k [e^{a_i K} e^{-\frac{K+\bar{\gamma}+1}{\bar{\gamma}} \gamma} A + e^{-a_i K} e^{-\frac{K+\bar{\gamma}+1}{\bar{\gamma}} \gamma} B] d\gamma}{k!} \right). \end{aligned} \quad (14)$$

The integral in (14) can be calculated by using [24, eq. (6.643-2)]. With the aid of the Whittaker function identity [24, eq. (9.220-2)] and after some algebraic manipulations an analytic expression for $\bar{P}_{d_{TWDP}}$ is derived in (15), shown in the footnote, where $\Phi(\alpha; \beta; z)$ denotes the Confluent Hypergeometric function, which is available in most of commonly used mathematical software packages such as MATLAB and MATHEMATICA [24]. The derivation of (15) is described in the Appendix.

In order to evaluate (15), truncation of the infinite series is required. Hence, the associated truncation error of (15) for N terms is bounded by [23, eq. (28)],

$$\begin{aligned} |E| \leq & 1 - e^{-\frac{\lambda}{2}} \sum_{n=0}^{u+N} \frac{\left(\frac{\lambda}{2}\right)^n}{n!} \left(1 - \frac{(K+1)e^{-K}}{2(K+\bar{\gamma}+1)} \sum_{k=0}^{N+1} \sum_{i=1}^M \right. \\ & e^{a_i K} \left(\frac{\gamma}{K+\bar{\gamma}+1}\right)^k \Phi\left(k+1; 1; \frac{K(K+1)(1-a_i)}{K+\bar{\gamma}+1}\right) \\ & \left. + e^{-a_i K} \left(\frac{\gamma}{K+\bar{\gamma}+1}\right)^k \Phi\left(k+1; 1; \frac{K(K+1)(1+a_i)}{K+\bar{\gamma}+1}\right) \right). \end{aligned} \quad (16)$$

By using (16), the truncation error of (15) can be computed. By way of example, Table I illustrates the minimum number of terms required to obtain a five figure accuracy when evaluating the detection performance of ED-based spectrum sensing over a TWDP fading channel with $K = 10$ dB and $\Delta = 1$ for a combination of different SNR and time-bandwidth product values, u .

$$\begin{aligned} \bar{P}_{d_{TWDP}} = & e^{-\frac{\lambda}{2}} \sum_{l=0}^{u-1} \frac{\left(\frac{\lambda}{2}\right)^l}{l!} + e^{-\frac{\lambda}{2}} \sum_{n=u}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^n}{n!} \left(1 - \frac{(K+1)e^{-K}}{2(K+\bar{\gamma}+1)} \sum_{k=0}^{n-u} \sum_{i=1}^M e^{a_i K} \left(\frac{\bar{\gamma}}{K+\bar{\gamma}+1}\right)^k \right. \\ & \times \Phi\left(k+1; 1; \frac{K(K+1)(1-a_i)}{K+\bar{\gamma}+1}\right) + e^{-a_i K} \left(\frac{\bar{\gamma}}{K+\bar{\gamma}+1}\right)^k \Phi\left(k+1; 1; \frac{K(K+1)(1+a_i)}{K+\bar{\gamma}+1}\right) \Bigg) \end{aligned} \quad (15)$$

Table I

NUMBER OF REQUIRED TERMS, N , IN ORDER FOR (15) TO ACHIEVE A TRUNCATION ERROR, $|E| \leq 10^{-5}$, FOR $P_{fa} = 0.1$ WITH $K = 10$ AND $\Delta = 1$.

SNR (dB)	u=1	u=10	u=100
0	11	14	19
5	15	17	22
10	21	29	35

B. Cooperative Spectrum Sensing

The detection performance of ED-based spectrum sensing may be affected by destructive channel conditions since the CR terminals are unable to distinguish between an unoccupied channel and one that is attenuated by deep fading. Therefore, cooperative detection that exploits spatial diversity among SUs has been proposed as an effective means of improving the detection performance by alleviating the effects of shadowing and multipath [25], [26]. For such a cooperative scheme with m collaborative SUs, the probabilities of false alarm, Q_{fa} and detection Q_d are given as [27],

$$Q_{fa} = 1 - (1 - P_{fa})^m \quad (17)$$

$$Q_d = 1 - (1 - P_d)^m. \quad (18)$$

Hence, the probability of detection over TWDP fading of a CR system with m cooperative users, $Q_{d_{TWDP}}$, is obtained by substituting (15) into (18) resulting in an analytic expression given by (19) (see footnote). Given that Q_{fa} is independent of the fading statistics it can be evaluated as [7],

$$Q_{fa} = \left[\frac{\Gamma(u, \lambda/2)}{\Gamma(u)} \right]^m. \quad (20)$$

C. Spectrum Sensing with Diversity Reception

Diversity is a well known technique used to compensate for signal fades in wireless communication channels. SLS is an effective diversity reception scheme that is highly regarded due to its simplicity. Its principle is based on selecting the branch with the maximum decision statistic, i.e., $y_{SLS} = \max[y_1, y_2, \dots, y_L]$ [28]. For an SLS diversity scheme with L independent and identically distributed (i.i.d) branches, the average probability of detection is given as [7],

$$\bar{P}_d^{SLS} = 1 - \prod_{j=1}^L \int_0^\infty \left[1 - Q_u(\sqrt{2\gamma_j}, \sqrt{\lambda}) f_{\gamma_j}(\gamma_j) d\gamma_j \right]. \quad (21)$$

$$\begin{aligned}
\bar{Q}_{d_{TWDP}} &= 1 - \left[1 - e^{-\frac{\lambda}{2}} \sum_{l=0}^{u-1} \frac{\left(\frac{\lambda}{2}\right)^l}{l!} + e^{-\frac{\lambda}{2}} \sum_{n=u}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^n}{n!} \left(1 - \frac{(K+1)e^{-K}}{2(K+\bar{\gamma}+1)} \sum_{k=0}^{n-u} \sum_{i=1}^M e^{a_i K} \left(\frac{\bar{\gamma}}{K+\bar{\gamma}+1} \right)^k \right. \right. \\
&\quad \times \Phi \left(k+1; 1; \frac{K(K+1)(1-a_i)}{K+\bar{\gamma}+1} \right) + e^{-a_i K} \left(\frac{\bar{\gamma}}{K+\bar{\gamma}+1} \right)^k \Phi \left(k+1; 1; \frac{K(K+1)(1+a_i)}{K+\bar{\gamma}+1} \right) \left. \right] \Bigg]^m \quad (19) \\
\bar{P}_{d_{TWDP}}^{SLS} &= 1 - \prod_{j=1}^L \left[1 - \left[e^{-\frac{\lambda}{2}} \sum_{l=0}^{u-1} \frac{\left(\frac{\lambda}{2}\right)^l}{l!} + e^{-\frac{\lambda}{2}} \sum_{n=u}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^n}{n!} \left(1 - \frac{(K+1)e^{-K}}{2(K+\bar{\gamma}_j+1)} \sum_{k=0}^{n-u} \sum_{i=1}^M e^{a_i K} \left(\frac{\bar{\gamma}_j}{K+\bar{\gamma}_j+1} \right)^k \right. \right. \right. \\
&\quad \times \Phi \left(k+1; 1; \frac{K(K+1)(1-a_i)}{K+\bar{\gamma}_j+1} \right) + e^{-a_i K} \left(\frac{\bar{\gamma}_j}{K+\bar{\gamma}_j+1} \right)^k \Phi \left(k+1; 1; \frac{K(K+1)(1+a_i)}{K+\bar{\gamma}_j+1} \right) \left. \right] \Bigg] \quad (22)
\end{aligned}$$

Therefore, by substituting (15) into (21), an analytic expression for $\bar{P}_{d_{TWDP}}^{SLS}$ is given by (22), in the footnote. The corresponding probability of false alarm, P_{fa}^{SLS} , is independent of the fading statistics and thus, it can be evaluated as follows [7, eq. (14)],

$$P_{fa}^{SLS} = 1 - \prod_{j=1}^L \left[1 - \frac{\Gamma(u, \lambda/2)}{\Gamma(u)} \right]. \quad (23)$$

D. Noise Uncertainty

In all previous cases perfect noise estimation is assumed. However, due to the fact that ED is based on hypothesis-testing with statistical assumptions of noise such as linearity, Gaussianity, and stationarity, perfect noise estimation is not always possible in real-world scenarios. Therefore, assuming a noise uncertainty of α , the estimated noise power, $\bar{\sigma}_w$, is modeled as $\bar{\sigma}_w \in [\sigma_w/\alpha, \alpha\sigma_w]$, where σ_w denotes the nominal noise power. Therefore, for the worst-case, $\bar{\sigma}_w = \alpha\sigma_w$, (5) becomes [29],

$$P_d = Q_u(\sqrt{2\gamma}, \alpha\sqrt{\lambda}). \quad (15)$$

Hence, by evaluating (15) over the TWDP SNR statistics using (8) as described in Section III.A, a closed-form expression for the average probability of detection over TWDP with noise uncertainty is obtained by scaling the detection threshold, λ , with the noise uncertainty term α , i.e., replacing λ with $\alpha\lambda$ in (15).

IV. NUMERICAL RESULTS AND DISCUSSION

This section analyzes the behavior of ED-based spectrum sensing over TWDP fading channels. The sensing performance is evaluated over different scenarios with respect to \bar{P}_d versus SNR and in terms of both Receiver Operating Characteristics (ROC) (P_d versus P_{fa}) and complementary ROC (P_{md} versus P_{fa})

curves. Furthermore, the effect of the fading parameters on P_{md} for negative SNR regions is quantified. The fading scenarios under investigation are considered to be representative of moderate, severe and worse than Rayleigh fading conditions within realistic operational environments.

The accuracy of the derived expressions is validated through comparison of results obtained by Monte Carlo simulation. The simulation model generates the received signal at the detector's front-end based on (1). Given that the derived expressions are independent of modulation, no modulation is considered for the transmitted signal. Furthermore, owing to the non-coherent nature of ED, no synchronization between the PU and SU is required. The noise samples are drawn from a Gaussian distribution, $\mathcal{N}(0, 1)$, whereas the channel fading coefficients are obtained from the TWDP PDF (6). The energy of the received signal is compared to the detection threshold in order to obtain the corresponding P_d and P_{fa} values, averaged over 10^6 iterations.

Figure 1 shows the complementary ROC curve for ED-based spectrum sensing over TWDP fading for different K and Δ values with a target P_{fa} of 0.1, an average SNR of 15 dB and a time-bandwidth product, $u = 1$ [29]. The complementary ROC curves for Rician and Rayleigh fading channels are provided to enable comparison between the derived expression and the corresponding expressions from [7].

The good match between these curves and between the simulation and numerical results validate the flexibility of the derived expression to account for Rician and Rayleigh fading. With reference to Figure 1, it can be seen that as the value of K increases, the ROC curve moves within the hyper-Rayleigh region towards the upper bound of two-ray fading ($K = \infty$, $\Delta = 1$). These results suggest that inferior detection performance is achieved when compared to the Rayleigh fading as a result of cancellation of two anti-phase specular waves and the low power of diffused components. Indicatively, for a $P_{md} = 0.1$, i.e., $P_d = 0.9$ the corresponding P_{fa} is 0.01, 0.03 and 0.27, under Rician, Rayleigh and hyper-Rayleigh fading, respectively. Thus, the required criterion $P_{fa} \leq 0.1$ is not met over "hyper-Rayleigh" fading conditions, which in turn may lead to low spectral utilization.

Figure 2 describes the sensing performance for severe fading scenarios where the diffuse components are of equal power whereas the total power of the specular waves is at least ten times that of the diffuse component. Under such fading conditions, as the value K increases, the product of K and Δ becomes large and the TWDP PDF becomes bimodal, thus exhibiting two maxima [20]. This in turn results in the order change that is observed in the $\bar{\gamma}$ versus \bar{P}_{dTWDP} curves at an SNR of 15 dB. The curve for the extreme case of $K = \infty$ and $\Delta = 1$ is also provided as the upper-bound of hyper-Rayleigh fading. These results suggest that the average detection performance deteriorates as K increases, (the PDF moves from Rician to two-wave fading). More specifically, for $\Delta = 1$, $K = 10$ dB, a minimum SNR of 18 dB is required to achieve a $P_d = 0.9$, while for $\Delta = 1$, $K = 20$ dB and $\Delta = 1$, $K = 30$ dB a minimum SNR of 21 dB and

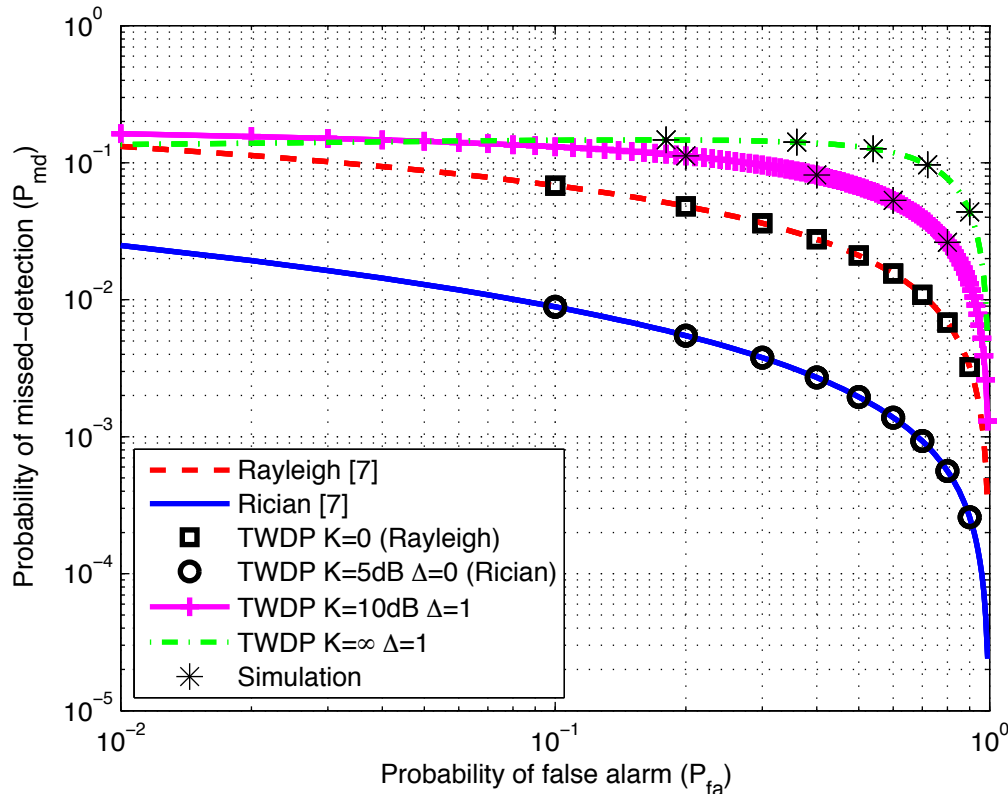


Figure 1. Complementary ROC curve for ED-based spectrum sensing over TWDP fading for $\bar{\gamma} = 15$ dB, $u = 1$, and different K and Δ values.

23 dB is required, respectively. On the other hand, ED-based spectrum sensing over Rayleigh requires an SNR of 13 dB. According to these SNR figures, it can be deduced that the SNR requirements for $P_d = 0.9$ are increased by up to 77% which would significantly affect the energy efficiency of energy-constrained CR nodes.

Figure 3 shows the ROC curve for ED-based spectrum sensing with up to six cooperative CR users over a TWDP fading channel with $K = 10$ dB and $\Delta = 1$ (i.e., Scenario 3) for an average SNR of $\bar{\gamma} = 5$ dB. It can be seen that the detection performance of the ED-based scheme improves substantially as the number of cooperative CR users increases. The Rayleigh curve for single-user detection is also provided for comparison. For this fading scenario, for a target probability of false alarm $P_{fa} = 0.1$, the probability of detection for six cooperative users is approximately 50% larger than for single user spectrum sensing.

Figure 4 shows the detection performance for and SLS diversity scheme with up to five branches. The average SNR for each branch is set to $\bar{\gamma}_1 = 5$ dB, $\bar{\gamma}_2 = 1$ dB, $\bar{\gamma}_3 = 2$ dB, $\bar{\gamma}_4 = 3$ dB and $\bar{\gamma}_5 = 4$ dB. With reference to Figure 4, it can be seen that as the number of diversity branches increases the detection performance improves as the average probability of detection increases by up to 38% for $L = 5$. More specifically, for a target probability of false alarm $P_{fa} = 0.1$, the probability of detection for $L = 5$ is

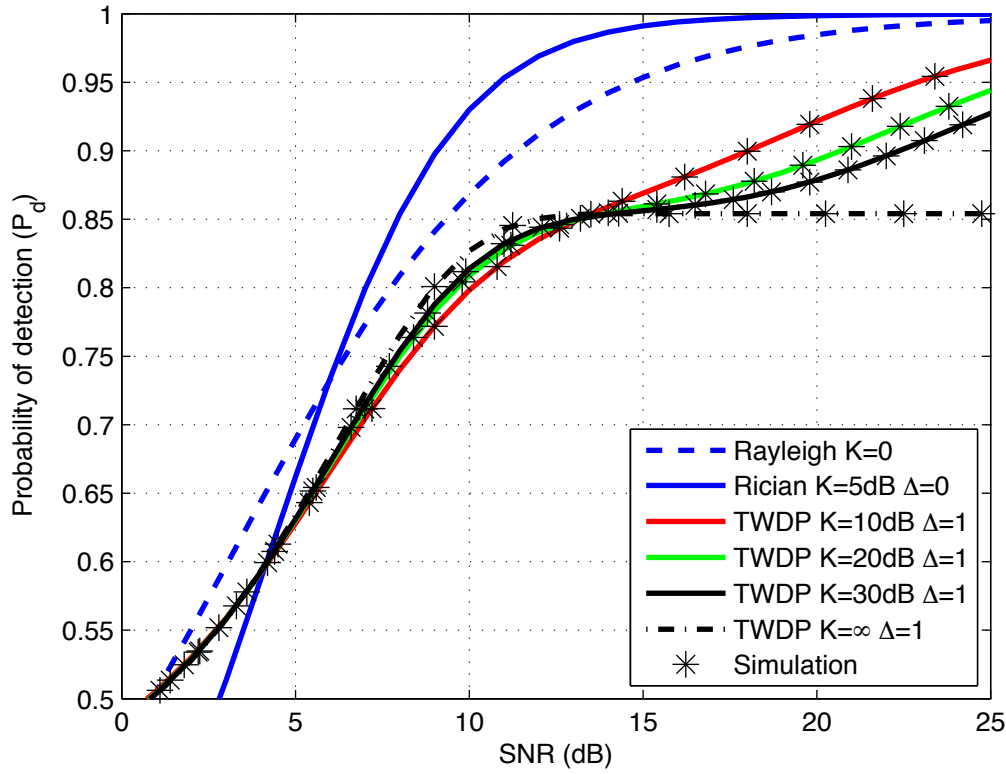


Figure 2. $\bar{P}_{d_{TWDP}}$ versus SNR for different K values, $\Delta = 1$ and $u = 1$.

85% larger than the corresponding value for $L = 1$. The highest diversity gain is observed for between the no diversity case to the dual branch ($L = 2$) scheme, where an increase of 21% is observed on the average probability of detection.

Signal detection in negative SNR regions is a major challenge of spectrum sensing since ED cannot distinguish between a deeply faded primary signal and actual noise due to noise uncertainty. However, as shown in Figure 5 the detection performance can improve as the number of independent signal samples within an observation time interval increases. Furthermore, in Figure 6 a performance analysis of ED-based spectrum sensing over TWDP in negative SNR regions is presented. It is shown that a gain of up to 16 dB can be achieved as u increases for a target performance of $P_d = 0.9$ and $P_{fa} = 0.1$ for an SNR of -22 dB. However, it is worth noting that such time-bandwidth values may not satisfy the real-time requirements. Nevertheless, they provide an insight into the detection performance of ED-based spectrum sensing over TWDP fading channels for low SNR regions.

Figure 7 shows how the detection performance varies with the average SNR over a TWDP fading channel with $K = 10$ dB $\Delta = 1$ under noise uncertainty. It is shown that the detection performance deteriorates as noise uncertainty increases. Indicatively, in order for ED-based spectrum sensing with

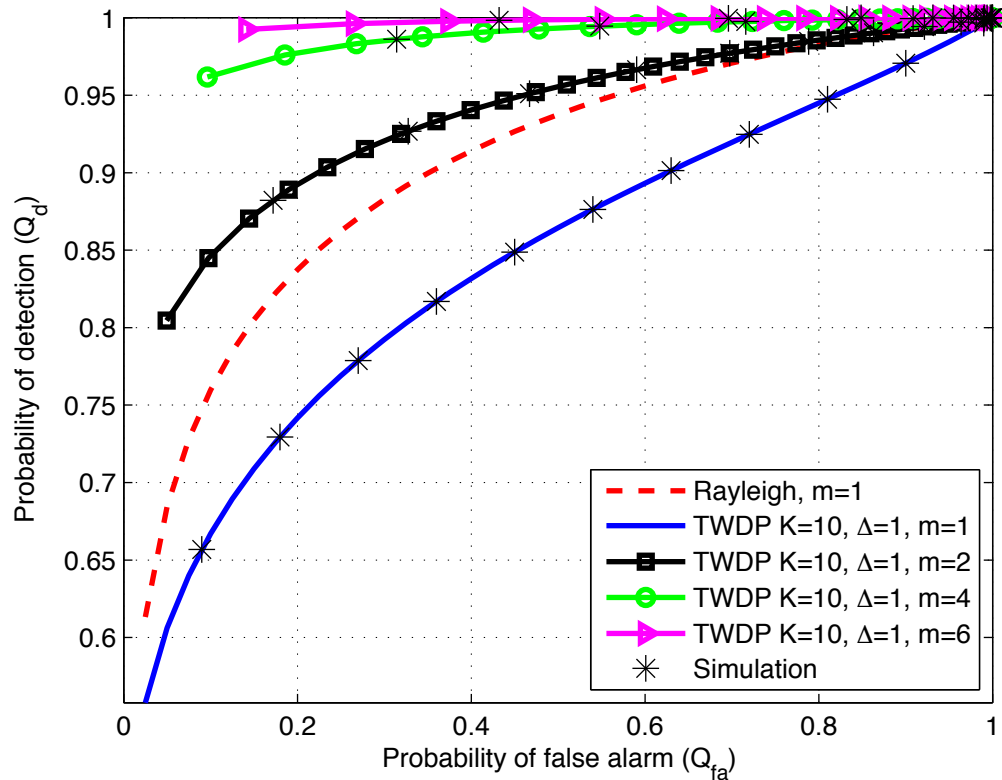


Figure 3. ROC curves for cooperative ED-based spectrum sensing over TWDP fading for different number of cooperative users with, $\bar{\gamma} = 5$ dB $u = 2$, $K = 10$ dB and $\Delta = 1$.

$P_{fa} = 0.1$ and $u = 10$ to achieve $P_d = 0.9$ a 5 dB SNR difference is observed from $a = 0$ dB, i.e., perfect noise estimation, to $a = 2$ dB.

V. CONCLUSION

This article studied the performance of ED-based spectrum sensing under moderate and severe fading scenarios modeled by the family of TWDP distributions. An novel analytic expression for the average probability of detection over TWDP fading channels is derived, which also encompasses Rayleigh and Rician fading as special cases. This expression has been extended to the case of SLS diversity reception scheme as well as to the case for when cooperative spectrum sensing is employed. These expressions allowed us to quantify the effects of fading in the performance of ED-based spectrum sensing for CR enabled applications that could not be adequately characterized by traditional fading models. Our results demonstrate that under such fading conditions, 46% and 137% higher SNR is required to achieve the same detection performance as in Rician and Rayleigh fading respectively. To this end, it is demonstrated that diversity reception and cooperative detection can significantly improve the sensing performance in severe fading conditions as the number of users or diversity branches increase. Our results provide a

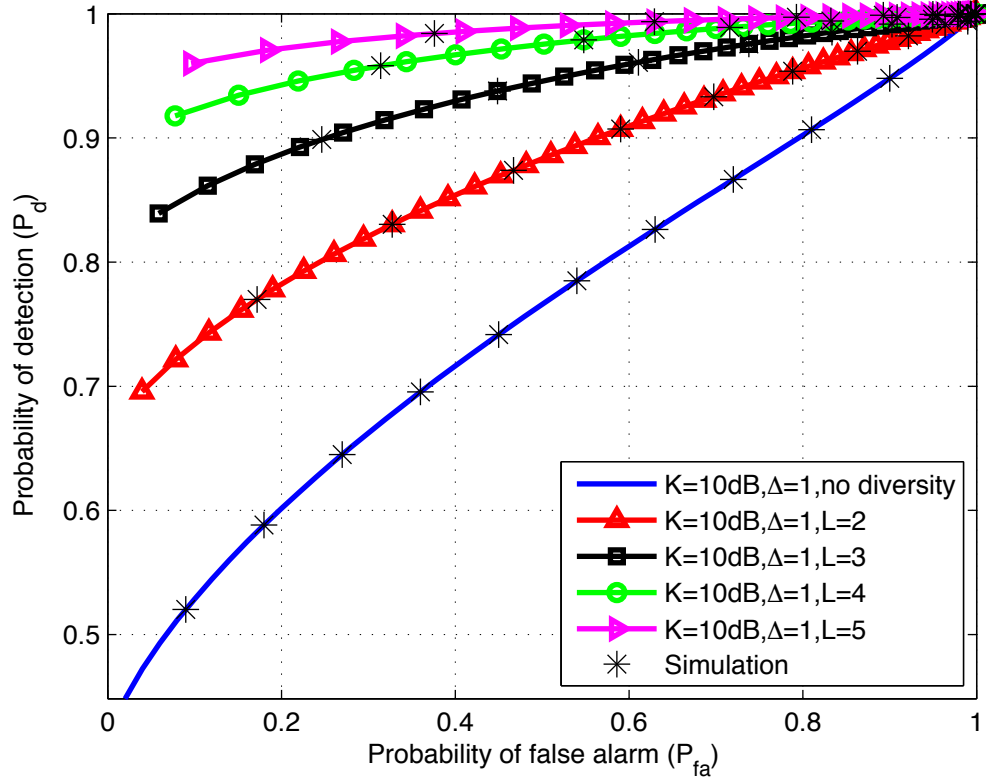


Figure 4. ROC curves for ED-based spectrum sensing with SLS diversity reception over TWDP fading for $K = 10$ dB, $\Delta = 1$ and $\bar{\gamma}_1 = 0$ dB, $\bar{\gamma}_2 = 1$ dB, $\bar{\gamma}_3 = 2$ dB, $\bar{\gamma}_4 = 4$ dB.

comprehensive performance evaluation of ED-based spectrum sensing for cognitive radio systems in more severe fading scenarios than Rayleigh. This may lead to improving the overall performance of cognitive radio communication systems in confined environments such as in-vehicular communications networks. Furthermore, it is shown that ED-based spectrum sensing requires a very large time-bandwidth product for robust sensing in low SNR regions over TWDP. Therefore, future research will focus on deriving a closed-form expression of reduced complexity for the case of large time-bandwidth product values by using appropriate expansions of the Bessel function for the limiting case $u \rightarrow \infty$.

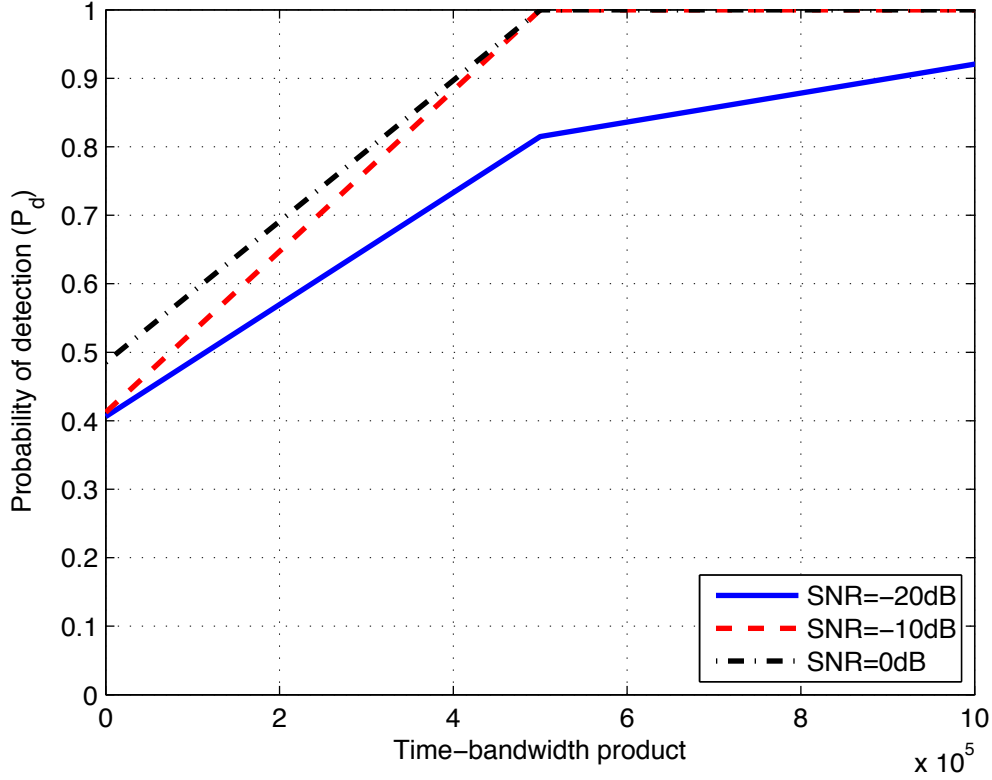


Figure 5. $\bar{P}_{d_{TWDP}}$ versus u over TWDP with $K = 10$ dB $\Delta = 1$ for $P_{fa} = 0.1$ for different SNR regimes.

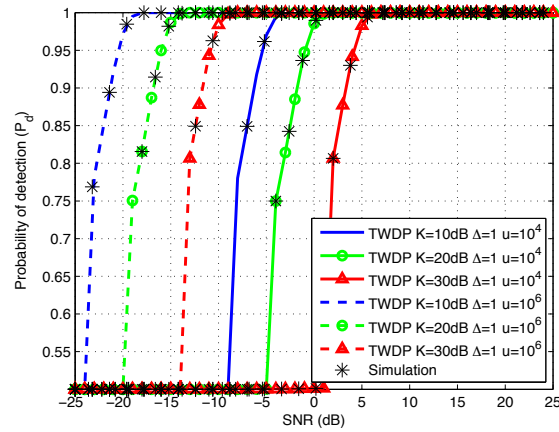


Figure 6. $\bar{P}_{d_{TWDP}}$ versus SNR for negative SNR regions with $P_{fa} = 0.1$ and different time-bandwidth values.

APPENDIX A

DERIVATION OF (15)

After expanding (15) it can be rewritten as,

$$\bar{P}_{d_{TWDP}} = e^{-\frac{\lambda}{2}} \sum_{l=0}^{u-1} \frac{\left(\frac{\lambda}{2}\right)^l}{l!} + e^{-\frac{\lambda}{2}} \sum_{n=u}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^n}{n!}$$

$$\left(1 - \frac{(K+1)e^{-K}}{2\bar{\gamma}} \sum_{k=0}^{n-u} \sum_{i=1}^M \frac{1}{k!} \right) e^{a_i K} \quad (\text{A.1})$$

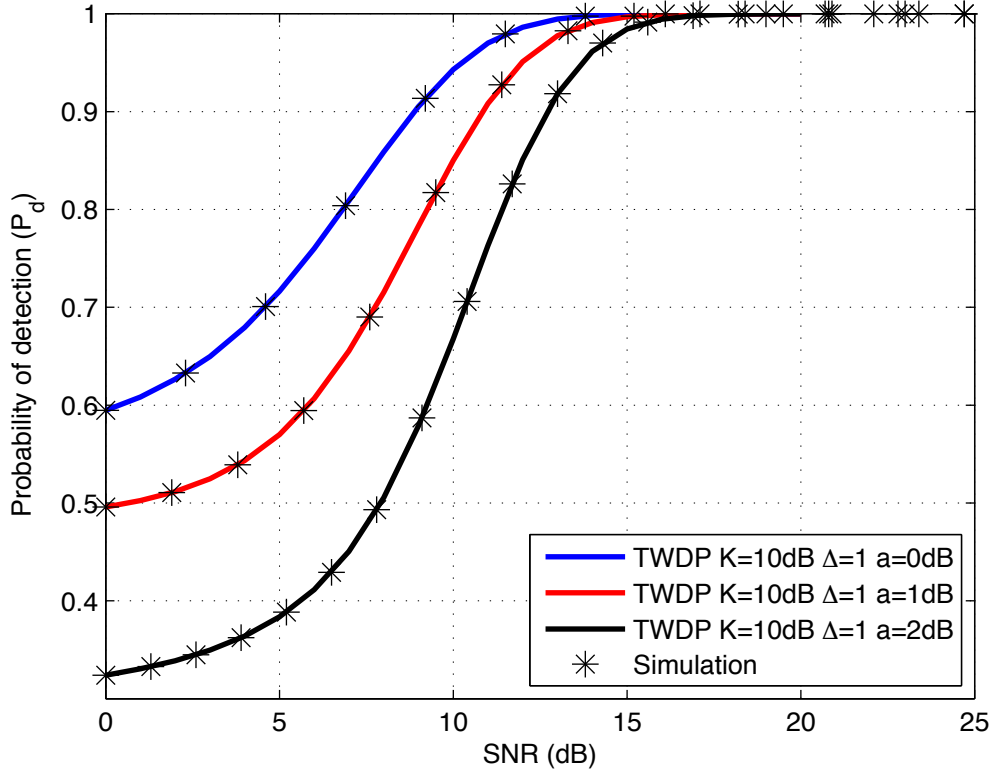


Figure 7. $\bar{P}_{d_{TWDP}}$ versus SNR over TWDP fading with $K = 10$ dB $\Delta = 1$ and noise uncertainty for $P_{fa} = 0.1$ and $u = 10$.

With reference to (9) and (10), I_1 and I_2 are given as,

$$I_1 = \int_0^\infty \gamma^k e^{-\frac{K+\bar{\gamma}+1}{\bar{\gamma}}\gamma} I_0 \left(2\sqrt{\frac{K(K+1)(1-a_i)\gamma}{\bar{\gamma}}} \right) d\gamma \quad (\text{A.2})$$

$$I_2 = \int_0^\infty \gamma^k e^{-\frac{K+\bar{\gamma}+1}{\bar{\gamma}}\gamma} I_0 \left(2\sqrt{\frac{K(K+1)(1+a_i)\gamma}{\bar{\gamma}}} \right) d\gamma \quad (\text{A.3})$$

Based on [24, eq. (6.643-2)] for $\mu = k + \frac{1}{2}$, $\alpha = \frac{K+\bar{\gamma}+1}{\bar{\gamma}}$, $\beta = \sqrt{\frac{K(K+1)(1\pm a_i)}{\bar{\gamma}}}$, and $\nu = 0$ (A.2) and (A.3) can be expressed as,

$$I_1 = \frac{\Gamma(k+1)}{\Gamma(1)} \left(\sqrt{\frac{K(K+1)(1-a_i)}{\bar{\gamma}}} \right)^{-1} e^{-\frac{K(K+1)(1-a_i)}{2(K+\bar{\gamma}+1)}} \times \left(\frac{K+\bar{\gamma}+1}{\bar{\gamma}} \right)^{-(k+\frac{1}{2})} M_{-(k+\frac{1}{2}),0} \left(\frac{K(K+1)(1-a_i)}{K+\bar{\gamma}+1} \right) \quad (\text{A.4})$$

$$I_2 = \frac{\Gamma(k+1)}{\Gamma(1)} \left(\sqrt{\frac{K(K+1)(1+a_i)}{\bar{\gamma}}} \right)^{-1} e^{-\frac{K(K+1)(1+a_i)}{2(K+\bar{\gamma}+1)}} \times \left(\frac{K+\bar{\gamma}+1}{\bar{\gamma}} \right)^{-(k+\frac{1}{2})} M_{-(k+\frac{1}{2}),0} \left(\frac{K(K+1)(1+a_i)}{K+\bar{\gamma}+1} \right) \quad (\text{A.5})$$

where M is the Whittaker function [24].

Given the Whittaker function identity [24, eq. (9.220-2)] yields,

$$M_{-\left(k+\frac{1}{2}\right),0}\left(\frac{K(K+1)(1-a_i)}{K+\bar{\gamma}+1}\right) = \left(\frac{K(K+1)(1-a_i)}{K+\bar{\gamma}+1}\right)^{\frac{1}{2}} e^{-\frac{K(K+1)(1-a_i)}{K+\bar{\gamma}+1}} \Phi\left(k+1;1;\frac{K(K+1)(1-a_i)}{K+\bar{\gamma}+1}\right) \quad (\text{A.6})$$

$$M_{-\left(k+\frac{1}{2}\right),0}\left(\frac{K(K+1)(1+a_i)}{K+\bar{\gamma}+1}\right) = \left(\frac{K(K+1)(1+a_i)}{K+\bar{\gamma}+1}\right)^{\frac{1}{2}} e^{-\frac{K(K+1)(1+a_i)}{K+\bar{\gamma}+1}} \Phi\left(k+1;1;\frac{K(K+1)(1+a_i)}{K+\bar{\gamma}+1}\right) \quad (\text{A.7})$$

By substituting (A.6) and (A.7) into (A.4) and (A.5) and taking into account that $\Gamma(k+1) = k!$ and $\Gamma(1) = 1$ the solution to (A.1) results in (15).

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