Big Effects of a Little Sector: The Structural Effects of Venture Capital on the Macroeconomy

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Contents

Abstract

We explore certain structural elements of venture capital investment, focusing on the role of venture capital as an asset class dedicated to technology investment. The structural role of technology as contributing to the total factor productivity is captured through the use of endogenous growth mechanisms as found in Romer (1990) and Rivera-Batiz and Romer (1991). In the first chapter, we explain certain elements of the two recessions in the first decade of the 21st century by combining these endogenous growth mechanisms with a financial accelerator in the market for production capital to capture the financial elements associated with decreased leverage after a financial crisis. In the second chapter, we assess the impact of policies in the late 1970s which largely created venture capital by encouraging technology investment to occur through debt contracts rather than equity contracts. We explain a set of stylized facts by contrasting a debt mechanism and an equity mechanism for an asset that derives its value from returns to technology goods in a stochastic endogenous growth model. Our final chapter deals with the disposition of venture capitalists towards Knightian uncertainty. We show that an uncertainty-loving behavior of venture capitalists leads to a Pareto improvement in the economy. However, the magnitude of the effect of changes in disposition towards uncertainty is small, implying that bubbles in the venture capital market caused by this type of uncertainty-loving behavior should not be a great concern for investors and policy makers.

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CHAPTER 1

Introduction

1. Introduction

The entirety of this thesis rests on a single premise: venture capitalists, through their role in investing mainly in unproven technology companies, have a significant structural impact on the economy. This premise should not be controversial—technology plays a very important role in the level of overall output in our economy. Indeed, mainstream analyses in both short-term business cycle fluctuations and longer-term growth rates rely strongly on the development of technology in both real business cycle (RBC) theory's reliance on technological fluctuations as a source of randomness and the focus of endogenous growth literature on technological development, respectively. Any sector or policy which impacts the development of technology could therefore potentially impact the progression of aggregate production rates in both the short term and the longer term. Venture capital is such a sector.

Venture capital firms serve as a financial intermediary focused on small technology-based firms. The value of such firms is usually difficult to define, as it is usually based heavily in intellectual property or market position in an unproven market. The potential profit from the monopolistic rents from patent rights, trade secrets, or first-mover advantage incentivize investment in this area.

The economics of venture capital is an active research field today. The economic literature on venture capital, however, is relatively young and underdeveloped. A good survey is Da Rin et al. (2011). Mostly, existing research in the area is done in a few targeted areas. First, empirical research on the returns to venture capital firms receives much of the attention of the financial community. Theoretically, the focus is on the microeconomic incentives faced by venture capitalists and the ability of venture capitalists to design mechanisms to deal with problems of moral hazard and information constraints.[1] Largely, the theoretical role of venture capitalists in the

[1]See Kaplan and Stramberg (2001), for example

macroeconomy is widely neglected, though empirical studies of the relation between venture capital and economic growth suggest a positive relationship.[2] This work deviates from that literature by abstracting away from describing a venture capitalist's role in overcoming issues with a particular asset class and instead considers the effect of this investment behavior on the progression of aggregate economic variables.

As an asset class based on technology investment, venture capital is a natural point of reconciliation between two complementary yet divergent areas in economics. Rational, incentivized investors allocating resources to the development of new technology is the focus of "new endogenous growth" theory.[3] This literature directly assesses the consequences of economic actions made by incentivized agents on aggregate growth rates of production. However, it largely ignores stochastic fluctuations and considerations of risk by the same rational agents, as well as many behavioral and short-term considerations by economic agents. Alternatively, real business cycle analysis, strongly based on the neo-classical growth model, treats these short-term considerations rigorously. Although technology plays a large role throughout this literature, its actual level and progression are generally treated as exogenous, and the analysis is firmly focused on analyzing the secondary effects of technology changes as they fluctuate through the economy.

Venture capitalists, while concerned with risk, uncertainty, moral hazard, and short-term price changes in the business cycle, also interact strongly in the overall growth rate of the economy. Thus, their short-term considerations are not independent of the growth rate. This interdependence should be viewed as a key transmission

[2]For example, Kortum and Lerner (2000) find in an empirical study that venture capital investment is positively correlated with aggregate indicators of innovation. They even go so far as to say that the RBC literature should be used in future exploration of the topic.
[3]Examples include Romer (1990), Rivera-Batiz and Romer (1991), and Jones (1995). For a complete overview of the literature on new endogenous growth theory, see Acemoglu (2008).

mechanism between the short-term economic fluctuations and longer-term economic growth. This thesis focuses on such a transmission mechanism.

The scope of this thesis, however, is more narrow than describing business cycles in the context of incentivized growth. We are directly concerned with specific behaviors of venture capitalists, and the impacts those behaviors have on the overall economy, given that venture capitalists are both incentivized by economic conditions and contribute to aggregate economic growth. That is, given the structural role venture capitalists have on the economy, we are concerned with the effects of their idiosyncratic behavior. Specifically, we assess how (1) aggregate credit cycles interact with this asset class through its focus on technology; (2) the asset structure used by venture capitalists in their investments affects aggregate variables; and (3) venture capitalists deal with the unknown potential of their investments, given that the markets and products in which they invest are often unproven.

The next section describes further the background leading to the research conducted in this thesis. Specifically, it describes venture capital in a fuller context, and it gives an overview on related literature. The following section describes the specific rationale for each of the three chapters and how those chapters fit together. We conclude with a discussion on some considerations specific to studying DSGE models with endogenous technological growth.

2. Background

For the most part, the endogenous growth literature and DSGE literature run as independent strands. Despite titles such as "Entrepreneurs, Moral Hazard, and Endogenous Growth," Reiss and Weinert (2005) and Fernandez-Villaverde et al. (2003) follow the RBC growth analysis described above rather than addressing any R&D type growth analysis. One brilliant exception, however, Azariadis and Chakraborty (1999), adds a Carlstrom and Fuerst (1997) type credit constraint to a model with stochastic constant returns to scale. Under certain conditions, the result is similar to that of an R&D model with complementarity between certain types of goods, as in Evans et al. (1998). Azariadis and Chakraborty show the possibility of multiple equilibria and sunspots in a dynamic endogenous growth model, which could be a potential topic for further research with my models. While interesting, this result takes the analysis in a different direction than this thesis.

While a combination of the two strands of literature is not widespread, it is not entirely without precedent. Kortum (1997) uses a simulation to assess the success of his endogenous growth model in capturing certain stylized facts. In his 1991 "Lab-Equipment" paper,[4] Paul Romer states his intention of creating a model that can eventually be used in calibration. Furthermore, a range of literature discusses the dynamic implications of endogenous growth models in a theoretical framework.[5] However, these models deal with the determinacy of the growth path, and stop short of discussing what the transition dynamics actually look like. One particularly useful preliminary to our specification is Jordi Gali's paper on the possibilities of "endogenous markups" on the interest rate due to market power in technological development, drawing specifically on Romer's reliance on the monopoly power of ideas producers.[6]

Meanwhile, there exists a wide literature studying the growth implications of the neo-classical RBC framework, starting with the well-known paper by Mankiw, Romer, and Weil.[7] In his formulation of an endogenous growth model, Rebelo (1991) avoids dealing with the non-rivalry of ideas, and, as a result, creates a model much more conducive to use in neo-classically based RBC models. For one, this approach does

[4]Rivera-Batiz and Romer (1991)

[5]See Arnold (2001), Benhabib et al. (1994) and Xie (1994)

[6]Gali (1994)

[7]Mankiw et al. (1992). Another interesting example of this literature (and a good overview of the other pieces) can be found in Chari et al. (1996) .

not require any market power, which would appeal to freshwater economists.[8] Antonio Fatas[9] shows that a simple endogenous AK model out-performs the RBC model in many respects, and argues that the endogenous growth type models are more intuitively pleasing for explaining the technology process in dynamic general equilibrium models. His following claim resets at the heart of the project at hand:

> Understanding all the implications that the endogeneity of growth has for business cycles is an open area for future research. One could imagine that, in these models, the transmission of shocks can lead to economic fluctuations that are quite different from the ones generated by a model where growth is treated as exogenous. The dynamics are possibly richer and could account for some of the empirical observations that are currently unexplained by business cycle models.

3. Organization

This thesis consists of three chapters. The first two chapters deal with the role of credit constraints and net worth on venture capital activity in the short and medium-term. We first discuss venture capital in an environment in which capital used in production is constrained. In addition to the obvious implications in the context of the recent financial crisis of 2008, understanding the effect of capital constraints in a framework containing the development of new technology is pertinent to the question of welfare costs of recessions.

The next chapter continues with the theme of credit constraints in the context of venture capital. However, the focus is now on the venture capital contract itself. The question posed is a specific one: how does the type of contract used by venture capitalists affect the macroeconomic fluctuations of the economy? We focus on a

[8] See Jones et al. (2005) for a recent example of simulations of this type of endogenous growth model.
[9] Fatas (2000)

specific natural experiment in which government policies actively encouraged shifting investment in new technology from debt contracts to equity contracts.

From the two chapters on debt and net-worth, we shift focus from the financial constraints to the more fundamental uncertainty surrounding the valuations of the new technologies.

The shift between credit constraints and uncertainty is not as stark as it might seem at first glance. The underlying motivation for the necessity of a debt contract is its efficacy in dealing with informational asymmetries between a lender and an entrepreneur. The consideration of debt contracts and the behavior of investors in the face of uncertainty rely on the intuition that information limitations are crucial in the venture capital industry.

The final chapter of this thesis deals widely with the implications of uncertainty in technology projects themselves. Specifically, we consider the possibility that venture capitalists might be "optimistic" in the face of uncertainty. Given the market imperfections central to the new endogenous growth framework, such optimistic behavior has the potential to be welfare-increasing. We show that this is indeed the case, as optimistic behavior leads to a Pareto improvement. In addition, we show that such behavior also has the benefit of increasing the utility of the venture capitalist, suggesting such behavior would be desirable if the venture capitalist could choose it.

4. Techniques used throughout this thesis

The three chapters in this thesis are similar in their use of endogenous growth theory to describe the venture capital industry. However, in contrast to the bulk of literature on new endogenous growth theory, we are concerned with dynamics at business cycle levels. In order to apply the new endogenous growth theory, there are two techniques used throughout this thesis which apply to the use of de-trending and the treatment of technology pricing. First, the way we treat non-stationary variables in our economy requires some explanation, given that the growth rate of the variables are no longer exogenous. Second, the treatment of pricing behavior for technological assets (patents) differs from that of Romer (1990),[10] and allows us to treat the model in a dynamic-stochastic general equilibrium setting.

4.1. De-trending. We follow King et al. (1988) in transforming our variables to ones that will be constant on a balanced growth path. Since consumption, capital, and output are all growing at the same rate as technology in the long run, this transformation is done simply by dividing these variables by the current level of technology. As a result, all non-stationary variables with a dynamic component contain the endogenous growth rate as a multiplier on all instances of the variable other than in the earliest period they are observed.

For example, consider a non-stationary variable, x_t. All occurrences of this variable are de-trended by dividing by the technology level, denoted by a tilde: $\tilde{x}_t \equiv \frac{x_t}{A_t}$. As we assign the technology level in the current period to the earliest occurrence of any trending variable, any occurrences of the variable in later periods will be multiplied by the growth rate: $\Xi_t \equiv \frac{A_{t+1}}{A_t}$. This gives us, $\frac{x_{t+1}}{A_t} = \frac{A_{t+1}}{A_t}\frac{x_{t+1}}{A_{t+1}} = \Xi_t \tilde{x}_{t+1}$.

This act of de-trending shows one of the major benefits of using a Cobb-Douglas formulation for our production function.[11] As we view labor as a stationary process already, the homogeneity of the production function in capital and technology taken together allows the de-trended form of the production function to appear identical to the trending one, absent any reference to the technology level. In fact, if the model itself is stationary, the technology level itself does not enter the model at all, as it

[10]When we refer to a "Romer" model without any citation in this thesis, the model in question is that from Romer (1990)

[11]As we will see in the next chapter, our production function actually contains a Dixit-Stiglitz multiplier, but this reduces to a Cobb-Douglas production function through an assumption of symmetry over intermediate capital goods.

would be a non-stationary variable in a stationary model. Instead, the entire impact of the endogenous technological change can be expressed through the technological growth rate. This deserves a word of caution: as the technological growth rate is used as a multiplier on latter period trending variables, an increase in the rate of growth will often lead to a decrease in earlier periods of such variables, despite the level of such variables increasing due to the shock. It is often necessary to consider the joint effects of the change in technology and the change in the trending variables together.

4.2. Forward-looking technology prices. Throughout this thesis we break from Romer's treatment of endogenous technological change in our treatment of the price of technology. For simplicity, Romer assumes a constant price of technology over time. For us, the price of technology is a crucial variable in the arbitrage between investing in capital for the production sector and investing in patents to fund technological projects.

We follow Romer in assuming that the price of a patent derives its value from the discounted income stream of future profit stream from the monopoly represented by the patent. However, rather than treating this asset in isolation, we incorporate it into our budget constraint:

$$k_{t+1} + P_t t_{t+1} = (1 - \delta + R_t)k_t + (P_t + \Pi_t)t_t + W_t l - c_t$$

This allows us to determine the progression of patent prices through a forward-looking Euler equation:

$$u'(c_t) = \beta u'(c_{t+1})(P_{t+1} + \Pi_{t+1})$$

The determination of the price of patents through a forward-looking Euler equation is important for a variety of reasons. First, it technically allows prices to deviate from the rate it would be on the balanced growth path. The Romer model, in contrast, pins down price to the balanced growth path, which effectively destroys the possibility for any transition dynamics away from steady-state. Allowing the price to adjust over time to a long run rate, rather than jumping to the rate immediately allows for our model to address short-term and medium-term fluctuations. Second, it allows for a richer interaction between capital and technology through arbitrage. In this case, the arbitrage requires expectations over both assets, requiring the household to consider correlations between consumption and patent prices in its deliberation. Finally, the assumption makes practical sense, as the time horizons for venture capital are considerably longer than for most other asset classes, requiring venture capitalists to consider the impacts of future events on their valuations of future assets. The value of a project is determined by the “exit,” in which the investor’s share is bought by another investor, either through private acquisition or by an initial public offering. These exit opportunities are a major consideration in the determination of valuations of companies at the time of investment. For example, if a company expects its exit to be through a public offering, the expected behavior of markets for such offerings is crucial in the determination of the initial offer value.

CHAPTER 2

Venture Capital in a Credit Crunch: A Structural Analysis

Abstract

We explain a series of stylized facts regarding the two recessions in the first decade of the 21st century, incorporating the venture capital industry through its role as an investor in technology. As with other chapters, we capture the technological focus of venture capital using an endogenous growth model. In this chapter, our endogenous growth mechanism is similar to the one found in Romer (1990), and we combine this with a financial accelerator in the market for production capital to capture the effects of leverage and financial constraints. We find that using this combined model, we can explain key facts about each recession through a technology shock in the 2001 recession and a shock to borrowers' net-worth in the 2008 recession.

1. Introduction

In the context of real business cycle analyses, in which fluctuations in aggregate production are largely attributed to shifts in the technological growth rate, the recent "great recession" of 2008 is an anomaly. In contrast to the downturn of 2001, the predominant explanation was not one of negative shock in the technology sector. Instead, the story is one of unstable debt and a sudden contraction of credit when banks tried to readjust to their seemingly sudden discovery that this debt was untenable.[1]

As an industry, venture capital fared relatively well during the recent economic downturn. Compared to the previous recession, few venture capital firms went bankrupt, and the returns to venture capital firms deviated only slightly from their longer-term upward trend. In addition, the drop in the amount of money invested in any given year was not nearly as prominent as the crash of the dot-com bubble in 2001, and this amount recovered relatively quickly. More interestingly, if we view the venture capital industry as an asset class focussed on technology investment, the fluctuations in total factor productivity (TFP) growth break from their usual correlation with overall production during this period.

Looking at these claims in more detail, Figure 1 depicts the median internal rate of return (IRR) for the venture capital sector according to data reported by the National Venture Capital Association as well as the number of firms in the database in a given year.[2] Interestingly, this rate of return shows a trend increase over the 12 year span shown. While there is a discernible deviation from the trend in the internal rate of

[1] Literature on the causes of the recent recession is abundant. While the details of causes of the recession are still being debated, most explanations rely on some combination of both corporate and household debt that increased due to the heated housing market and on the sudden financial contraction in 2008. Examples include Krugman (2009) and Lewis (2011) on the more populist economics side, and Campello et al. (2010) and Chodorow-Reich (2014) on the academic side.

[2] National Venture Capital Association (2014). The smoothness of the chart is hardly surprising given the long-term nature of the investments—projects spanning multiple years are included in the rate of return for each of the years of the project, although the reporting makes an attempt to update the value of the projects themselves.

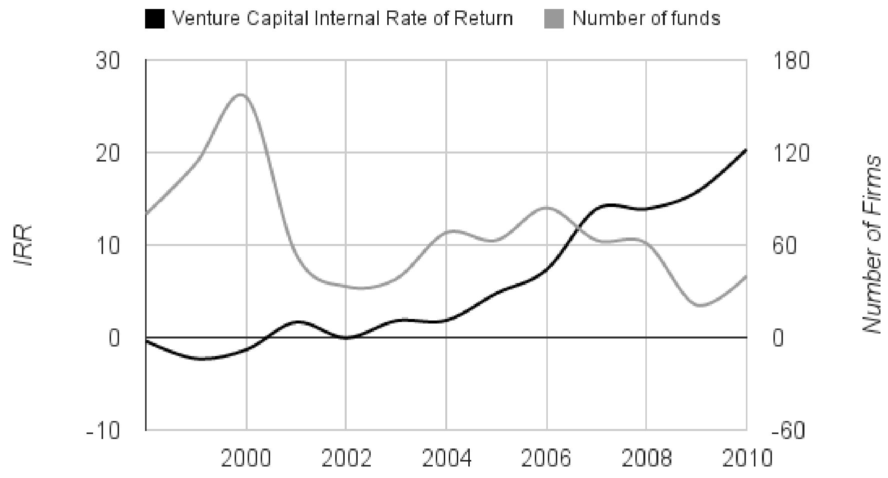


Figure 1. Median IRR and number of firms reported by the NVCA

return around the 2008 crisis, the rate never actually declines during this period, and the upward trend continues through the end of the period. In addition, the number of firms depicted in Figure 1 reduces moderately by 40 funds, compared to the precipitous decline in 2001 when 123 funds were lost.

The break in correlation between total factor productivity growth and output can be seen in Figure 2. This figure shows the annualized real quarterly productivity growth rates and utility-adjusted TFP growth rates.[3] One can see a strong co-movement between the two data sets until 2008. Indeed, from the beginning of the series until the beginning of 2008, the correlation coefficient between the two is 0.42. However, following the financial crisis, this correlation broke down; GDP in the final quarter of 2008 dropped to -8.2%, while adjusted TFP rose to 5.4% by the first quarter of 2009.

The purpose of this paper is to build a model of technology investment incorporating imperfect credit markets to describe the role of venture capital in a credit crunch. Most importantly, we show the break in correlation between TFP growth and growth

[3]Sources: International Monetary Fund World Data and Fernald (2014)

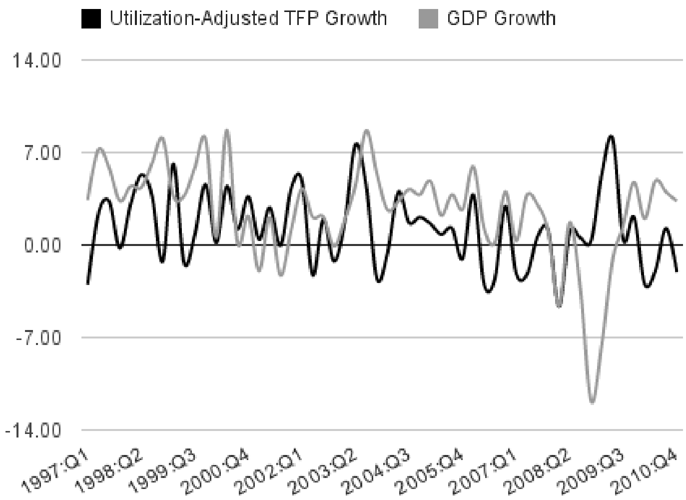


Figure 2. Quarterly growth in GDP and utilization-adjusted total factor productivity

in production can arise from a negative wealth shock to the financial sector, as we had in 2008 when financial institutions re-evaluated the mortgage-backed securities already on their books. In addition, our model replicates the differences in severity of the 2001 and 2008 recessions and their different effects on the venture capital sector. Specifically, our model corresponds with (1) the relatively moderate recession in 2001, (2) low economic growth in the periods following the 2001 recession, (3) a sharp increase in the price of capital in 2008-2009, and (4) a prolonged decline in both output and consumption following the 2008 financial crisis. Finally, comparing our models to reduced-form models without certain mechanisms to show that the inclusion of endogenous growth explains better the prolonged recession of 2008 and that the inclusion of a financial accelerator actually reduces the impact of a negative technology shock, corresponding to the mild recession of 2001.

We address the drivers of venture capital investment over the business cycle using a structural macroeconomic approach. This allows us to incorporate the underlying driver of value for venture capital, profits from patents, in a larger market that includes the return to capital dedicated to production. The structure of the economy is one with endogenous growth through human capital being devoted to R&D as modeled by Romer (1990). Use of this framework incorporates the focus of the venture capital industry on technology investments, a crucial aspect of the role of these investors. The use of such a highly stylized structural model allows for a full general equilibrium to be assessed.

In addition, we explore the possibility that the capital path is further complicated by the inclusion of agency costs in navigating the market for investment projects, where agency costs are modeled as in Carlstrom and Fuerst (1997). This arises through a process in which the household is beholden to an entrepreneur to invest his foregone consumption and assets in the production sector. The entrepreneur can extract rents from this process through an informational asymmetry. As a result, the entrepreneur accumulates assets of his own, leading to a second consumption and investment decision. The entrepreneur's spending decision leads to a second source of supply of capital, increasing the price the household must pay for that capital. Intuitively, this allows the overall level and value of capital in the market to fluctuate according to a different process from the household's optimization problem, reducing the ability for households to correct a capital imbalance. We refer to this as a "financial accelerator," as the lingering effects of net-worth of the entrepreneur create a protraction of a negative shock through terms of lending in the financial sector. In addition, the agency costs include a fixed monitoring cost, which further distorts the value of capital between periods.

This approach of combining an agency cost mechanism with an endogenous growth one is facilitated by the compatibility of the models. The endogenous growth model used in this paper allows for the growth of the economy to be determined by an endogenous allocation of resources to technological advancement. It does so, however, at the price of imperfect competition in the market for capital. As a result, the quantity of capital is reduced from the competitive level in order to provide compensation for the development of technology. This capital market behavior is central to determining the level of technology produced in a period, as it directly affects both the price paid to technology and the interest rate paid to the capital producers. In this basic formulation of the Romer R&D model, capital only enters the production process through the production of durable goods, so we can create a wedge between the household's supply of capital and the final goods producer's demand without worrying about any effects concerning the use of capital in the technology sector. Thus, we can add a basic agency cost with relatively little modification to the Carlstrom & Fuerst framework.

Combining the two mechanisms, endogenous growth and a financial accelerator, we are able to tell a story that differentiates the recent recession from its predecessor. Specifically, the existence of credit constraints in our capital market causes a breakdown in the normal arbitrage for our two assets. While holdings of technological assets progresses according to a standard Euler equation, the prevalence of debt in the production sector causes the relative price of capital to depend on "leverage," the ratio of our entrepreneurs' net-worth to the total amount of investment required in a project. This drives a wedge between the relative values of the two assets. In a negative productivity shock, the returns to both lenders and borrowers are affected, leading to a minimal change in leverage. This minimal change in leverage implies that the relative returns between the two sectors remains close to steady state. In contrast, a negative shock to the net-worth of borrowers, as we saw in our recent recession due to the sudden re-valuation of mortgage-backed derivatives, increases required leverage to maintain the previous level of investment. The borrowers react to this decrease in net-worth by increasing the price of capital and reducing the overall amount of capital borrowed. This has two related effects: (1) the change in price of capital in the production sector makes the the relative wage for labor in technology development higher, and (2) the reduction in capital borrowing causes more investors to turn to the technology sector in search of investment opportunities.

The discussion begins with an outline of the agency costs as they are modeled in this paper. The treatment of the standard debt contract in a dynamic setting is drawn from the work of Carlstrom and Fuerst (1997)(henceforth CF). We then incorporate this into a general equilibrium model with endogenous growth, as in Romer (1990) to assess the stability conditions under the restrictions imposed by agency costs from the debt contract. Using this combined model, we show how simulations under this model match the stylized facts mentioned above.

2. Related literature

While the literature on venture capital is vast, there are relatively few papers that incorporate any sort of macroeconomic structure in their explanation of the venture capital market. Keuschnigg (2004) is the closest to our purpose, incorporating the semi-endogenous growth of Romer (1987) with financial constraints. However, he stops short of a fully-endogenous growth model in which technology grows based on competitive investment in the technology process, as in Romer (1990). Furthermore, his financial constraints are different from ours, focusing on a double moral-hazard problem, which has less relevance to the recent fluctuations in economic and financial markets. Schwienbacher (2008) also incorporates considerations for economic growth in the technology focus of venture capital.

The impact of a financial accelerator has been assessed in several contexts. While we use the framework described in Carlstrom and Fuerst (1997), Bernanke and Gertler (1989) is arguably another seminal paper in this area, and Bernanke et al. (1999) is similarly prevalent for models incorporating a monetary mechanism. The only known effort similar to ours is Reiss and Weinert (2005) in that they incorporate explicit financial contracts into a model of endogenous growth. However, the focus of their paper is on the effects of inequality, and the financial contract is significantly different from ours. Similarly, Acemoglu and Zilibotti (1997) features an endogenous growth model in which the development of financial markets explains the differences in growth rates between countries. However, our purpose here is not to describe the development of a financial market, but to assess the impact of a known financial constraint in the context of medium-term risk.

3. Carlstrom and Fuerst's agency costs

We begin with a discussion of agency costs, using a formulation originally derived by Carlstrom and Fuerst, who put the financial accelerator of Bernanke and Gertler (1989) into a general equilibrium framework. The intuition is that borrowers' access to credit under a debt contract is limited by their net-worth. This net-worth fluctuates due to the fact that the borrower retains any returns to a project surplus to the pre-arranged repayment to the lender. The effect is that negative shocks to the economy will be protracted and propagate over a longer period of time. Bernanke has used this type of model to explain the breadth and depth of the Great Depression.[4]

This type of contract requires two types of heterogeneity. First, we distinguish between our standard household and a second type of agent, an entrepreneur. This terminology is a bit confusing in the context of a model with venture capital, where one would expect "entrepreneur" to refer to people creating new ideas. Instead, entrepreneurs here are simply financial intermediaries used for the creation of new production capital. The agency cost framework between lender and borrower is specific to the development of production capital in this model. The role of venture capitalists

[4]Bernanke (1983)

enters later, as dealers of patents. The intuition of this model is that it allows for an alternative asset in a credit contraction to contribute to incentivized technology development. As such, we abstract away from the specific behavioral elements of venture capitalists, such as syndication, moral hazard, and oversight.

We normalize the total size of the economy to 1, with η entrepreneurs and $(1-\eta)$ households. These entrepreneurs are financial intermediaries who identify investment opportunities in the production sector. Second, the entrepreneurs themselves are heterogeneous over their ability to identify investment opportunities for their clients, expressed through a privately known conversion rate of foregone consumption into capital goods, θ. This conversion rate is distributed randomly across entrepreneurs, i.i.d. across time and entrepreneurs, with a cumulative distribution of $\Phi(\theta)$, a density of $\phi(\theta)$, and a non-negative support. Each entrepreneur uses its signal to transform k_t units of foregone consumption into $k_t\theta$ units of capital, priced at q_t.

The entrepreneurs invest their own money, z_t, in these projects and borrow both consumption goods and capital from the household to augment their own net-worth to optimize the returns from the project.The overall size of the project is denoted, k_t. In equilibrium, we specify that $k_t > z_t > 0$, meaning that the entrepreneur's share is always strictly positive, but the entrepreneur can never completely fund the overall project himself.

Since the household does not know the amount of capital created by the project, it cannot claim a proportion of the project in line with its original contribution. Instead, it creates a contract with the entrepreneur, taking the form of the standard debt contract outlined by Townsend (1979). In effect, the lender is promised a fixed return on his contribution, which we index by the break-even project's signal, $\bar{\theta}_t$. If the entrepreneur claims not to be able to pay the fixed return, the household can claim the entire proceeds from the project by paying a monitoring cost μ to uncover the true return from the project. That is, the lender and entrepreneur create an incentive compatible scheme at the expense of a monitoring cost. This incentive compatibility and the specification that any windfall proceeds above the agreed rate go to the entrepreneur are sufficient to define a partial equilibrium that can be used to anchor "the price of capital," q_t.

We represent the proportion of the proceeds of a contract accruing to each agent through functions of the break-even signal. $f(\bar{\theta}_t)$ is the proportion of the returns to the capital project accruing to the entrepreneur, while $g(\bar{\theta}_t)$ accrues to the lender. If we follow CF by defining r^k as the interest rate accruing to the lender on the capital creation project (remember that all lenders will receive the same rate as they cannot observe the entrepreneurs cost without paying a monitoring cost), we can define the break-even signal as the one associated with this interest rate, where $\bar{\theta} \equiv (1 + r^k)(k_t - z_t)$. We follow CF in assuming that both entrepreneur and lender are risk-neutral with respect to intra-temporal considerations of the distribution of risk among projects.[5]

Defining the equilibrium through a break-even project rather than the interest rate allows us to write each agents' returns as a function of one variable:

$$\text{(entrepreneur)} \qquad f(\bar{\theta}_t) = \int_{\bar{\theta}}^{\infty} \theta\phi(\theta)d\theta - \left(1 - \Phi(\bar{\theta}_t)\right)\bar{\theta}_t$$

$$\text{(lender)} \qquad g(\bar{\theta}_t) = \int_0^{\bar{\theta}} \theta\phi(\theta)d\theta + (1 - \Phi(\bar{\theta}_t))\bar{\theta}_t - \Phi(\bar{\theta}_t)\mu$$

where we define $\int_0^{\infty} \theta\phi(\theta)d\theta \equiv 1$ to make the relationships above represent the proportion of capital independent of the level of capital generated. The right side of each of the two equations above begins with an integral. For the entrepreneur, this

[5] While we simplify the Carlstrom and Fuerst model by eliminating the risk-neutral financial intermediary in order to eliminate confusion, we appeal to the potential for including such middlemen as intuitive justification for allowing our households to be risk-neutral in this aspect while remaining risk-averse in their inter-temporal decisions.

represents "windfall" value, or the value of the returns from the contract above the cutoff rate which the entrepreneur keeps. For the lender, the expression under the integral represents the value the lender can recover in the case of default. The second term in each equation represents the probability of not defaulting multiplied by the interest payments by the entrepreneur from a successful project, $\bar{\theta}$. Finally, the lender pays a monitoring cost for the benefit of the payoff in the case of default. Due to the monitoring cost, these proportions do not sum up to one. Instead, we have $f(\bar{\theta}_t) + g(\bar{\theta}_t) = 1 - \mu\Phi(\bar{\theta}_t)$. This represents a net loss that increases proportionally to the value of $\Phi(\bar{\theta}_t)$, the cumulative probability of bankruptcy.

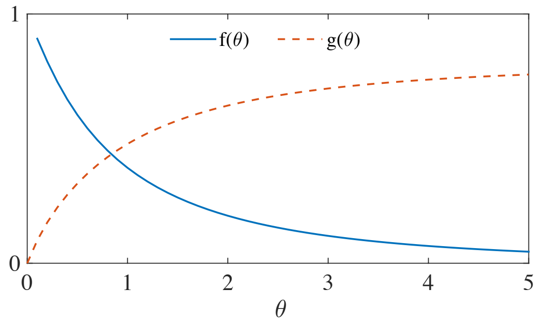


FIGURE 3. Proportion of capital going to the borrower ($f(\bar{\theta}_t)$) and the lender ($g(\bar{\theta}_t)$), as a function of $\bar{\theta}$

Figure 3 shows the distribution of proceeds accruing to each agent in our contract as a function of the break-even signal. Notice that the sensitivity to shifts in each agent's share is far greater towards the lower end of the distribution. The entrepreneur's ability to compensate for a change in net-worth by changing the allocation of returns from the project is much higher on this end of the probability distribution.

Since the entrepreneur has the private signal, we treat him as the optimizing agent, as he is in the unique position to manipulate the contract to benefit his superior information. He therefore maximizes his own return subject to a participation constraint for the lender:

$$\max_{k_t} q_t k_t f(\bar{\theta}_t)$$

$$\text{s.t. } q_t k_t g(\bar{\theta}_t) = k_t - n_t$$

where n_t is the entrepreneur's net-worth, consisting of his previous period's retained assets, z_t, and an endowment, x. We follow CF in allowing for a small arbitrary endowment to ensure that the entrepreneur's asset level is strictly positive. That is, the entrepreneur earns a small nominal amount at the beginning of each period in case his net-worth from the previous period is zero from bankruptcy.[6]

For an increase in household investment, the entrepreneur faces a decision of how best to pay back the household: by increasing the size of the contract, thereby reducing the price of contracts, or by increasing the proportion of the contract which will accrue to the household. The equilibrium conditions of the contract therefore are determined through the first order conditions of the entrepreneur's optimization of his share subject to a participation constraint of the lender. The first order condition and participation constraint, respectively, are:

$$1 = q_t \left(1 - \mu\Phi(\bar{\theta}_t) + \mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)}\right) \tag{1}$$

$$q_t k_t g(\bar{\theta}_t) = k_t - z_t \tag{2}$$

Conditions (1) and (2) completely describe the contract, $(q_t, \bar{\theta}_t)$, which depends entirely on the overall supply of foregone consumption to the project, and the proportion of that supply originating from the household.

[6]We differ from CF slightly by treating this as a capital endowment rather than a labor one in order not to disturb the labor markets in the model. We set it to 0.01 in our calibration. In addition, we calibrate the model and the shock to ensure that this quantity does not hit the boundary condition of $z_t = 0$ in our simulations.

The final element to the agency cost mechanism is the evolution of the entrepreneur's contribution to the project. Due to the entrepreneur's private signal, his proceeds from the contract will generally be positive. Thus, the household is no longer the only agent in the economy facing an intertemporal consumption decision. Despite arising from a first order condition of his utility function, the entrepreneur's Euler equation is markedly different from the household's, due to the different nature of his earnings and his preferences. Thus, we must first describe the path of the entrepreneur's assets in order to explain his intertemporal decisions.

For each period, an individual entrepreneur dedicates his own foregone consumption to an investment project. However, he only receives $q_t k_t f(\bar{\theta}_t)$ on average from this process, which we know from the participation constraint is equal to $\frac{z_t q_t f(\bar{\theta}_t)}{1-q_t g(\bar{\theta}_t)}$. In addition, we allow the entrepreneur to earn a market return from investing this capital in the production process and receiving the same market return as the household. Thus, at the end of the period, the entrepreneur has accumulated $\frac{z_t q_t f(\bar{\theta}_t)}{1-q_t g(\bar{\theta}_t)}\big(q_t(1-\delta)+r_t\big)$.

We can now describe the evolution of the entrepreneur's assets:

$$q_t z_{t+1} = (z_t + x)\left(q_t(1-\delta)+r_t\right)\left(\frac{q_t f(\bar{\theta}_t)}{1-q_t g(\bar{\theta}_t)}\right) - c_t^e \tag{3}$$

where z_t is the capital held by the entrepreneur at any given time, t.

Notice that the price of the contract enters twice on the right side of 3. The first occurrence is the value created by the project in a period. The household's equivalent does not have this occurrence of q_t, because the guarantee of receiving the value of its original loan from the participation constraint isolates it from the overall value of the project. However, both the household and entrepreneur treat the exclusive right to this project as a commodity in itself, which they can sell at any time for the market price of a project. This is represented by the term $q_t(1-\delta)$.

Finally, we follow CF in assuming a linear utility function, to isolate the path of q_t through the entrepreneur's Euler equation:

$$q_t = \beta\gamma \mathbf{E}_t(q_{t+1}(1-\delta) + r_{t+1}) \left(\frac{q_{t+1} f(\bar{\theta}_{t+1})}{1 - q_{t+1} g(\bar{\theta}_{t+1})} \right) \tag{4}$$

where γ is an extra impatience parameter. The existence of this parameter is necessary to keep the entrepreneur from saving his entire net-worth for as long as it takes for him to no longer require outside funding. It is set to a level that allows the entrepreneur's consumption to be stationary with respect to the long-term growth of household consumption and output growth.

This condition is derived using a utility function that is linear in entrepreneurial consumption, so that the actual level of consumption is left out of the first order condition. This allows us to consider the path of the capital price independently of the level of entrepreneurial consumption. That is, the intertemporal optimization of the entrepreneur is expressed entirely through evolution of the price of capital, which takes into account the terms of the contract.[7] This price of capital, in turn, feeds through to the contract and to the household's budget considerations.

Thus, our entrepreneur therefore has two behavioral differences from our standard household, both of which lead to less cautious behaviour. The existence of γ, the impatience parameter, causes the entrepreneur to consume more and save less than he otherwise would due to intertemporal preferences. While the linearity of utility also

[7]The linear utility function does not exhibit consumption smoothing, as the entrepreneur is indifferent between consuming one period and the next. However, the effect of increased consumption in one period for the entrepreneur has an effect on the price of capital, which enters into the household's consumption decision. An increase in entrepreneurial consumption in one period leads to a higher price of capital in the next periods, as the entrepreneur will have diminished his net-worth. As a result, his asset holdings will secure fewer consumption goods in the future. Thus, the household's consumption smoothing leads to implicit consumption smoothing for the entrepreneur. This is a further justification for the use of an entrepreneur with linear utility in consumption: consumption smoothing already exists in the model and feeds back to the entrepreneur.

increases the entrepreneur's consumption and reduces savings, the underlying reason is a lack of risk aversion.

4. Growth

We begin with a description of "producer variety," which serves as a microfoundation for the imperfect competition and increasing total factor productivity in the Romer model. It is here that venture capitalists enter the model. As mentioned above, the role of venture capitalists is essentially as middlemen distributing technological assets (patents), abstracting away from any industry-specific behavior. As such, we take the price of patents, p_t, as a general representation of the health of the sector, and the overall growth rate, Ξ_t, as the resulting technological activity arising from investment in this sector.

There are two sectors in this model—one produces consumption goods that bring utility to the household, and the other produces new ideas that can be used in future periods to expand the overall output of consumption goods. The general concept behind this model is that, despite a new idea being a public good in the sense that it contributes to the further formation of new ideas and intermediate goods of all developers of technology, households intentionally allocate resources to the development of new ideas because of market power for the immediate application of the idea once it has been created. For our purposes, we call the resource the household allocates, "specialized labor."[8] We represent the specialized labor as part of our overall labor allocation, L, changing the interpretation of labor from being in units of hours worked to units of *effective* hours worked. We limit ourselves to an inelastic supply

[8]Most models of this variety refer to this resource as "human capital." However, to avoid confusion with models in which human capital accumulates between periods, we refer to it by a different name. To quote Shakespeare, "a rose by any other name..."

of this variable in each period, as we are interested in the significance of capital market restrictions on growth rates. Instead, the significance of effective labor is in its allocation between creating final goods for consumption and creating new ideas; while the overall level of effective labor available to the household is supplied inelastically in each period, the distribution between production, $L_{Y,t}$, and the creation of new ideas, $L_{A,t}$, is chosen by the household in each period. We now describe each sector in turn.

4.1. Production. Capital enters the production process through a series of complimentary intermediate goods, x_i, aggregated through a CES operator,

$$Y_t = L_{Y,t}^{1-\alpha} \sum_{i=0}^{A_t} x_{i,t}^{\alpha} \tag{5}$$

where Y_t is production of consumption goods. By allowing for the intermediate capital goods, $x_{i,t}$, to enter as compliments through the CES aggregator, we enable market power in the provision of individual intermediate goods while allowing the producers of Y_t to act as price takers.

Here, the technological innovation leading to growth is the development of new types of producer durables, A_t. Market power for these intermediate goods is justified through the existence of patents over the design for a particular type of durable good. In each period, the owner of a patent maximizes the difference between revenue and costs given the demand function for his good. Since the final goods producer is a price taker, the demand for any intermediate good is simply the marginal benefit of that good. The cost of providing each unit of proprietary capital is one unit of general capital, K_t, which the intermediate producer can effortlessly convert to his proprietary good. We can therefore describe the monopolist's incentive as:

$$
\begin{aligned}
&\max \pi_t \\
&= \max_{x_t} \left[p(x_t)x_t - r_t x_t\right] \\
&= \max_{x_t} \left[\alpha L_{Y,t}^{1-\alpha} x_{i,t}^{\alpha} - r_t x_t\right] \qquad (6)
\end{aligned}
$$

This yields an interest rate, $r_t = \alpha^2 L_{Y,t}^{1-\alpha} x_{i,t}^{\alpha-1}$, which is less than the marginal product of capital. As a result, a non-negative return, $\pi_t = \alpha(1-\alpha)L_{Y,t}^{1-\alpha} x_{i,t}^{\alpha}$ can be allocated to the owner of a patent, yielding a market incentive for holding a patent. As creating intermediate goods is the only use of capital, and we are not concerned with the effects of any heterogeneity in capital, we can appeal to symmetry across capital goods to equate the amount of capital to the full amount of intermediate goods: $K_t = A_t x_{i,t}$. This gives us closed-form solutions for both capital and profit to the intermediate goods producer in each period:[9]

$$
r_t = \alpha^2 (A_t L_{Y,t})^{1-\alpha} K_t^{\alpha-1} = \alpha^2 \frac{Y_t}{K_t} \qquad (7)
$$

$$
\pi_t = \alpha(1-\alpha) L_{Y,t}^{1-\alpha} (A_t K_t)^{\alpha} = \alpha(1-\alpha) \frac{Y_t}{A_t} \qquad (8)
$$

4.2. Development of new ideas. The proportional increase of intermediate goods in a period is simply a linear function of the labor dedicated to this process:

$$
A_{t+1} = v_t(\zeta L_{A,t} + 1) A_t \qquad (9)
$$

where v_t is a stochastic process outlined below.

[9]Notice that with these expressions, $rK + \pi A$ gives the same result as we would normally get for K in the competitive model: $\frac{\partial Y}{\partial K} K$. Effectively, we are allowing the owners of patents to deprive the capital holders of some of their rents to provide an incentive for technology development.

We represent the right to monopoly profits from an idea as a separate asset, which we call a "patent." As a patent is simply the right to the monopoly profits from an idea, the number of patents grows with the number of ideas. We represent a patent as T_t, which is used below in the household's budget and Euler equations.

The amount of labor devoted to the R&D sector will be determined through an arbitrage condition between the returns in each sector to this factor of production. In both sectors, perfect competition allows the household the marginal benefit from the use of its labor. In the technology sector, this marginal benefit will be the marginal increase in technology multiplied by the price of the patent for that technology in terms of consumption goods, which we call, p_A.

$$w_t = \zeta A_t p_{A,t} = \gamma(1-\alpha)\frac{Y_t}{L_{Y,t}} \tag{10}$$

4.3. Stochastic elements. For the purposes of explaining shifts in venture capital spending and its relation to the rest of the economy, we allow two sources of risk to the economy.

First, as in a traditional real business cycle model, cyclical behavior of production and consumption arises through shocks to the technology process driving production. While this technology level is endogenous in this model, we allow fluctuations around the endogenously determined level:

$$\log(v_t) = \rho \log(v_{t-1}) + \varepsilon_{1,t} \tag{11}$$

where v_t is the multiplier on the technology growth path described in (15), and $\varepsilon_{1,t}$ represents a stochastic process normally distributed with mean zero, and ρ is a parameter representing the persistence of the shock. The shock itself represents the granular nature of scientific progress—while research and development follows endogenous incentivized behavior outlined in the growth rate, the actual progression of technology relies on the successes of individual projects and new discoveries in science. A positive value of $\varepsilon_{1,t}$ can be viewed as the creation of an individually large new technology. As the level of technology is growing at the same positive rate as the rest of the economy, a negative shock of a magnitude less than the overall growth rate of the economy can be interpreted as a slow year for the development of new ideas, as the overall growth will still be positive. This intuition also holds for values of $\varepsilon_{1,t}$ that cause the overall growth rate of technology to be negative if we also consider that the existing technology might have some element of depreciation through obsolescence. While we do not explicitly model the process of obsolescence,[10] we do acknowledge the existence of such considerations and treat them as implicit and exogenous.[11]

In addition, we allow for a second source of risk: a potential wealth shock to the net-worth of entrepreneurs. Ultimately, the motivation behind this source of risk is the information shock that financial intermediaries had to write-off a large portion of their assets due to the revaluation of risk in the subprime market in 2008. However, this event should not be viewed in isolation: the potential returns for any asset held by a financial intermediary is uncertain and will often require a write-off. Thus, this is not a single shock in an otherwise deterministic process, but a realization of a particularly bad shock in a process that has risk at all times. We call this process v_2, t, and it evolves with a similar AR(1) process as v_t:

$$\log(v_{2,t}) = \rho_t \log(v_{2,t-1}) + \varepsilon_{2,t}. \qquad (12)$$

[10]See Acemoglu (2008) for a description of models that explicitly use obsolescence in describing the growth process.

[11]In effect, we are saying that the random process $\varepsilon_{1,t}$ is actually biased with a mean greater than zero, but that this bias is exactly equal to the obsolescence of existing ideas. However, after adjusting for obsolescence, we can simplify our shock to zero mean.

We calibrate this second shock to allow for the variance decomposition of our consumption stream to be equal under the two shocks. This allows us to compare the relative reactions of other variables to the fluctuations of each of our stochastic processes. The results from this comparison are given in our results section.

4.4. Consumer behavior. As in a standard business cycle model, we postulate a representative consumer-household that optimizes discounted lifetime utility:[12]

$$(13)\quad \max_{c_t} \sum_{t=0}^{\infty} \mathbf{E}_t \beta^t u(c_t)$$

$$(14)\quad s.t.\ \left[k_{t+1} - n_{t+1}\right] q_t + p_t t_{t+1} = \left[(1-\delta) q_t + r_t\right](k_t - n_t) + (p_t + \pi_t) t_t + w_t l - c_t$$

with labor, l, supplied inelastically at the beginning of each period. We have two assets. The first, $k_t - n_t$, simply represents the capital provided by the household to the monopolists to create their intermediate goods. As we saw with our debt contract, the remainder of the capital is provided by the entrepreneur. The second, t_t, represents patents. As discussed above, these patents are monopoly rights to produce intermediate goods and receive the profit stream, π, each period. As these monopoly rights can be bought and sold, we treat them as an asset. The lifetime value of this profit stream, which we assume to be infinitely-lived, has positive value, p_t. Again, we are not concerned with any heterogeneity in the productivity of different types of intermediate goods, so we can represent the price of all patents as the marginal value of a new patent.[13]

However, the development of technology and patents in the decentralized equilibrium is an externality for the individual consumer. In a decentralized equilibrium,

[12] We use discrete time, as this is generally the most straightforward when using computational methods for numerical solutions. Many authors describe this model in continuous time, but the two formulations are conceptually equivalent.

[13] We use lower-case letters for k_t, t_t, c_t, and l above to represent the levels faced by individuals, in contrast to aggregate levels denoted by capitals.

the consumer's allocation of labor to the R&D sector and capital to patents are determined by arbitrage conditions, taking the wage paid to labor in each sector and the price of a patent as given. Instead, the household takes the level of technology growth as exogenous, allocating labor to the R&D sector based on the expected wage for labor in that industry, which, again, the consumer treats as exogenous. In effect, the household makes its decision using,

$$A_{t+1} = v_t[\Xi_t + 1]A_t \tag{15}$$

where Ξ_t is given in equilibrium by $\zeta(L - L_{y,t})$, where the household takes this aggregate variable as given in its optimization.

For our utility function, we specify a standard utility function with constant relative risk aversion, $u(c_t) = \frac{c_t^{1-\sigma}-1}{1-\sigma}$. That is, despite our earlier assumption that households are risk-neutral intra-temporally, they are risk-averse inter-temporally. We maintain inter-temporal risk aversion as it is the standard assumption in economics. Indeed, the risk-neutrality across projects in any given period can be justified through the inclusion of a financial intermediary, as Carlstrom and Fuerst do. As mentioned above, our model does not differ intuitively from allowing such an intermediary, but we do not explicitly include one in order to maintain simplicity.

The Euler equations describing the path of the price of patents and the level of capital held by the consumer are given respectively by:

$$c_t^{-\sigma} = \beta E_t\left[c_{t+1}^{-\sigma}\frac{p_{t+1}+\pi_{t+1}}{p_t}\right] \tag{16}$$

$$c_t^{-\sigma} = \beta E_t\left[c_{t+1}^{-\sigma}\frac{1}{q_t}((1-\delta)q_{t+1}+r_{t+1})\right] \tag{17}$$

In equilibrium, the expected returns in each of the Euler equations should be equal due to arbitrage. That is, if we call, $M_{t+1} \equiv \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}$, our stochastic discount factor, we can equate 17 and 16, through,

$$E_t\left[M_{t+1}\frac{p_{t+1}+\pi_{t+1}}{p_t}\right] = E_t\left[M_{t+1}\frac{1}{q_t}((1-\delta)q_{t+1}+r_{t+1})\right] \tag{18}$$

where π_t and r_t are given by 8 and 7, respectively, also through arbitrage. This will ensure an interior solution, as the relative value of r_t to π_t decreases as capital is allocated to the production sector.

4.5. Timing. To clarify the evolution of assets through the debt contract, we outline the timing conventions here.

The period starts when shocks are realized with assets left over from the previous period. For the household, these are bonds and patents, whereas the entrepreneur simply holds his version of bonds. The household and entrepreneur pool their bonds together to create an investment project, worth $q_t I_t(1-\mu\Phi(\bar{\theta}_t))$. The household and entrepreneur then use the proceeds from the project to invest in the production sector, and they also allocate their labor across the two sectors at this time. At the end of the period, the household and entrepreneur receive proceeds from their investments and wages from their labor, which they then decide whether to consume or invest. If they decide on a net increase in investment, they must convert consumption goods into either patents or projects, at prices p_t and q_t, respectively, which are then carried to the next period.

4.6. Equilibrium conditions. The framework above allows us to specify a rational expectations equilibrium.

Definition 1 (Decentralized Equilibrium). *A Decentralized Equilibrium consists of:*

(1) *Households choosing a consumption and asset bundle, c_t, k_t, and t_t, to optimize their discounted lifetime utility, (13), with respect to their budget, (14),*

(2) *Entrepreneurs choosing a consumption and asset bundle, c_t^e, and z_t, to optimize their discounted lifetime utility subject to their budget, 3,*

(3) *Intermediate goods producers choosing an optimum amount of their intermediate good, x_t, to produce, as given in 6, yielding a profit given by 8,*

(4) *Final goods producers choosing the optimum level of output, y_t, paying competitive prices, r_t, w_t, determined by (7), and the marginal returns to production for $L_{Y,t}$, respectively,*

(5) *The technology, expressed as the number of intermediate capital goods used in production, evolving according to (9) with a new patent created for each new intermediate capital good,*

(6) *Household labor and entrepreneurial endowments supplied inelastically at the beginning of each period, and*

(7) *The overall allocation of C_t, C_t^e, K_t, T_t, and Z_t, is feasible in that each element is non-negative and the allocation satisfies the market clearing condition:*

$$C_t + C_t^e + q_t K_{t+1} = (1-\delta) q_t^2 K_t + Y_t - K_t r_t \left(1 - \mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)}\right)$$

where C_t, C_t^e, and K_t are the sum of their respective individual variables of the same symbol.

The equilibrium conditions used in calibration and the derivation of the feasibility condition are outlined in Appendix A. Essentially, it says that the collective consumption and assets left in the next period are equal to the production in this period, the

Coefficient	Level	Description
α	0.3	Cobb-Douglas coefficient for capital
β	0.95	Discount factor
δ	0.15	Depreciation on capital
ϕ	0.2	Multiplier on the production of patents from labor
μ	0.2	Cost of state verification
σ	0.18	Risk aversion of consumer/household
γ	0.99	Impatience parameter of entrepreneur
μ_θ	-0.5	First parameter for lognormal distribution of θ
σ_θ	1	Second parameter for lognormal distribution of θ
ρ	0.1	Persistence of technology shock

Figure 4. Parameter values for our calibrated model

capital remaining from this period after depreciation, adjusted for price movements, and an adjustment for the deadweight loss from monitoring.

As in any other decentralized general equilibrium model under rational expectations, the household's knowledge of the endogenous variables outside of steady state is perfect, although the household's small size prevents it from considering its own influence over aggregate variables. This is equivalent to saying that out of the steady state, the model is still in equilibrium and that the household will anticipate markets clearing when making rational expectations over possible states of the model.

5. Calibration

We start our calibration by using standard values of 0.95, 0.15, and 0.3 for the discount factor, depreciation on capital, and the capital share of production, respectively. As these values are similar to those used by Carlstrom & Fuerst, we retain some degree of comparability by using these standards. In addition, we set the overall level of labor supplied inelastically by the household to one. The rest of our variables are therefore in units are of effective labor.

Variable	Value	Description
$\tilde{K}$	1.21	Capital
$\tilde{Y}$	1.012	Production
A	1.015	Technology growth rate
$\tilde{C}$	0.49	Consumption of households
$\tilde{C}^e$	0.11	Consumption of entrepreneurs
r	0.075	Interest rate (real)
q	1.12	Cost of Capital
L_Y	0.925	Proportion of labor dedicated to production
$\frac{n}{i}$	0.400	Entrepreneur's contribution to the capital project
$\bar{\theta}$	0.943	Signal associated with break-even project
$f(\bar{\theta})$	0.401	Share of capital going to entrepreneur
$g(\bar{\theta})$	0.532	Share of capital going to household

FIGURE 5. Steady-state values for our calibrated model.

We fix our value of σ, the coefficient for risk aversion in our utility function, to match long-term economic growth of 1.5% at an equilibrium return to capital at 7%, which is in line with long-term trends.

For the specifics of the debt contract, we follow CF where possible. Our value for μ is therefore 0.2, and the private signal $\bar{\theta}_t$ is distributed according to a lognormal distribution corresponding to a normal distribution with mean 0 and a σ of 0.18.[14] The first of these figures arises from the observed cost of bankruptcy. The specification of a zero mean allows for the relation $\int_0^\infty \theta\phi(\theta)d\theta \equiv 1$ specified above, which prevents bias from the distribution of the signal on the rest of the model. Finally, the value for σ_θ arises from an analysis of the bankruptcy rate from 1984-1990, but it also coincides with other agency cost models.[15]

Our impatience parameter for the entrepreneur is given by $\gamma = \frac{1-q^* g(\bar{\theta}^*)}{\beta(1+r^*-\delta)f(\bar{\theta}^*)}$ where asterisks indicate steady state values. This identity allows the entrepreneur's income to be stationary with respect to the household at the steady state values of $\bar{\theta}_t$.

[14]The codel would be made considerably simpler by using a uniform distribution for the private signal. The lognormal distribution, while more realistic, also allows for tractability with CF and improves the steady-state values for the calibrated model.
[15]Notably Boyd and Smith (1994)

Finally, the addition of the endogenous growth mechanism only requires one additional parameter: ϕ, the multiplier on labor for the production of new ideas. However, both the dynamics of the model and the steady-state values are highly sensitive to this parameter. The value ties the level of L_y to the growth rate in steady state through our technology process, but it also ties the steady state interest rate to the portion of capital going to technology. If ϕ is too high, our steady state interest rate will not be reasonably close to long-term trends, and we reach a corner solution in which all labor is allocated to production if ϕ is too low. We set the value to 0.2, as that keeps the steady state labor allocation to production at 92.5%, while allowing for reasonable steady state figures for the rest of our variables.

Our steady state value for $\bar{\theta}_t$ is determined by setting the entrepreneur's fraction of the initial contribution to the project to 40%. This allows for a return to the entrepreneur of roughly 40%, as well, meaning that the lender is left with 53% from an initial contribution of 60%. This is the cost of acquiring capital for the household. Effectively, the entire cost of monitoring, while originally paid by the entrepreneur, is passed to the household in the steady state values for the contract.

Our steady state values for the model are not far off what we observe in the overall economy. Our overall consumption comprises roughly 60% of production, which is not far off US figures of between 70% and 80%. While the ratio of production to effective labor is relatively low at 1, we should instead consider the ratio of production to effective labor in the production sector, which is 1.1. While still low, this is easily explained by the fact that we have built in two restrictions on capital in the model (monopoly power to intermediate goods producers and agency costs). Indeed, our steady state stock of capital is also lower than we would like at 1.2 times the level of effective labor.

All the results presented below are in terms of stationary variables. For variables that are non-stationary in the model, we have de-trended them as described in the introduction to this thesis.

6. Results

We now use simulated impulse response functions of the above model to describe the two recessions of the first decade of the new millennium through a single propagation mechanism. The differences between the recessions, under this explanation, are due to different types of shocks—the first recession is explained through a negative technology shock representing the collapse of the dot-com bubble, and the second is explained through a negative wealth shock to the entrepreneur, representing the negative information shock of financial managers learning their derivatives are worth less than previously thought. In the case of the technology shock, the propagation mechanism has the effect of reducing the drop in production relative to TFP while prolonging the lower production rate. In the case of the wealth shock, the propagation mechanism causes a break in the correlation between changes in production and TFP, allowing production to experience a precipitous decline while TFP increases, as we see in the data.

We also compare the impulse responses from our combined model to those of similar shocks in reduced models, to show that these effects require the combined inclusion of an endogenous growth mechanism and financial constraints. The inclusion of a financial accelerator dampens the decline in output caused by a negative technology shock compared to a model with only the endogenous growth mechanism. This is due to the entrepreneur's preferences and high burden of risk under the debt contract. Similarly, the inclusion of an endogenous growth mechanism magnifies and prolongs the movement away from steady state of output and household consumption following a negative net-worth shock, as the outside investment opportunity for the household prolongs the process for the entrepreneur to rebuild his net-worth.

We start by describing the response of our combined model to a negative shock to the technology growth rate, as represented by a negative realized value of the stochastic variable, ν_t. This is meant to represent the negative shock corresponding to the 2001 dot-com bubble. We then discuss the case of a negative net-worth shock, as expressed as a negative realized value of the stochastic variable, $\nu_{2,t}$. This, in turn, is meant to represent the negative wealth shock that corresponded to the revaluation of mortgage backed securities in the 2008 crisis, much of which was owned by highly leveraged financial institutions. While there are some interesting dynamics created by the financial sector in the case of the technology shock, the magnitude of the effect is minor when compared to the case of the negative wealth shock. Indeed, we show that the combination of the financial accelerator and the endogenous growth mechanism create a magnification of the original shock, while allowing TFP growth and a modest decline in patent prices when compared with production-based capital assets.

6.1. Technology shock. The immediate effect of an unexpected decrease in the technology growth rate in our model is the same as with any RBC model—a rational consumer faces the expectation of reduced lifetime income. Following a negative productivity shock, the consumption-smoothing household decreases consumption to offset the decrease in expected future earnings. The accompanying reduction in household income, through lower wages and capital interest, results in a lower demand for our two assets.

In addition, we see a protraction in the decrease in consumption, investment, and output that lasts for 3-4 periods, the last of which can be seen in Figure 6. This protraction can be explained through the workings of our growth mechanism: in the

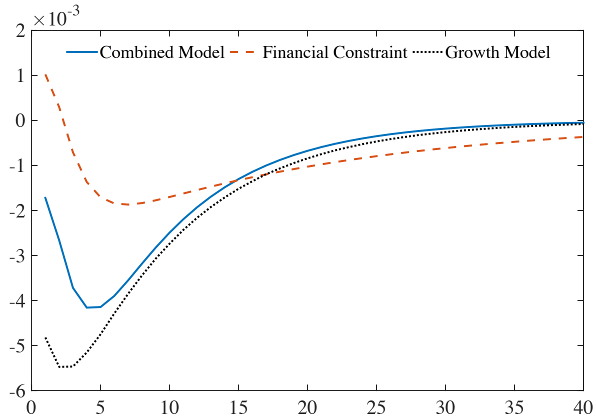


FIGURE 6. Response of technology-adjusted production to a negative technology shock

event of a negative technology shock, both the interest to capital and wages to labor in production decrease. This creates an incentive to allocate resources towards the development of new technology. The allocation of resources away from production decreases current production even further, resulting in the further decrease in output and consumption.

It is counter-intuitive that a negative shock to the technology level would lead to a temporary *increase* in resources allocated to technology, especially given the positive externalities from previous technology levels. However, arbitrage condition for labor relies on the relative wages between the sectors. The technology sector is fairly inelastic with respect to labor, as fluctuations are determined exclusively by movements in the price for technology due to the constant returns to scale in production. In contrast, the production sector depends directly on the amount of labor in production. Thus, as the

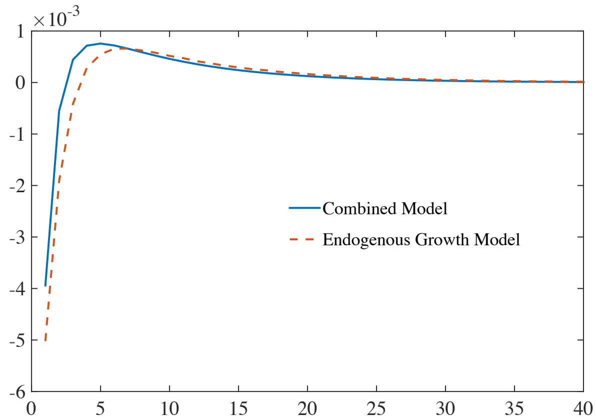
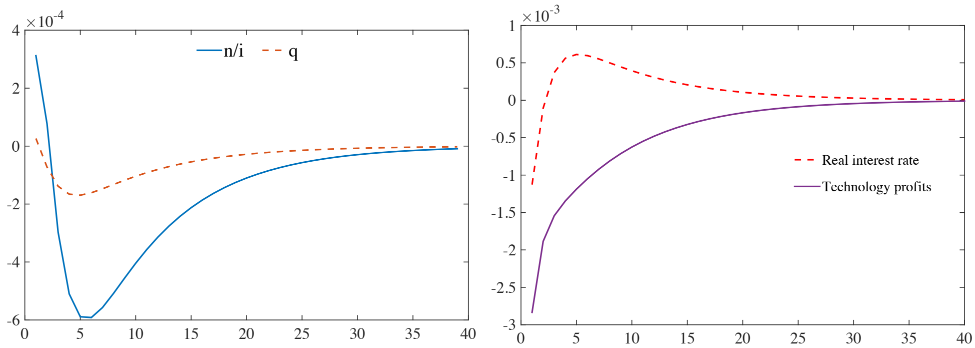


FIGURE 7. Response of labor allocation to production to a negative technology shock

real wage in the production sector decreases due to a drop in output, the technology sector exhibits a higher relative wage. The increased allocation of labor to technology quickly reverses, as the increased technology level leads to higher production in future periods. As a result, we see an "overshooting" effect as both the interest rate and the the allocation of labor to technology return to the steady-state balanced growth path.

We can see the strength of this effect by comparing the labor allocation between the endogenous growth and combined models, as shown in Figure 7. Here, we see that the overshooting effect of the endogenous growth model is prominent in the combined model, with a similar magnitude of overshooting. We also see that the addition of credit constraints has little effect on the response of labor, and therefore the progression of produced technology, to a negative technology shock; it only slightly reduces the magnitude response to the initial shock and causes the overshooting to occur two periods earlier.



(A) Leverage (inverse of n/i), and the impact it has on q_t

(B) The "wedge" between the returns to holding capital and technology

FIGURE 8. Response of capital markets after a negative tech shock

The standard financial accelerator also exhibits a protraction of the response to consumption, output, and investment in production to a negative shock, as this is the mechanism with which Bernanke explains the protraction of the Great Depression.[16] In contrast to the household's loss of income following the negative technology shock, the entrepreneur faces no such immediate loss of income, as his income is tied to his earnings as an intermediary. This leads to a temporary decrease in leverage (the ratio of overall investment to net-worth). As a result, the terms of the debt contract temporarily become more favorable to the entrepreneur, with a simultaneous value of $\bar{\theta}_t$ below steady state and q_t above steady state. However, this is short lived, as the entrepreneur's impatience leads the entrepreneur to increase his consumption while the terms of borrowing are good. Furthermore, the increased q_t causes the household

[16] Bernanke (1983)

to reduce consumption even further, so as the entrepreneur reduces his consumption, the terms of borrowing become more favorable to the household.

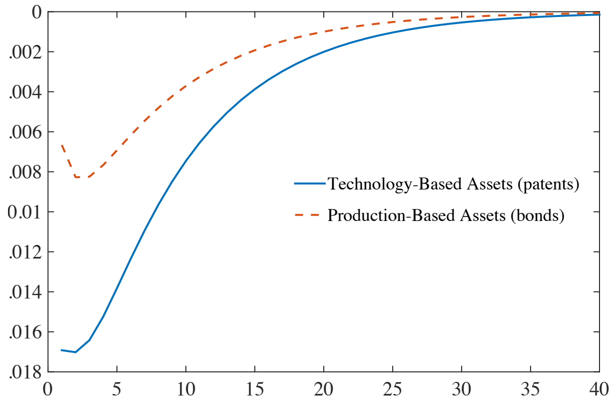


Figure 9. Response of household holdings of the two asset types to a negative technology shock

In this case, the financial accelerator actually reduces the impact of the negative shock on production, compared with the endogenous growth model without a financial accelerator (see Figure 6). This is because our entrepreneur, whose investment is focused on the production sector due to his privileged information in that sector, receives a windfall from the negative technology shock due to the lack of immediate loss of income and favorable terms described above. As this windfall is dedicated exclusively to the production sector, the entrepreneur boosts investment in production during the crisis. This helps improve output during a recession, while reducing the relative price of technology patents, as shown in Figure 9. As a result, the relative decline in allocation of labor to production is also slightly lessened, leading to a lower long-term growth rate, at the cost of an initial boost to production.

Overall, we see a disproportionate decline in the price of patents, coupled with a moderate recession, in accordance with the 2001 burst of the dot-com bubble. In addition, we see an increase in leverage after about four periods (Figure 8a), as the overshooting from the endogenous growth model provides investment opportunities in the production sector. Meanwhile, the level of production remains below the steady state growth rate for a considerable length of time, corresponding with the historically low growth rates in the US through the first decade of this millennium.

6.2. Net-worth shock. We now turn to our explanation of the 2008 financial crisis. Contrary to the technology shock, we see here that the effect of the financial constraint is the dominant mechanism following a disproportionate decrease in the wealth of the entrepreneur.

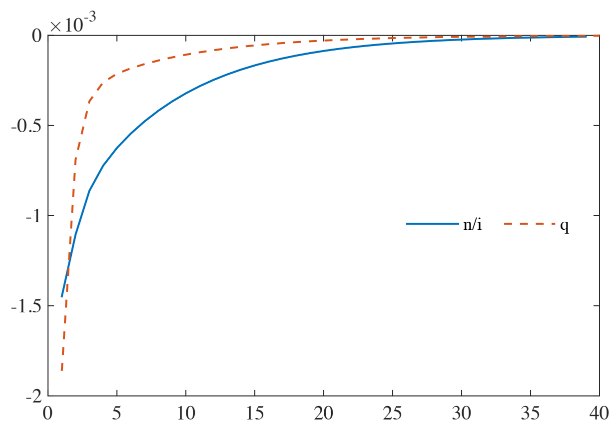


FIGURE 10. Response of leverage and price of capital to a wealth shock

Figure 10 captures the main mechanism at work in this shock. We see the negative shock to net-worth directly through a decrease in the ratio of net-worth to overall investment. This, in turn, forces the borrower to accept less favorable terms of borrowing while decreasing the price of capital, q_t. Remember, the borrower optimizes the tradeoff between promising a larger portion of proceeds to the lender and changing the overall returns to the project, represented through q_t. For large reductions in the net-worth, reducing the size of the project becomes more desirable than promising more to the lender, resulting in disproportionately large changes in q_t. This is the reason for the large yet short-lived response of q_t to the negative shock in Figure 10.

The low value of q_t acts as the main mechanism for propagating the net-worth shock through the rest of the economy. A lower value of q_t can be interpreted as a decrease in the overall size of capital projects as the borrower does not wish to borrow at the terms available at such high leverage. This reduction in the availability of capital causes an overall drop in household investment in production assets, which in turn causes a drop in production. We see this represented through a high value of the real interest rate as producers try to limit the effect of less capital by relying on labor, causing overall production to decrease more slowly than capital. This results in a high output-capital ratio, which determines the real interest rate in this model. In contrast to the technology shock, the real interest rate *immediately* shifts above the steady state, negating any potential for overshooting due to capital constraints. Instead, it remains above the steady-state level for a considerably long duration while the entrepreneur rebuilds his net-worth.[17]

Interestingly, arbitrage for labor forces the allocation of labor to production to behave similarly to the way it did in the technology shock—the sudden decrease in

[17]This variable seems to follow an exponential decay with a half life of roughly 7 periods. As our model is calibrated to represent annual data, this is a long return to steady state.

output reduces the real wage in the production sector, causing a shift of labor to the technology sector. This reverses for two reasons. First, the new technology causes an increase in labor productivity, allowing an increase in the real wage in the production sector. Second, the financial constraint in the production sector forces households to rely more on patents in their asset allocations. As the number of patents is fixed to the technology level, this search for yield forces the already depressed price of patents to adjust more slowly than it would otherwise. Both of these mechanisms are apparent in the technology shock, and this allocation decision reacts nearly identically to that case.

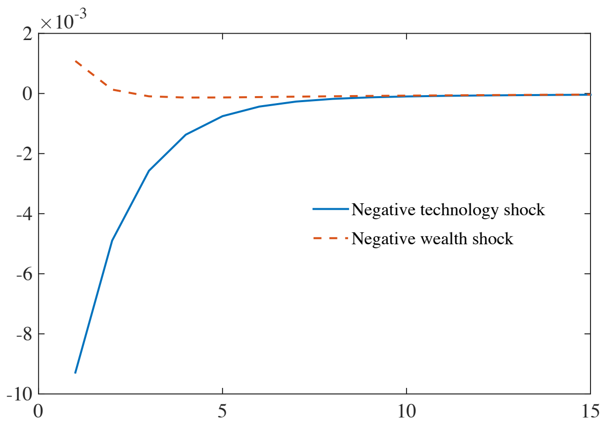
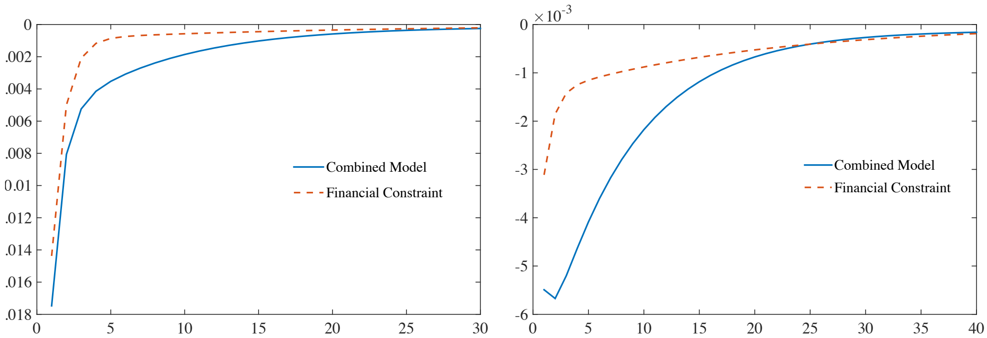


FIGURE 11. Impulse response of total factor productivity to each shock

Despite the similar behavior of labor allocation between sectors, the behavior of total factor productivity (TFP) does not. In Figure 11, we see the key dynamic of the model—the break in the correlation of total factor productivity with production. In both shocks, our economy experiences a significant decline in production, but the wealth shock results in a temporary increase in total factor productivity. The difference between the two shocks is that one shock *directly* affects the TFP level. It is this stochastic process that forces the correlation between movements in output and TFP in the case of a technology shock. Indeed, if we separate TFP into its two components, the stochastic process and the incentivized technological development from endogenous growth, the endogenous growth component does not correlate with changes in output in the technology shock either. However, this should not call into question the validity of the model in its fit with the stylized fact—TFP is the variable which correlates strongly with GDP changes in 2001 but not 2008, and our model exhibits that behavior for TFP.



(A) Output (B) Household consumption

FIGURE 12. Impulse responses for a wealth shock with and without endogenous growth

Finally, the addition of endogenous growth to the financial accelerator creates a prolonged reduction in both output and household consumption[18] compared to the financial accelerator alone. There are two reasons for this. First, the availability of

[18]This effect also exists for entrepreneurial consumption, although it is not as pronounced.

patents as an alternative asset to production capital allows for a slower adjustment of capital to the steady state, as the entrepreneur faces tougher competition for capital as output increases and returns to patents improve. Second, the allocation of labor away from the technology sector to the production sector decreases the production growth rate, causing capital growth to remain below the steady-state for a longer duration. This effect is especially evident in the kink in the impulse response for household consumption—as the allocation of labor moves away from technology development in the second period, the level of consumption decreases due to the household's expectation of lower lifetime income.

6.3. Magnitudes. In addition to describing the response of individual variables to each shock, we also consider the magnitudes of fluctuations caused by such stochastic processes. We do this by comparing the variance decomposition for each variable our simulations. These variance decompositions, given in Figure 13, describe the percentage of variance is caused by each stochastic process in our simulated variables. To explain the relative effects of each shock on the magnitude of perturbation of other variables, we calibrate the shocks to force an equal share for each shock in the variance decomposition of household consumption. While each shock can describe 50% by design, the wealth shock is responsible for 61% of the fluctuations in our simulated output and 74% of overall capital. This corresponds to the claim that a recession caused by a net-worth shock would lead to particularly large declines in output and capital levels, as we claim for the recent 2008 recession. In addition, we see that most of this deviation in capital levels comes from changes in the entrepreneur's capital holdings, Z, as variance in household holdings of capital, B, is evenly distributed between the two shocks.

Variable	Technology Shock	Wealth Shock
$\tilde{C}$	49.96	50.04
$\tilde{Y}$	38.63	61.37
$\tilde{K}$	26.01	73.99
$\tilde{B}$	51.37	48.63
$\tilde{Z}$	34.82	65.18
Ξ	32.58	67.42
L_y	32.58	67.42
p	44.70	55.30
$\tilde{C}_e$	2.87	97.13
π	38.63	61.37
r	36.51	63.49
n	13.44	86.56
$\frac{n}{i}$	28.89	71.11
q	4.58	95.42

FIGURE 13. Variance decomposition for each type of shock in the model.

As one would expect, the net-worth shock is responsible for the majority of fluctuations in leverage ($\frac{n}{i}$) and entrepreneurial consumption. Alternatively, while the technology shock is only responsible for 40% of the variance in the price of technology, this is a larger share of variance than the technology shock claims for overall capital. We interpret this as meaning that while the wealth shock leads to larger variances for most variables, the degree to which it is responsible for technology pricing is relatively lower than with other variables. This is consistent with our story that the 2001 recession was relatively moderate compared to 2008, but that the fluctuation of technology prices were a larger part of the story in 2001 than in 2008.

Furthermore, while the recession of 2001 was one marked by shocks to the technology sector, the 2008 recession is responsible for a large portion of the total variance, relative to consumption and output. This corresponds to the TFP figures shown in Figure 2, which shows relatively minor fluctuations in TFP growth during the 2001 recession.

7. Conclusion

This paper serves to explain a series of stylized facts regarding venture capital and the wider economy over the last two recessions. Specifically, the interaction between finance and technology investment requires a stylized model incorporating an endogenous growth mechanism with a financial accelerator based on a debt contract. Using this model, we explain the relatively moderate decline in output accompanied by a large decline in the price of technology-based assets in 2001, an exaggerated decline in 2008 with a prolonged depression from the steady state, and a break in the correlation between movements in GDP and TFP in the 2008 recession.

The last of these is especially interesting as such a movement would be extremely difficult to explain without an endogenous growth mechanism. RBC models are often criticized as relying heavily on technology shocks to produce business cycle fluctuations, and the recent financial crisis would be difficult to explain with such a shock. Trying to explain an increase in TFP with a wealth shock, when the causality between technology and business cycle fluctuations usually runs in the direction from technology to other variables, would require assumptions that would be difficult to justify.[19]

Our combined model is also better than either individual mechanism for explaining the minor production decrease during the 2001 recession and the prolonged recovery from our recent downturn. In the former, the existence of the entrepreneur with informational advantages in the capital market for production assets forced some foregone consumption to remain aimed at production capital when an endogenous growth model would have had investors flee the sinking ship. In the latter, this same temptation to flee a sinking ship allows investors to reduce their contribution to

[19]It would undoubtedly have to involve a connection between the financial market and utilization of the technology. This would then beg the question of why the TFP was actually increasing during this period, a question that incentivized investment in technology is designed to address.

production, decreasing the recovery for a longer period. In addition, the allocation of labor to production also slows future growth, compounding the length of the recovery.

Also interesting is that the model exhibits a period of increased leverage and reduced returns to capital projects during this same period. Obviously, it would be naive to attribute the extreme leveraging that led to the 2008 crisis to the residual credit constraints from the 2001 crisis, as regulatory changes are much more obvious culprits. However, regulation does not exist in a vacuum, and many of the regulatory attitudes that were blamed for the 2008 crisis were formed following the 2001 downturn. Constrained borrowing due to leveraging regulations would be particularly apparent at a time in which a negative shock reduced borrowers' net-worth, and it is at that time that such constraints would face political opposition. Of course, one does not need an endogenous growth mechanism to explain why we would have a reduction in net-worth following a negative technology shock, but it does correspond well with a sluggish recovery in which investors see tepid growth accompanied by low returns to financial projects (our q_t).

Finally, one limitation of this model was the relative response of patent prices to the two recessions. We sought to explain why a minor recession would have a disproportionate effect on such prices, and instead we found a relatively even effect between the two. The reason for this result was that the investment in patents was constrained by the growth of the overall technology level. As the quantity of technology is fixed, a fall in the price of patents would correspond to either a decrease in the expected returns from their profit stream or a decrease in the demand for alternative investments. However, as explained in our discussion of variance decomposition, while the overall magnitudes of changes in the price of technology are lower for the technology shock process, this shock can still account for a larger share of the variance in price than it can for overall capital and output. Thus, in the context of the net-worth shock resulting in higher variance for most variables, the effect of the technology shock on patent prices is relatively large, in line with our explanation of the 2001 recession.

To recap, our model is most successful in the sense that the incorporation of both mechanisms is useful in explaining the relatively moderate decline in 2001 compared to 2008 and the break in correlation between TFP growth and GDP growth in 2008. In addition, it successfully describes low economic growth in the periods following the 2001 recession, a sharp increase in the price of capital in 2008-2009, and a prolonged decline in both output and consumption following the 2008 financial crisis.

Appendix A: Equilibrium and Market Clearing Conditions

Below are the equilibrium conditions for the closed form model. These are the equations used in the simulation. These are the conditions *before* de-trending, and capital letters indicate aggregate variables.

Also included is the derivation of the market clearing condition expressed in Definition 1.

We begin by describing the aggregate evolution of technology and production.

$$A_{t+1} = \zeta L_{A,t} A_t + A_t$$

$$Y_t = L_{Y,t}^{1-\alpha} K_t^{\alpha}$$

$$1 = L_{Y,t} + L_{A,t}$$

where the final term defines $L_{Y,t}$ and $L_{A,t}$ as the proportions of labor going to output and technology development, respectively. In equilibrium, our prices for r_t and π_t are determined by arbitrage conditions for capital, given the market concentration in the sector for intermediate production goods. We also have an arbitrage condition for labor allocation between technology and production of consumption goods. These are given, respectively, by:

$$R_t = \alpha^2 \frac{Y_t}{K_t}$$

$$\Pi_t = (1-\alpha)\frac{Y_t}{A_t}$$

$$\zeta A_t p_t = (1-\alpha)\left(\frac{K_t}{L_{Y,t}}\right)^{\alpha};$$

Next, we have the terms of the contract, in aggregate. In order, we have the participation constraint, the optimal condition for the entrepreneur, and the resulting asset accumulation equation for the entrepreneur. For exposition, we define a new variable, $B_t \equiv K_t - \frac{(1-\eta)}{\eta} N_t$.

$$B_t = q_t \left(B_t + \frac{(1-\eta)}{\eta}(Z_t + X) \right) g(\bar{\theta}_t);$$

$$1 = q_t \left(1 - \mu\Phi(\bar{\theta}_t) + \mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)} \right);$$

$$Z_{t+1} = \left[q_t(1-\delta) + r_t\right](Z_t + X)\frac{f(\bar{\theta}_t)}{1 - q_t g(\bar{\theta}_t)} - \frac{C_t^e}{q_t}$$

Our two types of consumers have inter-temporally optimal Euler equations. There are two for the consumer and one for the entrepreneur, respectively:

$$q_t c_t^{-\sigma} = \beta \mathbf{E}_t c_{t+1}^{-\sigma}(q_{t+1}(1-\delta) + r_{t+1})$$

$$p_t c_t^{-\sigma} = \beta \mathbf{E}_t c_{t+1}^{-\sigma}(p_{t+1} + \pi_{t+1})$$

$$q_t = \beta\gamma \mathbf{E}_t \left[((1-\delta)q_{t+1} + r_{t+1}) \frac{q_{t+1} f(\bar{\theta}_{t+1})}{1 - q_{t+1} g(\bar{\theta}_{t+1})} \right]$$

Finally, we have a market clearing condition, stating that aggregate consumption bundles and assets held into the next period must not be greater than the combination of assets held from the previous period and the goods produced, minus a real cost adjustment for capital and a deadweight loss for monitoring of bad projects:

$$q_t(B_{t+1} + Z_{t+1}) + C_t C_t^e = (1-\delta)(B_t + Z_t + X)q_t^2 + Y_t - r_t(B_t + Z_t + X)q_t \mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)}$$

To achieve the market clearing condition represented through the last equation in the model above, we start by aggregating the individual budgets of the household and entrepreneur:

$$B_{t+1}q_t + p_t T_{t+1} = \left[(1-\delta)q_t + r_t\right] B_t + (p_t + \pi_t)T_t + w_t L - C_t$$

$$q_t Z_{t+1} = \left(q_t(1-\delta) + r_t\right) q_t(B_t + Z_t + X) f(\bar{\theta}_t) - C_t^e$$

Adding these together, we get:

$$q_t(B_{t+1} + Z_{t+1}) + p_t T_{t+1} =$$

$$= \left[(1-\delta)q_t + r_t\right] q_t(B_t + Z_t + X)(f(\bar{\theta}_t) + g(\bar{\theta}_t)) + (p_t + \pi_t)T_t + w_t L - C_t - C_t^e$$

Using the fact that patents are created one for one with new intermediate goods, we can say, $\frac{T_{t+1} - T_t}{T_t} = \zeta L_{A,t}$. In addition, the competitive wage in the technology sector is the marginal cost of labor in this sector, $\zeta A_t p_t$. Combining these two, we can say that $p_t T_{t+1} = p_t T_t + w_t L_{A,t}$. Thus, we can eliminate a few terms in the equation above:

$$q_t(B_{t+1} + Z_{t+1}) =$$

$$= \left[(1-\delta)q_t + r_t\right] q_t(B_t + Z_t + X)(f(\bar{\theta}_t) + g(\bar{\theta}_t)) + \pi_t T_t + w_t L_{Y,t} - C_t - C_t^e$$

Using the identity, $f(\bar{\theta}_t) + g(\bar{\theta}_t) = 1 - \mu\Phi(\bar{\theta}_t)$ and the optimal contract condition, $1 = q_t \left(1 - \mu\Phi(\bar{\theta}_t) + \mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)}\right)$, we can say that $q_t(f(\bar{\theta}_t) + g(\bar{\theta}_t)) = 1 - q_t\mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)}$ This allows us to write:

$$q_t(B_{t+1} + Z_{t+1}) = \left[(1-\delta)q_t + r_t\right](B_t + Z_t + X)\left(1 - q_t\mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)}\right) + \pi_t T_t + w_t L_{Y,t} - C_t - C_t^e$$

Given that our closed form versions of r_t, π_t and w_t yield the common identity, $r_t K_t \pi_t T_t + w_t L_{Y,t} = Y_t$, we arrive at the market clearing condition given in the text:

$$q_t(B_{t+1} + Z_{t+1}) = (1-\delta)(B_t + Z_t + X)q_t^2 + Y_t - C_t - C_t^e - r_t(B_t + Z_t + X)q_t\mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)}$$

CHAPTER 3

From Debt to Equity:

Did Policy Makers Get It Right?

Abstract

We explore the consequences of a series of policies enacted at the end of the 1970s which encouraged technology investment to move from a debt to an equity contract. As with other chapters in this thesis, we capture the technological focus of such investment through an endogenous growth model, this time using a model similar to the one in Rivera-Batiz and Romer (1991). We capture debt contracts through a net-worth multiplier, as in Bernanke and Gertler (1989), and equity contracts through an issuance cost, as in Covas and Haan (2011). We find that the change in contracts can account for a large increase in investment over the period and a break in the correlation between TFP growth and the returns to production capital. It can also partially explain the relative variance in TFP growth between decades, and the model corresponds to certain elements of the 1973 recession.

1. Introduction

Venture capital deals typically take the form of a private investment in a company in return for equity in the firm. However, during the period from 1959 to 1979, the United States federal government invested directly into small firm financing in the technology sector. In 1958, the Small Business Administration was given the authority to charter "Small Business Investment Companies." These SBICs financed firms through debt due to their bureaucratic structure. By 1963, these firms provided over 75% of investments into new technologies. The shift from SBICs to private venture capital firms is credited largely to the change in the ERISA's "prudent man" rule in 1979. This change explicitly allowed pension funds to invest in venture capital. Coinciding with a decrease in capital gains tax the previous year, funding in new technology companies shifted from debt to equity relatively quickly and can be attributed to an exogenous policy change.[1]

A number of authors have argued that the form of contract is a significant determinant of venture capital funding. Issues such as investment duration, non-financial contributions, moral hazard, and risk-return profiles have gained much attention. Specifically, both costly state verification and monitoring of current investments have been identified as significant contractual issues with venture capital, each having a special role in debt and equity contracts, respectively.[2]

This chapter investigates whether a change from a debt to an equity contract in funding new technology ventures around the year of 1980 had any effect on the general equilibrium behavior of the market. Similar to other chapters in this thesis, the role of venture capital in funding technological advancement is central to our analysis, in that it allows for growth rates of the economy to vary endogenously and based on

[1]The historical information above comes from Gompers (1994)

[2]Kaplan and Stramberg (2001) is one particularly strong example, and Da Rin et al. (2011) offers a complete survey of the use of contracts in venture capital on a micro level.

incentivized behavior. The general intuition behind this analysis is that the structural role of technology investment through its impact on total factor productivity causes a magnification and propagation of the effects of a change in contract type.

We build a model that incorporates R&D costs, similar to Rivera-Batiz and Romer (1991), into a dynamic stochastic general equilibrium framework. We then incorporate two types of financial contract in the technology sector, in turn. As we will see, both types of financial contacts have some sort of contractual cost to them, forcing a break in the Modigliani-Miller theorem.[3] Using the financial accelerator of Bernanke and Gertler (1989), we incorporate a debt contract into the technology sector using a formulation similar to Carlstrom and Fuerst (1997).[4] This debt contract is in response to moral hazard under asymmetric information, in which the returns to an investment are determined prior to the realization of private information to a borrower, and the cost of the contract comes through the state-dependent monitoring costs, where monitoring only happens when a borrower claims default.[5] For an equity contract, we appeal to Covas and Haan (2011) in incorporating quadratic equity costs to mimic the expense of selecting and monitoring investments.

We find that the models incorporating the different contracts each fit the data better for their respective periods than a model in which contractual limitations are not present. Furthermore, incorporating the contractual limitations for each period helps explain a number of stylized facts:

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[3]Modigliani and Miller (1958) show that the capital structure of a firm does not affect the returns to capital investment in the firm. The theorem fails to hold when financial contracts have costs associated with them, such as maintenance or information costs.

[4]The key differences being that CF used a debt contract in an economy in which technology was given as exogenous and that the limitations from debt contract applied universally across all assets. We focus the debt contract on the technological sector, where the returns to the asset act as the incentive for investment in new technology.

[5]Townsend (1979) shows that a debt contract is the optimal solution to the costly state verification problem.

Decade	Variance
50s	12.43
60s	8.93
70s	14.83
80s	11.76

Table 1. Variance of quarterly utilization-adjusted TFP over various decades. Source: Fernald (2014)

(1) the decreased variance in TFP from 1960-1969 relative to the periods immediately before and after the policy was implemented;[6]
(2) the depth and duration of the recession of 1973-74;
(3) the increased funding to technological assets in the years 1984–2008; and
(4) a reversal in the correlation between TFP growth and interest rates occurring around the year 1980.

First, in the decade immediately following the authorization of the Small Business Administration to issue debt contracts, the variance of utilization-adjusted total factor productivity (TFP) growth reduced significantly. As shown in Table 1, quarterly calculations of utilization-adjusted TFP growth are considerably lower in the 1960s than in the 1980s or 1950s. This is in line with the mechanics of our model, which allow a much larger deviation in resources allocated to technology development in an equity contract than when a debt contract is used, following a technology shock of similar magnitude. This is because the risk is borne entirely by the entrepreneur in the debt contract, with the lender only responding through changes in the terms of lending. As the entrepreneur is less risk-averse and more impatient than the lender, much of the brunt of the shock will be absorbed by the entrepreneur, rather than passed on through the terms of the loan.

[6]The exclusion of the 1970s from this stylized fact is explained below.

While the model corresponds with relative changes in measured variance of utilization-adjusted TFP between the 1950s, 1960s, and 1980s, the 1970s do not follow the predictions of our model. While the period which we characterize through a debt contract for technological investment includes the 1970s, this decade exhibits a high variance in the quarterly utilization-adjusted TFP growth level, which corresponds to an equity contract in our model. However, the 1970s was a decade characterized by a high-level of supply-side volatility, due to shocks in the oil supply and the ultimate end of the Bretton Woods international financial system. Therefore, we believe that the comparison of the 1970s was not like-for-like in the underlying shocks, making any comparison with surrounding decades tenuous. In addition, the high and volatile inflation of the 1970s begs an explanation that includes a monetary mechanism, which is not included in our model. Thus, while our model predicts the relative variances of technology growth in the 50s, 60s, and 80s, we believe the 1970s to be an outlier that can be explained through other means.

Despite the limitation of our model to explain the oil-related volatility of the 1970s, our model does bear some relevance to a particular incident in the decade. The recession of the early 70s in the United States was one marked by persistently low GDP growth, as shown in Figure 1. While the overall level of GDP growth never dipped below -5% in any given quarter, the period from the pre-recession peak in the first quarter of 1973 to a level above the long-term average in 1975 is particularly long and was marked by 5 quarters of negative real growth rates. Given a strong negative shock during a period in which technology investment occurs under a debt contract, our model predicts that the recession would be protracted and long in duration, similar to other models exhibiting the net-worth accelerator. In our model, the debt contract forces the returns to investment in technology to remain low in subsequent periods due to the borrowing constraints of the entrepreneurs in the tech sector, depressing any

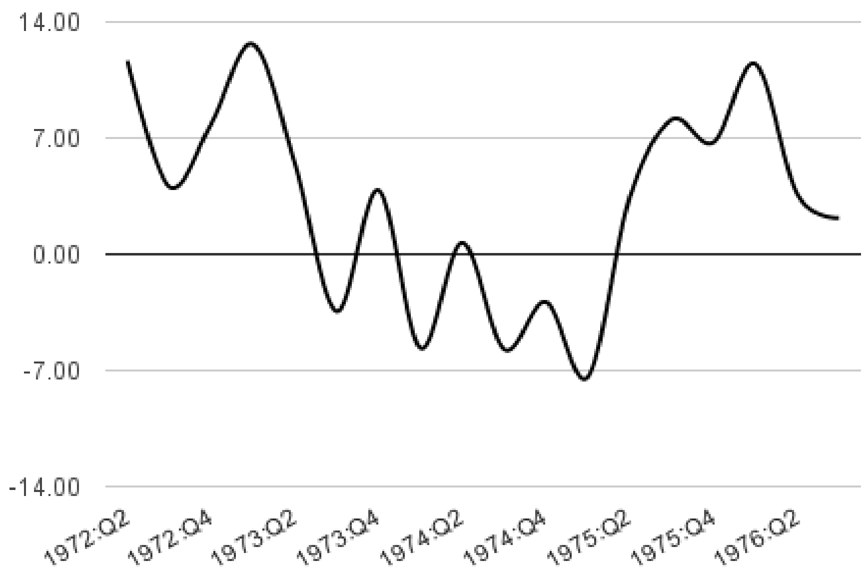


Figure 1. Real GDP Growth in the 1972 Recession

technologically-led recoveries. Indeed, the utilization-adjusted TFP growth referenced above shows below-average levels for 1972-74.

More importantly, our model predicts a large increase in the amount of private investment in technological development when an equity contract is used. This is due to the shared risk between entrepreneur and investor in a debt contract, which allows the investor to allocate his assets to the technology projects at exactly the time in which she wishes to increase investment.

Finally, the dynamics of our model match the observed break in correlation between returns to investment in production equities and total factor productivity growth following the change in policy encouraging technological investment to follow an equity contract. We see this first by considering the utilization-adjusted total factor productivity growth figures in Fernald (2014), and comparing them to the capital input levels from the same dataset. As theoretical competitive returns to capital in our model are $(1-\alpha)\frac{Y}{K_Y}$, we can compare these utilization-adjusted TFP growth rates with the theoretical returns to capital. These values are negatively correlated from 1950 to 1977, and positively correlated from 1980 to 2000, at values of -0.10 and 0.19, respectively. Moreover, comparing these same utilization-adjusted TFP values with the returns to equities found in Shiller (1992) and updated in Shiller (2014), this shift in correlations is even stronger between the same two periods, at -0.30 and 0.42, respectively.

There is a relatively large body of research loosely related to the analysis at hand. The relationship between type of finance and economic development is well-established, with more developed "frontier" economies relying more on equity markets than developing economies, which rely more on debt. Levine and Zervos (1998), Bencivenga and Smith (1991), and Bencivenga et al. (1995) discuss the relationship between the type of finance and the level of development in various economies, with the latter specifically focusing on transaction costs. In addition, Agénor and Aizenman (1997) and Boyd and Smith (1998) discuss a costly state verification model in the context of economic development. Another strand of related literature focuses on the broader economic implications of venture capital. Many of these focus on more narrow employment and economic activity of venture capital funded companies, while a few focus on the broader economic implications of venture capital. Two pertinent examples of this type of analysis are Kortum and Lerner (2000) and Keuschnigg (2004).

The only other paper known to this author dealing explicitly with the general equilibrium implications of contracting in the context of technology growth is Reiss and Weinert (2005). This paper is similar to ours in that it incorporates moral hazard into a general equilibrium endogenous growth model. However, the focus is not on venture capital; the type of growth described is different from ours, and the overall focus is on the effects on inequality of endogenous growth. Fernandez-Villaverde et al. (2003) deal with the implications of financial intermediation and entrepreneurship in the macroeconomic context, but they stop short of using endogenous technological change as the main mechanism for growth in their model.

2. The model

In this section, we build a baseline model from which we compare the effects of using a debt contract versus an equity contract in creating new technology. The debt contract is adapted from Carlstrom and Fuerst and exhibits a financial multiplier. The intuition is that the net-worth of an entrepreneur can fetch a proportional amount of funding from the market. However, the entrepreneur also absorbs the idiosyncratic risks and returns from the economy, causing his net-worth to fluctuate. As the level of overall funding is tied to the net-worth of the entrepreneur at any period, the absorbed losses and gains from aggregate shocks cause a magnification to the fluctuations in the overall funding for the entrepreneur's project, and shocks persist as they are propagated by fluctuations in the entrepreneur's net-worth. As the economy suffers, the market price of capital will also increase, hurting the terms of the contract for the entrepreneur, and further limiting the amount of capital available for the project.

The equity contract has some similar elements to the debt contract.[7] The entrepreneur's net-worth is still central to the terms of the contract—the entrepreneur determines how much equity he wishes to raise, given that he must provide a just compensation for any outside funding. However, a negative shock to the economy is now shared between both parties, as failing projects lead to commensurate losses to both the entrepreneur and lender.

[7]Our equity contract is related to that of Covas and Haan (2011).

The assumption that a debt contract necessitates the risk burden being shouldered by borrowers is central to the concept of debt, in which the repayment amount is determined at issuance. In contrast, the shared ownership of equity is fundamentally one of risk sharing as the costs and benefits of ownership are distributed across equity holders.

In addition, the switch from debt to equity also exhibits a move from a state-dependent monitoring cost to a quantity-dependent issuance or governance cost. This, coupled with the more equitable risk burden between parties described above, drives our results.

The cost structure of each asset class is less fundamental to the definition of the class, but the differences are still intuitively defensible. First, the role of costly state verification is important in debt, as the asymmetry between lender and borrower creates a moral hazard for the borrowers to lie. This, coupled with the observed costs of monitoring in bankruptcy and the result that this type of contract being optimal in the case of moral hazard makes monitoring costs an attractive form for modeling the cost of the contract. Likewise, in the absence of information asymmetries, the constraints to equity come in the form of issuance and governance costs, with regulatory compliance, the organization of shareholder voting, and the issuance of dividends as the dominating costs of issuing equity. The first two of these costs, regulatory compliance and the organization of shareholder voting, would arguably increase with the size of the number of investors, which we represent through a quadratic cost of issuance of equity.

Furthermore, the overall level of investment is closely related to the marginal productivity of the project itself, meaning that while decreases in the entrepreneur's net-worth will result in a lower overall investment due to the costs of the contract being related to the level of equity created, the terms of the contract may end up more favorable to the entrepreneur with a low net-worth (depending on the strength of the equity costs). As a result, the entrepreneur's returns are countercyclical (or less pro-cyclical than a debt contract), and the equity contract dampens the impact of a negative shock.

3. Production

To include the effects of venture capital, we build a model of increasing product variety, as in Rivera-Batiz and Romer (1991). This model incorporates incentivized growth through the development of new intermediate goods used in the production process. That is, our production process includes a variety of intermediate capital goods entering through a Dixit-Stiglitz aggregator:

$$Y_t = L_t^{1-\alpha} \int_0^{A_t} x_{i,t}^{\alpha} di \tag{19}$$

where Y_t is production, L_t is labor used in production, x_i is an intermediate capital good of type i, and A_t is the number of intermediate goods at time t. As in the previous chapter, this can be used to derive an equilibrium interest rate, r_t, and profits, π_t:

$$r_t = \alpha^2 \frac{Y_t}{K_{Y,t}} \tag{20}$$

$$\pi_t = \alpha(1-\alpha)\frac{Y_t}{A_t} \tag{21}$$

We assume, for simplicity, symmetry over the intermediate goods, so the overall level of capital dedicated to production, $K_{Y,t}$ can be divided among the intermediate capital goods. That is, for all i, we represent $x_{i,t}$ through a generalized $\bar{x}_t$, which is equal to $\frac{K_{Y,t}}{A_t}$. Thus, (19) can be simplified as:

$$\begin{aligned} Y_t &= L_t^{1-\alpha} A_t \left(\frac{K_{Y,t}}{A_t}\right)^{\alpha} \\ &= (A_t L_t)^{1-\alpha} K_{Y,t}^{\alpha} \end{aligned} \tag{22}$$

4. Technology and patents

At the heart of this model is a project for the development of a new intermediary good. For this, our economy contains a continuum of technological entrepreneurs, each of which has a proposed project for a new intermediate good to be used in production. Each project uses capital and the existing stock of technology to develop new intermediate goods. The projects differ in their effectiveness in creating new technology, using capital as the primary input.[8]

$$a_{i,t+1} = \zeta k_{A,i,t}^{1-\lambda} A_t^{\lambda} \theta_i \tag{23}$$

where θ_i represents the effectiveness of entrepreneur, i, and the subscript, a, on capital denotes that this is a portion of the capital stock allocated to the technology sector. λ represents the diminishing returns to capital and existing technology in the creation of new technology, and is bound by $0 < \lambda < 1$. Across this chapter, lower case letters represent individual variables and upper case variables represent aggregate variables.[9]

Furthermore, we assume that the timing of this investment requires that the level of capital investment made to a specific technology be determined before any of the agents realize the level of θ_i. This eliminates any concerns that private information can lead to signaling or adverse selection through the level of investment requested in the project.

[8]The growth model presented here differs from the rest of this thesis in that it uses capital as the primary input of production. The reason is that the efficiency of the size of the project is crucial in comparing the two types of finance. We require the input to technology to exhibit diminishing returns to scale in order for the project to have an efficient size, and a stationary variable such as human capital or labor could not lead to balanced growth with diminishing returns to scale. We could add a second input to production of technology, but this would also alter the model from the other chapters and add complexity not required for this model.

[9]The idea of an entrepreneur creating an individual technology is a technical but useful abuse of notation, as once the technology is created, it immediately and automatically contributes to the overall level of technology through the non-rival nature of ideas. However, the technology is individual in the sense that it is created by an individual entrepreneur and in that the entrepreneur can then create an exclusive right to that technology for the purposes of production.

In aggregate, the individual abilities of each entrepreneur aggregate to a constant θ_{agg}, which we normalize to one.[10] Our aggregate technology production function is therefore,

$$A_{t+1} - A_t = \zeta K_{A,t}^{1-\lambda} A_t^{\lambda}. \quad (24)$$

where new technology is now given as the difference between the future and current levels of A, and $K_{A,t}$ is the aggregate level of capital allocated to the development of new technology. Combined with $K_{Y,t}$, this gives our entire capital stock, K_t. At the individual level, the entrepreneur creates new technology using the overall stock of technology, A, so we can ignore the effects of previous technology created by any specific entrepreneur.

As capital is the only input for creating a new intermediate good, the cost of creating the good is the price of capital times the amount of capital dedicated to technology. We call this initial investment, i_t. Once the project is complete and a new intermediate good, $A_{t+1} - A_t$, has been created, $\zeta K_{A,t}^{1-\lambda} A_t^{\lambda} q_t$ assets are created in the form of patents, where q_t is the price of a *new* patent.

To pay for this capital, the entrepreneur provides his own net-worth, n_t, and borrows the remaining amount, $i_t - n_t$, from households in the market. The new assets are divided up between the lender and entrepreneur, depending on the type of contract, as described below.

The behavior of the entrepreneur drives the dynamics of the investment in the technology project for this model. Fluctuations in the entrepreneur's net-worth drive

[10] Individual realizations of this θ are central to the debt contract, while we are able to completely overlook the existence of theta in the equity contract due to normalizing the aggregate level to one. In the debt contract, normalizing the aggregate level of θ to one takes the same role as in the previous chapter: while agents are heterogeneous over their individual level of theta, the expected value of theta is normalized to one, implying the effects of this heterogeneity are distributional rather than contributing to aggregate levels.

the market price and level of capital in a way that is dependent on the type of contract used. However, the contracts are similar in that the entrepreneur faces a mechanism design problem in which she optimizes the terms of the contract to maximize her own return, subject to a participation constraint of the household providing the rest of the capital. As we will see, both contracts exhibit a participation constraint that requires returns to the household to be equal to the opportunity cost of investing in the productivity sector. Given this inelastic return to the household, the entrepreneur varies the proportion of returns she receives and the overall size of the project in order to maximize her return given the restraints of each contract. Thus, a lack of net-worth for the entrepreneur manifests itself in a higher proportion of proceeds going to the household, but also in a lower investment in the project. This lower investment determines the propagation mechanism, as the funding for the project determines the growth rate of the economy.

We discuss the debt contract and the equity contract in turn.

5. Debt

As with Bernanke et al. (1999), the existence of the entrepreneur is significant in that it provides a "net-worth multiplier," causing the aggregate investment to fluctuate with the net-worth of the entrepreneur, a binding restriction. The key is in the private information available to the entrepreneur, giving an edge in investing in this type of asset for this agent. As a result, the returns will be higher, and the entrepreneur will want to invest his entire net-worth into the asset in each period. However, the terms of the contract will fluctuate with the net-worth of the entrepreneur, which will drive cyclical fluctuations in the overall investment in this asset class.

At the heart of the debt contract is the idea that the effectiveness of the entrepreneur's technology project is private information known only to the entrepreneur.

The lender can discover the value of the entrepreneur's effectiveness level by paying a monitoring cost proportional to the size of the project. However, this monitoring cost is a deadweight loss, and the household and entrepreneur desire to build a contract in which the lender refrains from paying that cost as often as possible.

At the beginning of each period, the entrepreneur works and receives, x_t, in wages which he contributes to his stock of patents for a net-worth of,

$$n_t = x + z_t p_t \tag{25}$$

which he invests into his own technology project along with capital raised from the household. As in the previous chapter, the entrepreneur is endowed with a minimal amount of capital in each period to ensure that the net-worth at the beginning of a period is positive. In addition, our shocks are calibrated to ensure that aggregate net-worth never dips below zero.

At the end of the period, the technology project produces patents, according to (24), which are divided between the entrepreneur and the lender, with $\zeta k_{a,t}^{1-\lambda} A_t^{\lambda} q_t f(\bar{\theta})$ going to the entrepreneur, $\zeta k_{a,t}^{1-\lambda} A_t^{\lambda} q_t g(\bar{\theta}_t)$ to the lender, and the rest are lost in the process of monitoring the signal of the entrepreneur. We can express this through the identity, $f(\bar{\theta}) + g(\bar{\theta}) = 1 - \mu\Phi(\bar{\theta})$, where $\Phi(\bar{\theta})$ is the probability that the lender will pay the monitoring cost.

The entrepreneur's decision from period to period is to optimize his utility from consumption, similar to the household. However, since he has superior returns to the household from the technology enterprise, he is only concerned with one asset, patents.

Once the entrepreneur receives his own return from the project, he consumes part and holds the rest for the next period:

$$p_t z_{t+1} = q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} f(\bar{\theta}_t) - c_t^e \tag{26}$$

As it is useful to express transition functions in terms of state variables, we incorporate (25) to give us,

$$p_t z_{t+1} = (x + z_t p_t)^{1-\lambda} q_t \zeta \left(\frac{k_{a,t}}{n_t}\right)^{1-\lambda} A_t^{\lambda} f(\bar{\theta}_t) - c_t^e \tag{27}$$

Since the entrepreneur has superior information, he has the bargaining power in the formation of the contract. Specifically, we treat him as the optimizing agent in determining the terms of payment, choosing which level of repayment he promises to give in a successful project. There are two opposing effects on the entrepreneur increasing the threshold signal for a successful project. First, by increasing the promised return to the lender for a successful project, the entrepreneur is directly paying the lender a higher share of the returns. However, by increasing this level, the entrepreneur must also bear a larger share of the initial investment, lowering the potential value of the project. By promising a high return, the entrepreneur also increases the likelihood that any project will be unsuccessful, as it will be more difficult to meet the threshold for a project to return the promised investment.

Specifically, the entrepreneur optimizes his own return $\zeta k_{a,t}^{1-\lambda} A_t^{\lambda} q_t f(\bar{\theta})$, subject to a participation constraint of the lender. That is, the lender will only offer money if the expected return is greater than potential other uses. Thus, the maximization problem is:

$$\max_{k_{a,t}} q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} f(\bar{\theta})$$

$$q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} g(\bar{\theta}_t) \geq (k_{a,t} - n_t)(1 - \delta + r_t) \tag{28}$$

We limit ourselves to interior solutions in which this will always bind, allowing us to state the participation constraint as a necessary portion of the initial investment:

$$\frac{n_t}{k_{a,t}} = 1 - q_t \zeta k_{a,t}^{-\lambda} A_t^{\lambda} g(\bar{\theta}_t)(1 - \delta + r_t)^{-1} \tag{29}$$

The first order conditions from the optimization are:

$$(1-\lambda) q_t \zeta k_{a,t}^{-\lambda} A_t^{\lambda} f(\bar{\theta}) = -\frac{f'(\bar{\theta}_t)}{g'(\bar{\theta}_t)}((1 - \delta + r_t) - (1-\lambda) q_t \zeta k_{a,t}^{-\lambda} A_t^{\lambda} g(\bar{\theta}_t)) \tag{30}$$

where the symbol, $x'(*)$, represents a derivative. The left side of the equation above can be described as the benefit to the entrepreneur of increasing $k_{a,t}$ at a given level of $\bar{\theta}_t$. The right side represents the cost of increasing the capital level. The term inside the brackets represents the marginal cost of increasing the size of the project net of the benefit from the increase in the project. The fraction represents the change in the proportion going to each agent given a change in the size of the project.

Defining $\Omega(\bar{\theta}_t)$ as the marginal increase in the monitoring cost associated with an increase in the project size,[11] it is useful to re-arrange (30) to obtain the form,

$$q(1-\lambda)\zeta k_{a,t}^{-\lambda} A_t^{\lambda} - (1 + r_t - \delta) = \Omega(\theta) \zeta k_{a,t}^{-\lambda} A_t^{\lambda} q_t \tag{31}$$

Here, we see the optimal condition for the entrepreneur drives a wedge between the marginal return to capital from the technology project and the marginal return to

[11] Specifically, it is defined as $\mu \left(\Phi(\bar{\theta}_t) + \Phi'(\bar{\theta}_t) \frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)} \right)$

capital from the production sector. The left side is the difference between the two marginal productivities. As omega tends to zero on the right side, this difference disappears. Thus, the cost of monitoring effectively chokes off technological development in the model and lowers the growth rate.

Since the contract only lasts a single period, the entrepreneur does not consider the constraints of the contract explicitly in his optimization. Thus, combining the entrepreneur's transition function, (27), with the participation constraint, (29), we get a transition function entirely in terms of the entrepreneur's state variable, z, the choice variable, c^e, and variables considered exogenous to the entrepreneur.

$$p_t z_{t+1} = (x + z_t p_t) \frac{q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} f(\bar{\theta})}{1 - \delta + r_t - q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} g(\bar{\theta}_t)} - c_t^e \tag{32}$$

The entrepreneur's preferences are given differently from the consumer's. Instead of CRRA preferences, the entrepreneur is considered risk neutral, allowing his lifetime consumption to fluctuate more than a risk-averse consumer would. Also, we stipulate that the entrepreneur is more impatient in his consumption decisions, with a time preference given by $\tilde{\beta} \equiv \beta\gamma$. Some authors interpret this increased impatience as a higher probability of death or likelihood of switching from being an entrepreneur to a household.[12]

The entrepreneur's intertemporal decision is given by the optimization:

$$\begin{aligned} &\max_{c_t^e} \mathbf{E}_t \sum_{t=0}^{\infty} \beta^t c_t^e \\ &\text{s.t. } p_t z_{t+1} = (x + z_t p_t)^{1-\lambda} q_t \zeta \left(\frac{k_{a,t}}{n_t}\right)^{1-\lambda} A_t^{\lambda} f(\bar{\theta}_t) - c_t^e \end{aligned} \tag{33}$$

[12]For example, Bernanke et al. (1999)

As usual, his decisions are represented through an Euler equation:

$$p_t = \beta\gamma \mathrm{E}_t \left[p_{t+1} q_{t+1} \zeta \left(\frac{k_{a,t+1}}{n_{t+1}} \right)^{1-\lambda} A_{t+1} f(\bar{\theta}_{t+1}) \right] \tag{34}$$

We now show the explicit form of $f(\bar{\theta}_t)$ and $g(\bar{\theta}_t)$, given by the standard debt contract as initially proposed by Townsend (1979).

5.1. The debt contract. The lender offers to lend money to the project in return for receiving his money back with interest, r^k, at the end of the project. Any additional proceeds from the project go to the entrepreneur. However, some projects will not be profitable enough to pay back the interest. There will be a threshold below which the the project will not yield enough money to pay the lender the promised amount. We index this threshold by the signal corresponding to the marginal project, $\bar{\theta}_t$. For all projects with a signal below this critical level, the lender recoups the entire value of the project, less a monitoring cost, $\mu\Phi(\bar{\theta}_t)$, to verify the amount.[13]

$$q_t \zeta k_{A,t}^{1-\lambda} A_t^{\lambda} \bar{\theta}_t = (1 + r_t^k)(i_t - n_t) \tag{35}$$

As in the previous chapter, we follow CF in assuming that the household is risk-neutral in its consideration of intra-temporal risk across projects. In contrast, the household is risk-averse in its inter-temporal allocation across sectors. Again, we appeal to the same intuition of CF that there could be risk-neutral financial intermediaries that manage the debt contract with the entrepreneurs. We abstract away from these intermediaries for simplicity rather due to any intuitive distinction.

[13] Note that *all* projects still receive the same amount of funding, as the signal is not known to the lender until the time of default.

The entrepreneur therefore receives,

$$q_t\zeta k_{A,t}^{1-\lambda}A_t^{\lambda}f(\bar{\theta}_t) = q_t\zeta k_{A,t}^{1-\lambda}A_t^{\lambda}\int_{\bar{\theta}_t}^{\infty}\theta\phi(\theta)d\theta - (1-\Phi(\bar{\theta}_t))(1+r_t^k)(i_t-n_t)$$

$$= q_t\zeta k_{A,t}^{1-\lambda}A_t^{\lambda}\left[\int_{\bar{\theta}_t}^{\infty}\theta\phi(\theta)d\theta - (1-\Phi(\bar{\theta}_t))\bar{\theta}_t\right] \tag{36}$$

while the lender receives,

$$q_t\zeta k_{A,t}^{1-\lambda}A_t^{\lambda}g(\bar{\theta}_t) = q_t\zeta k_{A,t}^{1-\lambda}A_t^{\lambda}\int_{0}^{\bar{\theta}_t}\theta\phi(\theta)d\theta + (1-\Phi(\bar{\theta}_t))(1+r_t^k)(i_t-n_t) - \mu\Phi(\bar{\theta}_t)$$

$$= q_t\zeta k_{A,t}^{1-\lambda}A_t^{\lambda}\left[\int_{0}^{\bar{\theta}_t}\theta\phi(\theta)d\theta + (1-\Phi(\bar{\theta}_t))\bar{\theta}_t - \mu\Phi(\bar{\theta}_t)\right] \tag{37}$$

We define $\int_0^{\infty}\theta\phi(\theta)d\theta \equiv 1$ to make the relationships above represent the proportion of capital independent of the level of capital generated.

Adding the entrepreneur's share to the lender's, we obtain the useful result,

$$f(\bar{\theta}_t) + g(\bar{\theta}_t) = 1 - \mu\Phi(\bar{\theta}_t) \tag{38}$$

While $f(\bar{\theta}_t)$ and $g(\bar{\theta}_t)$ represent the proportions of the newly created patents going to the entrepreneur and lender, respectively, the shares themselves do not add to one. Instead, we have a monitoring cost $\mu\Phi(\bar{\theta}_t)$, which is lost in each period due to the need for the lender to verify the state in the case of bankruptcy.

Using (38), we can eliminate $g(\bar{\theta}_t)$ from (30):

$$1-\delta+r_t = p_t q_t\zeta k_{a,t}^{1-\lambda}\left(1-\mu\Phi(\bar{\theta}_t) + \mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)}\right) \tag{39}$$

$$\equiv p_t q_t\zeta k_{a,t}^{1-\lambda}h(\bar{\theta}_t)$$

In addition to allowing a reduced form of the first order conditions, (39) offers an intuitively appealing description of the behavior of q_t. The entrepreneur's optimal changes to the promised return to the lender are directly offset by the price of the new patent to allow the expected return from the project compete with the market in acquiring capital. While the entrepreneur isn't directly changing the price of new patents, his constraint on the supply of patents by altering the terms of the contract effectively decides this price. Conditions (39) and (28) completely describe the contract, $(q_t, \bar{\theta}_t)$, which depends entirely on the overall supply of foregone consumption to the project, and the proportion of that supply originating from the household.

6. Equity

We compare the contract to one in which the entrepreneur promises an investor a share of the overall return from the project, regardless of the outcome of the project. The structure of the equity contract is similar to that of the debt contract in that the entrepreneur chooses the size of the contract and the proportion of proceeds accruing to the household subject to the same participation constraint for the household. However, instead of a state-dependent contract, the entrepreneur now creates a more simple contract in which the share accruing to each agent is independent of the private signal of the entrepreneur. Similarly, the monitoring costs associated with the contract also are no longer state-dependent. Instead, there is a quadratic cost of issuing equity, similar to Covas & Den Haan (2012).

As mentioned above, this quadratic cost can be interpreted as the cost of governance of an enterprise given the amount of equity issued. Practically, this is quadratic to represent the increasing marginal cost of governance in issuing more shares. This can be due to disclosure requirements, the increased risk of demands from active shareholders, or the increased costs of holding shareholder meetings. In effect, the inclusion of having a linear marginal cost of equity is also to allow the models to be comparable, as the debt contract has a linear increased cost of monitoring for each unit of probability that the contract will be in default. This probability, in turn, is a function of the higher returns a borrower must pay to attract capital for a larger project. Thus, the tradeoff between both contracts is between a larger project and the marginal cost of attracting more capital. We have tried to keep this marginal cost as simple as possible.

Recall that the efficiency of a debt contract came through the existence of a private signal known by the entrepreneur. As such, the entrepreneur could expect a return from the project above market rates, and the efficiency of the debt contract arose in the ability for the household to receive market returns while monitoring only part of the time. By not conditioning the return to the household on the success of projects, the efficiency of this system is potentially gone. This poses two problems—it eliminates the market power of the entrepreneur to dictate the terms of the contract, and it potentially allows for the investor to increase his investment infinitely without any decrease in efficiency. Both these problems are eliminated by adding a cost of issuing equity to outside investors that is quadratic in the amount of equity issued.

As above, the entrepreneur brings a net-worth to the technology project, consisting of his assets from previous projects and his labor wage from the beginning of the period:

$$n_t = x + z_t p_t \tag{40}$$

where the labor is set-up the same way as in the debt contract.

Again, the entrepreneur enlists the help of the household to provide capital to the project, only now as an investment. At the end of the period, the technology project produces patents, which are divided between the entrepreneur and the lender. However, now the division of patents at the end of the period does not depend on the returns from individual projects. Instead, the household investor receives a fixed share, s_t, of the project, as agreed at the time of investment.

The entrepreneur's problem is now simply:

$$\max_{s_t}(1 - s_t)\left[q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda}\right] - \frac{a s_t^2}{2}$$

$$\text{s.t. } s_t\left[q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda}\right] = (1 + r_t - \delta)(k_{a,t} - n_t) \tag{41}$$

where $k_{a,t}$ and n_t remain the amount invested in a technology project and entrepreneur's net-worth, respectively. As with the debt contract, the contract is described by a pair of parameters (q_t, s_t), given by the participation constraint, (41), and the first order condition to the maximization, given by,

$$k_{a,t} - n_t = \frac{q_t(1 - \lambda)\zeta k_{a,t}^{-\lambda} A_t^{\lambda} - (1 + r_t - \delta)}{a} \tag{42}$$

This term is straightforward: the marginal cost of increasing the amount of equity issued, $a(k_{a,t} - n_t)$, must equal the marginal benefit of that equity, which is the numerator on the right side. As the share of returns accruing to the household investor only enters the optimization linearly, the point at which the marginal return from increasing the shares equals the marginal cost will be independent of the proportion of the project going to each person.[14]

Similar to (31), (42) represents a wedge driving apart the marginal returns from the different uses of capital. As a tends to zero, any difference between the marginal benefit between using capital for technology and production would result in the entrepreneur raising an infinite amount of capital for his project. As a increases, capital is restricted from the technology project, driving up the marginal return from the project. In this

[14] As mentioned before, the optimal condition would also be independent of the size of the overall project, $k_{a,t}$, if it also were not for the decreasing returns to scale in the input to technology project.

sense, the equity contract is similar to capital adjustment costs used in some RBC and New Keynesian models. However, in this model, the amount of equity required for a given level of investment depends on the level of net-worth of the investor. As the net-worth increases, these "equity issuance costs" decrease, allowing the gap between projects to close. Thus, in order to see the dynamic effects of this wedge, it is necessary to assess the progression of the entrepreneur's net-worth.

Progressing as we did with the debt contract, the entrepreneur uses the return from the project for his own consumption,

$$p_t z_{t+1} = (1-s)q\zeta k_{a,t}^{1-\lambda} A_t^{\lambda} - as^2 - c_t^e \tag{43}$$

As it is useful to express transition functions in terms of state variables, we incorporate (25) to give us,

$$p_t z_{t+1} = (x + z_t p_t)^{1-\lambda}(1-s_t)q_t\zeta \left(\frac{k_{a,t}}{n_t}\right)^{1-\lambda} A_t^{\lambda} - as_t^2 - c_t^e \tag{44}$$

From (41), we get the identity,

$$\frac{k_{a,t}}{n_t} = \frac{1 + r_t - \delta}{1 + r_t - \delta - s_t q_t \zeta k_{a,t}^{-\lambda} A_t^{\lambda}} \tag{45}$$

We can now set up our entrepreneur's inter-temporal problem as,

$$\max_{c_t^e} \mathbf{E}_t \sum_{t=0}^{\infty} \beta^t c_t^e$$
$$\text{s.t. } p_t z_{t+1} = (x + z_t p_t)^{1-\lambda}(1-s_t)q_t\zeta \left[\frac{k_{a,t}}{n_t}\right]^{1-\lambda} A_t^{\lambda} - as_t^2 - c_t^e \tag{46}$$

An interior solution is described by,

$$p_t = \beta\gamma(1-\lambda)\mathbf{E}_t t_{t+1}^{-\lambda} p_{t+1}^{1-\lambda} q_{t+1}(1-s_{t+1})\zeta \left[\frac{k_{a,t+1}}{n_{t+1}}\right]^{1-\lambda} A_{t+1}^{-\lambda} \tag{47}$$

A negative productivity shock to the economy would cause the rental price of capital, r_t, to increase. In order to raise the same amount of equity, the entrepreneur would have to increase the household's share of the returns from the project. The entrepreneur responds by issuing slightly less equity, as dictated by his optimality condition, (42). As a result, the entrepreneur restricts the creation of new technology in the next period and can issue fewer patents, causing the price of new patents, p_t, to rise.

7. Closing the model

We now close the model by describing the behavior of households and aggregation of the model to account for the distribution of agents between households and entrepreneurs.

7.1. The household. The value of a patent derives from the monopoly profits accruing to the holder of the patent. We see this represented in the evolution of the assets held by a representative household. At the beginning of a period, the household is endowed with labor, l_h, for which it receives a competitive wage, w_t. At the beginning of the period, the household also holds a portfolio of assets, held between capital, $(k_t - n_t)$,[15] and patents, valued at the end of the previous period, $p_{t-1}t_t$. These investment goods can be used for three purposes: investment in the production of consumption goods ($k_{Y,t}$), holding current patents (t_t), and investment in the production of new technology ($k_{a,t} - n_t$). Once the returns from each of these projects and the returns to labor have been realized, the household determines how

[15] k_t represents the overall level of capital, so the amount held by the household is $k_t - n_t$.

much of his return he wishes to consume (c_t) leaving the rest in the form of patents or capital. The evolution of the household's assets therefore evolve according to the equation:

$$\begin{aligned}
&(k_{t+1} - n_{t+1}) \left(\frac{\eta}{1-\eta}\right) + p_t t_{t+1} = \\
&\qquad = w_t l_h + (1 - \delta + r_t) k_{Y,t} + q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} g(\bar{\theta}_t) \left(\frac{\eta}{1-\eta}\right) + (p_t + \pi_t) t_t - c_t \\
&\qquad = w_t l_h + (1 - \delta + r_t) \left(k_{Y,t} + (k_{A,t} - n_t) \left(\frac{\eta}{1-\eta}\right)\right) + (p_t + \pi_t) t_t - c_t \qquad (48)
\end{aligned}$$

for the case of the debt contract. The second line takes into account the participation constraint, (28), which guarantees the household's share of the technology project to receive the market interest rate.

For the equity contract, the household's assets evolve similarly:

$$\begin{aligned}
&(k_{t+1} - n_{t+1}) \left(\frac{\eta}{1-\eta}\right) + p_t t_{t+1} = \\
&\qquad = w_t l_h + (1 - \delta + r_t) k_{Y,t} + s_t q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} \left(\frac{\eta}{1-\eta}\right) + (p_t + \pi_t) t_t - c_t \\
&\qquad = w_t l_h + (1 - \delta + r_t) \left(k_{Y,t} + (K_{a,t} - n_t) \left(\frac{\eta}{1-\eta}\right)\right) + (p_t + \pi_t) t_t - c_t \qquad (49)
\end{aligned}$$

The household makes its choices via a standard optimization problem:

$$\max_{c_t} \mathbf{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma} - 1}{1-\sigma}\right)$$

$$\text{s.t. } k_{Y,t+1} + p_t t_{t+1} + i_{h,t+1} = w_t l_h + (1 - \delta + r_t)(k_{Y,t} + i_{h,t}) + (p_t + \pi_t) t_t - c_t \qquad (50)$$

where we define $i_{h,t} \equiv (k_{a,t} - n_t)\left(\frac{\eta}{1-\eta}\right)$ as the household's loan to the entrepreneur. The optimization gives the following Euler equations:

$$c_t^{-\sigma} = \beta \mathrm{E}_t c_{t+1}^{-\sigma}(1 - \delta + r_{t+1}); \tag{51}$$

$$c_t^{-\sigma} = \beta \mathrm{E}_t c_{t+1}^{-\sigma} \frac{p_{t+1} + \pi_{t+1}}{p_t}; \tag{52}$$

where the first Euler equation is the result for optimizing with respect to $k_{Y,t}$. The quantity of i_h, t held by the household is determined not by optimization, but by the availability of this asset as determined by the entrepreneur's optimization, (39).

7.2. Aggregation. As we are concerned with the progression of the economy given the existence of venture capitalists, we stipulate a constant proportion of the population, η, as entrepreneurs. We are not concerned with the progression of how this sector transforms over time, but with the effect of the sector on the rest of the economy.

Patents are held only by households. Entrepreneurs' assets are technically patents and priced as such, but they are sold immediately in order to fund a new round of technology investment. As a result, the total number of patents accumulating profit streams is simply the aggregate of household patents:

$$T_t = (1 - \eta) t_t \tag{53}$$

However, each period, new patents are created by the entrepreneurs' projects. The patents at the end of the period for the debt contract are given by,

$$\begin{aligned} T_{t+1} &= \eta \left[q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} f(\bar{\theta}_t) \right] + (1-\eta) \left[t_t + q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} g(\bar{\theta}_t) \left(\frac{\eta}{1-\eta} \right) \right] \\ &= T_t + q_t \zeta K_{A,t}^{1-\lambda} A_t (1 - \mu \Phi(\bar{\theta}_t)) \end{aligned} \tag{54}$$

which is similar to (24), except that it incorporates the rate patents are created from new technology and the destruction of value through the costly state verification. We also have a similar equation for the equity contract:

$$T_{t+1} = \eta\left[q_t\zeta k_{a,t}^{1-\lambda}A_t^{\lambda}(1-s) - as_t^2\right] + (1-\eta)\left[t_t + q_t\zeta k_{a,t}^{1-\lambda}A_t^{\lambda}s_t\left(\frac{\eta}{1-\eta}\right)\right]$$

$$= T_t + q_t\zeta K_{A,t}^{1-\lambda}A_t(1-as_t^2) \tag{55}$$

We can write the overall level of labor as, $L_t = (1-\eta)l_t + \eta x_t$. Similarly, household consumption is given by $C_t = (1-\eta)c_t$. Since the entrepreneur holds his assets only in patents, household capital is divided between $K_{Y,t} = (1-\eta)k_{Y,t}$ and $K_{A,t} = (1-\eta)k_{A,t}$, with $K \equiv K_{Y,t} + K_{A,t} = 1$[16]

Finally, we are able to define a competitive equilibrium for both the debt and equity cases:

Definition 2 (Debt-based Decentralized Equilibrium). *A Debt-based Decentralized Equilibrium consists of:*

(1) *Households choosing a consumption and asset bundle, c_t, k_t, and t_t, to optimize their discounted lifetime utility with respect to their budget, as in (48),*
(2) *Entrepreneurs choosing a consumption and asset bundle, c_t^e, and z_t, to optimize their discounted lifetime utility subject to their budget, (33),*
(3) *Intermediate goods producers choosing an optimum amount of their intermediate good, x_t, to produce yielding a profit given by (21),*
(4) *Final goods producers choosing the optimum level of output, y_t, paying competitive prices, r_t, w_t, determined by (20), and the marginal returns to production for L_t, respectively,*

[16]Recall that an individual entrepreneur's project defined in (23) used k_i units of capital. We assume capital is distributed evenly across the entrepreneurs' projects. Thus $k_{i,t} = K_{A,t}/\eta = \frac{(1-\eta)}{\eta}(k_{A,t})$.

(5) *The technology, expressed as the number of intermediate capital goods used in production, evolving according to (24) with a new patent created for each new intermediate capital good,*

(6) *Household labor and entrepreneurial endowments supplied inelastically at the beginning of each period, and*

(7) *The overall allocation of $C_t, C_t^e, K_t, Y_t, T_t$, and Z_t, is feasible in that each element is non-negative and the allocation satisfies the market clearing conditions:*

$$K_{Y,t+1} + K_{A,t+1} = (1-\delta)K_{Y,t} + Y_t - C_t - C_t^e$$

and (54), where C_t, C_t^e, and K_t are the sum of their respective individual variables of the same symbol.

Definition 3 (Equity-based Decentralized Equilibrium). *A Equity-based Decentralized Equilibrium consists of:*

(1) *Households choosing a consumption and asset bundle, c_t, k_t, and t_t, to optimize their discounted lifetime utility with respect to their budget, as in (49),*

(2) *Entrepreneurs choosing a consumption and asset bundle, c_t^e, and z_t, to optimize their discounted lifetime utility subject to their budget, (46),*

(3) *Intermediate goods producers choosing an optimum amount of their intermediate good, x_t, to produce yielding a profit given by (21),*

(4) *Final goods producers choosing the optimum level of output, y_t, paying competitive prices, r_t, w_t, determined by (20), and the marginal returns to production for L_t, respectively,*

(5) *The technology, expressed as the number of intermediate capital goods used in production, evolving according to (24) with a new patent created for each new intermediate capital good,*

(6) *Household labor and entrepreneurial endowments supplied inelastically at the beginning of each period, and*

(7) *The overall allocation of* $C_t, C_t^e, K_t, Y_t, T_t,$ *and* Z_t, *is feasible in that each element is non-negative and the allocation satisfies the market clearing conditions:*

$$K_{Y,t+1} + K_{A,t+1} = (1-\delta)K_{Y,t} + Y_t - C_t - C_t^e$$

and (55), where $C_t, C_t^e,$ *and* K_t *are the sum of their respective individual variables of the same symbol.*

The system of equations that make up our equilibrium conditions in the two decentralized equilibria are outlined in Appendix A.

8. Calibration

The model has been calibrated to match characteristics of the US economy. We match the long-term growth rate of 2% and use an interest rate of 8%. Capital depreciates at an annual rate of 15%, and our discount factor is 0.95 to represent annual data.

More important is our calibration for aspects specific to growth and finance. Our multiplier on the production of new technologies is 0.04, to allow a non-negligible amount of investment in these new technologies.[17] This calibration allows for 15% of capital to be dedicated to the technology sector. Likewise, λ, our Cobb-Douglas

[17]Notice that this value is much lower than in previous chapters, as changing the factor of production from human capital/labor to capital requires an independent calibration.

parameter in the technology sector is 0.3, to provide a relatively low amount of curvature for this production function. This allows for large deviations in the amount of capital allocated to technology, increasing the effect of the mechanism we wish to model. For both equity and debt, we fix the steady state proportion of net-worth to overall investment in technology to 10%. This is roughly in line with US data, though the data tends to show a higher leverage for equity than for debt. We set them equal to isolate the effects of our mechanism from the effects of increased leverage.

Finally, all the results presented below are in terms of stationary variables. For variables that are non-stationary in the model, we have de-trended them as described in the introduction to this thesis.

9. Results

We study the effects of an unexpected positive technology shock. In a debt contract, the effect of such a shock will lead to higher returns for all projects. As the terms of the contract have already determined the level of $\bar{\theta}$, and the incentive compatibility constraint continues to bind, the returns going to the household from the successful technology projects will continue to be determined by the market interest rate. Thus, the majority of the windfall from a positive technology shock will accrue to the entrepreneur.[18] In contrast, the sharing of idiosyncratic risk in the equity contract also allows sharing of aggregate risk of technology fluctuations. The risk profile associated with the ability to share the windfall from technology fluctuations is the key difference between the two contracts.

Before discussing the transition following our shock, we first examine the magnitudes of the displacements caused by such technology shocks. To do this, we consider

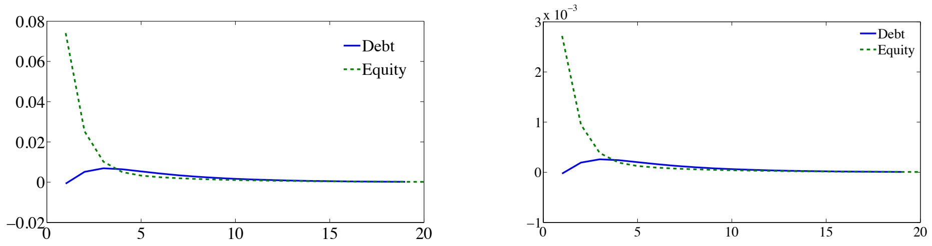
[18] There will be a slight increase to the household due to an unexpectedly low proportion of projects going bankrupt. However, given our calibration, the vast majority of the unexpected returns accrue to the entrepreneur. This also appears to be the case for most reasonable calibrations for the steady-state values of $\bar{\theta}_t$.

Debt		Equity	
Variable	Value of Second Moment	Variable	Value of Second Moment
e^Z	0.1061	e^Z	0.1061
$\tilde{K}_a$	0.0146	$\tilde{K}_a$	0.0775
A	0.0005	A	0.0029
$\tilde{n}$	0.0042	$\tilde{n}$	0.0028
$g(\bar{\theta})$	0.0048	s	0.0398
$\tilde{c}^e$	0.0780	$\tilde{c}^e$	0.0505
r	0.0083	r	0.0084
pa	0.0671	pa	0.0696
$\tilde{Y}$	0.1416	$\tilde{Y}$	0.1412

Table 2. Variances of key variables in debt and equity contracts

the variance of key variables given a simulation of 2,000 iterations of a technology shock for the debt contract and the equity contract, respectively. These magnitudes are given in Table 2. One can see that the magnitude of the shock, which we call e^Z, is equal for both simulations. This is by design, as we compare the responses to similar variations in technology growth. With this equivalent shock, the equity contract leads to a much higher deviation in the amount of capital dedicated to technology, which we can also see in Figure 2. In turn, this leads to a increase in the overall technology level roughly six times higher that of the debt contract. Note that we are referring to produced technology here, as the actual technology change will be a combination of the stochastic process, e^Z, and the endogenously produced technology, A. In our calibration, the stochastic element is stronger, while the relative comparison we make here is for the produced element alone. Thus, while we control for similar variances in the stochastic portion of total factor productivity, the different responses of the technology sector to these deviations in each contract imply that the overall deviations in total factor productivity are not equal between contracts.

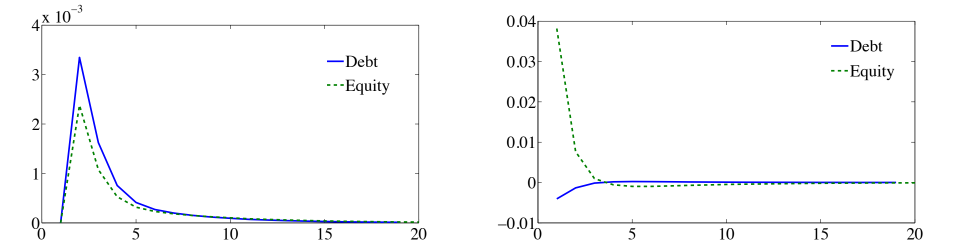
The entrepreneur is better off in the equity contract, too. While the entrepreneur receives more of the windfall in the debt contract, the increase in the amount raised from the household in the equity contract more than compensates for the increase in net-worth due to the windfall accruing to the entrepreneur under the debt contract.



(A) Impulse response for $K_{A,t}$ (B) Impulse response for A_t

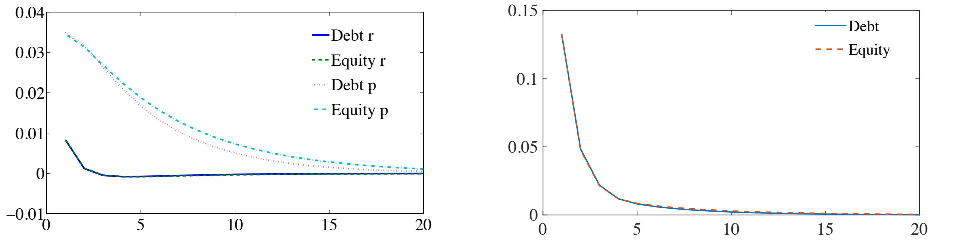
FIGURE 2. Response of contractual allocations to a technology shock

The left side of Figure 3 shows that the overall increase in net-worth for the investor is not much greater for the debt case, and the two contracts behave relatively similarly in the propagation of net-worth through time. In contrast, the share of the project accruing to the investor/lender varies greatly between the two contracts, as shown by the right side of the same figure. In the debt contract, an increase in the overall technology level and a subsequent increase in the entrepreneur's net-worth causes the entrepreneur to offer worse terms of investment for the lender. In contrast, the same fluctuations in the equity contract cause the entrepreneur to improve the terms of investment for the investor. This is because allowing for an improvement in the terms of investment for the household in the debt contract significantly changes the risk profile of the entrepreneur. Better terms for the household mean that the entrepreneur has to produce a higher amount before he even starts to make an income. After a positive technology shock, this effect is exaggerated by the increase in the market interest rate, due to an increased capital share in production, as shown in Figure 4. In contrast the equity contract allows the entrepreneur to share both the risks of technology shocks and the rewards. As a result, the entrepreneur will be more inclined to allow for a higher share to accrue to the household after a technology shock,



(A) Impulse response for $\tilde{n}_t$ (B) Response of investor's/lender's share

FIGURE 3. Progression of the entrepreneur's behavior after a technology shock



(A) Returns after a technology shock (B) Output after a technology shock

FIGURE 4. Entrepreneur's behavior after a technology shock

especially given that this improved terms of investment will allow for a much larger project overall. It is this risk-aversion in the debt contract that limits the increase in K_A to 19% of the increase in an equity contract following a positive technology development.

Finally, Figure 4 seems to imply that the production side of the economy is largely unaffected by the form of the contract in technology investment. While the price of patents remains slightly higher over time in an equity contract, the difference is very small relative to the overall deviation from the shock in the first place. More importantly, the interest rate and output rate are virtually identical between the two contracts.

However, the model simulated above has been de-trended to show how the economy returns to a steady-state after a stochastic shock. For non-stationary variables, we consider a ratio of variables with the same growth rate. Namely, we consider the ratio of the variable under question and the growth rate of technology, which drives the growth of the overall economy. Thus, Figure 4 actually implies that the model exhibits a co-movement of the ratio of output to technology. This is significant in that it means there is no extra growth in output under either contract due to the allocation of capital between sectors, but the gross growth in output itself is not equal under the two contracts. The equal co-movement in Figure 4 coupled with the higher increase in technology growth under the equity contract in Figure 2 imply that the output growth is actually five times greater in the equity contract. While a result of this magnitude is generally considered extreme in business cycle models, the economic growth associated with the development of personal computing and information technology in the 1980s and 1990s could correspond to such a large shift.

Furthermore, while we see that the returns to investment in the production sector is largely unchanged by the type of the contract in the technology sector, the response of the technology sector to such a shock changes direction when moving from an equity contract to a debt contract. As a result the correlation between the two reverses, in line with the fourth stylized fact in the introduction. This does come with one caveat: the variable which we use to represent technology development in this case only represents one composite element of total factor productivity; it does not represent the stochastic element. This caveat also applies to our explanation of total factor productivity variance. Thus, while we can show an underlying mechanism that would explain correspondence to such stylized facts, the magnitude of the mechanism relies on the calibration of the model to adjust the relative variance between the composite elements of total factor productivity. The key parameter in this regard is ζ, which we have set at 0.04, to dampen the effects of changes in the technology sector. The main justification of this was to allow a conservative result on the magnitude of the change from debt to equity, as 5x is already a startling result. In addition, the calibration of this model is very sensitive to changes in the value of ζ, so adjusting this parameter to match any particular stylized fact is inadvisable.

10. Conclusion

In a model in which technological investment occurs in a deliberate and incentivized way, we have shown that the type of financial contract plays a large role in both the dynamics of the development of new technology and in the magnitude of the response to independent technology shocks. A debt contract, characterized by borrowers' superior information of their own projects and their absorption of the macroeconomic risk, is characterized by relative stability at the cost of severely diminished quantities of capital raised. The equity contract, characterized by risk sharing with a preference for a small investor pool for efficient governance, allows total factor productivity to vary widely and closely follow the shocks to the economy.

These results are indeed in line with the stylized facts outlined in the introduction. First, the relative stability of total factor productivity during the 1960s corresponds to the use of the debt contract during this period, whereas the relatively high volatility in the 1980s corresponds with a move to equity.[19] Ultimately, our explanation for this is the absorption of risk by a relatively risk-loving and impatient agent through the debt contract in the 1960s.

Our model also corresponds with our second stylized fact, that the recession of 1973-74 was one characterized by relatively moderate yet persistent declines in utilization-adjusted total factor productivity. Again, the moderation is due to the absorption of risk by the entrepreneurs who borrow to fund their projects in the debt contract, while the duration is due to the reduction of these entrepreneurs' ability to

[19]We make no claim for the 1990s as we do not wish to make any claims about whether technology shocks were abnormally stable in the period of "great moderation." Obviously, the first decade of this millennium is characterized by two significant events in venture capital, which we have already discussed in the previous chapter.

lend following their absorption of a negative shock. This has the benefit of being the standard financial accelerator associated with previous analyses including this type of debt contract based on net-worth multipliers.

Most significantly, our model explains the large increase in investment in technological development that occurred in the 1980s. While the results of our model, a 5-fold increase in the investment to new technology, seems startling, it corresponds with the magnitude during this period. The lesson from this increase is simple: by allowing investors to share in the potential benefits from macroeconomic risk, their expected return from the growth created by investment in new technology spur an increase in the investment. Given the old adage, "a rising tide lifts all boats," we add the caveat, "only if you allow people to put their boats in the water." To abuse the metaphor, our model shows an additional effect that more boats in the water causes the rising tide to displace an even greater volume, benefiting each boat even further.

Finally, our model partially explains the divergence in total factor productivity and returns to investment in equity that occurred in the early 1980s. In our model, this is actually due to the negative correlation between produced technology and exogenous technology in the debt contract. As the real interest rate to investment in production ratio correlates positively with the exogenous technology process, a switch from a debt contract to an equity contract will result in a switch from a negative correlation between produced technology and return to production capital to a positive one. However, this explanation is only partial in that it only explains a switch in the correlation between return to production capital and *one* of the components of total factor productivity. The overall correlation would depend on the relative makeup of total factor productivity, which depends on the calibration of the model. This would require a more extreme calibration than the one we were prepared to offer.

Thus, we finally arrive at the question originally posed in this paper: were policy makers right in their decision to incentivize the move from debt to equity in technology investment? In the sense that the policy resulted in a dramatic increase in investment to technology, which had a positive effect on the development of digital technology, the answer is a resounding "yes." However, our model shows that this increased investment comes at the price of increased volatility in total factor productivity and relative investment in technology. While the 1990s and 2000s were marked by economic conditions that make it difficult to assess whether this increased volatility was a factor, we maintain that our model can explain some variance in TFP of the 1980s.

Appendix A: Equilibrium Conditions

Below are the conditions for the closed form model with debt and equity, respectively. These are the equations used in the simulation. These are the conditions *before* de-trending, and capital letters indicate aggregate variables.

Debt: The model starts with the aggregate production of technology, patents, and output:

$$A_{t+1} = \zeta(K_{A,t})^{1-\lambda}A_t^{\lambda} + A_t$$

$$T_{t+1} = q_t\zeta K_{A,t}^{1-\lambda}A_t^{\lambda}(f(\bar{\theta}) + g(\bar{\theta})) + T_t;$$

$$Y_t = e^{\Xi}L^{1-\alpha}K_{Y,t}^{\alpha}$$

where the difference between patents and technology is the change in value from the debt contract and the deadweight loss due to monitoring. We next have our market returns to capital and patents:

$$R_t = \alpha^2\frac{Y_t}{K_{Y,t}}$$

$$\Pi_t = (1-\alpha)\frac{Y_t}{A_t}$$

From here, we describe the contract through the participation constraint and the optimal conditions:

$$q_t\zeta K_{A,t}^{1-\lambda}A_t^{\lambda}g(\bar{\theta}) = (K_{A,t} - N_t)(1 + R_t - \delta)$$

$$1 + R_t - \delta = (1-\lambda)q_t\zeta K_{A,t}^{-\lambda}A_t^{\lambda}\left(1 - \mu\Phi(\bar{\theta}_t) + \mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)}\right)$$

Next, our entrepreneur's budget and Euler equation are:

$$p_t Z_{t+1} = (x + Z_t p_t)^{1-\lambda} q_t \zeta \left(\frac{K_{a,t}}{N_t}\right)^{1-\lambda} A_t^{\lambda} f(\bar{\theta}_t) - C_t^e$$

$$p_t = \beta\gamma \mathbf{E}_t \left[p_{t+1} q_{t+1} \zeta \left(\frac{K_{a,t+1}}{N_{t+1}}\right)^{1-\lambda} A_{t+1}^{\lambda} f(\bar{\theta}_{t+1}) \right]$$

And the household's Euler equation is:

$$1 = \beta \mathbf{E}_t \left(\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} (1 + R_{t+1} - \delta) \right)$$

$$p_t = \beta \mathbf{E}_t \left(\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} (p_{t+1} + \Pi_{t+1}) \right)$$

Finally, we close the model with an aggregate budget constraint (the household's budget would also fit this purpose) and the evolution of our stochastic element.

$$K_{Y,t+1} + K_{A,t+1} = (1-\delta)(K_{Y,t}) + Y_t - C_t - C_t^e$$

$$\log(\Xi_t) = \sigma_{\Xi} \log(\Xi_{t-1}) + e_t;$$

Equity: The model with the equity contract is essentially the same as the model with the debt contract. The only differences are the equations describing the contract and the behavior of the entrepreneur. Production and equilibrium returns are unchanged:

$$A_{t+1} = \zeta(K_{A,t})^{1-\lambda}A_t^{\lambda} + A_t$$

$$T_{t+1} = q_t\zeta K_{A,t}^{1-\lambda}A_t^{\lambda} - b\frac{K_{A,t}^2}{2} + T_t$$

$$Y_t = e^{\Xi}L^{1-\alpha}K_{Y,t}^{\alpha}$$

$$R_t = \alpha^2\frac{Y_t}{K_{Y,t}}$$

$$\Pi_t = (1-\alpha)\frac{Y_t}{A_t}$$

where the last two equations are determined by arbitrage. While the participation constraint differs from the debt contract, the only change is that the returns to the investor are no longer state dependent. Similarly, the optimality condition only differs in the second term on the right, which deals with the marginal cost of issuing more equity (the marginal benefits are the same in both cases, as are the opportunity costs for the funds).

$$s_t q_t\zeta K_{A,t}^{1-\lambda}A_t^{\lambda} = (K_{A,t} - N_t)(1 + R_t - \delta)$$

$$1 + R_t - \delta = (1-\lambda)q_t\zeta K_{A,t}^{-\lambda}A_t^{\lambda} - b(K_{A,t} - N_t)$$

Likewise, since the entrepreneur views $\bar{\theta}$ as exogenous when making inter-temporal decisions under the debt contract, the entrepreneur's Euler equations do not include any marginal changes in $\bar{\theta}$. As such, the entrepreneur's inter-temporal optimality condition is the same as under debt, only replacing $f(\bar{\theta})$ with s_t.

$$p_t Z_{t+1} = (X + p_t Z_t)^{1-\lambda}(1-s_t)q_t\zeta\left(\frac{K_{A,t}}{N_t}\right)^{1-\lambda}A_{t+1}^{-\lambda} - C_t^e$$

$$p_t = \beta\gamma\mathbf{E}_t\left[p_{t+1}q_{t+1}(1-s_{t+1})\zeta\left[\frac{K_{A,t+1}}{N_{t+1}}\right]^{1-\lambda}A_{t+1}^{-\lambda}\right]$$

Finally, the household's Euler equations, the aggregate budget, and the evolution of our stochastic variable are unchanged from the model under a debt contract.

$$1 = \beta \mathbf{E}_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (1 + R_{t+1} - \delta) \right)$$

$$p_t = \beta \mathbf{E}_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (p_{t+1} + \Pi_{t+1}) \right)$$

$$K_{Y,t+1} + K_{A,t+1} = (1 - \delta)(K_{Y,t}) + Y_t - C_t - C_t^e$$

CHAPTER 4

Optimism Under Uncertainty in Venture Capital

Abstract

We model uncertainty in the venture capital market through an investor's fear of model misspecification as in Hansen and Sargent (2008) using an endogenous growth model to incorporate the focus of venture capital on technology investment. We then alter the uncertainty aversion framework to allow for uncertainty-loving, or optimistic, behavior. We find that the market imperfections central to the endogenous growth model allow for perturbations to be welfare increasing. Furthermore, such perturbations could also increase the individual investor's value function, making it optimal to have certain non-rational preferences. Finally, the effects of this type of optimism cause minimal contributions to the price of technology, suggesting one should not worry about such optimism causing overheating in the venture capital market.

1. Introduction

Venture capital is an industry focused on private investment in new technology companies. Venture capitalists' focus on young companies and new technologies requires them to deal in assets and markets characterized by uncertainty, where uncertainty is defined in the Knightian sense. That is, they deal in environments in which events can occur with unknown probabilities: the assets in which venture capitalists invest are typically unproven technologies in which the probability of success is unknown.[1] Even as an aggregate, the risk and returns for the venture capital industry are problematic in assessing.

Furthermore, investors also face a large amount of uncertainty in their choice of funds for investment, allowing the disposition of the investors towards uncertainty to enter into investment decisions. Often, for example, these decisions are aided by consultants recommendations of funds, which are driven largely by "soft factors," rather than the funds' past performance.[2] Gompers and Lerner (1999) find that incentive compensation to venture capitalists is unrelated to the subsequent performance of the fund, suggesting that "neither the venture capitalist nor the investor knows the venture capitalist's ability. A venture capitalist who is just getting started will work hard even without explicit pay-for-performance incentives, because the fund can establish a good reputation."[3]

Recent evidence suggests that venture capitalists are overly optimistic in reporting their own returns. Analysis of venture capital databases have shown that roughly half of all venture capital funds report that their returns are in the "top quartile."[4] In addition, valuations and reported returns are inflated during fundraising, further

[1] Gompers and Lerner (2001)
[2] Jenkinson et al. (2013a)
[3] Gompers and Lerner (1999)
[4] Harris et al. (2012)

supporting the notion that investment decisions in the industry are made with an optimistic assessment of potential returns.[5]

One partner at a leading venture capital firm was recently quoted by the Wall Street Journal as saying, “Although we may have not reached the level of observing obvious greediness, there is certainly an absence of fear. Those who managed companies in 2008, or 13 years ago in 2001 know exactly how fear feels, and this is not it.”[6] Thus, even venture capitalists recognize the importance of fear and its alternative in their investment decisions. The partner continues, “you’re left with trying to advise someone to be pragmatically aggressive with some type of conservative backdrop or alternate strategy if the economy shifts.” This “pragmatic aggressiveness” is the focus of our investigation, with the conclusion that such behavior is advisable.

This chapter addresses the implications of fear and optimism in the face of uncertainty in technology investing. Specifically, given the uncertainty surrounding the venture capitalist’s decision, we allow the venture capitalist to alter her decision based on her disposition towards uncertainty. The venture capitalist can either display uncertainty-aversion or uncertainty-loving behavior. For uncertainty aversion, we use the maximin framework developed by Thomas Sargent and Lars Hansen, outlined in Hansen and Sargent (2008). Knowing that the venture capitalist’s best understanding of the price fluctuations, or “benchmark” model, is likely to be flawed, the overall investment level in technology is made by incorporating an venture capitalist’s desire to protect herself from particularly harmful deviations from that benchmark.

We follow Hansen and Sargent in describing uncertainty-averse behavior as a repeated game with a fictitious “evil agent” who alters the model in ways harmful to the household. Specifically, the venture capitalist entertains the belief that an evil

[5] Jenkinson et al. (2013b)
[6] Koh and Winkler (2014)

agent could alter her approximating model in a way that reduces her utility. However, models that are very different from the household's approximating model would be easier to detect, in that an econometrician would be more likely to distinguish these models from the approximating one given a dataset created by either process.[7] The household recognizes that repeated large deviations are less likely, and wishes to protect herself in a way that weighs the potential of single large deviations against persistent small deviations that are harder to detect. The weighting between such deviations is expressed as repeated game in which the evil agent weighs perturbations to the venture capitalist's model against the potential that such perturbations could be observed, expressed as entropy. A higher fear of misspecification means that the venture capitalist will allow the agent more room to maneuver around the benchmark model by reducing the agent's penalty for high-entropy perturbations of the model. Given the consideration for this evil agent's perturbation, the consumer then alters her own decision to account for this "worst case" distortion.

Optimism on the part of the venture capitalist is modeled through an analogous uncertainty-loving behaviour defined in the context of model misspecification. In both cases, a decision maker is concerned that the model on which she makes her decision might be inaccurate. As a result, a choice is made not just with respect to the model, but with respect to a series of models similar to the original one. In the robust control literature, fear coupled with the belief that the model might be wrong leads the decision-maker to act in a way that, for any circumstance, the correct action would be her best response to the worst possible model in the set she is considering.[8]

[7]Indeed, this likelihood that an econometrician could detect one model as the correct model given a dataset of a certain length is the basis behind "detection error probabilities," which we use to a sense of perspective of the extremity of our uncertainty aversion.

[8]Notice that we refer to the maximin framework both as a maximization within a set of models within a certain entropy distance from our approximating case and as a game with an evil agent who exacts a penalty for such deviations. Hansen and Sargent (2008) distinguish between the two descriptions and show that, under certain conditions, these treatments are equivalent. For our purposes, we treat the two descriptions as equivalent

However, for the optimistic venture capitalist, this fear is replaced with a hope that the correct model will be the best model in the set. Crudely, instead of protecting herself against worst case distortions of the model, the optimistic venture capitalist prepares for possible divine intervention.[9]

The optimism in this model is interesting in the context of the technology focus of venture capitalists, as the sector is fundamentally characterized by market imperfections making the decentralized equilibrium sub-optimal. We capture the technological focus of the venture capital sector through the framework of a basic endogenous growth model based on Romer (1990), allowing for transition dynamics and rational expectations in the price movements of patent rights over the new technology. This model exhibits three elements that violate the first fundamental theorem of welfare economics in a way that reflects actual practices in this investment class. First, the non-convex nature of the system leads to higher sensitivity of the model to agents' fear of potential misspecification. A higher sensitivity to deviations from the approximating model in ways that help the technological investment may lead to non-trivial differences in the impact of behavioral characteristics.[10] Second, the positive externality arising from the non-rival nature of ideas in creating new technology leads to a natural underinvestment in technological development in a competitive equilibrium. A tendency toward overvaluing such assets due to model misspecification might therefore lead to a social good. Finally, the model exhibits market concentration in one sector, as patent holders have monopoly rights over the intermediate goods covered by their patent. This is an artificial creation in the sense that this market power is artificially created by government policy and does not exist naturally through incentives of the market,

[9]Obviously, we refer to a loving God rather than a vengeful one.

[10]The non-convex nature also has the potential to lead to sunspot solutions in endogenous growth models, such as in Benhabib et al. (1994) and Xie (1994). However, the conditions for such equilibria are not fulfilled in this version of the endogenous growth model.

but it is real in the sense that such monopoly rights created by patents already exist in the real world.

Our findings are four-fold. First, the inclusion of an endogenous growth mechanism magnifies the impact of model uncertainty on consumption choices and allocation of labor between sectors. This is hardly surprising, as the choices in an endogenous growth model have an increased impact on the growth rate. Second, under endogenous growth, the economy is closer to the social optimum where the representative household is uncertainty-loving. While this result is somewhat limited in the sense that it requires all venture capitalists to act equally uncertainty-loving as each other, we discuss alternative interpretations of the representative agent.[11] In addition, the idea that all venture capitalists might want to behave in some uncertainty loving way is promoted by the third finding: the individual household could be better off exhibiting uncertainty-loving behavior in the sense that the discounted sum of realized consumption bundles would be higher. Finally, we comment on the possibility of the venture capital market becoming overheated due to this type of optimism in light of the comments by the fund manager quoted above. We find that the cyclical volatility in the market has a much higher impact on the price of technology than any reasonable effect of optimism due to uncertainty.

As robust control theory is relatively modern in the field of macroeconomics, the existing literature relevant to this exercise is scarce. For a full treatment of formulating behaviour under the fear of model uncertainty, Hansen and Sargent (2008) is the standard resource. Of the applications of the Hansen Sargent framework, only Cagetti et al. (2002) seem to consider the framework in the context of growth, which

[11]This is because the mechanism in which the disposition of venture capitalists towards uncertainty affects the growth rate is through dependence on investment in technology interacting with the labor allocation through the price of technology. As this is a pecuniary effect in a model in which individuals are small compared to the aggregate, an individual venture capitalist with a unique disposition towards uncertainty would not receive any benefit from such behavior.

is exogenously determined in their model. For the treatment of investment behavior under uncertainty, the most relevant reference is Hansen et al. (1999), who explore the impact of model uncertainty on asset prices. However, their model treats the underlying determinants of asset prices as exogenous, whereas the general equilibrium derivations are crucial for this analysis. Croce et al. (2012) is the closest project to our own, as it builds upon a fully endogenous growth model with robustness to price uncertainty in the market with respect to the market pricing of uncertainty in Bansal and Yaron (2004) endowment economy. However, the Croce paper limits itself to the replication of the Bansal-Yaron figures with an endogenous growth model, while this paper focuses more specifically on the effects of venture capital and risk-loving behavior.

Technically, Bidder and Smith (2011) is the closest to our formulation in this project, as they use a Ramsey model in a similar context to the standard real business cycle literature. The idea behind Bidder and Smith is that the beliefs based on uncertainty aversion create a type of Keynesian "animal spirits" in which the agents' behavior is self-fulfilling in that the actual transition path is altered by the implied probability density of the consumer's uncertainty aversion. The current project is an extension of the Bidder and Smith paper in the sense that it describes venture capitalists' uncertainty-loving behavior as self-fulfilling in the sense that it interacts with an externality to cause the market to move towards a more efficient equilibrium.

2. Approximating model

Throughout this paper, we discuss the implications of venture capitalists fearing that their understanding of the economy is wrong. To clearly describe that fear, they build an "approximating" model, around which they consider possible deviations against which they desire to protect themselves. This approximating model is given by a fully endogenous growth mechanism of increasing product variety, as in Romer (1990), altered to allow for endogenous price fluctuation of technology. However, we follow our previous chapters in referring to the factor of production allocated between production and technology development, $H_{Y,t}$, as "effective labor" rather than Romer's "human capital," as the total amount to be allocated is supplied inelastically at the beginning of each period, rather than accumulating as capital would.

Given a Cobb-Douglas production function for aggregate production,

$$Y_t = e^{Z_t} K_t^{\alpha} (A_t H_{Y,t})^{1-\alpha}, \tag{56}$$

we follow Romer (1990) in defining an *ad-hoc* endogenous transition equation for the progression of the Solow Residual:

$$\frac{A_{t+1} - A_t}{A_t} = \zeta(H - H_{Y,t}) \tag{57}$$

As the value of a patent derives from the monopoly rights to the technology represented by the patent, we can also say that the level of patents grows at the same rate as technology, with $T_t = A_t$.

The source of risk in this economy is in a stochastic process altering the effectiveness of output:

$$Z_t = \rho Z_{t-1} + \epsilon_t \tag{58}$$

where ε is a random shock to the economy, and Z follows a stochastic process with persistence parameter, ρ, and a stochastic shock, ϵ, normally distributed with mean zero and variance of σ_Z^2. Though Romer's (1990) paper focused on the growth implications of this model in a deterministic setting, other authors have used this model in a stochastic setting.[12]

The labor used for developing these new ideas is allocated according to an arbitrage condition equating the marginal benefit from labor between sectors:

$$\zeta A_t P_t = (1-\alpha)\frac{Y_t}{H_{Y_t}} \tag{59}$$

where P_t is the price of technology.

There are three non-standard elements to the endogenous growth mechanism arising from incentivized investment in research and development. First, we have non-convexities in the production function in the sense that the production function exhibits increasing returns to scale in A_t, K_t, and $H_{Y,t}$.[13] Notice that this also implies the returns to the factors of production are greater than the sum of their marginal benefits. This is a fundamental attribute to models of the "expanding variety" type of endogenous technological change. The level of technology is represented through the Solow residual on an otherwise homothetic production function.[14] In a competitive equilibrium, the homogeneity would imply that the revenue from production would all go to paying individual factors of production. Thus, there would be none left for paying for technological development.

In this model, these non-convexities are addressed through another market imperfection, namely the monopoly power of patent holders. One benefit of the Romer model is that the market power in intermediate goods yields a closed-form solution

[12]Most notably, Comin and Gertler (2006)

[13]Note that the increasing returns to scale are specific to aggregate production, as given by (56). Individual producers still treat technology as exogenous, and it is the increase in the number of products that leads to the expansion of aggregate production.

[14]As a break in homogeneity can be viewed as a return to scale, we can justify the assumption of a homogeneous production function by saying that in the general equilibrium, all possible returns to scale have been realized except those from the development of technology over time.

for a capital interest rate that is a function of already existing parameters. The effect is to reduce the returns to one factor of production, capital, in order to pay for the investment in future growth. Limiting the returns to capital in order to incentivize investment in technological development allows a portion of the output in the economy to go to the technology sector to pay for the labor necessary to create new technology.

Our interest rate is:

$$R_t = \alpha^2 \frac{Y_t}{K_t} \tag{60}$$

which is similar to the interest rate in a competitive equilibrium, with α^2 now replacing α in the competitive case. Since, α is less than one, the interest rate above is less than the equilibrium interest rate in a competitive equilibrium. As a result, we can allocate the rest of the resources that would ordinarily go to capital to paying off the patent holders in each period:

$$\Pi_t = \alpha(1-\alpha)\frac{Y_t}{T_t}, \tag{61}$$

In addition to increasing returns to scale in production and monopoly power in capital, our third non-standard element in this model is an externality arising from the development of technology. That is, the deliberate development of a new technology, A_{t+1}, by allocating labor to the effort, relies on previous similar efforts, as A_t enters explicitly into (57).[15] It is worth noting that this formulation is somewhat controversial.[16] However, rather than taking a stance on this issue, we simply use it as a standard baseline commonly used in the literature.

[15]The justification for this is generally through an argument relying on the non-rival nature of ideas, though the motivation for using this form in our context is admittedly for the desired stationarity of the right side of equation (57).

[16]See Jones (1999)

The equations (56) to (61) show the aggregate behavior of production in our two sectors and prices. This aggregate behavior is common knowledge across all our agents, and is given by the information set S_t. When our agents display a certain disposition towards uncertainty, they have access to this information set as their approximating model, but are doubtful of its accuracy.

2.1. Consumption and Investment. We now break from the standard treatment of the Romer model to address venture capitalists' behavior. There are two types of consumer-investors, each facing an optimal consumption problem. Households invest in the production sector through the purchase of production bonds. Also, the household is the sole provider of labor in our economy.[17] Central to the household's decision is the budget constraint:

$$k_{t+1} = (1 + R_t - \delta)k_t + W_{Y,t}h_{Y,t} + W_{A,t}h_{A,t} - ch_t \tag{62}$$

where k_t, ch_t, $h_{A,t}$, and $h_{Y,t}$ denote that these are household-level production capital, consumption, provision of labor to technology development, and provision of labor to production, respectively. Here, lowercase letters denote individual-level variables. In addition to the budget constraint containing variables within the household's control, the household also considers the progression of aggregate variables in its optimization, given by the information set S_t.

Venture capitalists (or VCs) face a similar optimization, in which they invest in patents, t_t, priced at p_t which return the monopoly profits from the patent, outlined in the previous section. The venture capitalist uses this asset to smooth her consumption,

[17]This assumption is not too extreme, as the labor is allocated through a static arbitrage condition, so the household's relative propensity to hold precautionary savings will not enter into the labor allocation. Allocating labor to the household will lead to increased wealth to the household, but the relative wealth of the two types of agents is determined directly through our calibration

ce_t, following the budget identity:

$$P_t t_{t+1} = (P_t + \Pi_t) t_t - ce_t \tag{63}$$

In the approximating case in which the venture capitalist is not concerned with the possibility that the model might be misspecified, both the household and the venture capitalist face the optimization:

$$\max_{c_t} \sum_{t=0}^{\infty} \mathbf{E}_0[\beta^t u(c_t)] \tag{64}$$

where c_t refers to either ch_t for the household or ce_t for the venture capitalist, and the maximization is subject to the budgets, (62) and (63), for the household and venture capitalist, respectively.

If we specify our point utility function, $u(c_t)$, as one with a constant relative risk aversion with a risk parameter σ,[18] this yields the standard Euler equations:

$$ch_t^{-\sigma} = \beta \mathrm{E}_t[ch_{t+1}^{-\sigma}(1 - \delta + R_{t+1})] \tag{65}$$

$$ce_t^{-\sigma} = \beta \mathrm{E}_t \left[ce_{t+1}^{-\sigma} \frac{P_{t+1} + \Pi_{t+1}}{P_t} \right] \tag{66}$$

2.2. Equilibrium. Given our description of production, technology development, and the behavior of each of our two types of agents, we are now ready to describe our decentralized equilibrium. We refer to this equilibrium as one incorporating endogenous growth.

Definition 4 (Decentralized Equilibrium). *The Decentralized Equilibrium is determined by:*

[18] i.e. $\frac{c_t^{1-\sigma} - 1}{1-\sigma}$

(1) *The household choosing a consumption level, ch_t, to optimize its utility, (64), subject to its budget constraint, (62), under the information set S_t,*

(2) *Intermediate goods producers choosing an optimum level of their intermediate good, x_t, also subject to the information set S_t to produce, yielding profit, π_t accruing to the patent holder given by (61),*

(3) *Final goods producers choosing the optimal level of output, y_t, also subject to the information set S_t to produce paying competitive prices, r_t and $w_{Y,t}$ for capital and labor, respectively. These levels are given by (60), and the right side of (59), due to arbitrage.*

(4) *The venture capitalist optimizing its utility, (64), by choosing a consumption level, ce_t, subject to its budget constraint, (63), and a rational understanding of the information set, S_t, and*

(5) *Technology producers choosing the optimal amount of technology to produce, a_t, given the prices, $w_{A,t}$, which equal the marginal productivity of labor in the technology sector shown in the left side of (59),*

(6) *Wages in both sectors being equal through arbitrage.*

(7) *Household labor supplied inelastically at the beginning of each period.*

(8) *Aggregate production given by the Cobb-Douglas function, (56),*

(9) *Markets clearing for aggregate variables $Ch_t, Ce_t, Y_t, T_t, A_t$ and K_t determined through the transition functions, (57), and*

$$K_{t+1} = (1-\delta)K_t + Y_t - Ch_t - Ce_t, \tag{67}$$

(10) *The evolution of patent price determined by the requirement that markets clear.*

2.3. Relation to Other Models. One benefit of this model is that it is relatively reducible to a standard real business cycle model in which growth is exogenous. To arrive at this model, we eliminate the arbitrage condition (59) and set ζ equal to zero. The households can still invest in patents, whose value continues to be represented by the profits from the patent returns. However, the growth rate of patents is no longer dependent on a household allocation of labor. More specifically, we define the equilibrium:

Definition 5 (Corresponding RBC Equilibrium). *The Decentralized Equilibrium in the model to which our approximating model reduces is given by:*

(1) *The household choosing a consumption level, ch_t, to optimize its utility, (64), subject to its budget constraint, (62), under the information set S_t,*
(2) *Intermediate goods producers choosing an optimum level of their intermediate good, x_t, also subject to the information set S_t to produce, yielding profit, π_t accruing to the patent holder given by (61),*
(3) *Final goods producers choosing the optimal level of output, y_t, also subject to the information set S_t to produce paying competitive prices, r_t and $w_{Y,t}$ for capital and labor, respectively. These levels are given by (60), and the right side of (59), due to arbitrage.*
(4) *The venture capitalist optimizing her utility, (64), by choosing a consumption level, ce_t, subject to its budget constraint, (63), and a rational understanding of the information set, S_t, and*
(5) *A and T grow exogenously at the long-term growth rate. (here, the asset, T, represents an equity in a rent-seeking venture, returning profit, Π, but not contributing to any production),*

(6) *The same level of labor as allocated to production in the steady state of the model with endogenous growth is supplied exogenously to the household and allocated entirely to the production sector,*

(7) *Aggregate production given by the Cobb-Douglas function, (56),*

(8) *Markets clearing for aggregate variables* $Ch_t, Ce_t, Y_t, T_t, A_t$ *and* K_t *determined through the transition functions, (57), and*

$$K_{t+1} = (1 - \delta)K_t + Y_t - Ch_t - Ce_t, \tag{68}$$

(9) *The evolution of the price of patents determined by the requirement that markets clear.*

Notice that we achieve this equilibrium through setting the labor level in production as equal to its steady state value in the case in endogenous growth. This also implies that there is no labor allocated to the technology sector, leading to a loss of income from the wages from that sector. This leads to a slight decrease in wealth in the steady state. However, the effects of this decreased labor income do not significantly affect the response of either agent to changes in the venture capitalist's disposition towards uncertainty.

3. Uncertainty aversion

We now allow for the possibility that the venture capitalist does not trust her approximating model. Specifically, we allow for the VC to fear that any of the functions nested in her transition function are incorrect. Instead, the VC treats the above transition as a benchmark, and entertains the possibility that the true evolution of the state variables, K_t, A_t, and T_t, arise from any one of a set of similar models to the one above. We represent these alternative models as perturbations of the original, indexing the extremity of the perturbation as a function of statistical entropy. The VC weighs the impact of possible deviations on her utility against this entropy in determining a policy at a given state. Larger deviations, i.e. those with a higher entropy, can potentially alter the decision of the VC if the impact on the household utility of such a deviation is particularly harmful to the household's utility.

As the consumer's decision is intertemporal, the specification of the "worst possible" model is not entirely straightforward, as one must specify how a perturbation today affects future decisions and perturbations. Specifically, we must address the possibility of compounded perturbations over time causing the approximated model and the distorted model to diverge over time.

3.1. Robust control and max-min specification. Hansen and Sargent (2008) outline a method for the dynamic consideration of multiple distortions over time using a "max-min" framework. Specifically, as noted above, the VC makes her decision "robust" by acting as if the model took the form of the worst possible deviation in the set of potential alternative models. According to Hansen and Sargent, she does this by imagining the existence of an "evil agent" which chooses the worst model in the set. The benefit of this specification is that the intertemporal considerations can be described through information constraints and strategic interaction between the two agents.

For the purpose of this paper, we specify the game as a repeated, sequential move game, with the fictitious agent moving second. The VC and evil agent both know the state of the economy and the incentives faced by each other. While in each period both agents are optimizing with an infinite horizon, they are not committed to their strategy past the current period. As a result both agents update their best response functions in each period. We search for stationary solutions in which the agents are non-cooperative and we limit ourselves to Markov-perfect equilibria for simplicity. The intuition behind these assumptions is tricky, and the timing conventions should largely be interpreted as the type of alternative models the venture capitalist is concerned with the most. By specifying Markov-perfect equilibria with non-cooperative strategies, we are essentially saying that the venture capitalists limit their considerations of uncertainty to cases that are not dependent on paths of previous states nor on paths of their previous decisions. While this is only one possible timing convention, assuming that the venture capitalists limit their considerations to non-path-dependent distortions seems like a reasonable starting point for simplicity.[19] As with standard Markov-perfect equilibria, we only consider payoff-relevant information.

The deviations themselves take the form of a multiplicative adjustment on the preference of the VC:

$$\max_{c_t} \min_{m_{t+1}} \sum_{t=0}^{\infty} \mathbf{E}[\beta^t M_t\{u(c_t) + \theta\beta\mathbf{E}(m_{t+1}\log m_{t+1})] \tag{69}$$

where the quantity M_t is the evil agent's "perturbation." This perturbation takes the form of a martingale with step increments, m_{t+1}, so that $M_{t+1} = m_{t+1}M_t$. To ensure that this martingale increment represents a normalized distortion of the transition probability of the VC's model, we also set $M_0 = 1$ and $\mathbf{E}(m_{t+1}|\epsilon^t, K_0, T_0) = 1$.

To see that martingale deviation of preferences directly implies a normalized distortion to the probability density, $p(X_{t+1}|X_t)$, we assign a value function, $\tilde{V}(X_t)$, to

[19]While we appeal to simplicity and refrain from defending any particular interpretation of this simplification, such conventions could have intuitive explanations. In the context of venture capital, the lack of commitment is particularly prescient, as venture capitalists often require capital commitments of multiple periods. Indeed, the reduced-form expression for our value function is similar to that of Epstein and Zin (1989), which has important implications for duration of investments and timing of returns. However, addressing the long-term commitments required by venture capital projects is outside the scope of this thesis. In addition, some papers (e.g. Levin et al. (1999)) specify models in which the uncertainty arises through a lack of confidence in the response of other agents in the economy to potential shocks. Thus, such limitations could have interpretations regarding the belief of venture capitalists concerning how they expect other venture capitalists to react to shocks.

the maximization in (69):

$$\tilde{V}(X_t) = \max_{c_t} \min_{m_{t+1}} \{M_t \left[u(c_t) + \theta\beta\mathbf{E}(m_{t+1}\log m_{t+1})\right] + \beta\mathbf{E}(\tilde{V}_{t+1})\} \tag{70}$$

As the evil agent is left to choose her martingale increment at any given time, the quantity, M_t, can be viewed as a pre-determined state variable. Thus, we can define a value, $V(X_{t+s}) \equiv \frac{\tilde{V}(X_{t+s})}{M_t}, \forall s \geq 0$, to write (70) independently of M_t:

$$V(X_t) = \max_{c_t} \min_{m_{t+1}} \{u(c_t) + \theta\beta\mathbf{E}(m_{t+1}\log m_{t+1}) + \beta\mathbf{E}(m_{t+1}V_{t+1})\} \tag{71}$$

where the last term on the right comes from the identity $\frac{\tilde{V}_{t+1}}{M_t} = \frac{\tilde{V}_{t+1}}{M_{t+1}}\frac{\tilde{M}_{t+1}}{M_t} = m_{t+1}V_{t+1}$

We follow Hansen and Sargent in using the parameter, θ, to represent the VC's concern that her model might be an incorrect representation of the economy. Higher values of θ represent lower concern for model uncertainty. As $\theta \to \infty$, the VC's behavior converges onto her behavior in the approximating model. We interpret θ as the weight the VC places on entropy caused by potential deviations. Placing a large weight on entropy is to place a relatively low weight on the effect a deviation would have on the VC's utility, causing the VC to concern herself less with such possible deviations.

The maximization is subject to the original transition function, (63), the VC's understanding of the transition of aggregate variables, (56) through (61), and the condition describing the evolution of the martingale process in the previous paragraph.

As we have set the timing of the game as a repeated Stackleberg game with the VC as a leader and the evil agent as a follower, we allow the evil agent to minimize without regard to the VC's policy decision. The VC then optimizes with the evil agent's expected best response in mind. In such a case, the evil agent's optimal condition is given by:

$$m_{t+1} = \frac{e^{\frac{-V(X_{t+1})}{\theta}}}{\mathbf{E}\left(e^{\frac{-V(X_{t+1})}{\theta}}\right)} \tag{72}$$

Substituting this into the VC's optimization, (71) becomes,

$$V(X_t) = \max_{c_t} u(c_t) - \theta\beta \log \mathbf{E}\left(e^{\frac{-V(X_{t+1})}{\theta}}\right) \tag{73}$$

Using the same CRRA utility of the approximating case, our Euler equations are given by,

$$c_t^{-\sigma} = \beta \mathrm{E}_t \left[c_{t+1}^{-\sigma}(1 - \delta + R_{t+1}) \frac{e^{\frac{-V(X_{t+1})}{\theta}}}{\mathbf{E}\left(e^{\frac{-V(X_{t+1})}{\theta}}\right)} \right] \tag{74}$$

$$c_t^{-\sigma} = \beta \mathrm{E}_t \left[c_{t+1}^{-\sigma} \frac{P_{t+1} + \Pi_{t+1}}{P_t} \frac{e^{\frac{-V(X_{t+1})}{\theta}}}{\mathbf{E}\left(e^{\frac{-V(X_{t+1})}{\theta}}\right)} \right] \tag{75}$$

4. Optimism and max-max

The motivation for the current paper relies on the premise that venture capitalists might have unique behavioral attitudes towards uncertainty. Specifically, we address the possibility that a venture capitalist might be more optimistic, or uncertainty-loving, than a neutral agent. We model this as a converse of the robust control framework outlined above. Again, the VC is concerned with the possibility that her model of the evolution of the state variables might be inaccurate. As before, this is modeled through the VC entertaining a set of possible alternative models closely related to its approximating model in making its decision. However, instead of being concerned with the worst possible deviation, the VC is optimistic that the best possible alternative specification of the model might be the one at play.

Technically, the optimistic VC looks very similar to the uncertainty-averse case above. We continue to model the VC as behaving as if she were in a repeated game with a fictitious agent. As before, the game takes the form of a sequential game, with no commitment and the fictitious agent moving second. However, instead of assuming an evil agent who minimizes over the set of possible alternate models, the VC assumes a benevolent agent that maximizes over a similar set. In addition, the penalty takes the opposite sign as well, as the objective of the fictitious agent is opposite that of the previous case. Again, we assume both agents know the state of the economy and the incentives faced by each other, and we limit ourselves to Markov-perfect equilibria.

Instead of (71), our value function is now,

$$OV(X_t) = \max_{c_t} \max_{m_{t+1}} \{u(c_t) - \theta\beta\mathbf{E}(m_{t+1}\log m_{t+1}) + \beta\mathbf{E}(m_{t+1}OV_{t+1})\} \tag{76}$$

where OV represents the "optimistic" VC. Note that this is not equivalent to the above value function using the identity, $\min x \equiv -\max x$, as that would lead to an inner optimization of $\max_{m_{t+1}} -\theta\beta\mathbf{E}(m_{t+1}\log m_{t+1}) - \beta\mathbf{E}(m_{t+1}V_{t+1})\}$, where the coefficient ahead of the second term is the opposite of the coefficient in the optimistic case.

The optimal value for the benevolent agent is,

$$m_{t+1} = \frac{e^{\frac{OV(X_{t+1})}{\theta}}}{\mathbf{E}\left(e^{\frac{OV(X_{t+1})}{\theta}}\right)} \tag{77}$$

leading to a simplified value function of the form,

$$OV(X_t) = \max_{c_t} u(c_t) + \theta\beta \log \mathbf{E}\left(e^{\frac{OV(X_{t+1})}{\theta}}\right) \tag{78}$$

4.1. The social planner's solution. As we have previously mentioned, the endogenous growth model has three elements that make the competitive equilibrium distinct from what one would expect from a social planner's solution. First, there are externalities to investment in technology. Investing in the level of technology increases the overall level of the TFP for production in the entire economy, which we attempt to allow the investor to recoup from some market power over the intermediate capital good created by a new patent. Second, there are economies of scale. This is the "shoulders of giants" effect, in which the level of technology also contributes to the potential development of technology in future periods. Finally, we have the monopoly power over intermediate capital goods.

Definition 6 (Social Planner's Solution). *The Social Planner's Solution is determined by:*

(1) *A social planner choosing the aggregate level of consumption, allocation of labor to output, and capital,* C_t*,* $H_{Y,t}$ *and* K_t*, respectively, to optimize over the same objective as the decentralized equilibrium, (64), determined without the restraints to information and competition by using (79) and (57).*
(2) *Aggregate labor supplied inelastically at the beginning of each period.*
(3) *Aggregate production given by the Cobb-Douglas function, (56),*
(4) *Markets clearing for aggregate variables* $Ch_t, Ce_t, Y_t, T_t, A_t$ *and* K_t *determined through the transition functions, (57), and*

$$K_{t+1} = (1-\delta)K_t + Y_t - Ch_t - Ce_t, \tag{79}$$

(5) *The evolution of patent price determined by the requirement that markets clear.*

The social planner's solution described above can be represented through the functional

$$SV(X_t, \epsilon_t) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbf{E}_t[SV(X_{t+1}, \epsilon_{t+1})] \tag{80}$$

$$\text{s.t. (57) and (79)}$$

where the function, $SV(X_t, \epsilon_t)$, describes the social value of arriving at any particular state, X_t, ϵ_t.

Schmidt[20] offers a strong explanation of how we can tie the price of technology at any time to the relative marginal products of labor in the technology and production sectors: if we interpret the shadow price of the technology path, (57), in our Lagrangian as the price of technology measured in utils, and the shadow price of the capital transition path as the price of capital or output, we can view the ratio of the two as the price of technology in terms of capital. We can then verify that the allocation of labor between the two sectors is determined by a condition in which the ratio of marginal products of labor from each sector is equivalent to the ratio of the shadow prices.

The social planner's solution gives us the Euler equations:

$$c^{-\sigma} = \beta E_t \left[c'^{-\sigma} \left\{ 1 - \delta + \alpha \frac{Y'}{K'} \right\} \right] \tag{81}$$

$$\lambda_t = \beta(\zeta H + 1) E_t \left[\lambda_{t+1} \right]$$

$$\text{where } \lambda_t = \frac{(1-\alpha) Y_t C_t^{-\sigma}}{\zeta A_t H_{y_t}}$$

[20]Schmidt (2001)

This social planner's solution has some interesting implications. If we view (81) as the intertemporal consumption decision, we can see that our consumption in each period should be higher than the decentralized equilibrium allows, since the interest rate in the decentralized equilibrium, $\alpha^2 \frac{Y'}{K'}$ is less than the marginal product of capital in the production function, $\alpha \frac{Y'}{K'}$.

Looking at (82), we can see that if consumption and output grow at the same rate as technology, Θ_t, the dummy variable, λ grows at the rate of Θ^{σ}. In a steady state de-trended by this growth rate, we can pin down the optimal growth rate to a function model parameters:

$$\Theta = \beta(\zeta H + 1)^{\frac{1}{\sigma}} \tag{82}$$

Two things are interesting about this. First, the parameter, ζ, can be viewed as the effectiveness of labor in producing new technology. As it approaches zero, even large amounts labor are useless in creating new technology. If we normalize the amount of overall labor to one, we see that values for ζ of even 0.1 will make the growth rate 5% under our benchmark calibration of σ. We view our benchmark calibration as rather high, and a lower value of σ would only make the this growth rate higher. Thus, the above equation implies that there is considerable room for improvement in our steady-state growth rate from our observed levels.

Second, it is interesting that the optimal growth rate above is dependent on the risk aversion parameter of the representative VC. A higher risk aversion implies a lower ideal growth rate. Intuitively, this makes sense: if we interpret (81) as determining the optimal level of consumption, then (82) determines the optimal allocation of labor across sectors. A higher risk-aversion parameter will force a higher proportion of labor to production under current technologies, as they provide a more immediate, and therefore less risky, return.

5. Calibration

We have already mentioned that the benchmark model above is reducible to a basic real business cycle. As such, we calibrate the model to the parameters of the corresponding business cycle model where possible. Our discount parameter, β, is set to 0.95 to represent annual data. Our risk aversion parameter, σ, is set to 2, which along with a steady state technology growth rate of 1.5%, implies our steady real return to production capital is 10.5%, which is high compared to historic risk-free rates, but not extreme compared to interest rates faced by manufacturing sectors.

More troublesome is our depreciation on capital of 5%, which is abnormally low for a calibration representing yearly data. Ultimately, a higher depreciation would lead to an interest rate that is even higher than our 10.5%, which would increase both the returns to capital and the profit streams to technology, effectively making capital a much larger share of overall production. As we will see that the predominant factor in the consideration for uncertainty is the extent of the market imperfection in the capital sector, increasing the impact of capital would magnify this effect. We deliberately lower the depreciation rate to take a conservative stance on the effect of uncertainty on capital.

We set our Cobb-Douglas parameter, α, to fix the steady-state consumption at 80% of total output, giving us a value of 0.455. While this is higher than standard RBC values, the adjustment is necessary given the requirement that the model divert some capital away from the production sector. As such, the capital stock is 3 times the annual output. This value is highly sensitive to the parameters, so we are willing to accept any calibration that allows a value between 2 and 20. Normalizing the amount

Coefficient	Level	Description
α	0.45	Cobb-Douglas parameter for capital share
β	0.95	Inter-temporal discount factor
δ	0.05	Capital depreciation
σ	2	CRRA utility parameter
H	1	Inelastic supply of labor, renewed in each period
ζ	0.2	Efficiency parameter of production of new technology

Table 1. Parameter values for both the endogenous growth model and the corresponding RBC model

of labor to one, we fix the multiplier on technology production, ζ, at 2 to allocate 92.5% of labor to the production sector. We are left with a steady-state ratio of $\frac{\pi}{P}$ of 0.085, implying a 8.5% real return to the technology sector.

For all of our simulations, we use an AR(1) shock process, z_t, with a persistence parameter of 0.5 and a volatility of 0.011. While this persistence parameter is much lower than most technology shocks in real business cycle models, the existence of endogenous growth acts as an extra source of persistence for our Solow residual, the combined $A_t e^{z_t}$. The volatility of this parameter is taken from Bidder and Smith (2011).

Finally, all the results presented below are in terms of stationary variables. For variables that are non-stationary in the model, we have de-trended them as described in the introduction to this thesis.

6. Results

Using the models and calibrations presented above, we now show that uncertainty aversion in an endogenous growth framework allows for stronger behavioral changes due to attitudes under uncertainty than a similar model with no endogenous growth mechanism. Furthermore, as the behavioral changes studied are particular to the entrepreneur in our model, the behavior of the household is unchanged in the case without endogenous growth. In contrast, the endogenous growth framework allows the entrepreneur's behavioral alteration to impact the decisions made by the household through the effects on the growth rate. This effect ends up increasing the realized utility of both the entrepreneurs and households for uncertainty-loving entrepreneurs, resulting in a Pareto improvement over the approximating, uncertainty-neutral, case. In addition, we interpret the increase in realized utility to the entrepreneurs as meaning it would be beneficial to their own interests if entrepreneurs could choose to be uncertainty-loving as a group.

We also show the overall effects of attitudes towards uncertainty on the price of technology in our model. For this, a perspective on the degree of uncertainty aversion and uncertainty-loving behavior is described through "detection error probabilities." We show that while uncertainty-loving behavior does lead to a rightward shift in the probability distribution of technology prices, the magnitude of this shift is small when compared to the variance of the distribution under all simulated attitudes to uncertainty. We interpret this as meaning that fear of bubbles due to "overly-optimistic," or uncertainty-loving, investors is probably misplaced; while we recognize that there may be some overvaluation of technology assets in the market, it is not due to the mechanism described in this paper.

As the model presented above extends beyond a linear-quadratic specification, we use numerical methods for computing the optimal policy and resulting states. For all the quantitative results presented below, we use a third-order Taylor approximation of the policy function, centered at the certainty-equivalent steady state. This technique, and the motivation for choosing it, is outlined in Appendix A.

6.1. Simulation results. We simulate both the endogenous growth model presented above and the RBC model to which it is reducible using a third-order Taylor approximation around a certainty-equivalent steady state. The simulation lasts for 100,000 periods, with a "burn-in" of 1,000 periods.

Variable	θ—lower values reflect a higher degree of uncertainty aversion					
	∞	400	20	10	8	4
$\tilde{C}$	1.1965	1.1965 (0.0)	1.1965 (0.0)	1.1965 (0.0)	1.1965 (0.0)	1.1965 (0.0)
$\tilde{C}e$	0.2710	0.2708 (-0.1)	0.2702 (-0.3)	0.2693 (-0.6)	0.2689 (-0.8)	0.2682 (-1.0)
$\tilde{K}$	2.0593	2.0593 (0.0)	2.0593 (0.0)	2.0593 (0.0)	2.0593 (0.0)	2.0593 (0.0)
$\tilde{Y}$	1.3294	1.3294 (0.0)	1.3294 (0.0)	1.3294 (0.0)	1.3294 (0.0)	1.3294 (0.0)

% deviation from approximating model in brackets

Table 2. Mean distortions of simulated variables with uncertainty-aversion in an RBC model with monopoly

Table 2 presents the distortion to the first moments of key variables for varying degrees of pessimism under uncertainty aversion. One can see that without endogenous growth, the venture capitalist's attitude toward uncertainty has no noticeable effect on the consumption behavior of the household. Furthermore, the overall production in the economy also seems unaltered. This is to be expected, as the venture capitalist is merely rent-seeking in the absence of endogenous growth. The money received from the profit stream on the patent does not go towards increasing the overall level of production. As there are no propagation mechanisms, the overall shift in venture capital consumption is low at 1%.

Variable	θ—lower values reflect a higher degree of uncertainty aversion					
	∞	400	20	10	8	6
$\tilde{C}$	0.8617	0.8615 (0.0)	0.8576 (-0.5)	0.8516 (-1.2)	0.8491 (-1.5)	0.8448 (-2.0)
$\tilde{C}e$	0.2685	0.2683 (-0.1)	0.2658 (-1.0)	0.2626 (-2.2)	0.2611 (-2.7)	0.2587 (-3.6)
$\tilde{K}$	2.0199	2.0188 (-0.1)	2.0014 (-0.9)	1.9758 (-2.2)	1.9649 (-2.7)	1.9467 (-3.6)
Hy	0.9271	0.9270 (0.0)	0.9254 (-0.2)	0.9239 (-0.3)	0.9231 (-0.4)	0.9217 (-0.6)
$\tilde{Y}$	1.3189	1.3185 (0.0)	1.3128 (-0.5)	1.3034 (-1.2)	1.2995 (-1.5)	1.2930 (-2.0)
Ξ	1.0146	1.0146	1.0149	1.0152	1.0154	1.0157

% deviation from approximating model in brackets

Table 3. Mean distortions of simulated variables with uncertainty aversion in an endogenous growth model

Once the venture capitalists' monopoly profits shift from purely rent-seeking to contributing toward the development of new technology, the story changes. Table 3 presents the analogous distortion to Table 2, showing the first moments of key variables for varying degrees of uncertainty for the "optimistic" household in the endogenous growth model. Not only is the effect of uncertainty larger on the venture capitalist's consumption than before, but the venture capitalist's aversion to uncertainty seems to have a pecuniary effect on the household. Moreover, despite the lower consumption for both the household and venture capitalist, increased uncertainty aversion seems to be conducive to a higher growth rate and a corresponding higher allocation of labor to the technology sector.

Both the increase in the growth rate and the reduction in consumption can be explained through a propagation mechanism centered around the arbitrage for labor. With increased uncertainty aversion, the venture capitalist is inclined to consume less in a given period. As the level of patents is determined by the growth rate of technology, the extra demand for savings among technology investors places upward pressure on the price of technology. Through the combination of (56) and (59), we know that $p = \frac{1-\alpha}{\zeta} e^Z \left(\frac{H_Y}{\tilde{K}}\right)^\alpha$, so the decrease in consumption causes an upward pressure on the ratio of capital to labor in production. Assume, temporarily, that this is corrected through allocating less labor for production. That reallocation would cause a decrease in the overall level of current output, decreasing the level of household consumption and capital together. The decrease in capital would cause more downward pressure on the amount of capital allocated to production in order to keep the price of technology higher, amplifying the decreases to output, consumption and capital. In addition, the decrease in output would also decrease the monopoly rent accruing to the venture capitalist, amplifying the decrease in venture capital consumption and causing the

Variable	θ—lower values reflect a higher degree of uncertainty-loving behavior					
	∞	400	20	10	8	6
$\tilde{C}$	0.8617	0.8620 (0.0)	0.8678 (0.7)	0.8728 (1.3)	0.8744 (1.5)	0.8787 (2.0)
$\tilde{C}e$	0.2685	0.2686 (0.1)	0.2717 (1.2)	0.2746 (2.3)	0.2758 (2.7)	0.2783 (3.7)
$\tilde{K}$	2.0199	2.0210 (0.1)	2.0460 (1.3)	2.0683 (2.4)	2.0759 (2.8)	2.0947 (3.7)
Hy	0.9271	0.9272 (0.0)	0.9286 (0.2)	0.9302 (0.3)	0.9311 (0.4)	0.9324 (0.6)
$\tilde{Y}$	1.3189	1.3193 (0.0)	1.3284 (0.7)	1.3361 (1.3)	1.3384 (1.5)	1.3449 (2.0)
Ξ	1.0146	1.0146	1.0143	1.0140	1.0138	1.0135

% deviation from approximating model in brackets

TABLE 4. Mean distortions of simulated variables with uncertainty-loving behaviour in an endogenous growth model

price of technology to drop overall (though the savings rate of the venture capitalist is still raised due to the uncertainty aversion).

Table 4 shows the analogous results for uncertainty-loving behavior, where we can see the mechanism described at work in reverse. Here, it is also worth noting that the monopoly power over intermediate capital goods implies a capital level far below the "golden rule" steady state as defined in the Solow model. That is, if we increase the level of capital in our economy, the steady state level of output will increase the steady-state level of consumption, as well. As a result, increases in the savings rate can cause increases in the overall level of consumption. Indeed, despite uncertainty aversion on the part of the venture capitalist, the average savings rate of the household increases by 0.7 percentage points from the approximating model to $\theta = 6$. While this equilibrium with a low value of productive capital magnifies the benefit from any increased savings on the part of the household, the increased savings rate from the venture capitalist's attitude to uncertainty is due to the mechanism described above.

6.2. Venture capitalists' optimality. Remembering that the utility of the venture capitalist was simply a discounted sum of positive monotonic transformations on consumption, a quick look at the tables above showing venture capitalist consumption increases under uncertainty loving behavior might lead one to think that the venture capitalist might be better off acting in such a way. While such a conclusion is not so straightforward, we find that such a result exists in this model.

In order to assess the individual optimality of uncertainty loving behavior, we must first find a useful measure for such optimality. One obvious choice would be the functional used by the household to determine its policy in the decentralized equilibrium, (64). However, by definition, the policy under this functional is the optimal choice under the functional, so any deviation from the policy would lead to a loss of utility as defined by the function $U(X_t)$.

However, the functional itself represents a large assumption through the inclusion of the expected utility operator, $\mathbf{E}_t$. This functional represents a mathematical expectation under a given probability distribution and transition function. In our case, the transition function is an inaccurate representation of the progression of the economy, so the expectational operator will lead to an inaccurate assessment of future returns. Our inclusion of behavioral assumptions can be interpreted as a distortion of the probability distribution in the expectation operator, which we use to compensate for the oversight in our transition functions.

A better representation of the benefit realized by the consumer would be the realized discounted utility of consumption values given by simulations of the model,

$$\sum_{t=0}^{n} \beta^t \frac{\hat{c}e_t^{1-\sigma}}{1-\sigma} \prod_{i=0}^{t} \Xi_i \tag{83}$$

where the $\hat{c}_t$ represents a realized value of de-trended consumption under simulations of the model. Using our simulations of 100,000 periods, dropping 5,000 for a "burn-in," we calculate this sum over sub-segments of 500 periods in length.[21] To minimize

[21]By the end of 500 periods, the discounted value of a period is of the order of $1x10^{-8}$. The contribution of any further periods to the utility calculation would be negligible compared to the statistical error from the simulation. For comparison, the variance of our calculated values of utility is 4.77 for the approximating model.

the effect of serial correlation on the calculation of discounted utility, we spaced the initial periods of subsegments by 10 periods. This reduces the autocorrelation of our discounted utility calculations from 0.9824 to 0.6830. Thus, while there is a large overlap between subsegments, we treat them as random subsamples of a stationary stochastic process. Furthermore, given a simulation of 100,000 periods, we have 9,450 observations of discounted utility.

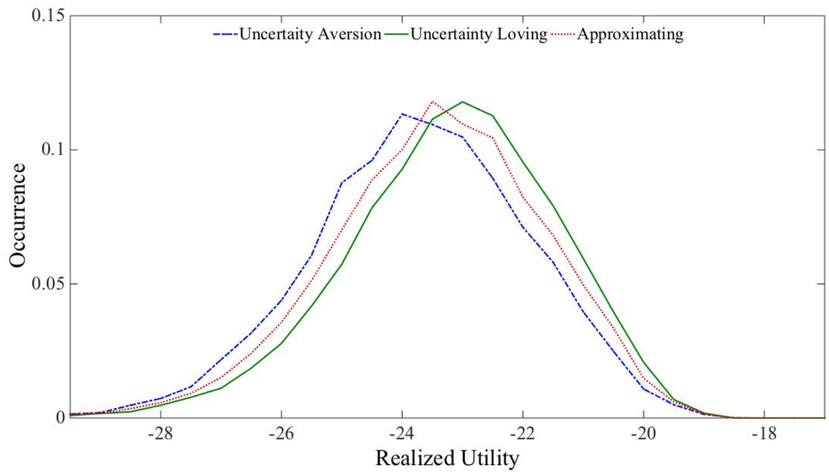


Figure 1. Distribution of utility realized over 50 periods by the venture capitalist under each behavioral assumption in the endogenous growth model. For each uncertainty model, we used a value of theta of 4.

Figure 1 shows the distribution of utility realized by the venture capitalist for each behavioral assumption—the uncertainty neutral VC/approximating model, the uncertainty averse VC, and the uncertainty-loving VC. As we see there is a shift towards the left as one goes from uncertainty-loving to uncertainty averse behavior, with the approximating case between the two.

Figure 2 shows the same calculations of realized utility for the venture capitalist as in Figure 1, only for the comparable model without any endogenous growth mechanism, as given by Definition 2.3. We see that a similar magnitude for our uncertainty parameter leaves us with a mean shift of our distribution, as with the

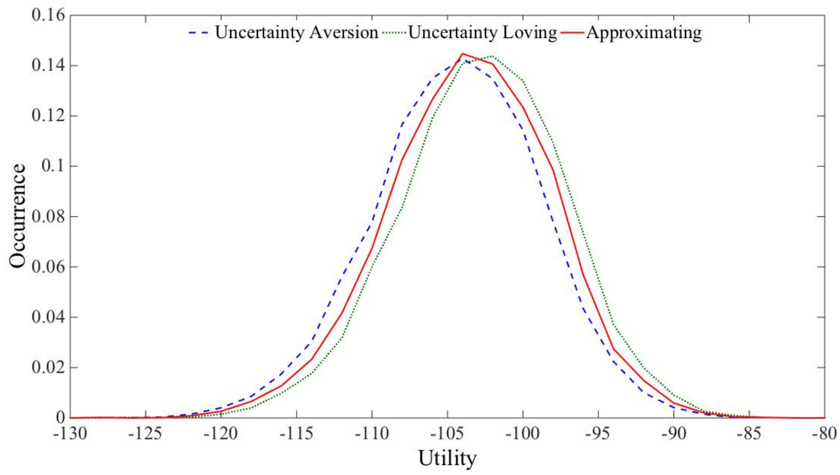


FIGURE 2. Distribution of utility realized over 50 periods by the venture capitalist under each behavioral assumption in an RBC model with market imperfections. For each uncertainty model, we used a value of theta of 4.

endogenous growth model, but with a shift of lower magnitude than our endogenous growth model. While comparability between the mean shifts is complicated by the large difference in the approximating mean utility under similar calibrations, one useful comparison would be in the relative changes between peaks. In the case without endogenous growth, this shift from the approximating case to either of the other two is ±1.9% the overall value in utility, whereas in the case with endogenous growth the shift is ±2.2%.[22] As the models are compatible in their market concentration, the increase in magnitude of this shift, as we explained above, is interpreted as the growth effect from increased investment by the household in the production sector following higher growth due to increased investment of the venture capitalist in the technology sector.

[22]The question of comparability between distribution shifts of utility when the original distributions are not immediately comparable is especially difficult given the fact that the underlying unit, a "util," does not have any immediate intuitive meaning. In this case, it is a weighted average of consumption streams, so we are merely stating that the overall stream of consumption has a larger shift in the uncertainty-loving case. A discussion of the interpretation of utility is a larger subject in welfare economics and is beyond the scope of this paper.

Interestingly, the utility increase we describe for optimism in the endogenous growth model also exists in the RBC model with the same monopoly power in the capital markets, albeit to a lower magnitude. This highlights one market imperfection of the endogenous growth model independently of the others—the market concentration in the capital market. Taking the comparison above of relative mean shifts in utility, the $\pm 1.9\%$ shift is quite large compared to 0.0%, shift in the same model without market imperfections.

Indeed, it seems that the distortion created by this one market imperfection is enough to create a break in the "rationality" of a rational expectations equilibrium. That is, the *ex-post* utility of venture capitalists would be higher if venture capitalists distorted their expectations in the way consistent with the optimistic case above. This serves to highlight the cost of distorting the capital markets to finance investment in technology: while investment in technology yields a positive benefit, the relaxation of the underinvestment in production due to the capital distortion also serves to increase utility under optimism.

While the venture capitalists as a group are better off acting in an uncertainty-loving way, the above analysis does not necessarily imply that an individual venture capitalist would be better off acting in such a way if other venture capitalists were to act as uncertainty-neutral. The analysis above relies on some reaction of market variables to the change in disposition towards uncertainty–namely, a response from the household to increase savings. This benefit would not accrue to any venture capitalist if she were acting alone in her optimistic behavior.

6.3. Social optimality. Due to the existence of both venture capitalists and households in our economy, social optimality cannot be determined simply through assessing the utility of a representative household. In order to refrain from making normative statements on interpersonal comparisons of utility, we refrain from establishing a social welfare function that weights the consumption streams of both individuals. Instead, we show an increase in social welfare due to uncertainty-loving behavior through (1) an appeal to Pareto efficiency, and (2), a comparison to the social policy function that would be achieved through the social planner's solution. The normative implications of the social planner's solution are debatable. Through the inclusion of a single representative agent instead of two, the social planner's problem implicitly weights the utility of both types of individuals equally. However, in the social planner's problem, there is no distinction in the behavior of the individuals either. As a result, it leaves the following question unaddressed, "how many extra units of consumption to a venture capitalist would be necessary to offset the social loss to one unit of consumption to a household?" As a result, we treat the social planner's decisions as ambivalent towards interpersonal utility comparisons.

A Pareto improvement is the increase in utility of one or more individuals without a decrease in utility of any other individual. We have already established that a representative venture capitalist's average realized utility increases in the case of optimistic disposition towards uncertainty. We now test the realized utility of the household to show that its realized utility does not decrease in the case of an optimistic venture capitalist, where realized utility is calculated in the same fashion as with the venture capitalist. From the simulation results above, we see that the economy with uncertainty-loving venture capitalists results in a mean increase in consumption. It is therefore not surprising that the realized utility of the household also exhibits a higher mean by roughly 9%. Thus, a venture capitalist acting optimistically in the face of uncertainty is Pareto optimal *on average*, as the distributions of realized utility for both venture capitalist and the representative household shift to the right. However, there is still the possibility that individual realizations of risk will allow for a decrease in utility as a venture capitalist acts in a more uncertainty-loving manner. To make this comparison, we run the simulation of the approximating model and the model with uncertainty-loving venture capitalists using the same series for realized shocks in both cases.[23] We find that the simulation with the uncertainty-loving venture capitalist uniformly yields an increase in the household consumption through the entire simulation.[24]

6.4. Detection Error Probabilities. Before moving on to discuss the pricing implications of entrepreneurs' disposition towards uncertainty, we provide some perspective on the magnitudes of such dispositions. For this, we use detection error probabilities, as outlined in Anderson et al. (2000), which describe the probability of correctly detecting which of our two models is responsible for a given data set. That is, it combines the likelihood of mistaking a data set created by an approximating model with one in which the entrepreneur is uncertainty-averse/loving with the likelihood that a data set generated by an uncertainty-averse/loving entrepreneur is mistaken for the approximating case. If the two data sets are close together, then the likelihood of mistaking one model for the other is 50%, and if the sets are completely distinguishable, the likelihood is 0%.

To determine these probabilities, we use a log-likelihood ratio test. For a simulation of a model of a specific length, we use conditional probability densities to calculate the likelihoods that (a) the model in question was responsible for the data and (b) the alternate model was responsible for the data. If this ratio is greater than one (or the log is greater than zero), the test would lead us to assume that the model in question

[23]For the uncertainty-loving case, we set θ at 6.

[24]There were a few instances in which the household consumption was greater in the approximating case during burn-in, but this can be overlooked as the burn-in is required to bring us to equilibria centered around the steady-state incorporating the behavior under uncertainty.

Sample size	Uncertainty averse	Uncertainty-loving
20	12.1%	12.6%
15	13.1%	15.2%
10	22.5%	20.0%
5	30.1%	27.0%

TABLE 5. Detection error probabilities, $\theta = 6$

was responsible for the data. Otherwise, the test would lead us to assume the other model was responsible for the data. To calculate the detection error probabilities, we repeat this calculation of log-likelihood ratio tests for a sufficient number of times to give us a probability of the log-likelihood ratio test giving us the incorrect solution. For our purposes, we use 100 repetitions. We do this using data from simulations from both models we wish to compare, and average the probabilities together to combine the type one and type two error, as described in the previous paragraph. A more detailed explanation of the method used to generate this likelihood is described in Appendix B.

We arrive at similar values for models with uncertainty-loving and uncertainty averse agents with similar uncertainty parameters. The values for the most extreme cases are given in Figure 5. In this figure, "sample size" refers to the length of the simulation used to calculate the likelihood in each repetition of the log-likelihood test. That is, the likelihood in question is the probability that a chain of length, "sample size," is responsible for the data. Again, this likelihood test is repeated 100 times for each model.

6.5. Pricing Implications. Finally, the motivation of this model is to describe the impact of venture capitalists' disposition towards inherent uncertainty in their investment class, due to the inherent nature of investing in untested technologies. We have seen that the interaction of the endogenous growth mechanism with uncertainty causes a magnification of the venture capitalists' change in behavior in output

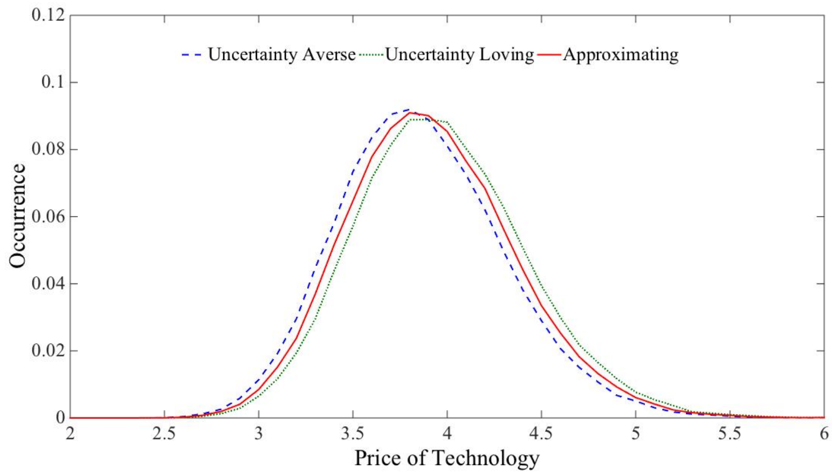


FIGURE 3. The price of technology over our simulation for extreme dispositions towards uncertainty. Both extremes use a value of 6 for theta.

and capital markets due to their disposition towards uncertainty. We now turn our attention to whether this magnification can lead to price bubbles in technology.

To do this, we consider the implications of our simulations above on the price of technology defined in our model. Figure 3 shows the distribution of prices over our simulation, as described above. First, consider the approximating model. While obviously not the optimal price in technology, this price distribution represents the general equilibrium values of technology prices over the business cycle. Realizations at the tail ends of the distribution are due to equilibrium decisions made under rational expectations. In that sense, a particularly high value of p does not represent an "overheating" of the market. Instead, it represents a high equilibrium price due to an incentive for the venture capitalist to invest in the asset more than she would in the steady state. We see that this rational deviation in the approximating model is relatively high compared to the price level, with a standard deviation of roughly 10% the value of the technology.

Comparing the distribution in the approximating model with the one with an uncertainty-loving venture capitalist allows us to see directly the implications of optimism on technology price. The distribution exhibits a mean shift compared to the approximating model, implying that an optimistic venture capitalist will lead to a higher equilibrium price of technology in all states of the economy. More importantly, though we see that the magnitude of this shift is small in comparison to the variance in the approximating model caused by rational responses to business cycle fluctuations.

To see this more clearly, consider an impulse response to a positive technology shock. In our approximating model, an unexpected shock of 0.1 causes a disturbance in the price of technology of 0.4854. In the model with the uncertainty-loving venture capitalist, this disturbance is 0.5017, a difference of 0.0163, or 3.4% of the displacement.

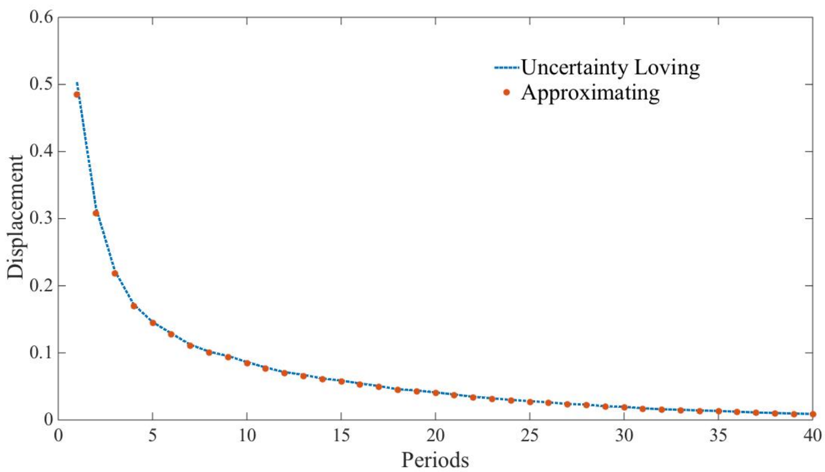


FIGURE 4. The response of the price of technology to a positive technology shock in our simulation for the uncertainty-loving and approximating cases. The uncertainty-loving case uses a value of 6 for theta.

However, this claim has two limitations worthy of note. First, the model above treats uncertainty as a fixed, static parameter for each simulation. This is important in that it implies that consumers and venture capitalists do not anticipate any changes in uncertainty in their consumption and investment choices. We therefore do not consider bubbles in asset markets that are driven by expectations of overheating. One can easily imagine a case in which a positive technology shock would cause an increase in investment due to the expectation that the rest of investors will be optimistic following the shock, in addition to the increases in investment due to the improved production capability and positive externalities arising from the new technology. The analysis below does not consider that possibility. While this seems limited, we believe that the possibility of such bubbles is limited by the ability of consumers and investors to anticipate such optimistic behavior. A purely backward-looking process for disposition toward uncertainty would simply cause the steady state to shift from one distribution shown above to another.

The second limitation to the model in its implications for technology pricing is the treatment of patent growth. We have assumed above that patents grow at the same rate as technology. Historically, this has not been the case—the number of patents tends to grow faster than the rate of technology during periods of rapid technological expansion. However, this would only strengthen the argument made below. An overextension in the number of assets available for a given aggregate technology level will depreciate the value of the average individual patent, dampening any claim of inflated valuations of such assets. In addition, we can interpret the value of T_t not as the number of patents, but as a patent index that adjusts for depreciation of the average patent due to accelerated patent issuance to assuage this concern.[25]

7. Conclusion

We have modeled venture capitalists' disposition towards uncertainty through their attitude towards the possibility that their model might be misspecified. The role

[25]A related issue for venture capitalists is the dilution of previous investments in a company due to revaluation of the technology. In later funding rounds, the value of technology represented by a share in a company can be diluted by the issuance of more shares. However, the potential for dilution relies on the strength of the original investment syndicate, leading to complicated repeated bargaining situations.

of venture capitalists as investors in technology is crucial in this context, as it allows a mechanism in which the venture capitalist's behavior feeds back to the household. We see that optimistic, uncertainty-loving venture capitalists can both be better-off in their own consumption streams and cause increases in overall utility by leading the household to increase its consumption, as well.

Ironically, the feedback mechanism here increases utility for both the household and the venture capitalist despite the overall growth rate decreasing in the model, albeit only slightly. Instead, the feedback comes through a rectification of an artificially low capital investment in production due to monopoly power over intermediate goods. This also contains an element of irony: it is the market imperfection that is otherwise welfare decreasing that causes the biggest increase in welfare in this case.

The large welfare improvements from higher steady-state capital expenditure in production is significant. It illustrates the magnitude of the distortion created by the monopoly over intermediate goods in this model. The capital sidelined for the venture capital sector is not purely rent-seeking: it goes to pay for current investment in technology by paying the wages for labor in the technology sector. Furthermore, we see from the social planner's solution that we have under-allocation of labor in technological development, to the point in which the growth rate is significantly lower than it could be. The implication of this exercise, that welfare improves from increasing optimism comes through increased investment in production despite a decreased allocation of labor to technology, is startling from a policy perspective. It suggests that the distortions in the capital sector created by patents could overwhelm the benefits of allocating more resources to technology development.

These concerns must be somewhat tempered. The model above arguably overemphasizes the distortion to capital, and the simplified specification of the technology sector could over emphasize the potential for increased investment in technology. First, our use of Romer's closed-form monopoly rate of interest requires the proportion of capital going to production to equal α. Higher levels of α are not credible in the Cobb-Douglas context, but lower values allocate a troublingly small proportion of capital to production. One obvious answer to this is that there already exists a limited scope on the lifetime of patents, reducing the current value of the patent and allowing a more efficient interest rate after the expiration date. Imperfect monopolies could also contribute greatly to reducing the distortion created by this closed form representation.

Furthermore, the level of labor allocated to technology is determined, in large part, by the arbitrage condition for labor. This equation equates the marginal benefit of labor in production to the marginal product of labor in technology development. Our technology development function is extremely simplified, with constant returns to scale and homogeneous labor. Obviously, neither of these assumptions are realistic. We can appeal to the standard macro argument of "low hanging fruit," that more deserving projects receive funding if resources are scarce, to argue that constant returns to scale are unrealistic. Decreasing returns to scale would decrease the potential for further growth from more allocation of labor to technology, as the better projects are already funded. Allowing for specialized labor could also reduce the distortion from the model above. The potential specifications for this type of alteration are many, but one could imagine a case in which labor is heterogeneous according to vintage, with older vintages being biased towards existing technologies. This would reduce the demand for labor of younger vintages in production, allowing for younger vintage graduates to be more affordable to new technology firms.

The findings in this chapter are significant, nonetheless. Despite the limitations of the simplifying assumptions discussed in the previous paragraphs, the analysis highlights the opportunity for a behavioral deviation form rational expectations to be utility-increasing for the venture capitalists exhibiting such behavior and welfare increasing overall. Furthermore, this exercise is useful to highlight the magnitude of the potential distortion from the monopoly power created by patents: such monopoly power is strong enough to make optimistic behavior welfare increasing despite a lower technological growth rate. Finally, the concern that venture capital markets overheat due to optimism under the face of uncertainty is a real concern among investors. While this model's implications are limited to a certain type of optimism, the result that the effect of such optimism is welfare increasing and only has a limited effect on market rates should help alleviate such concerns.

Appendix A: Simulation techniques

The data presented above derive from a perturbation-based simulation. Perturbation is used to find an approximation of a policy function around a steady state. These approximations usually follow a Taylor series, and we used a Taylor series to a third-order approximation. This method gives good approximations around the steady state, with the accuracy of the approximation declining with the distance from the steady state. This method is desirable in that it allows an increase in the number of variables with little difficulty, and it allows us to separate first, second, and third order effects of the model to describe the curvature of the policy function.

An alternate method would be to run simulations based on collocation and interpolation methods.[26] This method uses calculation of the optimal policy function at a series of representative nodes and fits curves through the nodes to describe, or "interpolate," each variable away from that node. Since the Euler equations rely on the mathematical expectation function, the actual location of the node will depend on expected future values away from the nodes. As a result, the process requires iteration of fitting the nodes and calculating the expectation to converge at a solution in which the expectations match what is predicted by the policy function.

This method is superior to the perturbation technique in that it gives a better approximation of the policy function, as the order of approximation is equal to the number of nodes in our state space, as opposed to just three from the third-order Taylor approximation. This improvement is mostly in the form of a better approximation away from the certainty-equivalent steady state, as the approximation is over a set of nodes in the state space, rather than simply at the steady state. This serves two purposes—first it is a good exercise to verify that the model does not deviate from

[26] We attempted to run simulations using this technique and found few differences in a simplified version of the model. Fuller versions were beyond the ability of our computers.

the approximation under perturbation methods away from the steady state. Second, the better approximation of the policy function over a larger grid allows for a more effective visualization of the policy function for an intuitive understanding of the mechanism at work.

The improved approximation of the collocation and interpolation method comes at a cost. First, the time to run such an approximation is much longer, as the degree of approximation is higher. In addition, this method does not lend itself to a large number of variables, due to the curse of dimensionality. Finally, the joint use of collocation and interpolation requires a stepwise convergence of estimated expectations and estimated policy functions, which often fails under higher degree approximations. For these reasons, the results from this chapter are purely from a simulation under perturbation around a steady state.

To run this simulation, we used the Dynare package in Matlab. This package is developed for economists to use Taylor approximations to solve and simulate models. We used a third-order Taylor approximation, following similar efforts by Bidder and Smith (2011), who observed that the full effect of uncertainty-aversion cannot be observed with a Taylor approximation of second-order or lower.

Appendix B: Detection error probabilities

The detection error probabilities in this chapter are calculated using the formula:

$$P(\theta) = \frac{1}{2}(P_A + P_B) \tag{84}$$

where P_A is the probability that one would assume the distorting model is more likely to generate data when the data is in fact generated by the approximating model, and P_B is the probability that one would assume the approximating model generated the data when the data was generated by the distorting model. In effect, we are taking the average of type I and type II errors, where type I error is the incorrect rejection of a true null hypothesis, while a type II error is the failure to reject a false null hypothesis.

We calculate whether one would assume the distorting model or the approximating model generates the data by the log-likelihood ratio of a series of simulated data. For each simulated state variable in the data, we calculate the probability that the next period will be in a particular state, conditional on the current state. This is done numerically, simulating 10,000 shocks of the same distribution as the shock in our model, and using the transition functions calculated through the same numerical methods used to create the simulation to determine the distribution of our future state variables given these shocks.[27]

Given the calculated conditional probability distribution of the state being in a particular value in the next period given the current state, we then extract the likelihood of the simulated progression of state variables based on how each state corresponded to the conditional probability of the previous state. For a simulation from a particular model, we calculate this likelihood both for that model and for the

[27] The large number of simulated shocks is to create a sufficiently smooth probability density function.

model against which we would like to compare it. We then conduct our log-likelihood ratio test, giving a 1 if the model responsible for the data is not the more likely of the two to create that string of data. We then repeat this process 100 times to give a probability of that model mistakenly yielding a false result in the log-likelihood test, calling this probability P_A. Assuming that the econometrician's null hypothesis is that this first model creates the data, P_A corresponds to the probability of type I error, or the incorrect rejection of the null hypothesis, in our log-likelihood ratio test. The entire process is then repeated for simulations from the alternate model in the comparison. This probability is labeled, P_B. Again assuming the econometrician's null hypothesis is that the first model is the correct one, P_B corresponds to the probability of type II error, or the incorrect acceptance of the null hypothesis. Finally, we apply P_A and P_B to the formula (84).

CHAPTER 5

Conclusion

The overarching theme of this dissertation can be found in the title: that venture capital, through its impact on technology development, has a large structural role in the economy. In our first full chapter, we showed that incorporating this role of venture capitalists allowed us to explain the two recessions of the first decade of the twenty-first century through a distinction between technology and wealth shocks. While the idea that a technology shock was responsible in 2001 while a wealth shock was predominant in 2008 is not unique to this thesis, we were able to show significant benefits from incorporating a mechanism through which development of new technology followed an incentivized structure. For one, including both the agency cost and endogenous growth mechanisms in one model allowed us to better capture the magnitudes and durations of the recessions in question. More importantly, our approach intuitively explains the break in correlation between the growth rates of total factor productivity and gross domestic product in 2008, a structural break in the relation between two fundamental economic indicators.

Likewise, our second chapter explained another shift in correlation between two indicators: total factor productivity growth and the interest rate for production capital. This time, moving from a debt to an equity contract lead from a negative correlation to a positive one. As with the previous model, the correlation of incentivized investment in technology with practical financial constraints provided a mechanism for describing this structural shift in correlations. However, in this case, the financial constraints were particular to the technology sector.

It fits with our overall theme that a similar mechanism applied to a different sector has such strong implications when incentivized technology investment is incorporated: the changes in this little sector have big effects. Traditional real business cycle theory taught us the strong structural impact of total factor productivity changes, primarily in the form of exogenous stochastic shocks. It is hardly surprising, then, that structural changes in the technology sector could have a large impact on the correlations of our central indicators. Indeed, in setting up these models, we found that small structural shifts, such as changing timing conventions, could significantly impact our results. Among other implications, this made calibration difficult.

However, the large implications of minor changes in our approach should be viewed as a strength of the approach. While such a sensitive calibration limits the precision of the claims we can make, it also provides a larger scope for discriminating between structural representations of the economy. To illustrate this, consider the alternative, that a large structural change results in a model that performs relatively similarly to the original model. Under this alternative, it is arguably easier to calibrate the model, as small errors in parameter values will not drastically worsen the predictive power of the model. As a result, the model is much more forgiving for small errors in calibration, allowing the modeler to make much more precise claims about the results of the model. However, there will also be a larger set of circumstances that such a correlation cannot address, as even large changes in parameter values and model specifications will only capture such large changes to a limited amount. Thus, having a model that is highly sensitive to structural specifications allows a greater degree of flexibility to explain a wide array of circumstances, as we do in this thesis.

Along these same lines, it should also be viewed as a strength that the models presented above are the simplest forms to represent the intuition of the argument. That is, we refrain from including many extra mechanisms that could allow the model to fit the data more closely. For example, we could have included technology adaptation mechanisms to provide a better fit for total factor productivity growth rates. We view the simplified model as stronger for our purposes due to the fact that our main aim as been predominantly to explain new mechanisms that lead to significant structural shifts, and the simplicity is useful for highlighting the impact of such mechanisms.

In is in the context of this sensitivity and simplicity that the final chapter has the greatest significance. In presenting a simple for of the Romer endogenous growth framework, the effects of market concentration seem stark and our results on welfare seem directly dependent on market imperfections. It is through this simplicity that the implications for rational expectations seem most profound: that a break from "rational behavior" leads to an increase in welfare for both types of agents in the model.

Moving forward, the incorporation of endogenous growth models in modern dynamic stochastic general equilibrium models should be an aspiration for the discipline as a whole. This thesis has drawn on the work of a handful of economists writing to that aim, but the practice of including an endogenous growth mechanism in more general business cycle analyses is still far from widespread. With this aim, our research agenda is threefold:

First, a model incorporating model uncertainty and endogenous growth in a cross-country analysis is already under development. The hope is the model can explain growth stagnation through a poverty trap in which economies close to a zero-technology-investment critical point are pushed over the edge by uncertainty aversion. This thesis helped pave the way for such analysis through the development of code for collocation methods with endogenous growth under uncertainty as outlined in the appendix for Chapter 4.

Second, a fully-derived "New Keynesian" model with endogenous growth could hopefully make dynamic stochastic endogenous growth models more accessible to monetary policy analysis. One benefit from such a project would be in the inherent reliance of New Keynesian models on market imperfections. Currently, many such models rely on ad-hoc monopoly power to allow prices to temporarily deviate from competitive levels for the desirable "stickiness." An endogenous growth model could partially provide motivation for such market power.

Finally, while reviewing the literature in preparation for this thesis, it was surprising that there have been no analyses on the relative performance of different endogenous growth mechanisms with respect to business cycle data. Indeed, in many of the papers cited in this thesis using endogenous growth mechanisms for business cycle analysis, the choice of the mechanism used is not explained. Thus, there remains a large amount of groundwork for establishing endogenous growth as a useful tool for business cycle analysis.

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