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New Keynesian Microfoundations Revisited: A Generalised Calvo-Taylor Model and the Desirability of Inflation vs. Price Level Targeting

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Abstract

Optimal monetary policy is sensitive to the Phillips curve specification used to represent the dynamics of inflation and output. Most recent literature has used a New Keynesian Phillips Curve based on Calvo pricing. This paper shows that this workhorse model is not robust to relatively minor changes in its microfoundations, in particular allowing for time varying probabilities of a firm being able to reset its price. We derive a general model that nests Calvo and the Taylor staggering model as special cases and analyse its implications for optimal policy, including the relative desirability of inflation and price level targeting.

Key Words

New Keynesian Phillips Curve, Stabilisation Bias, Forward Looking Expectations, Inflation Targeting, Price Level Targeting.

JEL Nos. E52, E58, E22, C61

Introduction

The monetary policy literature has made rapid progress in recent years in analysing the consequences for optimal policy of the presence of forward looking inflation expectations in the Phillips curve or aggregate supply relationship. When commitment is feasible optimal policy in response to a supply shock adjusts future policy to improve current outcomes through the intertemporal link of expected future inflation in the Phillips curve.[1] From the point of view of the future taken in isolation such a policy is costly (and hence is not generally carried out under discretion, resulting in stabilisation bias) but optimal policy balances these future costs against current benefits. Clearly the strength of the intertemporal links in the Phillips curve, primarily the size of the coefficient on expected future inflation, is important for optimal policy as well as more generally for our understanding of macroeconomic dynamics.

This paper analyses the microfoundations of the Phillips curve and the coefficient on expected future inflation in particular, showing that the standard value of close to unity used in the literature from Calvo pricing is not robust to plausible and relatively minor changes in its microfoundations. The paper presents a generalised version of the Phillips curves used in the literature that may provide a better basis for policy analysis and shows its implications for optimal policy. The common theme is fully optimising microfoundations but with different

[1]See Clarida, Gali and Gertler (1999) and Woodford (2000) for summaries. A growing literature assesses how improved stabilisation may be achieved under discretion (when intertemporal commitment is generally infeasible) by altering the loss function to be minimised under delegation by an independent central bank. These include interest rate smoothing (Woodford, 1999), nominal income growth targeting (Jensen, 1999, Rudebusch, 2002), targeting the change in the output gap (Soderstrom, 2001, Walsh, 2001), and allowing for changes in inflation expectations (Svensson and Woodford, 1999).

exogenous staggering structures, motivated in part by concerns about the robustness of the Calvo model (see Wolman, 1999, and Dotsey, King and Wolman, 1999, for recent discussion and results) and more generally by the empirical evidence presented in Wolman (2000) and Taylor (1999a) which emphasises the richness and variety of price and/or wage staggering structures and our comparative ignorance of how to model them most accurately. McCallum (1999) suggests that aggregate supply is the least well understood component of monetary policy models.

It is helpful to briefly review the use of the New Keynesian Phillips curve in the monetary policy literature before setting out the contribution of the current paper in more detail. The Calvo (1983) model, which gives rise to a New Keynesian Phillips curve of the form shown by (1), has become the workhorse for much recent research.[2] In (1) π is the rate of inflation, β the (real) discount factor which is close to unity, x the driving variable (such as the output gap or marginal cost), κ a constant and u a shock variable.

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa x_t + u_t \tag{1}$$

Price staggering in the Calvo model is introduced by firms only being able to reset their prices at stochastic times and a simplifying assumption is made that the probability of being able to reset price in each period is constant and unrelated to the time that has elapsed since the last price change. Clearly this is a strong assumption but use of the model has been encouraged by broad similarities between its properties and those of other models of price staggering. For example Rotemberg's (1987) model of convex price adjustment costs also leads to a Phillips curve of the Calvo form, though Rotemberg regards convex price adjustment costs as a simplifying rather

[2]For example see Clarida, Gali and Gertler (1999), McCallum and Nelson (1999, 2000) and Rotemberg and Woodford (1999).

than fundamental assumption in much the same way as Calvo (1983) presents the assumed constancy of the probability of price change.

A number of authors (for example Roberts, 1995, and Walsh, 1998) have also pointed to broad similarities between the Calvo model and that of Taylor (1979, 1980) where price changes are also staggered but are simply fixed for two (or more) periods at a time. If it was the case that both the Calvo and Taylor models, with their different staggering assumptions, predicted the same form for the Phillips curve it would be strong evidence that the details of staggering structures are not very important but we show that this is not the case, particularly in the presence of supply shocks which present the most acute problems for policy makers. Under perfect foresight the Taylor model is similar to the Calvo case (1), and has an identical coefficient on expected future inflation. With shocks, however, Taylor staggering gives the Phillips curve (2) in which $E_{t-1}[\pi_t]$ is present and the coefficient on $E_t[\pi_{t+1}]$ is approximately half its value in (1).[3].

$$\pi_t = \frac{\beta}{1+\beta} E_{t-1}[\pi_t] + \frac{\beta}{1+\beta} E_t[\pi_{t+1}] + f(x,u) \tag{2}$$

This has very different implications for optimal policy compared with (1) given the discussion

[3] The Taylor Phillips curve is sometimes presented (see Roberts, 1995, and Walsh, 1998, eqn. (5.45) p.217 for example) in the form of (1) but with an additional 'expectational error' term on the right hand side involving $E_{t-1}[\pi_t]-\pi_t$ (which is zero under perfect foresight but not otherwise). This error is sometimes combined with the shock term, u_t, to form a composite error. This is algebraically correct but it is misleading if this error is subsequently treated as exogenous. On average under rational expectations the expectational error should be zero but its size in any given time period, and thus its effect on the Phillips curve, is endogenous to time t policy because $E_{t-1}[\pi_t]$ is pre-determined at t whereas π_t is not. Hence treating the error as if it is zero or exogenous overstates the true coefficient on $E_t[\pi_{t+1}]$ in the Phillips curve. The driving variable and shock terms also take a more complex form in the Taylor model. These are explored thoroughly below but for now the focus is on the terms in expected inflation and hence we use the shorthand form f(x,u) in (2). Kiley (1998) draws attention to the general differences between the Calvo and Taylor models but without explicit reference to the different coefficients on expected inflation and their implications for optimal policy.

above about the role of this coefficient in influencing the optimal extent to which policy should commit to different future outcomes in order to affect the present.

Given that the Calvo and Taylor Phillips curves differ significantly in the presence of shocks it appears that we are in the uncomfortable position of having divergent predictions for the Phillips curve from different assumptions about price staggering when there seems little compelling reason to find one or other more plausible. In terms of these models it also appears to be a binary choice with strong implications for the policy conclusions that will follow. Given this a common stance is to choose the Calvo Phillips curve (1), primarily because the Calvo model has more explicit microfoundations than the standard derivation of Taylor (see Clarida, Gali and Gertler, 1999, and the discussion in Walsh, 1998, for examples). We show below, however, that the latter may be derived from the same microfoundations as Calvo.[4]

The main contribution of this paper is to show that both the Calvo and Taylor models may be viewed as special cases of the more general model that we derive below. This clarifies the reason for their different predictions for the Phillips curve, which arise solely from their different assumptions about price change staggering, and clarifies that their underlying microfoundations are the same apart from the staggering structure. The generalised model follows the Calvo approach of making firms' ability to change price stochastic, but rather than the probability of being able to change price remaining constant we allow it to take a different value one period after a price change (q_1) than the per period probability thereafter (q). This simple generalisation

[4]Ascari (2000) and Ascari and Rankin (2002) derive a microfounded Taylor model but without explicit comparison with Calvo.

(which by no means exhausts all possibilities[5]) encompasses the Calvo model for which q_1=q and the Taylor model where q_1=0 and q=1 (for two period fixed prices). Defining $q^*=q-q_1$, where q* is zero in Calvo and unity in Taylor, allows the generalised Calvo-Taylor Phillips curve (derived in full below) to be expressed by (3) where for the time being we focus on the terms in expected inflation and leave the driving variable and shock terms in general form.

$$\pi_t = \frac{\beta q*}{1+\beta q*}E_{t-1}[\pi_t] + \frac{\beta}{1+\beta q*}E_t[\pi_{t+1}] + g(x,u) \tag{3}$$

From (3) it is clear that the generalised model has coefficients on $E_{t-1}[\pi_t]$ and $E_t[\pi_{t+1}]$ that vary with q* ($0 \le q^* \le 1$) between approximately a half each in Taylor, and zero for $E_{t-1}[\pi_t]$ and close to unity for $E_t[\pi_{t+1}]$ in Calvo.[6] The generalised model clarifies the similarities and differences between the Calvo and Taylor models individually while suggesting that it is not appropriate, given our limited understanding of the most realistic way to model price staggering, to choose one or the other. Instead it appears that good practice requires monetary policy analysis to check the sensitivity of results to variation in the expected inflation coefficients in the Phillips curve at least between the ranges suggested above.

To explore the implications of different staggering structures for the Phillips curve further we also derive two other versions of the generalised model. In the first, wages rather than prices are staggered, the latter being fully flexible ex post in this case. We show that this reversal of the roles of the two key nominal variables makes no difference to the coefficients on the first two

[5]Wolman (1999) considers a richer structure of price change probabilities though without explicit results for the Phillips curve or optimal policy.

[6]Mankiw and Reis (2001) derive a Phillips curve from a different approach in which the term in $E_t[\pi_{t+1}]$ disappears altogether.

terms in (3). This is a standard result with Taylor staggering but less familiar with Calvo probabilities and new for the generalised model. Secondly we show the consequences, for the staggered prices version of the generalised model, of wages being partly set in advance rather than being flexible ex post. This is in the spirit of the models of Barro and Gordon (1983) and Rogoff (1985) except that here ex ante wage setting coexists with price staggering. In this case the coefficient on $E_{t-1}[\pi_t]$ increases relative to that in (3) and that on $E_t[\pi_{t+1}]$ falls further.

We also note two aspects of the literature which we do not address in the paper. The first is that it has become increasingly common to introduce inflation persistence (through a term in π_{t-1}) into the Phillips curve but as yet there is no consensus on optimising microfoundations for this.[7, 8] The focus of this paper is the different forms of the Phillips curve that emerge from different staggering structures with common optimising microfoundations. Hence we do not consider

[7]The approach to this issue of Christiano, Eichenbaum and Evans (2001), Sbordone (2001) and Woodford (2001), who introduce price or wage indexing, appears very promising but it appears to be too early to regard these models as widely accepted. If optimising microfoundations for inflation persistence emerge from time dependent staggering the generalised model of this paper (which could be extended to include indexing behaviour) will also contribute to this research program. See also Calvo et. al. (2001), Erceg and Levin (2000), Estrella and Fuhrer (1998), Gali and Gertler (1999), Gali et. al. (2001), Mankiw (2001), Nelson (1998), Rudd and Whelan (2001), (Rudebusch (2002), Rudebusch and Svensson (1999, 2000), Roberts (1997, 2001) and Soderlind et. al. (2002).

[8]The criticism in footnote 3 of some interpretations of the Taylor model extends to the usual presentation of the Fuhrer-Moore (1995) relative real wage contracting model also. This model is often used as the basis for introducing inflation persistence. With a discount factor of unity for simplicity and considering only terms in inflation, the Fuhrer-Moore Phillips curve is usually reported (for example see Walsh, 1998, eqn. (5.62) p. 225) as $\pi_t=(1/2)(\pi_{t-1}+E_t[\pi_{t+1}])+\eta_t/2$ where $\eta=E_{t-1}[\pi_t]-\pi_t$. The expectational error η_t is sometimes taken as exogenous or zero or rolled into a composite error term with the truly exogenous shock variable (see for example the definition of ϵ_{5t} in eqn. (8) of Batini and Haldane, 1999). As in footnote 3, $E_{t-1}[\pi_t]$ is pre-determined at time t whereas π_t is not and hence their difference should not be treated as exogenous at time t (except under perfect foresight which is not usually the case of interest). Making η_t explicit gives the inflation terms in the Fuhrer-Moore model by $\pi_t=(1/3)(\pi_{t-1}+E_{t-1}[\pi_t]+E_t[\pi_{t+1}])$.

inflation persistence directly while noting that, i) the generalised Calvo-Taylor model does not have any structural inflation persistence in the sense that π_{t-1} does not appear in the Phillips curve (except when wages are set partly ex ante and then as part of a time t-1 expectation error), but ii) under discretion inflation has some serial correlation in reduced form. The latter is not the case in the Calvo model but may contribute to observed inflation persistence in practice. An open question is whether the generalised model implies sufficient serial correlation in inflation to be data consistent without the addition of a term in lagged inflation in the Phillips curve.

The second restriction of scope is that we consider only time dependent pricing behaviour. While Ball and Ceccheti (1988) and Ball and Romer (1989) showed that staggering may emerge as an equilibrium[9] it might be argued that state dependent pricing models are more theoretically attractive. While sympathetic to this view, the stance taken in this paper is that state dependent pricing tends not to give tractable results and pending further progress (see Dotsey, King and Wolman, 1999, for a recent contribution), and given the use of time dependent pricing in most of the literature, it remains important for us to understand the robustness of the workhorse Calvo model and the impact of different time dependent staggering structures.

While the prime contribution of the paper is the derivation of the generalised Calvo-Taylor Phillips curve model we also derive optimal policy for the new model in the presence of inefficient supply shocks. This confirms the link between the coefficient on $E_t[\pi_{t+1}]$ in the Phillips curve and the extent to which policy should alter its future course in order to affect current outcomes. In particular we examine the result of Clarida, Gali and Gertler (1999) who

[9]Bhaskar (1999) summarises arguments questioning these results while providing an additional mechanism to support them.

showed that with a simple Calvo Phillips curve and a standard policy loss function involving fluctuations in the output gap and inflation about target, optimal policy involves a stationary price level and hence price level targeting type policy choices.[10] We show that this result no longer holds in the generalised Calvo-Taylor model.[11] This result complements the same finding by Jensen (1999) when inflation is persistent (which is also implicit in Steinsson, 2000) but shows that the optimality of price level targeting is not robust even without inflation persistence. Given the continued controversy about the degree of inflation persistence this result suggests very strongly that a cautious approach is required before advocating price level targeting (see King, 1999) and is also highly relevant for the ongoing debate about the appropriate objectives for the European Central Bank discussed by Alesina et. al. (2001) amongst others. More generally the policy results are suggestive of the mixed inflation-price level targeting approach of Nessen and Vestin (2000) and Batini and Yates (2001).

The paper is structured very simply in that Section 1 derives the generalised Phillips curve model when either prices or wages are staggered (but with the other flexible), Section 2 analyses the case of staggered prices with wages partly set ex ante, and Section 3 shows the implications of the analysis for optimal policy and inflation vs. price level targeting in particular. Section 4 concludes while the appendices contain supplementary material, including detailed microfoundations to emphasise that the models share a common optimising structure.

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[10]See also King and Wolman (1996), Dittmar and Gavin (2000) and Goodfriend and King (2001).

[11]An assessment of the various mechanisms that have been proposed to achieve appropriate 'policy inertia' (see footnote 1) in the new generalised model is beyond the scope of this paper but it seems highly likely that the reduced coefficient on expected future inflation in the Phillips curve reduces the optimal degree of inertia (though not to zero).

1. The Generalised Calvo-Taylor Model

This section derives the generalised Calvo-Taylor model, initially for the case where prices are staggered but wages are fully flexible, before showing that similar results obtain if these roles are reversed. We consider the price setting decision of a firm that is able to change its price subject to the exogenous probabilities of being able to change price again of q_1 the following period and q each period thereafter if the price has not already been reset. As noted above the special case of q_1=q recovers the Calvo model and q_1=0, q=1 corresponds to the Taylor model. Appendix A gives microfoundations for optimal single period behaviour upon which the multi-period optimisation under staggering constraints is based. Having derived and simplified the optimal price for a single firm we substitute for these individual prices into the appropriate expression for inflation given this staggering structure (from Appendix B) to generate the Phillips curve.

We follow the standard discrete time solution procedure for the Calvo model (as in Rotemberg, 1987, and summarised in Walsh, 1998, p.218-220) subject to the different probability structure noted above.[12] Based on a second order Taylor series for profits as a function of price this approximates the firm's optimisation by the minimisation of the expected discounted and probability weighted sum of a per period loss function that is quadratic in the difference between

[12]We also follow Rotemberg (1987) and the standard forms of the Calvo and Taylor models in assuming that new prices in a given time period are set ex post on the basis of information available in that time period. This contrasts with the original papers by Fischer (1977) and Taylor (1979) where new prices or wages were set the previous period. Rotemberg and Woodford (1997) allow for some agents (in an otherwise Calvo framework) to have to set prices on the basis of previous period information. This reduces the importance of time t dated expectations in the Phillips curve for π_t. In Section 2 we allow for a proportion of wages to be set ex ante while new prices remain set ex post which has a similar effect.

the logs of the firm's price and the ideal single period price. The latter is derived in Appendix A, denoted p*, and corresponds to the price which the firm would set in that period in the absence of constraints on changing prices in the future. The quadratic term for each period is discounted by the discount factor, β, and weighted by the probability of the price set at time t still being in place in each subsequent period, t+j. This probability is simply $(1\text{-}q_1)(1\text{-}q)^{j\text{-}1}$ for $j \geq 1$ and unity for j=0. The optimisation need not consider what happens after the firm has been able to reset its price since the choice of price at time t does not constrain that subsequent optimisation. Hence the firms choice problem may be expressed by (4) where L_i^f is the total loss function for firm i and x_{it} the price it sets.

$$Min_{x_{it}}\ L_i^f \ = \ E_t\ [\ (x_{it} - p_t*)^2 \ + \ \Sigma_{j=1}^{\infty}\beta^j(1-q_1)(1-q)^{j-1}\ (x_{it} - p_{t+j}*)^2\ ] \tag{4}$$

The first order condition for (4) is given by (5) which shows that the optimal price set by a firm depends on the current and expected future optimal single period prices, appropriately weighted by the discount factor and the probability of the time t price still being in effect in future periods. For convenience we use the notation $q^* = q - q_1$.

$$x_{it} \ = \ [\frac{1-\beta(1-q)}{1+\beta q*}][\ p_t* \ + \ \frac{(1-q_1)}{(1-q)}E_t[\Sigma_{j=1}^{\infty}\beta^j(1-q)^j p_{t+j}*]\ ] \tag{5}$$

From this point we may drop the i subscript due to symmetry across firms that are changing their prices at the same time. It is convenient to take period t+1 out of the last term in (5) to give (6).

$$x_{it} \ = \ [\frac{1-\beta(1-q)}{1+\beta q*}][\ p_t* + (1-q_1)\beta E_t[p_{t+1}*] + \frac{(1-q_1)}{(1-q)}E_t[\Sigma_{j=2}^{\infty}\beta^j(1-q)^j p_{t+j}*]\ ] \tag{6}$$

Next we shift (5) one period ahead to give the optimal x_{t+1} and take expectations of this at t to give (7).

$$E_t[x_{t+1}] = [\frac{1-\beta(1-q)}{1+\beta q*}]\ E_t[\ p_{t+1}* + \frac{(1-q_1)}{(1-q)}\Sigma_{j=2}^{\infty}\beta^{j-1}(1-q)^{j-1}p_{t+j}*\ ] \tag{7}$$

In turn (7) may be rearranged to give (8), the right hand side of which is the same as the last term in (6) so the left hand side may be substituted into (6) which gives (9).

$$\beta(1-q)[E_t[x_{t+1}]\frac{(1+\beta q*)}{[1-\beta(1-q)]} - E_t[p_{t+1}*]] = \frac{(1-q_1)}{(1-q)}E_t[\Sigma_{j=2}^{\infty}\beta^j(1-q)^j\ p_{t+1}*] \tag{8}$$

$$x_t = \beta(1-q)E_t[x_{t+1}] + \frac{1-\beta(1-q)}{1+\beta q*}[p_t* + \beta q* E_t[p_{t+1}*]] \tag{9}$$

In turn we may substitute for $E_t[x_{t+1}]$ in terms of $E_t[\pi_{t+1}]$ from (B7) and $p^*=p+\gamma y+\epsilon$ from (A11), where y is the (log) output gap and ϵ a shock term, to give (10) where k is defined by (11).

$$x_t = \begin{array}{l} q\Sigma_{r=1}^{\infty}(1-q)^{r-1}x_{t-r} + \dfrac{\beta(1+q*)}{q(1+\beta q*)}E_t[\pi_{t+1}] \\ +k\dfrac{(1+q*)}{q}[\gamma y_t + \epsilon_t + \beta q* E_t[\gamma y_{t+1} + \epsilon_{t+1}]\ ] \end{array} \tag{10}$$

$$k = \frac{q[1-\beta(1-q)]}{(1-q_1)(1+\beta q*)[1+\beta q*(1-q)]} \tag{11}$$

In turn we may lag (10) one period to give the optimal price set at t-1 by (12).

$$x_{t-1} = \begin{array}{l} q\Sigma_{r=2}^{\infty}(1-q)^{r-2}x_{t-r} + \dfrac{\beta(1+q*)}{q(1+\beta q*)}E_{t-1}[\pi_t] \\ +k\dfrac{(1+q*)}{q}[\gamma y_{t-1} + \epsilon_{t-1} + \beta q* E_{t-1}[\gamma y_t + \epsilon_t]] \end{array} \tag{12}$$

Appendix B derives the rate of inflation given the staggering structure and the prices set at different times; x_t, x_{t-1} etc. In particular (B6) may be rearranged to give (13) in which it may be seen that the two pairs of terms correspond to the left hand side minus the first term on the right hand side of (10) and (12). From (13) we can see that the origin of the Calvo special case is that with q*=0 the second pair of terms in (13) disappear so the prices set at t-1 become immaterial for π_t and $E_{t-1}[\pi_t]$, which appears in (12) but not (10), will be absent from the Phillips curve.

$$\pi_t = \frac{q}{1+q*}[x_t - q\Sigma_{r=1}^{\infty}(1-q)^{r-1}x_{t-r} + q*[x_{t-1} - q\Sigma_{r=2}^{\infty}(1-q)^{r-2}x_{t-r}]\ ] \tag{13}$$

Substituting (10) and (12) into (13) gives (14) which is the Phillips curve for the generalised Calvo-Taylor model with staggered prices and the key result of this section.

$$\pi_t = \begin{array}{l} \frac{\beta}{1+\beta q*}E_t[\pi_{t+1}] + k(\gamma y_t + \epsilon_t + \beta q* E_t[\gamma y_{t+1} + \epsilon_{t+1}]) \\ + \frac{\beta q*}{1+\beta q*}E_{t-1}[\pi_t] + kq*(\gamma y_{t-1} + \epsilon_{t-1} + \beta q* E_{t-1}[\gamma y_t + \epsilon_t]) \end{array} \tag{14}$$

From (14) it may be seen that the absolute values of q_1 and q matter only for the parameter k given in (11) whereas q* plays a more significant role. Firstly if q*=0, the Calvo special case, the whole of the second line and the last two terms of the first line disappear and (14) takes the same form as (1). This shows that the Calvo assumption of q*=0 is very restrictive for the form of the Phillips curve, effectively because with q*=0 it so happens that the prices set at t-1 are irrelevant for π_t. If q* is positive the coefficient on $E_t[\pi_{t+1}]$ is reduced, that on $E_{t-1}[\pi_t]$ becomes larger, and the output gap and shock terms have a richer structure involving both lags and expectations over these variables. In particular $E_t[y_{t+1}]$ appears so forward looking quantity expectations are relevant as well as forward looking inflation expectations, and the lagged terms y_{t-1} and ϵ_{t-1} are present also. Section 3 shows that these changes from Calvo matter for policy.

We briefly note further aspects of (14). Firstly the Phillips curve has unusual properties if q* is negative but this would only arise if the probability of being able to change price decreases rather than increases with the time elapsed since the last price change. This seems implausible and we restrict attention to $0 \leq q^* \leq 1$. Secondly, (14) shares the property of the Calvo and Taylor models individually of violating the weak form of the natural rate hypothesis in the sense that the sum of the coefficients on inflation terms on the right hand side of (14), which equals $\beta(1+q^*)/(1+\beta q^*)$, is (slightly) less than one unless $\beta=1$. Under perfect foresight (14) becomes (15) where the sum of inflation coefficients on the right hand side is simply β.

$$\pi_t = \beta\pi_{t+1} + (1+\beta q^*)k\gamma[q^* y_{t-1} + (1+\beta q^{*2})y_t + \beta q^* y_{t+1}] \tag{15}$$

From (15) we note that under perfect foresight the generalised model reduces to the Calvo form (1) with respect to the expected inflation coefficients though the richer structure of output gaps remains. As noted above the perfect foresight Phillips curve is not applicable when analysing optimal policy responses to shocks.

Lastly, while we derived the generalised model assuming that prices are staggered with wages set flexibly ex post these roles may be reversed and the terms in expected inflation are the same as in (14). This is shown in Appendix C which uses the same microfoundations as the model above while changing the staggering assumptions. Typically the Calvo model has been used only with staggered prices-flexible wages while the Taylor model has been used with either prices or wages staggered. Hence one may think of either of these approaches or their generalisation above, being combined with staggered prices or staggered wages.

2. Generalised Calvo-Taylor Model: Staggered prices with wages partly set ex ante.

The generalised Calvo-Taylor model of the previous section assumed that either prices or wages were staggered whereas the other variable was set with full flexibility ex post. This raises the question of what happens if there is some form of staggering or timing constraints on the setting of both variables. Clearly there are many possibilities including changes in both variables being stochastic (possibly with different probabilities) or one could be stochastic and the other set for fixed periods of time, and with each of these the variables could wholly or partly have to be set in advance rather than ex post. Exploring all of these is beyond the scope of the paper and we focus on a particular case where prices are set as in the previous section while wages are still set for a single period but with a constraint that they must be partly set in advance. We show that even with this relatively simple modification there are further significant changes in the Phillips curve. More precisely we assume that a fraction, μ, of wages must be set in advance or on the basis of previous period information while the remaining fraction, $1-\mu$, are set as before. As noted above, Rotemberg and Woodford (1997) make a similar assumption with respect to price setters and the approach is also in the spirit of Fischer (1977) and Taylor (1979) where wages were set ex ante as well as Barro and Gordon (1983) and Rogoff (1985) where wages are set entirely in advance.

The staggering/timing assumptions of this section are also of interest since in the limit where q_1 tends to unity (so prices become flexible) and μ tends to one, the model tends to that of Barro and Gordon (1983), Rogoff (1985) and the literature that follows them. We do not place particular emphasis on that special case and present this version of the model as i) potentially realistic if prices are staggered but wages are partly set in advance, and ii) more generally a further exploration of the possible form of the Phillips curve given different timing and staggering constraints, once again based on fully optimising microfoundations. From the above we assume that the aggregate nominal wage, W (in levels), is given by (16) where W^p is the ex post wage from (A11) as above and W^a the ex ante wage. In common with much of the literature we ignore possible aggregation issues and treat W in (16) as the common wage level across all firms.

$$W_t = (W_t^a)^{\mu}(W_t^p)^{1-\mu} \tag{16}$$

For the ex ante wage we follow the approach of Rogoff (1985) which amounts to workers setting W^a in advance to satisfy the expected value of (A11) which gives (17).

$$W^a = (\frac{W}{P})^* E_{t-1}[P_t(\frac{Y_t}{Y^*})^{\frac{1}{\eta[1+\theta[1-\alpha(1-\frac{1}{\eta})]]}}] \tag{17}$$

Using (A11), (16) and (17) the wage rate is given by (18) and substituting this into the optimal single period price for firm i, P_i given by (A10) gives us the log optimal single period price across all firms, p*, by (19) which may be compared with (A13) where the new constants γ_1 and γ_2 are defined in (20) and all other notation is the same as before.[13]

$$(\frac{W}{P})_t = (\frac{W}{P})^* \; [\frac{E_{t-1}(P_t)}{P_t}]^{\mu} \; [(\frac{Y_t}{Y^*})^{1-\mu}(\frac{E_{t-1}[Y_t]}{Y^*})^{\mu}]^{\frac{1}{\eta[1+\theta[1-\alpha(1-\frac{1}{\eta})]]}} \tag{18}$$

$$p_t* = p_t + \gamma y_t + \epsilon_t + \mu[\gamma_1(E_{t-1}[\pi_t] - \pi_t) + \gamma_2(E_{t-1}[y_t] - y_t)] \tag{19}$$

[13]The shocks in the model are all log-linear and all variables are log-linear in those shocks so arithmetic mean preserving spreads in logs and geometric mean preserving spreads in levels. Since the log of a geometric mean is the same as the arithmetic mean of a log we have the convenient result that here the log of an expectation is the same as an expectation of a log.

$$\gamma_1 = \frac{\alpha}{\eta[1-\alpha(1-\frac{1}{\eta})]} \quad ; \quad \gamma_2 = \frac{\gamma_1}{\eta[1+\theta[1-\alpha(1-\frac{1}{\epsilon})]]} \tag{20}$$

We now summarise the derivation of the Phillips curve for this version of the generalised model. Firstly it may be noted that the new constraint on wage setting enters the derivation of Section 1 only through p* in (19) and hence (4)-(9) above remain unchanged since p* appears in general form. Substituting the new p* from (19) into (9) gives (21) as the equivalent of (10).

$$x_t = \begin{matrix} q\Sigma_{r=1}^{\infty}(1-q)^{r-1}x_{t-r} + \frac{\beta(1+q*)}{q(1+\beta q*)}E_t[\pi_{t+1}] \\ +k\frac{(1+q*)}{q}\left[\begin{matrix}\gamma y_t+\epsilon_t+\beta q*E_t[\gamma y_{t+1}+\epsilon_{t+1}] \\ +\gamma_1\mu(E_{t-1}[\pi_t]-\pi_t)+\gamma_2\mu(E_{t-1}[y_t]-y_t)\end{matrix}\right]\end{matrix} \tag{21}$$

We lag (21) one period to generate the equivalent expression for x_{t-1} and substitute that and (21) into (13) to give the Phillips curve for this model by (22).

$$\pi_t = \begin{matrix} [\frac{\beta}{(1+\beta q*)(1+k\gamma_1\mu)}]E_t[\pi_{t+1}] + [\frac{\beta q*+(1+\beta q*)k\gamma_1\mu}{(1+\beta q*)(1+k\gamma_1\mu)}]E_{t-1}[\pi_t] \\ +\frac{kq*\gamma_1\mu}{1+k\gamma_1\mu}(E_{t-2}[\pi_{t-1}]-\pi_{t-1}) \\ +\frac{k}{1+k\gamma_1\mu}[\gamma y_t+\epsilon_t+\beta q*E_t[\gamma y_{t+1}+\epsilon_{t+1}]+\gamma_2\mu(E_{t-1}[y_t]-y_t)] \\ \frac{kq*}{1+k\gamma_1\mu}[\gamma y_{t-1}+\epsilon_{t-1}+\beta q*E_{t-1}[\gamma y_t+\epsilon_t]+\gamma_2\mu(E_{t-2}[y_{t-1}]-y_{t-1})]\end{matrix} \tag{22}$$

We briefly note the properties of (22) compared with (14): i) (22) reduces to (14) if μ=0; ii) the perfect foresight version of (22) is the same as the perfect foresight version of (14) given by (15); iii) more generally if wages are partly set in advance (μ>0) the coefficient on $E_t[\pi_{t+1}]$ in the Phillips curve (22) is smaller than in (14) and that on $E_{t-1}[\pi_t]$ is larger; iv) a special case of this is that if μ and q_1 tend to unity (so prices are flexible, which implies that k becomes large, and all wages are set ex ante) the coefficient on $E_t[\pi_{t+1}]$ tends to zero and that on $E_{t-1}[\pi_t]$ tends to unity (thus giving a "Barro-Gordon-Rogoff" form to the Phillips curve); v) the ex ante setting of some wages results in the expectation error terms in the second, third and fourth lines of (22); and vi) (22) also violates the natural rate hypothesis though once again the sum of inflation coefficients on the right hand side is close to unity (it is also increasing in μ).

3. Optimal Monetary Policy in the Generalised Calvo-Taylor Model

We analyse optimal monetary policy both with commitment and under discretion, focusing on the staggered prices/flexible wages generalised Calvo-Taylor model of Section 1.

Commitment

We first derive the optimal monetary policy rule when commitment is feasible by analysing the policy maker's minimisation of the (standard) loss function (23) subject to the Phillips curve constraint (14). In (23) λ is the relative weight on output gap fluctuations and for simplicity we assume that the target inflation rate is zero. The latter assumption does not affect the results except in so far as price level targeting would become targeting a price level trend with a positive inflation target. We also assume for simplicity that there is no serial correlation in the supply shocks and that the policy maker may be thought of as setting output directly.

$$L = \Sigma_{t=0}^{\infty}\beta^t[\lambda y_t^2 + \pi_t^2] \tag{23}$$

We note that $E_{t-1}[\pi_t]$ and $E_{t-1}[y_t]$ are present in (14) and hence to distinguish these variables from

π_t and y_t we make use of the reduced form expressions $\pi_s = E_{s-1}[\pi_s] + c_1\epsilon_s$ and $y_s = E_{s-1}[y_s] + d_1\epsilon_s$ where s can be any time period. Making use of these we form the Lagrangean (24) where ϕ_t is the multiplier on (14). We think of the policy maker optimising separately with respect to $E_{s-1}[\pi_s]$ and $E_{s-1}[y_s]$ (in each case based on period s-1 information) followed by c_1 and d_1.

$$L = \Sigma_{t=0}^{\infty}\beta^t \left[\begin{array}{c} \frac{1}{2}[E_{t-1}[\pi_t] + c_1\epsilon_t]^2 + \frac{1}{2}\lambda[E_{t-1}[y_t] + d_1\epsilon_t]^2 \\ \phi_t\left[\begin{array}{c} -k\epsilon_t - kq^*\epsilon_{t-1} + c_1\epsilon_t - d_1 k\gamma(\epsilon_t + q^*\epsilon_{t-1}) \\ + \dfrac{E_{t-1}[\pi_t] - \beta E_t[\pi_{t+1}]}{1+\beta q^*} - k\gamma(q^* E_{t-2}[y_{t-1}] + (1+\beta q^{*2})E_{t-1}[y_t] + \beta q^* E_t[y_{t+1}]) \end{array} \right] \end{array} \right] \tag{24}$$

The first order conditions for the minimisation of (24) are (25)-(28) where the expressions for c_1 and d_1 make use of the reduced form expression for the ϕ_t process, $\phi_s = E_{s-2}[\phi_s] + e_1\epsilon_s + e_2'\epsilon_{s-1}$ where e_1 and e_2' remain to be determined.

$$E_{s-1}[\pi_s] = \frac{\phi_{s-1} - E_{s-1}[\phi_s]}{1+\beta q^*} \tag{25}$$

$$E_{s-1}[y_s] = \frac{k\gamma}{\lambda}[q^*\phi_{s-1} + (1+\beta q^{*2})E_{s-1}[\phi_s] + \beta q^* E_{s-1}[\phi_{s+1}]\ ] \tag{26}$$

$$c_1 = -e_1 \tag{27}$$

$$d_1 = \frac{k\gamma}{\lambda}(e_1 + \beta q^* e_2') \tag{28}$$

From (14) and (25)-(28) we conjecture the MSV solutions (29). Substitution of these into (14) and (25)-(28) and solving for arbitrary ϵ_t, ϵ_{t-1}, y_{t-1} and ϕ_{t-1} generates twelve equations for the twelve unknown coefficients in (29). These are solved numerically.

$$
\begin{aligned}
\pi_t &= c_1\epsilon_t + c_2\epsilon_{t-1} + c_3 y_{t-1} + c_4\phi_{t-1} \\
y_t &= d_1\epsilon_t + d_2\epsilon_{t-1} + d_3 y_{t-1} + d_4\phi_{t-1} \\
\phi_t &= e_1\epsilon_t + e_2\epsilon_{t-1} + e_3 y_{t-1} + e_4\phi_{t-1}
\end{aligned}
\tag{29}
$$

We show the implications of the generalised Calvo-Taylor model for optimal policy graphically in Figures 1-3. These show the response of inflation, output and the price level under the optimal rule to a single shock (for illustrative purposes) in period 1. The simulations assume for simplicity that inflation and output (and expectations of their future values) are all zero prior to the shock. Assumed parameter values are α=0.67, β=0.98, η=6, θ=0.5, λ=0.2 which imply γ=0.16. The overall pattern of the results is robust to changing these parameter assumptions but for brevity we do not report multiple cases. We impose k=0.5 across all the simulations for comparative purposes while varying q*. The size of shock, ϵ, has a multiplicative effect on all the variables in all time periods so we do not show a scale on the vertical axes.

Figure 1 portrays the cumulative effect on the price level over time of the single shock in period 1 under the optimal rule for q*=0, 0.5, 1. This confirms the conjecture above that for q*>0 the optimal rule does not correspond to targeting the price level whereas the price level is stationary if q*=0. This result is robust to wide changes in the parameter assumptions as long as q*>0. Figure 2 shows inflation rates over time following the shock. It may be seen that when q*>0 inflation the period after the shock tends to be positive. This reflects two factors, first the shock in period 1 also affects the Phillips curve in period 2 through the $q^*\epsilon_{t-1}$ term in (14), and second the smaller coefficient on $E_t[\pi_{t+1}]$ in the Phillips curve with q*>0 implies less gain from the rule committing to a low inflation rate the period after the shock. The first of these factors matters for the optimal inflation path but is not decisive for the long run price level remaining above its initial value. If we imposed this "shock" in period 2 in a Calvo model the price level would still be stationary (since it is stationary in relation to a single shock and the effects of many shocks are additive). In period 3 the effect of the $E_t[\pi_{t+1}]$ term may be seen in that the strongly negative inflation rates when q*>0 will improve the inflation-output tradeoff in period 2. This linkage between periods 2 and 3 is not affected by the reduced coefficient on forward looking inflation in (14) because after period 1 the response to the period 1 shock is completely predictable and the perfect foresight Phillips curve (15) has a coefficient on forward looking inflation equal to β, as in the Calvo case, for any q*. Figure 3 shows the corresponding output gaps.

Discretion

Under discretion we continue to assume that the policy maker minimises the loss function (23) subject to the Phillips Curve constraint (14) but with the additional constraint that commitment is not possible. This means that expectations already formed and past values of variables are taken as given when policy choices are made though the policy maker may and should take into account the effect of current choices on future choices subject to that constraint given that y_{t-1} is an endogenous state variable. To derive optimal policy under discretion we form the Lagrangean (30) where ω_t is the multiplier for the discretion case.

$$L = \Sigma_{t=0}^{\infty}\beta^t \Big[-\tfrac{1}{2}(\lambda y_t^2 + \pi_t^2) + \omega_t \Big[-\pi_t + \frac{\beta}{1+\beta q^*}E_t[\pi_{t+1}] + \frac{\beta q^*}{1+\beta q^*}E_{t-1}[\pi_t] + k\epsilon_t + kq^*\epsilon_{t-1} + k\gamma[q^* y_{t-1} + \beta q^{*2}E_{t-1}[y_t] + y_t + \beta q^* E_t[y_{t+1}]\ ] \Big] \Big] \tag{30}$$

Given the presence of y_{t-1}, ϵ_t and ϵ_{t-1} in the Phillips curve we conjecture the reduced form solutions for inflation and output given by (31) and (32) such that differentiating (30) with respect to output will take account of the reduced form ρ parameters, including their effect on expectations, but without the policy maker being able to optimise their values since this would require a commitment to respond to past values (see McCallum and Nelson, 2000).

$$\pi_t = \rho_\pi y_{t-1} + a_1 \epsilon_t + a_2 \epsilon_{t-1} \tag{31}$$

$$y_t = \rho y_{t-1} + b_1 \epsilon_t + b_2 \epsilon_{t-1} \tag{32}$$

From (30)-(32) we find the first order conditions for maximising (30) shown by (33) and (34).

$$\pi_t = -\omega_t \tag{33}$$

$$y_t = \frac{1}{\lambda}\left[\frac{\beta\rho_\pi}{1+\beta q^*} + k\gamma(1+\beta q^*\rho)\right][\omega_t + \beta q^* E_t[\omega_{t+1}]\ ] \tag{34}$$

Substituting the first order conditions into (14) we find that ρ is given implicitly by (35) and ω_t by (36) where coefficients and constants are shown by (37)-(39).

$$0 = \rho(1-\beta\rho)^2 + \frac{k^2\gamma^2}{\lambda}(1+\beta q^*)^2(q^*+\rho)(1+\beta q^*\rho)^3 \tag{35}$$

$$\omega_t = f_1\epsilon_t + f_2(\epsilon_{t-1} + \rho\epsilon_{t-2} + \rho^2\epsilon_{t-3} + ...) \tag{36}$$

$$f_1 = \frac{-k}{E}\left[\frac{1-\beta\rho+\beta q^*}{1+\beta q^*} + \frac{k^2\gamma^2}{\lambda(1-\beta\rho)}(1+\beta q^*\rho)^3(1+\beta q^*)\right] \tag{37}$$

$$f_2 = \frac{-kq^*}{E} \tag{38}$$

$$E = \frac{\frac{1-\beta\rho}{1+\beta q^*} + [\frac{k^2\gamma^2}{\lambda(1-\beta\rho)}(1+\beta q^*\rho)^3(1+\beta q^*)]^2}{\frac{k^2\gamma^2}{\lambda(1-\beta\rho)}(1+\beta q^*\rho)(1+\beta q^*)[(1+\beta q^*\rho)^2+\beta q^{*2}(2+\beta q^*\rho)+\frac{1-\beta\rho+\beta q^*}{1+\beta q^*}]} \tag{39}$$

Having established the process for the multiplier we may substitute these results into the first order conditions above to give inflation and output explicitly. We show these results graphically in Figure 4 which compares rule and discretion outcomes for inflation and output for q*=0, 0.5, 1 (using a common vertical scale to facilitate comparisons). The top pair of figures repeats the standard Calvo results (see Clarida et. al., 1999) that optimal policy under discretion simply responds to the current value of the shock variable with inflation and the output gap returning to zero immediately if no further shock occurs the following period. Once q* is greater than zero, y_{t-1} and ϵ_{t-1} appear in the Phillips curve and policy both responds to ϵ_{t-1} and also the current choice of output, y_t, determines the value of "y_{t-1}" the following period. The latter effect is small, however, since in period 3 when the effect of the period 1 shock is no longer present, optimal discretionary choices are close to zero. Hence while the outcomes under discretion are fairly close to the rule in period 1 when the shock occurs the major difference lies in period 3 when the rule can commit to negative inflation (which benefits the period 2 outcome also) while discretion achieves only marginally negative inflation, in turn worsening the period 2 outcome.

Figures 5 and 6 give results for the effect of optimal policy in each case on the expected per period value of the loss function (which strictly speaking is the correct measure of welfare only if β=1). As expected the loss under the rule is less than that under discretion but both of them, and the relative gap between them shown by Figure 6, increase with q*. This follows from the fact that an increase in q* effectively raises the total variance of the shocks hitting the economy since a shock in one period has an additional effect the following period with weight q*. Lastly, Figure 7 examines the reduced form persistence properties of inflation and output under discretion, shown by the simple correlation coefficient between neighbouring values. These are zero when q*=0 (since the Calvo model has no intertemporal dimension under discretion) but rise significantly above zero as q* increases. It should be emphasised that we do not place a structural interpretation on these values, especially that for inflation since π_{t-1} is not present in the Phillips curve (14), but they show that observed inflation and output persistence can arise from the generalised Calvo-Taylor model without serial correlation in the shock process. An interesting empirical question is whether the model can generate realistic observed persistence without the addition of a lagged inflation term in the Phillips curve but in any event it is likely that the empirical coefficient on that term would be smaller.

4. Conclusion

This paper has analysed the microfoundations of the New Keynesian Phillips Curve, exploring the differences between the Calvo and Taylor models when prices or wages are staggered. It presented a generalised model which nested these as special cases by means of allowing the probability of firms changing price to vary between the period immediately after a price change and subsequent periods. While this change only affects one period, and is only one of many possible changes to the staggering structure that could be made, it shows that the coefficients on expected inflation and the structure of the output and shock terms in the Phillips curve vary significantly with the difference between these two probabilities. We leave to future research explicit consideration of further enrichment of the price change probabilities though in general this would give rise to longer lags and leads in the Phillips curve.

The model clarifies the source of the different predictions for the Phillips curve in an otherwise common framework, showing that they result solely from differences in staggering structures given optimising behaviour by agents. While not itself giving a unique prediction of the appropriate Phillips curve coefficients to use in policy modelling (since q* could take a range of values, and for which of course empirical evidence is crucial) the paper nevertheless has a strong conclusion that we should be wary of policy results that rest sensitively on particular staggering assumptions. This is in the spirit of the appeals for robustness in Taylor (1999b) for example. In particular, i) the stationary price level result of Clarida, Gali and Gertler (1999) is not robust to changes in the structure of staggering away from Calvo pricing; and ii) the generalised model contributes to our understanding of observed persistence in inflation (and output) even in the absence of a structural term in lagged inflation in the Phillips Curve.

Figure 1: Price Level Under the Optimal Rule (Single Shock in Period 1)

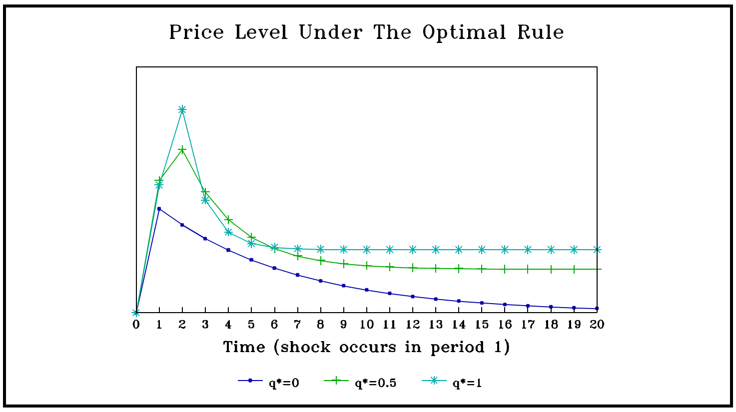


Figure 2: Inflation Under the Optimal Rule (Single Shock in Period 1)

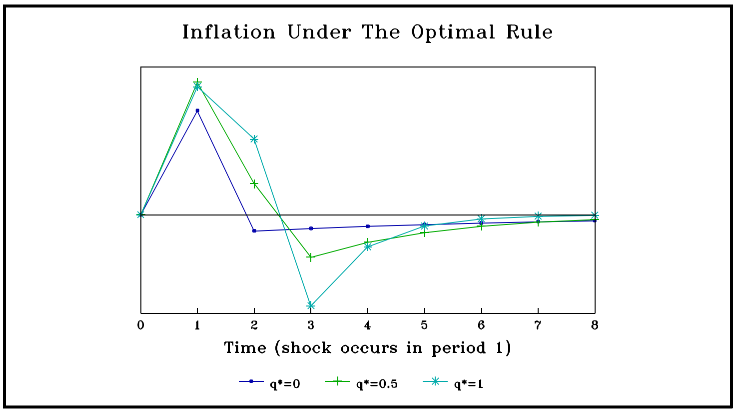


Figure 3: Output Gap Under the Optimal Rule (Single Shock in Period 1)

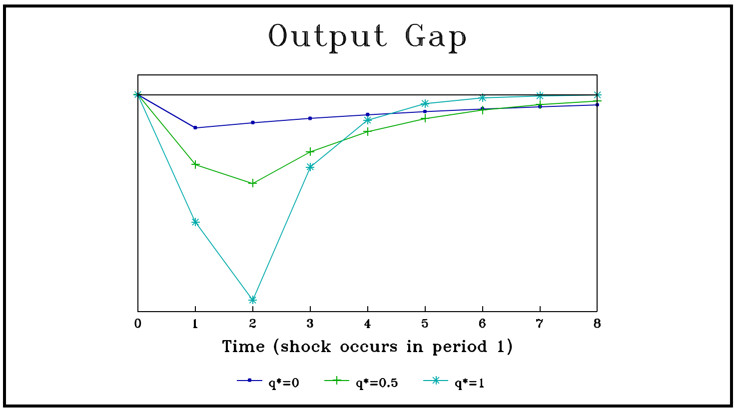


Figure 4: Rule and Discretion Outcomes Compared (Single Shock in Period 1)

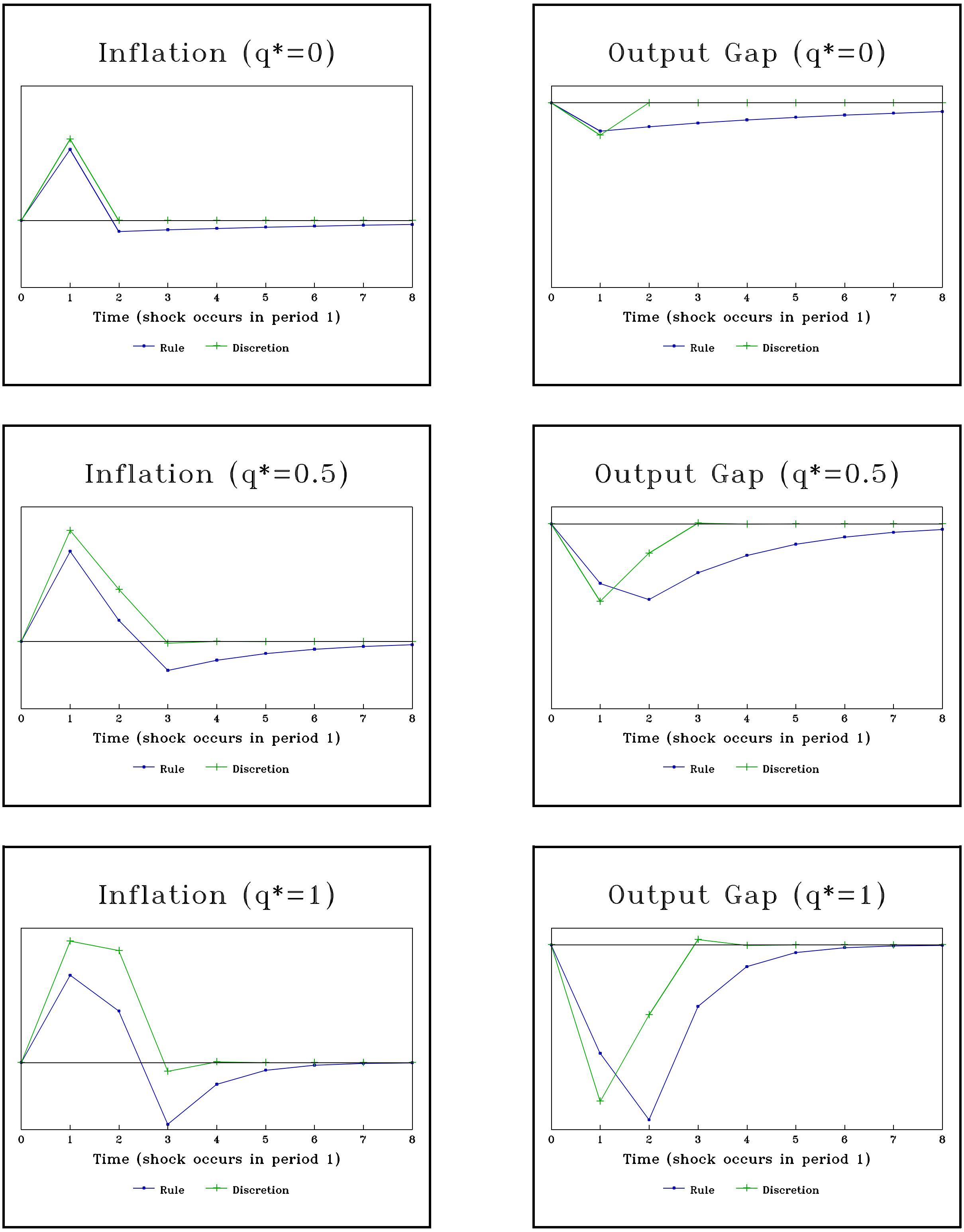


Figure 5: Expected Level of Per Period Loss Under Rule and Discretion

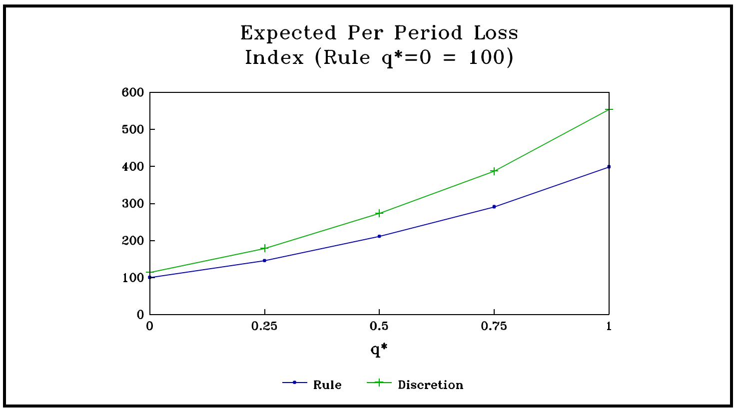


Figure 6: Expected Relative Loss Under Rule and Discretion

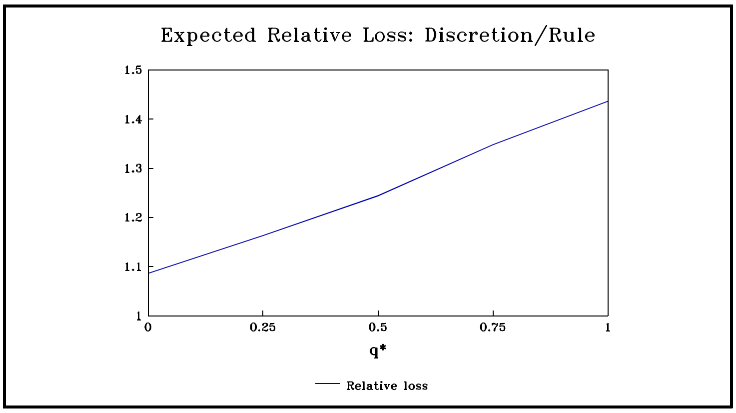
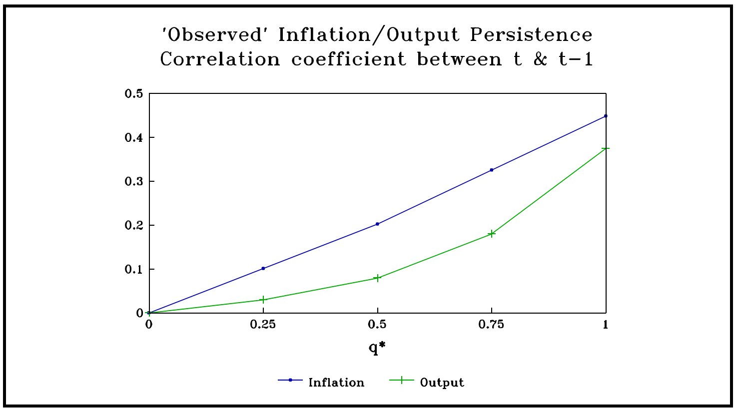


Figure 7: Simple Correlation Coefficients Between π_t & π_{t-1} and y_t & y_{t-1}



Appendix A: Microfoundations of Optimal Price and Wage Setting

We derive expressions for the prices and wages that would be set each period in a flexible price/wage environment in order to generate 'ideal' prices and wages per period which form part of the derivation of optimal price (later wage) setting once staggering constraints are imposed. The microfoundations of these choices are standard. We consider a large number of symmetric, monopolistically competitive firms, indexed by i, each with production function (A1), where Y_i is firm output, K_i the firm's capital stock which we hold constant and L_i firm level employment. Any multiplicative constant that may be present in (A1) is normalised to unity for convenience and without loss of generality. All expressions in this appendix refer to a single period and hence for simplicity we do not include time subscripts.

$$Y_i = K_i^{1-\alpha} L_i^{\alpha} \tag{A1}$$

Each firm faces the demand curve (A2) where P_i is the firm's price, P the general price level (defined as the weighted geometric mean of firm prices with weights summing to unity and equal to the proportion of all firms with each particular price), η the common price elasticity of demand (defined such that $\eta>0$) and Y_{di} an index of aggregate demand per firm.

$$Y_i = (\frac{P_i}{P})^{-\eta} \; Y_{di} \tag{A2}$$

We also make use of the notation W for the common nominal wage (labour is assumed mobile between firms) which is exogenous to firms individually, c_K for the per period cost of capital (which plays no part in the analysis since capital is fixed), and R_i for firm profits. Profits are given by $R_i=P_iY_i-WL_i-c_K K$ in terms of the three variables endogenous to the firm, P_i, Y_i and L_i. For given Y_{di} and P the choice of any of these three implies the other two through (A1) and (A2) and we substitute from those expressions for Y_i and L_i to give profits in terms of P_i by (A3).

$$R_i = P[(\frac{P_i}{P})^{1-\eta} Y_{di} - K_i^{-\frac{(1-\alpha)}{\alpha}} (\frac{W}{P})(\frac{P_i}{P})^{-\frac{\eta}{\alpha}} Y_{di}^{\frac{1}{\alpha}}] - c_K K_i \tag{A3}$$

Differentiating (A3) with respect to P_i gives the first order condition for price, (A4), which may be substituted into (1) and (2) to give optimal employment by (A5) and output by (A6). The second order condition for profit maximisation may readily be shown to be satisfied.

$$P_i = P[\frac{Y_{di}^{1-\alpha}(\frac{W}{P})^{\alpha}}{K_i^{1-\alpha}[\alpha(1-\frac{1}{\eta})]^{\alpha}}]^{\frac{1}{\eta[1-\alpha(1-\frac{1}{\eta})]}} \tag{A4}$$

$$L_i = [\frac{\alpha(1-\frac{1}{\eta})K_i^{(1-\alpha)(1-\frac{1}{\eta})} Y_{di}^{\frac{1}{\eta}}}{\frac{W}{P}}]^{\frac{1}{1-\alpha(1-\frac{1}{\eta})}} \tag{A5}$$

$$Y_i = [\frac{\alpha(1-\frac{1}{\eta})K_i^{\frac{1-\alpha}{\alpha}} Y_{di}^{\frac{1}{\eta}}}{\frac{W}{P}}]^{\frac{\alpha}{1-\alpha(1-\frac{1}{\eta})}} \tag{A6}$$

In (A4)-(A6) the powers on the firm level quantity variables, L_i, K_i and Y_{di} are such that we may multiply each of these by the number of firms which then cancels such that the P_i in (A4) may be expressed as a function of aggregate demand, Y_d, and the aggregate capital stock and (A5) and (A6) give aggregate employment and output simply by dropping the i subscripts. From this point K_i may be normalised to unity. P_i in (A4) may also be shown to be equal to nominal marginal cost divided by (1-1/η) but we keep the roles of output and the real wage separate for clarity.

We turn to wage setting behaviour, assuming that wages are set competitively by many small groups of workers (who by symmetry set the same wage) whose preferences may be summarised by the aggregate labour supply curve $L_s=(W/P)^{\theta}$ where without loss of generality a possible multiplicative constant is normalised to unity. Equating labour supply with aggregate labour demand from (A5) without i subscripts gives the equilibrium real wage by (A7). While we think of workers setting the nominal wage we express outcomes in terms of the real wage for convenience, noting that with contemporaneous wage setting rational workers will have full information about the real wage that will result from any given nominal wage. Using the labour supply curve and (A1) equilibrium employment and output are given by (A8) and (A9).

$$\frac{W}{P} = [\alpha(1-\frac{1}{\eta})Y^{\frac{1}{\eta}}]^{\frac{1}{1+\theta[1-\alpha(1-\frac{1}{\eta})]}} \tag{A7}$$

$$L = [\alpha(1-\frac{1}{\eta})]^{\frac{\theta}{1+\theta(1-\alpha)}} \tag{A8}$$

$$Y = [\alpha(1-\frac{1}{\eta})]^{\frac{\alpha\theta}{1+\theta(1-\alpha)}} \tag{A9}$$

Given that we have assumed complete within period price and wage flexibility and not yet introduced shocks (A7)-(A9) may be interpreted as flexible wage-price natural rates. Denoting these with a "*" we may re-express (A4) and (A7), each with their right hand side variables at their natural rates, compactly in terms of deviations from natural rates by (A10) and (A11).

$$P_i = P[(\frac{Y}{Y^*})^{1-\alpha}(\frac{\frac{W}{P}}{(\frac{W}{P})^*})^{\alpha}]^{\frac{1}{\eta[1-\alpha(1-\frac{1}{\eta})]}} \tag{A10}$$

$$\frac{W}{P} = (\frac{W}{P})^{*} \; (\frac{Y}{Y^{*}})^{\frac{1}{\eta[1+\theta[1-\alpha(1-\frac{1}{\eta})]}} \tag{A11}$$

We make further use of (A11) when we consider staggering of wages below but for the time being the core version of the model assumes complete wage flexibility while prices are staggered so (A11) may simply be substituted into (A10) which gives (A12) as the ideal single period price which would be set by an individual firm in the absence of staggering constraints.

$$P_i = P(\frac{Y}{Y^{*}})^{\frac{1+\theta(1-\alpha)}{\eta[1+\theta[1-\alpha(1-\frac{1}{\eta})]}} \tag{A12}$$

As a final step (see Walsh, 1998 p.219) we take logs of (A12) and assume a log linear shock to price setting (which could also arise from wage setting through (A11) substituted into (A10)), ϵ, to give (A13) as the single period ideal price in logs denoted p* (without an i subscript since it is symmetric across all firms). We also add time subscripts and y refers to the log of (Y/Y*), the output gap.

$$p_t{*} = p_t + \gamma y_t + \epsilon_t \quad ; \quad \gamma = \frac{1+\theta(1-\alpha)}{\eta[1+\theta[1-\alpha(1-\frac{1}{\eta})]} \tag{A13}$$

Appendix B: Inflation Given Optimal Prices or Wages

We first derive the aggregate inflation rate for given individual staggered prices set by firms for use in the derivation of the generalised Calvo-Taylor Phillips curve in Section 1. The new price set by firms in period t that are able to change their price at that time is x_t, and the probability of being able to change price again the following period is q_1 and, assuming that a new price has not already been set, q each period thereafter. We derive the distribution of prices existing in each period, distinguished by the time since they were set, on the assumption that the distribution is in its ergodic stationary state. Given that distribution the derivation of the inflation rate in terms of prices set at different times is straightforward.

We start by assuming that at time t-1 there is a given (for the time being unknown) proportion of firms, s, that have changed their price at t-1 with the remaining prices having been set at time t-r being distributed with (unknown) weights $(1\text{-}s)\omega_r$ such that p_{t-1} is given by (B1) where $\sum\omega=1$.

$$p_{t-1} \;=\; sx_{t-1} \;+\; (1-s)\Sigma_{r=2}^{\infty}\omega_r x_{t-r} \tag{B1}$$

Moving forward one period to time t, these prices will evolve as follows. Considering the first term in (B1), the probability of these prices changing again at time t is q_1 and the probability of remaining the same, $1\text{-}q_1$. Given the assumed large number of firms these probabilities translate into proportions and hence at time t these prices will become x_t with weight (ie. their proportion of all the prices that will exist at time t) $q_1 s$ and remain at x_{t-1} with weight $(1\text{-}q_1)s$. Of the prices in the second term in (B1), the probability of their changing is q and hence these prices will either change into x_t (with weight $(1\text{-}s)q$, noting that $\sum\omega=1$) or remain at x_{t-r} (with weights $(1\text{-}q)(1\text{-}s)\omega_r$ respectively). Hence p_t is given by (B2).

$$p_t \;=\; [q_1 s + q(1-s)]x_t \;+\; (1-q_1)sx_{t-1} \;+\; (1-q)(1-s)\Sigma_{r=2}^{\infty}\omega_r x_{t-r} \tag{B2}$$

Making use of the notation $q\text{-}q_1\text{=}q^*$ we first equate the proportions of newly changed prices at t-1, s in (B1), and t, $q_1 s+(1\text{-}s)q$ in (B2), which implies that $s=q/(1+q^*)$. Substituting for s in (B1) and (B2) gives (B3) and (B4) for p_{t-1} and p_t.

$$p_{t-1} = \frac{qx_{t-1} + (1-q_1)\Sigma_{r=2}^{\infty}\omega_r x_{t-r}}{1+q*} \tag{B3}$$

$$p_t = \frac{qx_t + q(1-q_1)x_{t-1} + (1-q_1)(1-q)\Sigma_{r=2}^{\infty}\omega_r x_{t-r}}{1+q*} \tag{B4}$$

Equating the proportions of prices set 1, 2, 3 etc. periods before in (B3) and (B4) gives ω_1=q and ω_r=q(1-q)$^{r-2}$ for r$\geq$2. Substituting these ω values into (B4) gives (B5) and subtracting the equivalent expression for p_{t-1} gives inflation, π_t by (B6).

$$p_t = \frac{q}{1+q*}[x_t + (1-q_1)\Sigma_{r=1}^{\infty}(1-q)^{r-1}x_{t-r}] \tag{B5}$$

$$\pi_t = \frac{q}{1+q*}[x_t - q_1 x_{t-1} - q(1-q_1)\Sigma_{r=2}^{\infty}(1-q)^{r-2}x_{t-r}] \tag{B6}$$

In Section 1 (B6) is re-expressed in a convenient form and we also make use of (B7) which is derived from shifting (B6) one period forward and take expectations at t.

$$E_t[\pi_{t+1}] = \frac{q}{1+q*}(E_t[x_{t+1}] - q_1 x_t - q(1-q_1)\Sigma_{r=1}^{\infty}(1-q)^{r-1}x_{t-r}) \tag{B7}$$

Appendix C: Staggered wages with flexible prices

We turn to the rate of inflation in the generalised Calvo-Taylor model of Section 2 where wages are staggered and prices are set flexibly period by period ex post. Workers are assumed to face the same probabilities over the ability to change their wages as firms did over their prices in the price setting version of the model set out in Section 1. Since the resulting structure of this version of the model is very similar we give a brief treatment. Firstly, from (A10) with P_i=P (given that all price setting is at the same time ex post) the log price is given by (C1).

$$p_t = w_t + \frac{(1-\alpha)}{\alpha} y_t - Ln[(W/P)*] + \epsilon_t \qquad \text{(C1)}$$

From (C1) the rate of price inflation, π, in terms of nominal wage inflation π^w is given by (C2).

$$\pi_t = \pi_t^w + \frac{(1-\alpha)}{\alpha}(y_t - y_{t-1}) + \epsilon_t - \epsilon_{t-1} \qquad \text{(C2)}$$

From (A11) the ideal wage that workers would set each period in the absence of any constraints on changing wages again, w*, is given by (C3) where γ^w is shown by (C4).

$$w_t* = p_t + \gamma^w y_t + LN[(W/P)*] \qquad \text{(C3)}$$

$$\gamma^w = \frac{1}{\eta[1+\theta[1-\alpha(1-\frac{1}{\eta})]]} = \gamma[1+\eta(\frac{1-\alpha}{\alpha})] > \gamma \qquad \text{(C4)}$$

We assume that workers' utility maximisation problem may be approximated by the minimisation of a loss function quadratic in the actual wage relative to the ideal wage given by (C3), the equivalent assumption to that for price setting. Given this the optimisation problem has the same form as (3) except with x^w, the wage set by all workers able to change their wage in a given time period, and w* in place of x and p*. Following this the first order condition (4) follows directly, as do (5)-(8) and the expressions in Appendix B with w and π^w in place of p and π. Substituting from the wage setting version of (B7) and w* from (C3) into the wage setting version of (8) gives x^w for t and t-1 by expressions with the same structure as (9) and (11) except with the change in notation outlined above and the coefficient γ^w+(1-α)/α in place of γ. Substitution of these into (12) for π^w gives (13) with the same changes as above. Use of (C2) then gives price inflation in the staggered wages version of the model by (C5) where the first two lines equal wage inflation and the last two lines the difference between price inflation and wage inflation. The coefficients on the output gap and shock terms in (C5) may be expressed more compactly but our focus is on showing that the coefficients on expected inflation in this staggered wages version of the model are the same as the staggered prices version (13).

$$
\pi_t = \begin{array}{c}
\frac{\beta}{1+\beta q*}E_t[\pi_{t+1}] \;+\; k[(\gamma^w+\frac{1-\alpha}{\alpha})y_t+\epsilon_t+\beta q*E_t[(\gamma^w+\frac{1-\alpha}{\alpha})y_{t+1}+\epsilon_{t+1}] \\
+\frac{\beta q*}{1+\beta q*}E_{t-1}[\pi_t] \;+\; kq*[(\gamma^w+\frac{1-\alpha}{\alpha})y_{t-1}+\epsilon_{t-1}+\beta q*E_{t-1}[(\gamma^w+\frac{1-\alpha}{\alpha})y_t+\epsilon_t]] \\
+\;(\frac{1-\alpha}{\alpha})(y_t-y_{t-1}) \;+\; \epsilon_t-\epsilon_{t-1} \\
-\frac{\beta}{1+\beta q*}[E_t[(\frac{1-\alpha}{\alpha})(y_{t+1}-y_t)+\epsilon_{t+1}-\epsilon_t]+q*E_{t-1}[(\frac{1-\alpha}{\alpha})(y_t-y_{t-1})+\epsilon_t-\epsilon_{t-1}]]
\end{array} \tag{C5}
$$

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