Supersymmetry Breaking in 4D String Theory

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Abstract

In this thesis we address the problem of supersymmetry breaking in four dimensional string theory. We derive an effective Lagrangian describing the low energy degrees of freedom including the Goldstone mode associated with the spontaneously broken R-symmetry when a gaugino condensate forms.

We show the equivalence between our approach and those previously used for studying gaugino condensate in 4D string theory but we also show the need to include quantum effects due to the strong coupling constant in the hidden sector.

We determine the vacuum structure of the complete scalar potential and show that supersymmetry is broken and a large mass hierarchy may develop with a single gaugino condensate. Realistic phenomenological values for the gauge coupling constant, unification scale and soft supersymmetric breaking terms can be obtained. Consistency with the minimal supersymmetric extension of the standard model requires the hidden gauge group to be $SU(6)$ or $SO(9)$.

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To my parents

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Contents

Chapter 1

Introduction

1.1 Introduction

The study of the elementary particles and their interactions is of primary importance for our understanding of observed physical phenomena. It also has important consequences on other fields of physics like cosmology. Both fields require a deep knowledge of the fundamental laws of nature and a major aim, in theoretical physics, is to obtain a theory that unifies all known interactions. The advantages of such a theory are considerable since it could resolve many unanswered questions.

Experiments suggest that there are only four interactions: i) the strong interaction, which binds the nucleus together, ii) the well-known electromagnetic force, which binds the atom together, iii) the weak one, which does not bind anything but is responsible for phenomena like radioactivity and iv) gravity. The first three have similar mathematical structure and the theory describing them is called the standard model. Gravity, on the other hand, is described by a geometrical theory and has a quite different mathematical structure.

The theory of elementary particles (excluding gravity) agrees remarkably well with the experimental results obtained at laboratory energies, but it contains far too many free parameters. This has led many physicists to believe that it must be the low-energy limit of a more fundamental theory.

Clearly a complete "theory of everything" must include gravity. It is now believed that a quantum theory of gravity for point-like objects is mathematically inconsistent and the only possible solution, up to now, is string theory [1]. The reason is that the fundamental object in string theory is a one-dimensional object. It contains only one free parameter, the string tension, which is taken as the Planck mass (we will set the reduced Planck mass to one, $m_p = M_{Planck}/\sqrt{8\pi} = 2.4 \times 10^{18}\,GeV = 1$) to ensure that the gravitational interaction has the correct physical strength. All other quantities, like the number of particles, their mass and the strength of the strong, electromagnetic and weak interactions, can be derived. Most consistent string theories require 10 space-time dimensions, so one has to postulate that 6 of the dimensions are "small" in order to test the theory. Unfortunately, by reducing from 10 to 4 dimensions, the uniqueness of string theory is lost because there are a large number of consistent 4D string vacua. Nevertheless, for every string vacuum, all physical quantities are again fixed in terms of the Planck mass.

For any unification theory (not necessarily string theory) the question arises as to why the observed particles, like the proton, have such a small mass compared to the unification energy (Λ_{gut}), the scale where the coupling constant become unified. The difference between these two scales, the mass of the proton and the unification scale, is at least 10^{14} orders of magnitude. This problem is called the mass hierarchy problem [2]. Even if the initial conditions were such that the mass of the proton were 10^{14} times smaller than Λ_{gut}, quantum corrections would give the proton a mass of the order of the unification scale unless some surprising cancelations at the quantum level took place. In any attempt to obtain a unification theory the mass hierarchy problem must be tackled. Solving this problem in string theory is one of the main motivations of the present work.

A considerable amount of work has been invested in the mass hierarchy problem and a very elegant solution is supersymmetry (SUSY) [3, 4]. This symmetry relates particles with integer spin (called bosons) to particles with half integer spin (called fermions). In a supersymmetric theory, for every fermion there is a corresponding supersymmetric boson with exactly the same mass. Due to the relationships between the couplings and masses of fermions and bosons in a SUSY theory, quantum corrections to the boson mass vanish solving the mass hierarchy problem. However, as mentioned above, SUSY predicts a supersymmetric state with the same mass for every known particle and since these extra states have not been observed SUSY cannot be an exact symmetry. If it is going to solve the hierarchy problem the supersymmetric states cannot be very massive, as we will show in chapter 2, and should be detectable in the particle accelerators in the near future. What is believed to happen is that, even though the theory is supersymmetric, the state of minimum energy is not, corresponding to a spontaneous symmetry breakdown. If the supersymmetric theory has an asymptotically free gauge group, the gauge interaction will become stronger for decreasing energies and may eventually bind fermions together. Such a binding condensate may result in a breakdown of supersymmetry and will generate a mass for all supersymmetric partners.

There are several string theories, and perhaps the best candidate to become the ultimate "theory of everything" is the heterotic string [32]. It is consistently formulated only in 10 dimensions and, once it is compactified to 4D, the gauge group is $E_6 \times E_8'$ or subgroups. The E_6 group allows for complex fermion representations needed to describe the physical fermion states and it is called the visible sector. The other gauge group E_8' (or subgroups) is the hidden sector and it could be responsible for supersymmetry breaking. Both gauge groups can be broken by Wilson lines at the compactification scale [77] and in many models the visible and hidden sector interact through gravity only.

We will attempt to extract the generic features of 4D string vacua and treat the model-dependent quantities as free parameters. In doing so we hope to restrict the number of phenomenologically viable string vacua and get a better understanding of string theory, going some way towards singling out the correct string vacua.

A generic feature of these 4D models is the existence of the dilaton field S, whose vacuum expectation value (v.e.v.) sets the gauge coupling constant at the unification scale, and moduli fields T_i that parameterize the geometry and complex structure of the compactified manifold. The moduli determine the unification scale and some of the Yukawa couplings as well. It has been recently observed that the 4D string theory is invariant under duality symmetry [6, 8]. This symmetry transforms the moduli fields in a non-trivial way and by demanding the effective 4D theory be duality invariant the number of string vacua is restricted. As we will see in chapter 5 this symmetry plays a crucial role in stabilizing the potential.

In order to determine the v.e.v. of the dilaton and moduli fields, SUSY must be broken, but it can not be arbitrarily broken since the mass splitting between scalars and fermions must be of order of $1\,TeV$ as discussed in section 2.1. The most common and perhaps the best way to break supersymmetry is via gaugino condensation [9], because it can easily lead to a large hierarchy. The reason is that the scale at which the condensate forms, the condensation scale, is exponentially suppressed relative to the Planck mass. The condensation scale is defined as the scale at which the gauge coupling constant becomes strong and the running of the coupling depends only logarithmically on energy. The advantages of gaugino condensation as the underlying supersymmetry breaking trigger is twofold. In the first place, it is generic in 4D-superstring theories to have a hidden sector with a gauge group given by E_6 or smaller. For such asymptotically free gauge theories, the coupling usually becomes strong at low scales, so gaugino condensation may be expected and there is no need to postulate an additional source of supersymmetry breaking. Secondly the scale of gaugino condensation and the associated supersymmetry breaking scale are dynamically determined and thus the mechanism offers an explanation for the magnitude of the hierarchy of masses scales. Thus, when combined with the radiative breaking of the electroweak symmetry, the resultant theory not only protects the electroweak and supersymmetry breaking scales from receiving large radiative corrections proportional to the unification or Planck scale (the usual hierarchy problem) but also predicts the supersymmetry and electroweak breaking scales in terms of the Planck scale and the multiplet content of the theory. In addition, after supersymmetry breaking, the moduli of the theory are fixed giving predictions for the value of the gauge couplings and Yukawa couplings.

A great deal of work has been invested in studying gaugino condensates in 4D string models and an effective interaction can be obtained by observing that, neglecting possible superpotential terms, the 4D string model is anomaly free under a generalized

R-symmetry under which the gauginos and dilaton field transform non-trivially. The transformation of the dilaton field determines an unique superpotential which is proportional to the condensation scale cubed. This effective superpotential is interpreted as the contribution coming from the gaugino bilinear below the condensation scale. It also agrees with dimensional analysis since it is expected that gaugino condensation is of the order of the condensation scale cubed. Analyzing the tree level potential of this effective interaction one finds that it is unbounded from below for $S \to 0$ and is a runaway potential for $S \to \infty$. Thus no stable solution is found[1]. Furthermore if one assumes that the v.e.v. of the dilaton is somehow fixed then the v.e.v. of the moduli is independent of S and is approximately one ($T \simeq 1.2$). The value of the unification scale is determined by the v.e.vs of the moduli and for $T \simeq 1.2$ the unification scale is of order of the string scale, 30-50 times larger than the required value by the minimal supersymmetric extension of the standard model. Therefore, we conclude that even though a gaugino condensate would break SUSY spontaneously, the details of the calculations show that there is no stable solution at tree level (for a single condensate) and the v.e.vs of the moduli are inconsistent with the minimal supersymmetric standard model.

In this thesis, we investigate the dynamics of the strong coupling leading to the formation of gaugino condensate. We will show that the results are quite different from the ones obtained at tree level for the effective superpotential describing the gaugino bilinear. We find that loop corrections of the strong coupling constant, play a crucial role in stabilizing the potential. Since the formation of a gaugino condensate is a non-perturbative effect, approximation methods must be introduced. A convenient approach to the study of the formation of fermion condensates was presented by Nambu and Jona-Lasinio (NJL) many years ago [10]. Starting with an underlying 4-Fermi interaction, there is no formation at tree level level of a fermion condensate, but once the bubble summation of the initial interaction is included a condensate is dynamically favoured, for a sufficiently large coupling constant.

[1] In order to obtain a stable solution for the dilaton, one needs to introduce two gaugino condensates and chiral matter fields with non-vanishing v.e.vs.

In a strongly interacting gauge theory, there is no primary 4-Fermi interaction, but such a gauge interaction may generate strong 4-Fermi force. We will model the strong interaction by this effective 4-Fermi coupling allowing the NJL to be used to study non-perturbative effects. The form and radiative corrections of the interaction are of great importance for determining the vacuum structure of the 4-D string model. In fact, for gauginos, in the absence of superpotential terms and anomalous terms, there is an R symmetry (under which the gauginos transform non-trivially) which is spontaneously broken if a gaugino condensate forms leading to a Goldstone mode. We derive an effective interaction for the low energy degrees of freedom of the 4D string theory which includes the Goldstone mode associated to the spontaneously broken R-symmetry. Using this interaction, we reproduce the effective potential derived previously, but we also show that it is necessary to go beyond tree level. Including loop corrections due to the strong binding coupling in the hidden sector, we show that the potential is now stable in the dilaton direction. Also, the splitting between the scalar and fermion masses can be of the order of $1\,TeV$ as required to solve the mass hierarchy problem. Furthermore, the v.e.v. of the dilaton that sets the gauge coupling constant at the unification scale is given in terms of the dimension of the hidden gauge group. In order to be consistent with the minimal supersymmetric extension of the standard model (MSSM), the gauge group has to be $SU(6)$ or $SO(9)$. The v.e.vs of the moduli are now much larger allowing for MSSM to work. Finally we also obtain the soft supersymmetric terms and find good agreement with the phenomenological constraints from electroweak symmetry breaking.

1.2 Outline

This thesis is arranged as follows. In chapter two we introduce supersymmetry and local supersymmetry or supergravity and we describe the NJL model. In chapter three we present string theory and derive the low energy 4D string model of the heterotic string. We also introduce the duality symmetry which is an important ingredient in the 4D model and discuss the threshold corrections to the gauge coupling constant. These two chapters summarize the relevant information needed for the presentation of our research given in the subsequent chapters. In chapter four, we introduce an effective 4-Fermi interaction in the context of 4D string theory and show its equivalence with other approaches. We analyse the full potential and find the stable solutions in chapter five. Finally, we present the phenomenological consequences of supersymmetry breaking via gaugino condensates in 4D string model in chapter six and show that by introducing a suitable superpotential it is possible to cancel the cosmological constant while having supersymmetry broken. The summary and concluding remarks are given in the final chapter.

Chapter 2

Supersymmetry and NJL model

In any unification theory, the mass hierarchy problem arises [2], because the unification mass is necessarily many orders of magnitude larger than the electroweak scale. Even if the initial conditions were such that the tree level mass were of the order of the electroweak scale, quantum corrections would render the mass close to the unification scale unless some cancelation takes place. One of the most promising solutions is given by introducing an extra symmetry called supersymmetry [3, 4]. This symmetry relates bosons to fermions and will be presented in section two of this chapter after discussing the hierarchy problem. In section three we discuss the R-symmetry and its anomaly. This anomaly is relevant in determining the supersymmetry breaking structure in the 4D string model. We consider in section four supergravity or local supersymmetry, which is necessary if we are to include gravity and must be present in any complete unified theory. In section five we discuss the necessary conditions for supersymmetry breaking and in section six we describe the NJL model. The NJL model is a convenient way to study the formation of condensates which may trigger supersymmetry breaking. Finally, we present a global supersymmetric extension of the NJL model. Contrary to the non supersymmetric case the formation of condensates is not dynamically favoured. We will leave for chapter four the discussion of the NJL in local supersymmetry in the 4D string model.

2.1 Hierarchy Problem

The study of the elementary particles in the last three decades has been quite extensive and has led to the well known standard model. The standard model describes the interaction of the elementary particles and it is in very good agreement with the experimental data. However, it is difficult to presume that it is a complete and final theory since it has far too many free parameters.

In the standard model, the breaking of the electroweak symmetry is achieved spontaneously via the Higgs field which acquires a vacuum expectation value and mass of the order of $250\,GeV$. Assuming that a more fundamental theory appears at a scale Λ_1, where Λ_1 could be the unification scale $O(10^{16})\,GeV$ or the Planck scale $M_p = 10^{19}\,GeV$, one can relate the mass of the Higgs particle at different scales. The low energy result is independent of the physics beyond Λ_1 and can be calculated in a model independent way using the renormalization group equation. The mass at the low energy scale Λ_0 is given by [11]

$$m_H^2(\Lambda_0) = m_H^2(\Lambda_1) + Cg^2 \int_{\Lambda_0^2}^{\Lambda_1^2} dk^2 + Rg^2 + O(g^4) \tag{2.1}$$

where g is the gauge coupling constant, C is dimensionless and R grows almost logarithmically with Λ_1 as $\Lambda_1 \to \infty$. The term proportional to C diverges quadratically for $\Lambda_1 \to \infty$. In order for $m_H^2(\Lambda_0) \ll \Lambda_1$, one has to fine tune $m_H^2(\Lambda_1)$ quite accurately (to one part in 10^{14}) to cancel the second term in eq.(2.1), which is of the order of Λ_1^2. This is called the fine tuning problem and the question of why $\Lambda_0 \ll \Lambda_1$ or why $m_H^2(\Lambda_0) \ll M_{Planck}^2$ is referred to as the mass hierarchy problem.

In a supersymmetric theory the quantity C in eq.(2.1) is zero because the quantum contribution from the fermions is canceled by the contribution from the scalar fields through diagrams such as the one shown in **fig.2.1** and no fine tuning is needed. However, supersymmetry predicts that scalar fields should have the same mass as the fermions but none has been detected. Therefore, supersymmetry must be broken, and, in such a case, the second term in eq.(2.1) is given by the difference between the fermion and scalar masses (Δm^2) multiplied by the coupling of the Higgs field with the fermions

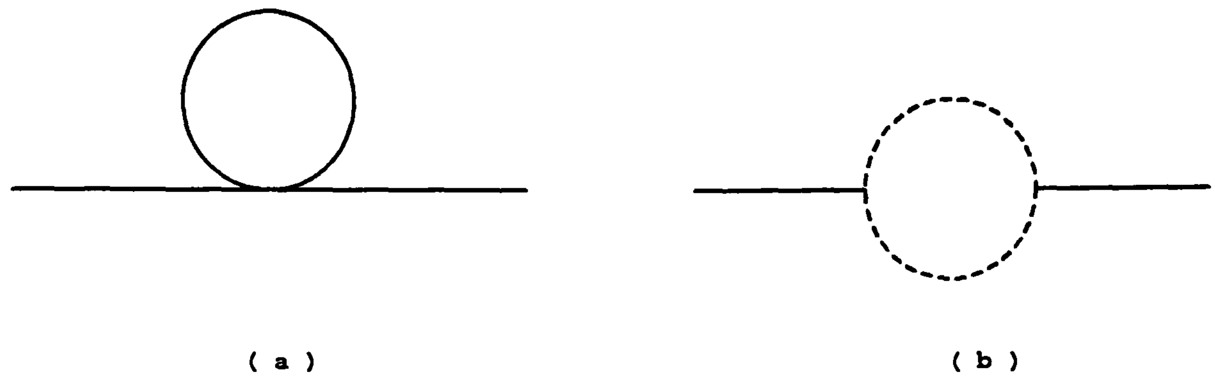

Figure 2.1: Graphs that contribute to the scalar mass. The dashed line represents a scalar field while the dotted line in fig.2.1b a fermion field.

or scalars. The condition that there is no fine tuning requires

$$g^2 \Delta m^2 \simeq m_H^2(\Lambda_0) - m_H^2(\Lambda_1) < 1\,TeV. \tag{2.2}$$

Thus, although supersymmetry can solve the hierarchy problem, it does so only if the masses of the superpartners are so low that they are accessible to experimental discovery. For this reason, considerable effort has been invested in constructing viable supersymmetric theories and their phenomenology.

Before presenting the supersymmetry breaking mechanism and its phenomenological consequences in 4D string theory we will introduce supersymmetry in the following sections.

2.2 Global Susy

In this section we will present supersymmetry (SUSY). This symmetry relates fermions to bosons and it is the only possible extension to the well known Coleman-Mandula's theorem [12]. The Coleman-Mandula theorem states that the only possible symmetries of a non-trivial S matrix for a relativistic quantum field theory in four dimensions are given by the generators of the Poincare group and generators of the internal compact Lie group. The theorem relies upon the assumptions that only a finite number of different particles associated with a one particle state of a given mass exist, and that there is a energy gap between the vacuum state and the one particle state. The proof of the theorem assumes that all generators are bosonic and, it can only be extended by the inclusion of fermionic generators leading to what is called supersymmetry [3, 4].

Supersymmetry (SUSY) is a symmetry that transforms bosons into fermions and vice versa, through a quantum operator Q_α,

$$Q_\alpha \,|\, fermions >= \,|\, bosons >, \tag{2.3}$$

and

$$Q_\alpha \,|\, bosons >= \,|\, fermions > .$$

Under the Lorentz group, Q_α transforms as a left handed Weyl spinor (1/2,0) while its hermitean adjoint transforms as a right handed one (0,1/2). Since the SUSY generators carry one half unit of spin, they obey anticommutation relations. In particular the anticommutation relations between Q_α and $\overline{Q}_{\dot\beta}$ must be proportional to the energy momentum tensor P_μ, since it is the only (1/2,1/2) conserved tensor due to the Coleman-Mandula theorem. The complete algebra for these generators is

$$[Q_\alpha, P_\mu] = 0, \tag{2.4}$$

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \tag{2.5}$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0 \tag{2.6}$$

where σ^μ denotes the Pauli matrices and $\alpha, \beta, \dot\alpha, \dot\beta = 1, 2$. Here there is only one Q_α and the algebra is called N=1 SUSY. For more then one Q^a_α, with $a = 1, 2, ..N$, then the algebra is called N-extended SUSY. The most interesting supersymmetric theory from a phenomenological point of view is N=1 SUSY, because it is the only one that allows for complex representations of fermions needed to describe chiral fermions.

Equation (2.4) shows (for $\mu = 0$) that Q_α commutes with the Hamiltonian and thus we expect the fermions and bosons related by SUSY to be degenerate in mass. Furthermore, from eq.(2.5) and using $\sigma_{\mu\alpha\dot\beta}\sigma^{\alpha\dot\beta}_\nu = 2g^\mu_\nu$ one derives that the Hamiltonian is given by

$$H = P^0 = \frac{1}{4}(Q_1\bar{Q}_1 + Q_2\bar{Q}_2 + \bar{Q}_1 Q_1 + \bar{Q}_2 Q_2) \tag{2.7}$$

which implies that the spectrum is semipositive definite, $H \geq 0$, i.e. the vacuum energy is non negative.

The simplest and most general way to obtain supersymmetric Lagrangians is by introducing the so called superfields. A superfield is a collection of fields with different spins that transforms among themselves under a SUSY transformation. Because the component field with the highest mass dimension does not usually get a kinetic term, it plays the role of a Lagrange multiplier and can be eliminated through its equation of motion.

Before presenting the relevant superfields used in this work, it is useful to introduce an anticommuting parameters θ_α and $\bar{\theta}_{\dot{\beta}}$ which are elements of the Grassman algebra

$$\{\theta_\alpha, \theta_\beta\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = \{\theta_\alpha, \bar{\theta}_{\dot{\beta}}\} = 0. \tag{2.8}$$

A finite SUSY transformation depends on the space-time coordinate x_μ and on θ_α and $\bar{\theta}_{\dot{\beta}}$ and it is defined by

$$S(x, \theta, \bar{\theta}) = e^{i[\theta Q + \bar{Q}\bar{\theta} - x_\mu P^\mu]}. \tag{2.9}$$

The multiplication of two successive SUSY transformations yields

$$S(x, \theta, \bar{\theta}) S(y, \alpha, \bar{\alpha}) = S(x + y - i\alpha\sigma_\mu\bar{\theta} + i\theta\sigma_\mu\bar{\alpha}, \theta + \alpha, \bar{\theta} + \bar{\alpha}). \tag{2.10}$$

Superfields

A scalar superfield $\Xi(x, \theta, \bar{\theta})$ is a mapping from points in superspace $(x_\mu, \theta, \bar{\theta})$ to complex numbers. Due to eq.(2.8), the Taylor expansion in $\theta, \bar{\theta}$ terminates after a finite number of terms and has the general form

$$\Xi = \phi + \theta\chi + \bar{\theta}\bar{\varphi} + \theta^2 F + \bar{\theta}^2 \bar{F} + \theta\sigma^\mu\bar{\theta}A_\mu + \theta^2\bar{\theta}\bar{\lambda} + \bar{\theta}^2\theta\psi + \theta^2\bar{\theta}^2 D, \tag{2.11}$$

where $\phi, \chi, ...etc$ denote the component fields. The mass dimension of the fields are obtained from $[\Xi] = [\phi]$ and $[\theta] = [\bar{\theta}] = -1/2$ where $[f]$ stands for the mass dimension of f. The SUSY transformation of Ξ is analogous to that of eq.(2.10),

$$S(y, \alpha, \bar{\alpha})\Xi(x, \theta, \bar{\theta}) = \Xi(x + y - i\alpha\sigma_\mu\bar{\theta} + i\theta\sigma_\mu\bar{\alpha}, \theta + \alpha, \bar{\theta} + \bar{\alpha}). \tag{2.12}$$

It is also useful to define covariant derivatives that anticommute with a SUSY transformation. They are given by

$$D_\alpha\Xi = (\partial_\alpha + i\sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu)\Xi \tag{2.13}$$

and

$$\overline{D}_{\dot{\beta}}\Xi = (-\bar{\partial}_{\dot{\beta}} - i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\beta}}\partial_{\mu})\Xi \tag{2.14}$$

where $\partial_{\alpha} \equiv \frac{\partial}{\partial\theta_{\alpha}}$.

Chiral Superfield

Different irreducible representations can now be obtained from the scalar superfield (2.11). One of the most common is the chiral superfield, which is defined by the condition

$$\overline{D}_{\dot{\alpha}}\Phi = 0. \tag{2.15}$$

It consists of a complex scalar field ϕ, a left handed Weyl spinor ψ and an auxiliary complex scalar field F. The mass dimensions of these fields are 1, 3/2 and 2 respectively, and their SUSY transformation are given by

$$\begin{aligned} \delta\phi &= \sqrt{2}\alpha\psi, \\ \delta\psi &= \sqrt{2}\alpha F + i\sqrt{2}\sigma_{\mu}\bar{\alpha}\partial^{\mu}\phi, \\ \delta F &= -i\sqrt{2}\,\partial^{\mu}\psi\,\sigma_{\mu}\,\bar{\alpha}. \end{aligned} \tag{2.16}$$

The SUSY variation of the highest component (F) of the superfield is a total derivative, a fact that will become important in constructing invariant Lagrangians. It is also clear that SUSY indeed transforms fermions into bosons and vice versa. Since the fermion field is a chiral fermion, it can describe chiral matter fields, and the ϕ component is its supersymmetric partner.

Vector Superfield

The chiral superfield does not include spin one particles. To include them, one must consider the vector supermultiplet which satisfies the reality condition $V^{\dagger} = V$. It contains 4 real spin zero fields C, M, N, D, two Weyl spinor ψ, λ and a spin one field A_{μ}. In the Wess-Zumino gauge, the fields ψ, M, N and C can be gauged away although their SUSY transformation is non-trivial. The fields λ, D and $F_{\mu\nu} \equiv \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ transform among themselves under a SUSY transformation and thus form an irreducible representation. These fields correspond to the gauge boson A_μ, its supersymmetric partner, the gaugino λ, and an auxiliary real scalar field D. One can define a chiral superfield W_α, called the gauge covariant chiral superfield, containing these fields as follows

$$W_\alpha = -\frac{1}{4}(\bar{D}\bar{D})D_\alpha V. \tag{2.17}$$

It is a chiral superfield since $\bar{D}W_\alpha = 0$. The field components are

$$W_\alpha = (-i\lambda_\alpha, \delta_\alpha^\beta D - i\frac{1}{2}(\sigma^l\bar{\sigma}^m)_\alpha^\beta F_{lm}, -\sigma^\mu_{\alpha\dot{\beta}}\partial_{\alpha\mu}\bar{\lambda}^{\dot{\beta}}) \tag{2.18}$$

where the terms in brackets correspond to an expansion of the superfield W_α in powers of the θ parameter with zero, one and two powers respectively.

In order to construct a supersymmetric Lagrangian, one needs to determine the kinetic terms for the chiral superfield and gauge covariant chiral superfield. They are given by

$$L_{ch} = [\bar{\Phi}\Phi]_D \tag{2.19}$$

and

$$L_{gau} = \frac{1}{4}[W_\alpha W^\alpha]_F + h.c. \tag{2.20}$$

where $\bar{\Phi}$ is an antichiral superfield. The subscript in eq.(2.19) denotes the D term of $(\bar{\Phi}\Phi)$, i.e. the $\theta^2\bar{\theta}^2$ term, while the F subscript refers to the F component of the chiral expression $(W_\alpha W^\alpha)$, i.e. the θ^2 term. In component notation these equations become

$$L_{ch} = FF^* + (\partial_\mu\phi)^2 + \frac{i}{2}[\bar{\psi}\sigma^\mu\partial_\mu\psi - (\partial_\mu\bar{\psi})\sigma^\mu\psi] \tag{2.21}$$

and

$$L_{gau} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{i}{2}[\bar{\lambda}\sigma^\mu\partial_\mu\lambda - (\partial_\mu\bar{\lambda})\sigma^\mu\lambda] + \frac{1}{2}D^2 \tag{2.22}$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual tensor of $F_{\mu\nu}$. From eq.(2.21) and (2.22) we recognize the usual kinetic terms for the scalar and fermion components of the chiral superfield Φ and for the gauge bosons and gauginos while the F and D fields do not get a kinetic term and therefore can be eliminated using their equation of motion.

Linear Superfield

Another relevant superfield is the linear superfield L. It is a real superfield and is defined by

$$L = \bar{L}, \tag{2.23}$$

and

$$(\bar{D}\bar{D})L = (DD)L = 0.$$

L has only three independent fields, a real scalar C, a majorana spinor ψ^L and a field strength of an antisymmetric tensor $b_{\mu\nu}$. The number of degrees of freedom is the same as for the Wess-Zumino multiplet, but here there is no auxiliary field since the number of fermionic and bosonic degrees of freedom matches off-shell. The kinetic term is given by

$$L_{lin} = \frac{1}{2}[L^2]_D. \tag{2.24}$$

The linear superfield L can be interpreted as a supersymmetric dualized chiral superfield. Furthermore, L_{ch} (2.19) and L_{lin} (2.24) are equivalent. To see this we introduce an extra term to eq.(2.24)

$$L_{lin} = -\frac{1}{2}[L^2]_D + [L(\phi + \bar{\phi})]_D \tag{2.25}$$

which is just a Lagrange multiplier. Eliminating L gives $L = \phi + \bar{\phi}$ and using the equation of motion for ϕ, one obtains $D^2L = \bar{D}^2L = 0$. This shows that L is a linear supermultiplet. Using these identifications one gets for the kinetic terms

$$L_{lin} = \frac{1}{2}[L^2]_D = -\frac{1}{2}[L^2]_D + [L(\phi + \bar{\phi})]_D = [\bar{\phi}\phi]_D = L_{ch}. \tag{2.26}$$

Though the linear and the chiral superfields are equivalent through a supersymmetric dual transformation, and the standard formulation of supergravity is given in terms of chiral superfields, the linear supermultiplet is needed for a consistent formulation of supergravity theories when one incorporates loop corrections because it allows for non-holomorphic kinetic terms for the gauge bosons.

The gauge interaction between the chiral and the gauge fields can be introduced by the interaction term (in the Wess-Zumino gauge)

$$[\bar{\Phi}e^{2gV}\Phi]_D = \mid D_\mu\phi \mid^2 + \frac{i}{2}\bar{\psi}\sigma^\mu D_\mu\psi + g\phi^* D\phi + ig(\phi^*(\lambda\psi) - \phi(\bar{\lambda}\bar{\psi})) + FF^* \tag{2.27}$$

where $D_\mu = \partial_\mu + igA_\mu$ is the usual gauge covariant derivative and V is the vector multiplet.

The interaction Lagrangian for the chiral fields is given by the F term of the superpotential W (plus hermitean conjugate), i.e. $[W]_F + h.c.$. The superpotential W is a function of chiral fields only. The mass terms and Yukawa interaction for the chiral fields are related to the quadratic and cubic terms of the superpotential,

$$W = \frac{1}{2}m_{ij}\Phi^i\Phi^j + \frac{1}{3}\lambda_{ijk}\Phi^i\Phi^j\Phi^k. \tag{2.28}$$

The F projection of the superpotential W is

$$[W]_F = F_i\frac{\partial W(\phi)}{\partial \phi_i} - \frac{1}{2}\frac{\partial^2 W(\phi)}{\partial \phi_i \phi_j}\psi_i\psi_j$$

where ϕ_i and ψ_i are the scalar and fermion component of the chiral superfield Φ_i respectively. It is easy to show that the scalar and fermion components of a given superfield are degenerated in mass. The tree level scalar potential is given by

$$V_0 = FF^* = \mid \frac{\partial W}{\partial \phi_i} \mid^2 \tag{2.29}$$

and it is semipositive definite.

In principle one could generalize eq.(2.20) by multiplying it by a gauge kinetic function f that should be chiral and transform as a symmetric product of the adjoint representation of the gauge group. One can, as well, generalize eq.(2.28) by allowing the superpotential to be an arbitrary polynomial of chiral superfields. But a renormalizable theory is only obtained if the gauge kinetic function is a constant and W does not include terms with degree higher than three. For these theories, a series of non-renormalization theorems [13] have been obtained. It can been shown that the F terms in the Lagrangian are not renormalized. As an example one has the masses and the Yukawa couplings of eq.(2.28). Thus, the bare mass and Yukawa coupling remain unchanged (up to wave function renormalization) after radiative corrections are included. It is precisely this property that makes SUSY so interesting, since it allows for particles with zero initial mass to remain massless even after loop corrections are included. This property is a consequence of the cancelation of loop diagrams between scalars and fermions since they give the same contribution but with opposite sign (cf. fig.2.1). Furthermore, since the vacuum is semipositive definite, if SUSY is not spontaneously broken at tree level then it will not be spontaneously broken (perturbatively). Even if SUSY is spontaneously broken, a supersymmetric theory has no quadratic divergences. The absence of quadratic divergences is a welcome property for any theory since it has then a better behavior in the ultraviolet limit. Not all theories without quadratic divergences are supersymmetric. There are only a finite number of non-supersymmetric terms which give no quadratic divergence. These terms are called soft supersymmetric breaking terms and they can naturally appear in the context of a more general theory like supergravity. Thus, a global supersymmetric theory with soft supersymmetric terms can be seen as a low energy limit of a more fundamental one. We will see that this is the case in the context of the 4D string model and will study the soft supersymmetric terms in chapter 6.

Finally it is important to keep in mind that the nonrenormalizable theorems are based on symmetries of the S matrix (the underlying theory could have more or less symmetry) and a flat Minkoswki space. For example, in local supersymmetry (or supergravity) the tree level potential is no longer semipositive definite. Non-perturbative effects may break SUSY spontaneously even in a theory with SUSY not spontaneously broken at tree level.

2.3 R-symmetry and anomaly

We will now present the R-symmetry . This symmetry transforms the phases of the component fields of a superfield differently. It is an additional symmetry that transforms the supersymmetric generators Q_α non-trivially, but leaves the supersymmetric algebra eqs.(2.4-2.6) unaltered. Under this symmetry the supersymmetric generators transform as $Q_\alpha \rightarrow Q'_\alpha = e^{-i\delta}Q_\alpha$ and $\bar{Q}_{\dot{\alpha}} \rightarrow \bar{Q}'_{\dot{\alpha}} = e^{i\delta}\bar{Q}_{\dot{\alpha}}$ while the Grassman parameters transform as $\theta \rightarrow \theta' = e^{i\delta}\theta$ and $\bar{\theta} \rightarrow \bar{\theta}' = e^{-i\delta}\bar{\theta}$. The generator of the R-symmetry does not commute with the supersymmetry generator but it does commute with the energy momentum tensor P_μ and with $M_{\mu\nu}$. The realization of this symmetry on the scalar superfield eq.(2.11) is

$$\Xi'(x,\theta',\bar{\theta}') = e^{2in\delta}\Xi(x, e^{-i\delta}\theta', e^{i\delta}\bar{\theta}') \tag{2.30}$$

where $2n$ is the chiral or R charge of the superfield Ξ. For a chiral superfield we find

$$\Phi' = (\phi', \psi', F') = e^{2in\delta}\Phi = e^{2in\delta}(\phi, e^{-i\delta}\psi, e^{-2i\delta}F) \tag{2.31}$$

while for the vector superfield we have

$$V' = V, \tag{2.32}$$

since it is a real superfield. Notice that under the R-symmetry, the transformation of the components fields is different, and eq.(2.32) implies that for the Wess-Zumino multiplet, the D and A_μ components have zero R charge while the λ component has R-charge one. The R-symmetry, if exact, imposes constraints on the possible interactions terms in the superpotential and the kinetic terms. For the chiral superfields eq.(2.19) is automatically invariant while eq.(2.20) requires the gauge covariant superfield to have R charge one (n=1). Thus, the gaugino field (which is the lowest component of W_α) has R charge one as well. R-invariance does not allow a mass term for the gauginos since the term $m\lambda\lambda$, where λ is a majorana spinor, is not R-invariant.

The R-transformation on the chiral fermion fields is equivalent to a chiral transformation that is anomalous [14]. The anomaly arises from triangle diagrams with gauge bosons as external particles and chiral fermions running in the loop. The divergence of the chiral or R current J_R is

$$\partial_\mu J_R^\mu = \Sigma_j 2im_j\bar{\psi}_j\gamma_5\psi_j + \frac{1}{3}\beta_0 F_{\mu\nu}\tilde{F}^{\mu\nu} \tag{2.33}$$

where m is the mass of the chiral fields, β_0 the one loop beta-function coefficient, $F_{\mu\nu}$ the field strength tensor for the gauge fields and $\tilde{F}^{\mu\nu}$ its dual. This current is not conserved even in the case $m = 0$ and the anomaly is called the chiral, axial or ABJ anomaly and it was shown that it is not affected by higher order radiative corrections [14].

2.4 Local Supersymmetry

In order to include gravity we study local supersymmetry or supergravity (SUGRA) [3] which is the gauge theory of supersymmetry. An infinitesimal SUSY transformation on a boson B gives a fermion F and applying a second SUSY transformation F is rotated back to ∂_μB. Thus, two successive SUSY transformations lead to a space-time rotation and one expects that local SUSY will necessarily include gravity. The global SUSY transformation eq.(2.9) becomes local when the supersymmetric parameter θ is a function of the space-time coordinates x_μ, i.e. $\theta = \theta(x_\mu)$.

The purely SUGRA multiplet in four space-time dimensions consists of a massless spin 2 particle, the graviton, and its supersymmetric partner the spin 3/2 gravitino. The Lagrangian is given by

$$L_g = -\frac{1}{2k^2}\sqrt{|\,g\,|}\,R - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma \tag{2.34}$$

where $|\,g\,|$ is the determinant of the metric $g_{\mu\nu}$ which should be written in terms of the vierbein e_μ^m as $g_{\mu\nu} = e_\mu^m e_\nu^n \eta_{mn}$, R is the curvature that depends on the vierbein and the spin connection w_μ^{mn}, ψ_μ is the gravitino, $D_\mu = \partial_\mu + \frac{1}{2}w_\mu^{mn}\sigma_{mn}$ the covariant derivative with respect to gravity and $k = 1/m_p$. The spin connection does not represent a new degree of freedom and can be eliminated by its equation of motion. The supersymmetry transformation that leaves eq.(2.34) invariant are

$$\begin{aligned} \delta e_\mu^m &= \frac{1}{2}k\bar{\xi}\gamma^m\psi_\mu, \\ \delta\psi_\mu &= \frac{1}{k}D_\mu\xi \\ \delta w_\mu^{mn} &= 0 \end{aligned} \tag{2.35}$$

where we have used four components notation with $\xi = (\theta_\alpha, \bar{\theta}^\alpha)$.

The most general global SUSY lagrangian coupling chiral Z and vector V superfields is

$$[\phi(Z, \bar{Z}e^{2V}]_D + Re[W(Z)]_F + Re[f(Z)_{ab}W^aW^b]_F. \tag{2.36}$$

The function ϕ transforms as a real vector, $W(Z)$ is the superpotential and f the gauge kinetic function. This theory is renormalizable only if W is a polynomial of degree less than or equal to three, f is a constant and $\phi = \bar{Z}e^{2V}Z$. To obtain a locally supersymmetric theory, one needs to couple Z and V to the supergravity multiplet. There are several ways of doing it. Since it is a lengthy calculation and the most general Lagrangian has been given in the literature [3, 15] we present the final version.

The standard form of N=1, D=4 supergravity coupled to Yang-Mills is described in terms of chiral superfields (with up to two derivatives in the bosonic fields) and it is completely specified by two functions; the Kahler potential G and the gauge kinetic function f. The functions ϕ and W of global SUSY are no longer independent, but they enter in the action through the Kahler potential G, a real and gauge invariant function, given by [15]

$$G = K + ln\,|\,W\,|^{\,2} \tag{2.37}$$

with $K = -3ln(-\frac{\phi}{3})$ a real function and W, the superpotential, an holomorphic function of the chiral superfields Z. The gauge kinetic function f is an holomorphic function of the chiral superfields Z and transforms as a symmetric product of the adjoint representation of the gauge group.

The SUGRA action is given by the superconformal Lagrangian [15]

$$e^{-1}L = -\frac{3}{2}[S_0\bar{S}_0 exp\{-\frac{1}{3}G(Z,\bar{Z}e^{2V})\}]_D + ([S_0^3]_F + h.c.) - \frac{1}{4}([f_{ab}(Z)W^aW^b]_F + h.c) \tag{2.38}$$

where $V = gV^aT^a$ is the Yang-Mills vector multiplet (in the Wess-Zumino gauge $V^a = (A_\mu^a, \lambda^a, D^a)), T^a$ are the group generators, g the gauge coupling constant, W^a the gauge covariant chiral multiplet. The conformal weights w of the different fields are: $w = 0$ for Z, $w = 1$ for S_0 and $w = 3/2$ for W^a. The two functions G and f have then conformal weight zero. The chiral multiplet S_0 is needed as a compensator which gauge fixes the superconformal symmetry down to super Poincare[1]. To obtain a canonical Einstein term -$\frac{1}{2}eR$, the compensator must be taken as [15, 16]

$$S_0 = (e^{G/6}, \frac{1}{3}e^{G/6}G_i\chi^i, h_o) \tag{2.39}$$

[1]In superconformal algebra the D and F projections include the contribution of the superfield E, which is the supersymmeteric generalization of the determinant of the metric $e = \sqrt{|\,g\,|}$

where h_o is the supergravity auxiliary field and $G_i \equiv \frac{\partial G}{\partial z^i}$ with z^i the scalar component of the chiral superfield Z^i.

To recognize the different terms in the action (2.38) we express it in terms of the component fields. The kinetic terms are given by (suppressing all gauge indices)

$$e^{-1}L_{kin} = G^i_j D_\mu z_i D^\mu z^{*j} - \frac{1}{4} Ref\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{4} Imf\, F_{\mu\nu}\tilde{F}^{\mu\nu} \tag{2.40}$$

$$+\frac{i}{2} Ref \bar{\lambda} \not{D}\lambda + iG^i_j \bar{\chi}^j \not{D}\chi_i - \frac{1}{4e}\bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma \epsilon^{\mu\nu\rho\sigma}$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\delta\rho}F_{\delta\rho}$ is the dual tensor of $F_{\mu\nu}$, χ_i is the fermion component of the chiral superfield Z_i and ψ_μ the gravitino. The gauge coupling constant can be read out of eq.(2.40) and it is given by $g^{-2} = Ref$, while Imf gives the strong CP violation term θ_c. The kinetic terms for the scalars and fermions are not in a canonical form but are multiplied by G^i_j. In string theory the scalar fields have a σ-like kinetic structure and cannot be put into a canonical form, while the gauge kinetic function f is just given by a chiral superfield S called the dilaton.

The tree level scalar potential is given by [15]

$$V_0 = h^j (G^{-1})^i_j h_i - 3e^G + \frac{1}{2}(Ref)^{-1} D^2 \tag{2.41}$$

where h_i is the auxiliary field of the chiral superfield Z_i and D_α is the auxiliary field of the vector supermultiplet V. Through their equations of motion these fields can be expressed in terms of the physical scalars and fermions and they read

$$h_i = -e^{G/2}G_i + \frac{f_i}{4}\bar{\lambda}_R \lambda_L + (\frac{1}{2}G^k_i G^j - G^{kj}_i)\bar{\chi}_{kR}\chi_{jL}, \tag{2.42}$$

$$D_\alpha = iRef^{-1}(-gG^i T^j_i z_j + \frac{i}{2}f^i \bar{\chi}_{iR}\lambda_L - \frac{i}{2}f_i \bar{\chi}^i_L \lambda_R). \tag{2.43}$$

Before analyzing the supersymmetry breaking mechanisms in supergravity theories we would like to comment on the renormalization of the gauge coupling constant. Explicit calculations of loop corrections to the gauge coupling constant [17]-[19] in four dimensional string theories show that some of the radiative terms are not holomorphic and thus, they can not be incorporated into the gauge kinetic function since this function must be holomorphic. The consistency between loop corrections and SUGRA is solved by considering not the standard form of SUGRA (cf. eq.(2.38)), but reformulating SUGRA [20, 21] in terms of a gauge kinetic function that depends on a linear multiplet L and not on chiral superfield S. This reformulation will allow for having a non-holomorphic gauge kinetic function. As we saw in section 2.2, these two fields are related through a duality transformation and both formalisms are equivalent.

A non-harmonic gauge kinetic function may be introduced by considering the term [20]

$$L_\Omega = -\frac{1}{4}[J(Z, \bar{Z}e^{2V})\Omega]_D \tag{2.44}$$

where J is a real gauge invariant function and Ω is the non-abelian Chern-Simons supermultiplet associated with the vector multiplet V (in the Abelian case it is just given by $\Omega = Tr[D^\alpha V W_\alpha + \bar{D}_\alpha V \bar{W}^\alpha + V D^\alpha W_\alpha]$). In component notation eq.(2.44) gives

$$L_\Omega = -\frac{1}{8}J(z,\bar{z})F_{\mu\nu}F^{\mu\nu} - \frac{1}{8}e^{-1}\epsilon^{\mu\nu\rho\sigma}E_\mu\Omega_{\nu\rho\sigma} \tag{2.45}$$

where

$$E_\mu = i(J^i\partial_\mu\bar{z}_i - \bar{J}_i\partial_\mu z^i) \tag{2.46}$$

and

$$\Omega_{\mu\nu\sigma} = Tr(V_{[\mu}F_{\nu\sigma]} - \frac{1}{3}V_{[\mu}V_\nu V_{\sigma]}) \tag{2.47}$$

is the Yang-Mills Chern-Simon form, for which $2\epsilon^{\mu\nu\rho\sigma}\partial_\mu\Omega_{\nu\rho\sigma} = Tr(F_{\mu\nu}\tilde{F}^{\mu\nu})$. If $J(z,\bar{z}) = f(z) + \bar{f}(\bar{z})$ is the sum of an holomorphic plus an antiholomorphic function then $E_\mu = i\partial_\mu(f - \bar{f})$ and by partial integration we recover the standard axionic coupling and kinetic term for the gauge fields (cf. eq.(2.40)) and both formalism are equivalent. But eq.(2.45) allows for an non-harmonic J consistent with SUSY.

In terms of the linear multiplet, the superconformal action for SUGRA theory with up to two derivatives is [20, 21]

$$e^{-1}L = -\frac{3}{2}[S_0\bar{S}_0\Phi(\frac{U}{S_0\bar{S}_0}, Z, \bar{Z}e^{2V}) + \frac{1}{6}(S+\bar{S})(U+\Omega)]_D + ([S_0^3]_F + h.c.) \tag{2.48}$$

where the field $U = L - \Omega$ is an unconstrained superfield and L is a linear multiplet. This action is equivalent to the standard supergravity Lagrangian (2.38) by choosing [21]

$$\Phi = \frac{2\sqrt{2}}{3}e^{-\hat{G}/2}(\frac{U}{S_0\bar{S}_0})^{-1/2} \tag{2.49}$$

where $\hat{G}$ is the Kahler potential excluding the contribution from the dilaton field S. The equation of motion for $S+\bar{S}$ only state that L must be a linear multiplet and using the equation of motion for U,

$$\frac{\partial}{\partial x}\Phi(x,Z,\bar{Z}e^{2V}) = -\frac{1}{6}(S+\bar{S}), \tag{2.50}$$

and substituting back into eq.(2.48) the standard SUGRA Lagrangian is obtained with

$$G = -ln(S+\bar{S}) + \hat{G} \tag{2.51}$$

and

$$f = S. \tag{2.52}$$

Explicit calculations of one-loop corrections to the gauge kinetic function show that these terms can be non-harmonic and one can introduce them in the SUGRA action by adding a term [20, 21]

$$-\frac{1}{4}[\triangle(L-\Omega)]_D \tag{2.53}$$

to eq.(2.48). The equation of motion for U (cf. eq.(2.50)) becomes

$$\frac{\partial}{\partial x}\Phi(x,Z,\bar{Z}e^{2V}) = -\frac{1}{6}(S+\bar{S}+\triangle). \tag{2.54}$$

We can now rewrite the supergravity Lagrangian in the standard form. The loop corrections (2.53) will appear in the Kahler potential G and not in the gauge kinetic function f with

$$G_{oneloop} = -ln(S+\bar{S}+\triangle) + \hat{G} \tag{2.55}$$

and f the same as in eq.(2.52). Thus, by formulating the SUGRA action in terms of a linear multiplet, we can introduce non-harmonic one-loop corrections in a consistent way with SUSY. Making a duality transformation gives a standard supergravity theory in terms of an holomorphic gauge coupling constant. The non-harmonic corrections appear now only in the Kahler potential G, which shows that they must be interpreted as a wave function renormalization of the dilaton field.

2.5 Supersymmetry breaking

The supersymmetric partners of the physical fermions have not been detected yet and therefore SUSY must be broken. It can be explicitly broken by introducing non-supersymmetric terms or it can be spontaneously broken. In general, a symmetry is spontaneously broken when the vacuum is not invariant under the symmetry generators. In the case of SUSY we require that the SUSY generators do not annihilate the vacuum

$$Q_\alpha \,|\, 0 >\neq 0. \tag{2.56}$$

Equivalently SUSY will be spontaneously broken if the anticommutator of $\{Q_\alpha, \psi\}$, where ψ is a fermion field, is different than zero. In such a case the SUSY transformation for the chiral fermion or gauge fermions is non vanishing and in local SUSY they are given by

$$\delta\psi_{Li} = \frac{1}{2}\hat{\not{D}} z_i \xi_R + \frac{1}{2} h_i \xi_L \tag{2.57}$$

and

$$\delta\lambda_L = \frac{1}{2}\sigma_{\mu\nu}\hat{F}^{\mu\nu}\xi_L - \frac{i}{2}D\xi_L. \tag{2.58}$$

A nonvanishing v.e.v. for a h_i or a D term implies that the supersymmetry transformation for the fermion fields is non zero and SUSY is spontaneously broken. The associated fermion field becomes a Goldstino and in local SUSY this field couples to the gravitino field through the mixed interaction term [15]

$$\bar{\psi}_L \cdot \gamma\,(\frac{1}{8} f_i \bar{\lambda}_L \lambda_R - e^{G/2} G_i)\chi^i_R \tag{2.59}$$

giving rise to the super-Higgs effect. The massless spin 3/2 gravitino combines with the spin 1/2 fermion giving a massive spin 3/2 gravitino. The mass of the gravitino can be deduced and it is given by

$$m_{3/2} = e^{G/2}. \tag{2.60}$$

The auxiliary field of a chiral fermion is (cf. eq.(2.42))

$$h_i = -e^{G/2} G_i + \frac{f_i}{4}\bar{\lambda}_R \lambda_L + (\frac{1}{2} G_i^k G^j - G_i^{kl})\bar{\chi}_{kR}\chi_{jL},$$

and thus for a non-trivial gauge kinetic function with $f_i \neq 0$, a non-vanishing gaugino condensate, $< \bar{\lambda}_R \lambda_L > \neq 0$, may lead to spontaneously broken SUSY.

From the structure of the tree level potential

$$V_0 = h^j (G^{-1})^i_j h_i - 3e^G + \frac{1}{2}(Ref)^{-1} D^2$$

we see that unlike in global SUSY, V_0 is no longer semipositive definite and one can have broken SUSY and vanishing cosmological constant (i.e. $h^j (G^{-1})^i_j h_i = 3e^G$ and $D = 0$). If the minimum of the tree level potential is at $h_i = D = 0$, SUSY is not broken though the gravitino mass and the scalar potential may be different than zero.

In global SUSY, the supertrace of the mass matrix, defined by $StrM^2 = \Sigma_J (-1)^{2J}(2J+1)M_J^2$ where J denotes the spin, vanishes even if SUSY is spontaneously broken. In local SUSY, this is no longer true, and in the simple case of having fields with canonical kinetic terms and assuming vanishing cosmological constant, the supertrace of the mass matrix is

$$StrM^2 = 2(N-1)m_{3/2}^2 \tag{2.61}$$

where N is the number of chiral fields. Thus we see that in SUGRA $StrM^2$ is in general different than zero. If we take the limit as $M_{Planck} \to \infty$ it can be shown that $m_{3/2} \to 0$ we recover the global SUSY result $StrM^2 = 0$. In eq.(2.61) one should not consider the mass of the goldstino since it has been already absorbed by the gravitino. The remaining $N-1$ chiral fermion remain massless while the $2(N-1)$ real scalar fields get a common mass $m_{3/2}$. It is the gravitino mass and not the supersymmetry breaking scale that gives the mass splitting between the scalar and the fermion fields.

The source of SUSY breaking is quite relevant because many of the low energy parameters, such as masses of the particles and gauge coupling couplings, are only fixed after SUSY is broken. One of the most common explanations for SUSY breaking is gaugino condensation. That a gaugino condensate indeed breaks SUSY can be seen from eq.(2.42). One would expect the gauginos to condense when the coupling constant becomes strong and binds them together in a similar way as for the chiral condensate in QCD. Since this phenomena is non-perturbative, approximation techniques must be

Figure 2.2: Diagrammatic representation of the mass gap equation for a 4-Fermi interaction

used to estimate the effects of gaugino condensation. Different mechanisms have been used to determine the effective interaction and they are very much restricted by the symmetries of the action. A very useful approach for investigating the condensation of fermions is due to Nambu-Jona-Lasinio (NJL) as described in the following section. We will leave for chapter 4 the analysis of the NJL model in the context of the 4D local supersymmetric string model.

2.6 NJL model

In order to study non-perturbative effects in field theory approximation methods must be used. In particular the NJL model [10], presented long ago, is a useful technique to address the question of whether a fermion condensate is dynamically favoured or not. Starting from a 4-Fermi interaction, one calculates the infinite number of fermion bubbles through the Schwinger-Dyson equation and derives the mass gap equation (see **fig.2.2**) [23],

$$1 = \frac{g^2\Lambda^2}{8\pi^2}(1 + \frac{m_F^2}{\Lambda^2}\, ln(\frac{m_F^2/\Lambda^2}{1 + m_F^2/\Lambda^2})) \tag{2.62}$$

where m_F is the mass of the fermion and Λ the cutoff. A non-trivial solution to this equation corresponds to a non-perturbative result since by equating a tree level result with a radiative correction one is necessarily in the non-perturbative region and is effectively summing an infinite number of fermion bubbles (see **fig.2.3**). A non-trivial solution signals that a condensate is dynamically favoured and through quantum corrections it will become a dynamical field, i.e. a kinetic term will be generated. This new field is the Goldstone boson of the axial chiral symmetry that is spontaneously broken when a condensate forms.

An alternative and useful approach to study this phenomena is to introduce an aux-

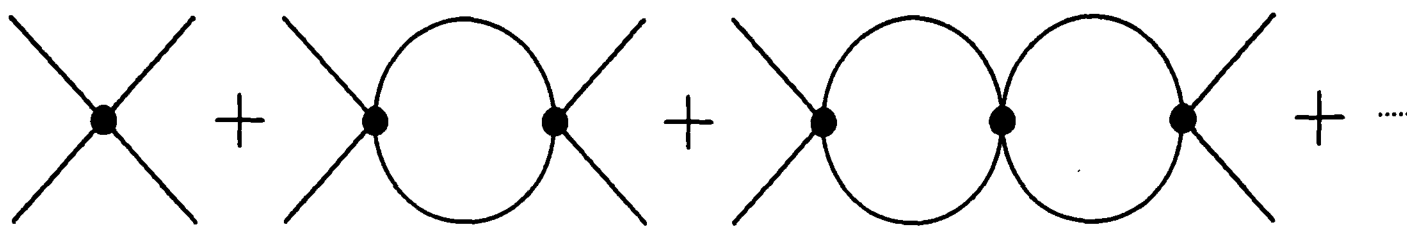

Figure 2.3: Bubble sum generated by the 4-Fermi interaction

iliary scalar field with no kinetic term that can be interpreted as the fermion bilinear by solving its equation of motion. The auxiliary field just plays the role of a Lagrange multiplier and if it is eliminated, one recovers the initial 4-Fermi interaction. After introducing this field one calculates the tree and one-loop scalar potential. The extremum equation with respect to the auxiliary scalar field gives the mass gap equation obtained in eq.(2.62). Since both formalisms are equivalent we choose to work with the auxiliary field method.

The non-SUSY NJL model starts with a four-fermion interaction described by the Lagrangian [10]

$$L = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{1}{4}g^2((\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2) \tag{2.63}$$

or in terms of right and left components notation

$$L = i(\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + \bar{\psi}_R\gamma^\mu\partial_\mu\psi_R) + g^2\bar{\psi}_L\psi_R\bar{\psi}_R\psi_L. \tag{2.64}$$

Here g^2 is a dimensional coupling, $g^2 = h^2/\Lambda^2$, where h is dimensionless and Λ is the mass scale at which the new physics generating the four-fermion interaction appears. The theory has a $U(1)_L \otimes U(1)_R$ chiral symmetry of independent phase rotations of the left and right handed fermion components. Equivalently, one can define the chiral $U(1)$ transformations

$$U_V : \psi_{L,R} \to e^{i\alpha}\psi_{L,R} \tag{2.65}$$

and

$$U_A : \psi_{L,R} \to e^{\pm i\beta}\psi_{L,R}. \tag{2.66}$$

If a condensate $\bar{\psi}_R\psi_L$ forms, the axial U_A transformation will be spontaneously broken.

Eq.(2.64) can be rewritten in terms of an auxiliary scalar field ϕ,

$$L = i(\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + \bar{\psi}_R\gamma^\mu\partial_\mu\psi_R) - |\phi|^2 + g\phi^*\bar{\psi}_R\psi_L + g\phi\bar{\psi}_L\psi_R \tag{2.67}$$

By its classical equation of motion, ϕ is identified with $g\bar{\psi}_R\psi_L$. Eliminating ϕ gives eq.(2.64).

The tree level potential

$$V_0 = |\phi|^2 \tag{2.68}$$

is semipositive definite. The minimum is at $\phi = 0$ and no condensation state is formed but once the radiative corrections are included a non-trivial result may be obtained. The one-loop corrections are properly taken into account by the Coleman-Weinberg result [25]

$$V_1 = -\frac{1}{8\pi^2}\int d^2p\, p^2 ln(p^2 + m_F^2) \tag{2.69}$$

where m_F is the tree level fermion mass. Integrating eq.(2.69) using a momentum-space cutoff one obtains

$$V_1 = -\frac{\Lambda^4}{16\pi^2}(x + x^2 ln(\frac{x}{1+x}) + ln(1+x)) \tag{2.70}$$

with

$$x = \frac{m_F^2}{\Lambda^2} = \frac{4g^2|\phi|^2}{\Lambda^2}. \tag{2.71}$$

The scalar potential is then the sum of V_0 and V_1 and it is given by

$$V = \frac{\Lambda^4}{16\pi^2}(\frac{2}{\alpha}x - x - x^2 ln(\frac{x}{1+x}) - ln(1+x)) \tag{2.72}$$

with

$$\alpha = \frac{g^2\Lambda^2}{8\pi^2}. \tag{2.73}$$

From eq.(2.72) it is easy to see that the extremum condition is

$$\frac{\partial V}{\partial \phi^*} = \phi\alpha(\frac{1}{\alpha} - 1 - x ln(\frac{x}{1+x})) = 0. \tag{2.74}$$

Provided V has a negative slope at the origin eq.(2.74) admits a non-trivial solution that is dynamically preferred. This is possible only for

$$\alpha = \frac{g^2\Lambda^2}{8\pi^2} > 1, \tag{2.75}$$

i.e. a strong coupling constant. In this case eq.(2.74) gives

$$1 = \frac{g^2\Lambda^2}{8\pi^2}(1 + \frac{m_F^2}{\Lambda^2} ln(\frac{m_F^2/\Lambda^2}{1 + m_F^2/\Lambda^2})) \tag{2.76}$$

which is the familiar mass gap equation that could have been derived from the interaction of eq.(2.63). Note that this solution is necessarily non-perturbative since it equates tree level and one loop contributions. It is straightforward to show that it amounts to a non-perturbative summation of fermion bubble graphs, which are dominant in the large N_c limit where N_c is the number of colours.

The solution corresponds to the case where ϕ acquires a v.e.v. and thus a fermion condensate is energetically favoured. The U_A chiral symmetry is spontaneously broken and the associated Goldstone mode may be identified with the field ϕ which, through quantum effects, acquires a kinetic term L_k and becomes a propagating field. It may be seen that $L + L_k$ is the effective Lagrangian describing the light (Goldstone) degrees of freedom, appropriate below Λ, together with the fermion field. The effective potential calculated using this Lagrangian just reproduces the results using the more familiar Schwinger-Dyson equation following from the original Lagrangian eq.(2.63). Thus the effective Lagrangian describing the would-be Goldstone mode provides a convenient way of implementing the NJL scheme for summing the leading terms in the large N_c limit.

2.7 Chiral Fermion condensate in Global SUSY

We will now present a SUSY extension to the chiral NJL model which shows that, contrary to the non-SUSY case, the formation of a condensate is not dynamically favoured. This may be expected from the non-renormalizable theorems of global SUSY which, among other properties, state that a global SUSY theory cannot be radiatively broken, although non-perturbative effects may violate such theorems. What happens in the global SUSY case is that the extra scalars needed to obtain a supersymmetric theory get the same mass as the fermions, and the one-loop potential vanishes. Therefore the tree level results remains valid, i.e. no fermion condensate. On the other hand, if global SUSY is explicitly broken by introducing a scalar mass, then the formation of a chiral condensate is energetically favoured.

In the globally SUSY NJL model [26] one has to introduce two auxiliary chiral fields ϕ_1 and ϕ_2, because otherwise one would obtain pathological kinetic terms with the wrong sign or with derivatives in the denominator. The appropriate Lagrangian is

$$L = \int d^4\theta[\bar{\phi}_+\phi_+ + \bar{\phi}_-\phi_- + \frac{1}{m^2}\bar{\phi}_2\phi_2] + \int d^2\theta[\phi_1\phi_2 - g\phi_1\phi_+\phi_-] + h.c. \qquad (2.77)$$

where ϕ_+, ϕ_- are the original chiral superfields whose fermion components yield the desired 4-Fermi interaction. The fields ϕ_+, ϕ_- and ϕ_1 carry mass dimension one while ϕ_2 has mass dimension two. The auxiliary field ϕ_1 is a Lagrange multiplier and its equation of motion constrains ϕ_2 to be

$$\phi_2 = g\phi_+\phi_-. \qquad (2.78)$$

In component notation

$$\begin{aligned} A_2 &= gA_+A_-, \\ \psi_2 &= g(A_+\psi_- + A_-\psi_+), \\ F_2 &= g(A_+F_- + A_-F_+ - \psi_+\psi_-). \end{aligned} \qquad (2.79)$$

Even though ϕ_2 has a kinetic term, the constraint (2.78) shows that ϕ_2 should be interpreted as an auxiliary field and not as a dynamically one. Using the equation of motion for ϕ_2, one can derive the constraint for ϕ_1 which in components reads,

$$\begin{aligned} A_1 &= \frac{g}{m^2}(A_+^*F_-^* + A_-^*F_+^* - \bar{\psi}_+\bar{\psi}_-) \\ \psi_1 &= -\frac{g}{m^2}i\sigma^\mu\partial_\mu(A_+^*\bar{\psi}_- + A_-^*\bar{\psi}_+) \\ F_1 &= -\frac{g}{m^2}\Box(A_+A_-). \end{aligned} \qquad (2.80)$$

To investigate the formation of condensates, we are interested in possible v.e.v. for A_1, A_2, F_1 and F_2. The tree level potential for these fields is

$$V_0 = -F_2F_2^* - A_1F_2 - A_1^*F_2^* - A_2F_1 - A_2^*F_1^*. \qquad (2.81)$$

Minimizing with respect to A_2 and F_2, one gets $< F_1 >= 0$ and $A_1 = F_2^*$. Thus, the tree level potential is

$$V_0 = A_1A_1^* \qquad (2.82)$$

At the minimum $< A_1 >= 0$, which shows (cf. eq.(2.80)) that there is no formation of fermion condensate. To determine the one-loop potential one calculates the tree level masses for the different physical fields A_-, A_+, ψ_- and ψ_+. The mass of these fields can be obtained from the interaction Lagrangian between ϕ_1 and $\phi_+\phi_-$

$$L_{int} = -gA_1(A_-F_+ + A_+F_- - \psi_+\psi_-) + h.c. \tag{2.83}$$

If one eliminates the $F_\pm$ fields through their equation of motion ($F_\pm = gA_1A_\mp$), eq.(2.83) becomes

$$L_{int} = -g^2 \mid A_1 \mid^2 \mid A_+ \mid^2 - g^2 \mid A_1 \mid^2 \mid A_- \mid^2 - (gA_1\psi_-\psi_+ + h.c.) \tag{2.84}$$

which shows that the scalar fields and fermion fields get a common supersymmetric mass $m = gA_1$. Hence the contribution to the one-loop potential between the scalar and the fermion fields cancels ($V_1 = 0$) and the complete scalar potential $V = V_0 + V_1 = V_0$ is given by the tree level one. Thus a zero v.e.v. for A_1 remains valid even after radiative corrections are included, and therefore no condensate is dynamically favoured.

Chapter 3

String Theory

String theories [1] are perhaps the best candidate to unify all interactions and provide the ultimate theory of everything. As is well known, a quantum gravity of point-like particles is mathematically ill defined and the only possibility up to now of incorporating a consistent quantum theory of gravity is to use string theory. The main observation is that in string theory the starting object, the string, is a one dimensional and not a point-like object as in field theory. This fact has enormous consequences; for instance all ultraviolet divergent terms present in field theory are absent in string theory.

Even though string theory in the critical dimension has only one free parameter, the Planck mass, the effective four dimensional field theory admits a large number of consistent vacua [27]-[31],[38]. In order to investigate the possible solutions and restrict the number of candidates, one may try to work in a model independent way and concentrate on general properties of 4D models. In this chapter we will present some of the general structure of the 4D effective theory.

The content of the chapter is the following. In the first three sections we present an overview of the bosonic, superstring and heterotic string, respectively. In section four we comment on the general properties of the compactification schemes and in section five we give the low energy spectrum. A symmetry of relevance to constrain the different string vacua is the duality symmetry presented in section six. The effective 4D string field model is given in section seven while radiative corrections to the gauge coupling constant are discussed in section eight.

3.1 Bosonic string

Unlike normal field theories the starting object in string theory is a one-dimensional object, the string. When the string evolves in space-time, it sweeps a two-dimensional world-sheet which is usually parameterized by the coordinates σ and τ. It is conventional to take σ as the space-like coordinate that runs from 0 to π and τ as the time-like one. The worldsheet is described by the position of the string $X(\sigma,\tau)$ at any given time. The action is just given by the area of the worldsheet and in Minkoswki space it is

$$S = T\int d\sigma d\tau\sqrt{\dot{X}^2X'^2 - (\dot{X}X')^2} \tag{3.1}$$

where

$$\dot{X} = \frac{\partial X}{\partial\tau} \quad , \quad X' = \frac{\partial X}{\partial\sigma},$$

and T is the string tension. In natural units ($\hbar = c = 1$) T has dimension of (mass)^2 and it is the only free parameter in the theory, which is chosen as the Planck mass ($M_{Planck} = 10^{19}GeV$). The action as written in eq.(3.1) is not simple to deal with since it contains a square root and an equivalent one is given by

$$S = -\frac{T}{2}\int d\tau d\sigma\sqrt{h}h^{\alpha\beta}\eta_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu \tag{3.2}$$

where $h^{\alpha\beta}$ is the metric of the world sheet with $\alpha,\beta = \tau,\sigma$ and $\eta_{\mu\nu}$ the space-time metric. The coordinates μ,ν go from $\mu,\nu = 1,...,D$ where D gives the dimension of space-time and, at the same time, the number of bosonic dimensions. Since h in eq.(3.2) has no derivatives, it is just a Lagrange multiplier that can be eliminated by its equation of motion and we would recover then eq.(3.1). The action (3.2) is conformally invariant and one can use this symmetry to fix the worldsheet metric $h_{\alpha\beta} = \eta_{\alpha\beta}$, where $\eta_{\alpha\beta}$ is the two dimensional Minkoswki metric, called the covariant gauge. With this choice, the Euler-Lagrange equation for the string is the wave equation

$$(\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\sigma^2})X_\mu = 0 \tag{3.3}$$

and the two dimensional energy momentum tensor is

$$\begin{aligned} T_{\alpha\beta} &= -\frac{2}{T}\frac{1}{\sqrt{h}}\frac{\partial S}{\partial h^{\alpha\beta}} \\ &= \partial_\alpha X^\mu \partial_\beta X^\nu - \frac{1}{2} h_{\alpha\beta} h^{\alpha'\beta'} \partial_{\alpha'} X^\mu \partial_{\beta'} X^\nu. \end{aligned} \tag{3.4}$$

The energy momentum tensor $T_{\alpha\beta}$ is automatically traceless, $h^{\alpha\beta}T_{\alpha\beta} = 0$, and the field equation $\frac{\partial S}{\partial h} = 0$ implies that $T_{\alpha\beta}$ must vanish.

As usual in two dimensions, the general solution to the wave equation can be written as the sum of two independent ones, the left and the right one,

$$X = X_R^\mu(\sigma^-) + X_L^\mu(\sigma^+) \tag{3.5}$$

where

$$\sigma^- = \tau - \sigma \quad , \sigma^+ = \tau + \sigma.$$

X_R describes "right-moving" modes while X_L "left-moving" ones. In terms of these coordinates, the Fourier expansion with boundary condition $X(\tau, \sigma) = X(\tau, \sigma + \pi)$ is

$$X_R^\mu = \frac{1}{2}x^\mu + \frac{1}{2}p^\mu \sigma^- + \frac{i}{2}\Sigma \frac{1}{n}\alpha_n^\mu e^{-2i\sigma^-} \tag{3.6}$$

and

$$X_L^\mu = \frac{1}{2}x^\mu + \frac{1}{2}p^\mu \sigma^+ + \frac{i}{2}\Sigma \frac{1}{n}\tilde{\alpha}_n^\mu e^{-2i\sigma^+} \tag{3.7}$$

where we have set the string tension to $T = 1/\pi$ and x^μ, p^μ may be interpreted as the center of mass position and momentum of the string respectively. The α_n^μ are the Fourier modes and obey the commutators

$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\delta_{m+n}\eta^{\mu\nu} \tag{3.8}$$

and

$$[\alpha_m^\mu, \tilde{\alpha}_n^\nu] = 0.$$

The α_m are therefore naturally interpreted as harmonic oscillators, raising and lowering operators for negative or positive m, respectively. The ground state $| 0 >$ is defined to be annihilated by the α_m for $m > 0$. However, this condition is not sufficient to fully specify the physical spectrum, because the Fock space is not positive definite due to the time component commutation relation in eq.(3.8). These extra states, called ghosts, have negative norm and must be removed from the Fock space to obtain the physical spectrum of the string.

In terms of the Fourier modes $L_n = \frac{1}{2}\Sigma\alpha_{m-n}\cdot\alpha_n$ of the energy-momentum tensor $T_{\alpha\beta}$, the condition that the energy tensor vanishes while acting on a physical state, $T_{\alpha\beta}\,|\,\phi >= 0$, is

$$(L_m - \delta_m a)\,|\,\phi >= 0 \quad ,m \geq 0 \tag{3.9}$$

where the a is a constant needed due to the normal ordering ambiguity in defining the zero Fourier mode L_0. The commutators relation among these modes gives the famous Virasoro algebra,

$$[L_m, L_n] = (m-n)L_{m+n} + A(M)\delta_{m+n} \tag{3.10}$$

where $A(m)$ is a c-number and arises due to quantum anomalies from the normal ordering ambiguities. In fact, with the inclusion of the anomaly term the algebra (3.10) is called the central extension of the Virasoro algebra. A consistent bosonic string without negative norm states nor anomaly is only possible if the dimension of space-time is $D = 26$ and $a = 1$.

The mass of the states for the closed string are

$$M^2 = 8\pi T(-1+N) = 8\pi T(-1+\tilde{N}), \tag{3.11}$$

where

$$N = \Sigma\alpha_n\cdot\alpha_n \tag{3.12}$$

and

$$\tilde{N} = \Sigma\tilde{\alpha}_n\cdot\tilde{\alpha}_n$$

are the number operators. The left and right parts of the string give the same contribution to the mass of the closed string. The ground state of the bosonic string is unfortunately a state with negative (mass)^2, i.e. a tachyon, $M^2 = -8\pi T$. The first excited level consists of massless particles and it contains a spin two particle (the graviton), a scalar field known as the dilaton and a state that transforms as a second rank antisymmetric tensor of SO(24). Higher excitations have masses quantized in Planck mass units. This is a generic feature in string theory, i.e. the states are either massless or they have a mass quantized in units of the Planck Mass. In order to eliminate the tachyon state and to introduce fermions one considers a supersymmetric extension of the bosonic string, called superstring.

3.2 Superstring

The bosonic string action describes a string moving in a 26 dimensional space-time and the coordinates X^μ can be seen as bosons living in the two dimensional worldsheet. A possible extension to this theory is to introduce more structure on the worldsheet. The superstring action contains bosonic and fermionic degrees of freedom and it is given by

$$S = -\frac{T}{2}\int d\sigma d\tau(\partial^\alpha X^\mu \partial_\alpha X^\mu - i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu) \tag{3.13}$$

where ψ are Majorana fermions which transform in the vector representation of SO(D-1,1) and ρ are the two dimensional Dirac matrices. The supersymmetric transformation which relates the bosons to the fermions is

$$\delta X^\mu = \bar{\epsilon}\psi^\mu \tag{3.14}$$

$$\delta\psi^\mu = -i\rho^\alpha \partial_\alpha X^\mu \epsilon \tag{3.15}$$

with ϵ a constant anticommuting spinor. As in the bosonic case one can define right and left modes for the fermion fields and the respective equations of motion are

$$\left(\frac{\partial}{\partial\tau} + \frac{\partial}{\partial\sigma}\right)\psi^\mu_- = 0 \tag{3.16}$$

$$\left(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial\sigma}\right)\psi^\mu_+ = 0. \tag{3.17}$$

One obtains two separate sectors with right or left movers only, i.e. ψ^μ_- and $\partial_- X^\mu$ (ψ^μ_- and $\partial_- X^\mu$) are both functions of σ^- (σ^+) only.

As a consequence of the boundary conditions

$$\psi^\mu_+ = \pm\psi^\mu_-, \tag{3.18}$$

each fermion can take either periodic values, called the Ramond (R) sector or antiperiodic values, referred as the Neveu-Schwarz (NS) sector. The Fourier mode expansions for the right and left modes for these two sectors are

$$\psi^{\mu}_{-} = \Sigma d^{\mu}_{n} e^{-2in\sigma_{-}} \tag{3.19}$$

$$\psi^{\mu}_{+} = \Sigma \tilde{d}^{\mu}_{n} e^{-2in\sigma_{+}}$$

for the Ramond sector and

$$\psi^{\mu}_{-} = \Sigma b^{\mu}_{r} e^{-2ir\sigma_{-}} \tag{3.20}$$

and

$$\psi^{\mu}_{+} = \Sigma \tilde{b}^{\mu}_{r} e^{-2ir\sigma_{+}}$$

for the Neveu-Schwarz one. In the R sector, the oscillators take integer values ($n \epsilon Z$), while in the NS they take half integer values ($r+1/2 \epsilon Z$). The anticommutation relation between the Fourier modes are

$$\{b^{\mu}_{r}, b^{\nu}_{s}\} = \eta^{\mu\nu}\delta_{r+s} \tag{3.21}$$

and

$$\{d^{\mu}_{n}, d^{\nu}_{m}\} = \eta^{\mu\nu}\delta_{n+m} \tag{3.22}$$

and similarly for the left moving ones. Physical states are expressed as tensor products of left and right movers. There are four different sectors referred as NS-R, R-NS,R-R,NS-NS depending on the boundary conditions of the states in either sector. The states in the Ramond sector give spinor representation in SO(D-1,1) as can be seen from the algebra of the d^{μ}_{0} modes eq.(3.22) which is just the Dirac algebra. Therefore the NS-R and the R-NS sectors describe space-time fermions while the R-R and NS-NS sectors describe space-time bosons.

As in the case of the bosonic strings, the superstring theory contains a ghost field which must be removed. This requires a 10 dimensional space-time and $a_{NS} = 1/2$ in the NS sector and $a_R = 0$ in the Ramond one, where the a is the normal order constant that appears when defining the zero component Fourier mode of the energy momentum tensor as for the bosonic string. But even for such values of the parameters D and a, the theory remains inconsistent at the quantum level and it still contains a tachyon state. The correct procedure is to truncate the spectrum with a GSO projection [73]. This projection consists in keeping only those states that are even under the (-1)^F operator. The eigenvalues of this operator are defined as +1 for Bose fields X^μ and -1 for Fermi fields ψ^μ. Thus, only states with an even number of Fermi fields are retained. The GSO operator is defined as

$$(-1)^F = -(-1)^{\Sigma b_r \cdot b_r} \tag{3.23}$$

in the NS sector and

$$(-1)^F = \Gamma^{11}(-1)^{\Sigma d_r \cdot d_r} \tag{3.24}$$

in the R sector. Γ_{11} is the chiral projection operator in 10 dimensions (the equivalent one to the γ_5 in four dimensions). This operator is needed in eq.(3.24) to ensure that the (-1)^F operator commutes with d_0^μ. For massless modes the Ramond sector contains only massless fermions with positive chirality (in the 10 dimensional sense). Another important consequence of these truncations is that it renders a supersymmetric theory in the 10 space-time dimensions and it removes the tachyon state since this state has negative GSO eigenvalue ($\Sigma b_r \cdot b_r = 0$ and $(-1)^F = -1$).

3.3 Heterotic string

Having presented the bosonic and superstring theories, we now consider the heterotic string which is a hybridization of the other two. Of all string theories it is the heterotic string [32] that has the most promising features to become the unification theory. It uses the fact that a closed string has left and right moving modes which are independent from each other. The right-movers are taken to be of the superstring type and provide the theory with fermions and ensure the absence of tachyon states. The left-movers correspond to the bosonic string. Since the right-modes live in a D=10 space-time, we take 10 of the 26 bosonic dimensions in the left sector to represent space-time and are left with 16 internal bosonic dimensions that provide the non-abelian gauge group. Compactifying these 16 bosonic dimensions on a 16-dimensional tori requires the internal momentum to be quantized and the lattice of this tori must then be a self-dual one. The only two self-dual lattice in 16 dimensions are SO(32) and $E_8 \times E_8$. It is the second one that is the most interesting from a phenomenological point of view, since it can easily contain the standard model group as a subgroup. The differences between the two theories result from boundary conditions. In two dimensions, one boson is equivalent to two Majorana fermions. The 32 fermions obtained from the 16 bosons in the left sector are in the vector representation of $SO(16) \times SO(16)'$ group and if all of them take the same boundary condition (periodic or antiperiodic) then one obtains the SO(32) gauge group. However, if they take opposite boundary values, they generate the $E_8 \times E_8$ gauge symmetry.

3.4 Compactification

As seen in the preceeding section, the heterotic string provides us with a unification theory with one free parameter, the string tension, but it is consistent only in 10 space-time dimensions. For any comparison with experiments, we clearly have to get "rid" of 6D to obtain a 4D effective theory. The procedure of getting "rid" of extra dimensions is called compactification. Unfortunately, there is no unique way of compactifying the extra dimensions, and many consistent string vacua arise in 4D [27]-[31]. If string theory is going to make any sense as a theory that describes the physical world, it should reduce, at low-energy, to the standard model of elementary particles. Although the number of candidates is enormous [29]-[31], there is no specific example to date that satisfies all constraints from the standard model. The main difficulty is that in string theory all quantities are in principle calculable, and the standard model imposes very heavy constraints on them. Anyhow, with the hope that such a model is contained in 4D strings, the search can be narrowed by imposing some generic properties.

As a first step one requires the 10D space to be a tensor product of $M_4 \times K_6$ where M_4 is the 4D space-time and K_6 the internal space to be compactified. Another necessary condition is that the 4D theory is described by an N=1 SUGRA theory to explain the mass hierarchy problem. This condition imposes non-trivial constraints on

K_6. If SUSY is unbroken, the variation of the fermion components must be zero

$$\delta\psi = 0. \tag{3.25}$$

The variation of the 10D fermions are [1]

$$\begin{aligned} \delta\psi_i &= \frac{1}{\kappa}D_i\eta + \frac{\kappa}{32g^2\phi}(\Gamma_i^{jkl} - 9\delta_i^j\Gamma^{kl})\eta H_{jkl} = 0, \\ \delta\chi &= -\frac{1}{\sqrt{2}\phi}\Gamma\cdot\partial\phi\eta + \frac{\kappa}{8\sqrt{2}g^2\phi}\Gamma^{ijk}\eta H_{ijk} = 0, \\ \delta\lambda &= -\frac{1}{4g\sqrt{\phi}}\Gamma^{ij}F_{ij}\eta = 0 \end{aligned} \tag{3.26}$$

where ψ, χ, η and λ are respectively the gravitino, dilatino, the ten dimensional SUSY spinor parameter and gluino; ϕ is the dilaton and H is the gauge-invariant field strength of the two form B. The indices i, j, k, l are 10 dimensional and we have dropped four Fermi interactions in eq.(3.26). In addition, we have the Bianchi identity

$$dH = trR\wedge R - \frac{1}{30}trF\wedge F. \tag{3.27}$$

If we assume, for simplicity, that

$$dH = H = d\phi = 0, \tag{3.28}$$

an unbroken SUSY implies that there is a covariant constant spinor,

$$D_i\eta = 0. \tag{3.29}$$

Thus, the K_6 space must be Ricci-flat

$$R_{ij} = 0$$

since $[D_i, D_j]\eta = 0$ implies that $R_{ijkl}\Gamma^{kl}\eta = 0$. Eq.(3.29) is a non-trivial result and shows that for a maximal symmetric space M_4, the Ricci scalar is zero. Thus, the cosmological constant vanishes in any 4D string model but it still remains to explain why the cosmological constant vanishes after SUSY is broken.

The condition $D_i\eta = 0$ states that the spinor η remains unchanged after being parallel transported around a closed path. In general, a spinor when transported around a closed curve transforms to $\eta' = U\eta$ where U are SO(n) matrices that form the holonomy group. The constraint of having one covariant constant spinor forces the holonomy group of K_6 to be SU(3). This can be understood by observing that the subgroup that leaves only one element of the fundamental representation of SU(4) (which is isomorphic to SO(6)) invariant is precisely SU(3). A manifold that admits a metric with SU(n) holonomy is called a Kahler manifold. If its first Chern class vanishes, then it is called a Calabi-Yau manifold [72]. Thus the requirement of one unbroken SUSY in 4D implies that the internal manifold K_6 must be a Calabi-Yau manifold with SU(3) holonomy group.

In order to solve the Bianchi identity eq.(3.27) with $dH = 0$, one could identify the SU(3) holonomy group with an SU(3) subgroup of the gauge group $E_8 \supset E_6 \times SU(3)$. With this identification, called the embedding of the spin connection on the gauge group, it can be shown that eq.(3.27) is automatically satisfied. Furthermore, the breaking of E_8 into $E_6 \times SU(3)$ is welcome from a phenomenological point of view since E_6 can accommodate complex fermion representations needed to describe chiral fermions.

Even though the Calabi-Yau manifold restrict the set of possible K_6 spaces and give a great deal of information, the complete spectrum and Yukawa couplings are difficult to obtain and only a few examples are known [33, 74]. A simpler compactification scheme would be to compactify on a 6D tori but such a manifold gives N=4 SUSY in 4D and is not acceptable. Another possibility are the so called Orbifolds [34]. These spaces do not correspond to manifolds since they contain singular points with infinite curvature. The great advantage of Orbifolds is that they are fairly simple to handle and many examples of three generations of standard model particles are known [27]-[31]. However, no realistic model that satisfies all standard model conditions has been found. Furthermore, the Yukawa couplings and the hole spectrum can be obtained [62]. An Orbifold is just a torus divided by the action of a discrete group. In 2D, for example, one can divide the torus by a Z_3 group whose elements are 120° rotations. Under the action of this group, points that are rotated by an angle of 120° are identified thus leaving 3 fixed points $z = \{0, \frac{1}{\sqrt{3}}e^{i\pi/6}, \frac{2}{\sqrt{3}}e^{-i2\pi/6}\}$. To construct a 6D Orbifold we could simply take the product of three 2D ones. The lattice generated by Z_3 is that of SU(3) and by identifying it with an SU(3) subgroup of K_6 one can get ride of three SUSY parameter leaving a 4D theory with N=1 SUSY parameter only. More general Orbifolds can be obtained [30, 34, 35] and one SUSY parameters in 4D restricts the Z_n [30, 34] group by which the 16D tori is divided to $n = 3, 4, 6, 7, 8$ or 12 (where n stands for n-root of 1).

3.5 Low energy spectrum

The low energy spectrum of the heterotic string in 10D consists of a super Yang-Mills multiplet (λ, A_μ) of gauge fields of the $E_8 \times E_8$ gauge group and the 10D SUGRA multiplet. The SUGRA multiplet w consists of a symmetric traceless tensor g_{ab} that contains the graviton, an antisymmetric tensor B_{ab}, a scalar field ϕ called the dilaton and their SUSY partners.

A massless particle in 10D obeys the wave equation $\Box_{10}\phi = 0$. For a 10D space given by $M_4 \times K_6$, massless states in 4D are in one to one correspondence with harmonic states of K_6,

$$\Box_4\phi = -\Box_6\phi = 0. \tag{3.30}$$

In a complex manifold, the number of independent harmonic (p,q) forms, where p counts the number of holomorphic forms and q the number of antiholomorphic forms, is given by the Hodge number $h_{p,q}$. For a manifold of three complex dimensions $(p, q \leq 3)$ with SU(3) holonomy group, one has $h_{0,0} = h_{3,3} = h_{0,3} = 1$, $h_{0,1} = h_{0,2} = h_{2,3} = 0$ and $h_{1,1}$, $h_{2,1}$ unspecified. Furthermore, it can be shown that the (1,1) forms transform under the SU(3) holonomy group as a 3 while the (2,1) forms as a $\bar{3}$. This result will be relevant for counting the number of generations, as we will shortly see.

The 4D spectrum of the supergravity multiplet is summarized in table 3.1. The massless states consist of the graviton, the dilaton field, $h_{1,1}$ complex scalar fields called the (1,1) moduli, $h_{2,1}$ complex scalar fields referred as (2,1) moduli and their SUSY partners. The (1,1) and (2,1) moduli arise from the 10D metric and antisymmetric tensor only and they parameterize the geometry and complex structure of K_6 [5]. In

D = 10	D = 4
$g_{\mu,\nu}$ $g_{\mu,k}$ $g_{M,N}\,(x,y)$ $g_{a,b}\left\{ g_{a,\bar{b}},\ g_{\overline{a,b}},\ g_{a,b} \right.$	$g_{\mu,\nu}$ (x) graviton $h_{1,1}$ real scalar fields $h_{2,1}$ complex scalar fields
$B_{\mu,\nu}$ $B_{\mu,k}$ $B_{M.N}\,(x,y)$ $B_{a,b}\left\{ B_{a,\bar{b}},\ B_{\overline{a,b}},\ B_{a,b} \right.$	$B_{\mu,\nu}$ (x) 1 real scalar $h_{0,1}$ = 0 spin 1 field $h_{1,1}$ real scalar fields $h_{0,2}$ = 0 complex scalar field
ϕ (x,y)	1 real scalar field
ψ_μ ψ_M (x,y) ψ_k	gravitino $h_{1,1}$ = Majorana spinors $h_{2,1}$ = Majorana spinors
ψ (x,y)	1 Majorana spinor

Figure 3.1: Dimensional reduction from 10 to 4 for the SUGRA multiplet. The indices M,N go from 1,2...10 while lower roman cases from 6,...,10 and greeks from 1,..4.

Orbifold compactification there are 3,5 or 9 (1,1) moduli [30, 35, 36]. Three of them are always present and they represent the size (radius) of each complex plane. The other (1,1) moduli correspond to the relative shape of the planes. The (2,1) moduli give the complex structure of K_6, and there are 0,1 or 3.

For the Yang-Mills multiplet the 4D spectrum as shown in table 3.2. One has gauge bosons that correspond to the $E_6 \times E_8$ gauge group, $h_{1,1}$ L-handed fermions in the (27,1) representation while $h_{2,1}$ L-handed fermions in the $(\overline{27},1)$ one and a number of unspecified L-handed singlet fermions plus their corresponding superpartners. The fact that $h_{1,1}$ and $h_{2,1}$ count the number of fermions in the fundamental representation of E_6 is due to the embedding of the spin connection into the gauge group as described in section 3.4 . The adjoint representation of E_8 decomposes into $E_6 \times SU(3)$ as

$$248 = (27,3) \oplus (\overline{27},\overline{3}) \oplus (1,8) \oplus (78,1). \tag{3.31}$$

Since the (1,1) and (2,1) transform respectively as a 3 and $\bar{3}$ under the SU(3) holonomy group (identified with the SU(3) gauge subgroup) the Hodge numbers $h_{1,1}$ and $h_{2,1}$ give the number of fundamental representation of E_6. The number of light generations is then $|\, h_{1,1} - h_{2,1} \,|$, since each 27 can accommodate a whole generation of standard model particles, and one assumes that the 27 and $\overline{27}$ representations will pair up and will get a mass of the order of the Planck mass. In fact the number of generations is given by half the Euler characteristic of the compactified manifold K_6 which is a topological number.

3.6 Duality

Recently considerable effort has been invested in determining the effective 4D string model [27, 28, 38]. As we have seen, the requirement of N=1 SUSY in 4D constrains the possible low energy models. However, there are still an enormous amount of consistent candidates and further study has to be carried out in order to restrict the number of string vacua. An interesting property of 4D string models is the appearance of the so called target space duality symmetry [6]-[8],[39]-[41]. This is a symmetry of the moduli space, it relates large to small values of the moduli and is intimately related with the contribution from the infinite number of Kaluza-Klein modes which are always present in string theory. As a consequence of duality symmetry, the compactified radius R can be transformed to $R \to 1/R$ [6], which suggests that there is a minimum distance, the Planck length, that can be probed in string theory. This minium distance is expected in any sensible quantum theory of gravity, so it is pleasing to see it emerge. What happens is that for $R \ll 1$ the momentum and winding quantum numbers of the string can be interchanged and R can be reinterpreted as $R' = 1/R \gg 1$. Although duality symmetry has only been shown to be an exact symmetry in perturbation theory [40], one expects that non-perturbative effects will respect it. One requires then the effective field theory to be duality invariant. This proves to be very useful (and necessary) in

D=10			D=4		
(E_8, E'_8)			(E_6, E'_8)		
		$A_\mu(x,y)$	(78,1)	$h_{o,o}$= 1	complex scalar field
(248,1)	$A_M(x,y)$		$(\overline{27},1)$	$h_{2,1}$	complex scalar fields
		$A_k(x,y)$	(27,1)	$h_{1,1}$	complex scalar fields
			(1,1)	n_E	complex scalar fields (not a topological number)
			(78,1)	$h_{o,o}$ = 1	Majorana spinor
			$(\overline{27},1)$	$h_{2,1}$	L-handed spinors
(248,1)	$\lambda_A(x,y)$		(27,1)	$h_{1,1}$	L-handed spinors
			(1,1)	n_E	L-handed spinors
(1,248)	$A_M(x,y)$		(1,248)	1	gauge field
(1.248)	$\lambda_A(x,y)$		(1,248)	1	Majorana spinor

Figure 3.2: Dimensional reduction from 10 to 4 for the Yang-Mills multiplet. The indices M,N go from 1,2...10 while a,b=6,...,10 and μ, ν=1,..4.

fixing the v.e.v. of the moduli fields once SUSY is broken and in constraining the different parameters in the superpotential, such as the Yukawa couplings as well as the possible string vacua [42, 43, 54, 58]. Furthermore, the contribution of these modes is also relevant in determining the gauge coupling constant and unification scale. The unification scale can be smaller than the string scale and thus allowing for minimal string model to work.

Before giving the general structure of duality symmetry we would like to present the properties of this symmetry in a simple example. Let us compactify one dimension of the 26D bosonic string on a circle [1]. For this coordinate, one identifies $x = x + 2\pi n R$, where R is the radius of the circle and n is an arbitrary integer. The most general configuration for the string coordinate $X(\sigma,\tau)$ satisfying the two-dimensional wave equation and the closed string boundary conditions then becomes[1]

$$X(\sigma,\tau) = x + p\tau + L\sigma + \frac{i}{\sqrt{2}}\Sigma\frac{1}{n}(\alpha_n e^{-2i\sigma^-} + \tilde{\alpha}_n e^{-2i\sigma^+}) \tag{3.32}$$

where

$$p = \frac{m}{R} \quad , L = nR. \tag{3.33}$$

The number m labels the allowed momentum eigenvalues and it must be an integer so that the quantum wave function $e^{ip\cdot x}$ is single-valued. The integer n gives the number of times that the string wraps around the circle. Such states exist only when the compactified manifold contains noncontractible closed curves, called Kaluza-Klein states. In terms of left and right components, one has

$$X_R = x_R + \frac{1}{2}(p - L)\sigma^- + \frac{i}{\sqrt{2}}\Sigma\frac{1}{n}\alpha_n e^{-2i\sigma^-} \tag{3.34}$$

and

$$X_L = x_L + \frac{1}{2}(p + L)\sigma^+ + \frac{i}{\sqrt{2}}\Sigma\frac{1}{n}\tilde{\alpha}_n e^{-2i\sigma^+}. \tag{3.35}$$

The contribution from the zero modes to the mass of the string is given by

$$M^2 = \frac{m^2}{2R^2} + \frac{1}{2}n^2R^2 \tag{3.36}$$

and it is symmetric under the exchange of

$$R \leftrightarrow 1/R \quad and \quad m \leftrightarrow n. \tag{3.37}$$

This is the duality symmetry. Thus we see that a space with $R < 1$ is equivalent to another space with $R > 1$ as long as we also interchange the integer n and m, i.e. the interpretation of momentum and winding states. For large R the winding states become very massive and do not contribute to the low energy limit but for small R these states become important and have to be taken into account in any effective theory.

In a more general case, the string action is [44]

$$\frac{1}{4\pi}\int d\sigma d\tau (G_{ab}\partial^\alpha X^a \partial_\alpha X^b + \epsilon^{\alpha\beta}B_{ab}\partial_\alpha X^a \partial_\beta X^b) \tag{3.38}$$

[1] Here we are taking the string tension $T = 1/2\pi$.

with G_{ab} the metric tensor of the compactified manifold and B_{ab} an antisymmetric tensor. They give the so called background fields. These two tensors parameterize the geometry and complex structure of the internal manifold. Again, one identifies $X^a(2\pi,\tau) = X^a(0,\tau) + 2\pi n^a$ with n^a an arbitrary integer. Single valuedness of the wave function requires that the momentum to be quantized in terms of integers m^a. The zero mode of the bosonic string is given by

$$X_0^a = x^a + n^a\sigma + G^{ab}(m_b - B_{bc}n^c)\tau \tag{3.39}$$

where G^{ab} is the inverse matrix of G_{ab}. Expanding the string in left and right sectors one obtains a similar expressions to eqs.(3.34) and (3.35) but with

$$p_R^a \equiv \frac{1}{2}(p^a - L^a) = \frac{1}{2}(-n^a + G^{ab}(m_b - B_{bc}n^c)) \tag{3.40}$$

and

$$p_L^a \equiv \frac{1}{2}(p^a + L^a) = \frac{1}{2}(n^a + G^{ab}(m_b - B_{bc}n^c)), \tag{3.41}$$

respectively. The zero mode contribution to the mass is

$$M^2 = \frac{1}{2}G_{ab}n^a n^b + \frac{1}{2}G^{ab}(m^a - B_{ac}n^c)(m_b - B_{bd}n^d). \tag{3.42}$$

Eq.(3.42) and the entire spectrum (and even the interaction theory) is invariant under the interchange $m^a \leftrightarrow n^a$ as long as

$$(G+B) \leftrightarrow (G+B)^{-1}. \tag{3.43}$$

As an example, for one compactified dimension X with radius R, then $G_{11} = R^2$ and there is no B field, and eq.(3.43) reduces to the first equation in (3.37).

The components of G and B are moduli fields and therefore these fields are related to the geometry and complex structure of the 6D complex space K_6. These moduli split into two classes, the (1,1) and the (2,1) moduli. The (1,1) moduli fields $T_i, (i = 1,..,h_{1,1})$, are associated to deformations of the Kahler class. The real part of the three (1,1) moduli always present in Orbifolds gives the radius of the compactified dimension R while the imaginary part is an axion like field b,

$$T_i = R_i^2 + ib_i, \quad i = 1,2,3. \tag{3.44}$$

The second class consists of (2,1) moduli fields $U_m, (m = 1, .., h_{2,1})$, which are related to the complex structure. The moduli space factorizes into two distinct subspaces $M_T \times M_U$ and the duality transformations for the subspaces are independent from each other. Depending on the Orbifold on which one compactifies the moduli space, $M_T \times M_U$ may vary [36, 37] but it contains a SL(2,Z) group as a subgroup for each of the three (1,1) moduli fields T_i which are always present and the (2,1) moduli U_i. To be as general as possible we will only consider moduli fields which transform under duality as elements of SL(2,Z) and thus we will drop the distinction between them and rename all moduli as T fields. They transform then as

$$T_i \to \frac{a_i T_i - i b_i}{i c_i T_i + d_i} \tag{3.45}$$

with $a_i, b_i, c_i, d_i \in Z$ and $a_i d_i - c_i b_i = 1$.

3.7 Effective 4D string model

The effective low energy 4D string model [46] that arises from the heterotic string corresponds, by construction, to an N=1 SUGRA model with a $E_6 \times E_8'$ gauge group. The gauge group can be broken after compactification down to the standard model in the observable sector and to any SU(N) subgroup, with rank less then eight, in the hidden sector. This can be easily done using Wilson lines [77] and is very much model dependent. The complete spectrum as described in section 3.5 is also model dependent although there are some generic features for all these models and it is on these properties that we will concentrate. Apart from the SUGRA and gauge supermultiplets it contains a gauge singlet dilaton S and moduli T_i superfields as well as an unspecified number of chiral matter superfields.

The bosonic part of the 10D SUGRA action is [45]

$$L^{10} = -\frac{1}{2}R - \frac{3}{4}\phi^{-3/2}H_{ijk}H^{ijk} - \frac{9}{16}(\frac{\partial_i\phi}{\phi})^2 - \frac{1}{4}\phi^{-3/4}TrF_{ij}F^{ij} \tag{3.46}$$

where R is the Ricci scalar, H is the field of the antisymmetric tensor B_{ij}, ϕ the 10D dilaton and F_{ij} the 10D Yang-Mills strength. By dimensional reduction to 4D, the kinetic terms for the scalar and boson fields are [46]

$$\begin{aligned} L^4_{kin} = & -3\partial_\mu\sigma\partial^\mu\sigma - \tfrac{9}{16}(\partial_\mu\phi/\phi)^2 - \tfrac{3}{2}e^{-2\sigma}\phi^{-3/2}(\partial_\mu b)^2 \\ & -\tfrac{3}{4}\phi^{-3/2}e^{6\sigma}H_{\mu\nu\rho}H^{\mu\nu\rho} - \tfrac{1}{4}\phi^{-3/4}e^{3\sigma}TrF_{\mu\nu}F^{\mu\nu} \end{aligned} \tag{3.47}$$

where we have chosen the metric of K_6 as $g_{IJ} = \delta_{IJ}e^\sigma$ and the components of the 4D metric $g^4_{\mu\nu}$ are related to the 10D ones by $g^4_{\mu\nu} = g^{10}_{\mu\nu}e^{3\sigma}$. The field b arises from the antisymmetric tensor B_{ij} of the 10D SUGRA.

The kinetic terms can be put into a standard SUGRA from by defining [46]

$$S = e^{3\sigma}\phi^{-3/4} + 3i\sqrt{2}D \tag{3.48}$$

and

$$T = e^\sigma\phi^{3/4} - i\sqrt{2}b \tag{3.49}$$

with D given by a duality transformation $\phi^{-3/2}e^{6\sigma}H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\delta}\partial^\delta D$. Thus we notice that the dilaton field comes originally as a linear multiplet and it is only after a duality transformation that one obtains the dilaton as a standard chiral superfield. As mentioned in chapter 2 it is in the context of a linear multiplet for the dilaton field that non-harmonic radiative corrections can consistently be incorporated in a SUSY theory.

The kinetic terms eq.(3.47) become then

$$L^4_{kin} = K^s_s \mid \partial_\mu S \mid^2 + K^T_T \mid \partial_\mu T \mid^2 - \frac{1}{4} ReS\, TrF_{\mu\nu}F^{\mu\nu} \tag{3.50}$$

with

$$K = -ln(S+\bar{S}) - 3ln(T+\bar{T}) \tag{3.51}$$

and $K^s_s \equiv \frac{\partial^2 K}{\partial S \partial \bar{S}}$ and similarly for K^T_T. From eq.(3.50) we observe that the gauge coupling constant at the unification scale is given by the real part of the dilaton field S and it is the same for all gauge groups. The gauge kinetic function is thus

$$f = S. \tag{3.52}$$

The kinetic terms for the superfields S and T are given by the Kahler potential eq.(3.51) and have a σ-model structure and thus cannot be put into a canonical form.

The form of the Kahler potential eq.(3.51) is not the most general one, since , by setting the internal metric as $g_{IJ} = \delta_{IJ}e^{\sigma}$, we have identified the three (1,1) moduli that give the size of each complex plane to be the same. For a generic dilaton and moduli fields the Kahler potential becomes

$$K = -ln(S + \bar{S}) - \Sigma_i ln(T_i + \bar{T}_i) \tag{3.53}$$

where i runs over all moduli. To specify fully the effective 4D string model we would need to determine the superpotential W and the Kahler potential for chiral matter superfields as well as the particle spectrum. In principle the Kahler potential and the superpotential can be completely determined for any specific compactification scheme. In practice it is not simple and most of the information so far obtained has been by expanding the superpotential and Kahler potential in powers of the chiral fields. Nevertheless, general results have been obtained relating derivatives of the Kahler potential with those of the superpotential by comparing string amplitudes with their equivalent ones using SUGRA [38]. One of these results is that the Yukawa couplings can be derived from the Kahler metric and vice versa. As a result of this connection, one can prove that the unrenormalized Yukawa coupling for the trilinear coupling of chiral superfields depends on (1,1) moduli while for the antichiral superfields it depends on (2,1) moduli only. All of these relationships between different unrelated terms in the SUGRA theory are obviously from a stringy nature and have no counterpart in field theory.

A generic property all 4D vacua share is invariance under duality symmetry. As we saw in section 3.6, this symmetry is observed in perturbation theory and is expected to remain valid after non-perturbative effects are included. In order to obtain a duality invariant theory, we have to demand the Kahler function,

$$G = K + ln \mid W \mid^2 \tag{3.54}$$

and the gauge coupling constant to be duality invariant. As we have seen, the moduli transform in a non-trivial way (cf.eq.(3.45)) and from eqs.(3.52) and (3.53) we have that the dilaton field is invariant,

$$S \to S \tag{3.55}$$

and that the superpotential must transform as a modular function of weight -1 for each moduli

$$W \rightarrow \frac{W}{\Pi_i(ic_iT_i + d_i)}. \tag{3.56}$$

If we demand that the T dependent part of the superpotential has no poles in the fundamental region then the choice is completely determined (up to a modular invariant function which we take it to be one) and it is given in terms of the Dedekind-eta $\eta(T_i)$ function with modular weight[2] 1/2, i.e.

$$\eta \rightarrow (icT + d)^{1/2}\eta.$$

It is defined by

$$\eta = q^{1/24}\Pi_{n=1}^{\infty}(1 - q^n), \quad q \equiv e^{-2\pi T}. \tag{3.57}$$

The superpotential can be expressed as

$$W = W_{cte}\Pi_i\eta^{-2}(T_i) \tag{3.58}$$

with W_{cte} modular invariant. Since the gauge kinetic function is given at tree level by $f = S$ and the dilaton field is duality invariant f and the gauge coupling constant $g^{-2} = Ref = ReS$ are as well.

Including chiral matter fields the Kahler potential becomes [38, 53]

$$K = -ln(S + \bar{S}) - \Sigma_i ln(T + \bar{T}_i) + K_i^i(T, \bar{T}) \mid \varphi_i \mid^2 \tag{3.59}$$

where the normalization of the kinetic term for the chiral matter superfields is given by $K_i^i(T, \bar{T})$ and it is a polynomial function of the moduli fields [38],

$$K_i^i = \Pi_j \; a_i \, T_{rj}^{n_{ij}} \tag{3.60}$$

with a_i a constant (in most cases $a_i = 1$) and n_{ij} the modular weight of the superfield φ_i with respect to the moduli T_j. The duality transformation on the chiral superfields with modular weight n_{ij} is

$$\varphi_i \rightarrow (ic_jT_j + d_j)^{n_{ij}} \, \varphi_i \tag{3.61}$$

[2]See Appendix C summary of modular functions.

and $K^i_i \,|\, \varphi_i \,|^2$ is then modular invariant. Eq.(3.59) was derived assuming that $K^i_i(T, \bar{T}) \,|\, \varphi_i \,|^2 \ll 1$.

Up to now we have only consider the tree level results for the Kahler function G and the gauge kinetic function f. But loop corrections to these quantities are important in determining the full effective potential and we will discuss in the next section the contribution to the gauge coupling constant from loop corrections.

3.8 Loop corrected and duality invariant gauge coupling constant and Kahler potential

String theory provides a natural framework for gauge coupling constant unification since, at tree level, all coupling constants are given in terms of the string coupling constant $g_s^2 = (ReS)^{-1}$ by $g_a^2 = \frac{g_s^2}{k_a}$, where S is the dilaton field and k_a is the level of the corresponding Kac-Moody algebra for the gauge group G_a. At scales below the string scale M_s, where[3]

$$M_s = 0.7\, g_s\, 10^{18}\, GeV, \tag{3.62}$$

these couplings will evolve according to their renormalization group equation. For a simple gauge group G_a the one-loop running gauge coupling constant is given by [17]-[19]

$$\frac{1}{g_a^2(\Lambda)} = \frac{k_a}{g_s^2} + b_a ln(\frac{\Lambda^2}{M_a^2}) + \Delta_a \tag{3.63}$$

where $b_a = \frac{1}{16\pi^2}(3C(G_a) - \Sigma_{R_a} h_{R_a} T(R_a))$ is the N=1 β-function coefficient and h_{R_a} the number of chiral fields in a representation R_a. M_a, as defined below, is the renormalization or compactification scale below which the coupling constants begins to run.

The existence of an infinite number of massive states (Kaluza and winding states), above the string scale, gives rise to a string threshold contribution Δ_a which is relevant to the determination of the coupling constant at the string scale. These threshold effects can be directly calculated by computing world-sheet string amplitudes involving external gauge fields and moduli [17]-[20], which has been done for (2,2) symmetric orbifold compactification.

[3] M_s is given in $\overline{MS}$ scheme [19].

Another possible way to calculate the threshold corrections is by imposing target space moduli invariance and the cancellation of target space modular anomalies. The existence of moduli fields in 4D superstring model is a generic feature for all vacua.

The scale M_a, below which the coupling constant starts to run, is in general a moduli dependent quantity

$$M_a^2 = \Sigma_i\,(T_{ri})^{\alpha_i}\,M_s^2 \tag{3.64}$$

where $T_{ri} = (T+\bar{T})_i$ and the constant α_i is model and gauge dependent. In the case of a single overall (1,1) moduli $T_i = T$, $i = 1,2,3$, with $\alpha = \Sigma\alpha_i = -1$ one obtains the field theoretical expression

$$M_a = (ReS\;ReT)^{-1/2}. \tag{3.65}$$

The contribution to the gauge coupling constant from the moduli fields in eq.(3.64) can be calculated from the anomalous triangle diagrams with two gauge bosons and several moduli fields as external legs and massless gauginos and charged (fermionic) matter fields circulating inside the loops.

The threshold term in eq.(3.63) is given by

$$\Delta_a = \Sigma_i(b_a'^i - k_a\delta_{GS}^i)ln\,|\,\eta(T_i)\,|^4, \tag{3.66}$$

$$b_a'^i = \frac{1}{16\pi^2}(C(G_a) - \Sigma_{R_a}h_{R_a}T(R_a)(1+2n_{R_a}^i))$$

where n^i is the modular weight for a chiral matter superfield with respect to the i-moduli, $\eta(T)$ the Dedekind-eta function (cf. eq.(3.57)). In the case of an overall moduli, one has $b_a' = \Sigma_i b_a'^i = \frac{1}{16\pi^2}(3C(G_a) - \Sigma_{R_a}h_{R_a}T(R_a)(3+2n_{R_a})) = b_a - 2\Sigma_{R_a}h_{R_a}T(R_a)(1+n_{R_a})$ with $n_{R_a} = \Sigma_i n_{R_a}^i$.

The universal Green-Schwarz coefficient δ_{GS} in eq.(3.66) is needed to cancel, using the Green-Schwarz mechanism [47], the gauge independent part of the target space modular anomaly. The threshold contribution of the massive fields, $b_a'^i - k_a\delta_{GS}^i$, is in general non-vanishing if at least one of the Orbifold twists leaves the i-plane unrotated. In this sector, the massive spectrum is N=2 space-time supersymmetric and $b_a^i - k_a\delta_{GS}^i$ is proportional to the N=2 β-function coefficient which is in general non zero. On the other hand, if all Orbifold twists rotate a specific plane, than the spectrum is N=4 supersymmetric and $b'^i_a - k_a\delta^i_{GS} = 0$ [19], giving no threshold contribution. In this case the gauge coupling constant is independent of the T_i moduli and one has that b'^i_a/k_a must be equal for all gauge groups, i.e. $b'^i_a/k_a = b'^i_b/k_b$.

To have a duality invariant gauge coupling constant, the α coefficient in eq.(3.64) has to be

$$\alpha^i = \frac{k_a\delta^i_{GS} - b'^i_a}{b_a}. \tag{3.67}$$

Note that only for an overall moduli T with $\delta_{GS} = 0$ and $n_{R_a} = -1$, $\alpha = -1$, M_a will correspond to the "naive" field theoretical result. The term proportional to b' in (3.67) is just the contribution to the gauge coupling constant from the anomalous triangle diagrams with two gauge bosons and one moduli as external states and massless fermions running in the loop.

Using eqs.(3.63) and (3.66), the one-loop gauge coupling constant can be written in an explicit duality invariant form

$$\frac{1}{g_a^2(\Lambda)} = \frac{k_a}{g_s^2} + b_a ln(\frac{\Lambda^2}{M_s^2}) + \Sigma_i(b'^i_a - k_a\delta^i_{GS})ln[T_{ri}\,|\,\eta(T_i)\,|\,^4] \tag{3.68}$$

where the gauge coupling constant at the unification scale is now defined as [17]-[20]

$$g_s^{-2} = ReS + \Sigma_i k_a\delta^i_{GS} lnT_{ri} \equiv \frac{Y}{2}. \tag{3.69}$$

For a non zero Green-Schwarz term ($\delta_{GS} \neq 0$) there will be a mixing term between the dilaton and the moduli fields in the Kahler potential. The dilaton will no longer be duality invariant but will transform in such a way that renders g_s duality invariant.

The condensation scale at which the coupling constant becomes strong (i.e. $\frac{1}{g_a^2} \ll 1$) is

$$\Lambda_c^2 = \Pi_i[T_{ri}\,|\,\eta(T_i)\,|\,^4]^{\alpha_i}\, M_s^2\, e^{-(\frac{k_a Y}{2b_a})}. \tag{3.70}$$

Thus, we have obtained a duality invariant gauge coupling constant and a condensation scale that contain the contribution from loop corrections of massless fermions states and moduli fields.

The loop contribution to the gauge coupling constant shows that the the tree level kinetic function given in eq.(3.52) must be corrected and it becomes

$$f = S + 2\Sigma_i(b'^i_a - k_a\delta^i_{GS})ln(\eta^2(T_i)), \tag{3.71}$$

while the S dependent part of the Kahler potential is [20]

$$K = -ln(S + \bar{S} + 2\Sigma_i k_a \delta^i_{GS} ln T_{ri}). \tag{3.72}$$

In order to render the theory duality invariant, the dilaton field must transform in a non-trivial way due to the mixing term between the S and T fields in eq.(3.72). The dilaton transforms as

$$S \to S + 2\Sigma_i k_a \delta^i_{GS} ln(ic_i T_i + d_i) \tag{3.73}$$

and the gauge coupling constant at the unification scale (cf. eq.(3.69)) is duality invariant.

Having presented some of the relevant information needed to study the breaking of supersymmetry in 4D string theory, we will proceed to analyse the dynamics that may trigger supersymmetry in the following chapters.

Chapter 4

Gaugino Condensation in 4D String Theory

In this chapter we will study the breaking of supersymmetry in 4D string theory [48]-[58]. As mentioned earlier, supersymmetry is needed to explain the mass hierarchy problem between the Planck and the electroweak scales. It is also interesting to note that the gauge coupling constants of the standard (non-supersymmetric) model do not unify at any given energy, but for the minimal supersymmetric extension of the standard model they meet at the unification scale $\Lambda_{gut} \simeq 10^{16} GeV$.

Of course SUSY cannot be an exact symmetry since no supersymmetric partner of the known elementary particles has yet been detected. Therefore it must be, at best, a spontaneously broken symmetry. To preserve the mass hierarchy, the mass of the supersymmetric partners cannot be much heavier than the present experimental energies obtained in the accelerators and, if they exist, they should be detected in the near future. There are several possibilities for breaking SUSY. Perhaps the best one is via gaugino condensates [9], because it can easily lead to a large mass hierarchy since the supersymmetric breaking scale is exponentially suppressed compared to the Planck mass. For any given asymptotically free gauge group, the gauge coupling constant becomes strong as energy decreases and may lead to the formation of fermion condensates as is the case in QCD [22, 23, 24]. The scale at which the condensate is formed is related to the so called condensation scale, defined as the scale where the gauge coupling constant becomes strong. Clearly it will vary for different gauge groups.

In string theory, one obtains an effective theory after compactification with two separate gauge groups $E_6 \times E_8'$ or subgroups there of. The group E_6, called the visible sector, is supposed to have the standard model as a subgroup while the other group E_8' (or a subgroup) is referred to as the hidden gauge group. Usually these two sectors interact through gravity only. Since in string theory all gauge coupling constants are unified at the string scale and are mainly given by the v.e.v. of the dilaton field, the condensation scale for each gauge group will be determined by its one-loop beta function coefficient β_0. The larger β_0 is, the larger the condensation scale will be, and we will then expect that SUSY is broken by the gauge group with largest β_0. That a gaugino condensate with a non-trivial gauge kinetic function does indeed break SUSY, as is the case in the 4D string model, can be seen from the expression of the auxiliary field of the chiral superfields (cf. eq.(2.42)). A non-vanishing gaugino condensate will render a non zero v.e.v. for the auxiliary field and this is the condition for spontaneously broken SUSY.

The non-renormalizable theorems for global SUSY theories state that global SUSY can not be perturbatively broken, i.e. if it is not broken at tree level, radiative corrections will not break SUSY. These theorems are evaded by non-perturbative effects, of which gaugino condensation is an example.

The breaking of SUSY via gaugino condensate has been extensively studied and there are primarily two different approaches: the "truncated" [48]-[51],[53] and the "effective Lagrangian" one [52, 57]. Both are uniquely determined by the symmetries and anomalies of the 4D string model and are basically equivalent. Unfortunately, no reasonable solution has been found for a single gaugino condensate. Recently it was shown that by including several gaugino condensate with slightly different one-loop beta function and matter fields (which should acquire a v.e.v.), it is possible to obtain a stable solution for the dilaton field and a large mass hierarchy [56].

Although these approaches give a general parameterization of the gaugino condensate, they do not address the dynamical question of why a condensate is energetically favoured. In practice this is an important consideration for the contribution to the vacuum energy from gaugino binding effects can play a significant role in determining the structure of the potential and the SUSY breaking effects in the visible sector. In order to study such effects, it is necessary to evaluate the non-perturbative effects giving rise to gaugino condensation. The complete solution is clearly beyond our present-day technology so we are forced to employ approximation methods. A convenient approach for obtaining non-perturbative information about the gaugino binding is to apply Nambu-Jona-Lasinio (NJL) techniques for summing the bubbles graphs that result from an underlying 4-Fermi interaction. In a strongly interacting gauge theory, there is no primary 4-Fermi interaction, but gauge interaction may generate strong 4-Fermi interaction. If we replace the strong gauge interaction by such a 4-Fermi interaction, NJL techniques may then be used to study the non-perturbative effects. In QCD, a 4-Fermi effective interaction for the quarks gives an acceptable phenomenology and reproduces some of the alternative low energy models of QCD [24]. Furthermore, the scalar potential in the "truncated" and "effective" approaches, used to describe gaugino condensation, also generate such a 4-Fermi interaction. With this model we will show that the form and radiative corrections of the interaction are of great importance in determining the vacuum structure of the 4D string model.

In supersymmetry (ignoring possible superpotential terms) there is an anomalous R-symmetry which is spontaneously broken if a gaugino condensate forms leading to a Goldstone mode. In this case the gaugino bilinear may be parameterized by an auxiliary scalar field ϕ that represents the Goldstone degrees of freedom of the spontaneously broken R-symmetry (under which the gaugino transform non-trivially). To have a consistent theory, the auxiliary field ϕ describing this would-be mode must be embedded in a chiral superfield Φ that is coupled in a supersymmetric way to the superstring model. Using this formalism we will show that, once non-perturbative loop effects are taking into account, a large mass hierarchy is indeed possible with only one gaugino condensate.

The outline of this chapter is as follows. In section one and two we present other parameterizations of the gaugino condensates referred to as the "truncated" and "effective Lagrangian" approaches, respectively. In section three we discuss the NJL model in the context of 4D string models and summarize it in section four. Finally we show in section five the equivalence between our approach and those used before.

4.1 Truncated approach

In the truncated approach the effects of the gaugino condensate [48]-[51] are introduced by hand by including a superpotential of the form $W = h\, e^{-3S/2b_0}$, where h is a constant, b_0 the one-loop beta function coefficient and S the dilaton field. This superpotential is proportional to the condensation scale cubed (cf. eq.(3.70)), the scale at which the gauginos condense. It is obtained by observing that in the absence of a superpotential, the complete Lagrangian is invariant under a generalized R symmetry

$$\lambda_L \rightarrow e^{-i\delta}\lambda_L \tag{4.1}$$

and

$$S \rightarrow S - \frac{i4b_0}{3}\delta \tag{4.2}$$

where λ_L are the left handed gauginos of the hidden sector. In terms of the gauge covariant chiral superfield W^a (cf. eq.(2.17)) and the dilaton S their coupling is given by eq.(2.38)

$$L_{int} = \frac{1}{4}\int d^2\theta S W^a W^a + h.c. \tag{4.3}$$

and the R transformation (cf. eq.(4.1)) for the gauge covariant chiral superfield reads

$$W^a \rightarrow e^{-i\delta} W^a(\theta e^{i\delta}). \tag{4.4}$$

To see that the transformation of eqs.(4.1) and (4.2) is a good symmetry of the theory, one first notes that the R current J^R_μ (cf. eq.(2.33)) is anomalous

$$\partial_\mu J^\mu_R = \frac{b_0}{3} F\tilde{F}, \tag{4.5}$$

and hence performing an R transformation on the gauge fields induces an extra term in the Lagrangian

$$\delta L = -\delta \frac{b_0}{3} F\tilde{F}. \tag{4.6}$$

However, this extra term is exactly canceled by the R transformation on the S field. Since ImS couples to $F\tilde{F}$ as in eq.(2.40) one gets another term

$$\delta L' = \delta \frac{b_0}{3} F\tilde{F} \tag{4.7}$$

and we obtain $\delta L + \delta L' = 0$. Although the R-symmetry is anomalous the Lagrangian remains invariant under the generalized R-symmetry eqs.(4.1) and (4.2) and this symmetry should still be valid after the gauginos condense. In order to obtain an anomaly free Lagrangian, the superpotential W must transform like

$$W \to e^{2i\delta} W \tag{4.8}$$

so that $\int d^2\theta W$ remains invariant ($d^2\theta \to e^{-2i\delta} d^2\theta$). The functional dependence of the superpotential on S is uniquely determined by eq.(4.8) and is given by

$$W = he^{-3S/2b_0} \tag{4.9}$$

where h is S independent but may depend on R-invariant fields such as moduli fields. The superpotential (4.9) is consistent with dimensional analysis where the superpotential W is proportional to the gaugino condensate $W =< S\lambda\lambda >$. By dimensional analysis (which may be supported by instanton calculations), $< \lambda\lambda >$ is of order Λ_c^3, where Λ_c is the condensation scale, and thus the S dependence is just that of (4.9). In the case of h being independent of moduli fields the tree level potential for W given in (4.9) and using $K_i (K^{-1})^i_i K^i = 3$ is

$$V_0' = \frac{1}{4} e^K \mid W \mid^2 \left(1 + \frac{3S_r}{2b_0}\right)^2 . \tag{4.10}$$

In a more general case where the moduli dependence of h is such that it renders a modular invariant Kahler function G, as discussed in chapter three, one must have $h = h_0 \Pi_i \eta(T_i)^{-2}$ with h_0 a modular invariant function. The tree level potential becomes

$$V_0 = \frac{1}{4} e^K \mid W \mid^2 B_0 \tag{4.11}$$

with

$$B_0 = (1 + \frac{3S_r}{2b_0})^2 + \Sigma_i \frac{T_{ri}^2}{4\pi^2} \mid \hat{G}_2(T_i) \mid^2 - 3. \tag{4.12}$$

4.2 Effective Lagrangian approach

We will now present the effective Lagrangian approach [52, 57]. In this approach, an effective superpotential P_{eff}, is derived by requiring that the trace, axial and superconformal anomalies cancel at the one-loop approximation. It is given by

$$P_{eff} = \frac{1}{4}Y\left(f_0 + \frac{2b_0}{3}ln(Y/\mu'^3)\right) \tag{4.13}$$

where the scalar component of Y is identified with the gaugino condensate and f_0 is the gauge kinetic function at the string scale. The functional dependence of Y on S is obtained by minimizing the scalar potential and it is proportional to Λ_c^3. A simplified expression for Y can be obtained by minimizing the superpotential $\frac{\partial W}{\partial Y} = 0$ yielding an effective superpotential of the form

$$P_{eff} = -\frac{b_0\mu'^3}{6e}\, e^{-3f_0/2b_0}. \tag{4.14}$$

If we take the gauge coupling constant to be given at tree level, then $f_0 = S$ and eq.(4.14) is equivalent to eq.(4.9) with

$$h_0 = -\frac{b_0\mu'^3}{6e}. \tag{4.15}$$

As we have seen the truncated and effective Lagrangian approaches yield almost equivalent theories, which are also consistent with dimensional and instanton calculations. The functional dependence on the dilaton field is the same in both approaches. Of course, it is not surprising since the derivation of both rely on anomaly cancelation.

After obtaining an effective model for incorporating the effects of gaugino condensation, one can study the behavior of the scalar potential and look for stable solutions to determine the scale of SUSY breaking. This scale is basically determined once the v.e.v. of S is obtained, but unfortunately both approaches yield a runaway potential for $S \to \infty$ and an unbounded potential for $S \to 0$. Thus no stable solution exists. The analysis has been extended to consider a hidden sector G that consists on two or more gauge groups $G = G_1 \times G_2 \times ....$ The superpotential takes the form

$$W = h^1 e^{-3f_0/2b_0^1} + h^2 e^{-3f_0/2b_0^2} + ... \tag{4.16}$$

where b_0^1, b_0^2... are the one-loop beta function coefficient for the first and second subgroups respectively and h^1, h^2,.. their corresponding constant parameters defined in eq.(4.15). However, no stable solution for reasonable values of S (i.e. $S = O(1)$) is found. This can be easily seen from the stationary condition

$$W - S_r W_S = 0 \tag{4.17}$$

which for two condensates yields

$$S_r = \frac{4}{3}\frac{1}{\frac{1}{b_0^1} - \frac{1}{b_0^2}} ln\left(\frac{h_1(3S_r/2b_0^1 + 1)}{h_2(3S_r/2b_0^2 + 1)}\right). \tag{4.18}$$

Thus, no solution with a large hierarchy, i.e. $3S_r/2b_0 \gg 1$, can be obtained.

A possible interesting solution is only found after matter fields are included. We will not attempt to give a complete analysis of these solutions but we will only sketch the arguments and refer the reader to the original works [55]. For a theory with massive matter fields, where their mass is given in terms of the v.e.v. of another chiral matter field, the coefficient in eq.(4.15) gets modified to

$$h_0 = -N(32\pi^2)^{\frac{M-N}{N}}[det\mathrm{M}]^{\frac{1}{N}} \tag{4.19}$$

where we have written the one-loop beta function coefficient as $b_0 = (3N - M)/16\pi^2$, $\mathrm{M}= \frac{\partial^2 W_m}{\partial\phi_i\phi_j}$ is just the mass matrix and $W_m = \Sigma_{i,j,k}\lambda^{ijk}\Phi_i\Phi_j\Phi_k$ is the trilinear superpotential for chiral matter superfields Φ_i. After integrating out the matter fields through their equations of motion the coefficient in (4.19) becomes

$$h_0 = -\frac{3N - M}{3}(32\pi^2 e)^{\frac{3(M-N)}{3N-M}}(\frac{M}{3})^{\frac{M}{3N-M}}. \tag{4.20}$$

For other gauge groups, the corresponding h_0 would have a similar form as that of eq.(4.20). Using eq.(4.18) and neglecting small logarithms, one gets for the v.e.v. of the dilaton, in the case of two condensates,

$$S_r \simeq 0.34\frac{N_2 M_1 - N_1 M_2}{(3N_2 - M_2)(3N_1 - M_1)} \tag{4.21}$$

and thus reasonable values of S can be obtained and a large hierarchy may develop.

In such a scenario SUSY is broken only if the auxiliary field of the moduli are different than zero since the auxiliary field of the dilaton vanishes. By minimizing the scalar potential with respect to the moduli, their v.e.v. is of order one ($T \simeq 1.2$) [53]. In the case of the moduli taking the dual invariant points ($T = 1, e^{i\pi/6}$), SUSY will not be broken since their auxiliary fields would then vanish. The values for the moduli do not allow for unifying the coupling constants, assuming the minimal string model, since the unification scale is of the same order as the string scale, which is much larger than the $\Lambda_{gut} = 10^{16} GeV$ needed [60]. Furthermore, a squeezed Orbifold, i.e. an Orbifold where the compactified radius take different values, seems to be required for phenomenologically viable solutions to the unification of gauge couplings [56]. It is not clear how such an Orbifold can be obtained in this context.

As mentioned before, neither of the two approaches used so far addresses the question of whether a gaugino condensation is energetically favoured nor do they address the study of the gaugino binding effects. In order to investigate this problem we will study the non-perturbative effects due to a strong coupling constant following NJL techniques presented in section 2.6. Using this method, we will show that a gaugino condensate is energetically favoured with a single gaugino condensate. A large mass hierarchy can be achieved with a reasonable value for the dilaton field and the v.e.v. of the moduli, which are much larger than one allowing for the minimal string unification scheme to work.

4.3 An effective 4-Fermi interaction in 4D string theory

In a theory with an asymptotic free gauge group, one expects the formation of fermion condensates at a scale where the gauge coupling constant becomes strong. Since it is a non-perturbative effect, we must use approximation techniques. We assume that the gauge interaction between gauginos and gauge bosons, for a strong gauge coupling constant, generates a point-like 4-Fermi interaction below the condensation scale (see **fig.4**) and it will vanish rapidly above this scale [24]. Using this 4-Fermi interaction, the formation of gaugino condensates can then be conveniently studied using the NJL model described in chapter 2.

As in the case of the NJL model, we look for a formulation that parameterizes the

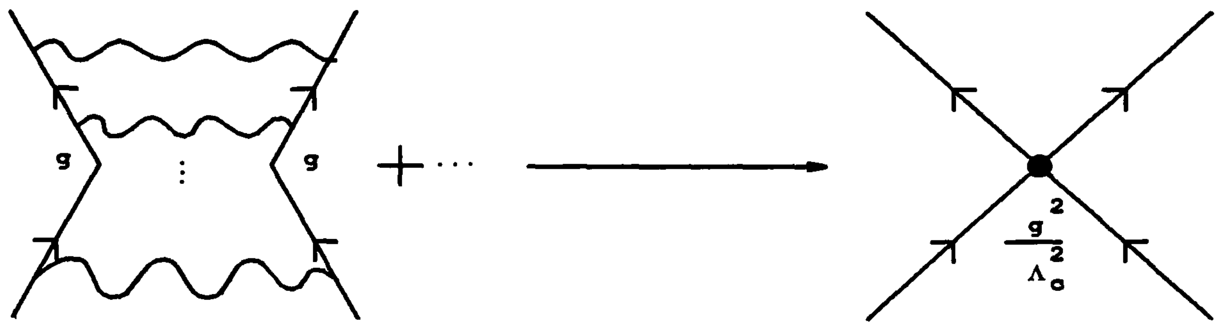

Figure 4.1: Effective 4-Fermi coupling generated by a strong gauge interaction below the condensation scale Λ_c

gaugino bilinear by a classically non-propagating auxiliary field, which at the quantum level becomes the Goldstone mode associated with the breaking of a continuous symmetry. In this case, however, the formation of the original theory in terms of an auxiliary field must be made consistent with local SUSY.

For gauginos, in the absence of superpotential terms, there is an R-symmetry which is spontaneously broken if a gaugino condensate forms leading to a Goldstone mode. In this case the auxiliary field ϕ describing this would-be mode must be embedded in a chiral superfield Φ which is coupled in a supersymmetric way.

In the standard formulation of SUGRA the theory is completely specified once the Kahler potential G and the gauge kinetic function f are given. We look for an effective ansatz for G and f that will describe the desired four gaugino interaction. As in the non-SUSY NJL model it is convenient to introduce an auxiliary field Φ that represents the gaugino bilinear through its equation of motion, and has no kinetic term at a classical level but rather develops one quantum mechanically.

For a superpotential W and gauge kinetic function f depending on this auxiliary chiral superfield $\Phi = (\phi, \chi_\phi, h)$, where ϕ, χ_ϕ, h are the scalar, fermion and auxiliary field components, the terms in the N=1 SUGRA Lagrangian involving these fields are (suppressing all gauge indices and using the gauge $\psi \cdot \gamma = 0$) [15]

$$e^{-1} L_{aux} = e^{K/6} (\frac{1}{2} e^{K/2} \frac{\partial W}{\partial \phi} - \frac{1}{4} \frac{\partial f}{\partial \phi} \bar{\lambda}_R \lambda_L) h - \frac{1}{2} e^{K/2} W^{ij} \bar{\chi}_{Ri} \chi_{Lj}$$
$$+ \frac{1}{4} f^{ij} \bar{\chi}_{Ri} \chi_{Lj} \bar{\lambda}_R \lambda_L + \frac{1}{2} f^i \bar{\chi}_{Ri} (-\sigma \cdot \hat{F} + iD) \lambda_L + h.c. \tag{4.22}$$

where λ_L represents the gaugino field[1] (with kinetic term $L_k = \frac{i}{2} Re f\, \bar{\lambda} \gamma^\mu \partial_\mu \lambda$), ψ the

[1] We define $\lambda_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\lambda$, $\bar{\lambda}_R \equiv (\lambda_R)^\dagger \gamma_0$

gravitino field, χ_i the fermion component of the chiral matter superfield φ_i, D the auxiliary field of the gauge covariant chiral multiplet (that has the gaugino bilinear as its scalar component), $\hat{F}_{\mu\nu} = F_{\mu\nu} - \frac{1}{2}\bar{\psi}_{L\rho}\sigma_{\mu\nu}\gamma^{\rho}\lambda_R$ with $F_{\mu\nu}$ the gauge field strength and the i, j indices in f and W represent derivatives with respect to the scalar component of chiral superfields (i.e. $W_i \equiv \frac{\partial W}{\partial z^i}$ with z^i the scalar component of φ_i).

The classical equation of motion for the component fields of Φ yield the constraints

i) $\frac{\delta L}{\delta h} = 0$

$$\frac{e^{K/2}}{2}\frac{\partial W}{\partial \phi} = \frac{1}{4}\frac{\partial f}{\partial \phi}\bar{\lambda}_R\lambda_L$$

ii) $\frac{\delta L}{\delta \chi} = 0$

$$\chi_{L\phi}(\frac{1}{2}f^{\phi\phi}\bar{\lambda}_R\lambda_L - e^{K/2}W^{\phi\phi}) + \frac{1}{2}f^{\phi}(-\sigma\cdot\hat{F} + iD)\lambda_L - \frac{1}{2}f^{\phi a}\bar{\lambda}_R\lambda_L\chi_{La} - e^{K/2}W^{\phi a}\chi_{La} = 0 \tag{4.23}$$

and

iii) $\frac{\delta L}{\delta \phi} = 0$

$$\frac{\partial V}{\partial \phi} = 0$$

with V the scalar potential (i.e. $V = L - L_k$ and L_k the kinetic Lagrangian) and the index a in eq.(4.23ii) runs over all scalar fields except for ϕ.

As mentioned above, the effects of gauge boson-gaugino interaction, for a strong gauge coupling constant, will be to generate an effective four fermion vertex, and the effective theory describing this interaction is specified once the Φ dependence of W and f are given.

If we demand that the effective theory, given in terms of the auxiliary field Φ, generates this 4-Fermi interaction then the forms of the W and f are uniquely determined (up to a constant). They are given by

$$W_0 = m^2\Pi_i\eta^{-2}(T_i)\ \phi \tag{4.24}$$

and

$$f = f_0 + \xi ln(\phi/\mu) \tag{4.25}$$

where m and μ are mass parameters independent of Φ, ξ is a dimensionless constant and f_0 is the gauge kinetic function at the string scale (cf. eq.(3.71)) and includes the contribution from the infinite number of massive Kaluza-Klein string modes. The T dependent part in eq.(4.24) is to ensure that the superpotential transforms under duality with weight -1 for each moduli as required by modular invariance (cf. eq.(3.56)).

From the classical equation of motion eqs.(4.23) the scalar component of the auxiliary field Φ is given in terms of the gaugino bilinear by

$$\phi = \frac{e^{-K/2}\xi}{2m^2\Pi_i\eta^{-2}(T_i)}\bar{\lambda}_R\lambda_L, \tag{4.26}$$

while the fermion component[2] is given in terms of the gaugino and chiral fermion fields by

$$\chi_{L\phi} = -\left(\frac{e^{-K/2}\phi}{m^2\Pi_i\eta^{-2}(T_i)}\right)\left(\frac{1}{2}f^{\phi}(\sigma\cdot\hat{F} - iD)\lambda_L + \frac{1}{2}f^{\phi a}\bar{\lambda}_R\lambda_L\chi_{La} + e^{-K/2}W^{\phi a}\chi_{La}\right). \tag{4.27}$$

The third equation of eqs.(4.23) is an extremal condition on the scalar potential and once the one-loop corrections are included, it is just the mass gap equation. As we will now show, these choices of W and f lead to an effective four-fermion interaction of the desired form once the auxiliary field is eliminated by its classical equation of motion, eq.(4.26). But before we proceed, we have to determine the parameters ξ, m, and μ of eqs.(4.24) and (4.25).

One can determine ξ by demanding the effective Lagrangian to be anomaly free under the R-symmetry under which the gauginos transform like $\lambda_L \to e^{-i\rho}\,\lambda_L$. As we saw in section 4.1 the effects of this transformation on the gaugino bilinear is to generate an anomalous term $\delta L = \frac{b_0}{3}\rho F\tilde{F}$ in the Lagrangian, where b_0 is the N=1 β-function coefficient for the hidden sector gauge group. To cancel this anomalous term, we note from eq.(4.26) that the field ϕ becomes

$$\phi \to e^{-i2\rho}\,\phi \tag{4.28}$$

and the gauge kinetic function eq.(4.25) transforms as

$$f \to f - i2\xi\rho. \tag{4.29}$$

[2] See appendix B for details about the interaction terms in the Lagrangian for χ_ϕ and D fields.

Therefore the R-symmetry is anomaly free if we identify the parameter ξ with $2b_0/3$, i.e.

$$\xi = \frac{2b_0}{3}. \tag{4.30}$$

Since the superpotential W given in eq.(4.23) must be chiral, due to supersymmetry the mass parameter m must be chiral as well. Thus, we are forced to identify it with the Planck mass, since it is the only field independent mass scale.

Finally, to determine the scale μ, we impose modular invariance on the gauge coupling constant eq.(4.25) and scalar potential. From eq.(4.25) we obtain that the ϕ field is given by

$$|\phi|^2 = |\mu|^2 \Pi_i(|\eta_i|^4)^{2(k_a\delta^i_{GS}-b'^i_a)/\xi}\, e^{-2ReS/\xi}\, e^{2Ref/\xi}$$

and in terms of the modular invariant quantity $Y = 2(ReS + \Sigma_i k_a \delta^i_{GS} lnT_{ri})$, one has

$$|\phi|^2 = |\mu|^2\, \Pi_i(|\eta_i|^4)^{-3b'^i_a/b_0}\Pi_i(T_r|\eta_i|^4)^{3k_a\delta^i_{GS}/b_0}\, e^{-3Y/2b_0}\, e^{2Ref/\xi}. \tag{4.31}$$

Modular invariance on the Kahler potential and gauge coupling constant requires ϕ to be modular invariant. The mass parameter μ must then be identified with $\Pi_i T_{ri}^{-3b'^i_a/2b_0}\, m$ with m a modular invariant mass scale, since $T_r|\eta|^4$ is a modular invariant function. We choose $m = M_s^3$ (in Planck mass units) so that in the field theoretical limit[3] μ is given in terms of the compactification scale M_a^3, defined in eq.(3.65), $\mu = M_s^3 \Pi_i T_{ri}^{-1/2} = M_a^3$. Other choices of μ are possible. None of them modify the analysis of the scalar potential significantly nor the hierarchical solutions.

The non-holomorphicity of the gauge kinetic function given through the μ term is due to the contribution from the light chiral matter fields of the hidden sector that couple to gauge bosons and moduli fields as discussed in section 3.8. It is interesting to notice that by imposing modular invariance one obtains an effective model which necessarily includes the contribution of these light fields. A non-holomorphic gauge kinetic function is consistent with local supersymmetry if the dilaton is given by a linear supermultiplet instead of a chiral superfield one (cf. section 2.4). Both formalism are equivalent through a duality transformation[4]. When the dilaton is represented by a

[3] In this limit $b'^i_a/b_0 = 1/3$.

[4] Duality here should not be confused with target space duality.

chiral superfield , i.e. in the standard formulation of SUGRA, the contribution from the light chiral matter fields should be seen as a wave function renormalization of S. They should appear in the Kahler potential and not in the gauge kinetic function (cf. eq.(4.25)). Doing so, the S dependent part of the Kahler potential becomes

$$K = -ln\left(S + \bar{S} + 2\Sigma_i(k_a\delta^i_{GS} - b'^i_a)lnT_{ri} + 2b_0 ln(M_s^2)\right). \tag{4.32}$$

With the inclusion of this term, the duality transformation for the dilaton must be such that it renders the Kahler potential eq.(4.32) modular invariant,

$$S \to S + 2\Sigma_i(k_a\delta^i_{GS} - b'^i_a)ln(icT_i + d). \tag{4.33}$$

The duality invariant quantity

$$Y = S + \bar{S} + 2\Sigma_i(k_a\delta^i_{GS} - b'^i_a)lnT_{ri} + 2b_0 ln(M_s^2) \tag{4.34}$$

gives the gauge coupling constant at the unification scale, $g_s^{-2} = \frac{Y}{2}$.

With this redefinition of Y and of the modular transformation of S, the parameter μ must be duality invariant and chiral. Thus, we identify it with the Planck mass, $\mu = m_{Planck}$.

We are now in position to calculate the tree level potential. It will be derived in chapter 5 but for completeness with the discussion in this section we will present it here. It is given by (see eq.(5.5))

$$V_0 = e^G B_0 = \frac{1}{4}e^K \Pi_i \,|\, \eta(T_i)\,|^{-4} \;|\, \phi\,|^2 B_0 \tag{4.35}$$

with

$$B_0 = (1 + \frac{Y}{\xi})^2 + \Sigma_i \frac{Y}{Y + a_i}(1 - \frac{a_i}{\xi})^2 \frac{T_{ri}^2}{4\pi^2}\,|\,\hat{G}_2(T_i)\,|^2 - 3 \tag{4.36}$$

and $a_i = 2(k_a\delta^i_{GS} - b'^i_a)$. If we eliminate the auxiliary field ϕ by its classical equation of motion (4.26), the tree level potential becomes

$$V_0 = (\frac{\xi}{4})^2\,|\,\bar{\lambda}_R\lambda_L\,|^2 B_0 \tag{4.37}$$

and we notice that the choice of superpotential and gauge kinetic function given in eqs.(4.24-4.25) reproduces the desired four gaugino interaction. Finally, we would like to point out that this 4-Fermi interaction is supersymmetric and the effective theory also contains related supersymmetric graphs like the ones shown in **fig.4.2.**

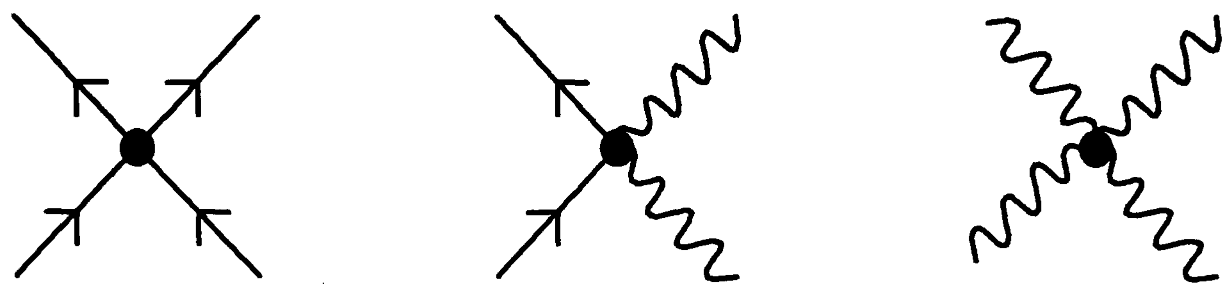

Figure 4.2: 4-Fermi interaction and related supersymmetric graphs. The full line represent a gaugino while the curly line a gauge boson.

4.4 Summary of the effective 4-Fermi interaction in 4D string theory

For convenience we summarize the effective model for the light degrees of freedom of the 4D superstring theory presented in section 4.3. It contains the dilaton field S, moduli fields T_i, chiral matter fields φ, gauge fields and the Goldstone mode Φ of the spontaneously broken R-symmetry, which is generated for a strong gauge coupling constant in the hidden sector.

The Kahler potential, superpotential and gauge kinetic functions are

$$K = -ln\left(S + \bar{S} + 2\Sigma_i(k_a\delta^i_{GS} - b'^i_a)lnT_{ri} + 2b_0 ln(M_s^2)\right) - \Sigma_i ln(T_{ri}) + K^i_i \mid \varphi \mid^2, \quad (4.38)$$

$$W_0 = \Pi_i \eta^{-2}(T_i)\ \Phi + W_m \quad (4.39)$$

and

$$f = f_0 + \frac{2}{3} b_0 ln(\phi) \quad (4.40)$$

respectively. W_m is the superpotential for the chiral matter superfields and the gauge kinetic function at the string scale is given by

$$f_0 = S + 2\Sigma_i(b'^i_a - k_a\delta^i_{GS}) ln\left[\eta(T_i)^2\right], \quad (4.41)$$

as in eq.(3.71). The Goldstone mode is a non-propagating field at tree level. Through the equation of motion of its auxiliary field, the scalar component is given in terms of the gaugino bilinear of the hidden sector (cf. eq.(4.26))

$$\phi = \frac{e^{-K/2}\xi}{2m_p^2 \Pi_i \eta^{-2}(T_i)} \bar{\lambda}_R \lambda_L$$

while its fermion component is given in eq.(4.27).

The model described in eqs.(4.38-4.40) is anomaly free and duality invariant. The duality transformation for the fields read

$$\begin{aligned} S &\rightarrow S + 2\Sigma_i(k_a\delta^i_{GS} - b'^i_a)ln(icT_i + d), \\ T_i &\rightarrow \frac{aT_i - ib}{icT_i + d}, \\ \phi &\rightarrow \phi, \end{aligned} \tag{4.42}$$

with $a, b, c, d \in Z$ and $ad-bc = 1$. As shown in eq.(4.37), this model generates the desired four-Gaugino interaction and reproduces the tree level scalar potential used by other parameterizations of the gaugino condensate. It also permits the determination of the radiative corrections and use of NJL technique to extract non-perturbative information in the regime of strong coupling. After minimizing the complete scalar potential (tree level plus one-loop potential), we will show that the vacuum structure is quite different than the tree level one, thus permitting us to find a stable solution for the dilaton with the inclusion of a single gaugino condensate. We will also show that the value of the v.e.v.'s of the dilaton and moduli fields (which are larger then the usual dual invariant ones) can give a good prediction of the fine structure constant at the unification scale and unification scale allowing for minimal string unification to work.

4.5 Connection with other parameterizations of the gaugino condensate

In the approach adopted here, when a gaugino condensate forms, ϕ will be the Goldstone mode associated with the spontaneous breaking of the R-symmetry. Since the construction leading to eq.(4.35) respects both the underlying R-symmetry and local SUSY, it must duplicate the results obtained in the "truncated" approach which relies on the R-symmetry. Thus, we may understand the origin of the highly constrained form of eqs.(4.24) and (4.25) leading to the potential of eq.(4.35) as following from consistency with the symmetries of the system.

It is then no surprise that a simple reparameterization of the auxiliary field ϕ gives the effective superpotential derived by requiring that the trace, axial and superconformal anomalies cancel at the one-loop approximation. This effective superpotential is (cf. eq.(4.13))

$$P_{eff} = \frac{1}{4}Y\left(f_0 + \frac{2b_0}{3}ln(Y/\mu'^3)\right)$$

where the scalar component of Y is identified with the gaugino condensate.

As we said before, we may cast a superpotential P in this form by defining

$$P = -m^2\Phi + f_{\alpha\beta}W^\alpha W^\beta \tag{4.43}$$

with f given in eq.(4.40) and W^α the gauge covariant chiral superfield (which has $\bar{\lambda}\lambda$ as its scalar component). Then, by using the equation of motion for Φ ($\Phi = \xi WW/m^2$) and rescaling the auxiliary field $\Phi' = \Phi/e$ (e is just exp(1)), one obtains

$$P = \frac{m^2 e}{\xi}\Phi'\left(f_0 + \xi ln(\Phi'/\mu)\right), \tag{4.44}$$

which is proportional to the superpotential used in the "effective" superpotential approach, eq.(4.13). The equivalence between the "truncated" and "effective" approaches were discussed in section 4.2.

Furthermore, by taking the gauge coupling constant at tree level as in section 4.1 (i.e. $f_0 = S$), the tree level scalar potential given in eqs.(4.11) and (4.12) obtained by the truncated approach is the same one as that of eqs.(4.35) and (4.36) calculated using the NJL parameterization of the Goldstone mode thus showing again the equivalence between the different approaches.

Chapter 5

Analysis of the Complete Scalar Potential

In this chapter we will determine the vacuum structure of the 4D superstring model and we will show that a large mass hierarchy may develop. As presented in chapter 4, a convenient way to extract information about possible fermion condensates in field theory with a strong coupling constant is to use the NJL model. In the NJL model, one has an initial four fermion interaction and one calculates the quantum corrections of an infinite sum of fermion bubbles. By finding a stable solution to the complete scalar potential, where by complete we mean the sum of the tree level and one-loop potential, one can determine whether a fermion condensate is dynamically favoured or not. The result obtained is necessarily non-perturbative since one is comparing the tree level with the one-loop scalar potential.

In this chapter we will first of all calculate the tree level potential for the model presented in chapter 4 and we will show that it describes the desired 4-Fermi interaction. We will then proceed to calculate in section two the one-loop scalar potential and we will discuss the symmetry breaking in section three. In section four we determine the v.e.v.'s of the dilaton and moduli fields while in section five we study the scalar potential including supersymmetric confinement masses for the gauge bosons and gauginos in the hidden sector.

We leave for chapter six the phenomenological analysis of the results obtained in this chapter.

5.1 Tree level potential

Having obtained the low energy 4D superstring model in chapter 4 we are now in position to determine the vacuum structure of the model and investigate whether a gaugino condensate is dynamically favoured or not. The relevant degrees of freedom are given by the gauge, moduli and dilaton fields as well as the Goldstone mode associated with the R-symmetry which is spontaneously broken when a gaugino condensate forms. We do not consider, in this chapter, chiral matter fields and set their v.e.v. to zero. We will include them in chapter 6.

As discussed in chapter 4, the 4D string model is described by an N=1 SUGRA theory with a Kahler potential and gauge kinetic function given by eqs.(4.38-4.40).

For any N=1 SUGRA model the tree level potential takes the generic form [15]

$$V_0 = h_i(G^{-1})^j_i h^i - 3e^G + \frac{1}{2}(Ref)^{-1}D^2 \tag{5.1}$$

where $(G^{-1})^j_i$ is the inverse matrix of $G^j_i = \frac{\partial^2 G}{\partial z^*_j \partial z^i}$ and h_i is the F-term of the i^{th}-chiral superfields and D the auxiliary field of the gauge vector. multiplet. In general these auxiliary fields are given by eq.(2.42)

$$h_i = -e^{G/2}G_i + \frac{1}{4}f_i\bar{\lambda}_R\lambda_L - G^{jk}_i\bar{\chi}_{Rj}\chi_{Lk} + \frac{1}{2}G^k_i\bar{\chi}_{Rk}G_j\chi^j_L \tag{5.2}$$

and D by eq.(2.43)[1]. As discussed in section 2.5 SUSY will be broken if either h_i or D develop a non-vanishing v.e.v. From the structure of V_0 eq.(5.1) one can see that in SUGRA it is possible to have broken SUSY and zero cosmological constant unlike in global SUSY.

For a pure gauge theory in the hidden sector of the 4D superstring model, the only chiral fields are the dilaton S and the moduli T_i. For the choices of superpotential and gauge kinetic function of eqs.(4.39) and (4.40) the F-terms for these fields are

$$h_S = -e^{G/2}G_S + \frac{1}{4}f_S\bar{\lambda}_L\lambda_R = \frac{1}{2}e^{K/2}W_0\frac{(1+Y/\xi)}{Y} \tag{5.3}$$

[1] For the contribution to V_0 from D and χ_Φ fields see appendix B.

gauge kinetic function of eqs.(4.39) and (4.40) the F-terms for these fields are

$$h_S = -e^{G/2}G_S + \frac{1}{4}f_S\bar{\lambda}_L\lambda_R = \frac{1}{2}e^{K/2}W_0\frac{(1+Y/\xi)}{Y} \tag{5.3}$$

and

$$h_{T_i} = \frac{1}{2}e^{K/2}W_0(-K_{T_i} - \frac{W_{T_i}}{W_0} + \frac{1}{\xi}f_{T_i}), \tag{5.4}$$

where Y is given by eq.(4.34).

Using the expressions for the auxiliary fields eqs.(5.3) and (5.4) and taking $D = 0$, the tree level scalar potential[2] is then

$$V_0 = m_{3/2}^2 B_0 \tag{5.5}$$

with

$$B_0 = (1 + \frac{Y}{\xi})^2 + \Sigma_i \frac{Y}{Y + a_i}(1 - \frac{a_i}{\xi})^2 \frac{T_{ri}^2}{4\pi^2} \,|\, \hat{G}_2(T_i) \,|^2 - 3, \tag{5.6}$$

where $\hat{G}_2$ is the Eisenstein modular form with weight 1/2 and $a_i \equiv 2(k_a\delta_i^{GS} - b'_{ai})$. The gravitino mass is given by

$$m_{3/2}^2 = \frac{1}{4}e^K \Pi_i \,|\, \eta(T_i) \,|^{-4} \,|\, \phi \,|^2. \tag{5.7}$$

The Kahler potential contains a mixing term between the S and T fields (cf. eq.(4.38)) and thus the matrix $(G^{-1})^i_j$ is not diagonal. The cross terms between the auxiliary field of the dilaton and moduli fields are proportional to the radiative correction of the dilaton wave function (cf.(4.34)) parameterized by a_i. These radiative corrections are small compared to the tree level value. In the limit $a_i \ll 1$, the Kahler metric is diagonal, and eq.(5.6) reduces to the standard tree level result

$$B_0 = (1 + \frac{Y}{\xi})^2 + \Sigma_i \frac{T_{ri}^2}{4\pi^2} \,|\, \hat{G}_2(T_i) \,|^2 - 3.$$

As mentioned before, SUSY is only broken if an auxiliary field gets a non-vanishing v.e.v., because in this case the supersymmetry transformation for the corresponding chiral fermions is non-vanishing. The chiral fermion component mixes then with the gravitino field through the interaction term [15]

$$\bar{\psi}_L \cdot \gamma\,(\frac{1}{8}f_i\bar{\lambda}_L\lambda_R - e^{G/2}G_i)\chi_R^i.$$

[2]See appendix C for details.

χ^i becomes the Goldstino field that is eaten by the gravitino acquiring a mass $m_{3/2}^2 = \frac{1}{4}e^K \mid W \mid^2$ as in eq.(5.7). Clearly a non-zero gravitino mass is only possible for a non-vanishing v.e.v. for ϕ in which case h_S (cf. eq.(5.3)) is also different from zero and SUSY will be broken.

If we eliminate ϕ by its equation of motion eq.(4.26), we can express the scalar potential in terms of the gaugino fields,

$$V_0 = (\frac{\xi}{4})^2 B_0 \frac{\mid \bar{\lambda}'_R \lambda'_L \mid^2}{(Ref)^2}. \tag{5.8}$$

The factor of $(Ref)^2$ in the denominator in eq.(5.8) appears because we have rescaled the gaugino fields appearing in this equation to have canonical kinetic terms. Thus, we have shown that a choice of W and f in eqs.(4.39) and (4.40) leads to a four-fermion interaction, as desired.

As we have seen (cf. chapter 4), the effective Lagrangian expressed in terms of the would-be Goldstone boson correctly parameterizes the form of the gaugino condensate derived by other methods [48]-[53]. What this connection shows is that these analyses give the "tree level" [3] form of the effective potential describing the gaugino condensate and that (cf. section 5.2) radiative corrections must be included. Indeed, the purpose of developing this formalism was to allow us to study non-perturbative effects in the strong hidden sector coupling using the NJL method. We proceed by calculating the ϕ dependence of the effective potential, V. If, at the minimum of V, ϕ develops a vacuum expectation value, it will signal that a gaugino condensate is dynamically preferred, corresponding to the breaking of supersymmetry. To the extent that expressing the theory in terms of the auxiliary field ϕ is a re-parameterization of the theory, our results will be exact.

As we have seen, eliminating ϕ leads to a Lagrangian involving a four-fermion interaction. The value of Ref in eq.(5.8) is not a free parameter for it defines the initial four fermion interaction used to *define* the strong binding interaction in the NJL approach that we have adopted to study gaugino condensation. Below the scale of gaugino condensation the effective four fermion interaction must have the form $\frac{c^2}{\Lambda_c^2}(\bar{\lambda}\lambda)^2$,

[3] By "tree level" we mean that it does not include the contribution from the gaugino bubbles due to the strong gaugino coupling in the hidden sector.

$c = O(1)$, where the condensation scale Λ_c is also the confinement scale. This will be true provided $[Ref(\phi)]^{-1}$ in eq.(5.8) reaches a maximum "frozen" value

$$[Ref]^{-1} = \frac{c}{\Lambda_c}. \tag{5.9}$$

As may be seen from eq.(5.8) the residual ambiguity in Ref parameterized by c corresponds to an ambiguity in determining $m^2\phi$ (and hence $\bar{\lambda}\lambda$) in terms of Λ_c^3, relatively unimportant when considering whether a condensate will form. We impose this physically motivated condition as a reasonable parameterization of the strong coupling effects which must eliminate the unphysical divergence associated with the vanishing of Ref and which we are presently unable to calculate .

In this thesis, we choose to parameterize the strong gauge interaction in terms of this four fermion interaction rather than the primary gauge and gaugino couplings. We are then able to perform the non-perturbative sum of these interactions corresponding to the sum of all fermion bubble graphs. In this way we can get, albeit incomplete, information about the dynamics of such non-perturbative effects. We find that they can have a dramatic effect on the structure of the effective potential allowing for a stable non-trivial minimum for ϕ corresponding to a supersymmetry breaking solution with a large mass hierarchy. This demonstrates the importance of including the binding effects and, at the very least, should encourage efforts to perform a more complete summation of such effects.

The tree level potential for ϕ, given by eqs.(5.5-5.7), has no stable solution in the dilaton direction. It is unbounded from below for S going to zero[4] and it is a runaway potential for S going to infinity. A partial solution is to include several gaugino condensates. As showed in section 4.2, one also needs chiral fields in the hidden sector to acquire a v.e.v. Thus, gaugino condensation as usually parameterized does not occur in models with a single hidden sector gauge group factor. However, we have argued it is essential to go beyond "tree level" to include non-perturbative effects in the effective potential which may allow for a non-trivial minimum even in the simple case of a single

[4]This results represents infinite gauge coupling constant at the unification scale. Therefore, the expansion in terms of the string coupling constant, used in deriving the effective string model, breaks down.

hidden sector gauge group. A similar result is obtained in the NJL model, where only after radiative corrections are included, a fermion condensate is dynamically favoured. This non-perturbative sum (equivalent to the NJL sum) is readily obtained simply by computing the one-loop correction to V. If these contribution stabilize the potential, the resultant minimum will correspond to a cancellation of tree level and one-loop terms which, as noted above, is necessarily non-perturbative in character [10, 23].

5.2 One-loop scalar potential

To determine the relevant quantum corrections we have to determine the one-loop scalar potential. They can be calculated using the Coleman-Weinberg one-loop effective potential [25, 59]

$$V_1 = \frac{1}{32\pi^2} Str \int d^2p\, p^2 ln(p^2 + M^2) \tag{5.10}$$

where M^2 represents the tree level field-dependent square mass matrices, i.e. the second derivative of the Lagrangian properly normalized to take into account for the non-canonical kinetic terms of the fields, and Str the supertrace. V_1 can be integrated to give

$$V_1 = \frac{1}{64\pi^2} \Lambda^4 Str J(x) \tag{5.11}$$

with

$$J(x) = x + x^2 ln(\frac{x}{1+x}) + ln(1+x) \tag{5.12}$$

and

$$x = \frac{M^2}{\Lambda^2}. \tag{5.13}$$

Since the 4-Fermi interaction is non-renormalizable, we regularize it by introducing a momentum space cutoff Λ that should be identified with the condensation scale eq.(3.70) for the gaugino loops. Above this scale the effective 4-Fermi vertex should vanish rapidly. For the gravitational interaction, which is also non-renormalizable, one should use the standard unification scale as the cut-off. The supertrace Str of a function $Q(M^2)$ is given by [59]

$$StrQ(x) = 3trQ(M_A^2) + trQ(M_S^2) - 2trQ(M_F^2) + 2Q(4m_{3/2}^2) - 4Q(m_{3/2}^2) \tag{5.14}$$

where $M^2_{A,S,F}$ are the tree level (mass)^2 matrices for vectors, scalars and spin=1/2 fields. The $2Q(4m^2_{3/2})$ term is the contribution of the spin=3/2 particle, the gravitino, and the term $-4Q(m^2_{3/2})$ is due to the gauge condition $\psi_R \cdot \gamma = 0$.

We have now to determine the tree level masses for the different fields in the model. These correspond to the S and T_i scalars, their fermion partners, the gravitino and the gauge fields. The Goldstino field that couples to the gravitino field through eq.(2.59) is mainly given by the fermion component of the dilaton. Therefore, its contribution to the supertrace in eq.(5.11) should not be taken into account since it is already present by giving the gravitino a mass. Otherwise, we would count it twice.

The tree level scalar masses for the S and T fields are obtained from the scalar potential

$$m^2 = (G^{-1})^i_j \frac{\partial^2 V_0}{\partial z^*_i \partial z^j}, \tag{5.15}$$

and in calculating the masses one has to take into account that the scalar kinetic terms are not (and cannot be) in a canonical form. The fermion masses can be read off of the N=1 SUGRA Lagrangian. The relevant terms are [15]

$$L_{FM} = \frac{1}{2}\bar{\chi}_{Ri} B^{ij} \chi_{Lj} + h.c. \tag{5.16}$$

with

$$B^{mn} = m_{3/2}(G^m_i G^n_j)^{-1/2}\left[D^{ij} + \frac{1}{4\xi}(4f^{ij} - 4G^{ij}_k (G^{-1})^k_l f^l - (Ref)^{-1} f^i f^j)\right] \tag{5.17}$$

and

$$D^{ij} = G^{ij} - G^i G^j - G^{ij}_k (G^{-1})^k_l G^l. \tag{5.18}$$

Finally the gaugino mass is obtained from

$$\begin{aligned} m_g &= \frac{2}{Ref}\frac{\partial^2 V_0}{\partial\bar{\lambda}_R \partial\lambda_L} \\ m_g &= \frac{2}{Ref}\frac{\partial V_0}{\partial W_0}\frac{\partial^2 W_0}{\partial\bar{\lambda}_R \partial\lambda_L} \end{aligned} \tag{5.19}$$

giving

$$m^2_g = m^2_{3/2}\frac{\xi^2 B_0^2}{4(Ref)^2}. \tag{5.20}$$

Neither the graviton nor the gauge bosons have a tree level mass although the gauge bosons, as discuss below, may acquire a supersymmetric mass due to confinement.

The complete scalar potential, tree level plus one-loop potential, is then given by

$$V_{tot} = \frac{B_0 \mid \phi \mid^2}{4Y\mathrm{H}_i T_{ri} \mid \eta_i \mid^4} - \frac{n_g}{32\pi^2}\Lambda_c^4 J(\frac{m_g^2}{\Lambda_c^2}) + V_1' \tag{5.21}$$

with J given in eq.(5.12). V_1' is the one-loop potential (cf. eq.(5.11)) for all fields excluding the gauginos and n_g is the dimension of the hidden sector gauge group.

The masses given in eqs.(5.15), (5.17) and (5.20) are the field-dependent supersymmetry breaking masses following from the gaugino condensate. They may be different from zero at the minimum only if supersymmetry is broken since all these masses are proportional to the gravitino mass eq.(5.7). In addition we should allow for a supersymmetric contribution to the mass of the hidden sector states generated by the strong hidden sector forces which (in analogy with QCD) may be expected to be confining. Of course, we are unable to determine these masses, and so we proceed by examining the various possibilities. The first possibility is that the gaugino condensate forms at a scale above confinement and there is a domain in which the states are correctly described by the gauge bosons and gauginos with the only mass coming from the gaugino condensate as calculated above (It is thought the equivalent situation may occur in QCD with chiral symmetry breaking occurring before confinement). In this case, we may now compute, using eqs.(5.15-5.20), the one-loop potential. Alternatively confinement may occur at, or above, the condensate scale. In this case, the radiative corrections should be computed using the confined spectrum of states. Lacking knowledge of this spectrum we may still try to estimate the result by using the average description of these states in terms of gluons and gluinos but allowing for the confinement effects by giving them a common (supersymmetric) mass. We will discuss both of these cases in the next sections.

5.3 Dynamical breaking of SUSY

We are now in a position to determine whether it is energetically favourable for a gaugino condensate to form. From eqs.(5.11-5.21) it is clear that to have a SUSY breaking solution to the gap equation [23]

$$\frac{\partial}{\partial \phi}(V_0 + V_1) = 0 \tag{5.22}$$

the negative contribution from the fermion loops must dominate. Note that the contribution to the one-loop scalar potential from each individual massive state is a monotonic function of the mass (for a fixed cutoff) being zero only for vanishing mass. In this limit since the (supersymmetry breaking) gaugino mass in eq.(5.20) is proportional to g^2 a strong gauge coupling constant will be dynamically preferred, i.e. $g^{-2} = Ref << 1$. In contrast with the NJL model, where the coupling constant is indeed a constant and the formation of fermion condensates is energetically favoured only for a sufficiently strong coupling, we have in this case that the coupling constant is field dependent and as it stands the one-loop potential will go to $-\infty$ for $Ref = 0$. This is an unphysical singularity which will be removed when non-perturbative effects are included for it corresponds to infinite coupling. As we discussed in section 5.1 the value of Ref is not a free parameter for it defines the initial four fermion interaction and it takes a maximum value of c/Λ_c (cf. eq.(5.9)).

It is interesting to note that duality invariance is needed in order to ensure a stable solution. In fact if we did not have a duality invariant theory the superpotential in eq.(4.39) would be independent of the moduli and the complete scalar potential would go to -∞ along the direction in which a gaugino condensate forms with T, the modulus setting the radius of compactification, in the limit $T \to 0$. Clearly this is physically unacceptable and the origin of this problem is that in the $T \to 0$ limit corresponding to the small radius (R) limit, Kaluza-Klein modes with masses αR^2 cannot be neglected in writing the low-energy effective Lagrangian. A convenient way of summing this contribution is by imposing duality invariance as discussed in section 3.6.

5.4 Extremum conditions

In this section we will minimize the complete scalar potential, $V_0 + V_1$. As discussed in the preceeding sections, in order to avoid the unphysical singularity when Ref approaches zero and the potential becomes unbounded from below, a cutoff for the coupling constant must be introduced consistent with the strength of the initial 4-Fermi effective interaction and the correct choice is $Ref = \Lambda_c$ (taking $c = 1$ in eq.(5.9)). After introducing this physically-motivated cutoff, the Goldstone mode can be eliminated in terms of the dilaton and moduli fields using eq.(4.40)

$$|\phi|^2 = M_s^6 \Pi_i (T_{ri} |\eta_i|^4)^{3\alpha_i} e^{-3Y/2b_0}. \tag{5.23}$$

The gravitino mass becomes

$$m_{3/2}^2 = \frac{1}{4Y} \Pi_i (T_{ri} |\eta_i|^4)^{-1} \Lambda_c^6 \tag{5.24}$$

with the condensation scale given by eq.(3.70)

$$\Lambda_c^2 = M_s^2 \Pi_i (T_{ri} |\eta_i|^4)^{\alpha_i} e^{-Y/2b_0}.$$

In order to minimize the scalar potential, it is useful to write eq.(5.21) as

$$V = \gamma \Lambda_c^4 (a\, x_g - J(x_g)) + V_1' \tag{5.25}$$

where

$$\begin{aligned} x_g &= \frac{m_g^2}{\Lambda_c^2} \\ x_g &= (\frac{\xi}{4})^2 \frac{B_0^2}{Y \Pi_i T_{ri} |\eta_i|^4} \Lambda_c^2. \end{aligned} \tag{5.26}$$

The parameters a and γ are defined by

$$a \equiv \frac{9}{\gamma b_0^2 B_0},$$

and $\gamma = n_g/32\pi^2$ with n_g the dimension of the hidden sector gauge group. V_1' in eq.(5.25) is the one-loop potential for all fields excluding the gauginos.

If there is a stable solution with a large mass hierarchy, then one necessarily needs the coefficient in the exponential of the condensation scale $Y/2b_0$ to be much larger than one which implies that $B_0 \simeq (3Y/2b_0)^2 \gg 1$. In this limit the main contribution to the one-loop potential comes from the tree level gaugino mass and as can be seen from the quadratic divergence $StrM^2 \simeq \Lambda_{gut}^2 m_T^2 - n_g \Lambda_c^2 m_g^2 = m_{3/2}^2 B_0 8 - m_{3/2}^2 n_g B_0^2$, where we have only considered the leading scalar masses. This shows that the dominant part of the potential comes from the gaugino bubbles ($n_g B_0 \gg 8$ and $\Lambda_{gut} < 1$), as expected, and to find an approximated analytic solution we can neglect the V_1' potential. This is done as a matter of convenience for presentational purposes and of course in any numerical solution the complete scalar potential must be used.

The v.e.v. of the dilaton and moduli fields are obtained by simultaneously solving the extremum equations with respect to the moduli and dilaton fields. The S dependence in the scalar potential comes only in Y so that $\frac{\partial V}{\partial S} = \frac{\partial V}{\partial Y}\frac{\partial Y}{\partial S} = \frac{\partial V}{\partial Y}$. Since at the extremum for the dilaton $\frac{\partial V}{\partial Y} = 0$, we do not have to worry about the moduli dependence of Y when looking for a solution to the extremum equation of the moduli. The extremum equations for these fields are

$$\frac{\partial V}{\partial T_i} = \gamma \Lambda_c^4 \left[\frac{\delta_{iT_i}}{\delta_i}(\alpha^i(3ax_g - 2J - x_g J') - ax_g + J'x_g) + \frac{B_{0T_i}}{B_0}(ax_g - 2x_g J') \right] = 0 \tag{5.27}$$

and

$$\frac{\partial V}{\partial S} = \gamma \Lambda_c^4 \left[ax_g(\frac{B_{0S}}{B_0} - \frac{3}{2b_0} - \frac{2}{Y}) + 2J(\frac{1}{2b_0} + \frac{1}{Y}) + x_g J'(\frac{1}{2b_0} - \frac{2B_{0S}}{B_0}) \right] = 0 \tag{5.28}$$

with

$$\delta_i \equiv T_{ri} \mid \eta(T_i) \mid^4,$$

$J' \equiv \frac{\partial J}{\partial x_g}, B_{0S} \equiv \frac{\partial B_0}{\partial S}$ and α^i given in eq.(3.67). Eq.(5.27) is satisfied either for T_i at the dual invariant points ($T_i = 1, e^{i\pi/6}$) where $\delta_{iT_i} = B_{0T_i} = 0$ or for

$$J' - a + \alpha^i(3a - 2\frac{J}{x_g} - J') = \frac{\delta_i}{\delta_{iT_i}}\frac{B_{0T_i}}{B_0}(2J' - a) \tag{5.29}$$

and only in the latter case may T_i be "large". We would like to emphasis that eq.(5.29) must be satisfied by at least one modulus to obtain a stable solution. The v.e.v. of the dilaton and moduli fields depend on the value of the N=1 β-function, the dimension of the hidden gauge group and on α_i (see below). For a phenomenological interesting solution, i.e. $m_{3/2} \simeq 1TeV$, one has $\frac{Y}{b_0} \gg 1$, $B_0 \simeq (\frac{3Y}{2b_0})^2 \gg 1$ and $\frac{B_{0T_i}}{B_0}\frac{\delta_i}{\delta_{iT_i}} \simeq -\frac{\pi T_r}{3B_0} \ll 1$. Therefore, if eq.(5.29) is satisfied it will be for a unique value of α_i, say α, since the values of x_g, J and a in eq.(5.29) are fixed at the minimum. This means that only if

$\alpha_i = \alpha_j = \alpha$ it is possible that different moduli T_i and T_j have a large v.e.v. and their v.e.v.s will necessarily be the same (up to modular invariant transformations), i.e. $T_i = T_j$. We see that it is thus only possible to have a large overall moduli T if all three α_i have the same value, as in the case for a Z_3 or Z_7 Orbifold where all α_i vanish. For an Orbifold with two completely rotated planes then only one α_i is (possibly) different from zero in which case the v.e.v. for the moduli attached to this plane will be different from those of the other two. A squeezed orbifold may thus naturally be obtained. These Orbifolds are better candidates for a minimal string unification [36].

Solving eqs.(5.28) and (5.29) up to leading order, one obtains

$$B_0 = \frac{9}{2\gamma b_0^2}\left(1 + \frac{2\alpha - 1}{3\alpha - 1}\epsilon\right)^{-1} \tag{5.30}$$

and

$$\epsilon \equiv x_g ln(x_g)\,|_{min} = \frac{4b_0}{Y}(3\alpha - 1) \tag{5.31}$$

where B_0 is given in eq.(5.6) and x_g in the eq.(5.31) should be evaluated at the minimum so that ϵ is just a (field-independent) negative number. Since at the minimum $x_g < 1$ eq.(5.31) requires $\alpha < 1/3$ and the exponent of $T_r\,|\,\eta\,|^4$ in the gravitino mass must be negative.

From eqs.(5.30) and (5.31) it follows that the dominant term in B_0 is given by the contribution from v.e.v. of the auxiliary field of the dilaton h_s and one can approximate $B_0 \simeq (\frac{3Y}{2b_0})^2$. The v.e.v. of Y and T_i are then given in terms of the dimension of the hidden gauge group, its one-loop N=1 β-function coefficient and α by

$$Y = 8\pi\sqrt{\frac{1}{n_g}\left(1 + \frac{2\alpha - 1}{3\alpha - 1}\epsilon\right)^{-1}}, \tag{5.32}$$

$$\Sigma_i(1 - \alpha_i)T_{ri} = \frac{3Y}{\pi b_0} - \frac{6}{\pi}ln(Q). \tag{5.33}$$

Eq.(5.33) is obtained from eqs.(5.26) and (5.31) and the sum is over all moduli that acquire a v.e.v. different from the dual points and $Q \equiv \frac{9Y^3}{64b_0^2}\frac{\Pi_i T_{ri}^{(\alpha-1)} M_s^2}{x_g}$. If the gauge group is broken down from E_8 to a lower rank group such as $SU(N)$ with $5 \leq N \leq 9$ [58], as can be easily done by compactifying on an Orbifold with Wilson lines [77], a phenomenologically interesting solution can be obtained with only one gaugino condensate. On the other hand, the v.e.v. of the moduli fields does depend on α_i, the v.e.v.

of the dilaton and b_0. The value of α is approximately $-1/3$ (satisfying the condition from eq.(5.31)) if either the modular weight of the hidden matter fields is $n^i_{R_a} = -1/3$ (i.e. untwisted fields) or if the dominant term in the N=1 β-function coefficient for the hidden gauge group b_0 is given by the quadratic Casimir operator $C(G_a)$ (cf. eq.(3.67)), in which case (taking $\delta_{GS} = 0$)

$$\alpha_i = -\frac{b'_{0i}}{b_{0i}} = -\frac{1}{3}\left(1 - \frac{2}{C}\Sigma_{R_a} h_{R_a} T(R_a)(\frac{1}{3} + n^i_{R_a})\right) \simeq -\frac{1}{3}. \tag{5.34}$$

If we only consider the three diagonal (1,1) moduli (which are always present in Orbifold compactification) then, for a gravitino mass of order $1TeV$ and $\alpha = -1/3$, $ReT \simeq 8, 12, 22$ depending on whether three, two or one moduli get a "large" v.e.v., respectively. In the case discussed above of a large overall modulus, $ReT = ReT_1 = ReT_2 = ReT_3 \simeq 8$. Note that this result is quite different from the previously obtained values of $T \simeq 1.2$ [53]. Clearly the discrepancy is due to the inclusion of the strong gaugino binding effects in V_1 (cf. eq.(5.11)). Interesting enough, these large values are necessary in the minimal string unification in order to predict the correct values of the weak angle $sin\theta_w$ and strong coupling constant α_{strong} as will be seen in section 6.2.

We can now verify if the radiative corrections to the wave function of the dilaton are small. For a phenomenological interesting solution with $m_{3/2} = 1TeV$ one needs $Y/b_0 \simeq 58(1-\alpha)$ and defining the radiative corrections as $\Delta Y \equiv 2\alpha b_0 lnT_r$ (cf. eq.(5.11)) one has using eq.(5.33) that $\Delta Y/Y = |\frac{\alpha}{25(1-\alpha)} ln(150/\pi)| < 8/100$. The maximum value is taken when $\alpha_{max} = 1/3$, which is its biggest value allowed by eq.(5.31), and the radiative corrections correspond to only 8% of the tree level value.

Using eqs.(5.25), (5.30) and (5.31), the scalar potential can be written at the minimum as

$$V = \frac{2b_0}{Y}(\alpha - 1)\, B_0\, m^2_{3/2} \tag{5.35}$$

while the gravitino mass is proportional to Λ_c^2 (cf. eq.(5.24)and (5.26)),

$$m_{3/2} = \frac{3\sqrt{x_g}}{b_0 B_0}\,\Lambda_c^2, \tag{5.36}$$

and not to Λ_c^3 as is the usual assumption. It is the T dependence in eq.(5.24) that accounts for the difference. The proportionality factor in eq.(5.36) is of order 10^{-4} and one requires $\Lambda_c \simeq 10^{12}GeV$ for phenomenologically viable solutions with $m_{3/2} \simeq 1TeV$.

Since $\alpha < 1/3$ and $B_0 > 0$, the scalar potential (cf. eq.(5.35)) is $O(\Lambda_c^4)$ and negative allowing for a non-trivial solution, but with a cosmological constant many orders of magnitude larger than the experimental bounds. This problem is shared by all attempts to generate spontaneously supersymmetry breaking. However, we will show that it is possible to cancel the cosmological constant by introducing an extra term in the superpotential and fine tuning it. We postpone this discussion and that of the implications of the soft supersymmetry breaking terms to chapter 6.

It is also interesting to compare this result with the one obtained at tree level (cf. section 4.2). In this case the scalar potential is just $V_0 = m_{3/2}^2 B_0$ and a non-trivial solution is found for $B_0 < 0$. A negative B_0 is obtained if the square of the auxiliary fields of the dilaton and moduli are smaller than $3m_{3/2}^2$ and for vanishing h_S and h_T then, although the value of the scalar potential at the minimum is different from zero, supersymmetry is not broken. As mentioned before, the order parameter for broken supersymmetry is a non-vanishing auxiliary field because only in this case the fermion component of this auxiliary field transforms non-trivially under supersymmetry. This field becomes a Goldstino and it is "eaten" by the gravitino field which acquires two extra degrees of freedom giving rise to the well-known superhiggs effect. Thus, once we include the contribution from the gaugino-loops, the result changes drastically and a non-trivial minimum is found with supersymmetry broken mainly due to the non-vanishing of the auxiliary field of the dilaton.

5.5 Confinement masses

As discussed above, confinement effects may cause the spectrum determining one-loop effects to differ from that used in section 5.2 so we turn to a consideration of how these effects may alter our conclusions. Since we do not know the details of this spectrum we will assume that, as an average of these effects, the states will get a common (supersymmetric) mass m_{con} which we take it to be proportional to the condensation scale ($m_{con} = k\,\Lambda_c$). The gauge bosons have a mass $m_A = m_{con}$ while the gaugino mass becomes $m_g' = m_g + m_{con}$ with m_g given by eq.(5.20).

We can now minimize the scalar potential including these confinement effects. At the minimum the value of Y and $x'_g = m'^2_g/\Lambda_c^2$ are given by

$$Y \simeq \frac{8\pi}{\sqrt{ng\left(1 - k\,ln(\frac{1+k}{k})\right)}} \tag{5.37}$$

and

$$x_g \simeq \frac{4b_0}{Y}(3\alpha - 1)\frac{(1+k)(1 - k ln\frac{1+k}{k})}{1 - (1+k)ln\frac{1+k}{k})} \tag{5.38}$$

where we have assumed that $m_g \ll m_{con}$ in eq.(5.38) to get an analytic expression. We see from eq.(5.37) that the effects of the confinement mass is to shift (increase) the v.e.v. of Y. Clearly it is still possible to obtain phenomenologically interesting results for reasonable values of k (i.e. $k \sim O(1)$).

Chapter 6

Phenomenological Consequences of SUSY Breaking in 4D String Theory

In this chapter we will analyze the phenomenological consequences of the 4D string model with SUSY broken via a gaugino condensate. As studied in the preceding chapters we have obtained an effective low energy scalar potential and we have found stable solutions. We will now show that a large hierarchy can be expected to occur in the 4D string model.

The content of the chapter is the following. In sections one and two we discuss the gravitino mass which sets the splitting between scalar and fermions and the minimal string unification scheme, respectively. Since some of the moduli get a large v.e.v., we study some of the phenomenological consequences in section three. In section four we include matter chiral fields and show that by introducing a suitable superpotential it is possible to have vanishing scalar potential with SUSY spontaneously broken. Finally, we present in section five the soft supersymmetric terms derived from the Lagrangian and discuss the electroweak symmetry breaking.

6.1 Gravitino mass

The main purpose of studying supersymmetry was to explain the mass hierarchy problem. In SUGRA, the splitting between the scalars and fermions is of the same order as the gravitino mass. In the absence of an explicit mass term in the superpotential, i.e. $m = 0$ in eq.(2.28), the fermions remain massless even if SUSY is spontaneously broken because the fermion fields are protected by chiral symmetry. On the other hand, there is no symmetry protecting the scalar fields, and once SUSY is spontaneously broken they acquire a mass proportional to the gravitino mass (cf. eq.(5.24)).

A large mass hierarchy is obtained for $m_{3/2}/m_{Planck} \ll 1$, and to get the correct electroweak breaking it must be between [60]

$$m_{3/2} = 10^2 - 10^4\, GeV, \tag{6.1}$$

depending on the relation between $m_{3/2}$ with m_0 and $m_{1/2}$. In 4D string theory, the gravitino mass is not a free parameter but is given as a function of the dilaton and moduli fields by eq.(2.27), $m_{3/2}^2 = \frac{1}{4Y}\Pi_i(T_{ri}\,|\,\eta(T_i)\,|^4)^{-1}\,\Lambda_c^6$. It can be written as

$$\begin{aligned} m_{3/2}^2 &= \frac{1}{4Y}\, M_s^6 \Pi_i(T_{ri}\,|\,\eta(T_i)\,|^4)^{3\alpha_0^i - 1}\, e^{-3Y/2b_0} && (6.2)\\ m_{3/2}^2 &= \frac{1}{4Y} M_s^6 \left(\frac{16\,Y\,x_g}{\xi^2 M_s^2 B_0^2}\right)^{\frac{1-3\alpha_0}{1-\alpha_0}} e^{-Y/b_0(1-\alpha_0)} && (6.3)\end{aligned}$$

where we have eliminated in eq.(6.3) the T dependent part using eq.(5.26). To get a large mass hierarchy one needs the exponent term in eq.(6.3) to be large, i.e. $Y/b_0(1-\alpha_0) \gg 1$. Since the v.e.v. of Y is given by the dimension of the hidden gauge group, $Y = 8\pi/\sqrt{n_g}$ (cf. eq.(5.32)), the gravitino mass is determined once n_g, b_0 and α_0 are fixed. In any string vacua these quantities are fixed and the mass hierarchy will be predicted, but in practice there are a large number of consistent string vacua, and we are not yet able to choose between them. Thus we can consider the parameters n_g, b_0 and α_0 as free parameters and a wide range of values for these quantities give a small gravitino mass of order $1\,TeV$. As an example, we will choose an $SU(6)$ gauge group in the hidden sector with a one-loop β-function coefficient $16\pi^2 b_0 = 16$ and $\alpha_0 = 0.3$. The value of the gravitino mass, condensation scale and gauge coupling constant at the unification scale are

$$\begin{aligned} m_{3/2} &\simeq 82\,GeV, \\ \Lambda_c &\simeq 4.2 \times 10^{11}\,GeV \end{aligned} \tag{6.4}$$

and

$$g^{-2} = \frac{Y}{2} = 2.1,$$

while the value of the moduli fields depends on how many of them get a "large" v.e.v., i.e. different than the dual invariant points. The v.e.v.'s are given by solving the equation (5.33) or equivalently

$$\Pi_i T_{ri} \mid \eta(T_i) \mid^4 = \frac{\xi^3 B_0^3 m_{3/2}}{32\, Y\, x_g^{3/2}}. \tag{6.5}$$

In the case of having one, two or three moduli with "large" v.e.v.'s we find

$$< T_r/2 >=< ReT >= 21.7, 12.1, 8.7 \tag{6.6}$$

respectively (this example is summarized in table 6.1). Thus, we see that reasonable values of n_g, b_0 and α_0 can give a large mass hierarchy and gauge coupling constant at the unification scale. Furthermore, some of the moduli acquire a "large" v.e.v., a fact that is important when we discuss the minimal string unification scheme in the following section.

6.2 Minimal string unification

In this section we study the possibility of having unification of the gauge coupling constants assuming the minimal string unification scheme (MSU) [36, 56]. This scheme consists of having the chiral matter content in the visible sector of the theory to be that of the minimal supersymmetric model (MSSM) [3, 4] while the hidden sector remains unspecified. The unification scale and gauge coupling constant in string theory are field dependent quantities and they are only fixed once a string vacuum is determined. At tree level the v.e.v. of the moduli are small ($< T > \simeq 1$) and the unification scale is of order of the string scale. However, once loop corrections are included, the v.e.v.'s of the moduli can be much larger and the unification scale can be reduced to $O(10^{16})\,GeV$.

Recent precision measurements of the weak coupling at LEP and strong coupling constant have permitted a refinement of the analysis of the unification of the gauge coupling constants. The $SU(5)$ prediction is now ruled out but the supersymmetric extension is in good agreement with the experimental values [60].

The gauge coupling constants in the standard model (SM) are measured at the electroweak energy scale. Using the well-known renormalization group equation these couplings can be determined for any other energy scale. The gauge coupling constants have a logarithmic evolution with respect to energy and at the one-loop level they are given by

$$g_a^{-2}(\Lambda) = k_a g_0^{-2}(\Lambda_0) + 2b_a ln(\frac{\Lambda}{\Lambda_0}). \tag{6.7}$$

Λ_0 is the energy scale at which the gauge coupling constant g_0 is measured, g_a is the gauge coupling constant at the arbitrary scale Λ, and b_a is the one-loop beta function coefficient for the G_a gauge group while k_a is its corresponding Kac-Moody level. In the SM the beta function coefficients are given by $16\pi^2 b_3 = 7, 16\pi^2 b_2 = -19/6$ and $16\pi^2 b_1 = -41/6$ with $k_3 = k_2 = k_1 3/5 = 1$ for $SU(3), SU(2)$ and $U(1)$ respectively.

As mentioned above, it has recently been shown that the evolution of the gauge coupling constants in the SM do not become unified, i.e. they do not meet for any given energy. However, interestingly enough they do meet if one includes supersymmetry (SUSY) and assumes the minimal supersymmetric standard model (MSSM), i.e. the minimal supersymmetric extension to the SM. The main reason is that in the latter case the one-loop beta function coefficients change due to the inclusion of the extra states and they become $16\pi^2 b_3 = 3, 16\pi^2 b_2 = -1$ and $16\pi^2 b_1 = -11$ and the strong coupling constant becomes less asymptotically free. For these values of the one-loop beta function coefficient and assuming that the supersymmetric breaking scale M_{ss} is of order $10^{3\pm1}\, GeV$, the gauge coupling constants are unified at a value for the fine structure constant given by (in the minimal subtraction scheme $\overline{MS}$) [60]

$$\hat{\alpha}_{gut}^{-1} \equiv \left(\frac{g_{gut}^2}{4\pi}\right)^{-1} = 25.7 \pm 1.7 \tag{6.8}$$

and unification scale

$$\Lambda_{gut} = 10^{16\pm0.3}\, GeV. \tag{6.9}$$

Though in the MSSM one has three free parameters M_{ss}, g_{gut} and Λ_{gut} to predict three coupling constants, meaning that there will always be a solution to the unification of the gauge coupling constant, this fact does not mean that the unification scheme is empty because the values of the parameters M_{ss}, g_{gut} and Λ_{gut} are very much restricted by phenomenological constraints. In fact, one requires $M_{Planck} > \Lambda_{gut} > 10^{15}\,GeV$ to avoid a fast decaying proton, M_{ss} must be at the most of order 1 TeV to explain the mass hierarchy problem and larger then $100\,GeV$ to prevent light SUSY states that would have been detected already. Finally g^2_{gut} must be positive and small so that we stay in the perturbative regime.

As mentioned above, in string theory the three parameters of MSSM (M_{ss}, g_{gut} and Λ_{gut}) are no longer free but are given in terms of more fundamental quantities and are completely specified for any string vacuum. The supersymmetric scale which gives the splitting of the scalar and fermion states is mainly given by the gravitino mass $M_{ss} \simeq m_{3/2}$, while the unification scale is given as a function of the moduli and it is fixed once the v.e.v. of the moduli fields are obtained. In string theory the running of the couplings was presented in section 3.8 and it is given by (cf. eq.(3.63))

$$\frac{1}{g_a^2(\Lambda)} = \frac{k_a}{g_s^2} + b_a ln(\frac{\Lambda^2}{M_a^2}) + \Delta_a \tag{6.10}$$

where $b_a = \frac{1}{16\pi^2}(3C(G_a) - \Sigma_{R_a} h_{R_a} T(R_a))$ is the N=1 β-function coefficient and h_{R_a} the number of chiral fields in a representation R_a (the latin indices of the beginning of the alphabet $(a, b, ..)$ represent gauge indices while those in the middle of the alphabet $(i, j, ...)$ refer to the type of moduli T_i). M_a is the renormalization scale given in eq.(3.64) and the threshold term Δ_a is given in eq.(3.66). The o-index refers only to the hidden sector gauge group while the indices a and b correspond to the gauge groups of the visible sector, i.e. b_0 is the one-loop β-function coefficient of the hidden sector gauge group while b_a and b_b are the coefficients for the visible sector groups (for the SM groups).

Eq.(6.10) can be rewritten in a similar form as eq.(6.7)

$$g_a^{-2}(\Lambda) = k_a g_{gut}^{-2}(\Lambda^a_{gut}) + 2b_a ln(\frac{\Lambda}{\Lambda^a_{gut}}) \tag{6.11}$$

with the unification scale defined by

$$\Lambda^a_{gut} = M_s \Pi_i (T_{ri} \mid \eta(T_{ri}) \mid^4)^{\alpha^i_a/2} \tag{6.12}$$

where $\alpha^i_a \equiv \frac{\delta^i_{GS} k_a - b'^i_a}{b_a}$ (cf. eq.(3.67)) and the gauge coupling constant at the unification scale is given by

$$g^{-2}_{gut} = \frac{Y}{2}. \tag{6.13}$$

If two gauge coupling constants become equal, i.e. $\frac{g^2_a(\Lambda_{gut})}{k_a} = \frac{g^2_b(\Lambda_{gut})}{k_b} = g^2_{gut}(\Lambda_{gut})$, at the unification scale Λ_{gut} the coefficients defined in eq.(3.67) for different gauge groups must then be equal, i.e. $\alpha^i_a = \alpha^i_b$ and hence we will drop the a,b-index.

From eq.(6.12), we note that a unification scale smaller than one (i.e. in Planck mass units) necessarily requires the exponent to be positive, $\alpha > 0$ since the modular invariant function $T_{ri} \mid \eta_i \mid^4 < 1$. One can eliminate the Green-Schwarz term in eq.(6.12) and the unification scale becomes [56]

$$\Lambda_{gut} = M_s \Pi_i (T_{ri} \mid \eta(T_{ri}) \mid^4)^{\frac{b'^i_a - b'^i_b}{2(b_b - b_a)}} \tag{6.14}$$

and is completely specified once the v.e.v. of the moduli and the b_0 and b'_i coefficients are determined. A positive α or equivalently a positive exponent in eq.(6.14) forces the compactified space to have chiral matter fields with modular weights different than -1/3 (untwisted fields have modular weights -1/3) otherwise $b'^i_a = b_a/3$.

In any given string vacuum the gauge groups and particle spectrum are entirely determined and therefore the gauge coupling constant, unification scale and gravitino mass are not free parameters. In practice, there are a large number of consistent vacua having the standard model gauge group and three generations of particles and until now there is has been no procedure to choose one. To avoid this problem, we work in a model independent form by allowing the coefficients n_g, b_0 and α_0 to be free parameters from which the the gauge coupling constant, unification scale and gravitino mass are obtained. Again, it would seem that there is not any predictive power, since we are replacing the three parameters of the MSSM with three other ones. Yet, this point of view is not entirely fair since the new parameters can take only a limited number of discrete values and in principle they are not free at all, as argued above. One could further restrict the minimal string unification by demanding that the gauge coupling constants of the visible sector and of the hidden sector become unified at the unification scale. In what follows we will consider both possibilities; (i) that all gauge coupling constants are unified at the same scale and (ii) that only the gauge coupling constants of the SM are unified.

Let us consider the (i) case first. From eq.(6.12), we notice that if all gauge coupling constants are to be unified, then the α parameters of the hidden and visible sector must be the same, i.e. $\alpha_0^i = \alpha_a^i = \alpha_b^i$. From the minimization condition (cf. discussion of eq.(5.29)) we obtained that $\alpha_0^i = \alpha_0^j$ for those moduli with "large" v.e.v.'s and thus all α's must have the same value (for moduli with dual invariant v.e.v. the quantity $(T_{rk} \mid \eta(T_k) \mid^4)^{\alpha_0^k} \simeq 1$ and we will neglect it).

Assuming that the gauge group in the hidden sector is of rank less or equal eight (i.e. a subgroup of E_8) then $n_g \leq 248$ (the dimension of E_8), $0 < 16\pi^2 b_0 < 90$ and $0 < \alpha_0 < 1/3$. If the hidden sector is broken down to an $SU(N)$ subgroup (with $N \leq 9$) then the coefficients are restricted to take values in the range

$$0 < 16\pi^2 b_0 \leq 3N, \tag{6.15}$$

$$0 < \alpha_0 < 1/3 \tag{6.16}$$

and

$$n_g = N^2 - 1. \tag{6.17}$$

Thus we see that a very small set of values for n_g, b_0 and α are allowed and that the values of the MSSM parameters (cf. eq.(6.8-6.9)) that can be obtained is far from trivial. Furthermore, the structure constant at the unification scale is given by [61]

$$\hat{\alpha}_{gut}^{-1} = \frac{16\pi^2}{\sqrt{n_g}}. \tag{6.18}$$

To gain a better insight into the solution obtained in eqs(5.30-5.31) we will trade the one-loop beta function coefficient of the hidden gauge group for the gravitino mass and we will rewrite the unification scale in terms of $m_{3/2}, n_g$ and α_0. From eqs.(6.2) and (6.12) we have

$$\Lambda_{gut} = M_s \Pi_i (T_{ri} \mid \eta_i \mid^4)^{\alpha_0/2}$$

$$\Lambda_{gut} \;=\; M_s \left(\frac{4Y m_{3/2}^2}{M_s^6}\right)^{\frac{\alpha_0}{2(3\alpha_0-1)}} e^{-\frac{3Y}{4b_0}\frac{1}{(1-3\alpha_0)}} \tag{6.19}$$

and the b_0 dependence is in the exponent only. Using eq.(6.3), one can eliminate this dependence and we get [61]

$$\Lambda_{gut} = M_s \left[\frac{\xi^3 B_0^3 m_{3/2}}{32\, Y\, x_g^{3/2}}\right]^{\alpha_0/2} \tag{6.20}$$

with b_0 given by

$$b_0 = -\frac{3Y}{2}\left[ln\left(4Y\frac{m_{3/2}^2}{M_s^6}\left[\frac{\xi^3 B_0^3 m_{3/2}}{32Y x_g^{3/2}}\right]^{1-3\alpha_0}\right)\right]^{-1}. \tag{6.21}$$

The r.h.s. of eqs.(6.20) and (6.21) still depend on b_0 through x_g, ξ and B_0 (cf. eq.(5.6), (4.30) and (5.26)) but the unification scale depends on b_0 now only linearly. It is not possible to solve eq.(6.21) analytically for b_0, although its dependence on the r.h.s. is only logarithmic. By setting $16\pi^2 b_0 = 3N$ on the r.h.s., one obtains a good approximation.

From eqs.(6.8) and (6.18), a coupling constant consistent with the range of MSSM requires a gauge group with $33.2 < n_g < 43.4$, which fixes the hidden gauge group to be $SU(6)$ or $SO(9)$. For these gauge groups, the fine structure constant would be $\hat{\alpha}_{gut}^{-1} \simeq 26.7$ and 26.3 respectively. We can now plot $\Lambda_{gut}\, vs.\, \alpha$ for a fixed value of the gravitino mass. In fig.6.1 we show the graph for an $SU(6)$ gauge group with $m_{3/2} = 82\, GeV$. The unification scale has a minimum at around $\alpha_{min} = 0.3$ with a value of

$$\Lambda_{gut} \simeq 2.8 \times 10^{16}\, GeV. \tag{6.22}$$

It is remarkable that the minimum value for the unification scale is just about the value required by the minimal unification of couplings in the MSSM. The value of α_{min} does not depend on the gravitino mass. For decreasing value of the gravitino mass, the unification scale is also reduced as it is clear from eq.(6.20). If we require that $m_{3/2} > 35\, GeV$, then $16\pi^2\, b_0 > 14.8$. Since b_0 can only take discrete values, we have set $16\pi^2\, b_0 = 15$ which gives $m_{3/2} \simeq 82\, GeV$.

The value of the moduli for this specific example were given in eq.(6.6). It is precisely the fact that the v.e.v. of the moduli get a "large" v.e.v. that permits the

Figure 6.1: Unification scale as a function of α, keeping $m_{3/2}$ fixed.

unification scale to be much smaller then the string scale (note that for $T_r \simeq 1.2$ as obtained at tree level the unification scale will always be of the same order of magnitude as the string scale). Furthermore, the solutions to eq.(5.31) allow for squeezed Orbifolds which where found better candidates for minimal string unification.

We will now consider the case in which the hidden sector gauge coupling constant does not necessarily become unified with the gauge coupling constants of the standard model. This would be the case when the threshold corrections to the gauge coupling constant differ for the hidden and visible sector gauge groups. In this case, there is no connection between the α parameters of the hidden sector and the ones in the visible sector. Thus, the unification scale is

$$\Lambda_{gut} = M_s \left[\frac{\xi^3 B_0^3 m_{3/2}}{32\, Y\, x_g^{3/2}} \right]^{\bar{\alpha}/2} . \tag{6.23}$$

The term in square brackets is independent of α_a and $\bar{\alpha}$ is the average of the α_a^i whose corresponding moduli T_i get a "large" v.e.v., i.e. $\bar{\alpha} \equiv \frac{1}{n_l}\Sigma_i^{n_l}\alpha_a^i = \frac{1}{n_l}\Sigma_i^{n_l}\alpha_b^i$, where n_l is the number of moduli with "large" v.e.v. It is easy to see that in this case a unification scale consistent with MSSM is possible. The value of α_0 is restricted only to be smaller then $1/3$ and it can take negative values. As an example, we could consider a hidden gauge group with matter field in the untwisted sector only and taking $\delta_{GS} = 0$ one has $\alpha_0 = -1/3$. For this value of α_0 and choosing $\bar{\alpha} = 0.32$, one obtains a gravitino mass and a unification scale of

$$m_{3/2} \simeq 632\, GeV \tag{6.24}$$

$$\Lambda_{gut} \simeq 2 \times 10^{16}\, GeV. \tag{6.25}$$

In this example the v.e.v of the moduli are $T_r/2 = ReT = 22,\ 12.3,\ 9$ for one, two or three with "large" v.e.v. and $16\pi^2 b_0 = 11$ (see table 6.3). The same value of Λ_{gut} can be obtained for larger gravitino mass and $\bar{\alpha}$.

To conclude this section, we have shown that after finding a stable solution to the scalar potential including the contribution from gaugino binding effects, the v.e.v. obtained for the moduli and dilaton fields give the required values of MSSM for the gauge coupling constant and supersymmetry breaking scale (i.e. gravitino mass). The unification scale can also be consistent with MSSM for specific values of α. Furthermore, since the value of the fine structure constant at the unification scale is fixed by the dimension of the hidden sector gauge group this group must be $SU(6)$ or $SO(9)$.

6.3 Moduli phenomenology

We will now study some of the consequences of having moduli fields with "large" v.e.v.'s. As mentioned before string theory has only one free parameter which may be taken to be the Planck mass. In principle, all other quantities can be predicted. As seen in the last section the coupling constant at the unification scale itself and the unification scale are two examples of such predictions. We turn now to the predictions for the Yukawa couplings. These couplings can be moduli dependent and their value is fixed once the dependence and v.e.v. of the moduli are determined. Furthermore, there is the possibility, in these models, of the existence of a light scalar which could be responsible for a fifth force. This force will compete with gravity and could be experimentally detected.

The functional dependence of the Yukawa couplings upon the moduli can be explicitly calculated for specific Orbifolds [62], but a generic and useful way to determine these couplings is to use target space modular invariance. As discussed in chapter three, the Kahler potential

$$G = K_0 + K^i_i \,|\,\varphi_i\,|^2 + ln\frac{1}{4}\,|\,W\,|^2 \tag{6.26}$$

must be duality invariant. K_0 is the contribution from the dilaton and moduli fields (cf. eq.(4.38)) and [38]

$$K^i_i = \Pi_j \, a_j \, T_{rj}^{n^{ij}}, \tag{6.27}$$

$$W = W_0 + W_3 \tag{6.28}$$

where a_j is a constant (which is usually one), W_0 is given in eq.(4.39) and W_3 is the chiral superpotential. Modular invariance is achieved if the chiral superfield φ_i and superpotential transform as

$$\varphi_i \rightarrow (icT+d)_j^{n^{ij}} \varphi_i, \tag{6.29}$$

$$W \rightarrow \Pi_i \frac{W}{(icT+d)_i} \tag{6.30}$$

where n^{ij} is the modular weight of the chiral superfield φ_i with respect to the T_j moduli and the superpotential transform as a modular function with weight -1 for each moduli.

For generic Abelian Z_N and $Z_N \times Z_M$ Orbifolds, the modular weights of the chiral superfields vary in an absolute range of $-9 < n < 4$ depending on which gauge group and Kac-Moody level they correspond [36]. Knowing the transformation properties of the chiral matter fields one can easily derive the transformation property of the Yukawa couplings Y_{ijk} of the trilinear superpotential $W_3 = Y_{ijk}\varphi_i\varphi_j\varphi_k$ and it is given by [42, 43]

$$Y_{ijk} \rightarrow \Pi_p(icT + d)_p^{-(1+n_{pi}+n_{pj}+n_{pk})}\, Y_{ijk} \tag{6.31}$$

where n_{pi} is the modular weight of φ_i with respect to T_{rp}.

As an application of this results let us consider a possible light scalar fields. Such a field would be phenomenologically important because if it couples to ordinary matter, a fifth force mediated by the exchange of this particle will be generated which may compete with gravity. The range of the interaction is inversely proportional to the mass of the exchanged particle. Only if it has a small mass would the range of this interaction be long enough to be experimentally detected. A good candidate for a light scalar field is the imaginary part of the moduli T_i, ImT, whose corresponding real part gets a "large" v.e.v., because its mass is exponentially suppressed relative to the gravitino mass. Using the scalar potential given in eq.(5.25) one can calculate the mass for this field (we have taken into account the fact that ImT does not have canonical kinetic term)

$$m_{ImT} = 2\pi^2\sqrt{\frac{2}{3}}\,T_r^2\, e^{-\pi T_r/2}\, m_{3/2}. \tag{6.32}$$

Clearly, a small mass requires a large v.e.v. for T_r. Using the example presented in section one where $T_r = 44.5, 24.2, 17.3$ for $n_l = 1, 2, 3$ respectively (n_l counts the number of moduli with large v.e.v.) the mass in eq.(6.32) is $m_{ImT} = 2 \times 10^{-24}$, 2.4×10^{-11}, $4.1 \times 10^{-7}\, GeV$. Therefore a long range fifth force is only possible for $n_l = 1$ since for $n_l = 2, 3$ the range $r \leq 8 \times 10^{-4}\, cm$ is too small to be experimentally detected.

The interaction between ImT and matter fields is given by the Yukawa interaction

$$L_Y = \frac{1}{2}e^{K/2}\, h_{ijk}\varphi_i\bar{\chi}_j\chi_k \tag{6.33}$$

where χ_i is the fermion component of the φ_i superfield and we have used the same symbol for the superfield and its scalar component. Without going into any specific Orbifold example, one can deduce the functional dependence of the Yukawa couplings on the moduli, up to a modular invariant function, by simply imposing duality invariance. Using the duality transformation in eq.(6.31) one has

$$h_{ijk} = \Pi_p\, \eta(T_p)^{-2(1+n_{pi}+n_{pj}+n_{pk})}. \tag{6.34}$$

To compare the strength of the Yukawa interaction relative to gravity, we take the two fermion fields in eq.(6.33) to be quarks while the scalar one we take as the Higgs doublet. The relative strength is then given by

$$\frac{G_5}{G_N} = \frac{|\,\tilde{h}_{ijk} < H > |^2}{M_n^2} \tag{6.35}$$

where $\tilde{h}_{ijk} = \frac{1}{2}e^{K/2}\sqrt{(K^{-1})^i_i(K^{-1})^j_j(K^{-1})^k_k}\ h_{ijk}$ is the normalized Yukawa coupling[1] and M_n the nucleon mass. Taking $< H >= 300\,GeV$ and $M_n = 1\,GeV$, one finds an upper limit for the allowed Yukawa couplings

$$\tilde{h}_{ijk} \leq 10^{-4}, \tag{6.36}$$

in order not to contradict the present experimental limit $G_5/G_N \leq 10^{-3}$ [64]. Of course eq.(6.36) only restricts those Yukawa couplings that depend on the light ImT and in this case the modular weights of the chiral matter fields are constrained. Defining $\delta_i = T_{ri}\,|\,\eta(T_i)\,|^4$ as in eq.(5.28) the normalized Yukawa coupling can be written as

$$|\,\tilde{h}_{ijk}\,| = \frac{1}{2\sqrt{Y}}\,\Pi_p \delta_p^{-\frac{1}{2}(1+n_{pi}+n_{pj}+n_{pk})} \tag{6.37}$$

using eq.(6.27) and setting $a_i = 1$. The constraint equation (6.36) for the Yukawa couplings using eq.(6.5) gives [63]

$$\Sigma_p^{n_l}(1 + n_{pi} + n_{pj} + n_{pk}) \leq -2\,\frac{ln(2\times 10^{-4}\sqrt{Y})}{ln(\frac{\xi^3 B_0^3}{32 Y x_g^{3/2}}\, m_{3/2})} \tag{6.38}$$

where the sum now is over the moduli that acquire a "large" v.e.v. For a coupling constant at the unification scale of $g_{gut}^{-2} = 2.1$, consistent with MSSM, and $m_{3/2} = 147\,GeV$ eq.(6.38) gives

$$\Sigma_p^{n_l}(n_{pi} + n_{pj} + n_{pk}) \leq -2 \tag{6.39}$$

for one "large" moduli. This last equation indicates that ordinary matter fields must have negative modular weights. Considering an overall modular weight defined by $N_i = \Sigma_p n_{ip}$ the allowed modular weights of the fields must satisfy $N_i + N_j + N_p \leq -2$. For abelian Orbifolds with standard choice of Kac-Moody levels the allowed range for

[1] $\tilde{Y}$ is given at the compactification scale and should be run down to the electroweak scale. We do not worry about this because one expects the change to be small

an overall modular weights for the standard model fields is given by $-3 \leq n_{Q,U,E} \leq 0$ and $-5 \leq n_{L,D,H,\bar{H}} \leq 1$ [36]. It is interesting to note that the permitted values of the modular weights from a fifth force consideration given in eq.(6.39) lie within the range of allowed values for the standard model fields and that this range is further restricted. For any particular Orbifold model, the modular weights of ordinary matter must satisfy eq.(6.39) for those Yukawa couplings that depend on moduli with "large" v.e.v. and this restricts the possible string vacua.

6.4 Matter superpotential

Up to now we have only considered the scalar potential for gauginos in the hidden sector, dilaton and moduli fields. We would now like to include the contribution from matter superfields. We will show in this section how the interaction between the gaugino bilinear and the matter fields stabilizes the scalar potential for vanishing v.e.v. of the matter fields up to radiative corrections in the visible sector. To compare the tree level potential with the scalar potential including the gaugino radiative corrections we will arrange the terms in powers of the gravitino mass. It will then be easy to see the main differences. As a consequence of the gaugino binding we will show that the soft trilinear terms are suppressed by a factor of 10^{-2} with respect to the tree level ones and we will calculate the supersymmetric masses for scalar fields and gaugino masses in the observable sector.

In general the matter superpotential is given as a power series expansion in the chiral superfields φ suppressed by the Planck mass [38]. In the absence of linear and quadratic terms, the leading term in the superpotential is cubic in the matter superfields. In what follows, it is assumed that the matter fields get at most a small v.e.v. compared to the Planck mass, to be consistent with the Kahler expansion in eq.(3.60), i.e. $|\varphi_i|^2 K_i^i(T,\bar{T}) < 1$. The gaugino condensate term W_0 gives the dominant contribution to the scalar potential and the v.e.v. of the dilaton and moduli fields obtained in chapter 5 remain valid.

For a generic matter superpotential W_m, using the Kahler potential of eq.(3.59), the tree level scalar potential is

$$V_0 = \frac{1}{4}e^K \left((K^{-1})^S_S \mid F_S W_0 + K_S W_m \mid^2 + \Sigma_i (K^{-1})^i_i \mid F_i W_0 + \beta_i \mid^2 - 3 \mid W_0 + W_m \mid^2\right) \tag{6.40}$$

where

$$\begin{aligned} F_S &= K_S - \frac{3}{2b_0} f_S = -\frac{1}{Y}(1 + \frac{3Y}{2b_0}), \\ F_i &= K_i + \frac{W_{0,i}}{W_0} - \frac{3}{2b_0} f_i \\ \beta_i &= K_i W_m + W_{m,i}, \end{aligned}$$

b_0 is the one-loop beta function coefficient for the hidden gauge group, f_i is the derivative of the gauge kinetic function with respect to the i-field and the index i runs over all matter and moduli fields. The first term in eq.(6.40) is just the square of the auxiliary field of the dilaton h_S, while the second term is a sum of the squares of the auxiliary fields of the moduli h_{Ti} and chiral matter fields h_{φ_i}.

To determine the vacuum structure, one has to include quantum corrections and the main contribution to the one-loop potential is given by the gaugino-loops. We would like to emphasize again that, although we are just calculating the one-loop potential, the final result is necessarily non-perturbative, since by solving the mass gap equation one is stabilizing the tree level potential with the one-loop potential, and is effectively summing an infinite number of gaugino-bubbles. One can easily determine the contribution of these loops by calculating the gaugino mass at tree level. Since all dependence on the gaugino bilinear in eq.(6.40) is given in W_0 one has,

$$m_g = \frac{b_0}{6Ref} e^{K/2} H \tag{6.41}$$

with

$$H \equiv F^S (K^{-1})^S_S (F_S W_0 + K_S W_m) + F^i (K^{-1})^i_i (F_i W_0 + \beta_i) - 3(W_0 + W_m). \tag{6.42}$$

The one-loop potential, considering the contribution from the gauginos-loops only, is (cf. eq.(5.11))

$$\begin{aligned} V_1 &= -\gamma \Lambda_c^4 J(x_g) \\ V_1 &= -\gamma \Lambda_c^4 (2x_g + x_g^2 ln(x_g)) \end{aligned} \tag{6.43}$$

where we have expanded the function J since, as we showed in section (4), $x_g = O(10^{-2})$ (cf. eq.(5.31)) and it is now given by

$$x_g = \frac{m_g^2}{\Lambda_c^2} = \frac{b_0^2\, e^K}{36\,\Lambda_c^4} \mid H \mid^2. \tag{6.44}$$

If $W_m = W_{m,i} = 0$ then $H = B_0 W_0$ and eq.(6.44) reduces to eq.(5.26), as should be the case. The complete scalar potential $V = V_0 + V_1$ is then

$$\begin{aligned} V = \frac{1}{4}e^K[(K^{-1})^S_S \mid F_S W_0 + K_S W_m \mid^2 + \Sigma_i (K^{-1})^i_i \mid F_i W_0 + \beta_i \mid^2 \\ - 3 \mid W_0 + W_m \mid^2 - \frac{2\gamma b_0^2}{9} \mid H \mid^2 (1 + \frac{1}{2} x_g ln(x_g))]. \end{aligned} \tag{6.45}$$

To obtain a more transparent expression for V, one can expand the scalar potential in powers of the gravitino mass ($m_{3/2}^2 = \frac{1}{4}e^K \mid W_0 \mid^2$). Making use of the solutions to the extremum equations eq.(5.30) and eq.(5.31) the scalar potential is [58]

$$V = m_{3/2}^2 B + (m_{3/2} A\, +\, h.c.) + C \tag{6.46}$$

with

$$\begin{aligned} B &= \left(F_S (K^{-1})^S_S F^S + F_i (K^{-1})^i_i F^i - 3\right) \left(\frac{2\alpha - 1}{3\alpha - 1}\epsilon - \frac{1}{2} x_g ln(x_g)\right), \qquad (6.47) \\ A &= \frac{1}{2}e^{K/2}\left(F^i (K^{-1})^i_i \beta_i + F^S (K^{-1})^S_S K_S W_m - 3W_m\right)\left(\frac{2\alpha - 1}{3\alpha - 1}\epsilon - \frac{1}{2} x_g ln(x_g)\right) \end{aligned}$$

and

$$\begin{aligned} C = \frac{1}{4}e^K (\beta^i (K^{-1})^i_i \beta_i - 2 \mid W_m \mid^2 \\ - \frac{1 + \frac{2\alpha - 1}{3\alpha - 1}\epsilon - \frac{1}{2}x_g ln(x_g)}{B_0} \left|F^S (K^{-1})^S_S K_S W_m - 3W_m + F^i (K^{-1})^i_i \beta_i\right|^2). \end{aligned}$$

It is worth comparing the expressions for A, B and C with those ones obtained at tree level. Defining A_0, B_0 and C_0 in a similar way as in eq.(6.47) one has

$$\begin{aligned} B_0 &= (F_S (K^{-1})^S_S F^S + F_i (K^{-1})^i_i F^i - 3), \\ A_0 &= \frac{1}{2}e^{K/2}(F^i (K^{-1})^i_i \beta_i + F^S (K^{-1})^S_S K_S W_m - 3W_m) \end{aligned} \tag{6.48}$$

and

$$C_0 = \frac{1}{4}e^K (\beta_i (K^{-1})^i_i \beta^i - 2 \mid W_m \mid^2).$$

One can easily see that A and B are suppressed by a factor of $\frac{2\alpha-1}{3\alpha-1}\epsilon - \frac{1}{2}x_g ln(x_g)$, which at the minimum it is just $\frac{1}{2}\frac{\alpha-1}{3\alpha-1}\epsilon = \frac{2b_0}{Y}(\alpha-1) = O(10^{-2})$, relative to A_0 and B_0, respectively. For the C term keeping the leading terms only, one has $C \simeq \frac{1}{4}e^K((K^{-1})^i_i\beta^i\beta_i - 3\mid W_m\mid^2)$ which differs from C_0 by $-\frac{1}{4}e^K\mid W_m\mid^2$.

The main contribution to the soft trilinear term A comes from h_S (i.e. from the $F_S = O(10^2)$ term) and the suppression factor in A and B is a consequence of canceling between the tree level and one loop diagrams. The field dependent term $m_g^4 ln(m_g^2)$ in V_1 (i.e. $x_g ln(x_g)$ in A and B) is essential in stabilizing the scalar potential for vanishing v.e.v.'s of the matter fields as will be shown in the next section.

Cosmological constant and stability for matter superfields

As we showed in section 5.4, in the absence of a matter superpotential, the scalar potential has a negative cosmological constant of order Λ_c^4, which is many orders of magnitude larger than experimental bounds. Here, we would like to show that it is possible to cancel the cosmological constant by a suitable choice of matter superpotential and we will also study the extremum solutions and stability of the scalar potential for matter superfields.

Let us begin with the study of the stability for the matter superfields. As was mentioned before the Kahler potential (cf. eq.(3.60)) was derived assuming small v.e.v. of the chiral matter superfields. Here we will show that $\varphi_i = 0$ is a stable solution to the scalar potential assuming that the superpotential is at least cubic in these fields.

If the gauge coupling constant (cf. eq.(4.25)) is independent of the chiral matter superfields, the condensation scale Λ_c and W_0 will be independent of these fields (here we will assume that this is the case, otherwise the generalization is straightforward) and the scalar potential will depend upon the chiral matter fields through the Kahler potential K and superpotential W_m only. Expanding the term $x_g ln(x_g)$ in powers of φ and keeping the leading term only ($x_g ln(x_g) \simeq (1 + K_i^i\mid\varphi_i\mid^2)\epsilon$) the expressions for A, B and C are

$$B \;=\; \frac{1}{2}\left(F_S(K^{-1})^S_S F^S + K_i(K^{-1})^i_i K^i - 3\right)\left(\frac{\alpha-1}{3\alpha-1} - K_i^i\mid\varphi_i\mid^2\right)\epsilon, \quad (6.49)$$

$$A = \frac{1}{4}e^{K/2}\left((K^{-1})^i_i K^i(K_i W_m + W_{m,i}) + F^S(K^{-1})^S_S K_S W_m - 3W_m\right) \times\left(\frac{\alpha-1}{3\alpha-1} - K^i_i \mid \varphi_i \mid^2\right)\epsilon$$

and

$$C = \frac{1}{4}e^K\left((K^{-1})^i_i \mid K_i W_m + W_{m,i}\mid^2 - 3\mid W_m\mid^2\right).$$

It is simple to see that, for a cubic superpotential, a zero v.e.v. for the matter fields corresponds to an extremum solution ($W_m = W_{m,i} = W_{m,i,j} = K_i = 0$). To determine whether it is a stable solution we expand the scalar potential in powers of φ_i. The leading quadratic term is

$$-\frac{1}{2}K^i_i\, B_0\,\epsilon\, m^2_{3/2} \mid \varphi_i \mid^2 \tag{6.50}$$

(using the fact that $B_0 \simeq F_S(K^{-1})^S_S F^S \simeq \epsilon^{-2} \simeq O(10^4)$) and eq.(6.50) is positive since $\epsilon < 0$. Note that e^K factors out in V eq.(6.45), and since the φ_i expansion of this term gives a contribution proportional to the cosmological constant, which (see below) we will arrange to vanish, we have not included it. Nevertheless, the inclusion of this term would not spoil the stability of the potential for $\alpha < 0$, since in this case the leading quadratic term is $\frac{\alpha}{1-3\alpha}K^i_i\, B_0\,\epsilon\, m^2_{3/2}\mid\varphi_i\mid^2$.

Thus, contrary to previous analyses we find that the scalar potential is indeed stable for $\varphi_i = 0$ [58]. Again the discrepancy lies in our inclusion of the radiative contribution from the strong gaugino binding for it is the $m_g^4 ln(m_g^2)$ term in V_1 that gives the leading contribution to the scalar mass and stabilizes the scalar potential in this direction. It is important to mention that, neglecting radiative effects due to the visible sector, the scalar potential is stable for vanishing v.e.v. of the matter fields. However, it is still possible to have the usual electroweak radiative breaking due to the top Yukawa coupling.

We would like now to study the possibility of cancelling the cosmological constant by introducing a suitable superpotential [58]. Although it is not clear how such a term arises, it is nevertheless interesting to see that it is in principle possible to have vanishing cosmological constant with supersymmetry broken at a phenomenologically realistic value. To achieve this we introduce a linear term in the superpotential

$$W_m = c\,D \tag{6.51}$$

where c is a constant that will be fine tuned to give zero cosmological constant and D is a chiral superfield. It is clear from eq.(6.47) that in this case a zero v.e.v. for D is not an extremum solution due to the linear term $F_S W_m$ in the soft supersymmetric term A. In fact, the coefficients A, B and C are now

$$\begin{aligned} B &= \frac{1}{2}\left(F_S(K^{-1})^S_S F^S + K_i(K^{-1})^i_i K^i + K^D_D \,|\, D\,|^2 - 3\right)\left(\frac{\alpha-1}{3\alpha-1} - K^D_D \,|\, D\,|^2\right)\epsilon, \\ A &= \frac{1}{4}e^{K/2}\left(D(K^D_D \,|\, D\,|^2 c + c) + F^S(K^{-1})^S_S K_S c D - 3cD\right)\left(\frac{\alpha-1}{3\alpha-1} - K^D_D \,|\, D\,|^2\right)\epsilon \end{aligned}$$

and

$$C = \frac{1}{4}e^K\left((K^{-1})^i_i \,|\, K_i W_m + W_{m,i}\,|^2 + (K^D_D)^{-1}\,|\, K^D_D \,|\, D\,|^2 c + c\,|^2 - 3\,|\, cD\,|^2\right) \tag{6.52}$$

where the i-index in eqs.(6.52) runs over all chiral superfields but for the D one. The leading contribution from the superpotential of eq.(6.51) is through the auxiliary field of D ($h_D = \frac{1}{2}e^{K/2}(K_D W + W_D)$). This contribution is semipositive definite and for vanishing v.e.v. of D the only term that survives is the one proportional to $|\, W_{m,D}\,|^2 = c^2$ and the one-loop potential is independent of it. It is in fact this term that gives the main contribution to the scalar potential even for non-vanishing v.e.v. of D. Since it is positive it allows for the cancellation of the cosmological constant by fine tuning[2] c. Since the value of h_D^2 is given by the difference between the tree level potential (which is mainly given by h_S^2) and the one-loop potential one necessarily has that $h_S \gg h_D$. This is important because then the masses of the scalar and the gauginos of the visible sector are insensitive to the details of h_D, relaying only on the fact that the cosmological constant vanishes.

The values of c and D that give a zero cosmological constant and solve the extremum equation of V (assuming that $K^i(K^{-1})^i_i K_i = 3$), corresponding to a stable solution,

[2] As a first impression one might think that the v.e.v. of D is such that it makes its auxiliary field vanish but this is not the case because the leading linear term in D is given by $F_S W_m$ coming from the auxiliary field of the dilaton instead of the one coming from h_D

are [58]

$$c^2 = -\frac{1-\alpha}{2(1-3\alpha)} K_D^D \, F_S (K^{-1})_S^S F^S \epsilon \,|\, W_0 \,|^{\,2} \tag{6.53}$$

and

$$K_D^D \,|\, D \,|^{\,2} = -\frac{(1-\alpha)^3}{(1-3\alpha)(3-5\alpha)^2} \frac{\epsilon}{2}. \tag{6.54}$$

Note that $K_D^D \,|\, D\,|^{\,2}$ is smaller than one, satisfying the Kahler expansion condition. From eqs.(6.53) and (6.54) and using that $\epsilon \, 3Y/2b_0 = 6(3\alpha - 1)$ the cD term in the superpotential W is different than zero and it is given by

$$cD = 3\frac{(1-\alpha)^2}{(3-5\alpha)} W_0 \tag{6.55}$$

and thus the superpotential W becomes

$$\begin{aligned} W &= W_0 + cD \\ W &= \frac{(3\alpha-2)(\alpha-3)}{(3-5\alpha)} W_0. \end{aligned} \tag{6.56}$$

Thus, we have seen that it is possible to cancel the cosmological constant by introducing a linear term cD in the superpotential and fine tuning c. We have also justified, a posteriori, not taking into account the term proportional to V in calculating the leading quadratic term in eq.(6.50).

6.5 Soft supersymmetric terms

We now consider the prediction for the soft SUSY breaking terms in the low energy Lagrangian. The phenomenological implications of supersymmetric extensions of the standard model depend sensitively on the nature of the supersymmetry breaking effects that must be included to give a viable theory. These effects can be parameterized through the introduction of the soft SUSY breaking terms which arise if supersymmetry breaking occurs spontaneously and which need not spoil the supersymmetric solution to the hierarchy problem. In supergravity theories these terms may be related in a model independent way if supersymmetry breaking occurs in the hidden sector with coupling to the visible sector via gravitational effects only.

The soft supersymmetric terms are the common supersymmetric scalar mass m_0, the gaugino masses in the visible sector $m_{1/2}$, the soft SUSY trilinear term A_t and the μ term associated with the higgsino masses. The MSSM is given by the supersymmetric extension to the standard model with the inclusion of the soft supersymmetric terms which break SUSY explicitly, but do not give any quadratic divergences. These terms may appear naturally in the context of a more fundamental theory as is the case of 4D string theory. However, for the electroweak breaking to take place and not to spoil the hierarchy solution, the different soft SUSY parameters have to satisfy some phenomenological constraints.

The MSSM is defined by the global SUSY superpotential [3]

$$W = h_t(UU^cH_2^0 - D'U^cH_2^+) + \mu(H_1^0H_2^0 - H_1^-H_2^+) \tag{6.57}$$

where, for simplicity, we have shown only the term associated with the top Yukawa couplings, h_t. The H_1 and H_2 are the two doublets needed in MSSM to give a mass to the standard model particles while the fermion components of the U and D' superfields are given by the *up* type quark and the *down* type quark, respectively. In addition to the terms obtained from eq.(6.57) one has the soft supersymmetric terms and the final form of the effective global SUSY potential is

$$\begin{aligned} V_{eff} &= m_1^2 \mid H_1 \mid^2 + m_2^2 \mid H_2 \mid^2 + [h_tA_t(UU^cH_2^0 - D'U^cH_2^+) + h.c.] \qquad (6.58) \\ &+[B\mu(H_1^0H_2^0 - H_1^-H_2^+) + h.c.] + m_0^2 \mid \varphi \mid^2 + \frac{\frac{3}{5}g_1^2 + g_2^2}{8}(\mid H_1^0 \mid^2 - \mid H_2^0 \mid^2)^2 \end{aligned}$$

where we have set all scalar masses to be equal and φ refers to the scalar components associated to the quarks and leptons. The last term in eq.(6.58) comes from the auxiliary D field associated to the gauge scalar superfield (cf. eq.(2.18)).

The masses of the two Higgs, taking $B = 0$ in eq.(6.58), are given by

$$m_1^2 = m_{H_1}^2 + \mu^2 \tag{6.59}$$

$$m_2^2 = m_{H_2}^2 + \mu^2 \tag{6.60}$$

with

$$m_{H_1}^2(\Lambda_{gut}) = m_{H_1}^2(\Lambda_{gut}) = m_0^2.$$

At the unification scale both masses are positive, but running them down to the electroweak scale one of them can become negative, triggering electroweak symmetry breaking. This is possible because the masses evolve differently as can be seen from the renormalization group equations [65]

$$\frac{dm_{H_i}^2}{dln(Q)} = \frac{1}{8\pi^2}(-3M_2^2 g_2^2 - \frac{3}{5}M_1^2 g_1^2 + 3h_t^2 \delta_{i2}(m_{\tilde{t}}^2 + m_{\tilde{t}c}^2 + m_{H_2}^2 + A_t^2)) \tag{6.61}$$

where the M_i are the gaugino masses and the tilde denotes supersymmetric state. For $i = 1$ in eq.(6.61) the term proportional to the top Yukawa coupling vanishes but for $i = 2$, if the top Yukawa coupling is large enough, it can drive m_2^2 negative. With the usual definitions

$$\frac{v_2}{v_1} = tan(\beta), \tag{6.62}$$

$$m_3^2 = B\mu, \tag{6.63}$$

$$sin2\theta = \frac{2B\mu}{m_1^2 + m_2^2} \quad (\theta = \frac{\pi}{2} - \beta) \tag{6.64}$$

we have

$$M_Z^2 = \frac{m_1^2 - m_2^2 - (m_1^2 + m_2^2)cos2\theta}{cos2\theta} \tag{6.65}$$

where the running masses are evaluated at M_z.

An acceptable electroweak breaking is achieved if the following conditions are satisfied [3, 4]:

i) The potential must be unbounded from below

$$\begin{aligned} 2\,|\,m_3\,|^2 &< m_1^2 + m_2^2 \\ i.e.\ |\,sin2\theta\,| &< 1 \end{aligned} \tag{6.66}$$

ii) Colour should be unbroken [70]

$$A_t^2 < 3(3 + \frac{\mu^2}{m_0^2}) \tag{6.67}$$

iii) Electroweak symmetry must be broken (i.e. $SU(2) \times U(1) \rightarrow U(1)_{em}$)

$$(m_1^2 m_2^2 < \,|\,m_3\,|^4)\,|_{M_Z} \tag{6.68}$$

iv) The top mass must be in the range $100\,GeV < m_T < 180\,GeV$ [71].

The first two conditions apply at all energy scales and the last two should be valid at the electroweak scale. From an experimental point of view the lower bounds on the supersymmetric particle masses are [66]

$$\begin{aligned} M_3 > 79\,GeV \quad &, \quad M_\chi > 45\,GeV, \\ m_{\tilde{q}} > 74\,GeV \quad &, \quad m_{\tilde{l}} > 45\,GeV, \end{aligned} \tag{6.69}$$

where M_3 is the gluino mass, χ is the lightest chargino, and $\tilde{q}, \tilde{l}$ denote the squarks and sleptons respectively.

As a matter of completeness, one should also require M_Z not to be too sensitive with respect to the different soft supersymmetric parameters. Otherwise there would be a fine tuning between these parameters in order to get the correct mass for the Z boson. This condition is conveniently introduced by demanding that [67]

$$\left| \frac{a_i}{M_Z^2} \frac{\partial M_Z^2(a_i, h_t)}{\partial a_i} \right| < \Delta \tag{6.70}$$

where a_i are the soft SUSY parameters. Δ measures the allowed degree of fine tuning needed; for $\Delta = 10$ the value of a_i must be fine tuned to $1/10$ of the Z mass.

The soft SUSY terms, as mentioned above, are free parameters in the MSSM and, interesting enough, there are solutions that satisfy all conditions including the constraints from gauge coupling unification as discussed in section 6.2. On the other hand, in 4D superstring theory these terms are no longer free and one cannot fix them arbitrarily. In section 6.2 we discussed predictions in 4D string theory for the gauge coupling constant at the unification scale, the unification scale and the supersymmetry breaking mass derived from strings. Now we will determine the predictions for the soft SUSY terms. As mentioned above these soft SUSY terms are: the common supersymmetric breaking mass m_0, the gaugino masses in the visible sector $m_{1/2}$, the trilinear term A_t and the μ term.

Common supersymmetric breaking mass m_0

The common supersymmetric mass is obtained by simply taking the second derivative of the scalar potential (6.45) with respect to the scalar fields. In the absence of explicit mass terms (quadratic terms in the superpotential), the scalar fields have a vanishing v.e.v. as shown in section 6.4 and the leading quadratic term in the scalar potential is given by expanding the Kahler potential K. From eq.(6.50) the leading quadratic term gives a mass

$$m_0 = \sqrt{-\frac{\epsilon B_0}{2}\frac{1}{4}e^K \mid W_0 \mid^2} \tag{6.71}$$

where we have included a $(K^{-1})^i_i$ factor to account for the noncanonical kinetic terms of the scalar field. If we express W_0 in terms of W using eq.(6.56) then eq.(6.71) becomes

$$m_0 = \sqrt{-\frac{\epsilon B_0}{2}\frac{(3-5\alpha)}{(3\alpha-2)(\alpha-3)}} m_{3/2}. \tag{6.72}$$

The auxiliary field of the dilaton is much bigger then the auxiliary field of D. Therefore, the scalar mass is only sensitive to the fact that one has no cosmological constant and not to the details of the cancelation of the vacuum energy.

Gaugino masses in the visible sector

The gaugino masses in the visible sector are obtained from the auxiliary field h_i of the chiral superfield given by eq.(2.42)

$$h_i = -e^{G/2}G_i + \frac{f_i}{4}\bar{\lambda}_R\lambda_L + (\frac{1}{2}G_i^k G^j - G_i^{kj})\bar{\chi}_{kR}\chi_{jL}. \tag{6.73}$$

To have a non-vanishing gaugino mass the gauge kinetic function must be non-trivial, $f_i \neq 0$. In 4D strings, f depends on the dilaton and the moduli fields and clearly the leading mass term will then come from the auxiliary field of the dilaton since $h_S \gg h_T$. Taking into account the fact that the gauginos do not have canonical kinetic terms the gaugino masses are

$$\begin{aligned} m_{1/2} &= \frac{2}{(Ref)_{vis}}\frac{1}{4} f_S(K^{-1})^S_S h^S[1-(1+\frac{\alpha}{1-3\alpha}\epsilon)] && (6.74)\\ m_{1/2} &= -6\alpha\frac{1}{2}e^{K/2}W_0. && (6.75)\end{aligned}$$

The first term in eq.(6.74) is the contribution from the tree level potential while the second term comes from the one-loop potential. Both equations are given to leading order in ϵ and for $\alpha = 0$ the mass would be of order $\epsilon\, m_{3/2}$. If we express eq.(6.75) in terms of W one has

$$m_{1/2} = -6\frac{\alpha(3-5\alpha)}{(3\alpha-2)(\alpha-3)}\, m_{3/2} \tag{6.76}$$

We have found that the gaugino masses for the different gauge groups of the visible sector are equal at the unification scale as expected because SUSY is mainly broken through the auxiliary field of the dilaton.

Trilinear terms A_t

The trilinear term, as defined in eq.(6.58), is given by

$$L_t = A_t\, m_{3/2}\, \tilde{h}_{123}\, \phi_1\phi_2\phi_3 \tag{6.77}$$

where $\tilde{h}_{123}$ is the physical Yukawa coupling and ϕ_i, $i = 1,2,3$ are arbitrary scalars with canonical kinetic terms. The physical Yukawa coupling is given by $L_{Yuk} = \tilde{h}_{123}\, \phi_1\bar{\chi}_2\chi_1$ where χ_i is the corresponding fermion component of ϕ_i and in 4D string theory one has $L_{Yuk} = \frac{1}{2}e^{K/2}\sqrt{\Pi_{i=1}^{3}(K^{-1})_i^i}\; h_{123}\, \phi_1\bar{\chi}_2\chi_3$ where the square root of K's is needed in order to have correctly normalized fields. The physical Yukawa coupling is then

$$\tilde{h}_{123} = \frac{1}{2}e^{K/2}\sqrt{\Pi_{i=1}^{3}(K^{-1})_i^i}\; h_{123}. \tag{6.78}$$

From eq.(6.48), assuming that one has only three moduli so that $K_i(K^{-1})_i^i K^i = 3$, the contribution to the trilinear term comes from the A and C terms. Eq.(6.77) becomes

$$L_t = \frac{1}{2}e^{K/2}\left[\frac{3Y(\alpha-1)}{4b_0(3\alpha-1)}\epsilon W_0 + cD(1+N+K^T(K^{-1})_T^T\frac{h_{123,T}}{h_{123}})\right]\tilde{h}_{123}\,\phi_1\phi_2\phi_3. \tag{6.79}$$

where N is the degree of the superpotential (in this case one has $N = 3$) and thus the trilinear term is

$$A_t = \frac{3(\alpha-1)}{(3\alpha-2)(\alpha-3)}\left[(3-5\alpha) + \frac{(\alpha-1)}{2}(1+N+K^T(K^{-1})_T^T\frac{h_{123,T}}{h_{123}})\right]. \tag{6.80}$$

with $N = 3$. In order not to have broken colour charge condition (ii) must be satisfied and the value of α is restricted to be positive, $0 < \alpha$, if we do not consider the contribution from the cD term used to cancel the cosmological constant (cf. eq.(6.51)) since in this case $A_T = 3(\alpha - 1)$. Otherwise eq.(6.80) requires $-3.5 < \alpha < 1/3$ which does not impose any heavy constraints since we expect $-1 < \alpha < 1/3$.

B term

Another soft SUSY term is the so called B-term. This term is defined by [69]

$$L_B = -m_{3/2}\, B\, \mu H_1 H_2 \tag{6.81}$$

where H's are the Higgs fields. This term could arise through various different mechanisms. One possibility is that there is a bilinear term $\mu H_1 H_2$ in the superpotential, as in MSSM term. In this case B is given by a similar expression as A_t in eq.(6.80), but with $N = 2$ and h_{123} replaced by μ. Taking μ independent of T we get

$$B' = \frac{3(\alpha-1)(3-7\alpha)}{2(3\alpha-2)(\alpha-3)}. \tag{6.82}$$

Another possibility is that the μ term can originate from a term in the Kahler potential proportional to $H_1 H_2$ [69]. After supersymmetry breaking this generates the $\mu H_1 H_2$ term in the superpotential with $\mu \propto m_{3/2}$. In this case the soft term is of the form of eq.(6.81) with

$$\begin{aligned} B'' &= \left(\frac{m_0}{m_{3/2}}\right)^2 \\ B'' &= \left(\frac{(3-5\alpha)}{(3\alpha-2)(\alpha-3)}\right)^2 \frac{\epsilon B_0}{2}. \end{aligned} \tag{6.83}$$

Yet another suggestion is that the μ term comes from a term in the superpotential proportional to $W_0 H_1 H_2$. In this case we find

$$\begin{aligned} B''' &= \frac{(\alpha-1)}{(3\alpha-1)} \left(\frac{m_0}{m_{3/2}}\right)^2 \\ B''' &= \left(\frac{(3-5\alpha)}{(3\alpha-2)(\alpha-3)}\right)^2 \frac{(\alpha-1)}{(3\alpha-1)} \frac{\epsilon B_0}{2}. \end{aligned} \tag{6.84}$$

This completes the derivation of the soft SUSY terms arising from gaugino condensation. It is illuminating to compute them for our realistic example of section 6.1. In this case one has an $SU(6)$ gauge group with $b_0 = 15/16\pi^2$ and a gravitino mass $m_{3/2} = 82\, GeV$. The soft supersymmetric terms are then (this example is summarized in table 6.1)

$$m_0 = 4.7\, m_{3/2},$$

$$\begin{aligned} m_{1/2} &= 0.9\, m_{3/2}, \\ A_t &= -0.07, \\ B' &= -0.32, \\ B'' &= 21.8, \\ B''' &= 153 \end{aligned} \tag{6.85}$$

These values suggest that the dominant supersymmetry breaking term is m_0 for the MSSM (i.e. for $m_0, m_{1/2}, A_t$ and B'). This is contrary to the analyses based on tree level results. There are two reasons for this. In the first place, the mechanism explored here, being nonperturbative, necessarily requires that the one-loop corrections to the scalar potential be as important as the tree level ones. Secondly the cancelation of the cosmological constant in gaugino condensate requires a contribution from the matter superpotential. Both effects are absent in [75] and explains the difference between the resulting soft terms. The value of B''' in the example shown in eq.(6.85) is large because $\alpha = 0.3$ and the denominator of eq.(6.84) is large. As discussed in section 6.2 this is not a generic case since α can be smaller without affecting the hierarchy solution (see table 6.3).

Finally we remark that for the specific example given in eq.(6.85) the conditions at the unification scale eqs.(6.66) and (6.67) are satisfied. The fine tuning parameter $\Delta \simeq 17$, since at a crude approximation $\Delta \simeq m_1^2/M_Z^2$ with m_1 evaluated at the electroweak scale and it is given by $m_1^2 = m_0^2 + \mu^2 + k m_{1/2}^2$ with $k \simeq 1/2$.

The values obtained for the soft supersymmetric terms satisfy the constraints required by the electroweak symmetry breaking. Therefore, the breaking of supersymmetry could generate the breaking of the electroweak symmetry and determine the electroweak scale dynamically.

For completeness, we summarize some of the phenomenologically relevant parameters for supersymmetry breaking, in **tables 6.1, 6.2** and **6.3**, for an $SU(6)$ gauge group with different matter content.

table 6.1 $SU(6)$ gauge group with $16\pi^2 b_0 = 15$ and $\alpha_0 = 0.3$.

n_l	Y	T_r	$m_{3/2}$	$\frac{m_0}{m_{3/2}}$	$\frac{m_{1/2}}{m_{3/2}}$	Λ_{gut}	$\hat{\alpha}_{gut}^{-1}$	A_t	B'	B''	B'''	Δ
3	4.25	17.3	55	4.3	0.9	2.8	26.7	-0.07	-0.32	18.6	130.5	6.7
2	4.22	24.2	82	4.7	0.9	2.8	26.5	-0.07	-0.32	21.8	153	17.5
1	4.15	44.5	147	5.3	0.9	2.8	26.1	-0.07	-0.32	26.1	196	71

table 6.2 $SU(6)$ gauge group with $16\pi^2 b_0 = 16$ and $\alpha_0 = 0.3$.

n_l	Y	T_r	$m_{3/2}$	$\frac{m_0}{m_{3/2}}$	$\frac{m_{1/2}}{m_{3/2}}$	Λ_{gut}	$\hat{\alpha}_{gut}^{-1}$	A_t	B'	B''	B'''	Δ
3	4.26	16.3	440	4.2	0.9	3.5	26.7	-0.07	-0.32	17.5	122.4	403
2	4.23	22.5	555	4.4	0.9	3.5	26.6	-0.07	-0.32	19.9	140	720
1	4.17	40.3	900	5.1	0.9	3.5	25.8	-0.07	-0.32	26.3	184	2508

table 6.3 $SU(6)$ gauge group with $16\pi^2 b_0 = 11$ and $\alpha_0 = -1/3$.

n_l	Y	T_r	$m_{3/2}$	$\frac{m_0}{m_{3/2}}$	$\frac{m_{1/2}}{m_{3/2}}$	Λ_{gut}	$\hat{\alpha}_{gut}^{-1}$	A_t	B'	B''	B'''	Δ
3	4.62	17.9	632	15.4	0.9	2.0	29.0	-0.8	-1.06	238	158	10^4
2	4.61	24.7	661	15.4	0.9	2.0	28.9	-0.8	-1.06	238	158	10^4
1	4.57	44.0	849	15.4	0.9	2.0	28.7	-0.8	-1.06	238	158	10^4

We show in **tables 6.1-6.3**, the values for phenomenologically relevant parameters determining the supersymmetry breaking. n_l is the number of moduli, T_r, with large v.e.v., $g_{gut}^{-2} = Y/2$ is the unification coupling, $m_{3/2}$ is given in GeV, Λ_{gut} in $10^{16} GeV$ and $\hat{\alpha}_{gut}$ is the fine structure constant at Λ_{gut}. A_t, B', B'' and B'' are explained in eqs.(6.80-6.84). Δ measures the tuning of the electroweak breaking (cf. eq.(6.70)). The unification scale is given for $\alpha_a = \alpha_0 = 0.3$ in **tables 6.1 and 6.2**, where α_a corresponds to the visible sector gauge groups. With this choice the $SU(6)$ and the standard model gauge groups are unified. In **table 6.3** $\alpha_a = 0.32$ and the $SU(6)$ gauge group does not become unified with the visible sector ones.

For a wide range of values for α, a large hierarchy can be obtained. However, the soft supersymmetric terms are more sensitive to α. In **fig.6.2** we show $m_0/m_{3/2}$, $m_{1/2}/m_{3/2}$ and the trilinear term A_t as a function of α. For increasing α, m_0 and A_t become smaller. The common supersymmetric mass, m_0, is always larger than $m_{3/2}$. On the other hand, the gaugino mass is always smaller than $m_{3/2}$ and it approximates the gravitino mass for $|\,\alpha\,| \to 1/3$. The ratio of $m_{1/2}/m_{3/2}$ is independent of the value of $m_{3/2}$, but $m_0/m_{3/2}$ is not. The reason is that $m_0/m_{3/2}$ depends on Y/b_0, which sets the hierarchy as can be seen from eq.(6.3).

Running the masses down to the electroweak scale, neglecting corrections due to possible large Yukawa couplings, the common supersymmetric mass remains the same while the gluino mass increases by a factor of three [65]. If we demand all supersymmetric masses to be in the range $100\,TeV \leq m_{ss} \leq 1000\,TeV$, then $m_0/m_{1/2}\,|_{gut} \leq 30$.

In **figs.6.3** and **6.4** we plot $m_0/m_{1/2}\,|_{gut}$ as a function of α. The ratio becomes larger for α going to zero, because the gaugino mass gets smaller. Even though we do not expect the limit $m_{1/2} \to 0$ for $\alpha \to 0$ to be valid[3], the gaugino mass does indeed decrease considerably.

To conclude, from the range of values of the parameter α, the preferred value is for α positive and close to $1/3$. For this value of α the difference between the supersymmetric masses of the scalar and gaugino fields is smallest. This allows for having a small fine tuning parameter Δ. At the same time, the trilinear term $|\,A_t\,|$ decreases. This is welcome since one-loop diagrams, associated to the complex phases of the trilinear terms, contribute to the electric dipole moment of the neutron. For $A_t = O(1)$, this contribution is about $10^2 - 10^3$ bigger than the experimental values [76]. Furthermore, as we saw in section 6.2, an $\alpha \simeq 0.3$ minimizes the unification scale and allows for unifying the standard model gauge groups with the hidden sector one. Thus, from all the range of possible values for the parameters n_g, b_0 and α, consistency with the

[3] The approximation used in calculating the gaugino mass breaks down for α going to zero.

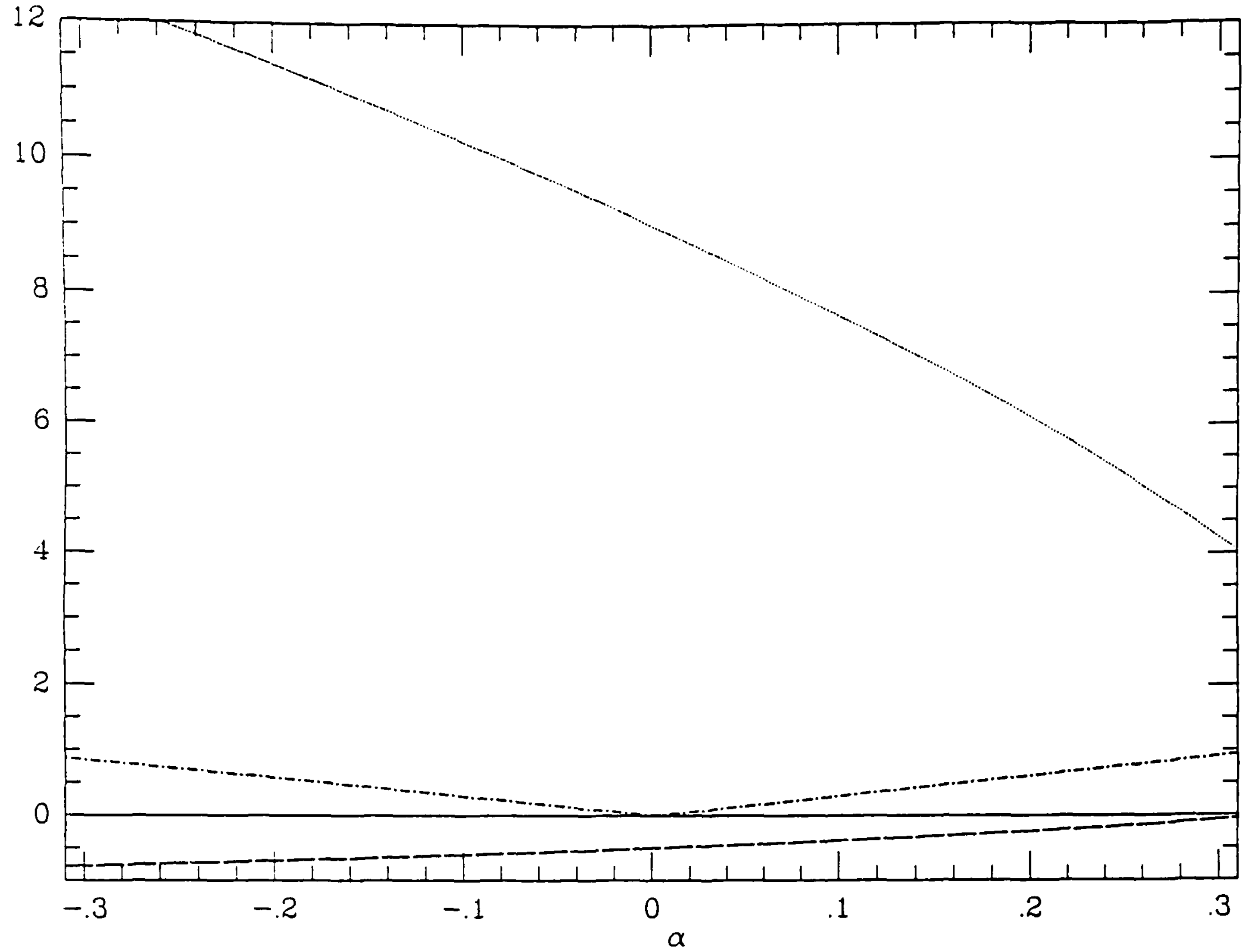

Figure 6.2: $m_0/m_{3/2}$, $m_{1/2}/m_{3/2}$ and the trilinear term A_t as a function of α. The dotted line represents $m_0/m_{3/2}$, the dot-dashed one $m_{1/2}/m_{3/2}$ while the dashed one A_t.

MSSM and electroweak breaking, selects a very narrow band. It is remarkable that the values of all soft supersymmetric terms improve within this band taking quite acceptable values.

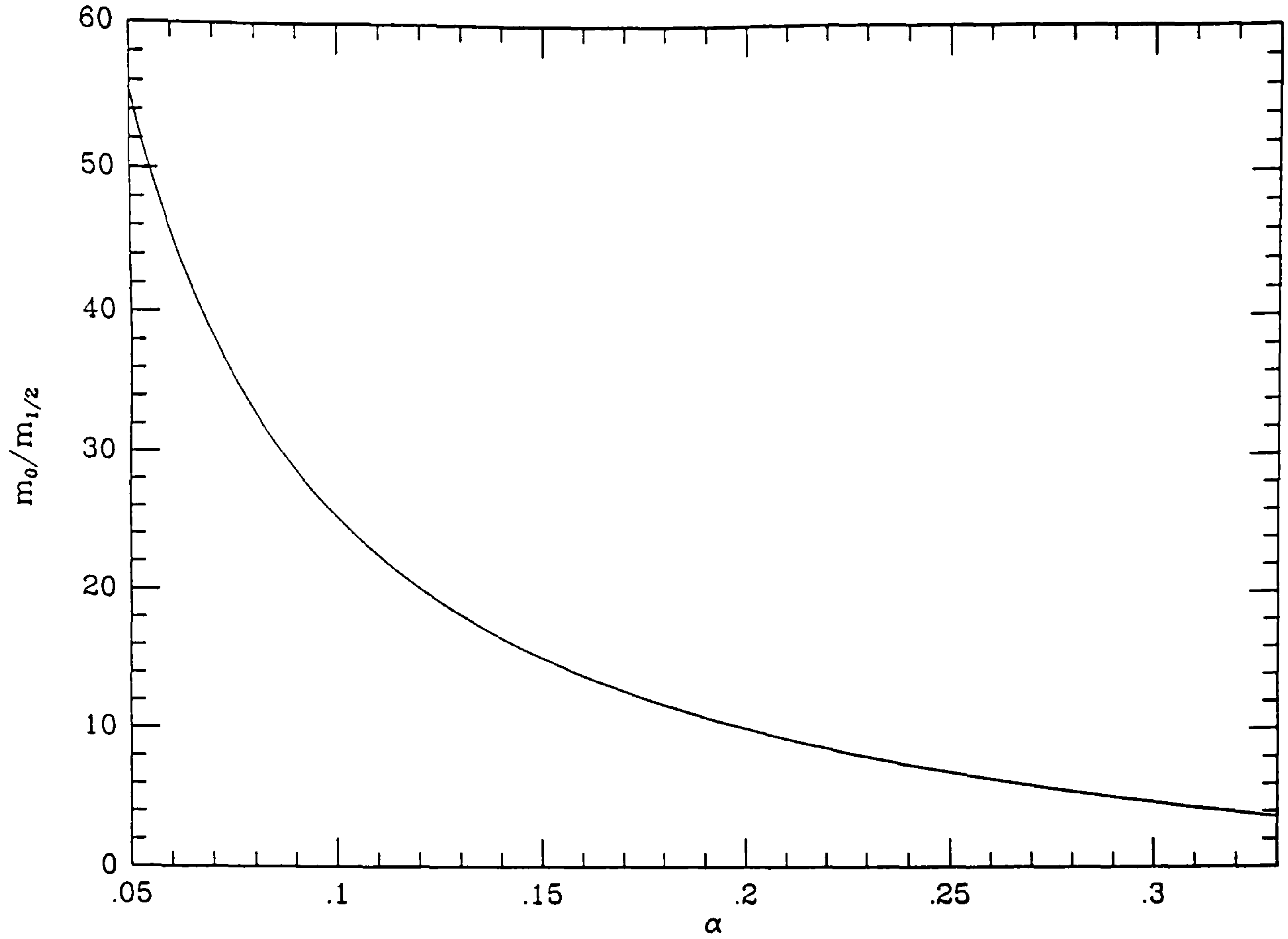

Figure 6.3: $m_0/m_{1/2}$ as a function of α

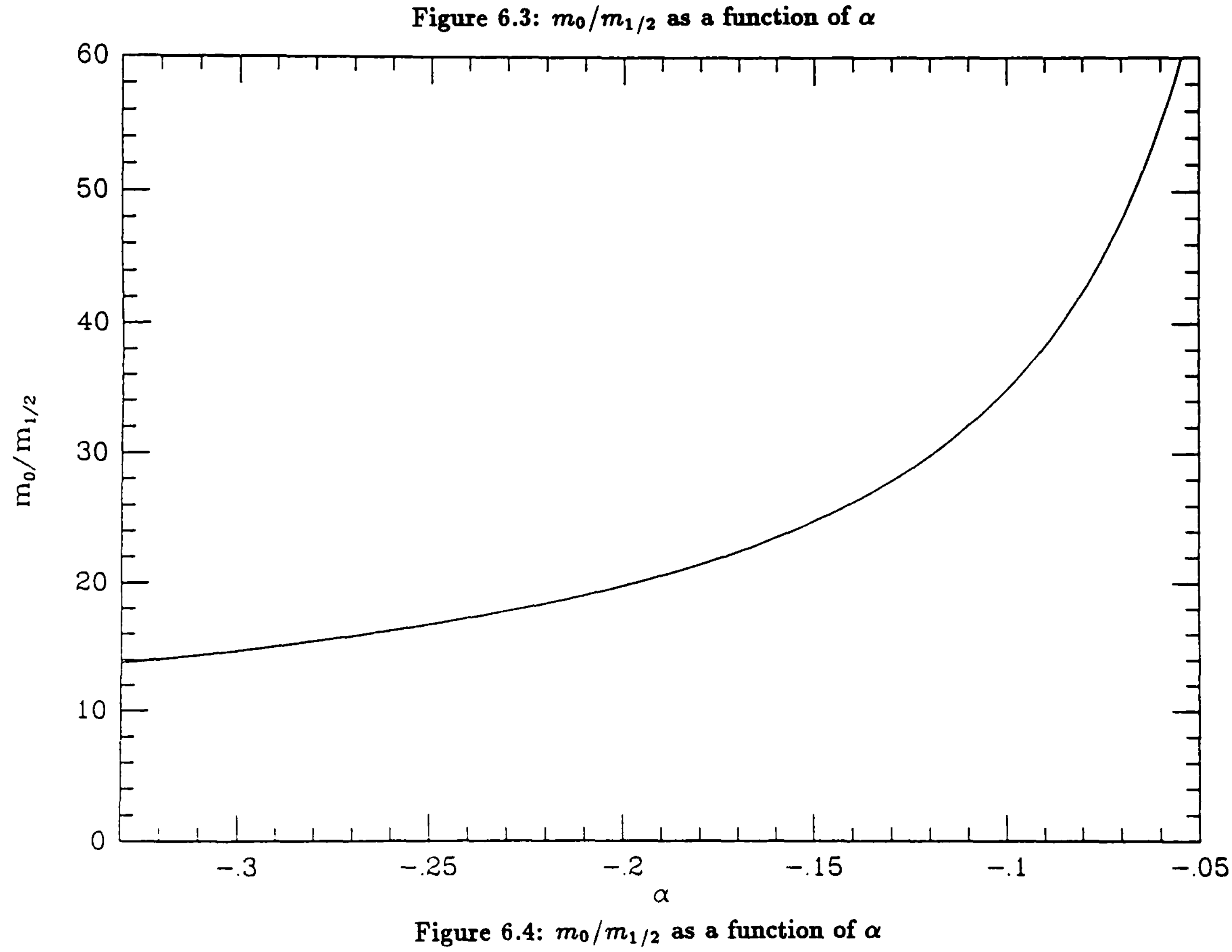

Figure 6.4: $m_0/m_{1/2}$ as a function of α

Chapter 7

Summary and Conclusions

In this thesis we have addressed the question of dynamical supersymmetry breaking in 4D string theory compactified in Orbifolds. We have argued that a strong coupling constant will generate a 4-Fermi pointlike interaction below the energy scale at which the coupling constant becomes strong. The presence of this 4-Fermi interaction is also supported by other parameterizations of gaugino condensates derived by demanding the effective theory to be anomaly free under the R-symmetry under which the gauginos transform non-trivially.

Our approach is equivalent to the ones used previously but we show that it is necessary to go beyond tree level in the strong coupling sector. The radiative corrections are conveniently introduced using NJL techniques and an infinite sum of gaugino bubbles is performed. Analyzing the full potential (including the radiative corrections which are non-perturbative in character) a stable solution is found for a single gaugino condensate. This result differs from the one obtained at tree level where no minimum is found.

At the minimum, supersymmetry is broken through the auxiliary field of the dilaton. We have shown that for very reasonable values of the gauge group one obtains a large mass hierarchy with a single gaugino condensate. The v.e.v. of the dilaton, which gives the gauge coupling constant at the unification scale, is fixed by the dimension of the hidden gauge group. Consistency with the minimal extension of the standard model restricts the gauge group, responsible for supersymmetry breaking, to be $SU(6)$ or $SO(9)$. The moduli can have large v.e.v.'s (of the order of 10), contrary to the value obtained at tree level (of the order of one). This allows the unification scale, which is moduli dependent, to be much smaller than the string scale and within the range required by MSSM. Furthermore, our solutions permit for squeezed Orbifolds, which are better candidates for minimal string unification.

In most theories with spontaneously broken symmetry the cosmological constant is negative and many orders of magnitude larger than the experimental bounds. To cancel the cosmological constant we introduced a suitable superpotential linear in a chiral superfield. We showed that by fine tuning the constant in the superpotential it is possible to have supersymmetry broken with a large mass hierarchy and vanishing cosmological constant.

In the models we have studied, the supersymmetry breaking is achieved in the hidden sector of the theory and it is transmitted to the visible sector through gravity only. The visible sector can then effectively be described by a globally supersymmetric theory with explicit soft supersymmetric breaking terms. Since the breaking of supersymmetry is by the auxiliary field of the dilaton the soft supersymmetric terms are universal. Introducing chiral matter fields we show that, neglecting radiative corrections due to possible large Yukawa couplings, they are stable for vanishing v.e.v. We then proceeded to calculate the soft supersymmetric breaking terms and we obtained reasonable values which permit the breaking of the electroweak symmetry through radiative corrections. Thus the breaking of supersymmetry through gaugino condensation not only explains the mass hierarchy between the electroweak and Planck scales but it also predicts the scale of supersymmetry and electroweak breaking.

To conclude, we have shown that gaugino condensation in the hidden sector could provide the necessary structure to dynamically break supersymmetry in a class of 4D string models. The values of the parameters determining the masses and interactions of the standard model are predicted and quite acceptable values are obtained.

Appendix A

Notation and Conventions

In this appendix we give the conventions and notation used in this thesis. We work in natural units $c = \hbar = 1$ and set the reduced planck mass

$$m_p = \frac{M_{Planck}}{\sqrt{8\pi}} = 1 \tag{A.1}$$

as well.

Indices can be raised by applying the two-dimensional ϵ tensor

$$\theta_\alpha = \epsilon_{\alpha\beta}\theta^\beta \tag{A.2}$$

where $\epsilon_{\alpha\beta} = \epsilon^{\alpha\beta}$ is antisymmetric in α and β ($\alpha, \beta = 1, 2$) and $\epsilon_{12} = 1$. The complex adjoint of θ is

$$\bar{\theta}^{\dot{\alpha}} = (\theta^\alpha)^\dagger. \tag{A.3}$$

The antisymmetric tensor $\epsilon_{\dot{\alpha}\dot{\beta}}$ with dotted indices has the same structure as $\epsilon_{\alpha\beta}$ and dotted indices can be lowered using $\epsilon_{\dot{\alpha}\dot{\beta}}$. The square of θ is defined by

$$\theta^2 = \theta^\alpha\theta_\alpha = \epsilon_{\alpha\beta}\theta^\alpha\theta^\beta. \tag{A.4}$$

The Pauli matrices carry a space-time index μ as well as a dotted and undotted index

$$\sigma_\mu^{\alpha\dot{\beta}} = (1, \sigma)_{\mu,\alpha\beta} \tag{A.5}$$

where σ_1, σ_2 and σ_3 are the usual Pauli matrices,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.6}$$

We choose a metric $g_{\mu\nu}$ with diagonal entries $(1,-1,-1,-1)$ and $\sigma = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$. The gamma matrices γ^μ satisfy

$$\{\gamma^\mu, \gamma^\nu\} = 2\, g^{\mu\nu} \tag{A.7}$$

with γ^0 hermitean, and γ^i antihermitean. We take $\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ with $\gamma_5^2 = 1$ and

$$\{\gamma_5, \gamma^\mu\} = 0. \tag{A.8}$$

From chapter 4 onwards we use 4 component notation for the fermion fields and defined

$$\psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi \tag{A.9}$$

and

$$\bar{\psi}_{L,R} = (\psi_{L,R})^\dagger \gamma_0. \tag{A.10}$$

A Majorana spinor satisfies the condition

$$\bar{\psi}_{L,R} = -\psi^T_{R,L}\, C^{-1} \tag{A.11}$$

where C is the charge conjugation matrix. In a chiral representation for the gamma matrices, one has $\gamma_5 = diag(1,-1)$ and $C = diag(-i\sigma^2, i\sigma^2)$ with the off diagonal entries being zero. In such a representation it is easy to see that the mass term for a Majorana particle is

$$m\, \bar{\psi}_R \psi_L = m\, i\psi_2^T \sigma^2 \psi_2 \tag{A.12}$$

where $\psi^T = (\psi_1^T, \psi_2^T)$. For a Dirac spinor the Weyl components ψ_1 and ψ_2 are independent but for a Majorana they are related through the majorana condition eq.(A.11).

Appendix B

Contribution from χ_ϕ to the Lagrangian

In this appendix we study the interaction generated by χ_ϕ, the fermion component of the Goldstone mode ϕ associated with the spontaneously broken R-symmetry described in chapter 4. We will show that these extra terms do not generate any tree level masses nor any contribution to the scalar potential, i.e. all terms vanish at the minimum.

As given in equation eq.(4.22), χ_ϕ is a Lagrange multiplier because it does not have a kinetic term and through its equation of motion eq.(4.23) it is given in terms of the gaugino and chiral fermion fields. It includes also the auxiliary field D of the gauge covariant chiral superfield which is itself a non-propagating field. In order to eliminate χ_ϕ and D, which are only Lagrange multipliers, one has to simultaneously solve the equations of motion of these fields. The interaction terms involving χ_ϕ and D are given by

$$e^{-1}L_{aux} = a\bar{\chi}_{R\phi}\chi_{L\phi} + \bar{\chi}_{R\phi}b_L + D\bar{\chi}_{R\phi}c_L + \frac{1}{2}eD^2 + \frac{1}{2}gD + h.c. \tag{B.1}$$

where

$$\begin{aligned} a &= \frac{1}{4}f^{\phi\phi}\bar{\lambda}_R\lambda_L - \frac{1}{2}e^{K/2}W^{\phi\phi}, \\ b_L &= \frac{1}{2}f^{\phi}(-\sigma\cdot\hat{F})\lambda_L - \frac{1}{2}f^{\phi i}\chi_{Li}\bar{\lambda}_R\lambda_L - e^{K/2}W^{i\phi}\chi_{Li}, \\ c_L &= \frac{i}{2}f^{\phi}\lambda_L, \end{aligned} \tag{B.2}$$

$$e = \frac{1}{2} Ref,$$

and

$$g = \tilde{g} G^i T_i^j z_j + \frac{i}{2} f^i \bar{\chi}_{Ri} \lambda_L - \frac{i}{2} f_i \bar{\chi}_L^i \lambda_R.$$

Using the equations of motion for χ_ϕ and D one obtains the constraints

$$\chi_{L\phi} = \frac{1}{2} \frac{(2g\bar{a} - \bar{c}_R b_L)c_L + (\bar{c}_R c_L + 4e\bar{a})b_L}{4e\,|\,a\,|^2 - \bar{a}\bar{c}_R c_L - a\bar{c}_L c_R}, \quad \text{(B.3)}$$

$$D = -\frac{2\,|\,a\,|^2 - a\bar{b}_L c_R - \bar{a}\bar{b}_R c_L}{4e\,|\,a\,|^2 - \bar{a}\bar{c}_R c_L - a\bar{c}_L c_R} \quad \text{(B.4)}$$

and substituting back eqs.(B.3) and (B.4) into eq.(B.1) the Lagrangian becomes

$$e^{-1} L_{aux} = -\frac{1}{4} \frac{4\,|\,a\,|^2 f^2 - 4f(a\bar{b}_L c_R + \bar{a}\bar{b}_R c_L) + 4e(a\bar{b}_L b_R - \bar{a}\bar{b}_R b_L)}{1}$$
$$\times \frac{2\bar{b}_L c_R \bar{b}_R c_L - \bar{b}_R b_L \bar{c}_L c_R - \bar{b}_L b_R \bar{c}_R c_L}{4e\,|\,a\,|^2 - \bar{a}\bar{c}_R c_L - a\bar{c}_L c_R}. \quad \text{(B.5)}$$

From the 4D string model defined in eqs.(4.24-4.25), one has $f^{\phi i} = W^{\phi\phi} = 0$, and if we further assume $W^{\phi i} = 0$ then eqs.(B.2) simplify considerably. We find that e and g do not change while the other quantities become

$$a = \frac{1}{4} f^{\phi\phi} \bar{\lambda}_R \lambda_L = -\frac{q}{4\phi},$$
$$b_L = i(\sigma \cdot \hat{F}) c_L \quad \text{(B.6)}$$

and

$$c_L = \frac{i\xi}{\phi} \lambda_L,$$

with $\xi = 2b_0/3$ and $q \equiv \frac{\bar{\lambda}_R \lambda_L}{\xi \phi} = 2e^{K/2} \Pi^i \eta_i^{-2}$.

We can now determine the contribution to the Lagrangian eq.(B.5) coming from the D and χ_ϕ fields. This contribution is given by

$$e^{-1} L_{aux} = -\frac{1}{2}(Ref - \xi)\, D^2 + \frac{\xi}{2}(\sigma \cdot \hat{F})^2 \quad \text{(B.7)}$$

with

$$D = -(Ref - \xi)^{-1}(g - i\xi(\sigma \cdot \hat{F})) \quad \text{(B.8)}$$

$$D = -(Ref - \xi)^{-1}(\tilde{g} G^i T_i^j z_j + \frac{i}{2} f^i \bar{\chi}_{Ri} \lambda_L - \frac{i}{2} f_i \bar{\chi}_L^i \lambda_R - i\xi(\sigma \cdot \hat{F})) \quad \text{(B.9)}$$

and χ_ϕ is

$$\chi_{L\phi} = -\frac{1}{q}(Ref - \xi)^{-1}(ig + Ref(\sigma \cdot \hat{F}))\lambda_L. \tag{B.10}$$

From eqs.(B.7) and (B.9) we notice that no tree level mass term is generated since at the minimum $D = 0$ and the gauge bosons have vanishing v.e.v.

If we compare eqs.(B.7) and (B.9) with the standard contribution of the D field to SUGRA given by

$$e^{-1}L_{aux} = -\frac{1}{2}Ref\, D^2 \tag{B.11}$$

with

$$D = -(Ref)^{-1}(\tilde{g}G^i T_i^j z_j + \frac{i}{2}f^i \bar{\chi}_{Ri}\lambda_L - \frac{i}{2}f_i \bar{\chi}_L^i \lambda_R) \tag{B.12}$$

we realize that the effects of the χ_ϕ field is to modify the expression of D, giving it an extra term (the second one in eq.(B.8)), and the coefficient of D^2 in the Lagrangian, i.e. $Ref \to Ref - \xi$. It also generates extra gauge bosons interaction (second term in eq.(B.7)) but non of these terms are relevant in determining the vacuum structure and do not generate any tree level masses.

Appendix C

Modular functions and tree level Scalar Potential

In this appendix we present the modular functions used in this work and some of their properties. We also derive the full tree level potential including the cross terms between the dilaton and the moduli fields in the Kahler potential K.

The modular group acting on the complex modulus T is defined as the group of linear fractional transformations

$$T \to \frac{aT - ib}{icT + d} \qquad a, b, c, d \in Z \qquad ad - bc = 1. \tag{C.1}$$

The relevant group for this transformation is $SL(2, Z)$ and it has two fixed points $T = 1$ and $T \equiv \rho = e^{i\pi/6}$. A modular form with weight n is defined by

$$F(\frac{aT - ib}{icT + d}) = (icT + d)^n F(T) \tag{C.2}$$

and the space of modular forms of a given weight is a finite dimensional vector space. A suitable basis is obtained by means of the Eisenstein series

$$G_{2k} = \Sigma_{n_1, n_2 \in Z}(icn_1 T + n_2)^{-2k}. \tag{C.3}$$

For $k > 1$, G_{2k} is a modular function of weight $2k$. For $k = 1$ the series does not converge unless the order of summation is specified. The function G_2 is an holomorphic function but transform as a connection $G_2 \to (icT+d)^2 G_2 - 2\pi ic(icT+d)$. On the other hand

$$\hat{G}_2 = G_2 - \frac{2\pi}{T_r} \tag{C.4}$$

with $T_r = 2ReT$ transforms as a weight two form, but is not holomorphic.

The Eisenstein functions ($k > 1$) have no poles and their zeros are at $T = 1$ for k odd and at $T = \rho$ for $k \neq 0$ mod 3. $\hat{G}_2$ has no poles and its only zeros are at the self-dual points $T = 1, \rho$. The derivative of any invariant function vanishes at the self-dual points. A useful function is the Dedekind-eta function defined as

$$\eta(T) = q^{1/24}\Pi_{n=1}^{\infty}(1-q^n), \qquad q = e^{-2\pi T} \tag{C.5}$$

which has modular weight 1/2. Its derivative is given by

$$\frac{\eta'(T)}{\eta(T)} = -\frac{G_2(T)}{4\pi} \tag{C.6}$$

$$\frac{\eta'(T)}{\eta(T)} = -\frac{\hat{G}_2(T)}{4\pi} - \frac{1}{2T_r}. \tag{C.7}$$

The functions $|T_r\hat{G}_2|^2$ and $\delta = Tr|\eta|^4$ are modular invariant functions since $T_r \to \frac{T_r}{|icT+d|^2}$. For large T_r one has

$$G_2 = \frac{\pi^2}{3}(1 - 24q + ...) \tag{C.8}$$

$$\eta = q^{1/24}(1 - q + ...) = e^{-\pi T/12}(1 - q + ...) \tag{C.9}$$

The low energy 4D string theory is given by an $N = 1$ SUGRA theory with Kahler potential G and gauge kinetic function f given by eqs.(4.38-4.40). The tree level potential in $N = 1$ SUGRA has the generic form

$$V_0 = h_i(K^{-1})^i_j h^j - 3e^G \tag{C.10}$$

where h_i, $i = S, T_i$ are the auxiliary fields of the dilaton and moduli fields. Since the Kahler potential mixes S and T the matrix $(K^{-1})^i_j$ is not diagonal. For simplicity, we will neglect the term proportional to $2b_0 lnM_s^2$ in the Kahler potential in eq.(4.38). Then, using eq.(4.38) we find

$$(K^{-1}) = \frac{1}{detK}\begin{pmatrix} K^T_T & -K^T_S \\ -K^S_T & K^S_S \end{pmatrix} \tag{C.11}$$

with

$$detK = \frac{Y+a}{T_r^2 Y^3} \quad , \quad K_S^S = \frac{1}{Y^2}, \tag{C.12}$$

$$K_S^T = K_T^S = \frac{a}{Y^2 T_r} \quad , \quad K_T^T = \frac{[Y(Y+a)+a^2]}{T_r^2 Y^2}. \tag{C.13}$$

The scalar potential is then

$$V_0 = Y^2 \mid h_s \mid^2 + \Sigma_i \frac{Y}{Y+a_i} \mid h_s a_i - T_{ri} h_{Ti} \mid^2 - 3e^G \tag{C.14}$$

where $Y = S_r + \Sigma_i a_i ln(T_{ri})$ and $a_i = 2(k_0 \delta_i^{GS} - b'_{0i})$. The auxiliary fields are given by eqs.(5.3) and (5.4), i.e.

$$h_S = \frac{1}{2} e^K W_0 \frac{(1+Y/\xi)}{Y}, \tag{C.15}$$

$$\begin{aligned} h_{T_i} &= \frac{1}{2} e^K W_0 \left[-K_{T_i} - \frac{W_{T_i}}{W} + \frac{f_{T_i}}{\xi} \right] \\ h_{T_i} &= \frac{1}{2} e^K W_0 \left[\frac{1}{T_{ri}} (1 + \frac{a_i}{Y}) + 2 \frac{\eta_{T_i}}{\eta} (1 - \frac{a_i}{\xi}) \right] \end{aligned} \tag{C.16}$$

where we have used eq.(4.26) to express the gaugino bilinear in terms of W_0 and $\xi = 2b_0/3$. Substituting these expression in eq.(C.14) we find

$$V_0 = m_{3/2}^2 \left[(1+\frac{Y}{\xi})^2 + \Sigma_i \frac{Y}{Y+a_i} \mid (1-\frac{a_i}{\xi})(1+2T_{ri}\frac{\eta_{T_i}}{\eta}) \mid^2 - 3 \right] \tag{C.17}$$

$$V_0 = m_{3/2}^2 \left[(1+\frac{Y}{\xi})^2 + \Sigma_i \frac{Y}{Y+a_i} (1-\frac{a_i}{\xi})^2 \frac{T_{ri}^2}{4\pi^2} \mid \hat{G}_2 \mid^2 - 3 \right] \tag{C.18}$$

where $m_{3/2}^2 = e^G$ is the gravitino mass. It is now easy to see that the tree level potential is duality invariant and in the limit $a_i \ll 1$, eq.(C.18) reduces to eq.(4.11).

Finally, we give the full solutions to eqs.(5.27) and (5.28) in the limit $x_g \ll 1$. The function $J(x)$, defined in eq.(5.12), behaves as

$$J(x) \simeq 2x + x^2 ln(x), \quad for \quad x \ll 1 \tag{C.19}$$

$$J(x) \simeq x, \qquad for \quad x \simeq 1 \tag{C.20}$$

and

$$J(x) \simeq ln(x), \quad for \quad x \gg 1. \tag{C.21}$$

From $V_s = 0$ one obtains $a \simeq 2$, where a is defined in eq.(5.26), and thus a solution to the full potential V (cf. eq.(5.25)) is for $x \ll 1$. Using eq.(C.19), the solutions to eqs.(5.27) and (5.28) are

$$a = 2\left(\frac{1+(1-5\alpha)c/2+d(2+c)}{1+(1-3\alpha)3c/2+d(4+3c/2)}\right) \tag{C.22}$$

and

$$\epsilon = xln(x) = \frac{c(3\alpha-1)-3d-dc}{1+(1-3\alpha)3c/2+d(4+3c/2)} \tag{C.23}$$

where $c \equiv 4b/Y$ and $d = -\frac{\delta B_{0T}}{\delta_T B_0}$. Taking $B_0 \simeq (3Y/2b)^2$ we can write $d = c^2 g$, with $g \equiv \pi T_r/36$, and eq.(C.23) becomes

$$\epsilon = c\left(\frac{(3\alpha-1)-gc(3+c)}{1+c((1-3\alpha)3/2+gc(4+3c/2))}\right). \tag{C.24}$$

Expanding in powers of c one obtains to leading order eq.(5.31).

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