

Price Discrimination, Competition and Welfare

Simon Cowan*

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Abstract

A monopolist in one market is also an output-setting oligopolist in a separate market with the same demand and cost conditions. Should it be allowed to set a price in its monopoly market that differs from the price in the competitive market? When there is a single rival the regulated firm under uniform pricing restricts its output in the competitive market so that its market share is one-third. Social welfare is higher with uniform pricing. Welfare is also higher with uniform pricing if there is more than one rival despite the regulated firm optimally choosing not to supply the competitive market. Uniform pricing may promote entry, and is socially preferred to discrimination in this case when demand is sufficiently convex. Similar results hold in a model where the competitive market has price-setting duopolists with differentiated products.

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*Department of Economics, University of Oxford, Manor Road Building, Oxford OX1 3UQ, UK. Email: simon.cowan@economics.ox.ac.uk. ORCID: 0000-0003-0704-2871. I am grateful to Flavio Toxvaerd, Paul Klemperer, John Vickers, and to participants of seminars in 2025 for their comments and questions.

1 Introduction

A monopolist in one market is an oligopolist in a separate market. The only difference between the markets is the amount of competition. Should this firm be allowed to set a different price in its monopoly market to the price in the competitive market or should it be required to ensure that the prices are the same in both markets? This paper explores the welfare effects of uniform pricing compared to discrimination when the markets have different amounts of competition.

The main model has firms that set outputs. Discrimination results in the monopoly outcome in the monopoly market and a symmetric Cournot equilibrium in the competitive market. This is a form of third-degree price discrimination: the firm that operates in both markets behaves differently in the two markets because of the exogenous difference in the amount of competition. Now suppose that the regulated firm faces a uniform pricing requirement. The regulated firm restricts its output in the competitive market in order to limit the reduction in profit in the monopoly market.

There is a welfare trade-off: a move to uniform pricing raises welfare in the monopoly market but reduces welfare in the competitive market as the regulated firm restricts its output there. Nevertheless total welfare is higher very generally. Under standard conditions total output is higher with uniform pricing and the output is efficiently distributed across the markets.

There are three variants of the model. In the first the rival incurs an entry cost, and its entry is profitable only if there is uniform pricing. The second allows the regulated firm to exit the competitive market under uniform pricing when there is a single rival there. In the third variant the markets are integrated, so uniform pricing arises endogenously. In each case the curvature of inverse demand matters for the welfare effects.

The second model has two price-setting firms in the competitive market with differentiated products and linear demand functions. Similar results hold. Uniform pricing is better for both total consumer surplus and total social welfare than discrimination. The benefit of competition in the duopoly market is transferred to the monopoly market, though the regulated firm raises its price in the duopoly market above the level with discrimination, as does its rival.

There is now a substantial literature on price discrimination and competition. Thisse and Vives (1988) have a symmetric Hotelling model, where each firm has perfect information about each customer's willingness to pay (in particular they

know each individual’s transport cost) and thus can make individual offers. Holmes (1989) models symmetric duopolists who compete in prices in two markets where the demand functions differ, and Corts (1998) develops that framework. Armstrong and Vickers (2001) model firms as competing over how much utility they offer customers. Stole (2007) presents a comprehensive survey of the literature. Armstrong and Vickers (2019) model price-setting firms selling identical goods and each with some captive customers. Rhodes and Zhou (2024) generalize the insights of Thisse and Vives (1988) in a general oligopoly model of personalized pricing.

Two papers are more closely related to the current one, as they model asymmetric opportunities for the regulated firm. Armstrong and Vickers (1993) consider a price-leading incumbent that operates in two markets while a price-taking competitor can enter in one market and can choose its scale of entry. Feng, Li, and Shuai (2023), in a model with differentiated products and two markets, show that a commitment to uniform pricing by an incumbent may induce the entrant to enter only one market, thus benefiting the incumbent. The Bertrand model in the current paper builds on their analysis.

Section 2 covers the model in which firms set outputs. The model of price-setting firms with differentiated products is in Section 3. Price-setting firms with identical products are briefly discussed in Section 4. Conclusions are in Section 5.

2 Output-setting firms

2.1 The Cournot model

Consider an n -firm Cournot oligopoly, where $n \geq 1$. The constant marginal cost of each firm is $c \geq 0$. Inverse demand for total output Q is $p(Q)$, which is strictly decreasing, twice-continuously differentiable and $\lim_{Q \rightarrow 0} p(Q) > c$. Direct demand is $D(p)$.

The curvature of inverse demand is $\sigma(Q) \equiv -\frac{Qp''(Q)}{p'(Q)}$ and the curvature of direct demand is $\alpha(p) \equiv -\frac{pD''(p)}{D'(p)}$. These are related via the price elasticity of demand, $\varepsilon(p) \equiv -\frac{pD'(p)}{D(p)}$ with $\sigma(Q) = \frac{\alpha(p(Q))}{\varepsilon(p(Q))}$. At least one of the following conditions is assumed to hold: (i) $\sigma(Q) < 2$ for all Q and (ii) $\alpha(p) < 2$ for all p . These limit how convex demand can be and ensure that the profit function is single-peaked. The profit function for firm i is $(p(q_i + Q_{-i}) - c)q_i$, where Q_{-i} is the output of the other firms. The first-order condition for positive output is $p(Q) - c + q_i p'(Q) = 0$. Writing $s_i = \frac{q_i}{Q} \leq 1$ as i ’s market share, the second-order condition for firm i holds

if and only if $2 - s_i\sigma > 0$. The arguments of the curvature measures and of the elasticity are omitted except where necessary or helpful. Under condition (i) i 's profit is strictly concave in q_i . To apply condition (ii) note that $2 - s_i\sigma = 2 - s_i\frac{\alpha}{\varepsilon}$. When firm i 's first-order condition holds with equality the Lerner condition applies: $\frac{s_i}{\varepsilon} = \frac{p-c}{p} \leq 1$. Thus $2 - s_i\frac{\alpha}{\varepsilon} = 2 - \left(\frac{p-c}{p}\right)\alpha > 0$ under condition (ii) and each firm's profit function is strictly quasi-concave in its own output.

The symmetric equilibrium is characterized by

$$p(Q^*) - c + \frac{Q^*}{n}p'(Q^*) = 0 \quad (1)$$

where the asterisk denotes the equilibrium value. Total output in the symmetric equilibrium is a function of the number of firms, denoted by $Q(n)$. For convenience the asterisk is used only where it is needed. Assume that $n + 1 - \sigma(Q(n)) > 0$. Seade (1980) calls this a stability condition. This holds automatically if $\sigma(Q) < 2$ for all Q . If $n + 1 - \sigma(Q)$ is positive for all Q then the equilibrium is unique because the expression on the left-hand side of (1) is strictly decreasing in Q (Schlee, 1993).

The slope of i 's best response to Q_{-i} is $\frac{dq_i}{dQ_{-i}} = \frac{s_i\sigma-1}{2-s_i\sigma}$, which has the same sign as the numerator. At the symmetric equilibrium each market share equals $\frac{1}{n}$ so the numerator has the same sign as $\sigma(Q(n)) - n$. Outputs are strategic substitutes at the symmetric equilibrium if $\sigma(Q(n)) - n \leq 0$. It is standard for outputs to be strategic substitutes, but the analysis to come does not require this. Sufficient conditions for outputs to be strategic substitutes globally are $\sigma(Q) \leq 1$ for all Q or $\alpha(p) \leq 1$ for all p .

Differentiating (1) with respect to n gives the effect of a marginal increase in the number of firms on total output

$$Q'(n) = \frac{Q(n)}{n(n + 1 - \sigma(Q(n)))} > 0. \quad (2)$$

The elasticity of output with respect to the number of firms, $n\frac{Q'(n)}{Q(n)} = \frac{1}{n+1-\sigma(Q(n))}$, is below 1 when outputs are strict strategic substitutes. An increase in the number of firms raises total output less than proportionally because the existing firms reduce their output in response to entry.

Social welfare is consumer surplus plus profits, or gross utility minus costs. Define gross utility as $U(Q)$, which is increasing and strictly concave, and satisfies $U'(Q) = p$. The analysis uses strict concavity of social welfare in the number of firms, which is assumed here, but holds under very general conditions.

Assumption 1. *Social welfare is strictly concave in the number of firms.*

Assumption 1 holds if, but not only if, total output is concave in n . Welfare is a strictly concave function of output because utility is strictly concave and costs are linear in output. If output, in turn, is concave in n then welfare is strictly concave in n . In fact welfare is strictly concave under a weaker assumption that does not require total output to be concave. Strict concavity of welfare implies that the welfare increase caused by the entry of a new firm is smaller than the welfare reduction from the loss of one firm. Full details are in Appendix A.

2.2 Discrimination and uniform pricing

In the monopoly market there is a single firm, called “the regulated firm” or “the firm” for short. In the competitive market there are $n \geq 2$ firms. The regulated firm is firm 1 in the competitive market and $i = 2, \dots, n$ indexes the rival firms. The two markets are separated: consumers in one market cannot buy from the other, and the rival firms cannot supply the regulated firm’s monopoly market. The markets have the same demand function and all firms have the same constant marginal cost.

When the regulated firm may discriminate it maximizes its profit in the monopoly market, with its monopoly output being denoted by $Q(1)$. For the competitive market it chooses q_1 to maximize its profit taking the output of the other firms as given. A symmetric Cournot equilibrium with n firms results. Industry profit in the competitive market, $\Pi(n) = (p(Q(n)) - c)Q(n)$, is decreasing in the number of firms. An additional firm raises total output and reduces total profit because industry marginal revenue is below marginal cost at each Cournot equilibrium with $n > 1$.

Suppose the regulated firm is subject to uniform pricing and that $n = 2$. Its output in the competitive market is q_1 . Output in the monopoly market is Q and in the competitive market the total output of the two firms is $q_1 + q_2$. Uniform pricing requires that $Q = q_1 + q_2$. The sum of the regulated firm’s outputs across the two markets is $2q_1 + q_2$, so its total profit function is

$$\pi_1(q_1, q_2) = (p(q_1 + q_2) - c)(2q_1 + q_2).$$

The first-order condition for the regulated firm, choosing q_1 , at the equilibrium with $q_1^* > 0$ is

$$2(p(Q^*) - c) + (2q_1^* + q_2^*)p'(Q^*) = 0. \quad (3)$$

The rival firm’s first-order condition is

$$p(Q^*) - c + q_2^*p'(Q^*) = 0. \quad (4)$$

The equilibrium has each firm setting a positive output. Suppose instead that $q_1^* = 0$ while firm 2 sets its output at the monopoly level $Q(1)$ with profit margin $p(Q(1)) - c$. Using (4) in the left-hand side of (3) the marginal profitability of output for firm 1 at $q_1^* = 0$ is $p(Q(1)) - c > 0$, so it prefers to set a strictly positive output. Similarly if $q_2^* = 0$ then the regulated firm would set q_1^* equal to the monopoly output, $Q(1)$, and marginal profitability for the rival would be $p(Q(1)) - c > 0$.

The profit functions are strictly quasi-concave under the same conditions as in the benchmark model.¹ To characterize output in each market, Q^* , add (3) and (4) and divide by 3 to give

$$p(Q^*) - c + \frac{Q^*}{1.5}p'(Q^*) = 0.$$

Compare this with (1). Total output in each market is $Q(1.5)$. In effect each market has one and a half firms. It is as if there has been a transfer of half a firm from the duopoly market to the monopoly market.

To show the effect of uniform pricing on the regulated firm's output in the duopoly market multiply (4) by 2 and subtract from (3) to give

$$q_1^* = \frac{q_2^*}{2}.$$

The regulated firm's output in the duopoly market is half that of its rival, and its share in that market is $\frac{1}{3}$. The output of each firm in the duopoly market with discrimination is $\frac{Q(2)}{2}$. The regulated firm's output is less than two-thirds of its discriminatory output:

$$q_1^* = \frac{Q(1.5)}{3} < \frac{2}{3} \frac{Q(2)}{2}.$$

The regulated firm reduces its output in the duopoly market to raise the price in its monopoly market.

What is the effect of uniform pricing on social welfare? Welfare rises in the monopoly market but falls in the competitive market. Welfare as a function of the number of firms is denoted by $W(n) = U(Q(n)) - cQ(n)$ where $U(Q)$ is the utility function, which is increasing and strictly concave. The effective transfer of half a firm from the competitive market to the monopoly market generates a larger welfare increase in the monopoly market than the reduction in welfare in the competitive market.

¹For firm 1 the second-order condition is $p'(Q)(4 - (1 + s_1)\sigma) < 0$. When $\sigma < 2$ this holds everywhere: the profit function is strictly concave. When the first-order condition holds the second-order condition is $2p'(Q) \left(2 - \left(\frac{p-c}{p}\right)\alpha\right) < 0$, which holds if $\alpha < 2$, so profit is strictly quasi-concave.

Proposition 1. *Suppose there are two firms in the competitive market. Uniform pricing yields higher social welfare than discrimination.*

Proof. With uniform pricing welfare is $2W(1.5)$. Welfare with discrimination is $W(1) + W(2)$. Strict concavity of welfare in n implies that $W(1.5) > \frac{W(1)+W(2)}{2}$. Thus welfare is higher with uniform pricing. \square

When output is concave there is a natural explanation for uniform pricing to be socially preferred. Total output with uniform pricing, $2Q(1.5)$, is weakly higher than that with price discrimination, $Q(1) + Q(2)$. At the same time uniform pricing ensures that the output is efficiently distributed across the markets because the profit margins, which are the marginal social values of output, are equal. Interestingly, though, welfare is higher even if total output falls when uniform pricing is required.

In the analysis of monopoly discrimination when demand functions differ across markets the benchmark has linear demand functions and all markets are served with uniform pricing. Pigou (1920) showed that in this case total output under discrimination equals that with uniform pricing (see also Varian (1985)). Here when $p(Q) = e^{-Q}$ and $c = 0$ each firm has the dominant strategy of setting output of 1 in the standard Cournot game. Total output, $Q(n) = n$, is linear in the number of firms and thus output with uniform pricing equals that with discrimination. This demand function was used by Hennessy (2000) to show that a merger in a symmetric Cournot industry can be profitable.

Now let $n \geq 3$. The uniform pricing constraint for the firm is $Q = q_1 + Q_{-1}$. The regulated firm's profit is $(p(q_1 + Q_{-1}) - c)(2q_1 + Q_{-1})$. The key positive result is that in the equilibrium the regulated firm chooses to set output of zero in the competitive market. Its Kuhn-Tucker conditions at the equilibrium are

$$\begin{aligned} \frac{\partial \pi_1(q_1^*, Q_{-1}^*)}{\partial q_1} &= 2(p(Q^*) - c) + (1 + s_1^*)Q^*p'(Q^*) \leq 0, & (5) \\ q_1^* &\geq 0, \\ \frac{\partial \pi_1(q_1^*, Q_{-1}^*)}{\partial q_1} q_1^* &= 0. \end{aligned}$$

The rival firms each set the same positive output. The first-order conditions for the rivals are

$$p(Q^*) - c + s_i^*Q^*p'(Q^*) = 0 \quad i = 2, \dots, n.$$

Adding the $n - 1$ first-order conditions for the rivals, and using the fact that $\sum_{i=2}^n s_i^* = 1 - s_1^*$, gives the equilibrium margin

$$p(Q^*) - c = \frac{-(1 - s_1^*)Q^*p'(Q^*)}{n - 1}.$$

Combining this with expression in (5) for the regulated firm yields the latter's partial derivative as

$$\frac{\partial \pi_1(q_1^*, Q_{-1}^*)}{\partial q_1} = \frac{Q^* p'(Q^*)}{n-1} (n-3 + (n+1)s_1^*).$$

First, let $n = 3$ and suppose that $s_1^* > 0$. The partial derivative is $2Q^* p'(Q^*) s_1^* < 0$. With complementary slackness it follows that $s_1^* = 0$, contradicting the assumption that $s_1^* > 0$. Both $\frac{\partial \pi_1(q_1^*, Q_{-1}^*)}{\partial q_1}$ and q_1^* equal zero. Second, let $n > 3$. It follows that $\frac{\partial \pi_1(q_1^*, Q_{-1}^*)}{\partial q_1} < 0$, as $n-3 + (n+1)s_1^* > 0$, so $s_1^* = 0$ by complementary slackness. Thus for $n \geq 3$ the regulated firm sets zero output in the competitive market. Perhaps surprisingly this is general and does not depend on the shape of demand.

Proposition 2. *Suppose there are at least three firms in the competitive market without regulation. Uniform pricing induces the regulated firm not to sell in the competitive market.*

Uniform pricing ensures that the common price equals that with $n-1$ firms, $p(Q(n-1))$. Again social welfare is higher with uniform pricing. The competitive market loses one firm, while it is as if the monopoly market gains $n-2$ firms. Because $n-2 \geq 1$ the gain in welfare in the monopoly market exceeds the loss in the competitive market.

Proposition 3. *Suppose the regulated firm is subject to uniform pricing and there are at least three firms in the competitive market. Social welfare is higher with uniform pricing than with price discrimination.*

Proof. Welfare is $2W(n-1)$ with uniform pricing and is $W(1) + W(n)$ with discrimination. Welfare with uniform pricing is higher:

$$2W(n-1) \geq 2W\left(\frac{1+n}{2}\right) > W(1) + W(n).$$

The weak inequality holds because $n \geq 3$ implies that $n-1 \geq \frac{1+n}{2}$. The strict inequality holds by strict concavity of welfare in n . \square

Uniform pricing ensures that some of the benefit of competition is transferred to the monopoly market. Concavity of welfare in the number of firms guarantees that welfare is higher when $n \geq 3$. The uniform pricing requirement leads to a weakly higher average number of firms and the effective number of firms in each market is the same. This is analogous to a lottery with a weakly higher mean and zero spread that second-order stochastically dominates another lottery with a weakly lower mean and positive spread.

It might be natural to assume that the uniform pricing requirement applies only if the regulated firm has a strictly positive output in the competitive market. This would be a regulatory mistake. When $n \geq 3$ the regulated firm would choose not to supply the competitive market, allowing it to set the monopoly output in the monopoly market. Welfare would be $W(1) + W(n-1)$ which is below discriminatory welfare of $W(1) + W(n)$.

The analysis in this section can be generalized to allow for different market sizes and a different number of competitive markets: see Appendices B and C. Concavity of social welfare in the number of firms is the key.

2.3 Uniform pricing and entry

Return to the case where $n = 2$ and suppose the rival has an entry cost, $F > 0$. Assume that the rival would be profitable with uniform pricing but would not be profitable with the symmetric duopoly, with

$$\frac{2}{3}\Pi(1.5) \geq F > \frac{1}{2}\Pi(2).$$

The regulated firm does not incur an entry cost. The move order is as follows. First, the regulator requires uniform pricing or allows discrimination; second, the rival firm decides whether to enter or not, incurring its entry cost F if it does so; third, the firms choose their outputs. In this subsection the opposite to uniform pricing will be called no regulation.

The equilibrium strategies of each firm are now characterized. When uniform pricing is required and the rival has entered the regulated firm sets output of $Q(1.5)$ in its monopoly market and $\frac{1}{3}Q(1.5)$ in the duopoly market. The firm sets the monopoly output, $Q(1)$, in each market if the rival has not entered, whether or not uniform pricing is required. If there is no regulation and the rival has entered then the firm would set $Q(1)$ in its monopoly market and $\frac{1}{2}Q(2)$ in the duopoly market.

Under uniform pricing the rival enters, setting output of $\frac{2}{3}Q(1.5)$. It does not enter if there is no regulation. If the rival were to enter at stage two under no regulation then it would set output of $\frac{1}{2}Q(2)$ in the final stage.

With no regulation the prices are the same in each market and equal to the monopoly price, despite there being no requirement for equal prices. The regulated firm sets the monopoly output in the potentially competitive market, but would set the symmetric Cournot output if the rival entered. This in turn ensures that the rival does not enter. In the language of the entry deterrence literature entry is

blockaded. With uniform pricing the regulated firm competes less aggressively to limit the reduction in profit in its monopoly market, and this gives room for the rival to enter.

In the first stage with no regulation there is no subsequent entry. Both markets are monopolized and total welfare is $2W(1)$. Uniform pricing would lead to entry with aggregate social welfare of $2W(1.5) - F$. Thus the regulator requires uniform pricing if $2(W(1.5) - W(1)) > F$ and has no regulation if $2(W(1.5) - W(1)) < F$.

Whether uniform pricing or no regulation is better for social welfare depends on the curvature of the inverse demand function. When demand is sufficiently convex uniform pricing is socially preferred. The aggregate welfare increase with the move from one firm to one and a half firms is larger than the maximum value of the fixed cost that is consistent with the rival entering. Concavity of welfare in n is again the key. This provides a lower bound to the welfare change that itself can be bounded below.

Proposition 4. *Suppose the rival has an entry cost such that entry is profitable if and only if uniform pricing is required. If $\sigma(Q(1.5)) \geq 1.5$ then uniform pricing is optimal for the regulator.*

Proof. By strict concavity of $W(n)$, $W(1.5) - W(1) > W'(1.5)(1.5 - 1)$, and thus

$$2(W(1.5) - W(1)) > W'(1.5).$$

From $W'(n) = (p - c)Q'(n)$ and (2) the lower bound is

$$W'(1.5) = \frac{2}{3}\Pi(1.5)\frac{1}{2.5 - \sigma(Q(1.5))}.$$

Uniform pricing is profitable, by assumption, so $\frac{2}{3}\Pi(1.5) \geq F$ and

$$W'(1.5) \geq F\frac{1}{2.5 - \sigma(Q(1.5))} \geq F,$$

with the final inequality following from the assumption that $\sigma(Q(1.5)) \geq 1.5$. Thus $2(W(1.5) - W(1)) > F$ and uniform pricing is optimal. \square

There are two natural applications of Proposition 4. First, when $p(Q) = e^{-Q}$ and $c = 0$ the equilibrium value of inverse demand curvature with uniform pricing is $\sigma(Q(1.5)) = 1.5$. Second, when demand has a constant elasticity, $\varepsilon > 1$, curvature is constant and is $\sigma = 1 + \frac{1}{\varepsilon}$. If the elasticity is at most 2 then $\sigma \geq 1.5$.

The condition in Proposition 4 is sufficient but not necessary. If direct demand is exponential, with inverse demand $p(Q) = -\ln(Q)$ for $Q < 1$ and thus $\sigma = 1$, then the

optimal policy is uniform pricing. Direct calculation shows that $2(W(1.5) - W(1)) > \frac{2}{3}\Pi(1.5)$.

With linear demand, in contrast, no regulation yields higher social welfare. Monopolization of each market is better than having more competitive markets, with effectively 1.5 firms, because of the high entry cost. The welfare gain is below the duopoly profit, $2(W(1.5) - W(1)) < \frac{\Pi(2)}{2}$, and because $\frac{\Pi(2)}{2} < F$ no regulation is the preferred policy.²

2.4 Uniform pricing and exit

Again assume that $n = 2$ and now allow for the possibility that the regulated firm might choose to exit the competitive market before outputs are chosen.

Suppose there is uniform pricing. If the regulated firm exits it can earn the monopoly profit $\Pi(1)$, because its exit from the competitive market allows the rival to become a monopolist there. The uniform pricing constraint is satisfied with the monopoly price being set in each market. Its total profit if it remains in the competitive market is $\frac{4}{3}\Pi(1.5)$, as it earns all the profit in the monopoly market and one-third of the profit in the duopoly market. The regulated firm does not exit if demand is log-concave, an assumption that is often made for Cournot models. See Appendix D for details. When, however, demand is not log-concave, it is possible that $\Pi(1) > \frac{4}{3}\Pi(1.5)$ so the regulated firm prefers to exit. This holds, for example, with a constant price elasticity of demand that is close to 1. The uniform pricing requirement has the perverse effect of inducing monopoly in each market and thus has a negative welfare effect. The circumstances, however, are limited: this works only if three conditions hold jointly. There must be no more than two firms, total profit with positive output in the competitive market must be lower than monopoly profit, and the regulated firm must be able to commit in advance not to supply the regulated market.

2.5 Uniform pricing as a result of market integration

Now suppose that uniform pricing arises because the markets have been integrated. Each firm can supply both markets. The monopoly market will be called A and the competitive market is B . Firm 1 already supplies both markets. With integration the

²Without loss of generality let $p(Q) = 1 - Q$ and $c = 0$. The welfare gain is $2(W(1.5) - W(1)) = 0.09$ which is strictly below the symmetric duopoly profit of $\frac{\Pi(2)}{2} = \frac{1}{9}$.

firms based in B also supply consumers in A . Each market has a Cournot equilibrium with n firms, with output $Q(n)$ and price $p(Q(n))$.

To assess the welfare effects of market integration suppose that firm 1 is wholly owned by the citizens of A , and the $n - 1$ firms in B are owned by the citizens of B . The welfare measure includes profits earned in the other market and subtracts profits earned in the local market by firms based in the other market. Denote consumer surplus as a function of the number of firms as $S(n) \equiv U(Q(n)) - p(Q(n))Q(n)$ and note that $U'(Q) = p$. The effect of an increase in the number of firms on consumer surplus is $S'(n) = -Qp'(Q)Q'(n) > 0$.

In A the price falls to the Cournot level $p(Q(n))$. In B the price is unchanged. Aggregate welfare must increase. Total output increases across the two markets and is now efficiently distributed. Total profit falls by the difference between monopoly profit and aggregate Cournot profits in one market.

Each firm in B earns twice the profits they did before integration because they now sell in A as well. Consumers in B are unaffected. In A the effects are more complicated. Consumers gain because the price falls. The profits of firm 1, owned by the citizens of A , are reduced. Welfare for the consumers and the firm in A is consumer surplus plus the profits of the firm, including its profit earned in market B . With discrimination and separate markets this welfare measure is $S(1) + \Pi(1) + \frac{\Pi(n)}{n}$ where $S(1)$ is consumer surplus with a monopoly, $\Pi(1)$ is the monopoly profit and $\frac{\Pi(n)}{n}$ is firm 1's Cournot profit earned in B . With market integration welfare for consumers and the firm is $S(n) + 2\frac{\Pi(n)}{n}$, which is consumer surplus plus the Cournot profits earned in the two markets. The welfare change for A is

$$G(n) = S(n) + \frac{\Pi(n)}{n} - S(1) - \Pi(1),$$

To determine the sign of $G(n)$ note first that $G(1) = 0$. Log-convexity of demand implies that the derivative $G'(n)$ is non-negative at $n = 1$ and is strictly positive for $n > 1$, so $G(n) > 0$ for $n > 1$.

Proposition 5. *Integration raises the welfare of consumers and the firm in the monopoly market if demand is log-convex for $n \geq 1$.*

Proof. The first-order condition (1), the output derivative (2), and the expression for $S'(n)$ together imply that

$$G'(n) = \frac{S'(n)}{n^2} [(n - 1)^2 + \sigma(Q(n)) - 1].$$

This has the same sign as the square-bracketed expression. Log-convexity for $n \geq 1$ means that $\sigma(Q(1)) \geq 1$ so $G'(1) \geq 0$. For $n > 1$ the square-bracketed expression is strictly positive so $G'(n) > 0$. Thus $G(n) = \int_1^n G'(\tilde{n})d\tilde{n} > 0$. \square

With log-convex demand the increase in consumer surplus exceeds the loss of profits whatever the number of firms. The condition in Proposition 5 holds for demands with constant $\sigma \in [1, 2)$, including those with constant elasticities, which are log-convex everywhere, and for $p(Q) = e^{-Q}$ with $c = 0$, for which $\sigma(Q(n)) = n$ so demand is log-convex for $n \geq 1$.

When demand is not log-convex the effect of market integration on welfare in A depends on the number of firms. With linear demand the welfare change can be calculated. For $p(Q) = 1 - Q$ and $c = 0$ output is $Q(n) = \frac{n}{n+1}$, consumer surplus is $S(n) = \frac{1}{2}Q(n)^2 = \frac{1}{2}\frac{n^2}{(n+1)^2}$, and total profit is $\Pi(n) = \frac{n}{(n+1)^2}$ so the welfare change is

$$G(n) = \frac{(n-5)(n-1)}{8(n+1)^2}.$$

This is negative for $n = 2, 3$ and 4 and zero when $n = 5$. For values of n above 5 the welfare change is positive. In general as n rises towards infinity consumer surplus increases towards the level that applies with perfect competition. This equals social welfare with perfect competition, which in turn is higher than welfare with monopoly, $S(1) + \Pi(1)$, so $G(n)$ is positive.

3 Price-setting with differentiated products

Similar results obtain in a standard model with two firms setting prices for differentiated products. The framework is similar to that used by Feng, Li, and Shuai (2023), who examine a model in which an incumbent firm commits to uniform pricing to induce an entrant only to enter one market. Here the regulator determines whether the firm that operates in both markets should price uniformly. There are two markets. The regulated firm, 1, sells its good in both markets, while firm 2 sells its good only in the competitive market. The utility function for the representative consumer in the competitive market is quadratic:

$$U(q_1, q_2) = q_1 + q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2)$$

where $b \in [0, 1)$ and q_1 and q_2 are the outputs of firms 1 and 2 for that market. The resulting direct demands, when both are positive, for prices p_1 and p_2 , are

$$q_1(p_1, p_2) = \frac{1}{1-b^2} (1-b-p_1+bp_2) \quad (6)$$

$$q_2(p_1, p_2) = \frac{1}{1-b^2} (1-b+bp_1-p_2). \quad (7)$$

The diversion ratio for firm 1 is the fraction of the sales lost when it raises its price that are transferred to its rival, i.e. $\frac{\partial q_2/\partial p_1}{-\partial q_1/\partial p_1} = b$. Marginal cost is zero for each firm. The standard Bertrand equilibrium in the competitive market has each firm $i = 1, 2$ choosing p_i to maximize $\frac{p_i(1-b-p_i+bp_j)}{1-b^2}$ with the best response being $p_i = \frac{1-b+bp_j}{2}$. The common price in the Bertrand equilibrium is $\frac{1-b}{2-b}$. This equals $\frac{1}{2}$ when $b = 0$ and goes to zero as b goes to 1. The diversion ratio is the natural measure of how competitive the duopoly market is. From now on assume that $b > 0$.

In the monopoly market only firm 1 operates. The utility function of the representative consumer becomes $U(Q, 0) = Q - \frac{1}{2}Q^2$ where Q is firm 1's output in that market. Marginal utility is $1 - Q$ for $Q \leq 1$. The demand function is $Q(p) = 1 - p$ for $0 \leq p \leq 1$. The monopoly price is $\frac{1}{2}$.

The firms set prices after the regulator has decided whether to require firm 1 to price uniformly or not, and only pure strategies are considered. When discrimination is allowed the regulated firm sets the monopoly price, $\frac{1}{2}$, in the monopoly market. In the competitive market there is a symmetric duopoly equilibrium, with $p_1 = p_2 = \frac{1-b}{2-b}$.

When uniform pricing is required the regulated firm chooses the common price p to maximize its concave profit function, for a given p_2 ,

$$p \left(1 - p + \frac{1}{1-b^2} (1-b-p+bp_2) \right).$$

Firm 1's best response is

$$p = \frac{(1-b)(2+b) + bp_2}{2(2-b^2)}.$$

For each value of $p_2 < 1$ firm 1's best price is higher than when there is no price constraint. The equilibrium prices, using firm 2's best response, are

$$p = \frac{(1-b)(4+3b)}{8-5b^2} \quad (8)$$

$$p_2 = \frac{(1-b)(4+2b-b^2)}{8-5b^2}. \quad (9)$$

Firm 1's equilibrium price, p , is above that of firm 2 and below the monopoly price. Each price is above the equilibrium price in the competitive market, $\frac{1-b}{2-b}$, that is set when there is no pricing constraint. Each firm earns higher profits in the competitive market than when the regulated firm discriminates. In the limit as $b \rightarrow 1$ the prices tend towards zero, i.e. marginal cost, despite the fact that firm 1 provides limited competition.

As in the Cournot model with two firms, uniform pricing induces higher prices in the competitive market and a lower price in the monopoly market. In the competitive market firms 1 and 2 set different prices, whereas the equilibrium prices with discrimination are equal. To compare aggregate consumer surplus with uniform pricing and discrimination use the lower bound to the consumer surplus change that follows from convexity of consumer surplus in prices. This lower bound is the initial output times the reduction in price, summed across the goods. The outputs with discrimination are $Q = \frac{1}{2}$ and $q_1 = q_2 = \frac{1}{(1+b)(2-b)}$, and in the move from discrimination to uniform pricing the lower bound to the change in consumer surplus is

$$\frac{1}{2} \left(\frac{1}{2} - p \right) + \frac{1}{(1+b)(2-b)} \left(2 \frac{(1-b)}{2-b} - p - p_2 \right).$$

Using the equilibrium prices in (8) and (9) it can be shown that for all values of $b \in (0, 1)$ the lower bound is strictly positive. Aggregate consumer surplus is higher with uniform pricing than with discrimination.

Varian (1985) adds the change in profits to the consumer surplus lower bound to yield the lower bound to the change in welfare. This equals the new profit margin times the change in output, summed across the goods. The output change in the monopoly market, in the move from discrimination to uniform pricing, is $1 - p - \frac{1}{2} = \frac{1}{2} - p$. The lower bound to the total welfare change is

$$p \left(\frac{1}{2} - p \right) + p \left(q_1(p, p_2) - \frac{1}{(1+b)(2-b)} \right) + p_2 \left(q_2(p, p_2) - \frac{1}{(1+b)(2-b)} \right),$$

where the prices are given in (8) and (9) and the outputs in the competitive market are found using (6) and (7). For all values of $b \in (0, 1)$ the lower bound is strictly positive. Aggregate social welfare is higher with uniform pricing than with discrimination.

Now suppose that the regulated firm can choose whether or not to supply the competitive market. If it decides not to supply then it earns its monopoly profit, 0.25, in market 1. It only supplies the competitive market if its total profit is at least equal to 0.25. For a high enough value of the diversion ratio, specifically for

$b > 0.827$, the regulated firm chooses not to supply the competitive market. This is intuitive: the higher the value of the diversion ratio the closer the equilibrium prices are to marginal cost and the lower the profits in the competitive market. For society the uniform pricing constraint has the undesirable effect when b is in this range of inducing both markets to be monopolized, with the price being $\frac{1}{2}$ in each market. As in the Cournot model if the regulated firm chooses to supply the competitive market the uniform pricing requirement is good for welfare, but if it can commit not to supply that market the welfare effect is negative when the diversion ratio is high enough. Both markets are monopolized. An alternative way to model this issue would be to allow for mixed strategies.

4 Price-setting with identical products

Now the goods are identical and the firms set prices. With discrimination firm 1 sets the monopoly price, p^m , in the monopoly market and each firm sets price equal to marginal cost for the competitive market. Suppose that uniform pricing is required and that $n \geq 3$, so firm 1 has at least two rivals in the competitive market. A unique pure-strategy equilibrium exists with the regulated firm setting the monopoly price for both markets, thus satisfying the uniform pricing regulation. It sells nothing, however, in the competitive market because each rival continues to price at marginal cost. Competition between the rivals keeps their prices equal to marginal cost. Thus there are no welfare consequences from the uniform pricing requirement. There are parallels with the Cournot model as firm 1 sells nothing in the competitive market and raises its price there, but this has no important effect.

When $n = 2$ there is no pure strategy equilibrium when uniform pricing is required. Firm 1 can guarantee to earn its monopoly profit by setting the monopoly price for both markets. There is a mixed strategy equilibrium with both firms randomizing over the interval $[\tilde{p}, p^m]$ where $\tilde{p} > c$ is defined by $2(\tilde{p} - c)D(\tilde{p}) = (p^m - c)D(p^m)$. Neither firm will price below \tilde{p} : firm 1, for example, would do better to jump to p^m as profit is increasing in price for $p \in [c, p^m)$. This is a special case of the duopoly model of pricing with captive customers of Armstrong and Vickers (2019). Here a single firm has captive customers. Proposition 1 of Armstrong and Vickers (2019) states that expected total profit with uniform pricing is at least as high as with discrimination.³

³The welfare results of Armstrong and Vickers (2019) for duopoly do not apply: they depend on symmetry of the firms or firm 1's captive market being relatively large.

5 Conclusion

In general a uniform pricing requirement is good for social welfare when the amount of competition differs across markets. The benefit from a lower price in the monopoly market more than offsets the loss from reduced competition in the market where the regulated firm is induced to compete less aggressively. The main results from the Cournot model are replicated in the Bertrand model with differentiated products. The Cournot duopoly model can be applied to consider the role of regulation in entry promotion. There are cases in which discrimination is socially preferred, but the conditions for this are special and limited.

The classic analysis of monopoly third-degree price discrimination has welfare effects that can go either way — see Varian (1985) and Aguirre, Cowan, and Vickers (2010). In contrast in the models in the current paper there is a general and robust argument for uniform pricing.

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Appendices

A Concavity of total output and welfare

Differentiating the output derivative (2) gives

$$\frac{Q(n)Q''(n)}{(Q'(n))^2} = -2n + \sigma(Q(n)) + n\sigma'(Q)Q'(n).$$

Output is concave if this is non-positive, which holds if

$$\sigma'(Q(n))Q'(n) \leq 2 - \frac{\sigma(Q(n))}{n}.$$

The right-hand side of this condition is strictly positive by the fact that the second-order condition holds. The sufficient condition for concavity is technical but straightforward to apply. It holds if inverse demand curvature is constant, so $\sigma'(Q) = 0$, and for the class of demand functions based on statistical distributions of valuations, such as logistic and probit demands, for which $\sigma'(Q) < 0$ (Fabinger and Weyl, 2018). The condition can also be shown to hold for demand functions with constant $\alpha \leq 1$.⁴

If output is concave then social welfare is strictly concave in the number of firms. Formally, utility is $U(Q)$, which is increasing and strictly concave, with first derivative $U'(Q) = p$. Social welfare is utility minus costs. As a function of the number of firms social welfare is $W(n) \equiv U(Q(n)) - cQ(n)$. This is increasing in n , as $W'(n) = (p(Q(n)) - c)Q'(n) > 0$, and strictly concave if $Q''(n) \leq 0$, as $W''(n) = (p(Q(n)) - c)Q''(n) + p'(Q(n))(Q'(n))^2$ with the first term being non-positive and the second term being strictly negative.

Now suppose that $\sigma'(Q(n))Q'(n) < 3 - \frac{\sigma(Q(n))}{n}$, which is a weaker assumption than the one that implies that output is concave. The second derivative of welfare is

$$W''(n) = -\frac{p'(Q)(Q'(n))^2}{n} \left(\frac{Q(n)Q''(n)}{(Q'(n))^2} - n \right)$$

which has the same sign as the expression in large brackets, which is $-3n + \sigma(Q(n)) + n\sigma'(Q(n))Q'(n)$. This is strictly negative under the weaker assumption. An example is when $p(Q) = (1 + Q)^{-\gamma}$ for $\gamma > n$ and $c = 0$. Amir and Lambson (2000) discuss this demand function, for which total output is strictly convex in the number of firms and outputs are strategic complements. The expression in large brackets is negative for $\gamma > 2$, which holds because $\gamma > n \geq 2$.

⁴For example when $c = 0$ and α is constant, the equilibrium value of inverse demand curvature is $\sigma(Q(n)) = \alpha n$, so $\sigma'(Q(n))Q'(n) = \alpha$, and output is concave when $\alpha \leq 1$.

B Different market sizes

Direct demand in the competitive market is $D(p)$, and demand in the monopoly market at the same price is $AD(p)$ for size parameter $A > 0$. At a common price social welfare in the monopoly market is A times welfare in the competitive market. With discrimination welfare in the competitive market with n symmetric firms is $W(n)$ and welfare in the monopoly market is $AW(1)$, so total welfare is $AW(1) + W(n)$. The uniform pricing requirement is $\frac{Q}{A} = q_1 + Q_{-1} = Q_c$ where Q is the output in the monopoly market and $Q_c = q_1 + Q_{-1}$ is output in the competitive market. The regulated firm chooses q_1 to maximize $(p(q_1 + Q_{-1}) - c)((1 + A)q_1 + AQ_{-1})$. The partial derivative of 1's profit with respect to its output in the competitive market, q_1 , has the opposite sign to

$$A(n - 2) - 1 + (n + A)s_1^*.$$

When $1 - A(n - 2) > 0$, for example when $n = 2$ or when $n > 2$ and A is sufficiently low, the interior solution for 1's share in the competitive market is

$$s_1^* = \frac{1 - A(n - 2)}{n + A}$$

which is decreasing in A . The first-order conditions may be added to give

$$p(Q_c) - c + \left(\frac{A + 1}{A + n}\right) Q_c p'(Q_c) = 0.$$

It is as if the number of firms is $\frac{A+n}{A+1}$. Welfare with the interior solution is $(A + 1)W\left(\frac{A+n}{A+1}\right)$. Strict concavity of welfare implies that when the regulated firm has positive output in the competitive market welfare is higher with uniform pricing: $(A + 1)W\left(\frac{A+n}{A+1}\right) > AW(1) + W(n)$, as in Proposition 1.

If $A(n - 2) - 1 \geq 0$, which requires $n \geq 3$, the regulated firm sets zero output in the competitive market, as in Proposition 2. Welfare with uniform pricing is $(A + 1)W(n - 1)$ and with discrimination welfare is $AW(1) + W(n)$. A similar argument to that used for Proposition 3 gives:

$$(A + 1)W(n - 1) \geq (A + 1)W\left(\frac{A + n}{A + 1}\right) > AW(1) + W(n),$$

with the weak inequality holding when $A(n - 2) - 1 \geq 0$ and the strict inequality holding by strict concavity of social welfare.

C More than one competitive market

Suppose that there is more than one competitive market, while there is a single monopolistic market. The total number of markets is $m \geq 3$. Suppose that $n = 2$ for each competitive market, the regulated firm faces a uniform pricing requirement, and chooses a positive output for the competitive markets. The partial derivative of the regulated firm's profit with respect to its competitive output has the opposite sign to

$$1 - m + (2m - 1)s_1^*,$$

so its optimal share in each competitive market is

$$s_1^* = \frac{m - 1}{2m - 1}.$$

The optimal share is below one half, and it tends to one half as m increases. Add the first-order conditions so that total output is characterized by

$$p(Q^*) - c + \frac{m}{2m - 1}Q^*p'(Q^*) = 0$$

so it is as if each market has $\frac{2m-1}{m}$ firms. Social welfare with discrimination is $W(1) + (m - 1)W(2)$. With the uniform price welfare is $mW\left(\frac{2m-1}{m}\right) = mW\left(\frac{1}{m} + \frac{(m-1)}{m}2\right)$. As before strict concavity of welfare in n yields the result that welfare is higher with the uniform price.

Now suppose that in each of the $m - 1$ competitive markets there are $n \geq 3$ firms. The partial derivative of the regulated firm's profit with respect to its competitive output has the opposite sign to

$$n - m - 1 + ((m - 1)n + 1)s_1^*,$$

so if $n \geq m + 1$ and $s_1^* > 0$ the regulated firm does better by setting $s_1^* = 0$, as in Proposition 2. With $n \geq m + 1$ welfare with uniform pricing is $mW(n - 1)$. With discrimination welfare is $W(1) + (m - 1)W(n)$. As in Proposition 3 uniform pricing is better for welfare:

$$mW(n - 1) \geq mW\left(\frac{1 + (m - 1)n}{m}\right) > W(1) + (m - 1)W(n),$$

with the weak inequality holding because $n \geq m + 1$ implies that $n - 1 \geq \frac{1 + (m - 1)n}{m}$. As usual strict concavity of welfare gives the strict inequality.

D Equilibrium profit and monopoly profit

Equilibrium profit when $n = 2$ is $\frac{4}{3}\Pi(1.5)$. The difference between equilibrium profit and monopoly profit is

$$\frac{4}{3}\Pi(1.5) - \Pi(1) = \Pi(1.5) \left(\frac{4}{3} - \frac{\Pi(1)}{\Pi(1.5)} \right).$$

The profit ratio, $\frac{\Pi(1)}{\Pi(1.5)}$, can be characterized using a similar technique to that used by Faulí-Oller (1997) to analyze merger profitability. The derivative of industry profit with respect to the number of firms is

$$\Pi'(n) = -\Pi(n) \frac{n-1}{n(n+1-\sigma(Q(n)))}.$$

Dividing by $-\Pi(n)$ and integrating over $n = [1, 1.5]$ gives

$$\int_1^{1.5} -\frac{\Pi'(\tilde{n})}{\Pi(\tilde{n})} d\tilde{n} = \ln \left(\frac{\Pi(1)}{\Pi(1.5)} \right) = \int_1^{1.5} \frac{\tilde{n}-1}{\tilde{n}(\tilde{n}+1-\sigma(Q(\tilde{n})))} d\tilde{n},$$

so the profit ratio is

$$\frac{\Pi(1)}{\Pi(1.5)} = \exp \left[\int_1^{1.5} \frac{\tilde{n}-1}{\tilde{n}(\tilde{n}+1-\sigma(Q(\tilde{n})))} d\tilde{n} \right].$$

When $\sigma(Q) \leq 1$ for all Q the integrand in the profit ratio expression satisfies

$$\frac{\tilde{n}-1}{\tilde{n}(\tilde{n}+1-\sigma(Q(\tilde{n})))} \leq \frac{\tilde{n}-1}{\tilde{n}^2} = \frac{1}{\tilde{n}} - \frac{1}{\tilde{n}^2}$$

so the integral is bounded above by

$$\int_1^{1.5} \left(\frac{1}{\tilde{n}} - \frac{1}{\tilde{n}^2} \right) d\tilde{n} = \ln 1.5 - \frac{1}{3}.$$

The resulting upper bound to the profit ratio is $1.5e^{-\frac{1}{3}} = 1.0748 < \frac{4}{3}$. Thus equilibrium profit is above monopoly profit when demand is log-concave.

When demand is not log-concave the profit comparison can go either way. With a high enough constant inverse demand curvature, specifically $\sigma \in [1.939, 2)$, the equilibrium profit is below monopoly profit. This applies, for example, if the price elasticity is constant and lies in the range $(1, 1.065]$. The profit ratio for constant $\sigma \neq 1$ is

$$\frac{\Pi(1)}{\Pi(1.5)} = 1.5^{\frac{1}{\sigma-1}} \left(\frac{2.5-\sigma}{2-\sigma} \right)^{\frac{2-\sigma}{1-\sigma}}.$$

For $\sigma = 1$ the profit ratio is $1.5e^{-\frac{1}{3}} = 1.0748$ which is the limit of the general expression as $\sigma \rightarrow 1$. The profit ratio for constant inverse demand curvature is an increasing and strictly convex function of curvature. The limit of the profit ratio, $\frac{\Pi(1)}{\Pi(1.5)}$, as $\sigma \rightarrow 2$ is 1.5, which exceeds $\frac{4}{3}$.