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François de Foix de Candalle: Euclidean authorship in the sixteenth century

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This article discusses the work of François de Foix de Candalle and its importance for the early modern Euclidean tradition. It provides an outline of Foix's life, education and career, and it describes in particular his editions of the *Elements* published in 1566 and 1578. Those editions made substantial additions to the text, increasing the number of 'books' from fifteen to eighteen: by far the largest addition to the Euclidean text since late antiquity. The article goes on to discuss Foix's intentions and his practices as an editor, and the reception of Foix's *Elements*. His book XVI remained in print in various languages well into the eighteenth century, making Foix an important and influential contributor to the early modern tradition of the *Elements*, whose work helps to illustrate the fluidity of that tradition as well as the character of early modern mathematical editorship more generally.

Introduction

First printed in Latin in 1482 and in Greek in 1533, Euclid's *Elements* appeared in over two hundred different versions during the sixteenth and seventeenth centuries, as editors and printers responded to an apparently insatiable demand not just for the *Elements of Geometry* but for new and improved versions of those elements.

Since Heiberg (1883–1916), versions of the *Elements* have included as authentic only Books I–XIII, although many earlier versions of the text contained the two supplementary books XIV and XV. Modern scholarship places their origin in late antiquity – respectively the second and the sixth–seventh centuries (Vitrac and Djebbar 2011–12) – but they were sporadically present in versions of the text that aspired to completeness throughout the surviving Greek manuscripts from the ninth century onwards and in print from the first (Latin) edition of 1482 through the following three centuries. In some texts they were clearly marked as of different or later authorship than what preceded them; elsewhere they were at least tacitly ascribed to Euclid himself. Their length varies enormously from one text to another (Wardhaugh 2024), but their intention is in every version to extend the treatment of solids undertaken in Book XIII with a number of additional propositions on the same subject.

Also present in a number of early modern printed texts of the *Elements* were further continuations, books numbered XVI–XVIII, whose origin, contents and trajectory have received less attention from modern scholarship. Books XVII and XVIII enjoyed only a small number of printed appearances: three in total, between 1578 and 1644. But Book XVI was a more long-lived and widespread piece of Euclideana. It was first printed in 1566 as part of the edition of the *Elements* by François de Foix de Candalle, and it remained visible in print for nearly two centuries, in translations into vernacular languages as well as in its original Latin.

François de Foix de Candalle

The author of *Elements* XVI–XVIII was François de Foix de Candalle (1512–1594). His name appears in a variety of forms in the modern scholarship, and a still wider in early modern sources: he is 'Flussas' to Billingsley (1570, fol 445v) and 'Francisco Flussate Candalla' to Clavius (1574, vol 2, 275). The two of his own title pages that are in French give his name as 'François, Monsieur de Foix, de la famille de Candalle', and one contemporary letter (see Larroque 1877, 13n3, 20) is addressed to 'Monseigneur François Monsieur de Foix de Candalle'. He will be 'Foix' for the remainder of this essay.

His family was descended from the ninth-century Dukes of Gascony, the comté de Foix originating in 1012 (Balteau et al. 1929, vol 14, col 210). François' ancestors fought on the English side during the Hundred Years' War and at Agincourt; his great-grandfather was created Earl of Kendal in 1446. He gave his allegiance to France a few years later, but his descendants retained the title, styling themselves Comtes 'de Candal(l)e'. Pierre de Foix, cardinal and papal legate, was involved with both the university of Avignon and the Toulouse Collège de Foix in the fifteenth century. Probably the most prominent member of the family was Françoise de Foix, Comtesse de Châteaubriand and companion of François I of France early in the sixteenth century.

François' elder brother Fréderic held the family's main titles: Comte de Candalle, d'Astarac et de Bénauges. For reasons that are not clear, modern reference books and library catalogues regularly refer to François himself as Comte de Candalle (Heath 1926, vol 1, 104; Thomas-Stanford 1926, 10; Faivre 2005). He was not; the title passed directly from his brother to his nephew. The only one of the family's titles he seems to have held was Captal de Buch (Larroque 1877, 15, 20; Harrie 1975, 29).

The family was a powerful one, connected by marriage to royal and noble houses including those of Valois, Bourbon, and Hapsburg (Harrie 1975, 29). François appears to have served at the court of Navarre, perhaps at least intermittently from his youth until his 40s, in roles including privy councillor to the King ('conseiller du Roy en son conseil privé') (Larroque 1877, 13n2, n3). His formal education was as a result rather limited, but he undertook on his own initiative a scholarly lifestyle and acquired a knowledge of Latin, Greek, and Hebrew, the liberal arts and theology, sufficent to impress his contemporaries greatly (Harrie 1975, 33). In this, he was assisted by the existence of the College de Guyenne at Bordeaux, which was responsible for a concentration of intellectuals in Bordeaux and which enabled Foix to turn his home at Puy-Paulin into a literary salon of wide reputation (Harrie 1975, 47). His circle included the mathematician Elie Vinet as well as Michel de Montaigne and Joseph Scaliger (Harrie 1975, *passim*).

Mathematics was evidently one of his most constant intellectual interests, and his invention of machines and other devices was sufficiently notorious and impressive to merit a visit from the King of Navarre in 1584 (Larroque 1877, 17). He invented or constructed wheels, pulleys, levers, a cannon and an organ stop. He interested himself in the movement of the tides and built a special clock to assist with the work; he determined the specific gravities of metals; he ascended one of the highest peaks of the Pyrenees in order to measure its height (Harrie 1975, 37). His opinion was solicited by the king about the Gregorian reform of the calendar, although his views, transmitted back to the Pope, were not taken up (Harrie 1975, 38–41).

Another side to Foix's intellectual curiosity was hermeticism (Faivre 2005). In 1471, Marsilio Ficino had published a Latin translation of the collection of treatises attributed to the legendary Hermes Trismegistus, known as the *Pimander* or *Poimandres*. The Greek text printed by Turnebus in 1554 was the basis for Foix's own Greek–Latin and Greek–French editions twenty years later. He also prepared a French-only edition (de Foix de Candale 1579) enlarged by long philosophical and theological discussions; the publication of this material seems to have owed something to his sister, Jacqueline de Foix (de Foix de Candale 1579, a4^{v}). This exposition of Christian hermeticism brought him fame in his own day and has been judged one of the most representative works of its kind: 'one of the most remarkable texts in Western esoteric currents since the early Renaissance, even until the present time' (Faivre 2005).

Foix also pursued the practical side of hermeticism; Montaigne mentions him as 'deeply immersed' in the 'quest for the philosophers' stone' (1588, 585), and at his chemical laboratory he also attempted to make silver ingots as well as to discover the composition of substances (Harrie 1975, 43, 182–183). His great success in this regard was the 'Eau magistrale' or 'Eau de Candalle', an alcoholic concoction brewed from plant and fruit matter – more than sixty ingredients in all – in spirits of wine (Balteau et al. 1929; Delaunay 1955; Harrie 1975, 43). After nine days of maceration, heating in a bed of horse dung, and finally distillation, the product was said to be effective against any malady. He was personally convinced of its efficacy; his will provided for its continued production and distribution to the poor after his death. The formula was eventually sold by the family in the eighteenth century for an unknown sum (Delaunay 1955, 96).

The duties of his family pursued Foix into later life, long after the period when he had established his lifestyle and his reputation as an intellectual. His younger brother Christophe died in 1569 or 1570; he had been bishop of Aire, in Burgundy, a position the family evidently considered its property (Harrie 1975, 55). Foix was nominated to the see and, after a series of delays, took nominal control of it in 1576. Actual responsibility was delegated to the three vicars general of the diocese, and a subsequent request to appoint Foix a cardinal was rebuffed by Pope Sixtus V (with some violence, according to a contemporary account) (Harrie 1975, 58). Foix's well-established interest in theology found final expression in a treatise on the Blessed Sacrament, composed in the 1580s but never published (Larroque 1877, 20; Faivre 2005).

In 1591, Foix founded a chair of geometry at the college of Guyenne, specifying that the holder must have discovered a new proposition in elementary geometry and a new proposition concerning the regular solids (Larroque 1877, 14n1; Harrie 1975, 202n44). Three years later he died, leaving bequests to his family, his household, and the relief of the poor, and his library to the Augustinian convent at Bordeaux. He was buried at St Augustine's in Bordeaux, in a mausoleum erected by his nieces (Harrie 1975, 63 with n182; but see Larroque 1877, 25, 30 for Foix's own intentions). In a drawing of this or a proposed replacement made in the 1630s, geometrical shapes surmount the sarcophagus itself while Foix's praying figure looks down on the whole (Figure 1).[1] The tomb does not seem to have survived the Revolutionary period, although Bordeaux does retain a Rue de Candale to the present day (Harrie 1975, 63).

[1]Bibliothèque nationale de France, département Estampes et photographie, EST RESERVE VE-26 (N): http://catalogue.bnf.fr/ark:/12148/cb40309846p.

Figure 1. Design for the tomb of François de Foix de Candalle. Hermann Van der Hem, 1638. Bibliothèque nationale de France / public domain.

The *Elements* 'restored'

Foix's most enduring monument was his geometry. Concurrently with the three editions he prepared of the *Pimander*, he published two of Euclid's *Elements*. The first appeared in 1566, when Foix was 54. It was printed at Paris by the king's printer Jean le Royer and dedicated to the King of France, Charles IX. It is reported that Catherine de Medici, the queen mother, had encouraged the preparation of this new Latin version for the nominal purpose of the young king's education (Harrie 1975, 234 with note 54). Jean Bodin and Jean Dorat (poète royal) among others contributed Latin verses in praise of Foix and his work (de Foix de Candale 1566, a3^{v}–5^{v}; Harrie 1975, 52).

By this date there had been more than thirty printings of the *Elements*, in versions including the Campanine text of 1482, Zamberti's new translation of 1505 and the fresh version divergent from either by Georg Joachim Rhäticus and Joachim Camerarius in 1549 (Wardhaugh et al. 2020; Wardhaugh 2024). The text had also been printed in Greek (1533) and Italian (1543). Certain authors of this period saw the Euclidean *Elements* as a 'perfect body' of geometry, virtually free of defects of any kind; others wished to use it merely as a spur to the creation of a substantially new text, 'rummaging' its bowels in search of nuggets of true geometrical content (Goulding 2010, 177, 170, citing respectively Henry Savile and Pierre de la Ramée). Foix's approach was in a sense a compromise between the two positions, asserting the inadequacy of the existing printed texts without denying the beauty, purity, and unfailing correctness of the Euclidean geometry itself. Foix's prefatory matter lamented the inadequacy of earlier printed editions, accusing them in general terms of infelicities, solecisms, and falsehoods which justified his new attempt (de Foix de Candale 1566, $a6^r$–$c1^v$; see Malet 2012, 207). He explicitly styled his version a 'restoration' of Euclid from the obscurity into which others had sunk him. He admired above all the logical structure of the *Elements*, and he sincerely wished to remove the accretions, obscurities, and downright errors that he found in other printed versions of the text. His priority, therefore, was not eloquence but accuracy, of both geometrical content and logical form. In a prefatory letter to the reader, Foix mentioned another danger: that excessive sophistication in manipulating the text would result in multiple, perhaps incompatible Euclids (de Foix de Candale 1566, $c1^r$; see Goulding 2010, 156–157 on Foix's 'textual anxiety'). His own desire was to present just one: the true one.

The methods Foix adopted in the pursuit of these ends were typical of those of Euclidean editors in this period, who frequently seem to have used multiple sources as well as their own initiative in order to arrive at a text recognizably part of the Euclidean tradition but distinct in detail from any given predecessor (Wardhaugh 2024). A main source was evidently the (1505) Latin translation by Zamberti: Foix took up more than 90% of his choices for the main geometrical terms, and the wording of his definitions and enunciations was often noticeably similar. He certainly also consulted the Greek text of Grynaeus (1533), whose system of axioms and postulates he adopted (De Risi 2016, 597), and he followed the example of Peletier (1557) in rejecting superposition as a geometrical procedure (Axworthy 2021, 101; see also Herremann and Vitrac for further assessments of Foix's sources).

Beyond this, he revised and amended where he believed it necessary. Restoration, for instance, took the form of detailed modifications to vocabulary: Foix's Latin contains two attempts to introduce terms that had not previously been used in a Euclidean translation: *alterolongius* at definition I.22, for Zamberti's *altera parte longius* (oblong), and at definitions X.3 and X.4 (*in*)*certa* for Zamberti's (*ir*)*rationalis.*

He revised the text of the proofs, and supplied them with a fresh set of diagrams not perfectly similar to any predecessor (Wardhaugh 2024): labelling was simplified compared with any of his models (and this had consequences in detail for the text of the proofs). D'Étaples' (1516) edition had characterized even the received proofs of the Greek tradition as a 'commentary' from the late antique hand of Theon of Alexandria, the authentically Euclidean text extending only to definitions and enunciations. Consistently with this view, Foix printed his proofs in a smaller type than the enunciations: at the same size, indeed, as his added notes and comments.

Occasionally he made larger changes. In several books, he introduced one or more extra propositions, where he believed the completeness of the geometrical exposition required it. In general, his proposition lists followed those of Grynaeus' (1533) Greek edition of the text (and therefore of Zamberti's influential (1505) Latin edition, evidently translated from Greek sources with similar characteristics). But in Book X he followed the 1552 edition of Mondoré in including 116 propositions (not Grynaeus' 118), and he introduced new propositions in books VI (five), IX (one), and XII (one).

All of this editorial intervention might reasonably fall under Foix's promise of revision for the sake of correctness and lucidity, although it is fair to say that his additional propositions pushed the boundaries of that category. His dedication to Charles IX of France set out another, and perhaps more important agenda: namely a philosophical one in the (neo)Platonic tradition (de Foix de Candale 1566, a2^{r}–3^{r}). As well as setting out geometry's supposed role in the training of the ancient philosopher kings in Egypt, Persia, and Plato's imagined republic, Foix noted the inadequacy of the senses to apprehend truth unaided, and the importance of the study of mathematical – and especially geometrical – entities for purifying the mind, detaching it from the merely sensual and training it to know that which is unchanging, universal and incorporeal. This much he could have found in the commentary of Proclus on Book I of the *Elements* – which Grynaeus had printed with his edition of the text – in various works of Plato himself and in other (neo)Platonic sources. Foix's agenda was a more specifically gnostic one, however, linked to his study of the hermetic corpus, in which spiritual progress positively required the development of the intellect and the acquisition of true knowledge (Harrie 1975, 168, noting that d'Étaples was one of the precedents in France for this line of thought about mathematics and salvation). This was a line of thought that Foix would develop further in commenting on the *Pimander*, where he made it clearer still that geometry was a privileged tool for moving from the confusion of the senses to the apprehension of a higher reality.

This philosophical agenda justified Foix in innovating in his Euclidean text beyond what could be justified on text-critical or purely geometrical grounds. In a Platonizing view of the structure of the *Elements*, the five regular solids were the culmination of the whole work, their status as objects of contemplation meriting the organization of hundreds of subsidiary propositions towards their eventual construction in book XIII. This was a tradition in which it was not unusual to style Euclid himself not as a geometer but as a philosopher: 'philosoph[us] platonic[us]' in the title of Zamberti's printing, taken up by Tartaglia among others, and reflected in the long-running identification of Euclid the geometer with the historical Euclid of Megara, a philosopher of Socrates's period.

To take this view was necessarily to problematize books XIV and XV, seeing them as an appendix to the main text, though one arguably consistent with its overall plan. In fact, Foix did not believe that these additions handled their subject completely enough (de Foix de Candale 1566, c4^{r-v}), in any of the existing versions of the text. In a process broadly similar to his modifications to certain earlier books, but going rather further in detail, he constructed new versions of these books containing respectively 20 and 21 propositions. The Greek text of Grynaeus, first printed in 1533, had just 4 and 5; the medieval Latin version of Campanus, first printed in 1482, was longer with 18 and 13, and was one of Foix's sources.

Even by this means, though, not enough could be made to appear in *Elements* XIV and XV to satisfy Foix's wish for completeness on this topic, crucial as it was for a philosophically fruitful reading of the whole work. He therefore took the natural next step and composed a book XVI.

Books XVI–XVIII

Foix's book XVI was concerned primarily to expand upon the subject of books XIII–XV: the regular polyhedra and their relationships, and in particular the sometimes complex and beautiful figures which resulted when one was placed inside another. He composed 37 propositions, which ranged across all possible pairs of figures thus inscribed/circumscribed. For example, proposition 30 considered a cube inscribed inside a regular tetrahedron, determining the ratios of their edge lengths, diameters, and volumes. As a virtuoso performance, this was reminiscent – possibly intentionally so – of the use by designers in this period of the regular and semi-regular solids as a means to display technical skill to potential patrons: as for instance in the well-known (1568) *Perspectiva Corporum Regularium* of Wenzel Jamnitzer.

The book culminated with a final proposition unrelated to most of what had come before but clearly echoing the culminating proposition of book I – Pythagoras' theorem – dealing with the relationship between the squares on the three sides of a triangle. Specifically:

> If, in a triangle having a certain proposed base, the sides being commensurable in power [i.e. in square] to the base, a perpendicular is drawn from the vertex cutting the base, then the sections of the base will be commensurable [in length] with the whole base, and will be commensurable in power with the perpendicular. (de Foix de Candale 1566, 101^{r} (*recte* 102^{r}))[2]

That is, take a triangle with the property that the squares of its three sides can be expressed as a set of whole numbers; draw a line through one vertex perpendicular to the opposite side; then the two new triangles thus created will have the same property.

As well as his Book XVI, Foix also added to his edition of Euclid a short treatise on other solids derived from the regular polyhedra. In outline, this asked what happens if one takes a regular polyhedron, finds the midpoint of each edge, and forms a new polyhedron whose vertices are those midpoints (the process now called rectification). This was a sign of what was in Foix's mind at the time, and of what would come in his later work on Euclid. The discussion was not a complete one; Foix gave names to the two new polyhedra which resulted from this process: the 'exoctahedron' derived from the cube or the octahedron, with six squares and eight triangles for its faces; it is now known as the cuboctahedron. And the 'icosidodecahedron', derived from the dodecahedron or icosahedron, with twelve pentagons and twenty triangles as faces (the modern name is the same) (Figure 2). He set out a few specimen results and constructions concerning their properties and the possibilities of inscribing them inside regular polyhedra. These two solids are of the type later known as semi-regular or Archimedean, that is with all edges equal and the same combination of faces around each vertex, but not all faces the same shape.

Together, book XVI and the treatise on solids were a remarkable piece of work, although they – and Foix's related additions to books XIV and XV – sit rather

[2]Si trianguli cer[t]am propositam basim habentis, latera basi potentia commensurabilia fuerint, Ab vertice autem demissa perpendicularis basim secuerit, Sectiones to[t]i basi longitudine, Perpendicularis verò po[t]entia commensurabiles e[r]unt.

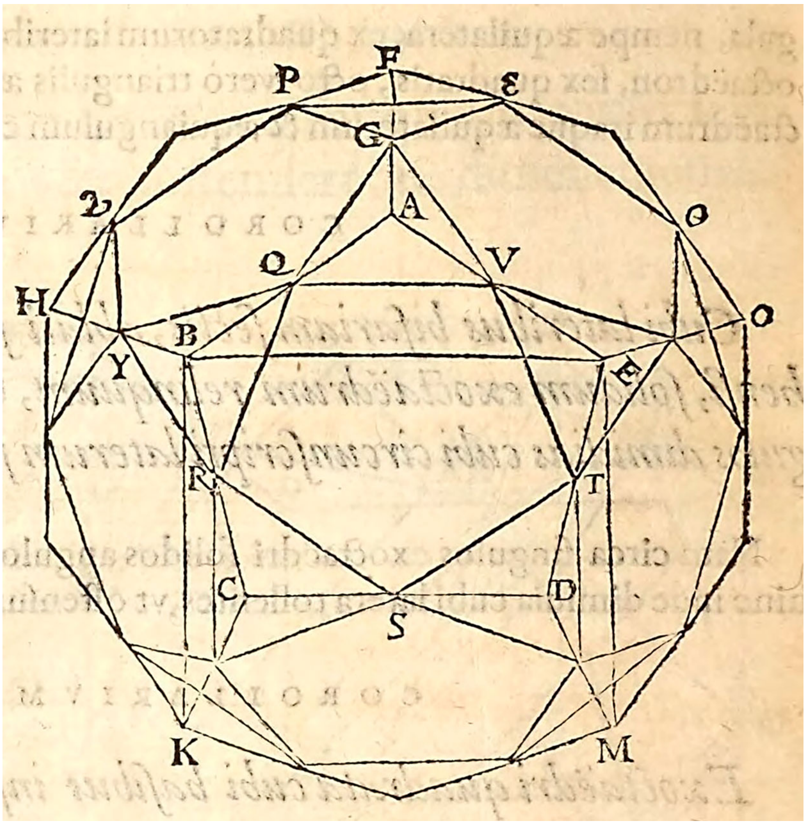

Figure 2. The icosidodecahedron: de Foix de Candale (1578, 512). Bayerische Staatsbibliothek, NoC-NC 1.0.

uneasily with his avowed aim to present the *Elements* stripped of modern accretions and complications.

The decade following Foix's *Elements* saw the appearance of two of the sixteenth century's most influential versions of the text: those of Commandino (1572) and of Clavius (1574). Each would be reprinted numerous times, and each had clear descendants to the end of the seventeenth century. Foix himself returned to the Euclidean text in 1578: it seems improbable that he was unaware of the work of his Italian-based colleagues, but he does not seem to have responded to it directly. In this second edition, the text was for most of its length substantially unchanged: indeed, the prefatory matter was not updated, retaining its original dedication to the now-dead Charles IX. Book VI had now gained yet another proposition, but the other books up to XV retained their lengths from 1566. Book XVI grew by nine propositions, to a total of 46. By this time named as bishop of Aire and about to be formally installed as such, and this in the exceedingly turbulent circumstances of the French Wars of Religion, Foix nevertheless prepared a substantial quantity of new material for this second edition, namely books XVII and XVIII. They replaced the short treatise on solids, and dealt at greater length with the manipulation of the exoctahedron and icosidodecahedron, considering this time exhaustively the possibilities for inscribing them in or about each of the five regular solids (Figure 3).

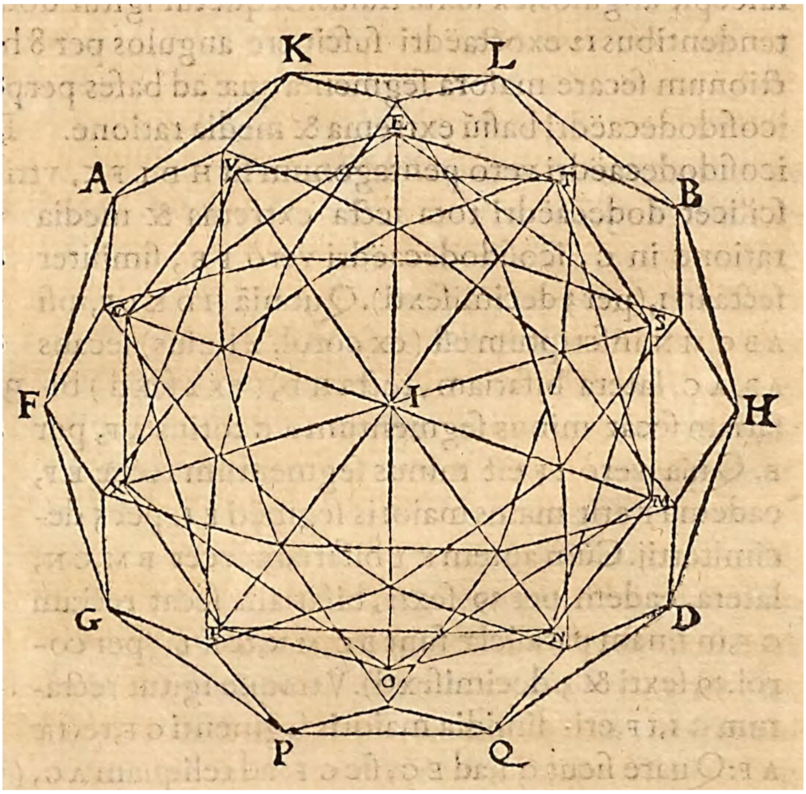

Figure 3. An icosahedron inscribed inside an icosidodecahedron: de Foix de Candale (1578, 531). Bayerische Staatsbibliothek, NoC-NC 1.0.

Foix worked through every possibility for inscribing a regular polyhedron in one of his two chosen semi-regular ones or vice versa, and of inscribing either of these semi-regular polyhedra inside the other, considering the orientation and the sizes of the figures that resulted. The first half of Book XVIII, for instance, was concerned almost exclusively with the ratios formed by the lengths of the edges of the two polyhedra in each case. Foix also showed how the two semi-regular polyhedra could be broken down into sets of pyramids, and gave occasional thought to the results of dividing some of the resulting constructions in two or four using planes. The final part of Book XVIII turned to the volumes of the polyhedra inscribed in one another, and the ratios those volumes formed. The language throughout was Euclidean in style; for example:

> Proposition 34: A dodecahedron exceeds its inscribed icosidodecahedron by the volume of the prism whose base is an equilateral triangle erected upon the edges of both solids, and whose height is two ninths of the diameter of the dodecahedron.[3] (de Foix de Candale 1578, 560)

[3]Dodecaëdron excedit sibi inscriptum icosidodecaëdron, solido prismate, cuius basis est æquilaterum triangulum, super binis eorum lateribus descriptum, vertex autem minoris segmenti dimetientis dodecaëdri duo nona.

Foix evidently believed he was making a real improvement to the *Elements* by his additions: in a preface to the new Books XVII and XVIII he explained at some length the benefits of having this material set out in full and in a proper order. These new books breathed a real excitement about the new solid shapes and constructions being described: Foix had already called them the 'most beautiful solids' (de Foix de Candale 1566, c4^{v}). He evidently believed that readers existed who would share his enthusiasm for an investigation of their properties in exhausting detail. Books XVII and XVIII also threw further emphasis on the high quality of Foix's own geometrical imagination.

Reception

The 1578 edition of Foix's *Elements* was printed by Jean de Tournes at Lyon according to its colophon, but the title page states that it was published in Paris by the bookseller Jacques du Puys. As Wardhaugh et al. (2020) remark, this at least hints that the disruptions of the period may have affected the publication of the book, as indeed they did the publication of the final version of Foix's *Pimander*, completed in 1572 but not published until 1579 (see Larroque 1877, 13; Harrie 1975, 228).

Later prefaces (Harrie 1975, 238–239 with n61; 261–262 n54) indicate that Catherine de Medici specifically requested Foix prepare a French version of his *Elements*, though to his visible embarrassment he never did so. Instead he dedicated his Greek–French Pimander to her: 'attendant le parachevement de vostre Euclide françois', and hoped that it would not be too long before he would be able to present to her 'tous mes labeurs, sur les elementz d'Euclide: qu'il vous a pleu me commander vous estre prepares & addresses' (Harrie 1975, 262).

Direct information about the print runs of the two editions of Foix's *Elements* is not known to exist, but an indirect hint about their success does. There are copies of the 1578 edition with a replacement ('cancel') title page asserting that they were published in 1602, either at Cologne or at Paris (see Wardhaugh et al. 2020, 88–89). This reasonably common subterfuge indicates that unsold sheets from the original printing had been acquired by another publisher and offered for sale. The fact that enough sheets remained unsold for this to be worthwhile indicates that the original edition did not sell as well as Foix or his printer expected. Given the strength of the Euclidean competition during the 1570s, that is not perhaps altogether surprising. Foix's editions were just two among twenty-odd Latin versions of the *Elements* offered for sale in the second half of the sixteenth century, and their distinctive features received in fact a mixed reception from other editors of the Euclidean text.

Foix's innovations with respect to vocabulary and the numbers of propositions in various books received little attention from later editors of the *Elements*: there was no uptake for his terminological novelties, none for his extensions to books VI, IX, and XII (Wardhaugh 2024). But his versions of book XIV and XV and his additional books were, by contrast, taken up by a number of others, some of them among the most visible and influential of Euclidean editors.

The famous 1570 English edition of the *Elements*, translated by Henry Billingsley, took up much of Foix's (1566) added matter. Billingsley gave two versions of book XIV, reflecting the diversity he found in his sources: one followed Zamberti and Grynaeus, the other Foix. He gave Foix's version of XV:

> *Flussas* also a diligent restorer of *Euclide*, a man also which hath well deserued of the whole Art of Geometrie, hath added moreouer in this booke … 9 propositions of his owne, touching the inscription, and circumscription, of these bodies, very singular vndoubtedly, and wittye. (Billingsley 1570, 431^{r-v})

(Flussas was Billingsley's best attempt to translate de Foix – 'Flussate' – back from Latin.) On both books he cited Foix's opinion about their authorship and original form. He included complete 'The sixteenth booke of the Elementes of geometrie, added by Flussas'. He also included the treatise on solids, to which Billingsley added extra matter of his own, 'For the better understanding' of the subject.

Still more important for the visibility of Foix's Book XVI was its inclusion in the great Latin edition of Christoph Clavius, printed in 1574 at Rome (see Price 2017, 168). For book XIV Clavius constructed a long composite text with 32 propositions; for XV he followed Foix. For XVI he printed not Foix's text but a paraphrase of it, as a sixteenth book on 'the comparison of regular solids', reduced to 31 propositions but supplemented by five propositions on the angles between adjacent faces in the regular solids. Foix's own treatise on solids did not appear. Clavius pointed out incidentally that some of his constructions were in fact incorrect. This version of Book XVI was included in the second edition of Clavius's *Elements* in 1589 (reprinted five times up to 1654): the main series of propositions 1–31 was unchanged, but the supplementary material was now extended with propositions from Pappus.

Many later editions of the *Elements* were more or less dependent on that of Clavius, but few included the later books. Of those that extended beyond book VI, Rhodius in (1609) rejected all books after XIII, whereas Herigone's (1634) version reverted to a shorter form of books 14 and 15 (8 and 5 propositions respectively) and included none of Foix's material. Similarly Nienrode's (undated) Dutch version gave 9 and 5 propositions in books 14 and 15, and rejected Foix's additional book. Near the end of the seventeenth century Astorinus (1691, reprinted 1701) did much the same, following Clavius in the main but reverting to a short list of propositions (7 and 5 respectively) for books XIV and XV, and omitting book XVI.

On the other hand, Richard in (1645) included books XIV–XVI in their full Clavian form, (without the treatise on solids). His Euclidean text largely followed that of Clavius, but also included a short version of book XIV, a collection of propositions taken from the commentary of Pappus, and a supplementary book containing fourteen constructions of lines in continued proportion (see Wardhaugh et al. 2020, 133). A century later, Caravelli (1750) edited books XI–XVI only (without the treatise on solids), apparently also following Clavius closely for the sequence of propositions. This appears to have been the last appearance of Foix's additions to the Euclidean text in Latin.

The influence of Clavius also extended to certain reimpressions of the Euclidean edition by Magnienus and Gracilis. This had first appeared at Paris in 1557 and went through a complex series of reimpressions both there and in Cologne; most retained the original series of fifteen books, but the 1607 Cologne version (itself reprinted in 1627) added propositions 6–21 to book XV, and included book XVI in the 31-proposition version of Clavius, plus the lemmas of Pappus from Clavius's second edition. The dependence on Clavius was noted on the title page.

In English, John Leeke and George Searle, mathematical practitioners, in (1661) followed Clavius for books I–XIII, but appear to have used Foix's 1566 text as their main source for books XIV–XVI, omitting just one proposition from book XIV and also – unlike Clavius – including the treatise of regular solids. The wording of their translation showed a dependence on Billingsley in the definitions and enunciations, but they do not seem to have followed him for the proofs or for decisions elsewhere in the book about the sequence of propositions. They called book XVI ‘The Sixteenth Element of Euclide’, and the treatise ‘a treatise of regular solids, by Campane and Flussas’.

In Dutch, the (1662) version of van Schooten is an unusual case. Citing Clavius as its main source (and giving enunciations only), van Schooten in fact omitted the nine propositions from book V that are not found in the Greek text, and similarly reduced books XIV and XV to 9 and 5 propositions respectively, evidently intending to remain closer to the Greek models than to what he found in Clavius’s copious text. Yet he included book XVI with the Clavian 31 propositions (plus the additions from Pappus). It is possible, in fact, that the 1607/1627 Cologne version of the text was the important model here.

Also in Dutch, and giving proofs as well as propositions, Voogt (1695, reprinted 1717) followed Clavius exactly as far as numbers of propositions were concerned, providing as a result extended versions of books XIV and XV, and a 31-proposition book XVI ‘toegevoegd van Francisco Flussate Candalla’.

Foix’s material made a final series of appearances in English, associated (somewhat loosely) with the Euclidean editorship of Isaac Barrow. Barrow’s Latin *Elements* first appeared in (1655), a hybrid text that followed Clavius in matters of vocabulary and most proposition counts, but provided new proofs using symbolic notation, and reduced books XIV and XV to 8 and 5 propositions respectively. Book XVI was not included. This version was reprinted in 1659, 1676 and 1678 and there were reissues of the final printing in 1685 and 1687. The text was translated into English in (1660), in a version which underwent various additions and changes after Barrow’s lifetime (he died in 1677). A reimpression of 1686 was unmodified, but that of 1705 added an English text of the Archimedean *Sphere and Cylinder*, and that of (1714) added the Euclidean *Data* and Foix’s treatise of solids, in Leeke and Serle’s English translation. No other Foix-inspired modifications to the text seem to have been made, and his Book XVI was neither included nor mentioned. Three more printings of the English version of Barrow’s *Elements* followed, in 1722, 1732 and 1751; the treatise of solids was included in all three. The 1751 printing seems to have been the final printed appearance of any part of Foix’s Euclidean material.

Foix’s books XVII and XVIII, meanwhile, had appeared four years too late for inclusion in Clavius’ first edition, and he did not see fit to add them to his second edition of 1589. Lacking this support, these two books saw no success to compare with Foix’s book XVI. Their only appearance after his lifetime was in a version printed at Paris in (1626), apparently under the editorship of Marin Mersenne. This enunciations-only text took Foix seriously as a source: it adopted three of his definitions in book XI, constructed a composite book XIV longer than any predecessor by editing together the texts of – in effect – Zamberti, Foix and Maurolico (see Harrie 1975, citing de Waard 1932, 1, 499–500; also Wardhaugh 2024). Book XV followed Foix; books XVI–XVIII were included complete, in the version of 1578. This version was reprinted in (1644) as part of Mersenne’s *Universae geometriae … synopsis*; this appears to have been the last printed appearance of books XVII or XVIII.

The printed appearances of Foix’s main sections of new Euclidean material are summarized in Table 1.

Table 1. Printed appearances of Foix's books XVI, XVII, XVIII and treatise on solids.

Version	XVI	Treatise on solids	XVII	XVIII
Foix 1566	1–37	included		
Billingsley 1570 (English)	1–37	included		
Clavius 1574	1–31 (+5)			
Foix 1578	1–46		1–28	1–45
Clavius 1589/1591/1603/1607/1611/1654	1–31 (+5 + Pappus)			
[Gracilis] 1607/1627	1–31 (+Pappus)			
Mersenne 1626/1644	1–46		1–28	1–45
Richard 1645	1–31			
Leeke and Searle 1661 (English)	1–37	included		
van Schooten 1662 (Dutch)	1–31 (+Pappus)			
Voogt 1695/1717 (Dutch)	1–31			
[Barrow] 1714/1722/1732/1751 (English)		included		
Caravelli 1750	1–31			

Readers

Beyond the work of later Euclidean editors, it is possible to show that certain mathematicians interested in the semi-regular solids and their classification consulted Foix's added books, although it is not usually possible to be certain which version of his additions they saw. He was by no means the first, in fact, to begin an investigation of the semi-regular solids: Archimedes wrote on the subject, though the text is lost and we have only a summary by Pappus (in the *Collection*, not printed until 1588). One manuscript of the fourteenth-century Euclidean *Tahrir* of al-Tusi contains an Arabic Book XVI 'describing techniques for constructing polyhedra within other polyhedra or within spheres' (De Young 2008, 133). The manuscript is dated 1593/4 (1003 AH); if De Young is correct that the text is the work of al-Khafrī (d. 1550), its composition must pre-date the work of Foix. There is no plausible route by which Foix could have been aware of this text; it is possible that both authors were aware indirectly of the work of Archimedes, but perhaps more likely that they were led by similar reasoning to the view that further propositions on regular solids and polyhedra derived from them were required at the end of the fifteen-book *Elements*.

It is possible, as Harrie notes, that Foix was in Paris in 1550 'when John Dee delivered a series of public lectures on Euclid at the College de Reims' (Harrie 1975, 53); and the catalogue of Dee's library shows that he later owned a copy of Foix's Euclid. 'In front of the entry, "Euclides p Flussates Candalla f° Paris", he placed a small triangle, his personal symbol to indicate a work he considered important' (54). Dee was of course a contributor to Billingsley's 1570 Euclid, and it is possible that his enthusiasm for Foix's work played a role in the inclusion of book XVI in the English translation and Billingsley's use of Foix more widely in his version. Foix himself, perhaps surprisingly, never mentioned Dee in his own work.

Also in an English context, Thomas Digges (1571) provided a much more systematic discussion of the solids resulting from 'transformation' of the regular solids, that is their truncation. This overlapped in part with Foix's treatment, and it is tempting to suppose that his inspiration was involved at least in a general way. Harrie (1975, 174) suggests that Billingsley's edition and its 'admonition to study carefully Foix-Candale's treatise for its "matter strange and delectable" and the "occasion of invention of greater things pertayning to the natures of the five regular solides"' were important here. Digges acknowledged no specific debt, however, and his title page in fact insisted that the new material was 'of his owne invention, hitherto not mentioned of by any Geometricians' (1571, title page: and compare the equally emphatic statement on the title page of the 1591 edition, by which time Foix's discussion had appeared in two more Latin versions and once in English). Even if this is not taken quite literally, it is not clear that the inspiration for Digges need have been Foix rather than some other model. Offusius (1570), for instance, is perhaps equally plausible as a source.

Another echo of Foix's extended *Elements* is to be found in the innovative cosmology of Johannes Kepler. The (1596) *Mysterium Cosmographicum* cited Foix's *Elements* by name; Field (1988, 197) suggests that 'while Kepler was writing this work he was, in the main, using the edition of the *Elements* by François Foix de Candale'. Westman (1972, 256) has suggested that his perusal of Foix's *Elements* was one of the 'preparatory conditions for that famous day in class, when he suddenly recognized that the regular solids could be inserted between the six planets'; thus that Foix's work 'may well have suggested the idea for his cosmographical model' (Westman 1971, 189). Harrie adds that 'Kepler, who was familiar with Foix-Candale's Latin translation of the Pimander, may have been induced by him to consider the study of the regular solids "the whole point of geometry"' (Harrie 1975, 174; cf. Mehl 2003, 447).

Finally, in the 1620s, René Descartes wrote a text entitled 'Exercices pour les éléments des solides', an examination of the semi-regular solids more systematic than that of Foix. Costabel (1987, 87–92) found evidence both in the text itself and in the circumstances in which it was written to suggest that Descartes may have taken inspiration from Foix's work, perhaps via the version of Mersenne.

Conclusion

As late as the 1640s, the French Jesuit Georges Fournier could refer to Foix as the 'restorer of mathematics in France' (Harrie 1975, 37). François de Foix de Candalle lived an unusual life in turbulent times, pursing a range of intellectual pursuits alongside his positions at court and in the episcopate. Among his varied activities, his edition of the Euclidean *Elements* stands out for its sheer size and for the quantity of intellectual effort it represents, as well as for its wide visibility. Despite Foix's stated intention to 'restore' the text he – ironically – enjoyed an unusual success as a Euclidean author: few other writers, if any, in his or the succeeding century inserted into Euclid's *Elements* new material of such bulk and longevity. Indeed, Foix was the most successful positive contributor to the Euclidean canon since the antique period which had seen the creation of Books XIV and XV. His emendations to the final two (spurious) books also achieved some visibility, but his contribution of three wholly new books concerning solid geometry included material that would remain in print until the mid-eighteenth century. This was in large measure due to the paraphrase published by Clavius and, therefore, to the enormous success of the Clavian *Elements* (equally distinctive and idiosyncratic, though in very different ways and with a rather different agenda from that of Foix). Some of the most notable mathematical writers of the decades around 1600 can be shown to have used his work. If Foix's legacy is in other ways elusive, he unarguably made a significant contribution to the study of polyhedra, and produced a quantity of quasi-Euclidean text that was read by generations of geometers and students over a period of nearly two hundred years.

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