

NUMERICAL SIMULATION OF TRANSPIRATION COOLING WITH A TWO-DIMENSIONAL SUBSTRUCTURE

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ABSTRACT

This paper presents a numerical model which assesses the coupled effect when transpiration cooling is applied to the external skin of a thermal protection system and the substructure of a high speed flight vehicle. The PIRATE transpiration cooling code has been extended and validated to account for quasi-two-dimensional lateral conduction effects allowing for analysis of more complex geometries. This enables very fast calculations of the two dimensional transient temperature response of a transpiration cooled thermal protection system, ideally suitable for large scale systems studies. To solve for the coupled transpiration cooled material and two-dimensional substructure, PIRATE has been coupled with the commercial finite element package COMSOL. This enables modelling of the longer term thermal effects of the integrated heat load over a flight trajectory. Transpiration cooling has been applied to a simplified blunt body model with an aluminium substructure. Results for the substructure temperature history for the Space Shuttle trajectory are obtained showing that transpiration cooling can lead to a 40% reduction in peak substructure temperature as well as a reduction in thermal stresses near the stagnation point.

Index Terms— Transpiration Cooling, Numerical, Systems Study

1. INTRODUCTION

Hypersonic vehicles are exposed to very high heat fluxes and heat loads during re-entry or sustained flight. This requires thermal protection systems to manage these heat fluxes and keep wall temperatures below critical values. Transpiration cooling is an active cooling technique that is envisaged for future hypersonic vehicles. It relies on a gaseous or liquid fluid which is fed through a porous wall material [10]. The coolant leads to internal convective cooling between the porous material and the fluid, and a protective film that keeps the hot external gas away from the surface [1]. Transpiration cooling provides alternative flexibility in the design of future hypersonic vehicles. The highlight of which is the ability to re-use

the entire thermal protection system, and activate cooling only when required.

To the author's knowledge, a complete systems level study accounting for the effects of transpiration cooling to the underlying substructure of a flight vehicle has never been conducted. Modlin et al [2] employed a finite difference model to model the effect of transpiration cooling to a generic hypersonic wing leading edge. Böhrk et al [3] developed the finite difference code HEATS to determine the transient wall heat flux to a transpiration cooled flat plate under laminar and turbulent conditions. Both of these models considered an adiabatic backside and did not consider the effects of the integrated heat load to the substructure of the vehicle. Hermann et al [4] applied the one-dimensional PIRATE numerical code to conduct systems studies of transpiration cooling for a blunt cone. This analysis considered radiation coupling between the transpiration cooled skin and the substructure, but assumed that the substructure is at constant temperature. Steelant et al [5] applied a constant heat flux to a finite element model of an uncooled SHEFEX II TPS panel showing that substructure temperatures can rise well after the outer TPS reaches radiative equilibrium, and potentially violate structural limits.

In this paper, the Oxford in-house PIRATE code is extended and validated in two important ways for more accurate modelling of the entire system for transpiration cooling. Firstly, the one-dimensional impulse response code has been expanded to model the quasi two-dimensional effects of lateral conduction within body of the porous material surface. A quasi-two-dimensional approach is employed so that the effects of lateral conduction can be approximated with acceptable degree of accuracy and minimal impact on computational time. Secondly, the thermal behaviour of a generalised substructure during a particular flight trajectory in the presence of a transpiration cooled system is assessed. This is achieved by coupling the numerical code PIRATE which solves for the thermal properties of the porous medium with the commercial FEM package COMSOL which solves for the thermal response of the substructure. Finally, this is then applied to a 30° half angle blunted cone with the Space Shuttle trajec-

tory where the substructure temperature reduction due to the application of cooling is demonstrated.

2. ONE-DIMENSIONAL PIRATE MODEL

The numerical code **Porous Impulse Response Analysis for Transpiration cooling Evaluation (PIRATE)** [4] calculates the transient temperature response for a one-dimensional porous material subjected to aerodynamic surface heating. By relying on a method based on impulse response convolution, significantly faster computation times are possible providing the opportunity to conduct optimisation studies for various flight trajectories and geometries. Figure 1 shows an outline of the code. For a given trajectory, the aerodynamic heat fluxes with no cooling are calculated from aerodynamic correlations or imported from an external database. The cooling effect of the local heat flux mitigation through blowing and downstream film effectiveness is taken into account using semi-empirical models.

The cooling within the inner structure of a porous material is carried out by convective heat transfer between the material and the coolant. Therefore, the full governing equations for the solid and fluid phases are [6]

$$(1 - \phi)\rho_s c_{p,s} \frac{\partial T_s}{\partial t} = k_{x,s}(1 - \phi) \frac{\partial^2 T_s}{\partial x^2} + k_{y,s}(1 - \phi) \frac{\partial^2 T_s}{\partial y^2} + h_v(T_f - T_s) \quad (1)$$

and

$$\phi \rho_f c_{p,f} \frac{\partial T_f}{\partial t} = \phi \rho_f c_{p,f} \frac{v_f}{\phi} \frac{\partial T_f}{\partial y} + h_v(T_s - T_f), \quad (2)$$

where (s) denotes the solid phase, (f) the fluid phase, k the thermal conductivity, ϕ the porosity, h_v the volumetric heat transfer coefficient, t the time, y the location medially through the material, and x the location along the body.

Figure 2 shows a schematic of the one-dimensional thermal model. The temperature of the solid and fluid phases are calculated based on a given coolant mass flow rate, porous

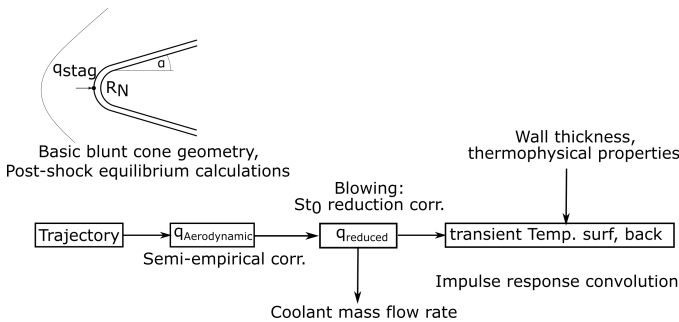


Fig. 1: Flow chart of the computational approach.

wall thickness and the thermophysical properties of the solid and fluid phases. The transient temperature calculations are based on an impulse response approach which solves the coupled fluid-solid heat conduction problem with radiation cooling to a constant temperature substructure. This approach is detailed further in Hermann et al [4]. The PIRATE model has been validated against a variety of experiments, both ground based and re-entry flight testing. In particular, experimental results from the SHEFEX II flight test campaign, and the RESPACE and IMENS+ campaigns are used [7, 8]. The different validation cases have demonstrated that this numerical model matches experimental data closely and agrees with experimental measurements to within 10%. The relevant effects occurring during transpiration cooled hypersonic flight, i.e. transient heating, local cooling through blowing, and downstream film coverage, are all replicated within acceptable accuracy.

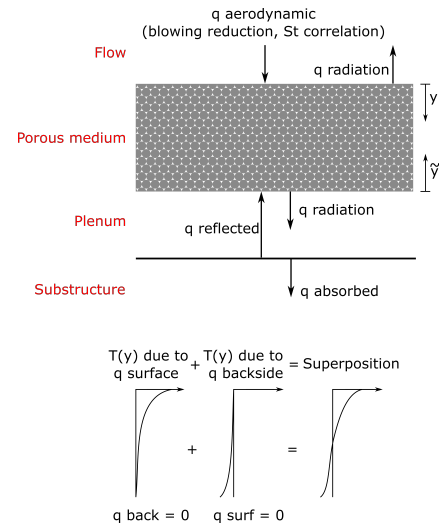


Fig. 2: Schematic of the one-dimensional thermal model.

3. EXTENSION OF PIRATE TO QUASI-TWO-DIMENSIONAL MODELLING

A key assumption made by the numerical simulation tool described is that only one-dimensional heat conduction through the porous medium is considered. Therefore, when a calculation is conducted across a full body, a series of one-dimensional models are solved. For many applications, especially where very blunt or flat plate geometries are considered this simplification is reasonable for fast system analysis. However, when considering complex geometries and materials with higher thermal conductivities, two dimensional effects are crucial. The following method of calculating lateral conduction is applicable for moderate thermal gradients where the effects of variable wall thickness and two-dimensional coolant mass flux are not considered.

Considering only the lateral term in Eq 1, the governing equation for one-dimensional transient conduction is

$$\frac{\partial T_s}{\partial t} = \frac{k_{x,s}}{\rho_s c_{p,s}} \frac{\partial^2 T_s}{\partial x^2}, \quad (3)$$

where $T_s(x, t)$ is the temperature of the solid material at spatial point x and time t .

To model lateral conduction, a quasi two dimensional approach is presented here where conduction medially through the material is calculated using the impulse response method of PIRATE [4]. Thereafter, the Crank-Nicholson scheme for one dimensional thermal conduction is applied to the existing temperature history. The advantage of using Crank-Nicholson is its known unconditional stability for both linear and non-linear heat conduction systems [9]. The Crank-Nicholson scheme can be expressed as

$$\frac{T_x^t - T_x^{t-1}}{\Delta t} = \frac{\alpha}{2} \left[\frac{T_{x-1}^t - 2T_x^t + T_{x+1}^t}{\Delta x^2} + \frac{T_{x-1}^{t-1} - 2T_x^{t-1} + T_{x+1}^{t-1}}{\Delta x^2} \right], \quad (4)$$

where x is the spatial step along the x axis (from $x = 0$ to L), t is the time step, Δt is the time step discretisation, Δx is the spatial discretisation and α is the thermal diffusivity defined as $\frac{k_{x,s}}{\rho_s c_{p,s}}$. Neumann boundary conditions are considered such that there is zero heat transfer across the sides (adiabatic boundary condition).

$$\left. \frac{\partial T}{\partial x} \right|_{x=0,L} = 0. \quad (5)$$

By applying this scheme to the existing transient temperature distribution from PIRATE ($T_s(y, t)$), where t is the time from $t = 0$ to t_{end} , this results in a new temperature distribution $T_s^{\text{lateral}}(y, t_1, t_2)$ where both t_1 and t_2 are from 0 to t_{end} . t_1 is the time step at which one-dimensional conduction through the material has occurred and t_2 is the time step for the lateral conduction. For example, at $t_2 = 0$, the previous one-dimensional temperature solution is recovered and $t_2 = t_{\text{end}}$ is the steady state lateral conduction solution.

To ensure that the transient change in temperature is taken into account, for each time step the temperature change due to lateral conduction is calculated

$$dT_{y,t_1,t_2}^{\text{lateral}} = T_{y,t_1+1,t_2+1}^{\text{lateral}} - T_{y,t_1,t_2}^{\text{lateral}}, \quad (6)$$

where $dT_{y,t_1,t_2}^{\text{lateral}}$ is the change of the entire temperature field between each time step. The changes in temperature are then summed up for each spatial location to give the full solution for temperature

$$T_{x,y,t}^{\text{lateral}} = dT_{x,y,1,t}^{\text{lateral}} + dT_{x,y,2,t-1}^{\text{lateral}} + \dots + dT_{x,y,t-1,2}^{\text{lateral}} + dT_{x,y,t,1}^{\text{lateral}} + T_{x,y,0}^{\text{lateral}}, \quad (7)$$

where t is the time step of interest, $T_{x,y,0}^{\text{lateral}}$ is the initial temperature distribution and $T_{x,y,t}^{\text{lateral}}$ is the new temperature distribution with lateral conduction applied.

The key advantage of this approach is the very quick computational time. In addition to the one-dimensional calculation time, this code solves for the two-dimensional field in only 30 s - 2 minutes. This enables large scale systems studies of the application of transpiration cooling for entire trajectories to be carried out. Henceforth, the quasi-two-dimensional version of PIRATE will be referred to as 'PIRATE Q2D' whilst the original code is 'PIRATE 1D'.

4. VALIDATION OF THE QUASI 2D MODEL

4.1. Uncooled - COMSOL FEA

This method to solve for lateral conduction has been validated by comparing the temperature response of the PIRATE Q2D code with the COMSOL commercial finite element package. For zero coolant mass flux, the differential equations for the solid and fluid phases of the porous material reduce down to the well known equations for two dimensional conduction. In this case, the temperature response via the quasi two dimensional method described here can be compared directly with a commercial finite element package.

Two high temperature porous materials are chosen for this validation; the main purpose of which is to confirm that the quasi two dimensional approach accurately models the transient change in temperature through the material. These were, Carbon-Carbon (C/C) ceramic composite [10], and the ultra high temperature ceramic (UHTC) ZrB₂ [4]. The material parameters used in this study are summarised in table 1. Both materials have been investigated in previous transpiration cooling studies for high speed vehicles [7, 11] and constitute representative materials for validating the lateral conduction model.

Material	UHTC	C/C
ϕ	0.41	0.1050
ϵ	0.7	0.85
$k_{x,s} / \text{Wm}^{-1}\text{K}^{-1}$	41.4	14
$k_{y,s} / \text{Wm}^{-1}\text{K}^{-1}$	41.4	2
$c_{p,s} / \text{J kg}^{-1}\text{K}^{-1}$	437	1650
$\rho_s / \text{kg m}^{-3}$	6100	1400
K_D / m^2	$3.4 \cdot 10^{-14}$	$7.5 \cdot 10^{-14}$
K_F / m	$15.7 \cdot 10^{-8}$	$8.5 \cdot 10^{-8}$
$h_v / \text{W m}^{-3} \text{K}^{-1}$	$5.5 \cdot 10^4$	$1 \cdot 10^6$

Table 1: Material properties used in this study

A two dimensional model described in Figure 3 is used as a validation case. A step change in heat flux of 0.1 MW/m² is applied at the centre (from $x = -0.05$ -0.05 m) of the model for 20 s and then allowed to thermally dissipate until 100 s.

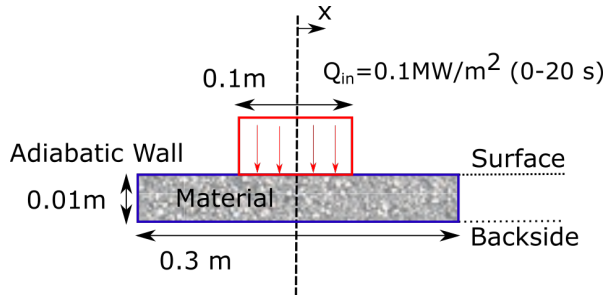


Fig. 3: Schematic of the validation case.

Adiabatic boundary conditions are assumed for the remainder of the boundaries. This represents a highly two-dimensional and transient heat transfer case where the front and backside temperatures initially strongly diverge, eventually approaching equilibrium for the steady state. COMSOL solves for heat conduction via the finite element method. For the validation model, the COMSOL mesh has a maximum element size of 0.003 m leading to 891 domain elements and 209 boundary elements. Time stepping is achieved with a relative tolerance of 0.0001. The PIRATE code applies a time discretisation of $dt = 0.1$ s for these validation studies. A one-dimensional simulation of the C/C material showed a peak temperature difference of 1.7% (or 7 K) between PIRATE Q2D and COMSOL. For the purposes of this simulation, the peak temperature difference is taken to be acceptable.

Figure 4 shows the evolution of the surface temperature over several time steps for the C/C. Comparing the PIRATE 1D to the PIRATE Q2D code, it is clear at later time steps more heat flux dissipates towards the sides demonstrating that PIRATE Q2D is correctly modelling the transient distribution of surface temperature.

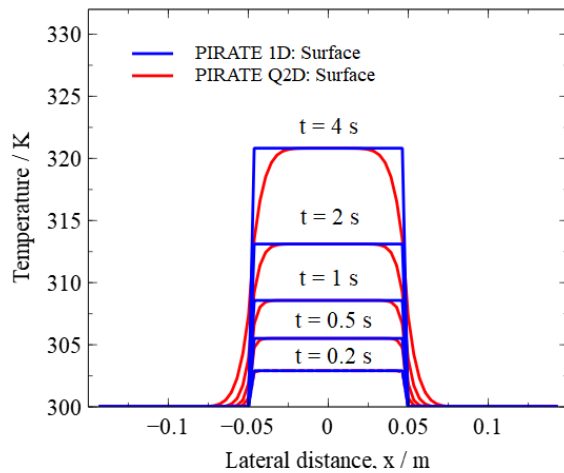
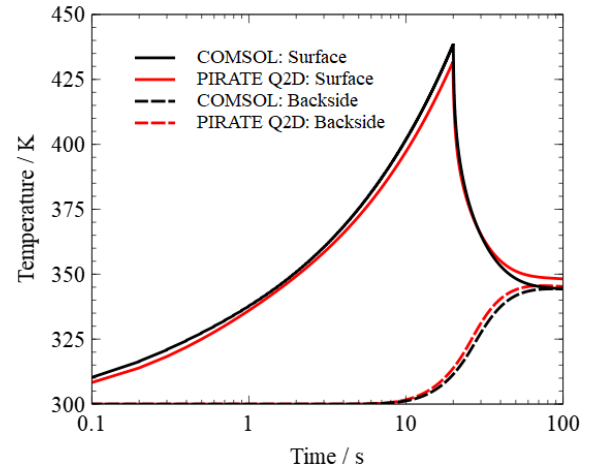
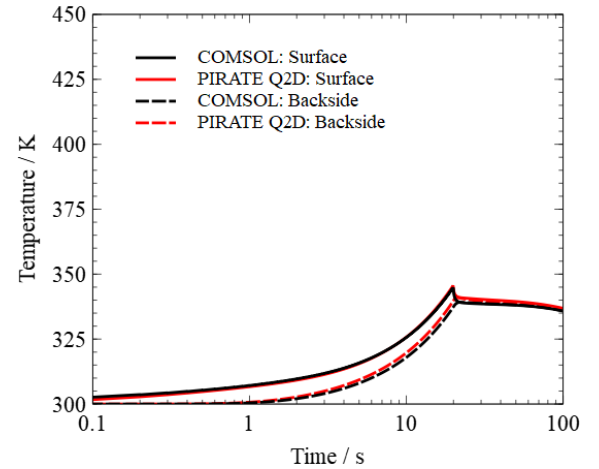


Fig. 4: Surface temperature evolution along body - C/C uncooled.

temperatures at the point $x = 0.05$ m for both materials. This point represents the point between the heat flux boundary input boundary condition and the adiabatic boundary around the model where the most significant two-dimensional effects are expected. For the entire time history, the PIRATE Q2D simulations agree within 5% of the COMSOL simulations, capturing the linear rise during the onset of heating and then the drop off as heat flux dissipates to rest of the material. For C/C, PIRATE Q2D under-predicts the temperature peak by 6 K which can be attributed to the time discretisation described before. In addition, there is a slight over prediction of 4 K of the final temperature. This is consistent with both materials and is likely to be due to the lateral discretisation ($dx = 0.02$ m for these simulations). As this point constitutes a very transient change in temperature, a small change in lateral discretisation leads to a large change in temperature.



(a) C/C



(b) UHTC

Fig. 5: Transient temperature evolution at $x = 0.05$ m - uncooled.

Figure 5 shows the transient surface and backside tem-

Figure 6, shows the temperature distribution along the body for C/C and UHTC at time 10 s. Reinforcing the results above, the general shape of the temperature distribution for PIRATE Q2D matches very closely with COMSOL, with the difference of 12 K in peak surface temperature for the $t = 10$ s C/C case attributed to the error due to time discretisation in the PIRATE Q2D simulation described above.

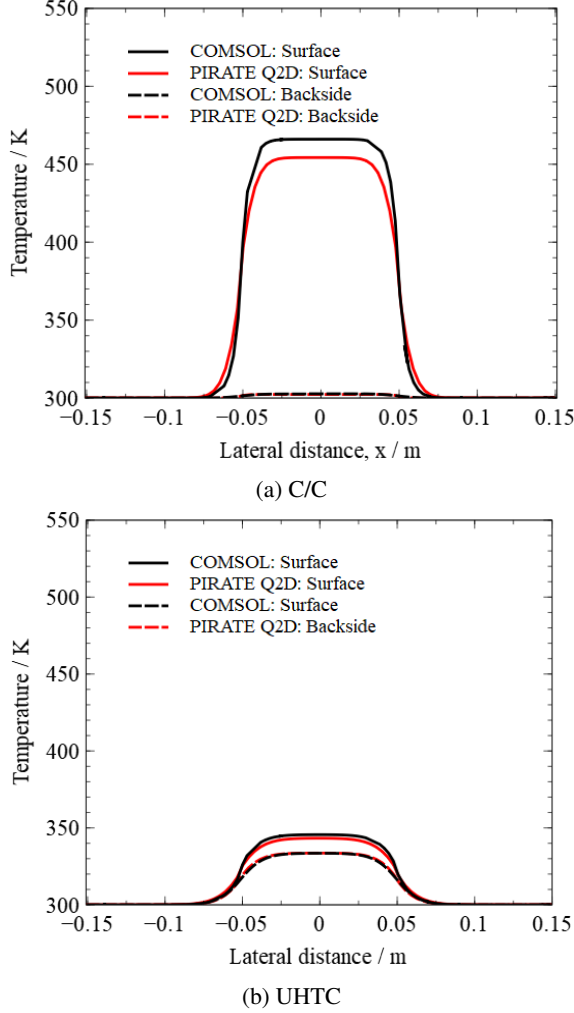


Fig. 6: Temperature distribution along the body at $t = 10$ s.

4.2. Cooled - SHEFEX II flight data

To validate the lateral conduction model with transpiration cooling, the PIRATE code with lateral conduction has been applied to the SHEFEX flight data [7]. The total flight time was roughly 500 s with 52 s of experimental time for the atmospheric re-entry at altitudes between 100 and 30 km. The SHEFEX II flight experiment implemented a based transpiration-cooled experiment named AKTiV. Porous C/C

samples (properties from Table 1) with a thickness of 5 mm were inserted in the centre of a 7 mm thick thermal protection panel made of C/C-SiC [7]). Nitrogen coolant gas is fed through the porous sample at a mass flow rate of 0.4 g/s with a stainless steel plenum. Figure 7 shows the locations of thermocouples for the AKTiV experiment at Panel C3. PIRATE simulations with the application of lateral conduction are compared to the surface mounted thermocouple measurements. The reservoir temperature is assumed to be 300 K [7]. Before the cooling is started at 431 s, the pre-heating of the model has been simulated by starting from the initial temperature distribution.

The temperature distribution for the AKTiV flight experiment is shown in Figure 8 for $t = 485.12$ s where the experimental results from the SHEFEX flight data, the two-dimensional finite difference HEATS simulation data and the two PIRATE solutions, namely the 1D and Q2D simulations, are plotted. The results show very good agreement within 2 K between PIRATE Q2D and the measured thermocouple data upstream of the porous sample (K35 & K36). The HEATS two-dimensional finite difference code follows a very similar trend to PIRATE Q2D. The over prediction of HEATS may be attributed to fact that PIRATE 1D and Q2D accounts for the radiation exchange between the backside and the plenum whilst HEATS considers an adiabatic backside. The prediction of the temperature reduction due to cooling is in good accordance with both HEATS and the experiment and lies within a margin of less than 10 K. There is a slightly different trend in the downstream cooling between HEATS and both PIRATE simulations. This may be attributed to the different models used between HEATS and PIRATE to calculate the downstream film effectiveness. Where HEATS uses a boundary layer heat balance, PIRATE uses a semi-empirical correlation [4]. Both PIRATE models follow the surface thermocouple data closely, within 10 K for K29-32. Finally, a comparison between PIRATE 1D and PIRATE Q2D simulations shows the increased accuracy attained by considering 2D effects. Near the porous sample, the effects of lateral conduction can be seen which follow closely the shape of HEATS, which is a two-dimensional finite-difference code, and the surface thermocouples.

5. COUPLED NUMERICAL CODE OUTLINE

One of the key assumptions made by the PIRATE solver is that the boundary condition at the backside of the porous material is represented as a radiative exchange between the back surface and a cold substructure (Figure 2):

$$\dot{q}_{\text{backside}} = -\frac{\sigma(T_{\text{backside}}^4 - T_{\text{sub}}^4)}{\frac{1}{\epsilon_s} + \frac{1}{\epsilon_{\text{sub}}} - 1} \quad (8)$$

where σ is the Boltzmann constant, ϵ_{sub} is the emissivity of the substructure and ϵ_s is the emissivity of the porous material. However, during a flight trajectory, the substructure

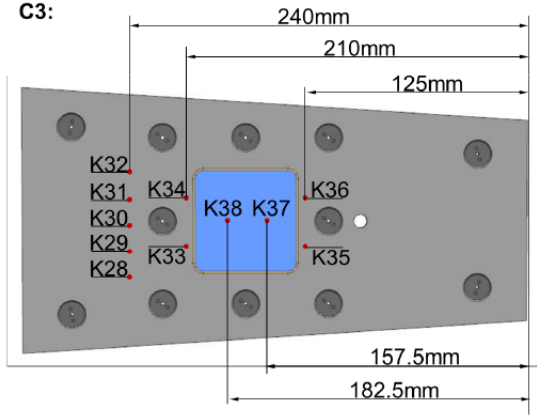


Fig. 7: AKTiV thermocouple locations on experiment [7].

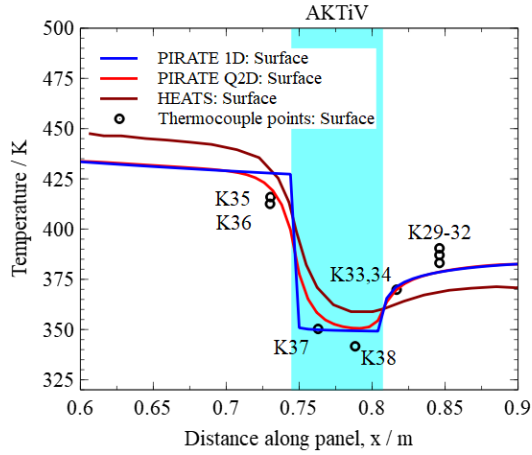


Fig. 8: Comparison of experimental to simulated temperatures along panel C3 at the surface of AKTiV at $t=485.12$ s [7].

temperature (T_{sub}) will increase due to the incoming radiative heat flux from the backside of the porous material. The increase in temperature of the substructure then leads to a reduction in the net negative backside heat flux. Even if critical material temperature limits are maintained for the TPS, this can still lead to overall failure for the vehicle if the substructure temperature exceeds critical material limits.

To model this effect, the PIRATE Q2D code has been linked to the COMSOL finite element package using LiveLink for MATLAB. Figure 9 shows a schematic of the scheme. For each time step, the backside temperature calculated by PIRATE Q2D is taken as an input into a backside material boundary condition in COMSOL. Thereafter, once COMSOL calculates the heat transfer to the substructure for a given time step, the backside radiative heat flux is taken as an input boundary condition for PIRATE Q2D. Henceforth, the

coupled version of PIRATE Q2D with substructure will be referred to as 'PIRATE Q2D+S'.

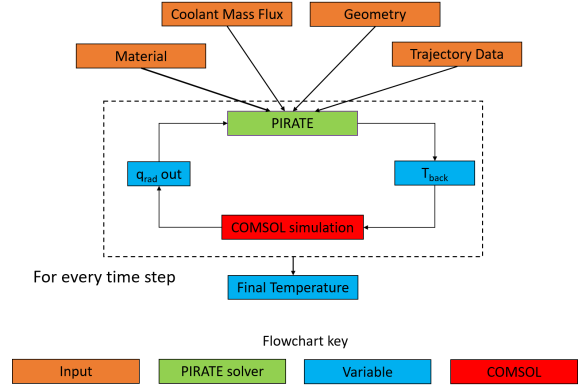


Fig. 9: Flowchart of coupled numerical model.

6. TRANSIENT FLIGHT ANALYSIS WITH SUBSTRUCTURE

To demonstrate an application of the coupled numerical simulation, a generalised case study of a 30° half angle blunted cone with a 0.5 m nose radius and a wall thickness of $L = 0.01$ m. The total length from the stagnation point along the body profile is 2 m which results in a rear base radius of 1.17 m. The Carbon-Carbon (C/C) ceramic composite material is chosen for this study of which the material parameters are summarised in Table 1. These simulations are all carried out for the Space Shuttle trajectory where the peak heat flux at the stagnation point is 0.5 MWm^{-1} .

For simplicity, this model considers a two-dimensional substructure of 10 mm aluminium with an emissivity, $\epsilon_{\text{sub}} = 0.07$; density, $\rho = 2700 \text{ kg m}^{-3}$; specific heat capacity, $c_p = 900 \text{ Jkg}^{-1}\text{K}^{-1}$ and thermal conductivity, $k = 234 \text{ Wm}^{-1}\text{K}^{-1}$ with a gap of 10 mm acting as a coolant reservoir. Heat transfer through any supports between the substructure and the outer porous material are ignored as well as any heat transfer between the coolant gas and the substructure. An adiabatic boundary condition is assumed for the backside of the substructure and a symmetric boundary condition at the stagnation point. A schematic of this model is shown in Figure 10 where the porous material surface, backside and the substructure surface will be compared with the application of transpiration cooling

6.1. Effect of Coupling

To understand how the substructure temperature can change over the course of a trajectory due to the integrated heat load accumulated over flight, the PIRATE Q2D code is compared with PIRATE Q2D+S. As the PIRATE Q2D model assumes that the substructure has infinite thermal capacity,

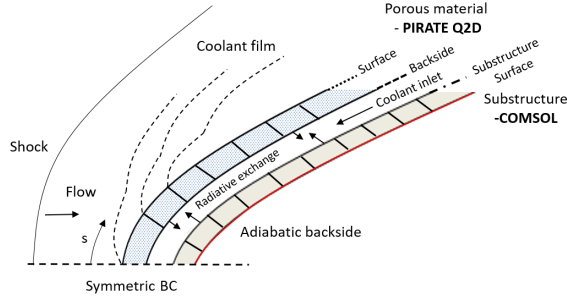


Fig. 10: Schematic of two-dimensional model with substructure.

this remains at a temperature of 300 K throughout the flight trajectory. PIRATE Q2D+S transiently calculates the change in substructure temperature due to the incoming radiative heat flux from the backside of the porous material.

Figure 11 shows the transient temperature history of the surface and the backside for the C/C material and the surface temperature of the substructure at the stagnation point. The results show a significant temperature increase of the substructure over the trajectory, reaching a peak of over 1000 K for C/C. The steady, linear rise in temperature is plausible as the backside absorbs the total integrated heat flux. The back face of the substructure is assumed to be adiabatic with no further heat transfer into internal structures. Aluminium has a melting temperature of ≈ 930 K, illustrating the clear risk of material failure for the substructure at this flight condition.

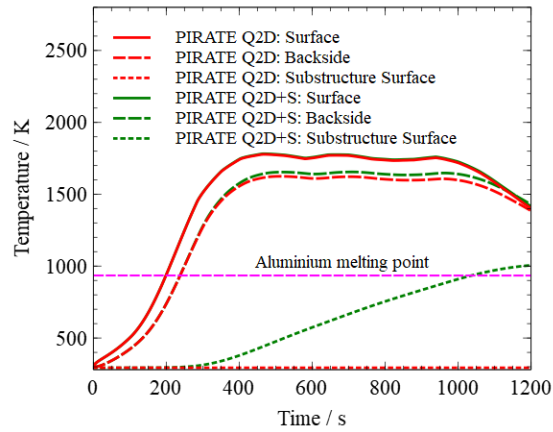
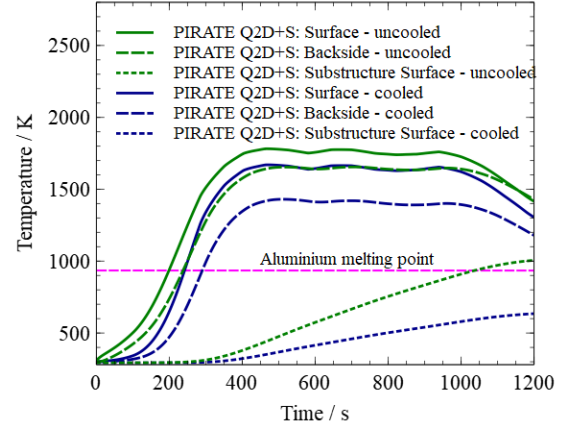


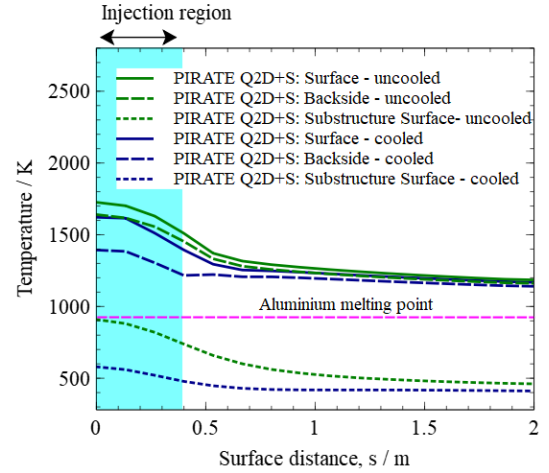
Fig. 11: Transient temperature evolution at the stagnation point, comparison of PIRATE Q2D and PIRATE Q2D+S - uncooled.

6.2. Effect of cooling

Transpiration cooling is applied to this model by injecting Helium coolant throughout the entire trajectory at a constant



(a) At the stagnation point



(b) At $t = 1000$ s

Fig. 12: Transient temperature evolution with the application of cooling

mass flux, $\frac{\dot{m}}{A} = 0.005 \text{ kg m}^{-2} \text{ s}^{-1}$ from $s = 0$ to 0.4 m. Coolant is applied locally near the stagnation point to also simulate downstream effects of the coolant injection. This represents a blowing parameter along the trajectory between 6.8-0.11.

Figure 12a illustrates the impact of adding transpiration cooling to the stagnation region. As expected, the coolant injection leads to a reduction in both the surface and backside of the porous material. Due to the reduction in backside temperature, this leads to a lower heat flux from radiative emission at the substructure and thereby a significantly lower substructure temperature rise. As the backside temperatures are reduced along the entire trajectory, a moderate reduction in the peak backside temperature of the outer TPS (≈ 100 K) leads to a > 400 K reduction in the substructure temperature at the final trajectory point. This effect shows that in addition to the use of transpiration cooling to reduce outer TPS sur-

face temperatures, there is an advantageous side effect where the integrated heat loads to the substructure can be reduced significantly.

By comparing the temperature laterally along the body for $t = 1000$ s (Fig 12b), applying coolant injection has the effect of significantly reducing the temperature gradient near the stagnation point of the blunt body. This is due both to high thermal conductivity of the aluminium which distributes the reduction in heat flux due to coolant mass injection near the stagnation point to locations downstream as well as the downstream effectiveness of the cooling detailed in Ref [4]. By reducing these temperature gradients, thermal stresses near the stagnation point are lowered.

7. CONCLUSIONS

This paper presents improvements to the Oxford in-house PIRATE code developed to perform rapid simulations of the transient temperature evolution of transpiration cooled heat shields for hypersonic vehicles. The Crank-Nicholson numerical scheme has been applied to the existing PIRATE code, providing the capability to accurately model lateral conduction for geometries with moderate thermal gradients and strictly one dimensional coolant mass flow. The quasi two-dimensional method allows for rapid calculations of the transient temperature history, suitable for systems studies.

To model the substructure of a vehicle where transpiration cooling is applied, the PIRATE code is coupled to the commercial finite element solver COMSOL to create a tool by which complete system studies can be investigated. Preliminary results show the heating up of the substructure is a major factor in choosing appropriate materials for a given mission trajectory. Applying transpiration cooling has the effect of not only reducing temperatures for the outer TPS but also significantly reducing the temperature of the substructure. By applying a moderate level of coolant injection, the maximum temperature of the plenum reduces from over 1000 K to under 600 K, making materials which are lighter (such as aluminium) feasible as a substructure for these flight conditions. In the future, this tool will be developed to perform optimisation studies for a complete transpiration cooling system.

8. ACKNOWLEDGEMENTS

This research is funded by the EPSRC grant "Transpiration Cooling Systems for Jet Engine Turbines and Hypersonic Flight" (reference: EP/P000878/1). The authors of this paper would also like to thank the Reaction Engines & EPSRC CASE reward for supporting the studentship of Imran Naved.

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