

Ultra-low Emittance Lattice Design for Advanced Synchrotron Light Sources

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Abstract

Storage ring based synchrotron light sources deliver high brightness radiation generated by high quality electron beam. The electron beam emittance plays an importance role in controlling the brightness and coherence of the radiation output. An extensive design effort is required to optimise the lattice to improve the beam emittance and machine performance. This thesis reports a series of investigations into lattice tuning, modification and full upgrade to improve the machine performance of third generation light sources, using the Diamond storage ring lattice as a model.

In the first part of this thesis, the optics functions of the existing lattice are optimised in order to reduce the natural beam emittance. A reduction of 27% is achieved and verified experimentally, although the effects of strong insertion devices (IDs) reduce this improvement. A second study was carried out with the aim of replacing one of the existing double bend achromat (DBA) cell with one and two double-double bend achromat (DDBA) cells providing additional straight sections for IDs. It was proven that the addition of one DDBA cell can be implemented without significant deterioration of the performance of the machine. The case with two DDBA cells however provides a beam lifetime which is only half of the value in the existing machine. A third study was carried out to consider a full lattice upgrade using multibend achromat (MBA) cells aiming for ultra-low emittance.

The last part of this thesis concentrates on the study of an improved beam injection scheme employing a pulsed multipole kicker (PM). The aim is to achieve an efficient beam injection while producing a lower perturbation to the stored beam during the injection. Based on particle tracking, the beam injection performance of the scheme is optimised for the existing Diamond and for the one DDBA lattice. The effects of the PM on the stored beam is proven to be negligible. The compatibility of the existing injection system with the new injection scheme is evaluated. Such scheme provides a feasible alternative to the existing injection scheme that could reduce the perturbation during the beam injection during Top-up injection.

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Chapter 1

Introduction

Since the first discovery by Röntgen in 1895, X-rays have been proven to be excellent probes to investigate properties of materials thanks to their high penetration and their wave length comparable to the atomic spacing in materials. X-ray tubes have been exploited until the late 1950s as the major source of X-rays until it became clear that the synchrotron radiation emitted by relativistic electrons ($\beta \approx 1$) in a circular accelerator could act as an intense source of X-rays.

In the early days, synchrotron machines were used for particle physics and nuclear physics experiments only. The application of synchrotron radiation has gained popularity after the first observation of intense visible synchrotron light from a donut-shape glass chamber of the 300 MeV electron synchrotron at the General Electric Co.[1]. The theoretical prediction of the property of synchrotron radiation was established initially by Liénard [2] in 1898 and later fully clarified by Schwinger (1949) [3] who explained the radiation from a moving charged particle on a circular path. The first purpose-built synchrotron radiation sources appeared in the early 1980s. The Synchrotron Radiation Source (SRS) at the Daresbury Laboratory is the world first second generation synchrotron built in 1981 which is dedicated to the generation of X-rays using the radiation from bending magnets. However, the radiation from dipole is broadband and has a wide angular distribution and only a fraction of the photon beam can be used. Since then, experiments always demand better photon beam quality, driving the demand for an increase in brightness of several orders of magnitude. Alternating periodic magnetic field of insertion devices (ID) have been introduced in order

to maximise the emission of synchrotron radiation with higher brightness and larger energy reach. Nowadays, third generation light sources provide highly collimated and narrow photon beams based on insertion devices. Synchrotron storage rings dedicated to the production of X-rays were designed to produce very powerful and tunable photons beams providing a wide range of intense radiation from infra-red to hard X-rays. The quality of the photon beam is determined by its brightness and the transverse coherence and it is far superior than the quality of most laboratory-based laser sources. Today more than 70 synchrotron facilities are operating globally serving a large number of users. In United States alone more than 9,000 users per year perform experiments in such facilities [4].

Modern synchrotron light sources strive to satisfy the users' demands for better beam quality to improve the accuracy and throughput of the experiments. Synchrotron radiation sources have developed and refined new designs in order to provide and ever increasing brightness in the last decade. The development of Free Electron Lasers (FELs), usually called the forth generation light sources based on linear accelerators improves dramatically the brightness compared to the third generation ring by many order of magnitudes. However, storage ring based light source can serve many experiments simultaneously and constitute a complementary research tool and will continue to provide unique radiation sources to an ever growing scientific community. In recent years, the study of ultra-low emittance (< 0.5 nm-rad) lattices has received much attention in storage ring light sources and damping rings community [5] in order to improve the photon brightness and luminosity for e^+/e^- colliders respectively.

This thesis presents study of the beam optics, the lattice design, and optimisations toward the upgrade of third generation light sources to ultra-low emittance lattice. Most of the design refers to the specific problem of the upgrade of the Diamond storage ring to a new ultra low emittance lattice for Diamond II. We have also investigated new beam injection schemes based on the concept of nonlinear injector kicker. The beam quality, capacity and reliability of radiation sources can be improved by lattice optimisation and advanced components to benefit the next generation users.

1.1 Single particle beam dynamics

In this chapter we review the basic formulae describing the motion of the electrons in the storage ring, the betatron oscillations and the equilibrium emittance derived from Refs. [6], [7], [8], [9], in order to introduce the nomenclature that will be used through out the thesis. In a circular accelerator, dipoles and quadrupoles are used to bend and focus the charged particles beam respectively. Because of inevitable momentum difference of the particles, chromatic aberration or focussing error is introduced and must be corrected with sextupoles magnets. We review here the equation of motion for an individual particle travelling in the accelerator ring.

1.1.1 Equations of motion

The equation of motion of a particle in a circular ring is usually given with respect to a reference trajectory and we generally assume small deviations in the positions ($x, y \ll 1$) and in the angles ($x', y' \ll 1$) which is the paraxial approximation. The ring is composed of dipoles with field B_0 , quadrupoles with gradient g and sextupoles with gradient g' giving the curvature of the trajectory as

$$\frac{1}{\rho} \approx x'' = \frac{eB}{cp} \quad (1.1)$$

with

$$\begin{aligned} B &= B_0 + gx + \frac{1}{2}g'x^2 + \dots \text{ or} \\ \frac{eB}{cp} &= \frac{1}{\rho_0} + kx + \frac{1}{2}mx^2 + \dots, \end{aligned} \quad (1.2)$$

where ρ_0 is the bending radius, k is the quadrupole strength, and m is the sextupole strength. The momentum dependent term $\frac{e}{cp}$ can be expand in term of momentum deviation from the nominal value ($\delta = \frac{\Delta P}{P_0}$) as

$$\frac{e}{cp} = \frac{e}{cp_0}(1 - \delta + \delta^2 - \delta^3 + \dots). \quad (1.3)$$

Considering the trajectory of particle with deviation from the ideal orbit Eq. 1.1 can then be rewritten

$$\frac{1}{\rho} - \frac{1}{\rho_0} \approx x'' = \frac{e}{cp}(B - B_0). \quad (1.4)$$

Thus we get the equation of motion with expansion of each term which is broadly associated with different dynamics effects as follows:

$$\begin{aligned} x'' = & -kx && \text{focussing term} \\ & + \frac{1}{\rho_0} \delta && \text{dispersion term} \\ & + kx\delta && \text{chromatic aberration} \\ & - \frac{1}{2}mx^2 && \text{chromatic and geometric aberration} \\ & + \mathcal{O}(3) && \text{higher order terms} \end{aligned} \quad (1.5)$$

The contribution of nonlinear terms have usually a great impact on the beam dynamics. Such terms increase rapidly with the particle amplitude (x) and lead to chaotic oscillation and further to fast particle loss, reducing the region of stability for the beam motion.

The linear terms (dipole and quadrupole) of the equation define the linear beam optics, and give the differential equation of motion (also known as Hill's equation)

$$x'' + k(s)x = \frac{1}{\rho_0} \delta. \quad (1.6)$$

where δ is the momentum deviation. Assuming the on-momentum particle ($x'' + k(s)x = 0$), the solution of the equation of motion can be found as harmonic oscillations in the form

$$x(s) = a_i \sqrt{\beta(s)} \cos(\mu(s) + \phi_i) \quad (1.7)$$

where $\beta(s)$ is the betatron function, $\mu(s)$ is the betatron phase, a_i and ϕ_i are integration

constants for particle i . The particle oscillations described by Eq. 1.7 are also called betatron oscillations with a position-dependent amplitude determined by $a_i\sqrt{\beta}$ and phase $\mu(s)$. While the beam envelope is a position-dependent quantity

$$E_{x,y}(s) = \pm \hat{a}_{x,y} \sqrt{\beta_{x,y}(s)} = \pm \sqrt{\varepsilon_{x,y} \beta_{x,y}(s)}, \quad (1.8)$$

which is the invariant of motion governed by the beam emittance ($\varepsilon_{x,y}$) corresponding to the area of the ellipse described by the particle in the phase space (x, x') . When dealing with distribution of electrons, this area is the RMS emittance of the beam. From Eq. 1.6 and 1.7, the betatron phase advance can be derived as a function of position along the machine (s)

$$\mu(s) = \int_0^s \frac{1}{\beta(u)} du. \quad (1.9)$$

In circular machines, the integral of the phase advance along the ring circumference (C) defines the betatron tune in both the horizontal and vertical planes

$$\nu_{x,y} = \frac{\mu_{x,y}(C)}{2\pi} = \frac{1}{2\pi} \oint \frac{du}{\beta_{x,y}(u)}. \quad (1.10)$$

Thus the betatron tunes give the number of betatron oscillations of the electrons in one revolution around the ring. Therefore, for betatron tunes close to integer numbers or resonance relations, small perturbation or errors could be sampled identically at every turn and drive beam loss. In the lattice optimisation, the betatron tunes have to be far from resonance line defined as

$$p\nu_x + q\nu_y = n \quad (1.11)$$

where p, q, n are integers, $|p| + |q|$ is the resonance order. Figure 1.1 shows the tune diagram produced by Eq.1.11 considering resonance up to the 4th order with $n = 1$. For an ideal machine, higher super-periodicity induces less resonance lines in the tune diagram thus improving the beam dynamics in the storage ring [10].

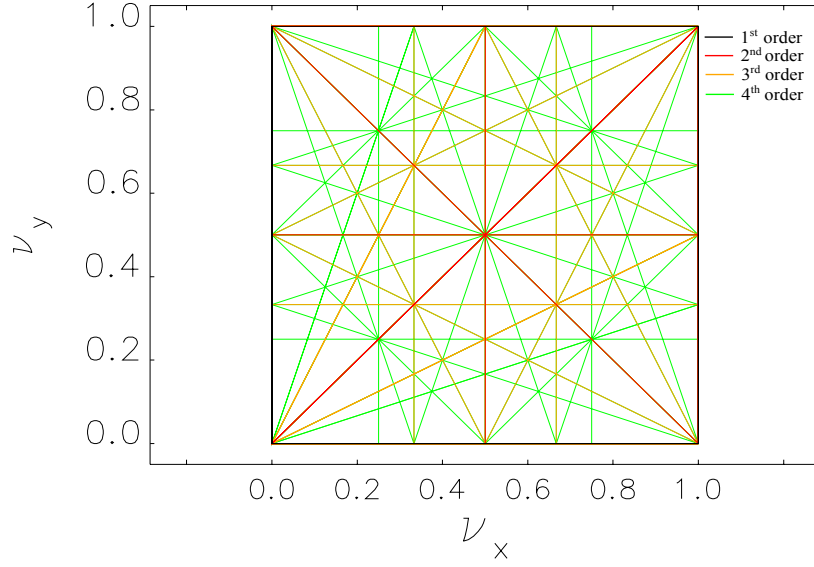


Figure 1.1: Betatron tune diagram with resonance line up to the 4th order in one integer unit.

1.1.2 Off-momentum equation of motion

For off-momentum particle motion, the total deviation of the particle from the nominal orbit is written as

$$x(s) = x_\beta(s) + x_\eta(s), \quad (1.12)$$

where $x_\beta(s)$ is the betatron motion and $x_\eta(s)$ is the off-momentum orbit. The deviation of the orbit for off-momentum particle is given by

$$x_\eta(s) = \eta(s)\delta, \quad (1.13)$$

where δ is the momentum deviation and $\eta(s)$ is the dispersion function. The betatron oscillations of off-momentum particles occur around the off-momentum orbit. Eq. 1.6 with $\delta = 1$ defines the dispersion function

$$\eta''(s) + k(s)\eta(s) = \frac{1}{\rho(s)}. \quad (1.14)$$

For a dipole with constant bending radius (ρ) the dispersion and its derivative can be expressed as

$$\eta(s) = \eta_0 \cos \frac{s}{\rho} + \eta'_0 \rho \sin \frac{s}{\rho} + \rho(1 - \cos \frac{s}{\rho}) \quad (1.15)$$

$$\eta'(s) = -\frac{\eta_0}{\rho} \sin \frac{s}{\rho} + \eta'_0 \cos \frac{s}{\rho} + \sin \frac{s}{\rho}. \quad (1.16)$$

In case of combined function dipole, the additional focussing from quadrupole gradient ($K = -k + \frac{1}{\rho^2}$) has to be considered, and the dispersion function and its derivative can be expressed as [11]

$$\begin{pmatrix} \eta_x \\ \eta'_x \\ 1 \end{pmatrix} = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_{x0} \\ \eta'_{x0} \\ 1 \end{pmatrix} \quad (1.17)$$

For $K > 0$

$$\begin{pmatrix} C & S & D \\ C' & S' & D' \end{pmatrix} = \begin{pmatrix} \cos(\varphi) & \frac{1}{\sqrt{|K|}} \sin(\varphi) & \frac{1}{\rho K} (1 - \cos(\varphi)) \\ -\sqrt{|K|} \sin(\varphi) & \cos(\varphi) & \frac{1}{\rho \sqrt{K}} \sin(\varphi) \end{pmatrix} \quad (1.18)$$

For $K < 0$

$$\begin{pmatrix} C & S & D \\ C' & S' & D' \end{pmatrix} = \begin{pmatrix} \cosh(\varphi) & \frac{1}{\sqrt{|K|}} \sinh(\varphi) & -\frac{1}{\rho |K|} (1 - \cosh(\varphi)) \\ \sqrt{|K|} \sinh(\varphi) & \cosh(\varphi) & \frac{1}{\rho \sqrt{|K|}} \sinh(\varphi) \end{pmatrix} \quad (1.19)$$

with $\varphi = s\sqrt{|K|}$, where C, S, D and C', S', D' are the cosine and sine like solutions of the Hill's equation, the dispersion function and their derivative respectively.

1.1.3 Chromaticity

Chromaticity or focusing error for off energy particles, is one of the major problems in the design of circular particle accelerators both for light sources and particle colliders. The

focusing error can be dramatic for particles collider whose final focusing at the interaction point (IP) requires extremely small beam which can be achieved by very strong quadrupoles. However even in modern light sources, where the low emittance is achieved by strongly focussing quadrupoles, the correction of chromatic errors is one of the most difficult problems in the design of the lattice.

In general, the distribution of charged particles stored in a bunch has a small momentum or energy deviation with respect to the nominal design. Particles with momentum deviation ($\Delta p \neq 0$) respond to external magnetic field differently leading to focusing errors in quadrupoles. Such focusing error consequently introduces a variation of the betatron tunes of the particles with their energy. This energy dependent focusing error is termed chromaticity in analogy with the energy dependent effects in ray optics. The higher energy the particle the less it can be deflected by the external magnetic field as shown in Figure 1.2. The focusing error, therefore, leads to a change in the betatron oscillations and in the betatron tunes from the nominal designed values. The off-momentum electrons in a quadrupole experience a different focussing depending on their momentum as follows:

$$\begin{aligned} k(p) &= \frac{e}{p}g \\ \Delta k &= \frac{dk}{dp}\Delta p = -\frac{eg}{p_0}\left(\frac{\Delta p}{p_0}\right) = -k_0\delta, \end{aligned} \tag{1.20}$$

For horizontal focussing quadrupole ($k_0 > 0$), higher momentum particle ($\delta > 0$) experiences weaker quadrupole strength and vice versa for particle with $\delta < 0$, whereas, for horizontal defocusing ($k_0 < 0$), the situation is opposite. These effects introduce a potentially dangerous betatron tune shift or tune spread across the resonance line especially for particles with large momentum deviation. The tune shift due to a focussing element error can be expressed by

$$d\nu = -\frac{1}{4\pi}\beta(s)\delta k_0 ds. \tag{1.21}$$

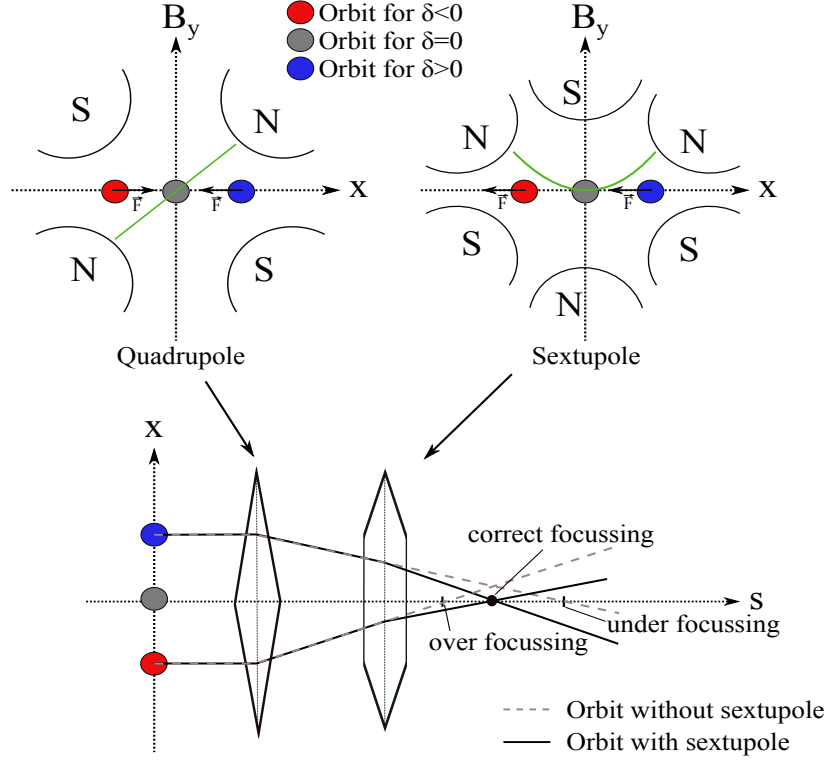


Figure 1.2: Above: Force on electrons moving out of the page affecting the dispersive orbit produced by quadrupole and sextupole field (green line) in a position with non-zero dispersion function. Both high and low momentum particles are kicked toward the centre by quadrupole. The sextupole kicks the high momentum particle toward the centre but kicks the low momentum particle outwards. Below: Focussing effect from pure quadrupole introducing focussing error (chromaticity) which can be corrected by sextupole.

Hence the total tune shift along the ring is given by

$$\Delta\nu = -\frac{1}{4\pi} \oint \beta(s) \frac{\Delta p}{p_0} k(s) ds = -\frac{1}{4\pi} \oint \beta(s) \delta k(s) ds, \quad (1.22)$$

From Eq. 1.22, chromaticity can be defined as the tune variation in the unit of momentum deviation as follows:

$$\xi_x = \frac{\Delta\nu_x}{\Delta p/p_0} = -\frac{1}{4\pi} \oint \beta_x(s) k_x(s) ds, \quad (1.23)$$

$$\xi_y = \frac{\Delta\nu_y}{\Delta p/p_0} = -\frac{1}{4\pi} \oint \beta_y(s) k_y(s) ds. \quad (1.24)$$

This is the so-called natural chromaticity and it depends only on the dipoles and quadrupoles

in the ring. From the equations of motions including the effect of sextupoles we have:

$$\begin{aligned} x'' + kx &= kx\delta - \frac{1}{2}m(x^2 - y^2), \\ y'' - ky &= -ky\delta + mxy. \end{aligned} \quad (1.25)$$

For an off-momentum particle, the dispersion trajectories with horizontal bending only are expressed as:

$$\begin{aligned} x &= x_0 + \eta_x \delta, \\ y &= y_0. \end{aligned} \quad (1.26)$$

Thus the equation of motions can be rewritten

$$\begin{aligned} x_0'' + kx_0 &= kx_0\delta - mx_0\eta_x\delta - \frac{1}{2}m(x^2 - z^2) + \eta_x^2\delta^2 + \mathcal{O}(3), \\ y_0'' - ky_0 &= -ky_0\delta + my_0\eta_x\delta + mx_0y_0 + \mathcal{O}(3), \end{aligned} \quad (1.27)$$

where m is the sextupole strength and $\mathcal{O}(3)$ is the higher order nonlinear terms. Considering only the linear perturbation of the equations we have

$$\begin{aligned} x_0'' + kx_0 &= (k - m\eta_x)x_0\delta, \\ y_0'' - ky_0 &= -(k - m\eta_x)y_0\delta. \end{aligned} \quad (1.28)$$

We can now calculate the chromaticities generated by quadrupole focussing error and by sextupoles in dispersive section around the ring

$$\begin{aligned} \xi_x &= -\frac{1}{4\pi} \oint \beta_x(s) \cdot [k(s) - m(s)\eta(s)] ds, \\ \xi_y &= -\frac{1}{4\pi} \oint \beta_y(s) \cdot [k(s) + m(s)\eta(s)] ds. \end{aligned} \quad (1.29)$$

Therefore, in horizontal and vertical planes the effect of the sextupole is opposite. To efficiently correct the chromaticity in each plane, chromatic sextupoles have to be placed

in large dispersion and well separated horizontal and vertical beta functions. Horizontal chromaticity correction needs large horizontal and small vertical beta function and vice versa for vertical chromaticity correction.

After a horizontal focusing quadrupole $k > 0$, a focussing sextupole with $m > 0$ is needed to correct the chromaticity next to the focusing quadrupole as shown in Figure 1.2. The sextupole compensates the focussing error by deflecting the high momentum particle ($\delta > 0$) toward the centre (focusing) and deflecting the low momentum particle ($\delta < 0$) outward (defocussing) which therefore counteract the effect of quadrupole and produce the same focal length for off-momentum particles.

For ultra-low emittance lattice design employing weak dipole field (larger bending radius ρ_0), the dispersion function in Eq. 1.15 will be smaller than that in the stronger dipole lattice. Although strong quadrupole will be used, small dispersion function is generally necessary to achieve ultra-low emittance. As a consequence stronger sextupoles will be required for chromaticity correction in such lattices. The presence of strong sextupoles produce strong nonlinear terms in the equation of motion Eq. 1.5 which are responsible for additional geometric aberrations. In conclusion, although a small dispersion function is required in order to achieve ultra-low emittance, it leads to a very challenging nonlinear beam dynamics optimisation.

1.2 Radiation effects

1.2.1 Radiation damping

Synchrotron radiation in electron circular machines generate a natural damping of the beam oscillation in all directions in 6D phase space coordinates (x, x', y, y', dE, t) which is called radiation damping. A brief explanation of this effect is reported in Figure 1.3: a particle randomly emits synchrotron radiation and loses momentum in both the longitudinal and transverse planes in the direction parallel to the original momentum before the emission

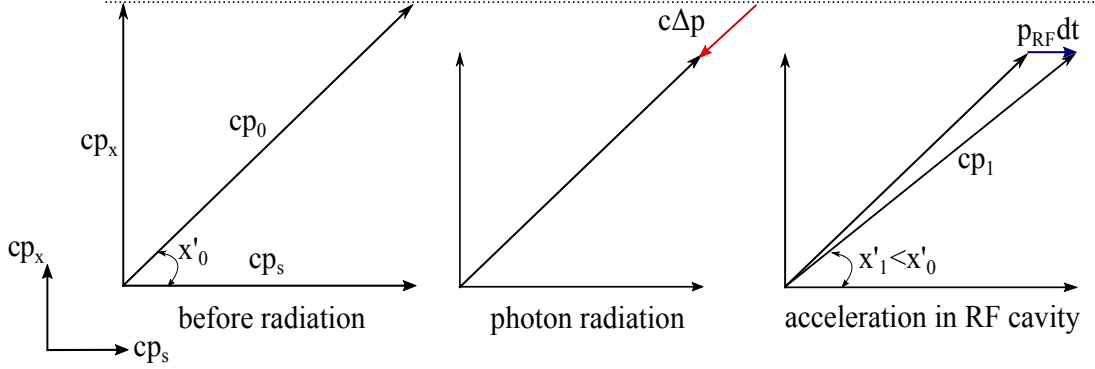


Figure 1.3: Transverse radiation damping process.

of the photon. To restore the energy lost, RF cavities are used, however they restore the momentum only in the longitudinal direction ($P_{RF}dt = P_{\gamma}dt$). The process continues and the transverse momentum becomes smaller every turn because of damping effect as a result of the radiation and acceleration.

From Figure 1.3, right after the photon radiation and longitudinal acceleration, the same transverse momentum gives

$$(cp_0 - c\Delta p)x'_0 = (cp_0 - c\Delta p + P_{RF}dt)x'_1, \quad (1.30)$$

where x'_0 and x'_1 are the angles of the particle trajectory before and after acceleration by the RF cavity respectively. Considering the variation in time, from $x' = \dot{x}/\beta c$, and the particle's energy E right after the emission:

$$\dot{x}_1 = \frac{E}{E + P_{\gamma}dt} \dot{x}_0 \approx \left(1 - \frac{P_{\gamma}dt}{E_0}\right) \dot{x}_0. \quad (1.31)$$

Now with very small change in energy $E \approx E_0$, the damping rate in horizontal and vertical planes are given by

$$\alpha_x = -\frac{1}{\dot{x}_0} \frac{d\dot{x}}{dt} = \frac{P_{\gamma}}{E_0}, \quad (1.32)$$

$$\alpha_y = -\frac{1}{\dot{y}_0} \frac{d\dot{y}}{dt} = \frac{P_{\gamma}}{E_0}. \quad (1.33)$$

As a result, the damping effect introduces a reduction of betatron oscillation amplitude ($A_0(t)$) from Eq.1.7 as a function of time:

$$A(t) = A_0(t)e^{-\frac{2P_\gamma}{E_0}t} = A_0(t)e^{-\alpha_s t}, \quad (1.34)$$

where $\alpha_s = 2\frac{P_\gamma}{E_0}$ is the longitudinal damping rate. The relationship between the damping rates in different planes can be written as

$$\alpha_s + \alpha_x + \alpha_y = 2\frac{P_\gamma}{E_0} + \frac{P_\gamma}{E_0} + \frac{P_\gamma}{E_0} = 4\frac{P_\gamma}{E_0}. \quad (1.35)$$

Thus the damping in a particular plane can be influenced by damping in the other planes.

1.2.2 Quantum excitation

Although the energy loss per turn is small compared to the beam energy for a few GeV but grows with the beam energy γ^4 and the perturbation to the beam itself has significant macroscopic effects. Consider an on-energy particle, as shown in Figure 1.4: the photon is emitted in quanta of discretized energy and introduces an oscillation of the electron around the off-momentum orbit. This process acts as noise source in phase space preventing the amplitude of the oscillation to be damped to zero, and acting in opposition to the radiation

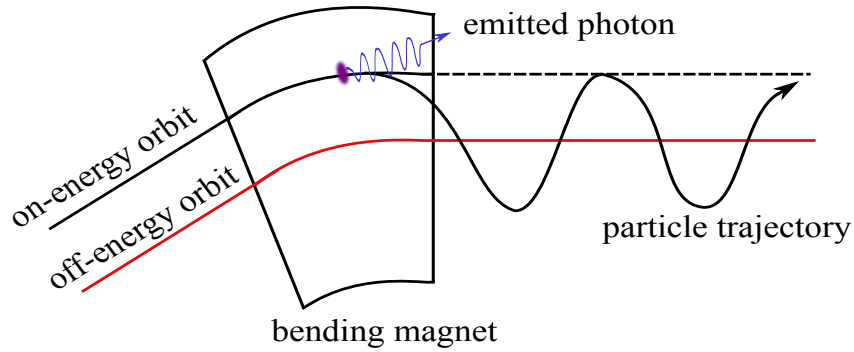


Figure 1.4: Particle trajectory during quantum excitation process. After photon emission, the particle move from on-energy orbit (black) and oscillate around the off-momentum orbit (red).

damping. It can be proven [6] that the oscillation amplitude variation rate for all photon energy emission along the ring is

$$\left\langle \frac{dA^2}{dt} \right\rangle = \int_0^\infty u^2 \dot{n}(u) du = \dot{N}_{ph} \langle u^2 \rangle_{tot} \quad (1.36)$$

where $\dot{n}$ is the time derivative of the number of emitted photon with energy u and $\dot{N}_{ph}$ is the total time derivative of all photon energy and

$$\dot{N}_{ph} \langle u^2 \rangle = \frac{55}{24\sqrt{3}} \frac{\gamma^7}{\rho^3} (cC_\gamma \hbar c (mc^2)^4),$$

where C_γ is $\frac{4\pi}{3} \frac{r_c}{(mc^2)^3} = 8.857 \cdot 10^{-5} \text{ m/GeV}^3$ for electron. The radiation is negligible for a heavier particle like proton because of much smaller C_γ and low γ . For TeV energy proton, on the other hand, the synchrotron radiation is not negligible.

1.3 Equilibrium beam parameters

The beam dynamics of the electrons in a storage ring is strongly influenced by the process of emission of synchrotron radiation. The two opposite effects described, radiation damping and radiation excitation, eventually reach an equilibrium. The corresponding distribution in phase space defines all the equilibrium beam parameters.

1.3.1 Equilibrium energy spread

The effects of radiation damping and quantum excitation were described in Eq.1.34 and 1.36 respectively and the equilibrium condition can be computed by

$$\left\langle \frac{dA^2}{dt} \right\rangle_{tot,damping} + \left\langle \frac{dA^2}{dt} \right\rangle_{tot,excitation} = 0, \quad (1.37)$$

where $\left\langle \frac{dA^2}{dt} \right\rangle_{tot,damping} = -2\alpha_s \langle A^2 \rangle$. Introducing the longitudinal damping time $\tau_s = 1/\alpha_s$, we can write

$$\langle A^2 \rangle = \frac{1}{2} \tau_s \langle \dot{N}_{ph} \langle u^2 \rangle \rangle_{tot}. \quad (1.38)$$

By assuming a Gaussian beam, the energy spread is defined by $\sigma_u^2 = \frac{\langle A^2 \rangle}{2}$, we can write

$$\sigma_u^2 = \frac{1}{4} \tau_s \langle \dot{N}_{ph} \langle u^2 \rangle \rangle_{tot}, \quad (1.39)$$

where

$$\dot{N}_{ph} \langle u^2 \rangle = \frac{55}{24\sqrt{3}} \langle P u_c \rangle$$

with the radiation power $P = \frac{e^2 c}{6\pi\epsilon_0} \frac{\gamma^4}{\rho^2}$ and critical radiated energy $u_c = \frac{3}{2} \frac{\hbar c \gamma^3}{\rho}$. The damping time definition is given by

$$\tau_s = \frac{2}{J_s} \frac{E_0}{\langle P \rangle},$$

where J_s is the longitudinal damping partition number. Finally, Eq. 1.39 gives the equilibrium energy spread

$$\left(\frac{\sigma_u}{E_0} \right)^2 = C_q \gamma^2 \frac{\langle 1/\rho^3 \rangle}{J_s \langle 1/\rho^2 \rangle}, \quad (1.40)$$

where

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar c}{m c^2} = 3.8319 \cdot 10^{-13} \text{m} \quad (1.41)$$

The bunch length can be determined by the equilibrium energy spread according to

$$\begin{aligned} \sigma_l &= \frac{c|\eta|}{\omega_s} \frac{\sigma_u}{E_0} \\ &= \frac{\sqrt{2\pi}c}{\omega_{rev}} \sqrt{\frac{-\eta E_0}{\hbar e V_{rf} \cos(\phi_s)}} \frac{\sigma_u}{E_0} \end{aligned} \quad (1.42)$$

where $\eta = \alpha_c - 1/\gamma$ is the phase slip factor, ϕ_s is the synchronous phase given by $V_{rf} \sin(\phi_s) = U_0$, ω_{rev} is the revolution frequency and h is the harmonic number.

1.3.2 Equilibrium emittance

The beam emittance is the area occupied by the beam in phase space (position and angle). Computing the change in the variation of horizontal coordinates due to the emission of a photon

$$\delta x_\beta = -\eta(s) \frac{u}{E_0}, \quad \delta x'_\beta = -\eta'(s) \frac{u}{E_0}. \quad (1.43)$$

And the corresponding change in the betatron oscillation invariant that represent the electron beam

$$A^2 = \gamma x^2 + 2\alpha x x' + \beta x'^2, \quad (1.44)$$

where γ_x , α_x , and β_x are the twiss functions and η and η' are the dispersion function and its derivative respectively, it is possible to prove that The average of the invariant increase can then be written as [6]

$$\left\langle \frac{dA^2}{dt} \right\rangle_{tot, excitation} = \left\langle \frac{\dot{N}_{ph} \langle u^2 \rangle \mathcal{H}}{E_0^2} \right\rangle_{tot} \quad (1.45)$$

with dispersive invariant

$$\mathcal{H} = \gamma \eta^2 + 2\alpha \eta \eta' + \beta \eta'^2 \quad (1.46)$$

Including the damping effect giving $\langle A^2 \rangle_{tot} = \frac{1}{2} \langle \dot{N}_{ph} \langle u^2 \rangle \mathcal{H} \rangle_{tot}$ and the RMS beam size $\sigma_x^2 = \frac{x^2}{2} = \frac{A^2 \beta_x}{2}$, the equilibrium emittance can be obtained

$$\varepsilon_x = \frac{\sigma_x^2}{\beta_x} = C_q \frac{\gamma^2 \langle \mathcal{H} / \rho^3 \rangle}{J_x \langle 1/\rho^2 \rangle}, \quad (1.47)$$

where J_x is the horizontal damping partition number. The equilibrium or natural beam emittance is a constant for a given energy and lattice. For a fixed beam energy the main contributor to the emittance is the bending radius (ρ) and the dispersive invariant ($\mathcal{H}$). For the ideal orbit, the Gaussian beam in phase space (x, x') as shown in Figure 1.5 follows the ellipse equation in Eq. 1.44. The ellipse with $A^2 = \varepsilon_x$ gives the RMS beam size (σ_x) and divergence (σ'_x) on the 1σ contour

$$\sigma_x = \sqrt{\varepsilon_x \beta_x(s)}, \quad \sigma'_x = \sqrt{\varepsilon_x \gamma_x} \quad (1.48)$$

As a result, the emittance can be measured via the beam size and divergence at a specific position in the machine if the optics functions are known.

In the vertical plane, for the ideal machine, there is no dispersion function $\eta_y = 0$ and $\mathcal{H}_y = 0$ and therefore, the vertical beam emittance is zero. However, synchrotron radiation can occur at the angle θ_y which changes the angle and momentum of the electron $\Delta p_\perp \neq 0$.

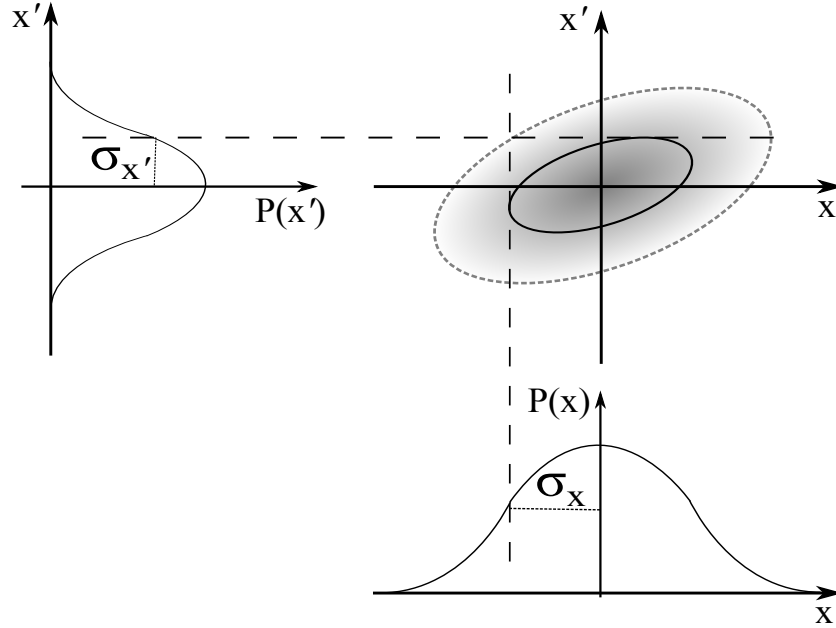


Figure 1.5: Electron density for a Gaussian distribution in horizontal phase space (x, x') . The projections of the beam profile on horizontal angle and position axes are shown on the left and bottom plot respectively.

Thus the vertical equilibrium beam emittance is

$$\varepsilon_y = \frac{\sigma_y^2}{\beta_y} = C_q \frac{\gamma^2}{J_y} \frac{\langle \beta_y \rangle \langle 1/\rho^3 \rangle}{1/\rho^2}, \quad (1.49)$$

where J_y is the horizontal damping partition number. The ε_y value is the quantum limit for the vertical emittance and it is measurable only when all other sources of vertical emittance have been corrected in the ring. These are due to the coupling of horizontal and vertical betatron motion caused by machine imperfections and skew quadrupole and vertical dispersion error caused by vertical bending field errors and misalignment of quadrupoles.

The above equilibrium beam dimension formulae can also be expressed in term of synchrotron radiation integrals [12]

$$\begin{aligned} I_1 &= \oint \frac{\eta}{\rho} ds, \\ I_2 &= \oint \frac{1}{\rho^2} ds, \\ I_3 &= \oint \frac{1}{\rho^3} ds, \\ I_4 &= \oint \eta \left(\frac{1}{\rho^3} + 2 \frac{b_2}{\rho} \right) ds, \\ I_5 &= \oint \frac{\mathcal{H}}{\rho^3} ds, \end{aligned} \quad (1.50)$$

where $b_2 = \frac{\partial B_y}{\partial x} / (B\rho) > 0$ is the quadrupole strength for an horizontal focusing gradient.

By using the above radiation integral, some of the beam parameters can be conveniently expressed as follows:

Damping partition numbers:

$$J_x = 1 - \frac{I_4}{I_2}, \quad J_y = 1, \quad J_s = 2 + \frac{I_4}{I_2}. \quad (1.51)$$

The sum of the damping partition number gives $J_x + J_y + J_s = 4$. This result is also known as Robinson theorem [13].

The damping times are given by:

$$\tau_x = \frac{3T_0}{r_0\gamma^3} \frac{1}{I_2 - I_4}, \quad \tau_y = \frac{3T_0}{r_0\gamma^3} \frac{1}{I_2}, \quad \tau_s = \frac{3T_0}{r_0\gamma^3} \frac{1}{2I_2 + I_4}, \quad (1.52)$$

where T_0 is the synchronous orbit period. The damping effect is governed by the integral I_2 and I_4 .

1.4 Photon source parameters

The main purpose of synchrotron light sources is to provide synchrotron radiation for experiments in beamlines. The main quality factors are the photon brightness and the transverse coherence.

1.4.1 Brightness

The photon brightness is a measure of the photon density in phase space and it is a conserved quantity in the transport of an optical system. The photon flux is the number of photons produced per second in a bandwidth and is given by

$$F(\lambda) = \frac{\text{number of photons}}{(0.1\% \text{bandwidth})(s)}. \quad (1.53)$$

The brightness is defined as the photon flux per unit area per unit solid angle at the source

$$B = \frac{F}{4\pi^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}}, \quad (1.54)$$

where $\Sigma_{x,y}$ and $\Sigma_{x',y'}$ are the effective photon beam size and divergence. The relationship between photon beam and electron beam (assuming a Gaussian beam) can be written as

$$\Sigma_{x,y} = \sqrt{\sigma_{x,y}^2 + \sigma_r^2} \quad (1.55)$$

$$\Sigma_{x',y'} = \sqrt{\sigma_{x',y'}^2 + \sigma_{r'}^2}, \quad (1.56)$$

where $\sigma_{x,y}$ are the horizontal and vertical electron beam sizes, σ_r and $\sigma_{r'}$ are the photon beam size and divergence respectively, $\sigma_{x',y'}$ are the horizontal and vertical electron beam divergence. The beam size and divergence of electron and photon beam with the wavelength of λ are defined as

$$\sigma_{x,y} = \sqrt{\varepsilon_{x,y}\beta_{x,y} + (\sigma_E\eta_{x,y})^2}, \quad \sigma_{x',y'} = \sqrt{\frac{\varepsilon_{x,y}}{\beta_{x,y}}}, \quad (1.57)$$

$$\sigma_r = \frac{1}{2\pi}\sqrt{\frac{\lambda L_u}{2}}, \quad \sigma_{r'} = \sqrt{\frac{\lambda}{2L_u}} \quad (1.58)$$

where L_u is the insertion device length. Similar to the electron beam, the emittance of the photon beam is given by

$$\varepsilon_r = \sigma_r\sigma_{r'} = \frac{\lambda}{4\pi}, \quad (1.59)$$

which depends only on the photon wavelength and is also known as the diffraction limit emittance for a Gaussian beam having the wavelength λ . For 1 Å X-rays (12 keV), the diffraction limit emittance is 8 pm·rad. The diffraction limit is a fundamental limit of the photon emittance due to the natural propagation of electromagnetic waves. Eq. 1.59 shows that there is no benefit in reducing the electron beam emittance below the diffraction limit since the brightness will be dominated by the photon emittance.

The beta function of the photon beam for a L_u long insertion device is defined as

$$\beta_r = \frac{\sigma_r}{\sigma_{r'}} = \frac{L_u}{2\pi}. \quad (1.60)$$

The brightness can be maximized by matching the electron beta function with β_r and minimizing $\varepsilon_{x,y}$ toward the diffraction limit emittance in Eq 1.59.

1.4.2 Coherence

For X-rays experiments, not only the brightness of the source is important but also its coherence which indicates the quality of the phase relations between different position of the wave front and the possibility of the source to produce interference patterns. The transverse coherent flux for a given photon wavelength is expressed in term of the brightness as [4]

$$F_{coh,\perp}(\lambda) = B(\lambda) \left(\frac{\lambda}{2} \right)^2. \quad (1.61)$$

The coherent fraction is the ratio of the coherence flux with respect to the total photon beam flux

$$\begin{aligned} f_{coh} &= \frac{F_{coh,\perp}(\lambda)}{F(\lambda)} \\ &= \frac{\lambda/(4\pi)}{\Sigma_x \Sigma_{x'}} \frac{\lambda/(4\pi)}{\Sigma_y \Sigma_{y'}} \\ &= \frac{1}{\sqrt{1 + \left(\frac{\sigma_x}{\sigma_r} \right)^2} \sqrt{1 + \left(\frac{\sigma_{x'}}{\sigma_{r'}} \right)^2} \sqrt{1 + \left(\frac{\sigma_y}{\sigma_r} \right)^2} \sqrt{1 + \left(\frac{\sigma_{y'}}{\sigma_{r'}} \right)^2}}. \end{aligned} \quad (1.62)$$

At the diffraction limit condition ($\varepsilon_{x,y} = \varepsilon_r$), the horizontal coherence is 25% thus there is potentially interest in improving further the electron emittance below the diffraction limit to increase the coherent fraction. In conclusion brightness and coherence are crucial factors in the performance of a light source and they can be maximized by minimising the electron beam emittance.

1.5 Diamond light source

The Diamond Light Source is the UK's third generation synchrotron light source located at the Harwell science and innovation campus in Oxfordshire. It has been operating since 2007 providing unique quality of synchrotron source for scientific research.

The Diamond lattice consists of six super periods which are composed of four Double

Bend Achromat (DBA) cells as shown in Figure 1.6. A total of 24 DBA cells are distributed over a circumference of 561.6 m. In the super period, there are two kinds of DBA cell: cell1 which has a long and short straight on each side (LS, DBA, SS) and cell2 which is a symmetric cell that has a short straight on both side (SS, DBA, SS). Thus the super period can be constructed by cell 1, cell 2, cell 2 and the reverse of cell 1. The optics functions of a super period are shown in Figure 1.7. In the long straight section the beta functions are larger than those in the short straight. This allows a large beam acceptance at the injection

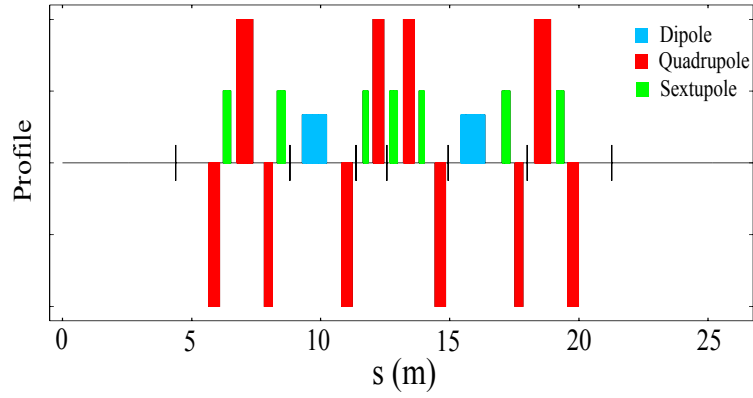


Figure 1.6: Magnets profile of the Diamond standard DBA cell. Quadrupoles above the middle line are horizontal focussing.

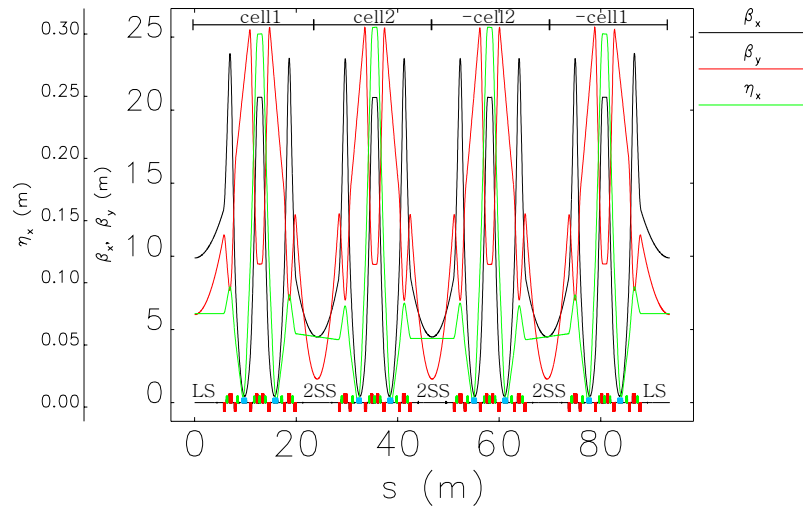


Figure 1.7: Optics functions of the Diamond super period (LS and SS is long and short straight section respectively).

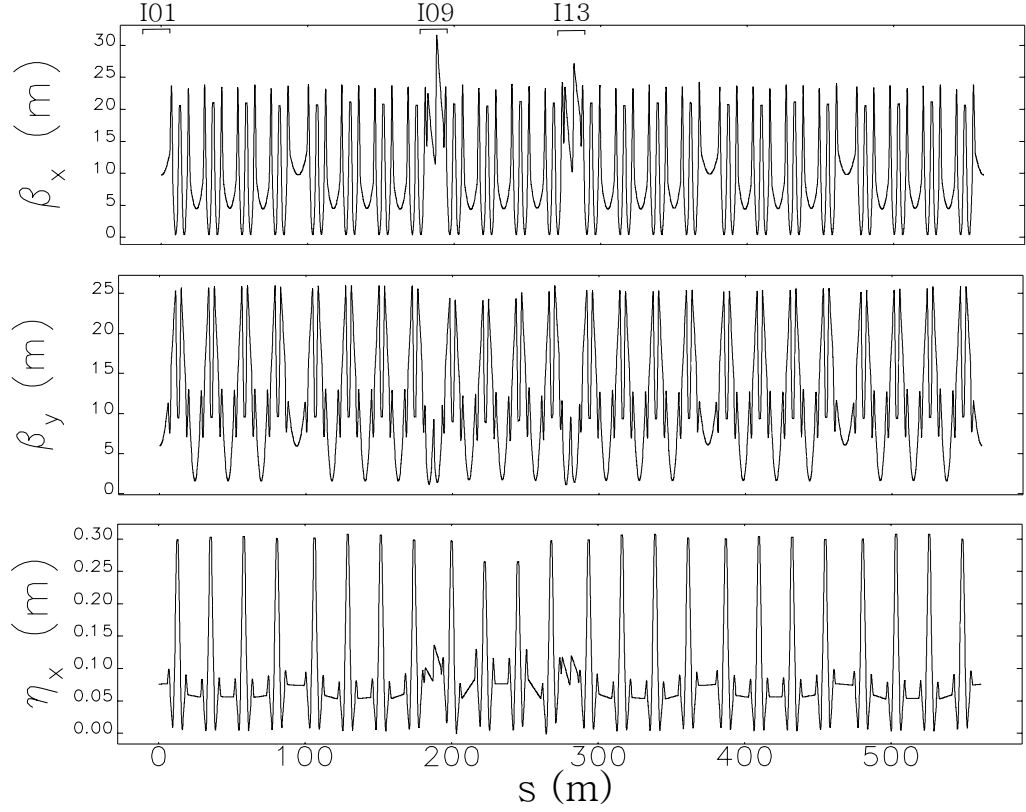


Figure 1.8: Optics functions of the Diamond storage ring.

point in the first long straight. In order to achieve a small emittance, the lattice has been optimised allowing the dispersion to leak in the straight sections. This has produced a DBA lattice with 2.7 nm·rad emittance. The conventional name of the beamlines for Diamond are Bn and In , where n is an integer number, for bending magnet and insertion device beamlines in the straight section respectively. The first straight section (I01) is used for beam injection with four kickers.

The optics functions of the Diamond storage ring are shown in Figure 1.8. Special customised straight sections have been implemented in the ring to satisfy special users' demand: double mini-beta sections which are used to accommodate two insertion devices in the long straight in straight I09 and I13 [14] and a planned small beta functions section in I21. Although these modifications broke the symmetry of the ring, intensive lattice optimisation and the flexibility given by independently powered magnets have allowed the machine to introduce these special straight sections without any deterioration of the machine perfor-

mance.

The performance of the standard Diamond storage ring lattice can be considered including machine imperfections. This allow us to compare the results with the presented lattice modifications. Misalignment as described in Table 3.4 was considered. Systematic and random multipole error in all magnets were also included. After introducing the imperfections, closed orbit, betatron tune and chromaticity corrections were performed. Then the error seeds giving the beam coupling of 0.3% is used in dynamic aperture (DA) and momentum aperture (MA) particles tracking. The dynamics apertures for 50 error seeds are shown in Figure 1.9. The DA about -12 to 15 mm is sufficient when the beam injection occurs at -8.3 mm. Similarly, the momentum aperture of 50 error machine is shown in Figure 1.10. Histograms in Figure 1.11 summarise the simulated Touschek lifetime and injection efficiency of the 50 cases. The calculation method for DA and Touschek lifetime will be explained in Chapter 2.

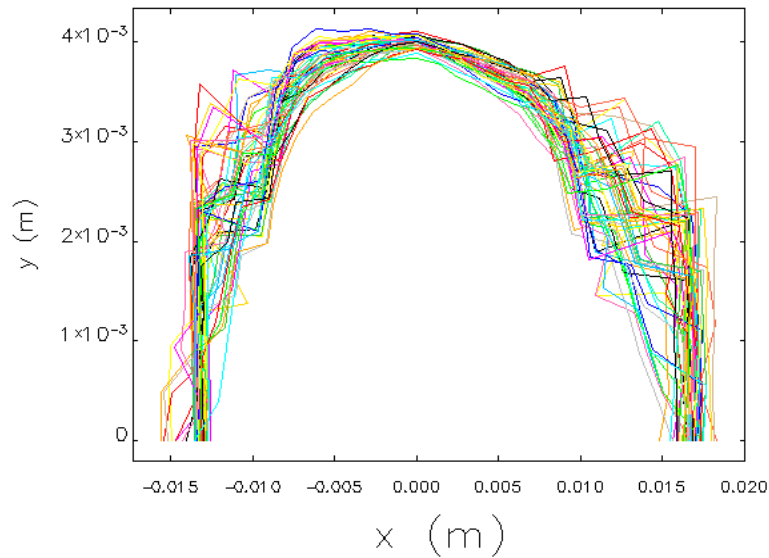


Figure 1.9: Dynamic aperture of Diamond storage ring (50 seeds errors.

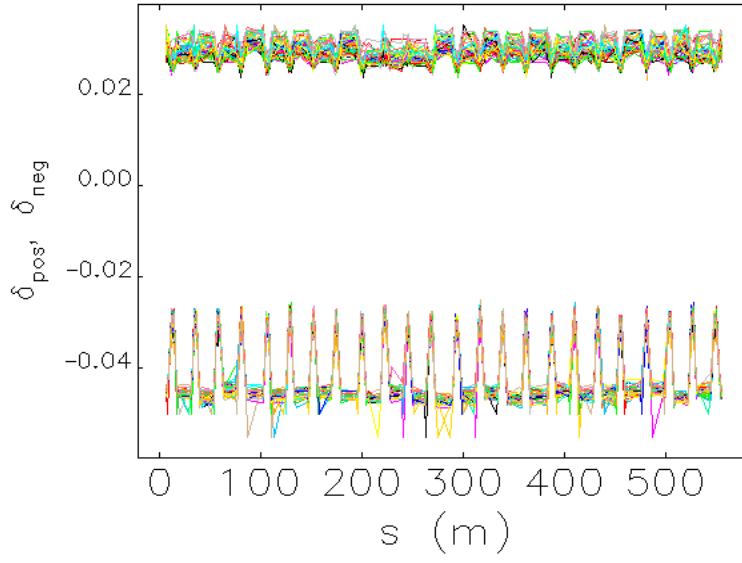


Figure 1.10: Momentum aperture of Diamond storage ring (50 seeds errors).

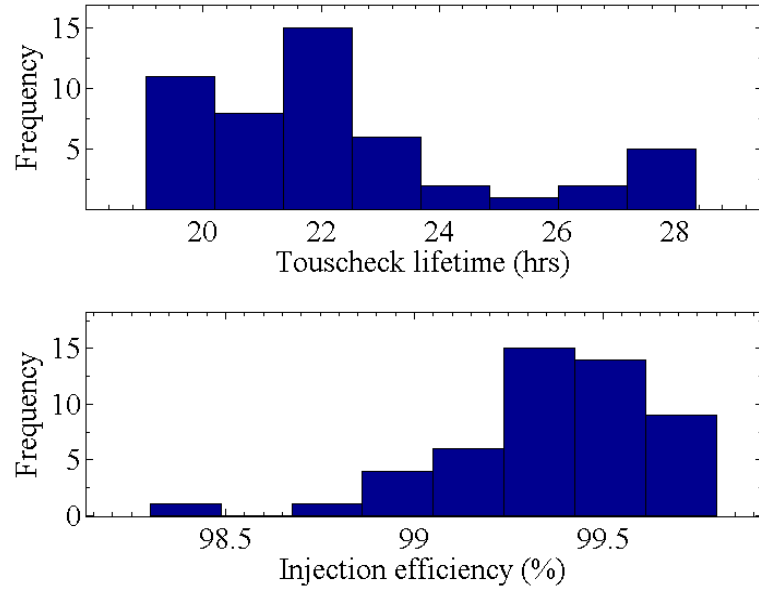


Figure 1.11: Histogram of Touschek lifetime and injection efficiency of the 50 seeds.

From the calculation of 50 error seeds, the average injection efficiency is $99.4 \pm 0.3\%$ and Touschek lifetime is 21.1 ± 1.9 hrs. These numbers will be used to compare with other lattice modifications to assess the solutions performance. Note that the measured injection efficiency

and Touschek lifetime in the machine are about 90% and 16 hrs. The possible sources of the discrepancy could be unknown machine imperfections and the effect of insertion devices.

While Diamond nominal beam parameters were state-of-the art at the beginning of operation, it is foreseeable that in the next decade Diamond will lose its competitiveness as compared to the new projects coming online. The next generation of storage rings aims at significantly smaller beam emittances below 1 nm·rad. In order to remain competitive, Diamond has also investigated the possibility of upgrading the lattice to minimize the beam emittance.

1.6 Ultra-low emittance lattices

A new generation of storage rings aiming for natural emittance below 0.5 nm·rad is currently studied by a large community worldwide for new machine projects as well as upgrade options for existing machine. Most of the new designs are based on the concept of Multi-bend achromats.

1.6.1 Multi-Bend Achromat (MBA) design

From Eq. 1.47, assuming an isomagnetic lattice (ρ is constant), the natural emittance can be written as [6]

$$\varepsilon_x \approx F \frac{C_q \gamma^2}{J_x} \Phi^3, \quad (1.63)$$

where Φ is the bending angle related directly to the total number of bending magnets by $\Phi = 2\pi/N_{bend}$ and F is the lattice design factor which depends on the optics functions in the specific lattice design. Thus the beam emittance depends strongly on the number of bending magnets (N_{bend}) and the beam energy E

$$\varepsilon_x \propto \frac{E^2}{N_{bend}^3}. \quad (1.64)$$

For a fixed beam energy, the beam emittance can be dramatically reduced by increase the number of bending magnets in the ring. A cell composed of multiple bending magnets also known as Multi-Bend Achromat (MBA) was proposed in the early 90s with a 4BA design [15] followed by an initial design of Swiss light source based on 7BA [16] whose emittance was about 490 pm-rad [17]. During these early times, the main difficulties in building such small emittance storage ring were the complexity of the strong and compact magnets and maintain vacuum properties in small aperture.

Generally, an MBA unit cell is composed of dipoles, quadrupoles and sextupoles grouped symmetrically together as shown in Figure 1.12. Then MBA can be constructed by centre the unit cells and two ends cells with one dipole each that allow the matching of the total MBA cell to the straight sections with additional quadrupoles.

MAX-IV [18], [19] is the first ultra-low emittance lattice to be built using an MBA cell with 7 bending magnets per cell (7BA) as shown in Figure 1.13a. The lattice is based on compact magnet designs and small vacuum chamber with NEG-coating (Non-Evaporable Getter). The design provides the emittance of 328 pm-rad at 3 GeV beam energy. The machine is due to give the first light in 2016. The possibility of obtaining even smaller beam emittance has also presented with a modified optics reducing the dispersion function in the

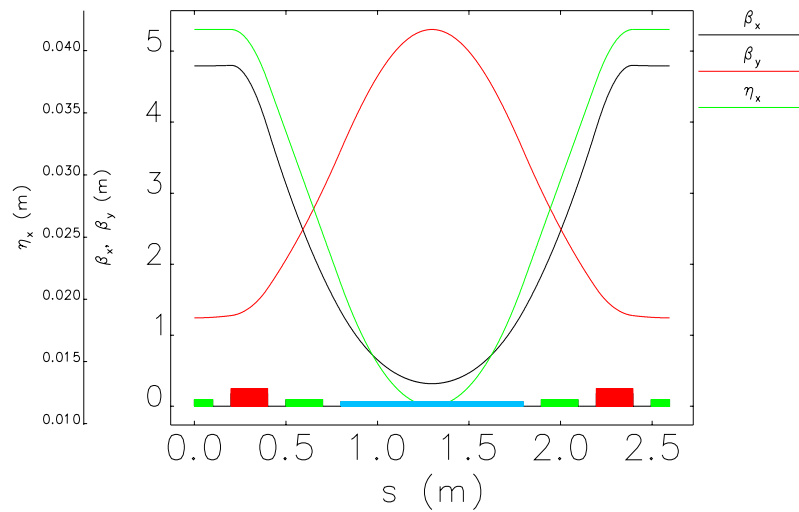


Figure 1.12: Optics functions of a unit cell for MBA lattice. Multiple of the unit cell will be used to construct an arc of the MBA cell.

cell giving 269 pm-rad beam emittance [20].

The same concept has been applied also by Sirius [21], [22], the Brazilian 3GeV light source under construction, that used 5BA lattice. The Sirius 5BA cell has a special modification at the centre of the cell where a longitudinal field gradient is created by sandwiching a thin 2 T super-bend (strong field bending magnet) in the low field middle dipole of 0.58 T. The design gives not only the low beam emittance of 280 pm-rad but also hard X-rad radiation (12 keV critical energy) from the super-bend in the middle.

A hybrid design which is currently gaining popularity is the hybrid-7BA design for ESRF-II as shown in Figure 1.13b. One of the major problems of an MBA lattices for ultra-low emittance is the small dispersion function in the cell and corresponding difficulties in correcting the chromaticity. The ESRF-II lattice design generates intentionally a dispersion bump between the outer two dipoles close to the straight section in order to allow the chromaticity correction locally, hence no sextupole in the arc is required. This allows small dispersion function in the dipoles which helps reduce the beam emittance further. The emittance for ESRF-II is about 150 pm-rad at 6 GeV beam energy. The hybrid 7BA design also motivated other facilities like Spring8-II (hybrid-5BA) [23] and APS-U (hybrid-7BA) [24] which employed similar cells. A complicated longitudinal gradient field has been used to

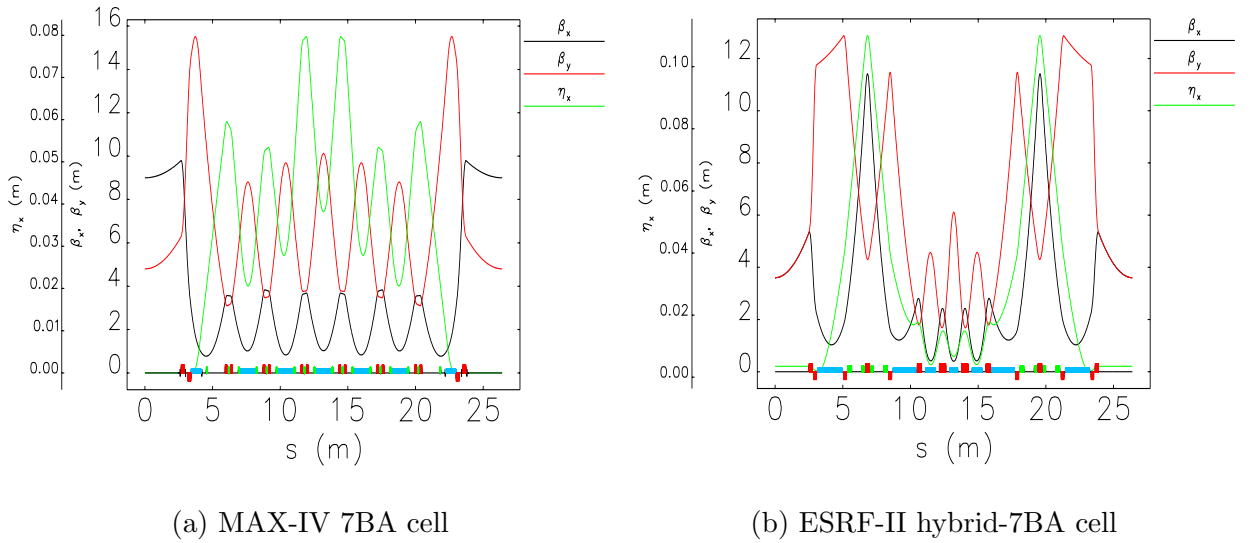


Figure 1.13: Optics functions of MBA cells

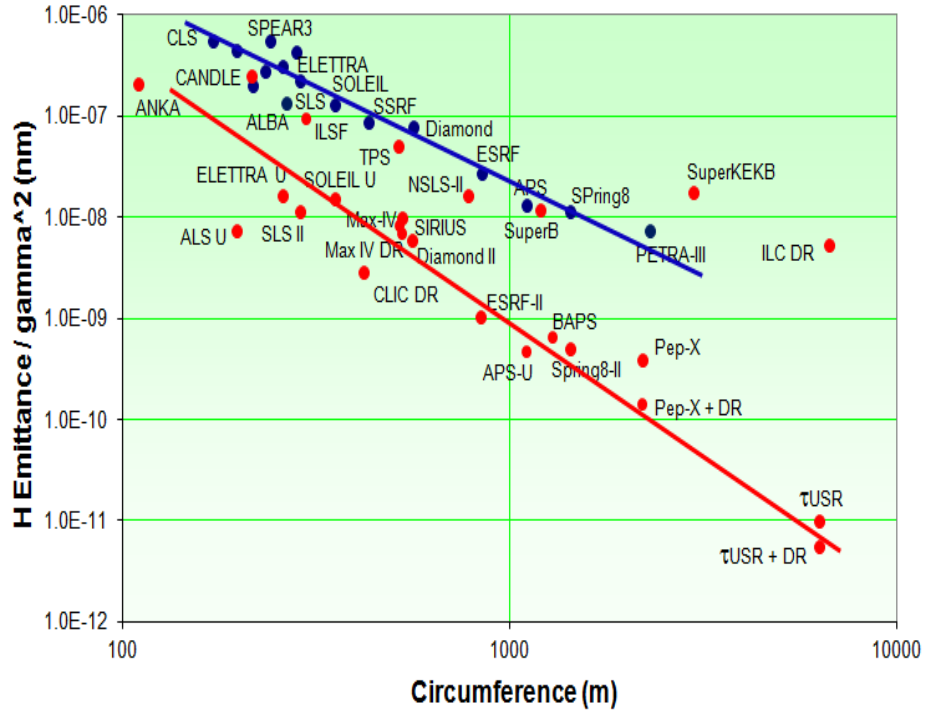


Figure 1.14: Normalised horizontal beam emittance as a function of ring circumference for existing machine (blue dot) and future machine (red dot) [26]. The lines show the trend of old design (blue) and MBA cell design (red).

optimise the radiation integral and reduce the emittance further in the design [25]. It should be highlighted that the dispersion bump technique is very useful for chromaticity correction in MBA lattice and the application will be discussed latter in Chapter 4.

Machine parameters of some selected MBA ring designs are presented in Table 1.1. In the table, the most popular choice for M is 7 considering the low emittance and minimum required straight section length of about 5 m for the total length of MBA cell of 25-30 m. Figure 1.14 shows the summary of the normalised beam emittance against ring circumference of operating and on-going design rings. There is a noticeable trend of old and new style machines, which use the MBA design and are able to reduce the emittance dramatically especially for those with large ring circumference. Diamond upgrade (modified-4BA), which will be discussed in Chapter 4, is also presented in the table with a unique characteristic of large available straight sections for IDs (44%) compared to the total ring circumference.

Table 1.1: Parameters for various MBA lattices [27]

Parameters	DIFL	MAX-IV	SIRIUS	ESRF-II	APS-II	DLS-II*	ALS-II	ELET-II	SLS-II
Energy[GeV]	3.0	3.0	3.0	6.0	6.0	3.0	2.0	2.0	2.4
Circumference[m]	404	528	518.4	844	1104	561.6	196.8	260	288
Emittance [nm-rad]	0.49	0.33	0.28	0.133	0.065	0.28	0.052	0.28	0.132
Jx	1.29	1.8	1.31	1.51	1.62	1.32	1.30	1.26	1.69
No. of cells	12	20	20	32	40	24	12	12	12
N_{bend} per cell	7	7	5	7	7	4	9	6	6
Bending field [T]	0.9375	0.5238	0.5835	0.7698	0.6065	0.8000		1.3615	2.0014
Gradient in bending [T/m]	9.166	8.566	7.805	37.025	46.832	14.41		14.677	30.981
Quadrupole gradient [T/m]	20.0	40.5	33.0	91.1	80.3	53.0	133.4	53.2	68.0
Nat. Chromaticity-X	-79.4	-50	-113.3	-97.12		-129		-79.5	-69.5
Nat. Chromaticity-Y	-38.5	-50.2	-80.2	-84.13		-93		-47.31	-34.16
Straight available [%]	15.4	17.9	27.1	20.5	21.1	44.6	41.2	24	19

*DLS-II is the Diamond upgrade option using modified-4BA lattice which will be discussed in Chapter 4.

1.6.2 Magnets design consideration

In most of the ultra-low emittance lattices, several dipoles are required for the fixed cell length. This makes the combined function dipoles, that integrate the quadrupole component, a desirable choice. A combined function dipole magnet with quadrupole gradient can reduce the space required for two quadrupoles in a symmetric unit cell. The bending magnet with defocusing quadrupole gradient also helps increase the horizontal damping partition number (J_x) and reduce the beam emittance (with the expense of increased energy spread). This will be discussed further in Chapter 2.

To introduce the quadrupole gradient G into a bending magnet the pole shape has to be specially designed. Similar to the weak focusing magnets used in the early synchrotron machine, the pole shape (half-gap height: $h(x)$) is varied as a function of horizontal position hyperbolically as

$$h(x) = \frac{h(0)}{1 - \frac{xG}{B_0}}, \quad (1.65)$$

where B_0 is the main dipole field. Figure 1.15 shows the pole shape for ALBA storage ring

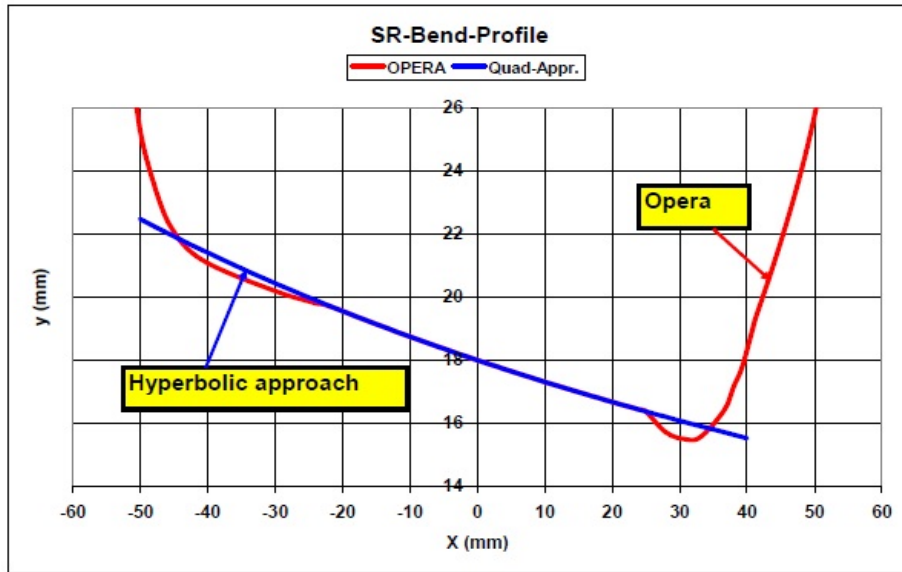


Figure 1.15: Combined function bending pole shape of ALBA storage ring [28]. The profile design using analytic formula and numerical program OPERA are plotted in blue and red respectively.

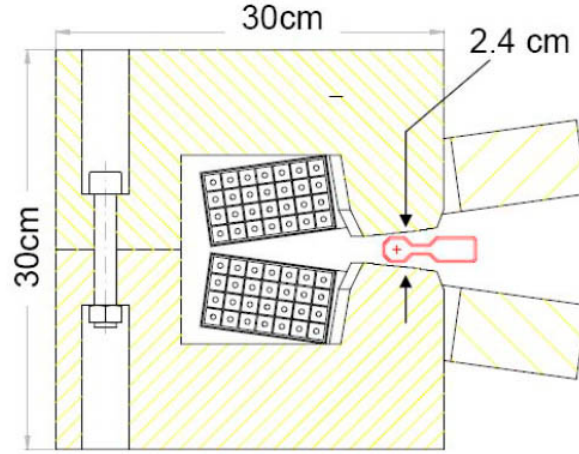


Figure 1.16: Max-IV combined bending magnet [29].

bending magnet.

The strength of the gradient in the dipole is limited by the pole shape, nominal dipole field and the required quality of both the dipole field and quadrupole gradient. The complicated pole shape introduces also other systematic multipole components which are associated with nonlinearity of the machine. The specified dipole field and gradient of some MBA lattices are presented also in Table 1.1. Figure 1.16 shows a compact designed combined function dipole of MAX-IV 7BA lattice which can achieved 0.52 T main field and 8.56 T/m quadrupole gradient. As shown in Table 1.1, the quadrupoles are unavoidably strong because of much stronger optics functions focussing which cause large natural chromaticity in MBA lattices and therefore, strong sextupoles for the chromaticity correction will be required.

1.7 Thesis layout

This thesis presents firstly an overview of ultra-low emittance ring designs aiming for better brightness and transverse coherence. Then it describes the works carried out to improve the existing lattice aiming for a better beam quality and accessibility to users which could be applied in the future.

The structure of the thesis is as follows. After the introductions, important criteria for lattice design, beam emittance optimisation techniques and useful algorithms for the lattice

optimisation will be discussed in Chapter 2. In Chapter 3, lattice tuning and implementation to achieve low emittance in Diamond without physical modification [30] and simulation for the existing cells replacements in order to achieve additional straight will be presented. Next, three different lattice options for the machine upgrade toward ultra-low emittance ring using Diamond as a model [31] will be covered in Chapter 4. Later in Chapter 5, a new injection scheme using a nonlinear pulsed kicker in Diamond will be presented. Eventually, conclusion and future work will be discussed in Chapter 6.

Chapter 2

Lattice design criteria and optimisation

In this chapter we review the main criteria for ultra-low emittance lattice design and optimisation. The minimisation of the equilibrium emittance is often in conflict with the need of achieving a good dynamic aperture for injection and a good momentum aperture for Touschek lifetime. In practice, the design is not driven exclusively by accelerator physics considerations, but also by engineering and capital cost that have to be taken into account. In the case of the upgrade of existing facilities further geometric constraints on the lattice must be considered, such as position and length of the straight sections to maintain the existing location of the beamlines. The upgrade tend to reuse the same tunnel, injection system and beamlines while the lattice components are replaced with the new designed lattice.

In this chapter, emittance optimisation and the main criteria which should be taken into account for lattice design and operation will be discussed. A set of effective optimisation methods and tools will be presented.

2.1 Review of beam emittance optimisation method

As discussed in Chapter 1, in order to achieve small emittance machine, a careful control of the optics functions is required. We will review here the main low-emittance options and the rationale for their choice.

The following emittance minimisation follows closely the treatment in [19]. From the definition in Eq 1.47, the beam emittance in practical unit can be expressed

$$\epsilon_x[\text{nm} \cdot \text{rad}] = 1470(E[\text{GeV}])^2 \frac{I_5}{J_x I_2}. \quad (2.1)$$

For homogeneous bending magnet with constant radius, the emittance can be rewritten as

$$\begin{aligned} \epsilon_x[\text{nm} \cdot \text{rad}] &= 1470(E[\text{GeV}])^2 \frac{\langle \mathcal{H} / \rho^3 \rangle}{J_x \langle 1/\rho^2 \rangle}, \\ &= 1470(E[\text{GeV}])^2 \frac{\langle \mathcal{H} \rangle}{J_x \rho}, \end{aligned} \quad (2.2)$$

therefore the beam emittance can be minimized by reducing the term $\langle \mathcal{H} \rangle / (J_x \rho)$ at the dipole considering the fixed beam energy. The beam emittance can also be reduced by lowering the beam energy, but generally the choice of the energy of the electron beam is dictated by the spectral range required by the user and this parameter will not be considered as part of the optimisation.

2.1.1 Emittance optimization by optics

The natural or equilibrium emittance of an electron beam in a storage ring is the result of the equilibrium between damping and quantum excitation effects as discussed in Chapter 1. It is possible to prove that we can minimise the contribution to the emittance for each dipole by considering the so called Theoretical Minimum Emittance (TME) cell. In general each lattice type has a theoretical minimum emittance value and the lattice design aim at reaching it to improve the photon brightness. To achieve the TME condition, from Eq. 1.47, the optics

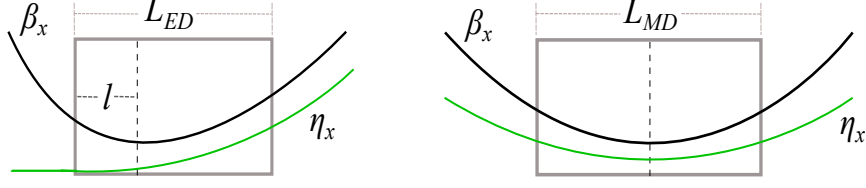


Figure 2.1: Schematic figure of edge and middle dipole (ED and MD) for an MBA cell. Horizontal betatron and dispersion functions are plotted in black and green respectively. Achromat condition gives zero dispersion function at the entrance of the ED and the minimal beta function is at l from the edge while for MD is at the middle of the dipole.

functions in bending magnets need to be optimised to minimise the dispersive invariant ($\mathcal{H}$). In simpler words, the horizontal dispersion and betatron function in bending magnets should be small.

We consider the minimisation of the contribution to the emittance in the dipole for two cases: the edge dipole (ED) which connects an arc with a straight the initial dispersion function is zero and the middle dipole (MD) at the centre of the cell [32] as shown in Figure 2.1. To obtain the minimum emittance in the middle dipole, consider the derivative of the dispersive invariant with respect to the beta and dispersion functions in these dipoles

$$\text{MD: } \frac{\partial \mathcal{H}}{\partial \beta_{x0}} = 0, \quad \frac{\partial \mathcal{H}}{\partial \eta_0} = 0. \quad (2.3)$$

where β_{x0} and η_{x0} are the smallest betatron and dispersion functions in centre of the dipole. Similarly for the edge dipole, we look for the conditions on the optics functions that minimise the emittance. Now, however, the dispersion function cannot be altered because it is fixed by the achromat condition, thus the beta function and its minimum position (l) in the dipole are considered

$$\text{ED: } \frac{\partial \mathcal{H}}{\partial \beta_{x0}} = 0, \quad \frac{\partial \mathcal{H}}{\partial l} = 0, \quad (2.4)$$

where β_{x0} is the minimum beta functions and l is the position of the minimum beta function in the edge dipole.

For the middle dipole magnet, the average value of the dispersive emittance over the

magnet's length of L_{ED} is given by

$$\langle \mathcal{H} \rangle_{mag} = \frac{1}{\rho^2 \beta_{x0}} \left[\frac{(L_{MD}/2)^4}{20} + \frac{(L_{MD}/2)^2}{3} (\beta_{x0}^2 - \eta_0 \rho) + (\eta_0 \rho)^2 \right], \quad (2.5)$$

and for the edge dipole of the length of L_{ED}

$$\langle \mathcal{H} \rangle_{mag} = \frac{1}{\rho^2 \beta_{x0}} \left[\frac{L_{ED}^4}{20} + \frac{L_{ED}^2}{3} (\beta_{x0}^2 + l^2) - \frac{l L_{ED}^3}{4} \right]. \quad (2.6)$$

Therefore, their derivatives give the TME conditions:

for the middle dipole magnet

$$\beta_{x0}^{min} = \frac{1}{2\sqrt{15}} L_{MD}, \quad \eta_0^{min} = \frac{L_{MD}^2}{24\rho}, \quad (2.7)$$

and for the edge dipole

$$\beta_{x0}^{min} = \sqrt{\frac{3}{320}} L_{ED}, \quad l^{min} = \frac{3}{8} L_{ED}. \quad (2.8)$$

Eventually, with the TME condition, the minimum average dispersive emittance can be obtained as

$$\text{MD} : \quad \langle \mathcal{H} \rangle_{mag}^{min} = \frac{L_{MD}^3}{12\sqrt{15}\rho^2} \times 1, \quad (2.9)$$

$$\text{ED} : \quad \langle \mathcal{H} \rangle_{mag}^{min} = \frac{L_{ED}^3}{12\sqrt{15}\rho^2} \times 3. \quad (2.10)$$

Generally for both cases, the minimum dispersive emittance can be summarised as

$$\langle \mathcal{H} \rangle_{mag}^{min} = \frac{L^3}{12\sqrt{15}\rho^2} F^{min} \quad (2.11)$$

where $F_{MD}^{min} = 1$ and $F_{ED}^{min} = 3$ are theoretical minimum factor for the middle and edge dipole respectively. More complicated cells can be built by considering one or more TME cells with centre dipoles, flanked by matching cells with edge dipole to connect the cell to the straight sections. This is the basic concept of the multi-bend achromat (MBA) cell. It is possible to prove that, in these cases, the condition for matching the dispersion function

and preserving the minimum emittance in the edge dipole requires that the length (L_{ED}) has to be adjusted to be shorter than that of the middle dipole by a factor of $3^{1/3}$ [33, 34]. Therefore, to minimise the emittance with achromatic condition, the MD and ED length can be related

$$L_{MD} = 3^{1/3} L_{ED}, \quad (2.12)$$

where L_{ED} and L_{MD} are the length of the edge and middle dipoles respectively. The edge dipole is shorter than the middle dipole for an isomagnetic ring. For the existing machine designed with the same bending magnets length, the emittance can be further reduced by allowing small dispersion function in the straight section in order to achieve the minimum dispersion at the centre of the bending magnets, as a result $\mathcal{H}$ function can be further reduced.

Lattice design targets not only the small emittance but also a sufficient length for the straight sections in order to accommodate insertion devices to produce high quality photons for the users. The most common lattice type for operating machines nowadays are Double Bend Achromat (DBA) and Triple Bend Achromat (TBA) which provide small achievable emittance and can accommodate long straight sections. MBAs lattice tend to have long arc cells compared to the available length of the straight sections. However in recent years new advanced compact magnet design has allowed the design of compact lattice with more bending magnet in a cell. For lattice composed of many small bending magnets with small angles ($\Phi < 20^\circ$) [19], substituting Eq.2.11 into 2.2, the beam emittance can be rewritten as

$$\epsilon_x[\text{pm rad}] = \frac{7.8}{J_x} (E[\text{GeV}])^2 (\Phi[^\circ])^3 \frac{F}{12\sqrt{15}} \quad (2.13)$$

where the factor F depends on the dispersion and betatron functions in the bending magnets. Figure 2.2 shows theoretical minimum emittance for lattices with different number of dipoles considering the beam energy of 3 GeV, 24 cells and $J_x = 1$. The minimum emittance achievable can be reduced dramatically by increasing the number of dipoles per cell.

In order to achieve the TME condition, the optics functions have to give the phase advance of 284.5° in a unit cell [19] which require too lengthy cells. In practice, optics requirements

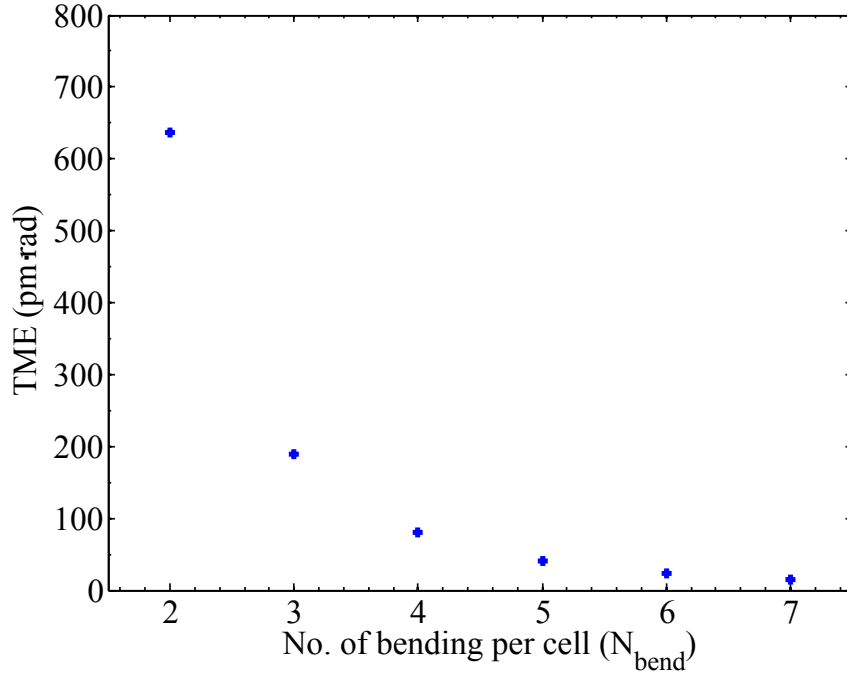


Figure 2.2: Theoretical minimum emittance (TME, $F = 1$) as a function of number of bending magnet per cell for total 24 cells in a ring.

for the beamline and space limit make most of the storage rings to have a smaller phase advance and a significantly larger beam emittance than the one given by the TME value. However, even if the TME condition is difficult to achieve, it can be used as a guideline on how to optimise the optics functions in dipoles in order to obtain the minimum emittance for a given set of constraints.

2.1.2 Damping wigglers

In electron storage rings or damping rings, strong field wigglers positioned in small dispersion function straights can introduce additional radiation loss. The stronger radiation damping and the effect of the wigglers on the beam emittance can be exploit to reduce further the designed emittance of the lattice. For light sources, the radiation from the wigglers can also be exploited to serve the beamline experiments providing high photon flux and higher photon energy than the dipole beamlines.

The equilibrium beam emittance in a storage ring with wigglers (ε_w) and the non-

perturbed equilibrium emittance (ε_0) can be written as

$$\varepsilon_w = C_q \frac{\gamma^2}{J_x} \frac{I_{5,w} + I_{5,0}}{I_{2,w} + I_{2,0}}, \quad (2.14)$$

$$\varepsilon_0 = C_q \frac{\gamma^2}{J_x} \frac{I_{5,0}}{I_{2,0}}, \quad (2.15)$$

where $I_{2,w}$ and $I_{5,w}$ are the synchrotron radiation integrals for the machine with wigglers, and $I_{2,0}$ and $I_{5,0}$ are the synchrotron radiation integrals for the bare machine which only consider the effect from dipoles. Thus the emittance growth ratio can be derived as

$$\frac{\varepsilon_w}{\varepsilon_0} = \frac{1 + \frac{I_{5,w}}{I_{5,0}}}{1 + \frac{I_{2,w}}{I_{2,0}}}. \quad (2.16)$$

Synchrotron radiation for a bare machine is given by

$$I_{2,0} = \oint \frac{ds}{\rho_0^2} = \frac{2\pi}{\rho_0}, \quad (2.17)$$

$$I_{5,0} = \oint \frac{\mathcal{H}}{\rho_0^3} ds, \quad (2.18)$$

where ρ_0 is the bending radius (7.1276 m for Diamond). The synchrotron radiation integrals $I_{2,w}$ and $I_{5,w}$ for a wiggler with the length of L are derived as follows [35]:

$$I_{2,w} = \oint \frac{ds}{\rho^2} = \int \frac{\cos^2(k_w s)}{\rho_w^2} ds = \frac{L}{2\rho_w^2} \quad (2.19)$$

$$I_{5,w} = \oint \frac{\mathcal{H}_w}{|\rho^3|} ds = \int \frac{\mathcal{H}_w |\cos^3(k_w s)|}{|\rho_w^3|} ds \quad (2.20)$$

where $\mathcal{H}_w(s) = \gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2$. This assumes a wiggler with a sinusoidal field, giving a local curvature

$$\frac{1}{\rho} = \frac{1}{\rho_w} \cos(k_w s) \quad (2.21)$$

where ρ_w is the wiggler bending radius from the peak field, k_w is the wiggler field wave number which can be calculated from $\frac{2\pi}{\lambda_w}$ where λ_w is the wiggler period (λ_w). The total

dispersion function including the self-dispersion generated by the wiggler is given by

$$\eta(s) = \eta_0 + \eta'_0 s - \frac{\sin k_w s}{\rho_w k_w^2} \quad (2.22)$$

$$\eta'(s) = \eta'_0 - \frac{\cos k_w s}{\rho_w k_w}. \quad (2.23)$$

where η_0 and η'_0 are the external dispersion and its derivative in the wiggler which are generated purely by the rest of the lattice. To determine the radiation integral $I_{5,w}$ in a wiggler, two extreme cases are usually considered [36].

In the first case, assuming that the natural dispersion function at the position of the wiggler is large, then the dispersion generated by the wiggler can be neglected, and we obtain

$$I_{5,w} = \frac{4 \langle \mathcal{H}_w \rangle L}{3\pi \rho_w^3}. \quad (2.24)$$

In the middle of the wiggler the optics functions are matched to $\eta'_0 = 0$, and $\alpha = 0$ resulting in $\langle \mathcal{H}_w \rangle \approx \eta_0^2 / \beta$.

In the second case, assuming that the natural dispersion function is zero, we have

$$I_{5,w} = \frac{4 \langle \beta \rangle L}{15\pi \rho_w^5 k_w^2}. \quad (2.25)$$

The Diamond lattice is designed with dispersive straight sections to reduce the beam emittance and the first condition used for large dispersion in the wiggler can be applied.

Additionally, to calculate $I_{5,w}$ another formula found in [37] is used to verify the calculation results. From the dispersive emittance in a wiggler the radiation integral is

$$I_{5,w} = \int_{-N_w \pi}^{N_w \pi} \frac{\mathcal{H}_w}{|\rho_w^3|} \quad (2.26)$$

$$\approx \frac{2N_w}{k_w \rho_w^3} \left\{ \frac{4}{3} (\beta \eta'_0 + \frac{\eta_0^2}{\beta}) + \frac{4}{15} \beta \frac{k_w^2}{(\rho_w k_w^2)^2} + (0.986 + \frac{1}{45} k_w^2 L^2) \frac{1}{(\rho_w k_w^2)^2 \beta} \right\}. \quad (2.27)$$

In order to reduce the beam emittance, as could be seen from Eq. 2.16, that the condition that gives $\varepsilon_w/\varepsilon_0 < 1$ is $\frac{I_{2,w}}{I_{2,0}} > \frac{I_{5,w}}{I_{5,0}}$. Therefore, η_0 and η'_0 have to be as small as possible.

2.1.3 Damping partition J_x

As described in Chapter 1, most of the upgrade designs employs combined function magnets, especially gradient dipoles in order to design compact cells allowing as many dipoles per cell as possible. The combined function dipole which introduces transverse quadrupole gradient allows not only a compact design but also reduces the beam emittance by increasing the horizontal damping partition number (J_x) as [38]

$$J_x = 1 - \frac{I_4}{I_2} = 1 - \frac{\oint \eta_x \rho^{-1} (\rho^{-2} + 2k) ds}{\oint \rho^{-2} ds} \quad (2.28)$$

where k is the transverse field gradient in the dipole, η_x is the horizontal dispersion function and ρ is the bending radius. From Eq. 2.28, it is clear that a horizontal defocusing gradient field ($k < 0$) is needed to increase the J_x . In the existing machines without combined function dipoles, a similar effect can be obtained by offsetting the beam orbit passing through quadrupoles along the ring with an orbit bump or shifting the frequency of the RF cavity, in these ways the beam experiences the combined function dipoles effect in the quadrupoles which increases the damping partition, and consequently reduce the beam emittance [39].

Although the increase of J_x produces a smaller beam emittance, using the relationship between longitudinal and horizontal damping partition number

$$J_s = 3 - J_x, \quad (2.29)$$

we see that it also produces a larger energy spread since from Eq.1.40, the energy spread for an isomagnetic ring is inversely proportional to the longitudinal damping partition (J_s)

$$\frac{\sigma_u^2}{E^2} = \frac{C_q \gamma^2}{J_s \rho}. \quad (2.30)$$

Another way to manipulate the damping partition number is to use a Robinson wiggler (a wiggler with the integrated gradient field) as proposed by Robinson [40]. The wiggler can intentionally reduce J_s or the damping of energy oscillation which results in stronger horizontal damping. The change of J_x in Eq. 2.28 is computed by

$$\frac{\Delta I_4}{I_2} = 2 \langle \eta_x \rangle \frac{1}{B_w} \frac{dB_w}{dx} \frac{4L_w \rho}{2\pi \rho_w^2} \left(1 + \frac{4L_w \rho}{2\pi \rho_w^2} \right)^{-1}, \quad (2.31)$$

where $\langle \eta_x \rangle$ is the average dispersion function in the wiggler, B_w , dB_w/dx , ρ_w and L_w are the peak field, the transverse gradient, the bending gradient and the length of the wiggler respectively, and ρ is the bending radius of the normal dipole in the ring. By introducing a magnet configuration that $B_w \cdot \frac{dB_w}{dx} < 0$ as shown in Figure 2.3, the horizontal damping partition number J_x can be increased resulting in smaller beam emittance. This solution comes with the prize of increasing energy spread because of the smaller J_s in Eq. 2.30.

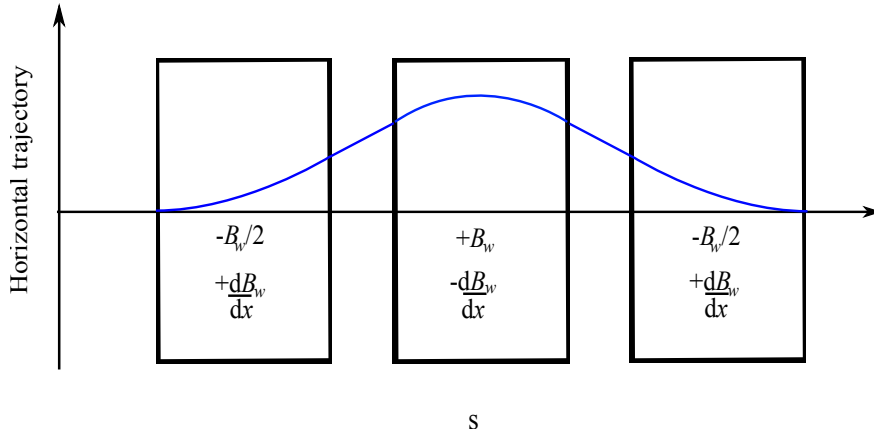


Figure 2.3: Schematic figure of a Robinson wiggler and particle trajectory. The configuration between the field (B_w) and gradient ($\frac{dB_w}{dx}$) is arranged in order to increase J_x and reduce beam emittance.

2.2 Lattice design criteria

As described in Chapter 1, brightness and luminosity are the main parameters which drive accelerator facilities to improve their machines. Other than the beam dimension, the lattice performance can be assessed by the ability to accumulate the particles efficiently and to store the beam as long as possible while the beam stability is maintained. Injection efficiency and beam lifetime have to be taken into account in the optimisation of the machine.

2.2.1 Dynamic Aperture and injection efficiency

The dynamic aperture (DA) is defined by the maximum amplitude in the transverse space (x, y) in which particles are stable or survive for a given number of turns. Normally, we consider the DA of a storage ring at the injection point which defines more directly the beam injection process. Therefore, DA is one of the most important criteria for lattice design impacting directly the capacity of storing the electron beam.

The DA is determined by solving numerically the equations of motion of the electron beam in the magnetic field of the lattice. This procedure, called particle tracking, can be performed with different methods: grids or lines search in the (x, y) space which begins at the origin and moves outwards or inversely. It is faster to start the tracking from large amplitude inwards since the DA search only looks for the points that particles survive while the unstable particles will be lost quickly. On the other hand, the DA search moving outwards in the (x, y) plane takes longer to search for unstable points since most of the particles within small amplitude survive but it is more accurate as it can highlight spurious DA areas formed by islands at large amplitude [41].

Another useful method to investigate the nonlinear beam dynamic and DA is the frequency map analysis [42]. From a set of initial coordinates in the (x, y) space, particle orbits are obtained by tracking for several hundred turns and the betatron tunes are extracted from the motion using Numerical Analysis of Fundamental Frequencies (NAFF) method [42]. An important indicator of the stability of the particle motion is given by the so-called diffusion

rate. The diffusion indicates the stability of the tracked particle based on the betatron tune variation during the tracking. To calculate the diffusion rate from the particle orbit with the total number of turns N_{tot} , the betatron tune will be calculated using the tracking information from the first to $N_{tot}/2$ turn and from $N_{tot}/2 + 1$ to N_{tot} turn. Then, the diffusion rate can be computed by [42]

$$d = \log(\Delta\nu_x^2 + \Delta\nu_y^2), \quad (2.32)$$

where $\Delta\nu_x$ and $\Delta\nu_y$ are the tune differences between two halves of the particle tracking in the horizontal and vertical plane respectively.

The frequency map of the existing Diamond storage ring is shown in Figure 2.4. The technique allows us to see not only the size of the DA but also how the particles behave inside the DA indicated by the diffusion rate and the particles footprint in the tune space. The particles at large amplitude experience stronger nonlinear kick and tend to be less stable because of the 4th (green) and 5th (blue) order resonance lines. The DA of about -15 mm is sufficient for beam injection at Diamond providing more than 90% injection efficiency.

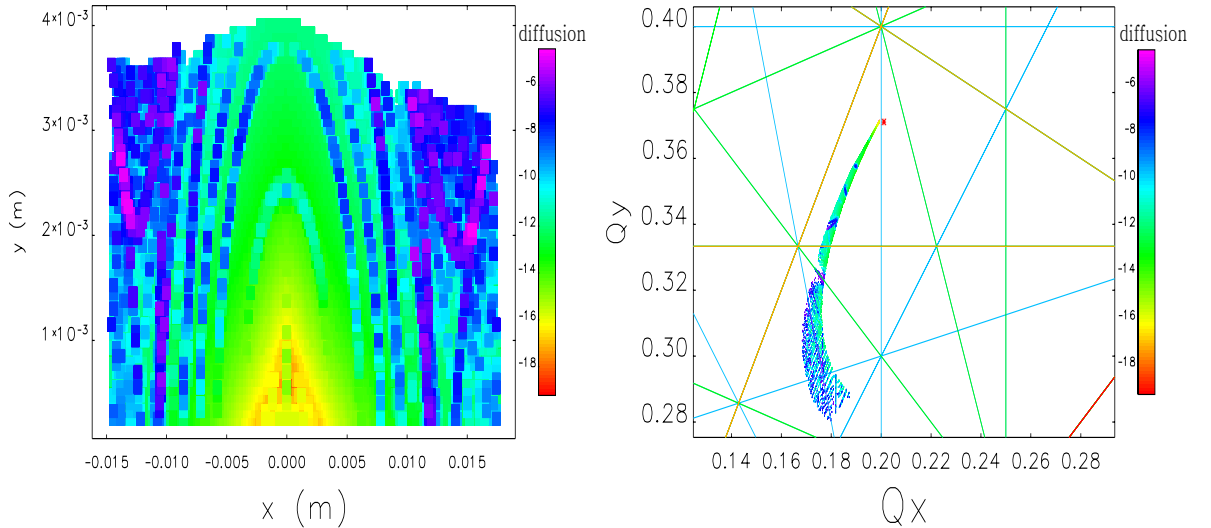


Figure 2.4: Frequency map analysis for the existing Diamond storage ring. Left: Diffusion plotted in amplitude space (x, y) representing the dynamic aperture. Right: Diffusion plotted in fractional tune space.

2.2.2 Resonance driving terms (RDTs)

Another important semi-analytical tool that can be used to characterize the non-linear beam dynamics is given by the resonance driving terms that provide the strength of the resonances excited in the ring. The following treatment follows closely the early work in [43]. To describe the non-linear effects in a particle accelerator, the complex Hamiltonian of the ring is normalised in resonance basis. The normalised one turn map for a ring which has n elements is given by

$$\mathcal{M}_{1 \rightarrow n} = \mathcal{A}_1^{-1} e^{i h} \mathcal{R}_{1 \rightarrow n} \mathcal{A}_1, \quad (2.33)$$

where $\mathcal{R}$ is a rotation operator, $\mathcal{A}_1$ is a normalising map where the subscripts 1 and n are the indices of the ring elements, and $e^{i h}$ is the exponential Lie operator defined as

$$e^{i h} g = g + [f, g] + \frac{1}{2!} [f, [f, g]] + \dots, \quad (2.34)$$

where

$$[f, g] = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial x_i} \right)$$

is the Poisson bracket of canonical variables. The normalised map $\mathcal{A}_i$ at the i^{th} element for a perturbed linear lattice without coupling can be expressed as

$$\begin{aligned} \mathcal{A}_i x &= \sqrt{\beta_{x,i}} x + \eta_{x,i} \delta, & \mathcal{A}_i p_x &= \frac{-\alpha_{x,i} x + p_x}{\sqrt{\beta_{x,i}}} + \eta'_{x,i} \delta, \\ \mathcal{A}_i y &= \sqrt{\beta_{y,i}} y, & \mathcal{A}_i p_y &= \frac{-\alpha_{y,i} y + p_y}{\sqrt{\beta_{y,i}}}, \end{aligned} \quad (2.35)$$

where $\beta_{x,i}$ and $\beta_{y,i}$ are the horizontal and vertical betatron functions and $\eta_{x,i}$ is the horizontal dispersion function. It is convenient to describe the dynamics in the so-called resonance basis, related to the action-angle variables (J, ϕ) and the phase space variable (x, p_x) by

$$h_x^\pm = \sqrt{2J_x} e^{\pm i \phi_x} = x \mp i p_x, \quad (2.36)$$

The rotation $\mathcal{R}_{i \rightarrow j}$ of the phase advance between i^{th} and j^{th} ring elements is given by

$$\mathcal{R}_{i \rightarrow j} h_x^\pm = \mathcal{R}_{i \rightarrow j} \sqrt{2J_x} e^{\pm i\phi_x} = e^{\pm i\mu_{i \rightarrow j, x}} h_x^\pm, \quad (2.37)$$

where $\mu_{i \rightarrow j, x}$ is the phase advance from the element i to j . For a non-linear element like sextupole with the strength k_2 , the potential can be given by

$$V(x, y) = \frac{k_2}{6} (x^3 - 3xy^2), \quad (2.38)$$

and the first order nonlinear driving term can be expanded in term of resonance driving basis

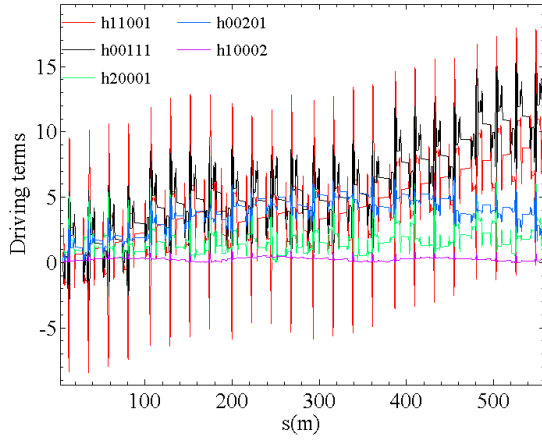
$$h^{(1)} = \sum_n [h_{abcde}] h_x^{+a} h_x^{-b} h_y^{+c} h_y^{-d} \delta^e. \quad (2.39)$$

The driving term h_{abcd0} drives resonance $(a - b)\nu_x + (c - d)\nu_y$. The last index e indicates the off-momentum or chromatic effects. The explicit formulae used to calculate the driving terms are given in Appendix A. Resonance Driving Terms (RDTs) can be used quickly to interpret the complexity of the system analytically and understand if particular resonances are excited. They cannot be used to predict analytically the extent of the dynamic aperture but usually a judicious minimisation of the strongest resonant driving terms provides a good guidance to improve the DA. The final results must always be checked with a full numerical tracking.

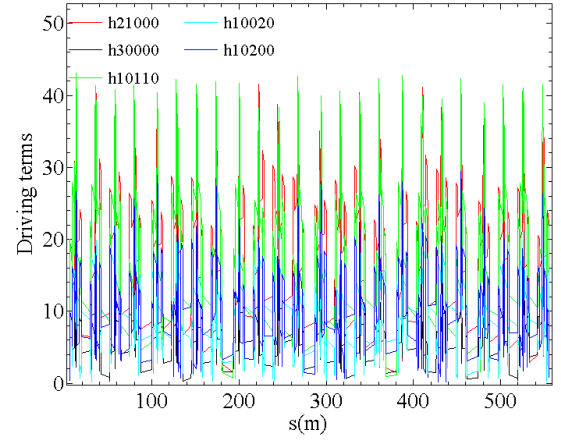
Because most of the optics design programs only provide the RDTs at the end of lattice, a script `drivingtermCal` was written to calculate the RDTs as functions of position along the given lattice. The twiss functions are read from Elegant output then the RDTs (h_{abcde}) as functions of position along the ring can be calculated. This feature gives us information on the development of the RDTs along the ring as shown in Figure 2.5 for Diamond storage ring. The differences of the calculated RDTs with respect to the Elegant outputs at the end of the lattice are very small as shown in Table 2.1. The RDTs depend on the optics functions, working points, magnet strengths in the ring.

Table 2.1: Comparison of the total first order chromatic and geometric RDTs calculations.

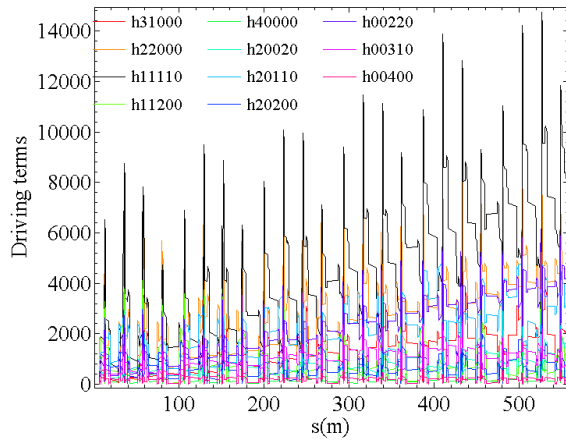
RDTs	Resonance driving effects	Elegant	drivingtermCal	Δ RDTs
h_{11001}	chromaticity ξ_x	9.38409	9.38409	0
h_{00111}	chromaticity ξ_y	12.44529	12.44528	1e-05
h_{20001}	synchro-betatron	2.47114	2.4711	4e-05
h_{00201}	synchro-betatron	2.54574	2.54576	-2e-05
h_{10002}	2^{nd} order dispersion	0.20242	0.20242	0
h_{21000}	ν_x	4.44416	4.44409	7e-05
h_{30000}	$3\nu_x$	1.88363	1.88327	0.00036
h_{10110}	ν_x	2.38546	2.38534	0.00012
h_{10020}	$\nu_x - 2\nu_y$	6.19094	6.19071	0.00023
h_{10200}	$\nu_x + 2\nu_y$	0.82408	0.82397	0.00011



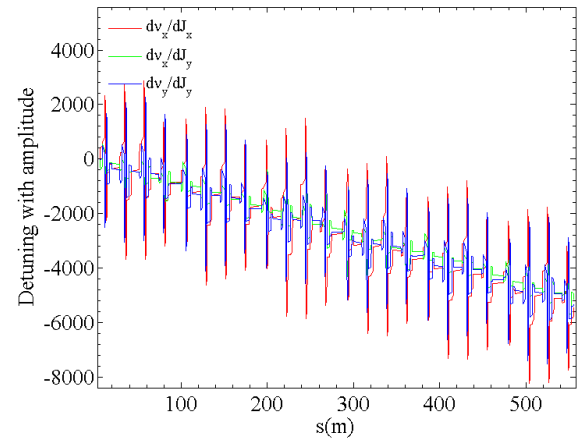
(a) First order chromatic terms



(b) First order geometric terms



(c) Second order geometric terms



(d) Detuning with amplitude

Figure 2.5: Resonance driving terms and detuning with amplitude as functions of position for the operating Diamond lattice.

The Diamond lattice was originally designed to compensate the third order geometric resonance at the end of each cell while operating with zero chromaticity. Figure 2.5 refers to the diamond lattice with positive chromaticity (2, 2), however the cancellation of the geometric resonance terms is still visible in Figure 2.5b.

2.2.3 Touschek lifetime

Charged particles in a bunch undergo elastic collision because of their natural motion around the ring. The elastic scattering can produce particle losses: this is the basic mechanism called Touschek effect. The event is a single Coulomb scattering leading to particles losses due to an energy transfer from the transverse to longitudinal plane which exceeds the momentum acceptance. As the beam is accelerated longitudinally to velocity close to the speed of light ($\beta \approx 1$), even a small energy transfer from the transverse plane to the longitudinal plane, it will be boosted by the relativistic Lorentz factor (γ) and might exceed the momentum acceptance. The phenomenon was first discovered by Touschek in the first electron storage ring [44]. Touschek lifetime is one of the most dominant beam lifetime because a couple of colliding particles losses occur simultaneously.

The analytic formula for Touschek scattering half lifetime for a non-relativistic transverse momentum flat beam is expressed as [45], [46]

$$\frac{1}{\tau} = \left\langle \frac{N_b c r_c^2}{8\pi\sigma_x\sigma_y\sigma_s} \frac{\sqrt{\xi} D(\xi)}{\gamma^2 \eta^3} \right\rangle, \quad (2.40)$$

with

$$\xi = \left(\frac{\eta\beta_x}{\gamma\sigma_x} \right)^2, \quad (2.41)$$

and

$$D(\xi) = -\frac{3}{2}e^{-\xi} + \frac{\xi}{2} \int_{\xi}^{\infty} \frac{\ln u}{u} e^{-u} du + \frac{3\xi - \xi \ln \xi + 2}{2} \int_{\xi}^{\infty} \frac{e^{-u}}{u} du, \quad (2.42)$$

where N_b is the number of particles per bunch, r_c is the classical electron radius, $\sigma_{x,y,s}$ are the

beam size in horizontal, vertical and longitudinal planes respectively, γ is the Lorentz factor and $\eta = \delta = \frac{dp}{p_0}$ is the momentum aperture of the machine. For a fixed beam dimension, the lifetime can be increased by enlarging the momentum aperture η .

Touscheck lifetime can be calculated using `touscheckLifetime` program [47] based on Piwinski formula [48], as described in Appendix A, which is distributed with Elegant. The program uses the simulated momentum aperture provided by the particles tracking in Elegant [41] following a procedure originally developed for SOLEIL [49]. At the entrance of each specified element, particles tracking is performed with increasing steps of momentum deviation until the particles are lost. The loss point in the lattice is recorded to study the loss distribution around the ring. Thus, at each selected element, the maximum momentum deviation that the particles still survive, defines the local momentum aperture. Figure 2.6 shows the local momentum aperture above 3% obtained from the particle tracking for the Diamond storage ring. The asymmetry of the momentum aperture which is the result of high order momentum compaction factor can be observed.

The Touscheck lifetime calculated for the operating Diamond storage ring considering 0.3% coupling, 900 bunches, 300 mA current with multipole error and misalignment is about

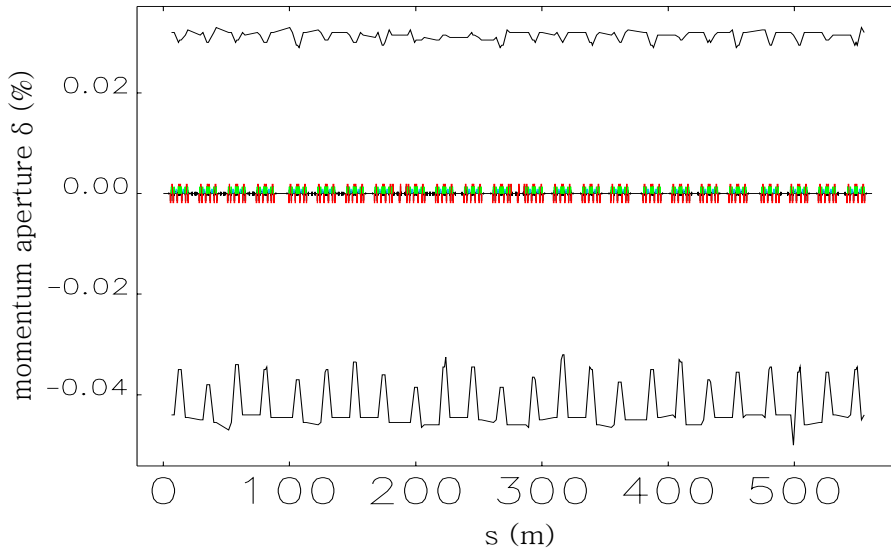


Figure 2.6: Momentum aperture for the existing Diamond storage ring.

21 hrs as discussed in Chapter 1 while the measured lifetime is 16 ± 1 hrs with IDs in operation. The origin of this discrepancy is still under investigation but it is thought to be due to the lack of information about possible errors sources in the existing diamond lattice.

2.2.4 Intrabeam scattering (IBS)

Unlike Touschek event, Intrabeam Scattering (IBS), also known as multiple Touschek effect, is a process where the particle undergoes multiple Coulomb scattering without acquiring a longitudinal momentum large enough to generate the particle loss. As more charged particles are stored in a bunch, the interactions between the particles also increases. The rate of multiple Coulomb scattering within a bunch of particles increases with reducing volume of the bunch leading to the increase of the energy spread and beam dimension without beam loss. Thus, the intrabeam scattering poses a limit on the maximum accumulated current in the storage rings, in particular for low emittance rings.

The IBS effect can be calculated by `ibsEmittance` [41] script which employs Bjorken and Mtingwas formula [50], [51] which is included in the `Elegant` package. The growth rate due to IBS effect in each plane is given by

$$\frac{1}{T_i} = 4\pi A(\log) \left\langle \int_0^\infty \frac{\lambda^{1/2} d\lambda}{\sqrt{[\det(L + \lambda I)]}} \left\{ Tr L^{(i)} Tr \left(\frac{1}{L + \lambda I} \right) - 3 Tr L^{(i)} \left(\frac{1}{L + \lambda I} \right) \right\} \right\rangle \quad (2.43)$$

with

$$A = \frac{r_c^2 c N_b}{64\pi^2 \sigma_s \sigma_p \sigma_x \sigma_y \sigma_{x'} \sigma_{y'} \beta^3 \gamma^4}, \quad (2.44)$$

where i can be x , y , or p , $(\log)$ is the Coulomb log factor, Tr and $\det$ are the trace and determinant of a matrix. Auxiliary matrices are given by

$$L = L^{(p)} + L^{(x)} + L^{(y)}, \quad (2.45)$$

$$L^{(p)} = \frac{\gamma^2}{\sigma_p^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2.46)$$

$$L^{(x)} = \frac{\beta_x}{\varepsilon_x} \begin{pmatrix} 1 & -\gamma\phi_x & 0 \\ -\gamma\phi_x & \gamma^2 \mathcal{H}_x / \beta_x & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2.47)$$

$$L^{(y)} = \frac{\beta_y}{\varepsilon_y} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \gamma^2 \mathcal{H}_y / \beta_y & -\gamma\phi_y \\ 0 & -\gamma\phi_y & 1 \end{pmatrix}, \quad (2.48)$$

where $\mathcal{H}_{x,y}$ is the dispersive invariant in horizontal and vertical plane respectively, and $\phi_{x,y} = \eta'_{x,y} - \frac{1}{2}\beta'_{x,y}\eta_{x,y}/\beta_{x,y}$, with $\beta_{x,y}$ and $\eta_{x,y}$ are the beta and dispersion functions in horizontal and vertical plane respectively.

From the denominator in Eq. 2.44, the IBS scattering rate is smaller at higher beam energies thus favouring high beam energy machines [4]. In electron storage ring the IBS effect can be compensated by radiation damping and can be neglected in most of the 3rd generation light sources. For Diamond, IBS causes only about 0.8% horizontal emittance increase at the current of 500 mA. However, in ultra-low emittance machines, the natural beam dimension is much smaller and stronger IBS effect can be expected.

2.2.5 Beam stability

The higher beam brightness can be fully exploited by the users only if all sources of instabilities, that cause the intensity variation of the photon source used in the experiments, are corrected.

One of the dominant instabilities in modern light sources operating with very narrow gap insertion devices is the resistive wall instability. The interaction between the beam and resistive vacuum chamber structure becomes stronger as the charge per bunch increases. The choice of betatron tune can also strengthen or weaken the resistive wall instability. The resistive wall instability growth rate is given by [52]

$$\tau^{-1} \approx -\frac{Nr_c c^2}{b^3 \gamma \omega_\beta T_0 \sqrt{\pi \sigma \omega_0}} f(\Delta_\beta) \quad (2.49)$$

with

$$f(\Delta_\beta) = \sum_{k=1}^{\infty} \sqrt{\frac{2}{k}} \sin(2\pi k \Delta_\beta) \quad (2.50)$$

where Δ_β is the non-integer part of the betatron tune ($\nu_\beta = \omega_\beta/\omega_0$), N is the number of particles, r_c is the classical radius of the particle, T_0 is the revolution period ($2\pi/\omega_0$), b is the half gap of the chamber, σ is the wall conductivity and γ is the Lorentz factor. To achieve negative growth rate ($\tau^{-1} < 0$) or damping effect, the condition $0 < \Delta_\beta < 0.5$ has to be met. For this reason the betatron tune has to be set below the half integer in order to suppress the resistive wall instability.

Another important collective effect is the so-called head-tail instability. From macro particles model, the growth rate of betatron oscillations for + and - modes are expressed as [52]

$$\tau_{\pm}^{-1} = \mp \frac{Nr_c W_0 c \xi \hat{z}}{2\pi \gamma C \eta}, \quad (2.51)$$

where W_0 is a constant parametrising the wake function integrated over the ring circumfer-

ence C , ξ is the chromaticity, η is the slippage factor and $\hat{z}$ is the synchrotron oscillation amplitude. Two macroparticles move up and down in phase for the $+$ mode and out of phase for the $-$ mode. The two $+$ and $-$ modes behave in the opposite way, i.e. if $+$ mode is stable (negative growth rate) the $-$ mode will be unstable (positive growth rate) and vice versa. The growth rate is proportional to N and ξ but reduces with the increasing beam energy γ .

The head-tail instability can be effectively compensated by adjusting the chromaticity of the ring. The term ξ/η indicates the sign of the growth rate, when $\xi/\eta > 0$ the growth rate becomes negative or $+$ mode damping and $-$ mode antidamping and the opposite is also true when $\xi/\eta < 0$. The ratio ξ/η has to be slightly positive considering to damp the head-tail instability [52]. For storage ring operating above transition ($\eta > 0$), the chromaticity ξ has to be small positive value. The chromaticities for the Diamond storage ring are 2 for both horizontal and vertical planes and these values are compatible with good injection efficiency and beam lifetime. There are more beam instabilities in practice but they will not be covered in this work.

2.3 Multi-objective genetic algorithm (MOGA)

In complicated optimisation problems, more than one objective has to be optimised with a large number of parameters. The design of ultra-low emittance lattices is an excellent example of such complicated problem. The new low emittance machine designs are moving from standard DBA or TBA cells to more complex multi-bend achromat (MBA) in order to achieve smaller beam emittance. Strong quadrupoles are required to focus the optics functions in very compact a MBA cell to minimise the beam emittance. The drawback of this strong focussing is the large natural chromaticity. Unavoidably strong sextupoles are required to correct the chromaticity which then generate strong non-linearity in the lattice and limit the dynamic and momentum apertures. In this sense the optimisation of the emittance is in conflict with the optimisation of the dynamics aperture and the momentum aperture. For these types of problems multi-objective genetic algorithms were proposed in

the field of accelerator physics about 10 years ago. They proved to be very effective in many cases and they have been applied to ultra-low emittance lattice optimisation in this thesis.

In general, multi-objective optimisation can be converted into single objective optimisation for a set of constraints, by using different weighted objectives and combining them into a single objective (scalarisation method). The objective function can then be written as

$$U = \sum_{i=1}^N w_i f_i(x), \quad (2.52)$$

where w_i is the weight factor and $f_i(x)$ is the objective function. The optimisation provides a set of optimised parameters giving the best objective functions. However, the drawback of this method is that the solutions depend strongly on the weight functions associated with the objectives. The choice of the weights of each objective is not simple to be made and there is no general applicable rule.

A better approach for multi-objectives problems is the search for the so-called Pareto optimal set (a set of solutions which are non-dominated to each other and provide the best trade-off between mutually conflicting objectives [53]). The corresponding objectives values in the objective space for a given Pareto optimal set are called Pareto front. By improving the Pareto front more than one objective can be improved simultaneously and the corresponding optimal solutions are unique and better than the other solutions in at least one of the objectives. In this way, a broader objective space can be explored and the desired solution can be selected from the Pareto optimal front.

In presence of conflicting objectives, the optimisation of one single objective at a time may deteriorate significantly the other conflicting objectives, for example optimisation of ultra-low emittance can drastically reduce the dynamic aperture. Multi-objective optimisation is suitable for the conflicting objective optimisation because it provides a set of solutions from which to choose the best one trading off between the different objectives.

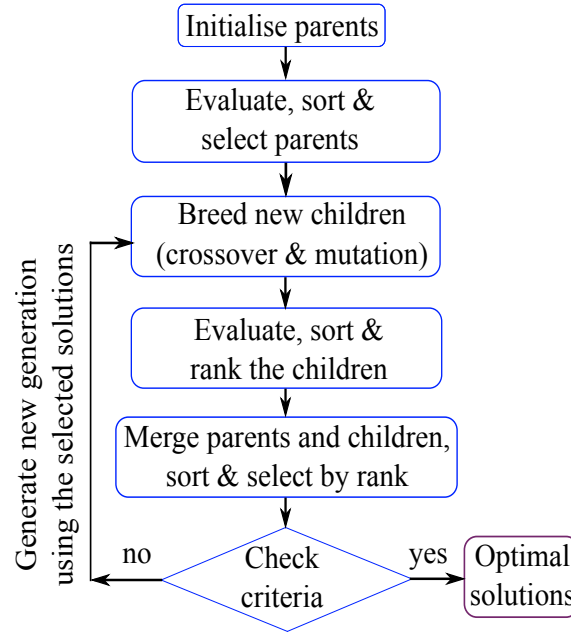


Figure 2.7: Diagram of genetic algorithm optimisation.

2.3.1 Genetic algorithm

The specific search algorithm that proved to be very effective in multi-objective optimisation is given by the class of genetic algorithms. Following the laws of natural evolution, genetic algorithms (GA) based on the exploration of the parameter space via selection, crossover, inheritance, mutation and evolution has been proposed by John Holland in 1975 [54]. Parallel processing can be applied to accelerate the objective functions calculation process and the selection of the populations in the algorithm. Over the last decades, genetic algorithms have been used extensively in broad fields: commerce, engineering and science. The first application of the algorithm in accelerator physics was used for DC gun photo injector optimisations [55].

Figure 2.7, summarise the basic step of the genetic algorithm. The first population (trial solutions) is initialized randomly and used to calculate objective functions. Then, good solutions are selected based on the objective functions and constraints or fitness. The selection operator creates a fixed size mating pool which contains solutions with good fitness. The next step is to generate offspring from the selected solutions by crossover or in some

case mutation operator which is used to diversify and perturb the offspring solutions. Then the children are evaluated, selected and generated repeatedly to produce better and better generation until the final criteria (for example the total number of population) are met. In the selection process, the solutions will be sorted and ranked based on their performance. The best solutions are assigned with rank 1. After sufficient generations, the descendants or solutions move toward the Pareto optimal front. A simple example of crossover and mutation operators is show in Figure 2.8. For the crossover process, from the selected parents (chromosomes) a crossing point is randomly selected and a part of the original chromosomes will be swapped to generate a couple of new chromosomes (offsprings) which inherit some characteristics of the parents as described in Figure 2.8a. For mutation, in Figure 2.8b, some random bits of the original chromosome will be flipped and produce a new chromosome. The physical chromosomes or variables for accelerator lattice optimisation could be betatron tunes, chromaticities or magnet strengths (e.g. quadrupole and sextupole).

The final solutions can be obtained from a solutions set with the lowest rank 1 or the Pareto optimal front of the last generation. From the Pareto optimal front representing the

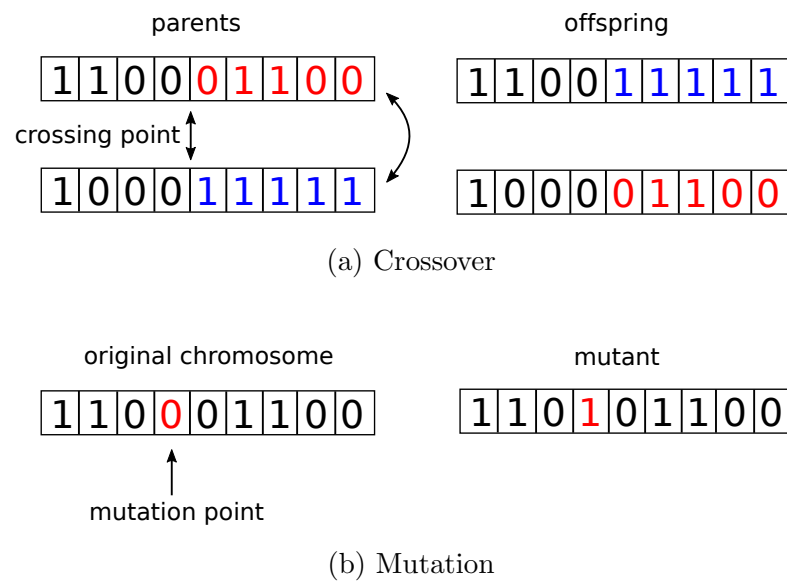


Figure 2.8: Genetic algorithm operators. Crossover operator produces two offsprings from parents chromosomes which are swapped at random crossing point while mutation generates a chromosome by randomly altering a bit of an original chromosome.

best solutions set in objective space we can select the best solution for our optimisation problem.

One of the most popular genetic algorithms is the non-dominated sorting genetic algorithm (NSGA) which was proposed by Srinivas and Deb in 1994. Later to speed up the algorithm and prevent the loss of good solutions, the upgraded algorithm NSGA-II was developed [56], [57]. NSGA-II is an elitist evolutionary algorithm, i.e. the best solution among both offspring and previous generation (parents) will always be passed on to the next generation. In other words, good solutions from ancestors will be considered in the subsequent reproduction and have opportunities to be in the mating pool. To ensure a well distributed or uniformly spread-out Pareto optimal front in the objective space, during the selection process, the crowded-comparison operator is introduced to guide the selection process in the algorithm [56]. In this way, the solutions will be selected from lower rank (better) and lower density region if the solutions have the same rank.

A genetic algorithm optimiser based on NSGA-II is provided with the Elegant program package [41]. ELEGANT-based MOGA using master-slave computing concept requires a cluster of several computing nodes which submits several jobs (population or lattices with initial random variables) to be calculated in each computing unit. The jobs distribution allows objective functions (dynamic aperture, Touschek lifetime, effective emittance etc.) calculations for each solution to be carried out simultaneously on the slave nodes while the master node collects the evaluated solutions, analyses them by rank-based sorting and selects a set of the best solutions (lowest rank: 1) in order to generate the next generation population (new lattice).

In lattice design, the MOGA algorithm was successfully tested in ALS giving new optics solution providing better brightness [58] [59] and also used for dynamic aperture optimisation [60]. Therefore MOGA appear to be the best suited tool for lattice optimisations especially in challenging MBA lattice design.

2.4 Summary

The main goal for lattice design and optimisation considered in this thesis is to achieve the lowest beam emittance while maintain good injection efficiency and good Touschek lifetime. The reduction of the emittance can be achieved by controlling the optics functions at the dipoles, increase the damping partition number but the most effective way is obviously the lattice upgrade using the MBA concept. In lattice designs, the choice of tunes and chromaticities should be made to avoid potential collective beam instabilities. Further constraints in the upgrade of existing machines are given by the existing geometry, constrained by the existing tunnel and beamlines position. Careful lattice design and advanced optimisation techniques are required to meet all the above criteria. MOGA has been applied widely in optimisation problems and proved to be a powerful algorithm for lattice optimisation with many objectives and will be the main optimisation tool used in this thesis.

Chapter 3

Low emittance studies at the Diamond storage ring

The search for an ultra-low emittance lattice upgrade at the Diamond storage ring has been supported by experimental studies at the existing storage ring and has eventually led to the proposal for the modification of one of the existing DBA cell.

In order to test the optimisation methods used in the lattice upgrade design in Chapter 4, we have applied the MOGA techniques to investigate the possible minimisation of the beam emittance of the present Diamond lattice without any modification of the existing hardware, using as parameter only the values of the existing quadrupoles and sextupoles.

The Diamond lattice optimisation was then verified experimentally during a number of dedicated machine shifts. The objective of the optimisation also included the injection efficiency and the beam lifetime.

In a second study we investigated a modification of the Diamond lattice where the original Double Bend Achromat (DBA) cells is replaced by the Double-Double Bend Achromat (DDBA) cell which constitutes one of the most promising options for the ultra-low emittance lattice upgrade in Chapter 4. The insertion of one single DDBA cell does not reduce significantly the emittance of the machine however the design of the cell allows the creation of

an additional straight section where an insertion device can be installed. The possibility of adding one straight section and therefore one more beamline was considered very favourable by the Diamond management and was successfully funded in 2014. The installation in the machine is foreseen by the autumn of 2016.

3.1 Diamond lattice optimisation with MOGA

To improve the photon beam brilliance, the electron beam emittance must be reduced. The Diamond storage ring was designed to operate with a natural emittance of 2.75 nm·rad. A smaller beam emittance can be achieved by allowing dispersion function leakage in the straight sections. However this route is limited by the simultaneous increase of the effective beam emittance due to the presence of dispersion in the straight section and the unavoidable energy spread, which concur in increasing the beam size and divergence. Furthermore, even if non-zero dispersion scheme provides further reduction of the emittance in the dipoles, a larger dispersion at the location of strong field insertion devices may deteriorate the emittance.

We present here the study performed to reduce the beam emittance of the existing machine using MOGA described in Chapter 2. The choice of the multi-objective optimisation technique is well suited to this problem as at least two conflicting objective functions can be optimised at the same time, namely lowering the emittance and increasing the dynamic aperture. The MOGA was used to retune the existing quadrupoles and sextupoles of the Diamond storage ring. The optimisation of the linear optics (i.e. twiss functions) required to achieve smaller beam emittance generates a lattice with large natural chromaticity. Strong sextupoles are necessary to correct the chromatic effects. The machine performance has to be further optimised to maintain a good dynamic aperture and momentum aperture.

To minimise the beam emittance, the horizontal beta and dispersion functions in the dipoles were controlled as described in Chapter 2. Linear optics matching gave new lattices with smaller beam emittance and new betatron tunes. Strong focussing generates a larger horizontal betatron tune (ν_x) and small horizontal beta function (β_x) in the dipoles to

reduce the beam emittance. In the optics matching, the integer part of the horizontal tune was increased from 27 to 28 while the fractional tune was kept the same as in the original Diamond lattice (28.20, 13.37). Maintaining the same fractional tunes minimises changes of the existing injection process. Notice that the fractional tunes are kept well below half-integer because Diamond had experienced strong instabilities due to resistive wall during a lattice modification of I13 optics due to several narrow gap in-vacuum IDs in the storage ring [14].

In this study, MOGA was used to optimise dynamic aperture, Touschek lifetime and effective emittance by setting them as objective functions. The lattice optimisations proceeded in two separate steps. The first step of the optimisation was focused on the combined minimisation of the beam emittance and dynamic aperture. The second step focused on the optimisation of the non-linear dynamics (dynamic aperture and Touschek lifetime) on the particular solution obtained from the first step of the optimisation. In the first step, emittance and dynamic aperture were optimised by MOGA using ten families of quadrupoles and six families of the harmonic sextupoles. Instead of using the natural emittance (ε_0), the effective emittance (ε_{eff}) at the centre of the first long straight section was used as objectives of the optimisation to control the increase of the dispersion function in the straights. The horizontal effective emittance ($\varepsilon_{x,\text{eff}}$) can be described as

$$\varepsilon_{x,\text{eff}} = \sigma_x \cdot \sigma_{x'} = \sqrt{\varepsilon_0 \beta_x + (\eta_x \sigma_\delta)^2} \cdot \sqrt{\varepsilon_0 \gamma_x + (\eta'_x \sigma_\delta)^2}, \quad (3.1)$$

where β_x and γ_x are the twiss functions, η_x and η'_x are the dispersion function and its derivative respectively, and σ_δ is the energy spread. Because of the non-zero dispersion in the straight section, the effective emittance is introduced to indicate the actual beams size at the straight sections in which insertion devices will be positioned. During linear optics matching for small emittance many constraints had been taken into account: the fractional tune has to be the same as the operating machine, the twiss functions in the straights have to be constrained: the horizontal beta function was kept below 15 m, maintain the same vertical beta function and the horizontal dispersion function was kept below 10 cm.

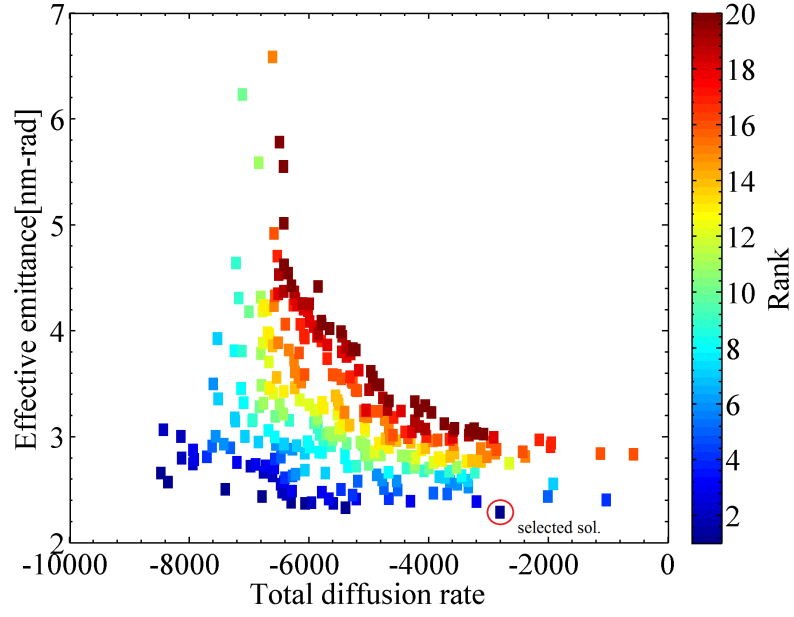


Figure 3.1: Objective functions from MOGA optimisation for effective emittance and total diffusion (dynamic aperture).

Furthermore, special optics sections for double-mini beta insertion in straight I09 and I13 had to be maintained unchanged.

For dynamic aperture optimisation, the sum of the diffusion term from the frequency map analysis in Eq. 2.32 was used rather than the area of dynamic aperture. It was proven that the area of the dynamic aperture and stability of the particle in the dynamic aperture can be optimised simultaneously using the sum of the diffusion [61]. MOGA optimised the lattice for the effective emittance and total diffusion providing several optimal solutions in the objectives space as shown in Figure 3.1. It is clear that the optimal Pareto front (solution with the rank 1), provides solutions with smaller effective emittance and lower total diffusion (larger dynamic aperture). The total diffusion will be optimised with Touschek lifetime in the next step, therefore we opted for selecting the smaller beam emittance in the optimal Pareto front. The selected solution with the smallest beam emittance indicated with the red circle in Figure 3.1 has a natural emittance of 2 nm·rad which is smaller than the operating emittance by 27%. The optics functions are shown in Figure 3.2. Despite the effective emittance was used as objective, the dispersion function in the straight section is slightly larger than the dispersion in the existing storage ring. The increased dispersion function

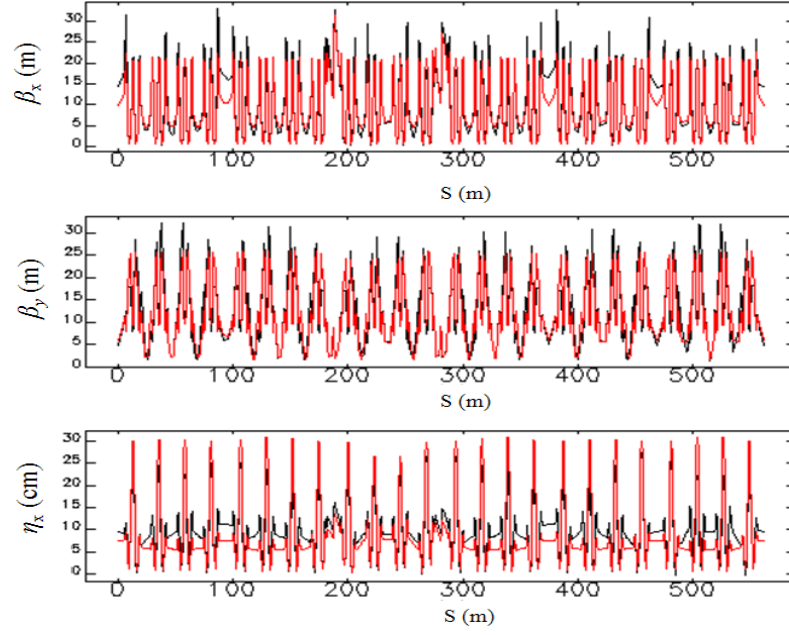


Figure 3.2: Twiss functions of optimal solution giving the natural emittance of 2 nm·rad (black) with respect to the original twiss functions (red).

can deteriorate the beam emittance in presence of strong ID magnetic fields as explained in Chapter 2. The relation between dispersion function in strong field IDs (I12 and I15) and beam emittance can be plotted as shown in Figure 3.3 for the solution found.

The subsequent optimisation for dynamic aperture and Touschek lifetime was performed using all available sextupoles as variables. No quadrupole was used so the linear optics was fixed in this step conserving the optimised beam emittance. After a new set of sextupoles was implemented in the lattice, the chromaticity was corrected to the operating value of 2 in both horizontal and vertical planes and the frequency map and momentum aperture were calculated. Similar to the previous study the sum of diffusion was again used to maximise the dynamic aperture. Touschek lifetime was calculated from the averaged of minimum momentum aperture from both positive and negative momentum [41]. The solutions found by MOGA after about a week of computation time on a 800 nodes cluster, are shown in Figure 3.4. The red asterisk represents the initial lattice taken from the previous optimisation of effective emittance and dynamic aperture. Blue dots represent the optimal lattice solutions with the rank 1. Both the dynamic aperture and Touschek lifetime can be improved (lattice solution with blue dots). The solution in the red circle in Figure 3.4 was selected.

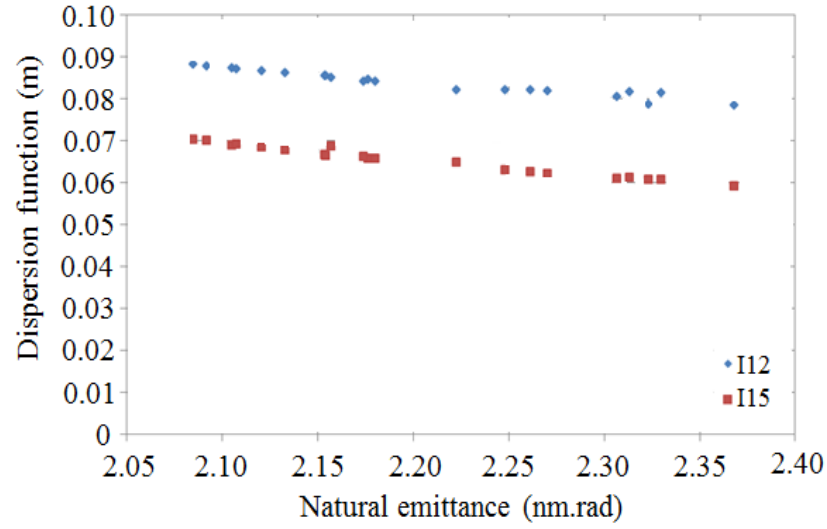


Figure 3.3: Dispersion function at the location of the strong wigglers in I12 (blue) and I15 (red) and natural emittance of optimal solutions.

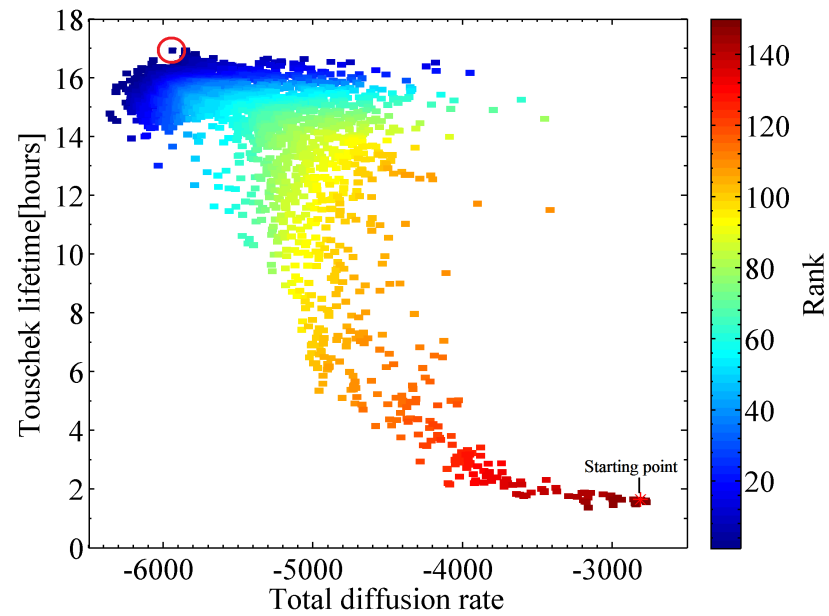


Figure 3.4: Touschek lifetime and total diffusion (dynamic aperture) as objective functions in MOGA optimisation (the color is the solution's rank).

3.1.1 New Lattice Implementation

The new set of magnets strengths of the solution selected in Figure 3.4 was tested in the Diamond machine [30]. Starting from the nominal operating machine configuration, quadrupoles and sextupoles' strengths were adjusted to the new optimised small beam emittance lattice. The betatron tunes and chromaticities were corrected to the model operating point. The linear optics of the new optimised lattice was corrected with Linear Optics from Closed Orbits code (LOCO) [62]. It was found that the injection efficiency was low without any adjustment to the injection system and booster to storage ring transfer line (BTS).

The beta function of the new lattice at the injection point is different from that of the standard lattice which the injection system has been optimised for. In particular the horizontal beta and dispersion functions in straight section is larger as a result of the beam emittance minimisation. The horizontal beta function is increased to 14.4 m for the small emittance lattice compared to the 9.8 m of the standard lattice. This larger beam size still allowed good injection efficiency and the last six quadrupoles in the BTS were tuned to match the injected beam with the new optics functions at the injection straight. At the exit of the BTS, the horizontal beta function (β_x) was changed from 3.2 m to 5.2 m [30]. The newly tuned twiss functions of BTS are described in Figure 3.5

After linear optics correction with LOCO, the injection efficiency was brought up to 50%. The injection angle was also tuned during the experiment and slight improvement was observed. It was found that the horizontal collimator at the beginning of the ring was too close to the beam. Finally after the BTS optics had been optimised and the horizontal collimator was opened, the injection efficiency reached 90%. In order to explore the possibility of improving further the injection efficiency, on-line optimisation of the betatron tunes was carried out. However, it was found that the tunes giving the best injection efficiency were the designed values obtained with MOGA confirming the quality of the optimisation. The tune scan around the nominal value betatron tune values is shown in Figure 3.6. It is clear that the third order resonance line $2\nu_y + \nu_x = 56$ (blue line) significantly deteriorates the injection efficiency.

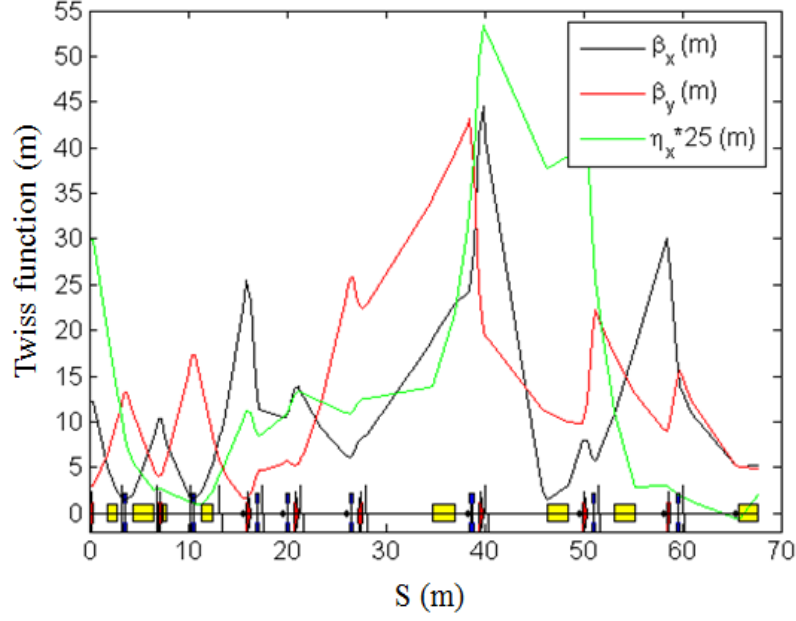


Figure 3.5: Twiss functions of the BTS after matching at injection point of the new low emittance lattice.

To further verify that the solution was implemented correctly in the real machine and produced the desired reduction in beam emittance, the beam dimension at a dipole were measured. The beam emittance can be extracted from the measured beam size using two X-ray pin hole cameras [63] [64]. From the relation

$$\begin{pmatrix} \sigma_{x1}^2 \\ \sigma_{y1}^2 \\ \sigma_{x2}^2 \\ \sigma_{y2}^2 \end{pmatrix} = \begin{pmatrix} \beta_{x1} & 0 & \eta_{x1}^2 \\ 0 & \beta_{y1} & \eta_{y1}^2 \\ \beta_{x2} & 0 & \eta_{x2}^2 \\ 0 & \beta_{y2} & \eta_{y2}^2 \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \sigma_E^2 \end{pmatrix} \quad (3.2)$$

where σ_{xn} and σ_{yn} are the calculated horizontal and vertical beam sizes from the pin hole camera images, β_{xn} , β_{yn} , η_{xn} and η_{yn} are the horizontal and vertical betatron and dispersion functions measured at the source points, the beam emittance (ε_x and ε_y) and the energy spread (σ_E) can be extracted from the inversion of Eq. 3.2. The beta and dispersion function were measured by LOCO rather than using the model values. This allows a more accurate measurement of the beam emittance. The measured beam emittance is in good agreement with the designed beam emittance as shown in Table 3.1. This lattice produces the expected

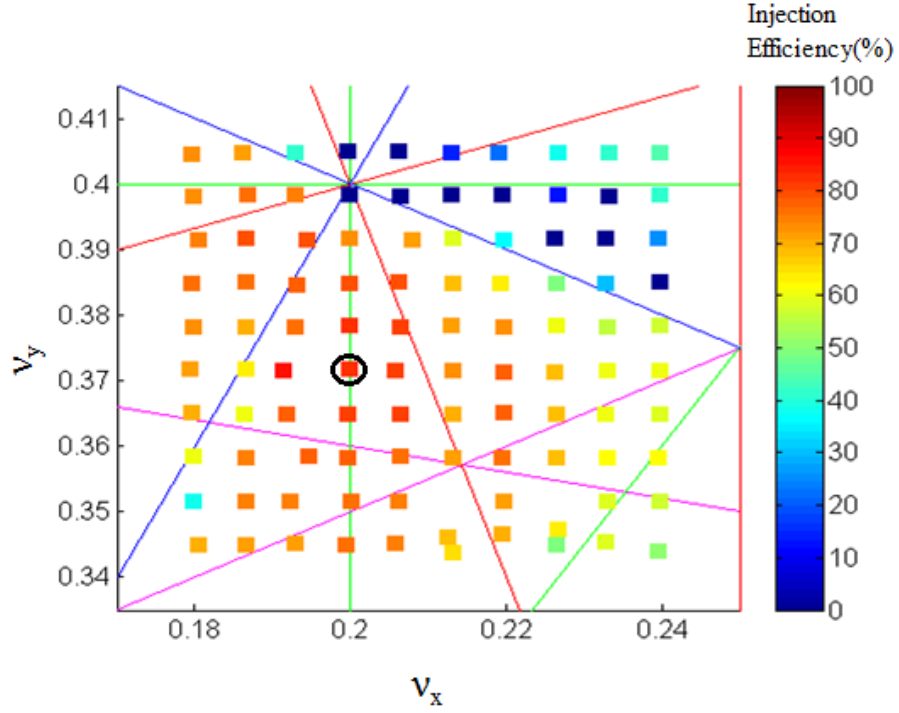


Figure 3.6: Variation of the injection efficiency with betatron tune for the low emittance lattice (designed tune in circle).

reduction of the beam emittance to 2 nm·rad for the bare machine without ID. An image of the beam from the pinhole camera is shown in Figure 3.7. The improved brightness from a standard Diamond 2 m long undulator (U23) is shown in Figure 3.8 indicating that about 25% brightness improvement with respect to the standard lattice is possible for the operation with 0.3% beam coupling with the new small emittance lattice.

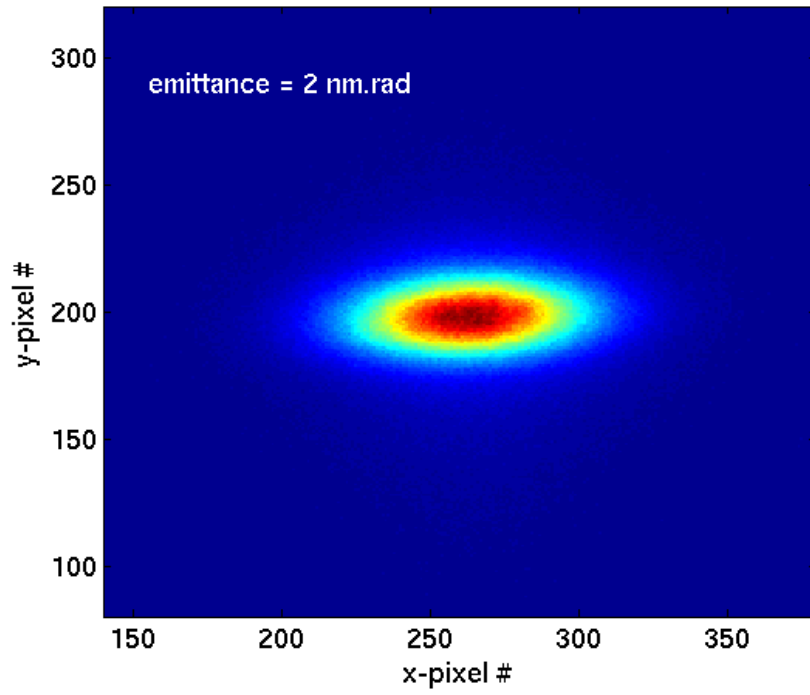


Figure 3.7: Snapshot of the small emittance beam from a pin hole camera.

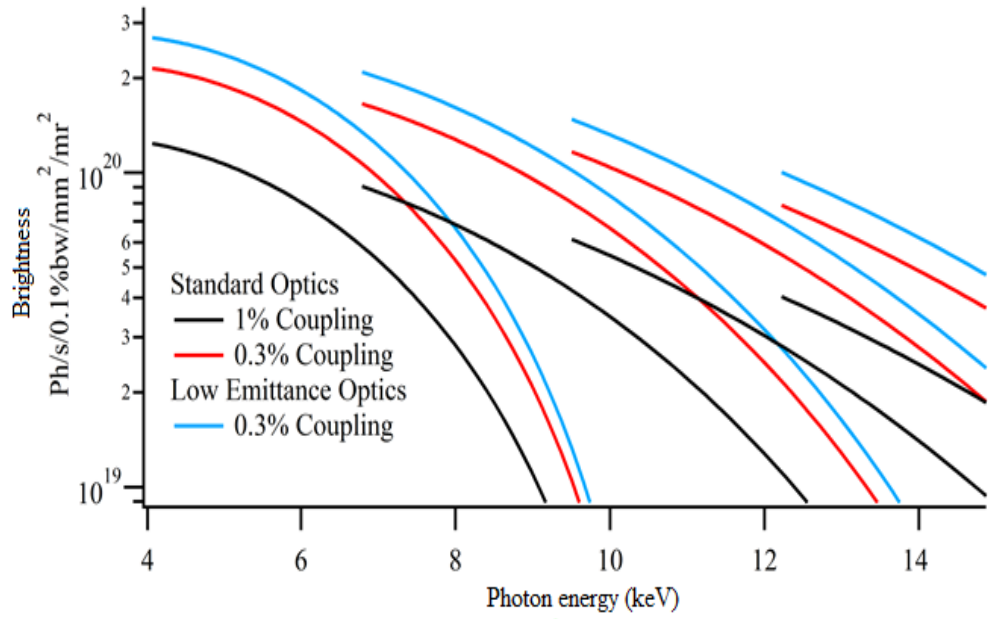


Figure 3.8: Comparison of brightness from the Diamond 2 m long ID (U23) [30].

Effect of wigglers on emittance growth

In the previous optimisation, the horizontal dispersion function was intentionally increased in straight sections, where insertion devices are placed, to achieve the emittance reduction. This could lead to emittance growth when strong field wigglers are in operation.

In the Diamond storage ring there are strong field wigglers operating with the peak field of 4.2 T and 3.5 T placed at straight section I12 and I15 respectively. As described in Chapter 2, if the wigglers are in non-dispersive straights, they can reduce the beam emittance. However, their effect can be opposite in presence of large dispersion function, leading to emittance increase. The emittance growth ratio can be calculated from Eq. 2.16 considering the radiation integral $I_{5,0} = 1.837607 \times 10^{-4} [\text{m}^{-1}]$ from the Diamond bare ring. Figure 3.9 describes the emittance growth ratio as a function of the dispersion function at the middle of wigglers in straight I12 and I15. The emittance ratio grows as the dispersion function is increased. In order for the wigglers to have no effect on the emittance, the emittance ratio has to be 1, from which we obtain the limits on the dispersion function of 3.43 cm and 3.57 cm in the straight I12 and I15 respectively.

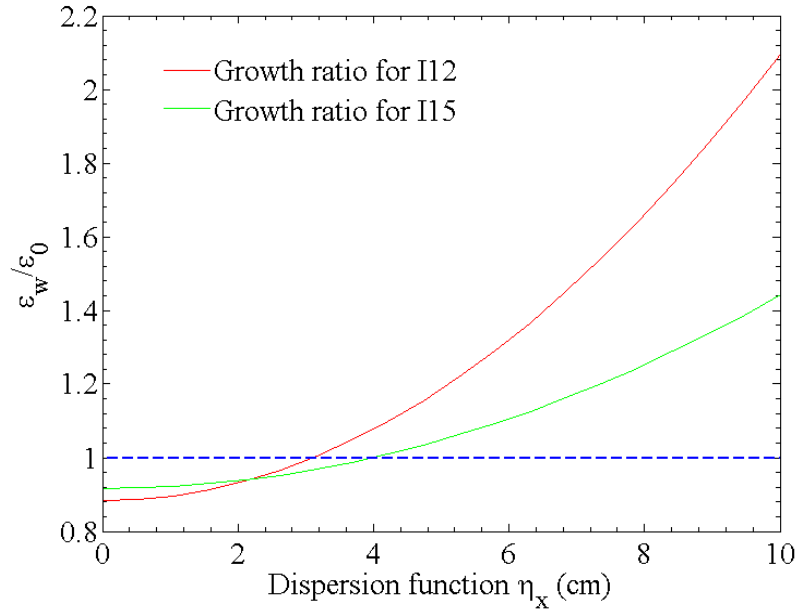


Figure 3.9: Emittance growth ratio as function of dispersion function in IDs for small emittance Diamond (broken blue line for $\epsilon_w/\epsilon_0=1$).

Table 3.1: Calculated and measured beam emittance

Study cases	Calculated emittance (nm·rad)	Matched coupling at two pin holes (nm·rad)	Different coupling at two pin holes (nm·rad)
Bare lattice	2.00	2.00	2.01
Lattice with I15 (3.5T)	2.77	2.90	2.92
Lattice with I12 (4.2T)	3.28	3.26	3.27

From Table 3.1, the measured beam emittance in the real machine nicely agrees with the calculated value. From the measurement the wigglers caused 45% and 63% emittance growth at I15 and I12 respectively. For the optimised small emittance Diamond lattice, it is clear that the operation of the wigglers will significantly deteriorate the emittance. This indicates that the effect of the wigglers need to be taken into account from the start or compensated a posteriori.

MOGA optimisation for emittance growth from wigglers

As a result of the study on the effect of the two wigglers on the optimised lattice, the emittance growth ratio was also included in the MOGA optimisation. The emittance growth ratio calculated from the effect of the wigglers (Eq. 2.16) was introduced as the third objective. The growth ratio is calculated from the newly matched optics functions at the position of wigglers provided by the optimiser assuming full field for the wigglers. All quadrupoles and sextupoles were used as the variables for the optimisation. The result of the optimisation with MOGA are reported in Figure 3.10. The Pareto optimal front, is clearly visible and provides the best compromise between effective emittance and emittance growth ratio. However, it turns out that the solutions with the emittance growth ratio below 1 produce only a marginal improvement on the existing initial emittance of the Diamond storage ring. It appears therefore that, within the existing hardware, the emittance reduction can only be achieved by increasing further the dispersion leak in the straight sections, as shown in Figure 3.3. and this is in conflict with the operation of strong wigglers.

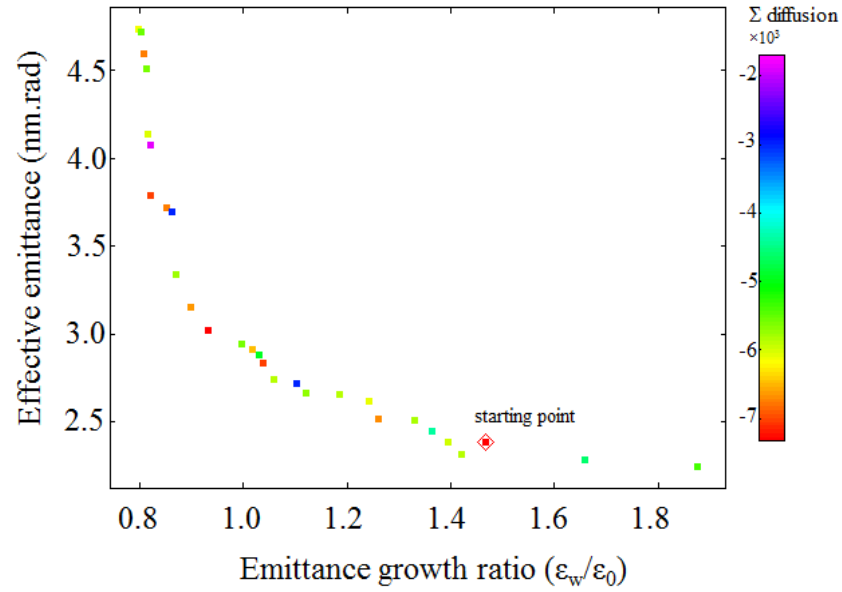


Figure 3.10: Pareto optimal front obtained with MOGA for the objective functions: effective emittance, emittance growth ratio and total diffusion (dynamic aperture).

Despite the lack of an operational solution for the existing Diamond ring these studies provided an experimental verification of the prediction of the MOGA optimisation. The new optimised solution, without wigglers, provides a record horizontal natural emittance of 2 nm·rad for Diamond's storage ring. It was proven to be an operable solution with about 90% injection efficiency with 9 hours lifetime considering 250 mA and 900 bunches. The emittance growth due to the strong field insertion devices has been compared to the analytic calculations and the strong dependences of emittance growth on the strong field insertion devices was confirmed experimentally. The optimisation including the effect of strong field wigglers shows that the improvement in beam emittance in comparison with the operating machine is small. Further development of this work will be the analysis of a local control of dispersion function targeting only the position of the wigglers to reduce the emittance growth induced by the wigglers on the whole lattice.

3.2 One DDBA lattice for Diamond

Diamond is currently operating more than twenty beamline and will reach its final complement of IDs at the end of 2017. Nevertheless there is still strong pressure from the users'

community to open new and diverse beamlines. The existing straight sections are either occupied or allocated for insertion devices. It would therefore be beneficial to find alternative solutions for providing additional space for new IDs and beamlines. One of the options considered for the full ring upgrade consists of a modified 4BA or double-double bend achromat (DDBA) lattice, where an additional straight section is introduced in the middle of the DDBA cell. This lattice doubles the number of straight sections per cell available for IDs and provides an ideal solution to allocate more IDs.

It was therefore proposed to study the possibility of replacing the existing DBA cell 2 with the DDBA cell to provide an additional straight for a versatile micro-focus and in-situ diffraction facility for macromolecular crystallography (VMX) beamline. In Figure 3.11, the comparison of the schematic layout of the original DBA and DDBA cells shows the additional straight section 3.4 m long which can accommodate a standard 2 m Diamond undulator. Considering the source point of the new ID in the middle of the DDBA cell, the radiation can be extracted through the existing second bending magnet port of the original DBA cell in the wall [65]. This gives the possibility of upgrading a standard bending magnet beamline to an insertion device beamline. The DDBA cell comprises two more dipoles designed with a weaker field of 0.8 T whereas the dipoles in the standard DBA cell have a magnetic field of 1.4 T. No dipole beamline is foreseen in the new DDBA cell. The new in-vacuum ID planned for the VMX beamline will operate with a full gap of only 5 mm, therefore a careful tuning of the optics functions is required in the mid-straight section.

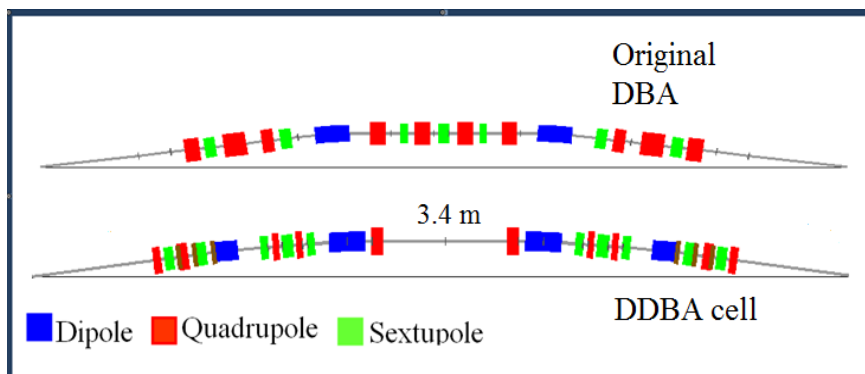


Figure 3.11: Schematic layout of the DDBA cell with respect to the original Diamond DBA cell.

3.2.1 Lattice optimisation for one DDBA cell in Diamond

The DDBA cell, shown in Figure 3.11, is the result of a trade-off between the length of the straight in the middle of the cell and the space and strength for magnets required to control the optics functions. By a careful adjustment of the drift sections and reducing the number of quadrupoles as much as possible, the middle straight between the two quadrupoles can be lengthened up to 3.4 m long. The design of the cell took also into account some engineering and cost constraints to reduce the manufacturing costs, the magnets lengths were eventually designed to be the same for each type of magnet (0.25 m for quadrupoles and 0.175 m for sextupoles). The combined function dipoles in DDBA were originally designed to have different gradient for the long and short dipoles. Then they were re-optimised to have the same gradient to ease the magnet production. The gradients in all magnets were constrained well within the values accessible within the existing technology of resistive magnets.

To install the new DDBA cell in the existing machine, the linear optics functions have to be rematched to connect the new cell with the standard short straight before and after the DDBA cell (I02 and I03 respectively). The ideal position of the waists of both horizontal and vertical beta function in the adjacent straight have to be conserved as well in order not to perturb the operation of the corresponding beamlines. Additionally, the dispersion function has to be similar to that of the existing lattice. The matching required all quadrupoles in the DDBA cell and quadrupole triplets in the adjacent DBA cells (cell 1 and 3). With the presence of the new DDBA cell, the betatron tunes were moved from their operating values. The horizontal tune was increased by half integer and placed in region above half integer. In order to reduce the sensitivity of the Diamond storage ring to the resistive wall instability [14], the horizontal tune was later matched to be one integer larger while the fractional tune was kept similar to the original fractional tune. The vertical tune change was not as significant as in horizontal plane, and was later matched to be slightly lower than the operating tune to avoid the third order resonances. The tune matching requires all the rest of quadrupoles in the standard DBA cell to distribute the change of the beta functions while the optics functions in the straights are conserved. Three special sections at Diamond

provide customised optics for the users: a double mini-beta sections in the straight I09 and I13 and a smaller beta functions straight in I21. The matching of the lattice with the single DDBA cell in cell2 had to guarantee minimum perturbation to these three special cells. The betatron functions in DDBA cell were kept small as the new section employs small aperture magnets. Also the optics functions in the middle of the DDBA cell have to be constrained to accommodate a new ID; they were matched to the same values as in the centre of the standard straight sections. Although the optics functions were successfully matched to the required value to accommodate ID, the phase advanced between two sextupoles in the DDBA cell is 1.95 and 0.66π for horizontal and vertical plane respectively which are not appropriate phase advance for driving terms cancellation especially in the horizontal plane. This leads to a challenging nonlinear optics optimisation.

The main parameters for one DDBA in Diamond are reported in Table 3.2. The comparison of the optics functions with the original Diamond lattice and one DDBA lattice is shown in Figure 3.12. A slight change of the overall optics functions is achieved with the matched tune to the value shown in Table 3.2. The optics functions in the DDBA cell are smaller than those of the DBA cell as shown in Figure 3.13. The optics functions in the new middle straight of the DDBA cell were matched to values similar to those in the standard short straight section with only a slight reduction in horizontal beta function.

Table 3.2: Parameters for Diamond DBA and one DDBA lattice

Parameters	Diamond DBA	one DDBA
Energy [GeV]	3.0	3.0
Circumference [m]	561.600	561.571
Emittance [nm.rad]	2.75	2.69
Betatron tunes Q_x/Q_y	27.20/13.37	28.18/13.28
Mom. compaction	1.6E-4	1.5E-4
Natural chromaticity ξ_{x0}/ξ_{y0}	-80.4/-35.6	-78.4/-41.3

We will show in Chapter 4 that when the DDBA cell is replicated 24 times for the whole lattice, the emittance of the ring is reduced by a factor 10, however the implementation of one single cell produces only a slight reduction in emittance. However, as stated, the main purpose of this study is to provide an additional straight section for an additional beamline.

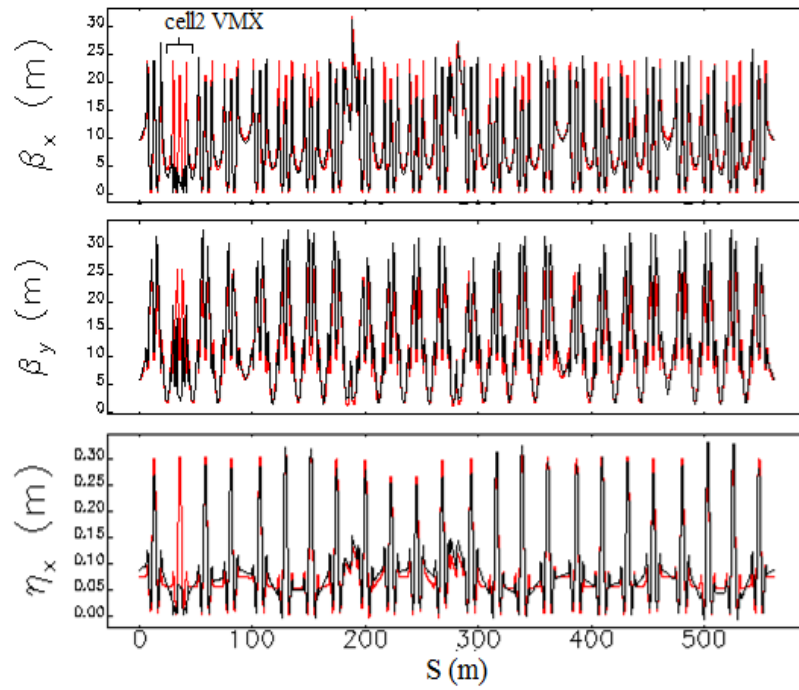


Figure 3.12: Twiss functions of Diamond with one DDBA cell (black) compare to the existing Diamond ring (red).

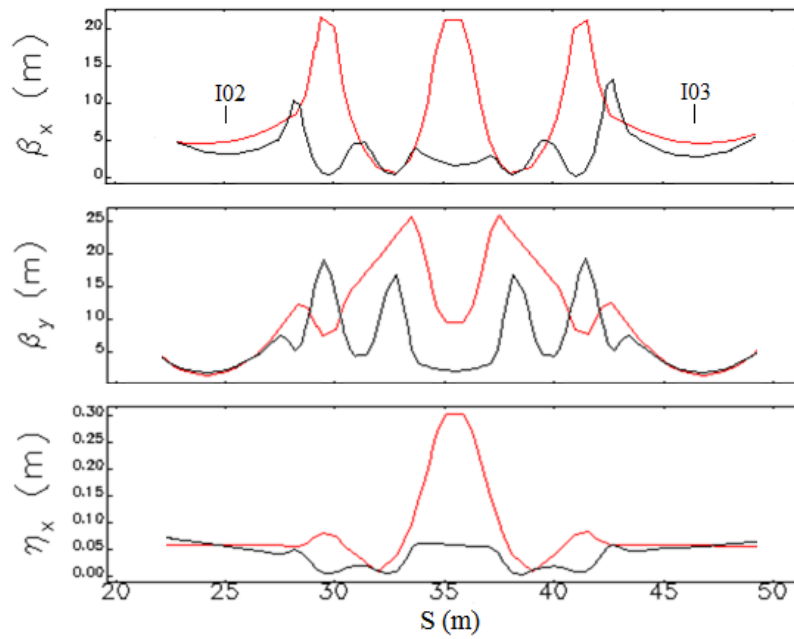


Figure 3.13: Twiss functions of the DDBA cell (black) compare to the existing Diamond DBA cell (red).

3.2.2 Magnets for DDBA

Due to the compactness of the DDBA cell, the lattice design has been carried out considering very small distances between the magnet elements allowing only 10 cm clearance. The cell design however allows the use of discrete magnets rather than more complex designs with integrated magnet block as used in MAX-IV [66]. All the magnets use the existing technology of resistive magnets reducing the R&D time to a minimum. Compared to the existing Diamond's magnets, the DDBA cell employs stronger magnets requiring a narrow pole gap designs to achieve the desired field strength and quality. All four dipoles for DDBA have to be combined function with quadrupole gradient thus eliminating the requirement of separate vertically focussing quadrupoles. A relatively high quadrupole gradient in the combined function dipole was achieved thanks to the low dipole field 0.8 T, in any case this is still smaller than the values proposed in other designs in Table 1.1. The magnet design parameters are reported in Table 3.3.

Table 3.3: Magnet specifications for DDBA cell [67]

Magnet	Parameter	Value
Dipole	Effective length	0.67 m / 0.967 m
	Physical length	0.65 m / 0.95 m
	Nominal field	0.8 T
	Gradient	14.385 Tm ⁻¹
Quadrupole	Effective length	0.25 m
	Physical length	0.32 m
	Gradient	70 Tm ⁻¹
	Bore diameter	30 mm
Sextupole	Effective length	0.175 m
	Physical length	0.232 m
	Gradient	2000 Tm ⁻²
	Bore diameter	30 mm
Integrated corrector	Strength (Hor./Ver.)	8 mT·m / 8 mT·m
Corrector	Physical length	0.102 m
	Aperture	70 × 70 mm
	Strength (Hor./Ver.)	8 mT·m / 8 mT·m

3.2.3 MOGA optimisation for one DDBA lattice

As discussed in Section 2.3.1, MOGA was used for lattice optimisation. Initially, similar to the optimisation for small emittance Diamond discussed in the Section 3.1, the Touschek lifetime and the sum of diffusion (dynamic aperture) were used as objective functions for the one DDBA lattice optimisation. The frequency map was computed with Elegant [41] and was used to calculate the diffusion with full 6D tracking including an RF cavity. All available sextupoles were used as variables in the optimisation including the sextupoles in the DDBA cell whereas the quadrupoles were kept untouched to fix the linear optics. Six harmonic sextupoles from the existing DBA cell and eight new sextupoles in the DDBA cell provides fourteen variables in total. The symmetry of the sextupoles in the DDBA cell was broken to give more freedom for the tuning. This symmetry breaking is not creating major consequences for the beam dynamics. In fact the Diamond storage ring's symmetry is already broken by the special optics sections (I09, I13 and I21). The optimal front provided by MOGA is shown in Figure 3.14. The selected solution with the red circle in Figure 3.14 gives a horizontal dynamic aperture above -10 mm which is sufficient for the Diamond storage ring since the injection occurs at -8.3 mm off-axis. The frequency map of the selected solution

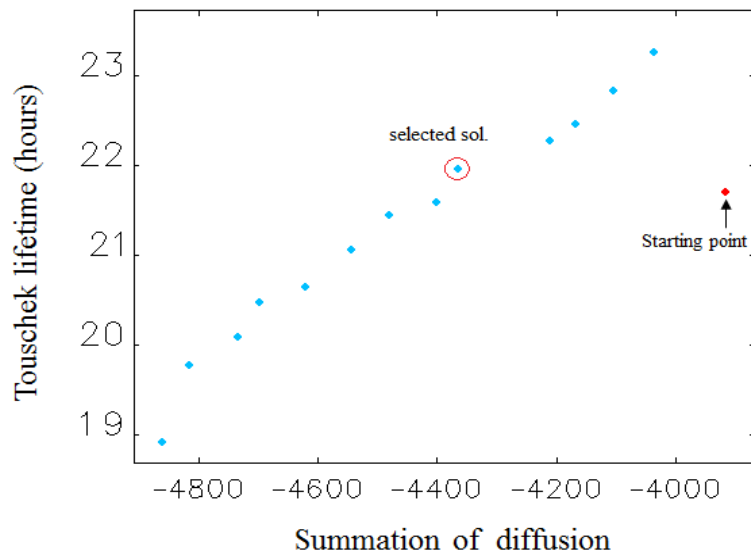


Figure 3.14: Optimal front of optimisation for Touschek lifetime and summation of diffusion.

is described in Figure 3.15. It is worth noticing here that in this optimisation only the ideal machine without any imperfection and misalignment was considered. The optimal solution is therefore further studied including magnets multipole errors and machine imperfections as discussed later in this chapter.

During the optimisation, it was realised that some solutions show particles with large momentum hitting the coupling resonance and generating losses, small momentum aperture and poor Touschek lifetime. The MOGA optimisation improved the detuning with momentum of the lattice by extending the crossing point of the two betatron tunes to larger momentum (from 2.8% to 3.5%) and improve the lifetime (from 22 hrs to 25 hrs) as shown in Figure 3.16. Additionally, the slope of detuning with momentum was also decreased indicating that higher order chromaticities are better compensated and particles with larger momentum are less likely to wander through dangerous resonances.

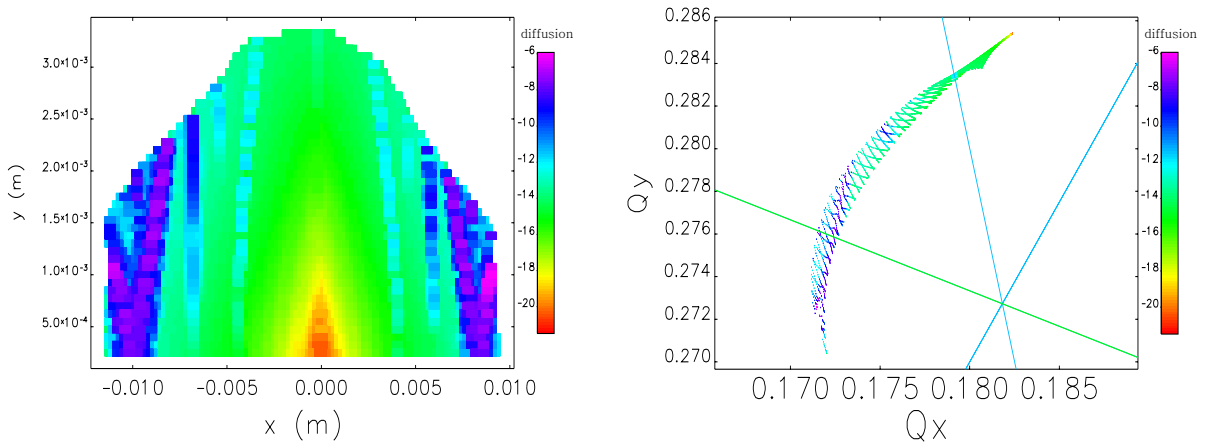


Figure 3.15: Frequency map of a selected solution from MOGA for Touschek lifetime and summation of diffusion (green and blue lines are fourth and fifth order resonances respectively).

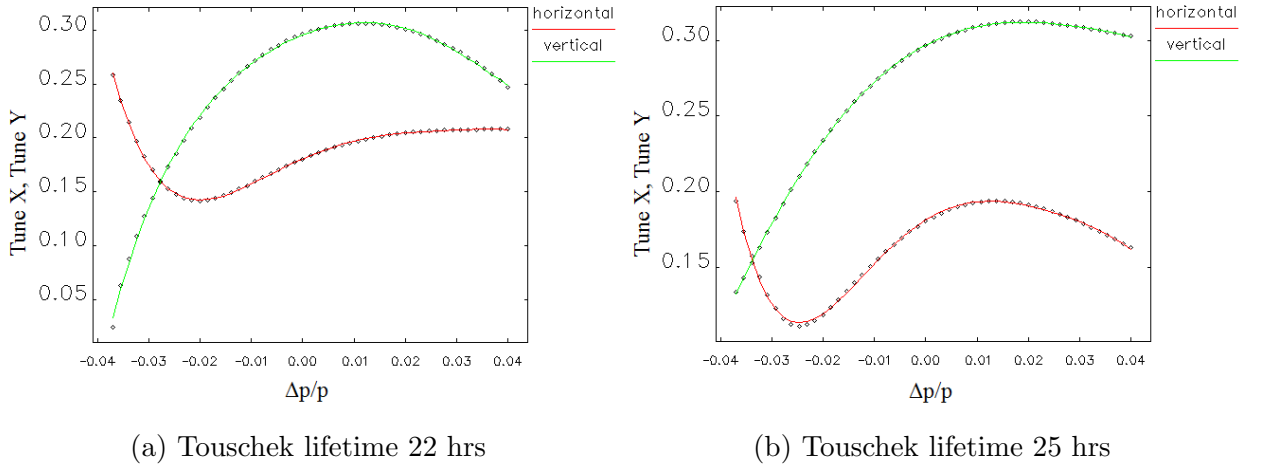


Figure 3.16: Detuning with momentum for different solution from MOGA for chromaticity of 2 in both planes.

3.2.4 Physical aperture and Touschek lifetime calculation

As discussed in Chapter 2, the Touschek lifetime was calculated with Elegant [41] using the computation of the momentum acceptance. To achieve the most realistic estimation of the Touschek lifetime, the actual physical aperture based on the engineering drawings was included in the tracking for the momentum aperture. The same calculation allows the computation of the loss distribution of the particles after a Touschek scattering event, however some comments are in order: Elegant considers the physical aperture at the end of each accelerator element, hence the description of the lattice elements has to be matched to every change of the physical aperture, otherwise the details of the vacuum pipe change will be missed. If the calculation is done with a physical aperture that is not matched to the lattice especially in dominant parts that affect the particle loss such as the collimator, the particle loss distribution would appear incorrectly in other places of the ring. The physical aperture of the first super-period including the DDBA section is shown in Figure 3.17. It is worth noticing that, due to the small aperture magnets, the physical aperture in DDBA section is much narrower (13.5 mm from 41 mm in horizontal plane) than that of the existing DBA section. Collimators were also introduced to control the particle losses at the injection straight section as in the original design. The simulations included both horizontal and

vertical collimators set at 12 mm and 3.5 mm respectively as in the standard operation. In the middle of the DDBA cell, the vertical aperture is 2.5 mm half gap to simulate the minimum gap when the new in-vacuum ID is closed.

A number of radiation bumps in the inner horizontal profile of the pipe is necessary (less than 3 mm high) to block the synchrotron radiation from hitting the uncooled pipe surfaces. They have negligible impact on the impedance and loss factor of the pipe [65]. Initially the physical aperture in the DDBA cell was designed to have these bumps on both side of the horizontal aperture, but eventually most of the bumps were placed on the side that will be hit by synchrotron radiation. To investigate the effect of the aperture on the beam dynamic, full detailed physical aperture including these bumps, horizontal and vertical collimators were used in dynamic aperture, momentum aperture and injection tracking studies. Realistic lattice errors and the operational values for beam coupling were introduced, and the collimators were set to the nominal value where they are most effective in capturing most of the losses from injection and Touschek losses as shown in Figure 3.18. The simulations show that apart from the collimators, for some error seeds, particles losses are predominantly occurring in the DDBA cell in narrow vertical aperture in the straight section I02 and I03.

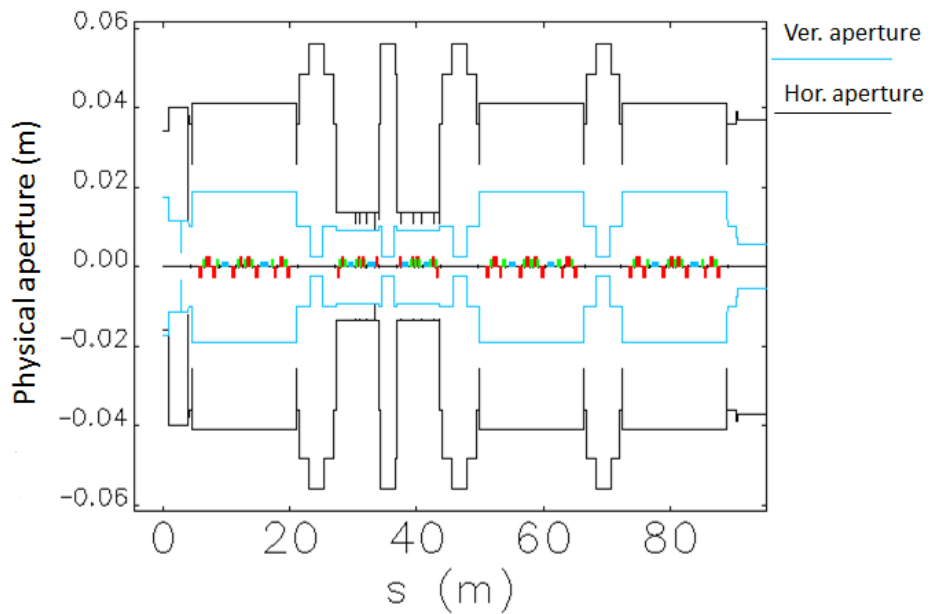


Figure 3.17: Physical aperture in the first super period.

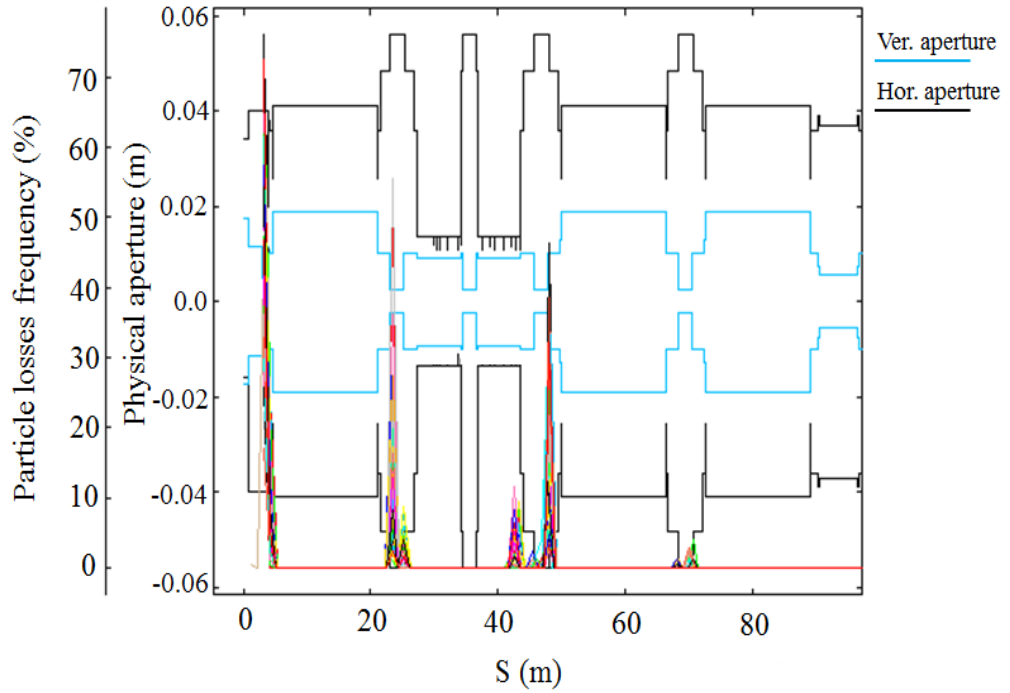


Figure 3.18: Particle loss during momentum aperture search with respect to physical aperture of 50 random error seeds.

3.2.5 Closed orbit correction

In practice, synchrotron light sources operate in presence of errors and imperfections from misalignments and magnets manufacturing. Misalignments and dipoles or IDs errors can cause Closed Orbit Distortion (COD) that can increase the beam size, generate fluctuation of the emitted photon beam as depicted in Figure 3.19, and these effects can be amplified along the beamline and eventually affect the users.

Obviously the closed orbit distortion has to be reduced as much as possible all around the ring and in particular at the source points. The Beam stability requirement at Diamond are set to less than 10% of the beam size and beam divergence for up to 100 Hz [68] implying an orbit control to sub-micron level. Considering the beam size and divergence of $\sigma_x = 123\mu\text{m}$, $\sigma_{x'} = 24\mu\text{rad}$ in horizontal plane and $\sigma_y = 6.4\mu\text{m}$, $\sigma_{y'} = 4\mu\text{rad}$ in vertical plane, the closed orbit has to be corrected down to the following level:

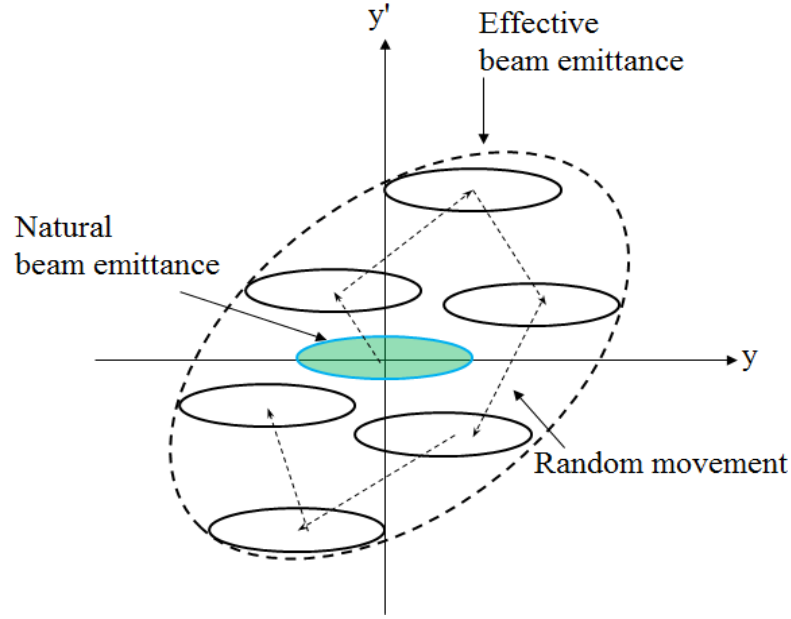


Figure 3.19: Beam size associated to closed orbit distortion.

$$\Delta x < 0.1 \times 123\mu\text{m} = 12.3\mu\text{m},$$

$$\Delta x' < 0.1 \times 24\mu\text{rad} = 2.4\mu\text{rad},$$

$$\Delta y < 0.1 \times 6.4\mu\text{m} = 0.64\mu\text{m},$$

$$\Delta y' < 0.1 \times 4\mu\text{rad} = 0.4\mu\text{rad}.$$

For sensitive experiments, the perturbed closed orbit is even more restrict to only a few percent of the beam size, especially for the beamlines who use the radiation in the infrared region. For this reason a closed orbit correction system and a fast orbit feedback (FOFB) system operating at 10kHz are used to eliminate static orbit distortion and any beam vibration to sub-micron scale in 1-100 Hz bandwidth including the slow feedback system based on X-ray beam position monitors giving $\pm 3\%$ beamsize stability over a week long operation [69].

The closed orbit correction scheme was designed to correct the orbit in the presence of misalignments in the DDBA cell including transverse and longitudinal random displacement amplitude of $100\mu\text{m}$ and rotation amplitude of $(100\mu\text{rad})$ for all magnets. Systematic and random multipole components of the magnets were also taken into account.

The field error of the dipoles in the DDBA cell was modelled allowing larger errors to

Table 3.4: Errors and misalignments amplitude

Type of errors	Dipoles	Quadrupole	Sextupole
Transverse displacement	0.1 mm	0.1 mm	0.1 mm
Roll error	0.2 mrad	0.2 mrad	0.2 mrad
Field error	0.1% (1% for DDBA)	0.1%	0.1%

take into account the fact that the dipoles are combined function magnets with a rather complex pole shape. The simulations of the closed orbit distortion used the rms error values reported in Table 3.4. The values used were increased with respect to what achievable with the present alignment technology in order to simulate possible worse case scenario at the beginning of the DDBA cell commissioning. An example of the horizontal displacement for quadrupole and sextupole for a random machine is shown in Figure 3.20. The random misalignment is well distributed within $100\ \mu\text{m}$.

The initial attempts at orbit correction relied on the available 10 independent horizontal and vertical correctors embedded in the sextupoles in the DDBA cell. It was noted however that with the 3.4 m long mid-straight section there is a poor sampling of the phase advance

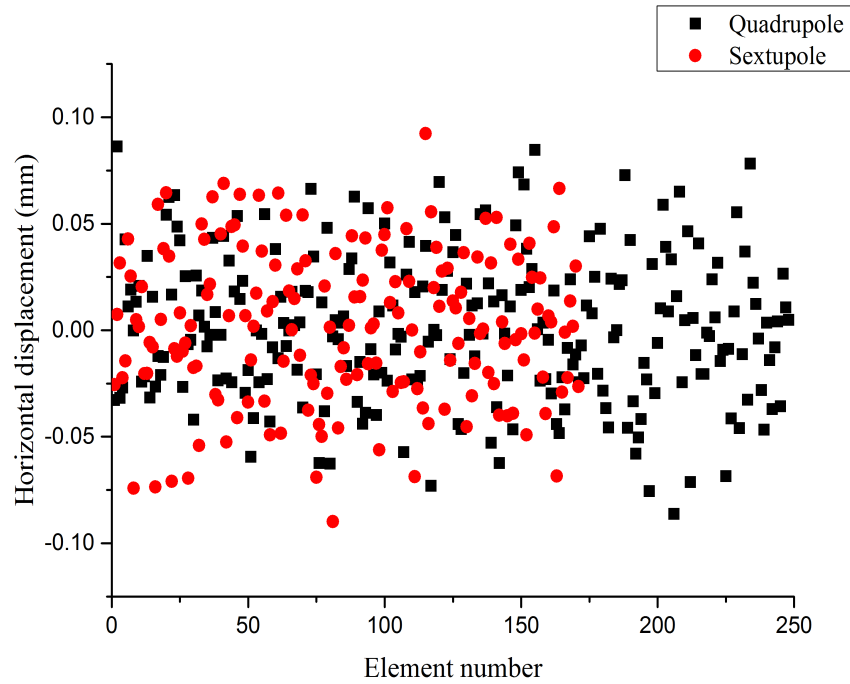


Figure 3.20: Horizontal displacement of quadrupoles and sextupoles in DDBA lattice for a random seed.

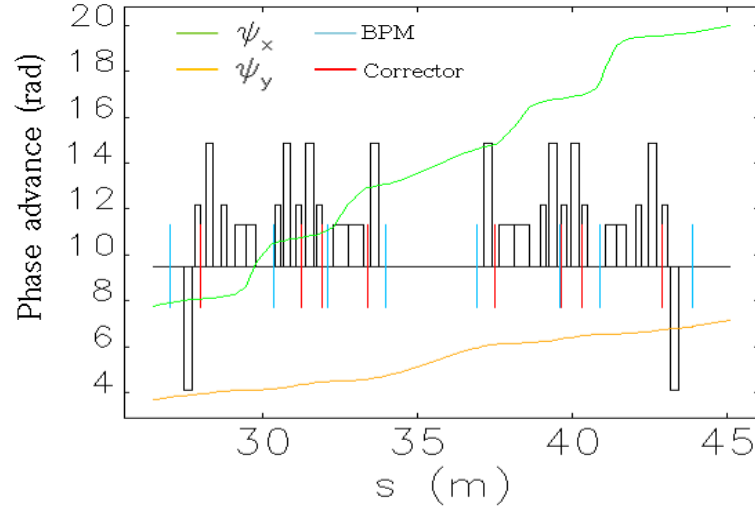
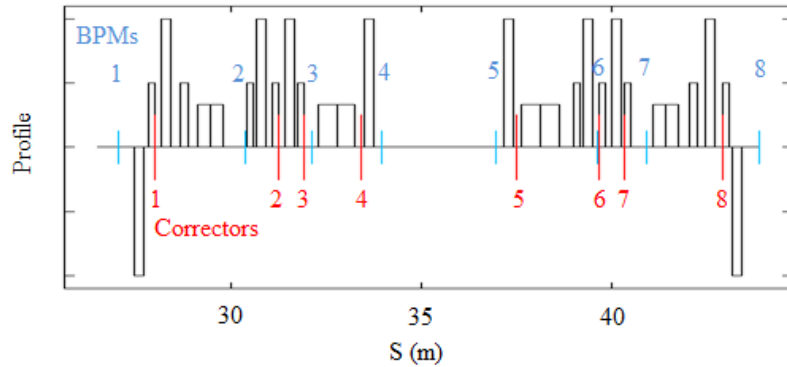


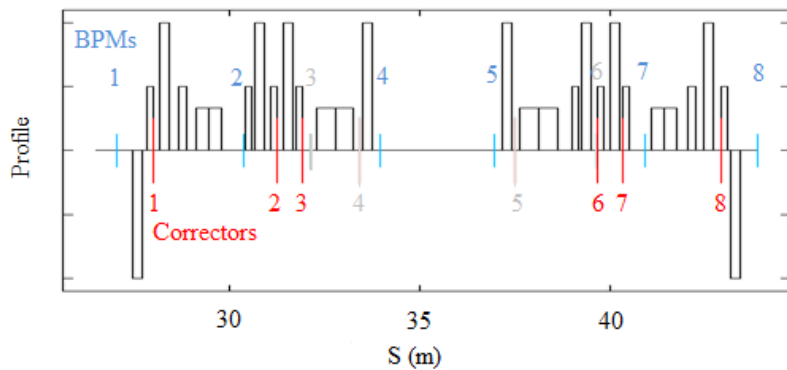
Figure 3.21: BPMs and correctors in DDBA cell and corresponding horizontal and vertical phase advance.

preventing a good orbit correction. The two correctors embedded in sextupoles adjacent to the mid-straight are separated by roughly a full betatron wavelength as shown in Figure 3.21, leading to a poor sampling of the betatron wave. It was found that the corrector's strength required for an acceptable closed orbit correction exceeded 1 mrad and was twice the hardware limit strength in some error seeds. For this reason, a couple of short dipole correctors have been introduced in the mid-straight section. The lack of space in the mid-straight forced a very compact design of short (102 mm) integrated horizontal and vertical correctors [67]. The short correctors were placed symmetrically between the long dipoles (2 and 3) and quadrupole in the middle-straight. These additional short correctors allow a good orbit correction even when four correctors embedded in the sextupoles close to dipoles are disabled as shown in Figure 3.22b. To allow precision steering of the electron beam through the ID, a couple of primary BPMs are also required on each side of the ID. These PBPMs have the same design as the existing one in the Diamond storage ring based on four buttons block of 40 mm long mounted on a separated stand isolating the PBPM from any unwanted vibrations and movements [65]. The orbit correction scheme is thus made of a system of eight correctors and eight BPMs shown in Figure 3.21. The strength limit on all new correctors was set at 0.8 mrad to be the same as the limit in the existing correctors. The orbit can be corrected perfectly down to nm level. Including misalignments in the whole ring, the orbit

before and after correction can be seen in Figure 3.23 and 3.24. Additionally, a case study for orbit distortion generated mainly by the new DDBA cell error was also conducted. This study assumes that the orbit in the rest of the ring is already well corrected: in fact the installation of a new cell² should not change significantly the misalignment already existing and corrected in the rest of the ring. This allowed us to investigate the orbit distortion after the DDBA cell installation. The same errors as shown in Table 3.4 were introduced in the DDBA cell component only. It was found that the beam coupling generated by such errors is insignificant with respect to the 0.3% beam coupling present in the existing ring. The closed orbits before and after correction were similar to the first case as shown in 3.23 and 3.24 and the correctors's strength used for the orbit correction assuming the misalignment distributed in the whole ring and concentrated in the DDBA cell are shown in Figure 3.25 and 3.26 respectively. The correctors' strength was well below the 0.8 mrad limit.

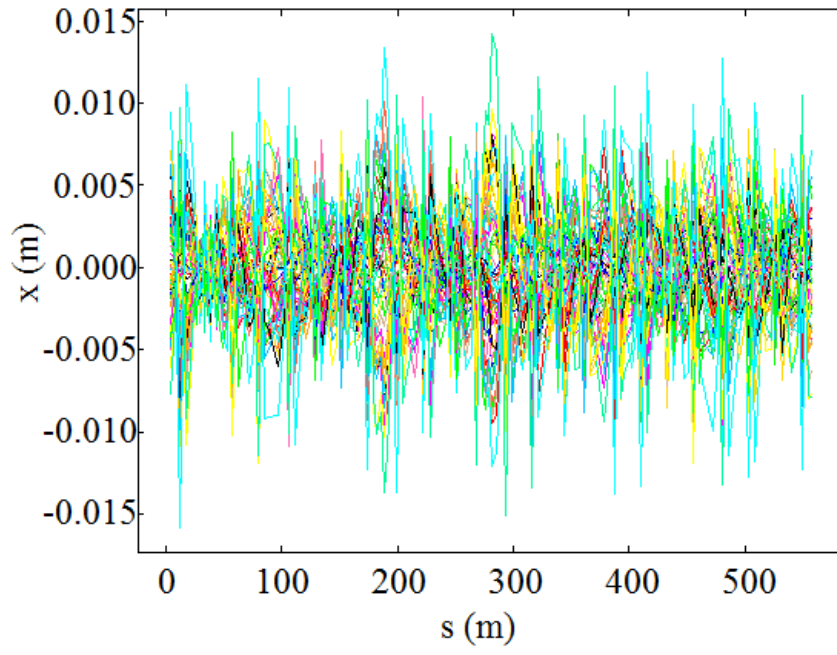


(a) Location of BPMs and correctors in the DDBA cell for static closed orbit correction.

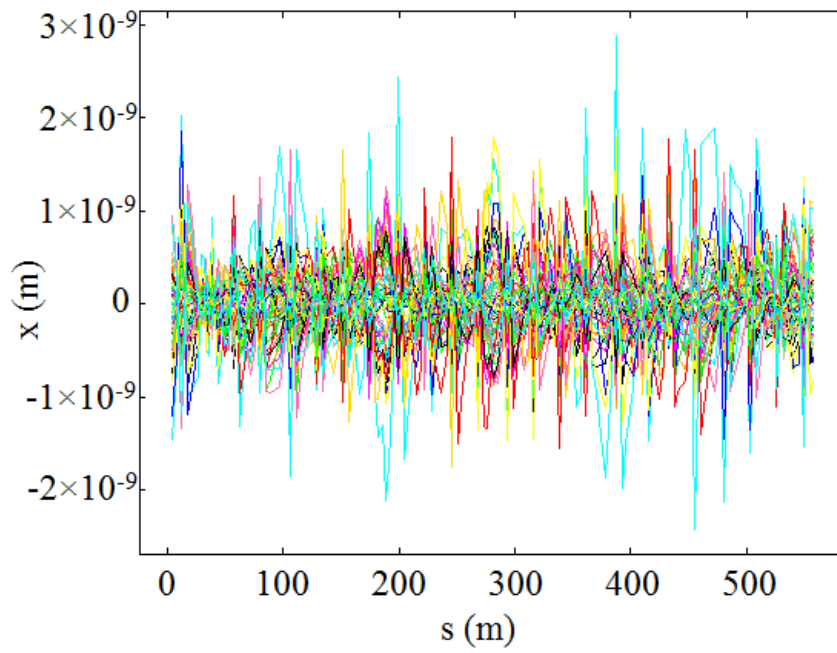


(b) Subset of correctors and BPMs used for fast orbit feedback.

Figure 3.22: BPMs and correctors in DDBA cell.

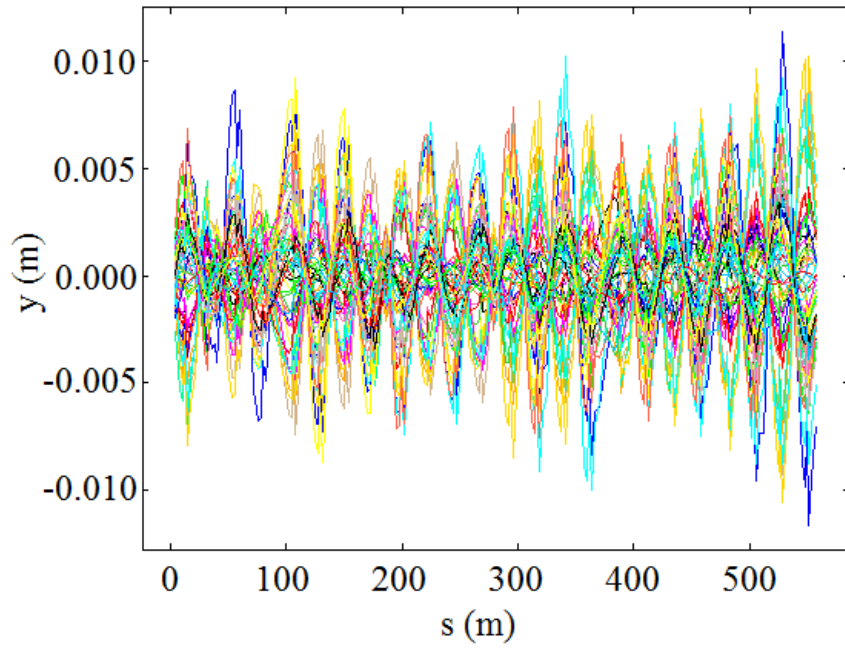


(a) Closed orbit distortion before orbit correction

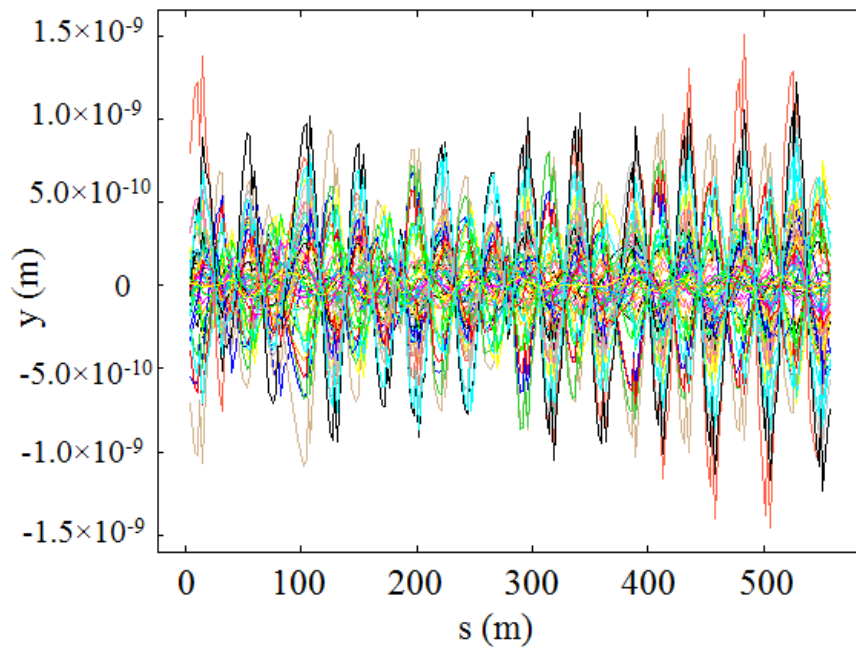


(b) Residual closed orbit distortion after orbit correction

Figure 3.23: Horizontal closed orbit at BPMs with and without correction of 50 random seeds.



(a) Closed orbit distortion before orbit correction



(b) Residual closed orbit distortion after closed orbit correction

Figure 3.24: Vertical closed orbit at BPMs with and without correction of 50 random seeds.

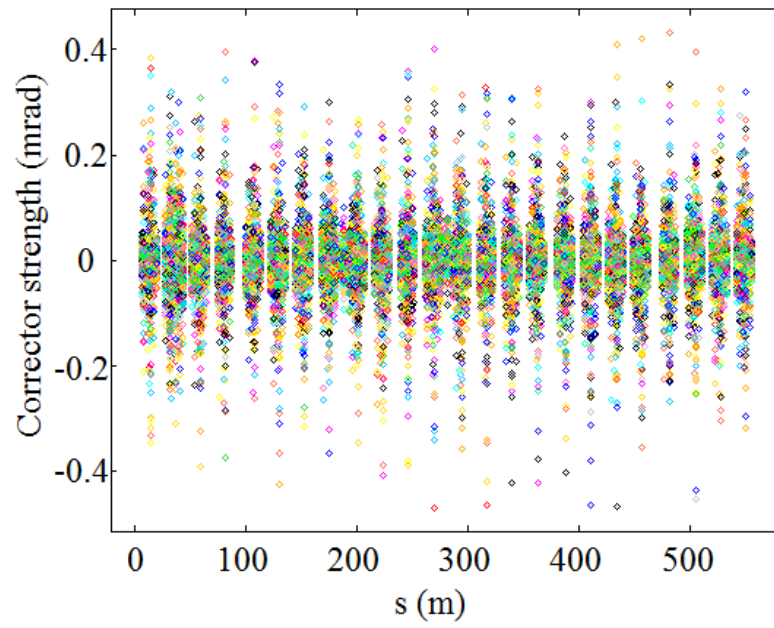


Figure 3.25: Correctors' strength for closed orbit correction of 50 random seeds.

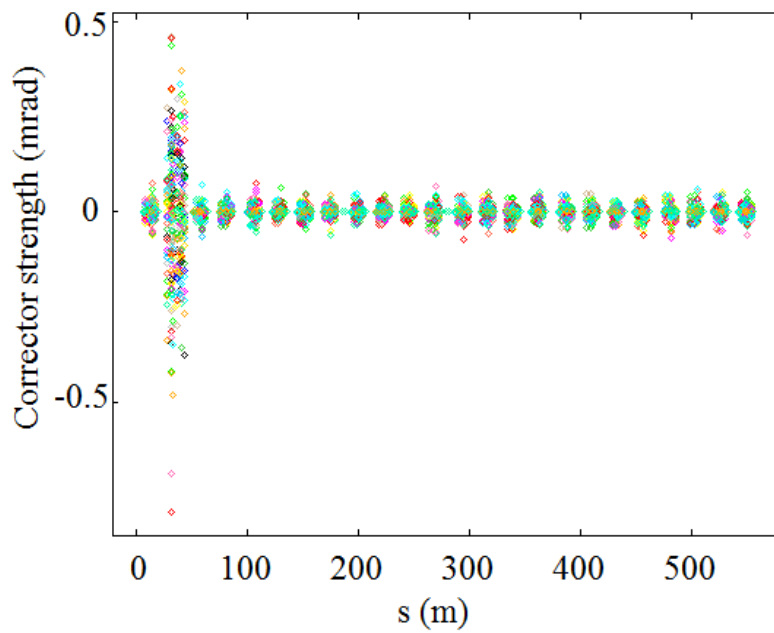


Figure 3.26: Correctors' strength for closed orbit correction for concentrating error in DDBA.

In the DDBA cell, a relatively small elliptical stainless steel chamber (outer diameter of 29×20.4 mm) is used as a consequence of the high gradient quadrupole magnet design with a small bore radius of 15 mm. The elliptical chamber was designed to concentrate 74% of the image currents on top and bottom parts of the pipe. This is beneficial because the side wall can be opened for the pumping port, gauging port and photo beam extraction [65] without impacting significantly on the impedance of the pipe. The vacuum pipe immediately downstream the dipoles in the DDBA cell is irradiated by synchrotron radiation that causes excessive heat and mechanical stress due to thermal expansion [70]. Adequate cooling is required in this location hence a cooled copper pipe is used due to its better thermal conductivity compared to stainless steel pipe. However, copper is a good conductor and the response of the copper pipe to high frequency variation of the external magnetic field from the correctors during Fast Orbit Feedback (FOFB) will produce undesirable eddy current which can distort the applied magnetic field. As a result, high frequency performance of the FOFB can be seriously deteriorated. For this reason, the orbit corrections have to be separated into a static (slow) orbit correction that uses all eight correctors and eight BPMs in the DDBA cell and FOFB working with the total six correctors and six BPMs in the cell without the pair of short correctors in the mid-straight which are positioned on the copper vessel as shown in Figure 3.22a and 3.22b respectively. The correctors, which are placed on the copper vessel, number 4 and 5 and the BPMs number 3 and 6 are disabled for the FOFB while the so-called primary BPMs at either ends of the mid-straight section of the cell are kept. After the static orbit correction, the peak-to-peak orbit distortion, generated by high frequency components, up to $\pm 100 \mu\text{m}$ can be successfully corrected by the FOFB scheme.

For a more realistic simulation, the effect of a new insertion device in the mid-straight on the closed orbit was also considered. Figure 3.27 shows the DDBA cell with a standard 2m undulator in the middle of the new straight. The COD introduced by the ID will be controlled using only the FOFB scheme after the slow orbit correction has been performed. A residual orbit of about $100 \mu\text{m}$ was generated by the ID misalignment error. The misalignment of the new ID was simulated assuming transverse displacement of $100 \mu\text{m}$ and roll error of $100 \mu\text{rad}$. The new ID in the DDBA cell was modelled with a full kickmap provided by the

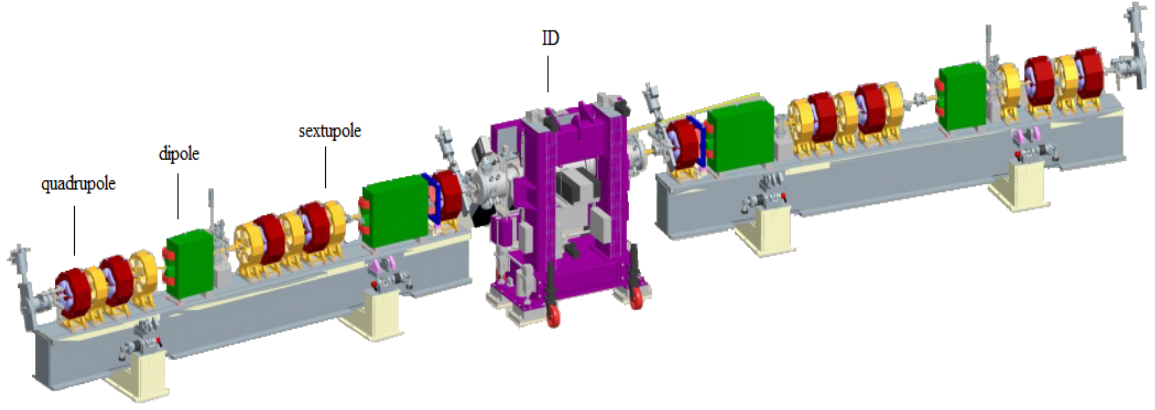


Figure 3.27: Engineering model for the DDBA cell with a standard Diamond in-vacuum undulator.

Diamond insertion devices group. The kickmap of a 2 m long Cryogenic Permanent Magnet Undulator (CPMU) operating in Diamond was used and is shown in Figure 3.28.

The effect of the ID misalignment can be successfully corrected using the FOFB scheme. As shown in Figure 3.29 and 3.30, the closed orbit was corrected to sub-micron level at the position of the new ID. The corrector strengths used are well below the limit of 0.8 mrad as shown in Figure 3.31. The small residual orbit distortion ($< 0.5 \mu\text{m}$) in the vertical plane at the ID is less than 10% of the beam size.

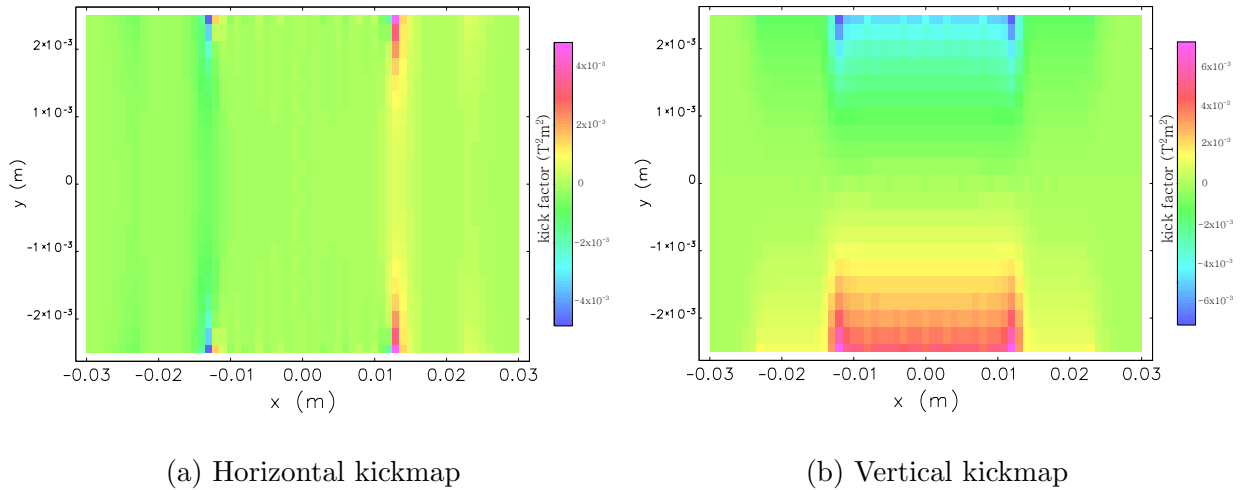


Figure 3.28: Kickmap of the Diamond CPMU considered for the DDBA cell.

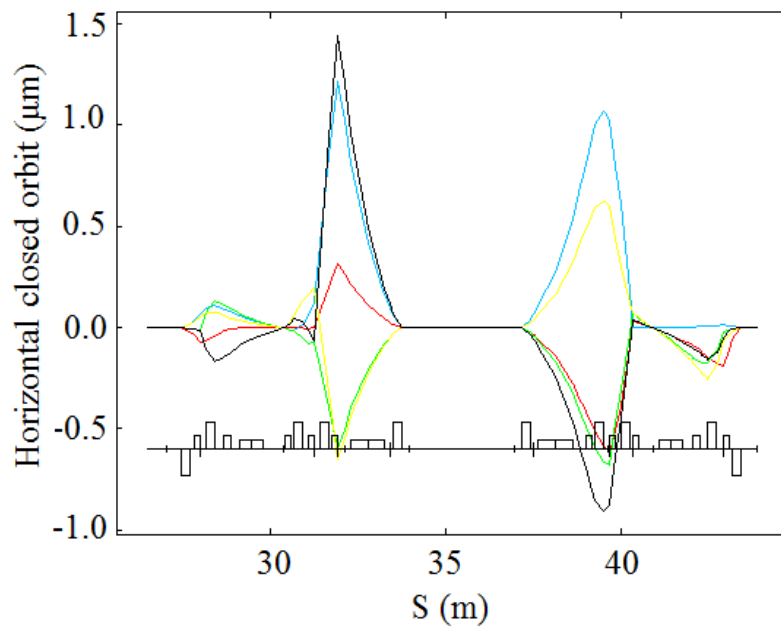


Figure 3.29: Horizontal closed orbit after correction in the DDBA cell for 10 error seeds.

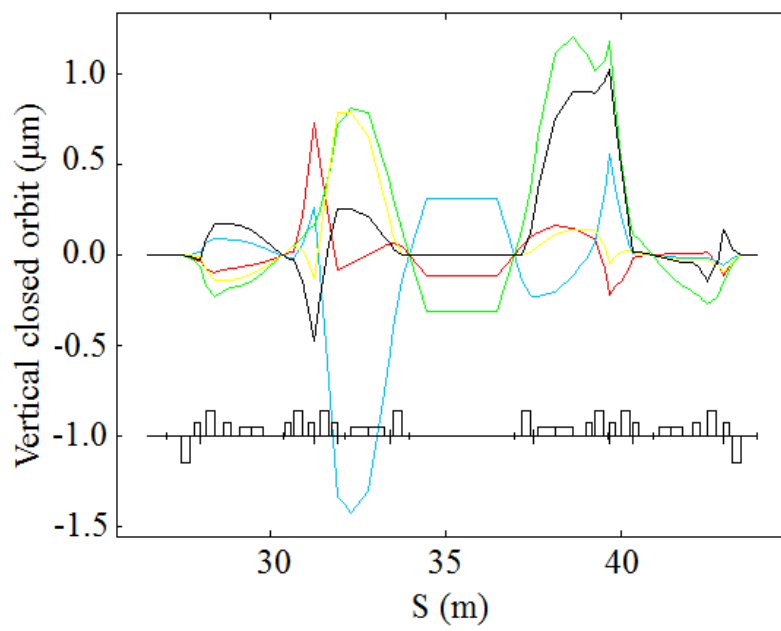


Figure 3.30: Vertical closed orbit after correction in the DDBA cell for 10 error seeds.

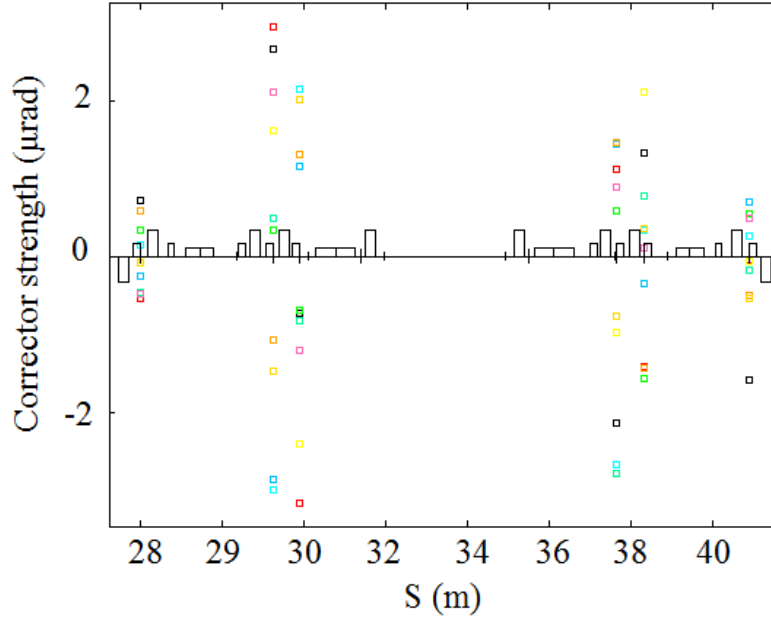


Figure 3.31: Corrector strength for the closed orbit correction in the DDBA cell for 10 random seeds.

Further study for orbit control inside the new ID were also considered with the aim of proving that slight adjustment of the source point position and angle are possible with the FOFB to cater for small requests from the beamline. Small closed orbit bump ($3.5 \mu\text{m}$) at the mid-straight section can be generated by the FOFB scheme to control the position and angle of the source point at the middle of the new ID. To generate the orbit bump in Elegant, the positions of two BPMs in the mid-straight were displaced by the desired offset and the closed orbit was corrected at the BPMs. Random errors were included in the DDBA cell to generate a maximum orbit about $\pm 100 \mu\text{m}$. Then the FOFB system using 6 correctors and 6 BPMs was used to correct the orbit, maintaining the orbit bump that produce the required offset and slope of the source point at the ID. Vertical orbit bumps using the displacement of BPMs in the mid-straight of $3.5 \mu\text{m}$ were demonstrated, within the limit of correctors' strength, as shown in Figure 3.32. Similarly, Figure 3.33 shows the orbit generated by forcing the beam to $-3.5 \mu\text{m}$ and $3.5 \mu\text{m}$ at the BPMs to create the vertical orbit slope. After the correction, the orbit leakage elsewhere in the cell is below $20 \mu\text{m}$ in both cases.

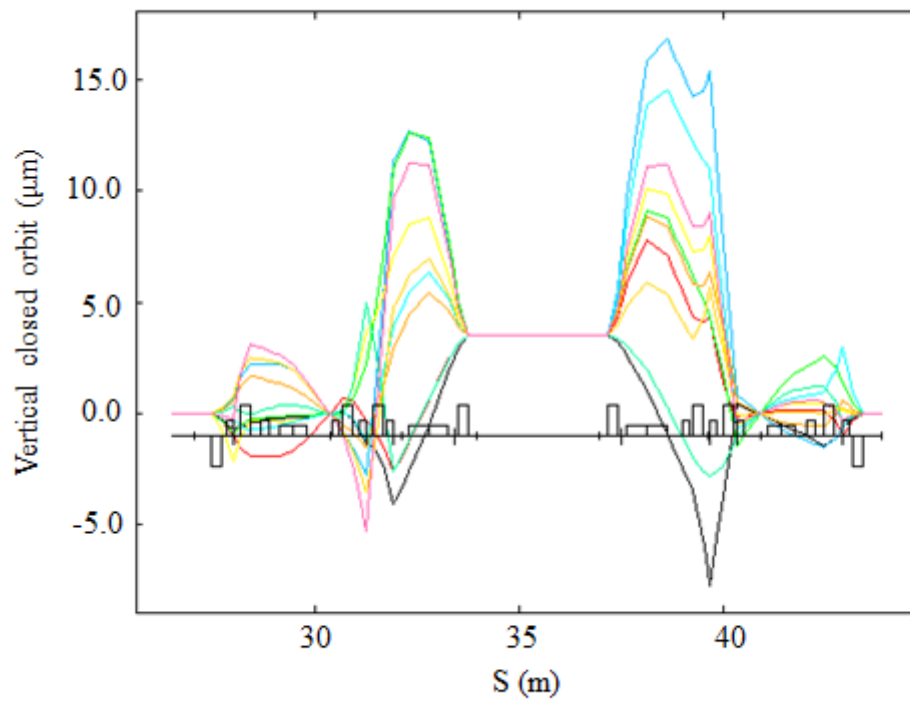


Figure 3.32: Vertical closed orbit bump in the DDBA mid-straight for 10 errors seeds.

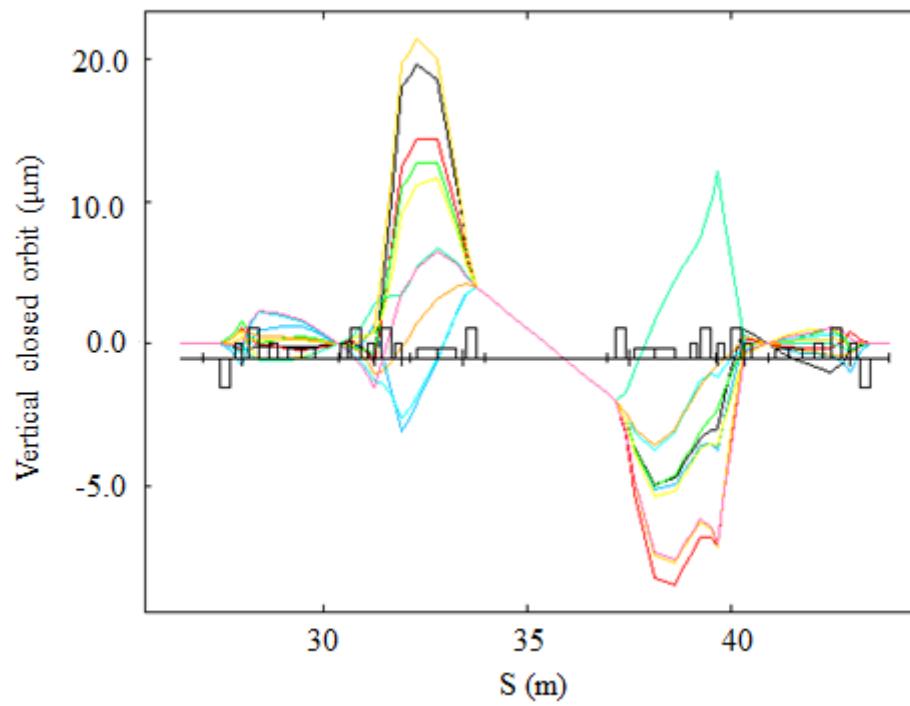


Figure 3.33: Vertical closed orbit slope in the DDBA mid-straight for 10 errors seeds.

3.2.6 Effect of magnets imperfections on the beam dynamic

Magnets imperfections due to the manufacturing errors and misalignments in Table 3.4 were introduced in the beam dynamics simulations to establish limits on the possible field errors and guide the design of the magnetic elements. Multipole components both from systematic and random errors were included. Systematic multipole errors can be extracted directly from the magnet model.

The Opera software [71] has been used for magnets design in the DDBA section by the magnet group at Diamond. Systematic harmonic errors in the magnetic elements were extracted from the model and tested in the accelerator model. The systematic multipole errors were given as input in Elegant. In the early stages of the magnet design, it was found that the systematic multipole errors in the quadrupoles and sextupoles of the new DDBA cell have small effect on the dynamic aperture. However the systematic errors in the dipoles shows a strong effect on dynamic aperture. Particularly the sextupole term in the dipoles deteriorates significantly the area of the dynamic aperture as shown in Figure 3.34

From the frequency map analysis in the Figure 3.34b, it is clear that the multipole errors in the DDBA dipoles drive particles with large amplitude towards a third order resonance line (orange) which cuts the area of the dynamic aperture compared to the ideal dipole case shown in Figure 3.34a. By analysing the different multipole terms in the dipoles, and comparing their effect on the dynamics aperture, it appears that the sextupole component is the main cause that drive tune points toward the resonance line. The dipole design was revisited to minimise the sextupole component. Figure 3.34c shows a better dynamic aperture in absence of any sextupole term in the dipole. Additionally, in Figure 3.34d shows that it is also possible to tolerate 50% stronger octupole term. This freedom was used to design the dipole to reduce the sextupole term at the expense of a slightly larger octupole term. The multipole components for the final DDBA magnet design are reported in Table 3.5 and 3.6. The systematic multipole errors in the main dipoles of the Diamond storage ring are shown in Table 3.7 and were obtained from existing measurements [72].

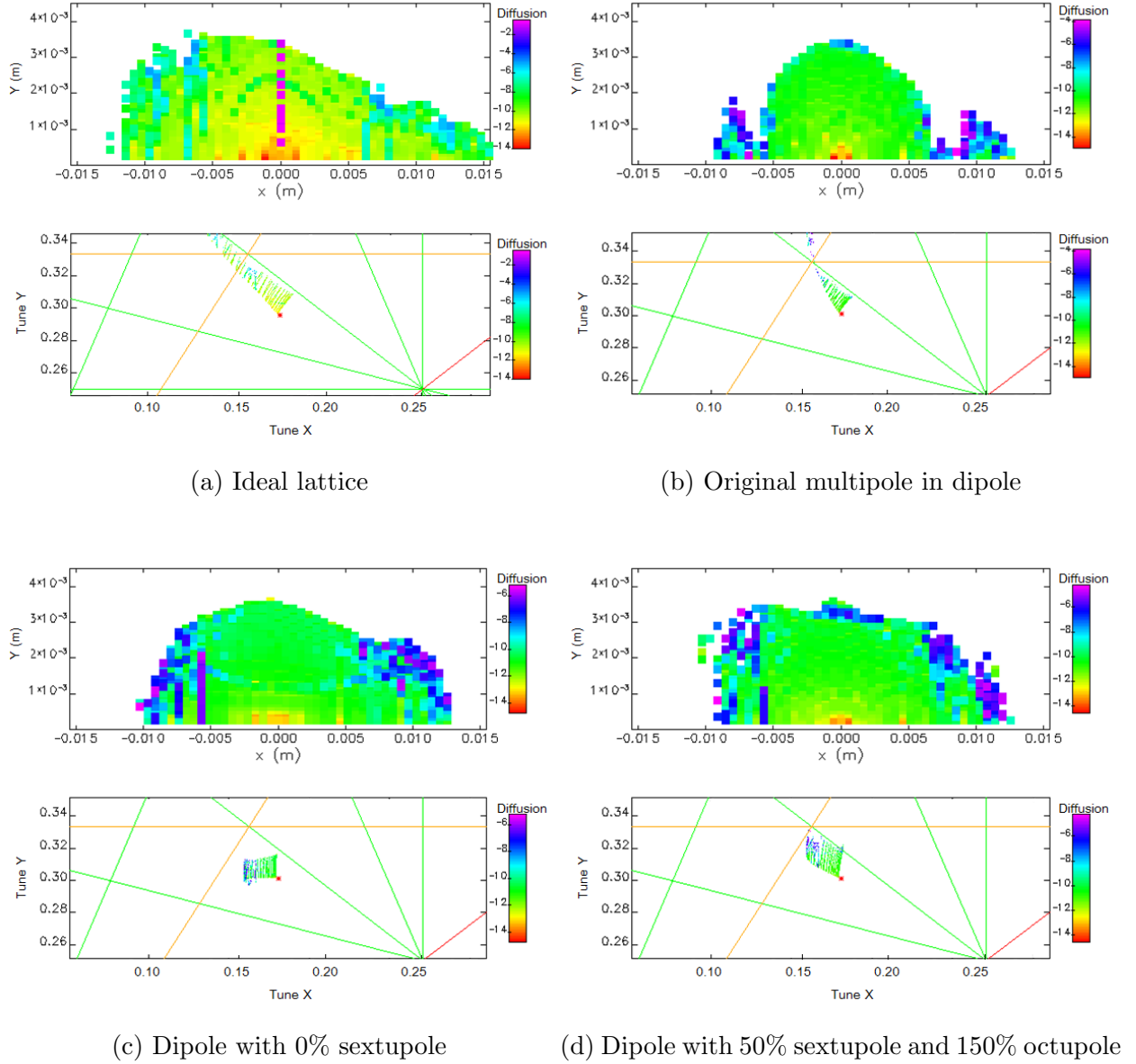


Figure 3.34: Frequency map analysis for the DDBA lattice with different multipole terms in the DDBA dipoles

The random multipole errors, associated with the manufacturing imperfection, were taken from the measured value of the magnets of the existing ring. For the new DDBA magnets, the random multipole components were scaled from the measured ones assuming as a worse-case scenario, a factor of two increase in errors since the new magnets are stronger than the existing ones. The rms values of the random multipole errors of the DDBA quadrupole and sextupole are reported in Table 3.8.

Misalignments and the final set of multipole errors in all magnets were extensively studied. Systematic multipole errors were taken from the magnets at maximum strength. To see how

Table 3.5: Multipole component of the DDBA dipole

n	Strength ($\text{m}^{-(n+1)}$)
1	-1.438
2	-7.940E-01
3	2.796E+01
4	-1.248E+05
5	-1.332E+07
6	7.776E+09
7	2.560E+12

Table 3.6: Systematic multipole component of the DDBA quadrupole and sextupole

n	Quadrupole strength ($\text{m}^{-(n+1)}$)		Sextupole strength ($\text{m}^{-(n+1)}$)	
	Skew (an)	Normal (bn)	Skew (an)	Normal (bn)
1	-5.601E-08	6.859	-1.617E-08	7.062E+00
2	-2.738E-08	7.170E-03	3.122E-13	3.082E-08
3	1.262E-03	-1.509E+02	3.485E-04	3.927E+01
4	-3.421E-03	-1.807E+03	8.763E-09	3.698E-03
5	6.667E+02	2.214E+06	1.404E+02	-1.061E+07
6	-3.657E+03	6.853E+07	2.388E-03	1.109E+03
7	7.655E+07	-9.973E+11	1.795E+07	-2.344E+12
8	-2.060E+09	5.508E+14	-2.109E+03	6.213E+08
9	9.116E+13	4.688E+18	3.044E+13	2.179E+18

the beam dynamics is affected by these imperfections and random errors, 50 seeds were generated, then the closed orbit, betatron tunes and chromaticities were corrected to the design values for each seed. The chromaticities are set to 2 in both horizontal and vertical plane as in the operating machine. The beam coupling was set to 0.3% (operating value) and for the dynamic aperture, injection efficiency and Touschek lifetime were calculated. The injection tracking was performed with beam of 1000 particles injected at -8.3 mm off-axis as done in the normal beam injection. The beam parameters at the end of the BTS were used and tracked for 1500 turns. The full detailed physical aperture was also included in all the tracking. The dynamic aperture for the 50 error seeds is about -10 mm as shown in Figure 3.35 and should provide sufficient room for the injected beam at -8.3 mm. However, the injection efficiency is only about $60.2 \pm 1.9\%$ and Touschek lifetime is 17.3 ± 0.9 hours compared to 21.1 hours in the original Diamond model (see Table 3.9). Such injection efficiency is too low and has to be optimised to be at least 90% in order for the machine to operate satisfactorily. The possibility of shifting the beam injection at -6.8 mm by moving

Table 3.7: Multipole component of Standard dipole

n	Strength ($\text{m}^{-(n+1)}$)
2	-0.6884
3	0.6930
4	-9859.2384

Table 3.8: Random multipole component (rms value) of the DDBA quadrupole and sextupole

n	Quadrupole strength ($\text{m}^{-(n+1)}$)		Sextupole strength ($\text{m}^{-(n+1)}$)	
	Skew (an)	Normal (bn)	Skew (an)	Normal (bn)
2	6.092E-2	-0.048	2.043	0.0
3	4.518	22.680	7.729E+2	7.448E+2
4	0.0	0.0	0.0	1.761E+8
5	0.0	3.301E+6	2.399E+7	6.622E+7
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	0.0	0.0	4.374E+15	6.939E+16
9	0.0	5.390E+17	0.0	0.0

the septum magnet closer to the stored beam was also considered but the injection efficiency improves only to $82.0 \pm 1.5\%$.

The deterioration of the dynamic aperture can be seen clearly in Figure 3.35 after introducing the systematic, random errors and misalignment compared to the ideal case optimised with MOGA in Figure 3.15. In order to investigate the causes of this reduction of the dynamic aperture, we have separated the imperfections to see the effect on the beam dynamic, and it was found that the systematic multipole terms in the main dipoles are responsible for the reduction of the dynamic aperture. A test including all the imperfections except the systematic errors in the main dipoles showed that the injection efficiency increase to above 90% from 82% with the injected beam at -6.8 mm and no significant change in Touschek lifetime can be observed. Since the design of the existing magnets (standard dipoles) can not be changed, we repeated the lattice optimisations including from the start all the systematic imperfections in all the other elements of the ring and the DDBA cell.

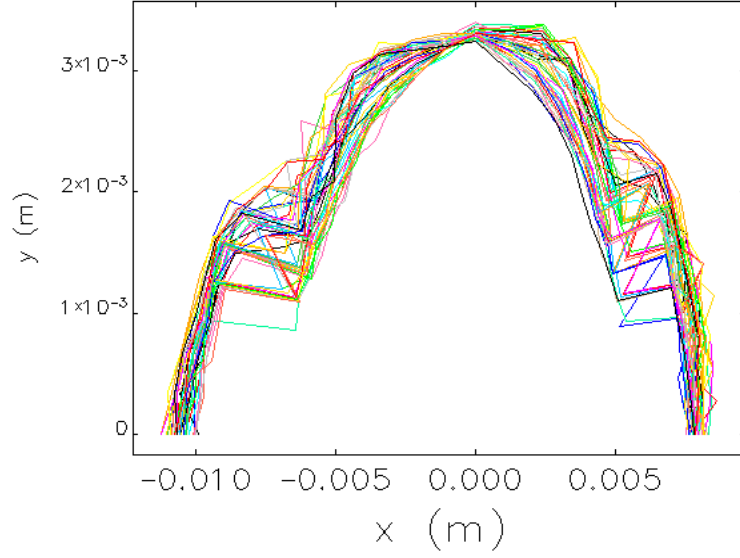


Figure 3.35: Dynamic aperture of the one DDBA lattice for different error seeds.

3.2.7 Lattice optimisation with imperfections

From the previous studies of lattice imperfections, we found that the Touschek lifetime and dynamic aperture (injection efficiency) were deteriorated compared to the ideal machine. Therefore, we considered the lattice optimisation including all systematic multipole errors in quadrupoles, sextupoles and dipoles. The difference from original MOGA optimisation is that the imperfections are introduced before objective functions calculation. In this optimisation, the injection efficiency was used directly rather than using the dynamic aperture as an objective. The beam was injected off-set at -8.3 mm then tracked through the ring. The injection efficiency was calculated from the number of particles that survive after 500 turns to speed up the optimisation. Tracking with more number of turns (1500) will be carried out as a final check only for the selected solution.

The optimal front for Touschek lifetime and injection efficiency is reported in Figure 3.36. The optimisation can successfully improve both the injection efficiency and Touschek lifetime. The initial starting point (red dot) with a low injection efficiency of about 70% can be increased to 90% while maintaining a Touschek lifetime.

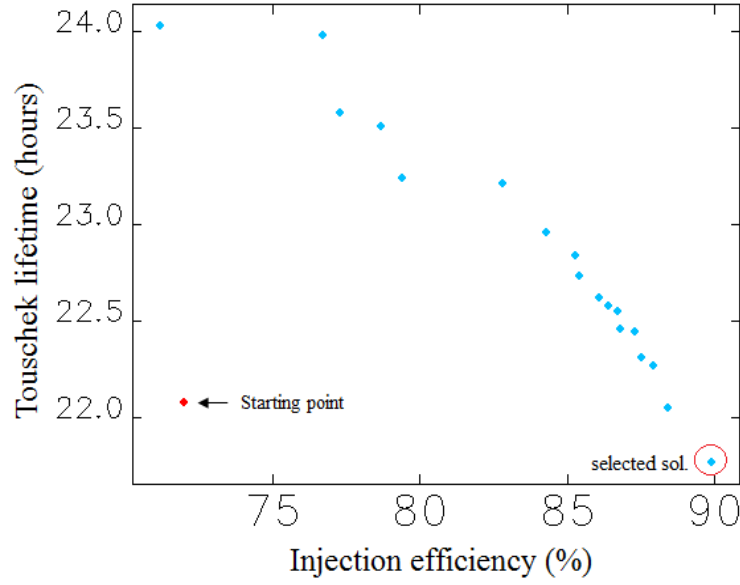


Figure 3.36: Optimal front of optimisation for Touschek lifetime and injection efficiency including multipole errors (solution in the red circle will be used for further optimisation).

We continued the optimisation further by including random multipole errors and misalignments in the optimisation. The optimal solution identified in Figure 3.36 was perturbed adding random multipolar errors and misalignments. Closed orbit correction, betatron tune correction and chromaticity correction were performed. The beam coupling is 0.3% and the injection efficiency remains about 90% while the lifetime drops from 22 hrs to 19 hrs) due to the random imperfections. MOGA was applied to increase the injection efficiency and Touschek lifetime of this lattice: the optimisation proceeded for about two weeks providing the solutions shown in Figure 3.37. The injection efficiency is more than 90% and the Touschek lifetime is restored to values close to the ideal case, containing only the systematic errors in the magnets (red circle in Figure 3.36).

The selected optimal solution had been checked again with 50 seeds of machine imperfections, including again misalignments and random multipole errors, to assure the robustness of the sextupole configuration devised by MOGA. The dynamic aperture of 50 errors seeds is shown in Figure 3.38 and indicates that the dynamic aperture of the Diamond lattice with one DDBA section in cell2, in presence of magnet imperfections, can be restored to the dynamic aperture of the ideal lattice, as shown in Figure 3.15. The new solutions after

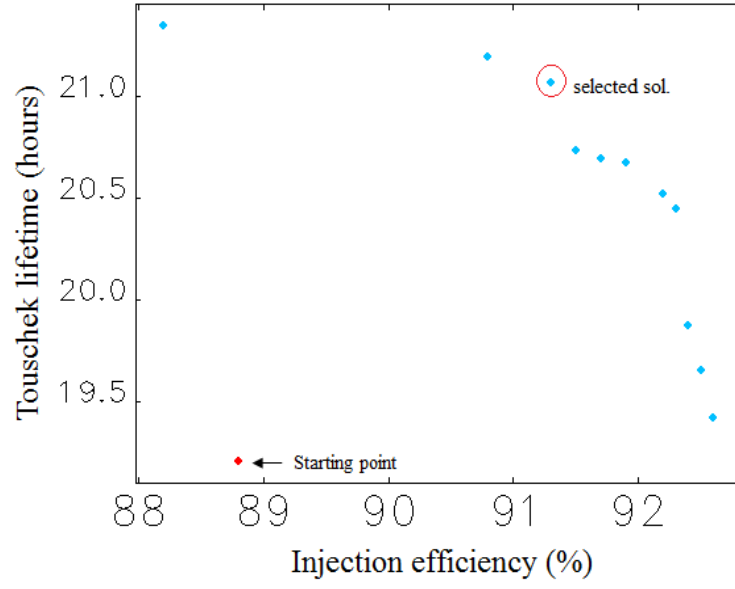


Figure 3.37: Optimal front for the optimisation of Touschek lifetime and injection efficiency including multipole errors and misalignments which generate 0.3% coupling.

introducing the imperfections in MOGA have wider Dynamic aperture at larger vertical positions and are not sensitive to the errors in the magnetic elements. The injection efficiency was improved from 60% to about 90% considering the beam injection at -8.3 mm and more

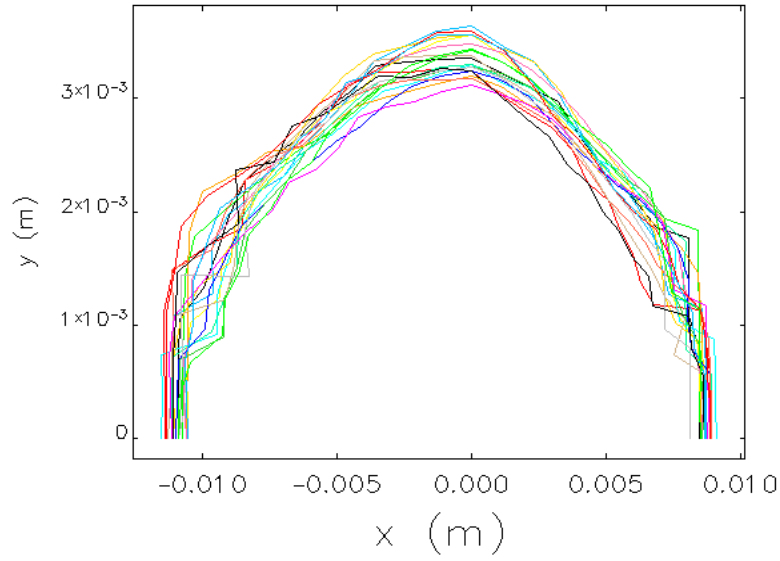


Figure 3.38: Dynamic aperture for 50 different error seeds (after including multipole error in the MOGA optimisation).

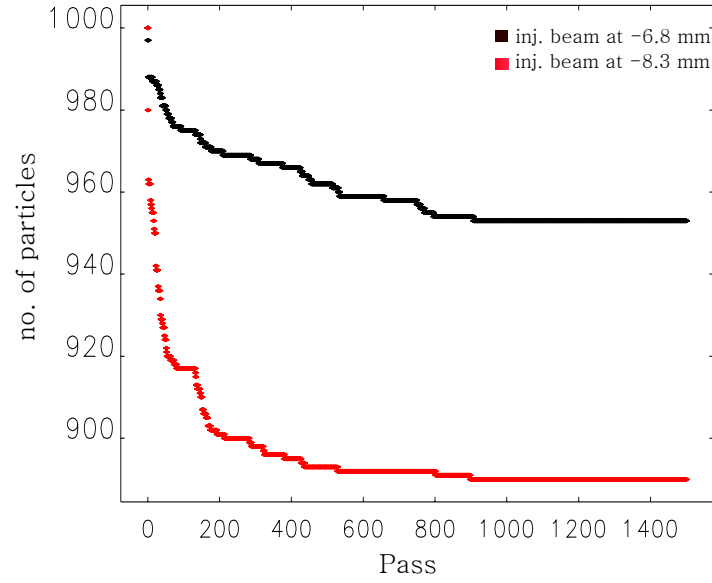


Figure 3.39: The number of surviving particles after injection tracking for 1500 turns for the injected beam at -6.8 mm (black) and -8.3 mm (red).

that 95% with the injected beam at -6.8 mm off-axis. Table 3.9 summarises the injection efficiency and Touschek lifetime for the different cases of the one DDBA lattice compared to the operating Diamond lattice with mini-beta sections (dlsi0913).

The particles surviving for one of the seeds during the injection tracking are shown in Figure 3.39 for the two cases when the beam is injected at -8.3 mm and -6.8 mm. The horizontal phase spaces of the injected particles after tracking the first 5 turns for off-axis injection at -8.3 mm and -6.8 mm are shown in Figures 3.40 and 3.41 respectively. It is clear

Table 3.9: Summary of injection efficient and Touschek lifetime

Lattice	inj. beam off-axis (mm)	injection efficiency (%)	Touschek lifetime (hrs)
dlsi0913	-8.3	99.4 ± 0.3	21.1 ± 1.9
	-6.8	99.8 ± 0.1	21.1 ± 1.9
oneDDBA	-8.3	60.2 ± 1.9	17.3 ± 0.9
	-6.8	82.0 ± 1.5	17.3 ± 0.9
oneDDBA (MOGA with multipole)	-8.3	87.2 ± 1.0	18.4 ± 1.7
	-6.8	94.7 ± 0.8	18.4 ± 1.7
oneDDBA (MOGA with multipole/misalignment)	-8.3	89.3 ± 4.0	20.2 ± 1.4
	-6.8	96.1 ± 0.9	20.2 ± 1.4

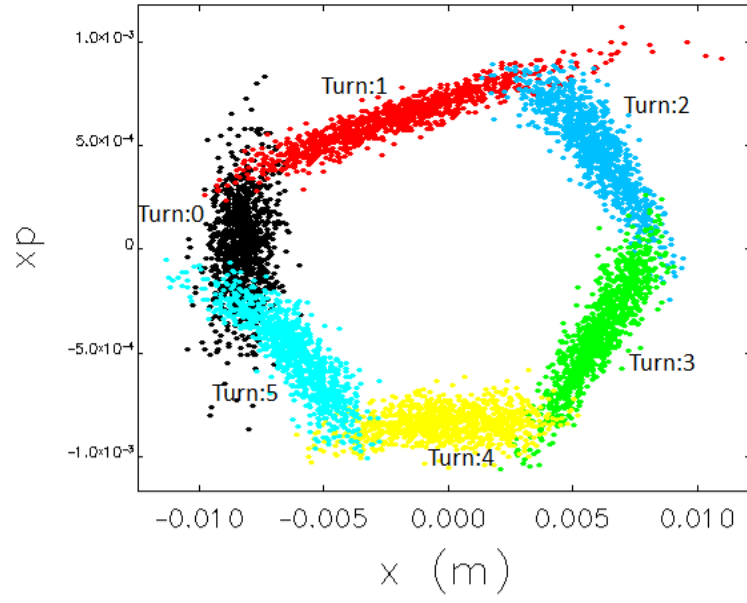


Figure 3.40: Horizontal phase space of the injected beam at -8.3 mm (black) after tracking the first 5 turns.

that the injection at smaller offset results in smaller beam oscillation closer to the center of the stored beam. The smaller amplitude injection is beneficial for injection efficiency since it keeps the beam away from small physical aperture and experience less nonlinear effects.

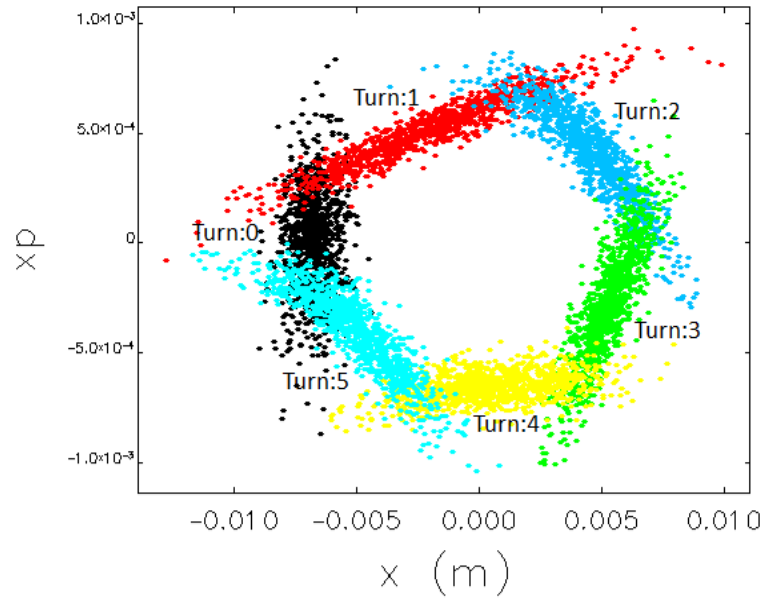


Figure 3.41: Horizontal phase space of the injected beam at -6.8 mm (black) after tracking the first 5 turns.

3.3 Two DDBA lattice for Diamond

In the previous studies, we proved that one DDBA cell can replace one of the existing DBA cell in Diamond providing an extra space for VMX beamline without significant deterioration of the machine performance. The possibility of replacing another DDBA cell in Diamond lattice cell 11 has been therefore considered. The extra 3.4 m long straight section is meant to provide space for a new beamline called Dual Imaging And Diffraction (DIAD) [73].

3.3.1 Linear lattice design

With the presence of the second DDBA cell in the Diamond storage ring, the horizontal betatron tune is increased by two integers compared to the original Diamond lattice (from 27 to 29). The vertical betatron tune was slightly changed and constrained to the same integer of 13. The same constraints to the optics functions, used in the addition of one DDBA cell, have to be applied to the second DDBA cell. The fractional betatron tunes have to be kept below half integer to avoid the resistive wall instability and small betatron functions have to be provided in the DDBA section to allow the use of small aperture magnets. For the second DDBA cell optics, we used the matched optics solution found for the single DDBA cell lattice as a starting point. All the existing special optics sections have to be kept unchanged in the Diamond lattice during the matching of the new DDBA cell in cell 11; these now include the double-mini beta sections in I09 and I13 and the recently modelled DDBA in cell 2. The comparison of the optics functions between the original lattice and the two DDBA lattice is plotted in Figure 3.42 and their parameters are given in Table 3.10.

Because of the lack of dispersion function in the DDBA cells, the chromaticity correction again will be performed by the remaining chromatic sextupoles families in the standard DBA cells. The over all chromatic sextupoles strengths are unavoidably stronger causing further difficulties in the non-linearities optimisation of the sextupoles.

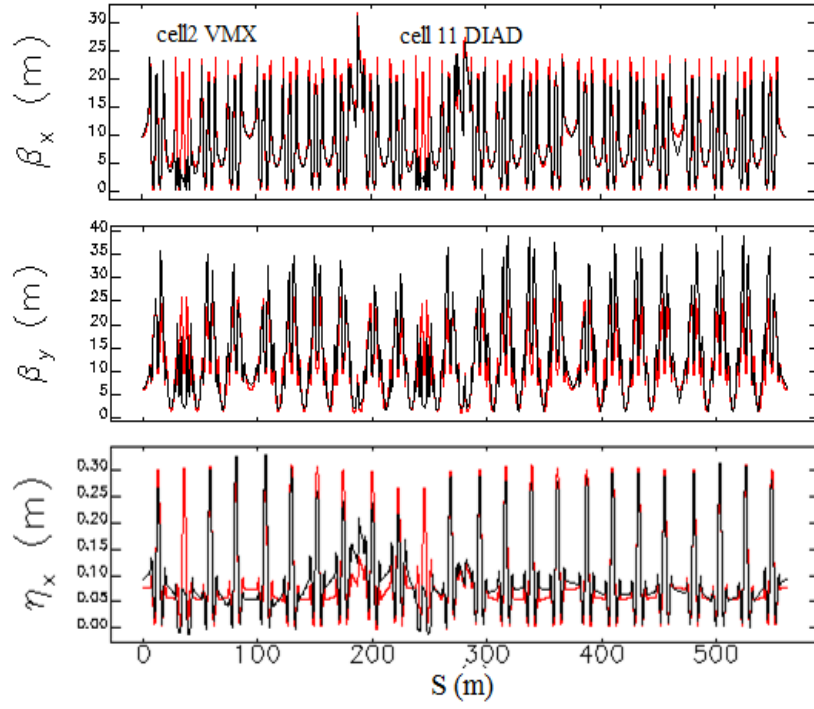


Figure 3.42: Twiss functions of Diamond with two DDBA cells (black) with respect to the existing Diamond DBA (red).

Table 3.10: Parameters for Diamond DBA and the two DDBA lattice

Parameters	Diamond DBA	Two DDBAs
Energy [GeV]	3.0	3.0
Circumference [m]	561.600	561.542
Emittance [nm·rad]	2.75	2.38
Betatron tunes Q_x/Q_y	27.20/13.37	29.2/13.37
Mom. compaction	1.6E-4	1.5E-4
Natural chromaticity ξ_{x0}/ξ_{y0}	-80.4/-35.6	-76.2/-45.0

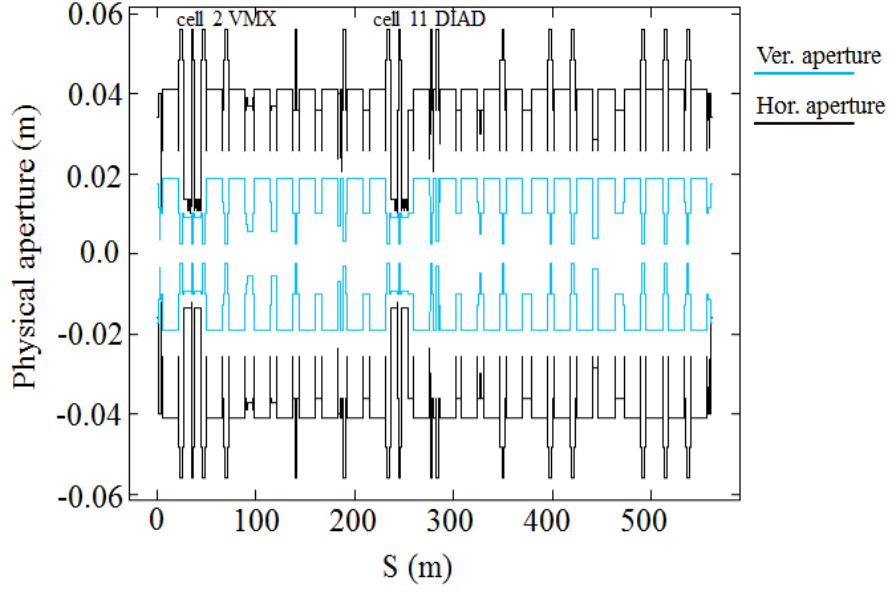


Figure 3.43: Physical aperture for two DDBA cell in Diamond.

3.3.2 MOGA optimisation for Two DDBA cell

MOGA was used for injection efficiency and Touschek lifetime optimisation for the two DDBA lattices. The particle tracking was carried out with the presence of the full detailed physical aperture as described in Figure 3.43. The physical aperture for the second DDBA cell in cell 11 is in all aspects similar to the physical aperture in cell 2 where the first DDBA cell was installed. Downstream the DIAD section, the vertical aperture in straight I12 is larger than the in-vacuum ID sections due to the presence of a superconducting wiggler. For the injection tracking in the two DDBA lattice, we used the same conditions as for the one DDBA lattice (injected beam at -8.3 mm off-axis).

As shown in Figure 3.44, the Touschek lifetime for the two DDBA lattice is not as good as the one DDBA lattice. The MOGA optimisation gave solutions with a Touschek lifetime of 12-13 hours and injection efficiency above 75%. Further linear optics optimisation found that a smaller horizontal beta function in the middle of the DDBA cell helps improving the Touschek lifetime. This effect was observed also during the optimisation of cell 2 but was not considered to be acceptable by the beamline. In fact, small horizontal beta function leads to large photon beam horizontal divergence which causes photon flux reduction at the first

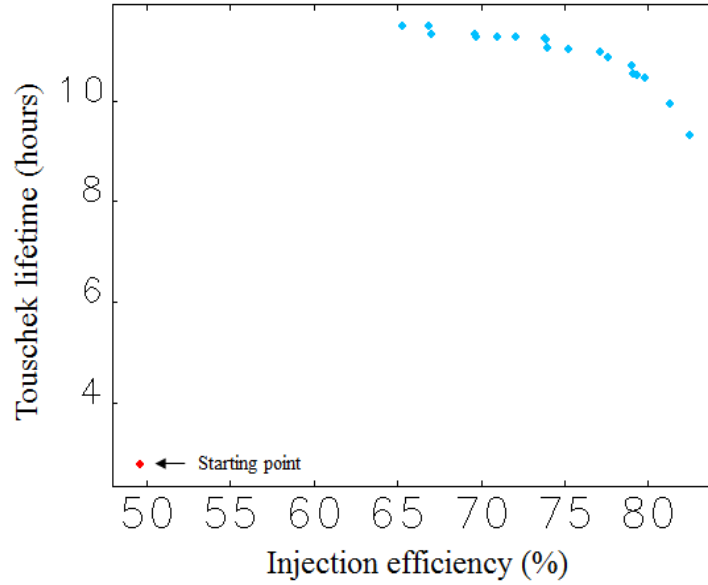


Figure 3.44: Optimal front for Touschek lifetime and injection efficiency for two DDBA lattice.

mirror in the beamline. Fortunately, in this case, the DIAD beamline can accept a larger horizontal divergence photon beam. The linear optics was therefore optimised allowing a reduced horizontal betatron function (from 2 m to 0.54 m) in the middle of the second DDBA cell while the optics elsewhere was kept as in the one DDBA lattice. The same procedure to optimise injection efficiency and Touschek lifetime was carried out. Figure 3.45 shows the Touschek lifetime and injection efficiency after MOGA optimisation of the two DDBA cells lattice. The Touschek lifetime and injection efficiency were about 16 hours and 90 % respectively showing clearly the benefit of smaller horizontal beta function. However, the lifetime is still low compared to the operating machine (29 hrs for ideal case). Further optimisation to improve the Touschek lifetime is required.

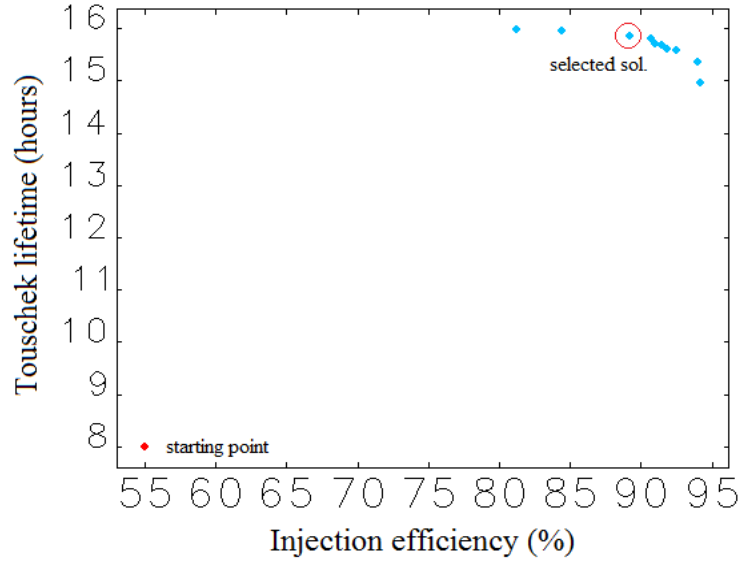


Figure 3.45: Optimal front for Touschek lifetime and injection efficiency for two DDBA lattice after improving optics matching.

3.4 Three DDBA lattice for Diamond

The possibility of installing a third DDBA cell to allow the insertion of a harmonic cavity for bunch lengthening has also been briefly considered. Such harmonic cavity is meant to improve the Touschek lifetime and allow an operation with higher bunch current. The possible place available for replacing the existing DBA with the third DDBA cell is in cell 20. The third DDBA cell provides again an extra 3.4 m in the middle straight which is sufficient for the high harmonic cavity.

The third DDBA cell matching is slightly different from the previous two DDBA cells because cell 20 connects two different straight sections: short straight I20 and long straight I21. The optics functions for the long straight section will be larger than in short straight to link with the rest of the ring. The linear optics was matched as show in Figure 3.46. For the three DDBA lattice, nonlinear beam dynamic will be investigated and optimised in the future. However, because the three missing dispersion bumps for chromaticities correction lead to even stronger chromatic sextupole in the rest of the ring and it is foreseeable that the optimisation of the nonlinear beam dynamics will be even more difficult.

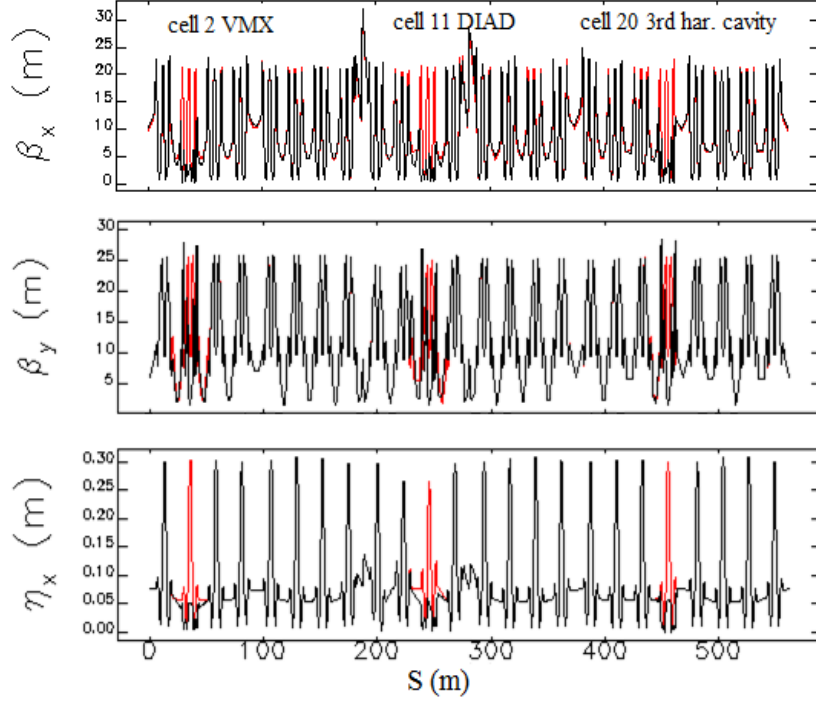


Figure 3.46: Twiss functions of Diamond with three DDBA cells (black) with respect to the existing Diamond DBA (red).

3.5 Summary

The possibility of reducing the emittance for the existing Diamond storage ring was investigated using MOGA. The optimised solution provides more than 20% reduction of the natural emittance. The implementation of this new lattice in the machine confirmed the reduction in the emittance. However, the emittance reduction is obtained by allowing a larger dispersion function in the straight section. This solution is perturbed by the effects of the wigglers and the corresponding beam emittance growth was measured and verified with the calculation with very good accuracy. Subsequent optimisation taking into account the effect of wigglers found that strong field wigglers are incompatible with larger dispersion leak in the straight section. These studies indicated that a possible route to achieve smaller emittance required reducing the dispersion to zero locally at the positions of strong IDs. This solution will be the object of forthcoming investigations.

The modifications to the Diamond lattice to increase the space available for IDs have also been discussed. A double-double bend achromat (DDBA) cell will be used to replace

the existing DBA cell at cell 2 in Diamond. Beam dynamics studies provide very promising results. For the single DDBA lattice, injection efficiency and Touschek lifetime are comparable to the one of the existing lattice. The cell commissioning is planned to start in autumn 2016. Additionally, the possibilities of the second and third DDBA cells at the locations of cell 11 and cell 20 respectively have been investigated as well. Poorer injection efficiency and Touschek lifetime compared to the one DDBA cell lattice was expected due to stronger chromatic sextupoles. For the two DDBA lattice, Touschek lifetime is reduced to about 50% of the operating value. Further optics matching and optimisation for the three DDBA lattice are also required.

With a few DDBA cells in Diamond, no significant reduction of the beam emittance is expected. In order to decrease significantly the emittance, the DDBA concept has to be extended to the full ring. This will be the object of the next Chapter.

Chapter 4

Diamond upgrade (Diamond-II)

In Chapter 3, we have discussed the modification to the existing Diamond lattice made in order to improve the emittance and to create an additional straight section for an insertion device beamline. In this chapter, we will present a full machine upgrade introducing major changes to the machine.

The Diamond light source has been operating since 2007 with many parameters that were world-leading at that time, e.g. the smallest emittance for medium energy machines.. In the last decade, the global attempts to improve the quality of synchrotron light sources by reducing the beam emittance drove Diamond to investigate upgrade options to lower the beam emittance.

New ring projects to be built on a greenfield can be designed with full freedom in term of space availability while the upgrade of existing machines usually tend to reuse the existing building and beamlines, so that the modification and the available space are strictly limited. In this chapter we present the design and optimization studies for the upgrade of the Diamond light source, where three different designs have been investigated based on MBA lattice with $M = 4, 5$ and 7 . The minimal straight section length of 5 m has to be preserved for the upgrade lattices to accommodate insertion devices and other instrumentation thus a compact cells design is required. In addition, magnets stronger than those used in the existing machines are required to focus the optics functions in more compact cells. In

a medium size storage ring (about 500 m ring circumference) like Diamond, it is more challenging to achieve very small beam emittance than in larger machines like PEP-X [74], ESRF-II [75], and Spring 8-II [23] because of the limited space that forces a larger bending angle per cell. In the design, linear optics functions have to be carefully tuned taking into account the required reduction in beam emittance and suitable space for IDs in the straight sections. As a result of these studies, a 4BA cell was modified specifically to achieve an extra straight section in the middle of the arc. This new design is called modified-4BA or Double-Double Bend Achromat (DDBA). Previously, in Chapter 3, one and two of the new designed DDBA cell have been modelled in the existing Diamond ring. The usefulness of the additional straight sections is mainly in the possibility of upgrading standard bending beamlines to ID beamlines. In this chapter, we prove that a full ring upgrade using the DDBA cells can also be optimized to reduce the beam emittance.

We present the results of the lattice optimisations for Diamond storage ring upgrade. Multi-objective genetic algorithm (MOGA) has again played important role in the nonlinear beam dynamic optimisations. The effects of strong field wigglers and intrabeam scattering will also be discussed.

4.1 Low emittance lattice options

For the machine upgrade, a multi-bend achromat (MBA) lattice has been used to achieve smaller beam emittance. As mentioned in Chapter 1, the relation between the natural beam emittance and the number of bending magnets in isomagnetic ring (constant bending radius) is $\varepsilon_0 \propto \frac{E^2}{N_{bend}^3}$, the natural emittance can be reduced by increasing the number of bending magnets. This requires the design of strong and compact magnets to be able to control the optics functions in the cell.

The Diamond storage ring, described in Chapter 1, is composed of six super periods. Each super period has four DBA cells which have mirror symmetry in the super period (cell 1, cell 2, -cell 2, -cell 1). Cell 1 separates a long straight and a standard straight while cell

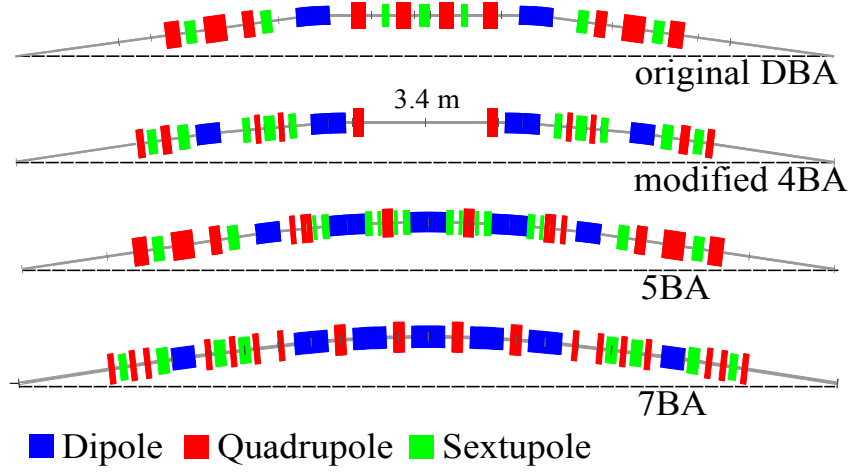


Figure 4.1: Comparison of standard cell layouts for the original DBA, modified-4BA, 5BA and 7BA.

2 symmetrically links two standard straights on both sides. For the lattice modification, we first optimise cell 2 to reduce the emittance and then the design was copied to cell 1 with the additional constraint of connecting the middle cells to the long straight sections at both end of the super period. In the MBA design, as discussed in Chapter 2, the length of the edge and middle dipoles were adjusted to minimize the beam emittance [34].

The Diamond upgrade should provide at least ten times smaller beam emittance than that of the operating lattice ($\varepsilon_x < 280 \text{ pm}\cdot\text{rad}$). The designs have to consider magnets strength constraints within the available technology. The layout of standard MBA cells proposed for the upgrade options for Diamond storage ring are shown in Figure 4.1. The main machine parameters for the MBA lattices design for the upgrade are described in Table 4.1 and compared to the operating Diamond DBA lattice. The lattices optimisations will be discussed in the following sections.

Table 4.1: Parameters for the Diamond DBA and MBA lattices

Parameters	Diamond DBA	Modified-4BA	5BA	7BA
Energy [GeV]	3.0	3.0	3.0	3.0
Circumference [m]	561.60	561.0	561.60	561.60
Emittance ε_{x0} [pm·rad]	2590	272	150	50
Betatron tunes Q_x/Q_y	27.20/13.37	51.21/17.31	53.66/28.87	75.42/52.17
Mom. compaction	1.6E-4	9.9E-5	1.30E-4	7.98E-5
Energy spread	9.6E-4	7.9E-4	7.4E-4	7.2E-4
Energy loss per turn [MeV]	1.0053	0.5728	0.4296	0.3222
Natural chromaticity ξ_{x0}/ξ_{y0}	-80.4/-35.6	-129/-93	-130/-50	-348/-119
Straight sections	$6 \times 11.3\text{m}/18 \times 8.3\text{m}$	$6 \times 9.1\text{m}/18 \times 6.7\text{m}/24 \times 3.4\text{m}$	$6 \times 9.5\text{m}/18 \times 6.5\text{m}$	$6 \times 8.0\text{m}/18 \times 5.0\text{m}$
Number of magnets per cell				
(Dipole/Quadrupole/Sextupole)	2/10/7	4/10/10	5/14/14	7/18/8
Bending field [T]	1.4	0.8	0.6	0.45
Max. quadrupole strength [m^{-2}]	1.9	5.3	6.8	10.0
Max. sextupole strength [m^{-2}]	45	400	800	1200
Dynamic aperture [mm]	-15, +15	-4, +7	-4, +4	-0.4, +0.5
Emittance ε_x with IBS* [pm·rad]	2630	290	200	118
Emittance ε_y with IBS* [pm·rad]	263	29.0	20.0	11.8
Beamline realignment distance [mm]	-	15	25	65

*IBS effect was calculated using 500 mA current, 900 bunches and 10% coupling.

For the lattice upgrade, it is necessary to compare the position of the straight sections of the newly designed machine with that of the original machine. Ideally, for the operating beamline, we want to minimise the displacement of the new straights with respect to the existing straights. This allows us to maintain the same positions of the exiting beamlines in the upgraded machine.

Generally, because of the larger number of dipoles, MBA lattices require weaker dipole field (larger bending radius, ρ_0) than the one in the existing ring. As a consequence the transverse position of the straight section is shifted with respect to the existing straights toward the outside of the ring. The impact of this shift on the ring circumference has to be analysed.

4.1.1 Seven Bend achromat lattice (7BA)

Considering the space limit in the Diamond ring, the MBA lattice option with the largest number of dipoles that could be reasonably fit in the existing arc dimension is $M = 7$. Compared to the MAX IV design based on a 7BA lattice, the Diamond ring circumference of 561.6 m is similar (MAX-IV has 528 m) [18]. However, the number of cells is larger, 24 against 20, and correspondingly the design is more challenging. Instead of distributing the dispersion function into the cell to provide suitable place for chromaticity correction between the dipoles, two dispersion bumps were generated just after the edge dipoles as shown in Figure 4.2, and these will be used for chromaticity correction. This technique has also been used in the ESRF-II design [75]. This solution can reduce the required strength of sextupoles for chromaticity correction by about 50%. The chromaticities correction is performed only in the dispersion bumps thus the requirement on the optics functions elsewhere in the cell can be simplified. Such design reduces the beam emittance to 50 pm·rad which is fifty times smaller than that of the existing machine. The dispersion function in the straight section is matched to zero and the beta function in the vertical plane is constrained to below 2 m in order to provide an ideal place for the narrow gap (5 mm) IDs. The minimum distance between magnets in the cell is forced to be larger than 10 cm.

To ease the nonlinear effect optimisation, linear optics with appropriate phase advance was taken into account in order to cancel the resonance driving terms (RDTs). As discussed in Chapter 2, RDTs describes the particle dynamics in the resonance basis. They are responsible for the excitation of resonances which can be minimised by suppressing the terms using suitable phase advances between the sextupoles. The phase advance matching achieved compensation of the RDTs over 4 cells (or one super period) with the horizontal and vertical tune of $\nu_x = 6.5\pi$ and $\nu_y = 4.5\pi$ per cell respectively. Thus the phase advance for one super period is 26π and 18π in the horizontal and vertical plane respectively which are the multiple of 2π . The RDTs calculated as functions of position is shown in Figure 4.3. Most of the RDTs were cancelled within one super period (4 cells) as shown in Figure 4.3a and 4.3b. However for higher order terms, shown in Figure 4.3c, there are some terms (h11110, h22000 and h00220) that cannot be cancelled as well as the detuning with amplitude in Figure 4.3d. Further optimisation is required using harmonic sextupoles which are placed in small dispersion function regions close to the straight sections. MOGA was used to optimise dynamic aperture using the harmonic sextupoles as parameters targeting the total of diffusion ($d = \log(\Delta\nu_x^2 + \Delta\nu_y^2)$) given by the frequency map analysis similar to the

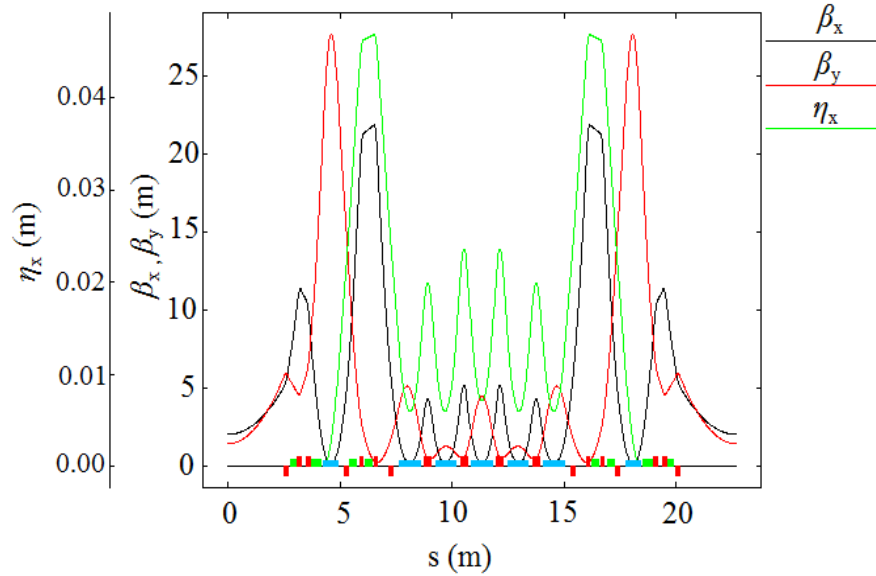


Figure 4.2: Optics functions in the 7BA cell. The dispersion bumps are generated at about 6 and 16 m.

optimisation performed previously in Chapter 3. Only four families of harmonic sextupoles were used as variables in the optimisation while the linear optics was fixed.

Although the 7BA lattice gives a very small emittance, the large natural chromaticities (-348 and -119 in horizontal and vertical plane respectively) require strong chromatic sextupoles which then leads to strong nonlinearities. As a consequence the 7BA lattice with the large detuning with amplitude has small dynamic aperture. The frequency map, reported in Figure 4.4, shows that particles move across several resonance line and less than 1 mm dynamic aperture can be achieved. Beam injection in these conditions will be very challenging and will require a new injection concept. However the solution shows the possibility

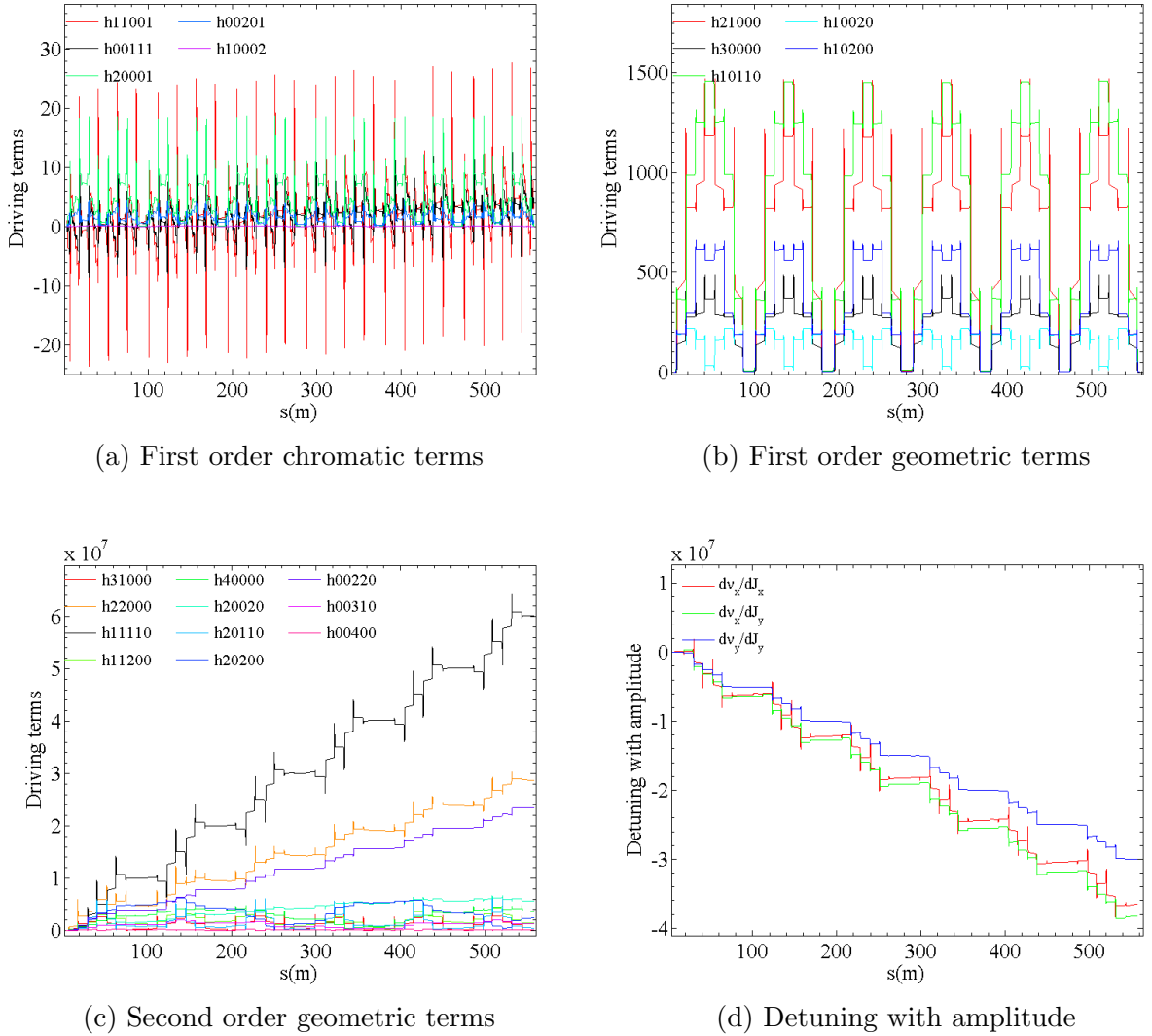


Figure 4.3: Resonance driving terms and detuning with amplitude as functions of position for the 7BA lattice after phase advance matching.

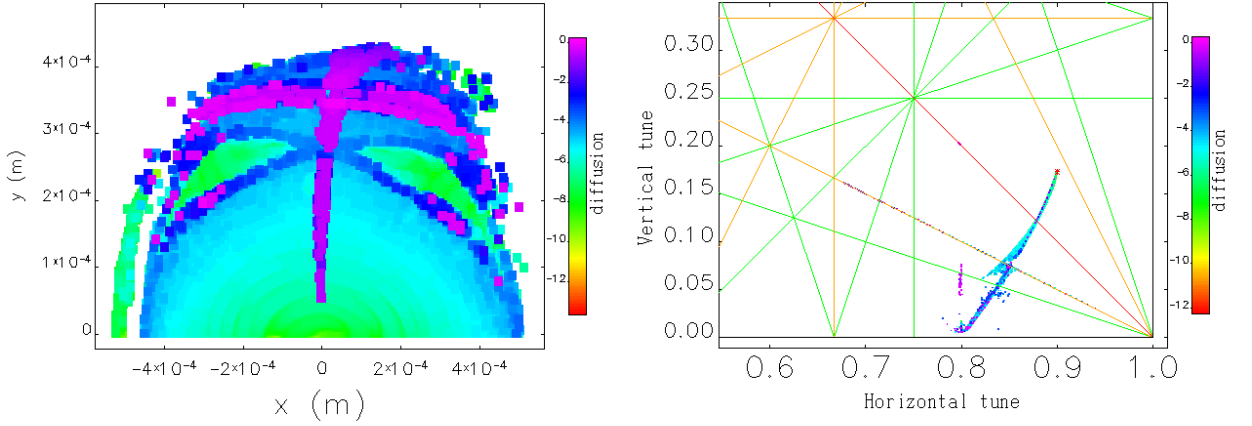


Figure 4.4: Frequency map for the 7BA lattice. Left figure shows the tracked particles in transverse amplitudes space (x, y). Right figure shows the tracked particles in tune space.

of achieving the minimal beam emittance of 50 pm·rad in the Diamond tunnel. Further optimisation is required if we want to pursue this option. Additionally, the magnet design is very challenging because of the required strong magnets as shown in Table 4.1.

4.1.2 Five Bend achromat lattice (5BA)

Though the 7BA lattice solution for Diamond can attain small beam emittance of 50 pm·rad, the requirement of strong magnets, and the difficulties in the optimisation of the dynamic aperture led to the investigation of a more relaxed solution based on a 5BA lattice. Unlike the 7BA solution, the 5BA lattice was designed based on the unit cell structure similar to that of MAX-IV. Chromatic sextupoles were distributed in the cell at positions of large dispersion function between dipoles. At the position of the chromatic sextupoles, the horizontal and vertical betatron function are well separated as shown in Figure 4.5 which provides an independent chromaticity correction in each plane. Zero dispersion and small vertical betatron function in the straight sections can be provided. The beam emittance for such design is about 150 pm·rad. The design is very compact and may need an advanced magnet design like in MAX-IV [18] where quadrupole, sextupole and dipole magnets are grouped into a unit and built on a single iron block.

The phase advance for the 5BA cell was matched in order to complete $2n\pi$ within four

cells where n is an integer. The phase advance per cell were $\psi_x = 4.5\pi$ and $\psi_y = 2.5\pi$ for the horizontal and vertical planes respectively. Hence, in one super period (4 cells), the total phase advances were $\psi_x = 18\pi$ and $\psi_y = 10\pi$. The betatron tunes were then adjusted using the quadrupoles triplet in the long straight. The RDTs for the 5BA after the phase advance matching are shown in Figure 4.6.

The harmonic sextupoles were adjusted by MOGA to optimise the dynamic aperture. The frequency map of the optimised solution is shown in Figure 4.7. With phase advance matching the DA can be improved from 2 mm to 4 mm. Figure 4.6 shows most of the RDTs cancellation within four cells. However, the higher order terms, h22000 and h11110 are still large. The detuning with amplitude, especially $d\nu_x/dJ_x$ increases along the ring as shown in Figure 4.6d and many resonances are crossed by the beam as shown in Figure 4.7 generating detrimental effect on the beam dynamics.

For the 5BA lattice, the transverse offset between the new straights with respect to the original straight section was calculated to be about 44 mm using the same ring circumference of 561.6 m. The distance can be reduced to 25 mm when the circumference of the 5BA lattice is reduced to 561.0 m by small reduction of all the straight sections. This however,

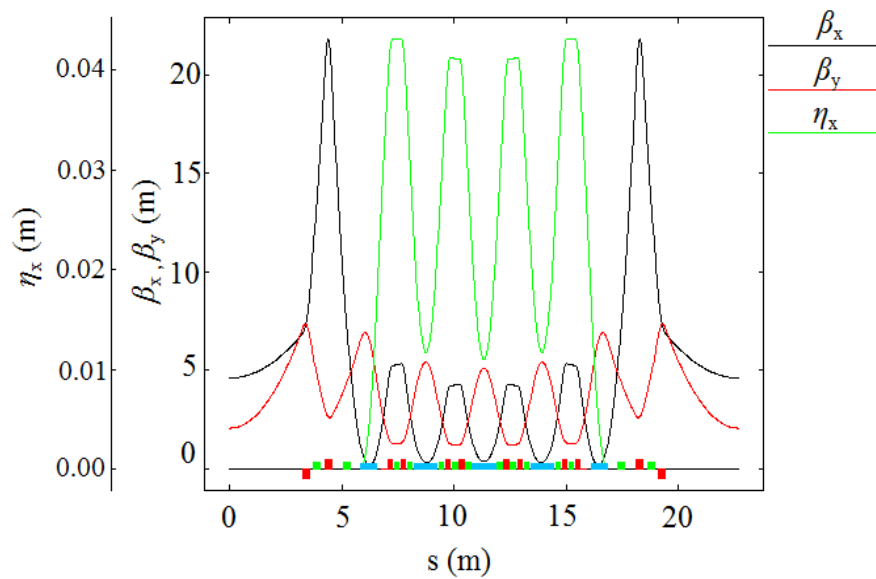
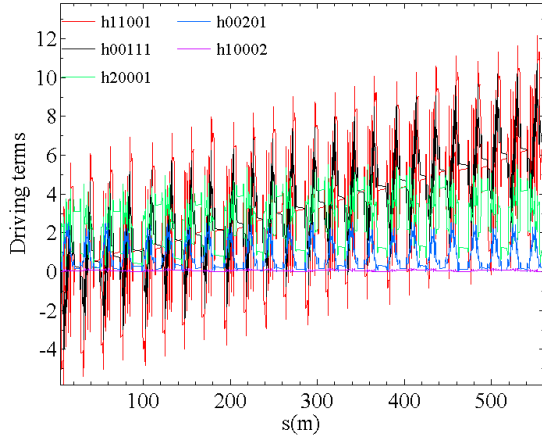
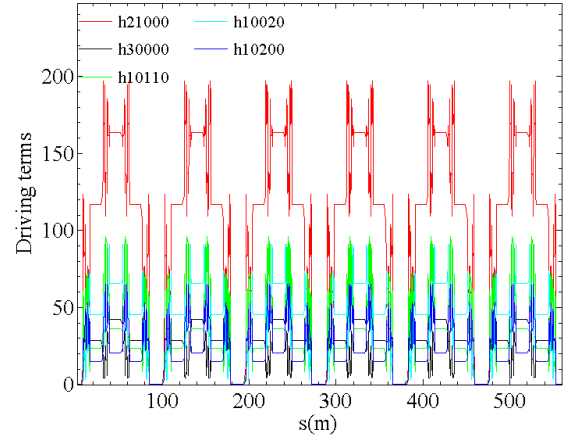


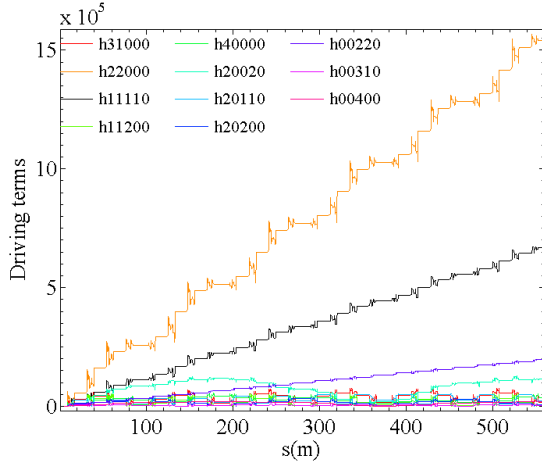
Figure 4.5: optics functions in the 5BA cell.



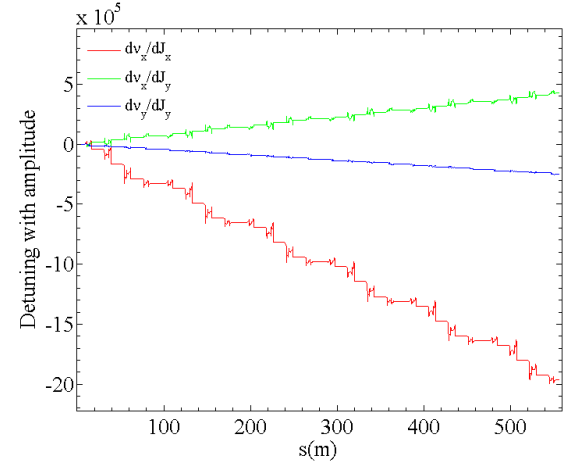
(a) First order chromatic terms



(b) First order geometric terms



(c) Second order geometric terms



(d) Detuning with amplitude

Figure 4.6: Resonance driving terms and detuning with amplitude as functions of position for the 5BA lattice after phase advance matching.

still exceeds the realignment limit of the existing beamline (15 mm) and requires them to realign their optics systems. The 5BA solution shows slightly better performance than the 7BA lattice with a more relaxed beam emittance. Further optimisation is still required to achieve a solution working with the off-axis injection.

4.1.3 Modified four Bend achromat lattice (DDBA)

In view of relaxing further the constraints on the compactness of the cell design, a 4BA cell based on the theoretical minimum emittance (TME) cell similar to the 5BA and 7BA designs was also considered for the Diamond upgrade. Subsequent modifications on the 4BA cell (so

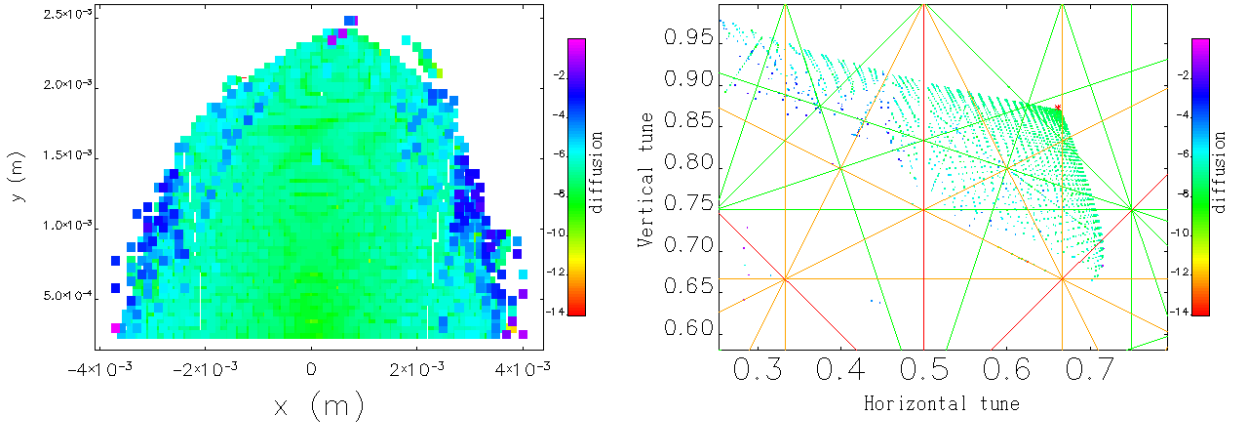


Figure 4.7: Frequency map for the 5BA lattice. Left figure shows the tracked particles in transverse amplitudes space (x, y). Right figure shows the tracked particles in tune space.

called modified-4BA) were carried out to allow an additional straight section in the middle of the cell as shown in Figure 4.1. The additional straight section per cell doubles the capacity of the ring to accommodate insertion devices. Compared to the Diamond original DBA cell, the modified-4BA is more compact because there are two more dipoles. The challenge of this design lies in the control the optics functions while conserving sufficient space for an insertion device in the middle. The length of the straight section should be as large as 3.4 m to accommodate the existing arrangement of the vacuum tube for the standard in-vacuum undulator at Diamond.

The compact design of the modified-4BA lattice was achieved using combined function dipoles which include a quadrupole gradient. There are two different dipole lengths: short (0.670 m) and long dipole (0.967 m). Although for the modified-4BA lattice design, the existing straight is slightly shortened, the total available straight sections' length with respect to the total ring circumference is 45.8% while only about 30% is available in the existing Diamond storage ring.

The positions of the straight sections of the new modified-4BA lattice were compared with those of the existing ring and large offsets (110 mm) were found when both rings have the same original circumference of 561.6 m. Thus the circumference of the modified-4BA ring was reduced to 561.0 m by decreasing all straight sections by 25 mm. Figure 4.8, shows

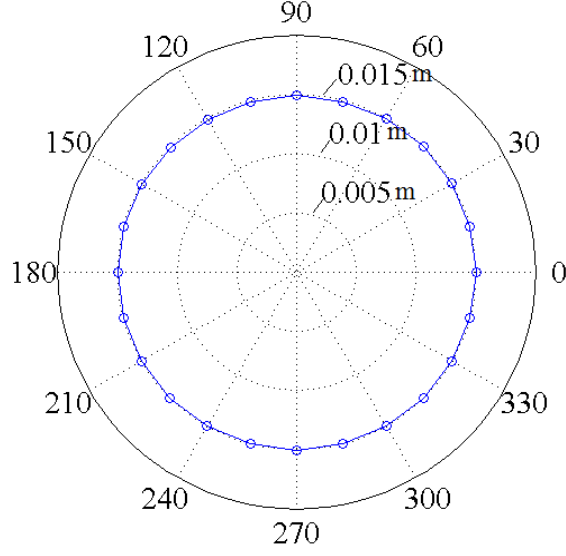


Figure 4.8: Distance between the modified-4BA straights and the existing Diamond straights at every 15° around the ring (total 24 straights). The distance is about 15 mm for all 24 straights.

the distance between the existing Diamond and the new upgraded ring straight after the circumference adjustment which is now only about 15 mm along the ring. The first straight beginning at zero degree, and it is seen that only minor adjustments of the existing beamlines are required.

The optics functions in the modified-4BA cell are described in Figure 4.9. The beta functions in the straight are constrained at 1.6 m in the vertical and between 2 and 5 m in the horizontal plane required by the beamlines. The dispersion function in the existing straight sections is kept to zero. In this condition, the IDs can reduce the beam emittance further. On the other hand, the dispersion function in the new middle straight cannot be zero if we want to maintain the desired beam emittance below 280 pm-rad. Non-zero dispersion in the middle straight of the modified-4BA cell allows minimal condition at the positions of the long dipole to keep the dispersive invariant ($\mathcal{H} = \gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2$) small. A minimum dispersion function of 2 cm compared to about 10 cm in the existing machine was achieved while the beam emittance is below the desired value (280 pm-rad).

In the existing Diamond lattice, there is one long straight for each super period (six

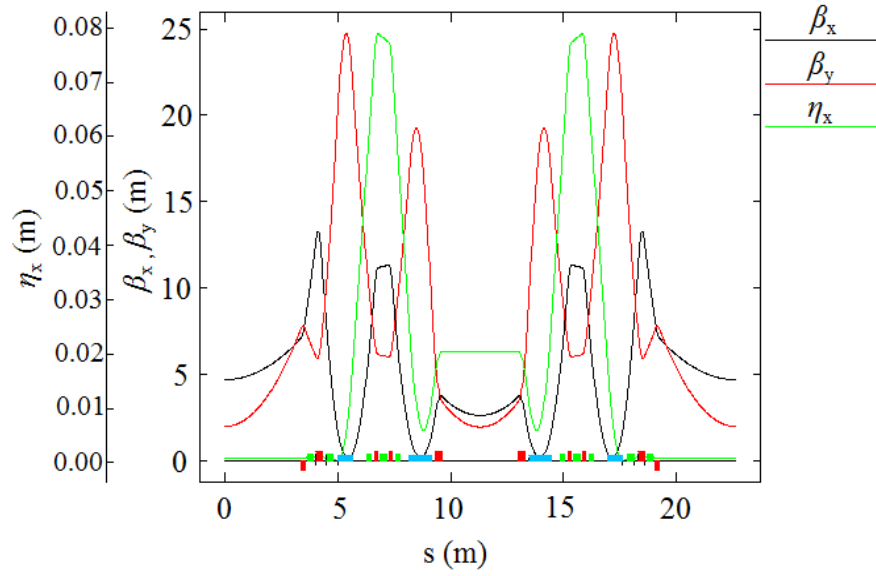


Figure 4.9: Optics functions in the modified 4BA cell (cell 2). The cell has a middle straight and a standard straight (two straights per cell).

super periods for the whole ring), and the requirements of the optics functions in the long straight are different from the standard straight. The horizontal betatron function in the long straight (also used for injection in the straight 1) is larger than that of the standard

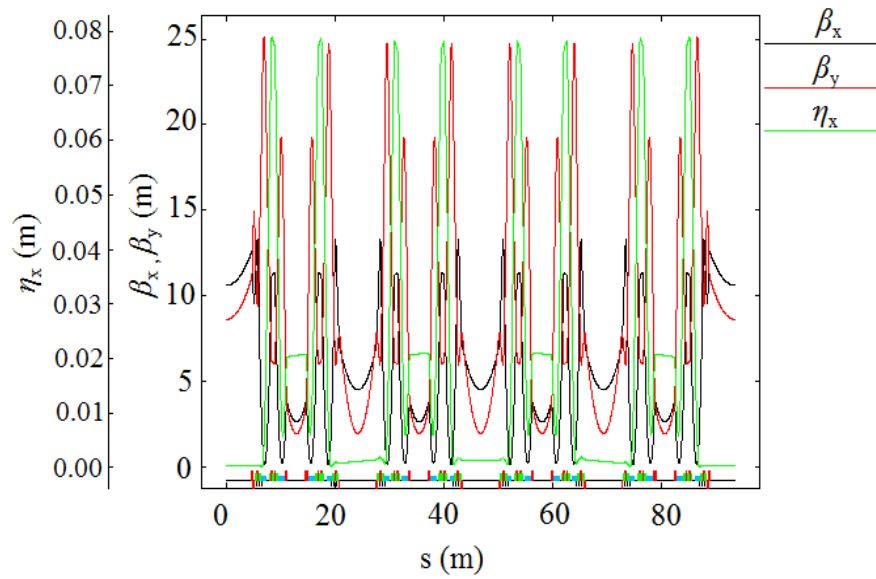


Figure 4.10: Optics functions in the modified 4BA super period composed of 4 cell. In the super period there are 4 middle straights, 3 standard straights and 1 long straight.

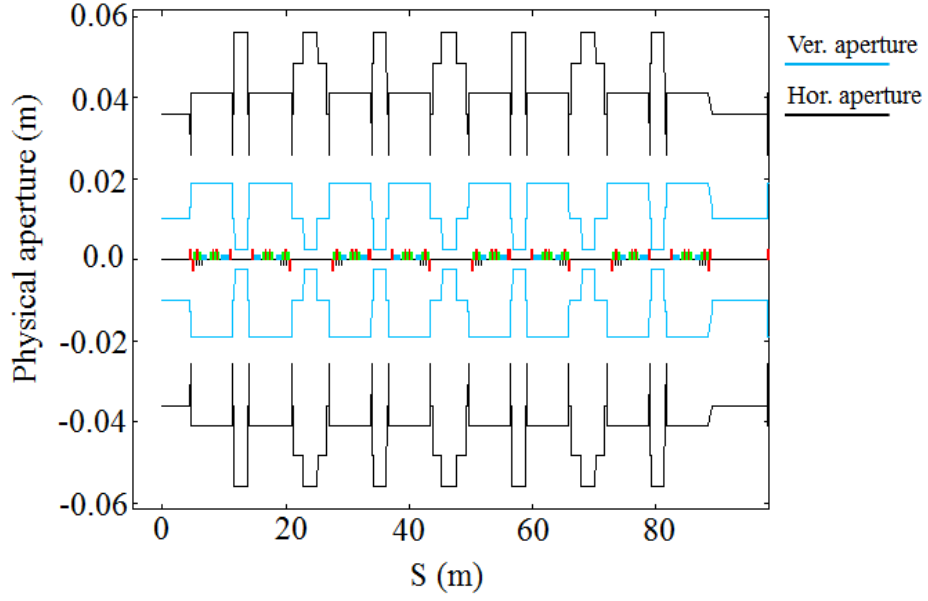


Figure 4.11: Physical aperture of the first super period in the modified-4BA lattice.

straight. The matched optics functions of the super period are shown in Figure 4.10. The betatron tunes were matched to be below half-integer for both planes to minimise the effect of the resistive wall instability due to many narrow gap in-vacuum insertion devices to be installed in Diamond II [31]. The correction of the betatron tunes over the whole machine should be made preserving the matched optics in cell 2. This was achieved by changing the quadrupoles triplet only in the long straight while all quadrupoles in the cell were kept untouched.

Once a satisfactory solution for the linear optics was found, we turned to the lattice optimisation for dynamic aperture and Touschek lifetime. MOGA was employed for the non-linear optimisation of the ring after the chromaticities correction to 2 in both horizontal and vertical planes by two families of chromatic sextupoles in the dispersion bumps. The six families of harmonic sextupoles were used as variables for the optimiser.

The optimisation also included the physical apertures, described in Figure 4.11 considering narrow full gap ID of 5 mm (vertical) in all straight sections including the new straights in the middle of the cell. This is a necessary ingredient to perform realistic simulations for

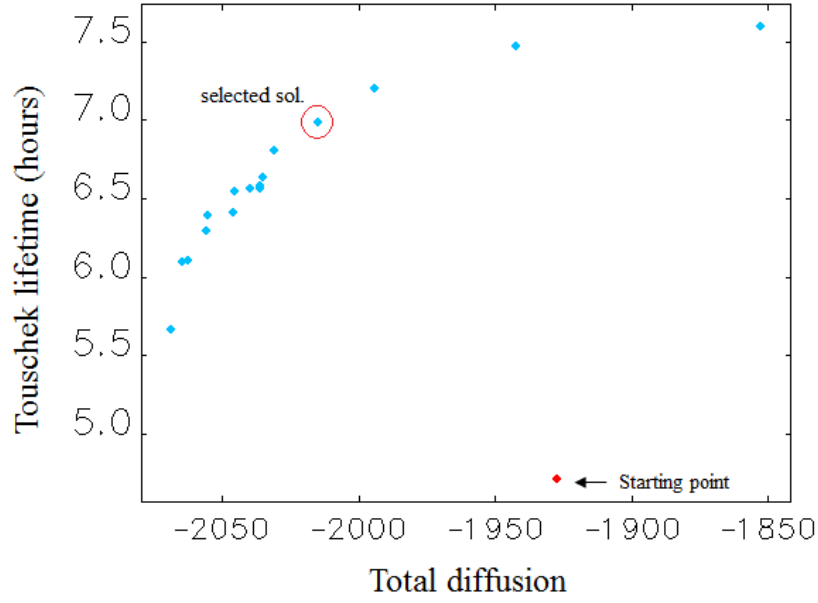


Figure 4.12: Optimal Pareto front for Touschek lifetime and total diffusion (DA) optimisation of the modified-4BA lattice. The selected solution (red circle) has good dynamic aperture and Touschek lifetime of 7 hrs

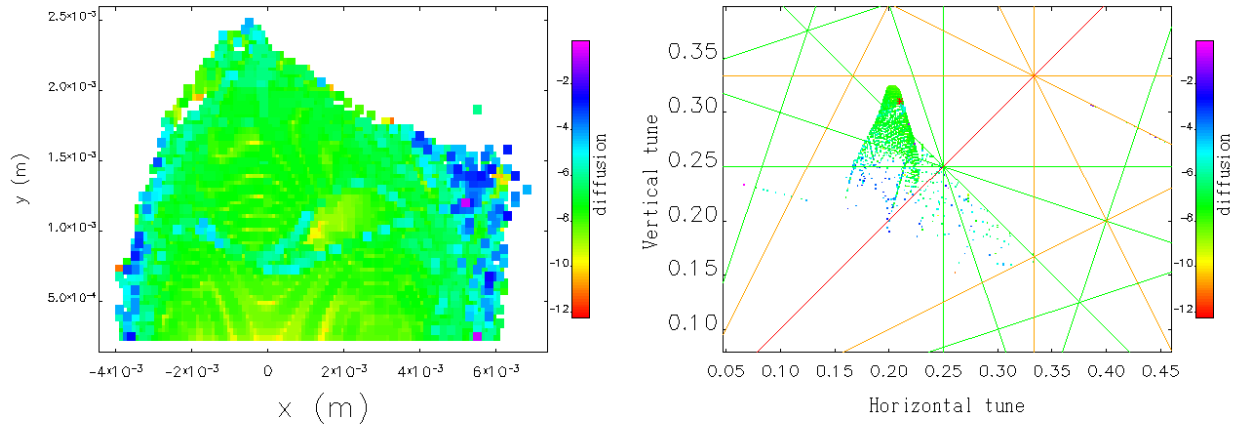


Figure 4.13: Frequency map for the modified-4BA lattice. Left figure shows the tracked particles in transverse amplitudes space (x,y). Right figure shows the tracked particles in tune space. The color code indicates the diffusion.

both dynamic aperture and momentum aperture tracking.

From the optimal front given by MOGA, shown in Figure 4.12, the selected solution gives dynamic aperture of 4-6 mm and Touschek lifetime of about 7 hrs. The lifetime was calculated assuming a bunch length slightly longer than the zero current limit in view of a

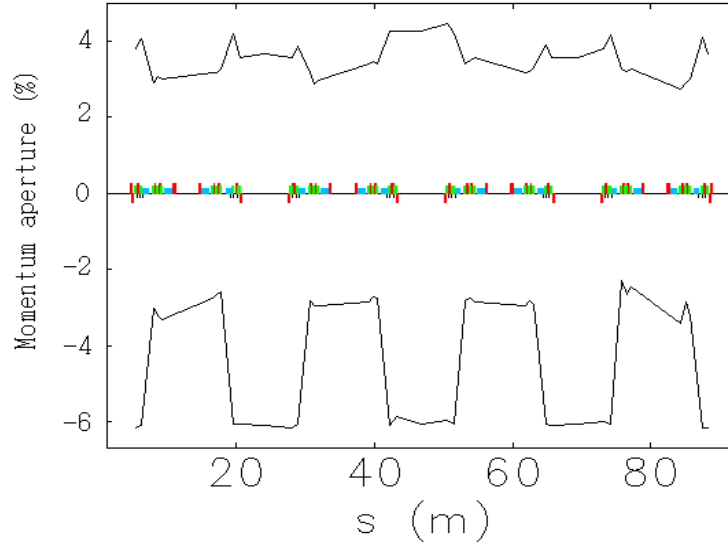


Figure 4.14: Momentum aperture of the modified-4BA lattice in one super period.

foreseeable bunch lengthening effect due to high current operation quantified to be a factor of 1.5 with 300 mA, 900 bunches, 10% coupling and 3.3 MV RF voltage. The frequency map for an optimised solution, reported in Figure 4.13, shows the foot print of the particles in the tune diagram. Forth and sixth order resonances, which appears at the same place as the second order resonance line (red diagonal line), potentially limit the dynamic aperture. The momentum aperture is about $\pm 3\%$ as shown in Figure 4.14.

To investigate a possible better working point, betatron tune scanning considering the total diffusion and Touschek lifetime was performed. The tune scan was made using only the quadrupoles triplet in the long straight sections. In such scans the linear optics is altered predominantly in the long straight section with negligible variation in the natural beam emittance. Figure 4.15a shows the dynamic aperture (DA) represented by the total diffusion rate of the tracked particles for each working point: better dynamic aperture implies lower diffusion rate (red dots). We found that even if the working point is moved away from the second order resonance, the dynamic aperture is only marginally improved because several third and fourth order resonances are crossed. Figure 4.15b shows a band of good Touschek lifetime around the horizontal tune of 51.20 to 51.25. These results indicate that the original working point is already in a good region for both dynamic aperture (total diffusion) and

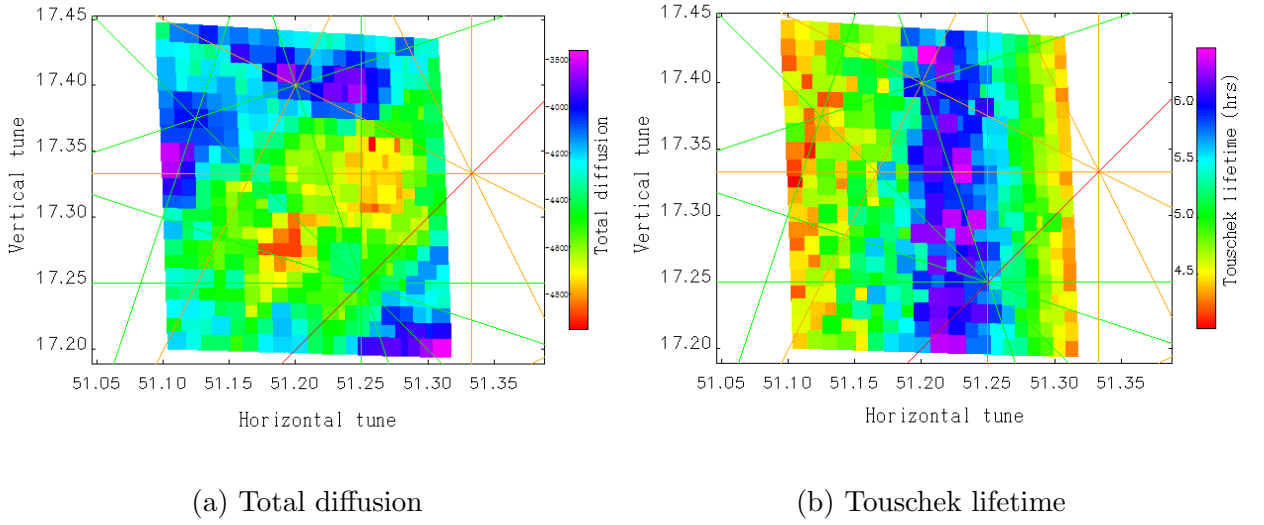
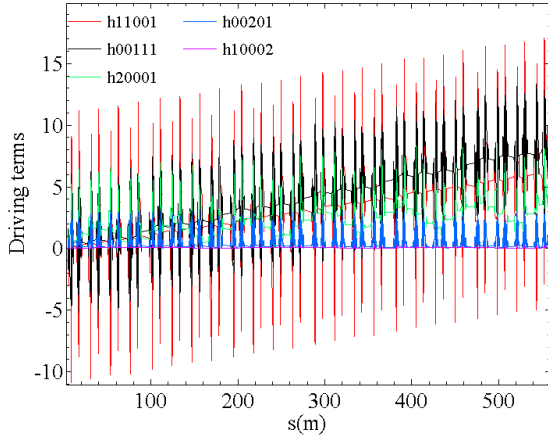


Figure 4.15: Modified-4BA total diffusion (a) and Touschek lifetime variation with working points (b).

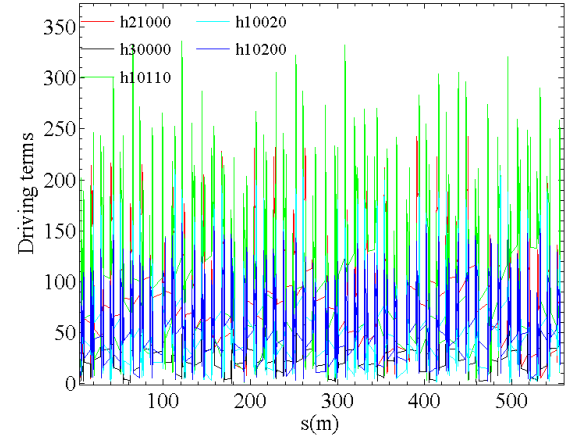
Touschek lifetime.

From the optimal solution, resonance driving terms as functions of the positions along the ring were calculated as shown in Figure 4.16. The first order RDTs for the modified-4BA lattice have similar value as those for the 5BA lattice. However, compared to the RDTs in the 5BA lattice, the second order geometric terms are about three times smaller and the detuning with amplitude terms are about ten times smaller in the modified-4BA lattice. Coherently with the improved RDTs, the dynamic aperture of the modified-4BA is larger than that of the 5BA lattice. The detuning with amplitude terms for the modified-4BA lattice, shown in Figure 4.16d, can be related to the behaviour of the tune spread as shown in Figure 4.13. The relatively large detuning in the vertical plane ($d\nu_y/dJ_y$) drives the particle with large amplitude away from the design tune quickly towards the dangerous coupling resonance (red diagonal line) which limits the size of the dynamic aperture.

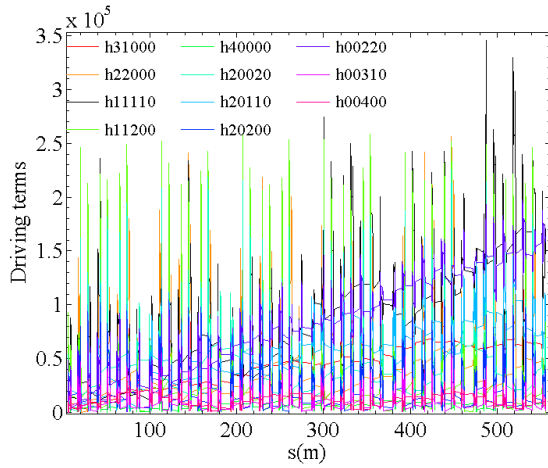
For the modified-4BA lattice, the position of the magnetic elements is limited and a proper phase advance matching to compensate the resonance driving terms (RDTs) between the dispersion function bumps within one cell is not achievable. This technique was used in ESRF-II 7BA (Hybrid MultiBend lattice design) that achieved 3π and π between the dispersion bumps for horizontal and vertical plane respectively [76] which provide the RDTs



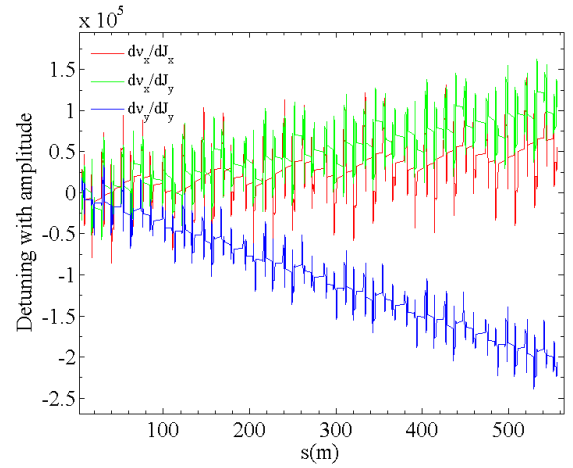
(a) First order chromatic terms



(b) First order geometric terms



(c) Second order geometric terms



(d) Detuning with amplitude

Figure 4.16: Resonance driving terms and detuning with amplitudes as functions of position for the modified-4BA lattice after MOGA optimisation.

cancellation within one cell. The phase advance between the sextupole in the modified-4BA cell is instead almost 2π and π in the horizontal and vertical plane respectively.

A further optimisation of the lattice was attempted by matching the phase advance to cancel RDTs in one super-period (4 cells). However not all the terms can be suppressed especially for the higher order (h11110, h22000 and h00220) which also drive the detuning with amplitude. Additionally, the driving terms h21000 and h10110 in Figure 4.17b that can be cancelled after one super period (four cells) are large in the middle of the super period compared to the optimal solution as shown in Figure 4.16b. These lead to a dynamic aperture of 3 mm smaller than the one found by MOGA.

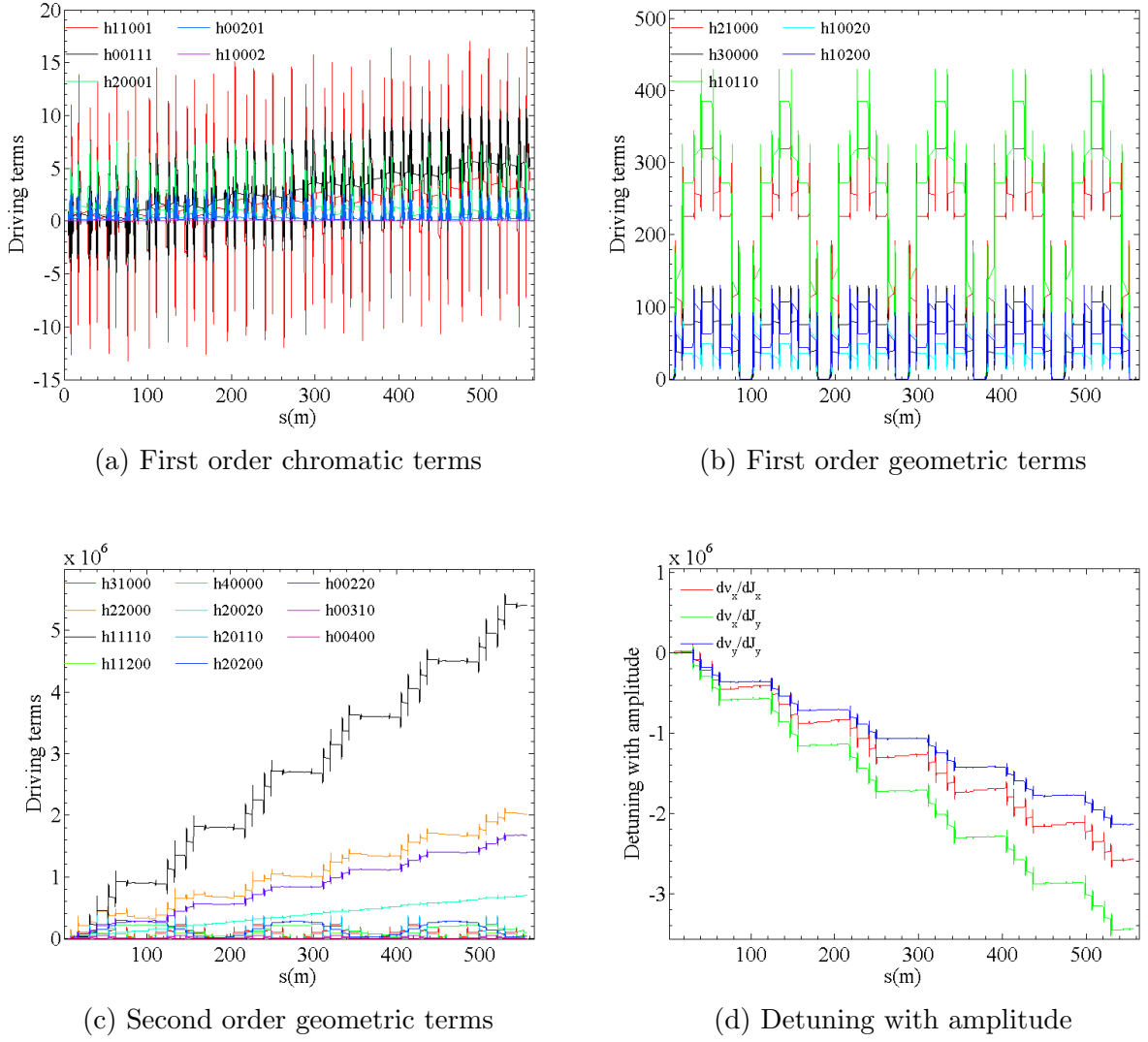


Figure 4.17: Resonance driving terms and detuning with amplitudes as functions of position for the modified-4BA lattice after phase advance matching.

Wigglers in the modified-4BA lattice

In Chapter 3, the effect of the wigglers in the existing Diamond lattice modified for operation with lower emittance was presented and we showed how the effect of large dispersion function resulted in the increase of the beam emittance. In Chapter 2, we also showed that a strong field wiggler in a small dispersion function straight can also be used to reduce the emittance. The zero-dispersion straight sections (standard straight) in the modified-4BA allow the use of strong IDs without deteriorating the beam emittance. In fact, the two existing wigglers in the Diamond ring at the straight section I12 (4.2 T peak field) and I15 (3.5 T peak field) can be used in the new upgrade ring to introduce stronger radiation damping effect

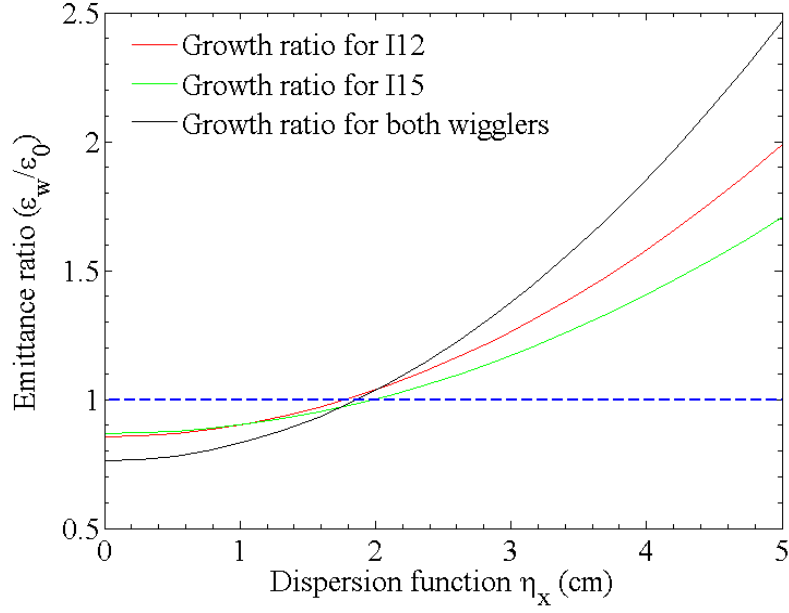


Figure 4.18: Emittance growth ratio as a function of dispersion function in standard straight for wigglers at the maximum operating field for modified-4BA ring.

and reduce the equilibrium beam emittance. For the modified-4BA the emittance ratio from Eq.2.16 provides the emittance reduction as shown in Table 4.2. The modified-4BA ring with two wigglers has about 30% smaller beam emittance than the designed value. Figure 4.18 describes the emittance growth ratio as a function of dispersion function in the straight. The new modified-4BA lattice can tolerate a dispersion function of about 1.8 cm in the straight without emittance growth.

Table 4.2: Calculated beam emittance with wigglers.

Study cases	Emittance growth ratio	Calculated emittance (pm.rad)
Bare lattice	-	276
Lattice with a wiggler in I15 (3.5T)	0.85	235
Lattice with a wiggler in I12 (4.2T)	0.83	229
Lattice with both wigglers	0.72	199

For ultra-low emittance lattices, the emittance growth ratio is more sensitive to the dispersion function in the IDs than that in the larger emittance rings because the radiation integral I_5 in Eq. 2.16 is much smaller. Further reduction of the emittance by means of dispersion leak in straight sections is thus not as effective as in standard DBA lattice if

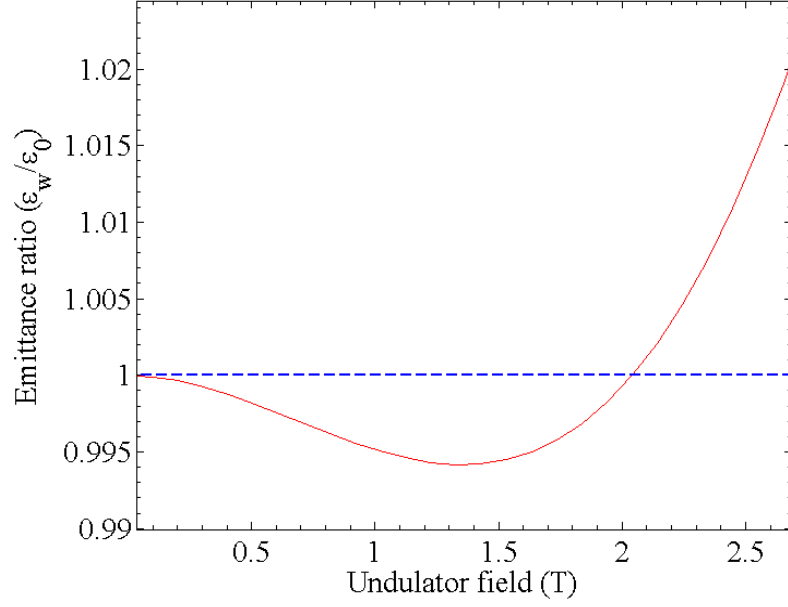


Figure 4.19: Emittance growth ratio as a function of the undulator field in the mid-straight for modified-4BA ring.

strong wigglers are located in the straight sections.

For the new special straight in the middle of the modified-4BA cell, insertion device has to be employed carefully to avoid emittance growth because of the non-zero dispersion function (2 cm) and smaller horizontal beta function than that of the standard straight. Considering one of the most common undulators used in Diamond ($B_{max}=2$ T, $N_{period}=85$, $\lambda_w=23$ mm) the emittance growth ratio as a function of the undulator field is shown in Figure 4.19. The optics in the middle straight was fixed as shown in Figure 4.9.

It is clear that the beam emittance is not deteriorated by the undulator in the mid-straight even at the maximum operating field of 2 T which still provides an emittance ratio below 1. Most of the existing undulators in Diamond operate below 1 T which actually reduce the emittance slightly (less than 1%) in the modified 4BA lattice. Therefore, the new 24 middle straight sections can comfortably accommodate the existing designed undulators. In conclusion, all the existing IDs can be used in the upgraded ring without the increase in the beam emittance.

4.1.4 Intrabeam scattering for Diamond upgrade

As discussed in Chapter 2, storage rings have to accumulate high average currents in the ring to provide the desired photon brightness and intrabeam scattering (IBS) effects are unavoidable. The effect of IBS can enlarge the beam emittance particularly in the ultra-low emittance lattices. The formula for the emittance growth due to IBS has been implemented in the script `ibsEmittance` [41] and has been used to calculate the IBS effect on the beam emittance. Assuming a maximum stored current of 500 mA, 900 bunches, 10% coupling ($\varepsilon_y/\varepsilon_x$) and the existing RF cavity operating with a voltage of 3.3 MV, the beam emittance for the MBA lattice options for Diamond was calculated as a function of the stored beam current.

Figure 4.20 shows that when the current increases, the emittance grows rapidly in 7BA and 5BA lattices. As expected for a very small emittance lattice, the emittance of the 7BA lattice grows more rapidly compared to the modified-4BA and 5BA lattice. At 300 mA the beam emittance for 7BA is about double of the designed natural emittance. For the modified-4BA lattice, the beam emittance increases to about 286 pm·rad at 500 mA current.

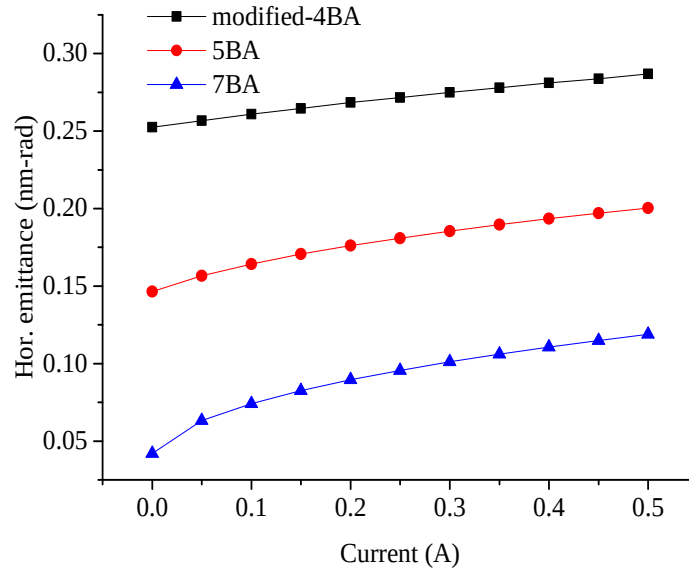


Figure 4.20: Beam emittance as a function of current considering IBS effect for the modified-4BA, 5BA and 7BA lattices Diamond upgrade considering 10% coupling.

At least in the modified-4BA case, the beam emittance increase due to the IBS effect, can be compensated by damping effect which is introduced by wigglers and undulators.

4.2 Summary

In this chapter, three options have been investigated to reduce the beam emittances for Diamond upgrade. The modified-4BA, 5BA and 7BA lattices provide emittance reduction factors by 10, 18 and 55 respectively compared to that of the existing machine. Linear optics were matched to optimise the emittance considering also the RDTs cancellation. As expected, very small emittance can be achieved by increasing the number of bending in the cell but the strong quadrupoles required lead to large natural chromaticities. Therefore, strong sextupoles for chromaticity correction have to be used leading to challenging nonlinear beam dynamics optimisation.

The modified-4BA lattice doubles the total number of beamlines while achieving a beam emittance is about 10 times smaller than that of the operating machine. The required magnet gradient are within the limits of the present technology. Unlike MAX-IV design, each magnet can be built as an independent element. Additionally, the modified-4BA lattice produces a tolerable offset of the beamline position of 15 mm well within the realignment capabilities of the beamlines.

The modified-4BA lattice is a promising solution for the future Diamond upgrade. Although the 5BA and 7BA provide smaller beam emittance, more challenging magnet design, major changes in the alignment of the beamlines, and lattice optimisation will have to be considered. If the required reduction in emittance for the future upgrade will be more aggressive, these options can be reconsidered. Up to now the modified-4BA solution has been deemed to be acceptable for an upgrade proposal and the optimisation has been focusing mostly on such design. MOGA gives some optimal solutions with about -4 to 6 mm dynamic aperture and 7 hr Touschek lifetime. The dynamic aperture is still small considering -10 mm requirement for the exiting off-axis injection scheme (at -8.3 mm) therefore further optimi-

sation is required. Linear optics matching to constrain the odd number of π phase advance between the dispersion bumps in order to achieve the RDTs cancellation in one cell as done in ESRF-II [76] is still challenging because of the short cell length and fewer components between the bumps. However, different injection methods based on on-axis injection can be used in such small dynamic aperture lattice if required. The most promising are swap out injection [77], longitudinal injection [78], or on-axis pulsed kicker injection [79].

For the modified-4BA lattice, the effect of the strong field existing wigglers in the non-dispersive straight 12 and 15 reduced the beam emittance by 30%. The special straight in the middle of the cell having 2 cm dispersion function can comfortably accommodate an undulator (2 m long) with the operating field below 2 T without the emittance growth.

Intra-beam scattering indicates the rapid increase of the emittance especially in the 7BA and 5BA lattice while in the modified-4BA lattice the emittance is still below 300 pm·rad at the maximum current of 500 mA.

The phase advance matching to compensate the RDTs every four cells is successful for every terms of the first order chromatic and geometric terms but cannot suppress some of the second order geometric and detuning with amplitude terms which cause the reduction of dynamic aperture. To be able to achieve also the RDTs cancellation in higher order, more number of cells in super period is required for example 8 cells for PEP-X lattice [74], [80], however such scheme cannot be implemented in the Diamond upgrade given the small number of cells. The challenging nonlinear effects, lattice optimisation were mostly dealt with MOGA based on direct particle tracking.

Chapter 5

Injection with a pulsed multipole kicker

Beam injection is one of the most critical processes in the operation of particle accelerators. Both light sources and colliders need high beam current to reach high brightness and high luminosity respectively. Most of the particle accelerators achieve good injection efficiency by a careful tuning the optic functions at the injection point and optimising the non-linear beam dynamics to maximise the dynamic aperture. High injection efficiency is beneficial for routine operation as it allows quick and efficient accumulation of the current minimising the time to refill the ring, e.g. after a beam trip.

Modern light sources operate in Top-Up mode, that allows a continuous injection. In this case, the transparency of the injection process is also important. The top- up injection allows continuous particle accumulation when the current decays below a fixed minimum threshold. In conventional injection schemes using orbit bump, the unavoidable imperfections of the injection system generate a noticeable change of the photon beam position and angle during injection. This also leads to temperature variation on the beamline optics system. As a consequence the photon beam stability is undermined and the users are forced to use temporary gating in their experiments during injection. Another disadvantage of the conventional injection consists in the relatively large dynamic aperture to accommodate the

orbit bump at the septum magnet. The combined requirements of transparent injection and smaller dynamic aperture, has triggered in the recent years a research on new possible injection schemes.

A seminal work was done at Photon Factory Advanced Ring (PF-AR) [81] where a new scheme of beam injection with a single pulsed quadrupole magnet with zero field at the center has been proven. Later designs, using higher order pulsed kicker [82], [83] providing less perturbation to the stored beam, have been successfully demonstrated. In view of the issues with Top-Up injection and the smaller dynamic aperture in the upgrade lattice, Diamond has also investigated the possibility of using a pulsed multipole kicker (PM) for the beam injection; the main candidate is an adapted version of the pulsed multipole kicker designed by Bessy-II [83]. As discussed in Chapter 3 one DDBA lattice will be commissioned in the near future and the pulsed multipole kicker injection option is also considered. In this chapter we review the basic principle of the standard multi-turn injection in storage rings. We then describe the specifications required for the PM and the expected performance. It is essential that the optimal PM position is found to achieve good injection efficiency. Particle tracking based optimisation for injection efficiency will be discussed. Besides the injection efficiency, PM injection should minimise the perturbation to the stored beam. Field errors and misalignment will be introduced to estimate the effect on the stored beam. Additionally, realistic PM design will be taken into account during the optimisation. The analysis is repeated for the existing Diamond ring and for the Diamond lattice with a single DDBA in cell 2.

5.1 Conventional beam injection

Pulsed dipoles kickers have been used for beam injection to accumulate the current in storage rings; an orbit bump is generated at the injection point to bring the stored beam closer to the injected beam. This scheme can be realised by two pulsed dipoles with π phase advance separation but usually three or four dipoles are more widely used. For an electron storage

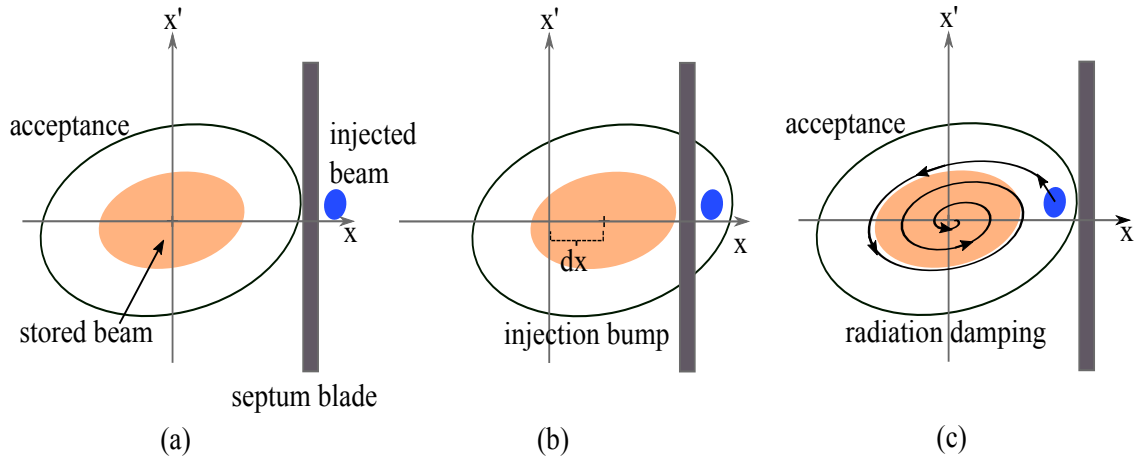


Figure 5.1: Phase space during the conventional injection with orbit bump: (a) before injection, (b) after the injection bump is created to capture the injected beam, (c) the amplitude of the captured injected beam reduces via synchrotron radiation.

ring, as shown in Figure 5.1, the beam is injected off-axis with respect to the stored beam and an injection bump is created to bring the stored beam closer to the injected beam. The injected beam is delivered at the end of septum magnet, and is captured in the machine acceptance. Because of synchrotron radiation, the oscillation amplitude of the injected electron is damped towards the stored beam at the centre and clears the back of the septum. Once the injected beam is fully damped and merges to the stored beam the injection process can start again and the current can be accumulated. The injection is repeated with a frequency of few Hz in typical third generation light sources.

5.1.1 Beam injection with four kickers

A schematic design of a four kickers injection system is shown in Figure 5.2. In such a system the first two kickers K1 and K2 move the closed orbit towards the septum magnet closer to the injected beam. The orbit bump is closed to the nominal orbit by the two downstream kickers K3 and K4. The injection process can be explained in normalised phase space as shown in Figure 5.3. The stored beam (red) arriving at the kicker K1 is kicked by θ_1 having large angle and travels along an arc of the new trajectory circle. Then similarly the kicker

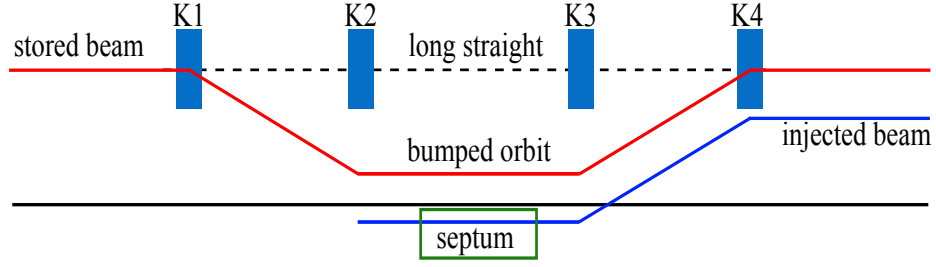


Figure 5.2: Schematic figure of the conventional beam injection with four kickers.

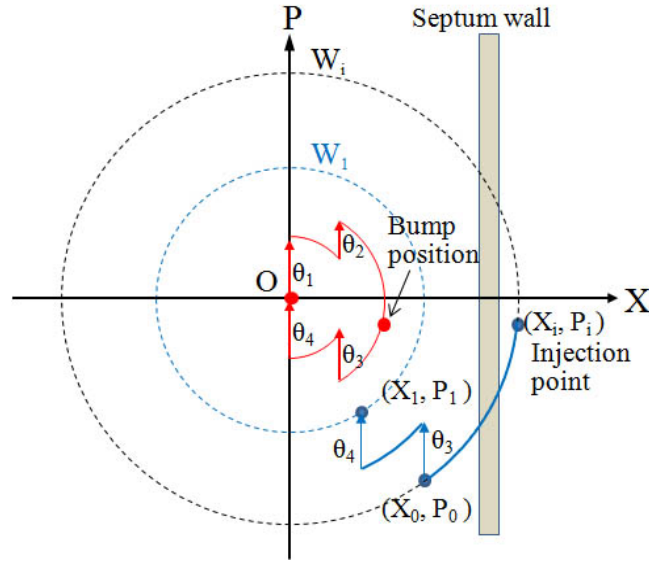


Figure 5.3: Injected and stored particle trajectories in a normalised phase space for four kickers injection scheme. Red and blue are the stored and injected beam respectively. The stored beam travels in the closed orbit bump while the injected beam invariant is reduced from W_i to W_1 .

K2 provides further kick θ_2 driving the stored beam towards the bumped position closer to the injection point. Then the beam from booster (blue) arriving at the end of septum magnet is injected off-axis at position X_i and angle P_i . It is convenient to associate to the injected beam the corresponding Courant-Snyder invariant W_i . Both stored and injected beams travel through the kicker K3 which is positioned downstream and are kicked toward a smaller invariant by the angle θ_3 . The angle coordinate is reduced again when both beams arrive at the kicker K4 by θ_4 which allows the stored beam to resume the nominal orbit and the injected beam to move to a smaller invariant W_1 at (X_1, P_1) simultaneously. The

normalised phase space coordinate is defined by

$$X_n = \frac{x}{\sqrt{\beta_x}}, P_n = \frac{\alpha_x x + \beta_x x'}{\sqrt{\beta_x}}, \quad (5.1)$$

where x and x' are horizontal position and angle of the particle; α_x and β_x are twiss parameters. Hence, the injection invariant is expressed by

$$W_i = X_i^2 + P_i^2 = \frac{\{x_i^2 + (\alpha_{x,i}x_i + \beta_{x,i}x'_i)^2\}}{\beta_{x,i}}, \quad (5.2)$$

where (X_i, P_i) is the normalised phase space coordinates at the injection point defined by Eq. 5.1.

For beam injection with four kickers bump, the injected beam phase space may not be matched to the acceptance of the ring and the injection efficiency might not be optimal; therefore, some particles of the injected beam are not captured inside the acceptance. To match the phase space of the injected beam, the optics functions of the beam transfer straight (BTS) have to be adjusted. The optimal condition for beta functions to be matched can be expressed as [84],[85]

$$3\beta_i^2 + 2\beta_i^{3/2} \frac{N_s \sqrt{\varepsilon_s \beta_s} + T}{N_i \sqrt{\varepsilon_i}} = \beta_s^2, \quad (5.3)$$

where β_i and β_s are the beta functions at the end of the transfer line at the septum and the injection point of storage ring respectively, N_s is an integer that gives the separation between stored beam and septum in the unit of rms size of the stored beam ($N_s=5$ for Diamond), N_i is an in that gives the separation between the injected beam and septum in the unit of rms size of the injected beam ($N_i=3$ for Diamond), ε_s and ε_i are the beam emittance of the stored and injected beam respectively, and T is the septum thickness.

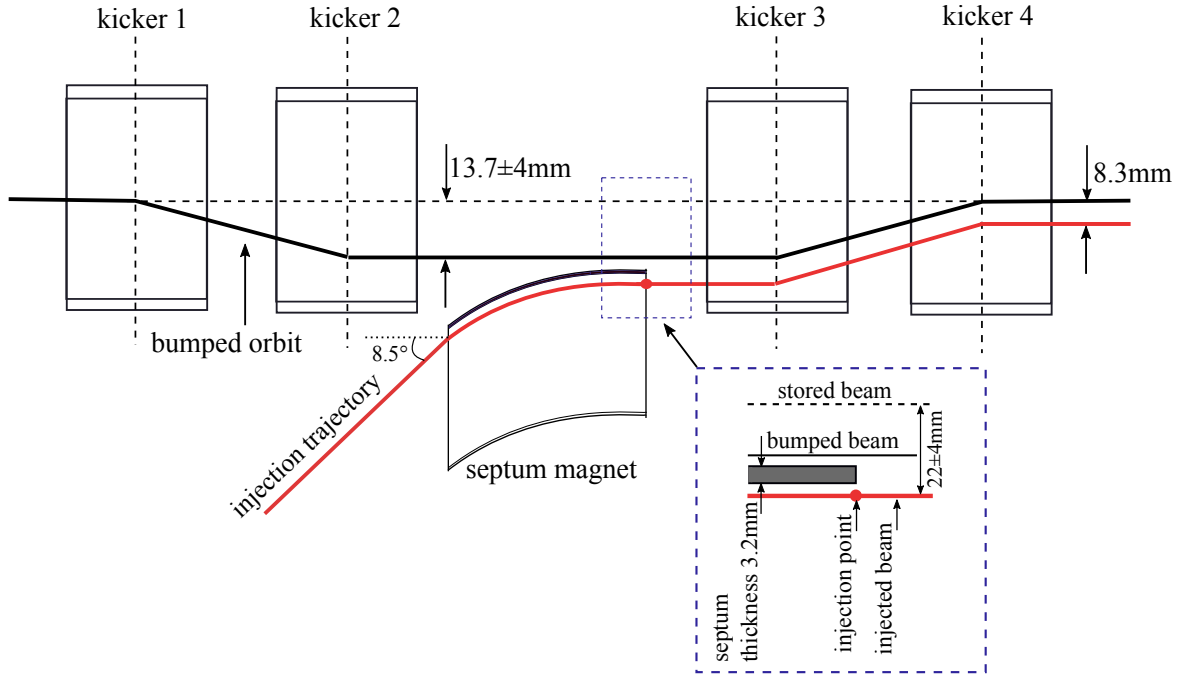


Figure 5.4: Schematic figure of Diamond four kickers injection.

5.1.2 Conventional injection at Diamond

Since the beginning of its operation in 2007, Diamond has used the design injection system with four kickers, all located in the 8 m long injection straight. The system deliver about 90% injection efficiency in routine operation.

The schematic view of the Diamond injection straight is shown in Figure 5.4. The injected beam delivered by the booster to storage ring transfer line (BTS) arrives at septum magnet with an initial angle of 8.5° . At the injection point the injected beam is separated from the stored beam by 22 mm. The kickers 1 and 2 generate an orbit bump towards the injected beam away from its center by 13.7 mm which reduces the distance between the stored and the injected beam from 22 mm to 8.3 mm. Then the kickers 3 and 4 close the orbit bump back to its original path while the injected beam is captured in the storage ring. The minimum distance between the injected beam and the bumped beam is mainly limited by the strength of the kickers and the septum thickness. The injection process can be simulated by tracking

the beam from the injection point off-axis at -8.3 mm for several turns: the amplitude of the injected beam is damped by the radiation damping towards the centre as shown in Figure 5.5.

Although most of the operating machines use the conventional injection with kickers, there are several disadvantages with this method:

- In order to avoid perturbation of the stored beam circulating in the machine, a perfect closed orbit bump is required at the injection straight. An ideal performance of the four kickers has to be achieved; however, several causes concur to perturb the kicker performance leaving an unclosed orbit bump. First, it is practically challenging to control the field quality of the four different magnets. Additionally, it is difficult to supply current to the kickers equally even when the kickers are connected in series: deviations during the pulse profile are unavoidable. Without a completely closed bump, the stored beam is perturbed during injection. About $50\text{ }\mu\text{m}$ injection residual orbits in both horizontal and vertical planes were achieved at Swiss Light Source (SLS) [86] after fine tuning of the kicker waveforms. Even with identical inductive loads, the generated pulses were still different.

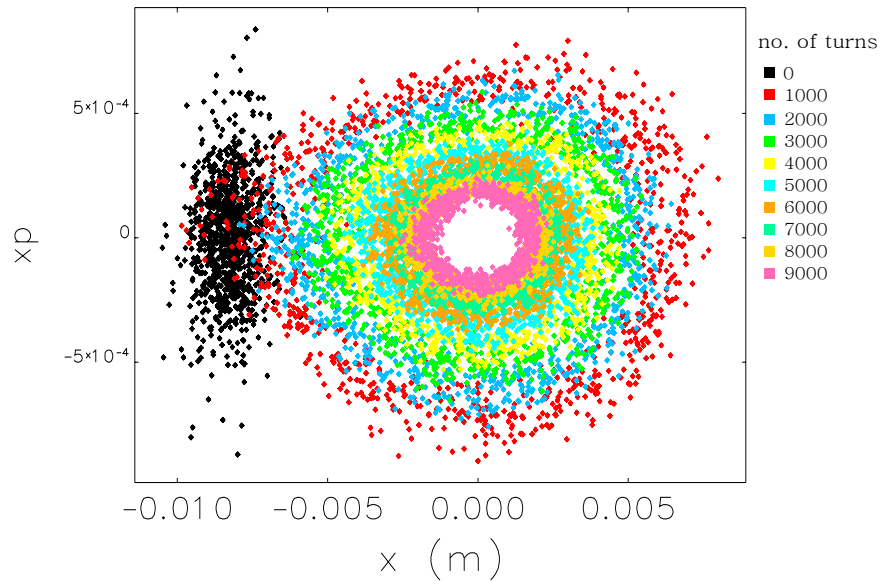


Figure 5.5: Beam injection at -8.3 mm (black) tracking in horizontal phase space for 9000 turns with radiation damping effect for Diamond storage ring.

- A sufficiently large dynamic aperture is required for injection considering closed orbit bump close to the septum magnet limiting the kicker strength and septum wall. At Diamond 10 mm of horizontal dynamic aperture is considered to be a safe margin.
- The conventional injection with four kickers requires large space in the storage ring for all the hardware to fit in and some machines cannot install the whole injection system in one straight section. In these cases the injected beam will travel at large amplitudes in nonlinear element in the cells and will experience nonlinear kick before the orbit bump is reduced to zero.

For these reasons a new injection scheme has been proposed. A single pulsed kicker providing sufficient kick to the injected beam off-axis appears to be significant simpler compared to the injection with kickers bumps.

5.2 Pulsed multipole magnet injection

The first example of pulsed injection kicker was proved at Photon Factory Advanced Ring (PF-AR) [81] using a pulsed quadrupole. Later development of the scheme focused on reducing the perturbation to the stored beam, a pulsed sextupole was used and was also successfully tested at Photon Factory storage ring (PF) [82]. Further progress using a higher order nonlinear pulsed injection kicker was proposed and is under development at Bessy-II [83]. This schemes is based on a single pulsed kicker with higher order multipole magnetic fields and holds the promise of solving many of the disadvantages of the conventional injection scheme.

The multipole kicker magnet has a zero magnetic field at the centre, which makes it transparent to the stored beam, and a sufficient field at large amplitudes to kick the injected beam into the machine acceptance. A schematic view of the stored beam at the zero field region of multipoles is described in Figure 5.6. The flatness of the magnetic field at the centre depends on the order of the multipole as seen in Figure 5.6; the higher order of the multipole magnetic field, the less likely the stored beam will be affected. The schematic view of

the beam injection with a single pulsed magnet, shown in Figure 5.7, is much simpler than the conventional injection scheme. Additionally, a small longitudinal space in the machine is required to install a single pulsed magnet. To understand how the injection with a pulsed kicker works, we should consider the normalised phase space in Figure 5.8. The injected beam at the end of septum magnet is placed at the injection point (X_i, P_i) then travels through the ring following a circle of the injection invariant (W_i) . The injected beam arrives at the position of the pulsed kicker (X_0, P_0) , which is separated from the injection point by phase advance (φ) . The kick applied by the PM reduces the angle of the injected beam (ΔP) in the first turn and as a result the injected beam moves to the new position (X_1, P_1) with a smaller invariant (W_1) and is captured into the machine. In this way, the invariant of the injected beam can be reduced by only one kick. Most importantly at the position of the stored beam (red dot) at the centre, the multipole kicker has zero field and therefore, the stored beam remains ideally unperturbed. Notice that the position of the injected beam and the PM can be optimised by a suitable choice of the injection angle (P_i) . The maximum injection angle is limited by the septum wall and available physical chamber. For the pulsed

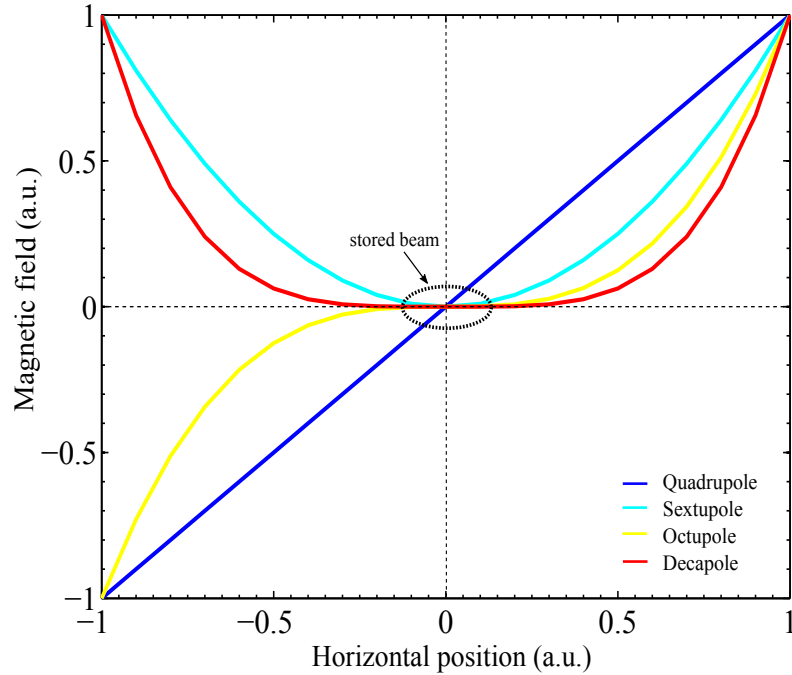


Figure 5.6: Multipole magnetic fields. Higher order multipole will have flatter magnetic field at the centre.

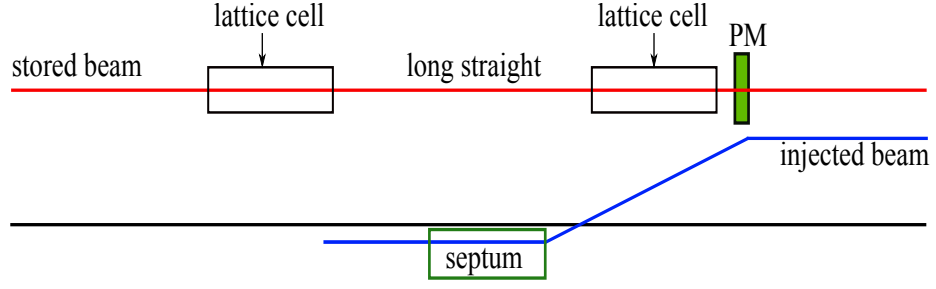


Figure 5.7: Schematic figure of pulsed multipole (PM) injection.

quadrupole injection scheme, the quadrupole strength used to reduce the invariant from W_i to W_1 is given by [81]

$$k_1 l = -\tan \varphi \pm \sqrt{\frac{r}{\cos^2 \varphi} - 1}, \quad (5.4)$$

where l is the length of the quadrupole, φ is the phase advance between the injection point

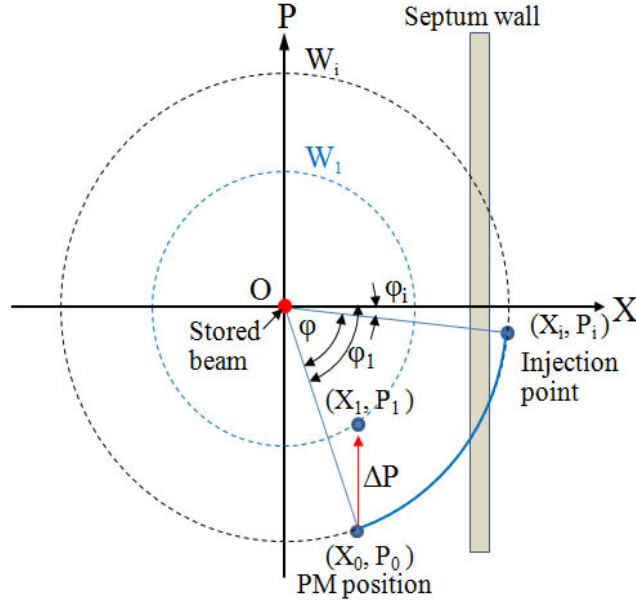


Figure 5.8: The injected beam (blue) trajectory in normalised phase space for Pulsed multipole injection scheme. From injection point, the injected beam travels along the machine and receives a kick by the PM which reduces the injected beam invariant from W_i to W_1 . The stored beam (red) experiences no kick because of the zero field at the centre.

and the position of the quadrupole and r is the invariant ratio defined by

$$r = \frac{W_1}{W_i}. \quad (5.5)$$

The smaller the invariant ratio (r) the closer the injected beam is moved toward the stored beam. From Eq. 5.4, we see that the reduction of the injection invariant depends on the phase advance between the position of the pulsed kicker and injection point and the strength of the kicker. Eq. 5.4 can be used to identify the suitable places of a pulsed kicker in a machine.

This work will concentrate on pulsed multipole magnet (PM). The Bessy-II pulsed multipole magnet [83] was used as a starting point for the diamond pulsed injection kicker. The design is based on four main parallel wires that generate the required off-axis kick angle. The magnetic field of the PM is similar to an octupole and provides a smaller perturbation to the stored beam compared to the injection with the pulsed quadrupole and sextupole. The residual effects of the pulsed magnets on the stored beam will be presented latter in this Chapter. Additionally, the PM provides almost a flat field off-axis which kicks the injected beam more uniformly.

5.3 Pulsed multipole injection at DIAMOND

The pulsed multipole magnet proposed for beam injection at Diamond [87], was studied in details with numerical simulations. The first point investigated was the definition of a suitable location in the storage ring that is currently available in the machine and as close as possible to the injection point. From Figure 5.8, the position for the PM needs to have a phase advance close to odd number of $\pi/4$ from the injection point and be such that the trajectory of the injected beam is sufficiently far from the stored beam so that the PM can reduce the beam invariant with a moderate kick angle. Particle tracking simulations were performed to identify the best position and pulsed multipole magnet (PM) required strength.

The position of the PM kicker was investigated preliminarily by using Eq. 5.4 for the

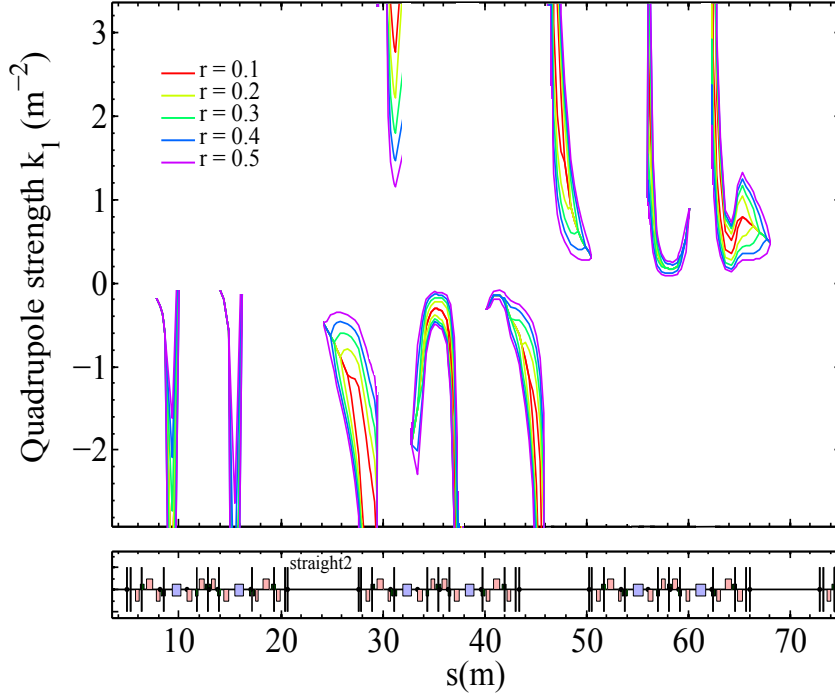


Figure 5.9: Pulsed quadrupole strength required for injection invariant reduction along the first super period in the Diamond storage ring.

pulsed quadrupole strength required to reduce the injection invariant. The required kick strength is plotted as a function of position along the Diamond ring in Figure 5.9. The possible positions for the PM can be selected from this graph and it transpired that the end of the straight 2 just before the cell 2 is available in the machine and has a suitable phase advance as shown in Figure 5.9. Most of the other straights downstream which have suitable phase advance with respect to the injection point are occupied by insertion devices. Straight 14 is also available, but it turned out to be too far from the injection point and not ideal for injection with large amplitude since the injected particles will experience strong nonlinear kicks from the sextupole in the ring. The results of the different positions for the PM injection scheme will be discussed later. The conclusion drawn from the analytical formula was verified with detailed simulations of the injection process by means of particle tracking.

Beam injection simulations without detailed physical aperture were carried out with the

Accelerator Toolbox (AT) [88] in Matlab. The AT tracking code provides full 6D coordinates of particles at each position which describe the beam evolution along the machine and the beam parameters at each element. To optimise the beam injection with the PM, tracking with a single particle and the full beam were performed.

5.3.1 Single particle tracking

The PM was located 26.8 m downstream from the injection point in the second straight. This provides a phase advance of $\frac{9\pi}{4} + \frac{3\pi}{20}$. To simplify the tracking study, initially a single particle representing the injected beam centroid was used. To investigate the dependence of the injection process on the initial condition of the injected beam, the injected angle (x'_{inj}) was varied while the injection position (x_{inj}) was fixed at -20 mm similar to the position used in standard operation (22 ± 4 mm) as shown in Figure 5.4. The position and angle of the injected particle arriving at the PM position (x_{pm}, x'_{pm}) were recorded as shown in Figure 5.10. At the PM position, the particle coordinates (x_{pm}, x'_{pm}) can be used to determine the required kick angle to reduce the beam invariant. The horizontal coordinate

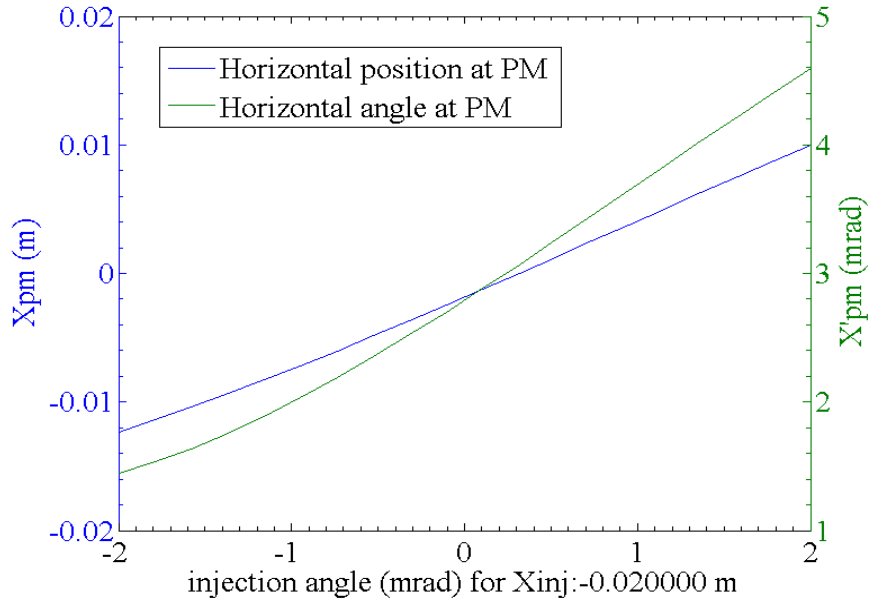


Figure 5.10: Horizontal position (x_{pm}) and angle (x'_{pm}) at the location of the PM in the 2nd straight of the Diamond lattice as a function of the injection angle (x'_{inj}) for the injection amplitude (x_{inj}) at -20 mm.

x_{pm} of the particle arriving at the position of the PM determines the position where the PM has the peak field. It is worth noticing that the position of the peak field should not be too close to the centre or the stored beam position otherwise the stored beam can be excited by the pulsed kicker during the injection and the flat field range of the pulsed kick in the transverse direction will be reduced. Practically, the peak position is also limited by the smallest distance of the wires and the narrowest vertical aperture that can be manufactured. For the injection angle of -1 mrad, the injected particle arrives at the PM location with the horizontal position $x_{pm} = -7$ mm and angle $x'_{pm} = 2$ mrad. In this condition the injected beam can be kicked effectively into the acceptance of the machine. This condition was used in further studies of the injection efficiency.

5.3.2 Multi-particle tracking

Using the selected initial position and angle of the injected beam, we verified the effect of the nonlinear kicker on the injection efficiency using multi-particle tracking. A beam with a Gaussian distribution was injected from the booster with the initial condition as shown in Table 5.1. For the tracking in AT, the only physical limit is the septum blade at the injection point with a thickness of 3.2 mm as shown in the Figure 5.4.

The injected beam in Figure 5.11, was tracked applying the pulsed kick in the first turn only. After the first turn, the pulsed kick was turned off and the tracking continued until the limit of 100 turns. The number of particles surviving after 100 turns was recorded as a

Table 5.1: Initial condition of the injected beam from the booster

Parameters	Symbole	Value
Number of particles	N_{tot}	1,000/10,000
Horizontal emittance	ε_x	150 nm·rad
Vertical emittance	ε_y	15 nm·rad
Energy spread	σ_E	0.00073
Bunch length	σ_z	29.8 mm
Horizontal injection position	x_i	-20 mm
Horizontal injection angle	x'_i	-1 mrad
Vertical injection position	y_i	0 mm
Vertical injection angle	y'_i	0 mrad

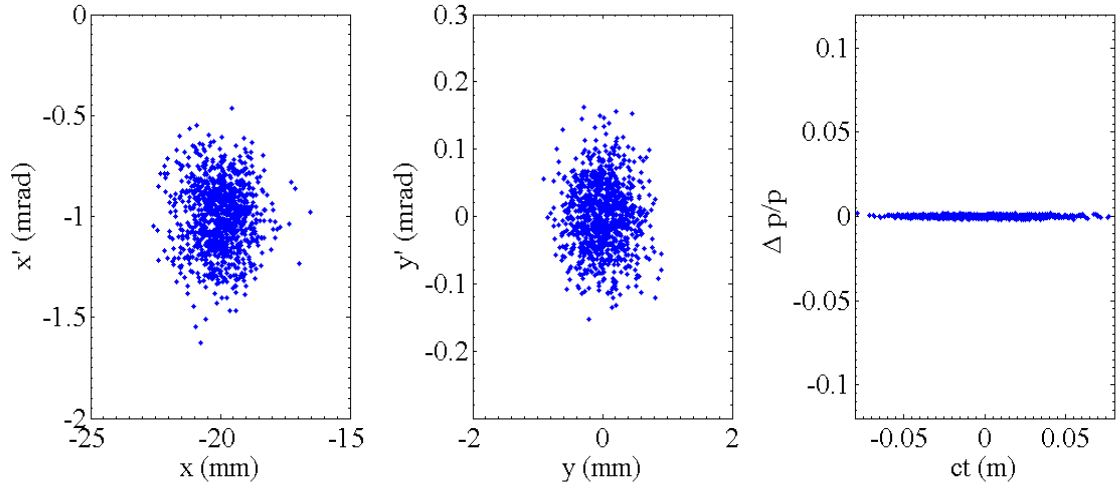


Figure 5.11: Injected beam of 1000 particles generated from the booster used for the tracking in AT.

function of the applied pulsed kick angle. Figure 5.12 shows the number of particle surviving for different kick angle varying from 0 to -3.0 mrad. The injection efficiency increases with stronger kick angles and becomes about 100% when the kick angle reaches -2.2 mrad. The injection efficiency remains 100% until the maximum kick investigated of -3.0 mrad. In order to minimise the pulsed current required for the PM, the kick angle of -2.2 mrad was selected.

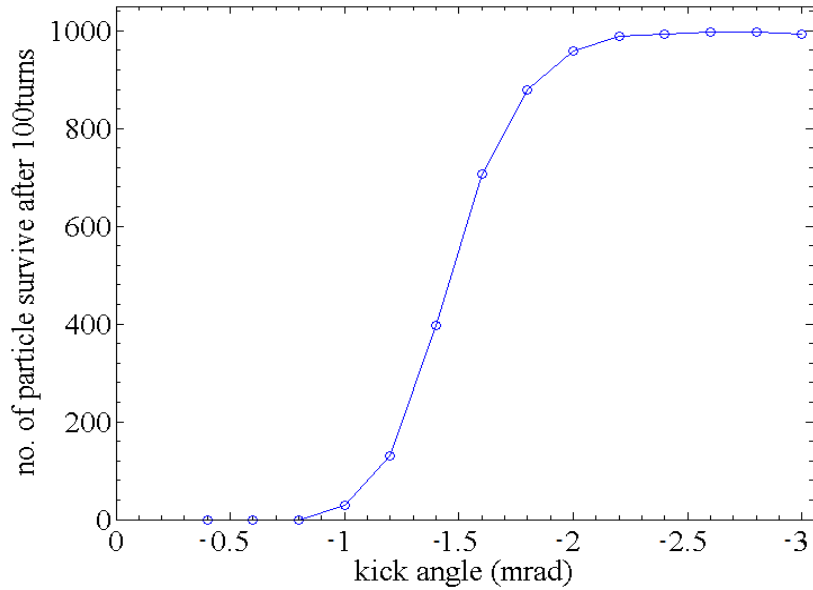


Figure 5.12: Number of surviving particles after 100 turns injection tracking as a function of the kick angle applied by the PM at the second straight of the Diamond lattice.

This value can be converted to the quadrupole strength from the relationship

$$k_1 = \frac{\theta_{pm}}{l \cdot x_{pm}}, \quad (5.6)$$

where θ_{pm} is the kick angle of the PM, l is the length of quadrupole and x_{pm} is the position of the injected beam centroid arriving at the PM. We found that the optimal kick angle of -2.2 mrad is equivalent to quadrupole strength (k_1) of -0.8 m^{-2} , and is in agreement with the quadrupole strength found in the analytic calculation as shown in Figure 5.9.

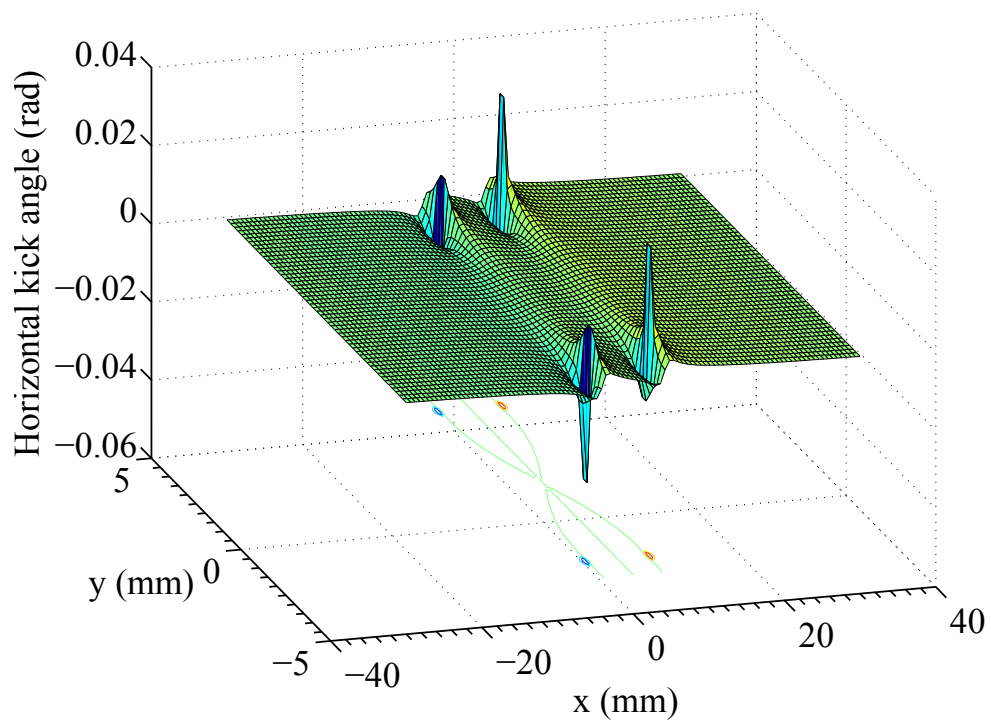
In more realistic simulation, the Bessy-II nonlinear pulsed kicker [83] was simply modelled with four parallel wires separated by the same distance both in horizontal and vertical planes. The magnetic field can be calculated from the integration of the magnetic field generated by the current along the wires. As the magnetic field from the PM is not a standard element type that exists in the simulation codes, a kick map [89] representing the effect of the PM was built. The kick angle can be calculated from the generated magnetic field as

$$\theta_{x,y} = \frac{B_{x,y}l}{B_0\rho_0}, \quad (5.7)$$

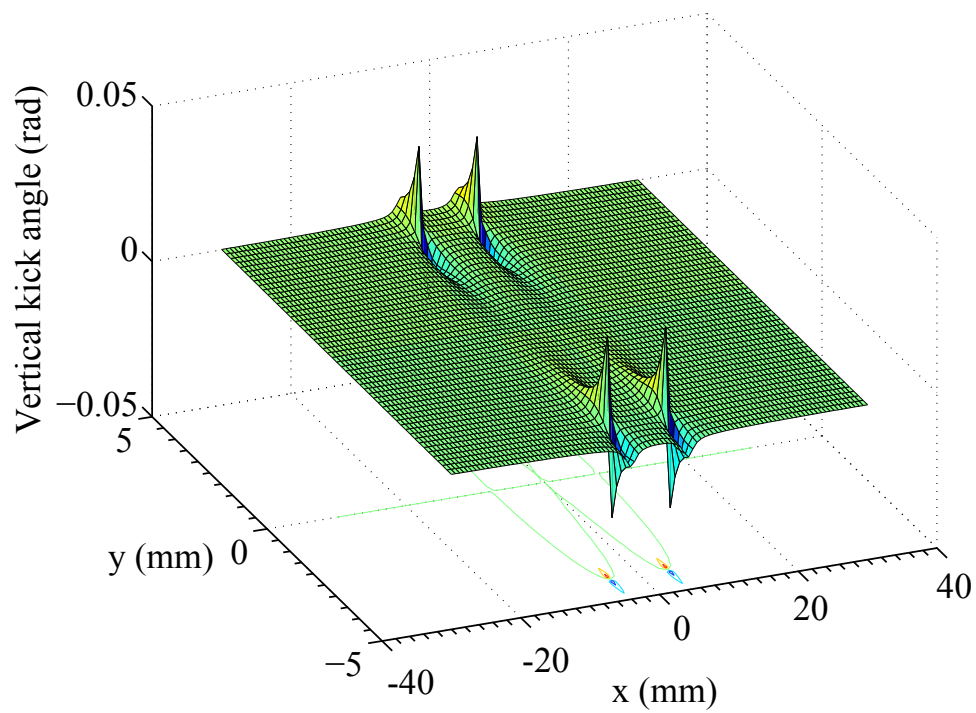
where $B_0\rho_0$ is magnetic rigidity of the ring (10.01 Tm for 3 GeV at Diamond) and l is the magnetic length. The kick map can be created from the kick factor

$$M_{x,y} = \theta_{x,y} \times (B_0\rho_0)^2. \quad (5.8)$$

The resulting kick map is shown in Figure 5.13. The distance between the wires was set to 8.2 mm and current to 710 A. The kick angle extracted from the kick map in Figure 5.13 will affect the beam only horizontally when $y=0$ as shown in Figure 5.14. The PM provides an almost flat magnetic field in the horizontal direction at the position of the injected beam, however, if the injected beam has a large horizontal rms size, the particles in the injected beam will experience different kicks. The peak kick angle was designed to be -2.2 mrad at the injected beam centroid arriving at the PM.



(a) Horizontal kick map.



(b) Vertical kick map.

Figure 5.13: Kick map generated from four wires pulsed multipole kicker .

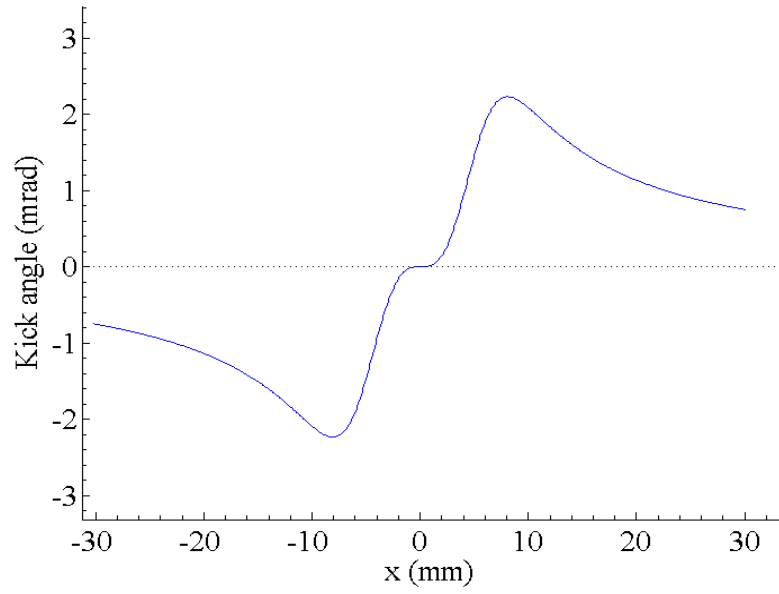


Figure 5.14: Kick angle of the PM along the horizontal axis when $y=0$.

The PM can be modelled at the selected position in the second straight using element kick map for ID (WigTablePass) in AT. Injection tracking with the beam shown in Figure 5.11 was performed for 1000 turns and the number of surviving particles was recorded for each turn as shown in Figure 5.15. After 1000 turn, the injection efficiency is more than 99%.

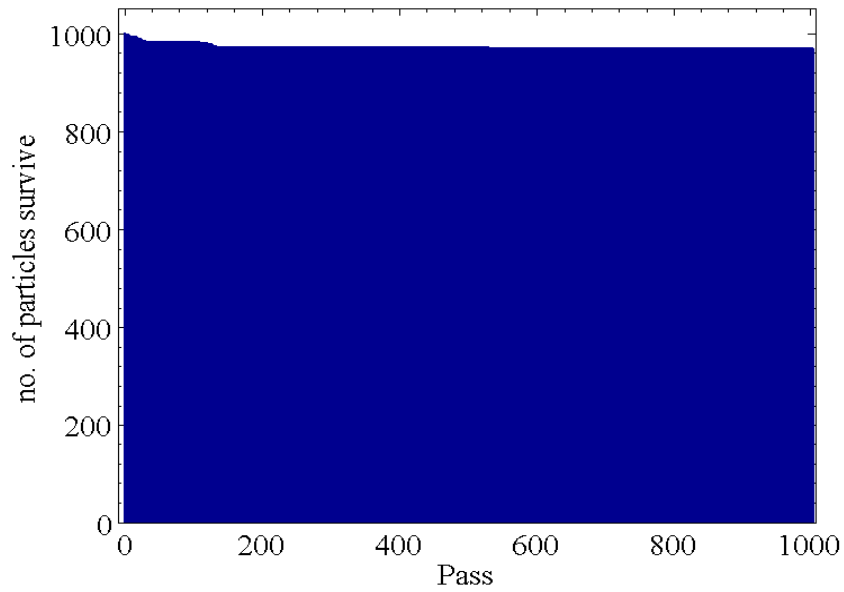


Figure 5.15: The number of surviving particles from the injection tracking for 1000 turns with the PM at the 2nd straight of the Diamond lattice.

Figure 5.16 shows the evolution of the injected beam in the horizontal phase space after five turns. The injected beam (pink) is kicked inside the machine acceptance after the first turn (red) and the beam after 100 turns is shown in Figure 5.17.

The main beam loss comes from the collision with the 3.2 mm thick septum blade which defines the horizontal acceptance (red ellipse). Figure 5.16 highlights also the effect of the transverse inhomogeneity of the magnetic field with the horizontal position that produces a deformation of the initial Gaussian distribution of the injected beam after the first turn. The non-linear effect of the sextupoles in the ring can also deteriorate the injected beam with large amplitude, however this effect is negligible if the PM is close to the injection point in straight 2 and small number of sextupoles is met by the injected beam at large amplitudes.

A comparison of the single particle tracking with PM on and off is reported in Figure 5.18 where we show the trajectories of the injected beam around the ring for the two cases. It was found that the trajectory of the injected particle right after the PM position is significantly reduced when the PM is turned on. The corresponding reduction of the injection invariant is shown in Figure 5.19 where we also show the single particle trajectory in the horizontal

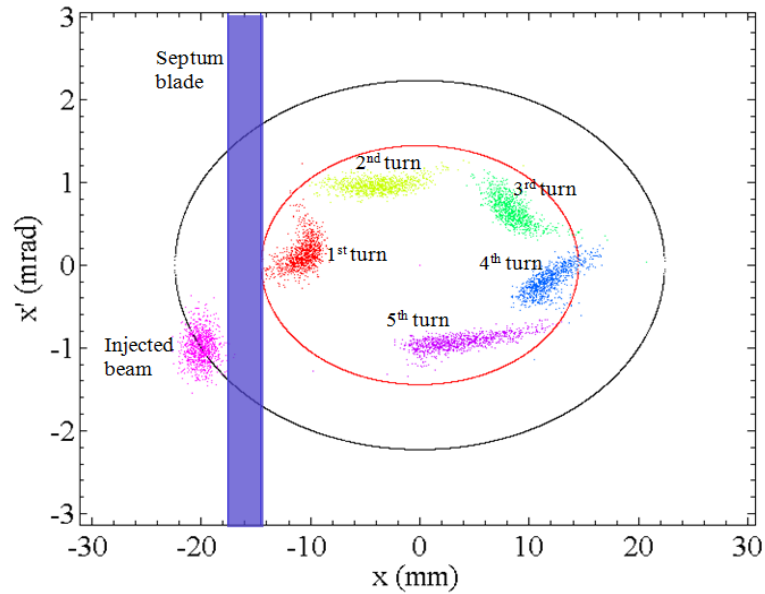


Figure 5.16: Beam injection with the PM at straight 2 of the Diamond lattice tracking for first five turns using 1000 particles. The black and red lines are the injection invariant and machine acceptance respectively.

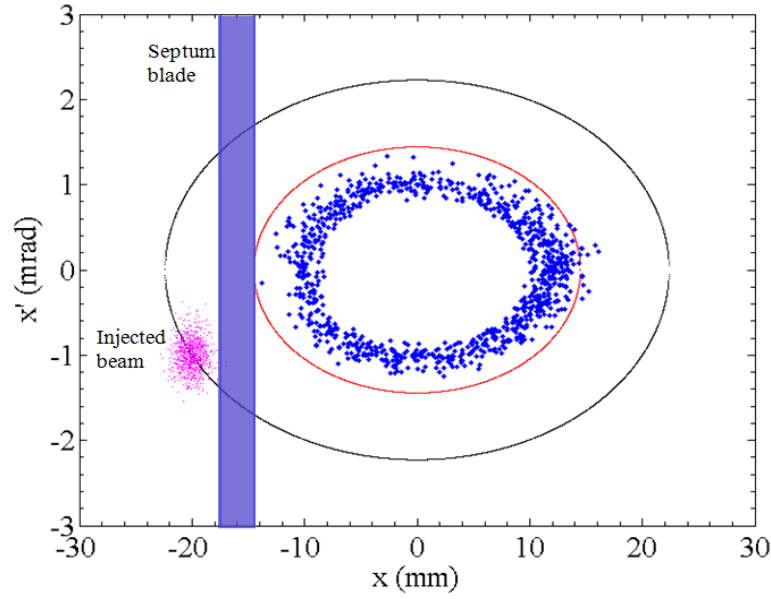


Figure 5.17: Beam injection with the PM at straight 2 of the Diamond lattice tracking for a hundred turns (blue dots) using 1000 particles. The black and red lines are the injection invariant and machine acceptance respectively.

phase space for a hundred turns and the kick angle (θ_{pm}) applied to the injected particle to reduce its angle to be inside the machine acceptance. We observe that the injected particle follows a distorted ellipse in phase space indicating the nonlinear effects from sextupoles.

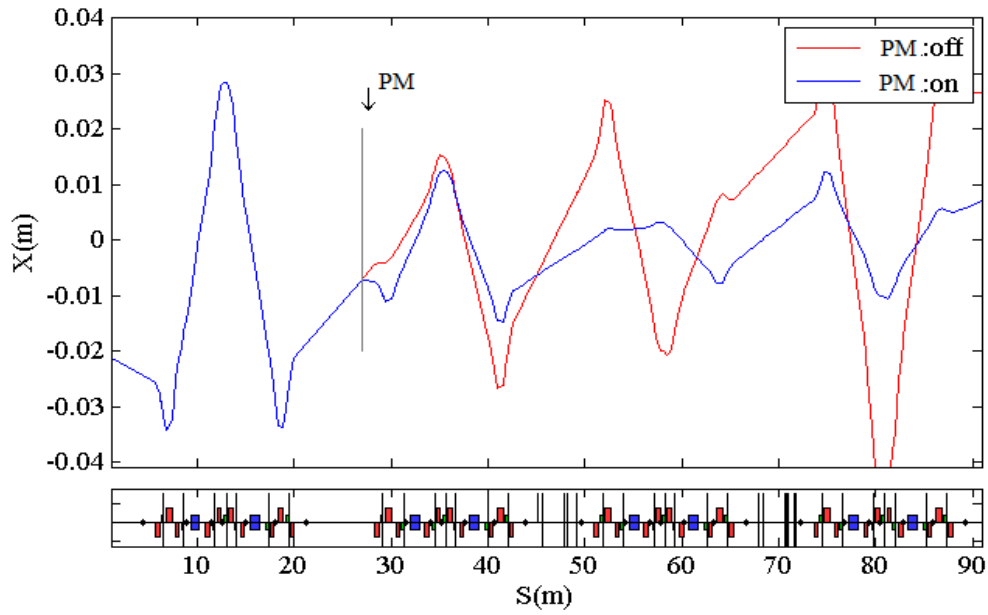


Figure 5.18: Single particle tracking for first turn with and without the PM.

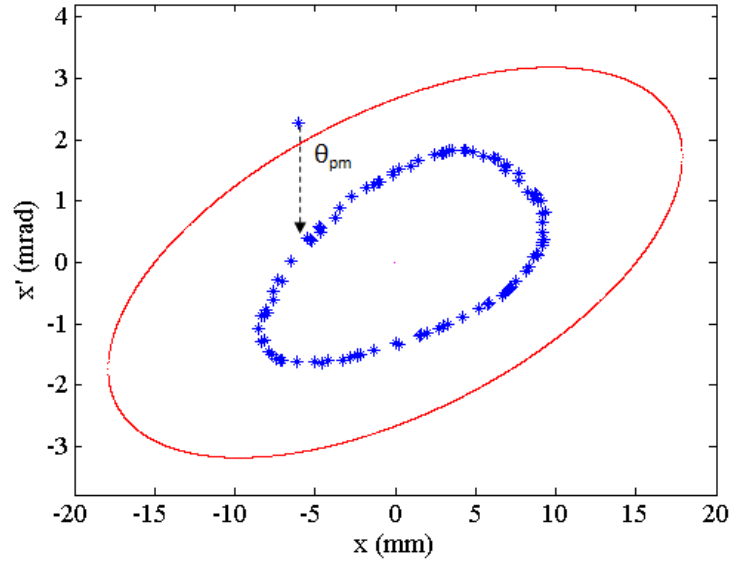


Figure 5.19: Single particle phase space at the PM after 100 turns.

The previous optimisation was verified with tracking of a larger number of particles (10,000 particles) performed in Elegant including the full engineering apertures. The same conditions described in Table 5.1 were used for the injected beam as shown in Figure 5.20.

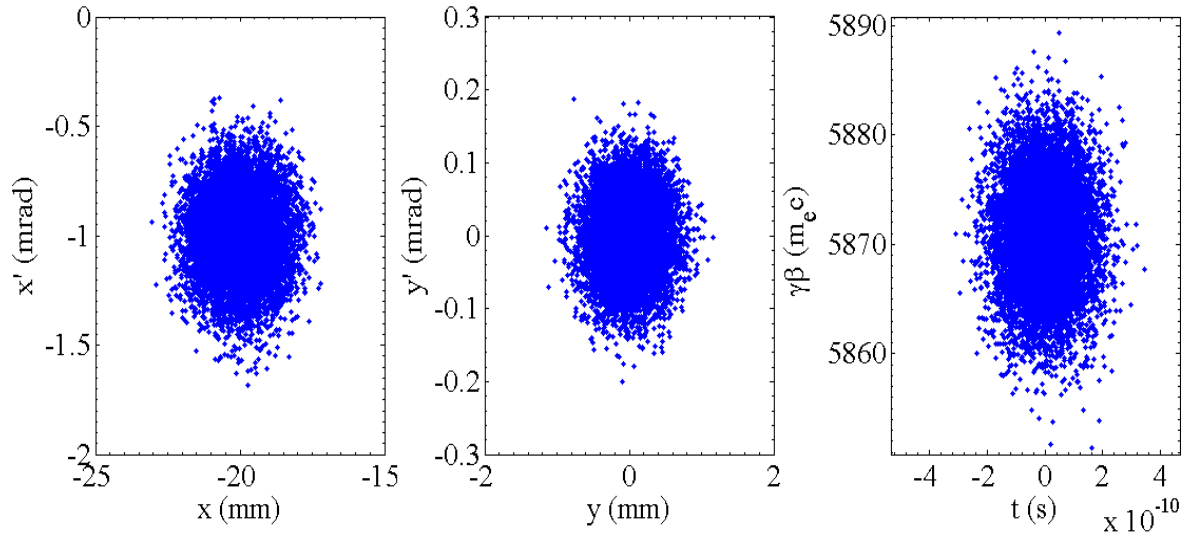


Figure 5.20: Injected beam of 10k particles generated from the booster used for tracking in Elegant. Elegant uses different longitudinal coordinates from AT ($\gamma\beta$ instead of $\Delta p/p$ and t instead of ct).

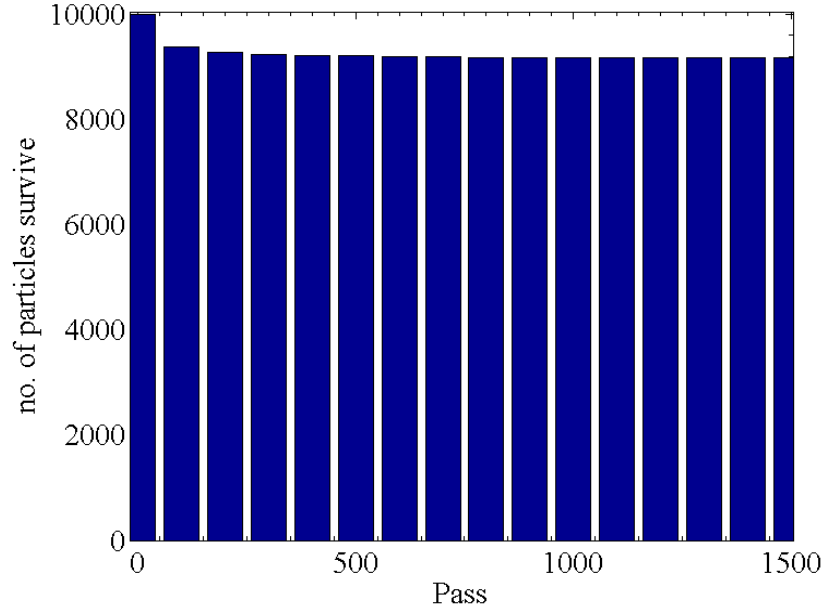


Figure 5.21: Number of surviving particles after tracking using the initial 10,000 particles with Elegant including physical aperture for 1500 turns.

The calculation was distributed to a cluster in order to speed up the particles tracking. The injection efficiency calculated from the number of particles surviving after 1500 turns is shown in Figure 5.21 and is larger than 90%. The particle losses distribution along the ring concentrates mainly in narrow vertical insertion devices' gaps as described in Figure 5.22, while horizontal losses occur mainly at the septum.

The schematic view of the cross section of the PM used for beam injection in Diamond is shown in Figure 5.23. It is based on modification of the design with four parallel wires operating in Bessy-II [83].

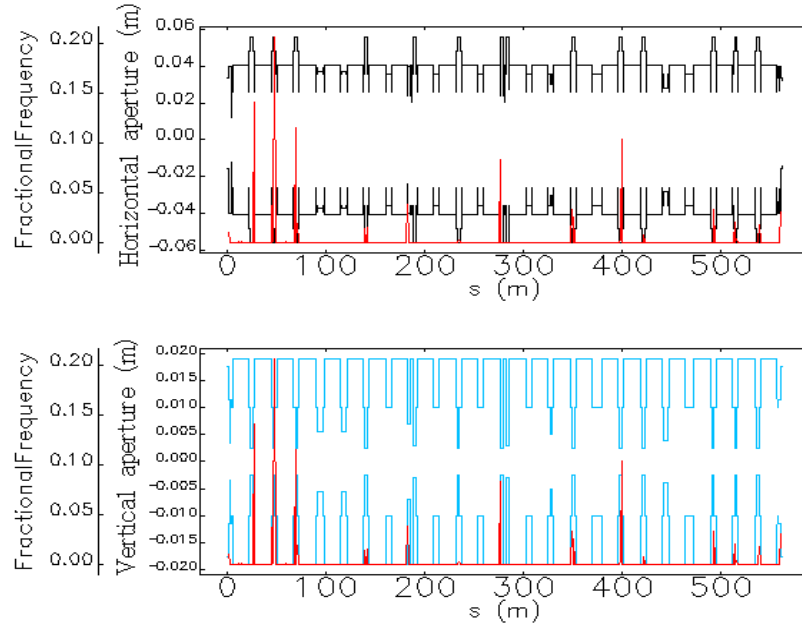


Figure 5.22: Particle loss distribution along the machine in the horizontal plane (top) and vertical plane (bottom). The fractional frequency is the local loss normalised by the total number of losses.

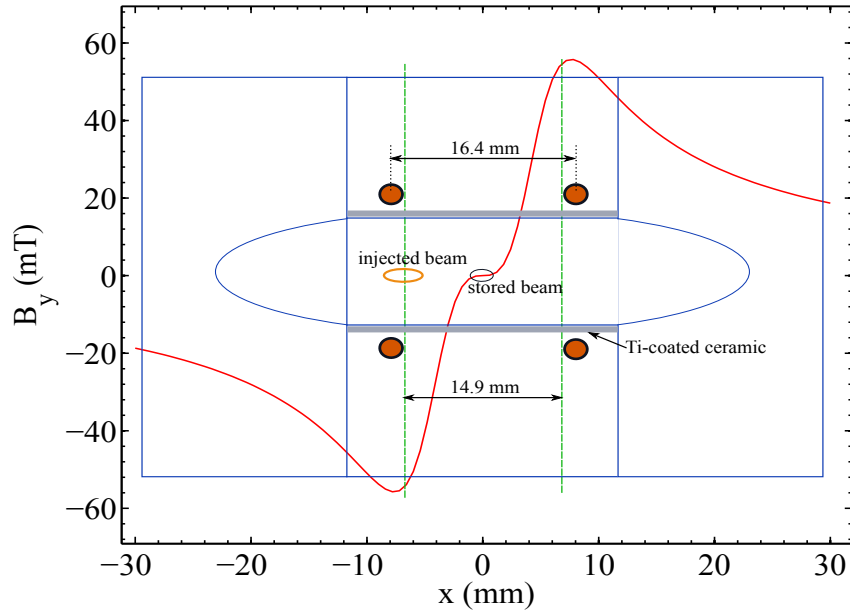


Figure 5.23: Vertical magnetic field with respect to the schematic view of the cross section of the PM which composed of 4 parallel wires.

Straight 14, which is located 308 m downstream the injection point, is also a possible location for the PM kicker according to the analysis of the relative phase advance. However, the long distance from the injection point leads to strong nonlinear effect due to the large number of nonlinear kicks which act on the injected beam at large amplitudes. The study was performed using the same injected beam injected with a horizontal offset at -20 mm. Choosing an injection angle of 0.3 mrad, the centroid of the injected beam arrives at the position of the PM at the straight 14 at the horizontal position (x_{pm}) of -6 mm. The most effective value for the PM kick was calculated to be 1.5 mrad. From the horizontal phase space of the first five turns shown in Figure 5.24, it is clearly seen that the injected beam was distorted because of the large nonlinear kicks due to the sextupoles located between the injection point and straight 14. In these conditions the effect of the PM kick enhances even further the beam distortion since it delivers a further inhomogeneous kick to the injected particles which depends non-linearly on their horizontal positions. The injection efficiency is reduced to about 70%. Furthermore, even in the case of an ideal flat dipole kick, the injection efficiency is only 80%. In conclusion, the injection with the PM in straight 14 results in worse injection efficiency with respect to the PM location closer to the injection

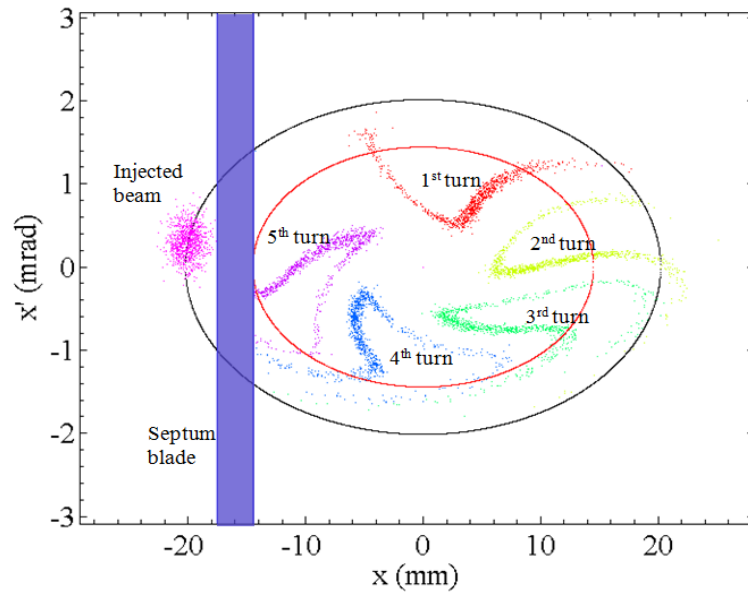


Figure 5.24: Injection with the PM at the straight 14 of Diamond lattice tracking for first five turns using 1000 particles.

point and was not investigated further.

5.3.3 Effect of pulsed multipole kicker on the stored beam

In order to operate with a transparent Top-Up injection the beam injection process should not perturb the stored beam. To investigate the effect of the new injection scheme, the stored beam was tracked while the pulsed kicker was excited as done during the injection study. The nominal stored beam was assumed to have a horizontal natural emittance of 2.75 nm-rad and a beam size of 0.45 mm at the position of the PM in straight 2. Three cases were investigated: pulsed quadrupole (PQM), pulsed sextupole (PSM) and pulsed multipole (PM) kicker. All the pulsed kickers were designed to provide the same kick of 2.2 mrad to the injected beam at the required position as discussed in the previous paragraph. Particles tracking was performed with 5000 particles only for the first turn after the injection kick as this is already sufficient to highlight the perturbation on the stored beam. The horizontal phase space obtained from 6D tracking at the position right after the pulsed kickers was recorded without any pulsed kicker, and with beam excitation with PQM, PSM or PM kicker. Figure 5.25, shows the horizontal phase space of the stored beam after pulsed kicker excitation (blue dots) with respect to the reference case (red dots) visually indicating the influence of the excitation to the stored beam.

As expected, it is clear that the pulsed kickers with lower order magnetic multipoles generate more disturbance to the stored beam as the comparison of the phase space coordinates with nominal beam shown. The PQM strongly perturbs the stored beam. The phase space of the stored beam in the case of PSM and PM shows a significant change in angle especially in the PSM for large amplitude. To clearly see the effect of the pulsed kicker on the stored beam, the difference of the phase space coordinate ($\Delta x, \Delta x'$) is also plotted on the right hand side of Figure 5.25. The PM is obviously the best choice providing the smallest perturbation on the stored beam as compared to the PQM and PSM.

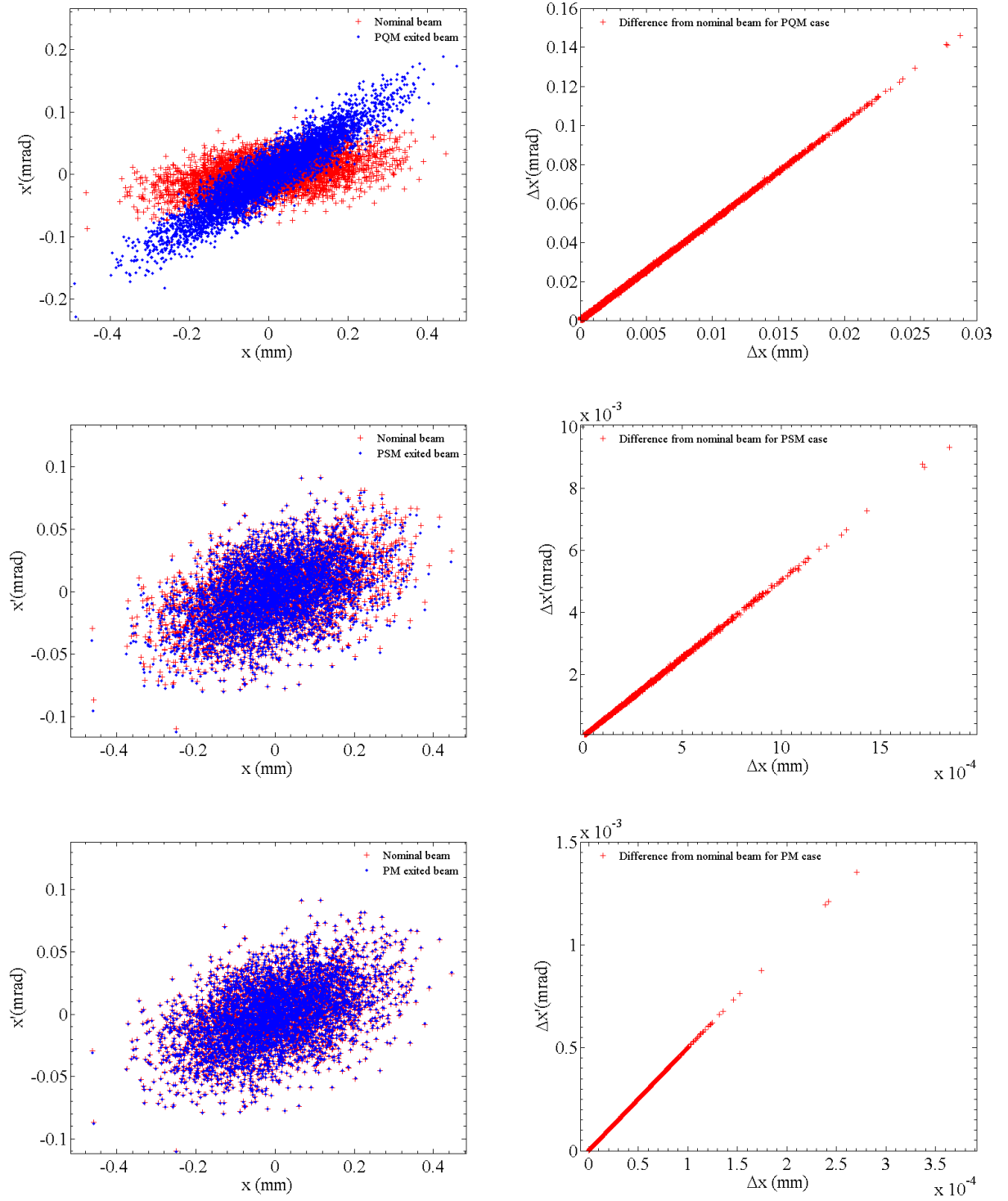


Figure 5.25: Pulsed kicker effects on the stored beam for the standard Diamond lattice, Left: Horizontal phase space of the stored beam affected by a pulsed kicker (PQM, PSM and PM) (blue) with respect to the nominal beam (red). Right: Phase space coordinates (absolute) difference caused by a pulsed kicker with respect to the nominal stored beam.

5.4 Pulsed multipole injection in the one DDBA lattice

We now want to consider the possibility of using a nonlinear injection kicker scheme for the modified Diamond lattice with the single DDBA cell. We have seen in Chapter 3 that with the existing conventional four kickers injection, the injection efficiency can be more than 90%. From the simulation with the standard Diamond ring, it can be proven that the pulsed multipole kicker (PM) provides comparable injection efficiency to the original injection scheme. This encourages the possibility of the beam injection using a PM in the new one DDBA lattice as well.

We started locating the PM as in the same position as in the previous study at the end of second straight section. As the new lattice has different operating tunes, the phase advance between the injection point and the position of the PM was only slightly changed. We have repeated the previous optimisation procedure for the case of the DDBA cell lattice.

5.4.1 Pulsed multipole optimization for the one DDBA lattice

PM injection at second straight of the one DDBA lattice

The PM was modelled at the end of the second straight just before the new DDBA cell in cell 2. Again Accelerator toolbox (AT) was used for PM injection optimisation considering ideal machine with septum blade as a physical aperture only. The position of the injected beam at the nonlinear kicker was evaluated as a function of the initial injection angle, with 1.15 mrad giving the horizontal position and angle of the injected particle at the PM of -8 mm and 1.2 mrad respectively. About 90% injection efficiency can be achieved with the optimised realistic PM kick angle of 2.2 mrad. However, more thorough studies carried out with Elegant including a full detailed physical aperture, showed a dramatic reduction of the injection efficiency to about 30%. The corresponding particles losses distribution of the tracked particles with respect to the physical aperture is shown in Figure 5.26.

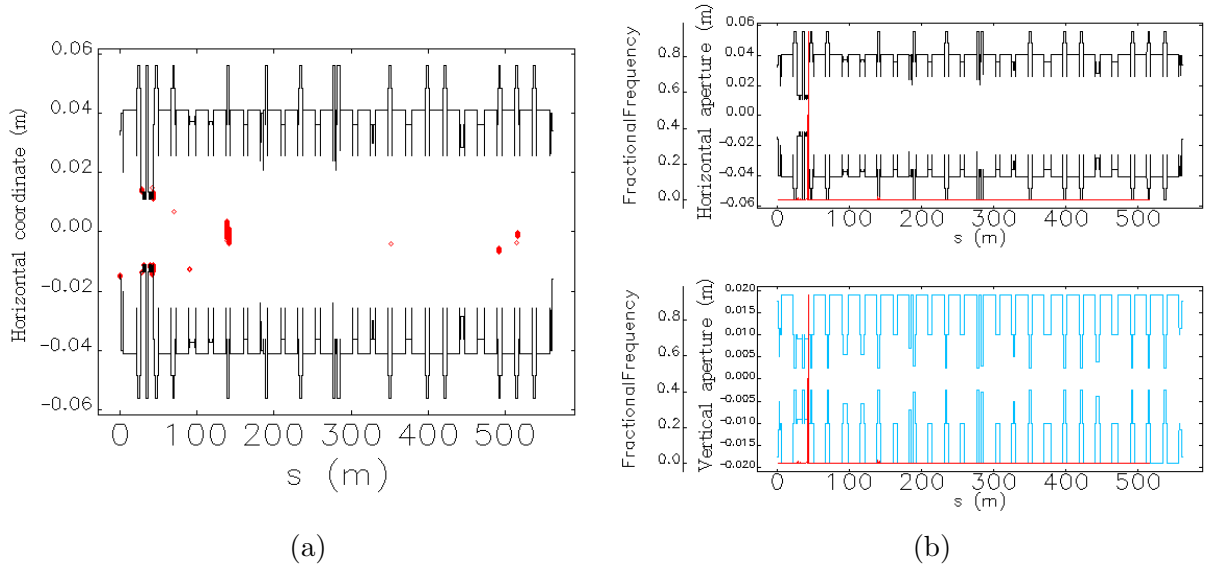


Figure 5.26: Particle loss distribution along the ring from injection tracking with the PM at the 2nd straight: (a) horizontal coordinate of the particles losses (red dot) with respect to the horizontal aperture, (b) particle loss histogram (red line) at each position with respect to horizontal (top) and vertical (bottom) aperture.

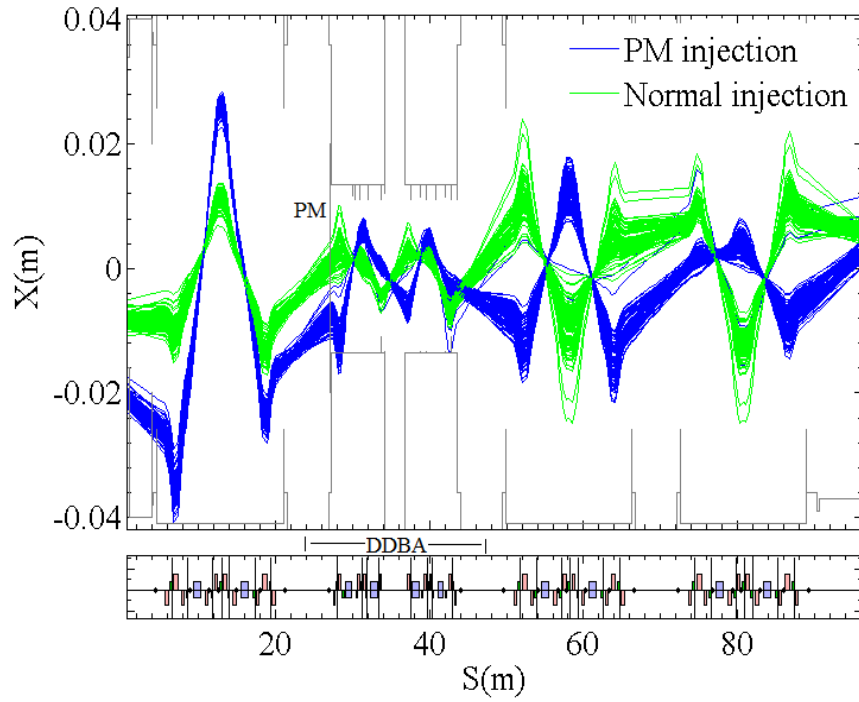


Figure 5.27: First turn injection trajectory for the normal injection (green) and the PM injection at the 2nd straight (blue) using 100 particles with respect to the horizontal physical aperture (grey).

Figure 5.26a shows that the loss is dominated by the horizontal aperture in the DDBA cell. The beam loss point of the injected particles is located at the DDBA section which has a narrow horizontal chamber as shown in Figure 5.26b with more than 90% of injected beam loss. Only a small fraction of losses occurs due to the vertical aperture in I07. To investigate the losses of injected beam, the trajectories of the injected particles along the machine were compared with the trajectory of the particles injected with the standard injection scheme with the injection point at -8.3 mm off-axis. We considered initially only the beam centroid; the trajectories from both injection schemes gave comparable oscillation well within the physical aperture after the PM in the second straight section.

To investigate the behaviour of the particles with large amplitude, the injected beam was represented by 100 particles and was tracked along the ring. The trajectories shown in Figure 5.27 indicate that some particles during the injection with the PM actually oscillate with large amplitude beyond the horizontal aperture limit in the DDBA cell (about 30 m downstream) especially in the negative side. The beam trajectory in the normal injection can instead pass through the aperture. This observation again suggests that PM injection critically requires injected beam of small transverse dimension to minimise the adverse effect of the nonlinear kick as a function of the horizontal position of the particles. An attempt to optimise the injection efficiency was performed using smaller injection amplitude and angle. Using -17 mm offset and -1 mrad injection angle, the injection efficiency can be improved to 60% but the majority of particles losses is still concentrated in the DDBA cell. This injection efficiency is too low to be proposed for Top-Up operation.

Optimisation with MOGA for PM injection in the second straight

Since the injection efficiency decreases dramatically when the realistic tracking studies including the narrow physical aperture are performed, we resorted to the optimisation of the injection efficiency based on 6D tracking with physical aperture Multi-objective genetic algorithm (MOGA). In this study, the main objectives were injection efficiency and initial injection angle. The variables used in the optimisation were the horizontal injection offset

(x_{inj}) , injection angle (x'_{inj}) and the PM current. In this optimisation, the distance between the PM's wires was fixed. The minimum horizontal injection position was -18 mm while the maximum injection angle was allowed to increase upto 2 mrad. The maximum current applied for the PM was also limited to 1.2 kA. Figure 5.28 shows the optimal front found by MOGA. The solution giving the best injection efficiency, about 60%, was obtained with an injection angle of -0.88 mrad and injection position at -18 mm and the PM current of about 720 A. From the shape of the optimal front, we infer that the injection efficiency decreases with the reduction of the injection angle. Because the injection angle determines the position of the injected beam at the location of the PM small injection efficiency is expected due to the fact that the injected beam does not hit the peak of the magnetic field in the PM. No significant improvement in the injection efficiency was found in these conditions.

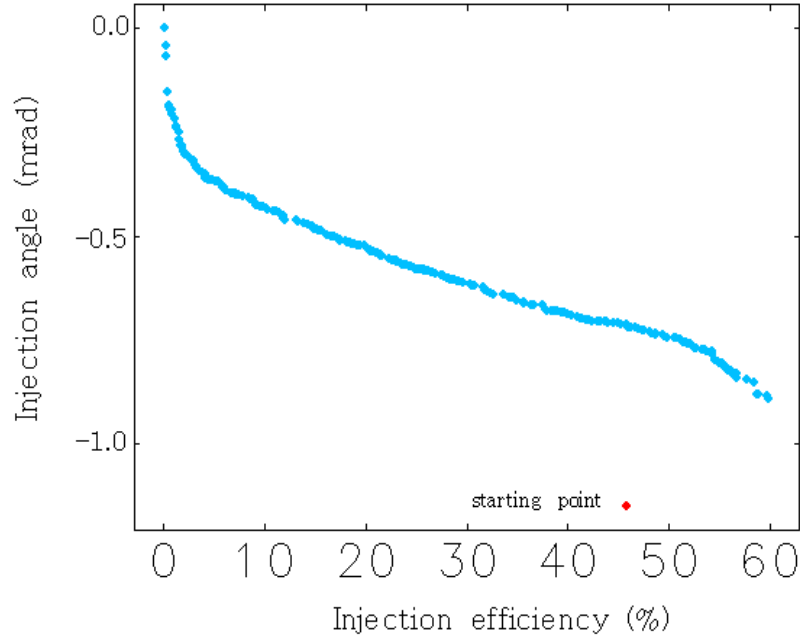


Figure 5.28: Pareto optimal front for the simulation of PM injection at the 2nd straight using angle and injection efficiency as objectives.

PM injection at the first straight of the one DDBA lattice

Given the difficulty of achieving good injection efficiency with the previous arrangement of the PM modelled in the straight 2, we consider other possible locations for the PM. It was found eventually that placing the PM upstream in the first straight section provides the best option. The first straight in Diamond storage ring hosts the four kickers used for conventional injection which are intentionally conserved. Due to these space constraints, the PM was modelled at 4 m downstream from the injection point after the kicker 4 and close to the end of the first straight section and the existing injection system was kept untouched.

The same process used in the PM optimisation for the standard Diamond lattice was employed allowing us to determine the conditions for the injected beam and the PM parameters required to achieve good injection efficiency. A scan of the injection angle was performed. The horizontal injection position was the same as in the previous cases at -20 mm. From Figure 5.29, the injection angle of 3 mrad provided a horizontal position of the injected particle arriving at the PM (x_{pm}) of about -7 mm and a horizontal angle about

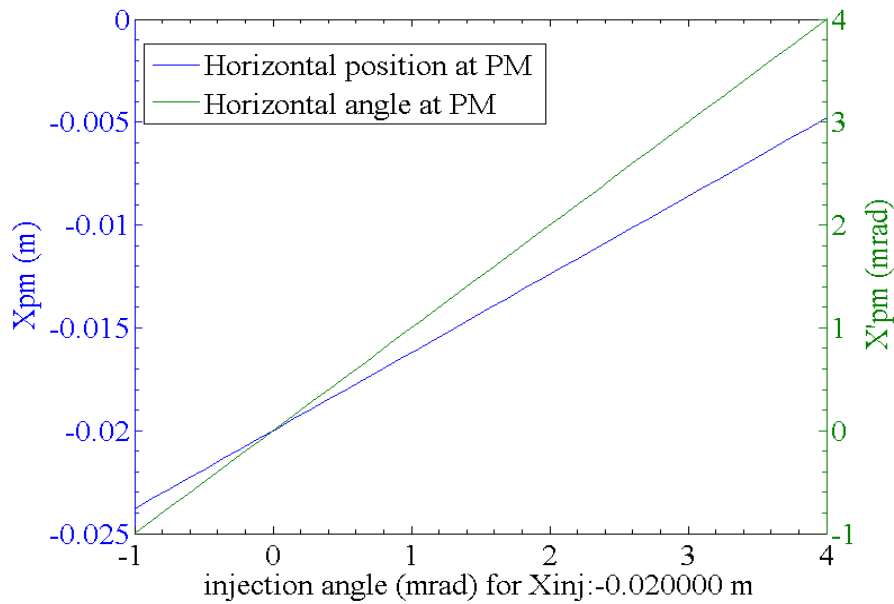


Figure 5.29: Horizontal position (x_{pm}) and angle (x'_{pm}) at the position of PM in the 1st straight of one DDBA lattice as function of injection angle (x'_{inj}) for the injection amplitude (x_{inj}) at -20 mm.

3 mrad. As x_{pm} determines the distance between wires of the PM, the value can indicate roughly the minimum half distance between the PM's wires. Ideally a smaller injection angle is preferable in order not to modify significantly the septum setting and the booster to storage ring transfer line (BTS). However, Figure 5.29 shows that small injection angle gives large horizontal position at the PM (x_{pm}) which limits the minimum reduction of injection invariant. Furthermore in these conditions, it is more likely for the injected particles to hit the narrow horizontal physical aperture in the DDBA cell.

Using the selected injection angle of 3 mrad, a Gaussian beam of 1,000 particles was tracked to study the injection efficiency variation as shown in Figure 5.30. The minimum required kick angle was found to be 3 mrad that achieved good injection efficiency. The injection efficiency dropped when the kick angle was increased at -4 mrad.

The PM was designed with the distance between wires of 7.5 mm with a nominal current of 990 A to generate the required kick of -3.2 mrad at the off-axis position of the injected beam of about -7 mm as shown in Figure 5.31. A test with the new generated PM was carried out with 1500 turn injection tracking of a thousand particles using the PM for the first turn

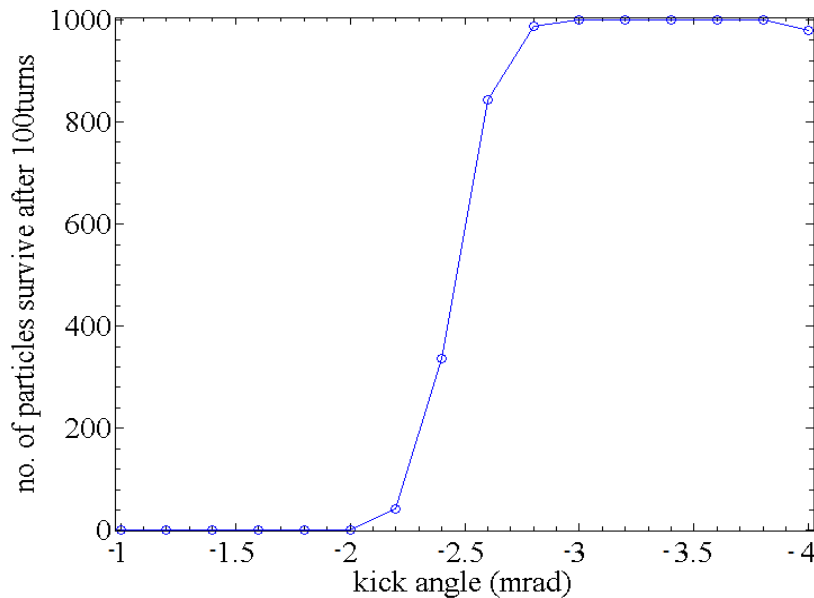
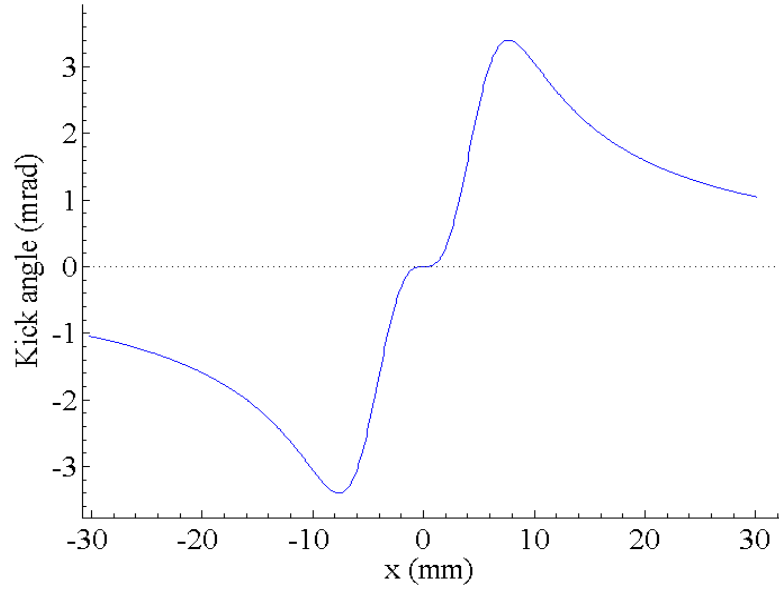


Figure 5.30: Number of surviving injected particles after tracking 100 turns for different kick angles applied by the PM in the 1st straight.

Figure 5.31: Kick angle along the horizontal axis at $y=0$.

only. As shown in Figure 5.32, the tracking with AT, considering only the septum blade as the physical limit, provides more than 99% injection efficiency. The horizontal phase space of the injected beam during the first five turns is shown in Figure 5.33. Starting with the injected beam (pink), using with the PM kick in the first turn only, we can verify that the

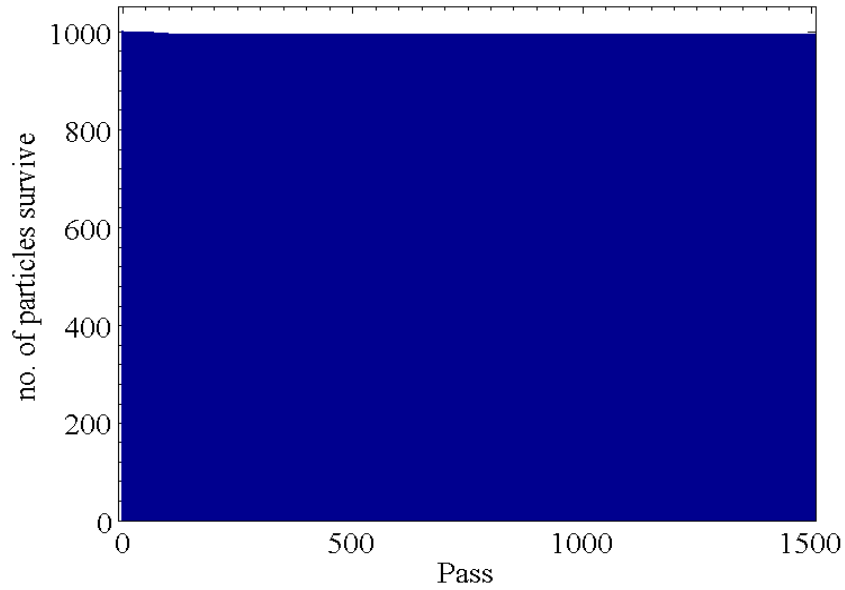


Figure 5.32: Injection with the PM at the 1st straight of one DDBA lattice tracking for 1500 turns using 1000 particles.

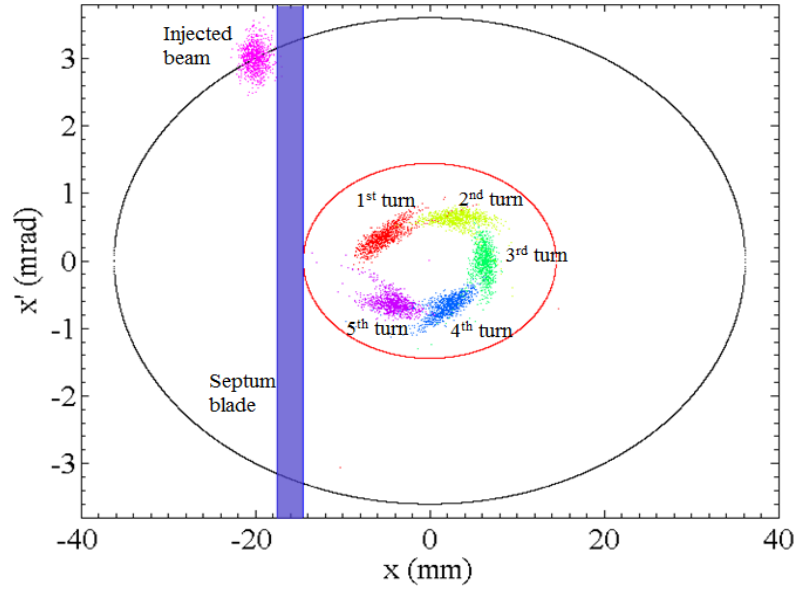


Figure 5.33: Injection with the PM at the 1st straight of the one DDBA lattice tracking for the first five turns using 1000 particles. The injected beam invariant (black line) can be reduced to be within the machine acceptance (red line) and the beam was captured

invariant can be reduced to be well within the machine acceptance (red ellipse). Although the shape of the beam after being excited by the PM kick was slightly changed because of its finite transverse size and the position dependence of the kick, most of the injected particles are captured.

To see clearly the trajectory of the injected beam, a single particle representing the beam centroid was tracked for one turn along the ring. The effect of the kick delivered by the PM is described in Figure 5.34. A dramatic reduction of the trajectory amplitude from red line (without the PM) to blue line (with the PM) is visible. The trajectory of the injected particle without the PM kick (red line) keeps increasing along the machine and the particle gets lost eventually. Notice that the amplitude of the injected beam trajectory in the DDBA section (at about 30 m from the injection point) is about 50% smaller compared to the previous case when the PM was located in the straight 2 in Figure 5.27. The phase space of a single particle tracking for a hundred turns at the position right after the PM, shown in Figure 5.35, demonstrates clearly the reduction of the angle of the injected particle introduced by the kick angle of the PM (θ_{pm}).

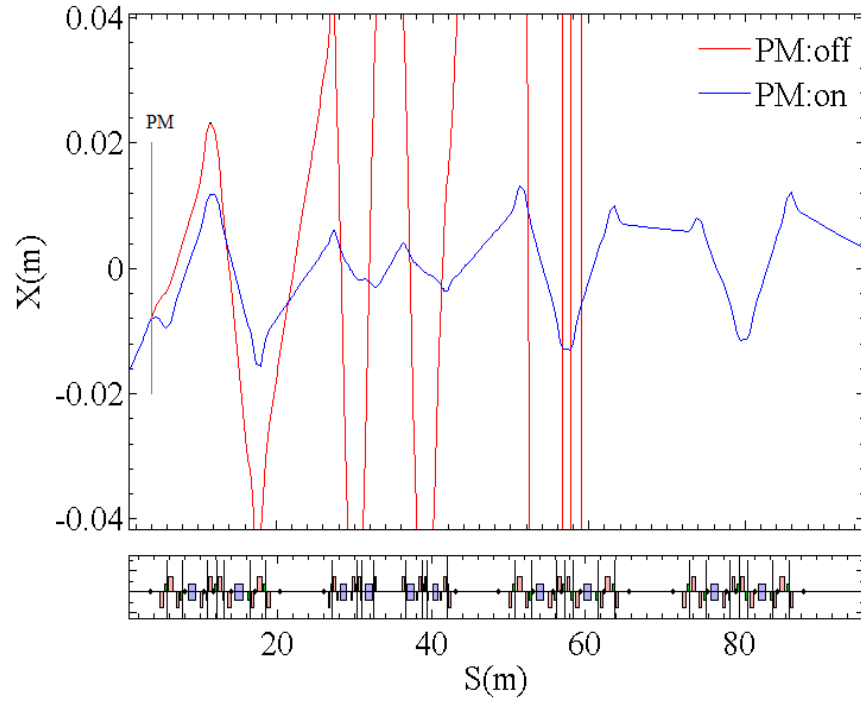


Figure 5.34: First turn injection trajectory with and without the PM in the 1st straight. The injected particle is lost in cell 3 without the PM (red) while the trajectory is reduced significantly when the PM was applied (blue).

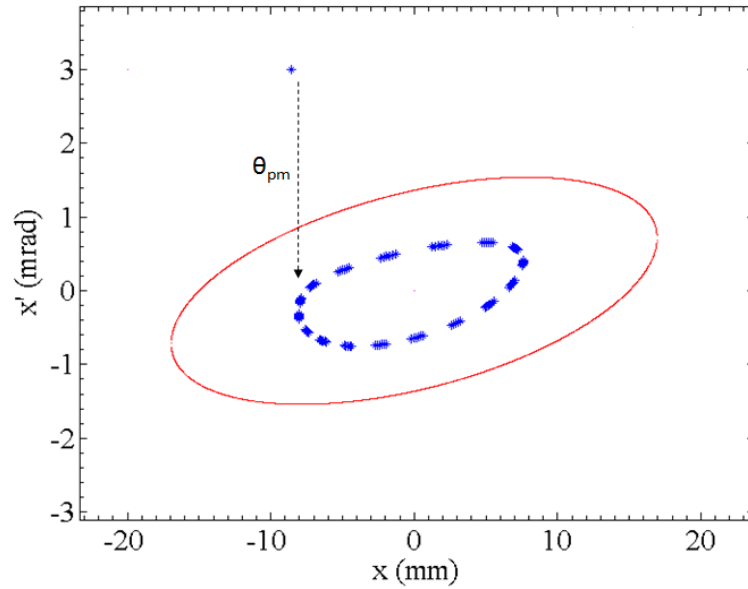


Figure 5.35: A single particle tracking for 100 turns at the position after the PM. The particle was successfully kicked inside the machine acceptance (red ellipse) by the angle θ_{pm} .

After the preliminary optimisation of the beam injection with the PM in the one DDBA lattice using AT, more realistic simulations with a larger number of particles and full detailed physical aperture have been performed with the 6D tracking in Elegant for 1500 turns. The injected beam was modelled with ten thousand particles generated with the previously optimised initial conditions: injection angle of 3 mrad and the injection position at -20 mm. The full detailed physical aperture was included and resulted in the injection efficiency of 83%. Unlike the previous study where the PM was placed in the straight 2, the majority of the particles losses now occur at the narrow gap ID I06 in the vertical plane not in the DDBA cell.

To check the improvement in the distribution of particle loss in the DDBA section, we repeated the first turn tracking with a hundred particles along the ring with a pulsed kick. The trajectories are reported in Figure 5.36 and indicate smaller oscillation of the injected particles compared to those in Figure 5.27 at the DDBA cell. The injected particles can pass through the narrow horizontal aperture in the DDBA cell, and as a consequence the

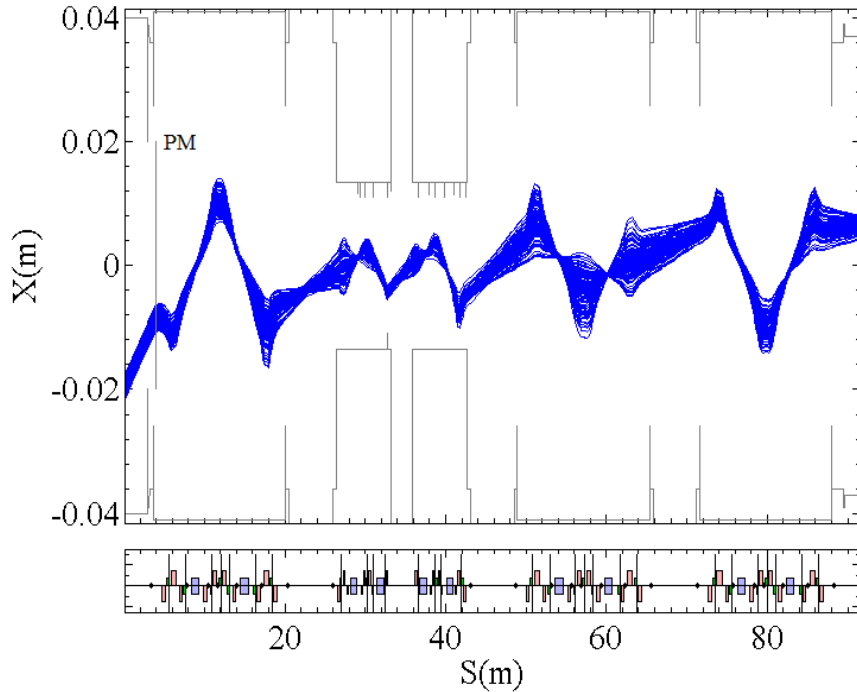


Figure 5.36: First turn injection trajectory of the injected beam for the PM located in the 1st straight using 100 particles. The horizontal physical aperture is plotted in grey.

injection efficiency is improved considerably to 83%. The effect of $\pm 1\%$ injected beam size variation on the injection efficiency was also considered and was found to have a negligible effect on the injection efficiency.

Optimisation with MOGA for the PM injection in the first straight

To further optimise the condition for beam injection, we use again MOGA as done in the previous case when the PM was placed in the second straight. In this study the aim was to find better injection efficiency while keeping the required injection angle small to reduce the requirement on the peak current of the kicker. The horizontal injection distance (x_{inj}), injection angle (x'_{inj}) and current for the PM were variables for the optimisation while the objectives were large injection efficiency and small injection angle. To optimise the injection with the PM, the injection position x_{inj} was allowed to be -22 to -18 mm and the current limit was set to 1.2 kA.

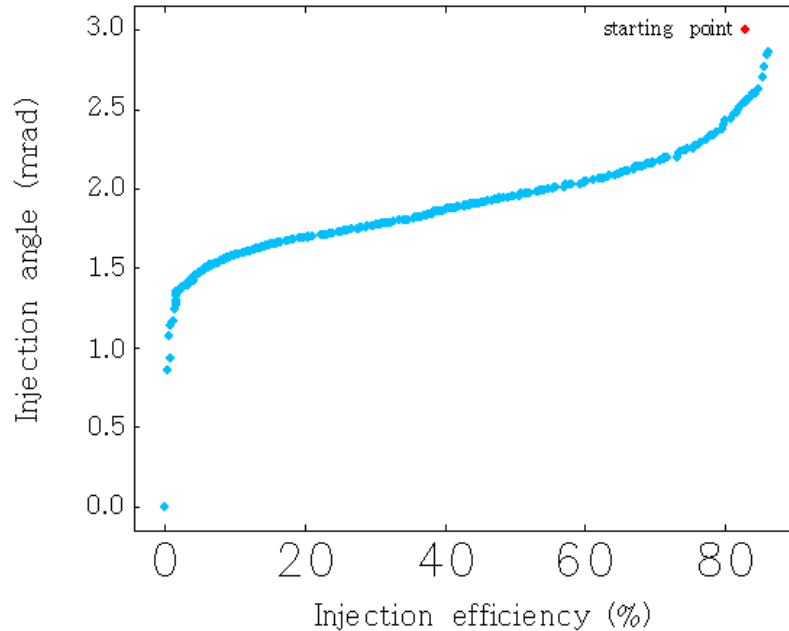


Figure 5.37: Pareto optimal front for injection angle and injection efficiency for the case where the PM injection kicker is located in the 1st straight section.

The optimal front shown in Figure 5.37, suggests that we can achieve an injection efficiency above 85%. The optimal solution with the injection efficiency of 86% needs an injection angle of 2.8 mrad which is smaller than the starting condition of 3.0 mrad. The injection position needs to be reduced to -19 mm from -20 mm while the current required is 990 A. These calculations assumed an ideal machine and the perfect PM magnetic field. Further simulation with imperfections were made to confirm the feasibility of this scheme.

5.4.2 Imperfections with the PM injection

The effects of machine imperfections on the conventional injection for the one DDBA lattice have been discussed already in Chapter 3. Similarly misalignments and errors were considered in the injection tracking with the optimised PM as described in Table 3.4. These include multipole errors in all magnets and rolls errors generating 0.3% coupling and random misalignments were also introduced later to the PM itself with transverse displacement amplitude of 100 m and roll error of 100 mrad. Fifty error seeds were used for comparing the cases where machine imperfections only and machine imperfections and PM misalignment were included in the simulations. The injection efficiency was found to be similar demonstrating the robustness against misalignments of the beam injection with the PM. Further imperfection of the PM where considered by assuming errors in the current supplied to the PM.

PM current error effect on injection efficiency

In the PM, current errors may occur because of temperature variation, induced heat in the wires and physical deformation of the structure. Two scenarios of the current error were considered. Initially the current was varied by assigning a random error of $\pm 1\%$ to one of the four wires of the PM. A kick map associated with a set of current including such errors was generated and used for the beam injection tracking for each of the different 50 seeds including misalignments and current errors. The worst case assumed that all four wires of the PM have different current errors within $\pm 1\%$. The distribution of the injection efficiency

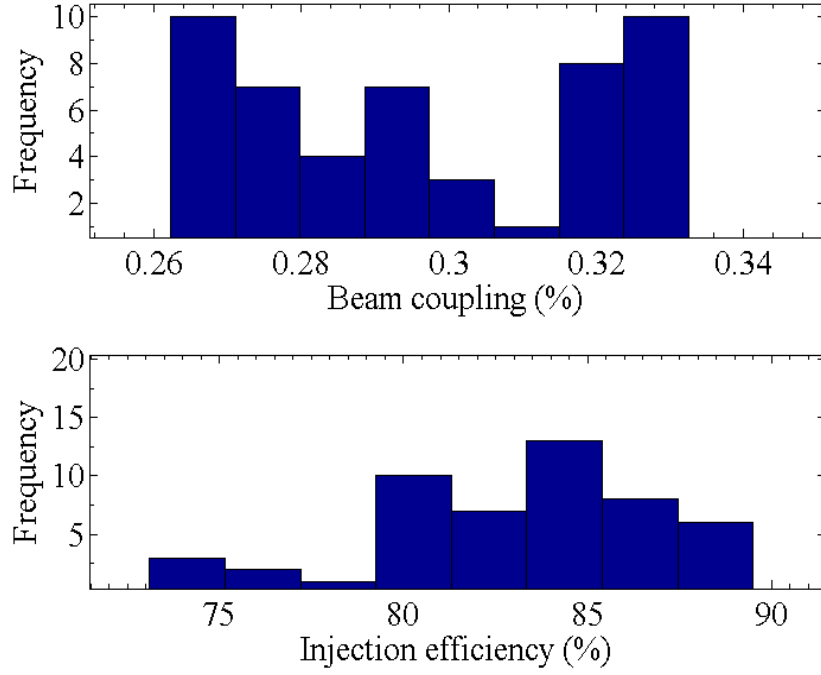


Figure 5.38: Beam coupling (top) and injection efficiency (bottom) for PM injection at the 1st straight for standard bunch length for different 50 error seeds.

for 50 error seeds, for beam coupling 0.3%, including all four wires current errors is shown in Figure 5.38. The injection efficiency was found to be $82.9 \pm 3.9\%$ which is only marginally lower than that of the ideal machine (86%).

One of the main reasons for the beam losses during injection is due to the large time duration of the electron bunches coming from the booster that are not matched to the storage ring RF bucket and therefore they undergo large synchrotron oscillations when they are injected in the storage ring. A possible upgrade of the booster RF cavity was considered by adding one more RF cavity to increase the RF voltage (2×0.9 MV) giving stronger phase focusing and shorter bunch length. As discussed in Chapter 1, the bunch length can be written as [90]

$$\sigma_l = \frac{\sqrt{2\pi}c}{\omega_{rev}} \sqrt{\frac{\eta_c E_0}{heV \cos \psi_s} \frac{\sigma_E}{E_0}}. \quad (5.9)$$

Thus, the bunch length of the injected beam can be reduced from 29.8 mm to 18.9 mm by increase the RF voltage (V). It was found that the injection efficiency can be improved to

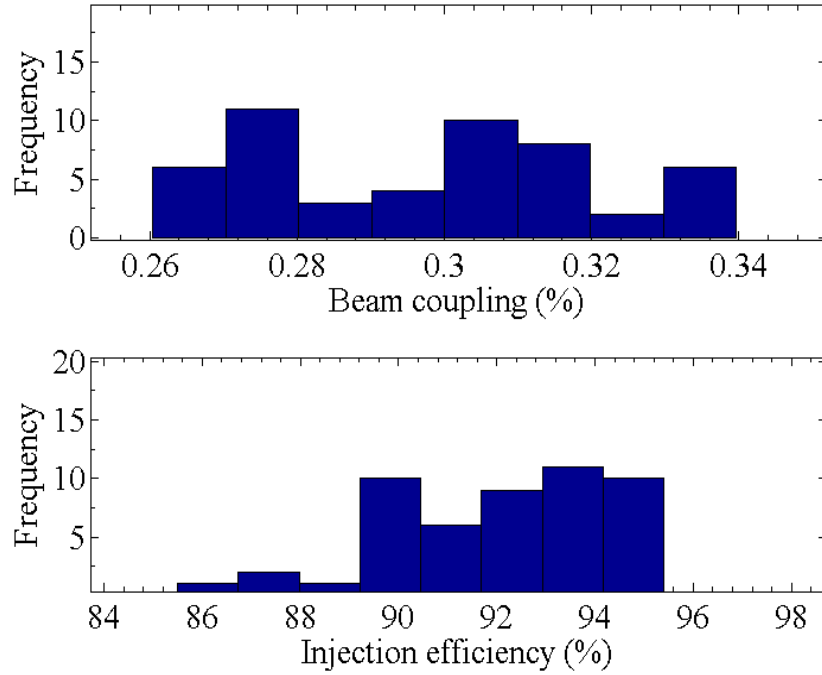


Figure 5.39: Beam coupling (top) and injection efficiency (bottom) for PM injection at the 1st straight for shorter bunch length for different 50 error seeds.

$91.9 \pm 2.3\%$, including all misalignment and $\pm 1\%$ PM current error, with the shorter booster bunch length as shown in Figure 5.39.

Effect of the PM on the stored beam

The effect of the PM kicker on the stored beam was investigated including three different pulsed magnets for injection (PQM, PSM, and PM) in the DDBA Diamond lattice. The stored beam was modelled with 5,000 particles with the emittance of $2.68 \text{ nm}\cdot\text{rad}$ and 0.3% coupling and was tracked through the pulsed magnet for the first turn only. The phase space coordinates were recorded right after the beam passed through the pulsed magnets. The size of the stored beam is $\sigma_x = 193 \text{ }\mu\text{m}$ at the location of the pulsed kicker. Misalignment and imperfection were not included in this study in order to put in evidence the pure effect of the different types of kick fields. All pulsed magnets were modelled to give the same kick to

the injected beam (3.15 mrad at 6 mm).

As seen in Figure 5.40, the largest effect on the stored beam due to the pulsed kick during injection is caused by PQM. The PM instead produced only a change in the horizontal phase space coordinate by $3\text{ }\mu\text{m}$ and $160\text{ }\mu\text{rad}$ giving the smallest effect to the stored beam. The beamsizes were changed by only 0.1% with respect to the stored beam without PM kick. The case of the PSM has a larger influence on the stored beam increasing the perturbation about 3 times with respect to the PM. The same conclusions are valid in the vertical plane where an initial beam size of $\sigma_y = 13\text{ }\mu\text{m}$ was considered at the position of the pulsed magnet.

In conclusion, the PM magnet for a single turn injection delivers the least perturbation to the stored beam and is more suitable for top-up operation which requires a transparent beam injection. This analysis confirms the result from the turn-by-turn measurement of beam oscillations during the injection with the PM made at Bessy-II [83].

During the operation of the PM injection at Bessy-II, it was found that the currents applied to the 4 wires of the PM were not perfectly identical. The asymmetry of the applied currents can introduce perturbation to the stored beam during the injection. Therefore, the effect of imperfection of the actual current applied to each wire has to be investigated.

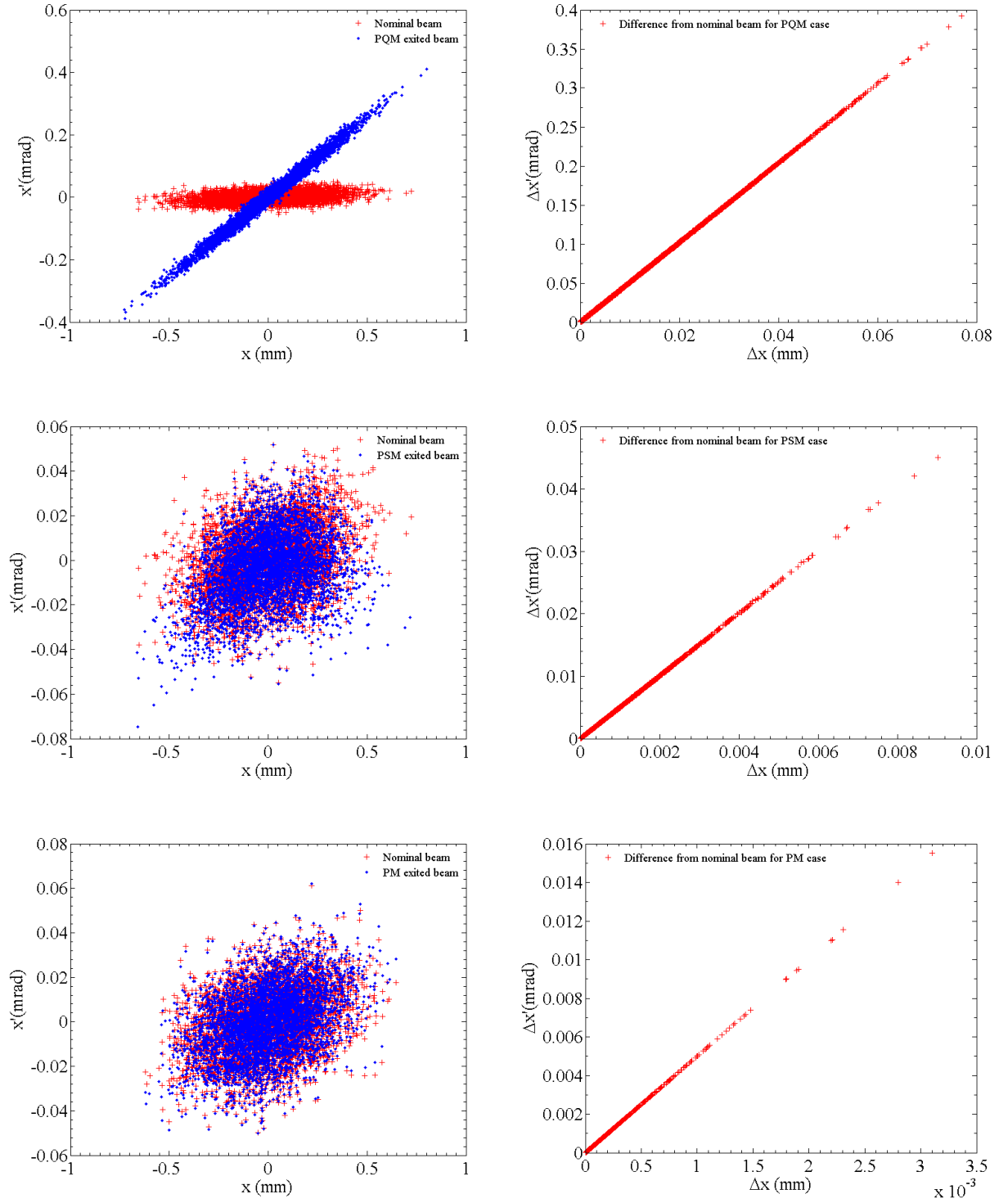


Figure 5.40: Pulsed kicker effects on the stored beam for the one DDBA lattice, Left: Horizontal phase space of the stored beam after the pulsed kicker (PQM, PSM and PM) (blue) with respect to the nominal beam (red). Right: Phase space coordinates (absolute) difference caused by a pulsed kicker with respect to the nominal stored beam.

Effect of the PM current error on the stored beam

The current error applied to the wires of the PM was also taken into account by numerical simulations. Although the effect of current errors of the wires in the PM is negligible on the injection efficiency, as described in the previous study, their effect on the stored beam has to be understood. Random current errors with a maximum of $\pm 1\%$ were applied to each of the four wires of the PM generating non-zero kick angle at the centre of the PM hence affecting the stored beam can be affected. As shown in Figure 5.41, 20 seeds errors generate different kick angle to the stored beam during injection.

Similar to the injection tracking study, the stored beam was observed during the injection after the PM kick. The PM was turned on only for the first turn, including the current wire errors to excited the stored beam of 10,000 particles and the tracking was followed without the PM for 100 turns. The results of the tracking of 20 error seeds, shown in Figure 5.42, give a beam centroid variation of $-1.3 \pm 42.1 \mu\text{m}$ and beam size variation of $1.35 \pm 1.60\%$ compared to the nominal stored beam.

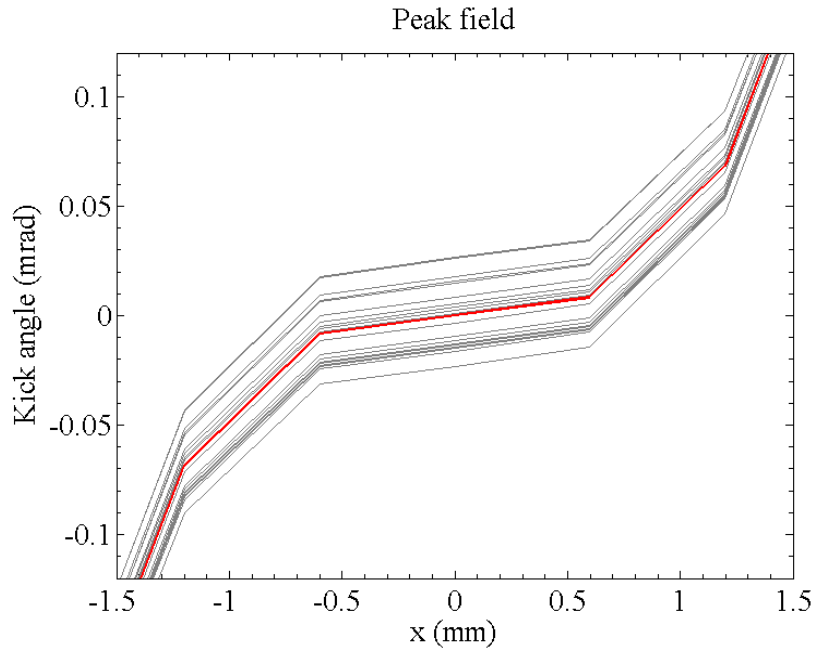


Figure 5.41: PM kick angle at the center due to random current error in four wires of 20 seeds (grey) compared to the ideal kick angle (red).

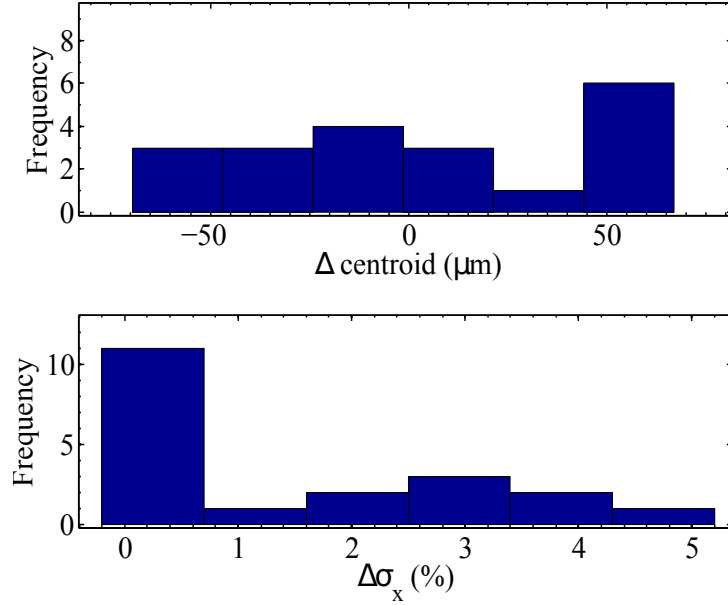


Figure 5.42: Beam centroid and beam size difference due to the PM random $\pm 1\%$ current error of 20 seeds compared to the nominal stored beam.

In practice, the pulse duration of the pulsed kicker is limited to a few μs and nearby buckets in the bunch train, separated by 2 ns (500 MHz) will be affected differently. In order to fully investigate the effect of time profile imperfection of the PM pulse, the effects of the time profile of the current applied to the PM excitation on the stored beam was considered. A half-sine wave pulse is normally used for injection kickers. The generated field rises from zero to the peak field then fall back to zero again to close a complete pulse cycle. To ease the design of the pulsed power supply, a half sine wave pulse with the pulse length of twice of the revolution period (T_{rev}) as shown in Figure 5.43 was used. In this way the injected beam is kicked only once and the beam will see zero field in the second turn [91]. For Diamond storage ring case, the pulse length can be $2 \times T_{rev} \approx 3.7\mu\text{s}$. Different bunches in the train will be kicked differently. In these conditions, part of the bunch train can actually be kicked twice, i.e. at two consecutive turns by the PM kicker.

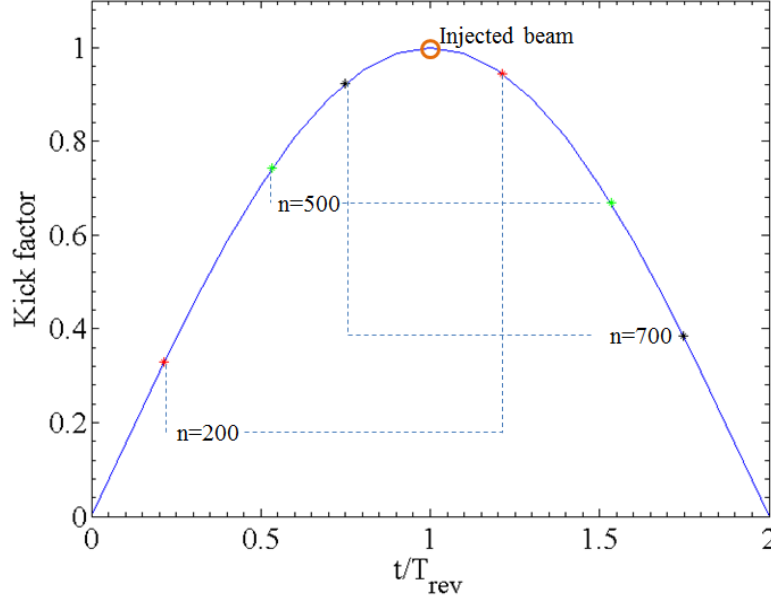


Figure 5.43: Schematic half-sine pulsed shape applied to the injected beam in term of kick factor exciting the stored beam in bucket 200, 500, and 700. The bucket number was selected to represent three possible different cases.

The kick factors for the first and second turns for each bucket ($K_{1,n}$ and $K_{2,n}$) can be calculated from:

$$\begin{aligned} K_{1,n} &= \sin\left(\frac{n}{h} \frac{\pi}{2}\right), \\ K_{2,n} &= \cos\left(\frac{n}{h} \frac{\pi}{2}\right), \end{aligned} \quad (5.10)$$

where n is the bucket number, and h is harmonic number (936 for Diamond).

We considered three different cases shown in the Figure 5.43: bucket 200 which gets kicked by small amount in the first turn and a large kick in the second turn, bucket 500 receiving similar kick for both turns and bucket 700 which experiences a large kick in the first turn and a small kick in the second turn. The magnetic field is generated including the current errors as shown in Figure 5.44 and can affect the stored beam differently depending on the bucket number. The stored beam simulated with 10,000 particles was tracked for 100 turns: the first two turns include the excitation of the PM kicker. The beam centroid and beam size were recorded and compared with the nominal beam without any excitation.

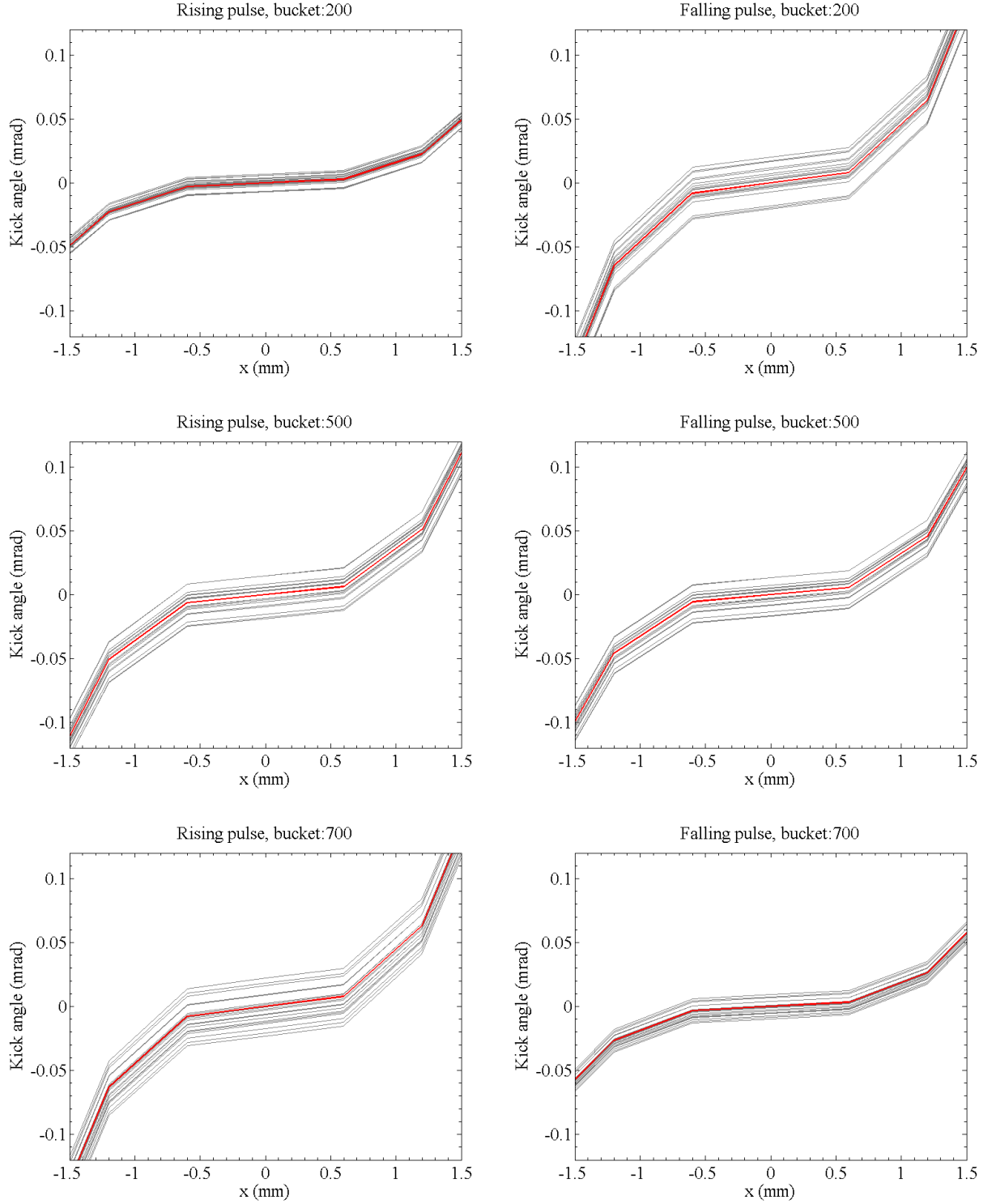


Figure 5.44: Horizontal kick angle including different random PM current error ($\pm 1\%$) of 20 seeds which can affect the stored beam in bucket 200, 500 and 700 during the pulse rising (left) and falling (right). The ideal kick angle is plotted in red.

Table 5.2: Beam centroid and beam size variations of the stored beam due to random current imperfections $\pm 1\%$ and pulse shape in the PM

Bucket number	Δ Beam centroid (μm)	Δ Beam size(%)
Peak field	-1.30 ± 42.10	1.35 ± 1.60
200	-2.05 ± 92.70	2.73 ± 5.11
500	-4.01 ± 95.65	1.45 ± 5.95
700	-4.75 ± 79.28	0.83 ± 5.53

It is clear that even if the different buckets receive different kicks because of the pulsed shape, the effect of random current error is similar as described in Table 5.2. Compared to the disturbance to the stored beam excited by the current error in the PM at the peak field, the beam centroid and beam size differences were doubled in the other buckets as they experienced the kicks twice. The maximum beam centroid and size variation are $100 \mu\text{m}$ and 7% respectively. This is a significant variation but still better than the residual kick measured during the standard injection with the four kickers injection system. The current errors in the wires have to be carefully avoided to keep the stored beam oscillation small during the injection. If such errors are unavoidable, the effect of the perturbation can be minimised by adjusting the stored beam closed orbit to align the stored beam with the magnetic centre of the PM.

5.5 Experimental test for initial injection angle

In preparation for the possible operation of a nonlinear kicker in the existing Diamond ring, we have performed a number of experiments to confirm the capability of fine controlling the angle and position of the injected electron beam at the location assigned to the nonlinear kicker. The existing hardware limit on septum physical aperture, beam pipe and septum magnet strength, were all thoroughly checked in these experiments. As shown in Figure 5.45, the current in the injection kickers was tuned to maximise the injection efficiency by keeping the injected beam trajectory as close as possible to the stored beam and of course the septum wall. This limits the possible injection angle before the beam will hit the septum wall. To be able to inject the beam with an angle, the injected trajectory, initially close

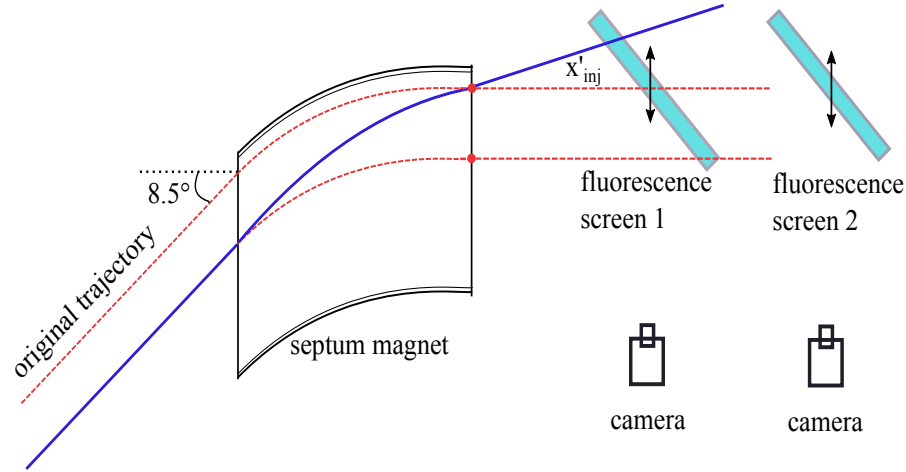


Figure 5.45: Schematic view for injection with angle experiment at Diamond. The injected beam trajectory to make an initial angle x'_{inj} (blue line) was controlled by the septum strength. The angle was verified by the position of the beam on the screen 1 and 2.

to the septum wall, was moved close to the centre of the septum while the angle was kept constant at zero. The last two correctors in the Booster to Storage ring transfer line (BTS) just before the septum were adjusted to move the injected beam trajectory. A matlab script (btsexitgui) was used to manipulate the position and angle of the injected beam at the end of the septum. The position of the injected beam was then controlled on a calibrated screen as shown in Figure 5.46. The main concern while using large injection angles is the limit

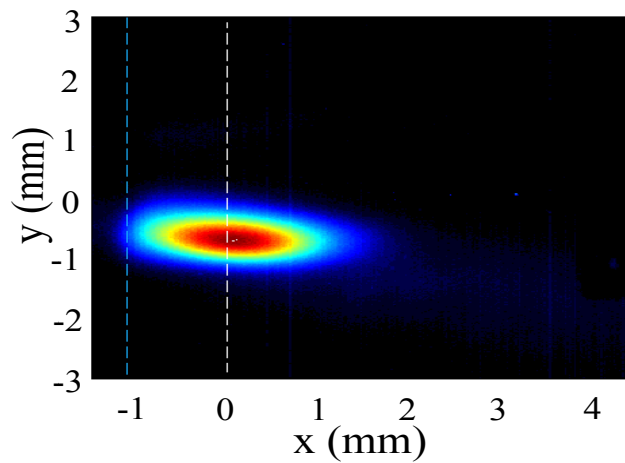


Figure 5.46: Injected beam on YAG screen.

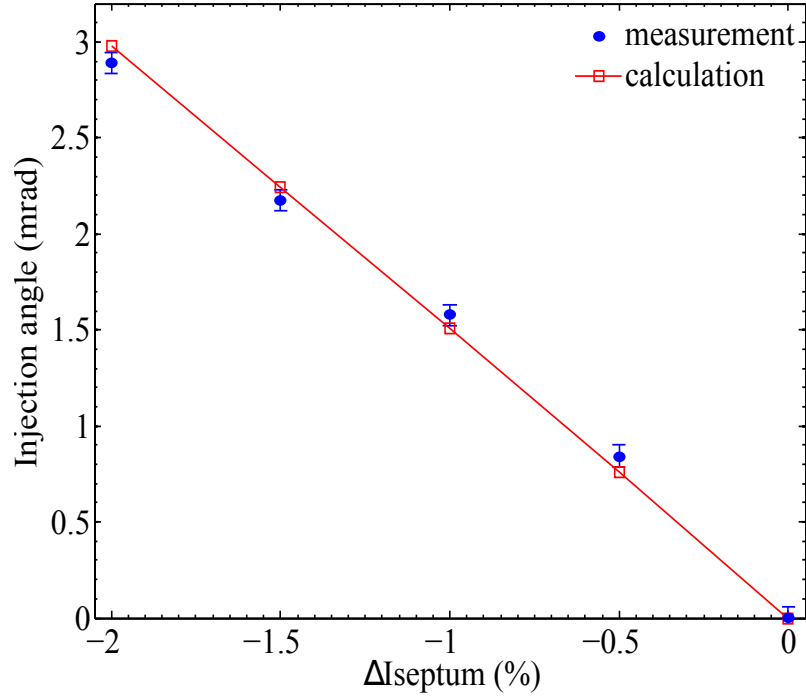


Figure 5.47: Comparison between measured and calculated injection angle as a function of septum current.

of the physical aperture of the septum. From the original trajectory, the injected beam was moved by about 2.5 mm towards the center of the septum considering the similar exit position at the edge of the septum but with an injection angle of 3 mrad. Then, by reducing the septum strength, a good control of the injected angle was achieved (positive injection angle means that the injected beam pointing towards the stored beam). To measure the angle of the injected beam (x'_{inj}), two fluorescent screens (YAG) were used. The positions of the injected beam on both screens were recorded as a function of the septum current as shown in Figure 5.47. The measurement confirmed that the required injection angle of 3 mrad can be achieved with the existing system.

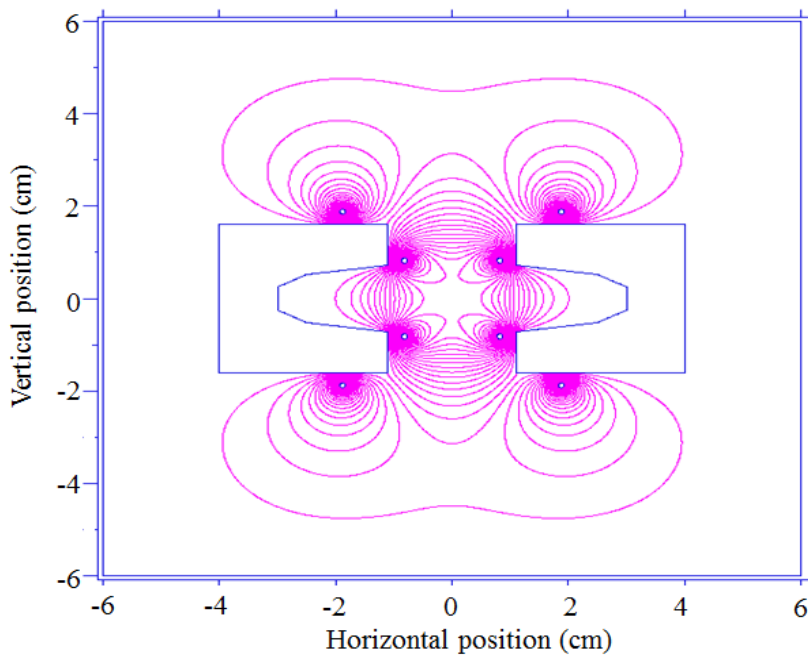


Figure 5.48: Magnetic field of the PM calculated by Poisson

5.6 Pulsed multipole hardware consideration

The magnetic field for the PM calculated from Poisson [92] is shown in Figure 5.48. The design is adapted from the Bessy-II pulsed multipole magnet [83]. Both lateral sides of the vacuum chambers are made of stainless steel and will be water cooled. In order to prevent the field generated by the circulating beam to penetrate and affect the pulsed magnet itself,



Figure 5.49: Nonlinear pulsed kicker used in Bessy-II (left), Ti-coated ceramic chamber (right).

Ti-coated ceramic vacuum chamber has been used. The thickness of the coating layer (d_{coating}) is required to be large enough compare to the squared of the skin depth (d_s) in the metal divided by the thickness of the ceramic chamber (d_{ceramic}) to completely block the field generated by the beam. This condition can then be written as

$$d_{\text{coating}} > \frac{d_s^2}{d_{\text{ceramic}}}, \quad (5.11)$$

with the skin depth in the metallic coating d_s defined by

$$d_s = \sqrt{\frac{2}{\omega\mu\sigma}}, \quad (5.12)$$

where ω is the frequency of the current, μ and σ is the magnetic permeability and conductivity of metallic coating. However the metallic coating itself affects the ohmic power losses associated with the beam leading to heat load in the coating [93].

The initial operation of the Bessy-II kicker showed severe issue with the heating of the chamber. As a consequence the design of the PM was improved to include a Ti-coated layer of 10 μm on the ceramic chamber to separate the PM wires from the beam and reduce the beam-induced over heating on the wires [83]. Figure 5.49 shows the PM of Bessy-II with the Ti-coated ceramic surface. Water cooling pipes are attached to the sides of stainless steel chamber.

5.7 Summary

We have investigated the performance of different types of pulsed injection kickers, both on the existing Diamond lattice and on the planned upgrade to a single DDBA cell. The simulations show that the PM at the end of the second straight can be optimised to provide an injection efficiency in the existing Diamond lattice in excess of 90%. Single particle tracking was used to determined suitable operating conditions, e.g. injection amplitude and angle at the PM location. Critical parameters such as the position of the injected beam at

the PM, the injection angle and offset have all been investigated with detailed multi-particle tracking to determine the optimal kick angle required to reach the best injection efficiency. The PM was modelled with four parallel wires considering the feasible size and strength similar to the Bessy-II design.

In addition, the PM was also modelled for the one DDBA lattice. The same process used in the previous optimisation was employed. Two positions for the PM were investigated in details. The first PM position was the same as used for the standard Diamond lattice in the second straight. While the optimisation of the injection in ideal machine conditions gave sufficient injection efficiency, the presence of the full detailed physical aperture, decreased the injection efficiency dramatically. In particular the much narrower horizontal aperture in the DDBA cell was the main cause of the injected beam loss. MOGA was also used to automatically retune the initial condition of the injected beam and current of the PM. However only 60% injection efficiency could be achieved. A new position for the PM was therefore investigated. It was found that the location before the DDBA cell, in the first straight (injection straight) produced significantly better results. The same method was used to optimise the injection efficiency. To control the optimal position of the injected beam at the location of the PM, the injection angle was set to 3 mrad. It was found that the injected particles trajectory is much smaller at the DDBA cell thus improving the injection efficiency significantly. The ability to control the injected beam to achieve these optimal conditions was tested experimentally. Further optimisation with MOGA improved the injection angle slightly raising the injection efficiency above 85%. The injection of shorter bunches obtained by increasing the RF voltage available in the booster raised the injection efficiency to above 90% even considering machine misalignments and imperfections and current errors in the PM. Additionally, the PM effect on the stored beam is less than PQM and PSM in all studies. Current errors applied to the PM can cause beam centroid and beam size disturbance to the stored beam. The pessimistic scenario with $\pm 1\%$ current error with long bunch length from the booster resulted in about $95 \mu\text{m}$ maximum beam centroid shift and 6% beam size variation which is still better than the conventional injection scheme. In practice, the effect from the PM current error can be cured by moving the stored beam closed orbit through

the magnetic centre of the PM using correctors.

The new beam injection scheme with a single pulsed multipole kicker is much simpler than the conventional injection scheme and the result of these studies show that the PM kicker can be a feasible injection system both for the existing Diamond ring or the newly upgraded Diamond lattice with the single DDBA cell . The PM injection scheme can be implemented as a backup injection system which deliver less perturbation to the stored beam.

Chapter 6

Conclusions

Brighter and better coherence synchrotron sources are required a large number of applications in Photon Science. With the development of free-electron laser, the competition of linac based fourth generation light source has forced further improvement in the performance of storage ring based 3rd generation synchrotron machines. Synchrotron light sources have the advantage of serving more users simultaneously and intrinsically more stable photon beam. The new route toward ultra-low emittance lattices will improve further the brightness and the transverse coherence, maintaining the competitiveness of these machines. The development in technology, particularly the availability of strong narrow bore magnet and high vacuum system in small pipe enables more challenging compact lattice design. Upgrade design of the existing machines and new design of fourth generation storage ring based on MBA lattice aiming for sub-nm beam emittance have been proposed in the recent years. Such designs require extensive linear and nonlinear optics optimisations in order to efficiently accumulate electron beam with good lifetime.

The works carried out in this thesis aim at extending the capability of the Diamond storage ring by retuning the existing lattice components, by proposing the modification of one cell, eventually leading to the full ring upgrade and a new injection scheme. This can serve as a template for the upgrade of any existing third generation light source.

6.1 Diamond modification

We studied the possibility of reducing the emittance of the Diamond lattice by purely re-tuning the exiting components. The optimisation was based on the use of Multi-objective genetic algorithms (MOGA) and has been performed by two steps: targeting first emittance and dynamic aperture and then optimising the dynamic aperture and Touschek lifetime. A reduction of 20% of the emittance was predicted and experimentally verified in the Diamond storage ring. Emittance growth due to the strong field wigglers is in good agreement between the calculation and measurement.

A second set of studies aimed at the optimisations of the Diamond lattice to increase the capability of the storage ring. The new design of the DDBA cell was devised to replace the existing DBA cell giving an additional straight section for the VMX beamline. Optics matching and optimisation were performed to restore and guarantee the same performance of the ring in terms of injection efficiency and lifetime. Closed orbit correction schemes were designed including all engineering constraints. MOGA was used to optimise Touschek lifetime and injection efficiency and increase the tolerance to the machine imperfection. As a result this project was approved and fully funded by Diamond. The DDBA cell installation is due to be completed by August 2016. Further extension to two DDBA cells providing an additional straight section for the DIAD beamline was also investigated with the same steps, however the lifetime dropped by 50% compared to the existing machine and further studies are needed to establish the feasibility of this new modification.

6.2 Diamond upgrade

In view of the full upgrade of the Diamond storage ring, an extensive analysis of the possible MBA lattice options was carried out. Cells with $M=4, 5$ and 7 dipoles were designed providing natural beam emittance of 272, 150 and 50 pmrad respectively. The 4BA cell was further modified to gain an additional straight section in the middle of the cell, thus doubling the ID capacity of the ring. The lattice optimisation was based on phase advance matching to

achieve cancellation of the resonance driving terms. Eventually MOGA was used to optimise the dynamic aperture and Touschek lifetime.

Among the solutions found, the modified-4 BA provides the best dynamic aperture and Touschek lifetime. At least ten fold beam emittance reduction can be achieved by the modified-4BA lattice with moderate magnets strengths compared to other upgrade projects. The positions of the existing straight sections is only slightly modified, with an offset of 15 mm from the original positions which allow the operation of the existing beamline after a minor realignment. The calculations of the effects of insertion devices in the straight sections indicates that a further 30% reduction can be achieved.

Intra beam scattering calculation considering the maximum current of 500 mA shows dramatic increase of the beam emittance in very small natural emittance solutions especially 7BA and 5BA as expected. For the modified-4BA lattice with the maximum current the beam emittance is still about 10 times smaller than that of the existing machine.

The dynamic aperture of the modified-4BA solution is good enough for on-axis injection, however further optimization is required to guarantee off-axis injection with the existing beam injection system. At least -10 mm dynamic aperture is required for such scheme. More aggressive solutions, providing much smaller beam emittance can be considered if ultra-low emittance is required, but they will have to rely to on-axis injection scheme as the dynamic aperture appears to be limited to few mm. The presence of strong non-linearity is a characteristic of all compact MBA designs and rises strong challenges to the lattice designer especially in upgrade project that are already constrained by the existing layout of the operating machines.

6.3 Pulsed multipole kicker injection

A new method for beam injection, based on the pulsed multipole kicker (PM) (Bessy-II design) was investigated in order to establish its performance in the existing Diamond lattice and the one DDBA lattice. It was found that in the Diamond lattice modified with one

DDBA cell, the narrow aperture limits the achievable injection efficiency and a thorough analysis of all possible location led to the identification of the best PM position upstream of the DDBA cell in order to reduce the injected beam amplitude before the aperture restriction in this cell. Injection optimisation was performed in order to achieve injection efficiency of about 90%. Fine adjustment of the injected beam at the position of the PM was crucial to devise the best injection efficiency. Moreover, MOGA was also applied not only to maximise the injection efficiency but also to minimise the required strength of the PM. The solution based on the PM provides minimal perturbation to the stored beam compared with the PQM the PSM and the existing four kickers injection system. The best injection is achieved with PM in the one DDBA lattice with a relatively large initial injection angle of 3 mrad. This condition was experimentally tested successfully.

The simulations proved that beam injection to storage ring with the PM is feasible and can be much simpler compared to the existing four-kicker injection scheme used in most of the modern light sources. The associated reduction of the perturbation to the stored beam make top-up transparent to the photon beam users. The technique can be used also in the upgrade ring and reduces slightly the requirement on the dynamics aperture necessary for injection.

Related Publications

R. Bartolini, M. Apollonio, C. Bailey, M. Cox, N. Hammond, R. Holdsworth, J. Kay, I. Martin, V. Smaluk, R. Walker, T. Pulampong “*Engineering Integration Constraints on the Beam Physics Optimisation of the DDBA Lattice for Diamond*”, Proc. IPAC 14, p. 322, (2014).

R. Bartolini, C. Bailey, N. Hammond, R. Holdsworth, J. Key, E. Rial, R. Walker, T. Pulampong, “*Magnet Design for the Diamond DDBA Lattice Upgrade*”, Proc. IPAC 14, p. 1319, (2014).

R. Walker, M. Apollonio, C. Bailey, M. Cox, R. Fielder, N. Hammond, M. Heron, J. Kay, I. Martin, S. Mhaskar, G. Rehm, B. Singh, V. Smaluk, A. Thomson, R. Bartolini, T. Pulampong, “*The Double-double Bend Achromat (DDBA) Lattice Modification for the Diamond Storage Ring*”, Proc. IPAC 14, p. 331, (2014).

B. Singh, M. Apollonio, R. Bartolini, E.C. Longhi, T. Pulampong, “*Investigation and Test of the Possibility of Reducing the Emittance of Diamond Storage Ring*”, Proc. IPAC 13, p. 234, (2013).

R. Bartolini, T. Pulampong, “*Ultra-low Emittance Upgrade Options for Third Generation Light Sources*”, Proc. IPAC 13, p. 237, (2013).

R. Bartolini, C. Bailey, M. Cox, N. Hammond, J. Kay, E. Longhi, S. Mhaskar, T. Pulampong, R. Walker, “*Novel Lattice Upgrade Studies for Diamond Light Source*”, Proc. IPAC

13, p. 240, (2013).

T. Pulampong, R. Bartolini, “*A Non-linear Injection Kicker for Diamond Light Source*”, Proc. IPAC 13, p. 2268, (2013).

R. Bartolini, N. Hammond, J. Kay, T. Pulampong, R.P. Walker, “*Ultra-low Emittance Upgrade Options for the Diamond Light Source*”, Proc. PAC 2013, p. 24, (2013).

Appendix A

Analytic formulae

A.1 Resonance driving terms formulas

Resonance driving terms (RDTs) were derived in the early works [43] and [94]. Each driving term drives different phenomenon or resonance of particles. The driving term h_{abcde} whose subscript indicates what resonance effect can be excited by the term. Generally, the driving term indicates the resonance

$$(a - b)\nu_x + (c - d)\nu_y = n \quad (\text{A.1})$$

where n is an integer number. The index e is related to dispersion or chromatic terms. The resonance driving term can be written explicitly in term of optics functions ($\beta_{x,y}$, η_x) and quadrupole and sextupole integrated strength (b_2 and b_3) as follows.

A.1.1 First order chromatic driving terms

$$h_{11001} = \frac{1}{4} \sum_{i=1}^N [(b_2 L)_i - 2(b_3 L)\eta_{xi}] \beta_{xi} + O(\delta^2), \quad (\text{A.2})$$

$$h_{00111} = -\frac{1}{4} \sum_{i=1}^N [(b_2 L)_i - 2(b_3 L)\eta_{xi}] \beta_{yi} + O(\delta^2), \quad (\text{A.3})$$

$$h_{20001} = \frac{1}{8} \sum_{i=1}^N [(b_2 L)_i - 2(b_3 L)_i \eta_{xi}] \beta_{xi} e^{i2\mu_{xi}} + O(\delta^2) \quad (\text{A.4})$$

$$h_{00201} = -\frac{1}{8} \sum_{i=1}^N [(b_2 L)_i - 2(b_3 L)_i \eta_{xi}] \beta_{yi} e^{i2\mu_{yi}} + O(\delta^2) \quad (\text{A.5})$$

$$h_{10002} = \frac{1}{2} \sum_{i=1}^N [(b_2 L)_i - (b_3 L)_i \eta_{xi}] \eta_{xi}^{(1)} \sqrt{\beta_{xi}} e^{i2\mu_{xi}} + O(\delta^3) \quad (\text{A.6})$$

In the chromatic terms, the effects of sextupoles can be used to counteract the effects generated by quadrupole after linear optics matching. h_{11001} and h_{00111} drives chromaticities in horizontal and vertical plane respectively. Chromatic sextupole can control the term effectively in non-dispersive sections. Normally, dispersion bumps are intentionally generated at the position of chromatic sextupoles to ease the correction.

A.1.2 First order geometric driving terms

$$h_{21000} = -\frac{1}{8} \sum_{i=1}^N (b_{3i}L) \beta_{xi}^{\frac{3}{2}} e^{i\mu_{xi}}, \quad (\text{A.7})$$

$$h_{30000} = -\frac{1}{24} \sum_{i=1}^N (b_{3i}L) \beta_{xi}^{\frac{3}{2}} e^{i3\mu_{xi}}, \quad (\text{A.8})$$

$$h_{10110} = \frac{1}{4} \sum_{i=1}^N (b_{3i}L) \beta_{xi}^{\frac{1}{2}} \beta_{yi} e^{i\mu_{xi}}, \quad (\text{A.9})$$

$$h_{10020} = \frac{1}{8} \sum_{i=1}^N (b_{3i}L) \beta_{xi}^{\frac{1}{2}} \beta_{yi} e^{i(\mu_{xi}-2\mu_{yi})}, \quad (\text{A.10})$$

$$h_{10200} = \frac{1}{8} \sum_{i=1}^N (b_{3i}L) \beta_{xi}^{\frac{1}{2}} \beta_{yi} e^{i(\mu_{xi}+2\mu_{yi})} \quad (\text{A.11})$$

The geometric driving terms are contributed by sextupole magnets in the ring. Although the sextupoles are used to control the chromatic term especially chromaticities, they also introduce geometric terms. To suppress these term, harmonic sextupoles can be introduced in straight section where dispersion function is small to avoid the increase of the chromatic terms.

A.1.3 Second order driving terms

For second order driving terms, the explicit formulae can be found in [94]. When sextupoles are introduced into a lattice high order resonance can be excited. The strengths of the driving terms increase with stronger sextupole strength.

The summation appears in the following formulae is defined as

$$\sum_{j>i} f(i, j) \equiv \sum_{j>i} [f(i, j) - f(j, i)] = \sum_{j=1}^N \sum_{i=1}^j [f(i, j) - f(j, i)] = \left(\sum_{j>i} - \sum_{i>j} \right) f(i, j) \quad (\text{A.12})$$

Hence the second order geometric terms can be expressed as follows:

$$h_{22000} = \sum \frac{i}{64} b_{3i} b_{3j} \beta_{xi}^{3/2} \beta_{xj}^{3/2} [e^{i3(\mu_{xi}-\mu_{xj})} + 3e^{i(\mu_{xi}-\mu_{xj})}], \quad (\text{A.13})$$

$$h_{31000} = h_{13000}^* = \sum \frac{i}{32} b_{3i} b_{3j} \beta_{xi}^{3/2} \beta_{xj}^{3/2} e^{i(3\mu_{xi}-\mu_{xj})}, \quad (\text{A.14})$$

$$h_{21010} = 0 \quad (\text{A.15})$$

$$h_{21100} = 0 \quad (\text{A.16})$$

$$h_{11110} = \sum \frac{i}{16} b_{3i} b_{3j} \sqrt{\beta_{xi} \beta_{xj}} \beta_{yi} \left\{ \beta_{xj} [e^{-i(\mu_{xi}-\mu_{xj})} - e^{i(\mu_{xi}-\mu_{xj})}] \right. \\ \left. + \beta_{yj} [e^{i(\mu_{xi}-\mu_{xj}+2\mu_{yi}-2\mu_{yj})} + e^{-i(\mu_{xi}-\mu_{xj}-2\mu_{yi}+2\mu_{yj})}] \right\}, \quad (\text{A.17})$$

$$h_{11200} = \sum \frac{i}{32} b_{3i} b_{3j} \sqrt{\beta_{xi} \beta_{xj}} \beta_{yi} \left\{ \beta_{xj} [e^{-i(\mu_{xi}-\mu_{xj}-2\mu_{yi})} - e^{i(\mu_{xi}-\mu_{xj}+2\mu_{yi})}] \right. \\ \left. + 2\beta_{yj} [e^{i(\mu_{xi}-\mu_{xj}+2\mu_{yi})} + e^{-i(\mu_{xi}-\mu_{xj}-2\mu_{yi})}] \right\}, \quad (\text{A.18})$$

$$h_{40000} = \sum \frac{i}{64} b_{3i} b_{3j} \beta_{xi}^{3/2} \beta_{xj}^{3/2} e^{i(3\mu_{xi}+\mu_{xj})}, \quad (\text{A.19})$$

$$h_{30010} = 0, \quad (\text{A.20})$$

$$h_{30100} = 0, \quad (\text{A.21})$$

$$h_{20020} = \sum \frac{i}{64} b_{3i} b_{3j} \sqrt{\beta_{xi} \beta_{xj}} \beta_{yi} [\beta_{xj} e^{-i(\mu_{xi}-3\mu_{xj}+2\mu_{yi})} \\ - (\beta_{xj} + 4\beta_{yj}) e^{i(\mu_{xi}+\mu_{xj}-2\mu_{yj})}], \quad (\text{A.22})$$

$$h_{20110} = \sum \frac{i}{32} b_{3i} b_{3j} \sqrt{\beta_{xi} \beta_{xj}} \beta_{yi} \left\{ \beta_{xj} [e^{-i(\mu_{xi}-3\mu_{xj})} - e^{i(\mu_{xi}+\mu_{xj})}] \right. \\ \left. + 2\beta_{yj} e^{i(\mu_{xi}+\mu_{xj}+2\mu_{yi}-2\mu_{yj})} \right\}, \quad (\text{A.23})$$

$$h_{20110} = \sum \frac{i}{32} b_{3i} b_{3j} \sqrt{\beta_{xi} \beta_{xj}} \beta_{yi} \left\{ \beta_{xj} [e^{-i(\mu_{xi}-3\mu_{xj})} - e^{i(\mu_{xi}+\mu_{xj})}] \right. \\ \left. + 2\beta_{yj} e^{i(\mu_{xi}+\mu_{xj}+2\mu_{yi}-2\mu_{yj})} \right\}, \quad (\text{A.24})$$

$$h_{20200} = \sum \frac{i}{64} b_{3i} b_{3j} \sqrt{\beta_{xi} \beta_{xj}} \beta_{yi} [\beta_{xj} e^{-i(\mu_{xi}-3\mu_{xj}-2\mu_{yi})} \\ - (\beta_{xj} - 4\beta_{yj}) e^{i(\mu_{xi}+\mu_{xj}+2\mu_{yi})}], \quad (\text{A.25})$$

$$h_{20200} = \sum \frac{i}{64} b_{3i} b_{3j} \sqrt{\beta_{xi} \beta_{xj}} \beta_{yi} [\beta_{xj} e^{-i(\mu_{xi}-3\mu_{xj}-2\mu_{yi})} \\ - (\beta_{xj} - 4\beta_{yj}) e^{i(\mu_{xi}+\mu_{xj}+2\mu_{yi})}], \quad (\text{A.26})$$

$$h_{10030} = 0, \quad (\text{A.27})$$

$$h_{10120} = 0, \quad (\text{A.28})$$

$$h_{10210} = 0, \quad (\text{A.29})$$

$$h_{10210} = 0, \quad (\text{A.30})$$

$$h_{10300} = 0, \quad (\text{A.31})$$

$$h_{00220} = \sum \frac{i}{64} b_{3i} b_{3j} \sqrt{\beta_{xi} \beta_{xj}} \beta_{yi} \beta_{yj} [e^{i(\mu_{xi}-\mu_{xj}+2\mu_{yi}-2\mu_{yj})} \\ + 4e^{i(\mu_{xi}-\mu_{xj})} - e^{-i(\mu_{xi}-\mu_{xj}-2\mu_{yi}+2\mu_{yj})}], \quad (\text{A.32})$$

$$h_{00310} = \sum \frac{i}{32} b_{3i} b_{3j} \sqrt{\beta_{xi} \beta_{xj}} \beta_{yi} \beta_{yj} [e^{i(\mu_{xi}-\mu_{xj}+2\mu_{yi})} - e^{-i(\mu_{xi}-\mu_{xj}-2\mu_{yi})}], \quad (\text{A.33})$$

$$h_{00400} = \sum \frac{i}{64} b_{3i} b_{3j} \sqrt{\beta_{xi} \beta_{xj}} \beta_{yi} \beta_{yj} e^{i(\mu_{xi}-\mu_{xj}+2\mu_{yi}+2\mu_{yj})}, \quad (\text{A.34})$$

$$h_{21001} = \sum \left\{ -\frac{i}{32} b_{3i} b_{2j} \beta_{xi}^{3/2} \beta_{xj} [e^{i\mu_{xi}} + e^{i(3\mu_{xi}-2\mu_{xj})} - 2e^{-i(\mu_{xi}-2\mu_{xj})}] \right. \\ \left. - \frac{i}{16} b_{3i} b_{3j} \beta_{xi} \beta_{xj}^{3/2} \eta_{xi} [e^{i\mu_{xj}} - 2e^{i(2\mu_{xi}-\mu_{xj})} + e^{-i(2\mu_{xi}-3\mu_{xj})}] \right\}, \quad (\text{A.35})$$

$$h_{11101} = 0, \quad (\text{A.36})$$

$$h_{30001} = \sum \left\{ -\frac{i}{32} b_{3i} b_{2j} \beta_{xi}^{3/2} \beta_{xj} [e^{i3\mu_{xi}} - e^{i(\mu_{xi}+2\mu_{xj})}] \right. \\ \left. - \frac{i}{16} b_{3i} b_{3j} \beta_{xi} \beta_{xj}^{3/2} \eta_{xi} [e^{i3\mu_{xj}} - e^{i(2\mu_{xi}+\mu_{xj})}] \right\}, \quad (\text{A.37})$$

$$h_{20011} = 0, \quad (\text{A.38})$$

$$h_{20101} = 0, \quad (\text{A.39})$$

$$\begin{aligned} h_{10021} = & \sum \left\{ \frac{i}{32} b_{3i} b_{2j} \sqrt{\beta_{xi} \beta_{xj} \beta_{yi}} [e^{i(\mu_{xi}-2\mu_{yi})} - e^{-i(\mu_{xi}-2\mu_{xj}+2\mu_{yi})}] \right. \\ & + \frac{i}{16} b_{3i} b_{2j} \sqrt{\beta_{xi} \beta_{yj} \beta_{yi}} [e^{i(\mu_{xi}-2\mu_{yi})} - e^{i(\mu_{xi}-2\mu_{yj})}] \\ & + \frac{i}{16} b_{3i} b_{3j} \beta_{xi} \sqrt{\beta_{xj} \beta_{yj} \eta_{xi}} [e^{i(\mu_{xj}-2\mu_{yj})} - e^{i(2\mu_{xi}-\mu_{xj}-2\mu_{yj})}] \\ & \left. - \frac{i}{8} b_{3i} b_{3j} \sqrt{\beta_{xj} \beta_{yj} \beta_{yi} \eta_{xi}} [e^{i(\mu_{xj}-2\mu_{yi})} - e^{i(\mu_{xj}-2\mu_{yj})}] \right\}, \end{aligned} \quad (\text{A.40})$$

$$\begin{aligned} h_{10111} = & \sum \left\{ \frac{i}{16} b_{3i} b_{2j} \sqrt{\beta_{xi} \beta_{xj} \beta_{yi}} [e^{i\mu_{xi}} - e^{-i(\mu_{xi}-2\mu_{xj})}] \right. \\ & + \frac{i}{16} b_{3i} b_{2j} \sqrt{\beta_{xi} \beta_{yj} \beta_{yi}} [e^{i(\mu_{xi}-2\mu_{yi}+2\mu_{yj})} - e^{i(\mu_{xi}+2\mu_{yi}-2\mu_{yj})}] \\ & + \frac{i}{8} b_{3i} b_{3j} \beta_{xi} \sqrt{\beta_{xj} \beta_{yj} \eta_{xi}} [e^{i\mu_{xj}} - e^{i(2\mu_{xi}-\mu_{xj})}] \\ & \left. - \frac{i}{8} b_{3i} b_{3j} \sqrt{\beta_{xj} \beta_{yj} \beta_{yi} \eta_{xi}} [e^{i(\mu_{xj}-2\mu_{yi}+2\mu_{yj})} - e^{i(\mu_{xj}+2\mu_{yi}-2\mu_{yj})}] \right\}, \end{aligned} \quad (\text{A.41})$$

$$\begin{aligned} h_{10201} = & \sum \left\{ \frac{i}{32} b_{3i} b_{2j} \sqrt{\beta_{xi} \beta_{xj} \beta_{yi}} [e^{i(\mu_{xi}+2\mu_{yi})} - e^{-i(\mu_{xi}-2\mu_{xj}-2\mu_{yi})}] \right. \\ & - \frac{i}{16} b_{3i} b_{2j} \sqrt{\beta_{xi} \beta_{yj} \beta_{yi}} [e^{i(\mu_{xi}-2\mu_{yi}+2\mu_{yj})} - e^{i(\mu_{xi}+2\mu_{yi}-2\mu_{yj})}] \\ & + \frac{i}{16} b_{3i} b_{3j} \beta_{xi} \sqrt{\beta_{xj} \beta_{yj} \eta_{xi}} [e^{i(\mu_{xj}+2\mu_{yj})} - e^{i(2\mu_{xi}-\mu_{xj}+2\mu_{yj})}] \\ & \left. + \frac{i}{8} b_{3i} b_{3j} \sqrt{\beta_{xj} \beta_{yj} \beta_{yi} \eta_{xi}} [e^{i(\mu_{xj}+2\mu_{yi})} - e^{i(\mu_{xj}+2\mu_{yj})}] \right\}, \end{aligned} \quad (\text{A.42})$$

$$h_{00211} = 0, \quad (\text{A.43})$$

$$h_{00301} = 0, \quad (\text{A.44})$$

$$\begin{aligned} h_{11002} = & \sum \left\{ \frac{i}{16} \beta_{xi} \beta_{xj} [(b_{2i} b_{2j} - 2b_{3i} b_{2j} \eta_{xi} + 4b_{3i} b_{3j} \eta_{xi} \eta_{xj}) e^{i2(\mu_{xi}-\mu_{xj})} + 2b_{3i} b_{2j} \eta_{xi} e^{-i2(\mu_{xi}-\mu_{xj})}] \right. \\ & \left. + \frac{i}{8} \sqrt{\beta_{xi} \beta_{xj}^3} \eta_{xi} (b_{3i} \eta_{xi} - b_{2i}) b_{3j} [e^{i(\mu_{xi}-\mu_{xj})} - e^{-i(\mu_{xi}-\mu_{xj})}] \right\}, \end{aligned} \quad (\text{A.45})$$

$$\begin{aligned} h_{20002} = & \sum \left\{ \frac{i}{16} \beta_{xi} \beta_{xj} [(b_{2i} b_{2j} - 2b_{3i} b_{2j} \eta_{xi} + 4b_{3i} b_{3j} \eta_{xi} \eta_{xj}) e^{i2\mu_{xi}} + 2b_{3i} b_{2j} \eta_{xi} e^{i2\mu_{xj}}] \right. \\ & \left. + \frac{i}{16} \sqrt{\beta_{xi} \beta_{xj}^3} \eta_{xi} (b_{3i} \eta_{xi} - b_{2i}) b_{3j} [e^{i(\mu_{xi}+\mu_{xj})} - e^{-i(\mu_{xi}-3\mu_{xj})}] \right\}, \end{aligned} \quad (\text{A.46})$$

$$h_{10012} = 0, \quad (\text{A.47})$$

$$h_{10102} = 0, \quad (\text{A.48})$$

$$\begin{aligned} h_{00112} = & \sum \left\{ \frac{i}{16} \beta_{yi} \beta_{yj} [(b_{2i} b_{2j} - 2b_{3i} b_{2j} \eta_{xi} + 4b_{3i} b_{3j} \eta_{xi} \eta_{xj}) e^{i2(\mu_{yi}-\mu_{yj})} + 2b_{3i} b_{2j} \eta_{xi} e^{-i2(\mu_{yi}-\mu_{yj})}] \right. \\ & \left. - \frac{i}{8} \sqrt{\beta_{xi} \beta_{xj} \beta_{yj} \eta_{xi}} (b_{3i} \eta_{xi} - b_{2i}) b_{3j} [e^{i(\mu_{xi}-\mu_{xj})} - e^{-i(\mu_{xi}-\mu_{xj})}] \right\}, \end{aligned} \quad (\text{A.49})$$

$$\begin{aligned} h_{00202} = & \sum \left\{ \frac{i}{16} \beta_{yi} \beta_{yj} [(b_{2i} b_{2j} - 2b_{3i} b_{2j} \eta_{xi} + 4b_{3i} b_{3j} \eta_{xi} \eta_{xj}) e^{i2\mu_{yi}} + 2b_{3i} b_{2j} \eta_{xi} e^{i2\mu_{yj}}] \right. \\ & \left. - \frac{i}{16} \sqrt{\beta_{xi} \beta_{xj} \beta_{yj} \eta_{xi}} (b_{3i} \eta_{xi} - b_{2i}) b_{3j} [e^{i(\mu_{xi}-\mu_{xj}+2\mu_{yj})} - e^{-i(\mu_{xi}-\mu_{xj}-2\mu_{yj})}] \right\}, \end{aligned} \quad (\text{A.50})$$

$$\begin{aligned} h_{10003} = & \sum \left\{ \frac{i}{4} b_{3i} b_{2j} \beta_{xi} \sqrt{\beta_{xj} \eta_{xi} \eta_{xj}} [e^{i\mu_{xj}} - e^{i(2\mu_{xi}-\mu_{xj})}] \right. \\ & \left. + \frac{i}{8} \sqrt{\beta_{xi} \beta_{xj} \eta_{xi}} [b_{2i} b_{2j} - b_{3i} \eta_{xi} (b_{2j} - 2b_{3j} \eta_{xj})] [e^{i\mu_{xi}} - e^{-i(\mu_{xi}-2\mu_{xj})}] \right\}, \end{aligned} \quad (\text{A.51})$$

$$h_{00103} = 0, \quad (A.52)$$

$$h_{00004} = \sum \frac{i}{4} \sqrt{\beta_{xi}\beta_{xj}\eta_{xi}\eta_{xj}} \left\{ [b_{2i}b_{2j} - b_{3i}\eta_{xi}(b_{2j} - b_{3j}\eta_{xj})] e^{i(\mu_{xi}-\mu_{xj})} + b_{3i}b_{2j}\eta_{xi} e^{-i(\mu_{xi}-\mu_{xj})} \right\}. \quad (A.53)$$

The coefficients $b_2 = K_1 L$ and $b_3 = K_2(L/2)$, where K_1 and K_2 are quadrupole and sextupole strength used in ELEGANT and MAD, and L is the magnet length.

A.2 Piwinski's formula for Touschek lifetime

From Piwinski's formula [48], Touschek lifetime can be written in simple form as

$$\frac{1}{T_l} = \left\langle \frac{R}{N_0} \right\rangle, \quad (A.54)$$

where R is the total of scattering occur and N_0 is the initial total number of particles in a bunch. The larger scattering rate the lower lifetime can be. The total scattering is given by

$$R = \frac{r_p^2 c \beta_x \beta_y \sigma_h N_b^2}{8 \sqrt{\pi} \beta^2 \gamma^4 \sigma_{x\beta}^2 \sigma_{y\beta} \sigma_s \sigma_p} F(\tau_m, B_1, B_2), \quad (A.55)$$

with

$$F = \int_{\tau_m}^{\infty} e^{-B_1 \tau} I_0(B_2 \tau) \frac{\sqrt{\tau} d\tau}{\sqrt{1+\tau}} \left(\left(2 + \frac{1}{\tau}\right)^2 \left(\frac{\tau/\tau_m}{1+\tau} - 1\right) + 1 - \frac{\sqrt{1+\tau}}{\sqrt{\tau/\tau_m}} - \frac{4\tau+1}{2\tau^2} \ln \frac{\tau/\tau_m}{1+\tau} \right), \quad (A.56)$$

$$B_1 = \frac{1}{2\beta^2 \gamma^2} \left(\frac{\beta_x^2}{\sigma_{x\beta}^2} - \frac{\beta_x^2 \sigma_h^2 \tilde{\eta}_x^2}{\sigma_{x\beta}^4} + \frac{\beta_y^2}{\sigma_{y\beta}^2} - \frac{\beta_y^2 \sigma_h^2 \tilde{\eta}_y^2}{\sigma_{y\beta}^4} \right), \quad (A.57)$$

$$B_2 = \sqrt{B_1^2 - \frac{\beta_x^2 \beta_y^2 \sigma_h^2}{\beta^4 \gamma^4 \sigma_{x\beta}^4 \sigma_{y\beta}^4 \sigma_p^2} (\sigma_x^2 \sigma_y^2 - \sigma_p^4 \eta_x^2 \eta_y^2)}, \quad (A.58)$$

$$\tau_m = \beta^2 \delta_m^2 = \beta^2 \left(\frac{\Delta p_m}{p} \right)^2, \quad (A.59)$$

$$\sigma_h = \frac{\sigma_{x\beta} \sigma_{y\beta} \sigma_p}{\sqrt{\tilde{\sigma}_x^2 \sigma_{y\beta}^2 + \tilde{\sigma}_y^2 \sigma_{x\beta}^2 - \sigma_{x\beta}^2 \sigma_{y\beta}^2}}, \quad (A.60)$$

$$\tilde{\eta}_{x,y} = \alpha_{x,y} \eta_{x,y} + \beta_{x,y} \eta'_{x,y}, \quad (A.61)$$

$$\tilde{\sigma}_{x,y}^2 = \sigma_{x\beta,y\beta}^2 + \sigma_p^2 (\eta_{x,y}^2 + \tilde{\eta}_{x,y}^2), \quad (A.62)$$

where r_c is the classical particle radius, $\beta_{x,y}$, $\alpha_{x,y}$, $\eta_{x,y}$ and $\eta'_{x,y}$ are the optics functions, N_b is the number of particles per bunch, $\sigma_{x,y,s}$ are the rms beam size in horizontal, vertical and longitudinal plane respectively, σ_p is the energy spread, β and γ are the Lorentz factor and $\sigma_{x\beta,y\beta}$ are the beam size without momentum spread.

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