ABSTRACT

Lewis, David K. 1976. PRINCIPLES AND PROCEDURES IN THE ECONOMIC ANALYSIS OF STAND TREATMENTS FOR TIMBER PRODUCTION. UNIVERSITY OF OXFORD. D. PHIL. THESIS 352 PP.

The objective of this study was to develop refined procedures for the economic analysis of stand treatments for timber production.

This objective has been achieved by a review and explicit statement of the economic principles relevant to the analysis of stand treatments. Based on these principles, procedures for the analysis of silvicultural investments are proposed and demonstrated.

The review of economic principles indicated that the appropriate class of criteria for the analysis of stand treatments are expected Present Certainty Equivalent Value. However, these criteria were also shown to be incomplete due to the failure of the Separation Theorem to hold under imperfect markets. This incompleteness and the theoretical requirement that the quantity and price of inputs and outputs, of investment ensembles, be measurable limits these criteria to defining efficient investment emsembles. Although these criteria were found to be incomplete, they do provide a means of organizing the measurable information associated with alternative stand treatments in a way that contributes to the selection of an optimum combination. The review also points out the necessity of correctly identifying the opportunities for productive transformation in the analysis of investments. In addition, the requirement to estimate real prices,

which define exchange opportunities in a manner consistent with the concepts of intertemporal choice is identified.

The maximization of expected Present Certainty-Equivalent Value of wealth gain is proposed as the rule and criterion for the economic analysis of stand treatments. The elements of the analysis procedure relating to (1) the level of analysis, (2) its duration, (3) the interval within the analysis period, and (4) treatment possibilities considered are discussed and a standard for each proposed. In addition, the requirements for estimates and measurements of the quantity and price of inputs and outputs are discussed. Standards for these estimates and measurements are identified in three categories; the requirements for the calculation of expected Present Certainty-Equivalent Value of wealth gain, the timber management requirement, and the measurement requirements.

The procedures for the analysis of stand treatments are demonstrated for a hypothetical compartment of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) in western Washington. The treatments considered in the analysis include thinning, fertilization, and regeneration.

The conclusion of the study is that the procedures proposed and demonstrated are feasible and practical under current conditions. Further, the information resulting from these analyses will contribute to a more efficient allocation of resources in timber production.

Additional study of the economic theory of investment choice is recommended with particular attention being concentrated on the theory of exchange in imperfect markets, and the role of information in the selection of uncertain alternatives. However, the major opportunity for additional study is in the collection of information and the development of estimation techniques required to forecast the quantity and price of inputs and outputs associated with investments in timber production.

PRINCIPLES AND PROCEDURES IN THE ECONOMIC ANALYSIS OF STAND TREATMENTS FOR TIMBER PRODUCTION

by

DAVID K. LEWIS

A thesis submitted to the University of Oxford in partial fullfilment of the requirements for the degree of Doctor of Philosophy

(HILARY) TERM 1976

LINACRE COLLEGE

OXFORD, ENGLAND

TO JUDY

WHOSE COURAGE AND INTEGRITY MAKE ALL THINGS POSSIBLE

ACKNOWLEDGEMENTS

I would like to express appreciation to my supervisor, Mr. Howard L. Wright, for his assistance in the computations required for this study and his constructive advice in the preparation of this dissertation. I am also indebted to Mr. Hywel G. Jones for his interest and patient guidance through the economic theory of investment choice.

I also want to acknowledge the support of the shareholders and management of the Weyerhaeuser Company, particularly Mr. George R. Staebler Director of Forestry Research, for the opportunity to carry on graduate studies while employed by the Weyerhaeuser Company.

I am also indebted to many of my associates at the Weyerhaeuser Company Forestry Research Center, especially Mr. James E. King, Manager Forest Management Research, for his enthusiasm and encouragement during the course of this study, Mr. David R. Bower for his assistance in the development of yield forecasts used in this study, and the secretaries for their assistance in preparation and typing of the dissertation manuscript.

For special support and encouragement, I thank my wife, Judith and our children, Lynn, Anne and Evan.

CONTENTS

CONTENTS (cont.)

CONTENTS (cont.)

CONTENTS (cont.)

CONTENTS (cont.)

CHAPTER I

INTRODUCTION

Estimates of the demand for timber contained in the current timber supply study for the United States (U. S. Forest Service, 1973) indicate that demand for timber will increase by 81 percent or 10.3 billion cubic feet between 1970 and the year 2000. Projections in this study also indicate increases in timber demand in Europe and Japan totaling 12 billion cubic feet or 70 percent of their 1970 consumption during the same period. These shifts in the world timber demand schedule are the result of anticipated increases in population and per capita gross national product.

The supply projections in this study (U.S. Forest Service, 1973) indicate that equilibrium will be achieved with price increases of 50 to 60 percent above 1970 levels during the thirty year period from 1970 to 2000. If the current relationship between product and stumpage prices continues these increases in product prices will result in stumpage price increases that average 100 percent above 1970 levels by the year 2000 (U.S. Forest Service, 1973). To achieve increases of this magnitude stumpage prices will have to increase at an average of 2.3 percent per year over the next 25 years relative to other goods.

While price estimates based on long term supply - demand equilibrium projections can be criticized for their lack of precision, the situation described in this timber supply study (U.S. Forest Service, 1973) indicates that the price of timber relative to other goods will probably increase at a significant rate. One of the results expected from these price increases is an expansion in the level of silvicultural activity directed toward the production of timber crops.

The increased investment required to intensify silvicultural practice is estimated at from 42 to 78 million dollars per year for farm, miscellaneous private, and National Forest classes of ownership (U.S. Forest Service, 1973) in the United States. This estimate includes approximately 62 percent of the commercial forest land in the United States so it is reasonable to estimate that nationally the additional investment required to intensify silvicultural practice might be as much as 67 to 122 million dollars per year.

In addition to these investment requirements, research in growth and yield has progressed to the point where forest managers can forecast yields resulting from a wide range of silvicultural treatments through the application of simulation techniques (Fries, 1974). This combination of investment activity and yield forecasting capability creates a need and opportunity for refined procedures to assist in the selection of optimum treatment sequences for timber production.

Objective of This Study

This study was planned with the objective of providing forest managers with refined procedures for the economic analysis of stand treatments, which will assist in the selection of optimum treatment sequences for timber production.

The basis for any procedure for economic analysis must be the principles of choice defined by economic theory. The principles for this study are developed by an explicit statement of the economic theory relevant to investments in timber production. These principles are used to develop refined procedures for the analysis of stand treatments for timber production. The procedures are illustrated by a demonstration analysis for a hypothetical compartment of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) in western Washington.

CHAPTER II

ECONOMIC PRINCIPLES

Economic science is concerned with how individuals and societies choose, with or without the use of money, to employ scarce productive resources to produce various commodities over time and distribute them for consumption now and in the future, among various people and groups in society (Samuelson, 1964). In order to understand complex reality, economic science, like all other science, has developed a body of theory which abstracts from the real world. Price theory is concerned with exchanges between individuals and groups within society and is the principle body of theory relevant to the analysis of stand treatments. Because timber production involves exchanges through time a special part of price theory, capital theory, is required to identify the relevant principles for the analysis of investments in stand treatments.

In addition to price and capital theory the analysis of investments also calls on concepts from the theory of economic equilibrium and welfare. However, these topics do not become explicitly central to the issue of investment choice and will receive limited emphasis in the development of principles for the analysis of stand treatments.

In application, economic theory can be described as positive or normative. Positive applications seek to describe how allocation problems are solved. Normative applications, on the other hand, describe how allocation problems should be solved (Friedman, 1953). The principles identified in this section will be developed as a unit of positive theory describing how individuals and groups solve a class of allocation problems similar to the problem of allocating resources for timber production. These principles will then be used in a normative application as a basis for analytical procedures, which if followed will yield information which will contribute to an optimum allocation of resources in timber production.

Because economic theory abstracts from the real world many simplifying assumptions are made in its presentation. These assumptions serve three roles. First as a framework for describing a theory. In this role assumptions serve as a means or criterion of classification for systematization of knowledge about economic phenomena.

The second role of assumptions is as an aid to the indirect test of a theory or hypothesis through their implications. One of the differences between economic science and the physical sciences is that controlled experiments involving economic phenomena are difficult if not impossible. For this reason hypothesis testing involves comparisons between theory and observed phenomena. In this role the assumption, which may or may not be observable, and its implications in terms of behavior which is observable facilitate hypothesis testing.

The third role of assumptions in economic theory is to specify the conditions under which the application of a theory can be expected to yield useful results. In this role assumptions describe the class of phenomena for which any theory is relevant. The important thing to remember about economic theory is that it exists as a body of tentatively accepted generalizations about economic phenomena which are used to predict the consequences of changes in circumstances (Friedman, 1953).

The theory of investment choice has been built up over a long period of time with the major modern contribution being due to Irving Fisher (1907, 1923, 1930). The discussion in this chapter follows the order of presentation and terminology of Jack Hieshleifer's book "Investment, Interest and Capital" (1970), and owes much to his presentation of the theory of investment choice.

Timeless Choice

The economic theory of individual choice rests on two postulates. The first of these, individual preference, describes the objects of choice in terms of their desirability to the consumer. The second, opportunity set, describes the environment in which choices are made in terms of a set of constraints.

Individual preferences and the nature of comparisons that individuals make in optimizing their choices among alternatives are systematically described by seven axioms of consumer preference. The first of these describes the completeness of preferences, and states that from a pair of alternatives the consumer either prefers one member of the pair to the other or is indifferent between them (Green, 1971). To put this another way, the theoretical consumer is said to behave as though all objects of choice are systematically related to each other in such a way that when any two are compared the consumer either prefers one to the other or is indifferent between them.

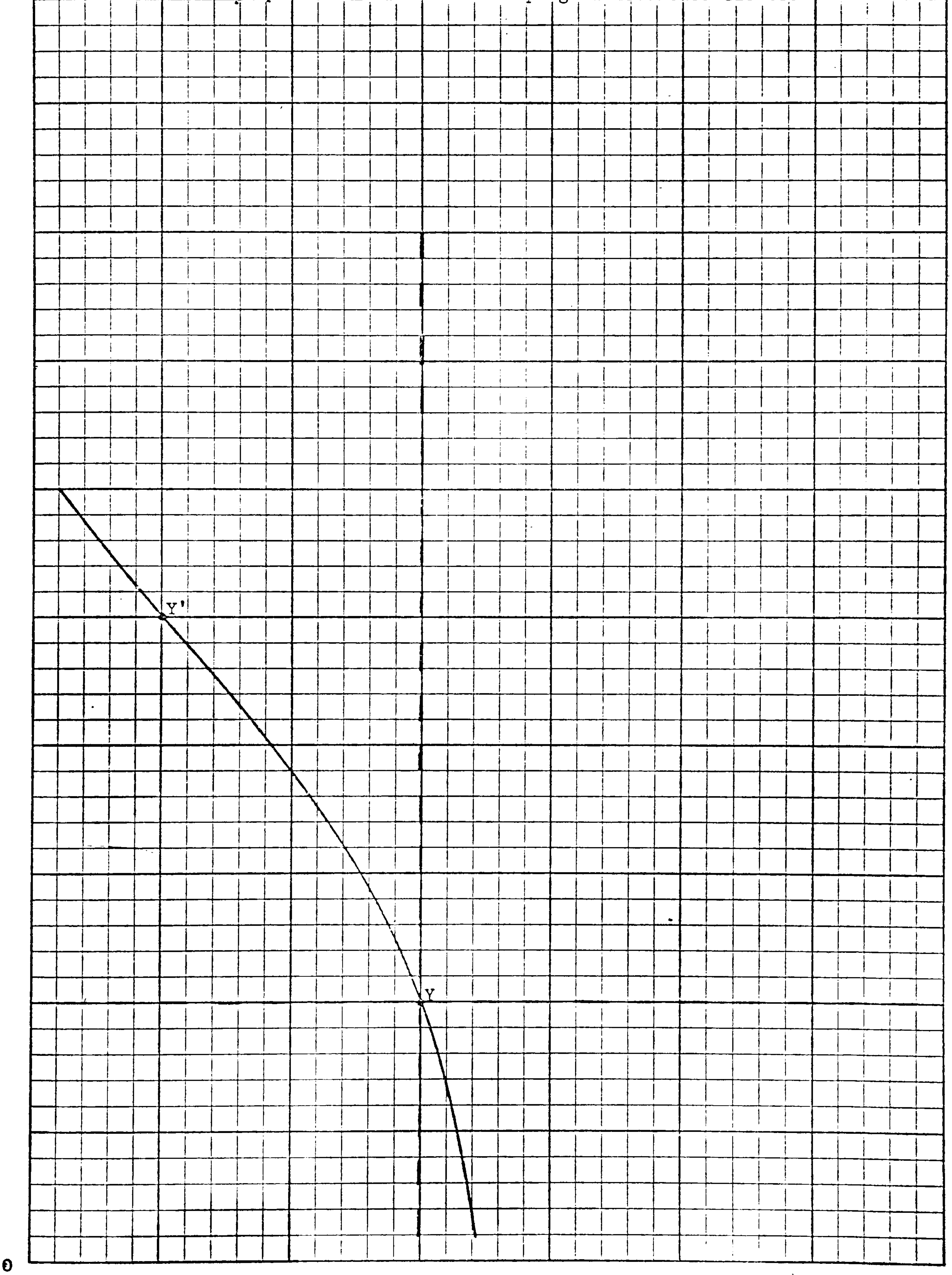
The second axiom describes the transitivity of consumer preferences, and states that if in any set of alternatives three elements are selected Y, Y', Y" and Y is regarded by the consumer as being at least as good as Y', and Y' is regarded as being at least as good as Y", then Y must be at least as good as Y" (Green, 1971). This axiom of transitivity allows the set of alternatives to be partitioned into indifference classes in which no object of choice is present in more than one class.

By applying the axioms of completeness and transitivity the third axiom can be stated. This axiom of rational choice states that if an alternative Y is selected from a set of alternatives by a consumer then Y is at least as good as all other alternatives in the set (Green, 1971). It is important to note that if there are alternatives in the set which are indifferent to the chosen alternative this axiom does not explain how the selection was made. This axiom is the basis for consumer sovereignty which is the core of both positive and normative applications of economic theory. Remember that in the context of economic theory this axiom does not say that consumers are rational only that they behave as though they are rational in the sense that they make consistent choices. Further no other axiom of consumer preference has yielded so many hypotheses of behavior that have been successfully tested emperically (Green, 1971). However, a variation from the standard theory of choice has been developed by Gary Becker (1962) which indicates that results consistent with the standard theory can be achieved under conditions of explicit irrational choice.

The fourth axiom of individual choice is called the axiom of non-saturation, and states that consumers prefer more of a good to less. A more precise way of stating this is, if the elements of alternative Y are at least as large as the identical element of Y', and one element of Y is larger than the identical element of Y', then Y is preferred to Y' (Green, 1971).

The proposition which follows from this states that indifference curves are downward sloping (negative slope). Formally this can be stated that any two alternatives in a consumption set Y and Y', which are in the same indifference class, cannot have a condition where all elements of Y are at least as large as the same element of Y' and have one element of Y larger than Y' (Green, 1971). To illustrate this proposition consider that there exist two sets of bundles containing equal quantities of all elements other than A and B as illustrated in Figure 2-1. Given the point Y (y_A, y_B) where may Y' (y'_A, y'_B) be located which is indifferent to Y, but not identical? By the axiom of non-saturation Y' cannot contain at least as much of each element and more of at least one, or it would be preferred to Y. That is, Y' cannot lie in or on either boundary of the quadrants northeast or southwest of Y. Therefore it must lie within, not on the boundary, of the quandrants northwest and southeast of Y. If in addition there exists a continuous line or curve consisting of points all indifferent to Y, the application of this proposition implies they must everywhere be downward sloping and to the right, as shown in Figure 2-1.

Figure 2-1: Individual preference axiom of non-saturation; the proposition of downward-sloping indifference curves.



The fifth axiom of consumer preference describes the continuity of preferences and allows the use of analytical techniques which require the assumption of continuity. Because this axiom is important for technical rather than conceptual reasons the formal statement of the axiom is left to Appendix A. Two technical considerations about curves of indifference points that follow from this axiom are that they have no thickness and that they cannot intersect.

The sixth axiom of consumer preference describes the convexity of indifference curves. The visual interpretation of convexity is that a curve is convex with respect to the origin of a two dimensional diagram. A more rigorous definition of convexity is that given a set of points C(Y), then for any two points Y and Y' within the set, all points on a straight line joining Y and Y' are also in the set. This set would be said to be strictly convex if every point between Y and Y' is in the interior of the set, not on the boundary. Formally the axiom of strict convexity states that for all alternatives Y, in C(Y), the set of alternatives C(Y') consisting of all Y' in C(Y) such that Y' is regarded by the consumer as at least as good as Y, is strictly convex (Green, 1971). This implies that no indifference curve or surface ever intersects an axis because the axes are straight lines and inconsistent with strict convexity. This axiom is usually justified by the observation that as any good becomes scarce more units of other goods are required to induce the owner of the scarce good to give up an additional unit.

Recently in the literature techniques have been presented for the analysis of choice in situations where consumer's preferences do not include a positive quantity of every scarce good. This condition allows a relaxation of the axioms of non-saturation and strict convexity. However, the application of the theory of investment choice to the analysis of stand treatments does not require a relaxation of these axioms and these techniques will not be developed in this discussion. For a presentation of these techniques see Green (1971) Chapter 7 or Baumol (1972) Chapter 7.

The observation of the changing rate at which individuals are willing to exchange goods is the basis for the concept of the marginal rate of substitution. This is defined as the amount of any good, A, that a consumer would be willing to give up to get an additional unit of good, B (Leftwich, 1966). The axiom of smooth indifference curves is the seventh axiom of consumer preference and states that for all alternatives in a consumption set the marginal rate of substitution is uniquely determined (Green, 1971).

Taken together the implications of the axioms of continuity of preferences, strict convexity, and smooth indifference curves can be summarized in the proposition that for any pair of commodities Y and Y' holding the amounts of all other commodities constant, the marginal rate of substitution of Y for Y' is a continuous and strictly decreasing function of Y (Green, 1971). Geometrically this is illustrated for the case of two goods by the indifference curves U, U' and U" in Figure 2-2.

A formal statement of the axioms of consumer preference, their corollaries and propositions is given at the beginning of Appendix A.

The postulate of an opportunity set, which defines the constraints on individual choice can be broken down into three elements. The first of these is the endowment of the individual. This is the combination of goods that provide a starting point for the optimizing choice. Endowment, in this context, includes the services of the individual's time as well as the physical goods under the individual's control. The second element is composed of the exchange opportunities available to the individual. Exchange opportunitites in this context are the combinations of goods attainable by the individual through exchanges with other members of society and can only exist when society has at least two individuals. The third element of the constraints on individual choice consists of the productive opportunities available to him. In the context of economic theory production is thought of as the physical transformation of one good into another or activities involving nature rather than other individuals (Hirshleifer, 1970). In this sense the act of growing trees is production rather than exchange because it involves dealing with nature.

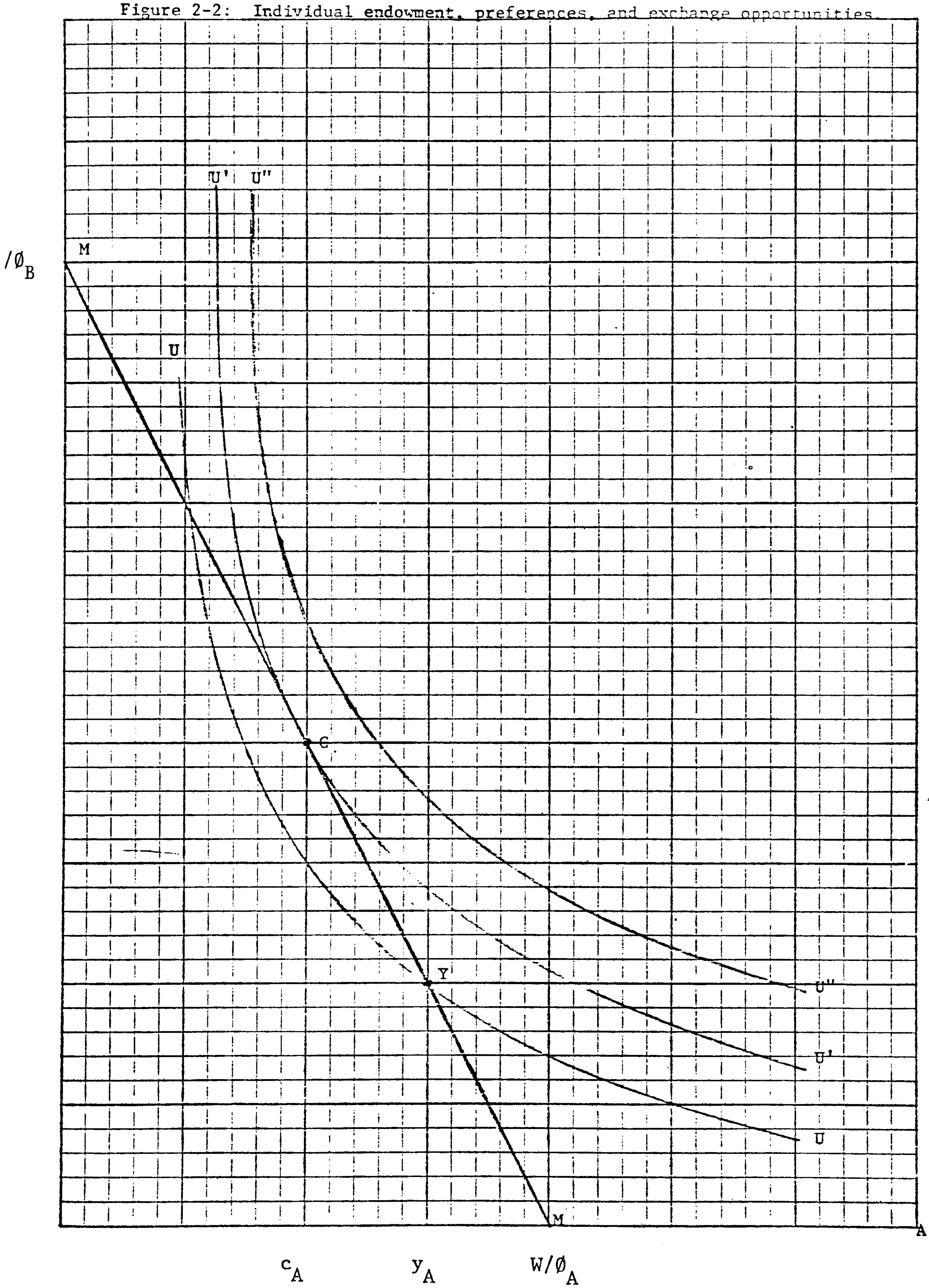
At the level of economic theory that describes how societies or aggregates of individuals function, the interaction of individuals, with their preferences and opportunity sets, produce an equilibrium through the mechanism of a market, which determines prices or ratios of exchange among various commodities. In the abstract world of economic theory the market mechanism acts as a means of transferring information about the conditions under which individuals are willing to exchange goods and as the means by which title to goods is transferred between individuals. However, the most important function is that of producing prices, or exchange ratios, under which an equilibrium may exist. In this context equilibrium is defined as a condition in which each individual is satisfied, according to his preferences, that he cannot attain, within his opportunity set, a combination of goods superior to the one now in his possession. If this condition is not met the dissatisfied individual will engage in market transfers or productive transformations until he is satisfied. In order for an equilibrium to exist all individuals within the society must be satisfied. Prices in this abstract system are the form of information transfer where the market expresses the exchange conditions under which equilibrium may occur, and may be expressed in real as well as the money terms to which we are accustomed in normal transactions.

Simple exchange. In order to develop an understanding of how these postulates operate in an economy, imagine a society consisting of a large number of individual economic agents and only two desired goods, A and B. The choices in this world are timeless. Furthermore, the individuals in this world can only alter their endowments through exchange (i.e., trade with other members of society).

Next, consider an individual in this society with an endowment Y (y_A, y_B) as illustrated in Figure 2-2. The exchange opportunities for this individual are defined by the market line MM which represents the outer limit of combinations of A and B attainable through exchanges with other members of society. To illustrate, observe in Figure 2-2 that for the individual to move from Y to C (c_A, c_B) he must exchange 5 units of A for 10 units of B. Therefore, we can say that the ratio of exchange between A and B is +10/-5 which reduces to +2/-1 or -2. The usual practice is to express prices in terms of the numerator of the ratio or a *numeriare*. The ratio +2/-1 can be expressed as -2 which, in turn, can be represented by a line with a slope equal to the price in terms of the *numeriare*. Since this individual has no productive opportunities the opportunity set is defined by the region OMM in Figure 2-2. The opportunities for consumption must lie within this region. Further at any point in the interior of OMM additional units of A or B can be consumed without giving up units of either commodity. These combinations are considered inferior and inconsistent with the axiom of non-saturation. Therefore, only combinations of A and B on the boundary of OMM along MM are considered for achieving a preferred combination of A and B for consumption.

The line UU in Figure 2-2 represents combinations of A and B to which the individual is indifferent. This is to say that the individual is indifferent between combinations of A and B represented by the line UU, and that the slope at any point on UU represents the marginal rate of substitution between combinations of A and B. The lines U'U' and U"U" also represent combinations of A and B to which the individual is indifferent. The curves UU, U'U', and U"U" represent the boundaries of

Figure 2-2: Individual endowment, preferences, and exchange opportunities.



consumption sets which are consistent with the axioms of consumer preference. Therefore, the alternatives represented by UU, U'U', and U"U" have the attribute of increasing preferability. To say this another way, the set bounded by U'U' is preferred to the set bounded by UU. Given the situation in Figure 2-2, and following the axioms of consumer preference, the individual would be expected to exchange A for B moving along the market line until a combination of A and B is achieved that is preferable to all other combinations attainable. This is represented by point C where the market line MM is tangent to the indifference curve U'U'. At this point of tangency the ratio of exchange is equal to the marginal rate of substitution. Point C in Figure 2-2 represents the highest possible state of preference achievable given the opportunity set OMM.

It should be noted that this presentation differs from the usual presentation of the theory of economic choice in that the constraint is in terms of wealth, a stock, rather than income, a flow. This difference is of no great significance here in timeless choice, but will become important later in intertemporal choice when the constraint on individual choice will be wealth rather than income.

The addition of an index of value to the above example will allow the accumulation of the amounts of A and B into a single measure of the endowment. This measure would be defined as wealth, and is illustrated in Figure 2-2 by points $W/\emptyset_A$ and $W/\emptyset_B$ which represent the amount of wealth for Y expressed in units of A and B respectively. Notice that these points are defined by the intersection of the market line MM with the A and B axes.

This discussion of the theory of economic choice has been made in terms of a situation considerably abstracted from the real world. Among the several elements omitted are considerations of more than two goods; production as well as exchange; the problems of choice over time; and market imperfections. Each of these components of reality will be developed later in this Chapter.

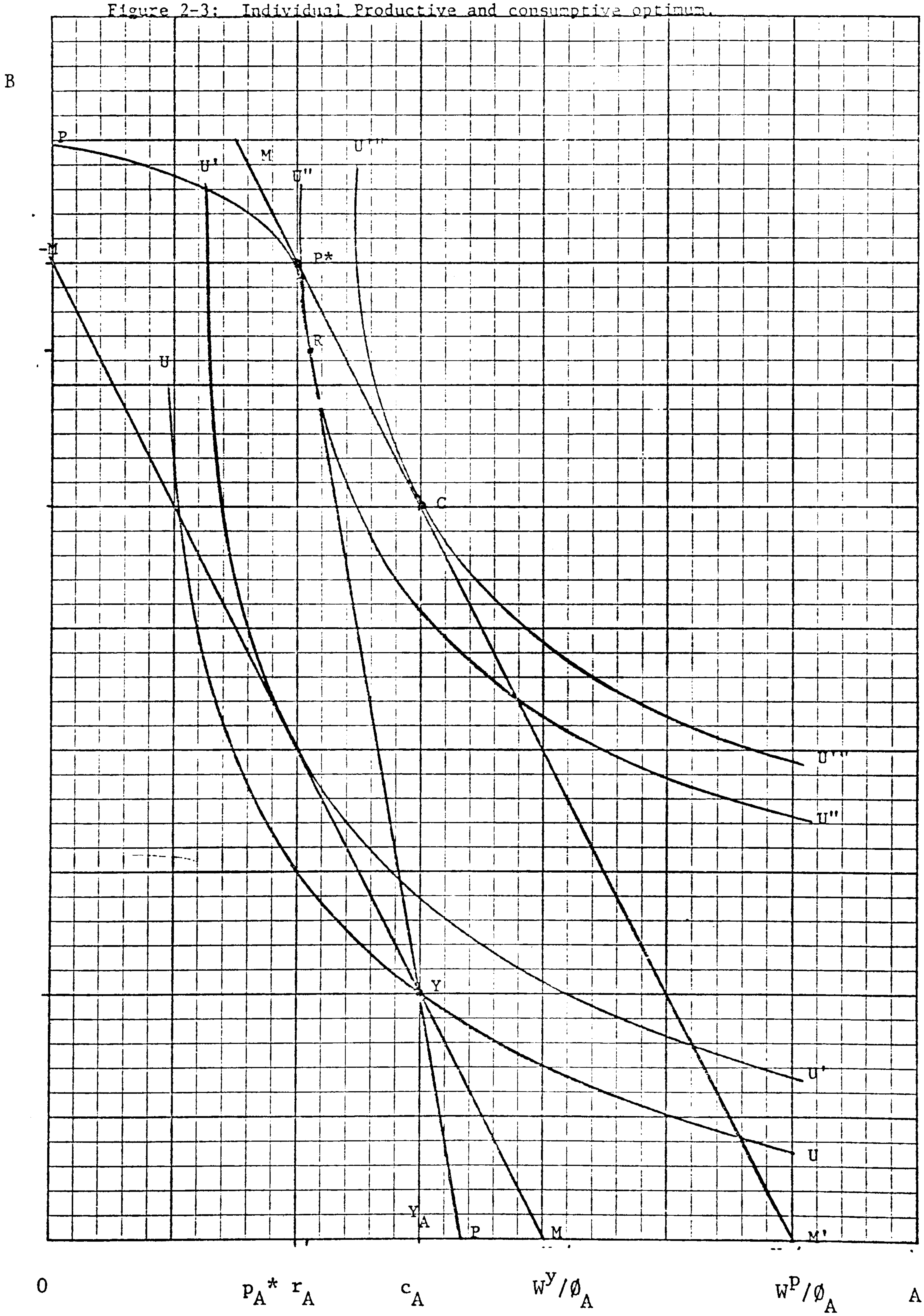
The first of these simplifications, that of two goods, can be examined in terms of a figure with a dimension for each good in the system. In this situation the optimum point would be on an n dimensional surface with n equalling the number of desired goods. The conditions at the optimum would remain, with the ratio of exchange in the market being equal to the marginal rate of substitution along an indifference surface in any direction.

It is important to observe at this point that by examining the real problem of choice in a system of exchange, in terms of the abstract and general concepts of economic theory, we are able to derive the important generalization about exchange that individuals will be expected to select to a commodity bundle for which their marginal rate of substitution is equal to the ratio of exchange. In addition, the theoretical structure provides a framework for predicting the effect of a perturbation on the system.

A mathematical formulation of the principles of simple exchange is given in Appendix A.

<u>Production and exchange</u>. Since the previous section showed that the extension of the economic theory of exchange to a large but finite number of goods was straightforward, nothing will be lost in returning to the simple case of two goods, A and B, to illustrate the paradigm of production and exchange. The situation is as in the example of simple exchange except for the addition of productive opportunities to the opportunity set. The combinations of A and B available to the individual through productive transformations in the two good cases are represented by OPP in Figure 2-3. The curved locus of points represented by PP is the constraint upon the individual's productive opportunities. This locus of points can also be thought of as the limit of productive transformations. The concavity of this curve with respect to the origin indicates diminishing returns as additional units of A are transformed into B or vice versa. The absence of a kink or corner in the production possibilities curve (PP) at Y or any other point indicates that the process of transformation is perfectly reversible. These idealized assumptions determine the shape of PP as it is drawn in Figure 2-3. However, in reality, the shape of the production possibility surface is determined by the available technology and may or may not conform to the idealizing assumptions implied in this example. These assumptions do not detract in any way from the generality of the solution which will be developed in the following paragraphs. The comment about combinations off the boundary of the opportunity set in the simple exchange example also hold for points off the boundary of the productive possibilities.

Figure 2-3: Individual Productive and consumptive optimum.



If the individual described in Figure 2-3 had an opportunity set limited to productive opportunities he would be expected to behave according to the axioms of consumer choice, transforming A to B or vice versa, and moving along the production possibility curve until a combination of A and B was achieved that is preferable to all other combinations attainable. This is represented by R in Figure 2-3 where the locus of points PP is tangent to the indifference curve U"U". At this point of tangency the marginal rate of production, represented by the slope of the curve PP, is equal to the marginal rate of substitution along the indifference curve U"U".

When there are both production and exchange opportunities the individuals overall opportunity set is expanded to the region represented by OM'M' in Figure 2-3. Notice that if the opportunity set were limited to either productive or exchange opportunities only, the individuals opportunity set would be limited to either OPP or OMM. The market lines MM and M'M' represent the exchange opportunities for different levels of wealth. In general, there are an infinite number of parallel market lines each representing the attainable combinations of goods for differing levels of wealth.

In this situation of production and exchange, the Separation Theorem (Hirshleifer, 1970) asserts that the individual achieves an overall optimum, represented by C in Figure 2-3, by a two stage process. First the individual maximizes his wealth, represented by $W^P/\emptyset_A$. This is equivalent to finding the highest possible market line. At this point the ratio of transformation, represented by the slope of PP, is equal to the ratio of exchange represented by the slope of MM and M'M'.

Second, the individual engages in exchanges along the market line, represented by M'M', until he reaches the most preferred bundle of goods for consumption, represented by C in Figure 2-3. At this point, as in the paradigm of simple exchange, the ratio of exchange is equal to the marginal rate of substitution along an indifference curve.

This examination of individual choice within the general framework of economic theory allows the development of the generalization that an individual will achieve a consumptive optimum in an economic environment that is composed of both productive and exchange opportunities by a two stage process. The first stage consists of achieving a position of maximum wealth within the constraint of the production opportunity set. At the point of maximum wealth, the ratio of transformation will be equal to the ratio of exchange. It is important to note that this point is determined by productive and market considerations, and that individual preference is not directly involved with wealth maximization, given the idealized assumptions we have been working under. This is the key feature that allows the delegation of productive decisions to firms and will be discussed in the next section.

The second stage in the achievement of an individual consumptive optimum consists of exchanges with other members of society to achieve a preferred bundle of goods for consumption. The condition at this point is equivalence between the ratio of exchange and marginal rate of substitution (Hirshleifer, 1970).

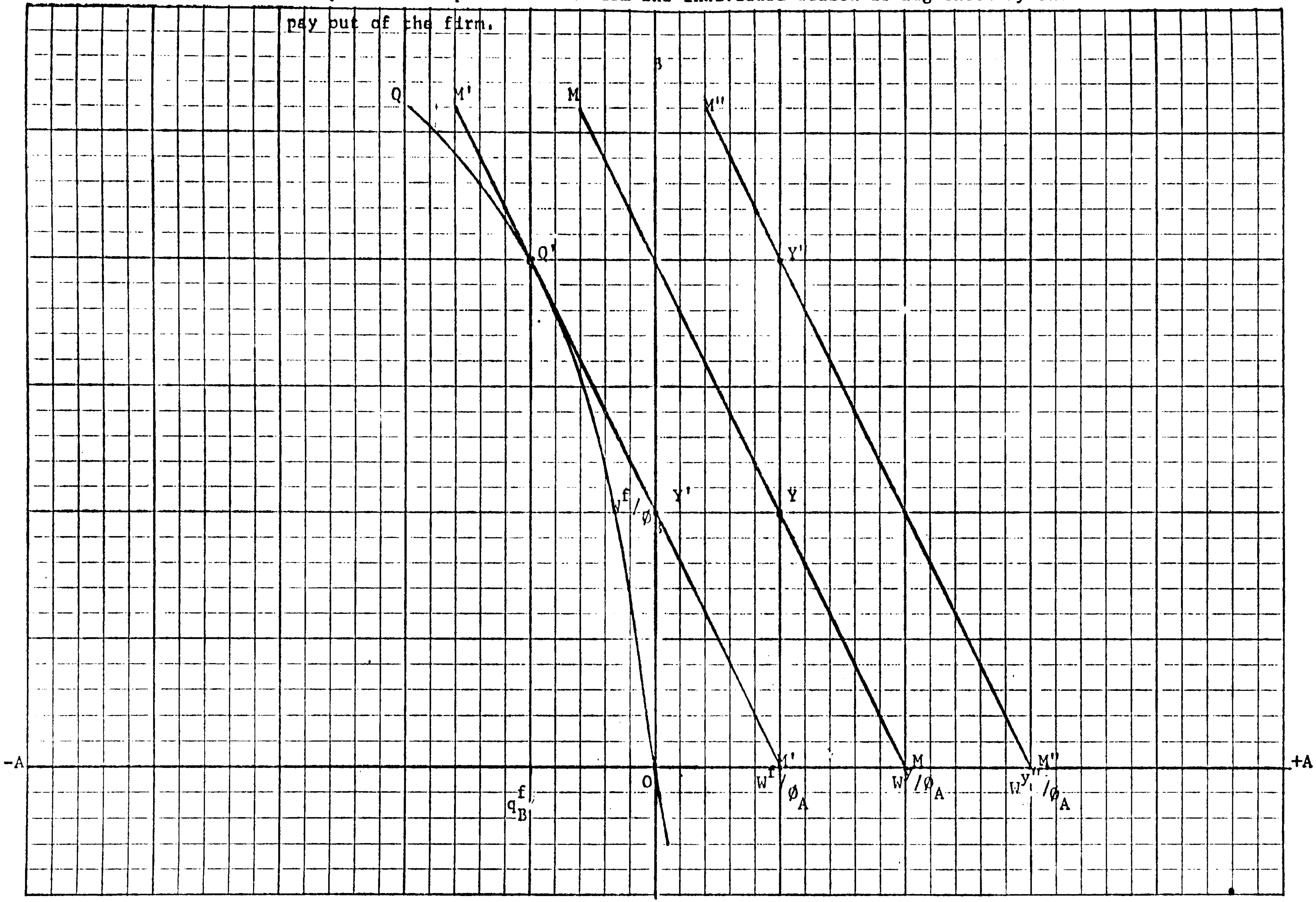
Production and exchange with firms. The exact definition and role of the firm is a subject about which economists are not in complete agreement (Hirshleifer, 1970). Coase (1937) discusses the role and existence of the firm emphasizing its efficiency and convenience as an institution for combining resources for the purpose of production. For this discussion firms will be defined as groupings of one or more individuals and their resources for the purpose of conducting productive activities. Furthermore, firms as decision making agents, will be treated under the assumption that only they are capable of production. Under this assumption, firms will have opportunities for production and exchange with other members of society. Individuals on the other hand will be treated as specialized agents of consumption and will have opportunities for exchange and initial endowments (Hirshleifer, 1970).

In the interest of simplicity, the explanation of production and exchange with firms will be in terms of the model of two goods. But as was shown previously, the extension to the multiple good case is straight forward.

For an individual firm the productive opportunities are represented by the locus of points QQ in Figure 2-4. This boundary of the firms productive opportunities has the same traits indicating diminishing returns, reversibility, and continuity as in the previous example. In addition, the locus of points QQ passes through the origin as the firm has no endowment and must use resources which are made available to it by individuals. Because production is the transformation of one good into another the boundary of the production set will have a negative slope and exist in the second and fourth quadrants of a two dimensional geometric diagram. As pictured in Figure 2-4, the opportunity set in the fourth quadrant describes the opportunities for transforming A into B, but again the extension to a diagram in the second quadrant is straightforward. The preferred production point for the firm is represented by Q'. This point, as in the prevous example, represents the highest market line attainable and the ratio of transformation is equal to ratio of exchange.

Since by definition the firm does not consume, it must distribute its output externally. The usual assumption is that this is done through exchanges to end up somewhere in or on the boundary of the positive quadrant. This is represented in Figure 2-4 by movement along the market line M'M'. At this point, it is most natural to assume that the firm distributes to its owners the net wealth created through the production activity, which is represented by the distance between 0 and $W^f/\emptyset_B$. This distribution increases the owners endowment to Y" placing them on the market line M"M" and increasing their opportunity set for consumption from OMM to OM"M" and making it possible for them to achieve a higher state of consumption preference.

Figure 2-4: The productive optimum for a firm and individual wealth as augmented by the pay out of the firm.



Notice in this example the conditions at the productive and consumptive optima remain as in the paradigm of production and exchange. The productive optimum being where the marginal rate of transformation is equal to ratio of exchange for the firm as well as the individual. The consumptive optimum for the individual remaining where the ratio of exchange is equal to the marginal rate of substitution along an indifference curve regardless of the presence or absence of firms in the economy. The key to the existence of equilibrium under these conditions lies in the distribution of the net wealth generated by the productive process to the owners and the resulting increase in their endowments.

Intertemporal Choice

As in the case of timeless choice, consumption will be postulated as the soile end of economic activity. This postulate is not as self-evident as it might appear. Economists are divided into two schools of thought with regard to whether or not consumption is the objective of economic activity. The first of these, the "work to live" school, treats wants as the end to human activity. The second the "live to work" school treats activities as the end toward which humans work (Friedman, 1962). For a more philosophical discussion of this division see Marshall (1920), Book III, Chapter II.

These divergent points of view are represented in the theory of intertemporal choice by the schools of thought which treat saving as deferred consumption (save to live), and saving and accumulation as an object of choice independent of consumption (live to save). However, the theoretical framework adopted for this discussion accepts the viewpoint that consumption is the objective of human activity.

Therefore, the objects to be dealt with in intertemporal choice are dated consumption claims. These dated claims to consumption are interpreted as titles to real generalized consumption. This simplifies the discussion of intertemporal choice by factoring out the decision among classes of consumption claims of the same date. The objective then of the consumer is to achieve a preferred pattern of consumption claims through time.

Exchange. The operation of the postulates of individual choice where the individual is limited to intertemporal exchanges is illustrated for a two period case in Figure 2-5. The point Y (yc_0, yc_1) in Figure 2-5 represents the individuals time-endowment with title to yc_0 units of consumption in period 0 and yc_1 units of consumption in period 1. The region OMM again represents the opportunity set with alternatives off the line MM being considered inferior as in the case of timeless exchange. The slope of the line MM again represents the ratio of exchange between titles to consumption in periods 0 and 1. The point W/C_0 where MM intersects the C_0 axis is the value of consumption in periods 0 and 1 in the endowment Y, measured in terms of consumption titles in period 0. This is a measure of wealth. The curves UU, U'U', and U"U" again represent combinations of consumption to which the individual consumer is indifferent. The curves U'U' and U"U" represent successively higher states of preference.

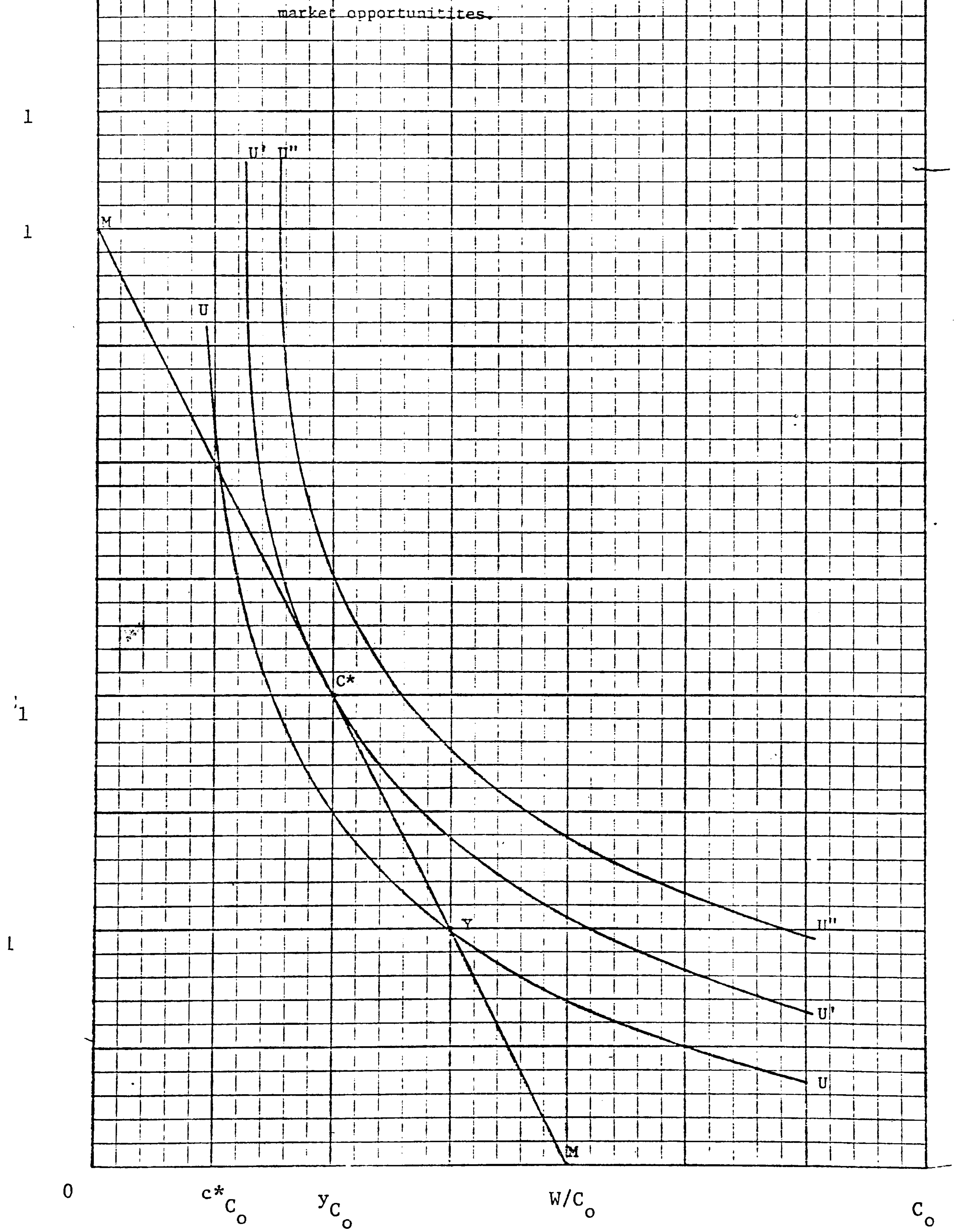
The slopes of the curves UU, U'U', and U"U" represent the marginal rate of substitution between titles to consumption in periods 0 and 1. Another term used to describe the intertemporal marginal rate of substitution is time preference.

As in the case of timeless exchange, the individual achieves a preferred combination of titles to consumption through exchanges along the market line MM. This is achieved which is represented in Figure 2-5 by movement to C*. The condition at this point is the same as in the example of simple exchange. The marginal rate of time preference is equal to the market ratio of exchange, and is represented by the tangency between the market line MM and the indifference curve U'U' (Hirschleifer, 1970).

As in the example of timeless exchange, this framework can be extended to multiple time periods through the addition of multiple dimensions. The optimum combination of titles to consumption would then be represented by the tangency in all directions between surfaces representing exchange opportunities and individual preferences.

Production and exchange. As in the example of timeless choice, the remaining discussion of intertemporal choice will be in terms of a two period example because the extension to a multiple period case is straightforward. The time preferences, time endowments, productive, and market opportunities for an individual whose opportunity set consists of both

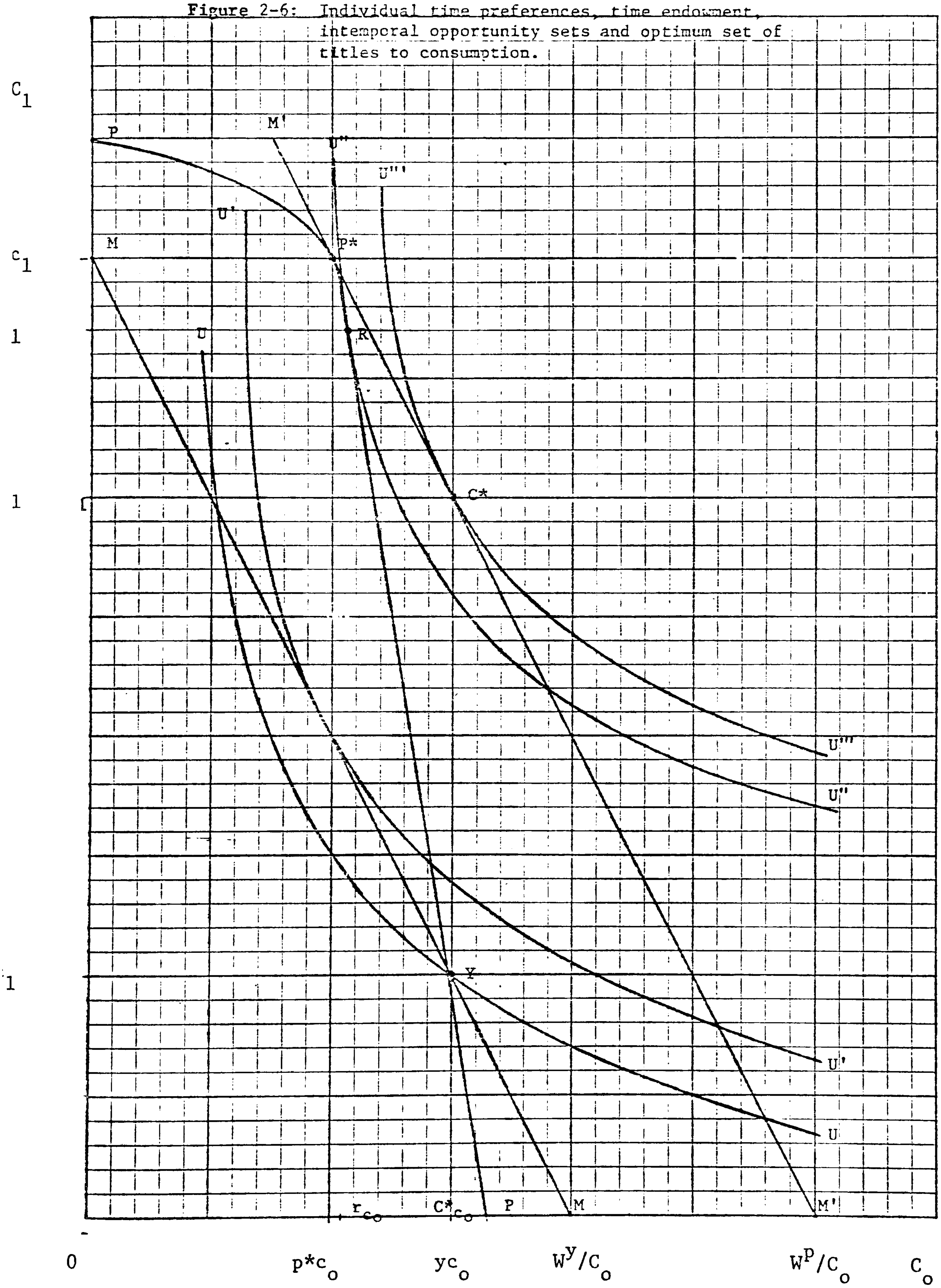
Figure 2-5: Individual time-preferences, time-endowments, and market opportunitites.



productive and exchange opportunities is illustrated in Figure 2-6. As in the example of production and exchange with timeless choice, the individual with only productive opportunities would be expected to engage in productive transformations until the highest possible state of indifference was achieved. This state is represented in Figure 2-6 at point R. The condition at this point, as in the similar example of timeless choice, is the tangency between the locus of points representing productive possibilities and a similar locus representing combinations of titles to consumption to which the consumer is indifferent. As in the timeless choice example, the condition implied by this tangency is that the marginal rate of transformation along the productive possibilities frontier is equal to the marginal rate of substitution along the locus of points representing combinations of titles to consumption to which the consumer is indifferent.

In the case where the individuals opportunity set consists of intertemporal exchange as well as production, the optimum levels of production and consumption will be defined by P* and C* respectively in Figure 2-6. The process by which this is achieved is similar to the example of timeless production and exchange. The individual engages in productive transformations until the marginal rate of transformation is equal to the ratio of exchange between titles to consumptio in periods 0 and 1. When this condition is met, the individual has achieved the maximum level of wealth that is attainable within the given opportunity set.

Figure 2-6: Individual time preferences, time endowment, intemporal opportunity sets and optimum set of titles to consumption.



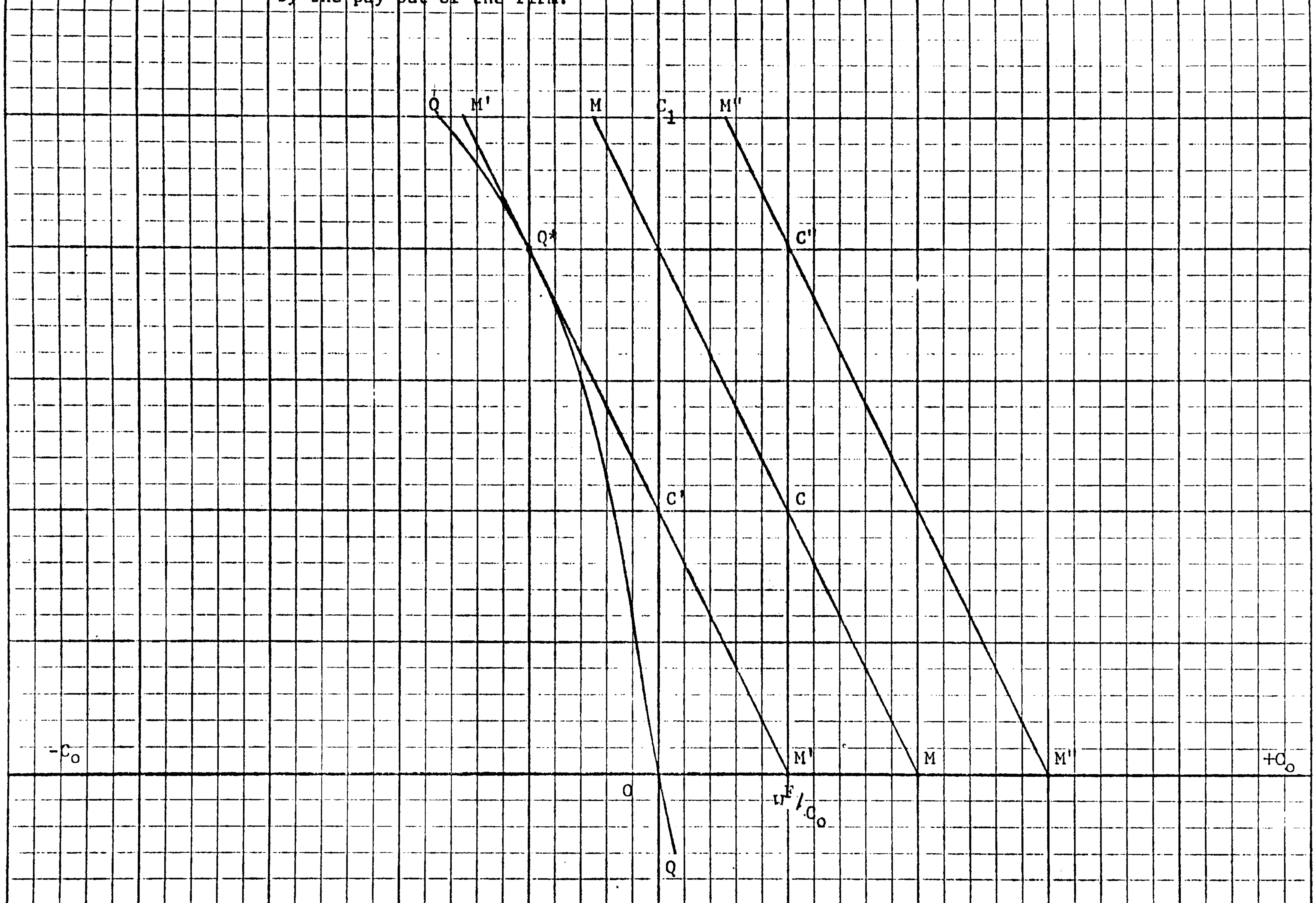
Having achieved the opportunity set defined by OM'M', in Figure 2-6, the individual optimizes the bundle of titles to cons-mption available through exchanges with other members of society until a most preferred combination is achieved which is represented by C*. The conditions at the point of tangency represented by C* are as in the case of intertemporal exchange, are that the marginal rate of substitution along an indifference curve will be equal to the ratio of exchange (Hirshleifer, 1970).

Production and exchange with firms. The operation of the postulates of individual choice in a situation with firms and individuals making choices through time is exactly parallel to timeless choice. Individuals give up claims to consumption to a firm represented by the movement from C to C' in Figure 2-7. Firms in turn transform titles to consumption in period 0 to titles to consumption in period 1 represented by Q*. At this point, the marginal rate of transformation is equal to the ratio of exchange.

The firm then distributes the proceeds of its production proportionately to the initial contributers resulting in an expansion of the individuals opportunity set. This is represented by the increase in the opportunity set from OMM to OM"M". Individual's then achieve preferred combinations of titles to consumption through exchange, as was illustrated in the example of intertemporal exchange (Hirshleifer, 1970).

Basic concepts in intertemporal choice. So far this section has concentrated on defining and illustrating the postulates of individual preference and opportunity set. In addition to these two postulates, the

Figure 2-7: The intertemporal productive optimum for a firm and individual wealth as augmented by the pay out of the firm.



theory of intertemporal choice draws on several other concepts to describe allocation decisions.

Interest, the first of these concepts, arises because in the usual treatment of optimization over time a departure is made from the ordinary notation of price. Instead of quoting the value of a title to consumption one time period from the present terms of titles to current consumption, the practice is to say that a unit of consumption today exchanges for (1+r) units of consumption one period from now. In this situation, r is the rate of interest. If the time period was one year (assuming no intermediate compounding), r would be the annual rate of interest (Hirshleifer, 1970).

This discussion of the concept of interest, which presents it as a price ratio between current and future claims to consumption, is not a complete presentation of the economic theory relevant to the subject of interest and the reasons for its existence. A discussion of this material is beyond the scope of this thesis. However, a concise discussion of the theoretical aspects of interest can be found in Samuelson (1964), in the Appendix to Chapter 28.

Compound interest expresses the price ratio between current and future claims involving multiple time periods as r in the geometric series $(1+r)^n$, where n equals the number of periods separating the present and future claims. This expression captures the essential nature of the exchanges in titles to consumption as a series of single period exchanges. For a brief review of the mathematics of interest see Appendix A.

This definition of interest, as the price ratio between present and future claims to consumption provides an index of value which allows the summing of consumption claims in various periods into an equivalent title to consumption in a single period. This is illustrated in Figure 2-5 by point W/C_0 which represents the equivalent title to consumption in period 0 of consumption titles yc_0 in period 0 and yc_1 in period 1. Following the same logic, the point W/C_1 is a measure wealth in terms of consumption titles in period 1 (Hirshleifer, 1970).

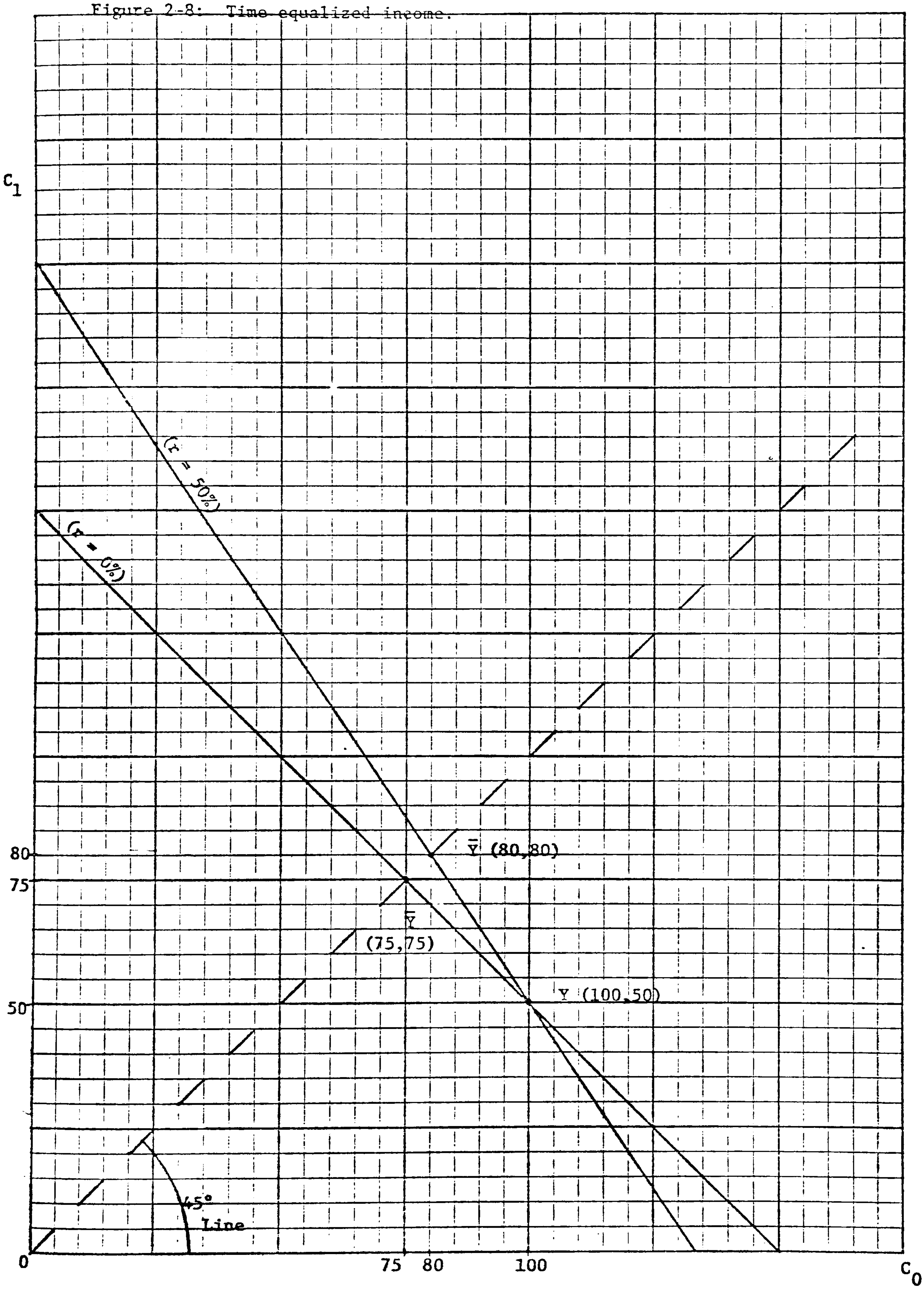
The concept of income, which has been avoided so far in this discussion, is difficult. Economic theorists generally agree that it represents potential consumption in the sense of the amount that could be consumed in any one period without impairing future consumption (Hirshleifer, 1970). However, this definition leads to difficulty in the sense that any consumption impairs future consumption as long as the opportunity set permits productive transformations or market exchanges through time. Hirshleifer (1970) proposes that this difficulty be over come by introducing the concept of a time equalized income or net income. To illustrate this, in a two period case with market opportunities only, consider an endowment vector of Y (100, 50) as shown in Figure 2-8. If in addition the interest rate r is 0 percent (current consumption titles and future consumption titles exchange at par) the time equalized or net income would be equal to 75. To say this another way, 75 units of consumption would be consumed in the first period and the remaining 25 units exchanged for an equal number of consumption units in the second period, bringing the total number of consumption units in that period to 75.

The disadvantage of the concept of time equalized income is that the amount is not independent of the rate of time conversion (rate of interest) between claims at different dates since the equalizing process requires explicit or implicit exchanges through time. Thus, if in the example illustrated in Figure 2-8 the interest rate were 50 percent, the time equalized flow of consumption claims would be 80.

A potential consumption or income concept which is independent of the rate of time conversion or interest rate is the elements of the endowment itself. In the example in Figure 2-8 this would say that 100 units of consumption could be consumed during the first period without impairing the consumption of 50 units in the second period. Hirshleifer (1970) calls this concept gross or endowed income. To return to the example in Figure 2-8, of the gross income of 100 units in the first period, at 0 percent interest, 25 units (20 units at 50 percent interest) must be deducted to arrive at a net or equalized income. These concepts of net and gross income are consistent with ordinary business and accounting practice (Hirshleifer, 1970).

The concepts of gross and net income lead directly into the concepts of gross and net savings. Gross savings being the excess between current endowed income and actual consumption. The excess of net (time equalized

Figure 2-8: Time-equalized income.



income) over actual consumption is current net savings. It is important to note that gross savings is an actual observable market magnitude while net savings is a mental construction rather than an observation.

The term investment is also often confusing. In this study the usage will follow Hirshleifer (1970), in the sense that investment will be limited to what may be called real investment or physical sacrifice of current consumption by productive transformation. These definitions do not require that an individual's gross savings be equal to investment.

The important theoretical concepts illustrated in this section are first, the idea of interest as the price of exchanges through time. Second, the fact that optimum points of production and exchange exist where the marginal rates of transformation and substitution are equal to prices (ratio of exchange). Third, that in the intertemporal case with production and exchange the Separation Theorm holds and the optimum series of consumption claims through time is achieved by a two stage process in which productive transformations are carried out which maximize wealth. This wealth, or series of consumptive claims is then exchanged until a series of consumptive claims which represents the highest achievable state of indifference is attained.

A mathematical presentation of the material in this section and a brief summary of compound interest formulas are given in Appendix A.

Investment Choice

The theory of investment choice is concerned with the application of the principles of intertemporal choice to the practical questions of: (1) What decision rule will correctly identify the optimum combination of investments to be undertaken? (2) Given any particular set of chosen investments, what is the best method of arranging consumption patterns to obtain the necessary investment inputs? The first question asks what production opportunities should be undertaken. The second asks how the necessary inputs are to be obtained taking account of market opportunities. The optimization involved can be examined at the level of a decision maker, either individuals, firms, or public agencies; and also at the level of the community as a whole. The point of view adopted for this discussion is that of the individual decision maker. Later in the discussion the rules for individual optimization will be examined in terms of the community as a whole.

Investment choice rules. The objective of investment choice rules is to define a test or series of tests which may be used to assist a decision maker in selecting investments.

In this discussion it is important to distinguish between rules and criteria. A criterion is the mathematical formula computed on the changes in claims to consumption associated with an investment. While a rule indicates the acceptability of an investment project by directing a comparison between the criterion computed and some other standard. The failure to distinguish between rules and criteria has been responsible for considerable confusion, since divergent rules can be proposed for the same criterion (Hirshleifer, 1970).

Some economists have discussed rules for ranking investments apart from rules specifiying the set of investments that should be adopted (Bierman and Smidt, 1966) (Van Horne, 1968). While this distinction may be useful in some applied work, theoretically it is only necessary to identify the combination of investments that will achieve the highest possible indifference curve or maximum possible level of wealth. It is also a characteristic of investments in stand treatments that they can not be uniquely ranked. The desirability of any treatment is usually influenced by the other treatments which have or will be imposed upon a stand. It follows from this that a correct ranking rule is not theoretically necessary, or generally possible, in the case of investments in stand treatments. Therefore, this discussion will limit its attention to rules for the adoption of investments.

The criteria most often used by economic theorists in the analysis of investment choice are present value or present worth. These criteria are defined as the value in terms of present consumption of a dated sequence of consumption claims, and are a wealth measure. There are several versions of present value criteria, all essentially equivalent, to which rules are applied in the analysis of investment choice.

The first of these, endowed wealth, is represented by the point W^y/C_0 in Figure 2-6 and is the present value of the endowment sequence represented by Y. The second criterion, attained wealth, is represented by the point W^p/C_0 in Figure 2-6, and is the present value of the sequence of consumption claims represented by P*. As an alternative to attained wealth the criterion of wealth gain, which for the two period case illustrated by Figure 2-6, would be the difference between W^p/C_0 and W^y/C_0 which will be represented by W^q/C_0, and is the difference between the endowed wealth and attained wealth of the investment ensemble adopted. If the decision making agent is a firm rather than a household the endowed wealth is assumed to be zero and the wealth gain and attained wealth are equivalent. For the two period case they are represented by W^f/C_0 in Figure 2-7.

Next consider an individual investment with changes in the sequence of consumption titles such as might be associated with an incremental movement along PP in Figure 2-6. The changes in attained wealth, W^p/C_0, and wealth gain, W^q/C_0, can be represented by the expressions $W^{\Delta p}/C_0$ and $W^{\Delta q}/C_0$, and will be called project present value.

From the previous discussion of intertemporal choice it follows that the rule by which the criteria of attained wealth, and wealth gain will be judged is to maximize these criteria. The rule by which the criterion of project present value is judged is to adopt the investment project as long as the project present value is greater than zero. This is equivalent to saying, adopt an investment project as long as it increases attained wealth or wealth gain.

One special case that arises is among mutually exclusive projects where the obvious rule is to select the project with the greatest project present value, as this is required to be consistent with the maximization of attained wealth and wealth gain.

Note that if the intertemporal productive possibilities curve is irregular, with multiple peaks, the investment alternatives analyzed must be such as to allow the selection of the alternatives which represent the maximization of attained wealth and wealth gain. For this reason, it is necessary to examine investment increments rather than just a marginal investment.

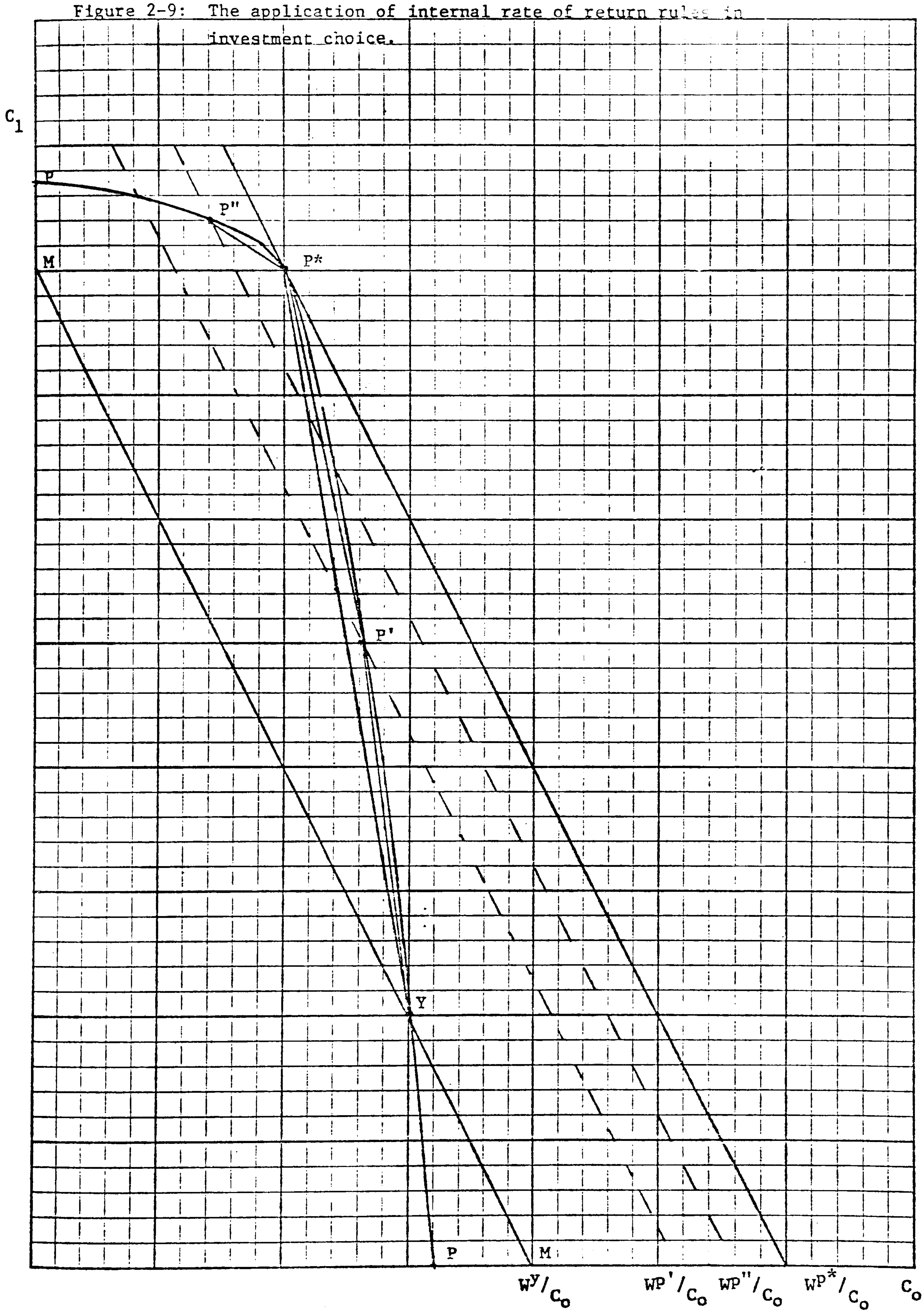
An entirely different class of criteria is that of the rate of return, rate of return over cost, marginal efficiency capital, or marginal efficiency of investment (Hirshleifer, 1970). This discussion will use the term internal rate of return for these criteria. The internal rate of return for an investment project or projects is defined as the discounting rate (irr) which makes the present value of the productive transformations equal to zero. A convenient image for the internal rate of return is to think in terms of a growth rate over time. As an illustration, consider a situation where an investment project or ensemble has an initial outlay in the first period o_o and no benefits except for the final receipt b_t, then the internal rate of return would be the rate of annually compounded growth that exchanges the initial input o_o into the terminal output b_t.

The investment choice rules employing the internal rate of return criteria fall into two different forms. First, those rules that direct that investment projects or combinations of projects be adopted that maximize the internal rate of return. Secondly, the class of rules which direct that an investment project or set of projects be adopted on the basis of a comparison between the internal rate of return and the market rate of interest.

The simple rule that calls for the adoption of an ensemble of investment projects that displays a maximum calculated internal rate of return does not seem to be endorsed by any theorists (Hirshleifer, 1970). The adoption of this rule can lead to absurd results. For example, if two independent investment projects yielding 20 percent and 15 percent were available, this rule would tell the decision maker to adopt only the 20 percent project because adopting the second must reduce the over-all internal rate of return for the combined set. This is regardless of the fact that the project with an internal rate of return of 15 percent may represent a desirable opportunity that should be adopted. An example of this situation is given for a two period example in Figure 2-9 where the adoption of project P* in addition to P' causes the internal rate of return to fall from the rate represented by the slope of P'Y to the rate represented by the slope of P*Y.

A second version of the rules, that call for the maximization of the internal rate of return, calls for the maximization of the internal rate

Figure 2-9: The application of internal rate of return rules in investment choice.



of return on the condition that the aggregate current outlay for investment is fixed (Lutz & Lutz, 1951) (Scitovsky, 1951). The argument in support of this rule is that for a given current sacrifice it is desirable to adopt the project ensemble maximizing the rate of growth into future titles to consumption. The failure of this rule is that it fails to indicate the optimal balance between current consumption and current investment (Hirshleifer, 1970). In addition, the following discussion of investment choice rules, involving comparisons between the internal rate of return and the market rate of interest, will show that this rule is not generally correct even if this objectionis waived.

Rules for investment choice, based on a comparison of a calculated internal rate of return and a market or external rate of interest, appear in three forms parallel to the three versions of the present value rules. The discussion of comparison rules, to be applied to internal rate of return criteria based upon attained wealth, wealth gain, and incremental projects, require the introduction of two additional concepts. First, the concept of a "Defender" and a "Challenger" (Hirshleifer, 1970). The "Defender" being defined as the attained gross income stream of the investment ensemble now adopted, and the "Challenger" being the gross income stream associated with an alternative ensemble. The second concept is that of an investment project or ensemble being "later" than a project or ensemble to which it is being compared. For example, a "Challenger" would be unambiguously "later" than a "Defender" if the differences between the gross income streams were all negative up to a certain date and after that all positive (Hirshleifer, 1970).

The criterion of an internal rate of return calculated from the differences between the gross attained income streams is known as "Fisher's rate of return over cost" (Fisher, 1930). The comparison rule applied to this criterion is to adopt the "later" if "Fisher's rate of return over cost" is greater than the market rate of interest, but to adopt the earlier if "Fisher's rate of return over cost is less than the market rate of interest." The need for the concepts of "earlier" and "later" can be seen from the fact that a reversal of the "Challenger" and "Defender" does not change the internal rate of return in the calculation of "Fisher's rate of return over cost" (See the mathematical formulation in Appendix A).

Prior discussion has demonstrated the relationship between attained wealth, and wealth gain. It follows from this that the rule to be applied to an internal rate of return criterion calculated from differences in the stream of gross incomes in wealth gain between two investment ensembles would be identical to that applied to "Fisher's rate of return over cost."

The rule for investment choice of a particular project is based on the internal rate of return calculated from the gross income stream associated with the project, which Fisher (1930) called "marginal rate of return over cost" is to adopt the project if the "marginal rate of return over cost" exceeds the market or external rate and the project is "later" than the previously adopted ensemble, or if the "marginal rate of return over cost" is less than the market or external rate to adopt the project only if it is "earlier" than previously adopted ensemble (Hirshleifer, 1970). The parallel between this rule and the previous one is obvious when the gross income stream of the project is considered as an income stream of wealth gain. The need for the conditioning of the rule in terms of "earlier" and "later" is also obvious from the ambiguous decisions that could arise if the incremental project and the existing ensemble were reversed.

The difficulties in the application of comparison rules to internal rate of return criteria are of two types. The first being the requirement to identify income streams as being either "earlier" of "later" than the income stream of a previously adopted ensemble. The fact that in only a limited class of comparisons will the "later" projects be unambiguously identified severely limits the operational use of these rules (Hirshleifer, 1970). In situation where "earlier" and "later" are clearly identified the internal rate of return comparison rules are equivalent with the present value rules. This is illustrated geometrically in Figure 2-9 for a two period case by the comparison of investment ensembles represented by P', P*, and P". Notice that P' is "later" than Y, P* is "later" than P", and P" is "later" than P*. The internal rates of return are represented by the slopes of the lines between the investment ensembles. The external or market rate of interest is represented by the slope of the lines which intersect the C_0 axis at W^y/C_0, $W^{p'}/C_0$, $W^{p'}/C_0$ and W^{p*}/C_0. Examination of the figure will show that regardless of the "Challenger", "Defender" order adopted, the application of comparison rules based upon internal rate of return criteria will result in the selection of investment project ensembles which maximize wealth and achieve the largest possible opportunity set as a basis for exchanges.

The second difficulty lies in the failure of rules based on internal rate of return criteria to indicate the optimal balance between current consumption and current investment. To illustrate this, imagine two mutually exclusive investment projects A and B such that the "marginal rate of return over cost" for both A and B are greater than the market rate and A is greater than B. Further, both A and B are later than the existing ensemble. However, project A may be of such a trivial size in comparison to B that B yields a much higher present value. Clearly, in this situation the rules based on comparisons applied to internal rate of return criteria do not lead to the adoption of the investment ensemble which creates the largest consumption opportunity set.

In summary, only in special cases where project ensembles can be identified as "earlier" or "later" do investment choice rules based on comparisons of internal rate of return criteria yield results which are consistent with the theory of intertemporal choice. In situations where project ensembles can be identified as "earlier" or "later" rules based on comparisons of internal rate of return criteria are equivalent to corresponding versions of present value rules.

In addition to present value and internal rate of return, several other criteria and associated investment choice rules have been proposed. One, terminal value, is based on the compounding of income flows forward to some future date rather than discounting them back to the present (Solomon, 1959). The parallel between this criterion and present value is complete and is demonstrated in Figure 2-5 for the two period case by the equivalence of W/C_0 and W/C_1. However, differences in project evaluation will arise if there is a difference between the reinvestment rate by which intermediate receipts are compounded to the terminal period and the discount rate employed in the calculation of present values. This situation cannot arise under the assumptionof perfect markets which has conditioned the discussion to this point. Later in this Chapter imperfections in the market that result in divergent borrowing and lending rates will be discussed and it will be shown that investment choice can be carried out without introducing terminal value concepts.

Solomon (1959) has also attempted to avoid the ambiguities of the internal rate of return concept while preserving the form of a rate of change per unit time that can be compared with the rate of interest by introducing teh concept of a reinvested rate of return. This proposal is based on the compounding forward to a terminal time of all intermediate receipts and payments at an external rate of interest. This terminal value is then divided by the current element (which is assumed to be an outlay) resulting in :

$$(1 + \widehat{rrr})^j$$

where: $\widehat{rrr}$ = the reinvested rate of return.

j = number of periods.

The important point to note about the concept of the reinvested rate of return is that it combines both internal and external elements and requires the implicit assumption that receipts and payments grow at the external rate. If correctly formulated this concept can be shown to be equivalent to the concept of present value (Hirshleifer, 1970) but it offers no theoretical superiority at the cost of increased complexity.

A third criterion, pay off period, is the length of time required for the summed receipts to equal the summed payments (usually undiscounted) (Bierman, and Smidt, 1966). This rule is defective in its failure to generally recognize timing in the gross income flows associated with an investment project. A second specific defect is its failure to recognize payments and receipts accruing at dates later than the payoff. However, survival of this rule in business practice may be due to its crude way of allowing for uncertainty in the future (Hirshleifer, 1970). The following discussion will develop rules to allow for uncertainty in a more defensible manner.

As was pointed out at the beginning of this section, the discussion of investment choice has been at the level of the individual decision maker and questions of the optimization of investment for the community as a whole have been ignored. The discussion while perfectly general has been set in a framework of choice that included perfect markets in the sense that no single individual could influence the price of exchanges, that desired goods were bought and sold at equal prices, and that all desired goods were traded and priced in the market. Investment choice decisions have also been made in a world of certainty. The question to be dealth with now is how does an investment hoice rule that correctly identifies optimum combinations of investment for an individual effect the well being of society as a whole.

This discussion of the well being of society will rest on the "Paretian" value judgement "that if one person is better off, and no one is worse off, welfare if increased" (Winch, 1971). It follows from this that under the conditions given if every member of society makes investment choices according to a rule which correctly identifies optimum combinations of investments then by the "Paretian" value judgement no improved state of welfare can be achieved. It is important to note that the previous statement requires the conditions of perfect markets and certainty to be met as well as acceptance of the "Paretian" value judgement as the standard by which the well being of society is judged. In addition, notice that the above statement does not say that perfect markets and certainty are necessary to achieve a "Paretian" optimum only that when they exist a "Paretian " optimum also exists (Winch, 1971).

In the following discussions of the Chapter, the theoretical formulation of investment choice rules under conditions of market imperfection and uncertainty will be discussed. However, one type of market imperfection will not be discussed in detail. This is the imperfection of incomplete markets. Markets are incomplete when goods are not exchanged and information is lacking on prices under which individuals are willing to exchange consumptive titles. Stands of trees are generally ascribed the attribute of producing multiple products for which there may not be market prices. The position of this thesis is that products of the forest or elements of an investment choice for which either no quantitative measurement or price exists are incommensurable. This condition introduces an irreducible element of subjective judgement to the investment decision which is beyond the scope of this thesis.

Before examining the influence of market imperfections on investment choice rules the application of these rules in several classes of general problems will be discussed.

Investment choice rules and nonregular opportunities. In previous examples, investment opportunities have been regular in the sense that they could be represented by smooth curves without inflection points. This condition will now be relaxed and the application of investment choice rules to non-regular investment opportunities examined.

The first class of nonregular investments to be considered can be described as interdependent investment oppotunities where poorer projects must be adopted to achieve better projects, and is illustrated in Figure 2-10. In this situation it is necessary to distinguish the true investment solution P* from the false solution P'. The point P' is a false solution because the consumption opportunity set OM'M' cannot possibly achieve a consumption set as desirable as those possible with the consumption opportunity set OM*M.

Figure 2-10: Interdependent investment opportunities: poorer projects prerequisite to better projects.

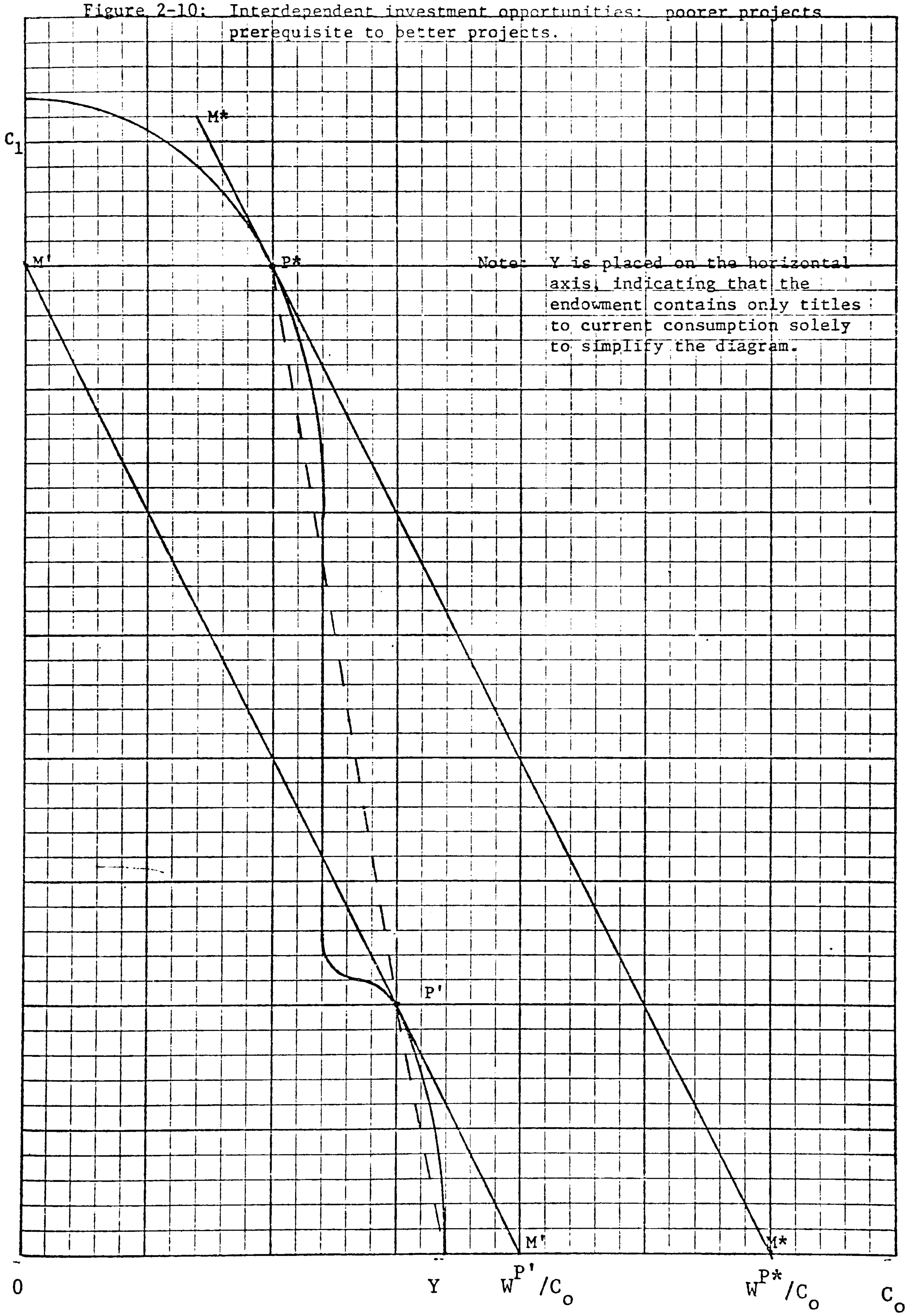


Figure 2-11 illustrates a situation where PP" and PP** represent two mutually exclusive investment opportunity sets. Again it is necessary to distinguish between the investment sets PP" and PP** when PP" represents the more productive set at high levels of investment and PP** the more productive at low levels. As in the previous example P* in set PP** represents the correct soluation because it results in the largest possible consumption opportunity set.

Nonregularity of a different sort is illustrated in Figure 2-12 where the lack of regularity is not due to interdependence but due to a limited number of discrete investment projects. As in the previous examples, the correct solution is the one that results in the largest possible consumption opportunity set, represented by P' + P*.

The fourth example of nonregularity is shown in Figure 2-13, which illustrates discrete and interdependent investment opportunities. As before, the correct investment choice is P* which creates the largest consumption opportunity set.

Examinzation of the previous four examples will reveal that the decision rule to maximize the present value of attained wealth always leads to a correct solution. The incremental present value rule also leads to correct solutions if the possible errors illustrated in Figure 2-10 and 2-11 are avoided by considering noninfinitesimal as well as infinitesimal investment increments.

Figure 2-11: Interdependent investment opportunities: alternative investment sets.

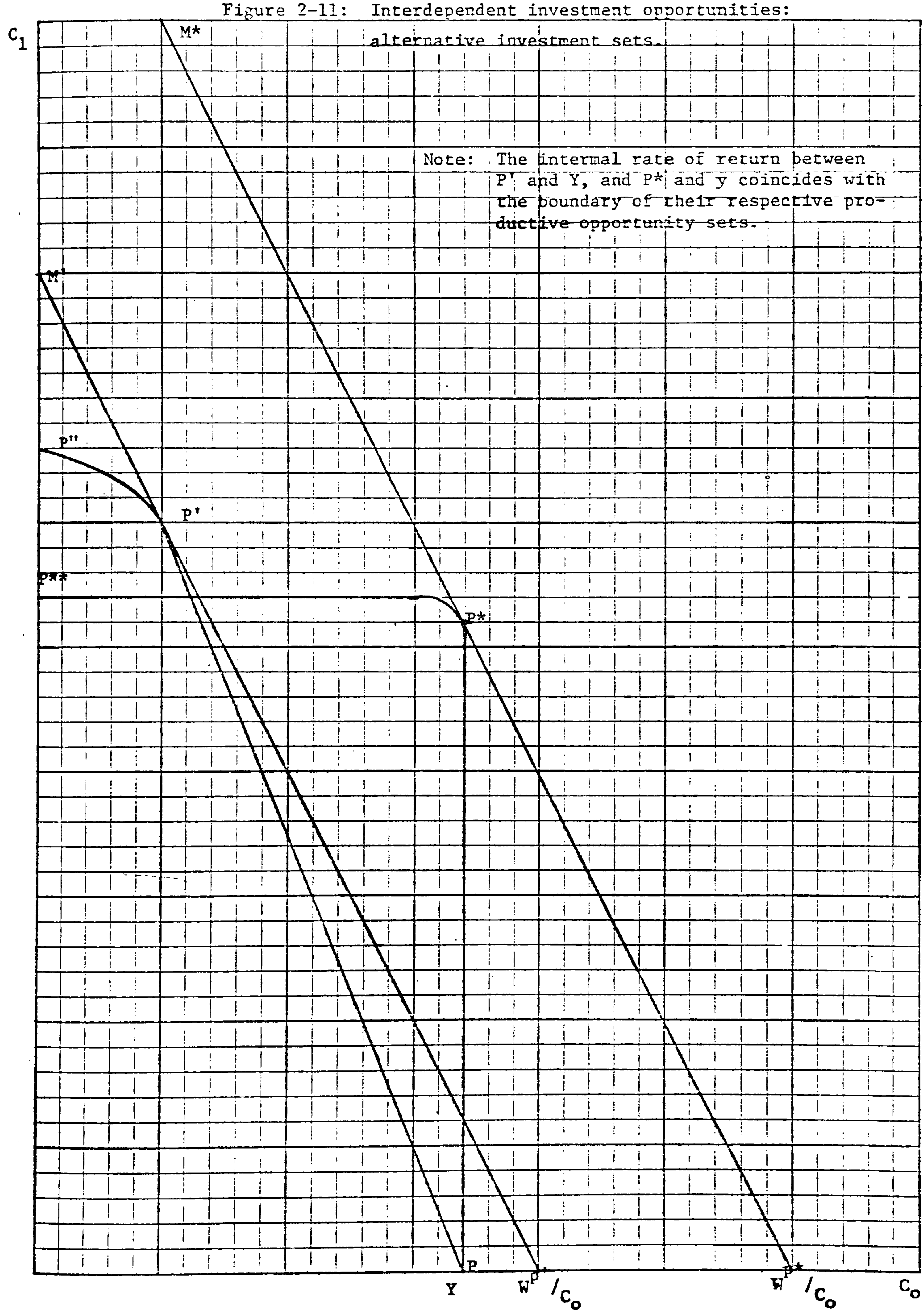


Figure 2-12: Discrete - independent investment opportunities.

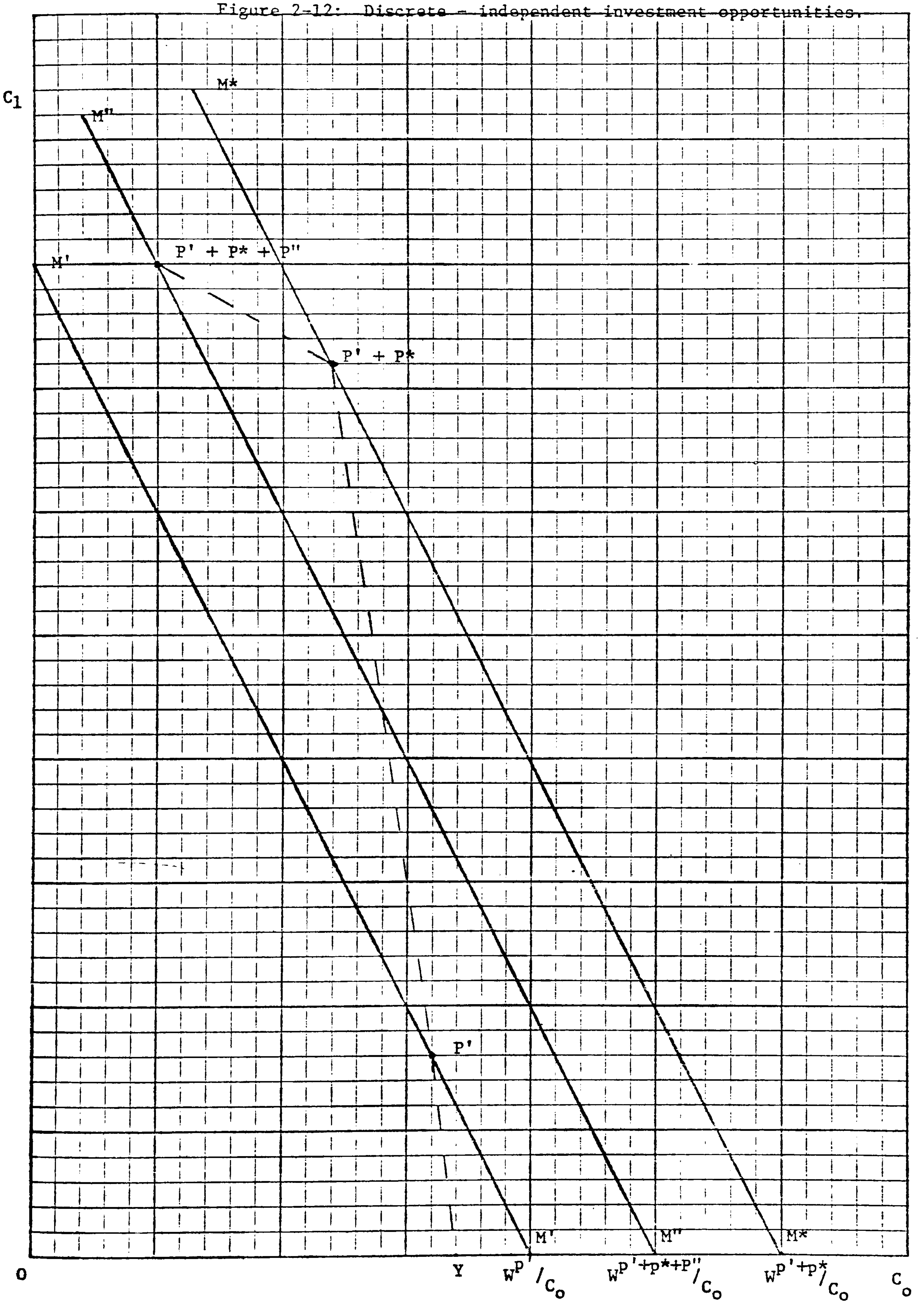
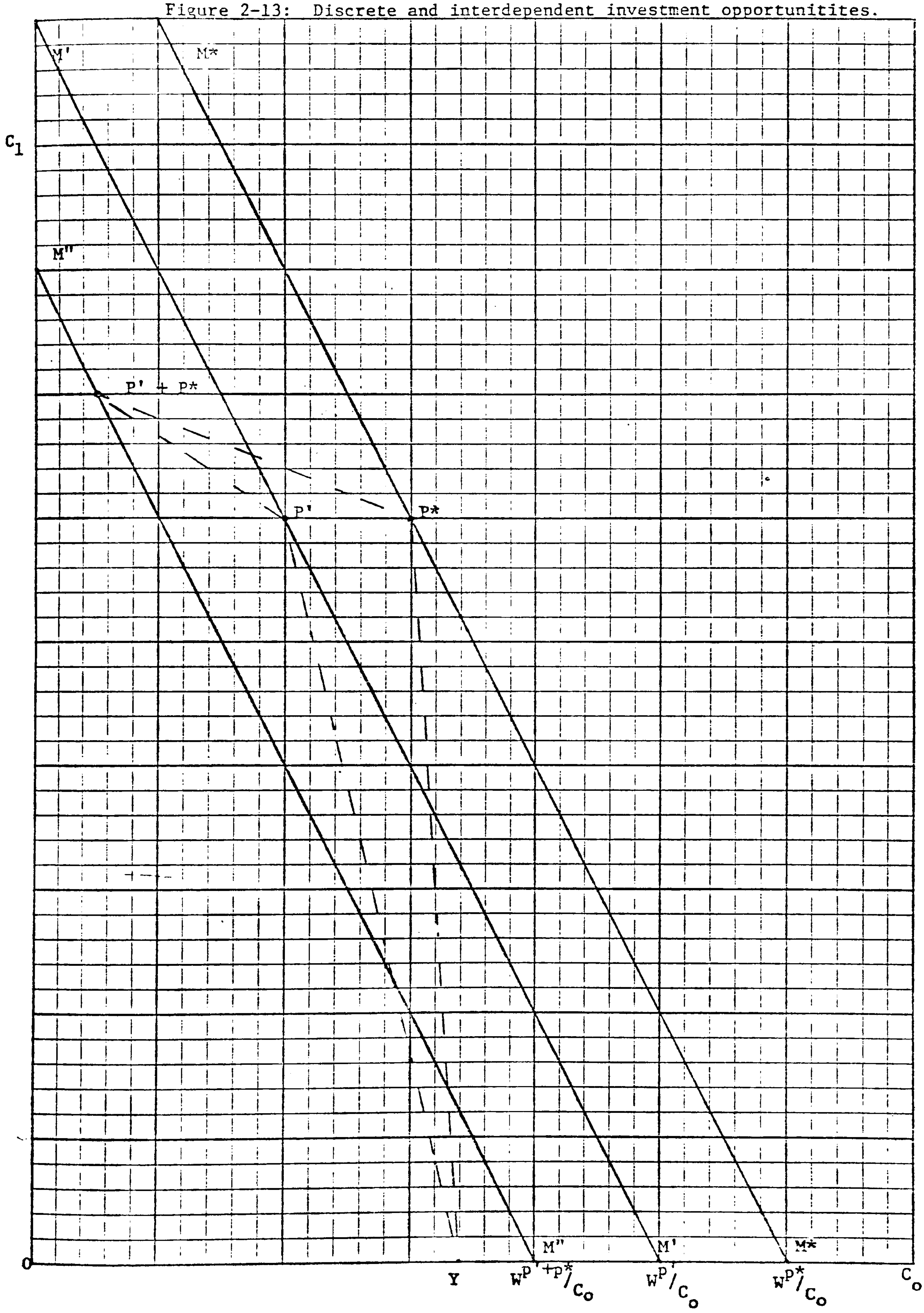


Figure 2-13: Discrete and interdependent investment opportunitites.



The examination of these examples also shows that internal rate of return comparison rules for attained wealth and wealth gain lead to correct solutions when "earlier" and "later" investment ensembles can be unambiguously identified. (The internal rate of return between investment choices in Figures 2-10 to 2-13 are illustrated by dashed lines.) The incremental rule for internal rate of return investments also requires consideration of jumps as well as infinitesimal movements along the investment possibilities frontier.

Investment choice rules and multiple time periods. The next step in this discussion is to examine the application of investment choice rules in an expanded paradigm of investment choice involving more than two periods. Geometrically this could be represented by a multidimensional figure in which each time period was represented by an axis. However, the application of investment choice rules in this multiperiod framework will be illustrated by a series of figures which will plot the relationship between present value and internal rate of return for a number of specified examples.

Figure 2-14 plots the present value of an investment opportunity involving the investment of two generalized consumption units at time 0 and receipt of four units at time 1 and the further receipt of two units in time 2 as a function of the discounting rate. The application of the present value rules would indicate adoption of this investment opportunity as long as the market rate was less than 140 percent.

The internal rate of return comparison rule would indicate adoption of the "Challenger" (-2,4,2), which is "later" than the "Defender", as long as the external rate was less than the internal rate of return (r) which is 140 percent.

Figure 2-15 illustrates a comparison of two investment projects, a and b, with gross income flows consisting of (-2,4,2) and (-2,0,8) respectively. Notice that opportunity b is superior in present value at low rates of interest while a is superior at high rates. The internal rate of return for a is higher than b (140 percent as compared to 100 percent), but at rates of interest less than 50 percent the adoption of b permits the attainment of a larger consumption set than a.

A more fundamental difficulty in the application of internal rate of return criteria is that there may not be a unique internal rate of return for an investment project. Situations of this type are illustrated in Figure 2-16 and 2-17. Generally the number of internal rates of return will be equal to the number of changes of sign in the gross income stream. These examples also serve to illustrate the difficulty in identifying "earlier" and "later" investment projects in the multiperiod case.

Hershleifer (1970) gives a version of internal rate of return comparison rules for multiple periods, but these rules are complicated and the calculations di-ficult so that for practical reasons the investment choice rules based on present value are superior.

Figure 2-14: Present value and internal rate of return for investment option (-2,4,2).

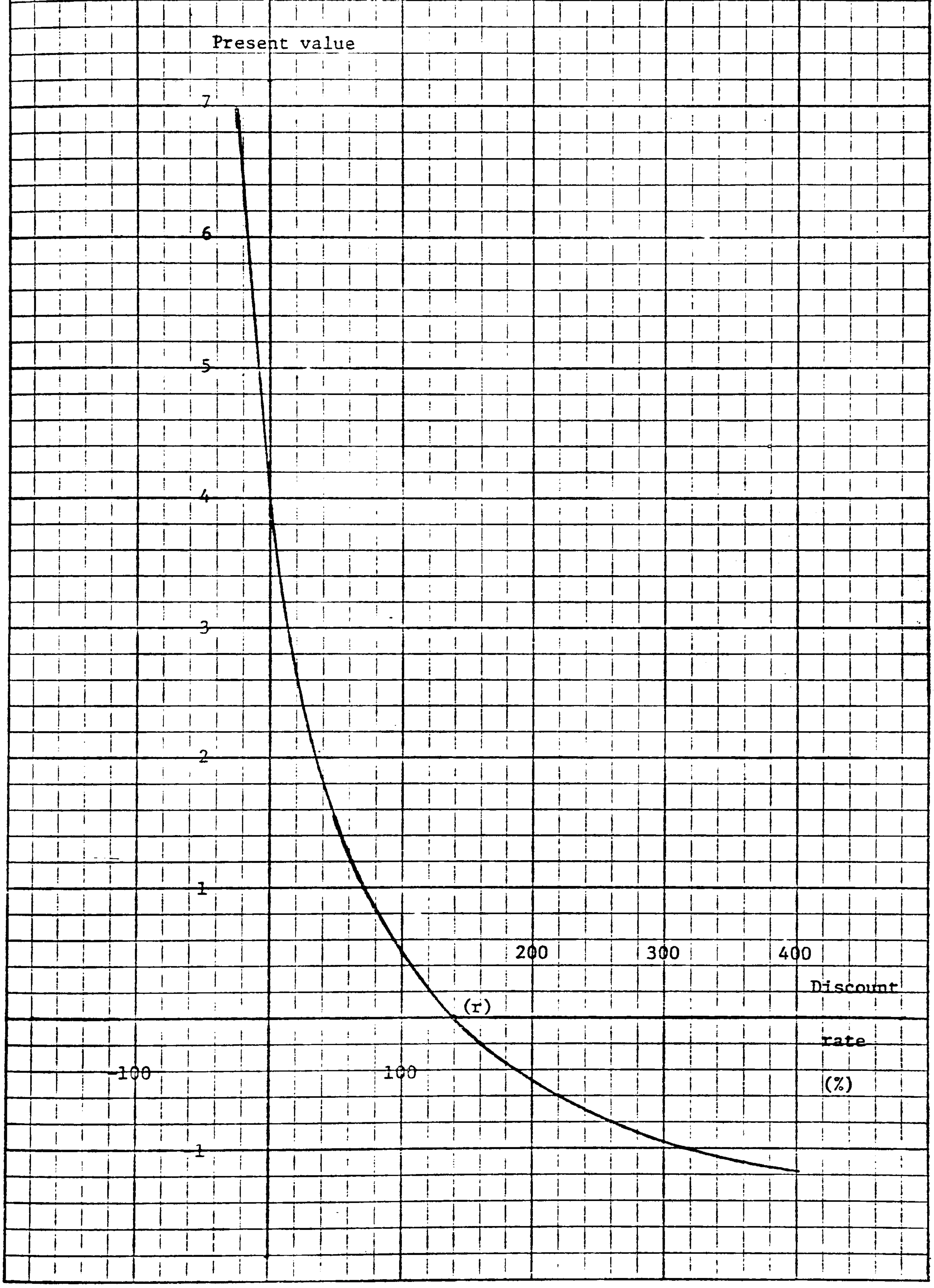


Figure 2-15: Comparison of investment options a (-2,4,2) and b (-2,0,8).

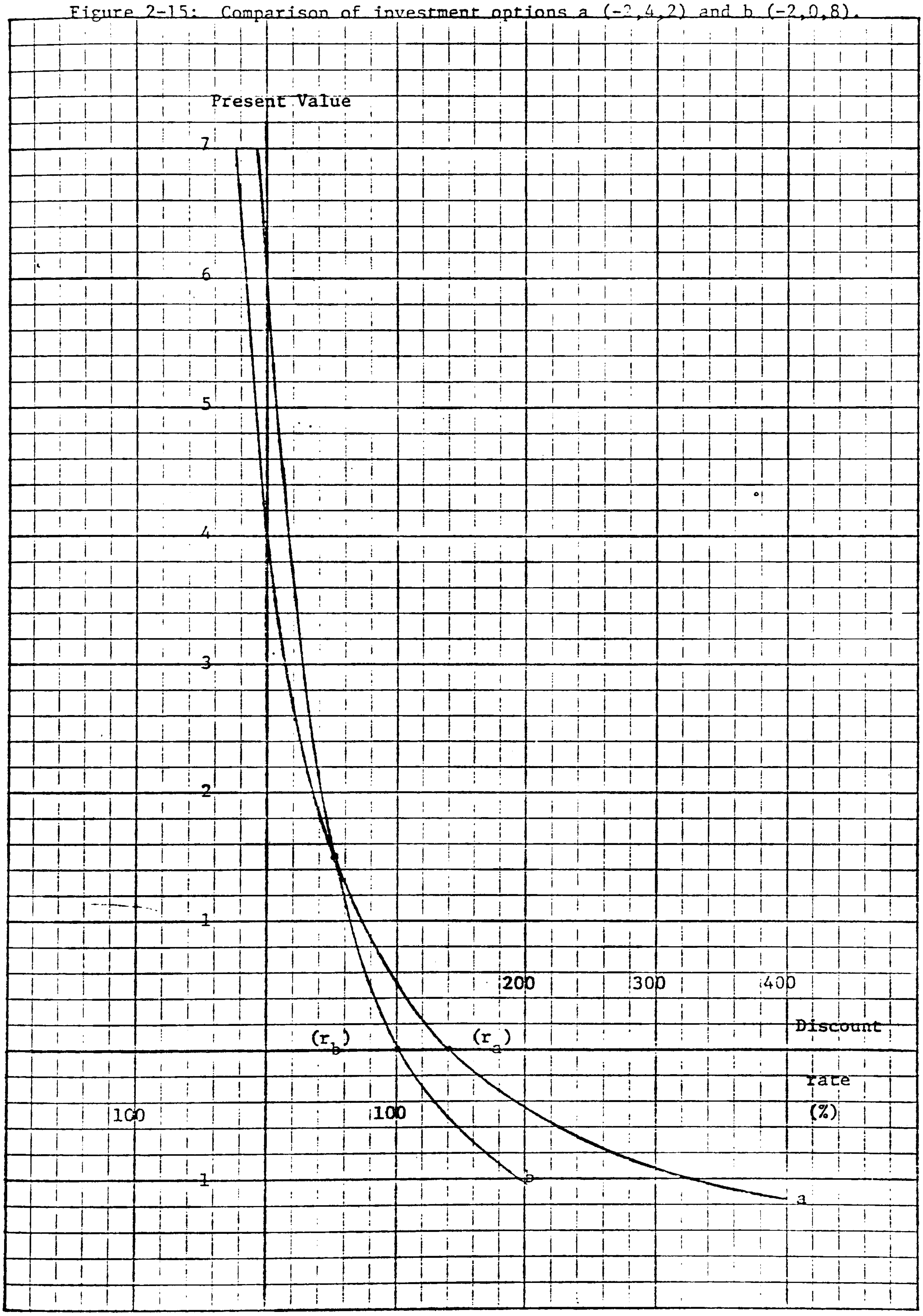


Figure 2-16: Present value and internal rates of return for investment option (-2,10,-12).

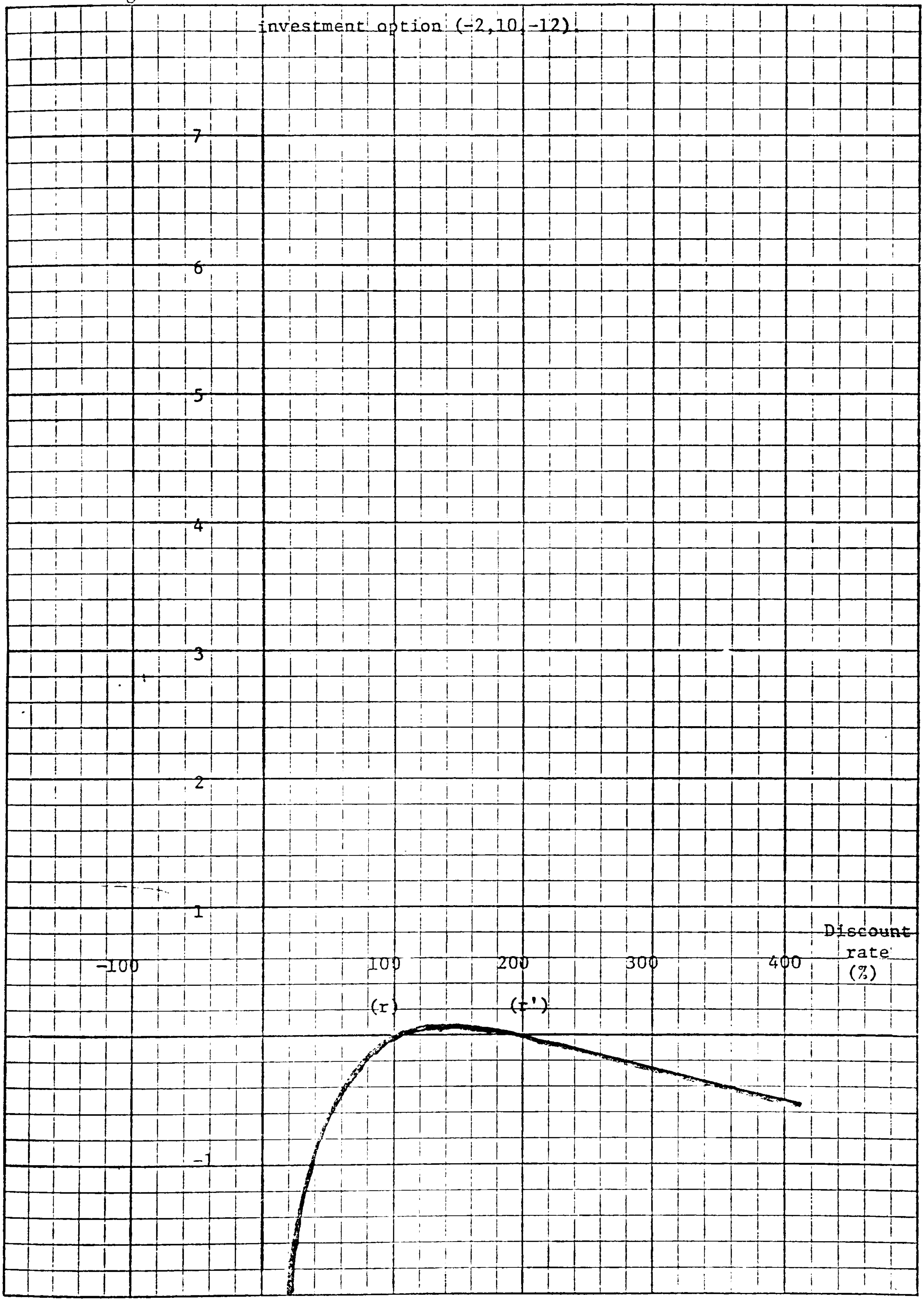
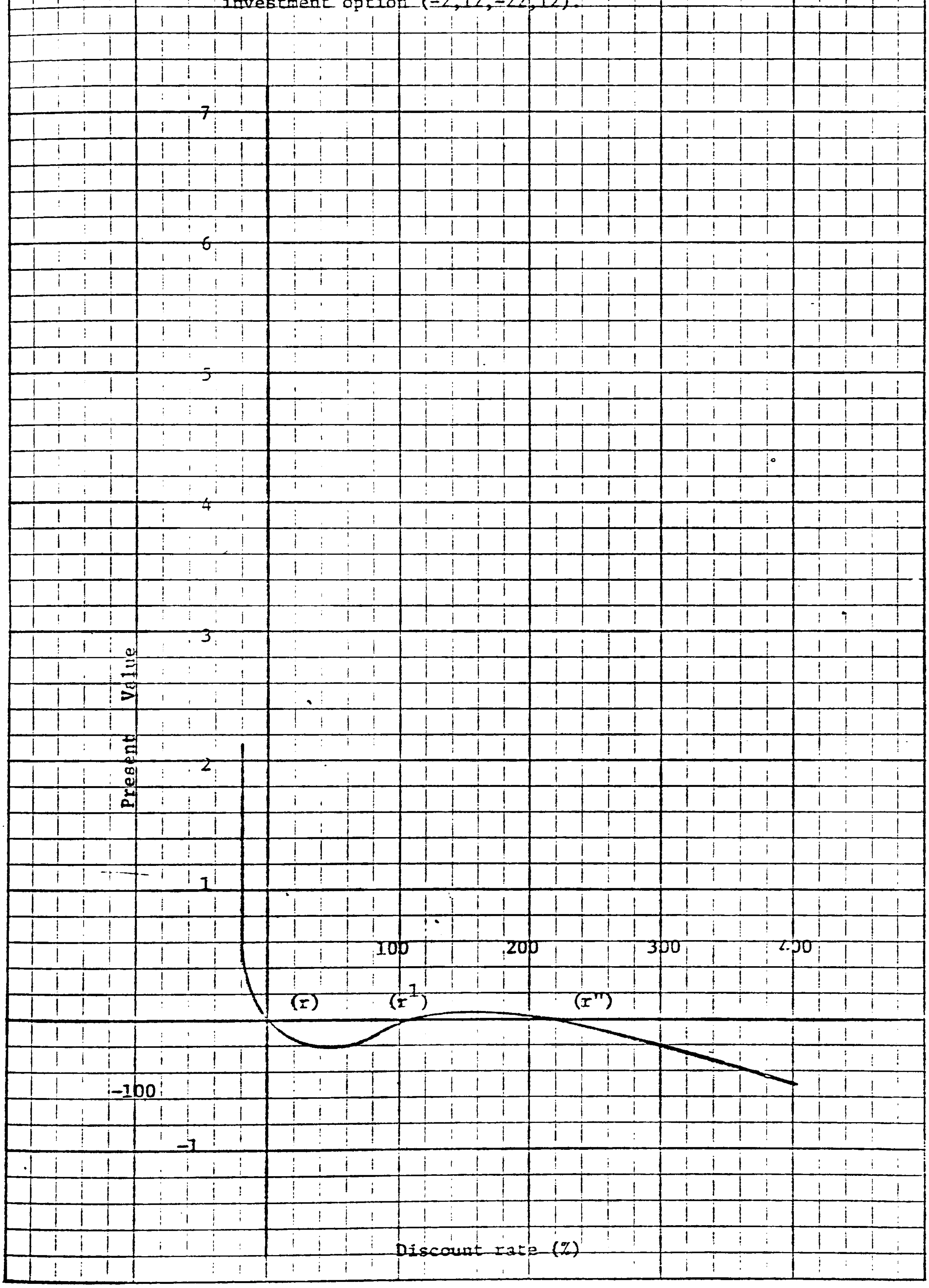


Figure 2-17: Present value and internal rates of return for investment option (-2,12,-22,12).



Investment choice rules and problems of duration and replacement. So far the discussion of intertemporal choice and investment choice has treated time in terms of discrete periods. This is the usual way in which continuous time is approximated. However, the special class of problems dealing with investment choice in situations involving the analysis of investment duration and replacement can be discussed most conveniently in terms of a continuous model of time.

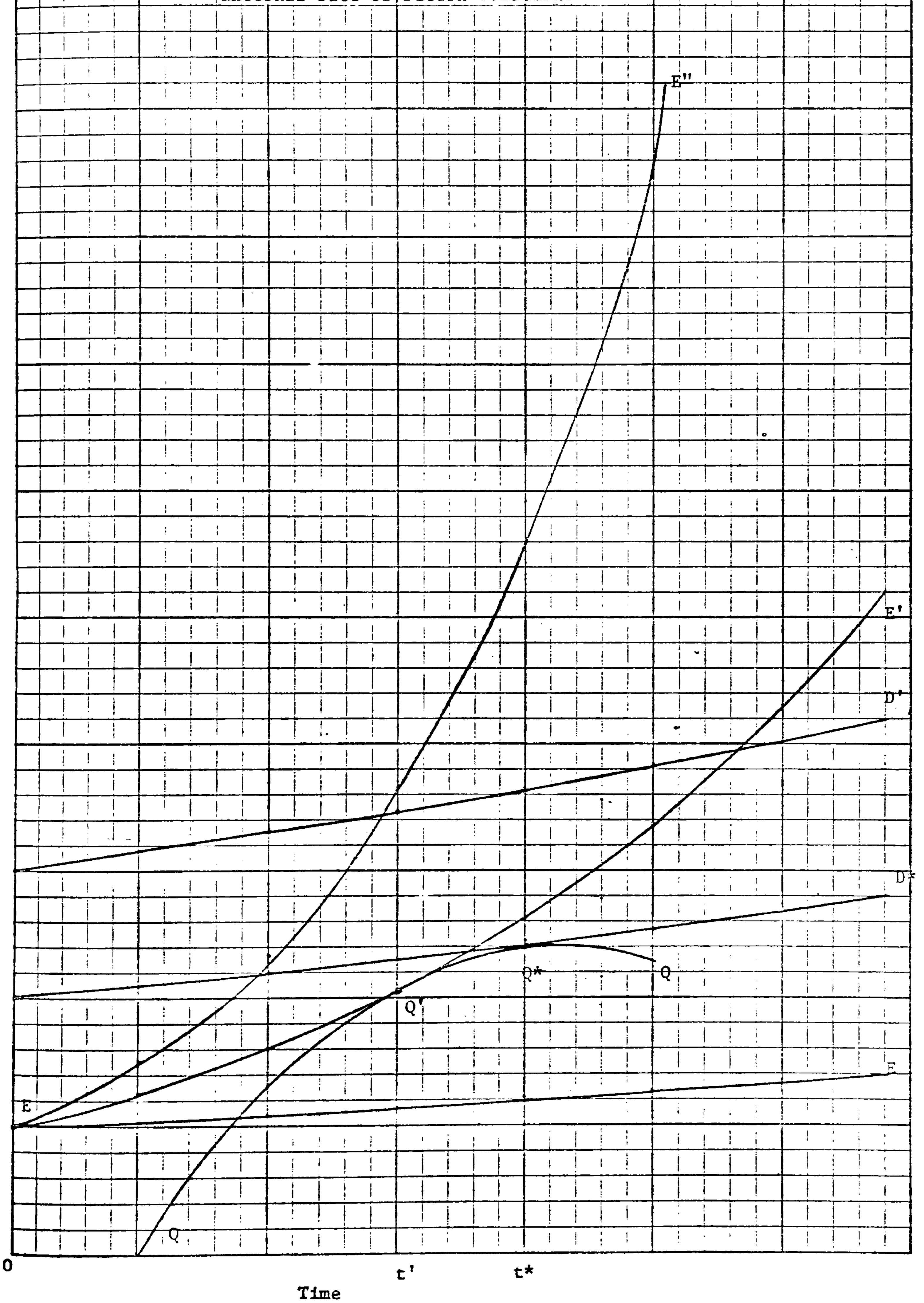
Historically economic theory has presented questions of investment duration in terms of problems involving how long to age a bottle of wine or how long to grow a tree. In its simplest form this class of problem involves only an initial current outlay and a terminating receipt, and are generally classed as point-input, point-output problems. The important theoretical question under these circumstances is: what is the optimum time interval between input and output? To further simplify the problem the initial input is taken as a constant which reduces the problem to a variationof the simple two period paradigm in the sense that there are two objects of choice, consumption claims now and consumptionclaims in the future. In the two period example the variables for a productive solution are the produced titles to consumption in periods 0 and 1, or their input-output transformations. For the problem of simple investment duration the variables are the optimal time interval and associated output. In the two-period model the problem was to select the optimum scale of investment, the duration being held constant; in this case the problem is to select the optimal duration, the scale being held constant. The problem of selecting the optimum scale of investment is sometimes referred to as "capital widening" and the problem of selecting the optimum duration is called "capital deepening" (Hirshleifer, 1970).

A simple investment duration problem is illustrated in Figure 2-18. The locus of points QQ shows the outputs as a function of time. The distance OQ along the time axis represents a period of time during which an investment has no output. The discounting curves V_0^* D* and V_0' D' represent the locus of points which would result from the continuous compounding of the present values V_0^* and V_0'. Similarly the opposite is true in that V_0^* and V_0' represent the present value of D* and D'. These loci of points also represent market lines, in the sense of the earlier analysis, in that any point on either V_0^* D* or V_0' D' can be attained from a point on the same line through market exchanges. QQ also represents the investment opportunity set of the previous examples.

The solution to the question of investment duration, under the rule of maximizing present value, is represented by Q* at time t* in Figure 2-18.

The alternative solution selects the duration that maximizes the internal rate of return (Boulding, 1966) and is represented by Q' in Figure 2-18. The loci of points EE, EE', and EE" represent the growth path under continusou compounding of the initial outlay 0_0 at different rates. The tangency between EE' and the productive possibilities curve QQ at Q' represents the maximum possible growth rate achievable.

Figure 2-18: Simple investment duration with present value and internal rate of return solutions.

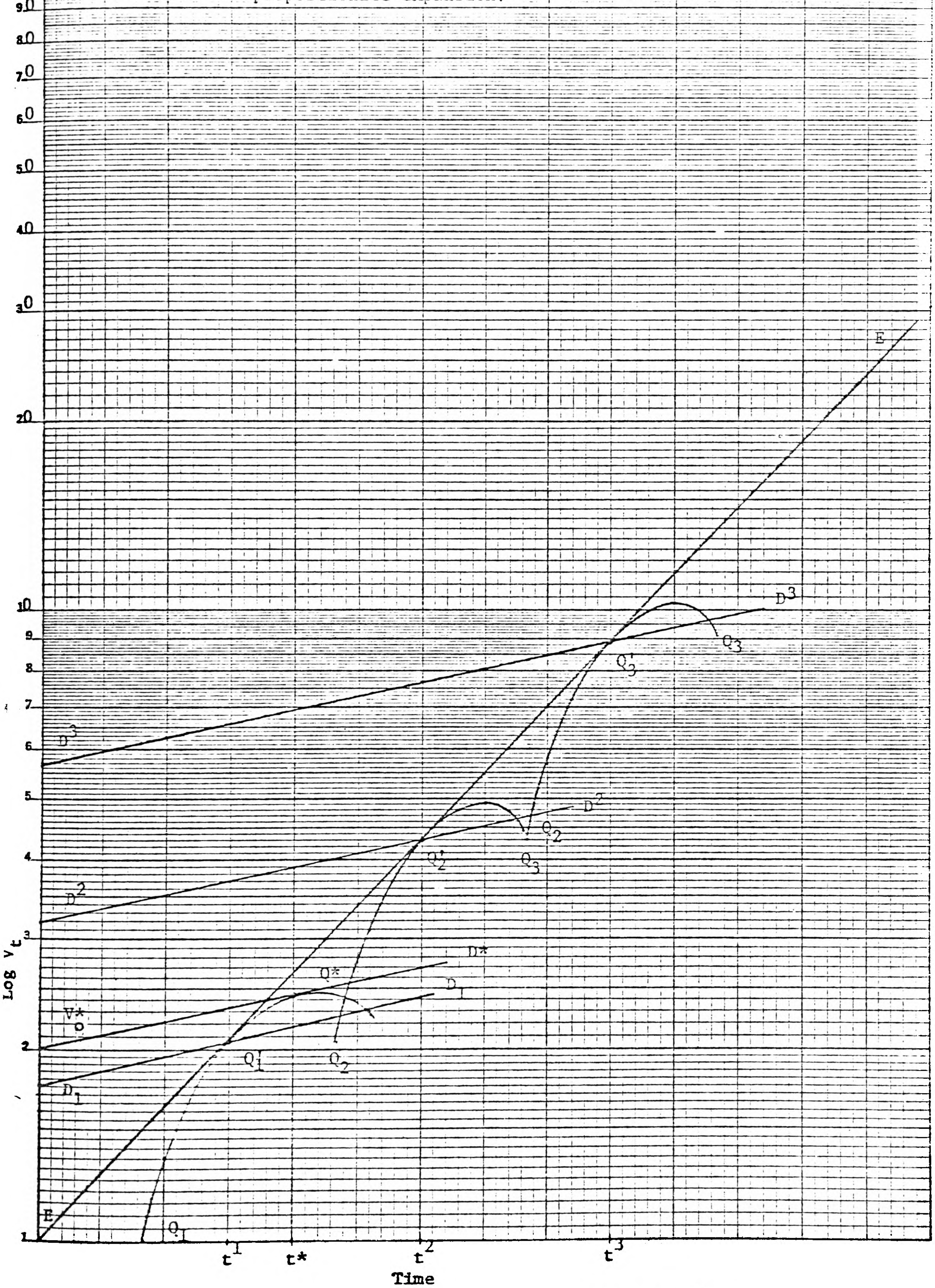


Examination of Figure 2-18 reveals that the largest possible consumption set is achieved by the selection of duration t* rather than t'. However, one of the primary arguments for the superiority of the internal rate of return rules is based on the premise that the productive opportunity QQ can be replicated with proportionate expansion of scale whenever the investment is terminated (Boulding, 1966). The replication, with proportionate expansion of scale, of an investment series is illustrated in Figure 2-19. This diagram is a transformation of Figure 2-18 to a vertical logarithmic scale which facilitates the explanation of the problem of investment duration with replication and proportionate expansion. Notice that the discount curves are now represented as a series of straight lines. The investment possibility curve Q_2 Q_2 represents what is possible if the initial investment possibility curve Q_1 Q_1, which is the semi-logarithmic version of QQ in Figure 2-18, is liquidated at t^1 and the proceeds immediately reinvested. The same is also true between Q_2 Q_2 and Q_3 Q_3. The points D_2 and D_3 represent the present value of the titles to consumption that would result from the termination of the investment series at Q_2' and Q_3'. Initial examination appears to indicate that the investment choice rule that selects the investment duration which achieves a maximum internal rate of return is superior to the present value rule. However, this interpretation is wrong in the sense that the present value rules must be interpreted in terms of the relevant investmentopportunity set. The problem represented by the investment opportunity set QQ in Figure 2-18 is not the same problem as represented by the irregular investment opportunity set defined by Q_1 Q_3 in Figure 2-19. It follows from this that a rule which selects Q*, at time t* as the duration for a problem of simple duration is not disproved by showing that a different solution would be correct for a different problem (Hirshleifer, 1970).

The question now is: do internal rate of return rules lead to solutions which are in conflict with present value rules in problems of investment duration where investments can be replicated with proportionate expansion? Under these conditions the investment opportunity set is bounded by the locus of points represented by Q_1 Q_3. The internal rate of return maximization rule would indicate the liquidation of investments and the reinvestment of the proceeds at Q_1', Q_2', Q_3'. However, under this course of action the consumptionof attained income from the investment series is deferred until some point infinitely in the future. Under these conditions the present value would rise without limit as long as the average internal rate of return of the investment series was greater than the external or market rate of interest. Thus, any investor with an investment opportunity of this type would have infinite wealth. The application of present value rules in this situation would dictate the attainment of the maximum, in this case infinite, wealth position. Therefore, in this situation, the two rules would coincide in recommending the investment decision along Q_1 Q_3.

However, observations in the real world do not indicate that individuals, firms, or governments possess infinite wealth. The reconciliation of the real world as observed and this paradigm lies in two conditions.

Figure 2-19: Investment duration with replication and proportionate expansion.



First the resources available for any investment, under today's technology, are available only in finite amounts ruling out any infinite solutions. Secondly, if infinite resources did exist, the operation of the price system, though imperfect, would increase the price of earlier titles to consumption relative to later titles until the external, or market rate of interest approached the internal rate of return. At this point the present value would also approach 0. Under either condition the present value rules lead to correct action in the sense of infinite wealth if possible or zero wealth if that is the best attainable (Hirshleifer, 1970).

This leads to the question of investment duration in a situation which permits infinite replication of the investment in the form of a constant scale of replication. The problem of rotation length in the management of timber crops is an example of a problem of this type. The original solution to this class of problems was formulated by Faustmann (1849), (Gaffney, 1960) (Linnard and Gane, 1968), and is a present value solution in which the investment opportunity set includes constant scale of replication (Hirshleifer, 1970). Because of the difficulty in representing this problem geometrically the following intuitive argument will be given.

Under the conditions of a single cycle investment opportunity (Figure 2-18) the optimum duration for the investment is defined by the point where the proportional change in the level of attained income is equal to the external or market rate of interest. If the investment opportunities include replication, it is possible on the next cycle to obtain an average proportional rate of increase in attained income greater than the external rate. (The average proportional rate of increase in attained income must be greater than the external rate for the opportunity to have a positive present value). However, only a portion of the attained income achieved by the initial investment can be reinvested in the second cycle and the remainder must be exchanged at the market rate of interest. This opportunity to reinvest a portion of the attained income would reduce the duration of the initial investment, in order to free the titles to consumption for the second cycle, to something less than t* but more than t' (Figure 2-18). The exact point being determined by the portion of the attained income required for investment in the second cycle (Hirshleifer, 1970).

In summary, this discussion of investment choice has illustrated the application of investment choice rules based on present value, and internal rate of return criteria. Further, these rules have been examined in terms of the theoretical paradigm of intertemporal exchange. This discussion has shown how present value rules generally yield results consistent with the conditions for optimum productive decisions. However, this examination has been made in terms of individual choice in a world of perfect markets where the decision makers opportunity set concists of both investment, and exchange opportunities. If the opportunity set does not include exchange the intra and inter-temporal prices are irrelevant to investment choice. The relationship between individual and collective choice was discussed and the sufficiency condition of perfect markets presented as an argument for individual optimization leading to collective optimization. The type of market imperfection, where desired goods were not exchanged and market prices do not exist was explicitly excluded from the discussion on the grounds that decisions of this type are essentially subjective and beyond the scope of this thesis. The application of investment choice rules under specific conditions of market imperfection will be discussed in subsequent sections.

Money and Investment Choice

One of the lements of reality which has been omitted up to this point is the existence of money as a distinct commodity separate from the dated consumption claims which have been the objects of choice in the previous discussion.

Money in the economy. Historically a number of useful commodities including cattle, tobacco, furs, slaves, wives, copper, gold and silver have been used as money (Samuelson, 1964). The primary reason for the existence, in all but the most primitive societies, of money is to facilitate exchange. To illustrate consider that in the absence of money an individual desiring to exchange one good for another must find another individual willing to make the exact reverse exchange. The introduction of money into the economic system promotes exchange by allowing individuals to exchange goods for money and money for goods.

The use of money as a medium of exchange leads to its use as an index of value. This comes about because goods exchange for money and the existence of money prices for goods constitutes a cardinal index of value (Dasgupta, and Pearce, 1972) which allows direct value comparisons between all goods in the system.

A third function of money is to act as a store of value. Individuals may choose to hold a portion of their consumptive claims in the form of money. This is generally possible because of money's characteristics of freedom from physical deterioration, ease of storage and physical protection (Samuelson, 1964).

These specialized functions of money in an economic system lead to its inclusion as a unique good in economic theory with distinct properties.

Money interest. One use of money which involves all of its functions is in intertemporal exchange where most exchanges are money-money exchanges between periods rather than the money-good exchanges which predominate within periods. The existence of interperiod money exchanges leads to the definition of a money rate of interest (r^D), distinct from the real rate of interest (r), which represents the premium on money in the current period. The monty rate of interest (r^D) is defined in the expression ($1 + r^D$) which is the number of monetary units one period from now which can be exchanged for (1) monetary unit in the current period.

Money prices. The use of money as a medium of exchange and as a store of value creates a demand for money as a desired good that would not exist if money functioned only as an index of value. The existence of this demand for money means that money prices are influenced by changes in taste and in the level of supply and demand in much the same way as other goods. As an illustration consider a timeless situation with only two goods, money and a consumption good. Under these conditions individuals would exchange money for the consumption good and vice-versa until everyone was satisfied with his or her bundle of money and the consumption goods. This would establish an equilibrium with an equilibrium price. If this equilibrium was modified either by increasing the supply of the consumption good or reducing the demand for money a new equilibrium would be established at a lower money-consumption good price. The opposite would occur if the supply of money were increased or the demand for the consumer good reduced. These changes in the level of money prices introduce an element of instability to the index of value based on money prices. This in turn leads to a divergence between the "real" and monetary rates of interest which must be recognized in the calculation of investment choice criteria.

Money and investment choice criteria. To illustrate how the monetary and "real" rates of interest diverge, imagine at the beginning of a particular year the "real" rate of interest is 5 percent and that the price level (price of all goods relative to money) is expected to rise by 5 percent.

Under these conditions the money rate of interest at the beginning of the year would be approximately 10 percent to adjust for changes in the purchasing power of money. If, however, the price level rose by 10 percent rather than the anticipated 5 percent the fully adjusted monetary rate of interest would be 15 percent. But because the price level increase was not fully anticipated the realized real rate of interest is 0 percent (Hirshleifer, 1970).

Fisher (1907) (1930) demonstrated that monetary interest rates adjust for anticipated changes in the level of prices. His later work also showed that the monetary rate of interest generally failed to adjust fully for actual changes in the level of prices resulting in some variation between "real" and realized "real" rates of interest.

The correctness of investment decision rules based upon present value criteria is due to their equivalence with the condition of wealth maximization in the paradigm of intertemporal choice.

One of the major elements in the calculation of present value criteria is the "real" rate of interest. This underlines the importance of determining the "real" rate of interest relevant to the calculation of present value criteria in the analysis of investments.

A mathematical summary of the relationship between real and monetary rates of interest is given in Appendix A.

Market Imperfections and Investment Choice

A second element of reality which has not been covered in this discussion of investment theory is that of imperfect intertemporal markets. Up to this point investment choice has been considered in the context of markets where a single, competitively determined, price defined the conditions of exchange. In this section three common classes of market imperfections will be discussed and their influence on investment choice rules examined.

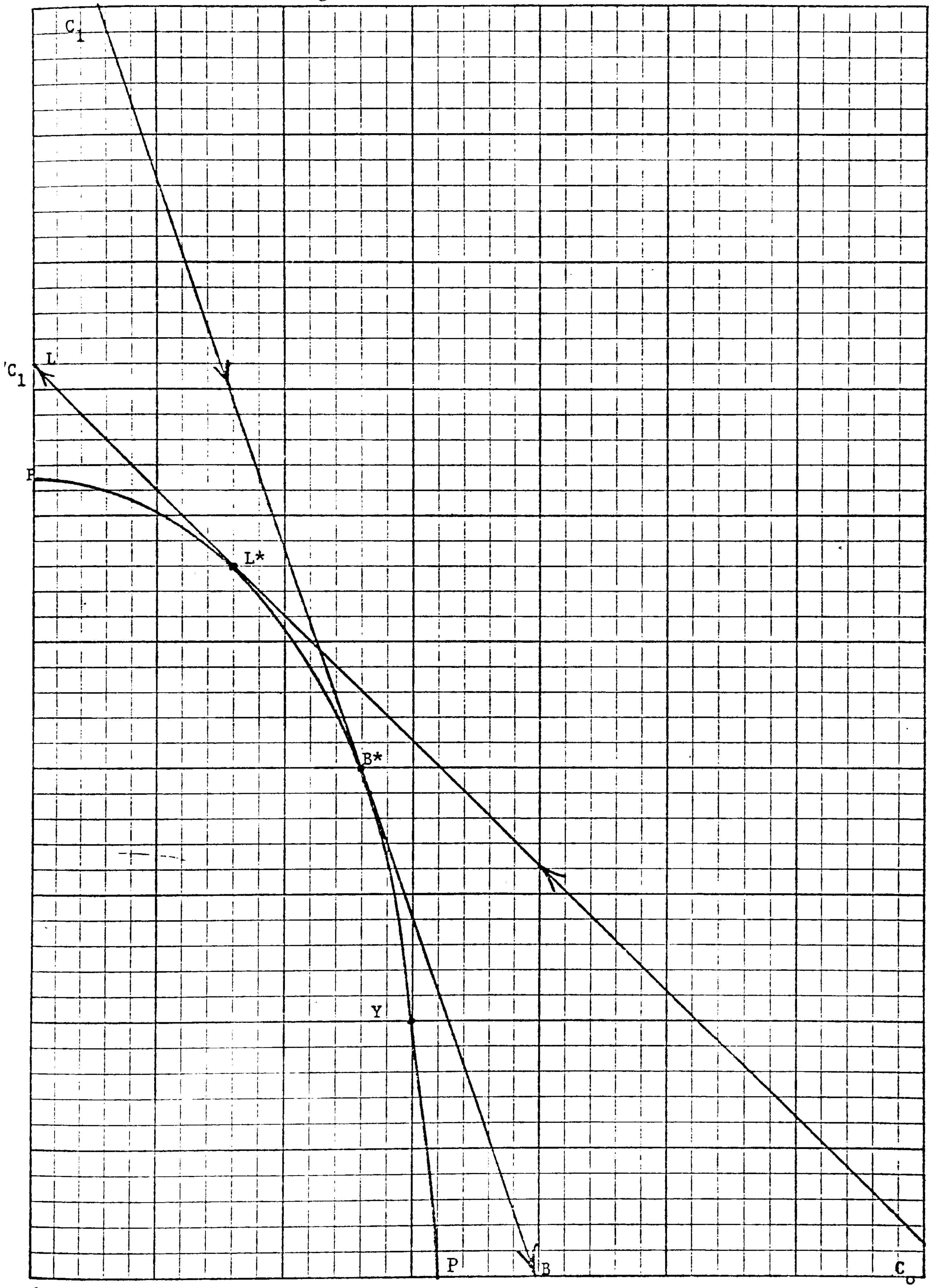
Divergent borrowing and lending rates. The first of these imperfections that of divergent borrowing and lending rates can best be thought of as a truncation of the market opportunity set. In Figure 2-6 the productive and consumptive optima for an individual with an opportunity set consisting of production and exchange are illustrated by P* and C*. The market opportunity set in this example is represented by OM'M'. The most extreme form of market imperfection is also illustrated in this Figure by OPP which represents the opportunity set for an individual with no exchange opportunities.

In Figure 2-20, the production and market opportunity sets for an individual facing divergent borrowing and lending rates are illustrated. The divergent borrowing and lending rates are illustrated by the slopes of BB* and LL*. BB* represents the consumptive bundles attainable through exchanges of consumption in period 1 for consumption in period 0. In the same manner line LL* represents the consumptive bundles attainable through exchanges of consumption in period 0 for consumption in period 1. The existence of the opportunities for borrowing and lending increase the size of the opportunity set, from OPP to OBB*L*L. The area PBB* represents the increase in consumption opportunities due to borrowing the area PL*L the increase due to lending.

The existence of the opportunity set OBB*L*L in Figure 2-20 creates a condition where three classes of solutions exist for the definition of the consumptive optimum. Depending on the shape of the indifference curves a point of tangency between an indifference curve and the opportunity set could exist in any one of regions LL*, L*B*, and B*B. A point of tangency with an indifference curve in the lending region LL* would entail a productive optimum at L* followed by lending exchanges along LL*. A tangency in the region B*B would entail a productive optimum at B* followed by borrowing exchanges along B*B. The third class of solution would result from an indifference curve tangency in the region L*B* which excludes the opportunity to benefit from either borrowing or lending exchanges.

The existence of three possible solutions leads to a question of what is the relevant intertemporal price to be used in calculating the level of wealth associated with any productive opportunity. By examining Figure 2-20 it can be seen that the relevant price is determined by the portion of consumption opportunity set boundary in which the solution falls.

Figure 2-20 Market imperfections: divergent borrowing and lending rates.



This is to say that if the indifference curve tangency is to the left of L* on the L*L curve then the lending rate is the appropriate price and L* is optimum productive solution. The logic for this is that since the decision maker iwll in fact be lending following the attainment of the productive solution productive projects should be adopted as long as the marginal rate of transformation exceeds the lending price.

Following the same logic if the indifference curve tangency is to the right of B* on the B*B curve then the borrowing rate is the appropriate price for calculating the wealth associated with any productive opportunity since after attaining the productive optimum B* the decision maker will be borrowing to attain a consumptive optimum.

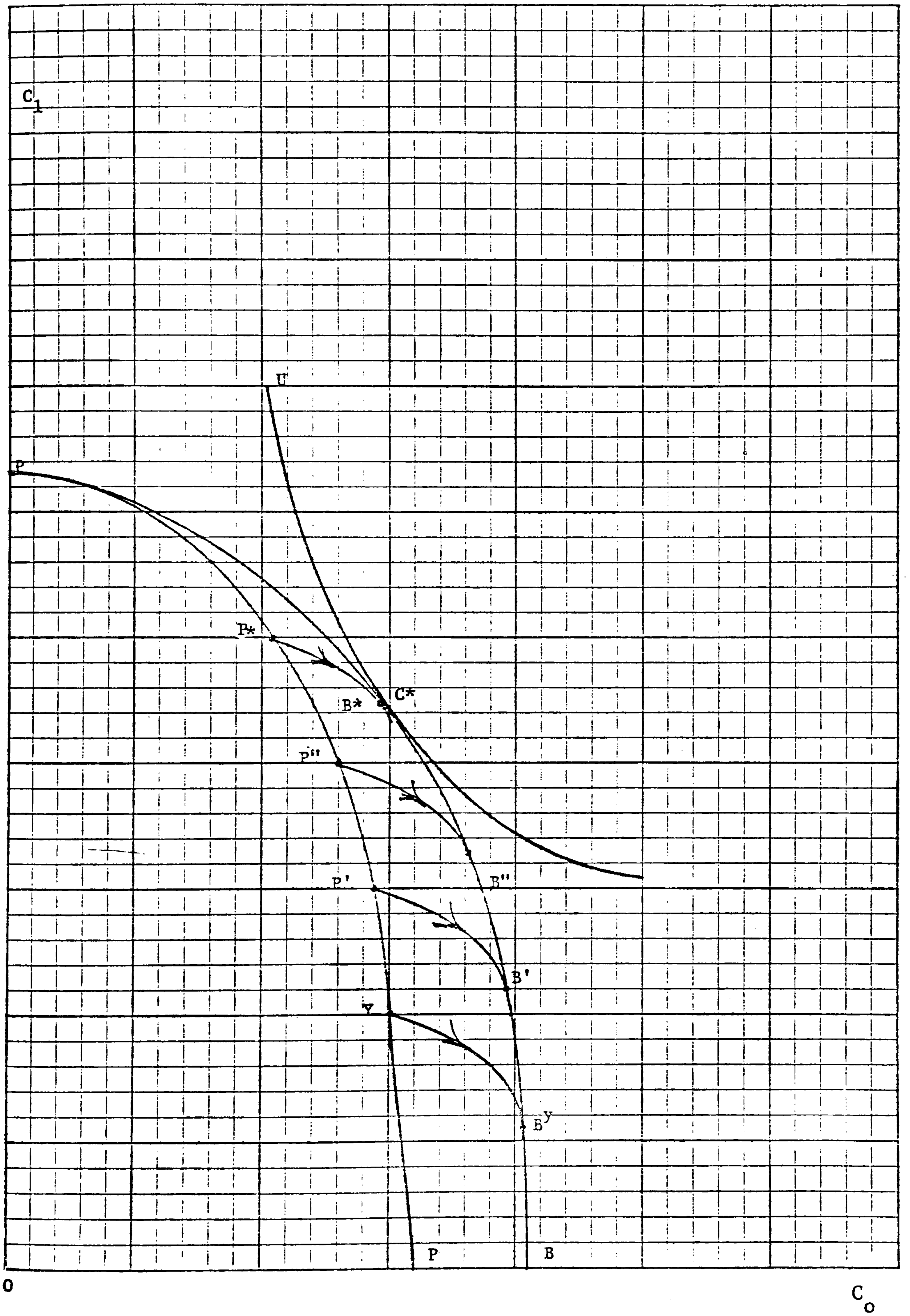
It is important to note that if the indifference curve tangency falls in the region between L* and B* wealth maximization is not the relevant criteria for production optimization since market exchanges cannot expand the opportunity set beyond that achieved through productive transformations.

Once the condition of perfect markets in intertemporal choice is relaxed, the Separation Theorem which allowed the definition of the productive optimum independently of preferences no longer holds (Hirshleifer, 1970).

Rising borrowing rates. A second type of market imperfection which is common in the real world is that of rising rates of interest as the level of borrowing is increased. As in the previous case this situation can be illustrated by the two period, two good diagram. In Figure 2-21 the productive opportunity set OPP is augmented by OBP representing the consumption bundles attainable through borrowing. The overall opportunity set OPB is hased on the condition that marginal borrowing costs are a function only of the scale of borrowing. As a result the slopes of the possible borrowing curves Y B^y, P'B', P"B" and P*B* are the same for equal distances from the boundary of the production opportunity set OPP. The boundary of the overall opportunity set OPB is made up of an envelope of the market curves. Under this condition the boundary is tangent to the possible borrowing curves at points B*, B", B' and B^y. The points B*, B", B' and B^y represent the maximum C_0 attainable by borrowing against a given C_1 that is not attainable from a point on the PP boundary with less C_1 and more C_0. The point C*, which represents the consumptive optimum, is attained where the opportunity set boundary is tangent to the highest possible indifference curves. The individual achieves this point by engaging in productive transformations up to P*, and then borrowing along P*B* to achieve a preferred consumption bundle C*.

The important question is: what is the relationship between the marginal rate of transformation, the marginal rate of exchange, and the marginal rate of substitution, all of which are represented by the slopes of PP, P*B*, and UU respectively? One of the conditions for tangency between UU and P*B* at C* is equal slopes indicating that the marginal rate of exchange and the marginal rate of substitution must be equal at the consumptive optimum. The economic logic of the previous examples leads

Figure 2-21 Market imperfections: increasing borrowing rates.



to the conclusion that the marginal rate of transformation and exchange must also be equal. Productive transformations will only be justified as long as the number of units transformed is less than the number of units which must be borrowed to achieve the preferred consumption bundle.

As in the previous examples of intertemporal production and exchange, the condition of wealth maximization under the relevant intertemporal price does define the productive optimum. However, this imperfect market condition causes the Separation Theorem to no longer hold as the relevant price is influenced by individual preferences (Hirshleifer, 1970). A formal economic proof of this is given by Hirshleifer (1958).

Interperiod variation in intertemporal price. The expansion of intertemporal choice with production and exchange to more than two periods with variation in interest rates adds considerable complexity to the problem. Because of the difficulty in illustrating this problem graphically, the following intuitive argument will be given:

If between each pair of successive periods there are distinct but constant borrowing and lending rates the productive solution will be associated with actual intertemporal exchanges. This means that in some periods the borrowing rate will be appropriate, while in others the lending or individual preference rates will be appropriate. Further, if the borrowing rate is non-constant the subjective marginal rate of substitution must be introduced in periods where borrowing occurs.

Quantitative limits on investment. Generally quantitative limits on investment take two forms. First, in the form of an imposed limit on the productive opportunities. This condition is illustrated in Figure 2-22 where the productive opportunity set is bounded by P LB*YP. This constraint on the productive opportunities reduces the consumption opportunity set from L'L*B*B to LLB*B.

The second form of limits to investment generally takes the form of a ban on access to the borrowing side of the intertemporal exchange market. This is illustrated in Figure 2-23 where the productive opportunity set bounded by PL*YP is augmented by lending opportunities only bounded by LL*P. The ban on borrowing condition acts to limit the consumption set to LL*YP.

In both situations the condition of equivalence between the marginal rate of substitution, the marginal rate of transformation and the ratio of prices holds as in the previous examples. The condition of wealth maximization again fails to define the productive optimum if the tangency between the opportunity set boundary and an indifference curve occurs coincident with the boundary of the productive opportunity set.

Market imperfections and investment choice criteria. The failure of wealth maximization as the general definition of optimum production under conditions of imperfect interperiod markets means that present value criteria are no longer universally correct for investment choice.

Figure 2-22 Market imperfections; limit on productive transformations.

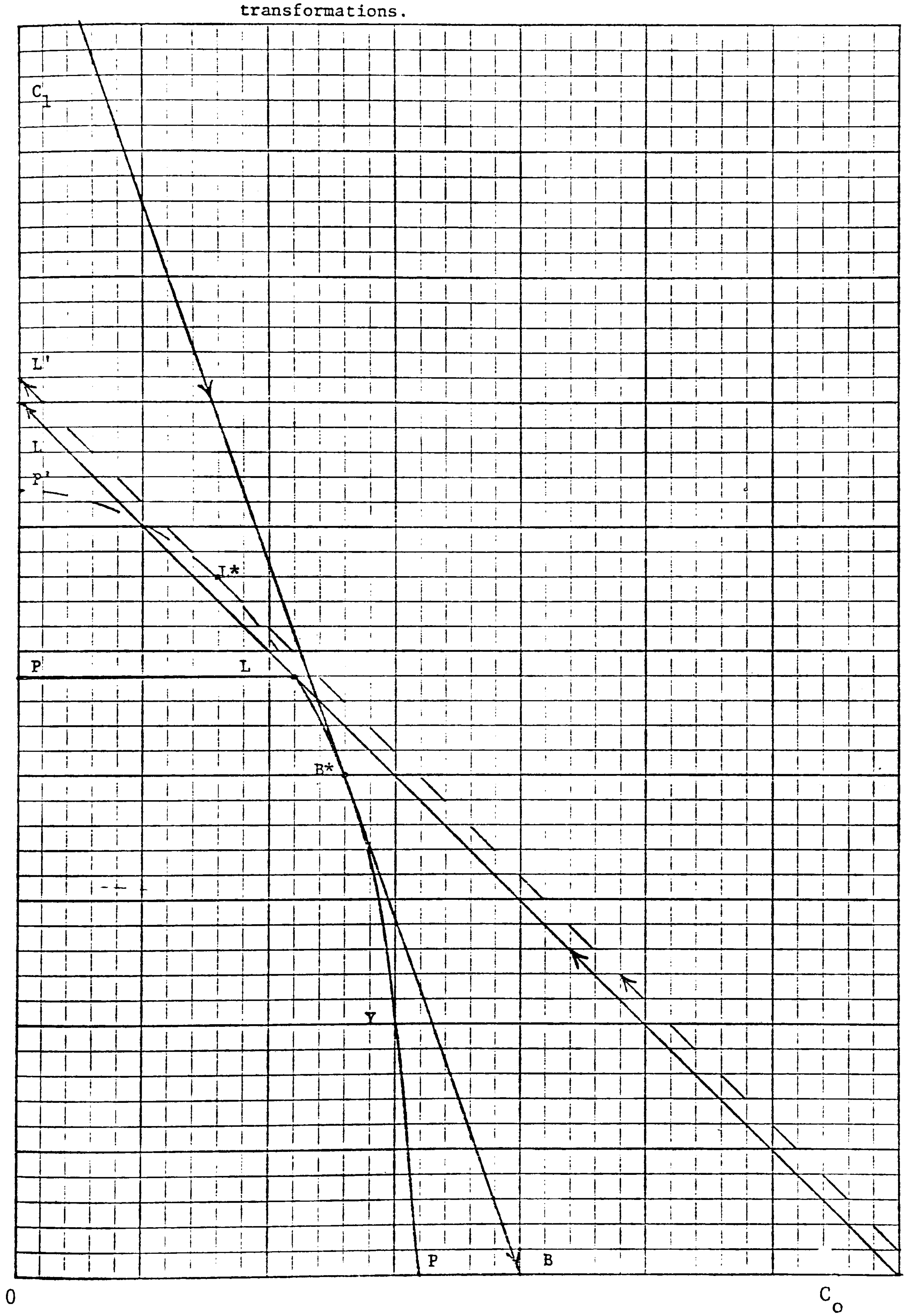
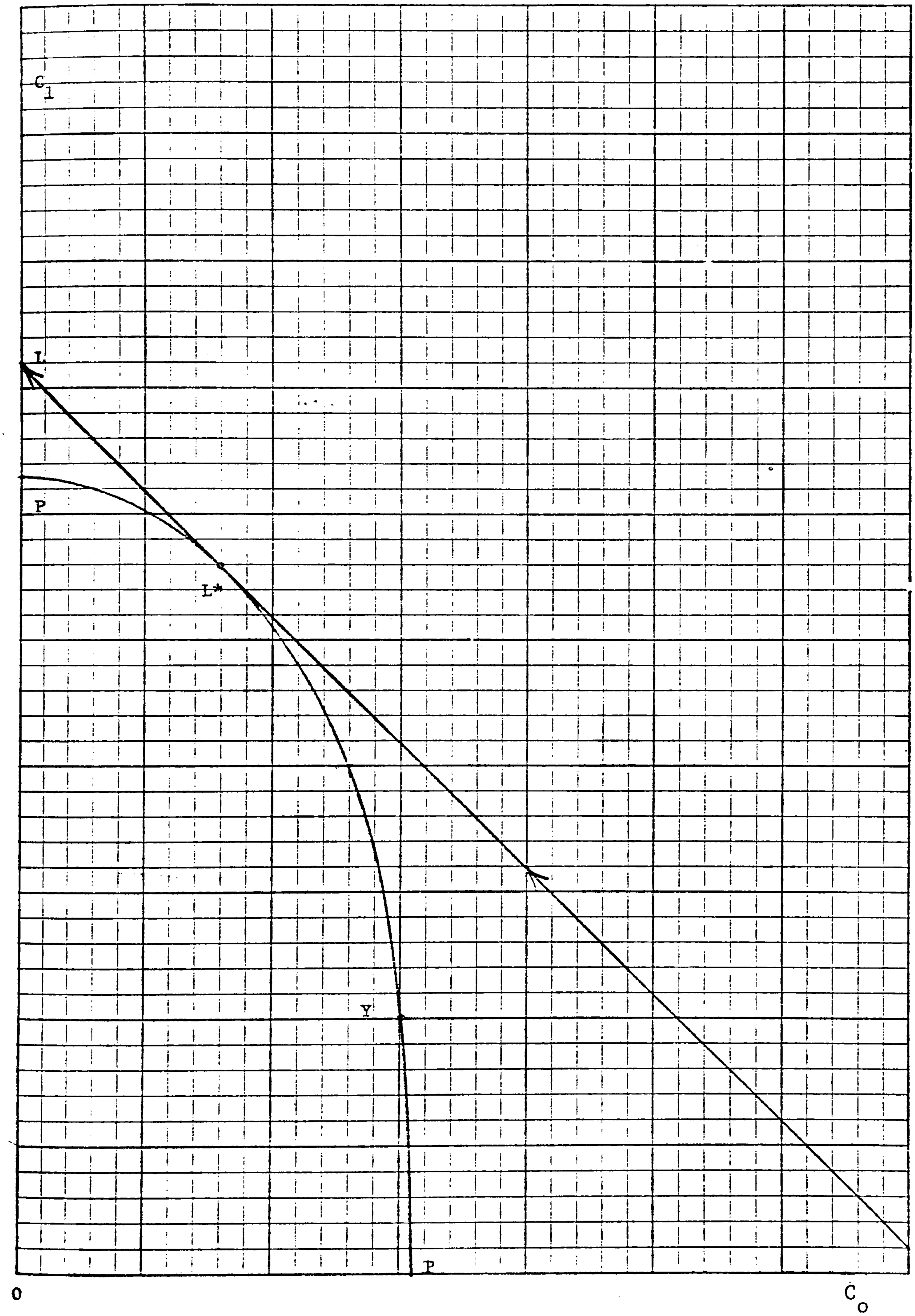


Figure 2-23 Market imperfections: a no borrowing condition.



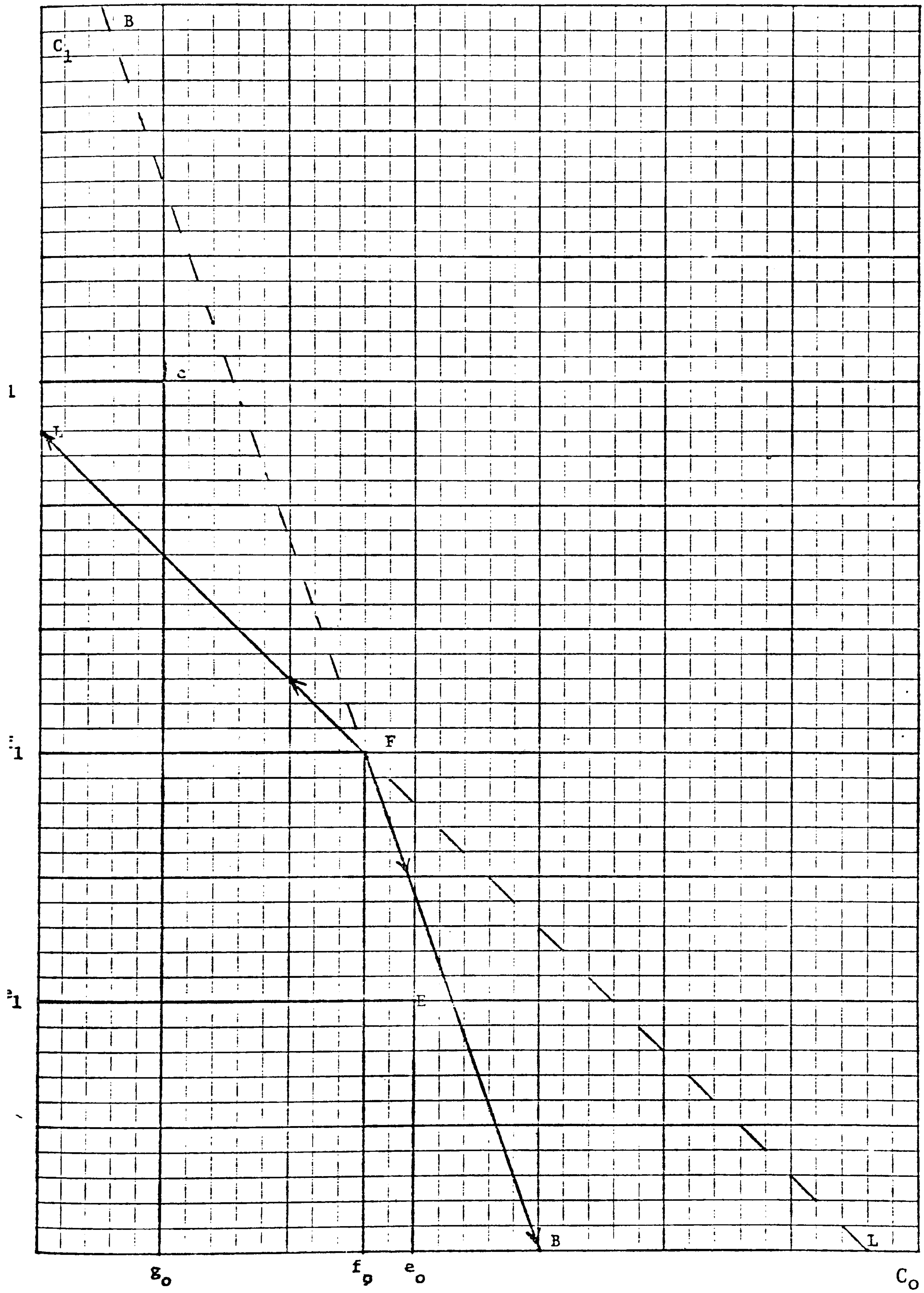
Correct in this context means that present value criteria fail to provide a measure independent of subjective preferences to which investment choice rules can be applied (HIrshleifer, 1970).

Under conditions of divergent borrowing and lending rates it is possible to state a present value rule for a limited class of cases where one income stream is dominant over another. In this "dominance" or specifically "simple dominance" refers to a condition where one income stream is never smaller, and has at least one element larger than another. A second type of dominance, "opportunity dominance" may occur once exchange opportunities have been defined (Hirshleifer, 1970).

In Figure 2-24 investment opportunity F is "opportunity dominant" over investment E because under the exchange conditions defined by the slope of BB and LL, consumption bundle E can be achieved from F through exchange with something to spare. On the other hand, comparing F with G reveals that indifference curves with standard properties could be constructed showing either one as preferred, and as a result, neither can be rejected on the basis of a rule free of subjective preferences.

"Opportunity dominance" is calculated by determining the attainable present consumption for an investment alternative by discounting the future income stream at the borrowing rate, and determining the attainable future consumption obtained by compounding the present income stream to the future. These attainable consumption titles are then

Figure 2-24 Investment comparisons free of subjective preferences.



compared for investment alternatives and if the present and future consumption titles for one investment alternative are greater than the equivalent consumption titles of another alternative the first is said to be "opportunity dominant" over the second. If either the present or future consumption titles are greater for one while the other set of consumption titles are equivalent the investment opportunity with the greater titles is considered "opportunity dominant." In situations where neither of the above conditions are met the test is said to be indeterminate, and a comparison free of subjective preferences is not possible (Hirshleifer, 1970).

Formally this rule can be stated as:

> If the wealth equivalent or present value of the income stream W_0 (F) is greater than or equal to W_0 (E), and the corresponding future value W_1 (F) is greater than or equal to W_1 (E), with the equality holding in at least one of the two comparisons, then F is "opportunity-dominant" over E (Hirshleifer, 1970).

In a situation characterized by rising borrowing costs the productive optimum is defined by the maximum level of wealth under the condition that the marginal rate of transformation, marginal rate of exchange, and marginal rate of substitution are equal. This is reflected in the requirement that present value calculations be based upon the marginal borrowing cost as the discount rate. This requirement that the discount rate in a present value calculation be equal to the marginal borrowing cost links present value criteria to subjective preferences (Hirshleifer, 1970).

The calculation of present value criteria in multiperiod situations with market imperfections increases the complexity of the problem several fold. The existence of distinct borrowing and lending rates between each pair of periods create a situation where the optimum investment ensemble will require borrowing in some periods, lending in others, and in a third class of periods there will be no attractive exchange opportunities. This leads to the use of a complex series of discounting rates for the calculation of present value criteria, based upon borrowing rates, lending rates, and subjective preference. If in addition the borrowing rate varies by the amount borrowed the subjective preferences must be introduced into the periods where borrowing is required.

In previous applications of present value criteria, where a single discount rate applied for either borrowing or lending, comparisons between attained wealth and wealth gain were equivalent. However, the introduction of subjective preference rates in the discounting sequence makes it necessary to compare the attained income sequence and not the project or ensemble income sequence (Hirshleifer, 1970).

The application of the concepts of "simple" and "opportunity dominance" to the attained income sequences associated with individual or sets of investment projects allows the elimination of some projects. The investment opportunitites remaining after the filtering process can only be selected by a subjective choice (Hirshleifer, 1970).

Under conditions where quantitative limits are imposed on the level of investment, it is generally assumed that limits are only imposed for the current period. The acceptance of this condition allows the application of a modified form of present value rules based upon the attained wealth, or wealth gain, in the first period after the current period. This allows the application of present value rules to the modified present value of investment ensembles possible under the constrained conditions of the current period.

A brief mathematical summary of this material is given in Appendix A.

Uncertainty and Investment Choice

The principles of investment choice discussed in the previous sections have been developed under an artificial assumption of certainty. In this section the assumption of certainty will be relaxed in order to develop additional principles relevant to the economic analysis of stand treatments in the real world.

The seven axioms of consumer choice which have been the basis of investment choices under the assumptionof certainty are supplemented by five postulates of choice under uncertainty. These postulates systematically describe the nature of comparisons made by individuals in optimizing their choices under uncertain conditions. The objects of choice under uncertainty are similar to those in previous discussions with the additional trait that they can be had with an associated probability. An example of this would be two goods, A and B, each with its associated probability, S and 1-S, the probabilities being expressed by real numbers between 0 and 1. In this discussion an object and its associated probability will be referred to as a prospect.

The first postulate is a restatement of the first three axioms of consumer choice for the objects of choice under uncertainty, and states that an individual has a complete and transitive weak preference ordering of a set of prospects and chooses rationally (Green, 1971).

The second postulate defines an individual's preference toward uncertainty and states that a higher probability to a prospect is preferred to a lower probability (Green, 1971). What this postulate is saying is that all other things being equal more certainty is preferred to less certainty.

The third postulate of choice under uncertainty defines continuity in much the same way as the fifth axiom of consumer choice. Formally the postulate says that given three objects of choice, Y, Y', Y", such that Y is regarded by the consumer as being at least as good as Y' and Y' is regarded as being at least as good as Y", then there exists some probability S such that the combination of prospects (Y, Y": S, 1-S) is indifferent to the certain object of choice Y' (Green, 1971). This is saying that there exists some combination of probabilities for the best and worst of any three prospects that will make the consumer indifferent between them and some certain intermediate object of choice.

The fourth postulate is the strong independence postulate. This postulate states that for any complex prospect (Y) any component object or prospect (X) can be replaced by an object or prospect indifferent to the component, and the resulting complex prospect is indifferent to the prospect as it was before the replacement was made (Green, 1971).

The fifth postulate states that any complex prospect (Y) involving the objects X, X', X", is indifferent to one of the form (X, X', X": S, S', S") when the probabilities S, S', S" are calculated by the usual rules for combining probabilities (Green, 1971).

The implication of this postulate is that an individual's preferences for a prospect composed in turn of other prospects is the same as though he or she had gone through all the calculations to determine the ultimate probabilities of the basic prospects.

Objections to the postulate on continuity have been raised by Alchain (1953). His objection is that one of the objects of choice may be so bad, in relation to the other two, that it is not possible for there to be a positive probability of the worst event occurring that will make the consumer indifferent between a prospect of the best and worst and a certain prospect of the intermediate choice. Generally this objection is countered by formulating a problem with Alchain's conditions and illustrating how the probability of the worst event can be made sufficiently small, but positive, so that it would be reasonable to expect an individual to be indifferent to the prospects (Green, 1971).

Baumol (1972) has objected to the postulate on independence. His objection has been that the desirability of a prospect is influenced by the context in which it is offered. Therefore, the condition of indifference following substitution in the fourth postulate may not hold.

Despite these objections many economists find these postulates useful in describing choice under uncertainty (Baumol, 1972). The position of this thesis is that in important wealth-oriented activities these five postulates generally reflect consumer choices.

A mathematical presentation of the postulates and illustrations of their implications is given in Appendix A. A fuller discussion of the postulates and their meaning is given in Green (1971) Chapters 13 to 15.

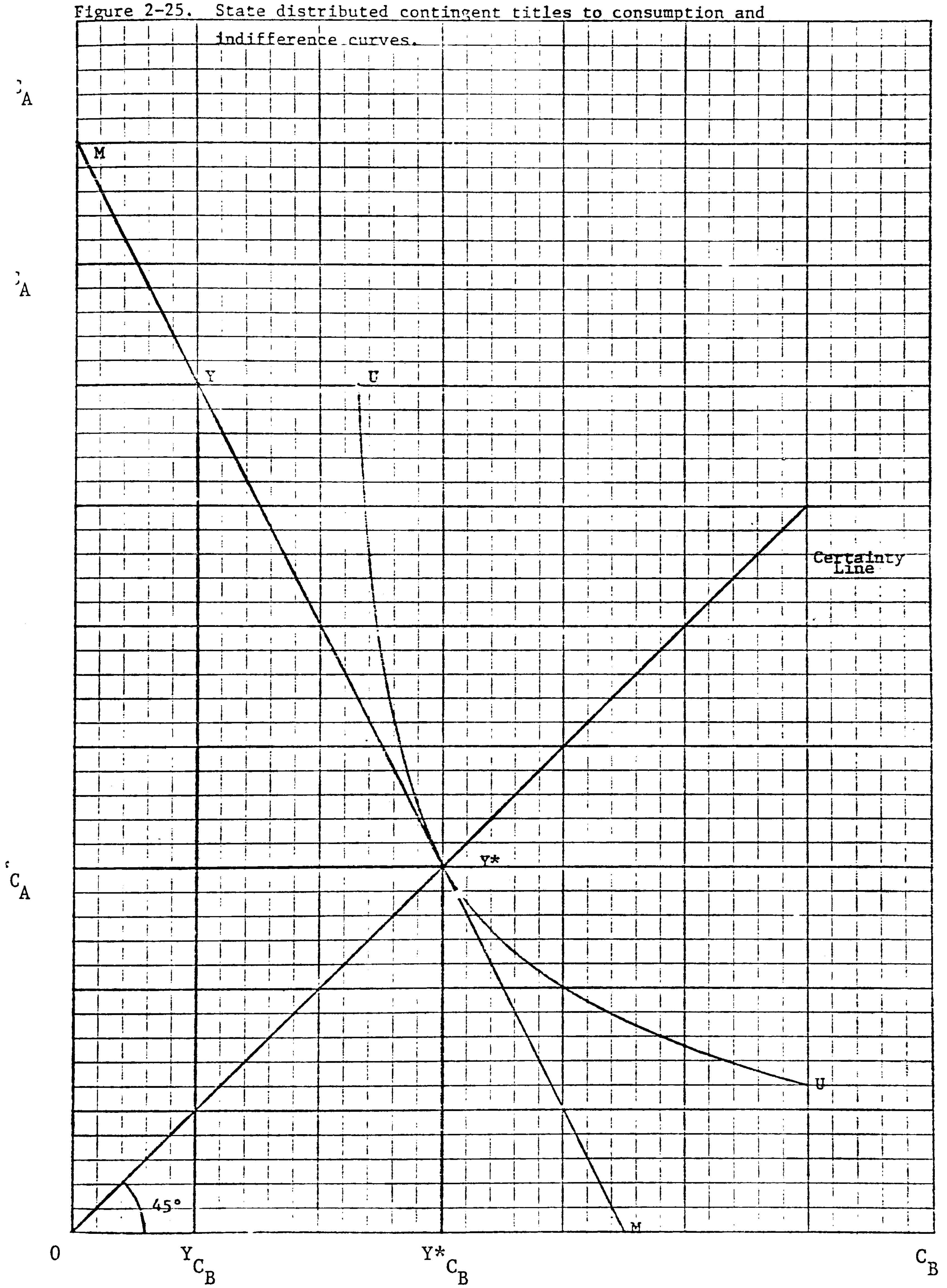
Timeless uncertainty. The application of these postulates of consumer choice under uncertainty has split into two branches. The first of these reduces the determinants of selection to the expected object of choice and the variance of the distribution of possible outcomes. Generally this approach is called the mean variance approach (Green, 1971) or the mean and standard deviation as objects of choice approach (Hirshleifer, 1970). In order for the mean and variance of an object of choice to function as the sole determinants of choice two restrictive assumptions must be accepted. The first of these is that the distribution of consumption titles must be normally distributed. The second is the basic requirement, that for the mean and variance to adequately function as the determinants of choice the individuals preferences must be such that as wealth is increased the willingness to accept uncertainty in asset holdings is decreased (Green, 1971).

The second framework for applying the postulates of choice under uncertainty is called "state preference theory", and is the theoretical framework for investment choice which will be adopted in this thesis. The objects of choice in state preference theory are contingent titles to consumption. These titles are contingent upon the state of the world. An example of a contingent title to consumption would be a win ticket on a two horse race. The contingent title is the pay out of the ticket which only pays if one of the horses wins, but not if the other horse comes in first. The second requirement of state preference theory is that there is no uncertainty as to whether the contingent title to consumption will be redeemed. All the uncertainty is limited to what state will occur and none is associated with the availability of the consumptive claim.

The following illustration of the application of state preference theory to a problem of choice under uncertainty will help to illustrate the operation of the theory. Consider that there exist two consumption prospects, c_A and c_B, each contingent upon states of the world A and B. These consumptive prospects each exist with probabilities S_A and S_B. Figure 2-25 illustrates the contingent claims c_A and c_B. The probabilities S_A and S_B are not shown explicitly but they influence the shape of the indifference curve UU. Any point in Figure 2-25 represents a particular prospect (c_A, c_B) for consumption. It is important to note the prospects represented in Figure 2-25 do not represent combinations of consumption good A and B. They rather represent a prospect of alternative consumption titles. The choices represented involve consumption of only one or the other of the elements (c_A or c_B) depending upon which of the two states (A or B) occur.

Consider the individual illustrated in Figure 2-25 with an initial endowment Y (yc_A, yc_B) and an indifference curve UU. The indifference curve UU is similar to the indifference curves in previous diagrams except that it represents the locus of prospects (c_A, c_B, S_A, S_B) to which this individual is indifferent. Now suppose our individual is offered a certain prospect Y* (y^*c_A, y^*c_B). Certain in this context means that the prospect Y* contains equal titles to consumption so that the individual is assured a given level of consumption regardless of which state A or B occurs. The transaction involving the exchange of Y for Y* can be thought of as

Figure 2-25. State distributed contingent titles to consumption and indifference curves.



giving up ($y^*c_A - yc_A$) in state A in exchange for ($y^*c_B - yc_B$) in state B, which is represented by the slope of the line MM. Under the conditions described in this figure prospect Y* would be preferred to all other prospects attainable given the initial endowment prospect Y, the market opportunity set OMM, the locus of indifferent prospects UU, and following the same logic as was used in the paradigm of simple exchange represented by Figure 2-2. This application of "state preference theory" to the question of timeless choice under uncertainty allows the statement of the principle that the optimum prospect will be defined where the ratio of prices between prospects is equal to the marginal rate of substitution between prospects along an indifference curve. Remember that a prospect consists of at least two contingent titles to consumption and their associated probabilities of occurrence (Hirshleifer, 1970).

The expansion of this example of two goods to a multiple goods paradigm is similar to the expansion followed in previous examples. The relationship between marginal rates of substitution and price ratios continues to hold in the multiple good multiple state case. A mathematical summary of this material is given in Appendix A.

Intertemporal uncertainty. The expansion of the paradigm of timeless choice under uncertainty to a framework allowing choices through time and over states requires that choices be applied to prospects which consist of dated conditional consumption sequences with associated probabilities of occurrence of the required state. Within this framework the probability of any sequence of titles to consumption is defined by the product of the conditional probabilities of the states required for the conditional titles. This theoretical construction retains indifference curves similar to those postulated in previous examples under uncertainty in the sense that they define combinations of prospects to which the individual is indifferent.

Given indifference curves and objects of choice of the type just described the operation of the choice theoretic system is exactly parallel to the system used to describe intertemporal choices. In a situation where an individual has only exchange opportunities time state consumptionclaims will be exchanged along the boundary of the market opportunity set until a preferred prospect is achieved. The marginal conditions associated with this preferred prospect will be that the ratio of prices between time state consumption claims will be equal to the marginal rate of substitution along an indifference curve. Following the same logic, in a situation with both productive and market opportunities, the individual would be expected to engage in productive transformations of the time state consumption claims in the endowment until the marginal rate of productive transformation between time state claims is equal to the ratio of prices. Having attained an optimum consumption set through intertemporal productive transformation an individual would be expected to achieve a preferred combination of consumptive prospects through exchange. The exchanges is this situation being similar to those in the example of intertemporal production and exchange except that the objects of choice again consist of conditional claims to consumption and associated probabilities of occurrence.

Uncertainty and investment choice criteria. The application of "state preference theory" to the reality of investment choice which involves both time and uncertainty requires a highly idealized version of the actual nature of the choices (Hirshleifer, 1970). One idealization which needs explicit discussion is the separation of production and transaction uncertainty. Production uncertainty refers to ignorance of the exogenous events which influence the physical outcome of the production process (Hirshleifer, 1970). This uncertainty exists even in situations with no exchange opportunities and is the type of uncertainty dealt with in this discussion. Transactions uncertainty refers to exchange opportunities, and represents ignorance of conditions under which consumptionclaims may be exchanged. In this context transactions uncertainty is a form of market imperfection in that a unique price at which transactions may take place is not known with certainty (Hirshleifer, 1970). Production uncertainty is the class of uncertainty which will be discussed in this section on investment choice. The analytical framework for transactions uncertainty involves questions of the form in which wealth is held and is considerably more difficult and beyond the scope of this thesis (Hirshleifer, 1970).

The previous example, illustrating the application of "state preference theory" to the problem of choice through time and over states, revealed the similarity between the theoretical frameworks for intertemporal choice under certain and uncertain conditions. Based upon this similarity Hirshleifer (1970) formulates the criteria of Present Certainty-Equivalent Value as being the appropriate generalization of the present value criteria to accomodate uncertainty. The investment choice rules associated with Present Certainty-Equivalent Value are equivalent to similar rules for the present value criteria. The rules, which call for the adoption of investment ensembles that maximize the present value of attained wealth, and wealth gain under certainty, call for the maximization of Present Certainty-Equivalent Value for the same parameters under uncertainty. The rules under certainty which calls for the adoption of an investment project as long as its present value is greater than zero, calls for the adoption of an investment project under uncertainty as long as its Present Certainty-Equivalent Value is greater than zero (Hirshleifer, 1970).

The Present Certainty-Equivalent Value of attained wealth (W^p/C_0) is defined as the discounted sum of conditional consumption claims through all time periods and over all states. Following the formulation of present value criteria for certainty the Present Certainty-Equivalent Value of wealth gain (W^q/C_0) is defined as the discounted difference between the endowment and attained wealth through all time periods and over all states. The project Present Certainty-Equivalent Value ($W^{\Delta p}/C_0$) ($W^{\Delta q}/C_0$) is defined in a similar manner as changes in attained wealth associated with a given investment project discounted through all time periods and over all states (Hirschleifer, 1970).

In the formulation of Present Certainty-Equivalent Value criteria, the rate at which conditional consumption claims are discounted, is uniquely determined for each state and time by the market price for the conditional consumption claims in terms of certain consumption in the current period. The relation between market price and the risky rate of interest, used to discount conditional consumption claims, is identical to the relationship between market price and the riskless rate of interest discussed in the section on intertemporal choice (Hirshleifer, 1970). An explicit formulation of the relationship between the market price of a risky prospect and the risky interest rate is given in Appendix A.

Once the risky rate of interest has been determined for all possible states in a time period, a riskless rate of interest can be defined as a function of the sum of all the risky rates used for discounting uncertain consumption titles (Hirshleifer, 1970). An explicit formulation of this riskless rate of interest is also given in Appendix A. Risky rates of interest and the resulting discount factors are usually greater than the equivalent riskless interest rates. However, this condition is not necessarily or universally true. Exceptions may occur when the risky prospect yields greater conditional consumption in a more valuable state (Hirschleifer, 1970). A numberical example of this is given in Appendix A.

In the application of Present Certainty-Equivalent Value criteria to problems of investment choice the usual approach has been to reduce the conditional consumption claims generated by any investment ensemble to a single expected consumption claim which is discounted to determine the Present Certainty-Equivalent Value. This reduction of the possible conditional consumption claims to a single expected value is necessary because of the enormous number of time state combinations possible with any investment ensemble, and the lack of visible prices for these time state claims. In order to overcome this obstacle two basic approaches have been adopted (Hirshleifer, 1970).

The first of these requires that the conditional consumption claims for any period be reduced to a single certain equivalent consumption claim which is then discounted using the risk free discount rate to determine the Present Certainty-Equivalent Value (Hirshleifer, 1970). The second approach is to calculate the mathematical expectation of the consumption titles generated by an investment ensemble at any point in time and discount this by the discount rate appropriate to the "risk class" for the investment (Hirshleifer, 1970). Because of the difficulty in determining the certainty equivalent of the conditional consumption claims associated with any investment ensemble the second approach to the calculation of Present Certainty-Equivalent Value is usually adopted (Hirshleifer, 1970).

This simplified theoretical framework allows the formation of the following additional principles for the economic analysis of stand treatments under uncertain conditions:

1. Present Certainty-Equivalent Value criteria and associated rules will identify correct sequences of stand treatments under perfect market conditions.
2. Because of incomplete information on possible yields associated with all possible states at any point in time the calculation of Present Certainty-Equivalent Value is based upon the mathematical expectation of yield at any point in time discounted by the discount factor appropriate for the risk class of the mathematical expectation.

A mathematical summary of this material on investment choice under uncertainty and numberical examples are given in Appendix A.

Conclusions From the Review of Investment Theory

This review of investment theory has led to the conclusion that the appropriate class of criteria for the economic analysis of stand treatments are Present Certainty-Equivalent Value. These criteria and associated rules will generally select optimum stand treatment sequences consistent with the economic theory of investment choice. The primary limitation of these criteria is their inability to define a unique optimum treatment sequence under imperfect intertemporal markets where the Separation Theorem fails to hold. Under this condition the criteria are limited to defining "opportunity dominant stand treatment sequences.

The estimation of Present Certainty-Equivalent Value for a stand treatment sequence requires forecasts of the expected yield and input series with their real prices. The appropriate discount rates for calculating Present Certainty-Equivalent Value are the series of real rates for the appropriate risk class based upon the anticipated intertemporal transactions of the decision making individual or organization.

The remainder of this thesis will be devoted to defining and illustrating procedures for estimating the Present Certainty-Equivalent Value of stand treatments. These criteria, rules, and associated procedures will assist in the selection of optimum treatment sequences for timber production.

CHAPTER III

REVIEW OF PREVIOUS WORK

Modern economic analysis of stand treatments has its origins in the solution proposed by Faustmann (1849) to the problem of forest valuation (Linnard and Gane, 1968). The principles for forest valuation have been supplemented and modified by contributions from economic science so that now there exists a large body of published work concerned with the economic analysis of stand treatments.

A selected bibliography of the published material on this subject is given in Appendix B.

For the purpose of this review, these studies will be considered in two classes. The first consists of studies which emphasize theoretical principles. The second consists of empirical studies of stand treatments.

Theoretical Studies

Criteria and rules. Initially the emphasis in theoretical studies was on selection of criteria for the determination of rotation. Studies by Gaffney (1960), Bentley and Tecguarden (1965), Pearse (1967) and Watt (1967) concluded that the appropriate criterion for the determination of rotation was soil expectation value as proposed by Faustmann (1849). Further, they concluded that the rule to be applied to soil expectation value was that it should be maximized to achieve an optimum rotation.

Davis (1965) and Haley (1966) have pointed out that soil expectation value is not generally consistent with modern investment theory. This is due to its failure to recognize the market value of land. Instead, a land value is calculated from the infinite stream of net monetary flows of successive timber crops. This failure to recognize the market value of one of the factors of production can lead to the choice of a rotation which is too long if the soil expectation value is less than the market value of land or too short if the soil expectation value is greater than the market value of land. This can be seen clearly in the previous discussion of investment choice rules and problems of duration and replacement.

In addition to studies on the appropriate criteria for rotation determination, several theoretical studies have been made on the broader question of investment criteria for forest management. These studies fall into two general classes: those recommending internal rate of return criteria based upon cash flows and those recommending forms of present value criteria based upon cach flows.

Duerr (1960), Duerr and Bond (1952), Kilkki (1968), Marty (1970), and Marty *et al.* (1966) have recommended forms of internal rate of return based upon cash flows as the appropriate criteria for the analysis of forestry investments. Ganser and Larson (1969) have summarized for forestry investments the reasons for the failure of rate of return criteria to generally define an optimum investment ensemble. These are identical to the reasons discussed in the previous section on rules for investment choice. Marty (1970) proposed the criterion of "composite internal rate of return" to overcome the weaknesses of ordinary rate of return criteria. However, this alternative offers no improvement over present value criteria at the cost of increased complexity in calculation and for these reasons it is not appropriate for the analysis of stand treatments.

The second class of theoretical studies on investment criteria for forest management which recommend forms of present value criteria includes work by Grayson (1962), Hamalainen (1973), Hughes and Post (1973), and Johnston *et al.* (1967). Lundgren (1971) has pointed out the importance of equating the ratio of prices between present and future cash flows for both consumption and investment in the formulation of investment ensembles.

In general, the application of present value criteria as formulated in these studies cannot be expected to define an optimum investment ensemble for timber production for the following reasons. First the formulation of the criterion in terms of cash flows ignores the basic constraint on investment which is the stock of wealth rather than the flow of income. Second, the use of money based criteria without explicit measures to overcome the distortions brought about by money in the real world, cannot define optimum investment ensembles over the time periods required for forestry investments. Third, with the exceptionof Lundgren (1971) all the studies ignore the problems of investment choice in imperfect intertemporal markets. Fourth, present value criteria as formulated in these studies ignore the problems of investment choice in an uncertain world.

An additional criterion has been proposed for investments in timber production by Lundgren (1973a). This criterion is called "cost-price" and is defined as the discounted cost of a unit of timber volume. The rule to be applied to this criterion would be to select the investment ensemble which minimized "cost-price". In addition to the general shortcomings of the previous cash flow based criteria this one has the added disadvantage of ignoring market prices for timber and concentrating exclusively on the cash inputs. This makes it impossible to define the optimum level of investment for timber production in relation to market prices.

Optimum treatment sequences. All of the work reviewed to this point has been a form of marginal analysis. The major drawback of marginal analysis is that it is limited to finding an optimum based upon stand characteristics and prices at a single point. This does not allow the definition of an optimum route to be followed in getting to and staying in an optimum condition. Three studies have been reported which apply the techniques of calculus of variation or dynamic programming to overcome the shortcomings of marginal analysis. Amidon and Akin (1968) demonstrated the technique of dynamic programming in determining optimum stocking levels and rotations of loblolly pine (*Pinus taeda L.*). In this analysis they used internal rate of return on additional growing stock as the criterion for determining the optimum level of growing stock. The rule applied to this criterion was to add growing stock or allow it to accumulate until the internal rate of return on the last unit of growing stock was equal to the alternative rate. The discussion of investment choice rules in the previous section illustrated why this criterion cannot be expected to generally define optimum investment ensembles. Amidon and Akin (1968) used the maximization of the soil expectation value as the rule and criterion for determining rotation. This rule and criterion are also not generally capable of defining optimum investment ensembles.

Naslund (1969) presented a theoretical argument for the application of the calculus of variation to the problem of optimizing harvest schedules involving thinning and clearcutting. However, Naslund failed to show an explicit solution to his formulation (Schreuder, 1971) and he used a present value criterion based upon cash flows.

Schreuder (1971) has also proposed a dynamic programming solution to the problem of optimum forest management schedules. Schreuder attempted to solve the problem through an application of the calculus of variation, but was unable to formulate a solution. This forced him to propose a solution using dynamic programming. Unfortunately, Schreuder has used a discounted net revenue criterion based upon cash flows which does not generally define optimum investment ensembles for reasons previously discussed (see also Lundgren, 1972).

Prices. There has been little theoretical work on the role of prices in the evaluation of stand treatments reported in the literature. The general rule has been to assume a set of constant prices and to ignore the problem of identifying the appropriate prices for use in analyses. Exceptions to this have been work by Buongiorno and Tecguarden (1973). Duerr (1960), Grayson (1962), Johnston *et al* (1967), and Lembersky and Johnson (1975). Duerr (1960) and Grayson (1962) explicitly recognize the possibility of prices changing through time and discuss in general terms the influence this will have on investments in timber production. Lembersky and Johnson (1975) recognize changes in the price of timber but ignore the possibility of price changes for other inputs or outputs associated with timber production. Johnson *et al* (1967) recognize the concept of real prices as opposed to market prices but fail to define the role of real and market prices in the analysis of forestry investments. Buongiorno and Tecguarden (1973) discuss the role and characteristics of prices in investment analyses made by governmental agencies. However, all these studies fail to identify the appropriate prices for use in the analysis of stand treatments.

The specific question of intertemporal prices and their role in the analysis of forestry investments has also received limited attention. Shepard (1925) has made an argument against the use of compound interest in forestry. His argument ignores the essential concept of interest as a means of expressing the ratio of exchange between consumption titles in different time periods. Expressing this ratio in the form of a geometric progression allows the ratio to be expressed independently of the number of periods between consumption titles.

Guttenburg (195) explicitly separates the pure or risk free rate of interest from the risky rate. He further proposes that the risky rate of interest for forestry investments be defined as the quotient between the risk free rate of interest and one minus the probability of loss (risky rate of interest = risk free rate of interest/(1 - probability of loss)) (Fisher, 1923). Aarestrup (1969) has carried the analysis of the rate of interest one step further by defining a risky real rate of interest for forestry in Denmark. This rate is defined as the long term effective first mortgage rate (4 1/2%) less the long term average rate of inflation (1%) and the average annual relative rise in forest products prices (1/2%).

Unfortunately none of these studies explicitly formulates a complete set of principles regarding the role of intertemporal prices in the analysis of stand treatments.

Uncertainty. As an element influencing the selection of alternative in forest management uncertainty has been approached from several directions. Flora (1964) proposed a set of restrictive assumptions under which he argued that uncertainty could be ignored. Johnston *et al* (1967) describe several criteria which may help in the selection of alternatives. However, they fail to present a unified consistent means of integrating these criteria into the main body of principles required for the analysis of forest management investments.

Marty (1964) and Thompson (1968) have proposed that uncertainty be recognized in the analysis of timber production investments by the assignment of subjective probabilities to possible discrete outcomes. The possible discrete outcomes which have been weighted by their subjective probabilities then become the objects of choice. There are two problems associated with this approach. First, the selection and assignment of subjective probabilities to discrete outcomes is a serious source of error. Secondly, the large number of possible outcomes to any timber management investment make the procedure difficult and complicated.

Lembersky and Johnson (1975) have proposed the application of a "Markov" decision process to the problem of defining optimum stand treatments. However, their solution requires the subjective assignment of probabilities of occurrence to discrete states and is subject to the same limitations described for the proposals of Marty (1964) and Thompson (1968).

Empirical Analyses of Stand Treatments

The procedures followed in empirical analyses of stand treatments have generally followed the principles formulated in theoretical studies.

Forecasts of growth and yield. The general procedure followed in forecasting or measuring physical performance in the evaluation of stand treatments has been to limit arbitrarily treatments or treatment sequences for analysis. This results in an analysis of pre-selected alternatives with no general reason to conclude that the optimum treatment or treatment sequence is among those selected for analysis. An exception to this generality has been a study of optimum harvesting sequences by Kilkki and Vaisanen (1969).

Several recent studies (Flora, 1966a,b, and 1970) (Sassaman, *et al*, 1973) have included the allowable cut effect (Lundgren, 1973b) (Schweitzer, *et al* 1972 and 1973) (Tecguarden, 1973) in estimates of the physical response to stand treatments. The use of this controversial procedure arises because of a confusion between the role of stocks and flows in the analysis of investments.

Criteria and rules. Several criteria in addition to those reviewed in the discussion of theoretical studies have been used to evaluate stand treatments. The most popular of these criteria have been those based upon forms current profit or revenue (Bond, 1940) (Bond, *et al*, 1937) (Bull, 1934) (Harlow, 1939) (Manogaran, 1973) (Osmond Smith, 1908) (Reynolds, 1939a,b) (Reynolds, *et al*, 1944) (Ward, 1958). The shortcoming of these criteria is their disregard for the results of a stand treatment which occur outside the current period. A closely related criterion that of current cost minimization (Lyford, 1934) (Morais, 1954) has also been reported which fails to recognize the results of any stand treatment beyond a single period. Marty and Allison (1960), and Worley Wheeland (1968) have evaluated stand treatments on the change in stumpage value based on current market prices. Again this criterion ignores the result of any treatment beyond the current period.

Several authors have used criteria which base evaluation of stand treatments on the difference between costs and revenues while ignoring the time periods in which they occur (Gentle, *et al*, 1968) (LIzardo, 1952) (Thomas, 1965) (Whiteley, 1971). These studies ignore the opportunities for market exchanges of consumption claims, and the prices which govern these exchanges.

Several authors have calculated or recommended the calculation of several different criteria which yield conflicting results (Callahan and Smith, 1974) (Crowe, 1967) (Goebel *et al*, 1964) (Grut, 1973) (Hiley, 1954 and 1956) (Huey, 1950) (Lerche and Saecd, 1967) (Lundgren, 1973a) (McCauley and Tremble, 1972) (VanLaar and Tingle, 1965) (Wambach, 1967) (Zillgitt, 1948). While the calculation of multiple criteria may provide useful information, the failure to specify rules for interpreting the various criteria and the possibility of different results with different criteria only confuse the analysis.

The failure to define decision rules applicable to the criteria calculated has been a shortcoming of most empirical studies. In many of the studies, the rules defined for calculated criteria generally would not define an optimum timber management investment sequence. Studies which have defined correct rules for calculated criteria include Buongiorno and Tecguarden (1973), Callahan and Smith (1974), Chapman and Baker (1954), Grut (1964 and 1967), Heiberg (1942), Hetherington (1969), Hosner and Lane (1953), Johnston *et al* (1967), Keipi and Kekkonen (1970), Lewis and Chappelle (1964), Lewis (1965), Linnard and Gane (1968), Malac (1966), Morais (1954), Osmond Smith (1908), Petrini (1953), Smith (1973), Stern (1970, 1971, and 1972), Sutton (1968), Walker (1969), Ward (1958), Whiteley (1971), and Yoho *et al* (1969).

Prices. The general practice in empirical analyses of stand treatments has been to assume a set or sets of market prices. This practice ignores the possibility of price changes through time and the possible distorting effects of money as a desired good in the economic system. The acceptance of unadjusted market prices in investment analyses also ignores the possibility of price distortions due to taxes and other factors. The possibility of changes in market price through time has been recognized and its influences on investments in timber management discussed by Callahan and Smith (1974), Duerr (1960), Grayson (1962), Johnston *et al* (1967), Kilkii and Vaisanen (1969), and Wikstram and Alley (1968). Manthy (1970) and Simmonds (1974) have included the influence of subsidies and taxes in their analyses of forest management investments. The social aspects of public forest investments have been incorporated in prices used by Burongiorno and Tecguarden (1973), Fenton and Dick (1972a,b, and c) and Fenton and Tustin (1972).

The procedure for selecting an interest rate to represent the ratio of prices between consumption claims in different periods has been similar to that followed in estimation of intratemporal prices. A single interest rate or several rates have been assumed based upon money rates of interest.

An exception to this was Cone (1972) who estimated a real rate which he defined as the money rate of interest less the prevailing rate of inflation.

Uncertainty. The existence of uncertainty has generally been ignored in previous analyses of stand treatments. Exceptions to this have been a study by Dowdle (1962) which applied a form of the "mean and standard deviation as objects of choice" analysis. The application of these criteria in forest management investment analyses requires the acceptance of the restrictive assumptions mentioned earlier. Cone (1972) and Schweitzer (1968) have calculated distributions of net present value based upon the probability of different outcomes. Investment ensembles were then selected according to the probability of a positive net present value. These methods have the same shortcomings as mentioned for the proposals by Marty (1964) and Thompson (1968).

In an analysis of reforestation decisions, Tecguarden (1969) proposed that a form of expected and present value be used as the criteria for investment decisions. In this study expected net present value was defined as:

$$ENPV = (R \times PSp) - I - (F \times PFp)$$

where: ENPV = Expected net present value

R = Discounted Revenue

PSp = Probability of plantation success

I = Initial plantation investment cost

F = Cost of plantation failure

PFp = Probability of plantation failure

This definition of expected net present value based upon the existence of only two states, success or failure, makes it unsuitable for the general class of forestry investments which includes stand treatments.

Without developing its theoretical foundations Heiberg (1942) applied a form of present certainty equivalent value to the problem of selecting leave trees in a partial harvest. The form he developed was very similar to the theoretical form introduced in the discussion of uncertainty and investment choice rules. However, Heiberg failed to expand this criterion beyond the problem of leave tree selection.

Conclusions From Review of Previous Work

Generally the previous work on the economic analysis of stand treatments provides information for decisions over a wide range of stand conditions, treatments, and price relationships. However, none of the studies reported in the literature is equal to the current need for the economic analysis of stand treatments for the following reasons.

1. The published studies fail to identify and estimate investment choice criteria which are consistent with the principles of economic theory which describes the mechanism of choice in investment decisions.

2. The published studies fail to estimate accurately the prices required to define an optimum investment ensemble.

3. The procedures followed in published studies are not generally able to utilize the flexibility of recently developed yield forecasting techniques.

CHAPTER IV

PROCEDURES

In the previous discussion of economic principles the central concept of intertemporal choice was presented. This is that under conditions which allow both production and exchange a preferred consumption set is achieved by a two stage process. The first stage of this process was shown to be the achievement of the largest possible market opportunity set through productive transformations. This was also shown to be equivalent to the maximization of wealth. The second stage in the achievement of a preferred consumption set was accomplished through market exchanges.

Outcome Criteria and Rules

The previous discussion also formulated a class of outcome criteria and rules for the analysis of investments which are consistent with the principle of wealth maximization in intertemporal choice.

Outcome criteria. The expected "present certainty-equivalent value" criteria consist of three criterion calculated from the wealth parameters of attained wealth, wealth gain and project wealth gain. All of these estimate wealth in units of current consumption.

The first expected Present Certainty-Equivalent Value of attained wealth, is an estimate of the total wealth of an individual or enterprise.

The second expected Present Certainty-Equivalent Value of wealth gain, is an estimate of the gain in wealth resulting from the adoption of an investment ensemble. The ensemble for this criterion can range from the total investment program of an enterprise or individual to a single investment.

If expected Present Certainty-Equivalent Value is based on the total investment ensemble of an individual or enterprise the sum of expected Present Certainty-Equivalent Value of wealth gain and the present value of the endowment sequence equal the expected Present Certainty-Equivalent Value of attained wealth.

The third criterion expected Present Certainty-Equivalent Value of a project wealth gain, is an estimate of the change in the expected Present Certainty-Equivalent Value for a single investment project.

The analysis of stand treatments involves the examination of an investment ensemble which is generally less than the total investment program of an individual or enterprise. In addition, stand treatments have the characteristic of interdependent investments. This is a result of the serial interdependence of growth and yield during the life of a stand. The condition that follows from this is that the yield following any treatment is influenced by other treatments applied to the stand during its life. For these reasons the appropriate criterion for the analysis of stand treatments is expected Present Certainty-Equivalent Value of wealth gain for the total investment sequence applied to a stand during its life.

Rules. The previous discussion also developed a series of rules indicating the acceptability of a project or ensemble by directing a comparison between a calculated criterion and a standard. These rules are to adopt the investment ensemble which maximizes the expected Present Certainty-Equivalent Value of attained wealth or wealth gain. The rule for determining the acceptability of an individual investment project is to adopt the project as long as it's expected Present Certainty-Equivalent Value is greater than zero. It follows from this that the rule to be applied in the analysis of stand treatments is to adopt the combination of treatments which maxmimze the expected Present Certainty-Equivalent Value of wealth gain.

Input - Output

Before going into a discussion of the requirements for estimates and measurements of the quantity and price of inputs and outputs, it is necessary to discuss the elements of the analysis procedure relating to:

1. The level of analysis.
2. The duration of analysis.
3. The interval within the analysis period.
4. Treatment possibilities for analysis.

The level of analysis for stand treatments can range from an individual stand to broad forest policy alternatives for a nation. However, the objective of this thesis is to develop refined procedures for the analysis of stand treatments at operational levels. This requires that the analysis be applied to the management unit to which stand treatments are applied. These units are generally designated as compartments (Committee on Forest Terminology, 1950). The essential management characteristics of a compartment are the existence of permanent boundaries, which can be identified on the ground, and its designation as the unit for silvicultural operations. The optimum treatment series for the total forest or enterprise is achieved through the sum of the treatments for the individual compartments and is dependent upon the accuracy of estimates of the quantity and price of the inputs and outputs to individual compartments.

The second element of an analysis procedure requiring discussion is how long a period of time to cover in the analysis? The calculation of Present Certainty-Equivalent Value does not require a specified length of time. The stock of goods and services associated with the investment at the end of the analysis are valued according to their prices at that time and this value is discounted to an equivalent present value. However, to fully evaluate any stand treatment, the analysis should be carried through the life of the timber crop to which it is applied. Since one of the treatments which should always be considered is the harvesting of the existing stand and its replacement with a new crop, the analysis should cover the full rotation period of a crop established during the current period. An exception to this may occur when the rotation of a species is very short, as in the case of some tropical species, and there exists a need for information concerning silvicultural operations for time periods in excess of one rotation. Under these circumstances, the duration of the analysis will be determined by the external requirements for information.

The interval at which the stock of wealth associated with any investment ensemble is evaluated should be determined by three considerations. The first of these covers the availability of estimates of wealth associated with an investment ensemble. Second, the ability to carry out treatments at frequent intervals. And third, the information requirements of the total timber management system. For example, if estimates of the quantity and price of inputs and outputs and are only available at five year intervals, there is no justification in evaluating the wealth of an investment sequence at more frequent intervals unless considerations associated with timber management operations or planning dictate otherwise.

In practice, the interval in the analysis should be variable during the analysis period. During the initial part of the analysis, they should be as short as the information and operational capability will allow. This is necessary to provide required information for the short range planning and scheduling of people, equipment and facilities. A second phase in the analysis would be the portion which provides information in support of mid-term plans. During this phase, the interval can be slightly longer because the need for detailed information on the timing of activities is reduced. The needs of timber management planning during this phase are to identify activities and estimate the requirements for manpower, equipment, and facilities far enough in advance to ensure that the capacity exists to carry out tasks when they are scheduled. The third phase of the analysis would support long range plans. The major emphasis in timber management planning during this phase is to identify substantial requirements for resources and the level of harbest well in advance.

The fourth element of an analysis procedure is concerned with the question of selecting the treatment alternatives for inclusion in the analysis. The general answer to this question is to include all treatments which can be physically carried out, for which yield forecasts following treatment can be made, and which do not violate laws or government regulations. An example of the error which can result from the incorrect specification of the production opportunity set is illustrated in the discussion of investment duration in the presentation of the economic principles.

Carron (1971) and Davis (1966) have discussed the general requirements for information used in forest management. However, the calculation of expected Present Certainty-Equivalent Value of wealth gain requires specific items of information with well defined characteristics. Further, this information must be integrated into, or extracted from, a large timber management information system, as well as measured initially in its collection. Since the application, compilation, and measurement of information are major considerations in choosing or developing a procedure for analysis, the standards for information will be summarized in these categories. Information on inputs and outputs required for tha analysis can also be divided into two general classes, the quantity of inputs and outputs and their associated prices.

Quantity. The requirements for information on the quantity of inputs and outputs in the calculation of the expected Present Certainty-Equivalent Value of wealth gain are:

1. Inputs and outputs or their characteristics must be quantitatively measurable. This means that the scale of measurement must conform to the characteristics of an interval scale (Husch, *et al*, 1972). Further, the unit of measure must be associated with an expression of real price such that the product of price and quantity is on a ratio (Husch, *et al*, 1972) or cardinal (Desgupta and Pearce, 1972) scale.

2. All inputs and outputs associated with an investment ensemble must be measured and included in the analysis. This is due to the fact that the information is being used to estimate a criterion which is based on the change in the total stock of wealth associated with an investment ensemble, and not just the incremental change usually considered in a marginal analysis.

The timber management requirements for information on the quantity of inputs and outputs are:

1. The measure of inputs and outputs should be applicable to all stands in an enterprise.

2. The measure of inputs and outputs associated with a class of stand treatments should be applicable to that treatment regardless of the circumstances in which it is applied.

3. All measures of inputs and outputs should be measurements that can be obtained and interpreted with equal accuracy over all types of stands and stáges of stand development.

4. The measures of inputs and outputs whould be based upon measured parameters and not hypothetical conditions.

5. The measures of inputs and outputs should be compatible with standard forest inventory and accounting methods.

6. The measures of inputs and outputs should contribute to the specification of silvicultural prescriptions.

7. The procedures and their supporting principles for measuring and interpreting the quantity of inputs and outputs should be clear, consistent, objective, and easy to apply.

8. The biometric techniques required to estimate inputs and outputs should conform to standard statistical procedures.

The mensurational requirements for the measurement of inputs and outputs are:

1. The measurements of inputs and outputs should be simple, objective, consistent, and economical.

2. The unit of measure for an input or output in the analysis of stand treatments should be the same as the unit of physical measurement or obtained by direct numerical conversion.

3. For a given input or output the same measurement unit should be used throughout the analysis.

4. The mensurational techniques and instruments required to measure inputs and outputs should conform to standard mensurational procedures and instruments available to all foresters.

Prices. The requirements for information on the price of inputs and outputs in the calculation of expected Present Certainty-Equivalent Value of wealth gain are:

1. The real price of inputs and outputs or their characteristics must be quantitatively measurable on a ratio (Husch, *et al*, 1972) or cardinal (Dasgupta, and Pearce, 1972) scale.

2. The unit in which prices are measured must be directly convert-

able into a quantity which defines the conditions of exchange for any other good in the economic system.

3. All prices should be expressed in units of constant value.

4. All prices should be real prices expressing the real conditions of exchange for the individual or enterprise. If the analysis is being carried out for a firm then prices must express the conditions of exchange in terms of the equity owners of the firm.

The timber management requirements for information on the price of inputs and outputs are:

1. All measures of price should be measurements that can be obtained and interpreted with equal accuracy for all inputs and outputs.

2. The measures of price should be based upon measured and estimated parameters and not hypothetical conditions or broad averages.

3. The measures of price should contribute to the estimation of the financial condition of the enterprise.

4. The procedures and supporting principles for the measurement of price should be clear, consistent, objective, and easy

to apply.

5. The econometric techniques required to estimate and apply input and output prices should conform to standard statistical procedures.

The measurement requirements for measures of the price of inputs and outputs are:

1. Price measurements should be simple, objective, consistent, and economical.

2. The unit of measurement for price should be the same as the price unit in the analysis or obtainable by direct numerical conversion.

3. Measures of price should be readily available from standard sources including market reports and accounting statements.

The final requirement of any procedure for the economic analysis of stand treatments is that it must define the optimum combination of treatments at each interval to ensure that the sequence of treatments selected results in the compartment following an optimum route in getting to and staying in an optimum condition. Failure to define an optimum sequence generally results in the adoption of changing treatment schedules as the compartment moves through time. This defeats the purpose of planning in timber management.

CHAPTER V

A DEMONSTRATION OF PROCEDURES FOR THE ECONOMIC ANALYSIS OF STAND TREATMENTS FOR TIMBER PRODUCTION

The following analysis of stand treatments for a compartment containing stands of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) is undertaken to demonstrate the analysis procedures for a realistic but hypothetical situation.

The subject of the analysis is a compartment of Douglas-fir in western Washington managed as part of an industrial tree farm by a forest products corporation. The compartment is defined with the aid of soil survey maps prepared by Steinbrenner and Gehrke (1964) and is selected to represent general operating and site conditions in the Douglas-fir Region of western Washington and Oregon. The stand conditions of age and density are also selected to represent conditions for extensive areas in the Douglas-fir Region.

General Procedure

The procedures, inputs, and outputs for this analysis are discussed in terms of the total analysis period for the compartment and treatments under consideration. However, the calculation of expected Present Certainty-Equivalent Value of wealth gain is limited to combinations of treatments considered for application during the initial interval. This simplifies the presentation of results, and allows attention to be focused on the essential elements of the analysis procedure.

Overall plan. The basic plan for the analysis is to select an optimum sequence of stand treatments by an iterative process involving the selection of an optimum combination of treatments for the initial interval. The optimum combination of treatments for the second and subsequent intervals are determined by repeating the selectionprocess given the stand conditions resulting from previous treatment combinations. The selection of the optimum combination of treatments at each interval is necessary to ensure that the procedure defines the optimum investment path for the compartment.

The optimum combination of treatments for each interval is defined by simulating all possible combinations of treatments for each stand in the compartment and forecasting the resulting stand yield at each interval in the analysis period. These stand yield forecasts are summarized on a compartment basis for each interval in the analysis period. The interval compartment summaries serve as the basis for estimates of the quantity of inputs and outputs associated with each potential treatment combination. The maximum duration for each potential treatment combination is determined by calculating the incremental expected discounted value of wealth gain for each interval. As long as this value is greater than 0 the additional interval adds to expected present certainty-equivalent value of wealth gain for the potential treatment. The formula for the calculationof incremental expected discounted value of wealth gain is:

$$IEW^{Q}/_{cm} = EIW^{Q}/_{cm} + (EW^{Q}/_{cm+1})(1 + r_{m,q}) \qquad (5\text{-}1)$$

where:

$IEW^{Q}/_{cm}$ = The incremental expected discounted value of wealth gain for interval m.

$EIW^{Q}/_{cm}$ = The expected investment required in period m to achieve $EW^{Q}/_{cm+1}$. (Note: The convention of indicating inputs to an investment sequence as negative quantities and outputs as positive quantities is adopted for this equation and all other equations in this thesis.)

$EW^{Q}/_{cm+1}$ = The expected attained wealth in interval m + 1.

$(1 + r_{m,q})^{-1}$ = The discount rate between consumption claims in interval m and m + 1 for the risk class appropriate to q.

Once the optimum duration for each potential treatment combination, for a single interval, is selected the expected discounted certinty-equivalent value of wealth is calculated for each potential treatment combination by the following.

$$EW^{QM}/_{cjm} = [\sum_{m=j}^{m*} (EW^{Q}/_{cm} + EIW^{Q}/_{cm})(1 + \bar{r}_{mq})^{-1}] - EW^{Q}/_{cj} \qquad (5\text{-}2)$$

when:

$EW^{Qm}/_{cjm}$ = The expected discounted certainty-equivalent value of attained wealth for any combination of treatments applied to the compartment during one interval.

m = The interval for which the treatments are being considered.

m^* = The interval of maximum duration for a treatment combination as defined by Equation 5-1.

$EW^Q/_{cm}$ = The expected attained wealth in interval m.

$EIQ^Q/_{cm}$ = The expected investment in interval m.

$$(1 + \bar{r}_{mq})^{-1} = [(1 + r_{jp})(1 + r_{j+1,q}) \ldots (1 + r_{m,q})]^{-1}$$

$(1 + r_{jq})$ = The discount rate between consumption claims in interval j and j + 1 for the risk class appropriate to q.

$EW^Q/_{cj}$ = The expected ensemble attained wealth at the beginning of interval j.

j = The interval for which the treatments are being considered.

The optimum treatment combination for any interval is defined by selecting the treatment combination with the greatest expected discounted certainty-equivalent value. The optimum combination of treatments for other intervals are determined by repeating the above procedure for each interval in the analysis period in sequence beginning with the current period.

Once the optimum combination of treatments has been identified for each interval the expected present certainty-equivalent value of wealth gain for the selected investment ensemble is calculated by the formula:

$$EW^{Q*}/_{co} = [\sum_{j=0}^{10} (EW^{Q}/_{cj} - EIW^{Q}/_{j})(1 + \bar{r}_{jq})^{-1}] - W^{Q}/_{co} \qquad (5\text{-}3)$$

where:

$EW^{Q*}/_{co}$ = Maximum expected present certainty-equivalent value of wealth gain.

$EW^{Q}/_{cj}$ = The expected attained wealth in interval j associated with the selected treatment sequence. (Equation 5-2)

$EIW^{Q}/_{j}$ = The expected investment in interval j associated with the selected treatment sequence. (Equation 5-2)

$$(1 + \bar{r}_{jp})^{-1} = [(1 + r_{0q})(1 + r_{1q})\ldots(1 + r_{jq})]^{-1}$$

$(1 + r_{jq})$ = The discount rate between consumption claims in one period and the next for the risk class appropriate to q.

$W^{Q}/_{co}$ = Initial endowment.

This is the maximum possible value for the criterion of expected present certainty-equivalent value of wealth gain by virtue of the maximization of the discounted certainty-equivalent value of individual investments in the sequence of the total ensemble.

A flow chart of the steps in this overall plan is given in Figure C-1 of Appendix C.

Length of analysis period. Preliminary calculations indicate that the maximum rotation age of stands created by a regeneration treatment in the first interval will be 50 years. Based on this information, the analysis period is set at 55 years. This maximum rotation age for a regeneration treatment in the first interval is estimated by application of Equation 5-1 to the investment sequence which would follow from selection of a regeneration treatment during the first interval.

Interval in analysis period. The interval selected for the demonstration analysis is five years. This length is selected because the accuracy of yield forecasts and price estimates do not justify a shorter interval. In addition, all estimates are made as point estimates at the midpoint of each interval. An exception to this is the first interval which is 2 1/2 years long and for which all estimates are made at the beginning of the interval. The intervals, their duration, and estimation points for the demonstration analysis are summarized in Table 5-1.

Treatments. The stand treatments considered in this analysis are limited to two regeneration treatments, five thinning treatments, and five fertilizer treatments. The considerations in selecting these treatments for analysis are first, the accuracy and precision of yield forecasting techniques available to forecast stand yields following treatment. Second, the precision and control with which stand treatments can be carried out operationally. Care must be taken in the analysis of stand treatments to ensure that the treatments under analysis are realistically defined. The accuracy and precision of yield forecasting techniques along with the operational ability to carry out combinations of treatments are primary considerations in the selection and specification of treatment combinations for analysis.

The regeneration treatments considered are no regeneration which is the continuation in some form of the existing stands, or natural regeneration which involves a clearcut harvest of the existing stands followed by a five year delay and the establishment of natural stands with Growing Stock Index 220 (King, 1970).

It is important to include at least one treatment involving the replacement of the existing stand in any analysis of stand treatments at every

Table 5-1

Interval, interval duration, and estimation point for demonstration analysis.

Interval	Duration			Estimation Point
	From		To	
Number	Month Year		Month Year	Year
0	1 - 1975	-	6 - 1977	1975
1	7 - 1977	-	6 - 1982	1980
2	7 - 1982	-	6 - 1987	1985
3	7 - 1987	-	6 - 1992	1990
4	7 - 1992	-	6 - 1997	1995
5	7 - 1997	-	6 - 2002	2000
6	7 - 2002	-	6 - 2007	2005
7	7 - 2007	-	6 - 2012	2010
8	7 - 2012	-	6 - 2017	2015
9	7 - 2017	-	6 - 2022	2020
10	7 - 2022	-	6 - 2027	2025
11	7 - 2027	-	6 - 2032	2030

interval. This is necessary to define the optimum duration for the existing stand and its associated investment ensemble. An example of this was discussed in Chapter II in the material on investment choice rules and problems of duration and replacement.

The five thinning treatments considered in the analysis are no thinning and removal of 10 to 40 percent of basal area in 10 percent increments as low thinnings. The low thinning treatments are simulated by removing trees from the smallest diameter breast height (DBH) classes until the required basal area for removal is accumulated.

Fertilizer treatments considered in the analysis are limited to applications of elemental nitrogen in the form of urea. The levels of nitrogen fertilization considered are 0, 100, 200, 300 and 400 pounds of elemental nitrogen per acre. The response duration of a fertilizer treatment is assumed to be five years regardless of the level of treatment.

Additional information on these treatments, the techniques used in their simulation, and procedures used to forecast stand yields following treatment are given in Appendix C.

Inputs and Outputs

Estimates of the quantity and price of inputs required for this analysis were made independently. The estimates of wealth required for the analysis were then calculated as part of the analysis procedure.

Quantity. The estimates of quantity and quality of land associated with the compartment in the analysis are based on information from the "Soil Survey of the Vail Tree Farm" (Steinbrenner, and Gehrke, 1964). The major items of information required for the analysis are the area of land within the compartment and an estimate of its site class. This information was necessary for the estimation of the quantity of several other inputs and outputs. The physical size of a compartment and its site class are also important determinates of the price of land. The other important characteristics of the compartment including location, operability of terrain, and soil properties available from the soil map which is given in Figure 5-1, which influence operations are the soil survey. The land area of the compartment is summarized by site class in Table 5-2.

It is important to include the land input in any analysis of stand treatments because this is the major component of the investment ensemble which is available for reinvestment. The importance of recognizing opportunities for reinvestment is illustrated and discussed in Chapter II in the material on investment choice rules and problems of duration and replacement.

The quantitative estimation of the quantity and quality of growing stock is one of the major problems in the economic analysis of stand treatments. Judgement must be exercised to ensure that the yield forecasts used in an analysis are appropriate to the conditions and treatments to which they are applied.

Estimates of the quantity and growing stock used in this analysis were developed using a modified system for the estimation of the yield of natural

Figure 5-1

Soil Map Showing Compartment Boundaries Location, and Terrain Features.

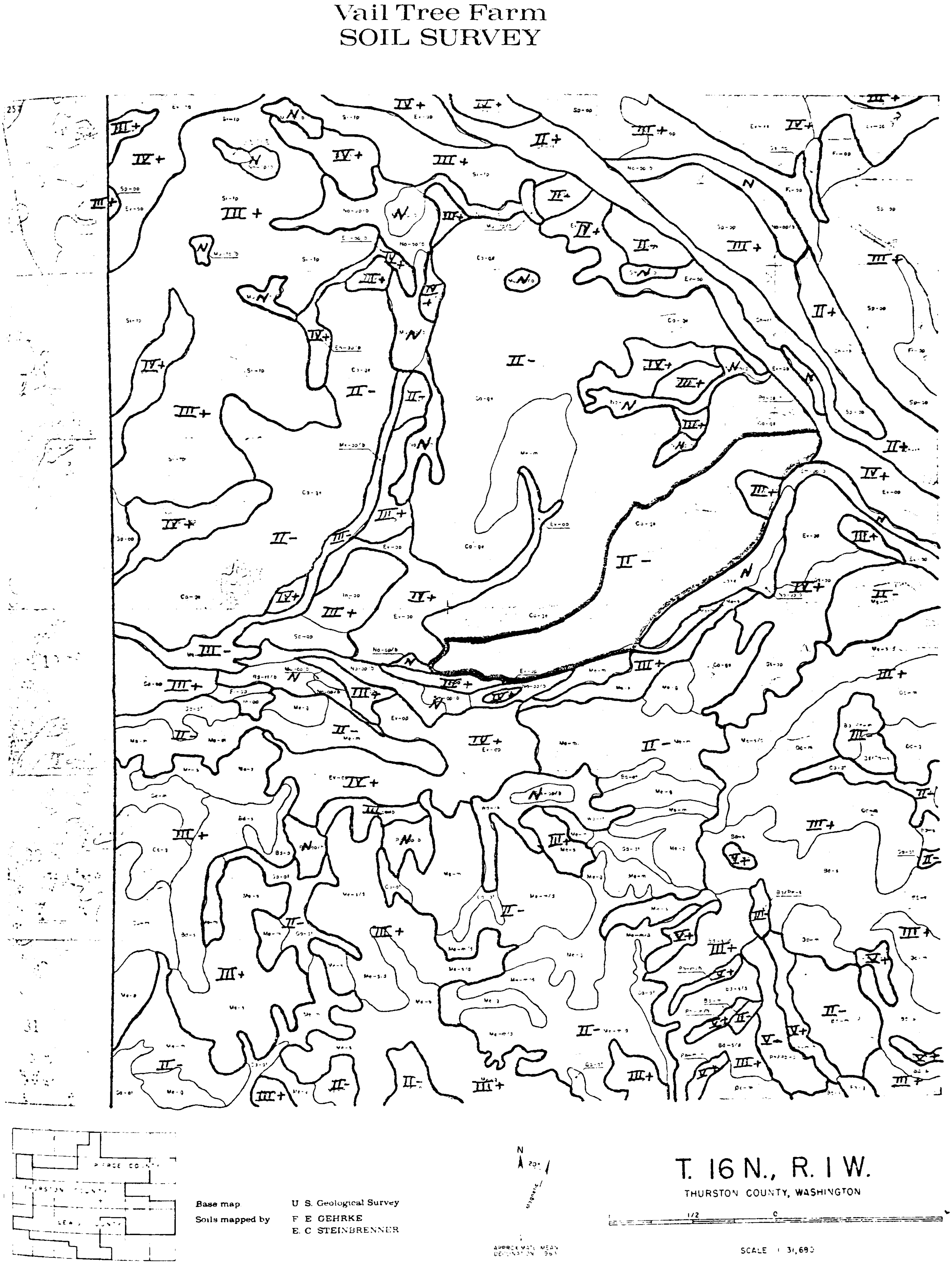


Table 5-2

Compartment area by Douglas-fir site class

Site Class[1]	Area
	Acres
110	63
120	2216
Total	2279

[1]Average site index to the nearest five feet based on King (1966).

stands of Douglas-fir developed by Bower and Shaw (1975). Estimates of the quality of the growing stock were made by estimating the distribution of volume within the stand by 1 inch DBH Classes. These distributions of volume by DBH class were based upon cumulative diameter distribution tables for Douglas-fir developed by King (1964).

Additional details on the yield forecasting procedures used in the analysis are given in Appendix C along with abbreviated yield tables for each of the stand treatment combinations considered in the analysis.

The next two inputs, ad valorem taxes and overhead are both important considerations in the analysis of timber production investment. While neither of these inputs may be changed as a result of a stand treatment they both have a significant influence on the level of wealth attained by any investment ensemble. Both part of the reinvestment element which is so important in questions of the replacement of existing stands.

The ad valorem tax relevant to this analysis is that imposed by the State of Washington. This tax is an annual tax of 1 1/2% of the market value of forest land (Washington Forest Protection Association, 1975). For this analysis the quantity of tax is estimated in annual acre units per interval. This resulted in an estimate of 5,697.5 units of tax for the first interval and 11,395 units for all subsequent intervals.

Overhead in this analysis is defined as the sum of inputs which are required to manage a timber crop but which are not influenced by the treatment applied to the crop. These would generally include overall supervision, maintenance of basic inventory information, and general activities related to the overall protection of the timber crop. In this study, the quantity of these overheads is estimated as two separate elements.

The first of these elements is estimated in terms of professional forester manyears per interval on a total compartment basis. The estimated quantities of professional forest manyears are given in Table 5-3. These estimates are based upon information compiled by the Industrial Forestry Association (1975) for industrial tree farms in the Douglas-fir Region. Detailed information on background to estimates given in Table 5-3 is given in Appendix C.

The second element of overhead inputs required by the compartment under analysis is the quantity of protection. This is also estimated from information compiled by the Industrial Forestry Association (1975) for the Douglas-fir Region. A summary of the estimates used in the analysis is given in Table 5-4. A more detailed explanation of the steps taken in the development of these estimates is given in Appendix C.

Neither of the regeneration treatments considered in this analysis required any inputs in their application. The output of growing stock associated with the replacement of the existing stands was measured in terms of clearcut stumpage volume.

Table 5-3

Estimated number of professional forester manyears required to manage the compartment by interval.

Interval	Estimation Point Year	Professional Forester Manyears Per Acre Per Year	Professional Forester Manyears Total Comp. Per Interval
		Manyears	
0	1975	0.00006328	0.3605
1	1980	0.00006885	0.7846
2	1985	0.00007443	0.8481
3	1990	0.00008000	0.9116
4	1995	0.00008557	0.9751
5	2000	0.00009114	1.0386
5+		0.00009114	1.0386

Table 5-4

Estimated quantity of protection required for the compartment by interval.

Interval	Estimation Point	Protection Units Per Acre Per Year	Protection Units Total Comp. Per Interval
	Year	Number	
0	1975	1	5,697.5
1	1980	1	11,395.0
1+		1	11,395.0

The inputs and outputs associated with thinning treatments are estimated in terms of the quantity of growing stock, measured in cubic feet of total volume, removed during the treatment. The convention of noting inputs as negative quantities and outputs as positive quantities was adopted for these estimates. In this case if the stumpage removed in a thinning treatment could be sold at a price which produces a positive net revenue, the quantity removed is recognized as an output and a positive quantity. Thinning treatments which required the removal of stumpage which could not be sold at a net revenue producing price are recognized in the analysis as an input and noted as a negative quantity.

The fertilizer input for fertilizer treatments is estimated as the number of pounds of elemental nitrogen required by the treatment being considered for the total compartment.

Price. In order to estimate prices used in the analysis, in units of constant value which define the conditions of exchange for all other goods in the system, all prices are estimated in 1975 dollars. The choice of this unit is arbitrary and it could just as well have been dollars of some other year. To keep the prices in units of constant value, in terms of the equity owners of the assumed firm, all conversions to 1975 dollars are based on the Consumer Price Index published by the U.S. Department of Commerce (1973, and 1975)(See Table C-5 in the Appendix).

In addition, all prices were estimated as after tax prices to define the real conditions of exchange for the equity owners of the firm. These price adjustments are made by adopting the simplifying assumption used by Grant and Ireson (1970) which is that the marginal tax rate on ordinary income for the firm is 50%. The marginal tax rate for capital gains income is assumed to be 25%. These assumptions on the marginal rate of tax on income allowed a price adjustment factor of 0.5 to be calculated for the price of inputs and outputs which influence ordinary income. An adjustment factor of 0.75 was calculated for the price of inputs and outputs which influence capital gains income. While these adjustment factors are based upon simplifying assumptions as to what the marginal rates of tax are, it is important to recognize the necessity in an actual analysis of estimating the marginal rate of tax for each interval in an analysis period, and to use this information to adjust prices so that they express the real conditions of exchange.

A more detailed discussion of the steps taken in adjusting observed prices to constant prices, and in the development of the price adjustment factors used in the analysis is given in Appendix C.

The price of land in the analysi is based on the market price of land of similar quality and general condition as compiled by the Washington State Department of Revenue for 1975. The prices used in the analysis are summarized in Table 5-5. In addition, it is assumed that the exchange of land will influence capital gains income and as a result, the prices in Table 5-5 were adjusted by the capital gains adjustment factor in the analysis. A more detailed discussion of the steps taken in the development of these estimates is given in Appendix C.

Table 5-5

Estimated market price for the land in the compartment by interval.

Interval	Estimation Point Year	Market Price Per Acre 1975 $
0	1975	77.00
1	1980	81.33
2	1985	85.90
3	1990	90.73
4	1995	95.83
5	2000	101.22
5+		101.22

The price of growing stock in the analysis is estimated as a clearcut stúmpage price. This is selected to isolate the growing of a crop of timber from its harvesting and transportation to a conversion facility. The prices are based on the historical price of Douglas-fir stumpage sold on National Forests on the west side of the Cascade Mountains in Washington and Oregon. The estimates of future prices are made by accepting the forecast of a 100% increase in stumpage price between 1970 and the year 2000 (U.S. Forest Service, 1973). After 2000, stumpage prices are assumed to remain constant.

In order to estimate the effect of treatments which modify the size distribution of trees in a stand, the estimated average stumpage price at each interval is estimated for 1 inch DBH classes. The resulting estimated stumpage prices by 1 inch DBH class and interval are given in Table 5-6.

These estimates of stumpage price by DBH class are applied to the calculated distributions of stand volume by DBH class to estimate a weighted average stumpage price which is applied to the total compartment volume. The exchange of timber in the form of stumpage generally receives income tax treatment as a capital gains. Therefore, the capital gain price adjustment is applied to average stumpage prices in the analysis.

As was discussed earlier, the ad valorem tax levied by the State of Washington which is relevant to the analysis of stand treatments is an annual tax levied on forest land. The 1 1/2 percent of market value of the tax sets the price of an annual tax unit. The price of taxes used in the

Table 5-6

Estimated Douglas-fir clearcut stumpage price per cubic foot total volume by interval and DBH class.

DBH Class Inches	Interval 0	1	2	3	4	5	5+
	1975 Dollars per Cubic Foot						
2	-0.0385	-0.0290	-0.0305	-0.0315	-0.0330	-0.0345	-0.0345
3	-0.0305	-0.0230	-0.0240	-0.0245	-0.0255	-0.0270	-0.0270
4	-0.0215	-0.0165	-0.0170	-0.0170	-0.0180	-0.0190	-0.0190
5	-0.0115	-0.0085	-0.0090	-0.0090	-0.0095	-0.0100	-0.0100
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.0125	0.0075	0.0080	0.0100	0.0110	0.0120	0.0120
8	0.0270	0.0175	0.0190	0.0210	0.0225	0.0270	0.0270
9	0.0430	0.0275	0.0310	0.0340	0.0350	0.0370	0.0370
10	0.061	0.0390	0.0430	0.0460	0.0480	0.0525	0.0525
11	0.0820	0.0525	0.0560	0.0610	0.0640	0.0680	0.0680
12	0.1050	0.0650	0.0710	0.0770	0.0810	0.0875	0.0875
13	0.1310	0.0810	0.0860	0.0940	0.0990	0.1080	0.1080
14	0.1500	0.0960	0.1040	0.1140	0.1200	0,1310	0.1310
15	0.1940	0.1140	0.1250	0.1350	0.1440	0.1560	0.1560
16	0.2310	0.1340	0.1450	0.1580	0.1680	0.1850	0.1850
17	0.2720	0.1540	0.1675	0.1840	0.1980	0.2160	0.2160
18	0.3220	0.1770	0.1930	0.2120	0.2280	0.2520	0.2520
19	0.3750	0.2020	0.2200	0.2420	0.2620	0.2900	0.2900
20	0.4390	0.2290	0.2500	0.2790	0.3000	0.3350	0.3350
21	0.5075	0.2570	0.2840	0.3140	0.3420	0.3800	0.3800
22	0.5850	0.2875	0.3200	0.3560	0.3900	0.4350	0.4350
23	0.6700	0.3240	0.3590	0.4000	0.4425	0.4925	0.4925
24	0.7700	0.3610	0.4010	0.4500	0.4975	0.5575	0.5575
25	0.8850	0.4010	0.4550	0.5100	0.5650	0.6325	0.6325
26	1.0069	0.4500	0.5050	0.5660	0.6350	0.7152	0.7152
27	1.1500	0.4950	0.5600	0.6350	0.7075	0.8000	0.8000
28	1.3200	0.5500	0.6200	0.7100	0.7950	0.9000	0.9000
29	1.4900	0.6050	0.6900	0.7900	0.8900	1.0200	1.0200
30	1.7000	0.6700	0.7650	0.8800	0.9900	1.1400	1.1400
31	1.9250	0.7400	0.8450	0.9750	1.1100	1.2700	1.2700
32	2.1900	0.8100	0.9300	1.0750	1.2300	1.4250	1.4250
33	2.4900	0.8900	1.0400	1.1900	1.3750	1.5900	1.5900
34	2.8100	0.9750	1.1400	1.3300	1.5400	1.7750	1.7750
35	3.2000	1.0800	1.2500	1.4750	1.7000	1.9900	1.9900
36	3.6100	1.1800	1.3800	1.6250	1.8900	2.2200	2.2200
37	4.0900	1.3000	1.5200	1.8000	2.1000	2.4750	2.4750
38	4.6000	1.4250	1.6750	1.9900	2.3400	2.7500	2.7500
39	5.2000	1.5500	1.8400	2.2000	2.5800	3.0750	3.0750
40	5.9000	1.7100	2.0250	2.4250	2.8800	3.4400	3.4400

demonstration analysis are given in Table 5-7. Ad valorem taxes are allowed as a deduction against ordinary income so the real price is calculated by application of the 0.5 adjustment factor.

The price of each of the elements of overhead are estimated separately. The price per professional forester manyear is based on income information compiled by the Society of American Foresters (see Table C-10 in the Appendix). It is further assumed that the total price of a professional forester is twice the salary received. The resulting prices which are used in the analysis are given in Table 5-8.

The price of an annual unit of protection is based on the average protection cost per acre per year for industrial tree farms in the Pacific Northwest (Industrial Forestry Association, 1975) (see Table C-12 in the Appendix). The estimated price of protection units used in the analysis are given in Table 5-9. Both items of overhead are treated as expenses against ordinary income for Federal Income Tax purposes; so real prices are calculated by application of the ordinary income adjustment factor.

A more detailed description of the sources of information and steps taken in preparation price estimates for ad valorem taxes and overhead are given in Appendix C.

The estimation of discount rates or the price of time is one of the most controversial topics in investment analysis. The criterion expected present certainty-equivalent value requires that the discount rate be a risky rate appropriate to the risk class of the investments under analysis.

Table 5-7

Estimated price of ad valorem taxes on the compartment per tax unit by interval.

Interval	Estimation Point Year	Price Per Tax Unit 1975 $
0	1975	1.1550
1	1980	1.2200
2	1985	1.2885
3	1990	1.3610
4	1995	1.4374
5	2000	1.5183
5+		1.5183

Table 5-8

Estimated price of professional forester manyears by interval.

Interval	Estimation Point Year	Price Per Manyear 1975 $
0	1975	36,007.37
1	1980	38,859.93
2	1985	41,712.48
3	1990	44,565.04
4	1995	47,417.59
5	2000	50,270.15
5+		50,270.15

Table 5-9

Estimated price per acre per year of a unit of protection.

Interval	Estimation Point Year	Price Per Acre Per Year 1975 $
0	1975	0.5071
1	1980	0.4281
2	1985	0.3491
3	1990	0.2701
4	1995	0.1911
5	2000	0.1121
5+		0.1121

For this demonstration, the discount rate is based upon the real rate of return achieved by marginal shares of common stock for a portfolio of forest products firms. This source of information can be criticized for several reasons. Most of these criticisms are due to various imperfections in security markets. This analysis accepts these criticisms but argues that real rates of return, achieved by marginal shares of common stock, do provide an unbiased estimate of the price individuals are willing to pay for intertemporal consumption claims of varying risk. The argument rests on the assumption that these markets in the United States are sufficiently competitive due to the large number of individual participants, the large number of shares traded, the ease of entry and exit to the market, and the inability of individuals or organizations to influence prices over long periods of time.

The discount rate for this analysis is based on the real rate of return earned by marginal shares of common stock for a group of 9 large forest product firms which were traded on the New York Stock Exchange between 1955 and 1975. All of the firms in this portfolio had the additional characteristic of owning, and managing for timber production substantial areas of forest land.

Examination of this sample indicated that there is no justification for an intertemporal price that varies through time. Therefore, a single price of 8.4 percent per year is estimated as the appropriate intertemporal price for investments in stand treatments for timber production in this analysis. This price is already after tax so no income tax adjustment was necessary. The discount factors used in the analysis are given in Table 5-10. See Appendix C for a more detailed description of the data and steps taken in the development of the estimate of the intertemporal price.

Neither of the two regeneration treatments considered in the analysis required any unique inputs or outputs. The price of harvested volume output for the natural regeneration treatment has already been discussed. The price of the time delay has also been discussed in the estimation of a discount rate.

The price of thinning treatments is based on the stumpage price of the volume removed by the treatment. This price is based on the average difference between the clearcut stumpage price and the thinning stumpage price for young growth Douglas-fir compiled by the Washington State Department of Revenue. The stumpage price of thinnings by one inch DBH classes was calculated following the same procedure used in the calculation of similar clearcut stumpage prices. The resulting prices for stumpage from thinnings are given in Table 5-11. These prices are applied to the volume removed in thinning following the same procedure used in the calculation of growing stock prices. The resulting weighted average price was applied to the total volume removed from the compartment in a thinning treatment. In application if the weighted average stumpage price of thinnings was a negative quantity the volume removed was noted as a negative quantity indicating an investment and the price noted as a positive value.

Table 5-10

Estimated discount factor for investments in the risk class of timber production by interval.

Interval	Estimation Point Year	Discount Factor $(1+r)^n$
0	1975	1.0
1	1980	1.49674
2	1985	2.24023
3	1990	3.5304
4	1995	5.01864
5	2000	7.51159
6	2005	11.2429
7	2010	16.8277
8	2015	25.1867
9	2020	37.6979
10	2025	56.4240
11	2030	84.4521

$r = 0.084$

Table 5-11

Estimated stumpage price of Douglas-fir thinnings per cubic foot total volume by interval and DBH class.

DBH	Interval						
Class	0	1	2	3	4	5	5+
Inches		1975 Dollars per Cubic Foot					
2	-0.0522	-0.0605	-0.9581	-0.9568	-0.0552	-0.0542	-0.0542
3	-0.0460	-0.0570	-0.0542	-0.0525	-0.0505	-0.0490	-0.0490
4	-0.0390	-0.0530	-0.0490	-0.0475	-0.0455	-0.0435	-0.0435
5	-0.0310	-0.0490	-0.0450	-0.0425	-0.0395	-0.0370	-0.0370
6	-0.0225	-0.0445	-0.0390	-0.0370	-0.0335	-0.0301	-0.0301
7	-0.0125	-0.0395	-0.0342	-0.0305	-0.0265	-0.0225	-0.0225
8	-0.0015	-0.0340	-0.0275	-0.0235	-0.0190	-0.0140	-0.0140
9	0.0120	-0.0280	-0.0210	-0.0160	-0.0100	-0.0045	-0.0045
10	0.0250	-0.0215	-0.0130	-0.0070	-0.0001	0.0060	0.0060
11	0.0420	-0.0145	-0.0055	0.0020	0.0100	0.0175	0.0175
12	0.0590	-0.0070	0.0040	0.0120	0.0220	0.0310	0.0310
13	0.0800	0.0020	0.0130	0.0240	0.0320	0.0450	0.0450
14	0.1040	0.0110	0.0240	0.0350	0.0475	0.0625	0.0625
15	0.1290	0.0210	0.0350	0.0490	0.0640	0.0810	0.0810
16	0.1580	0.0320	0.0475	0.0640	0.0810	0.1000	0.1000
17	0.1920	0.0430	0.0625	0.0800	0.1000	0.1240	0.1240
18	0.2280	0.0560	0.0775	0.0975	0.1210	0.1475	0.1475
19	0.2700	0.0740	0.0940	0.1175	0.1450	0.1760	0.1760
20	0.3180	0.0850	0.1150	0.1400	0.1710	0.2075	0.2075
21	0.3725	0.1010	0.1325	0.1650	0.2000	0.2410	0.2410
22	0.4325	0.1190	0.1550	0.1900	0.2325	0.2800	0.2800
23	0.5000	0.1390	0.1780	0.2180	0.2675	0.3225	0.3225
24	0.5750	0.1610	0.2050	0.2500	0.3100	0.3700	0.3700
25	0.6650	0.1850	0.2350	0.2850	0.3500	0.4250	0.4250
26	0.7669	0.2100	0.2650	0.3760	0.3950	0.4752	0.4752
27	0.8750	0.2380	0.3000	0.3650	0.4500	0.5450	0.5450
28	1.000	0.2690	0.3375	0.4100	0.5050	0.6200	0.6200
29	1.1400	0.3010	0.3780	0.4650	0.5700	0.6990	0.6990
30	1.3000	0.3390	0.4250	0.5200	0.6450	0.7900	0.7900

This unique feature of thinning treatments that they can be either exchanged with a positive effect on potential current consumption, or investments with a negative effect on potential current consumption, complicates the calculation of their real price. United States Federal Income tax regulations allow the deduction of thinning costs from ordinary income. However, the gain from revenue producing thinnings is allowed to be treated as capital gains for tax purposes. As a result of this tax treatment both adjustment factors are applied in the calculation of the real price for thinning treatment. If the thinning treatment is an investment, its price is adjusted using the ordinary income adjustment factor of 0.5. If the thinning treatment produces net revenue, as indicated by a positive quantity, the capital gains adjustment factor of 0.75 is applied.

The price of fertilizer treatments is based on historical per acre fertilization costs compiled by the Industrial Forestry Association (1975)(Appendix Table C-18). The price of fertilizer treatments used in the demonstration analysis is estimated as a single price of 0.1796 1975 dollars per pound of elemental nitrogen applied.

A more detailed discussion of the sources of information, and steps taken in the development of estimates of both the quantity and price of inputs and outputs is given in Appendix C.

Analysis

The comparison of treatment combinations, considered for application in the initial interval of the analysis, requires the screening of 26 possible treatment combinations. The expected discounted certainty-equivalent value of attained wealth for each treatment combination is calculated, according to Equation 5-2, and the combination with the greatest attained wealth is selected for application during the initial interval. The optimum combination of treatments for subsequent intervals in the analysis would be defined by repeating these steps for each interval.

Under the conditions of this analysis the optimum combination of treatments for the initial interval are the removal of 40% of the basal area in a low thinning and the application of 400 pounds of elemental nitrogen per acre in the form of urea.

A summary of the analysis for the initial interval is given in Table 5-12. This table illustrates the information required by and resulting from this analysis procedure. The summary for the total analysis would consist of a series of entries similar to those for interval 0 for each interval in the analysis period. The expected Present Certainty-Equivalent Value of wealth gain for the entire sequence of treatments would be given by the expected Present Certainty-Equivalent Value of wealth gain for interval 11.

In application the procedures illustrated in this analysis would be repeated for a given compartment as often as required by the planning activity in timber management. The major determinant of the frequency at which these procedures would be repeated is the availability of new information for an analysis.

Table 5-12

Optimum treatment sequence for compartment
(Secs. 13-15, 21-23; Twp. 16N; Rge 1W, Thurston County, Washington)

Interval 0 - 1975

Initial stand summary statistics per acre

	Stand	
	1	2
Area (Acres)	2216	63
Site Index	120	110
Growing Stock Index	220	220
Age (Years)	25	25
Trees (Number)	228	284
Ave. DBH (Inches)	7.0	6.0
Basal Area (Square Feet)	60.2	55.9
Total Volume (Cubic Feet)	1239	1036

Initial compartment summary statistics

	Quantity	Price Before Tax	Price Real	Value
		1975 Dollars		
Land (Acres	2279	77.00	57.75	131,612.25
Growing Stock (cu.ft.)	2,810,892	0.05908	0.04431	124,556.13
Initial Endowment (W^Q/C_0)				256,168.38

Table 5-12 (cont.)

Treatments:

Thinning: 40% Basal Area Removal
Fertilization: 400 Pounds of Nitrogen Per Acre

Post treatment stand summary statistics per acre

	Stand	
	1	2
Area (Acres)	2216	63
Site Index	120	110
Growing Stock Index	220	220
Age (Years)	25	25
Trees (Number)	52	46
Ave. DBH (Inches)	11.2	11.3
Basal Area (Square feet)	35.8	31.9
Total Volume (Cubic feet)	832	699

Post treatment compartment summary statistics

		Price		
	Quantity	Before Tax	Real	Value
		1975 Dollars		
Land (Acres)	-2,279	77.00	57.75	-131,612.25
Growing Stock (Cubic feet)	-1,887,749	0.08667	0.06500	-122,712.37
Property Tax (Ann. Ac. Units)	-5,697.5	1.155	0.5775	-3,290.31
Overhead				
Managers	-0.3605	36,007.37	18,003.68	-6,490.33
Protection	-5,697.5	0.5071	0.2536	-1,444.60
Fertilization (Pounds of N)	-911,600	0.1796	0.0898	-81,861.68
Thinning (Cubic feet)	-923,143	0.02013	0.01006	-9,290.17
Expected Investment (EIW^Q/cm)				-356,701.71

Table 5-12 (cont.)

Interval 1 - 1980

Initial stand summary statistics per acre

	Stand 1	Stand 2
Area (Acres)	2216	63
Site Index	120	110
Growing Stock Index	220	220
Age (Years)	30	30
Trees (Number)	52	46
Average DBH (Inches)	17.2	18.1
Basal Area (Sq. ft.)	83.6	81.8
Total Volume (Cu. ft.)	2007	1732

Initial compartment summary statistics

	Quantity	Price: Before Tax	Price: Real	Value
		1975 Dollars		
Land (Acres)	2,279	81.33	61.00	139,013.30
Growing Stock (Cu. ft)	4,556,628	0.1190	0.08926	406,726.43
Expected Ensemble Attained Wealth (EW^Q/C_1)				545,739.73
Expected Present Certainty Equivalent Value of Wealth Gain (EW^{Q*}/C_0)				7,917.22
Maximum Expected Present Certainty - Equivalent Value of Wealth Gain (Interval 5)				383,401.25

.
.
.

Interval 10 - 2030

.
.
.

Conclusions from Demonstration Analysis

The objectives of this analysis are to demonstrate a procedure for the analysis of stand treatments consistent with relevant economic principles, and to illustrate the standards proposed for analyses of this type. The analysis demonstrates that refined procedures which rely on a more consistent application of principles from economic theory are feasible. Further, that the information requirements and standards are attainable by any organization or individual considering timber production investments involving silvicultural treatments.

Although the standards proposed and information required by the procedures, illustrated in this analysis, are attainable, the analysis also demonstrates the importance of accuracy in the estimation of the quantity and price of inputs and outputs associated with treatment combinations under consideration. The importance of accurately forecasting the volume and price of growing stock is illustrated, in the example, by the size of these two parameters and the resulting estimate of value. In the demonstration analysis these two parameters account for 75% of the expected ensemble attained wealth in interval 1. The discount rate must also be estimated accurately because of its importance in determining the optimum growth path for stands of timber.

The investment of 356,701.71, 1975 dollars, in the example also indicates the magnitude of the opportunity to achieve an improved allocation of resources by the systematic evaluation of alternatives at operational levels.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR ADDITIONAL STUDY

A useful procedure for the economic analysis of stand treatments must provide a logically consistent method of combining the elements of information relevant to the selection of a preferred ensemble of treatments in a way that contributes to this selection. The logical consistency and identification of relevant information is ensured in the procedures proposed in this thesis by the adoption of economic principles from the theory of investment choice.

The review of the theory of investment choice led to the conclusion that the appropriate class of criteria for the economic analysis of stand treatments are Present Certainty-Equivalent Value. These criteria are calculated from the wealth sequence of investment ensembles and estimate the value of an investment ensemble in units of potential certain consumption in the current period. The application of these criteria to problems of investment choice in timber production requires forecasts of the expected yield and input series for each investment ensemble with its real prices.

A review of the published work concerned with the economic analysis of stand treatments led to the conclusion that the previous studies are not adequate for the current needs of timber management because: (1) They fail to identify and estimate investment choice criteria which are consistent with the economic principles of investment choice; (2) they fail to estimate accurately the prices appropriate to the analysis of timber production investments; (3) they do not utilize yield forecasting techniques with sufficient flexibility to allow the definition of an optimum investment path.

The procedures proposed in this thesis require the calculation of the expected Present Certainty-Equivalent Value of wealth gain in a manner consistent with the economic principles of investment choice. The rule, which calls for the adoption of the investment ensemble that maximizes this criterion, is also consistent with the economic principles of investment choice.

A demonstration of the analysis procedures, for an example of Douglas-fir in the Pacific Northwest of the United States, illustrates the sequence of steps to be taken in the analysis of stand treatments. The iterative sequence of steps which are demonstrated allow the procedure to be readily adapted to computer processing. In addition, the magnitude of the investment sequence, in the demonstration analysis, illustrates the opportunity for increased efficiency in the allocation of resources for timber production.

The major shortcoming of the procedures proposed in this thesis is the general failure of expected Present Certainty-Equivalent Value criteria to define a unique optimum investment ensemble under imperfect markets.

The limitations of these criteria can be reduced by testing the opportunity dominance of investment ensembles and recognizing the irreducably subjective element of investment choice in imperfect markets. Additional study of the theory of exchange in imperfect markets and the role of information in uncertain choice is needed to overcome the weakness of the theoretical foundation for the procedures proposed.

Regardless of the limitations due to theoretical principles major opportunities exist for additional research to define productive opportunities and the treatment sequences required for their achievement. Major improvements are also needed in forecasting the real prices which are needed to define optimum treatment sequences.

In conclusion, the procedures proposed in this thesis are a feasible and practical means of organizing some of the information relevant to the selection of preferred sequences of stand treatments. Improvements to the theoretical foundations and required information are desirable but not necessary for immediate application in the Douglas-fir Region of the United States. The use of the procedures proposed in this study will provide information that will assist in an improved allocation of resources in timber production through the application of the economic principles of investment choice.

LITERATURE CITED

Aarestrup, Jorgen. 1969. Average Rate of Interest on Sustained Yield Forestry Based on Historical Danish Material. Forestry. 42(1):83-92.

Alchian, Armen A. 1953. The Meaning of Utility Measurement. The American Economic Review. 43(1):26-50. ALSO IN: Readings in Micro economics 2nd. Ed. W. Breit & H. M. Hochman (Ed.). Holt, Rinehart and Winston Inc. New York, N.Y. 1968. Chapt. 4. pp. 57-76.

Amidon, Elliott L. and Akin, Garth S. 1968. Dynamic Programming to Determine Optimum Levels of Growing Stock. Forest Science. 14(3):287-291.

Bank and Quotation Record. 1956 to 1965. Volumes 29 to 38.

Bank and Quotation Record. 1975. 48(9).

Baumol, William J. 1972. Economic Theory and Operations Analysis, 3rd Ed. Prentice-Hall International, Inc. London, England. 626 pp.

Becker, Gary S. 1962. Irrational Behavior and Economic Theory. The Jour. of Political Economy. 70(1):1-13.

Bentley, William R. and Tecguarden, Dennis E. 1965. Financial Maturity: A Theoretical Review. Forest Science. 11(1):76-87.

Bierman, Harold Jr., and Smidt, Seymour. 1966. The Capital Budgeting Decision, 2nd. Ed. The Macmillan Co. New York, N.Y. 420 pp.

Bond, W. E. 1940. Dollars-and-Cents Control in Forest Management. Southern Lumberman. 161(2033):193-196.

Bond, W. E., Wahlenberg, W. G., and Kirkland, Bert P. 1937. Profitable Management of Shortleaf and Loblolly Pine for Sustained Yield. U.S.D.A. Forest Service. Southern Forest Experiment Station. Occasional Paper No. 70. 37 pp.

Boulding, K. E. 1966. Economic Analysis, 4th Ed. Vol. I. Harper & Row. New York, N.Y. 720 pp.

Bower, David R. and Shaw, Dale L. 1975. Estimation of Yield for Thinned and Fertilized Natural Stands of Douglas-fir. Weyerhaeuser Company Forestry Research Center, Centralia, WA. 98531. (Unpublished Report) 12 pp.

Bull, Henry. 1934. Profit from Improving a Second-Growth Forest of Loblolly and Shortleaf Pines and Hardwoods. U.S.D.A. Forest Service. Southern Forest Experiment Station. Occasional Paper No. 38. 8 pp.

Buongiorno, Joseph and Tecguarden, Dennis E. 1973. An Economic Model for Selecting Douglas-fir Reforestation Projects. Hilgardia. 42(3):35-120.

Callahan, John C. and Smith, Robert P. 1974. An Economic Analysis of Black Walnut Plantation Enterprises. Purdue Univ. Agricultural Experiment Station. Research Bulletin No. 912. 20 pp.

Carron, L. T. 1971. Analytical Aid for Management. IN: Pinus Radiate: Proceedings of a Symposium Held at the Australian National University, August 1970, Volume II. The Australian National University. Canberra, Australia. pp. 7A.1 - 7A.14.

Chapman, Gordon L. and Baker, Gregory. 1954. Planned Thinning in a Forest Stand Can Pay A Double Profit. Maine Farm Research. 2(3):6-8.

Coase, R. H. 1937. The Nature of the Firm. Economica. 4(16):386-405. ALSO IN: American Economic Association Readings in Price Theory. Richard D. Irwin. Homewood, Ill. 1952. pp. 331-351.

Committee on Forest Terminology. 1950. Forest Terminology: A Glossary of Technical Terms Used in Forestry. Society of American Foresters. Washington, D. C. 93 pp.

Cone, Bruce W. 1972. Economic Feasibility of an Integrated CoHonwood Plantation Utilizing A Nuclear Power Reactor. Journal of Forestry. 70(10):621-623.

Crowe, N. D. 1967. Growth, Yield and Economics of Pinus Patula in the Natal Midlands. Annale Universiteit Van Stellenbosch. 42A(2):71-152.

Dasgupta, Ajit K., and Pearce, D. W. 1972. Cost Benefit Analysis: Theory and Practice. The Macmillan Press Ltd. London, England. 270 pp.

Davis, Kenneth P. 1965. A Structural Analysis of Land, Income, and Cost Values in Timber - Production. Journal of Forestry. 63(6): 446-451.

Davis, Kenneth P. 1966. Forest Management Regulation and Valuation, 2nd Ed. McGraw-Hill Book Company. New York, N.Y. 519 pp.

Dowdle, Barney. 1962. Investment Theory and Forest Management Planning. Yale University. School of Forestry Bulletin. No. 67. 63 pp.

Duerr, William A. 1960. Fundamentals of Forestry Economics. McGraw Hill Book Co., Inc., New York, N.Y. 579 pp.

Duerr, William A. and Bond, W. E. 1952. Optimum Stocking of A Selection Forest. Journal of Forestry. 50(1):12-60.

Evans, T. C. 1965. Professional Income of Foresters - 1964. Journal of For. 63(9):681-686.

Eyre, F. H. 1960. Professional Income of Foresters - 1959. Jour. of For. 58(12):952-956.

Faustmann, Martin. 1849. Berechnung des Werthes, welchen Waldhoden sowic noch nicht haubarc Holzbestande fur die Waldwirtshaft besitzen. Allgemeine Forst-und Jagd - Zeitung 25:441-455.

Fenton, R. and Dick, M. Merle. 1972a. Profitability of Radiata Pine Afforestation For The Export Log Trade - On Site Index 80. New Zealand Journal of Forestry Science. 2(1):69-99.

Fenton, R. and Dick, M. Merle. 1972b. Profitability of Radiata Pine Afforestation for the Export Log Trade - On Site Index 110. New Zealand Journal of Forestry Science. 2(1):100-127.

Fenton, R. and Dick, M. Merle. 1972c. Profitability of "Normal" Afforestation for the Overseas Log Trade on Site Indexes 95-110. New Zealand Journal of Forestry Science. 2(3):298-312.

Fenton, R. and Tustin, J. R. 1972. Profitability of Radiata Pine Afforestation for the Export Log Trade - On Site Index 95. New Zealand Journal of Forestry Science 2(1):7-68.

Fisher, Irving. 1907. The Rate of Interest. Macmillan Co. New York, N.Y. 442 pp.

Fisher, Irving. 1923. The Nature of Capital and Income. The Macmillan Co. New York, N.Y. 425 pp.

Fisher, Irving. 1930. The Theory of Interest. The Macmillan Co. New York, N.Y. (Reprinted: 1965 by Augustus M. Kelly, Bookseller. New York, N.Y.) 566 pp.

Flewelling, Jim. 1975. Fertilizer Response Eqn. for Dave Lewis. Weyerhaeuser Co. Forestry Research Center. Centralia, WA. 98531. Inter-office Communication. 2 pp.

Flora, Donald F. 1964. Uncertainty in Forest Investment Decisions. Journal of Forestry 62(6):376-380.

Flora, Donald F. 1966a. Economic Guides for Ponderosa Pine Dwarf Mistletoe Control in Young Stands of the Pacific Northwest. Pacific Northwest Forest and Range Experiment Station. U.S. Forest Service Research Paper PNW-29. 16 pp.

Flora, Donald F. 1966b. Economic Guides for a Method of Precommercial Thinning of Ponderosa Pine in the Northwest. Pacific Northwest Forest and Range Experiment Station. U.S. Forest Service Research Paper PNW-31. 10 pp.

Flora, Donald F. 1970. Economics and Policy Environments for Forest Regeneration. IN: Regeneration of Ponderosa Pine, Symposium Held September 11-12, 1969. R. K. Hermann, Ed. Oregon State University. School of Forestry. 1970. pp. 62-63.

Friedman, Milton. 1953. The Methodology of Positive Economics. IN: Essays in Positive Economics by M. Friedman. The Univ. of Chicago Press. Chicago, Ill. pp. 3-43.

Friedman, Milton. 1962. Price Theory: A Provisional Text. Aldine Publishing Co. Chicago, Ill. 285 pp.

Fries, Joran (Ed.). 1974. Growth Models for Tree and Stand Simulation. Royal College of Forestry. Stockholm, Sweden. Department of Forest Yield Research. Research Notes. Nr. 30. 379 pp.

Gaffney, M. Mason. 1960. Concepts of Financial Maturity of Timber and Other Assets. North Carolina State College Dept. of Agric. Econ. A. E. Information Series No. 62. 105 pp.

Gasner, David A. and Larsen, David N. 1969. Pitfalls of Using Internal Rate of Return to Rank Investments in Forestry. Northeast Forest Experiment Station. U.S. Forest Service Research Note NE-106. 5 pp.

Gentle, S. W., Bamber, R. K. and Humphreys, F. R. 1968. Effect of Two Fertilizers of Financial Yield and Wood Quality of Radiata Pine. Forest Science 14(3):282-286.

Gessel, S. P., Stoate, T. N., and Turnbull, K. J. 1969. The Growth Behavior of Douglas-fir with Nitrogenous Fertilizer in Western Washington Univ. of Washington. Contributions from the Forest Products Institute. No. 7. 119 pp.

Goebel, N. B., Warner, J. R. and Van Lear, D. H. 1974. Periodic Thinnings in Loblolly Pine Stands: Growth, Yield, and Economic Analysis. Clemson Univ. Coll. Forestry and Recreation Resources. Dept. of Forestry. Forest Research Series No. 28. 11 pp.

Grant, Eugene L. and Ireson, W. Grant. 1970. Principles of Engineering Economy 5th Ed. The Ronald Press. New York, N.Y. 640 pp.

Grayson, A. J. 1962. Criteria of Profitability in Relation to Volume Production. Forestry Comm. London, England. Eighth British Commonwealth Forestry Conf. 9 pp.

Green, H. A. John. 1971. Consumer Theory. Penguin Books Inc. Baltimore, Md. 344 pp.

Grut, M. 1964. Comparisons of Various Rotations and Thinning Programms for Pinus Patula on Site Index 75. Forestry in South Africa. No. 4 pp. 67-77.

Grut, Mikael. 1967. Most Profitable Silvicultural Programme for Pinus Patula and Cost of Improving Timber by Adapting Other Programmes. Forestry in South Africa. No. 8. pp. 95-115.

Grut, Mikael. 1973. Methods of Estimating the Most Profitable Thinning Programme. Forestry in South Africa. No. 14. pp. 25-29.

Guttenberg, Sam. 1950. The Rate of Interest in Forest Management. Journal of Forestry. 48(1):3-7.

Hadley, G. 1961. Linear Algebra. Addison-Wesley Publishing Co., Inc. Reading, Mass. 290 pp.

Haley, David. 1966. The Importance of Land Opportunity Cost in the Determination of Financial Rotations. Journal of Forestry. 64(5):326-329.

Hamalainen, Jouko. 1973. Profitability. Comparisons in Timber Growing: Underlying Models and Empirical Applications. Communications Instituti Forestalis Fenniae. 77(4). 178 pp.

Hamilton, Thomas E. 1965. Production Prices, Employment and Trade in Pacific Northwest Forest Industries - 4th Quarter 1964. U.S.D.A. Forest Service. Pacific Northwest Forest and Range Experiment Station. 18 pp.

Harlow, C. M. 1939. Financial Rotation Versus Rotation of Highest Income. Indian Forester. 65(8):475-481.

Heiberg, Svend O. 1942. Cutting Based Upon Economic Increment. Journal of Forestry. 40(8):645-650.

Henderson, James M. and Quandt, Richard E. 1971. Microeconomic Theory A Mathematical Approach 2nd Ed. McGraw-Hill Book Co. New York, N.Y. 431 pp.

Hetherington, J. C. 1969. An Economic Evaluation of Alternative Stand Treatments in Relation to the Development of Understory Vegetation and Subsequent Regeneration Costs. Forestry 42(1):47-68.

Hiley, W. E. 1954. Woodland Management. Faber and Faber Ltd. London, England. 463 pp.

Hiley, W. E. 1956. Economics of Plantations. Faber and Faber Ltd. London, England. 216 pp.

Hirshleifer, Jack. 1958. On the Theory of the Optimal Investment Decision. The Journal of Political Economy. 66(4):329-352.

Hirshleifer, Jack. 1970. Investment, Interest and Capital. Prentice-Hall Inc. Englewood Cliffs, N.J. 320 pp.

Hosner, John F. and Lane, Richard D. 1953. Making Farm Woodland Improvement Pay. U.S.D.A. Forest Service. Central States Forest Experiment Station. Technical Paper No. 133. 12 pp.

Huey, Ben M. 1950. The Profit in Pruning Western White Pine and Ponderosa Pine. U.S.D.A. Forest Service. Northern Rocky Mountain Forest and Range Experiment Station. Research Note No. 85. 6 pp.

Hughes, Jay M. and Post, Boyd W. 1973. Economic Considerations in Forest Fertilization. IN: Forest Fertilization Symposium Proceedings. Northeast Forest Experiment Station. U.S. Forest Service General Technical Report NE-3. pp. 45-54.

Husch, Bertram, Miller, Charles I., and Beers, Thomas W. 1972. Forest Mensuration 2nd Ed. The Ronald Press Co. New York, N.Y. 410 pp.

Industrial Forestry Association. 1975. Summary of Industrial Tree Farm Performance 1949-1974. Industrial Forestry Assoc. 1220 S.W. Columbia, Portland, OR. 3 pp.

Johnston, D. R., Grayson, A. J. and Bradley, R. T. 1967. Forest Planning. Faber and Faber Ltd. London, England. 541 pp.

Keipi, Kari and Kekkonen, Oho. 1970. Calculations Concerning the Profitability of Forest Fertilization. Folia Forestalia. No. 84. 23 pp.

Kilkki, Pekka. 1968. Some Economic Aspects of Growing Forest Stands. Silva Fennica. 2(4):225-234.

Kilkki, Pekka, and Vaisanen, Unto. 1969. Determination of the Optimum Cutting Policy for the Forest Stand by Means of Dynamic Programming. Acta Forestalia Fennica. No. 102. 22 pp.

King, James E. 1964. Stand Diameter Distribution Tables for Douglas-fir Weyerhaeuser Co. Forestry Research Center. Centralia, WA. 98531 (Unpublished Report).

King, James E. 1966. Site Index Curves for Douglas-fir in the Pacific Northwest. Weyerhaeuser Co. Forestry Research Center. Centralia, WA. 98531. Forestry Paper No. 8. 49 pp.

King, James E. 1970. Principles of Growing Stock Classification for Even-Aged Stands and An Application to Natural Douglas-fir Forests. Univ. of Washington. Ph.D. Thesis. 91 pp.

King, James E., and Wiley, Kenneth N. 1963. (Reprint, 1973) Net and Gross Yield for Natural Stands of Douglas-fir. Weyerhaeuser Co. Forestry Research Center. Centralia, WA. 98531. (Proprietary) 50 pp.

Lembersley, Mark R. and Johnson, K. Norman. 1975. Optimal Policies for Managed Stands: An Infinite Horizon Markov Decision Process Approach. Forest Science. 21(2):109-122.

Leftwich, Richard H. 1966. The Price System and Resource Allocation, 3rd Ed. Holt, Rinehart, and Winston. New York, N.Y. 369 pp.

Lerche, Cai and Saeed, Akhtar. 1967. A Study on the Rotation and Economic Management of the Coniferous Forests in West Pakistan. Journal of Forestry. 17(1):81-118.

Lewis, Gordon D. and Chappelli, Daniel E. 1964. Farm Woodland Management Costs and Returns in the Southern Piedmont of Virginia. Southeast Forest Experiment Station. U.S. Forest Service Research Paper SE-15. 20 pp.

Lewis, N. B. 1965. Economics of Pruning for Long Timber in South Australia. Australian Forestry. 29(3):149-160.

Linnard, W. and Gane, M. 1968. Martin Faustmann and the Evolution of Discounted Cash Flow. Commonwealth Forestry Institute, Univ. of Oxford. Institute Paper No. 42. 55 pp.

Lizardo, Leonor. 1952. Benguet Pine (*Pinus insularis* Endl.) as a Reforestation Crop. The Philippine Journal of Forestry. 7(1st to 4th Quarter 1950):43-62.

Lundgren, Allen L. 1971. Ranking Investment Alternatives - A New Look. Journal of Forestry. 69(9):568-573.

Lundgren, Allen L. 1972. A Correct Expression for Continuous Discounting. Forest Science. 18(1):95.

Lundgren, Allen L. 1973a. Cost-Price A Useful Way to Evaluate Timber Growing Alternatives. North Central Forest Experiment Station. U.S. Forest Service Research Paper NC-95. 16 pp.

Lundgren, Allen L. 1973b. The Allowable Cut Effect: Some Further Extensions. Journal of Forestry. 71(6):357, 360.

Lutz, Friedrich, and Lutz, Vera. 1951. The Theory of Investment of the Firm. Princeton Univ. Press. Princeton, N.J. 253 pp.

Lyford, C. A. 1934. The Application of Economic Selection to Logging Operations in the Douglas-fir Region. Journal of Forestry. 32(7):716-724.

Malac, Barry F. 1966. Twenty-Year Old Slash Pine Plantation Responds to Fertilization. Union Bag-Camp Paper Corp. Woodland Research Note No. 15. 4 pp.

Manogaran, Chelvadurai. 1973. Economic Feasibility of Irrigating Southern Pines. Water Resources Research. 9(6):1485-1496.

Manthy, Robert S. 1970. An Investment Guide to Cooperative Forest Management in Pennsylvania. Northeast Forest Experiment Station. U.S. Forest Service Research Paper NE-156. 59 pp.

Marshall, Alfred. 1920. Principles of Economics. 8th Ed. Macmillan and Company Ltd., London, England. 731 pp.

Marty, Robert. 1964. Analyzing Uncertain Timber Investments. Northeast Forest Experiment Station. U.S. Forest Service Research Paper NE-23. 21 pp.

Marty, Robert. 1970. The Composit Internal Rate of Return. Forest Science 16(3):276-279.

Marty, Robert J. and Allison, Glenn R. 1960. Appraising White Pine Weevil Control Opportunities. Journal of Forestry. 58(3):203-206.

Marty, Robert, Rindt, Charles, and Fedkiw, John. 1966. A Guide for Evaluating Reforestation and Stand Improvement Projects in Timber Management Planning on the National Forests. U.S.D.A. Agriculture Handbook No. 304. 24 pp.

McCauley, Orris D. and Trimble, George R. Jr. 1972. Forestry Returns Evaluated for Uneven-Aged Management in Two Appalachian Woodlots. Northeast Forest Experiment Station. U.S. Forest Service Research Paper NE-244. 12 pp.

Moody's Investors Service, Inc. 1975. Moody's Industrial Manual 1975. Vol. I and II. Moody's Investors Service Inc. New York, N.Y. 3950 pp.

Morais, Roger. 1954. Research Project No. 14. Thinning in Balsam Fir Stands 40 to 50 Years of Age. Causapscal Forest Research Station. Research Note No. 4. 2 pp.

Nasland, Bertil. 1969. Optimal Rotation and Thinning. Forest Science 15(4):446-451.

Osmond Smith, F. A. 1908. The Conversion of Underwood and Coppice-With-Standards Into Highwood. Quarterly Journal of Forestry. 2(3):154-165.

Pearse, P. H. 1967. The Optimum Forest Rotation. The Forestry Chronicle. 43(2):178-195.

Petrini, Sven (Translated by M. L. Anderson). 1953. Elements of Forest Economics. Oliver and Boyd. Edinburgh, Scotland. 210 pp.

Reynolds, R. R. 1939. Improvement Cuttings in Shortleaf and Loblolly Pine. U.S.D.A. Forest Service. Southern Forest Experiment Station. Occasional Paper No. 81. 4 pp.

Reynolds, R. R. 1939. Improvement Cuttings in Shortleaf and Loblolly Pine. Journal of Forestry. 37(7):568-570.

Reynolds, R. R., Bond, W. E. and Kirkland, Burt P. 1944. Financial Aspects of Selective Cutting in the Management of Second-Growth Pine-Hardwood Forests West of the Mississippi River. U.S.D.A. Technical Bulletin No. 861. 118 pp.

Ruderman, Florence K. 1975. Production, Prices, Employment and Trade in Northwest Forest Industries - 2nd Quarter 1975. U.S.D.A. Forest Service. Pacific Northwest Forest and Range Experiment Station. 56 pp.

Samuelson, Paul A. 1964. Economics - An Introductory Analysis. 6th Ed. McGraw-Hill Book Co. New York, N.Y. 838 pp.

Sassaman, Robert W., Barrett, James W. and Smith, Justin G. 1973. Economics of Thinning Stagnated Ponderosa Pine Sapling Stands in the Pine Grass Areas of Central Washington. Pacific Northwest Forest and Range Experiment Station. U.S. Forest Service Research Paper PNW-144. (Revised) 17 pp.

Schreuder, Gerard F. 1971. The Simultaneous Determination of Optimal Thinning Schedule and Rotation for an Even-Aged Forest. Forest Science. 17(3):333-339.

Schweitzer, Dennis L. 1968. Evaluating Timber Production Investments Under Uncertainty. University of Minnesota. Ph.D. Thesis. 101 pp. (Abstract in Dissertation; Abstracts 29B(8):2669-2670. Feb. 1969).

Schweitzer, Dennis L., Sassaman, Robert W., and Schallan, Con H. 1972. Allowable Cut Effect: Some Physical and Economic Implications. Journal of Forestry. 70(7):415-418.

Schweitzer, Dennis L., Sassaman, Robert W., and Schallau, Con H. 1973. The Allowable Cut Effect: A Reply. Journal of Forestry. 71(4):227.

Scitovsky, T. 1951. Welfare and Competition. Richard D. Irwin Co., Chicago, Ill.

Shepard, Ward. 1925. The Bogey of Compound Interest. Journal of Forestry 23(3):251-259.

Simmons, Donald W. 1974. The Economics of Late Rotation Fertilization of Loblolly Pine Pulpwood Stands. North Carolina State Univ. School of Forest Resources. Technical Report No. 51. 44 pp.

Smith, H. Donnell. 1973. Economics of Hardwood Plantations. IN: Proceedings: Twelfth Southern Forest Tree Improvement Conference Baton Rouge, Louisiana, June 12-13, 1973. Louisana State Univ. Div. of Continuing Education. pp. 158-168.

Soloman, Ezra. 1959. The Arithmetic of Capital-Budgeting Decisions. IN: The Management of Corporate Capital. E. Soloman, Ed. The Free Press of Glenco. Glenco, Ill. pp. 74-79.

Steinbrenner, E. C. and Gehrke, F. E. 1964. Soil Survey of the Vail Tree Farm. Weyerhaeuser Co. Forestry Research Center. Centralia, WA. 98531

Stern, R. C. 1970. Rotation Lengths for Conifers. Quarterly Journal of Forestry. 64(4):297-302.

Stern, R. C. 1971. Pruning of Free Grown Hardwoods. Quarterly Journal of Forestry. 65(4):322-326.

Stern, R. C. 1972. Poplar Growing at Close Spacing. Quarterly Journal of Forestry. 66(3):230-235.

Sutton, W.R.J. 1968. Initial Spacing and Financial Return of Pinus Radiata on Coastal Sands. New Zealand Journal of Forestry. 13(2):203-219.

Tecguarden, Dennis E. 1969. Economic Criteria and Calculated Risk in Reforestation Investment Decisions. Journal of Forestry. 67(1):25-31.

Tecguarden, Dennis E. 1973. The Allowable Cut Effect: A Comment. Journal of Forestry 71(4):224-226.

Thomas, A. G. 1965. The Comparative Profitability of Larch and Beech in the Cotswolds. Quarterly Journal of Forestry. 59(1):56-60.

Thompson, Emmett F. 1968. The Theory of Decision Under Uncertainty and Possible Applications in Forest Management. Forest Science. 14(2):156-163.

Thompson, Emmett F., Sullivan, Alfred D., and Theoe, Donald R. 1974 Income and Employment of SAF Members - 1972. Journal of Forestry. 72(2):82-86.

Thompson, Emmett F., Sullivan, Alfred D., and Theoe, Donald R. 1975. Income and Employment of SAF Members - 1974. Journal of Forestry. 73(9):590-594.

U.S. Department of Commerce. 1973. 1973 Business Statistics. U.S. Dept. of Comm. Social and Economic Statistics Administration. Bureau of Economic Analysis. 282 pp.

U.S. Department of Commerce. 1975. Survey of Current Business. 55(6).

U.S. Forest Service. 1963. Pacific Northwest Quarterly Stumpage and Log Supply Report - 3rd Quarter 1963. U.S.D.A. Forest Service. Pacific Northwest Forest and Range Experiment Station. Misc. Pub. 27 pp.

U.S. Forest Service. 1973. The Outlook for Timber in the United States. Forest Service Forest Resource Report No. 20. 367 pp.

Univ. of Washington, College of Forest Resources, 1975. Regional Forest Nutrition Research Project, Biennial Report 1972-74. Univ. of Washington, College of Forest Resources.

Van Horne, James C. 1968. Financial Management and Policy. Prentice-Hall, Inc. Englewood Cliffs, N.J. 583 pp.

Van Laar, A. and Tingle, A. C. 1965. Economic Aspects of Poplar Growing for Matchwood Production. South African Forestry Journal No. 53. pp. 14-24.

Walker, Nat. 1969. Economic and Management Models for Cottonwood in Central Oklahoma. Oklahoma State Univ. Agricultural Research Bulletin B-664. 18 pp.

Wambach, Robert F. 1967. A Silvicultural and Economic Appraisal of Initial Spacing in Red Pine. Univ. of Minnesota. Ph.D. Thesis. 294 pp. (Abstract in Dissertation Abstracts. 28B(6):2210. Nov. 1967).

Ward, W. W. 1958. A Mensurational and Economic Study of Thinning in the Carbaugh White Pine Plantation. Penn. State Univ. The Penn. State Forest School. Research Paper No. 26. 16 pp.

Washington Forest Protection Association. 1975. Washington Forest Tax Handbook. Washington Forest Protection Association. Building Suite 1220, 1411 Fourth Avenue, Seattle, WA. 98101, 33 pp.

Watt, A. J. 1967. A Comparison of Some Basic Concepts of Rotation Age. Australian Forestry. 31(4):275-286.

Webster, Steve R. 1975. Emperical Fertilizer Trials - Progress Report: Preliminary Results 1971-1974. Weyerhaeuser Co. Forestry Research Center, Centralia, WA. 98531. Forestry Research Technical Report - Forest Management. 12 pp.

Whiteley, D. 1971. The Relationship of Savings in Harvesting Pruned Trees to the Cost of Pruning Australian Forest Research. 5(2):51-57.

Wikstrom, J. H. and Alley, Jack R. 1968. Ranking Treatment Opportunities in Existing Timber Stands on White Pine Land in the Northern Region. Intermountain Forest and Range Experiment Station. U.S. Forest Service Research Paper INT-46. 75 pp.

Winch, D. M. 1971. Analytical Welfare Economics. Penguin Books, Ltd. Harmondsworth, Middlesex, England. 208 pp.

Worley, David P. and Wheeland, Hoyt A. 1968. An Economic Evaluation of Cull-Tree Removal in Mixed Hardwood Stands. Northeast Forest Experiment Station. U.S. Forest Service Research Note NE-82. 5pp.

Yoho, James G., Chappelle, Daniel E. and Schweitzer, Dennis L. 1969. The Economics of Converting Red Alder to Douglas-fir. Pacific Northwest Forest and Range Experiment Station. U.S. Forest Service Research Paper PNW-88. 31 pp.

Zillgitt, W. M. 1948. Optimum Economic Stocking for Northern Hardwoods. U.S.D.A. Forest Service. Lake States Forest Experiment Station. Station Paper No. 10. 14 pp.

A P P E N D I C E S

CONTENTS

CONTENTS (cont.)

APPENDIX A

MATHEMATICAL FORMULATION OF ECONOMIC PRINCIPLES

The purpose of this appendix is to supplement the discussion of economic theory in the body of the thesis with a mathematical presentation. This presentation is based on Hirshleifer (1970), and follows the order of presentation and terminology used in the main text.

Timeless Choice

The starting point for a presentation of the theory of timeless choise is the concept of individual preference. This concept has a number of properties which allow mathematical techniques to be used in the solution of allocation problems. These properties are based on a series of axioms of consumer preference which systematically describe the framework within which economic theory describes the attributes of the objects of choice and the natur eof comparisons made by consumers. The material for this discussion is abstracted from Chapters 1 and 2 of Green (1971).

The starting point for a formal statement of the axioms of consumer prefererence is the binary comparison of two commodity bundles or objects of choice. The nature of this comparison can be expressed by the statement:

> Bundle Y is regarded by the consumer as at least as good as bundle Y'.

In order to develop a symbolic way of stating the binary comparison, "is regarded by the consumer as at least as good as", let:

R = the binary relationship, "is regarded by the consumer as at least as good as".

The previous statement may then be given by the expression:

$$Y\ R\ Y'$$

This statement now provides the basic symbolism required to state the consumer preference axiom of completeness.

Axiom 1 (Completeness): For all Y, Y' in set S either Y R Y' or Y' R Y or both.

The implication of this axiom is that there are three possible states where Y R Y' and Y' R Y must be true or false. These states are summarized as:

Y R Y'	Y' R Y	
True	True	Written as Y I Y'
True	False	Written as Y P Y'
False	True	Written as Y' P Y

Using the definitions of R, P, and I given above the following corollaries of the axiom of completeness can be given:

Corollaries: For all Y, Y' in S:

(a) one and only one of Y P Y', Y' P Y, Y I Y' holds;

(b) Y I Y;

(c) if Y I Y' then Y' I Y.

The next axiom of consumer preference describes the transitivity of preferences:

Axiom 2 (Transitivity): For all Y, Y', Y" in S: if Y R Y' and Y' R Y" (written as Y R Y' R Y"), then Y R Y".

There follows from this axiom and the previous one on completeness a series of corollaries which are as follows:

Corollaries: For all Y, Y', Y", in S:

(d) if Y P Y' P Y", then Y P Y";

(e) if Y I Y' I Y", then Y I Y";

(f) if Y R Y' P Y", then Y P Y";

(g) if Y P Y' R Y", then Y P Y";

(h) if Y R Y' I Y", then Y R Y";

(i) if Y I Y' R Y", then Y R Y";

(j) if Y P Y' I Y", then Y P Y";

(k) if Y I Y' P Y", then Y P Y".

The combination of the axioms of completeness and transitivity lead to the axiom of rational choice which is basic to both positive and normative applications of economic theory. This axiom can be stated as:

Axiom 3 (Rational choice): if Y is chosen from a set of alternatives S, then for all Y' in S, Y R Y'.

It is important to note that if there are alternatives in a set which are indifferent to the alternative chosen this axiom does not explain how the selection was made within the indifference set.

In order to further develop the systematic description of consumer preferences and elaborate the mechanisms of choice within a subset of alternatives it is necessary to examine the elements of alternatives. The usual way of representing the elements of an alternative is to adopt the mathematical concept of vectors and allow each element of an alternative to be represented by an element of a vector. Applying this notation to the general alternatives Y and Y' would yield:

$$Y = (y_1, y_2, \ldots, y_n)$$

$$Y' = (y_1', y_2', \ldots, y_n')$$

Given this vector notation the following definition can be developed:

$Y = Y'$ if $y_1 = y_1'$, $y_2 = y_2', \ldots, y_n = y_n'$,

and if any elements are not equal then;

$Y = Y'$.

However, the following unequal conditions can be defined as;

$Y > Y'$ if $y_1 > y_1'$, $y_2 > y_2', \ldots, y_n > y_n'$

and the two intermediate cases;

$Y \geqq Y'$ if $y_1 \geqq y_1'$, $y_2 \geqq y_2', \ldots, y_n \geqq y_n'$

$Y \geq Y'$ if $y_1 \geqq y_1'$, $y_2 \geqq y_2', \ldots, y_n \geqq y_n'$ and $Y \neq Y'$

Given these definitions the non-saturation axiom can be formally stated as:

Axiom 4 (Non-saturation): For all Y, Y' in the consumption set C, if $Y \geq Y'$ then Y P Y'.

There follows from this the proposition of the downward sloping indifference curve which can be formally stated as:

Proposition 1 (Downward sloping indifference curves): For all Y, Y' in C if Y I Y', then neither $Y \geq Y'$ nor $Y' \geq Y$.

This proposition is illustrated in Figure 2-1 of the text.

The axiom of continuity of preference is convenient in that it allows the use of mathematical techniques which require the assumption of continuity. In order to formally state this axiom the concept of the boundary set of Y, B(Y), is required. The boundary set is defined as the set of all points Y' on a ray from the origin through Y with the property that for any other point Y" on the ray OY', if y" $\geq$ Y' then Y" P Y, while if Y" $\leq$ Y' then Y P Y". This allows the axiom of continuity of preferences to be formally stated as:

Axiom 5 (Continuity of preferences): For all Y' in the boundary set B(Y) associated with Y, Y' I Y.

The axiom of strict convexity of consumer preferences requires a definition of convexity and strict convexity before it can be stated formally. The usual way to define convexity is to examine the set of points regarded by the consumer as at least as good as Y. This set R(Y) consists of all points Y' in C such that Y' R Y. This set is then said to be convex if, for all Y', Y" in the set, every point between Y' and Y" on the straight line joining Y' and Y" were in the interior of the set (not on the boundary). Formally, the axiom of strict convexity can be stated as:

Axiom 6 (Strict convexity): For all Y in C the set R(Y), consisting of all Y' in C such that Y' R Y, is strictly convex.

This axiom has the strong implication of ruling out points of tangency or intersection of the indifference curves and the axes in the analysis of economic choice.

The axiom of smooth indifference curves in consumer preference can be stated formally as:

> Axiom 7 (Smooth indifference curves): At all points Y in C, the marginal rate of substitution between any pair of commodities is uniquely determined.

The implications of the axioms of continuity of preferences, strict convexity, and smooth indifference curves can be partially summarized and formally stated as:

> Proposition 2: For any pair of elements or commodities y_i and y_j, given the quantities of all other commodities, the marginal rate of substitution of y_i for y_j is a continuous and strictly decreasing function of y_i.

This proposition expresses the fundamental property of consumer preferences which allows the systematic solution of the class of economic problems involving individual choice.

Simple exchange.

To illustrate the application of these axioms, their corollaries, and postulates in a mathematical formulation consider an individual (i) one of a large number of individuals in a world with only two goods

A and B. This world is also timeless and an optimum consumption bundle can only be achieved through exchange. The endowment of i is given by:

$$Y = (y_A, y_B) \tag{A-1}$$

where: Y = endowment

y_A = number of units of A;

y_B = number of units of B.

The wealth represented by the initial endowment is given by:

$$W_y = \bar{p}_A\, y_A + \bar{p}_B\, y_B \tag{A-2}$$

where: W = wealth

$\bar{p}_A$ = price of A in terms of a money of account

$\bar{p}_B$ = price of B in terms of a money of account.

The preferences of (i) can be represented by the expression:

$$U = u(c_A, c_B) \tag{A-3}$$

where: U = an absolute level of preference;

c_A = number of units of A available for consumption;

c_B = number of units of B available for consumption.

However, only the ordinal properties of the preference function are required to solve this choice theoretic system. The particular property required is the rate at which the individual is willing to give up a unit of one good to gain a unit of the other good or the slope of the indifference curves which is given by the expression:

$$\frac{\partial u/\partial c_A}{\partial u/\partial c_B} = \frac{dc_B}{dc_A}\bigg|_u = U'\ (c_A,\ c_B) \qquad \text{(A-4)}$$

which is the ratio of the partial derivatives of the preference function.

The opportunity set for (i) is formulated as a wealth constraint where:

$$\bar{p}_A\ c_A + \bar{p}_B\ c_B = \bar{p}_A\ y_A + \bar{p}_B\ y_B \qquad \text{(A-5)}$$

A standard technique for solving this problem is to form a "Lagrangian" expression of the form:

$$\text{"L"} = U(c_A,\ c_B) - \lambda(\bar{p}_A\ c_A + \bar{p}_B\ c_B - \bar{p}_A\ y_A - \bar{p}_B\ y_B) \qquad \text{(A-6)}$$

where:

λ = some undetermined "Lagrangian" multiplier.

Then taking the partial derivatives of "L" with respect to c_A, c_B, and λ, and setting these equal to 0 results in the following first order conditions.

$$\frac{\partial \text{"L"}}{\partial c_A} = \frac{\partial u}{\partial c_A} - \lambda \bar{p}_A = 0 \qquad \text{(A-7)}$$

$$\frac{\partial \text{"L"}}{\partial c_B} = \frac{\partial u}{\partial c_B} - \lambda \bar{p}_B = 0 \qquad \text{(A-8)}$$

$$\frac{\partial \text{"L"}}{\partial \lambda} = \bar{p}_A c_A + \bar{p}_B c_B - \bar{p}_A y_A - \bar{p}_B y_B = 0 \qquad \text{(A-9)}$$

Shifting the λ terms to the right hand side of the equations and dividing A-7 by A-8 yields.

$$\frac{\partial \text{"L"}}{\partial c_A} = \frac{\partial u}{\partial c_B} = \lambda \bar{p}_A \qquad \text{(A-10)}$$

$$\frac{\partial \text{"L"}}{\partial c_A} = \frac{\partial u}{\partial c_B} = \lambda \bar{p}_B \qquad \text{(A-11)}$$

$$\frac{\partial \text{"L"}/\partial c_A}{\partial \text{"L"}/\partial c_A} = \frac{\partial u/\partial c_A}{\partial u/\partial c_B} = \frac{\lambda \bar{p}_A}{\lambda \bar{p}_B} \qquad \text{(A-12)}$$

Equation A-12 can be simplified by incorporating the expression from Equation A-4 which yields:

$$U' (c_A, c_B) = \frac{\partial u/\partial c_A}{\partial u/\partial c_B} = \frac{\lambda \bar{p}_A}{\lambda \bar{p}_B} \quad \text{(A-13)}$$

which in turn simplifies to:

$$U' (c_A, c_B) = \frac{\lambda \bar{p}_A}{\lambda \bar{p}_B} = \frac{\bar{p}_A}{\bar{p}_B} \quad \text{(A-14)}$$

$$c_B = \frac{\bar{p}_A}{\bar{p}_B} U' (c_A) \quad \text{(A-15)}$$

and substituted into Equation A-9 yielding:

$$\bar{p}_A c_A + \bar{p}_B [\frac{\bar{p}_A}{\bar{p}_B} U' (c_A)] - \bar{p}_A y_A - \bar{p}_B y_B = 0 \quad \text{(A-16)}$$

which can be solved for c_A and once c_A is known can be solved for c_B in the original form:

$$\bar{p}_A c_A + \bar{p}_B c_B - \bar{p}_A y_A - \bar{p}_B y_B = 0 \quad \text{(A-9)}$$

In order to ensure that this solution is a constrained maximum the bordered Hessian determinant of the matrix of second derivatives must be positive (Henderson and Quandt, 1971). The matrix of second and cross derivatives from Equations A-7 and A-8 is:

$$\begin{bmatrix} \dfrac{\partial^2 \text{"L"}}{\partial c_A{}^2} & \dfrac{\partial^2 \text{"L"}}{\partial c_A c_B} & -\bar{p}_A \\ \dfrac{\partial^2 \text{"L"}}{\partial c_A c_B} & \dfrac{\partial^2 \text{"L"}}{\partial c_B{}^2} & -\bar{p}_B \\ -\bar{p}_A & -\bar{p}_B & 0 \end{bmatrix} \quad \text{(A-17)}$$

By substituting

$$\bar{p}_A = \left(\frac{\partial u}{\partial c_A}\right) \Big/ \lambda \quad \text{(A-18)}$$

(from Equation A-7)

and

$$\bar{p}_B = \left(\frac{\partial u}{\partial c_B}\right) \Big/ \lambda \quad \text{(A-19)}$$

(from Equation A-8)

in A-17 the matrix can be rewritten as:

$$\begin{bmatrix} \dfrac{\partial^2 \text{"L"}}{\partial c_A{}^2} & \dfrac{\partial^2 \text{"L"}}{\partial c_A \, c_B} & \dfrac{\partial u}{\partial c_A} \Big/ \lambda \\ \dfrac{\partial^2 \text{"L"}}{\partial c_A \, c_B} & \dfrac{\partial^2 \text{"L"}}{\partial c_B{}^2} & \dfrac{\partial u}{\partial c_B} \Big/ \lambda \\ \dfrac{\partial u}{\partial c_A} \Big/ \lambda & \dfrac{\partial u}{\partial c_B} \Big/ \lambda & 0 \end{bmatrix} \qquad \text{(A-20)}$$

The expansion of this matrix (A-20), and multiplying through by λ^2 yields the following expression which must be positive for the previous solution to represent a maximum level of preference.

$$2\left(\frac{\partial^2 \text{"L"}}{\partial c_A \, c_B}\right)\left(\frac{\partial u}{\partial c_A}\right)\left(\frac{\partial u}{\partial c_B}\right) - \left(\frac{\partial^2 \text{"L"}}{\partial c_A{}^2}\right)\left(\frac{\partial u}{\partial c_B}\right)^2 - \left(\frac{\partial^2 \text{"L"}}{\partial c_B{}^2}\right)\left(\frac{\partial u}{\partial c_A}\right)^2 > 0$$

(A-21)

The remaining formulation of economic principles will assume that this second order condition is positive and its explicit determination in other examples will not be discussed.

To expand this model of two goods to a model of multi-goods let the level of preference be represented by:

$$U = u\ (c_1,\ c_2,\ldots,\ c_n) \tag{A-22}$$

where $(c_1,\ c_2,\ldots,\ c_n)$ is on n dimensional vector which can be written:

$$c = (c_1,\ c_2,\ldots,\ c_n) \tag{A-23}$$

so Equation A-3 can be written as:

$$U = u(c) \tag{A-24}$$

using vector notation. The same notational principles can be followed for representing the initial endowment and wealth if:

$$y = (y_1,\ y_2,\ldots,\ y_n) \tag{A-25}$$

where $(y_1,\ y_2,\ldots,\ y_n)$ is an n dimensional vector representing the physical amounts of goods 1 to n in the initial endowment. Prices can also be represented by the vector $(\bar{p}_1,\ \bar{p}_2,\ldots,\ p_n)$ where $\bar{p}_1$ to $\bar{p}_n$ represent the prices for goods 1 to n. This price vector can then be represented as:

$$\bar{p} = (\bar{p}_1,\ \bar{p}_2,\ldots,\ \bar{p}_n) \tag{A-26}$$

Using this notation wealth can be represented by the expression:

$$Wy = \sum_{i=1}^{n} \bar{p}_i \; y_i \qquad \text{(A-27)}$$

As in the two good case the problem remains to maximize the individuals consumption bundle subject to the constraint of the opportunity set, and as in the two good case this is done by forming a "Lagrangian" expression:

$$\text{"L"} = u(c) - \lambda \; [(\sum_{i=1}^{n} \bar{p}_i \; c_i) - (\sum_{i=1}^{n} \bar{p}_i \; y_i)] \qquad \text{(A-28)}$$

To maximize "L" calculate the partial derivatives for "L" with respect to the variables c_i (i = 1 to n) and λ. Set these derivatives equal to 0 yielding n + 1 equations for the n + 1 variables, which can be written:

$$\frac{\partial \text{"L"}}{\partial c_1} = \frac{\partial u}{\partial c_1} - \lambda \bar{p}_1 = 0$$

$$\frac{\partial \text{"L"}}{\partial c_2} - \frac{\partial u}{\partial c_2} - \lambda \bar{p}_2 = 0 \qquad \text{(A-29)}$$

$$\frac{\partial \text{"L"}}{\partial c_n} = \frac{\partial u}{\partial c_a} - \lambda \bar{p}_n = 0$$

$$\frac{\partial \text{"L"}}{\partial \lambda} = (\sum_{i=1}^{n} \bar{p}_i \; c_i) - (\sum_{i=1}^{n} \bar{p}_i \; y_i) = 0 \qquad \text{(A-30)}$$

Equations A-29 can be written in a more condensed form as:

$$\frac{\partial \text{"L"}}{\partial c_i} = \frac{\partial u}{\partial c_i} - \lambda \bar{p}_i = 0 \qquad i = 1 \text{ to } n \qquad \text{(A-31)}$$

As in the case of two goods the condition that the marginal rate of substitution between any two goods must be equal to the ratio of prices between those goods can be obtained by transposition of the second term in Equation A-31 to the right hand side and dividing any one equation by any other yielding:

$$u' (c_i, c_{\bar{i}}) = \frac{\bar{p}_i}{p_{\bar{i}}} \qquad \text{(A-32)}$$

This results in a system of equations consisting of n - 1 equations in the form of A-32 plus Equation A-30 which can be solved as a system of simultaneous equations yielding the optimum consumption vector c.

For additional information on vectors and mathematical operations with vectors see Hadley (1961). The remaining material in this section on timeless choice will use the vector notation followed in the multi-good example of simple exchange.

Production and exchange.

In the paradigm of production and exchange the locus of points representing the constraint upon the individuals productive opportunities (PP in Figure 2-3 can be expressed as:

$$P = p(p) \tag{A-33}$$

where:

(p) = the vector $(p_1, p_2, \ldots, p_n)$

For the purpose of developing the mathematical framework of production and exchange it is convenient to define the productive opportunities in terms of the input-output transformations such that:

$$q = p - y \tag{A-34}$$

where:

p = an n dimensional vector

q = an n dimensional vector

y = an n dimensional vector

It follows from this that the productive locus can equally well be expressed as:

$$P = p\ (q + y) \tag{A-35}$$

To define the optimum consumption bundle for an individual whose opportunity set is limited to production the productive solution will be equivalent to the consumptive optimum so that:

$$u\ (c) = u\ (q + y) \tag{A-36}$$

The optimum consumption vector can then be defined by forming the "Lagrangian" expression:

$$\text{"L"} = u\ (q + y) - \lambda\ (p(q + y)) \qquad \text{(A-37)}$$

Taking the partial derivitives of this expression with respect to q_i and λ and setting these equal to 0 yielding n + 1 equations of the form:

$$\frac{\partial\ \text{"L"}}{\partial\ q_i} = \frac{\partial\ u}{\partial\ q_i} = \lambda \frac{\partial\ p}{\partial\ q_i} = 0 \qquad i = 1 \text{ to } n \qquad \text{(A-38)}$$

and:

$$\frac{\partial\ \text{"L"}}{\partial\ \lambda} = p(q + y) = 0 \qquad \text{(A-39)}$$

In this case, the optimum point of production and consumption will be where the marginal rate of substitution between any two goods is equal to the marginal rate of transformation between the same two goods. This can be represented mathematically by transposing the second term of A-38 to the right hand side of the equation and dividing any one equation by any other yielding:

$$u'\ (q_i + y_i,\ q_{\bar{i}} + y_{\bar{i}}) = \frac{\partial\ p/\partial\ q_i}{\partial\ p/\partial\ q_{\bar{i}}} = p'\ (q_i + y_i,\ q_{\bar{i}} + y_{\bar{i}}) \qquad \text{(A-40)}$$

This again results in a system of n unknowns with n - 1 equations in the form of Equation A-40 and one in the form of Equation A-39 which can be solved as a system of simultaneous equations yielding the optimum production and consumption vectors.

In the paradigm of production and exchange the variables are retained and a price vector:

$$\bar{p} = (\bar{p}_i, \bar{p}_2, \ldots, \bar{p}_n) \quad \text{(A-26)}$$

Reintroduced from the simple exchange model. In this situation the individual will engage in productive transformations that maximize wealth. This can be represented by the "Lagrangian" expression:

$$\text{"L"} = [\sum_{i=1}^{n} \bar{p}_i \ (q_i + y_i)] - \lambda \ (p \ (q + y)) \quad \text{(A-41)}$$

Taking the partial derivatives of the expression with respect to q_i and λ, and setting these equal to 0 yields n + 1 equations of the form:

$$\frac{\partial \text{ "L"}}{\partial \lambda} = \bar{p}_i - \lambda \frac{\partial p}{\partial q_i} = 0 \qquad i = 1 \text{ to } n \quad \text{(A-42)}$$

$$\frac{\partial \text{ "L"}}{\partial \lambda} = p \ (q = y) = 0 \quad \text{(A-43)}$$

As in the previous examples the condition for the maximization of wealth is that the marginal rate of transformation be equal to the ratio of prices between the goods concerned. This can be represented by transposing the second term of Equation A-42 to the right hand side and dividing any one equation by any other yielding:

$$\frac{\bar{p}_i}{\bar{p}_{\bar{i}}} = p' \; (q_i + y_i, \; q_{\bar{i}} + y_{\bar{i}}) \qquad \text{(A-44)}$$

This results in a system of n variables with n - 1 equation in the form of A-44 and one equation in the form of A-43 which can be solved as a system of simultaneous equations yielding the optimum production vector.

The optimum consumption vector is then defined as in the first example of simple exchange with a "Lagrangian" of the form:

$$\text{"L"} = u \; (c) - \lambda[\sum_{i=1}^{n} \; (\bar{p}_i \; c_i) - \sum_{i=1}^{n} \; (\bar{p}_i \; p_i^*)] \qquad \text{(A-45)}$$

when: $p_i^* = (q_i^* + y_i)$ defined as optimum level of production in equation (A-43) and (A-44).

and solved in a similar manner.

Note that these examples of production and exchange for the individual could have been formulated in terms of the variable p. However, the formulation in terms of the transformations q is more general and simplifies the extension of the paradigm to production and exchange with firms.

<u>Production and exchange with firms.</u>

The extension of this to the paradigm of production and exchange with firms is straight forward only requiring a modification in notation so that:

$$p\ (p^f) = p\ (q^f) \qquad \text{(A-46)}$$

ommitting y because firms are assumed to have no endowment.

The firms productive optimum is then defined by the expression:

$$\text{"L"} = [\sum_{i=1}^{n} (\bar{p}_i\ q_i^f)] - \lambda\ (p\ (q^f)) \qquad \text{(A-47)}$$

which can be solved as in the previous example defining the productive optimum for the firm. The second modification in notation requires the results of production by the firm be distributed to the owners in proportion to their contribution to the firms initial resources. This can be represented by the expression:

$$q_i^f = \sum_{i=1}^{n} q_i^I \qquad \text{(A-48)}$$

Then the optimum consumption bundle for the individual is defined by:

$$"L^{I}" = u(c) - \lambda[\sum_{i=1}^{n} (\bar{p}_i\ c_i) - \sum_{i=1}^{n} (\bar{p}_i(q_i^I + y_i))] \qquad \text{(A-49)}$$

which can be solved as in the previous example defining the optimum consumption vector for the individual whose endowment is supplemented by the results of production and exchange of firms.

Intertemporal Choice

The mathematical formulation of intertemporal exchange is similar to simple exchange except that variables must be redefined to express the terms of exchange through time. Therefore, let the level of consumption preference of each of the n goods in each of the m periods be represented by:

$$u = u(c_{11},\ldots,c_{12},\ldots,c_{n2},\ldots,c_{1m},\ldots,c_{nm}) \qquad \text{(A-50)}$$

where: $(c_{11},\ldots,c_{nm})$ is an n x m dimensional vector which can be written:

$$\hat{c} = (c_{11},\ldots,c_{n1},\ c_{12},\ldots c_{n2},\ldots c_{1m},\ldots c_{nm}) \qquad \text{(A-51}$$

so that equation A-50 can be written:

$$U = u(\hat{c}) \qquad \text{A-52)}$$

Following the same notational principles let the vector of gross incomes in the endowment be given by:

$$(y_{11},\ldots,y_{n1},y_{12},\ldots,y_{n2},\ldots,y_{1m},\ldots,y_{nm}) \tag{A-53}$$

for the n goods in m time periods. The intra time period prices are represented by the vector:

$$(\bar{p}_{11},\ldots,\bar{p}_{n1},\bar{p}_{12},\ldots,\bar{p}_{n2},\ldots,\bar{p}_{1m},\ldots,\bar{p}_{nm}) \tag{A-54}$$

for the n goods over m time periods. The ratio of exchange or rate of discount applied to future claims to consumption which equates them with present consumption is given by the expression:

$$(1 + \hat{r}_j)^{-1} = [(1 + r_0)(1 + r_1)\ldots(1 + r_j)]^{-1}$$

Using this notation wealth expressed in terms of the index of value for the present time period is defined by the expression:

$$W_0 = \sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ij}\; y_{ij})(1 + \hat{r}_j)^{-1} \tag{A-56}$$

This expression of wealth leads to the following expression of wealth constraint;

$$\sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ij}\; y_{ij} - \sum_{i=1}^{n} \bar{p}_{ij}\; c_{ij})(1 + \hat{r}_j)^{-1} \tag{A-57}$$

which operates in exactly the same manner in intertemporal choice as it does in timeless choice, and allows the formulation of a "Lagrangian" expression which can be solved as in the previous examples.

$$"L" = u(\hat{c}) + \lambda \sum_{j=0}^{m} (\sum_{i=1}^{m} \bar{p}_{ij}\, y_{ij} - \sum_{i=1}^{n} \bar{p}_{ij}\, c_{ij})(1 + \hat{r}_j)^{-1} \qquad \text{(A-58)}$$

$$\frac{\partial\, "L"}{\partial\, \hat{c}} = u_{ij} + \lambda\, \bar{p}_{ij}\, (1 + r_j)^{-1} = 0 \qquad i=1 \text{ to } n \quad j=0 \text{ to } m \qquad \text{(A-5)}$$

$$\frac{\partial\, "L"}{\partial\, \lambda} = \sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ij}\, y_{iu} - \sum_{i=1}^{n} \bar{p}_{ij}\, c_{ij})(1 + \hat{r}_j)^{-1} \qquad \text{(A-60)}$$

Resulting in (n x m) -1 equations of the form;

$$\frac{-\partial\, \hat{c}_{\hat{ij}}}{\partial\, \hat{c}_{ij}} = \frac{\bar{p}_{ij}\, (1 + \hat{r}_j)^{-1}}{\bar{p}_{\hat{ij}}\, (1 + \hat{r}_j)} \qquad \text{(A-61)}$$

and one of the form of Equation A-60 which can be solved as a system of simultaneous equations for the (n x m) variables identified as $\hat{c}_{ij}$.

Production and exchange.

In addition to the variables already developed the paradigm of intertemporal production and exchange requires an expression defining the limits of the productive opportunities available through time. This can be expressed as a vector:

$$\hat{p} = p\ (\hat{p}_{11},\ldots,\hat{p}_{n1},\hat{p}_{n12},\ldots,\hat{p}_{n2},\ldots,\hat{p}_{1m},\ldots,\hat{p}_{nm}) \qquad \text{(A-62)}$$

As in the paradigm of timeless production and exchange it is convenient to define the productive opportunities in terms of input-output transformations so that:

$$\hat{q}_{ij} = \hat{p}_{ij} - y_{ij} \qquad i=1 \text{ to } n \qquad j=0 \text{ to } m \qquad \text{(A-63)}$$

To formulate the mathematical solution for a condition where the opportunity set consists of only productive opportunities it is convenient to formulate the level of consumptive preference as a function of the input-output relationship, yielding:

$$U = u(\hat{c}) = u(\hat{q} + y) \qquad \text{(A-65)}$$

Using these variables the problem can be solved as in the previous examples by formulating the "Lagrangian" expression:

$$\text{"L"} = u(\hat{q} + y) - \lambda[p(\hat{q} + y)] \qquad \text{(A-66)}$$

for which a constrained optimum for $u(\hat{q} + y)$ can be found by taking partial derivatives of "L" with respect to $(\hat{q}_{ij} + y_{ij})$ and solving as in the previous examples, yielding:

$$\partial "L"/(q_{ij} + y_{ij}) = \frac{\partial u}{\partial(\hat{q}_{ij} + y_{ij})} - \frac{\lambda \, \partial \, p}{\partial(\hat{q}_{ij} + y_{ij})} = 0 \qquad \text{(A-67)}$$

$i=1$ to n $\quad j=0$ to m

$$\partial "L"/\partial\lambda = p(\hat{q} + y) = 0 \qquad \text{(A-68)}$$

As in the example of timeless production the condition of the optimum level of production and consumption is where the marginal rate of transformation is equal to the marginal rate of substitution, and is given by:

$$U'(\hat{q}_{ij} + y_{ij}, \hat{q}_{ij} + y_{ij}) = \frac{\partial p/\partial(\hat{q}_{ij} + y_{ij})}{\partial p/\partial(\hat{q}_{\overline{ij}} + y_{\overline{ij}})} = p' \; (\hat{q}_{ij}+y_{\overline{ij}}, \hat{q}_{ij}+y_{\overline{ij}}) \qquad \text{(A-69)}$$

The $(n \times m) -1$ equations in the form of Equation A-67) and Equation A-68 can be solved as a system of simultaneous equations yielding optimum values of $\hat{q}_{ij}$.

Notice that this formulation of intertemporal production does not include any prices, either intra or intertemporal. The lack of exchange opportunities makes these irrelevant and the intra-intertemporal marginal parameters with which the marginal rates of transformation are equivalent at the point of optimum production.

If the paradigm is expanded to include exchange as well as productive opportunities the Separation Theorem (Hirshleifer, 1970) requires that the optimization be carried out in two steps. As in the example of timeless production and exchange the first step is to engage in productive transformations in such a way as to maximize wealth. Examination of Figure 2-6 in the text, and Equation A-56 shows that a change in gross income at any point in time is reflected in a change in wealth regardless of which periods consumption claims are chosen as the measure of wealth. However, it is common practice to use current consumption claims as the measure of wealth to be maximized. Adopting this convention the intertemporal expression for the level of production which maximizes wealth subject to the constraints of the opportunity set can be given by the "Lagrangian" equation:

$$"L" = \sum_{j=0}^{m} [\sum_{i=1}^{n} \bar{p}_{ij} (\hat{q}_{ij} + y_{ij})(1 + \hat{r}_j)^{-1} - \lambda [p(\hat{q} + y)] \qquad \text{(A-70)}$$

which can be solved as in the previous examples by taking partial derivatives with respect to $(\hat{q}_{ij} + y_{ij})$ and λ, setting these equal to 0 and calculating the ratio between partial derivatives yielding:

$$\partial"L"/\partial(\hat{q}_{ij} + y_{ij}) = \bar{p}_{ij}(1 + \hat{r}_j)^{-1} - \frac{\lambda\, \partial\, p}{\partial(\hat{q}_{ij} + y_{ij})} = 0 \quad \begin{matrix} i=1 \text{ to } n \\ j=0 \text{ to } m \end{matrix}$$

(A-71)

$$\partial"L"/\partial\lambda = P(\hat{q} + y) = 0 \qquad \text{(A-72)}$$

$$\frac{\bar{p}_{ij} \ (1 + \hat{r}_j)^{-1}}{\bar{p}_{i\bar{j}} \ (1 + \hat{r}_{\bar{j}})^{-1}} \quad = \quad p'(\hat{q}_{ij} + y_{ij}, \ \hat{q}_{i\bar{j}} + y_{i\bar{j}}) \qquad \text{(A-73)}$$

These equations can be solved as a system of simultaneous equations defining the level of production which will yield the maximum wealth attainable within the opportunity set.

The optimum series of consumption titles through time can then be defined as in the previous example of intertemporal exchange with the "Lagrangian" expression:

$$\text{"L"} = u(\hat{c}) + \lambda \sum_{j=0}^{m} \left(\sum_{i=1}^{n} \ \bar{p}_{ij}, \ p^*_{ij} - \sum_{i=1}^{n} \ \bar{p}_{ij} \ c_{ij}\right)(1 + \hat{r}_j)^{-1} \qquad \text{(A-74)}$$

where:

$p^*_{ij} = \hat{q}_{ij} + y_{ij}$
(the optimum level of production from equation A-70).

Production and exchange with firms.

The extension of this to the paradigm of intertemporal production and exchange with firms is straight forward only requiring a modification in the notation used in the paradigm of timeless production and exchange with firms so that;

$$p(p^f) = p(\hat{q}^f) \qquad \text{(A-75)}$$

again omitting y_{ij} because firms are assumed to have no endowment.

The productive optimum for the firm is then defined by the expression:

$$"L" = \sum_{j=0}^{m} [\sum_{i=1}^{n} \bar{p}_{ij} (\hat{q}^f_i)(1 + \hat{r}_j)^{-1}] - \lambda [p(\hat{q}^f)] \qquad \text{(A-76)}$$

which can be solved as before defining the optimum productive vector for the firm. As in the previous examples firms are not allowed to consume by definition and the proceeds of production are distributed to the owners where it expands their opportunity sets for consumption.

Compound interest formulations.

This mathematical discussion of the economic principles of intertemporal production and exchange has required the introduction of compound interest as an expression of the ratio of exchange between claims to consumption in different time periods. The following summary of compound interest formulas is intended as a brief review and is abstracted from Grant and Ireson (1970).

The following symbols will be used in this explanation:

r (interest)	= the interest rate per interest period (a one year interest period is used in most illustrations but the formulas presented apply to interest periods of any length).
j (number)	= the number of interest periods.

P (present) = the value of titles to consumption in the present period.

F (future) = the value of titles to consumption at the end of n periods from the present that is equivalent to P with interest i.

A (annual) = the value of titles to consumption received in a uniform series (gross or endowed income) continuing for the coming n periods, the entire series equivalent to P at interest rate i.

The fundamental interest formulas that express the relationship between P, F, and A in terms of r and j are:

Given P to find F.

$$F = P(1 + r)^j \tag{A-76}$$

Given F to find P.

$$P = F(1/(1 + r)^j) \tag{A-78}$$

Given F to find A.

$$A = F[r/((1 + r)^j-1)] \tag{A-79}$$

Given P to find A.

$$A = P[r(1 + r)^j/(1 + r)^j-1] = P[(r/((1 + r)^j-1)+r)] \tag{A-80}$$

Given A to find F.

$$F = A[(1 + r)^j-1/r] \tag{A-81}$$

Given A to find P.

$$P = A[(1 + r)^j-1/r(1 + r)^j] = A[1/((r/(1 + r)^j-1)+r)] \tag{A-82}$$

The previous material in this section of the appendix and in the text section on economic theory has treated the interest period as one year. However, this conventional period may not correspond to the relevant transaction or decision period for consumptive, productive or exchange purposes. In these situations it may be necessary or convenient to compound intertemporal exchanges at period lengths other than one year. Under these circumstances it may be necessary to establish equivalence between the nominal rate of interest used for compounding at periods of other than one year and the effective annual rate.

Let interest be compounded p times a year at a rate r/p per compounding period. (A-83)

The nominal interest rate per annum $= p(r/p)=r$ (A-84)

The effective interest rate per annum $= (1 + r/p)^{p}-1$ (A-85)

In circumstances where consumptive, productive, or exchange decisions are continuous it may be desirable to apply techniques involving continuous compounding. The formula for present and future titles to consumption will be given by the following:

when e = the base of natural or "Napierian" logarithms.

$$F = Pe^{rj} \quad \text{(A-86)}$$

$$P = Fe^{-rj} \quad \text{(A-87)}$$

For the derivation and additional discussion of this material on compound interest formulas see Chapter 4 of Grant and Ireson (1970).

Investment Choice Present Value Criteria.

The mathematical formulation of present value criteria is given by the following formulae for a series of j descrete time periods.

$$W^P_0 = (\sum_{i=1}^{n} \bar{p}_{io}\, p_{io}) + \frac{(\sum_{i=1}^{n} \bar{p}_{i1}\, p_{i1})}{(1+r_1)} + \frac{(\sum_{i=1}^{n} \bar{p}_{i2}\, p_{i2})}{(1+r_1)(1+r_2)} + \ldots +$$

$$\frac{(\sum_{i=1}^{n} \bar{p}_{im}\, p_{im})}{(i+r_1)(1+r_2)\ldots(1+r_m)} \qquad \text{(A-88)}$$

where: W_0^P = attained wealth

$\sum_{i=1}^{n} \bar{p}_{ij}\, p_{ij}$ = the sequence of potential consumption flows of p_i goods or attained gross income resulting from an investment ensemble.

Equation A-88 can be simplified to:

$$W_0^P = \sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ij}\, p_{ij})(1+\bar{r}_j)^{-1} \qquad \text{(A-89)}$$

Note the equivalence between the present value criterion of attained wealth and the term of the "Lagrangian" to be maximized in the paradigm of intertemporal choice with production and exchange formulated in equation A-70).

$$W_0^P = \sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij} \left(q_{ij} + y_{ij}\right)\right)\left(1 + \bar{r}_j\right)^{-1} \qquad \text{(A-90)}$$

The rule to select the investment ensemble which maximizes attained wealth follows from this equivalence. The maximized value of attained wealth will be symbolized by W_0^* which is the present value of the optimum sequence of potential consumption flows or attained gross income.

An alternative present value criterion is based on the sequence of productive transformations associated with an investment ensemble and is defined as:

$$W_0^Q = \left(\sum_{i=1}^{n} \bar{p}_{io}\, q_{io}\right) + \frac{\left(\sum_{i=1}^{n} \bar{p}_{i1}\, q_{i1}\right)}{(1 + r_1)} + \frac{\left(\sum_{i=1}^{n} p_{i2}\, q_{i2}\right)}{(1+r_1)\,(1+r_2)} + \ldots + \frac{\left(\sum_{i=1}^{n} \bar{p}_{im}\, q_{im}\right)}{(1+r_1)(1+r_2)\ldots(1+r_m)} \qquad \text{(A-91)}$$

which can be simplified to:

$$W_0^Q = \sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij}\, q_{ij}\right)(1+\bar{r}_j)^{-1} \qquad \text{(A-92)}$$

where W_0^Q = wealth gain.

The rule that calls for the maximization of wealth gain follows from its definition which is:

$$W_0^P = W_0^Q + W_0^Y \tag{A-93}$$

Since endowed wealth is a constant in any problem of investment choice it is evident that maximizing wealth gain is equivalent to maximizing attained wealth.

The third present value criterion is the present value of a particular investment project and is represented by:

$$W_0^{\Delta Q} = (\sum_{i=1}^{m} \bar{p}_{i0} \ \Delta q_{i0}) + \frac{(\sum_{i=1}^{n} \bar{p}_{i1} \ \Delta q_{i1})}{(1+r_1)} + \frac{(\sum_{i=1}^{n} \bar{p}_{i2} \ \Delta q_{i2})}{(1+r_1)(1+r_2)}$$

$$+ \ . \ . \ . + \frac{(\sum_{i=1}^{n} \bar{p}_{im} \ \Delta q_{im})}{(1+r_1)(1+r_2)\ldots(1+r_m)} \tag{A-94}$$

which simplifies to:

$$W_0^{\Delta Q} = \sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ij} \ \Delta q_{ij})(1 + \bar{r}_j)^{-1} \tag{A-95}$$

when: $W_0^{\Delta Q}$ = a change to the wealth gain of an existing investment project ensemble associated with a particular investment project.

Δq_{ij} = are changes resulting from a particular investment project in the series of productive transformations associated with the existing investment project ensemble.

The rule to adopt an investment project if its present value is greater than 0 follows from its effect of increasing W_0^Q and W_0^P.

Internal rate of return criteria.

The mathematical formulation of internal rate of return criteria are as follows for the sequency of productive transformation associated with an investment ensemble and an individual investment project:

$$0 = \sum_{i=1}^{n} \bar{p}_{i0} q_{i0} + \frac{\sum_{i=1}^{n} \bar{p}_{i1} q_{i1}}{(1+irr)} + \frac{\sum_{i=1}^{n} \bar{p}_{i2} q_{i2}}{(1+irr)^2} + \ldots + \frac{\sum_{i=1}^{n} \bar{p}_{im} q_{im}}{(1+irr)^m} \tag{A-96}$$

which simplifies to:

$$0 = \sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij} q_{ij} \right)(i+irr)^{-j} \tag{A-97}$$

and:

$$0 = \left(\sum_{i=1}^{n} \bar{p}_{io} \, \Delta q_{io} \right) + \frac{\left(\sum_{i=1}^{n} \bar{p}_{i1} \, \Delta q_{i1} \right)}{(1+irr)} + \frac{\left(\sum_{i=1}^{n} \bar{p}_{i2} \, \Delta q_{i2} \right)}{(1+irr)^2} + \ldots + \frac{\left(\sum_{i=1}^{n} \bar{p}_{im} \, \Delta q_{im} \right)}{(1+irr)^m} \qquad \text{(A-98)}$$

which simplifies to:

$$0 = \sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij} \, \Delta q_{ij} \right) (1+irr)^{-j} \qquad \text{(A-99)}$$

where irr = the internal rate of return.

Notice that for Equations A-96 and A-98 to have a solution there must be at least one sign reversal among the elements on the right hand side of the equations. Since the endowment sequence $y_{io}, \ldots, y_{im}$ contains positive elements only it has no internal rate of return and the attained wealth sequence $p_{io}, \ldots, p_{im}$ may also have only positive elements and no internal rate of return. However, the transformation sequences $q_{io}, \ldots, q_{im}$ and $\Delta q_{io}, \ldots, \Delta q_{im}$ must have a sign reversal, if not, either the endowment vector y or the attained vector p must have been inefficient (Hirshleifer, 1970).

Notice that the internal rate of return criteria are only equivalent to the present value criteria and the paradigm of intertemporal choice with production and exchange when the internal rate of return (irr) is equal to the external rate of interest (r).

$$\sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij} \; q_{ij} \right) (1+\bar{r}_j)^{-1} = \sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij} \; q_{ij} \right) (1 + irr)^{-j} \qquad \text{(A-100)}$$

$$\sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij} \; \Delta q_{ij} \right) (1 + \bar{r}_j)^{-1} = \sum_{j=0}^{m} \left(\sum_{i=1}^{m} \bar{p}_{ij} \; \Delta q_{ij} \right) (1 + irr)^{-j} \qquad \text{(A-101)}$$

only where r = irr

(see equation A-55)

This failure of internal rate of return criteria to be equivalent to present value criteria for the same variables except when the internal rate of return is equal to the external rate of interest is the basic cause for inconsistencies resulting from application of investment choice rules based upon the two criteria.

The formulation of an internal rate of return based upon the differences in the attained gross income, "Fisher's rate of return over cost" (Fisher, 1930) is:

$$0 = \sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij} \; (pd_{ij} - pc_{ij}) \right) (1 + irr)^{-j} \qquad \text{(A-102)}$$

where: pd_{ij} = the attained gross income sequence of the "Defending" investment ensemble.

pc_{ij} = the attained gross income sequence of the "Challenging" investment ensemble.

Since $p_{ij} = q_{ij} + y_{ij}$ and the endowment sequence y_{ij} is the same for both the "Challenging" and "Defending" investment ensembles the formulation.

$$0 = \sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij} \; (qd_{ij} - qc_{ij}) \right) (1 + irr)^{-j} \qquad \text{(A-103)}$$

where: qd_{ij} = the productive transformation sequence of the "Defending" investment ensemble.

qc_{ij} = the productive transformation sequence of the "Challenging" investment ensemble.

is equivalent to equation (A-102).

The need for the concepts of "Defender" and "Challenger", and "earlier" and "later" is obvious from equations (A-102) and A-103) when the inconsistent choices which would result from a reversal of a "Challenger" and "Defender" are considered.

Money and Investment Choice

In the previous formulations of economic principles prices $(\bar{p}_{ij})$ have been expressed in terms of a money of account which functioned as an index of value. In this section money as a distinct good will be introduced into the formulation through money prices $(\bar{p}^{d}_{ij})$.

As was pointed out in the body of the thesis, the predominance of money-money exchanges in intertemporal exchange leads to the existence of a money rate of interest which can be defined as:

$$\frac{Dj}{D\bar{j}} = \frac{1}{(1 + r^{D}_{\bar{j}})} \qquad \text{(A-104)}$$

where: $\frac{Dj}{D\bar{j}}$ = the price of next year's money in terms of this year's money.

The relation between the monetary and "real" rates of interest is:

$$(1 + r^{D}) = (\sum_{i=1}^{n} \bar{p}d_{i1} / \sum_{i=1}^{n} p\text{-}d_{i0}) - (1 + r) \qquad \text{(A-105)}$$

which can be simplified to:

$$1 + r_{j}^{D} = (1 + a_{j})(1 + r_{j}) \qquad \text{(A-106)}$$

or

$$r^D_j = a_j + r_j + a_j r_j \qquad \text{(A-107)}$$

where: a_j = is the proportionate anticipated or realized change in the prices of consumptive goods relative to money between period j and j-1 (Hirshleifer, 1970).

Given the existence of money and money prices the variables $(\bar{p}_{ij})$ and (r_j) in the formulation of present value criteria Equations A-89, A-92, A-95 must be determined from their monetary counterparts $\bar{p}_{ij}$ and r^D_j. The relation of r^D_j to r_j is given in Equation A-106 and A-107. The relation between $\bar{p}^d_{ij}$ and $\bar{p}_{ij}$ is:

$$\bar{p}_{ij} = \bar{p}^d_{ij} \, (1 + a_j)^{-1} \qquad \text{(A-108)}$$

It is important to recognize that for a present value criterion to be equivalent to the wealth parameter, in intertemporal choice, monetary rates of interest, and prices must be adjusted to their "real" counterparts. Further, because of the failure of money rates of interest to fully adjust to changes in purchasing power of money (Fisher, 1930), rate of return investment criteria should be calculated in terms of "real" values of $\bar{p}_{ij}$.

Market Imperfections and Investment Choice

The failure of the Separation Theorem to generally hold under conditions of imperfect markets requires that the productive optima be defined either conditionally upon individual preferences, or as a function of individual preferences.

An example of conditional optimization can be illustrated under divergent borrowing and lending rates. On the condition that an individual's preferences are such that attained titles to consumption would be lent until a final date the productive optima would be defined by the "Lagrangian":

$$"L"^L = \sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij}\, p_{ij} \right) (1 + \bar{r}_j^{\,L}) - \lambda(p - (p)) \qquad \text{(A-109)}$$

where:

$$(1 + \bar{r}_j^{L}) = (1 + r_o^{L})(1 + r_1^{L}) \ldots (1 + \bar{r}_j^{L}) \qquad \text{(A-110)}$$

r_j^L = the lending rate between the j and the j + 1 periods.

This equation can be solved, as in the previous example, yielding the condition that the marginal rate of transformation is equal to the ratio of lending prices:

$$\frac{\bar{p}_{ij}\ (1 + \bar{r}_j^L)}{\bar{p}_{\bar{ij}}(1 + \bar{r}_j^L)} = P'\ (p_{ij},\ p_{ij}) \qquad \text{(A-111)}$$

In the same manner the productive optimum under the condition that attained consumption will be borrowed against to secure consumption titles in the current period would be formulated as:

$$"L"^B = \sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij}\ p_{ij} \right) (1 + \bar{r}_j^B)^{-1} - \lambda(p - (p)) \qquad \text{(A-112)}$$

where:

$$(1 + \bar{r}_j^B)^{-1} = [(1 + r_0^B)(1 + r_2^B)\ \ldots\ (1 + r_j^B)]^{-1} \qquad \text{(A-113)}$$

r_j^B = the borrowing rate between j and the j + 1 period.

which can be solved, yielding the condition that the marginal rate of transformation is equal to the ratio of the borrowing prices:

$$\frac{\bar{p}_{ij}\ (1 + \bar{r}_j^B)^{-1}}{\bar{p}_{ij}\ (1 + \bar{r}_j^B)} = P'\ (p_{ij},\ p_{\bar{ij}}) \qquad \text{(A-114)}$$

The existence of an optimum level of production under a lending condition and an optimum level of production under a borrowing condition creates a situation where the consumptive optimum will be defined on the boundary of an opportunity set which is described by three discontinuous functions. The expression:

$$"L" = u(c) - \lambda[\sum_{j=0}^{m} [[(\sum_{i=1}^{n} p_{ij} c_{ij}) - (\sum_{i=1}^{n} \bar{p}_{ij} p_{ij}^{*L})][1+\bar{r}_j^{L}]]] \qquad \text{(A-115)}$$

where: p_{ij}^{*L} = the optimum level of production under the lending condition, defined by equation A-109.

defines the preferred level of consumption under the condition that the marginal rate of substitution along an indifference curve at p_{ij}^{*L} was less than the marginal rate of transformation. Following the same pattern:

$$"L" = u(c) - \lambda[\sum_{j=0}^{m} [[(\sum_{i=1}^{n} \bar{p}_{ij} c_{ij}) - (\sum_{i=1}^{n} \bar{p}_{ij} p_{ij}^{*B})]$$

$$[1 + \bar{r}_j^{B}]^{-1}]] \qquad \text{(A-116)}$$

where: p_{ij}^{*B} = the optimum level of production under the borrowing condition defined by equation A-112.

defines the preferred level of consumption under the condition that the marginal rate of substitution along an indifference curve at p_{ij}^{*B} was greater than the marginal rate of transformation. In the event that neither of the previous conditions are met the opportunity set is not expanded, in terms of the individual preferences, by the market opportunities available and the preferred consumption bundle is defined by:

$$"L" = u(c) - \lambda(p(p)) \tag{A-66}$$

The discontinuity in the boundary of the consumption opportunity set resulting from the divergence of the borrowing and lending rates results in the general failure of the Separation Theorem to hold. This failure means that the optimum level of production cannot be defined independently of individual preferences.

A similar but not identical condition arises under rising borrowing rates. Under the assumption that marginal borrowing costs are a function only of the scale of borrowing, the following system of equations can be developed. First, the condition that the optimum level of production is where the marginal rate of transformation is equal to the ratio of prices as illustrated by Equation A-117).

$$p'(p_{ij}, p_{\bar{i}j}) = \frac{\bar{p}_{ij}\ (+ \bar{r}_j^B)^{-1}}{\bar{p}_{ij}\ (1 + \bar{r}_j^B)^{-1}} \tag{A-117}$$

Secondly, the condition that the preferred consumption bundle is defined by the condition that the marginal rate of substitution is equal to the ratio of prices.

$$u'(c_{ij}, c_{\bar{i}j}) = \frac{\bar{p}_{ij}\ (1 + \bar{r}_j^B)^{-1}}{\bar{p}_{\bar{i}j}\ (1 + \bar{r}_j^B)^{-1}} \tag{A-118}$$

Equations A-117 and A-118 can be combined to yield:

$$p'(p_{ij}, p_{\bar{i}j}) = \frac{\bar{p}_{ij}\ (1 + \bar{r}_j^B)^{-1}}{\bar{p}_{\bar{i}j}\ (1 + r_j^B)^{-1}} = u'(c_{ij}, c_{\bar{i}j}) \tag{A-119}$$

which in turn can be simplified to:

$$p'\ (p_{ij}, p_{\bar{i}j}) = u'(c_{ij}, c_{\bar{i}j}) \tag{A-120}$$

which defines the essential condition that the marginal rate of transformation be equal to the marginal rate of substitution. In this respect the paradigm of production with rising borrowing costs as a function of borrowing is identical to production without exchange. The difference lies in the opportunity to expand the consumption opportunity set through borrowing with the condition described by Equation A-116).

The failure of the Separation Theorem to hold means that present value criteria for investment choice are no longer universally correct in the sense that an objectively calculated criteria free of individual preferences is adequate for investment decisions (Hirshleifer, 1970). However, as was pointed out in the text, present value - future value criteria can be combined with the concept of "dominance" to eliminate investment alternatives which are not on the boundary of the consumption opportunity set. The present value of an investment ensemble is defined by:

$$W_0^P = \sum_{j=0}^{m} (\bar{p}_{ij}\ p_{ij})\ (1 + \bar{r}_j^B)^{-1} \qquad \text{(A-121)}$$

The future value of an ensemble is defined by:

$$W_0^P = \sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ij}\ p_{ij})\ (1 + \bar{r}_j^L) \qquad \text{(A-122)}$$

It is against these two criteria that the "dominance" rule given in the text can be applied to eliminate inefficient investment ensembles.

Under conditions of quantitative limits on investment, present value rules are applied to modified present value criteria. The modified criteria consist of the discounted sequence of attained wealth, wealth gain, or project present value as of the period after the period in which the quantitative limits are imposed.

The formulae for these criteria are:

$$W^P/c_0 = \sum_{i=1}^{n} \bar{p}_{i1} p_{i1} + W^P/c_1 \qquad \text{(A-123)}$$

$$W^P/c_1 = \sum_{j=1}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij} p_{ij} \right) (1 + \bar{r}_j)^{-1} \qquad \text{(A-124)}$$

$$W^q/c_0 = \sum_{i=1}^{n} \bar{p}_{i1} q_{i1} + W^q/c_1 \qquad \text{(A-125)}$$

$$W^w/c_1 = \sum_{j=1}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij} q_{ij} \right) (1 + \bar{r}_j)^{-1} \qquad \text{(A-126)}$$

$$W^{\Delta q}/c_0 = \sum_{i=1}^{n} \bar{p}_{i1} \Delta q_{i1} + W^{\Delta q}/c_1 \qquad \text{(A-127}$$

$$W^{\Delta q}/c_1 = \sum_{j=1}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ij} \Delta q_{ij} \right) (1 + \bar{r}_j)^{-1} \qquad \text{(A-128)}$$

Uncertainty and Investment Choice

As in the earlier presentation of investment choice the basis for a mathematical discussion of choice under uncertainty is five postulates. These postulates augment the seven axioms of consumer preference by systematically describing the framework with in which consumers choose between risky alternatives. The notation used to represent objects of choice must be modified so that each object of choice or prospect is also associated with a probability of occurrence and is written:

$$Y;S$$

Where:

Y = the object of choice

S = the probability that Y will occur.

A complete set of prospects for any event would be represented by the expression:

$$(Y_1, Y_2, \ldots, Y_g; S_1, S_2, \ldots, S_g)$$

when:

$$S_1 + S_2 + \ldots S_g = 1$$

$$0 < S_k < 1, K = 1 \text{ to } g$$

The first postulate of consumer choice under uncertainty which states that individuals have a complete and transitive weak preference ordering and choose rationally, confirms that the first three axioms of choice hold for uncertain prospects as well as certain choices and does not require restatement (Green, 1971).

The second postulate formalizes the nature of the choice regarding levels of uncertainty and can be stated as:

if (y p y') and,

Y = (y, y'; s, 1-s) and

Y' = (y, y'; s', 1-s')

then: (Y P Y') if and only if s > s' (Green, 1971)

The third postulate defines the existence of continuity between prospects and may be stated as:

If y R y' R y" then there exists as s such that

(y, y"; s, 1-s) I y'

(Green, 1971)

The fourth postulate defines the quality of independence to prospects and is stated as:

If Y = (y, y'; s, 1-s) and

y I $(x, x'; s_x, 1-s_x)$ then

Y I $((x, x'; s_x, 1-s_x), y'; s, 1-s)$

The fifth postulate describes the relationship between complex prospects made up of simple prospects and states that:

If Y = $[(x^a, x^b; s_x, 1-s_x),(x^a, x^b; s_x, 1-s_x); s, 1-s$

then:

$$Y\ I\ [x^a,\ x^b;\ (s)\ x\ (s_x) + (1-s)\ s'_x,\ s(1-s_x) + (1-s)(1-s'_x)]$$

Before formulating the paradigm of timeless exchange in state preference theory the notation used in the previous example must be modified to accept uncertainty. This will be done by adding a subscript (k) to each of the variables to represent the state in which the variable exists and the probability of its existence. Further (k) will be allowed to exist from 1 to g subject to the condition that the sum of the probabilities of (k) in any time period equal 1. This is represented by the expression:

$$\sum_{k=1}^{g} s_k = 1 \qquad \text{(A-129)}$$

Using this notation, the paradigm of timeless exchange under uncertainty can be formulated in terms of "state preference theory" in a familiar "Lagrangian" expression:

$$\text{"L"} = u(c) - \lambda[(\sum_{i=1}^{n} \sum_{k=1}^{g} \bar{p}_{iok}\ c_{iok}) - (\sum_{i=1}^{n} \sum_{k=1}^{g} \bar{p}_{iok}\ c_{iok})] \qquad \text{(A-130)}$$

where:

$$c = (c_{101},\ c_{102},\ldots,\ c_{10g},\ c_{201},\ldots,\ c_{20g},\ldots,$$
$$c_{n01},\ldots,\ c_{n0g};\ s_{101},\ s_{102},\ldots,\ s_{10g},\ s_{201},\ldots,$$
$$s_{20g},\ldots,\ s_{n01},\ldots,\ s_{n0g})$$

and solved as in the previous examples yielding:

$$\frac{\partial \text{"L"}}{\partial c_{iok}} = \frac{\partial u}{\partial c_{iok}} - \lambda \bar{p}_{iok} \qquad i = 1 \text{ to } n; \quad k = 1 \text{ to } g \tag{A-131}$$

$$\frac{\partial \text{"L"}}{\partial \lambda} = [(\sum_{i=1}^{n} \sum_{k=1}^{g} \bar{p}_{iok}\, c_{iok}) - (\sum_{i=1}^{n} \sum_{k=1}^{g} \bar{p}_{iok}\, y_{iok})] \tag{A-132}$$

This results in the general marginal condition that:

$$U' (c_{iok}, s_{iok}; c_{iok}, s_{iok}) = \frac{\bar{p}_{iok}}{\bar{p}_{\overline{iok}}} \tag{A-133}$$

Intertemporal uncertainty.

The expansion of this formulation to accommodate choice through time and over states requires an additional modification in notation. This modification is based upon the relationship between the price of a risky prospect in terms of certain consumption claims in the current period and a risky rate of interest. This is defined in exactly the same manner as the riskless rate of interest used in previous examples:

$$\bar{p}_{o,j,k} = \frac{1}{(1 + r_k)^j} \tag{A-134}$$

where:

$$\bar{p}_{o,j} = \frac{1}{(1 + \bar{r})^{j}} = \sum_{k=1}^{g} \frac{1}{(1 + r_{k})^{j}} = \sum_{k=1}^{g} \bar{p}_{o,j,k} \qquad \text{(A-135)}$$

Given the existence of the risky discount rate as defined in Equation A-134 the discount factor applied to uncertain future consumption claims can be formulated in a manner similar to the discount factor for certain claims given in Equation A-54:

$$(1 + r_{jk})^{-1} = [(1 + r_{0k})(1 + r_{1k}) \; \ldots \; (1 + r_{jk})]^{-1} \qquad \text{(A-136)}$$

$$j = 0 \text{ to } m$$
$$k = 1 \text{ to } g$$

This formulation of the risky discount rate for future conditional consumption claims allows the paradigm of intertemporal exchange to be formulated as a "Lagrangian" similar to Equation A-58).

$$\text{"L"} = U\ (c) - \lambda[\sum_{k=1}^{g} [\sum_{j=0}^{m} [[(\sum_{i=1}^{n} \bar{p}_{ijk}\, c_{ijk}) -$$

$$(\sum_{i=1}^{n} \bar{p}_{ijk}\, y_{ijk})]\ [1 + \bar{r}_{jk}]^{-1}]]] \qquad \text{(A-137)}$$

This equation can be solved as in the previous examples yielding the marginal conditions of the optimum uncertain consumption set as:

$$U'(c_{ijk}, c_{\overline{ijk}}) = \frac{\bar{p}_{jk}\ (1 + \bar{r}_{jk})^{-1}}{\bar{p}_{\overline{ijk}}\ (1 + \bar{r}_{\overline{jk}})^{-1}} \tag{A-138}$$

The paradigm of intertemporal production and exchange with uncertainty is also formulated by two "Lagrangian" equations exactly parallel to similar equations for certain conditions (Equations A-70 and A-74) and are:

$$"L" = \sum_{k=1}^{g} \left(\sum_{j=0}^{m} \left(\sum_{i=1}^{n} \bar{p}_{ijk}\ (q_{ijk} + y_{ijk})\right)(1 + \bar{r}_{jk})^{-1}\right) - \lambda$$

$$(p\ (q_{ijk} + y_{ijk})) \tag{A-139}$$

which defines marginal conditions and level of production which maximize the consumption opportunity set as:

$$\frac{\bar{p}_{ijk}\ (1 + \bar{r}_{jk})^{-1}}{\bar{p}_{ijk}\ (1 + \bar{r}_{jk})-1} = p'\ (q_{ijk} + y_{ijk} + q_{ijk} + y_{ijk}) \tag{A-140}$$

and

$$P\ (q_{ijk} + Y_{ijk}) = 0 \tag{A-141}$$

The optimum consumption set is then defined by the expression:

$$"L" = U(c) - \lambda\left[\sum_{k=1}^{g} \left[\sum_{j=0}^{m} \left[\left[\left(\sum_{i=1}^{n} \bar{p}_{ijk}\ c_{ijk}\right) - \left(\sum_{i=1}^{n} \bar{p}_{ijk}\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\overset{*}{p}_{ijk}\right)\right]\left[1 + \bar{r}_{jk}\right]^{-1}\right]\right]\right] \tag{A-142}$$

when $\overset{*}{p}_{ijk} = (q^*_{ijk} + y_{ijk})$ = the optimum level of production defined by Equation A-139.

$$W^Y/c_0 = \sum_{k=1}^{g} [\sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ijk}\, y_{ijk})(1 + \bar{r}_{jk})^{-1}] \qquad \text{(A-143)}$$

for endowed wealth:

and solved in the same manner as equation A-137.

It is important to note that the term in the "Lagrangian" equation which defines the optimum level of production (equation A-139) is a measure of wealth. This in turn leads to the definition of the "present certainty-equivalent value" as a measure of wealth which is formulated as:

$$W^P/c_0 = \sum_{k=1}^{g} (\sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ijk} p_{ijk})\ (1 + \bar{r}_{jk})^{-1}) \qquad \text{(A-144)}$$

for attained wealth:

$$W^Q/c_0 = \sum_{k=1}^{g} (\sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ijk}\, q_{ijk})(1 + \bar{r}_{jk})^{-1}) \qquad \text{(A-145)}$$

for wealth gain, and:

$$W^{\Delta Q}/c_0 = \sum_{k=1}^{g} (\sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ijk}\, \Delta q_{ijk})\ (1 + \bar{r}_{jk})^{-1}) \qquad \text{(A-146)}$$

for individual projects.

Before continuing the discussion on the application of Present Certainty-Equivalent Value criteria in investment choice the following numerical example adapted from Hirshleifer (1970) may clarify the material already introduced.

Consider a simple economy made up of a large number of individuals and a single consumption good, timber. The representative individual in which we are interested has an initial endowment consisting of 100 cunits of timber in the current period and contigent claims to 200 cunits in state A one year from now or 80 cunits in state B one year from now. This individual's opportunity set is limited to exchange and the prices for timber in state A and B are .3 and .5 respectively. These prices mean that one contingent cunit of timber in the future period in state A or B exchanges for 30 or 50 cubic feet of timber in the current period. By applying Equation A-134 the risky rates of interest for contingent consumption claims in state A and B can be found:

State A

$$.3 = \frac{1}{(1 + r_1^A)}$$

$$r_1^A = 2.33 = 233\%$$

State B

$$.5 = \frac{1}{(1 + r_1^B)}$$

$$r_1^B = 1.0 = 100\%$$

In addition, if the limiting assumption is made that A and B are the only possible states that can occur in the next period, the risk free rate of interest can be defined by equation A-135 as:

$$\bar{p}_1 = \frac{1}{(1 + r_1)} = \frac{1}{(1 + 2.33)} + \frac{1}{(1 + 1)} = .3 + .5$$

$$r_1 = .25 = 25\%$$

$$\bar{p}_1 = .8$$

Up to this point the objects of choice have been simple prospects consisting of a conditional consumptive claim and the associated probability of the existence of the required state. However, the more realistic situation is one where the objects of choice are complex prospects consisting of several possible conditional claims and probabilities. If the market price of the simple prospects which make up the complex prospect are known the price of the complex prospect is:

$$\bar{p}_x = {}_k\Sigma_{in\ x}\ (\bar{p}_k\ c_k) \qquad \text{(A-147)}$$

In order to generalize the discussion of complex prospects it is useful to introduce the concept of a unit expectation and its associated price which is defined as:

$$\bar{p}_{Ex} = \frac{\bar{p}_x}{E_x} = \frac{\sum_{k \text{ in } x} (\bar{p}_k \; c_k)}{\sum_{k \text{ in } x} (c_k \; s_k)} \qquad \text{(A-148)}$$

From Equation A-148 it can be seen that $\bar{p}_{Ex}$ is the unit price of consumption with a returns distribution of prospect (X). This unit price of prospects with a given returns distribution can be used in the case of intertemporal choices to define an interest rate for prospects with similar distributions of returns by application of Equation A-134 yielding:

$$\bar{p}_{Ex_{0,j}} = \frac{1}{(1 + r_x)^j} \qquad \text{(A-149)}$$

To illustrate this with a numerical example return to the previous example and imagine a particular prospect being offered which consists of 10 cunits of timber in state A or 6 cunits of timber in state B. The prices of the elementary claims making up the prospect are .3 and .5 respectively and applying Equation A-147 the price of this prospect would be:

$$(.3 \times 10) + (.5 \times 6) = 6$$

which is to say that the prospect under discussion would cost 6 cunits of certain current timber available for consumption. Further if the probabilities associated with states A and B are .6 and .4 respectively the expected timber available for consumption in the next period resulting from the purchase of the prospect is:

$$(.6 \times 10) + (.4 \times 6) = 8.4 \text{ cunits.}$$

The expected consumption and price of the prospect can then be used in Equation 149 to determine the unit price of expected consumption with the level of uncertainty of the prospect which is:

$$\frac{6}{8.4} = .714$$

This price in turn is used in Equation A-149 to determine the rate of discount for this prospect with its associated distribution of consumption claims:

$$.714 = \frac{1}{(1 + r_{1x})}$$

$$r_{1x} = .4$$

This is equivalent to an interest rate of 40 percent which compares with the certain rate of interest calculated earlier of 25 percent. In the text it was pointed out that the certain rate of interest is usually less than the uncertain but that this is not universally true. All that is required for the uncertain rate of interest to be less than the certain rate is for the uncertain prospect to yield more consumption in the more valued state (Hirshleifer, 1970). As an illustration consider a second prospect in the numerical example yielding 6 cunits of timber in state A and 10 in state B. The price of this prospect would be:

$$(.3 \times 6) + (.5 \times 10) = 6.8$$

cunits of certain timber in the current period. The expected consumption resulting from this prospect would be:

$$(.6 \times 6) + (.4 \times 10) = 7.6$$

cunits of timber in the next period. The unit price of expected consumption for this prospect would be:

$$\frac{6.8}{7.6} = .835$$

The rate of discount for this prospect would be:

$$.835 = \frac{1}{(1 + r_{1z}}$$

$$r_{1z} = .114$$

or 11.4 percent which is less than the certain rate of 25 percent. It is important to note that, in order for a risky prospect to have a lower rate of discount than a certain one of the same expected consumption, it must yield more consumption in the more valuable state (Hirshleifer, 1970).

Because of the need to reduce the number of time, state consumption claims associated with an investment ensemble to a single estimate of value Present Certainty-Equivalent Value criteria are usually calculated on the basis of expected value and the discounting factor appropriate to the distribution of consumption claims associated with an investment ensemble. This results in Equation A-143 to A-146 being rewritten as:

$$EW^Y/c_0 = \sum_{j=0}^{m} (\sum_{i=1}^{m} \bar{p}_{ij} \; Ey_{ij})(1 + \bar{r}_{jy})^{-1} \tag{A-150}$$

$$EW^P/c_0 = \sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ij} \; Ep_{ij})(1 + \bar{r}_{jp})^{-1} \tag{A-151}$$

$$EW^Q/c_0 = \sum_{j=0}^{m} (\sum_{j=0}^{n} \bar{p}_{ij} \; Eq_{ij})(1 + \bar{r}_{jq})^{-1} \tag{A-152}$$

$$EQ^{\Delta Q}/c_0 = \sum_{j=0}^{m} (\sum_{i=1}^{n} \bar{p}_{ij} \; E\Delta q_{ij})(1 + \bar{r}_{j\Delta q})^{-1} \tag{A-153}$$

APPENDIX B

THE ECONOMIC ANALYSIS OF STAND TREATMENTS - A SELECTED BIBLIOGRAPHY

Aarestrup, Jorgen. 1969. Average Rate of Interest on Sustained Yield Forestry Based on Historical Danish Material. Forestry. 42(1): 83-92.

Adams, D. M. and Ek, A. R. 1974. Optimizing the Management of Uneven-Aged Forest Stands. Canadian Journal of Forest Research. 4(3): 274-287.

Allen, Peter H. 1964. Should We Prune New Hampshire White Pine? How High? Fox Forest Notes No. 108. 3 pp.

Amidon, Elliott L. and Akin, Garth S. 1968. Dynamic Programming to Determine Optimum Levels of Growing Stock. Forest Science. 14(3): 287-291.

Anderson, Walter C. 1968. Rating Investments for Improving Georgia Timber Stands. Land Economics. 44(1):121-127.

Anderson, Walter C. and Gutterberg, Sam. 1971. Investor's Guide to Converting Southern Oak-Pine Types. Southern Forest Experiment Station. U.S. Forest Service Research Paper. SO-72. 10 pp.

Anderson, Sven-Olof. 1964. A Calculation Regarding Economics of Silviculture Involving Only a Few Thinnings. University of British Columbia. Faculty of Forestry Translation No. 26. 16 pp.

Bandstrom, Axel J. F. 1940. An Economic Basis for Silviculture. Journal of Forestry. 38(2):182-188.

Bennett, Frank A. 1956. Financial Aspects of Pruning Planted Slash Pine. U.S.D.A. Forest Service. Southeast Forest Experiment Station. Station Paper No. 64. 9 pp.

Bentley, William R. and Fight, Roger D. 1966. A Zero Interest Comparison of Forest Rent and Soil Rent. Forest Science. 12(4):460.

Bentley, William R. and Tecguarden, Dennis E. 1965. Financial Maturity: A Theoretical Review. Forest Science. 11(1):76-87.

Bond, W. E. 1940. Dollars-and-Cents Control in Forest Management. Southern Lumberman. 161(2033):193-196.

Bond, W. E., Wahlenberg, W. G., and Kirkland, Bert P. 1937. Profitable Management of Shortleaf and Loblolly Pine for Sustained Yield. U.S.D.A. Forest Service. Southern Forest Experiment Station. Occasional Paper No. 70. 37 pp.

Brace, L. G. and Calvert, W. W. 1969. How Investment in Pruning of White Pine Raised Lumber-Value $9.50 per 1,000 bf. Canadian Forest Industries. 89(12):36-37, 39.

Brown, G. S. 1965. The Yield of Clearwood from Pruning: Some Results with Radiata Pine. Commonwealth Forestry Review. 44(3):197-221.

Brown, G. S. 1969. Another Test of Pruning Results - Radiata Pine from Napier, New Zealand. Commonwealth Forestry Review. 48(2):144-150.

Buckman, Robert E. and Lundgren, Allen L. 1962. Three Pine Release Experiments in Northern Minnesota. U.S.D.A. Forest Service. Lake States Forest Experiment Station. Station Paper No. 97. 9pp.

Buckman, Robert E. and Wambach, R. F. 1966. Physical Responses and Economic Implications of Thinning Methods in Red Pine. Proceedings: Society of American Foresters Meeting - 1965. Society of American Foresters. Washington, D.C. pp. 185-189.

Buell, Bruce. 1937. Some Financial Aspects of Selective Logging. Journal of Forestry. 35(1):59-62.

Bull, Henry. 1934. Profit from Improving a Second-Growth Forest of Loblolly and Shortleaf Pines and Hardwoods. U.S.D.A. Forest Service. Southern Forest Experiment Station. Occasional Paper No. 38. 8pp.

Buongiorno, Joseph and Tecguarden, Dennis E. 1973. An Economic Model for Selecting Douglas-fir Reforestation Projects. Hilgardia. 42(3):35-120.

Burgers, Th. F. 1968. The Importance of O'Connor's Correlated Curve Trend (C.C.T.) Method for Use Outside South Africa. Nederlands Bosbouw-Tijdschrift. 40(10):381-392.

Callahan, John C. and Smith, Robert P. 1974. Purdue University. Agricultural Experiment Station. Research Bulletin No. 912. 20pp.

Calvert, W. W. and Brace, L. G. 1969. Pruning and Sawing Eastern White Pine. Canadian Forestry Service. Dept. of Fisheries and Forestry. Publication No. 1262. 22pp.

Campbell, Robert A. 1956. Profits from Pruning Appalachian White Pine. U.S.D.A. Forest Service. Southeast Forest Experiment Station. Station Paper No. 65. 9pp.

Carvell, K. L. and Goodspeed, A. W. 1962. From Brush to Plantation an Economic and Silvicultural Study. West Virginia Univ. Agricultural Experiment Station. Bulletin 469. 15pp.

Chapman, Gordan L. and Baker, Gregory. 1954. Planned Thinning in a Forest Stand Can Pay a Double Profit. Maine Farm Research 2(3):6-8.

Chapman, H. H. 1940. Comments on "Rotations". Journal of Forestry. 38(10):790-791.

Chapman, H. H. and Meyer, Walter H. 1947. Forest Valuation. McGraw-Hill Book Co. Inc. New York, N.Y. 521pp.

Chappelle, Daniel E. and Nelson, Thomas C. 1964. Estimation of Optimal Stocking Levels and Rotation Ages of Loblolly Pine. Forest Science. 10(4):471-502.

Cone, Bruce W. 1972. Economic Feasibility of an Integrated Cottonwood Plantation Utilizing a Nuclear Power Reactor. Journal of Forestry. 70(10):621-623.

Corty, F. L. and Struins, J. J. 1959. Pine Planting and Profits in North Louisiana. Louisiana State Univ. Agriculture and Mechanical College. Agricultural Experiment Station. Bulletin No. 525. 27pp.

Crowe, N. D. 1967. Growth, Yield and Economics of Pinus Patula in the Natal Midlands. Annale Universiteit Van Stellenbosch. 42A(2):71-152.

Davis, Grant. 1958. Profits from Pruning Ponderosa Pine in Relation to Pruning Diameter. Journal of Forestry. 56(12):905-908.

Davis, Kenneth P. 1965. A Structural Analysis of Land, Income, and Cost Values in Timber Production. Journal of Forestry 63(6):446-451.

Delaney, Richard. 1939. Selective Logging Costs Less. Journal of Forestry. 37(7):522-524.

DeVilliers, P. C., Marsh, E. K., Sonntag, A. E. and Van Wyk, J. H. 1961. The Silviculture and Management of Exotic Conifer Plantations in South Africa. Forestry in South Africa. No. 1. pp. 13-20.

Donald, D. G. M. 1961. The Effect on Yield and Profitability of Methods of Establishment Adopted for Pines Grown in the Midland Conservancy. Forestry in South Africa. No. 1. pp. 105-121.

Doran, Samuel M., Buhaly, Joseph, and Curry, Loren. 1971. Red Alder Costs and Returns for Western Washington. Washington State University. College of Agriculture. Cooperative Extension Service. EM 3461. 33pp.

Dowdle, Barney. 1962. Investment Theory and Forest Management Planning. Yale Univ. School of Forestry. Bulletin No. 67. 63pp.

Duerr, William A. 1960. Fundamentals of Forestry Economics. McGraw-Hill Book Co. Inc. New York, N.Y. 579pp.

Duerr, William A. and Bond, W. E. 1952. Optimum Selection of a Selection Forest. Journal of Forestry. 50(1):12-60.

Duerr, William A., Fedkiw, John, and Guttenberg, Sam. 1956. Financial Maturity: A Guide to Profitable Timber-Growing. U.S.D.A. Technical Bulletin No. 1146. 74pp.

Duerr, William A. and Guttenberg, Sam. 1968. Conversion Surplus as a Measure of Value. The Forestry Chronicle. 44(4):24-27.

Dutrow, G. F., McKnight, J. S., and Guttenberg, S. 1970. Investment Guide for Cottonwood Planters. Southern Forest Experiment Station. U.S. Forest Service Research Paper SO-59. 15pp.

Farquhar, John D. 1970. Financial Criteria for the Forest Manager - Another Viewpoint. Commonwealth Forestry Review. 49(1):78-85.

Fasick, Clyde A. 1965. An Evaluation of Thinning Methods by the Financial Maturity Principle. Duke Univ. Ph.D. Thesis. 318pp. (Abstract in Dissertation Abstracts. 27A(9):4933. 1966).

Faustmann, Martin. 1849. Berechnung des Werthes, welchen Waldboden sowie noch nicht haubare Holzbestande fur die Waldwirtschaft besitzen. Allgemeine Forst-und Jagd-Zeitung. 25:441-455.

Fedkiw, John and Yoho, James G. 1960. Economic Models for Thinning and Reproducing Even Aged Stands. Journal of Forestry. 58(1):26-34.

Fenton, R. 1972. Economics of Radiata Pine for Sawlog Production. New Zealand Journal of Forestry Science. 2(3):313-347.

Fenton, R. 1972. Economics of Sawlog Silviculture Which Includes Production Thinnings. New Zealand Journal of Forestry Science. 2(3):348-368.

Fenton, R. 1973. Profitability of Second Log Pruning. New Zealand Journal of Forestry Science. 3(3):313-322.

Fenton, R. and Brown, C. H. 1963. An Economic Analysis of Tending Pinus Radiata in Southland. New Zealand Journal of Forestry. 8(5):793-813.

Fenton, R. and Dick, M. Merle. 1972. Profitability of Radiata Pine Afforestation for the Export Log Trade - On Site Index 80. New Zealand Journal of Forestry Science. 2(1):69-99.

Fenton, R. and Dick, M. Merle. 1972. Profitability of Radiata Pine Afforestation for the Export Log Trade - On Site Index 110. New Zealand Journal of Forestry Science. 2(1):100-127.

Fenton, R. and Dick, M. Merle. 1972. Profitability of "Normal" Afforestation for the Overseas Log Trade on Site Indexes 95 and 110. New Zealand Journal of Forestry Science. 2(3):289-312.

Fenton, R. and Tustin, J. R. 1972. Profitability of Radiata Pine Afforestation for the Export Log Trade - On Site Index 95. New Zealand Journal of Forestry Science. 2(1):7-68.

Ferell, Raymond S. and Bentley, William R. 1969. Plantation Investment Opportunities in Black Walnut. Journal of Forestry. 67(4):250-254.

Flora, Donald F. 1964. Uncertainty in Forest Investment Decisions. Journal of Forestry. 62(6):376-380.

Flora, Donald F. 1966. Economic Guides for Ponderosa Pine Dwarf-mistletoe Control in Young Stands of the Pacific Northwest. U. S. Forest Service. Pacific Northwest Forest and Range Experiment Station. Research Paper PNW-29. 16pp.

Flora, Donald F. 1966. Economic Guides for a Method of Precommercial Thinning of Ponderosa Pine in the Northwest. Pacific Northwest Forest and Range Experiment Station. U.S. Forest Service Research Paper PNW-31. 10pp.

Flora, Donald F. 1966. A Method of Forecasting Returns from Ponderosa Pine Dwarfmistletoe Control. Pacific Northwest Forest and Range Experiment Station. U.S. Forest Service Research Paper PNW 32. 17pp.

Flora, Donald F. 1970. Economics and Policy Environments for Forest Regeneration. IN: Regeneration of Ponderosa Pine, Symposium held September 11-12, 1969. R. K. Hermann, Ed. Oregon State Univ. School of Forestry. pp. 62-63.

Forrest, W. G. 1974. Profitability of Thinning in Radiata Pine Plantations. New Zealand Journal of Forestry Science. 4(3): 529-551.

Fortson, James C. 1972. Which Criterion? Effect of Choice of the Criterion on Forest Management Plans. Forest Science. 18(4):292-297.

Gaffney, M. Mason. 1960. Concepts of Financial Maturity of Timber and Other Assets. North Carolina State College. Dept. of Agricultural Economics. A. E. Information Series No. 62. 105pp.

Ganser, David A. and Larson, David N. 1969. Pitfalls of Using Internal Rate of Return to Rank Investments in Forestry. Northeast Forest Experiment Station. U.S. Forest Service Research Note NE-106. 5pp.

Garver, R. D. 1928. Cutting the Farm Woods "Profitwise". U.S.D.A. Forest Service. Leaflet No. 30. 4pp.

Gentle, S. W., Bamber, R. K., and Humphreys, F. R. 1968. Effect of Two Fertilizers on Financial Yield and Wood Quality of Radiata Pine. Forest Science. 14(3):282-286.

Goebel, N. B., Warner, J. R. and Van Lear, D. H. 1974. Periodic Thinnings in Loblolly Pine Stands: Growth, Yield and Economic Analysis. Clemson Univ. Coll. of Forestry and Recreation Resources. Dept. of Forestry. Forest Research Series No. 28. 11pp.

Goossens, R. 1964. Method of Determining the Economics of Pruning Oak. Forestry Commission (British) Translation No. 221. 16pp.

Grah, Rudolf F. 1960. Effects of Initial Stocking on Financial Return Young-Growth Douglas-fir. Hilgardia. 29(14):613-679.

Grayson, A. J. 1962. Criteria of Profitability in Relation to Volume Production. Forestry Commission. London, England. Eighth British Commonwealth Forestry Converence. 9pp.

Grayson, A. J. 1967. Species Growth Rate and Profitability. Timber Grower No. 23. pp. 20-27.

Grut, M. 1964. Comparison of Various Rotations and Thinning Programmes for Pinus Patula on Site Index 75. Forestry in South Africa. No. 4. pp. 67-77.

Grut, Mikael. 1967. Most Profitable Silvicultural Programme for Pinus Patula and Cost of Improving Timber by Adopting Other Programmes. Forestry in South Africa. No. 8. pp. 95-115.

Grut, Mikael. 1970. Pinus Radiata Growth and Economics. A. A. Balkema. Capetown, South Africa. 234pp.

Grut, M. 1973. Methods of Estimating the Most Profitable Thinning Programme. Forestry in South Africa. No. 14. pp. 25-29.

Gunter, J. E. and Rudolph, V. J. 1968. Economics of Red Pine Release on the Fife Lake State Forest. Michigan State Univ. Agricultural Experiment Station. Quarterly Bulletin. 50(4): 507-519.

Guttenberg, Sam. 1950. The Rate of Interest in Forest Management Journal of Forestry. 48(1):3-7.

Guttenberg, Sam. 1953. A Discussion of Financial Maturity: Financial Maturity vs. Soil Rent. Journal of Forestry. 51(10):714.

Haley, David. 1966. The Economics of Forest Fertilization. Forestry Chronicle. 42(4):390-394.

Haley, David. 1966. The Importance of Land Opportunity Cost in the Determination of Financial Rotations. Journal of Forestry. 64(5):326-329.

Hallin, William E. 1956. Planting Ponderosa Pine is a Good Investment. U.S.D.A. Forest Service. California Forest and Range Experiment Station. Forest Research Note No. 104. 5pp.

Hamalainen, Jouko. 1973. Profitability Comparisons in Timber Growing: Underlying Models and Emperical Applications. Communicationes Instituti Forestalis Fe-naie. 77(4):178pp.

Harlow, C. M. 1939. Financial Rotation Versus Rotation of Highest Income. Indian Forester. 65(8):475-481.

Hawley, Ralph C. and Clapp, Robert T. 1935. Artificial Pruning in Coniferous Plantations. Yale Univ. School of Forestry Bulletin No. 39. 60pp.

Heiberg, Svend O. 1942. Cutting Based Upon Economic Increment. Journal of Forestry. 40(8):645-650.

Herrick, Owen W. and Morse, J. Edwin. 1968. Investment Analysis of Stand Improvement and Reforestation Opportunities in Appalachian Forests. Northeast Forest Experiment Station. U.S. Forest Service Research Paper NE-111. 43pp.

Hetherington, J. C. 1969. An Economic Evaluation of Alternative Stand Treatments in Relation to Understory Vegetation and Subsequent Regeneration Costs. Forestry. 42(1):47-68.

Hiley, W. E. 1927. A Critical Note on Some Recent Literature on Forest Economics. Forestry. 1:97-107.

Hiley, W. E. 1939. The Most Paying Rotation in Indian Forestry. Indian Forester. 65(8):471-475.

Hiley, W. E. 1954. Woodland Management. Faber and Faber Ltd. London, England. 463pp.

Hiley, W. E. 1956. Economics of Plantations. Faber and Faber Ltd. London, England. 216pp.

Hiley, W. E. 1956. Underplanting of Japanese Larch. Quarterly Journal of Forestry. 50(3):189-196.

Hosner, John F. and Lane, Richard D. 1953. Making Farm Woodland Improvement Pay. U.S.D.A. Forest Service. Central States Forest Experiment Station. Technical Paper No. 133. 12pp.

Hoyer, Gerald E. 1971. Fertilization Response Estimates 1969: How Much? What is it worth? What Does it Mean? State of Washington. Dept. of Natural Resources Report No. 19. 13pp.

Huey, Ben M. 1950. The Profit in Pruning Western White and Ponderosa Pine. U.S.D.A. Forest Service. Northern Rocky Mountain Forest and Range Experiment Station. Research Note No. 85. 6pp.

Hughes, Jay M. and Post, Boyd W. 1973. Economic Considerations in Forest Fertilization. IN: Forest Fertilization Symposium Proceedings. Northeast Forest Experiment Station. U.S. Forest Service General Technical Report NE-3. pp. 45-54.

Johnson, Norman E. 1963. Some Economic Considerations in Planning Control of Insects Affecting Young Trees. Journal of Forestry. 61(6):426-429.

Johnson, D. R., Grayson, A. J., and Bradley, R. T. 1967. Forest Planning. Faber and Faber Ltd. London, England. 541pp.

Joubert, A. F. 1966. Some Economical Aspects of Growing Pulpwood. South African Forestry Journal No. 58. pp. 3-11.

Kearns, R. S. 1940. Rotations. Journal of Forestry. 38(4):339-340.

Keipi, Kari and Kekkonen, Oho. 1970. Calculations Concerning the Profitability of Forest Fertilization. Folia Forestalia. No. 84. 23pp.

Keister, Thomas D. 1972. Thinning Slash Pine Plantation Results in Little Growth or Economic Gain after 40 Years. Louisiana State Univ. Agricultural and Mechanical College. School of Forestry and Wildlife Management. Agricultural Experiment Station. Research Release L.S.U. Forestry Notes. Note #102. 3pp.

Kidd, W. E. 1969. Forest Regulation and the Alternative Rate, A Case Illustration. Journal of Forestry. 67(12):885-887.

Kilkki, Pekka. 1968. Some Economic Aspects of Growing Forest Stands. Silva Fennica. 2(4):225-234.

Kilkki, Pekka. 1971. Optimization of Stand Treatment Based Upon the Marginal Productivity of Land and Growing Stock. Acta Forestalia Fennica. No. 102. 22pp.

Kilkki, Pekka and Vaisanen, Unto. 1969. Determination of the Optimum Cutting Policy for the Forest Stand by Means of Dynamic Programming. Acta Forestalia Fennica. No. 102. 22pp.

King, D. B., Stoltenberg, C. H., and Marty, R. J. 1960. The Economics of White Pine Blister Rust Control in the Lake States. U.S.D.A. Forest Service. 91pp.

Laurie, M. V. 1939. The Most Paying Rotation in Indian Forestry. Indian Forester. 65(7):393-403.

Lembersky, Mark R. and Johnson K. Norman. 1975. Optimal Policies for Managed Stands: An Infinite Horizon Markov Decision Process Approach. Forest Science. 21(2):109-122.

Lerche, Cai and Saecd, Akhtar. 1967. A Study on the Rotation and Economic Management of the Coniferous Forests in West Pakistan. The Pakistan Journal of Forestry. 17(1):81-118.

Lewis, Gordon D. and Chappelle, Daniel E. 1964. Farm Woodland Management Costs and Returns in the Southern Piedmont of Virginia. Southeast Forest Experiment Station. U.S. Forest Service Research Paper SE-15. 20pp.

Lewis, N. B. 1965. Economics of Pruning for Long Lumber in South Australia. Australian Forestry. 29(3):149-160.

Linnard, W. and Gane, M. 1968. Martin Faustmann and the Evolution of Discounted Cash Flow. Commonwealth Forestry Univ. of Oxford. Institute Paper No. 42. 55pp.

Lizardo, Leonor. 1952. Benquet Pine (*Pinus insularis* Endle.) as a Reforestation Crop. The Philippine Journal of Forestry. 7(1st to 4th quarter 1950):43-62.

Lugton, G. S. 1963. Economics of Plantations. IN: Seminar on Management of Low Site Quality Plantations, Moss Vale, March 1963. Australia, New South Wales Forestry Commission. Technical Paper No. 1. pp. 8-18.

Lugton, G. S. 1963. The Use of the Faustmann Formula in Determining the Comparative Profitability of Different Treatments in Plantations. IN: Seminar on Economics of Forestry. A. J. Leslie. New South Wales (Australia) Forestry Commission. Technical Paper No. 3 pp. 50-66.

Lundgren, Allen L. 1961. Investment Opportunities in Regenerating Black Spruce are Greatly Affected by Site. U.S.D.A. Forest Service. Lake States Forest Experiment Station Technical Note No. 611. 2pp.

Lundgren, Allen L. 1963. An Economic Analysis of Three Pine Release Experiments in Northern Minnesota. Forest Science. 9(2):242-256.

Lundgren, Allen L. 1965. Thinning Red Pine for High Investment Returns. Lake States Forest Experiment Station. U.S. Forest Service Research Paper LS-18. 20pp.

Lundgren, Allen L. 1966. Estimating Investment Returns from Growing Red Pine. North Central Forest Experiment Station. U.S. Forest Service Research Paper NC-2. 48pp.

Lundgren, Allen L. 1971. Ranking Investment Alternatives - A New Look. Journal of Forestry. 69(9):568-573.

Lundgren, Allen L. 1972. A Correct Expression for Continuous Discounting. Forest Science. 18(1):95.

Lundgren, Allen L. 1973. Cost-Price: A Useful Way to Evaluate Timber Growing Alternatives. North Central States Forest Experiment Station. U.S. Forest Service Research Paper NC-95. 16pp.

Lundgren, Allen L. 1973. The Allowable Cut Effect: Some Further Extensions. Journal of Forestry. 71(6):357, 360.

Lyford, C. A. 1934. Application of Economic Selection to Logging Operations in the Douglas-fir Region. Journal of Forestry. 32(7):716-724.

MacKenzie, J. M. D. 1946. Some Financial Speculations. The Scottish Forestry Journal. 60:29-37.

Malac, Barry F. 1966. Twenty-Year Old Slash Pine Plantation Responds to Fertilization. Union Bag - Camp Paper Corp. Woodland Research Note No. 15. 4pp.

Mann, William F. Jr. 1951. Profits from Release of Loblolly and Shortleaf Pine Seedlings. Journal of Forestry. 49(4):250-253.

Manogaran, Chelvadurai. 1973. Economic Feasibility of Irrigating Southern Pines. Water Resources Research. 9(6):1485-1496.

Manthy, Robert S. 1970. An Investment Guide for Cooperative Forest Management in Pennsylvania. Northeast Forest Experiment Station. U.S. Forest Service Research Paper NE-156. 59pp.

Manthy, R. S., Rannard, C. D., and Rudolph. V. J. (No Date). The Profitability of Red Pine Plantations. Michigan State Univ. Agricultural Experiment Station Research Report 11. 11pp.

Markus, Rudolfs. 1956. Stand Establishment Costs and the Theory of Relative Forest Rent. Journal of Forestry. 54(8):503-505.

Marty, Robert. 1964. Analyzing Uncertain Timber Investments. Northeast Forest Experiment Station. U.S. Forest Service Research Paper No. NE-23. 21pp.

Marty, Robert. 1970. The Composite Internal Rate of Return. Forest Science. 16(3):276-279.

Marty, Robert J. and Allison, Glenn R. 1960. Appraising White Pine Weevil Control Opportunities. Journal of Forestry. 58(3):203-206.

Marty, Robert and Mott D. Gordon. 1963. Evaluating and Scheduling White Pine Weevil Control in the Northeast. Northeast Forest Experiment Station. U.S. Forest Service Research Paper No. NE-19. 56pp.

Marty, Robert, Rindt, Charles, and Fedkiw, John. 1966. A Guide for Evaluating Reforestation and Stand Improvement Projects: In Timber Management Planning on the National Forests. U.S.D.A. Agriculture Handbook No. 304. 24pp.

McCauley, Orris D. and Marquis, David A. 1972. Investment in Precommercial Thinning of Northern Hardwoods. Northeast Forest Experiment Station. U.S. Forest Service Research Paper NE-245. 12pp.

McCauley, Orris D. and Trimble, George R. Jr. 1972. Forestry Returns Evaluated for Uneven-Aged Management in Two Appalachian Woodlots. Northeast Forest Experiment Station. U.S. Forest Service Research Paper NE-244. 12pp.

McClay, T. A. 1955. Economic Considerations in Douglas-fir Stand Establishment. U.S.D.A. Forest Service. Pacific Northwest Forest and Range Experiment Station. Research Paper No. 15. 10pp.

McConnen, Richard J. and Amidon, Elliot L. 1970. A Computer-Based Approach for Evaluating Plantation Alternatives - A Case Study of *Pinus contorta* in Ireland. Forestry. 43(1):31-43.

McKillop, William. 1971. Land Value, Logging Costs, and Financial Maturity. The Forestry Chronicle. 47(4):210-214.

Morais, Roger. 1954. Research Project No. 14 Thinning in Balsam Fir Stands 40 to 50 Years of Age. Causapseal Forestry Research Station. Research Note No. 4. 2pp.

Murphy, Paul A. 1973. Choice of Criteria for Forest Planning Under Imperfect Capital Market Conditions. Univ. of Georgia. Ph.D. Thesis. 97pp. (Abstract in Dissertation Abstracts International B. 34(6):2394-B. Dec. 1973).

Naslund, Bertil. 1969. Optimal Rotation and Thinning. Forest Science 15(4):446-451.

Osmond Smith, F. A. 1908. The Conversion of Underwood and Coppiee-with Standards into Highwood. Quarterly Journal of Forestry. 2(3):154-165.

Payandeh, Bijan. 1973. Analysis of a Forest Drainage Experiment in Northern Ontario II: An Economic Analysis. Canadian Journal of Forestry Research. 3(3):399-408.

Pearse, P. H. 1967. The Optimum Forest Rotation. The Forestry Chronicle. 43(2):178-195.

Pennefather, M. and MacGillivray, F. 1971. Fertilizing of *Eucalyptus grandis* (Saligna) on Old Wattle Soils. South African Forestry Journal. No. 76. pp. 1-5.

Petrini, Sven (Translated by Mark L. Anderson). 1953. Elements of Forest Economics. Oliver and Boyd. Edinburgh, Scotland. 210pp.

Porterfield, Richard Lee. 1972. Financial Returns from Managing Southern Hardwood Stands for Pulpwood. Journal of Forestry. 70(10):624-627.

Price, Colin. 1973. To the Future: With Indifference or Concern? - The Social Discount Rate and Its Implications in Land Use. Journal of Agricultural Economics. 24(2):393-398.

Reynolds, R. R. 1939. Improvement Cuttings in Shortleaf and Loblolly Pine. U.S.D.A. Forest Service. Southern Forest Experiment Station. Occasional Paper No. 81. 4pp.

Reynolds, R. R. 1939. Improvement Cuttings in Shortleaf and Loblolly Pine. Journal of Forestry. 37(7):568-570.

Reynolds, R. R., Bond, W. E., and Kirkland, Bert P. 1944. Financial Aspects of Selective Cutting in the Management of Second-Growth Pine-Hardwood Forests West of the Mississippi River. U.S.D.A. Technical Bulletin No. 861. 118pp.

Rickard, Wesley M., Hughes, Jay M., and Newport, Carl A. 1967. Economic Evaluation and Choice in Old-Growth Douglas-fir Landscape management. Pacific Northwest Forest and Range Experiment Station. U.S. Forest Service Research Paper PNW-49. 33pp.

Riley, L. F. 1973. Operational Trials of Techniques to Improve Jack Pine Spacing. Canadian Forestry Service. Dept. of Environment. Great Lakes Forest Research Center. Information Report O-X-180. 29pp.

Sassaman, Robert W. 1972. Economic Returns from Planting Forage in National Forests. Journal of Forestry. 70(8):487-488.

Sassaman, Robert W., Barrett, James W., and Smith, Justin G. 1973. Economics of Thinning Stagnated Ponderosa Pine Sapling Stands in the Pine Grass Areas of Central Washington. Pacific Northwest Forest and Range Experiment Station. U.S. Forest Service Research Paper PNW-144 (Revised). 17pp.

Schlich, William. 1895. A Manual of Forestry - Volume III Forest Management. Bradbury, Agnew, and Company Ltd. London, England. 397pp.

Schreuder, Gerard F. 1971. The Simultaneous Determination of Optimal Thinning Schedule and Rotation for an Even-Aged Forest. Forest Science. 17(3):333-339.

Schweitzer, Dennis L. 1968. Evaluating Timber Production Investments Under Uncertainty. Univ. of Minnesota. Ph.D. Thesis. 101pp. (Abstract in Dissertation Abstracts. 29B(8):2669-2670. Feb. 1969).

Schweitzer, Dennis L. 1970. The Impact of Estimation Errors on Evaluation of Timber Production Opportunities. North Central Forest Experiment Station. U.S. Forest Service Research Paper NC-43. 18pp.

Schweitzer, Dennis L., Sassaman, Robert W., and Schallan, Con H. 1972. Allowable Cut Effect: Some Physical and Economic Implications. Journal of Forestry. 70(7):415-418.

Schweitzer, Dennis L., Sassaman, Robert W., and Schallan, Con H. 1973. The Allowable Cut Effect: A Reply. Journal of Forestry. 71(4):227.

Sertmehmetoglu, Z., Acar, O., and Birler, A. Sencer. 1972. Some Investigations on Financial Maturity of Poplar Plantations: Applied to Cultivation of (*P.* x *euramericana* cu. 1-214). Turkey. Poplar and Fast Growing Forest Trees Research Institute. 24pp.

Shaw, Elmer W. and Staebler, George R. 1950. Financial Aspects of Pruning. U.S.D.A. Forest Service Pacific Northwest Forest and Range Experiment Station. 45pp.

Shepard, Ward. 1925. The Bogey of Compound Interest. Journal of Forestry. 23(3):251-259.

Sherry, S. P. 1967. Is High Pruning of Pinus Patula Economically Justifiable? Forestry in South Africa. No. 8. pp. 87-94.

Simmons, Donald W. 1974. The Economics of Late Rotation Fertilization of Loblolly Pine Pulpwood Stands. North Carolina State Univ. School of Forest Resources. Technical Report No. 51. 44pp.

Sinden, John A. 1964. An Economic Analysis to Aid the Marginal Decision on Rotation Length: I. Presentation of the Principle. Forestry. 37(2):161-178.

Sinden, John A. 1965. An Economic Analysis to Aid the Marginal Decision on Rotation Length: II. Development of the Technique. Forestry. 38(2):201-217.

Sinden, J. A. 1968. Technical and Financial Maturity in Radiata Pine. Australian Forestry. 32(1):15-25.

Smith, H. Donnell. 1973. Economics of Hardwood Plantations. IN: Proceedings: Twelfth Southern Forest Tree Improvement Conference Baton Rouge, Louisiana, June 12-13, 1973. Louisiana State Univ. Division of Continuing Education. pp. 158-168.

Smith, J. Harry G. 1954. The Economics of Pruning. The Forestry Chronicle. 30(2):197-214.

Stern, R. C. 1970. Rotation Lengths for Conifers. Quarterly Journal of Forestry. 64(4):297-302.

Stern, R. C. 1971. Pruning Free Grown Hardwoods. Quarterly Journal of Forestry. 65(4):322-326.

Stern, R. C. 1972. Poplar Growing at Close Spacing. Quarterly Journal of Forestry. 66(3):230-235.

Sutton, W. R. J. 1968. Initial Spacing and Financial Return of Pinus Radiata on Coastal Sands. New Zealand Journal of Forestry. 13(2):203-219.

Tecguarden, Dennis E. 1968. Economics of Replacing Young-Growth Ponderosa Pine Stands ... A Case Study. Pacific Southwest Forest and Range Experiment Station. U.S. Forest Service Research Paper PSW-47. 16pp.

Tecguarden, Dennis E. 1969. Economic Criteria and Calculated Risk in Reforestation Investment Decisions. Journal of Forestry. 67(1): 25-31.

Tecguarden, Dennis E. 1973. The Allowable Cut Effect: A Comment. Journal of Forestry. 71(4):224-226.

Thomas, A. G. 1965. The Comparative Profitability of Larch and Beech in the Cotswolds. Quarterly Journal of Forestry. 59(1):56-60.

Thompson, Emmett F. 1968. The Theory of Decision Under Uncertainty and Possible Applications in Forest Management. Forest Science. 14(2):156-163.

Thompson, Roy B. 1940. Are Similar Results Always Obtained by use of Soil Rent and Forest Rent Procedures? Journal of Forestry. 38(10):792-793.

Tingle, A. C. and VanLaar, A. 1972. The Economics of Thinning in Poplar Plantations. Univ. Stellenbosch. Faculty of Forestry. Communication No. 31. 18pp.

VanLaar, A. and Tingle, A. C. 1965. Economic Aspects of Poplar Growing for Matchwood Production. South African Forestry Journal No. 53. pp. 14-24.

Walker, Nat. 1969. Economic and Management Models for Cottonwood in Central Oklahoma. Oklahoma State Univ. Agricultural Research Bulletin B-664. 18pp.

Wambach, Robert F. 1967. A Silvicultural and Economic Appraisal of Initial Spacing in Red Pine. Univ. of Minnesota. Ph.D. Thesis. 294 pp. (Abstract in Dissertation Abstracts. 28B(6):2210. Nov. 1967).

Ward, W. W. 1958. A Mensurational and Economic Study of Thinning in the Carbaugh White Pine Plantation. Penn. State Univ. The Penn State Forest School. Research Paper No. 26. 16pp.

Wardle, P. A. 1967. Spacing in Plantations: A Management Investigation. Forestry. 40(1):47-69.

Watt, A. J. 1967. A Comparison of Some Basic Concepts of Rotation Age. Australian Forestry. 31(4):275-286.

Webster, Henry H. 1965. Profit Criteria and Timber Management. Journal of Forestry. 63(4):260-266.

Whaley, Ross S. 1969. Economic Guidelines for Timber Management Investments in Michigan. Univ. of Michigan. Ph.D. Thesis. 130pp. (Abstract in Dissertation Abstracts. 30B(5):1976, Nov. 1969).

Whiteley, D. 1971. The Relationship of Savings in Harvesting Pruned Trees to the Cost of Pruning. Australian Forest Research. 5(2): 51-57.

Whyte, A. G. D. 1965. The Influence of Thinning on Taper, on Volume Assortment Outturn and on Economic Return of the Bowmont Spruce Sample Plots. Commonwealth Forestry Review. 44(2):109-122.

Wiksten, Ake. 1970. An Economic Guide to Silviculture. The Forestry Chronicle. 46(3):214-216.

Wikstrom, J. H. and Alley, Jack R. 1968. Ranking Treatment Opportunities in Existing Timber Stands on White Pine Land in the Northern Region. Intermountain Forest and Range Experiment Station. U.S. Forest Service Research Paper INT-46. 75pp.

Wikstrom, John H. and Willner, Charles A. 1961. The Opportunity to Thin and Prune in the Northern Rocky Mountain and Intermountain Regions. U.S.D.A. Forest Service. Intermountain Forest and Range Experiment Station. Research Paper 61. 14pp.

Williston, Hamlin L. 1967. Thinning Desirable in Loblolly Pine Plantations in West Tennessee. Southern Forest Experiment Station. U.S. Forest Service Research Note SO-61. 7pp.

Worley, David P. 1970. The "Let-It-Grow" Treatment for Timber - Is It Economically Worthwhile? Northeast Forest Experiment Station. U.S. Forest Service Research Paper NE-157. 37pp.

Worley, David P. and Wheeland, Hoyt A. 1968. An Economic Evaluation of Cull-Tree Removal in Mixed Hardwood Stands. Northeast Forest Experiment Station. U.S. Forest Service Research Note NE-82. 5pp.

Worrell, Albert C. 1953. A Discussion of Financial Maturity: Financial Maturity: A Questionable Concept in Forest Management. Journal of Forestry. 51(10):711-714.

Worthington, Norman P. 1957. Some Economic Considerations in Thinning Douglas-fir. U.S.D.A. Forest Service. Pacific Northwest Forest and Range Experiment Station. Research Note No. 137. 8pp.

Worthington, Norman P. and Fedkiw, John. 1964. Economic Considerations in Management of Douglas-fir Growing Stock. Pacific Northwest Forest and Range Experiment Station. U.S. Forest Service Research Paper PNW-88. 31pp.

Zasada, Zigmond A. 1952. Does It Pay to Thin Young Aspen. Journal of Forestry. 50(10):747-748.

Zillgitt, W. M. 1948. Optimum Economic Stocking for Northern Hardwoods. U.S.D.A. Forest Service. Lake States Forest Experiment Station. Station Paper No. 10. 14pp.

APPENDIX C

SUPPLEMENTARY DETAILS FOR THE DEMONSTRATION ANALYSIS

The material in this appendix is to supplement the demonstration analysis in the body of the thesis. This material consists of:

1. A detailed description in the form of a flow chart, of the sequence of steps in the analysis procedure.

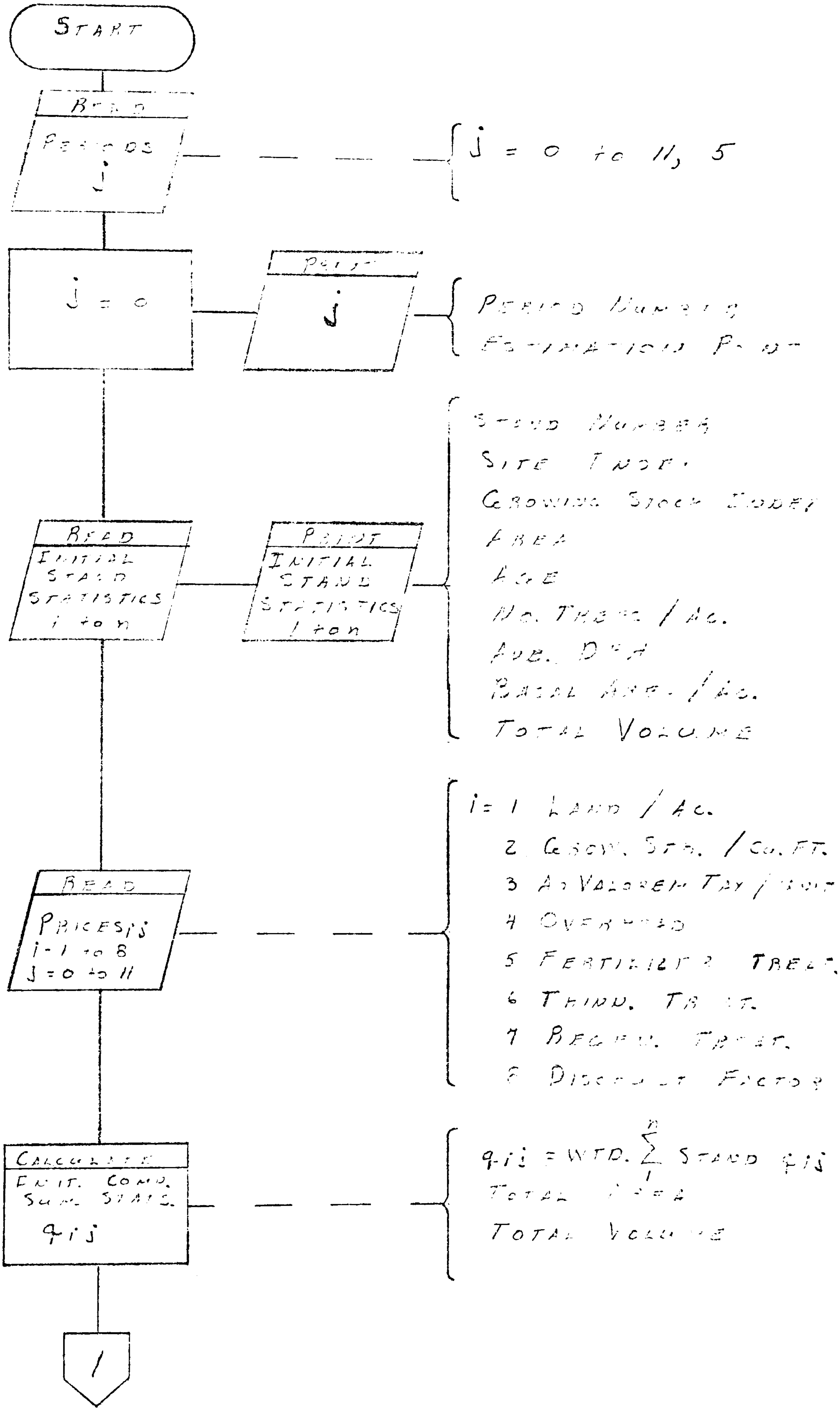
2. A discussion of the sources of information, and steps taken in the development of quantity and price estimates used in the analysis.

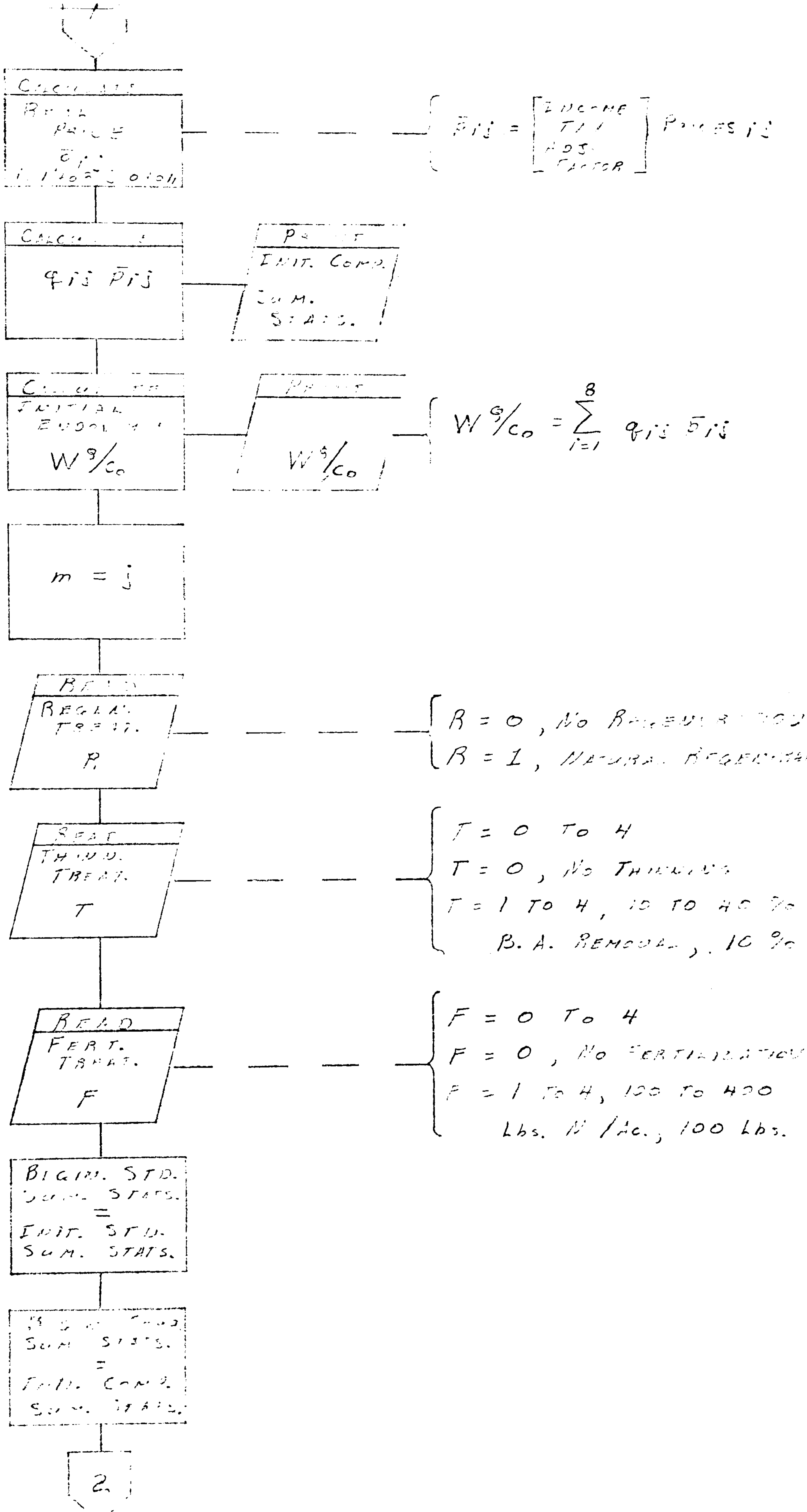
3. Abbreviated yield tables for Douglas-fir which are the basis for estimates of the quantity of growing stock, and yield in the demonstration analysis.

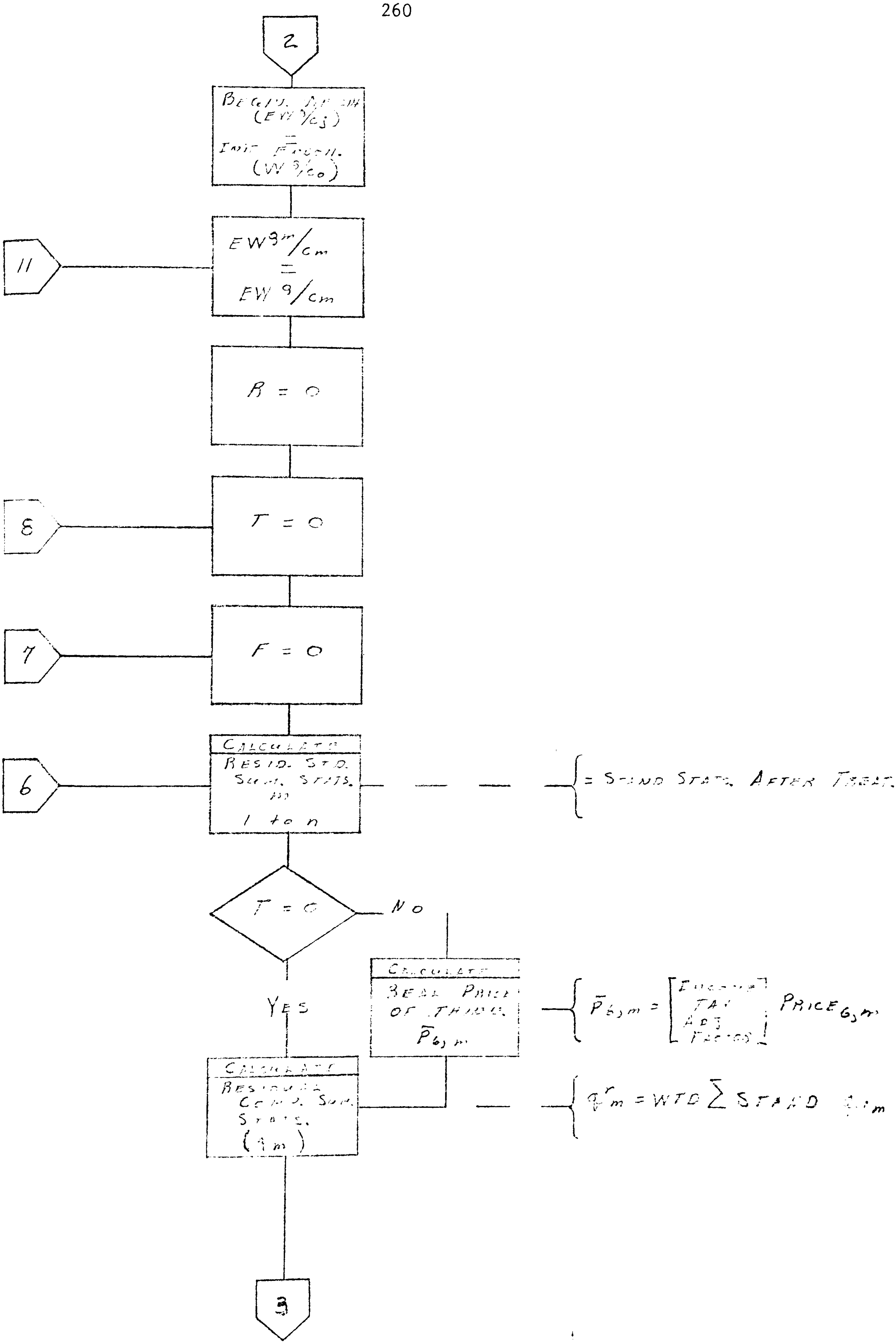
Flow Chart of Analysis Procedure

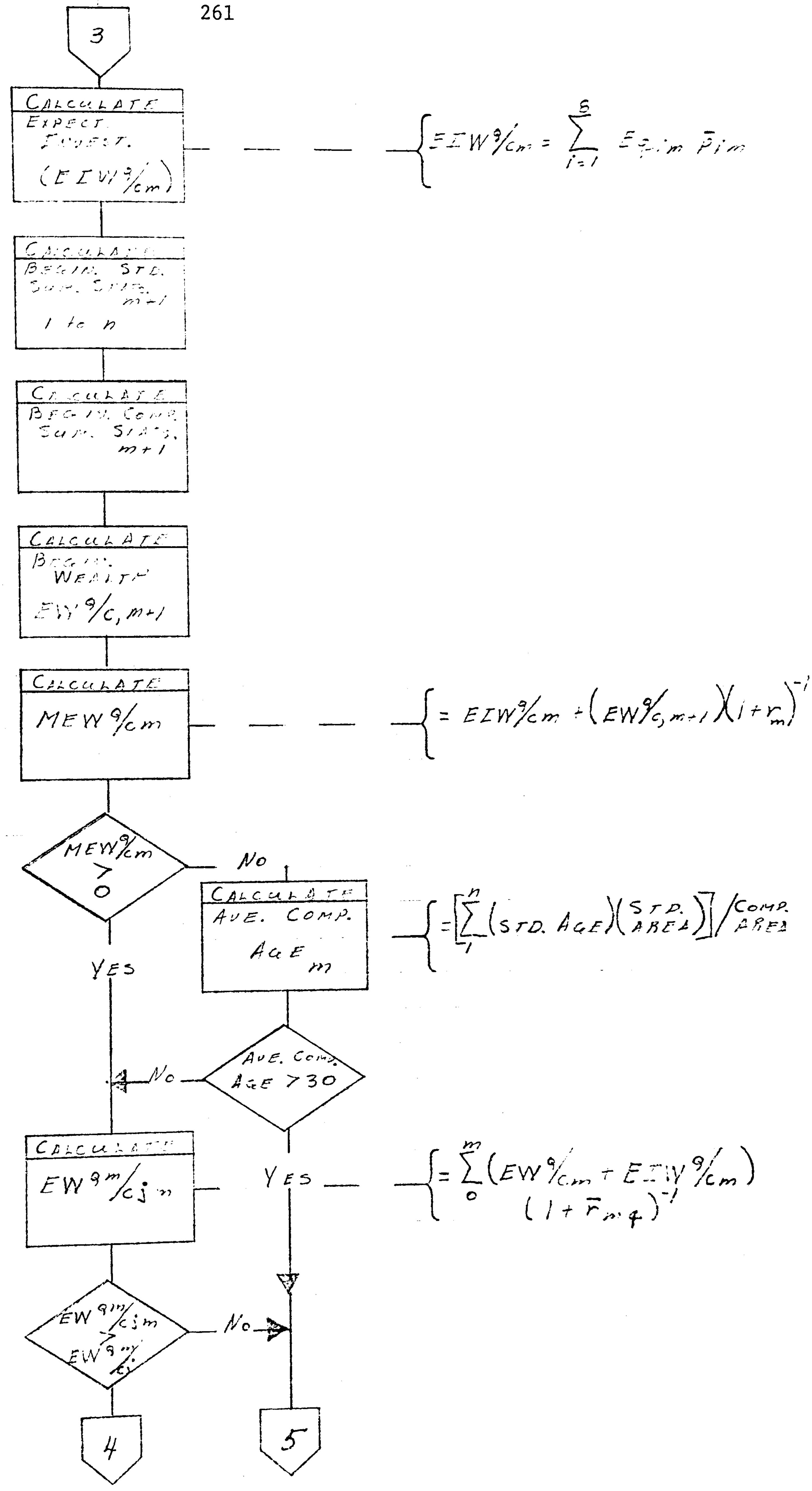
The flow chart of the analysis procedure given in Figure C-1 is a detailed description of the sequency of steps followed in the demonstration analysis. This flow chart can also be used as a guide for the development of a computerized system for the economic analysis of stand treatments for timber production.

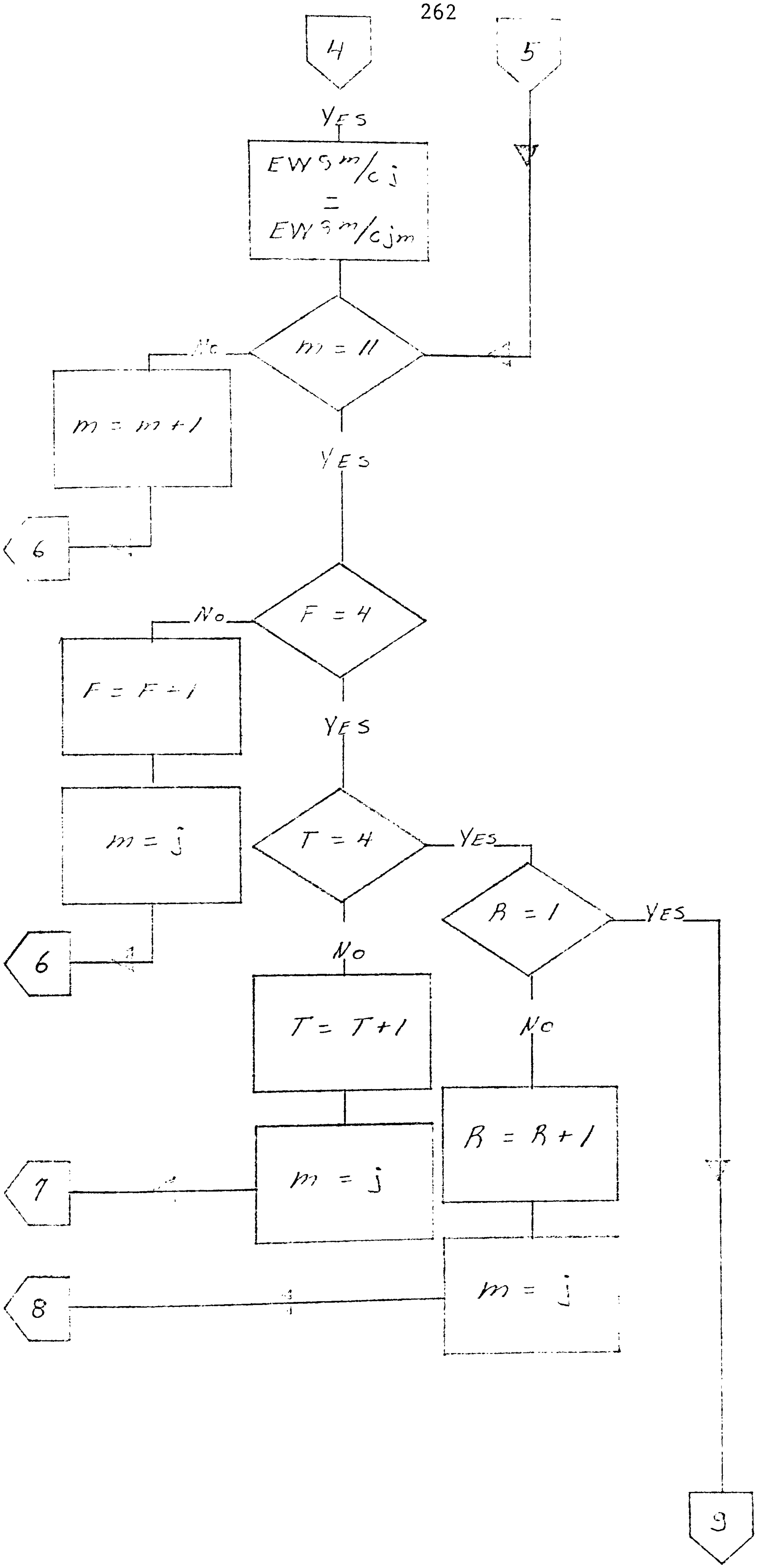
Figure C-1

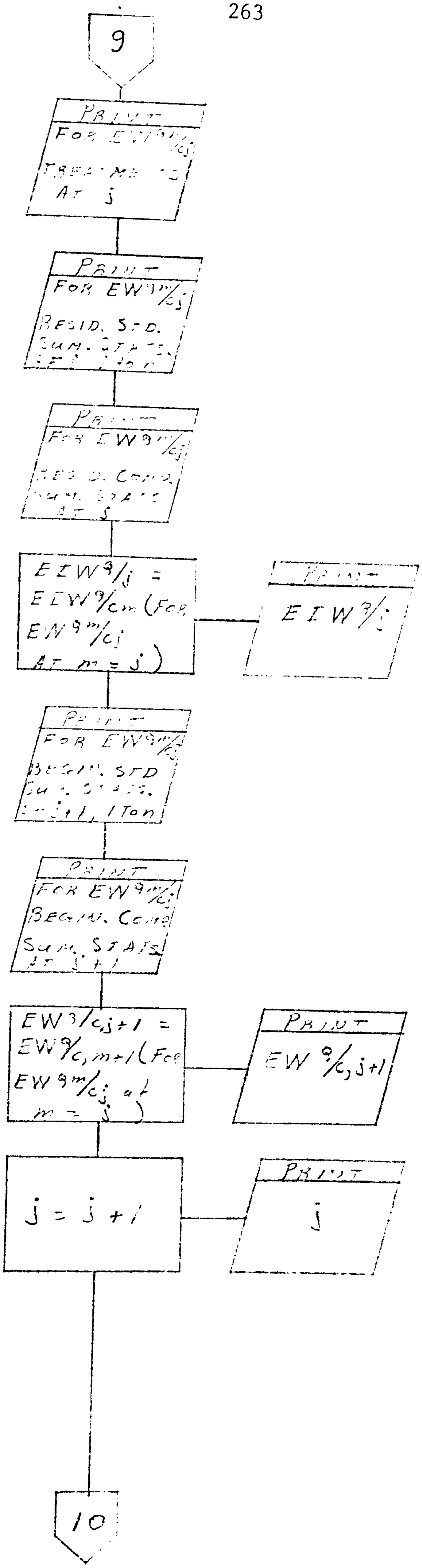

START
READ
PERIODS
j
j = 0 to 11, 5
j = 0
PRINT
j
PERIOD NUMBER
READ
INITIAL
STAND
STATISTICS
i to n
PRINT
INITIAL
STAND
STATISTICS
i to n
STAND NUMBER
SITE INDEX
AREA
AGE
NO. TREES / AC.
AVE. DBH
TOTAL VOLUME
i = 1 LAND / AC.
READ
PRICES$_{ij}$
i = 1 to 8
j = 0 to 11
2 GROW. STK. / CU. FT.
3 AD VALOREM TAX
4 OVERHEAD
5 FERTILIZER TREAT.
6 THINN. TREAT.
7 REGEN. TREAT.
8 DISCOUNT FACTOR
CALCULATE
INIT. COND.
SUM. STATS.
q_{ij}
TOTAL VOLUME
1

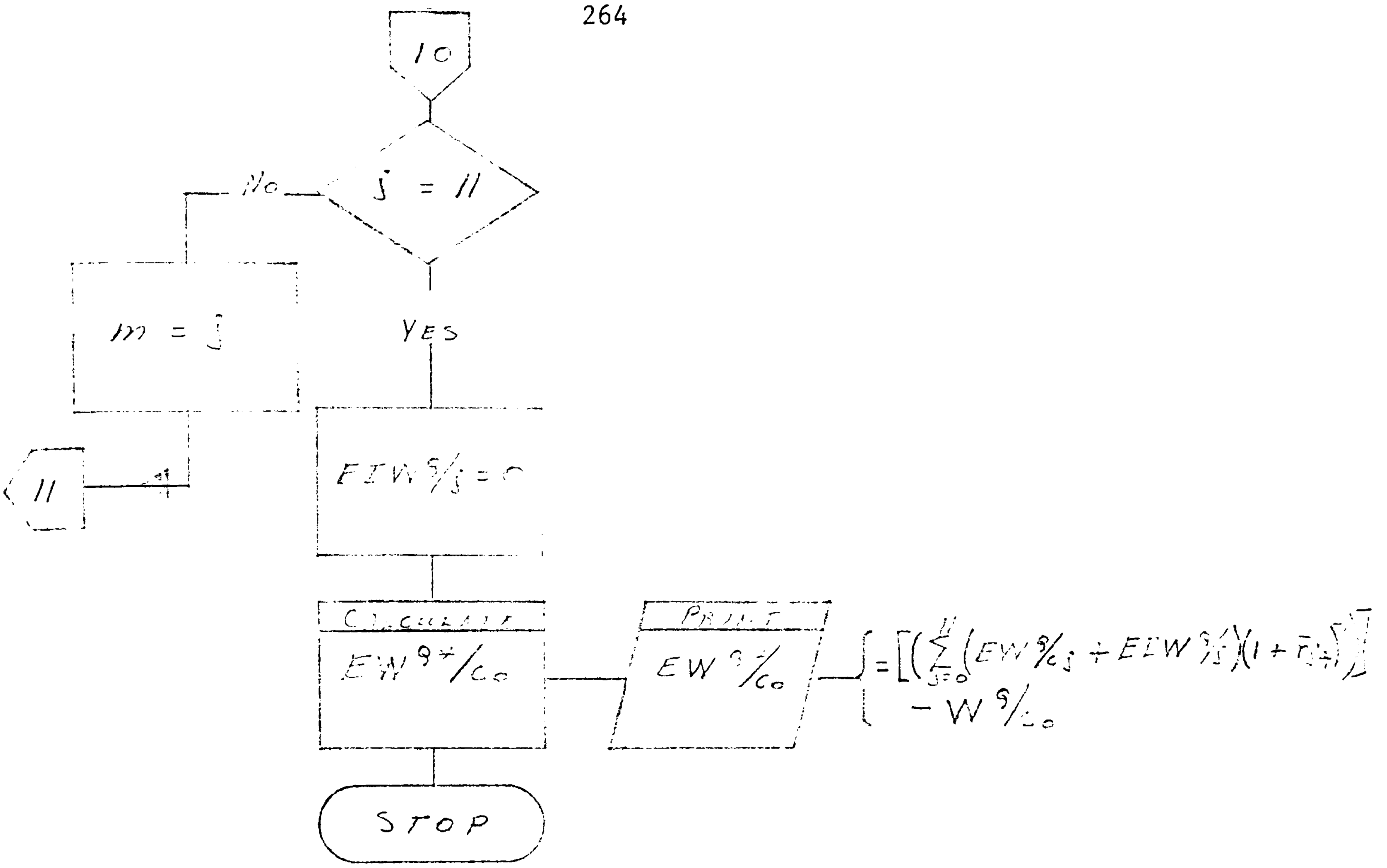

qis Pis
Init. Comp.
Sum.
Stats.
W g/Co
$W\,g/C_o = \sum_{i=1}^{8} q_{is}\,P_{is}$
m = j
R
T = 0 To 4
T = 1 To 4 , 10 To 40 %
B. A. Removal , 10 %
Read
Fert.
Treat.
F
F = 0 To 4
F = 1 To 4, 100 To 400
Lbs. N /Ac., 100 Lbs.
Begin. Std.
Sum. Stats.
=
Init. Std.
Sum. Stats.
Sum. Stats.
=
Sum. Stats.
2


2
EW9m/cm = EW9/cm
11
R = 0
8
T = 0
7
F = 0
6
CALCULATE
RESID. STD. SUM. STATS. 1 to n
T = 0
NO
YES
CALCULATE
REAL PRICE OF THINN. P̄6,m
CALCULATE
RESIDUAL COMP. SUM. STATS. (qm)
= WTD Σ STAND
PRICE6,m
3


3
CALCULATE
EXPECT.
INVEST.
(EIW g/cm)
EIW g/cm = Σ (i=1 to 5) E g im P̄ im
CALCULATE
BEGIN. STD.
SUM. STATS.
m+1
1 to n
CALCULATE
BEGIN. COMP.
SUM. STATS.
m+1
CALCULATE
BEGIN.
WEALTH
EW g/c,m+1
CALCULATE
MEW g/cm
= EIW g/cm + (EW g/c,m+1)(1+r m)^-1
MEW g/cm > 0
No
YES
CALCULATE
AVE. COMP.
AGE m
= [Σ (1 to n) (STD. AGE)(STD. AREA)] / COMP. AREA
AVE. COMP. AGE > 30
No
YES
CALCULATE
EW gm/cjm
= Σ (0 to m) (EW g/cm + EIW g/cm) (1 + r̄ m4)^-1
EW gm/cjm > EW gm/cj
No
4
5


4
5
YES
EWSm/cj = EWSm/cjm
m = 11
No
YES
m = m+1
6
F = 4
No
YES
F = F+1
m = j
6
T = 4
YES
R = 1
YES
No
T = T+1
No
R = R+1
m = j
7
m = j
8
9


9
PRINT
FOR EW9m/cj
AT j
PRINT
FOR EW9m/cj
RESID. STD.
PRINT
FOR EW9m/cj
AT j
EIW9/j =
EIW9/cm (FOR
EW9m/cj
AT m = j)
PRINT
EIW9/j
PRINT
FOR EW9m/cj
BEGIN. STD
j+1, ITon
PRINT
FOR EW9m/cj
BEGIN. COMB
SUM. STATS
AT j+1
EW9/c,j+1 =
EW9/c,m+1 (FOR
EW9m/cj at
m = j)
PRINT
EW9/c,j+1
j = j + 1
PRINT
j
10


10
j = 11
No
YES
m = j
11
EIW%j = 0
CALCULATE
EW%/Co
PRINT
EW%/Co
= [(Σ (EW%cj + EIW%j)(1 + rj)] − W%/Co
STOP

Inputs and Outputs

The material on the sources of information and preparation of estimates will be considered in two classes. The first will cover material pertaining to estimates of the quantity of inputs and outputs. The second with material pertaining to price estimates.

Quantity. The sources of information and methods of estimation used to forecast the quantity of inputs and outputs in the analysis will be discussed for each class of input or output.

The information on the quantity and quality of the land component in the compartment under analysis is derived from the soil survey of the Weyerhaeuser Company Vail Tree Farm (Steinbrenner, and Gehrke, 1964). A compartment for the analysis was selected with the aid of the soil survey based upon the existing road system, topographic features and soil characteristics. The compartment used for the analysis is located in sections 13-15, 21-23; township 16 north; range 1 west; Thurston County, Washington (Figure C-2). The area of the compartment and the stands within the compartment are measured by optical planimeter. The area of the compartment and its component stands are summarized in Table C-1.

The estimates of growing stock and yield used in the analysis are developed using a computerized system for the estimation of the yield of natural stands of Douglas-fir developed by Bower and Shaw (1975). The following

Figure C-2

Soil map showing compartment boundaries, location, and terrain features.

Vail Tree Farm
SOIL SURVEY

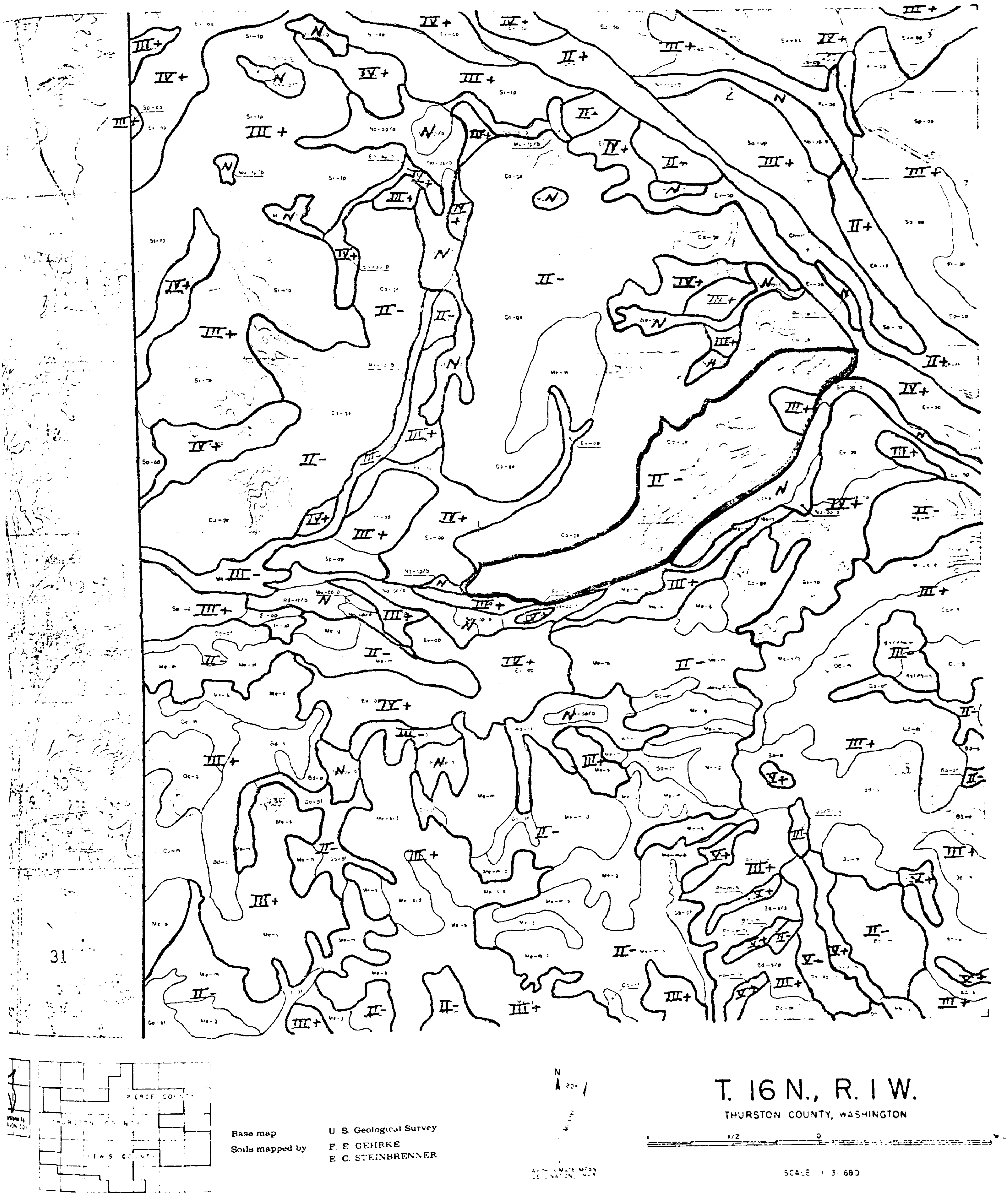


Table C-1

Compartment Area by Douglas-fir Site Class

Site[1] Class	Area
	Acres
110	63
120	2,216
Total	2,279

[1]Average site index to the nearest 5 feet based on King (1966).

is an abstract from their report. The yields for untreated stands are from "Net and Gross Yield for Natural Stands of Douglas-fir" by King and Wiley (1963).

The growth and yield of stands following fertilization is estimated by the addition of 7%, 13%, 20%, 27% of the height increment and 0.5, 0.7, 1.0, 1.3 inches in DBH increment for 100, 200, 300, and 400 pounds of elemental nitrogen, in the form of urea, for each five year period. The duration of the growth response to fertilization is limited to a single five year period. The growth and yield of stands for periods following the period immediately after fertilization is estimated by applying the equation for basal area growth developed by King (1970). The estimates of number of trees, average DBH, and total volume are estimated from basal area using modified equations from King and Wiley (1963). These responses to fertilization and those for thinned stands are based upon experiments and analyses reported by Flewelling (1975), Gessel, *et al* (1969), Webster (1975) and University of Washington (1975).

All the thinning treatments considered in the analysis are low thinnings. The treatments considered were a removal of 10, 20, 30 and 40 percent of the basal area in a single thinning. These thinnings are simulated by distributing the trees in unthinned stands according to cumulative stand diameter distributions developed by King (1964) and accumulating trees to be removed in a thinning from the smallest diameter classes until the required basal area removal is achieved. The trees remaining in the simulated stand are grown using a system of equations developed by Bower and Shaw (1975). Thinning treatments in subsequent periods are simulated by repeating the steps for the initial thinning to diameter distributions of the stand at the time of treatment, and growing the stand following thinning through application of the previously mentioned equations (Bower and Shaw, 1975).

The yield of stands that are thinned and fertilized is estimated by adding 13, 27, 40 and 53 percent to basal area and height growth for 100, 200, 300 and 400 pounds of nitrogen respectively during the five year period following fertilization. As in the growth estimates for fertilization of unthinned stands, the response to fertilization was limited to a single five year period.

In addition to being used in the simulation of thinning treatments distributions of volume over diameter class are calculated for the yield at each interval and used in the estimation of stumpage prices.

Yield tables for combinations of treatments considered in the analysis are given at the end of this appendix.

Ad Valorem taxes used in the analysis are based upon similar taxes in the State of Washington. The taxes levied by the State of Washington which pertain to the production of timber crops are an annual tax of 1-1/2%

excise tax on the market value of the stumpage which is paid at the time the trees are cut (Washington Forest Protection Association, 1975). For this analysis all transactions are on stumpage and the yield tax is considered an element of the harvesting operation and not explicitly included in the analysis.

However, the ad valorem tax on forest land is relevant to the analysis of stand treatments. In the analysis the quantity of tax on land is measured in annual units. Therefore, the quantity of ad valorem tax is estimated as 2.5 per acre units for period 0 and 5 per acre units for each subsequent period.

Estimates of the quantity of overhead required by the compartment in the analysis are made up of two components: (1) the number of professional man years per year required to manage the compartment; and (2) the number of units of protection required to ensure that the crop is not destroyed.

Estimates of the number of professional forester manyears per year required to manage the compartment are based upon statistics compiled by the Industrial Forestry Association (1975) and are summarized in Table C-2.

To determine if a significant trend exists between the number of professional forester manyears required to manage industrial forests and time, the data for period and professional forester manyears per acre are fitted to a single linear regression of the form:

Table C-2

Acres per Professional Forester Manyear by Year[1]

Year	Period	Acres per Professional Forester Manyear	Professional Forester Manyears per Acre
	Number	Acres	Manyears
1949	-26	36,193	0.00002763
1950	-25	28,931	0.000034565
1951	-24	25,369	0.000039418
1952	-23	24,709	0.000040471
1953	-22	24,664	0.000040545
1954	-21	23,483	0.000042584
1955	-20	22,383	0.000044677
1956	-19	22,238	0.000044968
1957	-18	22,937	0.000043598
1958	-17	23,560	0.000042445
1959	-16	21,487	0.00004654
1960	-15	22,563	0.00004432
1961	-14	23,880	0.000041876
1962	-13	21,596	0.000046363
1963	-12	19,740	0.000050658
1964	-11	21,485	0.000046544
1965	-10	20,823	0.000048024
1966	-9	17,749	0.000056341
1967	-8	17,057	0.000058627
1968	-7	16,392	0.000061005
1969	-6	18,628	0.000053683
1970	-5	17,087	0.000058524
1971	-4	18,148	0.000055102
1972	-3	16,764	0.000059652
1973	-2	16,919	0.000059105
1974	-1	14,958	0.000066854

[1]From: Industrial Forestry Association (1975)

Manyears per acre = $b_0 + b_1$ period

b_0 = 0.000063281

b_1 = 0.0000011145

The square of the linear correlation
coefficient (R^2) = 0.8629

The standard error of estimate for manyears per acre = 0.000032174

The calculated F-ratio for the regression indicated that the coefficient b_1 was significantly different from 0.

Based on this information the number of professional forester manyears required to manage the compartment is estimated from this equation by extrapolation through the year 2000. Beyond the year 2000, the professional forester manyears are estimated to remain constant at the level of the year 2000 to the end of the analysis period. The result of this estimation procedure is given in Table C-3

The units of protection are estimated as annual acre units and are summarized in Table C-4.

The quantity of fertilizer required as inputs for the five fertilizer prescriptions considered are measured in terms of the number of pounds of elemental nitrogen prescribed on a total compartment basis.

Table C-3

Estimated Number of Professional Forester Manyears

Required to Manage the Compartment by Interval

Interval	Estimation Point	Professional Forester Manyears	
		Per Acre Per Year	Total Comp. Per Interval
		Manyears	
0	1975	0.00006328	0.3605
1	1980	0.00006885	0.7846
2	1985	0.00007443	0.8481
3	1990	0.00008	0.9116
4	1995	0.00008557	0.9751
5	2000	0.00009114	1.0386
5+		0.00009114	1.0386

Table C-4

Estimated Quantity of Protection Required

for the Compartment by Interval

Interval	Estimation Point	Protection Per Acre Per Year	Units Total Compartment Per Interval
			Number
0	1975	1	5,697.5
1	1980	1	11,395.0
1+		1	11,395.0

The quantity of thinning associated with any thinning treatment is estimated as the amount of stumpage removed measured in cubic feet total volume. When a thinning treatment involves an input, as would be the case if the sale of the volume to be removed created a cost, the convention was adopted of describing the volume removed as an input to the investment sequence. If the sale of stumpage associated with a thinning treatment creates revenue the volume of timber removed by the treatment was treated as an output.

The regeneration treatments which are considered do not require any measured inputs. The yields following the natural regeneration treatment are estimated by assuming a five year delay in the establishment of regeneration followed by a stand with Growing Stock Index 220.

Price. All price estimates used in the demonstration analysis are in 1975 dollars. For this reason the first step in the development of price estimates for inputs and outputs is the conversion of recorded market prices to their equivalence in 1975 dollars. This is accomplished through the use of the "Consumer Price Index" published by the U.S. Department of Commerce (1973 and 1975). The index of consumer prices used in estimation of prices for this analysis is given in Table C-5.

The calculation of expected Present Certainty-Equivalent Value requires that distortions associated with taxes be removed from external prices.

Table C-5

Consumer Price Index[1] 1947-1975

Year	Consumer Price Index	
	1967 = 100	1975 = 100
1947	66.9	42.4
1948	72.1	45.7
1949	71.4	45.2
1950	72.1	45.7
1951	77.8	49.3
1952	79.5	50.4
1953	80.1	50.8
1954	80.5	51.0
1955	80.2	50.8
1956	81.4	51.6
1957	84.3	53.4
1958	86.6	54.9
1959	87.3	55.3
1960	88.7	56.2
1961	89.6	56.8
1962	90.6	57.4
1963	91.7	58.1
1964	92.9	58.9
1965	94.5	59.9
1966	97.2	61.6
1967	100.0	63.4
1968	104.2	66.0
1969	109.8	69.6
1970	116.3	73.7
1971	121.3	76.9
1972	125.3	79.4
1973	133.1	84.3
1974	147.7	93.6
1975[2]	157.8	100.0

[1]From: U.S. Department of Commerce (1973 and 1975)

[2]Based upon index for January-May

To accomplish this the approximation of United States corporate income taxes used by Grant and Ireson (1970) is adopted. This approximation is the assumption that the marginal rate of income tax on ordinary income for corporation is 50% and the marginal rate on capital gains income is 25%. The tax adjustment calculations are further simplified by ignoring the purchase price of the land and application of the external stumpage prices to natural stands with no allowance for depletion. These simplifications allow the prices to be adjusted to after tax prices by the application of factors calculated from the following formulae:

Ordinary income after tax price.

After tax price = (1 - marginal tax rate) external price

After tax price = .5 external price

Capital gains income after tax price

After tax price = .75 external price

For a more detailed discussion of the impact of taxes on forestry in particular and investment in general see Duerr (1960) Chapter 27 and Grant and Ireson (1970) Chapters 16 and 17.

The external price of land used in the analysis is based on the market price for forest land of average site quality and favorable access and topography as compiled by the Washington State Department of Revenue for 1975. This price was increased at the rate of 1.1 percent per year, to reflect the general rate of increase in population (U.S. Forest Service, 1973) to the year 2000. After the year 2000, the price of land is estimated as constant at the level of the year 2000. A summary of land prices used for the demonstration analysis is given in Table C-6.

The estimated external price of growing stock used for the analysis is based upon the average price for west side Douglas-fir saw timber sold on National Forests (U.S. Forest Service, 1963; Hamilton, 1965; and Ruderman, 1975). The stumpage price to the year 2000 was estimated by applying the 100 percent increase forecast by the U.S. Forest Service (1973) to the 1970 price. These prices and the resulting forecasts are summarized in Table C-7 and Figure C-3.

Stumpage prices were estimated for 1 inch DBH classes for the purpose of forecasting more accurately the prices associated with the stand structures which follow stand treatments. This was done by plotting a straight line on semi-logarithmic paper between the 6 inch DBH class, which was assumed to have a 0 price and the 26 inch DBH class which was assumed to have the average stumpage price for the interval. The results of this forecasting procedure are given in Figures C-4 to C-9 and Table C-8.

The annual ad valorem tax on forest land in the State of Washington is 1-1/2% of market value. The price of ad valorem taxes used in the demonstration analysis is summarized in Table C-9.

Table C-6

Estimated market price for the land in the compartment by interval

Interval	Estimation Point	Market Price Per Acre
	Year	1975 $
0	1975	77.00
1	1980	81.33
2	1985	85.90
3	1990	90.73
4	1995	95.83
5	2000	101.22
5+		101.22

Table C-7

Average stumpage price for westside douglas-fir sawtimber sold on national forests[1]

Year	Price		
	\$/MBF	\$/Cu. Ft.[2]	1975\$/Cu.Ft.
1952	21.00	0.1321	0.2621
1953	15.50	0.0975	0.1919
1954	18.00	0.1132	0.222
1955	38.20	0.2403	0.4729
1956	37.80	0.2377	0.4607
1957	26.20	0.1648	0.3086
1958	21.80	0.1371	0.2497
1959	36.80	0.2314	0.4185
1960	32.00	0.2013	0.3581
1961	27.60	0.1736	0.3056
1962	24.80	0.1560	0.2717
1963	28.00	0.1761	0.3031
1964	38.10	0.2396	0.4068
1965	42.60	0.2679	0.4473
1966	50.00	0.3145	0.5105
1967	41.70	0.2623	0.4137
1968	61.20	0.3849	0.5832
1969	82.20	0.5170	0.7428
1970	41.90	0.2635	0.3576
1971	49.10	0.3088	0.4016
1972	71.70	0.4509	0.5679
1973	138.10	0.8686	1.0303
1974	202.80	1.2755	1.3627
1975[3]	160.10	1.0069	1.0069

[1]From: U.S. Forest Service, 1963. Table 2
Hamilton, 1965. Table 13
Ruderman, 1975. Table 34

[2]1 MBF. = 159 cu. ft.

[3]1st and 2nd quarters only

Figure C-3

Actual and Estimated Stumpage Prices

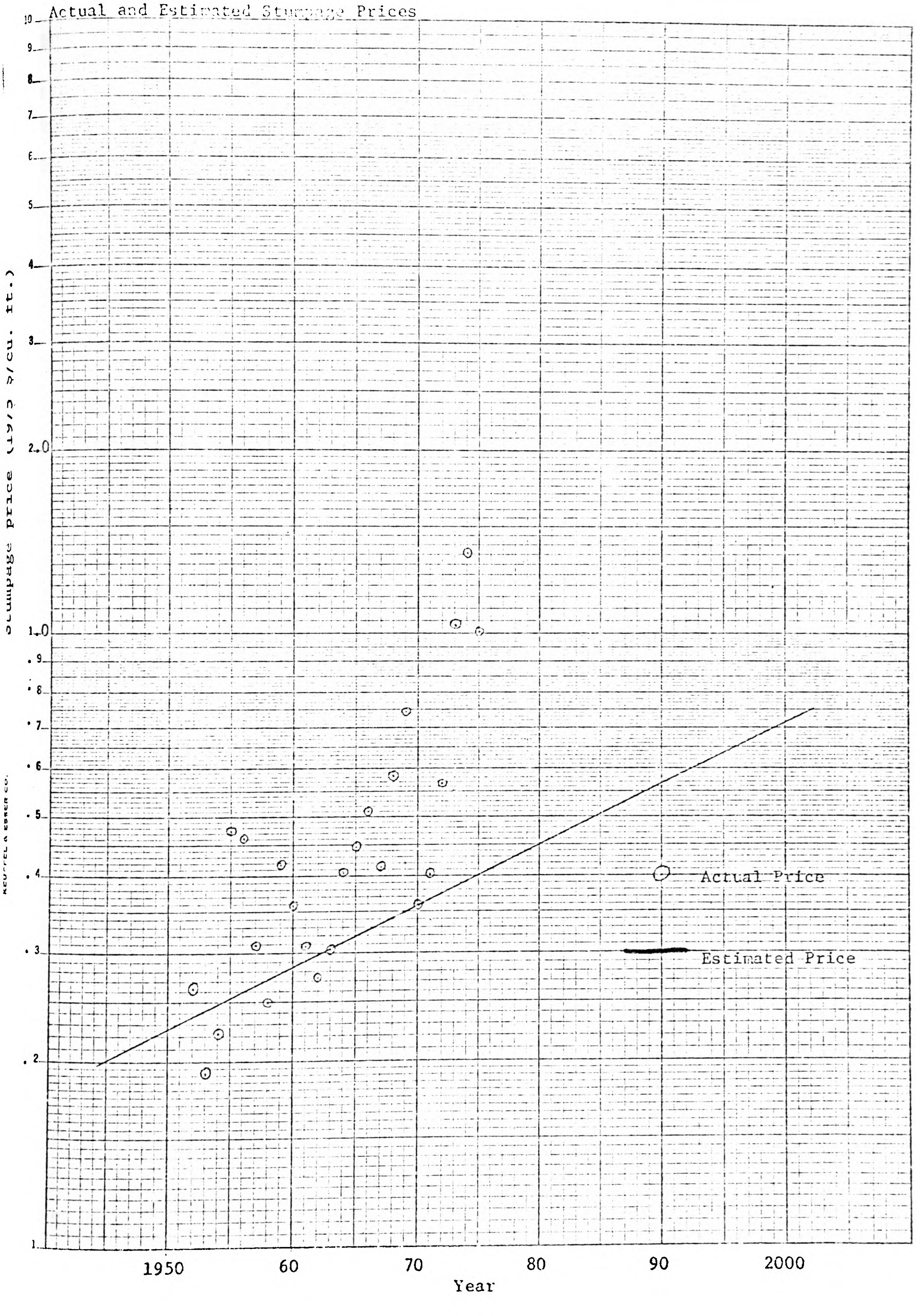


Figure C-4

Estimated Clearcut and Thinning Stumpage Price by DBH Class in 1975-

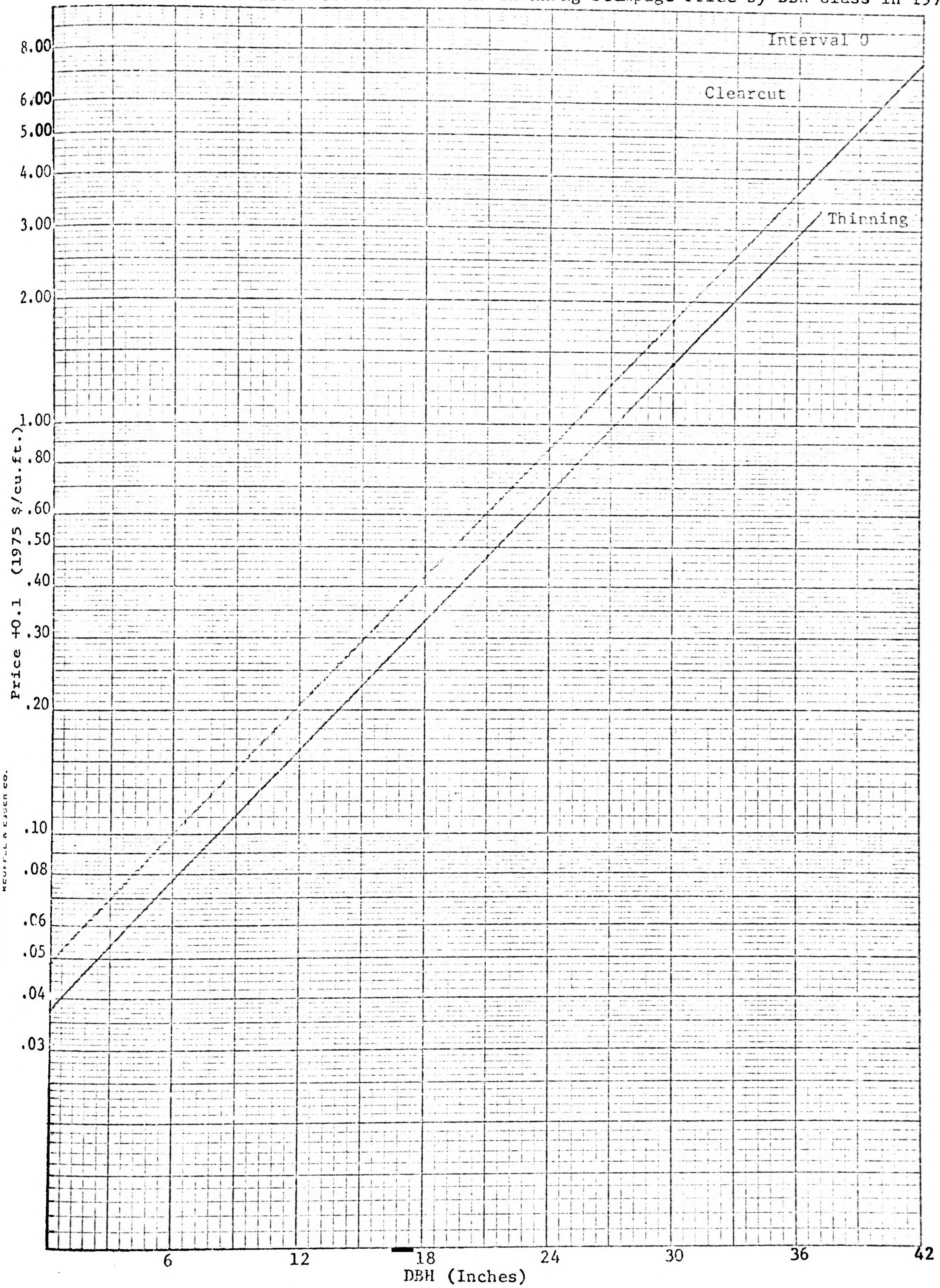


Figure C-5 Estimated Clearcut and Thinning Stumpage Price by DBH Class in 1980 - Interval 1

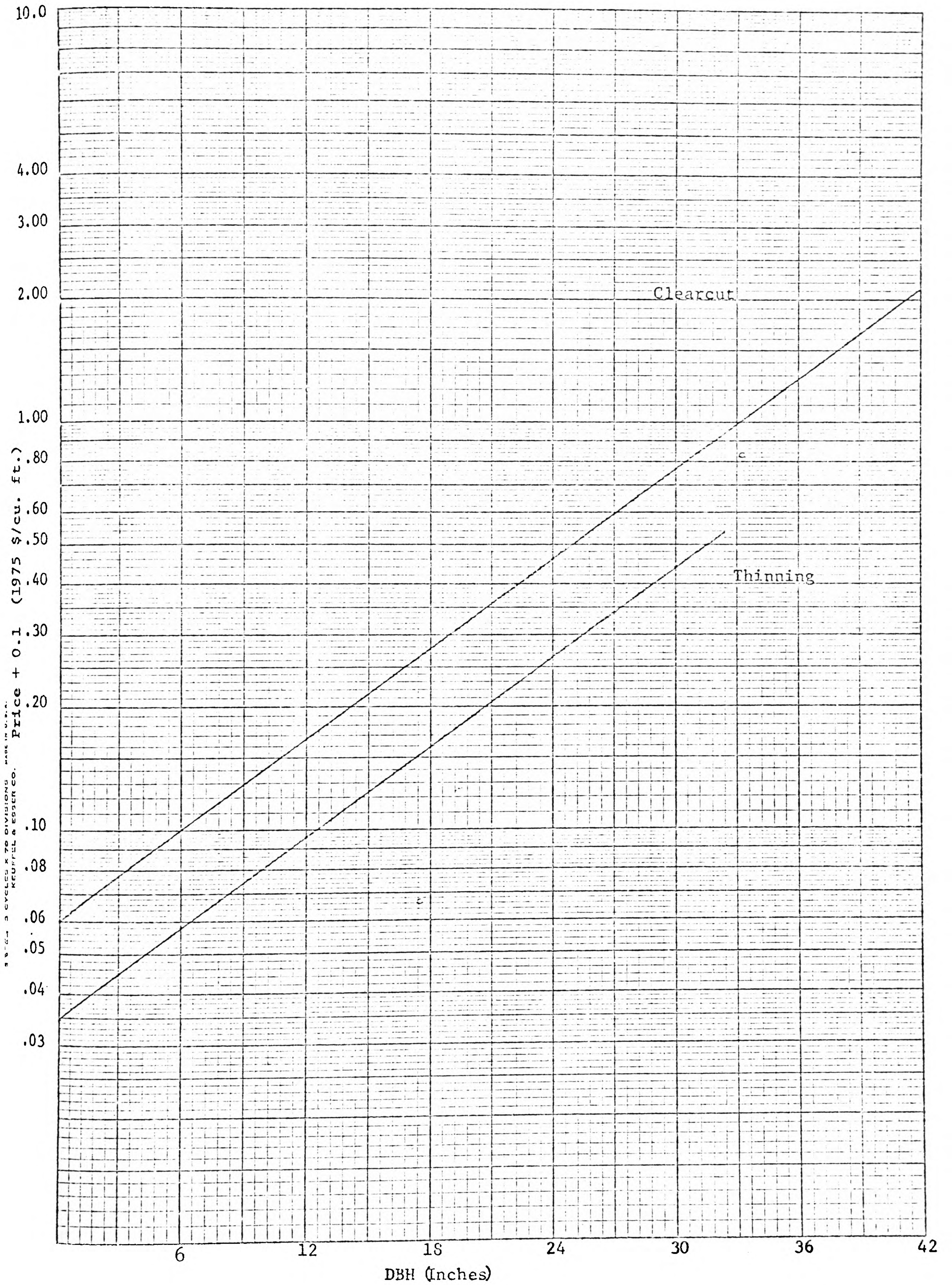


Figure C-6

Estimated Clearcut and Thinning Stumpage Price by DBH Class in 1985 - Interval 2

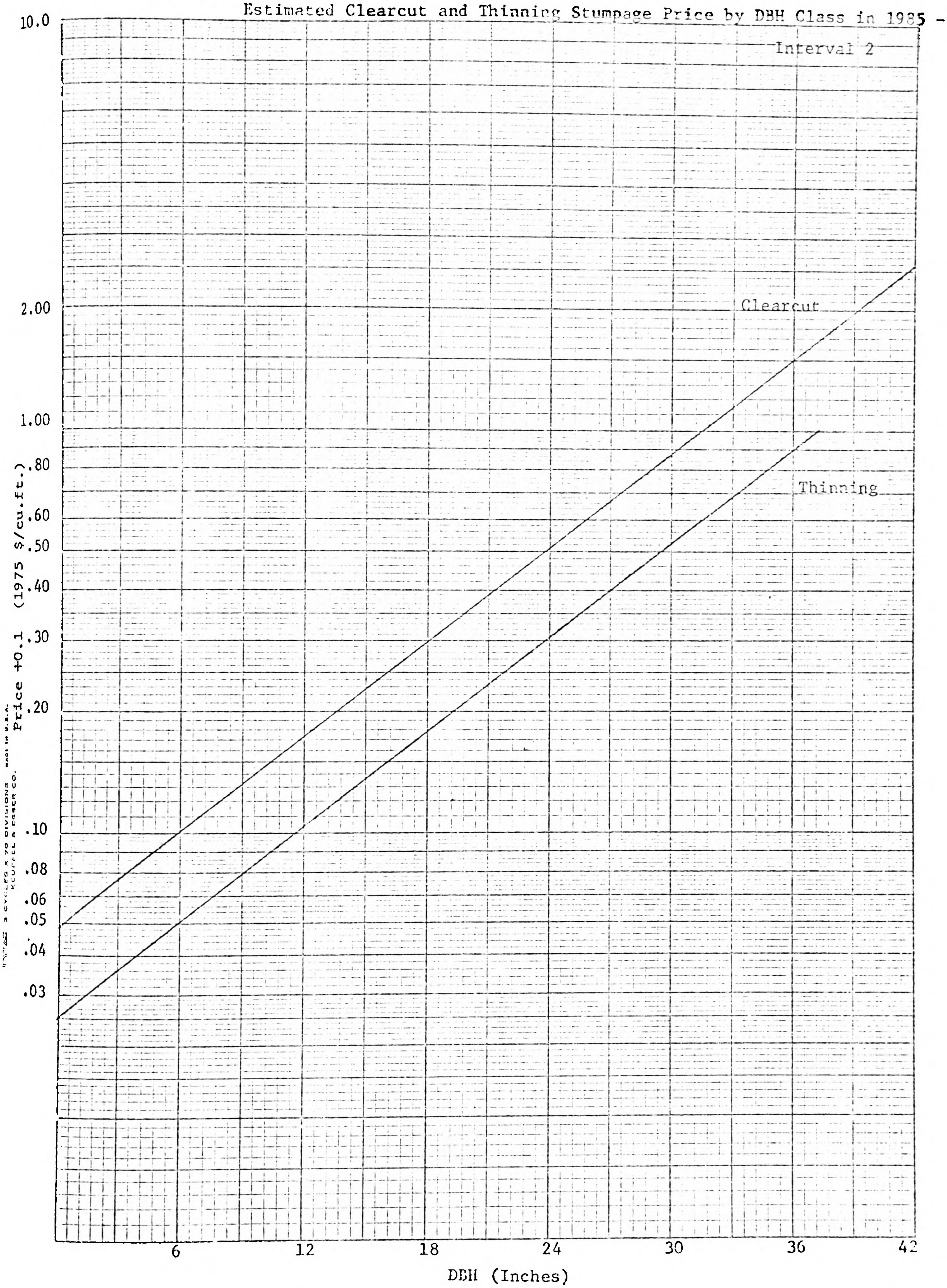
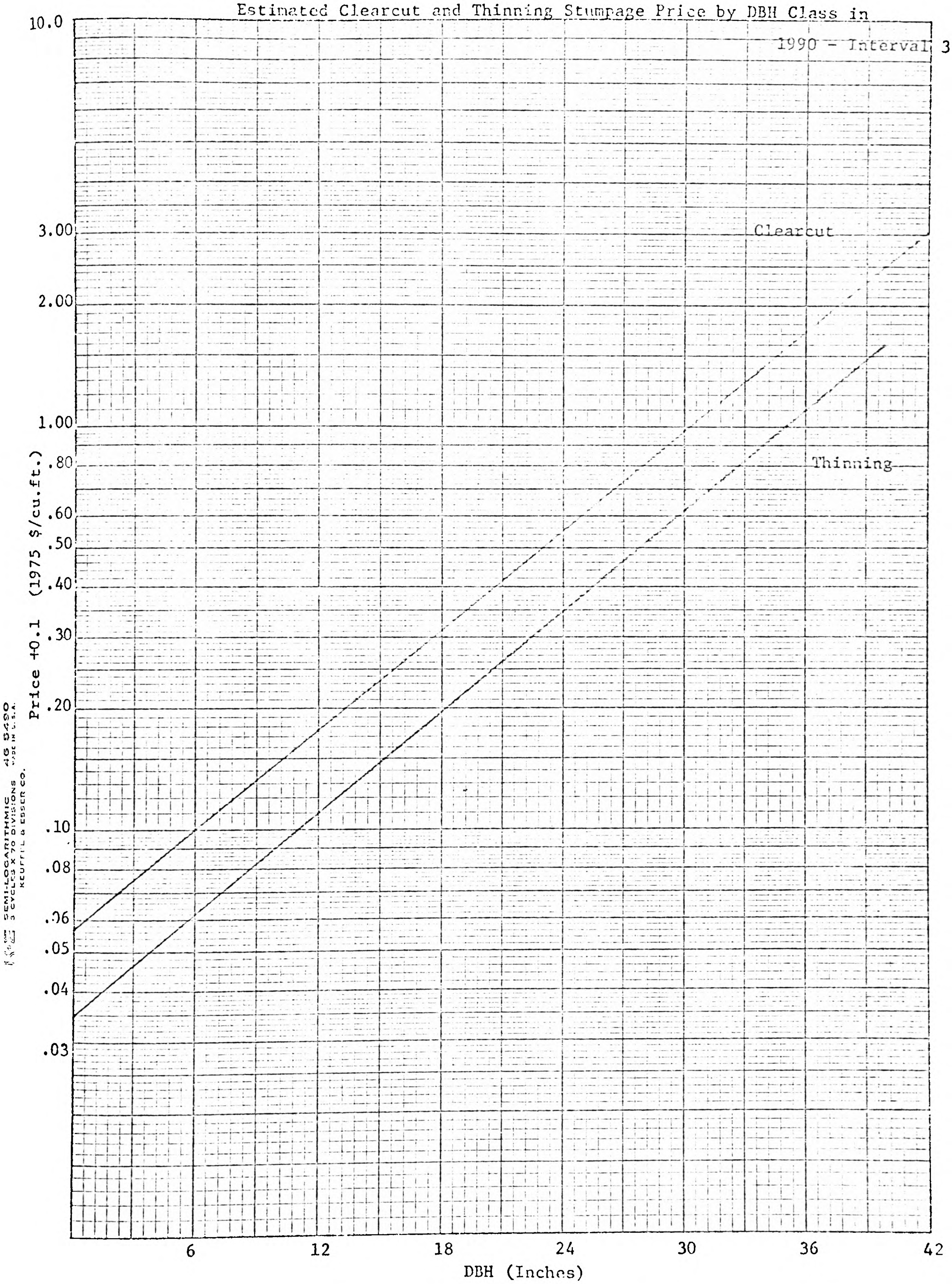


Figure C-7



Estimated Clearcut and Thinning Stumpage Price by DBH Class in 1990 - Interval 3
10.0
3.00
2.00
1.00
.80
.60
.50
.40
.30
.20
.10
.08
.06
.05
.04
.03
Price +0.1 (1975 $/cu.ft.)
Clearcut
Thinning
6
12
18
24
30
36
42
DBH (Inches)
SEMI-LOGARITHMIC 46 5490
3 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.

Figure C-8

Estimated Clearcut and Thinning Stumpage Price by DBH Class in 1995 - Interval 4

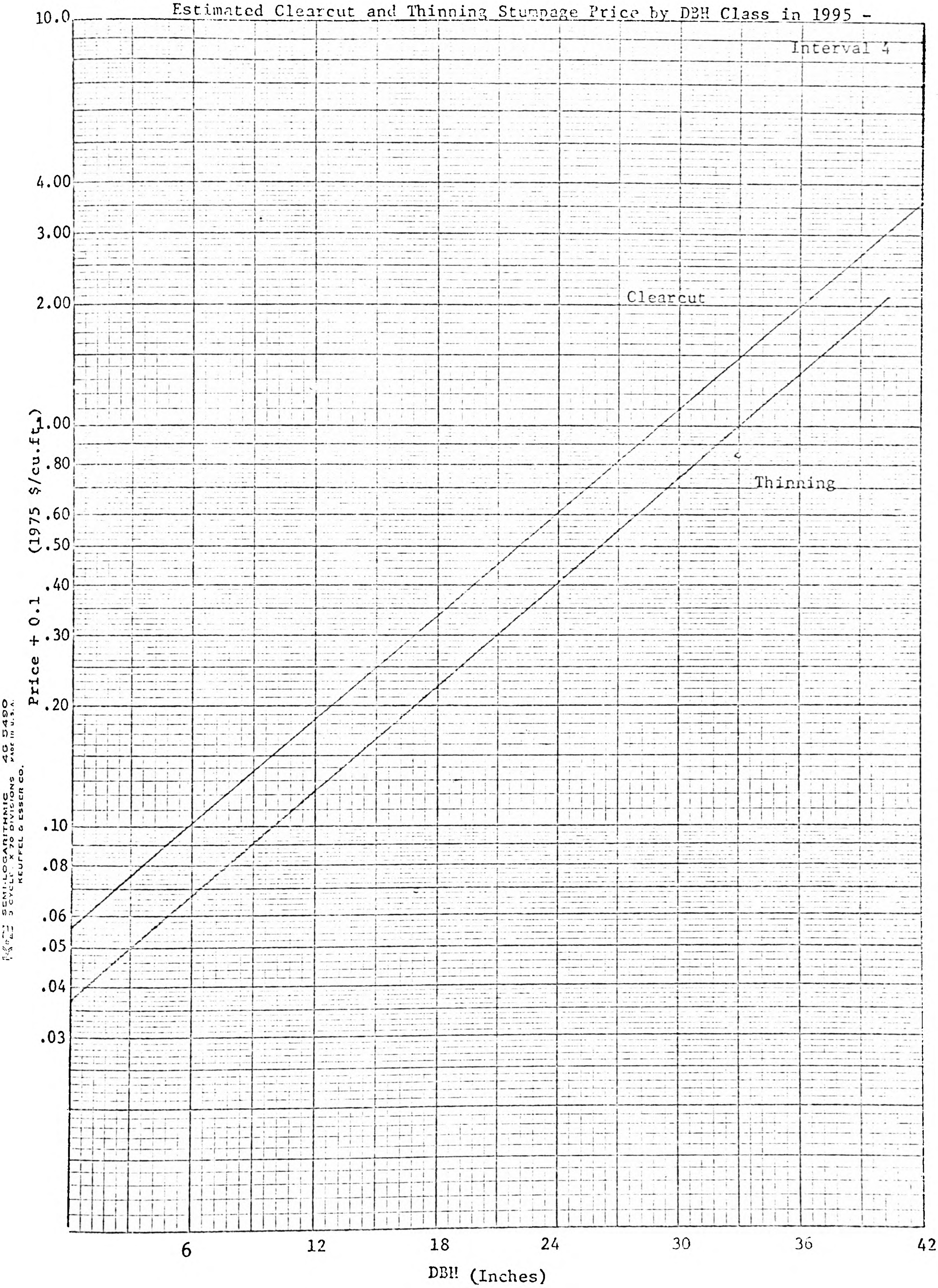


Figure C-9

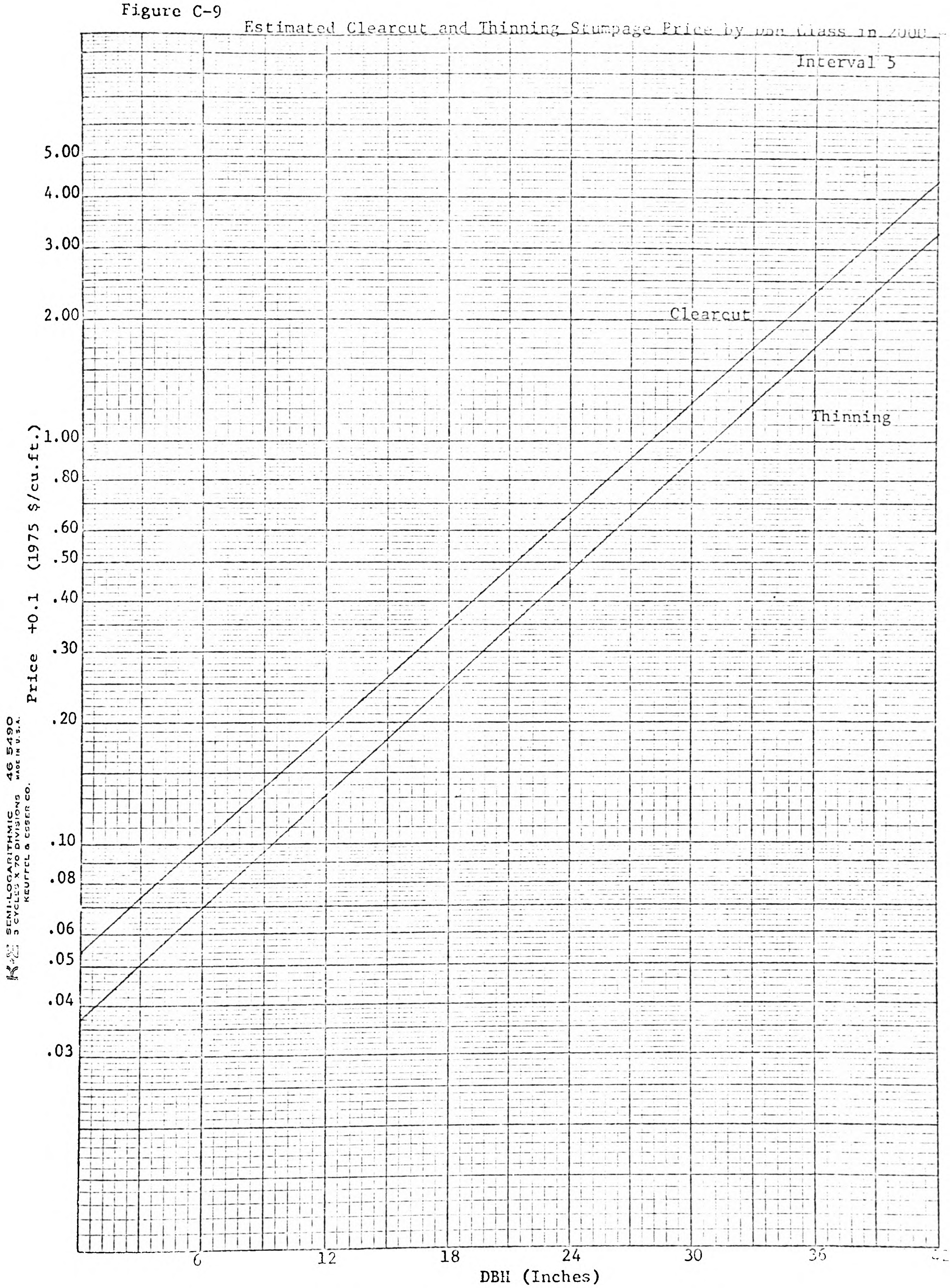

Estimated Clearcut and Thinning Stumpage Price by DBH Class in 2000
Interval 5
Clearcut
Thinning
5.00
4.00
3.00
2.00
1.00
.80
.60
.50
.40
.30
.20
.10
.08
.06
.05
.04
.03
Price +0.1 (1975 $/cu.ft.)
6
12
18
24
30
36
DBH (Inches)
SEMI-LOGARITHMIC 46 5490
3 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.

Table C-8

Estimated Douglas-fir clearcut stumpage price per cubic foot by interval and DBH class

DBH Class	Interval 0	1	2	3	4	5	5+
Inches	1975 Dollars per Cubic Foot						
2	-0.0385	-0.0290	-0.0305	-0.0315	-0.0330	-0.0345	-0.0345
3	-0.0305	-0.0230	-0.0240	-0.0245	-0.0255	-0.0270	-0.0270
4	-0.0215	-0.0165	-0.0170	-0.0170	-0.0180	-0.0190	-0.0190
5	-0.0115	-0.0085	-0.0090	-0.0090	-0.0095	-0.0100	-0.0100
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.0125	0.0075	0.0080	0.0100	0.0110	0.0120	0.0120
8	0.0270	0.0175	0.0190	0.0210	0.0225	0.0270	0.0270
9	0.0430	0.0275	0.0310	0.0340	0.0350	0.0370	0.0370
10	0.0610	0.0390	0.0430	0.0460	0.0480	0.0525	0.0525
11	0.0820	0.0525	0.0560	0.0610	0.0640	0.0680	0.0680
12	0.1050	0.0650	0.0710	0.0770	0.0810	0.0875	0.0875
13	0.1310	0.0810	0.0860	0.0940	0.0990	0.1080	0.1080
14	0.1500	0.0960	0.1040	0.1140	0.1200	0.1310	0.1310
15	0.1940	0.1140	0.1250	0.1350	0.1440	0.1560	0.1560
16	0.2310	0.1340	0.1450	0.1580	0.1680	0.1850	0.1850
17	0.2720	0.1540	0.1675	0.1840	0.1980	0.2160	0.2160
18	0.3220	0.1770	0.1930	0.2120	0.2280	0.2520	0.2520
19	0.3750	0.2020	0.2200	0.2420	0.2620	0.2900	0.2900
20	0.4390	0.2290	0.2500	0.2790	0.3000	0.3350	0.3350
21	0.5075	0.2570	0.2840	0.3140	0.3420	0.3800	0.3800
22	0.5850	0.2875	0.3200	0.3560	0.3900	0.4350	0.4350
23	0.6700	0.3240	0.3590	0.4000	0.4425	0.4925	0.4925
24	0.7700	0.3610	0.4010	0.4500	0.4975	0.5575	0.5575
25	0.8850	0.4010	0.4550	0.5100	0.5650	0.6325	0.6325
26	1.0069	0.4500	0.5050	0.5660	0.6350	0.7152	0.7152
27	1.1500	0.4950	0.5600	0.6350	0.7075	0.8000	0.8000
28	1.3200	0.5500	0.6200	0.7100	0.7950	0.9000	0.9000
29	1.4900	0.6050	0.6900	0.7900	0.8900	1.0200	1.0200
30	1.7000	0.6700	0.7650	0.8800	0.9900	1.1400	1.1400
31	1.9250	0.7400	0.8450	0.9750	1.1100	1.2700	1.2700
32	2.1900	0.8100	0.9300	1.0750	1.2300	1.4250	1.4250
33	2.4900	0.8900	1.0400	1.1900	1.3750	1.5900	1.5900
34	2.8100	0.9750	1.1400	1.3300	1.5400	1.7750	1.7750
35	3.2000	1.0800	1.2500	1.4750	1.7000	1.9900	1.9900
36	3.6100	1.1800	1.3800	1.6250	1.8900	2.2200	2.2200
37	4.0900	1.3000	1.5200	1.8000	2.1000	2.4750	2.4750
38	4.6000	1.4250	1.6750	1.9900	2.3400	2.7500	2.7500
39	5.2000	1.5500	1.8400	2.2000	2.5800	3.0750	3.0750
40	5.9000	1.7100	2.0250	2.4250	2.8800	3.4400	3.4400

Table C-9

Estimated price of ad valorem in the compartment per tax unit by interval

Interval	Estimation Point	Price Per Tax Unit
	Year	1975 $
0	1975	1.155
1	1980	1.22
2	1985	1.2885
3	1990	1.361
4	1995	1.4374
5	2000	1.5183
5+		1.5183

Estimates of the price of overhead are made for each of the components discussed earlier. The price of professional foresterman years is based on the median income of professional foresters employed in private industry as reported by Eyre (1960), Evans (1965), Thompson *et al.* (1974) and (1975). This data is summarized in Table C-10. To estimate the price of a professional forester manyear the salaries, in 1975 dollars, are doubled to estimate the full cost of a manyear. The data are then fitted to a simple linear regression of the form;

$$\text{Price per manyear} = b_0 + b_1 \text{ period}$$

to determine if a significant trend exists between the price per professional forester manyear and time. Values of the constant and coefficient are:

b_0 = 36,007.3721

b_1 = 570.5109

The square of the linear correlation coefficient (R^2) = .9509

The standard error of estimate for price per manyear = 31,377.06

resulted and the calculated F-ratio indicated that the coefficient b_1 was significantly different from 0. Based on this information the price of professional foresterman years is estimated from this equation by extrapolation through the year 2000. Beyond the year 2000 the price is estimated to remain constant at the level of the year 2000. The results of this estimation procedure are summarized in Figure C-10 and Table C-11.

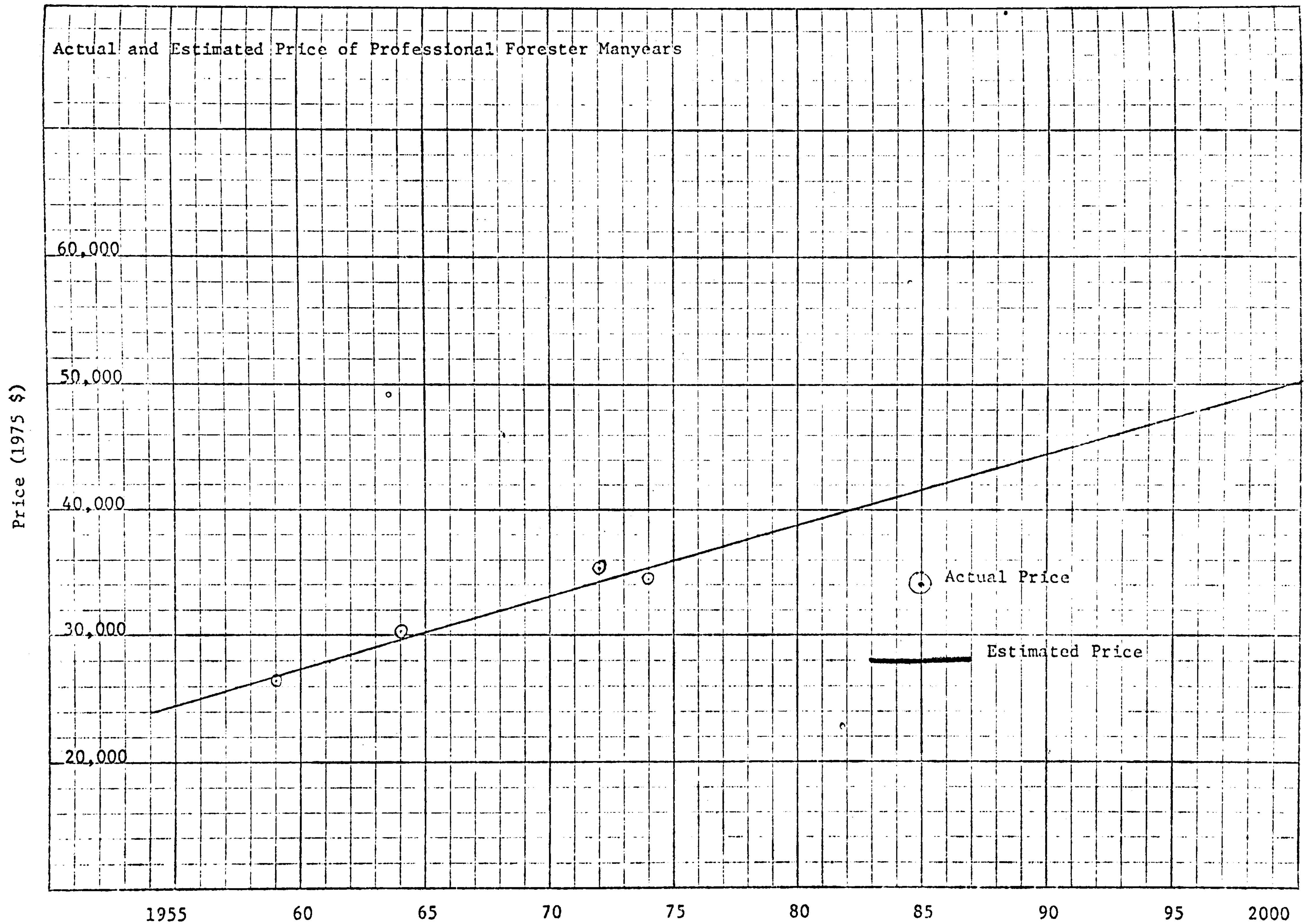
Table C-10

Median Income of Professional Forestors Employed in Private Industry[1].

Year	Income: Dollars	Income: 1975 $
1959	7,300.00	13,200.72
1964	8,910.00	15,127.33
1972	14,000.00	17,632.24
1974	16,110.00	17,211.54

1 From: Eyre (1960), Evans (1965), and Thompson et. al. (1974) and (1975).

Figure C-10



Actual and Estimated Price of Professional Forester Manyears
Price (1975 $)
60,000
50,000
40,000
30,000
20,000
1955
60
65
70
75
80
85
90
95
2000
Actual Price
Estimated Price

Table C-11

Estimated price of professional forester manyears by interval

Interval	Estimation Point	Price/ Manyear
	Year	1975 $
0	1975	36,007.37
1	1980	38,859.93
2	1985	41,712.48
3	1990	44,565.04
4	1995	47,417.59
5	2000	50,270.15
5+	2000+	50,270.15

The price of an annual unit of protection is based upon the average protection cost per acre per year for industrial tree farms compiled by the Industrial Forestry Association (1975) and given in Table C-12. This data was also fitted to a simple linear regression of the form:

Price of an acre protection unit per year = $b_0 + b_1$ period

$b_0 = 0.5071$

$b_1 = 0.0158$

The square of the linear correlation coefficient (R^2) = 0.5122

The standard error of estimate for price per acre per year of protection = 0.12.

The calculated F-ratio indicated that the coefficient b_1 was significantly different from 0. Based upon this information the price of an annual per acre protection unit is estimated from this equation by extrapolation through the year 2000. Beyond the year 2000 the price of protection units used in the analysis are given in Table C-13.

The price of time or the discount rate used in the demonstration analysis is based upon the real rate of return earned by marginal shares of a portfolio of common stocks of forest products firms traded on the New York Stock Exchange between 1955 and 1975. The data upon which the estimate of the discount rate is based are summarized in Tables C-14 to C-16.

Table C-12

Average protection cost per acre per year[1]

Year	Average Protection Cost/Acre/Year	
	Dollars	1975 $
1949	0.31	0.73
1950	0.35	0.77
1951	0.46	0.93
1952	0.48	0.95
1953	0.47	0.93
1954	0.46	0.90
1955	0.47	0.93
1956	0.52	1.01
1957	0.52	0.97
1958	0.46	0.84
1959	0.43	0.78
1960	0.43	0.77
1961	0.38	0.67
1962	0.36	0.63
1963	0.32	0.55
1964	0.32	0.54
1965	0.34	0.57
1966	0.37	0.60
1967	0.40	0.63
1968	0.29	0.44
1969	0.35	0.50
1970	0.39	0.53
1971	0.47	0.61
1972	0.47	0.59
1973	0.50	0.59
1974	0.75	0.80

[1]From: Industrial Forestry Association (1975)

Table C-13

Estimated price per acre per year
of a unit of protection

Interval		Price
	Year	1975 $
0	1975	0.5071
1	1980	0.4281
2	1985	0.3491
3	1990	0.2701
4	1995	0.1911
5	2000	0.1121
5+	2000+	0.1121

Table C-14

Mean prices[1] and dividends[2] for common stocks of selected forest products firms traded on the New York Stock Exchange and "Moody's Composit Share"

Year	Boise Cascade Corp.		Crown Zellerbach Corp.		Georgia Pacific Corp.		International Paper Co.		Kimberly Clark Corp.		St. Regis Paper Co.		Scott Paper Co.		Westvaco Corp.		Weyerhaeuser Co.		Moody's Composit	
	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend
	Dollars																			
1955			55.50	29.85	34.188	3.0512	99.75	7.9875	47.25	1.80	41.688	1.80	67.125	1.80	42.00	1.40			117.36	4.75
1956			60.00	1.80	45.156	5.5662	121.50	6.645	49.188	1.80	50.50	1.90	66.375	1.85	52.875	1.55			130.55	5.31
1957			49.312	1.80	29.812	3.385	92.938	5.7881	45.50	1.80	35.875	1.55	56.25	2.00	39.75	1.60			125.46	5.43
1958			51.125	1.80	39.50	4.16	103.875	5.0775	58.50	1.80	36.562	1.40	65.062	2.00	41.125	1.50			132.02	5.29
1959			55.312	1.80	54.625	17.9096	126.375	5.5275	66.875	1.80	49.75	2.395	80.14	2.05	51.50	1.20			163.47	5.41
1960			47.125	1.80	51.812	0.50	70.875	185.7334	76.038	1.80	43.125	2.2625	82.375	2.20	43.438	1.20			155.46	5.59
1961			59.125	1.80	64.312	0.50	34.844	1.7469	84.75	1.80	36.438	2.1288	79.312	123.5599	39.062	1.20			185.66	5.70
1962			48.688	6.669	44.188	0.50	30.375	1.6575	62.415	1.80	30.938	2.0188	33.875	0.80	36.25	1.20			177.87	5.99
1963			52.812	1.80	50.188	0.50	31.188	1.6738	62.562	1.95	31.75	2.035	34.688	0.825	37.938	1.20	33.688	1.20	202.32	6.42
1964			58.625	1.85	57.312	5.8426	34.188	1.1625	63.375	2.00	32.812	2.0562	37.125	0.90	41.25	1.225	38.375	1.20	235.08	7.05
1965	53.375	0.40	53.625	2.00	58.625	0.50	32.188	1.25	53.812	2.00	34.50	2.09	36.562	0.975	48.062	1.35	43.938	1.20	250.31	7.65
1966	46.406	28.006	46.375	2.00	48.406	146.2188	29.375	1.2375	51.75	2.00	35.312	2.1062	32.625	1.00	46.562	1.55	35.438	1.40	230.88	8.25
1967	33.938	0.20	48.062	2.20	53.29	3.1316	28.562	1.35	59.875	2.15	31.562	2.0312	28.75	1.00	41.188	1.75	40.50	1.40	246.54	8.16
1968	56.125	1.322	52.625	2.20	80.00	1.00	33.062	1.3875	64.375	2.20	36.625	2.1325	28.625	1.00	40.062	25.9107	62.062	1.40	264.62	8.53
1969	65.875	1.578	48.875	26.538	69.281	68.4799	40.29	1.50	70.25	2.20	40.875	1.55	30.688	1.00	31.312	1.0125	60.312	71.1053	262.77	8.68
1970	58.438	1.419	29.50	1.60	47.812	1.7562	34.188	1.50	50.875	66.8963	32.812	1.60	28.062	1.00	23.50	1.05	46.50	1.20	226.70	8.99
1971	32.438	0.899	32.625	1.20	50.312	1.8062	34.562	1.50	30.688	1.20	36.50	1.60	20.312	0.875	20.938	1.05	51.688	0.80	261.43	8.91
1972	14.828	0.1875	29.50	1.20	43.25	1.665	37.695	1.50	34.062	1.20	41.25	1.60	15.00	0.50	22.688	1.05	50.188	0.83	290.65	8.92
1973	13.312	0.0625	33.688	1.20	37.812	1.556	45.00	1.75	40.50	1.20	58.562	20.9312	15.062	0.545	30.125	1.0625	50.531	33.4625	285.44	9.58
1974	14.438	0.375	29.625	1.70	33.812	1.4762	43.812	1.75	27.00	1.44	27.625	1.25	13.872	0.62	28.75	1.175	34.938	0.80	220.35	10.63
1975[3]	18.875	0.25	32.375	0.90	46.562		47.375	1.00	29.25	0.80	25.688	0.70	15.625	0.17	25.125	0.70	35.438	0.40	230.57	10.54

From: Moody's Investors Services Inc. (1975), Bank and Quotation Record (1956 to 1965) and (1975), U.S. Department of Commerce (1973) and (1975)

[1] Mean Price = (Maximum Price + Minimum Price)/2
[2] Stock splits and stock dividends are treated as cash dividends for year of issue
[3] Price based on period from January to August 1975

Table C-15

Mean prices and dividends for common stocks of selected forest products firms traded on the New York Stock Exchange and "Moody's Composit Share" expressed in 1975 Dollars

Year	Boise Cascade Corp.		Crown Zellerbach Corp.		Georgia Pacific Corp.		International Paper Co.		Kimberly Clark Corp.		St. Regis Paper Co.		Scott Paper Co.		Westvaco Corp.		Weyerhaeuser Co.		Moody's Composit	
	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend	Price	Dividend
	1975 Dollars																			
1955			109.252	58.7598	67.299	6.0063	196.358	15.7234	93.012	3.5433	82.063	3.5433	137.136	3.5433	82.677	2.7559			231.02	9.35
1956			116.279	3.4884	87.512	10.7872	235.465	12.8779	95.326	3.4884	97.868	3.6822	128.634	3.5853	102.471	3.0039			253.00	10.29
1957			92.345	3.3708	55.828	6.339	174.041	10.8391	85.206	3.3708	67.1816	2.9026	109.082	3.7453	74.438	2.9963			234.94	10.17
1958			93.124	3.2787	71.949	7.5774	189.208	9.2486	106.557	3.2787	66.597	2.5501	118.51	3.643	74.909	2.7322			240.47	9.64
1959			100.022	3.255	98.779	32.3863	228.526	9.9955	120.931	3.255	89.9638	4.3309	144.9186	3.7071	93.128	2.17			295.61	9.78
1960			83.852	3.2028	92.192	0.8897	126.112	330.4865	135.299	3.2028	76.7349	4.0258	146.575	3.9145	77.292	2.1352			276.62	9.95
1961			104.933	3.169	113.225	0.8803	61.345	3.0755	149.208	3.169	64.151	3.7479	139.634	217.535	68.771	2.1127			326.87	10.04
1962			84.322	11.6185	76.9826	0.6711	52.918	2.8876	108.737	3.1359	53.899	3.5171	59.016	1.3937	63.153	2.0906			309.88	10.44
1963			90.898	3.0981	86.332	0.8606	53.680	2.8899	107.68	3.3563	54.647	3.5026	59.704	1.42	65.298	2.0654	57.983	2.0654	348.23	11.05
1964			99.533	3.1409	97.304	9.9195	58.044	1.9737	107.598	3.3956	55.708	3.491	63.031	1.528	70.034	2.0798	65.153	2.0374	399.12	11.97
1965	89.107	0.6678	89.524	3.3389	97.871	0.8347	53.736	2.0868	89.836	3.3389	57.596	3.4891	61.038	1.5442	80.237	2.2538	73.352	2.0033	417.88	12.77
1966	75.334	45.4642	75.284	3.2468	78.581	237.3682	47.687	2.0089	84.01	3.2468	57.325	3.4192	52.963	1.6234	75.588	2.5162	97.529	2.2727	374.81	13.39
1967	53.530	0.3155	75.808	3.47	84.054	4.9394	45.050	2.1293	94.44	3.3912	49.782	3.2038	45.347	1.5773	64.965	2.7603	63.88	2.2082	388.86	13.03
1968	85.038	2.003	79.735	3.3333	121.212	1.5151	50.094	2.1023	97.538	3.3333	55.492	3.2311	43.371	1.5151	60.70	39.2586	94.033	2.1212	400.94	12.92
1969	98.958	2.2672	70.223	38.1293	99.542	98.3907	57.888	2.1552	100.934	3.1609	58.728	2.227	44.092	1.4368	44.989	1.4547	87.374	102.1628	377.54	12.47
1970	79.292	2.0243	40.027	2.171	64.874	2.3829	46.388	2.0352	69.030	90.7684	44.521	2.171	38.076	1.3569	31.886	1.4247	63.094	1.6282	307.60	12.20
1971	42.182	1.1691	42.425	1.5605	65.425	2.3488	44.944	1.9506	39.906	1.5605	47.464	2.0806	26.414	1.1378	27.228	1.3654	67.215	1.0403	339.96	11.46
1972	18.675	0.2361	37.154	1.5113	54.471	2.097	47.475	1.8892	42.899	1.5113	51.952	2.0151	18.892	0.6297	28.574	1.3224	63.209	1.0453	366.06	11.23
1973	15.791	0.0741	39.962	1.4235	44.854	1.8458	53.381	2.0759	48.043	1.4235	45.744	24.8294	17.867	0.6465	35.735	1.2604	59.942	39.6945	338.60	11.36
1974	15.425	0.4006	31.651	1.8162	36.1239	1.5771	46.808	1.87	28.846	1.5385	29.514	1.3355	14.821	0.6624	30.716	1.2553	37.327	0.8547	235.42	11.36
1975	18.875	0.25	32.375	0.90	46.562		47.375	1.00	29.25	0.80	25.688	0.70	15.625	0.17	25.125	0.70	35.438	0.40	230.57	10.54

Table C-16

Real rate of return for marginal shares of common stock for selected forest products firms traded on the New York Stock Exchange and "Moody's Composit Share"

Year	Boise Cascade Corp.	Crown Zellerbach Corp.	Georgia Pacific Corp.	International Paper Co.	Kimberly Clark Corp.	St.Regis Paper Co.	Scott Paper Co.	Westvaco Corp.	Weyerhaeuser Co.	Mean for all Shares	Moody's Composit
1955		60.2	39.0	27.9	6.3	23.7	0.1	27.3		26.4	13.6
1956		-17.6	-23.9	-20.6	-7.0	-27.6	-12.4	-24.4		-19.1	-3.1
1957		4.5	40.2	14.9	29.0	3.5	12.1	4.7		15.6	6.7
1958		10.9	47.8	25.7	16.6	38.9	25.4	28.0		27.6	26.9
1959		-12.9	26.1	-40.4	14.6	-9.9	3.7	-14.7		-4.8	-3.1
1960		29.0	23.8	210.4	12.6	-11.2	-2.1	-8.3		36.3	21.8
1961		-16.1	-31.2	-8.7	-25.0	-10.1	98.1	-5.1		0.2	-2.1
1962		20.7	13.9	6.9	1.9	7.9	3.5	6.7		8.8	15.7
1963		12.9	13.1	13.5	3.0	8.4	8.0	10.4	15.9	10.6	17.8
1964		-6.9	10.8	-4.0	-13.4	9.7	-0.7	17.5	15.7	3.6	7.7
1965	-14.7	-12.2	-18.9	-7.4	-2.8	5.6	-10.7	-3.0	-18.8	-9.2	-7.3
1966	31.4	4.8	309.0	-1.3	16.3	-7.2	-11.3	-10.7	15.0	40.8	7.3
1967	59.4	9.8	50.1	15.9	6.9	17.9	-0.9	-2.3	50.7	23.1	6.5
1968	18.7	-7.7	-16.6	19.8	6.9	11.7	5.2	38.8	-4.8	8.0	-2.6
1969	-17.6	11.2	64.0	-16.1	-28.5	-20.4	-10.4	-25.9	89.1	5.0	-15.2
1970	-44.2	11.4	4.5	1.2	89.3	14.9	-27.6	-10.1	9.1	5.4	14.5
1971	-54.0	-8.7	-13.2	10.0	11.4	13.8	-24.2	10.0	-4.4	-6.6	11.0
1972	-17.1	11.6	-13.8	16.4	15.5	-8.1	-2.1	29.7	-3.5	3.2	-4.4
1973	-1.8	-17.2	-15.3	-8.4	-37.0	18.8	-13.4	-10.5	28.5	-6.3	-27.1
1974	24.8	8.0	33.3	5.2	6.7	-8.4	9.9	-14.1	-2.8	7.0	2.7
1975											
Mean for all Years	-1.5	4.8	27.1	13.0	6.2	3.6	2.5	2.2	15.8	8.4	4.4

$$\text{Rate of Return}_o = [(\text{Price}_1 + \text{Dividend}_o)/\text{Price}_o] - 1$$

This data was also fitted to a simple linear regression to determine if a time trend could be identified. However, the statistical tests indicated that the coefficient for the time variable is not significantly different from 0. Based on this information the discount rate is estimated to be constant through the period of the analysis. A discount rate based on the mean annual rate of return of the stocks in the portfolio is the basis for the discount rate appropriate to the risk class of investments in timber production. The discount rate for each interval in the analysis is given in Table C-17.

The price of fertilizer treatments is based upon historical per acre costs of fertilization on industrial tree farms in the Douglas-fir region (Industrial Forestry Association, 1975). This data is summarized in Table C-18. There is insufficient data to justify the estimation of different prices for fertilizer treatments for different intervals in the analysis period. Further, the cost data are assumed to be the result of the application of a standard treatment of 150 pounds of elemental nitrogen per acre in the form of urea. The application of this assumption results in an estimated price of 0.1796, 1975 dollars, per pound of elemental nitrogen for fertilizer treatments.

The price of thinning treatments is determined by applying an estimated stumpage price to the estimate of volume removed in the treatment. The stumpage price of volume removed in thinning is based on the average difference of $.24 per cubic foot reported for 1975 between the stumpage

Table C-17

Estimated discount factor for investments in the risk class of timber production by interval

Interval	Estimation Point	Discount Factor
	Year	$(1 + r)^{*n}$
0	1975	1.0
1	1980	1.49674
2	1985	2.24023
3	1990	3.5304
4	1995	5.01864
5	2000	7.51159
6	2005	11.2429
7	2010	16.8277
8	2015	25.1867
9	2020	37.6979
10	2025	56.4240
11	2030	84.4521

$r = 0.084$

Table C-18

Average cost of forest fertilization by year of treatment.[1]

Year	Average cost of forest fertilization per acre	
	Dollars	1975 $
1968	22.02	33.36
1969	18.47	26.54
1970	15.53	21.07
1971	17.97	23.37
1972	16.72	21.06
1973	22.21	27.06
1974	33.82	36.13
Mean		26.94

[1]From: Industrial Forestry Association (1975)

price paid for young growth Douglas-fir and Douglas-fir thinnings reported by the Washington State Department of Revenue. This difference was applied as a reduction to the stumpage price of trees with a 26 inch DBH, and the stumpage price of trees of other DBH classes was estimated from a line parallel to the base line for clearcut stumpage prices. This is illustrated in Figures C-4 to C-9. The resulting stumpage prices for volume removed in thinnings are given in Table C-19.

The natural regeneration treatment considered in the analysis required no inputs other than those involved in waiting for the next stand to become established. Therefore, there are no unique prices associated with this regeneration treatment.

Yield Tables For Treatment Combinations Considered In The Demonstration Analysis

The abbreviated yield tables which follow present estimates in the form of per acre summaries of stand yield for all combinations of treatments considered in the analysis. The tables are presented in the order in which treatments are considered. The underlined entries in the table represent the stand conditions existing at the start of each interval.

The description of the column headings in the tables is as follows:

Site Index: Height of site trees at breast height age 50 years. Site trees are the ten trees of largest DBH from a sample of 50 trees (King, 1966).

Table C-19

Estimated stumpage price of douglas-fir thinnings per cubic foot by interval and DBH class

DBH Class	Interval 0	1	2	3	4	5	5+
Inches	1975 Dollar Per Cubic Foot						
2	-0.0522	-0.0605	-0.0581	-0.0568	-0.0552	-0.0542	-0.0542
3	-0.0460	-0.0570	-0.0542	-0.0525	-0.0505	-0.0490	-0.0490
4	-0.0390	-0.0530	-0.0490	-0.0475	-0.0455	-0.0435	-0.0435
5	-0.0310	-0.0490	-0.0450	-0.0425	-0.0395	-0.0370	-0.0370
6	-0.0225	-0.0445	-0.0390	-0.0370	-0.0335	-0.0301	-0.0301
7	-0.0125	-0.0395	-0.0342	-0.0305	-0.0265	-0.0225	-0.0225
8	-0.0015	-0.0340	-0.0275	-0.0235	-0.0190	-0.0140	-0.0140
9	0.0120	-0.0280	-0.0210	-0.0160	-0.0100	-0.0045	-0.0045
10	0.0250	-0.0215	-0.0130	-0.0070	-0.0001	0.0060	0.0060
11	0.0420	-0.0145	-0.0055	-0.0020	0.0100	0.0175	0.0175
12	0.0590	-0.0070	0.0040	0.0120	0.0220	0.0310	0.0310
13	0.0800	0.0020	0.0130	0.0240	0.0320	0.0450	0.0450
14	0.1040	0.0110	0.0240	0.0350	0.0475	0.0625	0.0625
15	0.1290	0.0210	0.0350	0.0490	0.0640	0.0810	0.0810
16	0.1580	0.0320	0.0475	0.0640	0.0810	0.1000	0.1000
17	0.1920	0.0430	0.0625	0.0800	0.1000	0.1240	0.1240
18	0.2280	0.0560	0.0775	0.0975	0.1210	0.1475	0.1475
19	0.2700	0.0740	0.0940	0.1175	0.1450	0.1760	0.1760
20	0.3180	0.0850	0.1150	0.1400	0.1710	0.2075	0.2075
21	0.3725	0.1010	0.1325	0.1650	0.2000	0.2410	0.2410
22	0.4325	0.1190	0.1550	0.1900	0.2325	0.2800	0.2800
23	0.5000	0.1390	0.1780	0.2180	0.2675	0.3225	0.3225
24	0.5750	0.1610	0.2050	0.2500	0.3100	0.3700	0.3700
25	0.6650	0.1850	0.2350	0.2850	0.3500	0.4250	0.4250
26	0.7669	0.2100	0.2650	0.3260	0.395	0.4752	0.4752
27	0.8750	0.2380	0.3000	0.3650	0.4500	0.5450	0.5450
28	1.0000	0.2690	0.3375	0.4100	0.5050	0.6200	0.6200
29	1.1400	0.3010	0.3780	0.4650	0.5700	0.6990	0.6990
30	1.3000	0.3390	0.4250	0.5200	0.6450	0.7900	0.7900

Growing Stock Index: Level of growing stock expressed as basal area per acre when the age of a stand is 50 years at breast height, based on the age of site trees (King, 1970).

Age: Average total age of site trees.

Height of Site Trees: Average total height of site trees.

Trees: Number of living trees.

Average DBH: Diameter of tree of average basal area.

Basal Area: Sum of the breast-height cross-sectional areas (including bark) of all living trees larger than 1.5 inches DBH.

Total Volume: Total stem volume inside bark of all trees larger than 1.5 inches DBH.

Table C-20

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220
Treatments: None

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
25	46	284	6.0	55.9	1036
30	59	326	7.1	89.5	2024
35	70	332	8.1	119.8	3143
40	80	323	9.1	146.5	4308
45	90	307	10.1	170.0	5479
50	98	291	11.0	190.9	6630

Table C-21

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220
Treatments: Fertilization - Age 25, 100 pounds of elemental nitrogen per acre

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
25	46	284	6.0	55.9	1036
30	60	238	7.5	72.4	1643
35	71	255	8.5	101.0	2655
40	82	257	9.5	126.8	3733
45	91	251	10.5	149.7	4828

Table C-22

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220
Treatments: Fertilization - Age 25, 200 pounds of elemental nitrogen per acre

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
25	46	284	6.0	55.9	1036
30	60	266	7.4	79.0	1801
35	71	280	8.4	108.4	2861
40	82	278	9.4	134.5	3975
45	91	269	10.4	157.7	5105

Table C-23

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220
Treatments: Fertilization - Age 25, 300 pounds of elemental nitrogen per acre

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
25	46	284	6.0	55.9	1036
30	61	287	7.4	85.8	1989
35	72	298	8.4	115.8	3102
40	83	293	9.4	142.3	4261
45	92	282	10.4	165.7	5428

Table C-24

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220
Treatments: Fertilization - Age 25, 400 pounds of nitrogen per acre

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
25	46	284	6.0	55.9	1036
30	62	310	7.4	92.9	2190
35	73	317	8.5	123.6	3360
40	84	309	9.4	150.4	4564
45	93	295	10.4	174.0	5769

Table C-25

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments:

Thinning: Age 25, 10% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	117	1.6	1.6	65	167	7.7	54.3	971
30	59	167	9.8	86.6	1800								
35	70	167	11.1	113.0	2780								
40	80	167	12.3	137.8	3850								
45	90	167	13.3	161.2	4971								
50	98	167	14.2	183.4	6119								

Table C-26

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index:220

Treatments: Fertilization: Age 25, 100 pounds of elemental nitrogen per acre

Thinning: Age 25, 10% basal are removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	117	1.6	1.6	65	167	7.7	54.3	971
30	61	167	10.0	90.3	1911								
35	74	167	11.1	112.1	2885								
40	86	167	12.3	136.8	3949								
45	95	167	13.3	160.1	5064								
50	104	167	14.1	182.2	6207								

Table C-27

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 | Growing Stock Index: 220

Treatments: | Fertilization: Age 25, 200 pounds of elemental nitrogen per acre

Thinning: Age 25, 10% basal area removal

		Before Thinning				Thinning				After Thinning			
Stand Age	Ht. Site Trees	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	117	1.6	1.6	65	167	7.7	54.3	971
30	62	167	10.2	94.1	2021								
35	75	167	11.3	116.3	3025								
40	86	167	12.5	141.5	4118								
45	95	167	13.5	165.2	5262								
50	104	167	14.4	187.8	6432								

Table C-28

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 300 pounds of elemental nitrogen per acre

Thinning: Age 25, 10% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	117	1.6	1.6	65	167	7.7	54.3	971
30	63	167	10.4	97.8	2132								
35	76	167	11.5	120.3	3165								
40	86	167	12.7	145.9	4286								
45	95	167	13.7	170.1	5456								
50	104	167	14.6	193.0	6653								

Table C-29

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments:

Fertilization: Age 25, 400 pounds of elemental nitrogen per acre

Thinning: Age 25, 10% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	117	1.6	1.6	.65	167	7.7	54.3	971
30	64	167	10.6	101.6	2242								
35	76	167	11.7	123.9	3301								
40	86	167	12.8	150.0	4447								
45	95	167	13.8	174.6	5643								
50	103	167	14.7	197.8	6864								

Table C-30

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments:

Thinning: Age 25, 20% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	182	3.5	12.0	145	102	8.9	43.9	891
30	59	102	12.1	81.6	1677								
35	70	102	13.9	107.3	2617								
40	80	102	15.4	131.4	3647								
45	90	102	16.7	154.3	4729								
50	98	102	17.8	175.9	5840								

Table C-31

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 100 pounds of elemental nitrogen per acre

Thinning: Age 25, 20% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	182	3.5	12.0	145	102	8.9	43.9	891
30	61	102	12.4	85.2	1782								
35	75	102	13.8	105.3	2707								
40	86	102	15.2	129.2	3723								
45	95	102	16.5	151.8	4792								
50	104	102	17.6	173.3	5890								

Table C-32

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 200 pounds of elemental nitrogen per acre

Thinning: Age 25, 20% basal area removal

		Before Thinning				Thinning				After Thinning			
Stand Age	Ht. Site Trees	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	182	3.5	12.0	145	102	8.9	43.9	891
30	62	102	12.6	88.9	1887								
35	75	102	13.9	107.5	2828								
40	86	102	15.4	131.7	3860								
45	95	102	16.7	154.6	4944								
50	104	102	17.8	176.2	6057								

Table C-33

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 | Growing Stock Index: 220

Treatments: | Fertilization: Age 25, 300 pounds of elemental nitrogen per acre

Thinning: Age 25, 20% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	182	3.5	12.0	145	102	8.9	43.9	891
30	63	102	12.9	92.6	1992								
35	76	102	14.1	111.2	2960								
40	86	102	15.6	135.8	4017								
45	95	102	16.9	159.1	5127								
50	104	102	18.0	181.1	6264								

Table C-34

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 400 pounds of elemental nitrogen per acre

Thinning: Age 25, 20% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	182	3.5	12.0	145	102	8.9	43.9	891
30	64	102	13.2	96.2	2097								
35	76	102	14.4	115.0	3092								
40	86	102	15.9	140.0	4176								
45	95	102	17.2	163.7	5311								
50	104	102	18.3	186.1	6473								

Table C-35

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments:

Thinning: Age 25, 30% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	214	3.9	18.0	238	70	10.0	37.9	798
30	59	70	14.0	75.2	1531								
35	70	70	16.2	100.0	2418								
40	80	70	18.0	123.4	3398								
45	90	70	19.5	145.5	4432								
50	98	70	20.9	166.4	5495								

Table C-36

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 | Growing Stock Index: 220

Treatments: | Fertilization: Age 25, 100 pounds of elemental nitrogen per acre

Thinning: Age 25, 30% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	214	3.9	18.0	238	70	10.0	37.9	798
30	61	70	14.4	78.7	1628								
35	75	70	16.2	99.6	2513								
40	86	70	17.9	123.0	3490								
45	95	70	19.5	145.0	4521								
50	104	70	20.8	165.9	5582								

Table C-37

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 200 pounds of elemental nitrogen per acre

Thinning: Age 25, 30% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	214	3.9	18.0	238	70	10.0	37.9	798
30	62	70	14.7	82.3	1726								
35	75	70	16.4	103.2	2637								
40	86	70	18.2	127.0	3639								
45	95	70	19.8	149.4	4694								
50	104	70	21.1	170.6	5778								

Table C-38

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 300 pounds of elemental nitrogen per acre

Thinning: Age 25, 30% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	214	3.9	18.0	238	70	10.0	37.9	798
30	63	70	15.0	85.8	1824								
35	76	70	16.7	107.1	2763								
40	86	70	18.5	131.3	3792								
45	95	70	20.1	154.1	4873								
50	104	70	21.5	175.7	5984								

Table C-39

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 | Growing Stock Index: 220

Treatments: | Fertilization: Age 25, 400 pounds of elemental nitrogen per acre

Thinning: Age 25, 30% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	214	3.9	18.0	238	70	10.0	37.9	798
30	64	70	15.3	89.4	1922								
35	76	70	17.0	110.7	2886								
40	86	70	18.8	135.2	3940								
45	95	70	20.4	158.4	5045								
50	104	70	21.7	180.4	6179								

Table C-40

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments:

Thinning: Age 25, 40% basal area removal

		Before Thinning				Thinning				After Thinning			
Stand Age	Ht. Site Trees	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	238	4.3	24.0	337	46	11.3	31.9	699
30	59	46	16.5	68.2	1373								
35	70	46	19.2	92.1	2203								
40	80	46	21.4	114.6	3127								
45	90	46	23.3	135.8	4107								
50	98	46	24.9	155.9	5118								

Table C-41

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 100 pounds of elemental nitrogen per acre

Thinning: Age 25, 40% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	238	4.3	24.0	337	46	11.3	31.9	699
30	61	46	16.8	71.2	1463								
35	75	46	19.4	94.4	2310								
40	86	46	21.6	117.1	3250								
45	95	46	23.5	138.6	4245								
50	104	46	25.2	159.0	5271								

Table C- 42

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 200 pounds of elemental nitrogen per acre

Thinning: Age 25, 40% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	238	4.3	24.0	337	46	11.3	31.9	699
30	66	46	17.3	75.0	1553								
35	75	46	19.8	98.3	2428								
40	86	46	22.0	121.5	3396								
45	95	46	23.9	143.4	4417								
50	104	46	25.6	164.1	5469								

Table C-43

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 300 pounds of elemental nitrogen per acre

Thinning: Age 25, 40% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	238	4.3	24.0	337	46	11.3	31.9	699
30	61	46	17.7	78.4	1642								
35	73	46	20.1	101.6	2541								
40	84	46	22.3	125.2	3532								
45	94	46	24.2	147.4	4576								
50	103	46	25.9	168.5	5650								

Table C-44

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index:220

Treatments: Fertilization: Age 25, 400 pounds of elemental nitrogen per acre

Thinning: Age 25, 40% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	46	284	6.0	55.9	1036	238	4.3	24.0	337	46	11.3	31.9	699
30	64	46	18.1	81.8	1732								
35	76	46	20.5	105.2	2657								
40	86	46	22.7	129.2	3672								
45	86	46	22.7	129.2	3672								
50	104	46	26.3	173.2	5838								

Table C-45

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 110 Growing Stock Index: 220
Treatments: Natural regeneration

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
15	19	193	3.4	12.4	123
20	32	238	4.3	24.3	342
25	46	284	6.0	55.9	1036
30	59	326	7.1	89.5	2024
35	70	332	8.1	119.8	3143
40	80	323	9.1	146.5	4308
45	90	307	10.1	170.0	5479
50	98	291	11.0	190.9	6630

Table C-46

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220
Treatments: None

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
25	53	228	7.0	60.2	1239
30	66	257	8.2	93.4	2322
35	79	261	9.3	123.4	3539
40	90	254	10.4	149.8	4795
45	100	242	11.4	173.2	6061
50	109	230	12.4	194.1	7307

Table C-47

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220
Treatments: Fertilization: Age 25, 100 pounds of elemental nitrogen per acre

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
25	53	228	7.0	60.2	1239
30	65	210	8.2	76.9	1859
35	79	212	9.5	105.3	2990
40	90	213	10.6	130.9	4152
45	100	207	11.7	153.9	5340

Table C-48

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220
Treatments: Fertilization: Age 25, 200 pounds of elemental nitrogen per acre

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
25	53	228	7.0	60.2	1239
30	66	225	8.2	83.2	2044
35	79	230	9.4	112.3	3201
40	90	228	10.5	138.2	4399
45	100	220	11.6	161.3	5615

Table C-49

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220
Treatments: Fertilization: Age 25, 300 pounds of elemental nitrogen per acre

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
25	53	228	7.0	60.2	1239
30	67	241	8.3	89.7	2238
35	79	246	9.4	119.3	3430
40	90	242	10.5	145.6	4671
45	100	232	11.6	168.9	5922

Table C-50

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220
Treatments: Fertilization: Age 25, 400 pounds of elemental nitrogen per acre

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
25	53	228	7.0	60.2	1239
30	68	258	8.3	96.6	2448
35	80	261	9.4	126.8	3696
40	91	254	10.5	153.3	4978
45	101	242	11.6	176.8	6267

Table C-51

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Stand Density Index: 220

Treatments:

Thinning: Age 25, 10% basal area removal

		Before Thinning				Thinning				After Thinning			
Stand Age	Ht. Site Trees	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	84	2.0	1.9	80	204	7.2	58.3	1159
30	66	204	8.9	88.2	2080								
35	79	204	10.2	114.7	3146								
40	90	204	11.2	139.5	4297								
45	100	204	12.1	162.8	5495								
50	109	204	12.9	184.8	6715								

Table C-52

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 | Growing Stock Index: 220

Treatments:

Fertilization: Age 25, 100 pounds of elemental nitrogen per acre

Thinning: Age 25, 10% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	84	2.0	1.9	80	204	7.2	58.3	1159
30	67	204	9.1	92.0	2202								
35	81	204	10.3	117.3	3288								
40	91	204	11.3	142.4	4457								
45	101	204	12.2	166.0	5672								
50	111	204	13.0	188.2	6909								

Table C-53

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120

Growing Stock Index: 220

Treatments:

Fertilization: Age 25, 200 pounds of elemental nitrogen per acre

Thinning: Age 25, 10% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	84	2.0	1.9	80	204	7.2	58.3	1159
30	69	204	9.2	95.8	2325								
35	81	204	10.4	120.2	3432								
40	92	204	11.4	145.6	4621								
45	102	204	12.3	169.5	5856								
50	111	204	13.1	192.0	7112								

Table C- 54

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 300 pounds of elemental nitrogen per acre

Thinning: Age 25, 10% basal area removal

		Before Thinning				Thinning				After Thinning			
Stand Age	Ht. Site Trees	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	84	2.0	1.9	80	204	7.2	58.3	1159
30	70	204	9.5	99.6	2448								
35	82	204	10.6	124.2	3583								
40	92	204	11.6	150.0	4801								
45	102	204	12.5	174.3	6062								
50	111	204	13.3	197.2	7345								

Table C-55

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220

Treatments:

Fertilization: Age 25, 400 pounds of elemental nitrogen per acre

Thinning: Age 25, 10% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	84	2.0	1.9	80	204	7.2	58.3	1159
30	71	204	9.6	103.4	2570								
35	85	204	10.7	128.2	3734								
40	95	204	11.8	154.5	4979								
45	105	204	12.7	179.2	6268								
50	114	204	13.5	202.5	7577								

Table C-56

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220

Treatments:

Thinning: Age 25, 20% basal area removal

		Before Thinning				Thinning				After Thinning			
Stand Age	Ht. Site Trees	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	130	4.1	12.2	181	98	9.5	48	1058
30	66	98	12.4	82.8	1934								
35	79	98	14.3	108.6	2957								
40	90	98	15.8	132.7	4065								
45	100	98	17.1	155.7	5221								
50	109	98	18.2	176.8	6401								

Table C-57

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 | Growing Stock Index: 220

Treatments: | Fertilization: Age 25, 100 pounds of elemental nitrogen per acre

Thinning: Age 25, 20% basal area removal

		Before Thinning				Thinning				After Thinning			
Stand Age	Ht. Site Trees	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	130	4.1	12.2	181	98	9.5	48.0	1058
30	67	98	12.7	86.5	2051								
35	81	98	14.3	108.8	3075								
40	91	98	16.1	132.9	4185								
45	101	98	17.1	155.6	5343								
50	111	98	18.2	177.1	6524								

Table C-58

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 | Growing Stock Index: 220

Treatments: | Fertilization: Age 25, 200 pounds of elemental nitrogen per acre

Thinning: Age 25, 20% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	130	4.1	12.2	181	98	9.5	48.0	1058
30	69	98	13.0	90.2	2167								
35	81	98	14.5	112.7	3219								
40	92	98	16.0	137.2	4356								
45	102	98	17.3	160.4	5540								
50	111	98	18.5	182.2	6747								

Table C-59

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120

Growing Stock Index: 220

Treatments:

Fertilization: Age 25, 300 pounds of elemental nitrogen per acre

Thinning: Age 25, 20% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	130	4.1	12.2	181	98	9.5	48	1058
30	70	98	13.3	93.9	2284								
35	82	98	14.8	116.8	3366								
40	92	98	16.3	141.8	4531								
45	102	98	17.5	165.3	5743								
50	111	98	18.7	187.5	6977								

Table C- 60

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 400 pounds of elemental nitrogen per acre

Thinning: Age 25, 20% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	130	4.1	12.2	181	98	9.5	48.0	1058
30	71	98	13.5	97.5	2401								
35	85	98	14.9	119.4	3502								
40	95	98	16.5	144.7	4686								
45	105	98	17.8	168.5	5915								
50	114	98	18.9	191.0	7166								

Table C- 61

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220

Treatments:

Thinning: Age 25, 30% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	156	4.6	18.3	291	72	10.3	41.9	948
30	66	72	14.0	76.6	1771								
35	79	72	16.1	101.4	2743								
40	90	72	17.8	124.8	3801								
45	100	72	19.3	146.7	4909								
50	109	72	20.7	167.5	6042								

Table C- 62

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120

Growing Stock Index: 220

Treatments:

Fertilization: Age 25, 100 pounds of elemental nitrogen per acre

Thinning: Age 25, 30% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	156	4.6	18.3	291	72	10.3	41.9	948
30	67	72	14.3	80.1	1881								
35	81	72	16.3	104.7	2876								
40	91	72	18.1	128.4	3958								
45	101	72	19.6	150.7	5087								
50	111	72	20.9	171.7	6242								

Table C- 63

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 200 pounds of elemental nitrogen per acre

Thinning: Age 25, 30% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	156	4.6	18.3	291	72	10.3	41.9	948
30	69	72	14.6	83.7	1990								
35	81	72	16.6	108.5	3013								
40	92	72	18.4	132.6	4121								
45	102	72	19.9	155.3	5277								
50	101	72	21.2	176.8	6456								

Table C- 64

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120

Growing Stock Index: 220

Treatments:

Fertilization: Age 25, 300 pounds of elemental nitrogen per acre

Thinning: Age 25, 30% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	156	4.6	18.3	291	72	10.3	41.9	948
30	70	72	14.9	87.2	2100								
35	82	72	16.8	111.2	3142								
40	92	72	18.6	135.6	4268								
45	102	72	20.1	158.6	5442								
50	111	72	21.4	180.2	6639								

Table C- 65

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 400 pounds of elemental nitrogen per acre

Thinning: Age 25, 30% basal area removal

		Before Thinning				Thinning				After Thinning			
Stand Age	Ht. Site Trees	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	156	4.6	18.3	291	72	10.3	41.9	948
30	71	72	15.2	90.8	2210								
35	85	72	17.1	115.4	3283								
40	95	72	18.9	140.4	4439								
45	105	72	20.4	163.8	5642								
50	114	72	21.8	185.8	6867								

Table C- 66

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220

Treatments:

Thinning: Age 25, 40% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	176	5.0	24.4	407	52	11.2	35.8	832
30	66	52	15.7	69.9	1599								
35	79	52	18.2	93.9	2516								
40	90	52	20.3	116.4	3522								
45	100	52	22.0	137.6	4579								
50	109	52	23.6	157.6	5662								

Table C- 67

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 100 pounds of elemental nitrogen per acre

Thinning: Age 25, 40% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	176	5.0	24.4	407	52	11.2	35.8	832
30	69	52	16.1	73.3	1701								
35	83	52	18.3	94.5	2623								
40	95	52	20.3	117.1	3633								
45	106	52	22.1	138.4	4694								
50	116	52	23.6	158.4	5782								

Table C- 68

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120

Growing Stock Index: 220

Treatments:

Fertilization: Age 25, 200 pounds of elemental nitrogen per acre

Thinning: Age 25, 40% basal area removal

		Before Thinning				Thinning				After Thinning			
Stand Age	Ht. Site Trees	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	176	5.0	24.4	407	52	11.2	35.8	832
30	69	52	16.4	76.7	1803								
35	81	52	18.6	98.4	2753								
40	92	52	20.7	121.4	3791								
45	102	52	22.5	143.1	4878								
50	111	52	24.0	163.5	5991								

Table C- 69

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 | Growing Stock Index: 220

Treatments: | Fertilization: Age 25, 300 pounds of elemental nitrogen per acre

Thinning: Age 25, 40% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	176	5.0	24.4	407	52	11.2	35.8	832
30	70	52	16.8	80.2	1905								
35	82	52	19.0	102.2	2882								
40	92	52	21.0	125.6	3946								
45	102	52	22.8	147.7	5059								
50	111	52	24.4	168.5	6197								

Table C- 70

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220

Treatments: Fertilization: Age 25, 400 pounds of elemental nitrogen per acre

Thinning: Age 25, 40% basal area removal

Stand Age	Ht. Site Trees	Before Thinning				Thinning				After Thinning			
		Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume	Trees	Ave. DBH	Basal Area	Total Volume
Years	Feet	Number	Inches	Sq.Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.	Number	Inches	Sq. Ft.	Cu. Ft.
25	53	228	7.0	60.2	1239	176	5.0	24.4	407	52	11.2	35.8	832
30	71	52	17.2	83.6	2007								
35	85	52	19.3	106.0	3012								
40	95	52	21.4	129.9	4102								
45	105	52	23.2	152.3	5241								
50	114	52	24.7	173.5	6404								

Table C- 71

Abbreviated yield table for Douglas-fir

Stand data per acre for trees 1.6 inches DBH and larger

Site Index: 120 Growing Stock Index: 220
Treatments: Natural regeneration

Stand Age Years	Ht. Site Trees Feet	Trees Number	Ave. DBH Inches	Basal Area Sq. Ft.	Total Volume Cu. Ft.
15	23	126	4.1	11.8	. 131
20	38	159	5.7	27.8	440
25	53	228	7.0	60.2	1239
30	66	257	8.2	93.4	2322
35	79	261	9.3	123.4	3539
40	90	254	10.4	149.8	4795
45	100	242	11.4	173.2	6061
50	109	230	12.4	194.1	7307