ISSN 1471-0498

DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

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Number 171

October 2003

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[1] I thank Frank Scherer for helpful discussion, encouragement and for providing valuable archival material on procurement, especially the copy of Moore (1962). I am also grateful for the financial support from the ESRC Research Fellowship Award R000 27 1065.

Abstract

This paper presents a positive theory about the contractual form of procurement contracts under cost uncertainty. While the cost of manufacture is uncertain it can be controlled, to an extent depending on the effort exerted by the agent. The effort exerted by the agent is not contractible but causes disutility to the agent. Hence, the amount of effort exerted depends on the power of incentives built into the terms of reimbursement agreed to in the contract. The analysis in the paper explicitly models the possibility that the belief about the cost uncertainty is ambiguous, in the sense that belief is described by a set of probabilities, rather than by a single probability. This allows us to incorporate ambiguity aversion (behavior of the kind seen in Ellsberg's "paradox") into the players' objective functions. The paper finds that, provided the agent is more averse to ambiguity than the principal, the more the ambiguity of belief the lower the power of the optimal incentive scheme. The fix-price contract is optimal if there is no ambiguity, but if the ambiguity is high enough a cost-plus contract is optimal; in between, a cost-share scheme is optimal. It is contended that the finding is particularly useful in explaining facts about the wide use of cost-plus and similar low powered contracts in research and development (R&D) procurement by the U. S. Department of Defense.

JEL Classification Numbers: D800, D810, D820, D890

Keywords: Procurement contracts, incentive contracts, uncertainty aversion, Ellsberg's paradox, cost reimbursement contracts, cost-plus contracts, fixed price contracts

1 Introduction

This paper presents a positive theory about the contractual form of procurement contracts under cost uncertainty. The two parties to the generic procurement contract considered are the principal, say the Department of Defense (DoD) which wishes to buy a unit item, and the agent, say a defense contractor firm which will manufacture and supply the item. The parties agree to terms of reimbursement and delivery in the contract. The contractor undertakes manufacture of the item *after* signing the contract and at the time of signing the cost of production is not known for sure to either party. However, while the cost of manufacture is uncertain it can be controlled, to an extent depending on the effort exerted by the agent. The effort exerted by the agent is not contractible but causes disutility to the agent and hence, its extent depends on the power of incentives built into the terms of reimbursement agreed to in the contract. As is usual in the literature, we assume that both the government and the firm are risk neutral. What is different in this paper is the analysis of the consequences of the possibility that the parties' beliefs are *ambiguous* and that they are *ambiguity averse*. The principal finding is that the greater the ambiguity and greater ambiguity aversion of the agent *relative* to that of the principal, the lower the power of the incentive scheme in the optimal contract.[1] Indeed, if the ambiguity is high enough or the relative ambiguity aversion of the agent high enough, then the form of the optimal contract is a cost-plus contract. It is contended that the finding is particularly significant in understanding the wide use of cost-plus and similar low powered contracts in research and development (R&D) procurement.

Savage's theory (Savage (1954)) of subjective expected utility maximization (SEU) is the received paradigm used for modelling decision-making under subjective uncertainty. A main implication of SEU is that a decision maker (DM) behaves as if his subjective assessment of likelihoods of uncertain events may be described by a precise and unique probability distribution. It is often the case, however, that a DM's knowledge about the likelihood of contingent events is consistent with more than one probability distribution, i.e., beliefs are *ambiguous*. But, does how precisely he knows the relevant odds influence the *choice* of the typical DM? Ellsberg's classic contribution (Ellsberg (1961)) laid the ground for showing that it does: imprecise information about odds affected behavior in a pervasive way: most preferred to bet on events with unambiguous rather than ambiguous odds.[2] People adjusting their decisions depending on how well they know the relevant odds and acting with greater wariness the more vague their knowledge of the odds, is a commonly observed attitude, and has been termed *ambiguity aversion*.[3] Ceteris paribus,

[1] The result, strictly interpreted, does not require either the principal or the agent to be ambiguity averse, but just the agent be *more* averse than the principal. Hence, the analysis applies to the case of a ambiguity neutral principal and ambiguity averse agent.

[2] Ellsberg presented a pair of examples of thought experiments, now famously called Ellsberg "paradoxes", which showed that there were circumstances where it would seem reasonable for decision makers to let their behavior be affected by their knowledge of how well they knew the relevant odds. Ellsberg's conjecture about how most people would behave in such experiments have been borne out in subsequent formal experimental work. See Camerer and Weber (1992) for details.

[3] An alternative terminology for ambiguity aversion is *uncertainty aversion*, a term used in the pioneering papers of this literature, Gilboa (1987), Gilboa and Schmeidler (1989), Schmeidler (1989). The term ambiguity aversion goes back a long way too, at least to Ellsberg (1961); recently, many researchers have returned to this term, as for instance, Epstein and Zhang (2001) and Ghirardato and Marinacci (2002).

an ambiguity averse DM will be averse to acts with payoffs that are crucially contingent on events about whose odds the DM has a relatively poor idea of, just as a risk averse agent shies away from risky acts.

If a DM's belief is ambiguous his knowledge is consistent with more than one probability distribution (over the relevant state space). Hence, formally, ambiguous belief is represented by a (convex) *set* of priors, rather than a single prior. Therefore, given an act with state contingent payoffs, there would be an *interval* of expected payoffs of the act consistent the DM's knowledge. Gilboa and Schmeidler (1989), was one of the pioneering contributions that showed how one may in incorporate ambiguity aversion into decision making. In Gilboa-Schmeidler's model, the DM evaluates an act by the minimum expected payoff consistent with his knowledge. In this paper we invoke a generalization of that model, axiomatized in Ghirardato, Maccheroni, and Marinacci (2002), wherein an act is evaluated by a weighted sum of its minimum and maximum possible expected payoffs, given the set of priors. The DM is considered to be more ambiguity averse the greater the weight he puts on the minimum as opposed to the maximum expected payoff.

Reimbursement schemes incorporated in procurement contracts vary in the power of incentives delivered. In the "cost-plus-fixed-fee" contract or, more simply, the "cost-plus" contract, the government pays the contractor the realized cost and plus a fee fixed ex ante. The cost-plus contract lies at the lower end of the spectrum delineating the power of commonly used incentive schemes. Because in such a contract the full realized cost is reimbursed the agent has no incentive to expend effort at reducing the cost. At the opposite end of the spectrum is the "firm fixed-price" contract or, more simply, "fixed-price" contract. A fixed-price cost incorporates very high powered incentives since it effectively makes the agent a residual claimant for the entire amount of the cost reduction achieved through the application of effort. In between these two extremes lie what are sometimes called "incentive" or "cost-share" contracts, in which the contractor and the government share the realized costs according to some predetermined sharing rule.

In this paper we consider contracts in which the reimbursement is linear in the realized cost: the reimbursement $P = \delta + \gamma C$; C is the realized cost and δ,γ are constants specified in the contract. Notice, $\gamma = 0$ corresponds to a fixed price contract while $\gamma = 1$ gives the cost-plus contract; when $0 < \gamma < 1$ we have the incentive contracts. The principal chooses δ and γ to maximize the surplus between the value of the item and the (expected) reimbursement. We assume that the principal and agent are symmetrically informed about the cost uncertainty at the time of writing the contract; their respective belief is described by a (possibly non-singleton) set of priors. The main result in the paper establishes, assuming that the agent is more averse to ambiguity than the principal, that the optimal γ is zero when there is no ambiguity, i.e., belief is described by a single prior, but increases monotonically to one as ambiguity increases, in the formal sense that the set of priors used to describe beliefs expands. Thus fixed price contracts are optimal when there is no ambiguity and cost-plus contracts become optimal when ambiguity is high enough. Results in the paper also show that if the principal and agent have (quantitatively) identical attitudes to ambiguity, the optimal contract is a fixed price contract. More generally, the more averse the agent's attitude to ambiguity relative to the principal's the larger the γ, and hence, the greater the role of ambiguity in reducing the power of the contracted incentive scheme.

An intuition for the results is as follows. Assuming the cost uncertainty is ambiguous, there is an interval of expected cost values (to fix ideas, it might help to think of this as a range of cost estimates). The key point to note (assuming that the agent is more averse to ambiguity than the principal) is that the principal's (ex ante) evaluation of the surplus from a contract incorporating a particular cost reimbursement scheme, will put a comparatively smaller weight on the high end of the interval of estimated costs, compared to the agent's evaluation. The next point to note is that the *smaller* the γ the *greater* the reimbursement amount that is fixed up front and based on ex ante evaluation of costs. Under a fixed price contract, i.e., $\gamma = 0$, the agent has the maximum possible incentive to apply effort to reduce costs; the payment being fixed up front, the agent gets keep every cent of cost reduced by his effort. However, under a fixed price contract, given that the agent's participation constraint has to be satisfied, the agent has to be reimbursed corresponding to a cost evaluation that is considered excessive by the principal. This is so because the principal, being less ambiguity averse than the agent, evaluates costs less pessimistically than the agent (the agent applies a relatively higher weight, than the principal, to more pessimistic/conservative beliefs). By reducing the fixed price portion and increasing the proportion paid ex post, i.e., increasing γ, the principal reduces the amount he has to pay based on ex ante cost estimates and increases the amount paid base on realized costs. Hence, by increasing γ the principal reduces the "ambiguity premium", the "extra" he has to pay corresponding to the relatively more pessimistic ex ante evaluation by the agent. Of course, this comes at the expense of decreasing the cost reducing effort. The trade off goes increasingly in favor of effecting a reduction in ambiguity premium as ambiguity increases (or as the ambiguity aversion of the agent relative to the principal increases). Indeed, with ambiguity high enough the contract is fully cost-plus, i.e., $\gamma = 1$.

The rest of the paper is organized as follows. The next section summarizes some institutional detail, empirics and findings reported in the related economic literature, relating to procurement contracting, that motivate this exercise. Section 3 briefly explains some technical details of the model of ambiguity aversion applied in the contacting model. Section 4 presents the formal contracting model and its analysis. Section 5 concludes the paper.

2 The motivation

Why might this exercise, showing the connection between ambiguity and cost-plus contracts (and more generally, between ambiguity and low-powered cost reimbursement contracts) be of interest? Principally, because it provides a good explanation for the long and widely observed link between cost uncertainty and the intensive use of cost-plus (and similar low powered contracts) in R&D procurement, especially by the U.S. Department of Defense (DoD).[4] The view that this link is very strong is prominent amongst analysts.

[4] In 1960, 40.9% of U.S. military procurement dollars involved cost-plus contracts, 13.6% incentive contracts, and 31.4% fixed price contracts (Laffont and Tirole (1993), page 12). While use of cost type contracts has declined somewhat over the years, the usage continues to be more than significant. In 1998, fairly representative of the trend in the 90s, the cost reimbursement type accounted for just over 40% of the total value of DoD Prime Contract Awards while about 16% were cost plus fixed fee type.

The total dollar value of R&D awards (procured domestically) amounted to $19.6 billion in 1998, which was 17.3% of the total value of defense procurement. Of the $19.6 billion, "educational and non-profit

Consider, for instance, the following remarks (on page 145) in the standard reference on defense procurement, Scherer (1964):

> R&D work is generally accompanied by significant cost uncertainties, and rightly or wrongly, cost reimbursement contracts have traditionally been chosen to cover such activities. For example, in the major hard goods category for fiscal year 1960, roughly 93% by dollar volume of Navy R&D contracts, but only 17% of production contracts were of the cost reimbursement type. ... A time-series analysis for fiscal years 1956 through 1962 suggests that the percentage of all military procurement obligations made under cost reimbursement contracts (Y) was an increasing linear function of the percentage of all awards going to experimental, developmental, test, and research work (X):
>
> $$Y = 19.29 + \underset{(.214)}{.917}X;$$
>
> with r^2 of .85.

A more modern day classic on the same subject, Laffont and Tirole (1993) reiterates this view thus (page 12):

> One must be careful to distinguish among stages of the procurement life cycle in this respect. A stylized fact is that low-powered contracts are employed much more in the early phase of the life cycle. Another stylized fact is that low-powered contracts are also more employed for high technology than for nonstandard equipment.

Other analysts of defense procurement policy have observed similarly, (see e.g.: Baron (1993), page 8; Fox (1974), pages 230-31; Reichelstein (1992), page 713; Rogerson (1994), page 67). Finally, it is worth noting how deeply entrenched this view is in practitioners' minds as well. For instance, the Federal Acquisitions Regulation (FAR) guidelines (followed by DoD), regarding choice of contract type in procurement, notes:[5]

> *Subpart 16.301-2*: Cost-reimbursement contracts are suitable for use only when uncertainties involved in contract performance do not permit costs to be estimated with sufficient accuracy to use any type of fixed-price contract.

institutions" and "small business firms" accounted for only \$4.5 billion; hence, approximatedly 77% of the total dollar value fo R&D procurement in 1998 went to large firms. In 2000, the most recent date for which these statistics are publicly available, about 80% (in value terms) of the "cost-plus fixed award fee" contracts were priced under \$20 million (approximately 99.7% of the number of cost-plus fixed award fee contracts awarded).

The top three companies taking in the most DoD procurement dollars in 2002 were Lockheed Martin ($\simeq$ \$17 billion), Boeing ($\simeq$ \$17 billion) and Northrop Grumman ($\simeq$ \$9 billion). These three also happened to be top three firms in R&D procurement in 2002, together accounting for more than 41% (in value) of contracts awarded. (Statistical information on recent procurement activity by DoD, including details mentioned in this footnote may be found on the Procurement Home Page, http://www.dior.whs.mil/peidhome/PEIDHOME.HTM).

[5] The documentation may be found at the following URL: http://www.arnet.gov/far/loadmainre.html

> *Subpart16.306*: A cost-plus-fixed-fee contract permits contracting for efforts that might otherwise present too great a risk to contractors, but it provides the contractor only a minimum incentive to control costs.
>
> (b) Application.
>
> (1) A cost-plus-fixed-fee contract is suitable for use when the conditions of 16.301-2 are present and, for example-
>
> (i) The contract is for the performance of research or preliminary exploration or study, and the level of effort required is unknown; or
>
> (ii) The contract is for development and test.
>
> (2) A cost-plus-fixed-fee contract normally should not be used in development of major systems (see Part 34) once preliminary exploration, studies, and risk reduction have indicated a high degree of probability that the development is achievable ... .

The FAR guidelines on R&D contracting practice (*Subpart 35.006: Contracting methods and contract type*), note:

> (c) Because the absence of precise specifications and difficulties in estimating costs with accuracy (resulting in a lack of confidence in cost estimates) normally precludes using fixed-price contracting for R&D, the use of cost-reimbursement contracts is usually appropriate (see Subpart 16.3). The nature of development work often requires a cost-reimbursement completion arrangement.
>
> (e) Projects having production requirements as a follow-on to R&D efforts normally should progress from cost-reimbursement contracts to fixed-price contracts as designs become more firmly established, risks are reduced, and production tooling, equipment, and processes are developed and proven. ... .

Hence, it would appear that analysts and practitioners alike are of the view that (cost) uncertainty is the key reason for the intensive use of cost-type contracting in R&D procurement. What is a less settled issue, and perhaps more to the point, is the question of the precise mechanism by which uncertainty exerts its significant influence on contract design in the case of R&D procurement. To put it more simply why, precisely, does uncertainty matter? The answer that this paper provides is that effect of uncertainty, at least in the instance of R&D contracting, works through the mechanism of ambiguity aversion. Projects involving new and untried technologies as well as projects located at the early part of the product life cycle are very good examples where cost uncertainty can be described as ambiguous. In such projects the uncertainty is one borne out of lack of substantive experience; where the novelty of the situation, lack of relevant data, makes it difficult to confidently assign "firm" probabilities to the possible (cost) outcomes. Hence the view that ambiguity aversion has potential to play an important role in such contexts.

There are alternative theories of the link between uncertainty and cost-plus (and other cost type) contracts. One theory is based on the hypothesis of asymmetry of information; i.e., ex ante firms have more information than the principal, the government, and can forecast the costs better because of this private information. Thus the principal has to trade off (information) rent extraction concerns with the concern about provision of incentives for efficient allocation of cost reducing effort. It may be optimal for the principal to provide low powered incentives if the information rent is substantial, as it would be if information asymmetry is high. (See Laffont and Tirole (1993) Chapter 2 for an account of the theory.) A second theory works on the logic of risk sharing. It rests on the hypothesis that firms are risk averse and the government is risk neutral (or, at least less risk averse than the firms), and that firms seek insurance when forecast errors are substantial. A cost-plus contract provides this insurance. A third and rather novel theory, advanced recently in Bajari and Tadelis (2001), explains cost plus contracting on the basis that it minimizes time to completion (compared to fixed price contracts) in complex construction projects where periodic changes in design are only to be expected. While cost uncertainty is not an explict parameter here, it could be argued that project complexity and construction change orders do contribute significantly to cost uncertainty. Next, we discuss how these theories apply to the case of R&D procurement.

The first alternative, which hinges on asymmetry of information rather than uncertainty per se, would appear to be better at explaining the cost-plus contracting seen at later stages of the production cycle than that at the initial research and development phase. (While cost-plus is not the predominant contracting form at the later stages, as one might guess recalling the second stylized fact mentioned by Laffont and Tirole (1993), there are instances of it being used.) The asymmetry of information about costs, between the government and the firm, is arguably minimum at the initial pilot phase when relevant information is scarce *all around*. Obviously, the information asymmetry should increase as the firm works through the research and development phase and hence would be a good explanation for the low powered incentives that sometimes remain in place in follow-on contracts *after* the research and development work has been undertaken. (Though, Bajari and Tadelis (2001) argue that this is not even a good explanation for cost-plus contracts in later stages of the procurement cycle.) In any case, it is a lot harder to argue that asymmetric information is more influential factor in the initial phase, when the firm has not had the time/opportunity to acquire significant private information.

The problem with the second theory, based on risk aversion of firms, is that it fails to account for the substantial proportion of relatively low priced cost-plus contracts in R&D that are taken on by large corporate firms. Evidently, the vast majority of cost-plus contracts are priced relatively low, below $20 million (see footnote 4). R&D contracts are usually cost-reimbursement type, as the evidence in Scherer (1964), quoted earlier, and the FAR guidelines suggest. And, R&D cost-plus contracts are typically lower priced than cost-plus contracts in other categories.[6] Also, a substantial proportion of R&D contracting is undertaken by large firms.[7] There are a couple of instances in which it

[6] Tables 9-12 (pages 36-39) in Moore (1962) show the size distribution of a statistical sample of 2501 cost-plus-fixed-fee contracts (completed between 1956-58) in four product fields. The four are: airframes and engines, missiles, electronics and R&D contracts. Clearly, the preponderance of small-sized CPFF contracts is most marked in the case of R&D: 88% of R&D contracts were priced below $1 million.

[7] The statistics quoted in footnote 4 provides reasonable, albeit indirect, indication of this. More direct evidence may be obtained by looking through the records of contract announcements in DefenseLINK News Contract Announcements (http://www.defenselink.mil/contracts/). Indeed, even a casual perusal reveals significant numbers of R&D cost plus contracts priced under $20 million awarded to large corporate firms. Looking at R&D cost-plus contract awards reported in April 2003, we find individual such awards going to contractors such as Lockheed Martin ($8 million; #F33615-03-C-1400), Boeing ($18million; #F33657-98-D-0002-RJ12-09), Northrup Grumman ($15 million; N00014-03-C-0209). Recall, annual turnover of each of these firms from defense contracting alone runs into several billion dollars.

might be reasonable to assume that a firm may act risk averse. One, if the risks are large relative to firm size and two, if there are inadequate opportunities for diversification. But many instances of R&D cost-plus contracting would not match either of these instances. The facts, as already noted, suggest many defense contractors are among the biggest of the corporate giants there are and most R&D cost-plus contracts involve relatively small sums. Finally, the risks involved in these projects (i.e., high tech projects) are, in the most part, risks that are idiosyncratic to the project (these are not risks following from shocks to aggregate economic activity) and therefore fulfill the conditions required by the standard arguments for diversification.

The final theory, as suggested in the paper itself, is arguably more appropriate for projects involving long-drawn, big budget construction, a feature that is but rare for R&D/early pilot phase projects, which usually involve researching techniques/designing prototypes/drawing up blue prints. Actual construction/implementation typically follows in the later stages of the procurement life cycle.

These reasons suggest that the alternative theories do not explain the link between cost uncertainty and the intensive use of cost type contracts in R&D procurement very well and hence, the "room" for the ambiguity theory. Of the three theories, clearly, the ambiguity based explanation is closest in spirit to the risk based theory. Hence, the case for the ambiguity theory would be strengthened if it could be shown that the theory is not susceptible to the diversification argument in the way the risk based theory is. Indeed, as it happens, it is not. But this is easier to see after gaining some familiarity with the technicalities of the ambiguity aversion model and the way it applies to the contracting context. Hence, we postpone discussion on this till (the end of) Section 4.

3 Modeling ambiguity aversion

A classic experiment illustrating how ambiguity aversion may affect *behavior*, due to Daniel Ellsberg (1961), runs as follows:

> There are two urns each containing one hundred balls. Each ball is either red or black. The subjects are told of the fact that there are fifty balls of each color in urn I. But no information is provided about the proportion of red and black balls in urn II. One ball is chosen at random from each urn. There are four events, denoted IR, IB, IIR, IIB, where IR denotes the event that the ball chosen from urn I is red, etc. On each of the events a bet is offered: \$100 if the events occurs and \$0 if it does not.

The modal response is that a subject prefers every bet from urn I (IR or IB) to every bet from urn II (IIR or IIB). That is, the typical revealed preference is $IB \succ IIB$ *and* $IR \succ IIR$. (The preferences are strict.) Clearly, DM's beliefs about the likelihood of the events, as revealed in the preferences, is not consistent with a unique probabilistic prior. The story goes: People dislike the ambiguity that comes with choice under uncertainty; they dislike the possibility that they may have the odds wrong and so make a wrong choice (*ex ante*). Hence they go with the gamble where they know the odds — betting from urn I.

Two pioneering contributions, Gilboa and Schmeidler (1989) and Schmeidler (1989), developed the maxmin expected utility (MEU) and the related Choquet expected utility model (CEU), respectively, as possible formalizations of decision behavior that incorporates ambiguity aversion. In these models, roughly put, the agent's belief is captured not by a unique probability distribution, in the standard Bayesian fashion, but instead by a set of probabilities. The agent's belief is *ambiguous* in the sense that more than one probability is consistent with his knowledge. Thus not only is the outcome of an act uncertain but also the expected payoff of the action, since the payoff may be measured with respect to more than one probability. In these models an *ambiguity averse* decision maker evaluates an act by the minimum expected payoff that may be (subjectively) associated with it.

More formally, let $\Omega = \{\omega_i\}_{i=1}^N$ be a finite state space, and suppose that the decision maker (DM) chooses among acts with state contingent payoffs, $f : \Omega \to \mathbb{R}$. Like with SEU, payoffs are given by a *utility function* $u : \mathbb{R}_+ \to \mathbb{R}$, $u'(\cdot) \geq 0$, which incorporates the DM's attitude to risk and wealth. The DM's subjective belief is described by a convex set of probabilities Π, with the generic element $\pi : 2^\Omega \to [0,1]$. The ambiguity of belief about an event $E \subseteq \Omega$ is given by $\max_{\pi\in\Pi} \pi(E) - \min_{\pi\in\Pi} \pi(E)$. The MEU functional evaluates an act f as follows:

$$\mathbb{E}_\Pi u(f) \equiv \min_{\pi\in\Pi} \left\{ \sum_{\omega\in\Omega} u(f(\omega))\, \pi(\omega) \right\}. \tag{1}$$

It is instructive to note how this functional accommodates the modal behavior seen in the Ellsberg experiment. For instance, let $\pi(IR) = \pi(IB) = 0.5$ for all $\pi \in \Pi$; but $\min_{\pi\in\Pi} \pi(IIR) = \min_{\pi\in\Pi} \pi(IIB) < 0.5$. It follows that the expected payoff from betting on $IR \equiv E_\Pi(u(IR)) = E_\Pi(u(IB)) = u(50)$; while, $E_\Pi(u(IIR)) = E_\Pi(u(IIB)) = \min_{\pi\in\Pi} \pi(IIR) \times u(100) = \min_{\pi\in\Pi} \pi(IIB) \times u(100) < u(50)$.

A generalization of the maxmin functional, axiomatized recently in Ghirardato, Maccheroni, and Marinacci (2002), allows the modeler to vary parametrically the DM's attitude to ambiguity while holding the DM's information, given by Π, constant. The so called α-maxmin functional evaluates an act f as follows:

$$\mathbb{E}_{\Pi,\alpha} u(f) \equiv \alpha \min_{\pi\in\Pi} \mathbb{E}_\pi u(f) + (1-\alpha) \max_{\pi\in\Pi} \mathbb{E}_\pi u(f), \tag{2}$$

where $\alpha \in [0,1]$ is the parameter describing the DM's attitude to ambiguity. The interpretation is the greater the α, the more averse the DM is to the ambiguity in the information described by Π. We use this α-maxmin functional to model ambiguity aversion in our contracting model.

In the remaining part of this section, looking ahead to the contracting model, we show how acts are evaluated given the way beliefs are specified in that model. In the contracting model the state space is a doubleton, $\Omega = \{\omega_L, \omega_H\}$. Closed, convex sets of probabilities on a doubleton state space can be represented parametrically in a way that provides useful perspective. Let $\bar{\pi} \equiv (\bar{\pi}_L, \bar{\pi}_H)$ be a probability vector on Ω, i.e., $\pi_L + \pi_H = 1$. The set Π describing the DM's belief is specified as follows ($\triangle$ denotes the two dimensional unit simplex):

$$\Pi = \Pi(\bar{\pi}, \mathcal{A}) \equiv \{(\pi(\omega_L), \pi(\omega_H)) \in \triangle \mid \pi(\omega_L) \geq (1-\mathcal{A})\,\bar{\pi}_L; \pi(\omega_H) \geq (1-\mathcal{A})\,\bar{\pi}_H\}. \tag{3}$$

Notice, the ambiguity of belief about an arbitrary state $\omega \in \Omega$, $\max_{\pi\in\Pi} \pi(\omega) - \min_{\pi\in\Pi} \pi(\omega)$, works out to be $\mathcal{A}$. Let $\mathbb{E}_{\bar{\pi}} u(f)$ denote the expected utility evaluated with respect to the probability $\bar{\pi}$. To fix ideas it might help to think of Π as a function of the parametric specification $(\bar{\pi}, \mathcal{A})$, where $\bar{\pi}$ denotes the "true probability law" and $\mathcal{A} \in [0,1]$ the parameter that describes the ambiguity of belief, the DM's "perception" of the true probability, as it were.[8] With belief specified as in (3), first note that the MEU functional (1) simplifies to:

$$\begin{aligned}
\mathbb{E}_{\Pi} u(f) &\equiv \min_{\pi\in\Pi} \left\{ \sum_{\omega\in\Omega} u(f(\omega))\, \pi(\omega) \right\} \\
&= u(f(\omega_L)) \times \min_{\pi\in\Pi} \pi(\omega_L) + u(f(\omega_H)) \times \min_{\pi\in\Pi} \pi(\omega_H) \\
&\quad + (1 - \min_{\pi\in\Pi} \pi(\omega_L) - \min_{\pi\in\Pi} \pi(\omega_H)) \times \min_{\omega_i} \{u(f(\omega_i))\}_{i=L,H} \\
&= (1-\mathcal{A}) \left[u(f(\omega_L))\, \bar{\pi}_L + u(f(\omega_H))\, \bar{\pi}_H \right] + \mathcal{A} \min_{\omega_i} \{u(f(\omega_i))\}_{i=L,H} \\
&= (1-\mathcal{A})\, \mathbb{E}_{\bar{\pi}} u(f) + \mathcal{A} \min_{\omega_i} \{u(f(\omega_i))\}_{i=L,H}\,.
\end{aligned}$$

Now it is easy to see that given the same belief, the α-maxmin functional works out as follows:

$$\begin{aligned}
\mathbb{E}_{\Pi,\alpha} u(f) &\equiv \alpha \min_{\pi\in\Pi} \mathbb{E}_{\pi} u(f) + (1-\alpha) \max_{\pi\in\Pi} \mathbb{E}_{\pi} u(f) \\
&= (1-\mathcal{A})\, \mathbb{E}_{\bar{\pi}} u(f) + \alpha\mathcal{A} \min_{\omega_i} \{u(f(\omega_i))\}_{i=L,H} \\
&\quad + (1-\alpha)\, \mathcal{A} \max_{\omega_i} \{u(f(\omega_i))\}_{i=L,H}\,. \qquad (4)
\end{aligned}$$

Notice, the α-maxmin model (just as the MEU model) nests the SEU model. For instance, with belief specified as in $\Pi(\bar{\pi}, \mathcal{A})$ (3), if the DM were to perceive no ambiguity at all, i.e. $\mathcal{A} = 0$, his preferences would be represented by the functional $\mathbb{E}_{\bar{\pi}} u(f)$.

4 The model and analysis

4.1 Specification of the model

This is a model of an optimal contract between a principal who wants to procure a unit item from an agent. The contract requires the delivery of the specified item by the agent to the principal at a future date. The contract also specifies a contingency specific amount P, that the principal is required to pay the agent in exchange. The item will be produced by the agent after the contract is agreed on. The cost of producing the item is uncertain at the time the contract is signed, though the agent may reduce the cost of production by exerting effort. The cost function is denoted by $C(\omega, e)$, where $\omega \in \{\omega_L, \omega_H\}$ is a state of the world, and $e \in (0, \hat{e})$ is the effort exerted by the agent and is assumed to take the simple form:

$$C(\omega, e) = \begin{cases} C_L - e \text{ if } \omega = \omega_L \\ C_H - e \text{ if } \omega = \omega_H \end{cases}$$

[8] Given that Ω is a doubleton, any closed convex set of probabilities on Ω may be represented by an appropriate pair of parameters $(\bar{\pi}, \mathcal{A})$.

where, $0 < \hat{e} < C_L < C_H$. However, exerting effort gives rise to a disutility, $\psi(e) = \frac{e^2}{2}$. The value (to the principal) of the item to be delivered is not subject to uncertainty and set equal to V. We assume that $V > C(\omega, 0)$, i.e., tradeable surplus is positive under all circumstances.

Given our motivation to provide an explanation for contractual form that does not rely on either asymmetric information or risk aversion, we assume that the principal and the agent share the same belief and both are risk neutral. The DMs' common belief is described by the set of probabilities Π defined in (3). Both the principal and the agent's preferences over acts with state contingent payoffs are represented by the α-maxmin functional, which simplifies given our specification of beliefs to the form described in . In the α-maxmin functional representing the principal's preferences we denote the principal's ambiguity aversion attitude by α_p. Similarly, the agent's ambiguity aversion parameter is denoted α_a, and we will assume that $\alpha_a > \alpha_p$. The assumption may be interpreted as saying that given a range of possible cost estimates for the procurement project, the government, or at the least the bureaucratic authority negotiating the contract on behalf of the government, would evaluate costs relatively *less* conservatively (i.e., obtain the preferred cost evaluation by putting a relatively smaller weight on the high end of the interval of estimated costs and relatively higher weight on the lower end) than the contractor firm. Finally, the assumption of risk neutrality implies the utility function $u(\cdot)$ in (4) is just the identity function.

Obviously, the key question of contract design we are interested in, namely, the provision of financial incentives to induce exertion of cost reducing effort, would not even arise if the effort could be directly contracted on. Hence, it is assumed that the agent's effort cannot specified and verified in requisite detail and/or cannot be monitored by the principal, and therefore not contractible. Notice, since cost is only a function of the state and effort, if it were possible to contract both on costs *and* ω then it would be (formally) equivalent to making effort directly contractible, and therefore uninteresting for our purposes. Hence, we assume that the only verifiable/contractible variable is the realized (accounting) cost of production, ruling out the possibility of contracting on the state, ω. (If we made ω contractible and C not contractible the analysis would be very similar.) Finally, for simplicity, we assume that the payments are linear in the realized cost and write the payment as $P(C) = \delta + \gamma C$, where δ and γ are terms to be fixed in the contract. The coefficient of realized cost, γ, may be interpreted as the share of "cost overrun" (the amount by which the realized costs exceed some fixed, uncontingent target incorporated in δ) to be reimbursed by the principal. Being a cost *share*, $\gamma \in [0, 1]$. Hence, in formulating the optimal (linear) contract the principal will solve the following problem:

$$\max_{\gamma \in [0,1], \delta} \left[V - \mathbb{E}_{\Pi, \alpha_p} P(C(\omega, e^*)) \right]$$

$$s.t. \begin{array}{l} i) \ \mathbb{E}_{\Pi, \alpha_a} \left[P(C(\omega, e^*)) - C(\omega, e^*) \right] - \psi(e^*) \geq 0 \\ ii) \ e^* \text{ solves } \max_{e \in (0, \hat{e})} \mathbb{E}_{\Pi, \alpha_a} \left[P(C(\omega, e)) - C(\omega, e) \right] - \psi(e) \end{array} \tag{5}$$

The first constraint in the principal's problem is a participation constraint while the second fixes the endogenous choice of effort. Since e^* is the effort chosen by the agent given the terms of the contract, the second constraint is, in its spirit, an incentive constraint.

4.2 Characterizing the optimal contract

We first analyze the endogenous choice of effort by the agent. Let

$$\bar{C} \equiv \mathbb{E}_{\bar{\pi}} C(\omega, 0) = \pi_L C_L + \pi_H C_H, \tag{6}$$

which for the purpose of fixing ideas may be interpreted as the expected cost evaluated by applying the "true probability" with effort set at its lowest level, 0. Given the functional forms assumed for $P(C)$ and $\psi(e)$, the agent's (expected) remuneration from the contract may be written as:

$$\begin{aligned}
& \mathbb{E}_{\Pi,\alpha_a}\left[P(C(\omega,e)) - C(\omega,e)\right] \\
= \ & \mathbb{E}_{\Pi,\alpha_a}\left[\delta + (1-\gamma)(-C(\omega,e))\right] \\
= \ & \delta + (1-\gamma)\begin{bmatrix} (1-\mathcal{A})\left((-1)(C_L - e)\pi_L + (-1)(C_H - e)\pi_H\right) \\ +\mathcal{A}\alpha_a \min\left\{(-1)(C_L - e), (-1)(C_H - e)\right\} \\ +\mathcal{A}(1-\alpha_a)\max\left\{(-1)(C_L - e), (-1)(C_H - e)\right\} \end{bmatrix} \\
= \ & \delta + (1-\gamma)\left[-(1-\mathcal{A})\bar{C} - \alpha_a \mathcal{A} C_H - (1-\alpha_a)\mathcal{A} C_L + e\right]
\end{aligned} \tag{7}$$

Hence, the optimal effort maximizes the following expression:

$$\max_{e\in(0,\hat{e})}\left\{(1-\gamma)\left[-(1-\mathcal{A})\bar{C} - \alpha_a \mathcal{A} C_H - (1-\alpha_a)\mathcal{A} C_L + e\right] - \frac{e^2}{2}\right\} \tag{8}$$

Applying the first order condition, the optimal effort obtains as:

$$e^* = 1 - \gamma \equiv e(\gamma). \tag{9}$$

Clearly, the optimal effort, $e(\gamma)$, is decreasing in γ and given that $0 \leq \gamma \leq 1$, it must be that $0 \leq e(\gamma) \leq 1$. An economic intuition for $e'(\gamma) \leq 1$ is that as γ increases a greater portion of the realized cost is reimbursed by the principal thereby reducing the incentive to lower the cost by the application of effort since the disutility from reducing effort is reimbursed less, at the margin.

Notice in any solution of the principal's problem (5) constraint (i) must hold as an equality, since otherwise $P(C)$ could be decreased uniformly across states without violating the constraint while increasing the value of the maximand simultaneously. Hence, we may rewrite (5) as:

$$\begin{aligned}
& \max_{\gamma\in[0,1],\delta}\left[V - \mathbb{E}_{\Pi,\alpha_p} P(C(\omega, e(\gamma)))\right] \\
& s.t.\ \mathbb{E}_{\Pi,\alpha_a}\left[P(C(\omega,e(\gamma))) - C(\omega,e(\gamma))\right] - \psi(e(\gamma)) = 0
\end{aligned} \tag{10}$$

We may actually write the principal's problem completely in terms of "standard" expectations operators, thereby converting it into a problem solvable by standard techniques. Given that the agent's ex post (state contingent) remuneration is

$$P(C(\omega,e)) - C(\omega,e) = \delta + (1-\gamma)(-C(\omega,e)), \tag{11}$$

and that $\gamma \geq 0$, the "worse" state for the agent is ω_H, the high cost state, since,

$$\arg\min_{\omega\in\Omega}\left[\delta + (1-\gamma)(-C(\omega,e))\right] = \omega_H.$$

Hence, we may rewrite the agent's expected remuneration as

$$\begin{aligned} & \mathbb{E}_{\Pi,\alpha_a}\left[P(C(\omega,e(\gamma)))-C(\omega,e(\gamma))\right] \\ = \ & (1-\mathcal{A}))\,\mathbb{E}_{\bar{\pi}}\left[P(C(\omega,e(\gamma)))-C(\omega,e(\gamma))\right] \\ & +\mathcal{A}\alpha_a\left[P(C(\omega_H,e(\gamma)))-C(\omega_H,e(\gamma))\right] \\ & +\mathcal{A}(1-\alpha_a)\left[P(C(\omega_L,e(\gamma)))-C(\omega_L,e(\gamma))\right]. \end{aligned} \tag{12}$$

The high cost state is, of course, also the worse state for the principal. Hence, we may rewrite the principal's remuneration as:

$$V-(1-\mathcal{A})\mathbb{E}_{\bar{\pi}}P(C(\omega,e(\gamma))-\mathcal{A}\alpha_p P(C_H-e(\gamma))-\mathcal{A}(1-\alpha_p)P(C_L-e(\gamma)) \tag{13}$$

Therefore, we may rewrite the principal's problem in (10) with expectation operators by substituting (13) for the maximand and (12) into the constraint as in (14), below,

$$\begin{aligned} & \max_{\gamma\in[0,1],\delta}\left[V-(1-\mathcal{A})\mathbb{E}_{\bar{\pi}}P(C(\omega,e(\gamma))-\mathcal{A}\alpha_p P(C_H-e(\gamma))-\mathcal{A}(1-\alpha_p)P(C_L-e(\gamma))\right] \\ & s.t.\left\{\begin{array}{c}(1-\mathcal{A})\mathbb{E}_{\bar{\pi}}\left[P(C(\omega,e(\gamma)))-C(\omega,e(\gamma))\right]+\mathcal{A}\alpha_a\left[\delta+(1-\gamma)(e(\gamma)-C_H)\right] \\ +\mathcal{A}(1-\alpha_a)\left[\delta+(1-\gamma)(e(\gamma)-C_L)\right]-\frac{[e(\gamma)]^2}{2}=0.\end{array}\right. \end{aligned} \tag{14}$$

Solution of the program displayed in (14), details of which are in the Appendix, yields a full and explicit characterization of the optimal contract, i.e., explicit expressions for δ and γ in terms of primitive parameters. The following proposition reports the characterization of γ, arguably the more significant finding.

Proposition 1 *In the optimal contract, obtained by solving the constrained maximization problem shown in (14), the cost share coefficient, γ, is given by*

$$\gamma=\left\{\begin{array}{cc}\mathcal{A}(\alpha_a-\alpha_p)(C_H-C_L) & \text{if } \mathcal{A}<\frac{1}{(\alpha_a-\alpha_p)(C_H-C_L)} \\ 1 & \text{if } \mathcal{A}\geq\frac{1}{(\alpha_a-\alpha_p)(C_H-C_L)}\end{array}\right.$$

where $0\leq\mathcal{A}\leq 1$.

Clearly, the proportion of realized cost reimbursed by the principal increases as ambiguity increases, as the difference between the ambiguity aversion of the agent and that of the principal, $(\alpha_a-\alpha_p)$, becomes larger, and the range of the cost variable increases.[9] As

[9] In a pamphlet, "Guide for Selection of Types of Contracts," the U.S. Air Force suggested some guidelines for relating the range of cost estimates to the type of contract. The table below (from Moore (1962), Table 18, page 66) summarizes these suggestions. There is a progression from cost-plus-fixed-fee (CPFF) to firm-fixed-price contracts, and these are related directly to the decrease in the range of cost estimates, which to quote Moore, (is) "a range presumably determinable in part by differences between the estimates of the Service and the contractor."

If the range of cost estimates is:	**Then a suitable contract type would be:**
± greater than 15%	CPFF
± 15% – ± 3%	sundry incentive type contracts
± 3%	firm-fixed-price

Observe if, following Moore, we interpret "a range of cost estimates" as the difference between the (ex ante) evaluation of cost preferred by DoD (using the relatively optimistic weight α_p) and the evaluation preferred by the contractor (using the less optimistic weight α_a), our formula for γ (in Proposition 1) would appear to be very consistent with the advice in the "Guide".

noted in the introduction, the key to the intuition of the result is the trade-off between the amount of "ambiguity premium" the principal has to pay and the reduction in cost effected by the agents effort exertion. The ambiguity premium decreases, ceteris paribus, as γ increases. On the other hand, the agent expends more effort into cost reduction the smaller the value of γ. If $\mathcal{A} = 0$, with the implication that $\gamma = 0$, then (from equation (24) in the Appendix) we have a fixed price contract with $\delta = \bar{C} - 1/2$, where $\bar{C}$ (defined in (6)) is the the expected cost evaluated by applying the "true probability". Given that the contract is fixed price, the agent's incentive to lower the cost by putting in effort is at its maximum, leading to a reduction in cost by 1/2 since the agent puts in the maximal optimal effort, $e(\gamma)|_{\gamma=0} = 1$. When ambiguity, given by the parameter $\mathcal{A}$, is high enough, the "pure" cost-plus contract is optimal. Further, fixing the level of ambiguity, the greater the agent's ambiguity aversion relative to the principal's, more the move toward the cost-plus form. Notice, the extent to which ambiguity has "bite" depends (positively) on the difference $(C_H - C_L)$. Effectively, the greater the spread of costs in the support, the greater the "scope of play" of ambiguity aversion, because the cost uncertainty is greater.

While ambiguity does not have any bite if the principal and the agent have an exactly identical attitude to ambiguity, the qualitative conclusions do not rest on the principal being ambiguity averse, all that is required is that the agent be more ambiguity averse than the principal. For instance, if the principal were an SEU maximizer with belief given by $\bar{\pi}$ (i.e., ambiguity neutral) and the agent were an MEU maximizer (i.e., ambiguity aversion coefficient $\alpha_a = 1$) with belief given by the set $\Pi(\bar{\pi}, \mathcal{A})$, then it can be shown[10] that

$$\gamma = \begin{cases} 0 & \text{if } \mathcal{A} = 0 \\ \mathcal{A}\left(C_H - \bar{C}\right) & \text{if } 0 < \mathcal{A} < \frac{1}{\left(C_H - \bar{C}\right)} \\ 1 & \text{if } \mathcal{A} \geq \frac{1}{\left(C_H - \bar{C}\right)} \end{cases}$$

Qualitative conclusions are also not altered by adopting more general specifications for the disutility of effort, so long as the disutility is increasing and convex. Of course, the form assumed in the model does have the advantage of facilitating a simple explicit characterization of γ.

Before closing this section we return to examining why the ambiguity explanation of cost-plus contracts may not be undermined by a diversification argument in the way risk-aversion based explanation is. The reason why the diversification argument does not work has to do with the way the law of large numbers works for ambiguous risks, as shown in Mukerji and Tallon (2001).[11] Specifically, let us consider an i.i.d. sequence $\{X_n\}_{n\geq 1}$ of $\{0,1\}$ −valued random variables. Suppose, that all the DM's knows is that the probability of each X_n lies in the interval $[1/4, 3/4]$, i.e., more formally,

$$\frac{1}{4} \leq \Pr\left(\{X_n = 0\}\right) \leq \frac{3}{4} \text{ and } \Pr\left(\{X_n = 1\}\right) = 1 - \Pr\left(\{X_n = 0\}\right) \text{ for all } n \geq 1.$$

As is usual with laws of large numbers, the question is about the limiting distribution of

[10] One way to show this would be to amend the program in (14) by setting $\mathcal{A} = 0$ in the maximand and $\alpha_a = 1$ in the constraint.

[11] Laws of large numbers for ambiguous beliefs have been studied by, among others, Walley and Fine (1982) and Marinacci (1999).

the sample average, $\frac{1}{n}\sum_{i=1}^{n} X_i$. The law implies:

$$\Pr\left(\frac{1}{4} \leq \liminf_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i \leq \limsup_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i \leq \frac{3}{4}\right) = 1. \tag{15}$$

In words, (15) says that the DM has a probability 1 belief that the limiting value of the sample average lies in the (closed) interval $[1/4, 3/4]$. However, unlike in the case belief is described by a single probability, the DM is not able to further pin down its value. Thus, even with ambiguous beliefs, where probability of an event is given by an interval, the law of large numbers works in the usual way in the sense that here too the tails of the distribution are 'canceled out' and the distribution 'converges on the mean'. But of course here, given that the DM's knowledge is consistent with more than one prior, there is more than one mean to converge on; hence, the convergence is to the *set* of means corresponding to the set of priors consistent with the DM's knowledge. Now, to fix ideas, think of $\{X_n\}_{n\geq 1}$ as a sequence of i.i.d. payoffs from n "projects", representing n idiosyncratic risks; i.e., each project pays off either 0 or 1 independent of the outcomes in other projects. Correspondingly, we may think of the sample average, $\frac{1}{n}\sum_{i=1}^{n} X_i$, as the payoff from an portfolio consisting of $1/n$ share of each project. If the investor owning this diversified portfolio has an α-maxmin preference (as in (2)) then it follows from (15) that in the limit as $n \to \infty$, the investor will evaluate the payoff from his portfolio as α-weighted average of the endpoints of the interval $[1/4, 3/4]$:

$$\frac{\alpha}{4} + \frac{3(1-\alpha)}{4} = \frac{3-2\alpha}{4}.$$

Suppose a firm, in charge of one of the projects with payoff X_k (say), is owned by shareholders all of whom have identical α-maxmin preferences, where $\alpha = \alpha_a$, and all of whom hold diversified portfolios. Then it follows that the firm, given the preferences of its owners, may evaluate the particular risky prospect that it controls, X_k, as equal to $(3-2\alpha_a)/4$. To put in perspective what the ambiguity is doing to the calculations, suppose the shareholders were risk averse expected utility maximizers and were "informed" that the i.i.d. random variables $\{X_n\}_{n\geq 1}$, have expected value μ_x. The law of large numbers implies $\Pr\left(\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i = \mu_x\right) = 1$. Accordingly, a typical expected utility shareholder, irrespective of his risk aversion, will evaluate ex ante payoff from his perfectly diversified portfolio as $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i = \mu_x$. Correspondingly, the firm may evaluate a particular project according to its expected value and hence, risk aversion does not remain an issue given the diversification. In contrast, with ambiguity, diversification allows for a non-degenerate interval of expected values, even in the limit; thus ambiguity retains its "bite" even when passing to this limit.

5 Conclusion

The formal analysis showed that ambiguity has, at least theoretically, determinate effect on the power of the optimal incentive scheme incorporated in procurement contracts. We found that the more the ambiguity the lower the power of incentives incorporated, with high enough ambiguity resulting in the pure cost-plus form. On the basis of institutional and statistical detail presented in Section 2, it seems possible to argue that this theoretical effect is a better explanation for cost-plus contracts, compared to the alternative theories, *in the specific context* of R&D procurement.

Finally, while there is a vast literature on ambiguity aversion (see Camerer (1995)), and indeed of the many other departures from SEU, that convincingly establishes both, the theoretical sophistication of the ideas as well as their importance in laboratory settings, this work has had little impact on the way that economics is done. In large part this is because there have been relatively few demonstrations that economically important phenomena that are only poorly understood on the basis of standard theory, can be understood better by using models other than the standard one (SEU).[12] This paper is a contribution that hopes to go some way in bridging this gap, especially as a complement to the finding in Mukerji (1998). That paper showed that ambiguity aversion could cause incentive contracts facilitating vertical purchaser/supplier relationships to have too little power, thereby providing an understanding of the link between uncertainty and vertical integration. There is empirical evidence suggesting that provision of incentives for R&D may be a particularly difficult problem in purchaser/supplier relationships. For example, Monteverde and Teece (1982) show that large automotive firms are much more likely to produce a component in-house if it involves significant R&D. It is of interest to note that ambiguity aversion explains this empirical regularity about R&D production (on the basis of results in Mukerji (1998)), just as it explains weak incentives incorporated in R&D procurement contracts.

6 Appendix: Solution of the optimization program as given in (14)

The Lagrangian, $\mathcal{L}$, for the maximization program, with λ denoting the Lagrange multiplier, may written as follows:

$$\begin{aligned}
&\mathcal{L}(\delta,\gamma,\lambda) = V - (1-\mathcal{A})\mathbb{E}_{\bar{\pi}}P(C(\omega,e(\gamma)))\\
&+\mathcal{A}\alpha_p \min\{[-P(C_L - e(\gamma))],[-P(C_H - e(\gamma))]\}\\
&+\mathcal{A}(1-\alpha_p)\max\{[-P(C_L - e(\gamma))],[-P(C_H - e(\gamma))]\}\\
&-\lambda[(1-\mathcal{A})\mathbb{E}_{\bar{\pi}}[P(C(\omega,e(\gamma))) - C(\omega,e(\gamma))] + \mathcal{A}\alpha_a[\delta + (1-\gamma)(e(\gamma) - C_H)]\\
&+\mathcal{A}(1-\alpha_a)[\delta + (1-\gamma)(e(\gamma) - C_L)] - \frac{[e(\gamma)]^2}{2}]
\end{aligned}$$

or equivalently,

$$\begin{aligned}
\mathcal{L} \;=\;& V - (1-\mathcal{A})\mathbb{E}_{\bar{\pi}}P(C(\omega,e(\gamma)) - \mathcal{A}\alpha_p P(C_H - e(\gamma))\\
&-\mathcal{A}(1-\alpha_p)P(C_L - e(\gamma)) - \lambda[(1-\mathcal{A})\mathbb{E}_{\bar{\pi}}P(C(\omega,e(\gamma))\\
&-(1-\mathcal{A})\mathbb{E}_{\bar{\pi}}C(\omega,e(\gamma)) + \mathcal{A}\alpha_a[\delta + (1-\gamma)(e(\gamma) - C_H)]\\
&+\mathcal{A}(1-\alpha_a)[\delta + (1-\gamma)(e(\gamma) - C_L)] - \frac{[e(\gamma)]^2}{2}]
\end{aligned} \tag{16}$$

[12]Though, lately, the list of papers exploring the implications of the idea of ambiguity aversion in economic context, has been growing. For an account of the literature see Mukerji and Tallon (2003).

The first order conditions are obtained by taking derivative of the expression for $\mathcal{L}$ in (16) with respect to δ and γ. Recall, $P(C(\omega, e(\gamma))) = \delta + \gamma C(\omega, e(\gamma))$. Hence,

$$\frac{\partial \mathcal{L}}{\partial \delta} = -1 - \lambda = 0 \Rightarrow \lambda = -1. \tag{17}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \gamma} &= -(1-\mathcal{A}) \frac{\partial \mathbb{E}_{\bar{\pi}} P}{\partial \gamma} - \mathcal{A}\alpha_p \frac{\partial \left[P\left(C_H - e(\gamma)\right)\right]}{\partial \gamma} - \mathcal{A}(1-\alpha_p) \frac{\partial \left[P\left(C_L - e(\gamma)\right)\right]}{\partial \gamma} \\
&\quad + (1-\mathcal{A}) \frac{\partial \mathbb{E}_{\bar{\pi}} P}{\partial \gamma} - (1-\mathcal{A}) \frac{\partial \mathbb{E}_{\bar{\pi}} C(\omega, e(\gamma))}{\partial \gamma} + \mathcal{A}\alpha_a \frac{\partial \left[(1-\gamma)\left(e(\gamma) - C_H\right)\right]}{\partial \gamma} \\
&\quad + \mathcal{A}(1-\alpha_a) \frac{\partial \left[(1-\gamma)\left(e(\gamma) - C_L\right)\right]}{\partial \gamma} - \frac{\partial \left[\frac{[e(\gamma)]^2}{2}\right]}{\partial \gamma}] = 0
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \; & -\mathcal{A}\alpha_p \frac{\partial \left[\delta + \gamma\left(C_H - e(\gamma)\right)\right]}{\partial \gamma} - \mathcal{A}(1-\alpha_p) \frac{\partial \left[\delta + \gamma\left(C_L - e(\gamma)\right)\right]}{\partial \gamma} \\
& - (1-\mathcal{A}) \frac{\partial \mathbb{E}_{\bar{\pi}} C(\omega, e(\gamma))}{\partial \gamma} + \mathcal{A}\alpha_a \frac{\partial \left[(1-\gamma)\left(e(\gamma) - C_H\right)\right]}{\partial \gamma} \\
& + \mathcal{A}(1-\alpha_a) \frac{\partial \left[(1-\gamma)\left(e(\gamma) - C_L\right)\right]}{\partial \gamma} + (1-\gamma) = 0
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \; & -\mathcal{A}\alpha_p \frac{\partial \left[\gamma\left(C_H - e(\gamma)\right)\right]}{\partial \gamma} - \mathcal{A}(1-\alpha_p) \frac{\partial \left[\gamma\left(C_L - e(\gamma)\right)\right]}{\partial \gamma} \\
& - (1-\mathcal{A}) \frac{\partial \mathbb{E}_{\bar{\pi}} C(\omega, e(\gamma))}{\partial \gamma} - \mathcal{A}\alpha_a \frac{\partial \left[(1-\gamma)\left(C_H - e(\gamma)\right)\right]}{\partial \gamma} \\
& - \mathcal{A}(1-\alpha_a) \frac{\partial \left[(1-\gamma)\left(C_L - e(\gamma)\right)\right]}{\partial \gamma} + (1-\gamma) = 0 \\
\Rightarrow \; & -\mathcal{A}\alpha_p \frac{\partial \left[\gamma\left(C_H - e(\gamma)\right)\right]}{\partial \gamma} - \mathcal{A}(1-\alpha_p) \frac{\partial \left[\gamma\left(C_L - e(\gamma)\right)\right]}{\partial \gamma} \\
& - (1-\mathcal{A}) \frac{\partial \mathbb{E}_{\bar{\pi}} C(\omega, e(\gamma))}{\partial \gamma} - \mathcal{A}\alpha_a \frac{\partial (C_H - e(\gamma))}{\partial \gamma} + \mathcal{A}\alpha_a \frac{\partial \left[\gamma(C_H - e(\gamma))\right]}{\partial \gamma} \\
& - \mathcal{A}(1-\alpha_a) \frac{\partial (C_L - e(\gamma))}{\partial \gamma} + \mathcal{A}(1-\alpha_a) \frac{\partial \left[\gamma(C_L - e(\gamma))\right]}{\partial \gamma} + (1-\gamma) = 0
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \; & \mathcal{A}(\alpha_a - \alpha_p) \frac{\partial \left[\gamma\left(C_H - e(\gamma)\right)\right]}{\partial \gamma} - \mathcal{A}(\alpha_a - \alpha_p) \frac{\partial \left[\gamma\left(C_L - e(\gamma)\right)\right]}{\partial \gamma} \\
& - (1-\mathcal{A}) \frac{\partial \mathbb{E}_{\bar{\pi}} C(\omega, e(\gamma))}{\partial \gamma} - \mathcal{A}\alpha_a \frac{\partial (C_H - e(\gamma))}{\partial \gamma} \\
& - \mathcal{A}(1-\alpha_a) \frac{\partial (C_L - e(\gamma))}{\partial \gamma} + (1-\gamma) = 0
\end{aligned} \tag{18}$$

Note,

$$\frac{\partial \mathbb{E}_{\bar{\pi}} C(\omega, e(\gamma)))}{\partial \gamma} = 1, \tag{19}$$

$$\frac{\partial\left[\gamma\left(C_H-e\left(\gamma\right)\right)\right]}{\partial\gamma} = C_H-1+2\gamma, \tag{20}$$

$$\frac{\partial\left[\gamma\left(C_L-e\left(\gamma\right)\right)\right]}{\partial\gamma} = C_L-1+2\gamma. \tag{21}$$

Hence, substituting (19), (20), (21) in (18), we obtain,

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\gamma} &= \mathcal{A}\left(\alpha_a-\alpha_p\right)\left(C_H-1+2\gamma\right)-\mathcal{A}\left(\alpha_a-\alpha_p\right)\left(C_L-1+2\gamma\right)\\ &- \left(1-\mathcal{A}\right)-\mathcal{A}\alpha_a-\mathcal{A}\left(1-\alpha_a\right)+\left(1-\gamma\right)=0\\ &\Rightarrow \gamma=\mathcal{A}\left(\alpha_a-\alpha_p\right)\left(C_H-C_L\right)\end{aligned}$$

Notice, if it is the case that $\mathcal{A}>\frac{1}{(\alpha_a-\alpha_p)(C_H-C_L)}$, then $\frac{\partial\mathcal{L}}{\partial\gamma}\left(=\mathcal{A}\left(\alpha_a-\alpha_p\right)\left(C_H-C_L\right)-\gamma\right)$ is strictly positive when evaluated at $\gamma=1$. Hence, taking into account that $\gamma\in[0,1]$ we conclude,

$$\gamma=\begin{cases}\mathcal{A}\left(\alpha_a-\alpha_p\right)\left(C_H-C_L\right) & \mathcal{A}<\frac{1}{(\alpha_a-\alpha_p)(C_H-C_L)}\\ 1 & \text{if } \mathcal{A}\geq\frac{1}{(\alpha_a-\alpha_p)(C_H-C_L)}\end{cases} \tag{22}$$

And, on simplifying the constraint equation in (14) we obtain,

$$\begin{aligned}&(1-\mathcal{A})\mathbb{E}_{\overline{\pi}}P(C\left(\omega,e\left(\gamma\right)\right)-(1-\mathcal{A})\mathbb{E}_{\overline{\pi}}C\left(\omega,e\left(\gamma\right)\right)\\ &+\mathcal{A}\alpha_a\left[\delta+\left(1-\gamma\right)\left(e\left(\gamma\right)-C_H\right)\right]\\ &+\mathcal{A}\left(1-\alpha_a\right)\left[\delta+\left(1-\gamma\right)\left(e\left(\gamma\right)-C_L\right)\right]\\ &-\frac{\left[e\left(\gamma\right)\right]^2}{2}=0\\ \Rightarrow\ &(1-\mathcal{A})\left[\mathbb{E}_{\overline{\pi}}\left(\delta+\gamma(C\left(\omega,e\left(\gamma\right)\right)\right)-\mathbb{E}_{\overline{\pi}}C\left(\omega,e\left(\gamma\right)\right)\right]\\ &+\mathcal{A}\alpha_a\left[\delta+\left(1-\gamma\right)\left(\left(1-\gamma\right)-C_H\right)\right]\\ &+\mathcal{A}\left(1-\alpha_a\right)\left[\delta+\left(1-\gamma\right)\left(\left(1-\gamma\right)-C_L\right)\right]\\ &-\frac{\left[1-\gamma\right]^2}{2}=0\\ \Rightarrow\ &(1-\mathcal{A})\left[\delta-\left(1-\gamma\right)\left(\bar{C}-\left(1-\gamma\right)\right)\right]\\ &+\mathcal{A}\alpha_a\left[\delta+\left(1-\gamma\right)\left(\left(1-\gamma\right)-C_H\right)\right]\\ &+\mathcal{A}\left(1-\alpha_a\right)\left[\delta+\left(1-\gamma\right)\left(\left(1-\gamma\right)-C_L\right)\right]-\frac{\left[1-\gamma\right]^2}{2}=0\end{aligned}$$

$$\begin{aligned}\Rightarrow\ &(1-\mathcal{A})\left[\delta-\left(1-\gamma\right)\left(\bar{C}-\left(1-\gamma\right)\right)\right]+\mathcal{A}\alpha_a\left[\delta+\left(1-\gamma\right)\left(\left(1-\gamma\right)-C_H\right)\right]\\ &+\mathcal{A}\left(1-\alpha_a\right)\left[\delta+\left(1-\gamma\right)\left(\left(1-\gamma\right)-C_L\right)\right]-\frac{\left[1-\gamma\right]^2}{2}=0\end{aligned} \tag{23}$$

Solving the simplified constraint equation (23) for δ we obtain:

$$\begin{aligned}\delta=-\tfrac{1}{2}+\bar{C}+-\tfrac{1}{2}\gamma^2-\gamma\bar{C}+\mathcal{A}C_L-\mathcal{A}\gamma C_L-\mathcal{A}\alpha_a C_L\\ +\mathcal{A}\alpha_a\gamma C_L+\mathcal{A}\alpha_a C_H-\mathcal{A}\bar{C}-\mathcal{A}\alpha_a\gamma C_H+\mathcal{A}\gamma\bar{C}\end{aligned} \tag{24}$$

where, γ is as found in (22).

The second order conditions may be checked by considering the determinant of the relevant bordered Hessian :

$$\begin{vmatrix} 0 & \mathcal{L}_{\lambda\delta} & \mathcal{L}_{\lambda\gamma} \\ \mathcal{L}_{\lambda\delta} & \mathcal{L}_{\delta\delta} & \mathcal{L}_{\delta\gamma} \\ \mathcal{L}_{\lambda\gamma} & \mathcal{L}_{\delta\gamma} & \mathcal{L}_{\gamma\gamma} \end{vmatrix} = \begin{vmatrix} 0 & -1 & \mathcal{L}_{\lambda\gamma} \\ -1 & 0 & 0 \\ \mathcal{L}_{\lambda\gamma} & 0 & -1 \end{vmatrix} = 1 > 0$$

■

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