Comparing Minimum Codeword Distances and Error Performance for Index Modulation and Maximum Distance Separable Coded Modulation

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***Abstract*—Index modulation (IM) that embeds information into combinations of activated codeword elements have desirable properties in terms of error performance, low-complexity implementation, and compatibility with existing communication techniques. However, a recent modulation concept based on a simple maximum distance separable (MDS) code, i.e., MDS modulation, achieves better performance than IM while preserving a low-complexity structure. In this paper, we conduct further numerical comparisons among the MDS and IM techniques to comprehend to what extent the MDS methods can outperform the IM methods in terms of distance properties and bit error rate (BER) performance. Our numerical distance comparisons show that the MDS techniques are more beneficial than the IM techniques when $\eta > 2$, where η is spectral efficiency (SE) per codeword element. Moreover, our BER results support the distance results and show the effectiveness of the MDS techniques against the IM techniques for a wide range of SE.**

I. Introduction

In index modulation (IM), constellation symbols related to the combinations of the activated (deactivated) codeword elements are index symbols, and such symbols have a higher minimum Hamming distance than those of the conventional modulation symbols, e.g., QAM or PSK symbols [1]. More specifically, the minimum Hamming distance between index symbols is two. However, such a distance is only one for the conventional modulation symbols, limiting the minimum Hamming distance of an IM codebook to one. Thus, IM maps incoming data bits to two different symbols with different distance properties. The main advantage of such a mapping is that the minimum Euclidean distance between the conventional modulation symbols becomes higher in IM than in classic modulation concepts since IM distributes total symbol energy among only the activated elements. One can improve such an advantage without sacrificing the low-complexity structure by employing IM over the in-phase and quadrature components of the codeword elements, separately [2], [3]. The resulting method is called in-phase and quadrature IM (IQ-IM), and IQ-IM doubles the number of index symbols compared to IM. Apart from implementing IM over the in-phase and quadrature parts, one can employ coordinate interleaving over these parts to introduce additional diversity to the IM systems [4]. In coordinate interleaved IM (CI-IM), as a result of rotating constellation points and locating the in-phase and quadrature parts of a codeword element on different codeword elements, the minimum Hamming distance becomes two. Besides IM, IQ-IM, and CI-IM, one can find many systems utilizing IM-like combinatorial structure in the literature [5], [6].

Dual-mode IM (DM-IM) in [7] and Multi-mode (IM) (MM-IM) [8] activate all codeword elements and embed information into the combinations and permutations of the disjoint constellations, respectively, as well as the modulation symbols drawn from these constellations.[1] Similar to IM, the minimum Hamming distance between the index symbols related to the combinations in DM-IM and permutations in MM-IM is two. However, DM-IM and MM-IM can employ a lower order modulation to achieve the same SE as conventional modulation concepts. That means incoming information bits are mapped to two different symbols: one has a higher minimum Hamming distance than the classic modulation symbols, whereas the other has a higher minimum Euclidean distance than these symbols due to employing lower order modulation. Because the asymptotic performance of a codebook in a fading channel is limited by the minimum Euclidean distance between the codeword pairs which have the minimum Hamming distance, DM-IM and MM-IM enable better asymptotic performance in a fading channel than the conventional modulation techniques. The reason is that they have a higher minimum Euclidean distance between unit Hamming distance codewords. Besides, MM-IM can produce more index symbols. Therefore, it can provide a better error performance than DM-IM. There are also other techniques using multiple disjoint constellations in a combinatorial fashion to form codeword elements [9]–[11]. In [9], the authors generalize the DM-IM technique in a way that the number of codeword elements drawn from the same disjoint constellation is alterable. The authors of [10] propose super-mode IM (Su-IM) that incorporates repetition coding with IM to obtain additional diversity. Instead of nulling codeword elements, they exploit two disjoint constellations over codeword elements. In [11], unlike the IM techniques that use combinations and permutations of the codeword elements, we use set partitions of a set to form the constellation points. In this scheme, we exploit disjoint constellations to specify the subsets of the set partitions and obtain more index symbols than the MM-IM and DM-IM techniques.

Motivated by the advantages of the techniques using disjoint constellations, we design a modulation concept based on using disjoint constellations in [12], [13]. Unlike these techniques, we use disjoint constellations to form the codewords of a simple maximum distance separable (MDS) code. By exhibiting the MDS coding mechanism, we arrive at the maximum number of symbols whose minimum Hamming distance is two. In this way, our proposed schemes achieve the same SE by employing less disjoint constellations or lower

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[1]Here, we mean by disjoint constellations that these constellations have no constellation points in common.

order modulation on the disjoint constellations than the IM techniques with multiple disjoint constellations. That makes our concepts more beneficial than these techniques in terms of error performance. Moreover, unlike most of these techniques, the proposed concepts can employ only the codewords whose minimum Hamming distance is two and achieve an additional diversity.

Against this background, in this paper, we focus on showing the effectiveness of modulation concepts based on a well-known MDS code against the IM techniques. In this regard, we compare the distance properties of the MDS and IM methods and demonstrate by our numerical results that the MDS techniques can achieve better distance properties. We further make numerical bit error rate (BER) comparisons between the MDS coded modulation and IM techniques to support the distance comparisons. Our comparisons indicate that the MDS concepts outperform the IM, QI-IM, ICM, and CI-IM concepts in error performance.

II. Overview of the MDS and IM Concepts

A. MDS Modulation

MDS modulation uses a simple MDS code along with Q disjoint M-ary constellations to construct the N-dimensional complex constellation points. The MDS mechanism generates the first $N-1$ elements of an N-tuple codeword, i.e., $I_1, I_2 \ldots, I_{N-1}$, by using the integers, $1, \ldots, Q$ as symbols, i.e., $I_\tau \in \{1, \ldots, Q\}$ and $\tau \in \{1, \ldots, N-1\}$. The last element, I_N, is chosen from the same set of integers by ensuring the sum of those N-tuples is zero under modulo-Q arithmetic, i.e., $(I_1 + I_2 + \ldots + I_N) \bmod Q = 0$ [14]. Therefore, we can construct Q^{N-1} N-tuples by using the MDS mechanism. Moreover, the MDS techniques use the integers in an N-tuple to determine which element of the related constellation point will be chosen from which disjoint constellation among available disjoint M-ary constellations [13]. After deciding the disjoint constellations, we draw each element of the constellation points from these constellations. In this way, one can form M^N different constellation points. Thus, the SE of the MDS modulation techniques is given by

$$\eta = \frac{\lfloor \log_2 Q^{N-1} \rfloor + N \log_2 M}{N}. \tag{1}$$

We can determine disjoint constellations by using different approaches. One can obtain the disjoint constellations by partitioning well-known PSK and QAM constellations. MDS with PSK (MDS-PSK) uses PSK constellations to form the disjoint constellations. In this method, the initial disjoint constellation is a regular M-ary PSK constellation, whereas the remaining disjoint constellations are obtained by rotating the initial constellation. In MDS with QAM (MDS-QAM), on the other hand, we use the well-known set partitioning technique to carve Q disjoint M-ary constellations from an (QM)-ary QAM constellation.

Like IM, we can apply the MDS method on separately the in-phase and quadrature parts of a codeword element. That is performed in MDS with in-phase and quadrature modulation (MDS-IQM). Considering N-tuples of the same MDS code for in-phase and quadrature components and assuming we have Q disjoint M-ary PAM constellations, MDS-IQM achieves twice the SE of the former MDS schemes, i.e.,

$$\eta = \frac{2\lfloor \log_2 Q^{N-1} \rfloor + 2N \log_2 M}{N}. \tag{2}$$

One can carve Q disjoint M-ary PAM constellations for this technique by set partitioning a (QM)-ary PAM constellation [8]. It is important to note that the size of each disjoint constellation, M, can be equal to one in the MDS concepts. In this case, since the MDS techniques do not carry information on the conventional modulation symbols, the resulting MDS codebook contains only constellation points whose minimum Hamming distance is two [13].

B. Index Modulation and Related Schemes

1) Index Modulation: In IM, K out of N elements of constellation points are nonzero elements, and the combinations related to the locations of these elements are used to form the N-dimensional constellation points [1]. Since each combination corresponds to a different constellation point, this structure produces a total of $\binom{N}{K}$ constellation points.[2] Moreover, IM produces different constellation points by choosing each nonzero element from an M-ary two-dimensional constellation diagram such as M-PSK or M-QAM constellation diagrams. Therefore, assuming M is a power of two, a further $K \log_2 M$ bits are mapped to the related constellation points, resulting in spectral efficiency (SE) of

$$\eta = \frac{\lfloor \log_2 \binom{N}{K} \rfloor + K \log_2 M}{N}. \tag{3}$$

On the other hand, IM can be employed separately over the in-phase and quadrature parts of the codeword elements. This special version of IM is called in-phase/quadrature index modulation (IQ-IM) [2], [3]. In this technique, K nonzero in-phase and quadrature components of the codeword elements are located independently to form the combinations related to the constellation points. Thus, a total of $2\lfloor \log_2 \binom{N}{K} \rfloor$ bits are mapped to the corresponding constellation points. Moreover, each nonzero component is chosen from an M-ary PAM constellation. Thus, the overall SE of IQ-IM is

$$\eta = \frac{2\lfloor \log_2 \binom{N}{K} \rfloor + 2K \log_2 M}{N}. \tag{4}$$

One can also construct the constellation points in a way that no element is zero [7], [8]. In this case, one can use disjoint constellations to form the combinations or permutations related to the constellation points. DM-IM employs disjoint constellations over the elements of a constellation point, and it exploits the combinations of elements chosen from these constellations to form different constellation points [7]. Assuming K and $N-K$ out of N elements are chosen from the first and second disjoint constellation, respectively, the SE of DM-IM is

$$\eta = \frac{\lfloor \log_2 \binom{N}{K} \rfloor + N \log_2 M}{N}. \tag{5}$$

Moreover, another IM concept called MM-IM chooses each element of a constellation point from a disjoint constellation to

[2]By using a simple fixed-length bit mapping scheme, the IM mapper can only map $\lfloor \log_2 \binom{N}{K} \rfloor$ information bits to these combinations.

form the permutations [8]. Thus, we need N disjoint constellations to form the constellation points in MM-IM. By permuting the elements selected from N disjoint constellations, one can obtain $N!$ different constellation points. Hence, assuming each disjoint constellation is an M-ary constellation, the SE of MM-IM is given by

$$\eta = \frac{\lfloor \log_2 N! \rfloor + N \log_2 M}{N}. \tag{6}$$

In multi-mode IQ-IM (MM-IQ-IM), on the other hand, in-phase and quadrature parts of the elements are drawn from N disjoint M-ary PAM constellations. By permuting these parts independently, one can obtain $2N!$ different constellation points for MM-IQ-IM. Thus, the SE of MM-IQ-IM is

$$\eta = \frac{2\lfloor \log_2 N! \rfloor + 2N \log_2 M}{N}. \tag{7}$$

BER curves of the aforementioned IM techniques cannot have a diversity order higher than one due to the use of conventional modulation techniques. However, one can achieve a diversity order for the BER curves of the IM and MM-IM techniques by employing coordinate interleaving. In coordinate interleaved IM (CI-IM) and MM-IM (CI-MM-IM), the points in the M-QAM/PSK constellations are rotated by an angle in a way that the imaginary parts of the points will not be equal. Then, the imaginary parts are interchanged between the $(2k-1)$th and $2k$th activated subcarriers for CI-IM and between the $(2l-1)$th and $2l$th subcarriers for CI-MM-IM where $k = 1, 2, \ldots, K$, $K = 2I_1$, $l = 1, 2, \ldots, N$, $N = 2I_2$, I_1, and I_2 are positive integers. In this way, the real and imaginary parts of the complex data symbols are transmitted over different active codeword elements. It is important to note that the minimum Hamming distance between the codewords of CI-IM and CI-MM-IM is equal to two.

Besides employing coordinate interleaving, one can achieve an additional diversity gain with the IM schemes by organizing the codewords elements the way that the minimum Hamming distance between the codeword pairs is two. Super-mode IM (SuM-IM) carries out this by exploiting repetition coding, i.e., repeating the IM symbols over zero elements [10]. In SuM-IM, two out of Q disjoint constellations and $N/2$ out of N elements are jointly selected according to the information bits. Then, one of M-ary constellation symbols related to the first disjoint constellation is located on the selected elements, while one of M-ary constellation symbols related to the second disjoint constellation is located on the remaining elements. Thus, the SE of SuM-IM is given by

$$\eta = \frac{\lfloor \log_2 \left(\binom{Q}{2} \binom{N}{N/2} \right) \rfloor + (N/2) \log_2 M}{N}. \tag{8}$$

It is important to note that the IM methods can be regarded as particular cases of permutation modulation (PM) in [15] since they use combinations or permutations of the codeword elements to form their constellation points as in classical PM. However, different from the classical PM concept, the codeword elements are chosen from a well-known M-ary constellation.

2) Index and Composition Modulation: One can use compositions to embed more information into the energy levels of classical IM. The resulting concept is called index and composition modulation (ICM) [5]. ICM activates K out N codeword elements first as in IM. Then, it embeds information into M-PSK symbols and the energy levels of the activated elements by using the compositions of an integer I. Considering element combinations, compositions, and M-ary PSK symbols on the activated subcarriers, the SE of ICM is given by

$$\eta = \frac{\lfloor \binom{N}{K} \rfloor + \lfloor \binom{I-1}{K-1} \rfloor + K \log_2 M}{N}. \tag{9}$$

3) Set Partition Modulation: In set partition modulation (SPM), unlike the IM techniques that use combinations and permutations of the elements, set partitions of an N-element set are used to form the constellation points. Moreover, M-ary disjoint constellations are used to distinguish between subsets in a set partition. For example, let us assume we have an N-element set $\mathcal{X} := \{x_1, x_2, \ldots, x_N\}$ and a set partition of that set where the first element is in the first subset, and the remaining elements are in the second subset, i.e., $\mathcal{X}_{1|2\ldots N} := \{\{x_1\}, \{x_2, \ldots, x_N\}\}$. In this case, considering bijective mapping between the elements of an SPM constellation point and a set partition, the first element of the related constellation point is drawn from the first disjoint constellation, whereas the remaining elements are drawn from the second disjoint constellation. Although there are different variants of SPM, ordered full SPM (OFSPM) achieves the highest SE as well as the best error performance. In OFSPM, the order of the subsets in a set partition is permuted, and the number of subsets in a set partition ranges from 1 to N to form different constellation points. Hence, the SE of the OFSPM scheme is

$$\eta = \frac{\lfloor \log_2 \breve{B}(N) \rfloor + N \log_2 M}{N} \tag{10}$$

where $\breve{B}(N)$ is the ordered Bell number.

III. MDS Modulation versus Index Modulation

In this section, we conduct comprehensive comparisons among the MDS and IM techniques. In this regard, we define the distance measures in Subsection III-A. Then, we provide our numerical distance and BER results in Subsection III-B.

A. Distance Measures

In our comparisons, we provide two different distances between the codewords of the modulation techniques. Let us start with defining the *minimum L-product distance* as distance

$$d_1 = \min_{\substack{\mathbf{s}, \hat{\mathbf{s}} \in \mathcal{S} \\ \mathbf{s} \neq \hat{\mathbf{s}}}} \prod_{s_n \neq \hat{s}_n}^{L} |s_n - \hat{s}_n| \tag{11}$$

between any two constellation points **s** and **ŝ** in the vector space $\mathcal{S}$ where L is the modulation diversity,[3] and s_n and $\hat{s}_n$ are the nth elements of the vectors **s** and **ŝ**, respectively. Note that d_1 is of paramount importance for coding gain of a codebook in Rayleigh fading channels. On the other hand, the second

[3]This is also defined as the minimum Hamming distance between the codewords of a codebook.

distance that we provide is the *minimum Euclidean distance* between the N-element codewords, i.e.,

$$d_2 = \min_{\substack{\mathbf{s},\hat{\mathbf{s}} \in \mathcal{S} \\ \mathbf{s} \neq \hat{\mathbf{s}}}} \sqrt{\sum_{n=1}^{N} |s_n - \hat{s}_n|^2}. \quad (12)$$

Note also that this distance is one of the main factors affecting the performance of a codebook when the channel is additive white Gaussian noise (AWGN) channel.

Remark. One can find an MDS scheme that achieves higher d_1 and d_2 than the techniques employing multiple disjoint constellations such as DM-IM, MM-IM, SuM-IM, and SPM. This is because the MDS technique is capable of achieving the same SE as those of these techniques with a lower number and smaller size of disjoint constellations. Moreover, the MDS approach is much more flexible than those schemes as it is applicable for a much broad range of the number of disjoint constellations and codeword length. These are why we do not include any numerical results related to these benchmarks in our comparisons below. We also do not incorporate any numerical outcome regarding the CI-MM-IM method of [16] since coordinate interleaving can also be employed in the MDS coded modulation methods to obtain better error performance than CI-MM-IM.

B. Distance and Bit Error Rate Comparisons

In tables and figures of this section, "IM $(N, K, M$-QAM)" and "CI-IM $(N, K, M$-QAM)" stand for the conventional IM and CI-IM[4] techniques, respectively, which activate K codeword elements out of N elements and employ M-QAM symbols on the activated elements. Moreover, "IQ-IM (N, K, M)" is the IQ-IM technique that has K nonzero in-phase and quadrature components out of N elements and chooses each nonzero element from an M-ary PAM constellation. "ICM (N, K, I, M)" is the ICM technique having K nonzero codeword elements out of N elements where the nonzero elements are drawn from M-ary PSK symbols and their energies are chosen according to the compositions of an integer I into K parts.[5] "MDS-IQM (N, Q, M)" and "MDS-QAM (N, Q, M)' are the MDS-IQM and MDS-QAM schemes that have N-element codewords with Q disjoint M-ary PAM constellations related to the in-phase and quadrature components and N-element codewords with Q disjoint M-ary QAM constellations, respectively.[6] In our simulations, we assume all schemes operate over a Rayleigh fading channel, whose elements are independent and identically distributed.

In Table I,[7] we compare the distance results of MDS-IQM and MDS-QAM with conventional modulation, IM, and IQ-IM when $N = 2$. It is clear from the table that the MDS concepts provide higher d_1 and d_2 than the IM and IQ-IM benchmarks when the SE per codeword element is higher than 2 bits. However, in the case where we have $\eta = 1$ and $\eta = 2$, one can find MDS schemes that achieve a higher diversity order than the IM and IQ-IM schemes. Moreover, the MDS techniques outperform conventional modulation in terms of d_1 for almost all values of SE. As a result, the MDS concepts can attain a superior error performance against the benchmarks. Such superiority is best illustrated with a BER figure.

In Fig. 1, we compare the BER performance of the MDS concepts with conventional modulation, IM, and IQ-IM when $N = 2$ and $\eta \in \{2, 4\}$. To obtain the BER curves, we employ the optimum ML detectors for all concepts as in Eq.(10) of [1]. As seen from the figure, IQ-IM (2, 1, 2) outperforms MDS-QAM (2, 4, 2) at both low and high SNR, since the IQ-IM scheme has better distance properties, or in other words, higher d_1 and d_2, than the MDS-QAM scheme as shown in Table I. Although MDS-IQM (2, 4, 1) is outperformed by 4-QAM, IM (2, 1, 8-QAM), and IQ-IM (2, 1, 2) at low SNR

Table I: Distance comparison of the MDS modulation techniques with the QAM, IM, and IQ-IM techniques when $N = 2$.

SE per codeword element	System	Diversity Order (L)	d_1	d_2
1 bit	BPSK	1	2.0000	2.0000
	IM (2, 1, BPSK)	1	2.8284	2.0000
	MDS-QAM (2, 4, 1)	2	2.0000	2.0000
2 bits	4-QAM	1	1.4142	1.4142
	IM (2, 1, 8-QAM)	1	1.1547	1.1547
	IQ-IM (2, 1, 2)	1	2.0000	1.4142
	MDS-QAM (2, 4, 2)	1	1.8257	1.1547
	MDS-IQM (2, 4, 1)	2	0.4000	0.8944
3 bits	8-QAM	1	0.8165	0.8165
	IM (2, 1, 32-QAM)	1	0.6325	0.6325
	IQ-IM (2, 1, 4)	1	0.8944	0.6325
	MDS-IQM (2, 2, 2)	1	1.2649	0.8944
4 bits	16-QAM	1	0.6325	0.6325
	IM (2, 1, 128-QAM)	1	0.3123	0.3123
	IQ-IM (2, 1, 8)	1	0.4364	0.3086
	MDS-IQM (2, 4, 2)	1	1.2344	0.4364

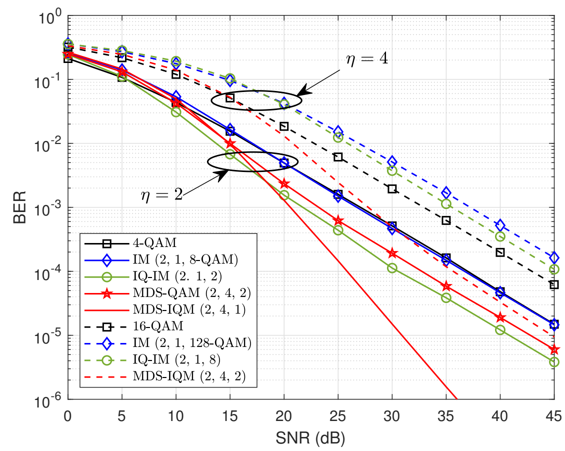

Figure 1: BER comparison of the MDS-IQM schemes with the QAM, IM, and IQ-IM techniques when $N = 2$ and $\eta \in \{2, 4\}$.

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[4] We rotate the constellation points for the CI-IM techniques by the optimum angles provided in [4].

[5] Note that the ICM constellation has a similar structure to that of star-QAM where the energies or regarding amplitude levels of the ICM symbols can be viewed as rings/circles like in star-QAM. Thus, for the ICM techniques in this paper, we rotate the consecutive circles by $\frac{\pi}{M}$ to increase the minimum Euclidean distance between the constellation points [5].

[6] Since the MDS mechanism enables us to perform simple Gray coding between information bits and constellation points, we employ the simple Gray-like coding mechanism in [17] for the MDS techniques in this paper.

[7] Note that we use bold characters in our tables to emphasize the methods with the highest d_1 among benchmarks achieving the same diversity order.

Table II: Distance comparison of the MDS techniques with the QAM, IM, IQ-IM, ICM, and CI-IM techniques when $N = 4$.

Groups	System	SE per Codeword Element	Diversity Order (L)	d_1	d_2
Group 1	4-QAM	2	1	1.4142	1.4142
	IM (4, 3, 4-QAM)	2	1	1.6330	1.6330
	IQ-IM (4, 3, 2)	2.5	1	1.6330	1.1547
	ICM (4, 3, 6, 2)	2	1	1.6330	0.7655
	MDS-QAM (4, 4, 2)	2.5	1	1.8257	1.1547
	CI-IM (4, 2, 16-QAM)	2.5	2	0.1169	0.4880
	MDS-IQM (4, 3, 1)	2	2	0.7500	1.2247
Group 2	8-QAM	3	1	0.8165	0.8165
	IM (4, 3, 8-QAM)	2.75	1	0.9428	0.9428
	IQ-IM (4, 2, 4)	3	1	0.8944	0.6325
	ICM (4, 3, 8, 4)	3	1	1.0000	0.5464
	MDS-QAM (4, 8, 2)	3.25	1	1.7889	0.8944
	CI-IM (4, 2, 64-QAM)	3.5	2	0.0149	0.1954
	MDS-QAM (4, 32, 1)	3.75	2	0.2000	0.6325
Group 3	32-QAM	5	1	0.4472	0.4472
	IM (4, 3, 64-QAM)	5	1	0.3563	0.3563
	IQ-IM (4, 2, 16)	5	1	0.2169	0.1534
	ICM (4, 3, 13, 16)	5	1	0.2164	0.2164
	MDS-IQM (4, 4, 2)	5	1	1.2344	0.4364
	CI-IM (4, 2, 512-QAM)	5	2	-	-
	MDS-IQM (4, 12, 1)	5	2	0.0420	0.2897

Table III: Distance comparison of the MDS techniques with the QAM, IM, IQ-IM, ICM, and CI-IM techniques when $N = 8$.

Groups	System	SE per Codeword Element	Diversity Order (L)	d_1	d_2
Group 1	4-QAM	2	1	1.4142	1.4142
	IM (8, 6, 4-QAM)	2	1	1.6330	1.6330
	IQ-IM (8, 4, 2)	2.5	1	2.0000	1.4142
	ICM (8, 4, 10, 4)	2.5	1	1.2649	0.6375
	MDS-QAM (8, 4, 2)	2.5	1	1.8257	1.1547
	CI-IM (8, 6, 4-QAM)	2	2	0.6667	1.1547
	MDS-IQM (8, 3, 1)	2.75	2	0.7500	1.2247
Group 2	8-QAM	3	1	0.8165	0.8165
	IM (8, 6, 16-QAM)	3.5	1	0.7303	0.7303
	IQ-IM (8, 4, 4)	3.5	1	0.8944	0.6325
	ICM (8, 6, 10, 8)	3.5	1	0.6846	0.6846
	MDS-IQM (8, 2, 2)	3.75	1	1.2649	0.8944
	CI-IM (8, 6, 16-QAM)	3.5	2	0.0780	0.3984
	MDS-IQM (8, 4, 1)	3.5	2	0.4000	0.8944
Group 3	32-QAM	5	1	0.4472	0.4472
	IM (8, 6, 64-QAM)	5	1	0.3563	0.3563
	IQ-IM (8, 5, 8)	5	1	0.3904	0.2760
	ICM (8, 6, 10, 8)	5	1	0.2677	0.2677
	MDS-IQM (8, 4, 2)	5.5	1	1.2344	0.4364
	CI-IM (8, 6, 64-QAM)	5	2	0.0099	0.1595
	MDS-IQM (8, 8, 1)	5.25	2	0.0952	0.4364

due to its relatively low d_2, it considerably outperforms these benchmarks at high SNR as it does achieve an additional diversity gain. On the other, when $\eta = 4$, the MDS-IQM technique outperforms the IM and IQ-IM techniques at both low and high SNR because MDS-IQM (2, 4, 2) has higher d_1 and d_2 than IM (2, 1, 128-QAM) and IQ-IM (2, 1, 8).

In Tables II[8] and III,[9] we compare the distance results of the MDS schemes with those of the QAM, IM, IQ-IM, ICM, and CI-IM techniques for $N = 4$ and $N = 8$, respectively. In these tables, we group the systems in a way that those with similar SE would be in the same group. As seen from the tables, one can find an MDS system that achieves a higher d_1 than the benchmarks for any group except for Group 1 in Table III where IQ-IM (8, 4, 2) attains higher d_1 and d_2 than MDS-QAM (8, 4, 2). Moreover, the gap between d_1 related to the MDS and benchmark techniques increases in favor of the MDS techniques when the SE increases. In this case, the MDS methods are expected to achieve increasingly better asymptotic error performance compared to the benchmark schemes. On the other hand, we observe from the tables that the MDS approach exhibits a d_2 which is comparable to that of the QAM techniques and higher than that of the IM techniques except for Group 1. These results demonstrate the clear advantage of such an approach against the IM method in error performance for relatively high SEs. Although the IM method seems more beneficial for low SEs, the MDS approach still achieves the best asymptotic error performance at low SEs by exhibiting additional diversity gain compared to the IM methods and better distance properties than the CI-IM methods.

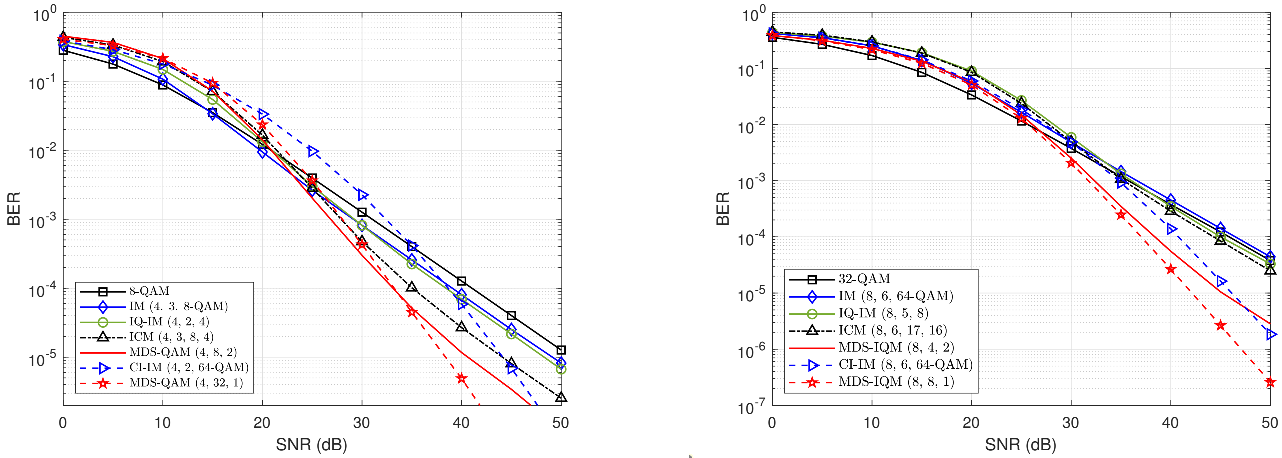
In Figs. 2a and 2b, we compare the BER performance of the MDS-IQM and MDS-QAM systems with that of the QAM, IM, IQ-IM, ICM, and CI-IM systems for $N = 4$ and $N = 8$, respectively. We employ low-complexity detectors of the IM and MDS techniques to obtain the corresponding BER curves. In this regard, we adopt LLR detectors in [1], [2], and [4] for the IM, IQ-IM, and CI-IM schemes, respectively, whereas we operate the LC-ML detectors in [5] and [13] for the ICM and MDS systems, respectively. As seen from Fig. 2a, MDS-QAM (4, 8, 2) exhibits the best error performance at medium-to-high SNR among the systems having unit diversity order. That verifies the results of Table II as MDS-QAM (4, 8, 2) has the highest d_1 in Group 2 of the table. One can also expect MDS-IQM (4, 8, 2) to achieve better BER performance at low SNR than most IM techniques as it has the second-highest d_2 in the group. However, this is not the case due to the suboptimal performance of the LC-ML detector and the number of codeword pairs having the minimum Euclidean distance, i.e., d_2. On the other hand, Fig. 2b demonstrates the superiority of the MDS techniques where the MDS techniques outperform the IM techniques at almost all SNR values.

In Fig. 3, we conduct more general distance comparisons among the MDS and IM methods. We also provide distance results related to the QAM method for benchmarking. Here, the distance results are provided as a function of η, i.e., SE. For the IM technique, we assume QAM symbols employed by the activated codeword elements and use the upper-bound on the L-product distance calculated in [18].[10] For MDS modulation, we consider two extreme cases where M is large and small, i.e., $M = 2^{\eta-1}$ and $M = 2$, respectively. MDS-QAM $(N, 4, 2^{\eta-1})$ can achieve a SE greater than or equal to those of other schemes, and the distance results of MDS-QAM $(N, 4, 2^{\eta-1})$ are valid for any value of N. For MDS-IQM $(16, Q, 2)$, we alter Q to achieve a target SE, i.e., $Q = \lceil 2^{\frac{N(\eta-2)}{N-1}} \rceil$ where $\lceil . \rceil$ is ceiling operation. As seen from the figure, the MDS techniques can deliver considerably higher L-product distance, d_1, than IM and QAM. Moreover, MDS-IQM $(16, Q, 2)$ outperforms IM and QAM in terms of d_2 when

[8]Here, we do not provide d_1 and d_2 for CI-IM (4, 2, 512-QAM) as the searching space for the optimum rotation angle related to this system is exhaustive.

[9]We provide upper-bound distance results on d_2 for the ICM techniques in Groups 2 and 3 since it is not straightforward to calculate the minimum Euclidean distance for these techniques when the codebook size is large.

[10]Here, we do not provide any numerical results for the minimum Euclidean distance of the IM method as is equal to the L-product distance for most values of SE.



(a) $N = 4$

(b) $N = 8$

Figure 2: BER comparison of the MDS modulation techniques with the QAM, IM, IQ-IM, ICM, and CI-IM techniques.

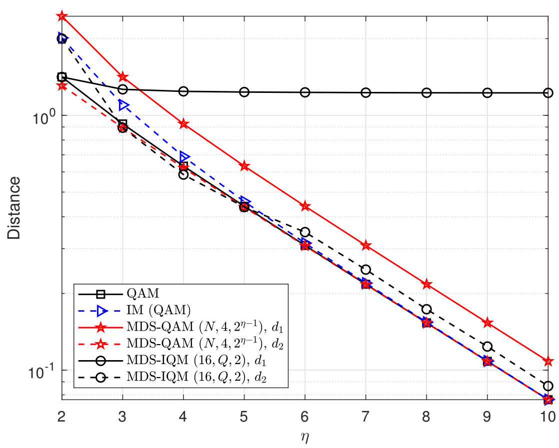


Figure 3: Distance vs. η for the MDS modulation, QAM, and IM techniques.

$\eta \geq 6$ in addition to considerably exceeding other systems in terms of d_1.

IV. Conclusion

In this paper, we provided extensive numerical results to prove the effectiveness of the MDS methods against the IM methods in terms of minimum codeword distances and error performance. Our numerical distance and BER results showed that the MDS coded modulation schemes surpass the IM schemes for a wide range of SE and codeword length.

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