ISSN 1471-0498

DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

THE MAINSTREAM NICHE

Renaud Foucart

Number 670
July 2013

Manor Road Building, Manor Road, Oxford OX1 3UQ

The Mainstream Niche

Renaud FOUCART*†

February 5, 2013

Abstract

Consumption in the time of Internet is characterized by extremely low search costs, leading to increased product diversity (the long tail) and large mainstream products (superstars). In this paper, I show how the mainstream taste can be catered by a niche product, while minority tastes can be mainstream. A competitive market with arbitrarily small search costs supplies a product design corresponding to the mainstream taste at a price that only mainstream buyers are willing to pay. Products corresponding to the taste of the minority are offered at lower price, and bought by several types of buyers.

1 Introduction

The Internet has dramatically decreased search costs in many industries, leading to what economists and business analysts denote by 'long tail' and 'superstar' effects (Anderson (2009), Bar-Isaac, Caruana and Cunat (2012), Yang (2012)). The long tail corresponds to the idea that lower search costs allow offering many specific products. The superstar effect characterizes the emergence of very competitive products sold to the masses. Mainstream products are generally described as designs that suit the preferences of a majority of people, while niche markets cater with the tastes of the minority. Minority buyers are expected to receive lower surplus (Yang (2012)).

*Nuffield College and Dept. of Economics, Oxford University - renaud.foucart@economics.ox.ac.uk

†A previous version of this work has circulated under the title 'On goods and premises'. I am grateful to Micael Castanheira, Julio Davilà, Patrick Legros, Mark Armstrong, Dominique Chariot, Quentin David, Alexandre de Corniere, Alice Duhaut, Jana Friedrichsen, Thomas Gall, Marjorie Gassner, Hannes Kammerer, Georg Kirchsteiger, Raphaël Levy, Alice McCathie, Margaret Meyer, Andras Niedermayer, Alexandre Petkovic, Martin Peitz, József Sákovics, Nicolas Sahuguet, David Sauer, Anastasia Shchepetova, Vincent Vannetelbosch, Elu Von Thadden and participants to conferences and seminars in Brussels, Louvain La Neuve, Tilburg, Montpellier, Marseille, Stockholm, Mannheim, Lima and Edinburgh for useful discussions and comments. All remaining errors are mine.

In this paper, I show that when the tastes of consumers are polarized (when buyers have strong horizontally diverging preferences), the market outcome challenges the classic understanding of niche and mainstream products. The product design preferred by the majority is a 'mainstream niche', sold at high price to buyers of the majority type only. The designs preferred by fewer consumers are cheaper and sold to several types of buyers. Buyers of the minority types search until they find a good match and are the only ones receiving positive surplus in equilibrium. This market has all the appearances of being well-functioning: one can observe search, and different designs are offered at different prices. It is not: a share of the buyers is mismatched, while small search costs would have been expected to make the matching process efficient. The share of sellers offering the mainstream niche is comprised between 0 and 1, and depends on a condition of iso-profit among sellers. In particular, the design preferred by the majority can be sold in equilibrium by a minority of sellers, to a minority of buyers.

I build on a simple competitive framework with two types of consumers, two categories of product design, endogenous category choice by ex-ante identical sellers and sequential search. I assume that information about the distribution of prices and the categories offered are common knowledge.[1] I find that it is enough to consider arbitrarily small search costs for the decentralized market to be inefficient at matching sellers with the tastes of buyers. The mismatch is independent of search costs. This specification can easily be generalized to a larger set of products. As long as there is a 'mainstream' taste sufficiently large for sellers to make high profit by specializing in it, the 'mainstream' category is the only one to be sold in a niche market, while the others are sold to two types of buyers: the ones who like this product (and search for it), and the mainstream buyers who never search.

I find price dispersion in equilibrium: the mainstream niche is sold at a more expensive price. Price dispersion is a well-documented fact, both in conventional and Internet markets (Baye, Morgan and Scholten (2004), Sorensen (2000)). Economists successfully managed to explain how ex-ante identical buyers may end up paying different prices for identical goods (see Baye, Morgan and Scholten (2005) for a survey). But existing models always find that buyers of the same quality (for instance, sharing the same information) face the same expected price and surplus before entering the market. The novelty of my results is that there is discrimination from the compet-

[1] Similar results would be obtained using rational expectations.

itive market: some buyers (of identical quality, with identical information) would benefit from exchanging their positions with others even before entering the market.[2]

The structure of the model is as follows. Sellers choose a product design of either category A or B, at no cost, and with no capacity constraint. There are buyers of type a and type b in the market, in a proportion that is common knowledge, with a strict majority of type a. Buyers buy either zero or exactly one unit of the good. A buyer awards a higher valuation to a good match ($A\&a$ or $B\&b$) than to a mismatch ($A\&b$ or $B\&a$). Buyers are price takers but have the option of not buying, and of searching for product of the different category or for a lower price. In the absence of search costs, all buyers would be correctly matched and firms would make zero profit. However, any strictly positive search cost makes the competitive market inefficient: some buyers are not correctly matched. When search costs are arbitrarily small and tastes are sufficiently polarized, the typical [3] equilibrium involves price dispersion, product diversity and full extraction of the surplus of the majority type a by the sellers. The buyers of the minority type b keep searching until they find a good match. As long as search costs are not exactly zero, the inefficiency remains and is independent of the size of these costs.

The price mechanism behind this equilibrium is the following: (i) sellers of either category set a price that corresponds to the participation constraint of type a buyers. Therefore (ii) category A sellers do not meet the participation constraint of type b buyers, and sellers of category B leave surplus strictly above their reservation utility to type b buyers. As a consequence (iii) buyers of type a accept any offer as they get exactly their reservation utility from both categories of firms, (iv) buyers of type b keep searching until they find a seller of category B, (v) the expected profit of sellers of category B is decreasing in their number (since they share the buyers of type b that search) while the expected profit of category A sellers is independent of their number (since there is no search component in their demand). Hence in equilibrium (vi) the share of category B sellers

[2] While some papers tend to explain price dispersion by differences in the quality of shops or in the information of buyers (Clemons, Hann and Hitt (2002), Salop and Stiglitz (1977), Varian (1980)), several contributions show how price dispersion can arise when buyers are ex-ante identical (Burdett and Judd (1983), Camera and Selcuk (2009)). However, there is no systematic discrimination of buyers. The expected surplus of buyers is identical; difference in treatment is only a feature of the equilibrium. For instance, in Camera and Selcuk (2009), the surplus of a buyer can be low because he is part of an excess demand for a given seller and faces high costs to find another one. But, before entering the market, this buyer is indifferent between his position and the one of any other buyer.

[3] The equilibrium concept used here is Coalition-Proof Nash Equilibrium, as defined by Bernheim, Peleg and Whinston (1987). Others Nash equilibria exist, but are not robust to self-enforcing deviations of a mass of sellers. Using this concept allows identifying a market failure that is not caused by a coordination failure.

is such that their expected profit is exactly equal to that of category A sellers.

Product diversity is a well-documented topic. The idea that the characteristics of goods can be valued differently by different types of individuals has been formalized by Lancaster (1966). The novelty of my results comes from a combination of 3 elements. First, the level of horizontal diversity is endogenously determined by the sellers, as opposed to random utility models (Perloff and Salop (1985), Deneckere and Rotschild (1992), Anderson and Renault (1999)). This implies that a continuum of firms may choose homogeneous design in equilibrium if it is a best response for them. Second, search costs are independent of the localization of the sellers, as opposed to models à la Hotelling (Salop (1979), Stahl (1982), Dudey (1990), Gabszewicz and Thisse (1986)). That is, in my model, sellers do not become more accessible to some buyers by changing their category (position in a Hotelling/Salop model). Third, I model a market, as opposed to the many models using a representative agent (as in Dixit and Stiglitz (1977)). Hence, the inefficiency is not related to the cost structure, but to the failure of the market mechanism itself.

The price structure in horizontal matching models has been studied by Besley and Ghatak (2005), Clark (2007) and Klumpp (2009), but all assume diversity to be exogenously given. Endogenous diversity in oligopoly is studied among others by Chen and Riordan (2008), Kuksov (2004) and Bar-Isaac, Caruana and Cunat (2008). The market failure in my model comes from the presence of nontransferabilities in the matching process (buyers are price takers and there is no bargaining). Legros and Newman (2007) study how non-transferable utility affects matching when differentiation is vertical.

The intuition behind the result that competitive firms can extract consumer surplus is close to Diamond (1971). In the classical formulation of price competition, firms set a price equal to the marginal cost in equilibrium. Introducing search costs in the specification yields the so-called 'Diamond Paradox': a model of search with a large number of buyers and a large number of sellers does not converge to a competitive equilibrium à la Bertrand. In finite time, the price becomes the one that maximizes joint profit. For a given price level in the market, sellers always have an incentive to slightly increase their price until they reach the monopoly level. Indeed, by charging a little more than their competitors, sellers make sure that a buyer who enters their shop will not keep searching for a lower price elsewhere. In the present paper, since there is no capacity constraint for the sellers, and as buyers buy either zero or exactly one unit, a Diamond Paradox would be an efficient outcome (with the entire surplus extracted by the sellers). However, product differentiation changes the picture and, in equilibrium, profit maximizing sellers are not maximizing joint profit. Hence, while the price mechanism is closely related to the one of Diamond (1971), my results do not imply a similar paradox. Once horizontal differentiation is desirable for the sellers, there is search and price dispersion in equilibrium. But another inefficiency arises, due to the mismatch of a share of buyers.

I present the setup of the model in the next section. Section 3 characterizes the equilibrium results. I first show that there is no Nash Equilibrium exhausting the gains from trade: even if search costs are arbitrarily small, not all buyers benefit from a good match. Two cases may arise.[4] When tastes are not very polarized, only one category is produced and price is homogeneous. When preferences are stronger, there is product diversity and search in the (unique) Coalition-Proof Nash Equilibrium. I extend the results to a continuous valuation, a larger number of types, and higher search costs in Section 4. Section 5 concludes.

2 Setup

The economy is composed of two groups, each of them being a continuum of mass 1. The first group is the buyers, with an exogenous fraction α of type a and $1-\alpha$ of type b. In this presentation of the model, I consider $\alpha \in (\frac{1}{2}, 1)$.[5] The second group is composed of sellers, who endogenously choose a category A or B. The choice of category is costless: price dispersion does not come from switching costs. A 'good' match (a&A or b&B) generates surplus V and a 'bad' match (a&B or b&A) generates surplus $v < V$.[6] The surplus is received by the buyer if she accepts the price set by the seller. The outside option is set to $r \in (0, v)$.[7] Both categories are produced with no fixed cost and marginal cost normalized to 0. A buyer can buy either 0 or exactly 1 unit of either good. Parameters α, V, v, r and discount factor δ are common knowledge. The stages of the game are as

[4] I characterize necessary and sufficient conditions for both cases.

[5] The results for $\alpha \in (0, \frac{1}{2})$ are symmetric. I exclude the non-generic possibility of having exactly $\alpha = \frac{1}{2}$. Heterogeneous buyers implies $\alpha < 1$.

[6] The fact that only two values exist for the surplus is not crucial for my results. I show in section 4.3 that the necessary condition is to have a sufficiently high density of buyers sharing close preferences.

[7] The outside option cannot be normalized to zero, as it would imply that any positive outcome, even if discounted a large number of times, is always higher than r.

follows:

1. Sellers simultaneously choose a category (either A or B) and price offer ;
2. Buyers learn the share γ of sellers of category A, and the distribution of prices ;
3. Each buyer is randomly matched with a seller. This first match is assumed to be costless.[8] She observes the price and the chosen category of the seller she is matched with. Each buyer decides whether to **Accept** the offer, **Leave** the market and receive the outside option r or to **Search** for another seller. If a buyer searches, she is randomly matched with another seller, but her payoffs are discounted with a parameter $\delta < 1$. There is no limit for search, but the cumulated discount factor decreases to δ^s after s searches.[9]

3 Equilibria

In this Section, I assume arbitrarily small search costs ($\delta \to 1$).[10] I first show that there are only two prices that are played with strictly positive probability in some Nash Equilibrium and that the market outcome is never efficient. I concentrate on the case where preferences are sufficiently polarized for a niche product to be viable, show there are multiple Nash Equilibria and that only one is Coalition Proof. This equilibrium is the mainstream niche; where the good of category A is sold to buyers of type a only, while category B is sold to both types. I conclude the section by briefly summarizing the other case (no product diversity and a single mainstream product).

3.1 Main characteristics of the Nash Equilibria

The equilibrium price only takes two values that I denote by 'high price' $p = V - r$ and 'low price' $p = v - r$. The results of this subsection are mainly driven by a mechanism that can be related to the one used by Diamond (1971) while introducing search costs in a homogeneous market.

[8] Making it costly would generate extra equilibria, not robust to the strongest concept of equilibrium used below.

[9] The search cost is supported only by the buyer (wasting time a given day) and not as postponed sales. Therefore, only the surplus of buyers is discounted. When search costs are arbitrarily small, none of the results is affected by this assumption. When search costs are arbitrarily small, similar results could be obtained with linear search costs (and a first search for free). However, discounting search costs simplifies the general exposition in section 4.2.

[10] This assumption is relaxed in section 4.2. The main equilibrium presented in this section is robust to an increase in search costs.

The difference here comes from the heterogeneous tastes of the buyers. The low price corresponds to the participation constraint of mismatched buyers. A seller that has positive demand from those buyers when the price is exactly $v-r$ certainly loses the demand from the whole group by slightly increasing the price. Therefore, there can be an incentive for sellers not to increase the price above this threshold. Similarly, the high price corresponds to the participation constraint of buyers with a good match, and any price above this value implies zero demand for the seller.

Lemma 1 *There are only two possible prices in a Nash Equilibrium: $p = V - r$ and $p = v - r$.*

Sketch of the Proof. The formal proof is given in Appendix 6.2. Sellers are free to choose their category at no cost. Therefore, if the expected profit of a seller of category i is higher than the expected profit of a seller of category j, this is not an equilibrium. While deciding whether to accept an offer or to search for another, a buyer considers the distribution of prices in the market $\hat{p}$. As there is a continuum of sellers, a single seller has no influence on $\hat{p}$. However, any seller knows $\hat{p}$, and can set her price in order to make buyers of a given type accept her offer. It is always a best response for a seller to slightly increase her price as long as she does not lose consumers by doing so. This can only happen for two levels of price: $v-r$ and $V-r$. At those levels, any increase in the price implies that one of the participation constraints is no longer satisfied. ■

The impossibility of having an equilibrium price different from those two values is the key factor that drives the inefficiency of any Nash Equilibrium of this model. Indeed, either sellers sell to both types of buyers - this implies that some buyers are not correctly matched - or they specialize in only one type and set a high price, such that search never occurs - this implies either a share of mismatches or some buyers leaving the market.

Proposition 1 *The market outcome never exhausts all gains from trade: a share of buyers is always mismatched.*

Proof. *(by contradiction)* Assume both types of buyers search until they find a good match. This implies that each seller is specialized in one type. Hence, as shown in Lemma 1, it is a Best Response for every seller to set a price slightly above the market level, even when it is exactly $p = v - r$. The only price in a Nash Equilibrium is therefore $p' = V - r$. At this level of price, the expected surplus on the market is at most r and buyers never search. This is a contradiction. ■

In any equilibrium, a share of buyers is not correctly matched. This implies that the total gains from trade are strictly lower than $V-r$. If there is a mass of sellers of each type and if the price is strictly lower than $V-r$, all buyers search and the total gains from trade tend to $V-r$ (as δ goes to 1). This Proposition shows the existence of a market failure. Indeed, consider instead that a monopoly owns all the sellers, and can commit to a price. It is easy to show that, by setting a price slightly below $V-r$ and producing both categories, all buyers search and all the potential surplus of the economy is extracted. However, this monopoly would eventually let the buyers with zero surplus. As will be made clear below, competition leaves some buyers with surplus, at the cost of an inefficiency in the matching process.

3.2 When preferences are polarized: a niche market is profitable

When buyers do not derive a sufficiently high surplus from a good match for a niche to be profitable for sellers, the market provides only one of the two categories. The good is sold at a low price, there is no search and no one is excluded from the market (I develop this result in Section 3.3).

When the importance of a good match increases sufficiently, or when the majority is sufficiently large, the incentive for sellers to extract the surplus of a good match also increases and product diversity starts to become a Nash Equilibrium. Condition 1 is necessary and sufficient to be in this case.

Condition 1 $\alpha > \frac{v-r}{V-r}$

In this Section, I assume condition 1 is fulfilled. This implies that the expected profit made by specializing in the mainstream buyers ($\alpha(V-r)$) is higher than the expected profit made by selling to everyone at lower price ($v-r$).

I first show that there are four Nash Equilibria in this game. Three of them coexist, depending on the values of the parameters. I list them below. Then, I consider a more restrictive concept of equilibrium: Coalition-Proof Nash Equilibrium (CPNE). This is to show that the market failure I identify is not the result of a coordination failure. The only CPNE, the Mainstream Niche, is the one that yields the highest profit to sellers.

Definition 1 ***Tyranny of the majority (high price):*** *TM_H. All sellers choose the category desired by the majority and sell it at the high price.*

In this equilibrium, the buyers of the minority are excluded from the market. There is no search.

Definition 2 ***Tyranny of the majority (low price):*** *TM_L. All sellers choose the category desired by the majority and sell it at the low price.*

Therefore, all buyers accept the offer. There is no search.

Definition 3 ***The Classic Niche*** *CN. There are sellers of both categories in the market. The sellers of category A (corresponding to the majority type a) sell at the low price and the sellers of category B sell at the high price.*

Here, the buyers of the minority type accept any offer, while the buyers of the majority type search until they find a good match.

Definition 4 ***The Mainstream Niche*** *MN. There are sellers of both types in the market. The sellers of category B (corresponding to the minority type b) sell at the low price and the sellers of category A sell at the high price.*

This equilibrium is the only CPNE. The buyers of the majority type accept any offer, while the buyers of the minority type search until they find a good match.

Lemma 2 *If condition 1 is true, there are exactly four Nash Equilibria in this game: TM_H, TM_L, CN and MN. For all parameters, either TM_L or CN exist but never both.*

Sketch of the Proof. The formal proof is given in Appendix 6.3. Here is the intuition for each of the equilibria.

- TM_H: it is not a best response for sellers to lower the price unless it is at most $p\prime = v - r$. If the price is exactly p', the expected profit is $\pi' = v - r$. This is lower than the equilibrium profit $\pi = \alpha(V - r)$ by condition 1. It is not a best response for a seller to sell the other category, as the profit of the deviating seller is at most $\pi'' = (1-\alpha)(V - r)$. Which is lower than π, since $\alpha > \frac{1}{2}$.

- TM_L: a seller that slightly increases the price never increases her profit, as she automatically loses a large share of the buyers. The profit of each firm is $\pi' = v - r$. The only possibility to increase profit is to sell the category desired by the minority at price $p = V - r$. Therefore, TM_L is a Nash Equilibrium if and only if $\pi'' < \pi'$.

- *CN*: the share γ^* of firms of category A is such that their profit is exactly the same as the firms of category B, $\pi'' = (1-\alpha)(V-r)$. A firm can increase its profit by deviating and selling category A at the low price if and only if $\pi'' < \pi'$. Therefore, if TM_L is a Nash Equilibrium, *CN* is not.

- *MN*: the share $(1-\gamma^*)$ of firms of category B is such that their profit is exactly the same as the firms of type A, $\pi = \alpha(V-r)$. Therefore, a deviating firm can at most receive profit π' or π'' which are lower by definition.

■

Considering the general definition of Nash Equilibrium, this economy displays a multiplicity of equilibria, and there is no way to predict which one is expected to be realized in practice. In the next Proposition, I use an alternative concept of equilibrium: Coalition-Proof Nash Equilibrium, as defined by Bernheim, Peleg and Whinston (1987). This more restrictive definition implies that there is no *self-enforcing* deviation by a coalition of sellers that can gain from deviating. Take for instance the first equilibrium, TM_H, which is a Nash Equilibrium because no single seller can make buyers search for it. However, there is a coalition of mass $(1-\gamma')$ that would benefit from selling the category desired by the minority at a low price. If $(1-\gamma')$ is not too high, those sellers can attract a sufficiently large number of buyers of type b to increase their profit. This deviation is *self-enforcing*, as none of those deviating sellers has any incentive to change her price or category. And there is no *self-enforcing* deviation by a sub-coalition that can increase her profit by doing so.

By definition, in every CPNE, the sellers get the highest possible equilibrium profit. Thus, the market failure identified here does not come from a coordination failure among sellers. Moreover, one can argue that this concept is much more realistic in the context of this paper. Indeed, firms can communicate and exchange ideas, even if they do not explicitly coordinate. Some sellers can for reasons unrelated to profit maximization try to sell the other category. This can even be created from the demand side: a coalition of buyers of the minority type could start its own business, or simply give a certification or a label to firms who accept to sell their preferred category. All of those effects would make the equilibrium TM_H disappear.

Proposition 2 *When condition 1 is fulfilled, the only Coalition-Proof Nash Equilibrium of this game is the Mainstream Niche: the sellers of category B (corresponding to the minority type b) set a low price and the sellers of category A set at a high price. The buyers of the majority type accept any offer, while the buyers of the minority type search until they find a good match.*

Sketch of the Proof. The formal proof is provided in Appendix 6.4. In TM_H there exists a *self-enforcing* coalition that can increase its profit by selling the type desired by the minority at the low price. In TM_L and CN, the profit of sellers is strictly lower than in MN. Therefore, a *self-enforcing* deviation of a coalition of mass 1 increases its profit by playing MN. ■

A key factor to understand the equilibrium outcome is that, as sellers are free to choose their category at no cost, the expected profit of all sellers is equivalent. The profit of sellers of category A is independent of their number and is $\pi = \alpha(V-r)$. Hence, the number of firms of category B is determined by the difference between high and low price (how much extra surplus a seller can extract by specializing in the majority type) and the share of buyers of the minority (how many buyers will search to reach a seller of category B). This equilibrium value is given by

$$\gamma^* = 1 - \frac{1-\alpha}{\alpha}\frac{v-r}{V-v}. \tag{1}$$

It should be noted that the category desired by the majority of buyers can be produced by a minority of sellers in equilibrium.

This result implies that if a majority of buyers prefer A, they are indifferent between paying a high price for category A and a low price for category B. Buyers of type b however, would never consume category A at high price. They keep searching until they find a product design corresponding to their preferences. If high profit can be made by sellers specializing in customers of type a, the model predicts the market to reflect the preferences of buyers. But this is not always true, and the market outcome can perfectly display a large majority of products of category B, patronized by all types of buyers, and only a few of category A, only patronized by buyers of type

a. Sellers of category B have no incentive to increase their price, as they would lose buyers of type a. Sellers of category A have no incentive to decrease their price, as it would not be sufficient to attract consumers with other preferences.

3.3 When preferences are not polarized: no product diversity

Assume now that condition 1 is not satisfied, i.e.

$$\alpha < \frac{v-r}{V-r}$$

This corresponds to assuming that the relative surplus generated by a good match with respect to a mismatch is quite small: the expected profit is higher by selling to everyone. This is a classic story of technological standard (think of VHS *versus* Betamax for instance): there is only one category sold in equilibrium, at the low price.

Proposition 3 *When condition 1 is not fulfilled, there is only one category provided in equilibrium. It is sold at low price, there is no search and no buyers are excluded from the market.*

Sketch of the Proof. The formal proof is given in Appendix 6.5. The profit of each firm is $\pi' = v - r$. There is no incentive for any firm to increase the price, as the profit would be at most $\pi = \alpha(V - r)$. This is lower than π' as condition 1 is not fulfilled. As the potential surplus from specializing in one category is not high enough, all sellers attract both types. Product diversity is not a Nash Equilibrium. If sellers set the low price but attract only one type of buyers, each seller has an incentive to slightly increase the price. And if one category is sold at the high price and the other at the low price, the profit of the latter sellers is at least $\pi' = v - r$, higher than what she can get by specialization. ■

As in the previous case, the market alone fails to provide an efficient level of product diversity. Note that a market where only category B, preferred by the minority, is produced is also a Nash Equilibrium.

4 Extensions

4.1 Generalized condition for N types of buyers and categories

Consider a more general specification with N types of buyers and N categories of goods. Denote by α_i the share of buyers of type $i = 1, ..., N$. Define $\alpha_m = \max_i \alpha_i$, the category m is the 'mainstream' taste. The reasoning to determine the CPNE of the game is similar as for the two-types game. If the mainstream product is large enough to be a profitable niche for some sellers, the following condition must be fulfilled:

Condition 2 $\alpha_m > \frac{v-r}{V-r}$

Thus, for condition 2 to fulfilled, a seller needs to have higher expected profit from the Mainstream Niche m than from selling to all types of buyers at price $v-r$. When this condition is fulfilled, the share of sellers of the mainstream niche is given by :

$$\gamma_m = 1 - \frac{1-\alpha_m}{\alpha_m} \frac{v-r}{V-v}. \tag{2}$$

And the share γ_i, $i \neq m$, of sellers of each of the other categories is proportional to the respective share of buyers of this type, and given by:

$$\gamma_i = \frac{\alpha_i}{\alpha_m} \frac{v-r}{V-v}. \tag{3}$$

An important difference with the two-types game is that it is much more complex for the Classic Niche to be an equilibrium. Indeed, the minority taste is not 'what is not mainstream', every minority taste has to be considered separately. Thus, for the Classic Niche to be a Nash Equilibrium for a product i, one needs $\alpha_i > \frac{v-r}{V-r}$.

When condition 2 is not fulfilled, the only CPNE is to sell the mainstream product m at a price $p = v-r$ to all types of buyers.

4.2 Increasing search costs

The objective of this extension is to present the additional conditions on discount factor δ for the existence of the various equilibria when search costs are not arbitrarily small. The computations are provided in the formal proof of each of the equilibria in Appendix 6.3.

a. When preferences are polarized.

When condition 1 is fulfilled:

- TM_H is always an equilibrium.
- TM_L is an equilibrium if $(1-\alpha) < \frac{(v-r)}{(V-r)}$ and $\delta > \frac{\alpha V-(v-r)}{\alpha(V-(v-r))}$. The first condition excludes specialization in the minority type, the second condition excludes the possibility for a seller of category A to increase the price, still have demand from majority buyers and increase its profit.
- The Mainstream Niche MN is an equilibrium if $\delta > \frac{\alpha r}{(1-\alpha)(v-r)+\alpha r}$. This condition ensures that there is a sufficiently large number of sellers of category B for buyers of type b to actually search.
- The Classic Niche CN is an equilibrium if $1-\alpha > \frac{v-r}{V-r}$ and $\delta > \frac{(1-\alpha)r}{\alpha(v-r)+(1-\alpha)r}$. The first condition ensures there are enough buyers of the minority type and that enough surplus can be extracted from them. The second condition ensures that there is a sufficiently large number of sellers of category A for buyers of type a to actually search.

The main result is that for values of α not too close to 1, the equilibria are robust to increases in search costs. When search costs increase too much, the number of equilibria decreases. One can show that if TM_L is a Nash Equilibrium, the Mainstream Niche MN is also a Nash Equilibrium (the reverse is not true). Thus, if search costs are low enough for all types of buyers to be served, the (unique) CPNE is the mainstream niche.

Example 1 *Consider values of r, v, V such that TM_L exists, and therefore CN does not: $r = 1$, $v = 2$, $V = 3$. TM_H is always an equilibrium, TM_L is an equilibrium for any $\delta > \frac{3\alpha-1}{2\alpha}$ (the right-hand side is always lower than 1 and increasing in α), MN is an equilibrium for any $\delta > \alpha$.*

Example 2 *Consider values of r, v, V such that CN exists, and therefore TM_L does not: $r = 1$,$v = 2$, $V = 4$. TM_H is always an equilibrium, CN is an equilibrium (iff $\alpha < \frac{2}{3}$) for any $\delta > 1-\alpha$ and MN is an equilibrium for any $\delta > \alpha$.*

b. When preferences are not polarized

When condition 1 is not fulfilled, both Nash Equilibria presented in Section 3.3 hold for any value of δ. Equilibria with product diversity can arise when δ decreases. One can show that, as long as $\delta < \frac{2r}{2r+V-v}$, there exists values of $\gamma^- < \gamma^+$ such that for any $\gamma \in (\gamma^-, \gamma^+)$ buyers buy any type without search. If buyers accept the offer regardless of the category, sellers are indifferent between both categories. The higher the search cost, the broader the interval in which those equilibria exist.

4.3 The value of a mismatch is drawn from a distribution

The specification of my model relies on assuming two different values of consumer surplus. The objective of this extension is to give a continuous interpretation of the main equilibrium of my model, the Mainstream Niche. As what drive the result that the Diamond Paradox does not hold is the existence of an equilibrium price $p = v - r$, I focus on a continuous valuation for the mismatch only.

As in the basic specification, there is a continuum of buyers and sellers of mass 1 and a fraction $\alpha > \frac{1}{2}$ of buyers of type a. A good match yields surplus V and buyers have an outside option of value r. Assume now that the valuation of a mismatch is drawn, for each buyer, from a continuous distribution f with support $[r, V]$. We try to characterize an equilibrium where sellers of categories A and B extract all the surplus of buyers of type a, and where buyers of type b search for sellers of category B that leave them some surplus.

First, I assume this equilibrium exists. I define the total profit for a firm of category A, and the total profit (and the maximization problem) for a firm of category B. Then, I derive the new value of γ. Finally, I show under which conditions it is actually an equilibrium.

The total profit for a firm of category A is

$$\pi_A = \alpha(V - r)$$

as in the discrete case.

To compute **the total profit of a firm of category** B**,** with price $p_B \in (r, V - r)$, one has to distinguish:

- *The demand from buyers of type a* who pick up the seller first. In this equilibrium, firms of type A give no more surplus than the outside option. This demand can be rewritten as $D_a(p_B)$, with $D_a(V-r)=0$ and $D_a(r)=\alpha$

- *The demand from buyers of type b who pick up the seller first, plus the demand from buyers of type b who actually search* (as $p_B^* < V-r$): $\frac{1-\alpha}{1-\gamma}$,

$$\pi_B = (D_a(p_B) + \frac{1-\alpha}{1-\gamma})p_B.$$

Hence, the maximization problem yields

$$D_a(p_B^*) + D_a'(p_B^*)p_B^* + \frac{1-\alpha}{1-\gamma} = 0$$

This rewrites in terms of p:

$$p_B^* = -\frac{D_a(p_B^*) + \frac{1-\alpha}{1-\gamma}}{D_a'(p_B^*)}$$

Thus, increasing the price increases total profit as long as the gain due to the higher price more than compensate the loss coming from the mismatched buyers that stop accepting the offer. This implies that, for such an equilibrium to exist, one needs a point with sufficiently high density (so that the elasticity of the demand is high enough), and a share of buyers of type a sufficiently high (so that the relative importance of this part of the demand is high enough).

Thus, one needs a sufficiently small value of γ, such that the 'search' part of the demand, which is constant as long as $p_B < V-r$, is not too high. Other things being equal, the smaller γ, the highest the elasticity of total demand for a seller of category B. However, even if γ is taken as exogenous in the maximization problem of an individual seller, it is still determined by the expected profit of a seller of category B. The expected profit of both categories of sellers being equal, γ must satisfy.

$$\gamma^* = 1 - \frac{(1-\alpha)p_B}{\alpha(V-r) - D_a(p_B)p_B}$$

which, for any $p < p_B^*$ is strictly decreasing in p_B. This is quite intuitive, as for any $p < p_B^*$ increasing the price increases the profit of a firm of category B, it also increases the number of firms of category B, and decreases the 'search' component of the demand.

Those prices correspond to a Nash Equilibrium if there exists a solution p_B^* associated with a share of category A sellers $\gamma^* \in (0,1)$. This is true if there is a sufficiently high concentration of types, at a point that yields a sufficiently high surplus of a mismatch, and with a sufficiently large majority of buyers of type a. This corresponds to the same intuition as the conditions of existence of this equilibrium in the discrete case.

If those conditions are satisfied, setting another price is not a best response for a seller of category A (a price higher then $V - r$ yields zero demand, a lower price does not increase demand but decreases profit). The price p_B^* set by firms of category B is an equilibrium by definition, as it is the result of individual profit maximization.

5 Concluding comments

This model relies heavily on two assumptions: each seller has to choose a unique category, and no seller has sufficient market power to attract buyers by changing her price. This first assumption is a common feature of the recent literature on the long-tail effect, using examples such as books, DVD or music. Another interpretation of the unique choice to be made by sellers is the appearance and the organization of the shop itself. The second assumption is the reason for the differences between the results of this model - with competitive markets - and the recent oligopoly models. It is worth noting that, if the basic pricing mechanism of sequential search is close to the work of Diamond (1971), the results I present here do not recover the Diamond Paradox. The possibility of a simple horizontal difference in tastes destroys the possibility to have an equilibrium corresponding to the strategy of a profit-maximizing monopoly. Even in the absence of product diversity, the strategy of sellers is not the one that maximizes joint profit. The market mechanism fails to exhaust all the gains from trade: a share of the buyers is not correctly matched. As long as search costs are not too high, this market failure is independent of search costs, and so is the average price on the market. Therefore, the perfectly competitive price is a knife-edge case, since any arbitrarily small search cost makes it disappear. The Mainstream Niche is the idea that, in the presence of product diversity, the design preferred by the largest group of people will be consumed by this group only. All other designs will be consumed by two types of consumers: the one that like it the most, and some mainstream consumers. The condition for this Mainstream Niche to exist is that preferences are sufficiently polarized, and the share of mainstream buyers is high enough. Thus, what could be intuitively considered as a strength (to be part of a large group) is in fact a weakness for consumers. The only buyers that extract surplus from the market above their reservation utility are the buyers of the minority types.

Bibliography

Simon P. Anderson and Regis Renault (1999), "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model", The RAND Journal of Economics, Vol. 30, No. 4 (Winter, 1999), pp. 719-735

Chris Anderson (2009), "The Longer Long Tail: How Endless Choice is Creating Unlimited Demand", 2009, London: Random House Business Books.

Heski Bar-Isaac, Guillermo Caruana, Vicente Cunat, (2008), "Information gathering externalities in product markets", Journal of Industrial Economics, March 2012, Vol. LX, 1, pp.162-185

Heski Bar-Isaac, Guillermo Caruana, Vicente Cunat, (2012), "Design, Search, and Market Structure", The American Economic Review, vol.102, 2, April 2012, pp. 1140-1160

Michael Baye and John Morgan (2004), "Price dispersion in the lab and on the Internet: theory and evidence", The RAND Journal of Economics, 35, 3, pp. 449-466

Michael Baye, John Morgan and Patrick Scholten (2004), "Price Dispersion in the Small and in the Large: Evidence from an Internet Price Comparison Site" ; The Journal of Industrial Economics, 52, 4, pp.463-496

Michael Baye, John Morgan and Patrick Scholten (2005), "Information, Search, and Price Dispersion" ; Handbook on Economics and Information Systems ; Elsevier ; T. Hendershott ed. ; 60pp

Douglas Bernheim, Bezalel Peleg and Michael Whinston (1987), "Coalition-Proof Nash Equilibria. I. Concepts", Journal of Economic Theory, Vol. 42, pp. 1-12

Timothy Besley and Maitreesh Ghatak (2005), "Competition and Incentives with Motivated Agents", The American Economic Review, Vol. 95, No. 3 (Jun., 2005), pp. 616-636

Kenneth Burdett and Kenneth Judd (1983), "Equilibrium Price Dispersion", Econometrica, 51, 4, pp. 955-969

Gabriele Camera and Cemil Selcuk (2009), "Price Dispersion with Directed Search", Journal of the European Economic Association, 28 (2009), 1193-1224

Chen, Yongmin and Michael H. Riordan (2008), "Price-Increasing Competition," The RAND Journal of Economics, 39, pp. 1042-1058

Simon Clark (2007), "Matching and Sorting when Like Attracts Like", ESE Discussion Papers

Eric Clemons, Il-Horn Hann and Lorin Hitt (2002), "Price Dispersion and Differentiation in Online Travel: An empirical Investigation", Management Science, 48, 4, pp.534-549

Raymond Deneckere and Michael Rothschild (1992), "Monopolistic Competition and Preference Diversity", The Review of Economic Studies, Vol. 59, No. 2 (Apr., 1992), pp. 361-373

Peter A. Diamond (1971) "A Model of Price Adjustment" Journal of Economic Theory 3, 156-168

Avinash K. Dixit and Joseph E. Stiglitz (1977) "Monopolistic Competition and Optimum Product Diversity", The American Economic Review, Vol. 67, No. 3, pp. 297-308

Marc Dudey (1990), "Competition by Choice: The Effect of Consumer Search on Firm Location Decisions", The American Economic Review, Vol. 80, No. 5 (Dec., 1990), pp. 1092-1104

J. Jaskold Gabszewicz and J.-F. Thisse (1986) "On the Nature of Competition with Differentiated Products" The Economic Journal Vol. 96, No. 381 pp. 160-172

Tilman Klumpp (2009), "Two-sided matching with spatially differentiated agents", Journal of Mathematical Economics 45 (2009) 376–390

Dmitri Kuksov, (2004), "Buyer Search Costs and Endogenous Product Design", Marketing Science, Vol. 23, No. 4, pp. 490-499

Kelvin J. Lancaster (1966), "A New Approach to Consumer Theory", Journal of Political Economy, Vol. 74, No. 2, pp. 132-157

Patrick Legros and Andrew Newman (2007), "Beauty is a Beast, Frog is a Prince – Assortative Matching with Nontransferabilities", Econometrica, vol.75, pp.1073-1102

Jeffrey M. Perloff and Steven C. Salop (1985), "Equilibrium with Product Differentiation" The Review of Economic Studies, Vol. 52, No. 1 (Jan., 1985), pp. 107-120

Steven C. Salop (1979), "Monopolistic Competition with Outside Goods", The Bell Journal of Economics, Vol. 10, No. 1 (Spring, 1979), pp. 141-156

Steven Salop and Joseph Stiglitz (1977), "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion", The Review of Economic Studies 44, 3, pp. 493-510

Alan T Sorensen (2000), "Equilibrium Price Dispersion in Retail Markets for Prescription Drugs", Journal of Political Economy, 108, 4 pp. 833-850

Konrad Stahl (1982), "Differentiated Products, Consumer search and locational oligopoly", The Journal of Industrial Economics, vol. 31, No. 1/2, pp. 97-11

Hal Varian (2000), "A Model of Sales", The American Economic Review, 70, 4, pp.651-659

Huanxing Yang (2012), "Targeted Search and the Long Tail Effect", unpublished working paper

6 Technical Appendixes

6.1 Two preliminary lemmas

The two following lemmas are used in several of the following proofs:

Lemma 3 *Both categories of sellers have the same expected profit in equilibrium.*

Proof. Sellers are free to choose their type at no cost. Therefore, if the expected profit of a seller selling a service of category i is higher than the expected profit of a seller of category j, this is not an equilibrium. It is a best response for a seller of category j to sell category i. ■

Lemma 4 *For a given level of price, a seller sets her price in a way to decide what kind of buyers accept her offer, but has no influence on who searches for her.*

Proof. This Lemma is close to Diamond (1971). While deciding whether to accept an offer or to search for another, a buyer considers the distribution of prices in the market $\hat{p}$. As there is a continuum of sellers, a single seller has no influence on $\hat{p}$. However, a seller knows $\hat{p}$, and can set her price in order to make buyers of a given type accept her offer. ■

6.2 Proof of Lemma 1

Consider first the following definitions:

Definition 5 ***The participation constraint for a seller of category*** j ***is satisfied for a buyer of type*** *i if the seller offers a price leaving the buyer of type i utility higher than the reservation utility r. I denote this by* PC_i^j*, with* $i \in \{a,b\}$, $j \in \{A,B\}$.

Definition 6 ***The incentive compatibility constraint for a seller of type*** j ***is satisfied for a buyer of type*** *i if the seller offers a utility higher than her (discounted) expected utility if she stays in the market. I denote this by* IC_i^j*, with* $i \in \{a,b\}$, $j \in \{A,B\}$.

Definition 7 *A buyer of type i accepts the offer of a seller of category j if and only if* PC_i^j *and* IC_i^j *are satisfied.*

This is the proof of Lemma 1:

Proof. PART 1: The probability of a price offer $p > V - r$ is zero in any (mixed) Nash Equilibrium.
(i) Consider a pair i, j with $i \neq j$.For a firm i setting price p, PC_i^i is given by $V - p \geq r$, $\Leftrightarrow p \leq V - r$ and PC_j^i is given by $v - p \geq r \Leftrightarrow p \leq v - r$.
(ii) A seller always makes a positive profit. Assume all firms sell at price zero and make zero profit. Slightly increasing the price is a profitable deviation for a firm of type i, as there exist $p' > 0$ such that IC_i^i is satisfied, i.e. $V - p' \geq \delta V$.
(iii) As a corollary of (ii), in equilibrium, it is never a best response for a firm to have zero demand. Hence, $p > V - r$ is never a best response, because no buyer ever accepts the offer.

PART 2: The probability of a price offer $v - r < p < V - r$ is zero in any (mixed) Nash Equilibrium. Consider a firm of category i. Denote the expected surplus resulting from the offer of a firm of category i for a buyer of type j by S_j^i. When $p > v - r$, PC_j^i is not satisfied. Thus, the firm only considers buyers of type i. We can therefore focus on conditions IC_i^i and PC_i^i. As $p < V - r$, PC_i^i is already satisfied.

IC_i^i is always satisfied for a firm setting a price p' when

$$S_i^i(p') \geq \max(\delta S_i^i(\hat{p}_i), \delta S_i^j(\hat{p}_j)).$$

It is thus enough to show that there exists a profitable deviation satisfying IC_i^i. (i) If $\max(\delta\ S_i^i(\hat{p}_i), \delta\ S_i^j(\hat{p}_j)) = \delta\ S_i^i(\hat{p}_i)$, there exists a price $p' > \hat{p}_i$ such that IC_i^i is satisfied. Indeed, $S_i^i(p') \geq \delta\ S_i^i(\hat{p}_i)$ for some $p' > \hat{p}_i$. Hence, it is always a best response for a firm to increase the price as long as PC_i^i is satisfied.

(ii) If $\max(\delta\ S_i^i(\hat{p}_i), \delta\ S_i^j(\hat{p}_j)) = \delta\ S_i^j(\hat{p}_j)$, then it is a BR for the firm to change and sell category j at price $p' > p_j$. Indeed:

a) By Lemma 3, in equilibrium, $\pi_i = \pi_j$.

b) As $\max(\delta\ S_i^i(\hat{p}_i), \delta\ S_i^j(\hat{p}_j)) = \delta\ S_i^j(\hat{p}_j)$, IC_j^j is satisfied and from PART 1, $p < V - r$.

c) For the same reason, if there exist some firms selling at price $\hat{p}_i$ in equilibrium, they must have non negative demand. Hence, IC_i^i is satisfied. Then, as $S_i^i(\hat{p}_i) < S_i^j(\hat{p}_j)$, IC_i^j is also satisfied, with $S_i^j(\hat{p}_j) > \max(\delta\ S_i^i(\hat{p}_i), \delta\ S_i^j(\hat{p}_j))$.

d) Hence, there exists some $p' > p_j$ such that IC_i^j is still satisfied.

PART 3: The probability of a price offer $p < v - r$ is zero in any (mixed) Nash Equilibrium.

When $p < v - r$, for both categories of firms and both types of buyers, PC is satisfied by definition.

(i) For both categories of firms selling to both types of buyers to be a Nash Equilibrium, profit must be the same. As expected demand is 1 for any firm, it is only possible if p is the same for any firm. Also, IC_B^a and IC_a^B have to be binding. Indeed, if it is not satisfied, a type of buyer searches. And if it is not exactly binding, slightly increasing the price is a best response, i.e. for IC_B^a

$$\begin{aligned} v - p &= \delta\gamma(V - p)\sum_{i=0}^{\infty}\delta^i(1-\gamma)^i \\ v - p &= \frac{\delta(1-\gamma)(V-p)}{1-\delta(1-\gamma)}. \end{aligned}$$

And similarly, IC_a^B

$$v-p=\frac{\delta(1-\gamma)(V-p)}{1-\delta\gamma}.$$

This is only possible if $\gamma=\frac{1}{2}$. But then, both equalities yield

$$\begin{aligned}(1-\delta\gamma)(V-p) &= \delta\gamma(V-p)\\ (1-\delta\gamma) &= \delta\gamma\\ 1-\frac{\delta}{2} &= \frac{\delta}{2}\\ \delta &= 1\end{aligned}$$

this is, no search cost at all.

(ii) If a firm is interested in only one type of seller, the reasoning becomes the same as in PART 2, there is always an incentive to increase the price. ■

6.3 Proof of Lemma 2

To prove the Lemma, I first prove that each of the four potential Nash equilibrium exist under some conditions, and then that no other equilibrium can exist.

a. TM$_H$ is a Nash Equilibrium:

Proof. (i) It is not a best response for a seller to sell category B and set price $v-r<p'\leq V-r$, because the profit will be at most $(1-\alpha)(V-r)<\alpha(V-r)$.

(ii) It is not a best response for a seller to sell category B and set price $p'\leq v-r$. By Lemma 4 a seller cannot make people search for it. So, the profit will be at most $v-r<\alpha(V-r)$ (by condition 1).

(iii) It is not a best response for a seller to sell category A at price $p'\leq v-r$. On her own, a seller can't make buyer search for her. So, the profit will be at most $v-r<\alpha(V-r)$ (by condition 1).

(iv) It is not a best response for a seller to sell category A at price $v-r<p'<V-r$. By condition 4 this yields demand α and therefore profit strictly lower than $\alpha(V-r)$. ■

b. TM$_L$ is a Nash Equilibrium iff $(1-\alpha)(V-r) < v-r$:

Proof. Profit in equilibrium is $\pi_A = v-r$.

(i) It is not a best response for a firm to sell category B and set price $p = v-r$ as it will lose all buyers of type a.

(ii) It is not a best response for a firm to sell category B and set price $p' = V-r$, because the profit will be $(1-\alpha)(V-r) < v-r$.

(iii) It is not a best response for a firm of category A to increase the price. Consider $\tilde{p}$, the threshold price such that for any $p'' > \tilde{p}$ buyers of type a start searching. There is no incentive to set $p > \tilde{p}$ as it leads to zero profit. Neither is it an incentive to set $V-r < p < \tilde{p}$. If $\tilde{p} < V-r$, it is not a BR to set $p''' < \tilde{p}$ as this yields profit $\alpha p''' < \alpha\tilde{p}$. Therefore, we only consider an increase of price to exactly $\tilde{p}$. Define $\tilde{p} = v-r+\varepsilon$. Buyers of type a do not search as long as

$$
\begin{aligned}
& V-(v-r+\varepsilon) \geq \delta(V-(v-r) \\
& \Leftrightarrow \varepsilon = (1-\delta)(V-(v-r))
\end{aligned}
$$

since the condition is binding. Therefore, it is a best response for a firm to increase the price iff

$$
\begin{aligned}
& \alpha(v-r+\varepsilon) \geq v-r \\
& \Leftrightarrow \delta > \frac{\alpha V-(v-r)}{\alpha(V-(v-r))} \qquad .
\end{aligned}
$$

■

c. The Mainstream Niche *MN* is a Nash Equilibrium:

Proof. A buyer waits if her expected surplus is higher than the reservation utility, i.e.

$$
\begin{aligned}
\Leftrightarrow \quad & r < \delta(1-\gamma)\sum_{i=0}^{\infty}\gamma^i\delta^i(V-v+r) \\
\Leftrightarrow \quad & r < \frac{\delta(1-\gamma)}{1-\gamma\delta}(V-v+r).
\end{aligned}
$$

this simplifies to

$$V - v > \frac{r}{1-\gamma}\frac{1-\delta}{\delta}.$$

The profit in equilibrium is given by:

For a firm of category A: $\pi_A = \alpha(V - r)$.

For a firm of category B: $\pi_B = \alpha(v - r) + \frac{1-\alpha}{1-\gamma}(v - r)$.

I want to find γ such that $\pi_A = \pi_B$. Write:

$$\begin{aligned}
& \alpha(V-r) = \alpha(v-r) + \frac{1-\alpha}{1-\gamma}(v-r) \\
\Leftrightarrow\quad & (1-\gamma)\alpha(V-v) = (1-\alpha)(v-r) \\
\Leftrightarrow\quad & \gamma^* = 1 - \frac{1-\alpha}{\alpha}\frac{v-r}{V-v}.
\end{aligned}$$

It is actually an equilibrium: (i) At isoprofit, consumers of type b actually search. We know $\gamma^* = 1 - \frac{1-\alpha}{\alpha}\frac{v-r}{V-v}$. I want $V - v > \frac{r}{1-\gamma}\frac{1-\delta}{\delta}$ for buyers of type b to wait. Replacing γ by γ^* yields $\frac{1-\alpha}{\alpha} > \frac{r}{v-r}\frac{1-\delta}{\delta}$. This is always true when $\delta \to 1$. The condition on δ can be conveniently rewritten as $\delta > \frac{\alpha r}{(1-\alpha)(v-r)+\alpha r}$.

(ii) In equilibrium, it is not a best response to sell category A at price $p'_A \leq v - r$. This yields at most profit $\pi_A = v - r$ which is lowering the profit by condition 1.

(iii) In equilibrium, no one wants to produce category B at price $v - r < p'_B \leq V - r$. Increasing the price make consumers of type a lose, and yields at most profit $\pi'_B = (1-\alpha)(V - r)$ which is lower than π_B as I have assumed $\alpha \geq \frac{1}{2}$.

(iv) In equilibrium, it is not a best response to sell category A at price $v - r < p_A < V - r$. Demand is at most α and the firm therefore makes lower profits.

(v) It is not a best response to change of category. There is isoprofit at equilibrium and, for any $\gamma > \gamma^*$ the best response of any firm is to supply category B (as π_B is an increasing function of γ).

■

d. The Classic Niche *CN* is a Nash Equilibrium iff $(1-\alpha)(V-r) > v-r$:

Proof. (i) Selling category A at price $p' > V-r$ yields lower profit as, by definition buyers of type a reject the offer and search.

(ii) Selling category A at price $p' = V-r$ is not a best response as long as buyers of type a reject the offer and search.

(iii) Selling category A at price $v-r < p' < V-r$ is not a best response, as by Lemma 4 it does not increase the demand, but, by lowering the price it lowers the profit.

(iv) Selling category A at price $p' < v-r$ is not a best response as by Lemma 4 it does not increase the demand, but, by lowering the price it lowers the profit.

(v) Selling category B at price $p' < v-r$ is not a best response as by Lemma 4 no one specifically search for the firm and therefore profit is at most $v-r$, which is lower than $(1-\alpha)(V-r)$.

(vi) Selling category B at price $v-r \leq p' < V-r$ is not a best response as by Lemma 4.

it does not increase the demand from buyers of type b and as long as buyers of type a reject the offer and search.

(vii) Selling category B at price $p' > V-r$ yields lower profit as, by definition buyers of type a reject the offer and search.

(viii) Using the same reasoning as for MN, (ii) and (iv) are true when $\delta > \frac{(1-\alpha)r}{\alpha(v-r)+(1-\alpha)r}$. ■

Now, to complete the proof, it only remains to show that no other equilibrium may exist.

e. No other equilibria exist under condition 1

Proof. I want to show that the previous equilibria are the only existing ones when condition 1 is true. Therefore, I still have to get rid of the following alternatives.

(1) All sellers sell category B at price $p_B = V-r$. Selling category A at price $p_A = V-r$ is a profitable deviation as it yields profit $\alpha(V-r) > (1-\alpha)(V-r)$, by $\alpha > \frac{1}{2}$.

(2) All sellers sell category B at price $p_B = v-r$. Selling category A at price $p_A = V-r$ is a profitable deviation as it yields profit $\alpha(V-r) > v-r$, by condition 1.

(4) A fraction γ of sellers sell category A at price $p_A = v-r$ while a fraction $(1-\gamma)$ sells category B at price $p_B = v-r$. (i) If buyers don't search for the seller of their category then, by setting $p_A = V-r$ a seller of category A does not lose buyers of type a and therefore increases profit (ii) If buyers do search then, a seller only serves buyers of its category, and there must exist a price

$p' > v - r$ such that buyers still accept the offer. Thus setting p' is a profitable deviation.

(5) A fraction γ of sellers sell category A at price $p_A = V - r$ while a fraction $(1-\gamma)$ sells category B at price $p_B = V - r$. As $\alpha \geq \frac{1}{2}$ one can never have the same profit when $\gamma \neq 1$.

(6) Given Lemma 1, I have exhausted all the potential Nash Equilibria. ■

6.4 Proof of Proposition 2

Proof. The Mainstream Niche *MN* is coalition-proof. By Lemma 1, any Nash-Equilibrium implies either $p = v - r$ or $p = V - r$. Selling category B at $p = V - r$ is not a profitable deviation, as demand would be zero. Selling category A at $p = v - r$ can increase joint profit of a coalition of sellers, but is not *self-enforcing*. Indeed, as the demand for such firms only comes from buyers of type A, each seller has an incentive to slightly increase the price - the participation constraint of buyers of type b is non-binding.

TM$_H$ is not coalition-proof. There exist a coalition of size $(1-\gamma') < (1-\gamma^*)$ that would increase her profit by selling a good of category B at price $p = v - r$. Such a deviation is *self-enforcing*, as this price is the best response of any member of the coalition given that all the other members play the same strategy. Hence, Tyranny of the majority at high price is not a coalition-proof Nash Equilibrium.

TM$_L$ and *CN* are not coalition-proof. As the profit of each firm is higher in *MN*, and as *MN* is coalition-proof, a coalition of mass 1 always has an incentive to choose the strategy associated with the equilibrium *MN*, and this strategy is *self-enforcing*. ■

6.5 Proof of Proposition 3

There is a Nash Equilibrium where all sellers sell the good desired by the majority at the high price:

Proof. (i) Buyers of type a buy without search (surplus $V - v + r \geqslant r$).

(ii) Buyers of type b buy without search (surplus r).

(iii) It is not a BR to sell category A at price $v - r < p'_A \leq V - r$. This decreases the profit to at most $\pi'_A = \alpha(V - r) < v - r$

(iv) It is not a BR for a firm to sell category B at price $p_B \leq v - r$. This yields at most profit

$\pi_B = (1-\alpha)(v-r)$. Why? Because buyers of type a wait[11] to match a buyer of their type, while buyers of type b do not search for the deviating firm.
(v) It is not a BR for a firm to sell category B at price $v-r < p'_B \leq V-r$. This yields at most profit $\pi'_B = (1-\alpha)(V-r)$. This is smaller because condition 1 is not fulfilled and $\alpha \geq \frac{1}{2}$.
■

There is a Nash Equilibrium where all sellers sell the good desired by the minority at high price:

Proof. (i) Buyers of type b buy without search (surplus $V-v+r \geqslant r$).
(ii) Buyers of type a buy without search (surplus r).
(iii) It is not a BR for a firm to sell category A at price $v-r < p'_A < V-r$. This decreases the profit to at most $\pi_A = \alpha(V-r)$ [profit loss by the fact that condition 1 is not fulfilled].
(iv) It is not a BR for a firm to sell category A at price $p'_A \leq v-r$. This yields profit at most $\pi'_A = \alpha(v-r)$. Why ? Because consumers of type 2 wait[12] to match a buyer of their type, while buyers of type a do not search for the deviating firm.
(v) It is not a BR for a firm to sell category B at price $v-r < p'_B \leq V-r$. This yields profit $\pi'_B = (1-\alpha)(V-r)$. This is smaller because condition 1 is not fulfilled and $\alpha \geq \frac{1}{2}$.
■

Consider the following condition:

Condition 3 $\delta < \frac{2r}{V-v+2r}$

If condition 3 is true, there exist threshold values (γ^-, γ^+) such that, for any $\gamma^- < \gamma < \gamma^+$, buyers accept any offer without search and sellers are indifferent between both types. Any $\gamma \in (\gamma^-, \gamma^+)$, with $\gamma^- = \frac{V-v+r}{V-v} - \frac{r}{\delta(V-v)}$ and $\gamma^+ = 1-\gamma^-$, with price $p_B = p_A = v-r$ is a Nash Equilibrium.

Proof. (i) Matched buyers buy without search (surplus $V-v+r \geqslant r$).
(ii) Mismatched buyers buy without search as long as $\gamma \in (\gamma^-, \gamma^+)$.
(iii) It is not a BR for a firm to sell category A at price $v-r < p'{}_A \leq V-r$. This decreases the profit to at most $\pi'_A = \alpha(V-r)$ [profit loss by the fact that condition 1 is not fulfilled].
(iv) It is not a BR for a firm to sell category B at price $v-r < p'_B \leq V-r$. This yields at most

[11] If condition 3 is false, sellers are indifferent.
[12] If condition 3 is false, sellers are indifferent.

profit $\pi'_B = (1-\alpha)(V-r)$. This is smaller because condition 1 is not fulfilled and $\alpha \geq \frac{1}{2}$.
(iv) Firms are indifferent between producing category A or category B at price $p_A = p_B = v-r$, as, for any value of γ,we have $\pi_A = \pi_B = v-r$, by (i) and (ii), which satisfy isoprofit.

■

Proof that no other equilibria exist when condition 1 is not fulfilled

Proof. I want to show that the previous equilibria are the only existing ones when condition 1 is not fulfilled. Therefore, I still have to get rid of the following alternatives:
(1) All sellers sell category B at price $p_B = V-r$. Selling category A at price $p_A = V-r$ always yields higher profit since $\alpha \geq \frac{1}{2}$.
(2) All sellers sell category A at price $p_A = V-r$. As condition 1 is not fulfilled, reducing price to $p'_A = v-r$ increases profit.
(4) A fraction γ of sellers sell category A at price $p_A = V-r$ while a fraction $(1-\gamma)$ sells category B at price $p_B = v-r$. Sellers selling category A do not make buyers of type a search (yields surplus 'r'). Then, profit is at most $\alpha(V-r) < v-r$ since condition 1 is not fulfilled.
(5) A fraction γ of sellers sell category A at price $p_A = V-r$ while a fraction $(1-\gamma)$ sells category B at price $p_B = V-r$. Such a price is too high to attract. Hence, as $\alpha \geq \frac{1}{2}$ one can never have the same expected profit for both categories.
(6) A fraction γ of sellers produces category A at price $p_A = v-r$ and a fraction of sellers $1-\gamma$ produce category B at price $p_B = V-r$. This means sellers of category B make profit $\pi_B = (1-\alpha)(V-r)$. Then, if isoprofit is satisfied, it is a BR for any seller to produce category A at price $p_A = V-r$ and get profit $\pi_A = \alpha(V-r) > \pi_B$.
(7) Given Lemma 1, I have exhausted all the potential Nash Equilibria. ■