Linguistic demands and more: A framework for analysing mathematics lessons

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We present a framework developed as a tool for analysing rich episodes of mathematics lessons in ways that reflect this richness rather than summarise it through coding. Our project supports language-responsive teaching. This framework was developed to address our concerns that through code-based analysis of lessons we were losing too much of the complexity, particularly relating to mathematical connections and aspects of the interactional context that build, evolve, and adapt during an episode, task or lesson. The framework involves identifying structure within lesson episodes, based on shifts in mathematical focus, and analysing how key dimensions of language-responsive mathematics teaching are at work within this structure. We draw on data from one analysed lesson episode to illustrate the use and potential of the framework, but also to underscore some of the challenges and reflections that the framework has brought to our work within the project.

Keywords: Language-responsive teaching, mathematics classroom discourse, noticing.

Introduction

In 2022, in the context of a broader team including secondary-school mathematics teachers and other researchers and teacher educators, we started a project aimed at examining and discussing language-responsive ways of developing mathematics teaching. We have faced several challenges since then, one of which has been the thinking and use of a framework that allowed us to "see" and investigate some of the features of the mathematics lessons conducted by the teachers engaged in the developmental activities of the project. Videotaped lesson data is always incredibly rich in many respects and decisions need to be made for the prioritisation and analysis of some features over others. Decisions such as splitting up lessons for the analysis and how to do the splitting, for example, are not trivial. As posed by Adler (2017): "A major task for any lesson analysis is how to chunk or divide the lesson into analytic units." (p. 130). Classroom research in mathematics education with a focus on language and communication has provided a range of alternatives for breaking down lessons. Adler (2017) suggests identifying the object of learning and creating related episodes; O'Halloran (2004) interprets mathematics lessons as sequences of curricular content microgenres; and Clarke (2001) proposes the structure consisting of *Lesson*, *Activity*, *Episodes*, *Negotiative event*, *Turn and Utterances*. What comes from the study of the lesson data can be hard to put together again to produce some understanding of how the teaching and learning developed, and more so if the criteria for chunking the lessons and analysing each separate lesson piece are not consistently shared and applied.

In this paper we present a framework that was elaborated as a tool for the analysis of mathematics lessons with experienced teachers and learners of diverse linguistic repertoires. Support for these teachers was planned through participation in a professional development program on language-responsive mathematics teaching, which involved the collaborative design of mathematical tasks on linear equations, angle properties and probability and the discussion of instances of teaching moves, amongst other key moments of practice. We draw on data from one of the videoed lessons to illustrate the use and potential of the framework, but also to underscore some of the challenges and reflections that the framework has brought to our work within the project.

Background

The authors are members of a research team working on the above-mentioned project on mathematics teaching in linguistically diverse classrooms in England, aiming to support language, communication and mathematics together (for further details of the project see Ingram and Lee, 2023). We view language and communication in mathematics lessons as consisting of words, images, diagrams, and other ways of communicating about mathematical objects, ideas, and practices. As described in Ingram et al. (2024), members of the team have experience with diverse interpretive approaches for researching language and communication in mathematics classrooms, including ethnomethodology, sociocultural theory and systemic functional linguistics. Our collaboration is mediated by this range of theoretical approaches and the belief that the understanding of lesson data can benefit from the adoption of a complementary perspective.

Design of the framework

Our intention was to develop a framework for analysing rich episodes of mathematics lessons that we have identified in our project data, in ways that reflect this richness rather than summarise it through coding. The framework we present (see Figure 2 for an example in use) was developed by the project team. It aims to provide a structure to support analysis of lesson discourse within lesson episodes for the purposes of following the mathematical story of the lesson. We identify episodes as sections of a lesson involving one type of activity in work on a task, e.g., a discussion or students working on a task. The framework uses as a unit of analysis the sub-parts of such an episode, which we call *stanzas*. A stanza could be as small as an utterance or span several minutes of a lesson. We recognise changes between stanzas as a shift in focus, as agreed by more than one member of the team, and adopted the term *stanza* because it has resonances of a smaller part of a whole and having a particular focus.

We name stanzas (column 1, Figure 2) by what we identify as the theme of the interaction in the stanza and give a qualitative characterisation of the mathematics in focus (column 2). Our intention for these columns is to help keep the mathematics foregrounded: how the interaction relates to this mathematics and how this relates to the story of the whole episode. This includes how the teacher or students return to a focus, for example in a developing argument, which we propose to indicate by overlaying arrows on the completed table. The four remaining columns involve analysis of key dimensions of language-responsive teaching (see below), which are a focus of the wider project. These four dimensions were developed by the project team based on prior work identifying design principles and teacher moves that enhance language for mathematics learning (Erath et al., 2021).

Our analysis of lesson episodes focuses on what we noticed during the stanzas, where what we notice individually is influenced both by the theoretical background we each work with and by the noticing of others in the analysis discussions, and may draw on one or more of these dimensions of language-responsive teaching. By focusing on what we noticed in the stanzas in relation to these dimensions, we aim to develop our understanding of the nature of language in classroom discourse and its role in students' learning.

Dimension 1: Linguistic demands of the mathematics. This dimension draws attention to linguistic demands by noting the language teachers or students say or write in relation to key concepts of the lesson, as this reflects the opportunities students have to hear, read, write or speak this language and how language use is modelled (Prediger, 2019). For example, choosing to name or label objects, relationships or processes can support communication about these and can emphasise their importance. Furthermore, choices of names or labels can foreground certain meanings in a particular context (Mason, 2002).

Dimension 2: Student reasoning, explanations and argumentation. Reasoning, explanations and argumentation in mathematics have distinctive features that differ from everyday language or other subjects (Erath et al., 2021; Schleppegrell, 2007). This dimension captures students' explanations, how they share their reasoning or build arguments, and what aspects of these are focused on, developed or evaluated. We note here that explanations or arguments that draw false conclusions may nonetheless contain mathematical features, such as conjectures, connections to previous mathematical tasks or use of mathematical language.

Dimension 3: Listening and feedback. This dimension includes how feedback on explanations draws attention to the mathematics, offers opportunities for students to listen to each other or be supported in this and teachers actively listening for mathematical features of what students say even when it is not obviously mathematical. Listening can serve different roles in mathematics lessons, for example, to evaluate or interpret students' responses, or to make connections with other ideas. Distinctions between these can be indicated by the response that follows.

Dimension 4: Making connections. Making connections is a key part of teaching and learning mathematics, but there are many different types of connection that could be made, for example between ideas, tasks, methods, or processes. In our study we also consider connections that relate to communicating mathematics, including between representations, between families of words and between informal and more technical ways of describing mathematics.

We note that the dimensions are ordered for the purposes of arranging them in the framework table (Figure 2), but we consider them to be intertwined and may occur throughout a stanza. In particular, like Adler and Ronda (2015), we caution that this ordering does not suggest a teaching sequence. This is one design reason for not locating *Listening and feedback* in the final column, which might suggest that it happens 'last' in classroom interaction (as noted by the teachers in the project).

The framework described above is therefore designed to support noticing and a focused analysis of each dimension separately and also consider how they are at work within each stanza and over the course of the episode, including whether and when some dimensions move out of focus.

Method

In the first stage of using the framework, two members of the project team used it to analyse one lesson episode together, then another episode separately. One of these team members and another team member then analysed a further episode separately to assess the potential for using the framework reliably by different researchers. This raised anticipated issues of grain size for the stanzas and time for use. As a group we then analysed two further lesson episodes from different topics, in person and then online (see below for details of episode selection). Our choice to collaborate at this stage was based on our goal of understanding more about the potential of working with the framework, rather than reliability of coding of our project data. Collaborating enabled us to surface and debate issues the framework raised for us, through which we drew on and examined how our different backgrounds in analysing classroom language and communication may have affected what we noticed in the data through differently developed sensitivities (Mason, 2002) and what we chose to report. For example, one author noticed pauses and intonation and others noted slight shifts in word choices and grammar. These differences raised questions about what level of detail to report in the framework while keeping the mathematics foregrounded.

Episodes were selected for analysis through a two-stage process that is part of the wider project work. In the first stage, lesson episodes were identified by considering the types of activity around a task, for example students working on the task or a discussion. In the second stage, a reduced version of the Global Teaching InSights coding scheme (OECD, 2020) was used to identify episodes that scored more highly in terms of the quantity and quality of interaction between the teacher and the students. We watched the video of the episodes multiple times during analysis, re-watching short sections to confirm our *accounts of* (Mason, 2002) or as our discussion drew attention to different features.

Results: Illustration of our use of the framework to analyse a lesson episode

We now illustrate our use of the framework using data from our analysis of a 5-minute episode from a lesson on angles in parallel lines for students aged 12-13 in England. For practical reasons our analysis took place in a recorded online meeting (approximately 1 hour 40 minutes) after previous work in person on other episodes. The episode involves a whole-class discussion that followed up on a task in which students were shown the diagram in Figure 1 and asked to discuss and "write down as many statements or equations about these angles as possible". In the discussion the teacher selected relationships that students had identified, wrote them on the board and asked for reasons for them.

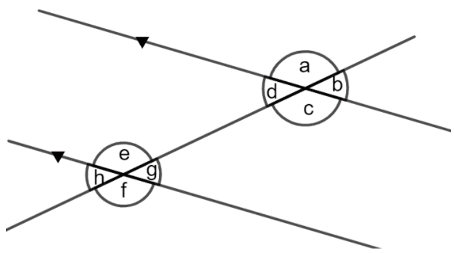

Figure 1: Parallel lines task (reproduced from lesson video)

Figure 2 illustrates our shared analysis of the first three of five stanzas in the episode, presented here in the form produced during our shared analysis session. We now describe some of our deliberations and choices made in this analysis. For reasons of space, we do not describe all details for each stanza.

In our first two meetings, we had first identified a stanza and then analysed each dimension before continuing to identify further stanzas. In contrast, in our analysis of this episode we first focused on identifying stanzas, and then analysed across the dimensions, working stanza by stanza. One challenge in our analysis was the level of detail to include in cells. Figure 2 illustrates our choices, which took into account the goals of our analysis and therefore combine verbatim quotes and summary comments.

In our shared analysis, we quickly reached agreement on the shifts in focus that defined boundaries between stanzas and the core mathematical ideas or activities within the stanzas. For example, we interpreted the two geometric results in stanza 1 as forming one stanza together, rather than separate

Stanza	Core mathematical ideas and activities	Linguistic demands of the mathematics	Student reasoning, explanations and argumentation	Listening and feedback	Making connections
1. Angles prior knowledge (0.00–1.06)	Sum of angles round point and on straight line. Using this as a basis for relationship between angles.	Angles around a point Angles on a straight line (preposition) Degrees, Equals, Equation, point Reason (why, and what you write after the answer), what a reason is in mathematics	Modified from angles in a 'complete circle' Students completing missing words in mathematical statements started by the teacher	Accepting answers as correct but requesting rephrasing and structuring statements into conventional forms Referencing routine of how to give a reason when making an argument	Students communicating using written notation, copied by teacher, but speaking the notation, e.g. equals not = Connecting to prior knowledge by asking for reasons for equations Complete circle given as reason for the equation
2. Straightness of line (1.06–1:28)	Raising assumption that lines are straight in the diagram	Straight, line, straight line Not necessarily, Assuming Parallel lines cut by…			Connecting representations (tracing along line) This diagram being a representation that tells them a line is straight (basis of assumption)
3. Comparing angles (1.28–3:30)	a>d is inequality, a=c is equation. When is it sometimes true, or always true?	Is greater than Equation, Expression, Inequality If… would …always Still the same Opposite angles True Cut line (naming transversal) Same, different Same place' to describe angles on new line In this scenario, case	Is the relationship always true or only sometimes true?	Validates appropriate mathematical responses, repeats appropriate answers	Inequalities are a type of expression, but it's not an equation Speaking 'greater than' connecting with symbol Using dynamic language to explore the static diagram.

Figure 2: Example of the framework in use

stanzas, because they both involve the core mathematical activity of students using prior knowledge in an explanation of a relationship they had identified between angles. We identified the move from stanza 1 to stanza 2 by the teacher drawing attention to an assumption that the transversal line was straight in response a student's use of *straight line* in their explanation in stanza 1. The shift in focus that marks a move from stanza 2 to stanza 3 was indicated by introducing the inequality relationship $a > d$, which holds for the given diagram, and using this as a starting point for the core mathematical activity of contrasting a relationship that is sometimes true with one that is always true, e.g. $a = c$.

In stanza 1 we identified *Linguistic demands* that included statements involving prepositions, such as *angles around a point* and specialised vocabulary such as *degrees*, *equation* and *point*. We included the teacher's clarification that a *reason* is what should be written after stating the relationship. We interpreted this as a linguistic demand because it set out what a reason looks like in mathematics, and because in England there is an expected form for these reasons in geometric arguments. There was an explicit interplay between the dimensions *Student reasoning, explanations and argumentation* and *Listening and feedback* in stanza 1, as *Listening and feedback* was characterised by restructuring students' arguments. For example, the teacher accepted "because it's a complete circle" as a correct reason but then requested "Angles around a WHAT equals 360?" In this way, the teacher's feedback also referenced a routine from general mathematical practice of giving reasons.

Connections that we considered to be made in stanza 1 include students connecting to prior knowledge of geometric results as reasons for their equations. The teacher also connected words with the symbols students used to communicate relationships, for example by speaking "=" as "equals". We discussed whether stanza 1 involved connecting algebraic representations with the diagram. Some students are likely to have made such connections before this episode by forming equations to express relationships between angles in the diagram, but we decided that this connection was not made during stanza 1 itself because neither the teacher nor the students connected the equations with the diagram in speech or through other means such as gestures (Alibali et al., 2014), which, by contrast, the teacher did in stanza 2. This discussion highlights the challenges of using the framework for individual episodes: connections made available previously may later be left tacit and we may only be able to record connections that are articulated in the episode.

Stanza 2 focuses on the assumption that the transversal line is straight (which we consider to develop the statement introducing angles on a straight line in stanza 1 and indicate by a connecting arrow in Figure 2). In stanza 2 we did not identify any student speech, so there are no examples of *Student reasoning* or *Listening and feedback*. *Linguistic demands* include classifying lines, e.g. *straight line*, *parallel lines*, and phrases associated with assumptions, e.g. *not necessarily*, which are important for precise communication within mathematical arguments as well as being everyday expressions. One focus of our discussion was the teacher's use of the diagram to make connections. First, she connected speech with the diagram by asking "How do I know this line is straight?" while tracing along the transversal line. She also connected the assumption that the line is straight with what students could "see from the diagram". We noted that use of the diagram therefore extends beyond a point of reference: its visual properties are used as a basis for a claim for properties of a line. This contrasts with properties made explicit through notation, such as arrows on parallel lines, or stated in the task.

In stanza 3 the mathematical focus is the inequality "$a > d$" and whether it is always true, which the teacher contrasts with the relationship $a = c$ which is always true. In addition to specialised terms, and giving the name *cut line* to afford communication about the transversal, *Linguistic demands* here particularly include conditional statements such as "If I moved this cut line, would a always be greater than d?" One particular type of connection that we interpret the teacher making is between the related specialised terms *equation, inequality* and *expression* as she identifies the *inequality* as a type of *expression* and contrasts *inequality* with *equation*. A further type of connection is between the static diagram and a range of possibilities that it could represent. The teacher makes these connections through dynamic and temporal language, e.g. *moved, always, still the same*, and drawing an alternative transversal for which $a < d$ could be interpreted visually from the relative size of the angles in the diagram and connecting this with an algebraic representation. As a form of *Listening and feedback,* we considered the teacher's repetition of mathematically appropriate student statements such as "They are the same" to validate these responses. Although opportunities for *Student reasoning, explanation and argumentation* were framed by the teacher questioning whether the result $a > d$ was always or only sometimes true, we did not identify examples of students explaining why this was the case, or being pushed to explain this.

In summary, these descriptions of our analysis of stanzas 1-3 illustrate how our use of the framework supports us to draw out details of how the teacher and students sustain or change the mathematical focus of discussions. For example, we identified a range of topic-specific and mathematical linguistic demands that differed between stanzas, support for more general mathematical practices such as giving reasons and considering whether relationships hold always or only sometimes. We also identified differences in attention to how students make their arguments through the teacher's listening and feedback and the variety of connections articulated in these short sections of discourse.

Conclusion

Data analysis requires the summarisation of data, which reduces the complexity of the situation. The act of videoing or observing a lesson also necessarily results in data reduction. However, classroom interactions are complex, multifaceted and dynamic in ways that intertwine pedagogic decisions, the mathematical content and the interactional context. We lose so much information when we reduce classroom data to a set of codes. This framework was developed to address a concern that we were losing too much in the earlier coding, particularly connections, themes and aspects of the interactional context that build, evolve and adapt during an episode, task or lesson, or even across lessons. Furthermore, many of the coding systems we considered seemed to lose sight of the way in which interactions are intertwined with the mathematical content because they focus on particular pedagogic teacher moves or structures of interaction, rather than how these moves or structures might be influenced by or influence the mathematics in focus. Other systems, such as Adler's MDI framework (Adler & Ronda, 2015), consider these relationships across lessons rather than at the level of interaction. We have illustrated here how the framework has supported our analysis of lesson episodes in our project and now raise the question of whether it could be used beyond our project to capture quality in language-responsive teaching.

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